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ABSTRACT

The Importance of the Distribution of Consumers in Horizontal Product Differentiation*

This Paper examines the importance of the distribution of consumers in Hotelling's circle on the comparison between the optimal and the market equilibrium levels of diversity. It finds that when most consumers are located very close to the firms, the result of Salop that the equilibrium number of firms is larger than the optimal one (surplus maximizing) can be reversed.

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1. Introduction

Salop (1979) introduces a model of spatial (or product) differentiation on a circle and charaterizes the different equilibria of the corresponding market.[1] When consumers are located uniformly on the circle, he shows that the number of firms in the market economy is higher than the number of firms at the social optimum. In other words, the market generates too many firms.[2] The reason why optimal variety is less than equilibrium variety for the circular market is that private and social incentives have no reason to coincide. Indeed, as Tirole (1988) points out, the social optimum is justified by the greater product diversity offered to the consumers whereas the market equilibrium is linked with 'stealing the business' of other firms while still being able to impose a mark-up.

Although the existence of excess variety at equilibrium from a social viewpoint is now taken for granted by most economists, Salop was skeptical about the general validity of his result:

> "This result of too many brands is not robust, but rather depends crucially on the distribution of consumers and preferences (...) That there is excess variety at equilibrium is not robust" [Salop (1979, pp. 152 and 156)]

The aim of this paper is to explore the implications of a general consumers' distribution on the market equilibrium, and to compare the optimal with the market product variety. Several papers have introduced general distributions of consumers in models of horizontal differentiation à la Hotelling (1929) where firms and consumers are located on a line or a circle. Shilony (1981) and Neven (1986) were the first to characterize the subgame perfect equilibrium of

[1] In fact, Vickrey (1964) was the first to introduce the circular model of differentiated products. As it is standard now, we use the model of Salop (1979) to describe the circular model.

[2] This result holds both for quadratic and linear transportation costs.

the location-price game with non-uniform distributions, and to determine its existence and uniqueness. Different applications have then been implemented. Economides (1984) constructs a model of trade in differentiated products in which the volume and value of trade are increasing in the degree of similarity of preferences between the trading economies. By introducing a land market in a standard Hotelling model, Fujita and Thisse (1986) endogeneize the location and thus the spatial distribution of consumers. Jehiel (1992) addresses the implications of price collusion on the degree of horizontal product differentiation. Finally, Bloch and Manceau (1999) analyze the effect of persuasive advertising in a model where consumers differ in their tastes for two competing products. In the latter, advertising by one firm shifts the distribution of consumer tastes towards one of the products (firm's location).

Section 2 presents the variant of Salop's circular model and characterizes the unique symmetric Nash equilibrium in prices. We determine in Section 3 how the number of firms at a symmetric zero profit equilibrium depends on the distribution of consumers on the circle, and relate it to the socially optimal number of firms. Section 4 contains all the proofs.

2. The model

The model is a variant of Salop (1979)'s circular model. Consider an industry formed by n firms and a continuum of consumers. Consumers are differentiated with respect to their location on the circumference of a circle $\mathcal{C}$ of length 1. Firm i's location is denoted by x_i, $i \in \{1, \dots, n\}$ and firms are equally spaced along the circumference $\mathcal{C}$.[3] Therefore, for all $i \in \{1, \dots, n\}$ we have $x_i = \frac{i}{n}$.

[3]Using a circle model, both Economides (1989), who assumes quadratic transportation costs, and Kats (1995), who assumes linear transportation costs, show that the equidistant configuration of firms is the equilibrium outcome of a game in which firms would choose their location prior to their prices. Furthermore, studying a two-stage (location, price) game within the framework of Hotelling's competition (the bounded line) with non-uniform customer distributions, Neven (1986) shows that there is a range of distributions (concave)

In particular, $x_1 = \frac{1}{n}$, $x_2 = \frac{2}{n}$, ..., $x_n = 1$. Transportation cost along the circle is linear in distance at rate t per unit of good.

There are many potentially active firms in the (imperfectly competitive) differentiated industry producing goods with marginal cost $c > 0$. Consumers have an outside option which comes from the existence of a (perfectly competitive) industry producing a non-differentiated good. The reservation price of consumers in the differentiated industry is then constant and equal to $v > 0$. We assume that v is large enough so that in equilibrium the market is always covered, i.e. all consumers buy goods.

