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ABSTRACT

Debt and Deficit Fluctuations and the Structure of Bond Markets*

The aim of this Paper is to test for the extent of incompleteness in the market for US Government debt. We show that when a government pursues an optimal tax policy and issues a full set of contingent claims, the value of debt has the same or less persistence than other variables in the economy and declines in response to higher government expenditure shocks. Examining US data, however, reveals that debt is substantially more persistent than other variables and increases in response to adverse expenditure shocks. We show that this behaviour is best accounted for by a model of incomplete markets, where governments only issue one-period risk-free bonds. We discuss the implications of this for the optimality of debt limits, debt management and assessing the sustainability of fiscal policy.

JEL Classification: E62 and H63
Keywords: bond markets, debt, deficits, fiscal policy and optimal taxation

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1 Introduction

In this paper we study the evidence that market incompleteness is an important feature determining the behavior of the US fiscal deficit and the evolution of government debt. We do so by answering the question 'What financial market structure justifies the way that US government debt responds to fiscal shocks?'. More precisely, we characterize the response of debt to fiscal shocks under both complete and incomplete markets and we use this analysis to test empirically for the nature of US government bond markets. While our analysis refers explicitly to the government sector the analysis is entirely general and also pertains to the debt of the personal and corporate sector.

We maintain throughout the paper the assumption that the government behaves as a benevolent social planner. A substantial literature exists characterizing the stochastic properties of optimal tax rates under this assumption[1] but surprisingly little focus has been placed on characterizing the behaviour of debt. Modelling the behaviour of debt is important not just for reasons of theoretical completeness. Policy discussion is increasingly focused on placing restrictions on government debt e.g. the Growth and Stability Pact in Europe, the Balanced Budget Amendment debate in the US, the Code for Fiscal Stability in the UK. Presumably these constraints are considered in order to prevent governments from shifting a heavy tax burden to future years and to avoid increasing incentives to default or utilizing the inflation tax. However, without articulating the constraints faced by actual governments choosing a debt policy it is impossible to assess the relative merits of alternative fiscal rules.

Whether market incompleteness is an important feature of US bond markets cannot be immediately discovered by casual theorizing or casual empiricism. In many models the Euler equation is identical for complete and incomplete markets so that the issue cannot be resolved by examining orthogonality conditions as in GMM estimation. Advocates of incomplete markets might say that due to moral hazard, limited commitment and transaction costs, it is clear that governments cannot issue a full set of contingent claims, so that incomplete markets should be introduced into the analysis. But this is not enough to rule out complete markets as a reasonable assumption. The

[1] See, *inter alia,* Lucas and Stokey (1983), Zhu (1992), Chari, Christiano and Kehoe (1994), Klein and Rios-Rull (2000) for complete markets versions and Marcet, Sargent and Seppala (2000) and Scott (2000) for incomplete markets.

literature in financial economics has a number of examples where a limited number of securities may generate the same equilibrium as a full array of contingent claims. Further, the recent literature introducing incomplete markets into asset pricing models suggests this does not substantively influence variables in the model[2]. In addition a number of authors point out how through the choice of maturity, using fixed or floating rate debt, price indexed debt, nominal debt or domestic or foreign currency denominated debt, the government has access to a wide range of securities, so it might appear that the complete market model is most appropriate.[3] By providing a simple test for the existence and extent of market completeness our analysis enables a more definitive perspective on this debate.

By focusing on the *behavior* rather than the *structure* of debt[4] we arrive at a remarkably straightforward test for whether government bond markets are complete: under a benevolent government, if bond markets are complete, the market value of government debt is *no more* persistent than the real variables in the economy. Further, under complete markets the market value of debt *decreases* in response to an *increase* in government spending. By contrast, under incomplete markets government debt is *more* persistent than other variables in the economy and *increases* in response to adverse government expenditure shocks.