For any list of f.o.b. prices $(p_1, \ldots, p_n)$ simultaneously set by firms, a consumer located at $x \in \mathcal{C}$ determines his/her preferred brand and buys one unit of this brand only if he/she obtains a surplus greater than the outside option v. Formally, consumer x solves:

$$\max \left\{ 0, \max_{i \in \{1,\ldots,n\}} \left(v - p_i - t \left| x - x_i \right| \right) \right\}$$

We assume that consumers are spread out along the circle with a density $f(x)$. The cumulative of $f(x)$ is denoted $F(x)$ and by normalization, we set $F(1) = 1$. For simplicity, we restrict to symmetric density functions such that $f(x_i + x) = f(x_i - x)$ and $f(x + \frac{1}{n}) = f(x)$. In words, we require that consumers are symmetrically distributed around firms and that the distribution of consumers between two adjacent firms be the same whatever the pair of firms considered. Hence, knowing the shape of the density function on the segment between two adjacent firms or, more generally, on any connected segment of length $1/n$, fully characterizes the density function on $\mathcal{C}$. We also assume that f is continuous, differentiable and strictly positive.

The timing of the events is as follows. Once the number of firms n is chosen, the consumer distribution endogenously adjusts to have n peaks and n troughs. Two polar cases may then arise, and we restrict attention to these

for which firms locate at opposite ends of the market, as in the case of a uniform density. Based on these results, it is reasonable to assume (as in Salop, 1979) equal spacing of firms in the linear-transportation-costs model of this paper.

cases. Either peaks of the consumer distribution coincide with firms' location so that most of the consumers are located close to firms, or troughs of the consumer distribution coincide with firms' location so that most of the consumers are located far away from firms. Formally, let $\theta = f(\frac{1}{2n})$ denote the density of consumers at the midpoint between two adjacent firms.[4] When θ is low, each firm has a body of consumers close to its own location, and the mass of consumers near the border between the two markets is relatively small. When θ is high, the bulk of consumers is located on the border between two firms. The parameter θ thus captures the different possible distributions of consumers, and $1/\theta$ reflects the adequation between consumers' distribution and firms' location (see Figure 1).

Even though this is not explicitly modeled, the first case (θ low) can arise for at least two reasons: (*i*) advertising and (*ii*) land market. (*i*) Using the same argument as in Bloch and Manceau (1999), advertising can be viewed as a mean by which a firm can shift the distribution of consumers towards its own product. Thus, in our model, for a given number n of firms, advertising can lead to a distribution of consumer tastes concentrated around each firm. (*ii*) Using the model of Fujita and Thisse (1986) and the more recent literature on agglomeration (see the surveys by Fujita and Thisse, 1996, 2001) in which a land market is explicitly introduced in an Hotelling framework, most consumers can be attracted to locate close to firms to reduce their transportation costs. However, not all consumers will locate exactly next to firms because of the increasing competition in the land market there.

The other case (θ high) can in fact arise using the opposite argument of Fujita and Thisse (1986). If transportation costs are sufficiently low and competition in the land market between consumers and firms is sufficiently fierce so that it implies high land prices for locations close enough to firms, then consumers can find optimal to locate as far as possible from firms to benefit from a lower land price. In our context, consumers locate in the middle of two

[4] With uniform distribution $f(x) = 1$, $\forall x \in \mathcal{C}$. In particular, $\theta = 1$.

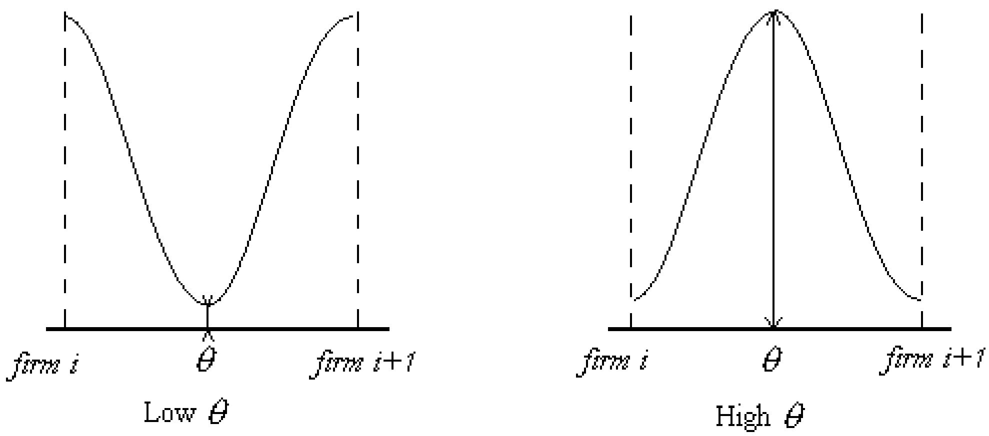

Figure 2.1:

firms to avoid the competition in the land market with firms.[5]

Proposition 1. *If $\theta \geq 2/3$, there exists a unique symmetric Nash equilibrium in prices where all firms charge*

$$p^* = c + \frac{t}{\theta n}$$

If each firm bears a fixed cost K, the equilibrium per-firm profit is equal to

$$\Pi^* = \frac{t}{\theta n^2} - K$$

The following comments are in order.

First, let us explain the intuition of the proof (the complete and formal proof is given in section 4). As it is well-known, the product demand in the circular model of Salop (1979) is not differentiable nor continuous. In fact, we can distinguish three different regions.[6] First, a *monopoly* region where each firm attracts the consumers located nearby and adjacent markets do not cover the whole consumer distribution. Second, a *competitive* region where the whole distribution of consumers is covered by the market and adjacent firms

[5] See Fujita and Ogawa (1980) for a similar result.

[6] See for instance Figure 1, page 143, in Salop (1979).

compete for the consumers located between them. Finally, a *supercompetitive* region where (at least) one firm sets a price low enough so as to attract the market of its immediate competitors. When we plot the product demand function of one firm, the monopoly and the competitive regions are separated by a kink (which undermines differentiability) whereas the competitive and the supercompetitive region are separated by a gap (which undermines continuity).

Our proof proceeds in two steps. First, we restrict to the competitive region where the per-firm product demand is both differentiable and continuous. In this region, the profit function is also strictly concave. The unique maximum of the profit function in this subdomain constitutes a candidate for the symmetric Nash equilibrium price. Computation gives $p^* = c + t/\theta n$. At this candidate equilibrium, the whole market is covered. Second, we check whether unilateral deviations from this candidate equilibrium are profitable. The only deviations to consider are those that switch to a different region. Switching to the monopoly region is never profitable. However, switching to the supercompetitive region by price undercutting (sometimes called mill price undercutting) may be profitable, unless $\theta \geq 2/3$. Indeed, when firms charge price $p^* = c + t/\theta n$, per-firm profits gross of fixed costs are $\Pi^* + K = t/\theta n^2$. If one firm unilaterally lowers its price to slightly below $p^* - t/n$, it steals all of the consumers from the two nearest competitors (supercompetitive region) and gets brut profits equal to $3t/\theta n^2 - 3t/n^2$. This deviation is profitable whenever $\theta < 2/3$.

Second, let us explain this last result. It is well known that price undercutting is a problem for the original model of Hotelling (1929) with two firms on a line segment and linear transportation costs (see d'Aspremont *et al.*, 1979). It is also well known that, in a circular model with equally spaced firms, price undercutting is *never* profitable (Salop, 1979 and Economides, 1989). However, in our model, this not true when consumers tend to concentrate around firms' locations. Indeed, when $\theta < 2/3$, a symmetric Nash equilibrium in mill prices fails to exist. Braid (1993) also provides a model with non-uniform consumer distributions and equally spaced firms where the competitive price equilibrium can be undermined by mill price undercutting.[7] What is the intuition of this result? If $\theta = f(1/2n)$ is too low (less than 2/3), then most consumers tend to locate close to firms. The peak of consumers is too high because it gives too much market power to each firm by isolating them from the other firms. This implies that, in this case, a *competitive* price equilibrium p^* fails to exist.

Third, our results share some features with the ones obtained by Salop (1979) with a uniform distribution. For instance, the equilibrium price p^* must exceed the marginal cost c to prevent firms from making a negative profit. Moreover, the equilibrium price p^* increases with the unit transport cost t (because firms have more market power on consumers locating close to them) and with the marginal cost c. On the contrary, it decreases with the number of firms n (since the average distance between two adjacent firms decreases with the number of firms). Observe also that p^* converges to the competitive price c when the number n of firms tend to infinity (since all firms lose their market power).