We establish these features by mixing analytic and simulation results with some intuitive descriptions of the model behavior. We provide a general theorem describing a recursive formulation for debt which has strong implications about the persistence of debt under complete markets. We provide the analytic solution to an example under incomplete markets to describe how debt behaves. These analytic results (in sections 2 and 4.2) are of independent interest. Finally, we provide simulations of all models considered to demonstrate the quantitative importance of the effects that we have found.

Examining US data using some measures of persistence and estimated

[2]See, for example, Tellmer (1993), Heaton and Lucas (1996), Krusell and Smith (1998) and Marcet and Singleton (1999).

[3]See, *inter alia*, Bohn (1990), Calvo and Guidotti (1990), de Fontenay, Milessi-Feretti and Pill (1995), Missale (1999), Buera and Nicolini (2000) and Lloyd-Ellis and Zhu (2000).

[4]Hansen, Sargent and Roberds (1991) discuss some difficulties in testing the budget constraint without observations on debt and without assumptions on the behaviour of the government. Our study has a very different focus, since we try and establish whether complete or incomplete markets is most appropriate, we use data on debt, and we start out by imposing behavioral assumptions on the government from the beginning.

VARs we find overwhelming evidence that government debt is best characterized by models with incomplete bond markets.We also show that incomplete markets are likely to lead an observer to mistake the optimal (and sustainable) debt policy with an insolvent policy. However, under complete markets the optimal policy differs strongly from unsustainable policies. This has strong implications for the role of debt limits and debt management: the better is debt management e.g the more complete the market for government debt, the more efficient debt limits are in discriminating against unsustainable fiscal policies. .

The plan of the paper is as follows. In Section 2 we outline the behaviour of government debt in the case of complete markets and we introduce the models that we will discuss in the paper, Section 3 examines postwar US data. In section 4 we examine the implications of an incomplete market model where the government can only issue one-period risk-free bonds and compare them with the behaviour of US government debt and our complete market model. In section 5 we consider the implications of our results for sustainability, debt limits and debt management and a final section concludes.

2 Debt under Complete Markets

We start by studying the behavior of debt under complete markets. We show that, in a very large class of models, equilibrium debt is a fixed function of the state variables determining the primary deficit and interest rates. As a consequence, the persistence of government debt will be no greater than any of the variables which determine deficits and interest rates. We also show that if the primary deficit process follows an autoregressive process with positive serial correlation, as in US data, then debt *decreases* in response to an increase in the government deficit.

Assumptions on the fundamentals of the model.

The variables in the economy are $\{x_t, s_t\}$, where s_t are exogenous shocks and x_t endogenous variables. The process $\{s_t\}$ is Markov and, without loss of generality can be partitioned into two subvectors $s_t = (s_t^1, s_t^2)$ with s_t^2 including those shocks known one period ahead and s_t^1 contains the rest[5]. For simplicity, assume that the distribution of the exogenous shocks conditional

[5]More precisely, s_t^2 contains the elements of s_t that are measurable with respect to s_{t-1}, and s_t^1 those that are not.

on the past has a density. There are I agents and each agent at time t chooses consumption and obtains net income from several sources. We denote by ω_t^i the value of agent i's deficit in period t (in units of the numéraire consumption good), i.e. expenditure minus income at time t.

At time t there exists a spot market for claims contingent on all possible values of s_{t+1}^1. A bond contingent on a value $\overline{s}^1 \in S^1$ will pay one unit of the consumption numéraire if $s_{t+1}^1 = \overline{s}^1$ occurs, and zero otherwise. Here $S^1 \equiv \cup_{t=1}^{\infty} \{\text{support of } s_t^1\}$. We denote by $b_t^i(\overline{s}^1)$ the quantity of bonds contingent on the occurrence of $\overline{s}^1$ purchased by agent i at time t. Hence, agent i has to choose a function $b_t^i : S^1 \rightarrow R$ in each period, this function may depend on the period and the realization. The payoff of this portfolio next period is $b_t^i(s_{t+1}^1)$ and the budget constraint of each agent $i = 1, ..., I$ satisfies

$$b_{t-1}^i(s_t^1) = \omega_t^i + \int_{S^1} b_t^i(\overline{s}^1)\ p_t^b(\overline{s}^1)\ d\overline{s}^1 \tag{1}$$

for all agents, periods and realizations. Here $p_t^b(\overline{s}^1)$ is the price at time t of a bond contingent on $s_{t+1} = \overline{s}^1$. All agents are prevented from defaulting and from running Ponzi schemes.