Finally, it is interesting to see that the equilibrium price p^* decreases with θ. Indeed, when θ increases, firms are less able to impose a mark-up since most consumers are 'located' at the fringe of their market pool. Price competition becomes fiercer and, as a result, equilibrium prices are lower. This result is in accordance with the one of Neven (1986). With a non-uniform distribution, he finds that a firm located in a dense area (i.e. low θ) will enjoy a competitive advantage because the market retention will be stronger. This implies that firms set higher prices.

To summarize, when $2/3 \leq \theta < 1$, there exists a unique price equilibrium with consumers agglomerated around firms whereas when $\theta > 1$, there exists a

[7] Braid (1993) analyzes a model of price competition with two-dimensional distribution of consumers. However, a special case of a one-dimensional consumer distribution is also included. His formula for the equilibrium price can be recovered from our more general formula in Proposition 1. The conditions for price equilibrium not to break down due to mill price undercutting are also similar.

unique price equilibrium with consumers located far away from firms (to avoid for example the resulting land competition). It should be clear that prices and profits are higher in the first case since firms enjoy more market power. Finally, when $\theta = 1$, consumers are uniformly distributed between firms and the unique price equilibrium lies in between the two other ones.

3. Symmetric zero profit equilibrium and welfare analysis

As in Salop (1979), we concentrate on symmetric zero profit equilibria (SZPE) where n equally spaced firms set a Nash symmetric equilibrium price p^* such that each firm's profit is zero. Observe that we do not focus on a free-entry equilibrium (since the symmetry of consumers around or far away from firms is not likely to hold). Rather, we take the zero-profit equilibrium as a benchmark and compare it with the social optimal outcome.

Denote respectively by n_u^*, n_{peak}^* and n_{trough}^* the number of firms at the SZPE for the uniform distribution, the non-uniform distribution when firms are located at peaks of the consumer distribution (θ low), and the non-uniform distribution when firms are located at troughs (θ high). The following result is then straightforward to establish.

Proposition 2. *At the symmetric zero profit equilibrium* $n_{peak}^* > n_u^* > n_{trough}^*$.

This result is quite intuitive since when consumers are agglomerated around firms (θ low), most consumers are close to their ideal product. Therefore, firms have high market power implying that both prices and profits are high: the market attracts a large number of firms. On the contrary, when consumers are located away from firms (θ high), firms have to steal the business of neighboring firms to attract enough consumers (now far from their ideal product). In this case, profits are low and there are few firms at the SZPE.

We now compare the number n^* of firms at the SZPE with the number n° of firms at the social optimum that minimizes the sum of fixed costs and commuting costs. The number n^* of firms at the SZPE is simply obtained by setting $\Pi^* = 0$, where the competitive equilibrium per-firm profit is given in Proposition 1. We have, $n^* = \sqrt{t/\theta K}$. In particular, when the consumer distribution is uniform that is, $\theta = 1$, we obtain $n^* = n_u^* = \sqrt{t/K}$ as in Salop (1979). Recall that in the original circular model with uniform consumer distribution, the zero profit equilibrium has twice as many firms as the social optimum that is, $n_u^* = 2n^\circ$.

Proposition 3. *If $\theta < \sqrt{2}$, the number n^* of firms at the SZPE is greater than the number n° of firms at the social optimum that is, $n^* > n^\circ$. If $\theta > 2$, this inequality is reversed that is, $n^\circ > n^*$.*

When consumers concentrate around firms ($\theta < 1$) or, at least, when consumers are not too far away from their ideal product ($1 \leq \theta < \sqrt{2}$), firms face a mild price competition and are able to impose a high mark-up. Therefore, as in Salop (where $\theta = 1$), there is an excessive product variety from the social viewpoint. On the contrary, when the market is distant from the firms ($\theta > 2$), competition is fierce and there are not enough firms at the SZPE compared to the social optimum. When $\sqrt{2} \leq \theta \leq 2$, the comparison between the optimal and the equilibrium diversity is ambiguous and depends on the details of the consumer distribution function. In this case, θ is not a sufficient statistic to fully characterize the shape of the consumer distribution whose details now matter for the determination of the optimal product variety.

4. Proofs

Proof of Proposition 1

Let i be a representative firm and denote by p_{i-1} and p_{i+1} the two prices for adjacent firms $i-1$ and $i+1$ respectively. Denote by $\overline{x}_i$ and $\overline{y}_i$ the outer boundaries of firm i's market, where $\overline{x}_i \leq x_i \leq \overline{y}_i$. Firm i's market area is then $[\overline{x}_i, \overline{y}_i]$. Whenever $[\overline{x}_i, \overline{y}_i] \subset [x_{i-1}, x_{i+1}]$ (strict inclusion), the outer boundaries $\overline{x}_i$ and $\overline{y}_i$ are determined by the location of the marginal consumers for whom the net price is identical between firms $i-1$ and i on the one hand, and firms i and $i+1$ on the other hand. We thus have

$$\begin{aligned}\overline{x}_i &= \frac{p_i - p_{i-1}}{2t} + \frac{x_{i+}x_{i-1}}{2} \\ &= \frac{p_i - p_{i-1}}{2t} + x_i - \frac{1}{2n}\end{aligned}$$

and

$$\begin{aligned}\overline{y}_i &= \frac{p_{i+1} - p_i}{2t} + \frac{x_{i+}x_{i+1}}{2} \\ &= \frac{p_{i+1} - p_i}{2t} + x_i + \frac{1}{2n}\end{aligned}$$

The condition $[\overline{x}_i, \overline{y}_i] \subset [x_{i-1}, x_{i+1}]$ characterizes the competitive region where firm i's product demand is differentiable (thus continuous). This condition is equivalent to

$$\max\{p_{i-1}, p_{i+1}\} < p_i + \frac{t}{n}$$

Assume that the previous condition holds for all firms in $\mathcal{C}$. Then, it is readily verified that $\overline{x}_{i+1} = \overline{y}_i$, for all firm i.

Firm i's profit net of fixed costs K is $\Pi_i = \int_{\overline{x}_i}^{\overline{y}_i} (p_i - c) f(x)dx - K$. Π_i is clearly continuous in (p_{i-1}, p_i, p_{i+1}). We now check for strict concavity of Π_i in p_i. Differentiating with respect to p_i we have:

$$\frac{\partial \Pi_i}{\partial p_i} = F(\overline{y}_i) - F(\overline{x}_i) - \left(\frac{p_i - c}{2t}\right)[f(\overline{y}_i) + f(\overline{x}_i)]$$

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -\frac{1}{t}[f(\overline{y}_i) + f(\overline{x}_i)] + \left(\frac{p_i - c}{4t^2}\right)[f'(\overline{y}_i) - f'(\overline{x}_i)]$$

We know that $\overline{x}_{i+1} = \overline{y}_i$, implying that $f'(\overline{x}_{i+1}) = f'(\overline{y}_i)$. Moreover, by symmetry of the firms' location and the periodicity of the density function we have $f'(\overline{x}_{i+1}) = f'(\overline{x}_i)$. Therefore, $f'(\overline{x}_i) = f'(\overline{y}_i)$, implying that

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -\frac{1}{t}[f(\overline{y}_i) + f(\overline{x}_i)] < 0$$

Hence, Π_i is strictly concave in p_i. The candidate Nash equilibrium in prices is then given by the unique local maximum of Π_i in the competitive subdomain that we determine by solving the first order condition:

$$F(\overline{y}_i) - F(\overline{x}_i) - \left(\frac{p_i - c}{2t}\right)[f(\overline{y}_i) + f(\overline{x}_i)] = 0$$

Note also that at a symmetric candidate equilibrium $p_{i-1} = p_i = p_{i+1} = p^*$. Therefore, $\overline{x}_i = x_i - \frac{1}{2n}$ and $\overline{y}_i = x_i + \frac{1}{2n}$ that is, the outer boundaries of i's market area are given by the midpoint between this firm and its adjacent neighbors. Hence, $f(\overline{y}_i) = f(\overline{x}_i) = \theta$ and $F(\overline{y}_i) - F(\overline{x}_i) = \frac{1}{n}$, implying that $p^* = c + \frac{t}{\theta n}$. The per-firm profit net of fixed costs is $\Pi^* = \frac{p^*-c}{n} - K = \frac{t}{\theta n^2} - K$.