Assumptions on the behavior of the model in equilibrium.

In equilibrium deficits and interest rates can be formulated recursively in the following sense: there is a set of state variables z_t, with z_{-1} a constant. We assume that the state variables, the deficits of all agents, and bond prices are given by time-invariant functions of the state variables; i.e., there exist time-invariant functions h, f^i, p such that

$$z_t = \mathrm{h}(z_{t-1}, s_t) \tag{2}$$

$$\omega_t^i = \mathrm{f}^i(z_{t-1}, s_t) \tag{3}$$

$$p_t^b(\overline{s}^1) = \mathrm{p}(\overline{s}^1, z_{t-1}, s_t)\ \mu(\overline{s}^1, s_t) \tag{4}$$

for all i,t, $\overline{s}^1$,where μ is the density of s_{t+1}^1 conditional on $(s_t, s_{t-1}, ...)$. We also assume the limiting condition

$$\lim_{T\rightarrow\infty} E_t \left(b_{t+T}(s_{t+T+1}^1) \prod_{j=1}^{T} \mathrm{p}(s_{t+j+1}^1, z_{t+j-1}, s_{t+j}) \right) = 0 \tag{5}$$

for all t and all realizations with probability one.

A large variety of models satisfy these assumptions, including models with multiple agents, public goods, distorting taxation, time-non-separable utility function, externalities, monopolistic power, market frictions, non-rational expectations or credit constraints.

Most models in the literature guarantee that a version of (4) holds as follows: if at least one agent $\bar{i}$ chooses an interior solution for all bonds in all periods, behaves competitively in the bond market, has full information, rational expectations and a utility function $E_0 \sum_{t=0}^{\infty} \beta^t \; u^{\bar{i}}(z_t, z_{t-1}, s_t)$, then

$$p_t^b(\overline{s}^1) = \beta \; \frac{u_c^{\bar{i}}(\mathrm{h}(z_t, \overline{s}^1, s_{t+1}^2), z_t, \overline{s}^1, s_{t+1}^2)}{u_c^{\bar{i}}(z_t, z_{t-1}, s_t)} \; \mu(\overline{s}^1, s_t) \tag{6}$$

where $u_c^{\bar{i}}$ is the derivative of agent $\bar{i}$'s utility function with respect to his/her own numéraire consumption, so that (4) is satisfied.[6]

At this point, we do not know much about the behavior of the bond portfolio b_t^i. This portfolio could depend on time, or the realizations of the process, or state variables other than z and s. In particular, equation (1) suggests that bond holdings depend on the realized value of $b_{t-1}^i(s_t^1)$, so it might seem that this variable should be included in the set of state variables. However, the following theorem shows that bond holdings are a time-invariant function of the same set of variables that allow for the recursive formulation of prices and deficits.

Proposition 1 *For a model satisfying the above assumptions, given functions* h, f, p, *the equilibrium portfolio of contingent bonds is given uniquely by a time invariant function* D^i

$$b_t^i(\cdot) = \mathrm{D}^i(\cdot, s_{t+1}^2, z_t) \tag{7}$$

for all realizations and all t, i, *where*

$$\mathrm{D}^i(s_t, z_{t-1}) \equiv \omega_t^i + E_t \sum_{j=1}^{\infty} \; \omega_{t+j}^i \prod_{\tau=0}^{j-1} \mathrm{p}(s_{t+\tau+1}^1, z_{t+\tau-1}, s_{t+\tau}) \tag{8}$$

[6]Note that, under the above assumptions, (s_{t+1}^2, z_t) are exact functions of (z_{t-1}, s_t) so that (5) is a function of $(\overline{s}^1, z_{t-1}, s_t)$ alone. Also note that, in this case, $\prod_{j=t}^{T} \mathrm{p}(s_{t+j+1}^1, z_{t+j-1}, s_{t+j}) \;= \beta^T \; \frac{u_c^{\bar{i}}(z_{t+T}, z_{t+T-1}, s_{t+T})}{u_c^{\bar{i}}(z_t, z_{t-1}, s_t)}$ and (4) is satisfied as long as the ratio of utilities is guaranteed to be bounded.