We now need to check whether this candidate equilibrium is robust to unilateral deviations that switch either to the monopoly or to the supercompetitive region. A deviation to the monopoly region is never profitable. We analyze mill price undercutting that is, a deviation to the supercompetitive region.

Suppose that all firms set prices equal to p^* and consider an unilateral deviation by firm i. Suppose that i undercuts prices and sets a price $p_i < p^* - \frac{t}{n}$. Now, the outer boundaries of i's market area are not $[\overline{x}_i, \overline{y}_i]$. Rather, marginal consumers located both at $i-1$ and $i+1$ prefer buying from i than from $i-1$ and $i+1$, respectively. As a result, firm i faces the market area $\left[\overline{x}_{i-1}, \overline{y}_{i+1}\right]$, three times larger than the initial market area $[\overline{x}_i, \overline{y}_i]$, and obtains a net profit $\Pi_i(p_i, p_{-1} = p^*) = \frac{3(p_i-c)}{n} - K$. This deviation is profitable whenever

$$\Pi_i > \Pi^* \Leftrightarrow 3(p_i - c) > p^* - c \Leftrightarrow p_i > \frac{p^*}{3} + \frac{2c}{3}$$

Recall that switching to the supercompetitive region requires that $p_i < p^* - \frac{t}{n}$. Therefore, an unilateral profitable deviation to the supercompetitive region exists if and only if

$$p^* - \frac{t}{n} > \frac{p^*}{3} + \frac{2c}{3} \Leftrightarrow \theta < \frac{2}{3}$$

■

Proof of Proposition 2

At the symmetric zero profit equilibrium, the number of firms present in the market is determined by solving the equation (up to integer problems) $\Pi^* = 0$, that leads to $n^* = \sqrt{\frac{t}{\theta K}}$. The parameter θ is the density of consumers at the midpoint between two adjacent firms. It takes low values (resp. high values) with respect to the uniform case when firms are located at peaks (resp. at troughs), implying that $n^*_{int} > n^*_u > n^*_{noint}$. ■

Proof of Proposition 3

The optimal number of firms at the social optimum minimizes the sum of fixed costs and commuting costs. Since the market is covered, the socially optimal number of firms n minimizes $SC(n) = 2nt\int_0^{1/2n} xf(x)dx + nK$. Differentiating with respect to n and recalling that $f\left(\frac{1}{2n}\right) = \theta$, we obtain after some algebra:

$$\frac{\partial SC(n)}{\partial n} = 2t\int_0^{1/2n} xf(x)dx - \frac{\theta t}{2n^2} + K$$

$$\frac{\partial^2 SC(n)}{\partial n^2} = \frac{\theta t}{2n^3} > 0$$

Therefore, $SC(n)$ is strictly convex and the social optimum n° is obtained by solving the first order condition $\frac{\partial SC(n^\circ)}{\partial n} = 0$. We now compute $\frac{\partial SC(n)}{\partial n}$ at $n^* = \sqrt{\frac{t}{\theta K}}$. We find

$$\frac{\partial SC(n^*)}{\partial n} = 2t\int_0^{1/2n^*} xf(x)dx + K\left(1 - \frac{\theta^2}{2}\right)$$

The first term on the RHS is positive and bounded above by $\frac{t}{n^*}\int_0^{1/2n^*} f(x)dx = \frac{t}{2n^{*2}} = \frac{\theta K}{2}$. Therefore,

$$1 - \frac{\theta^2}{2} \leq \frac{1}{K}\frac{\partial SC(n^*)}{\partial n} \leq 1 - \frac{\theta^2}{2} + \frac{\theta}{2}$$

We have $1 - \frac{\theta^2}{2} > 0$ whenever $\theta < \sqrt{2}$. Therefore, $\theta < \sqrt{2}$ implies $\frac{\partial SC(n^*)}{\partial n} > 0$ which in turn implies (by strict convexity) that $n^* > n^\circ$. If $\theta > 2$, $1 - \frac{\theta^2}{2} + \frac{\theta}{2} < 0$. Hence, $\theta > 2$ implies that $\frac{\partial SC(n^*)}{\partial n} < 0$ implying in turn that $n^* < n^\circ$. ■

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