Proof

First of all notice that, since s_t is Markov, the conditional expectations involving future deficits conditional on the past are a function only of (s_t, z_{t-1}); therefore D^i satisfying (8) is time-invariant.

To show that any equilibrium bond portfolio satisfies (7) notice that

$$\begin{aligned} b^i_{t-1}(s^1_t) &= \omega^i_t + \int_{S^1} b^i_t(\overline{s}^1)\ \mathrm{p}(\overline{s}^1, z_{t-1}, s_t)\ \mu(\overline{s}^1, s_t)\ \ d\overline{s}^1 \\ &= \omega^i_t + E_t\ \left(b^i_t(s^1_{t+1})\ \mathrm{p}(s^1_{t+1}, z_{t-1}, s_t)\right) \\ &= \omega^i_t + E_t \sum_{j=1}^{\infty}\ \omega^i_{t+j} \prod_{\tau=0}^{j-1} \mathrm{p}(s^1_{t+\tau+1}, z_{t+\tau-1}, s_{t+\tau}) \end{aligned} \tag{9}$$

for all t, i. The first equality uses (1) and (4), the second equality uses the definition of conditional expectation, and the third equality is obtained by recursive forward substitution of the random variable $b^i_t(s^1_{t+1})$ and (5).

To show that $\mathrm{D}^i(\cdot, s^2_{t+1}, z_t)$ is an equilibrium portfolio it is enough to show that this portfolio satisfies the budget constraint (1) for all t, i and all realizations, and for the equilibrium prices (4). Note that

$$\mathrm{D}^i(s^1_t, s^2_t, z_{t-1}) = \omega^i_t + \tag{10}$$

$$E_t \left[\mathrm{p}(s^1_{t+1}, z_{t-1}, s_t)\ \left(\omega^i_{t+1} + E_{t+1} \sum_{j=1}^{\infty}\ \omega^i_{t+j+1} \prod_{\tau=0}^{j-1} \mathrm{p}(s^1_{t+\tau+2}, z_{t+\tau}, s_{t+\tau+1})\right)\right]$$

$$\begin{aligned} &= \omega^i_t +\ E_t\ \left[\mathrm{p}(s^1_{t+1}, z_{t-1}, s_t)\ \mathrm{D}^i(s^1_{t+1}, s^2_{t+1}, z_t)\ \right] \\ &= \omega^i_t + \int_{S^1} \mathrm{p}(\overline{s}^1, z_{t-1}, s_t)\ \mathrm{D}^i(\overline{s}^1, s^2_{t+1}, z_t)\ \mu(\overline{s}^1, s_t)\ d\overline{s}^1 \\ &= \omega^i_t + \int \mathrm{D}^i(\overline{s}^1, s^2_{t+1}, z_t)\ p^b_t(\overline{s}^1)\ d\overline{s}^1 \end{aligned}$$

Here, the first equality follows from the definition of D^i and the law of iterated expectations, the second equality again from the definition of D^i, the third equality from the definition of conditional expectation and the fact that s^2_t is known one period in advance, and the last equality from the formula for equilibrium bond prices.

Therefore, taking $b^i_t(\cdot) = \mathrm{D}^i(\cdot, s^2_{t+1}, z_t)$, the budget constraint of all agents is satisfied, so that this gives the equilibrium portfolio.■

Proposition 1 says that the bond portfolio at time t is independent of the realization of s_t^1. It follows that if z_t includes only *real* variables, as is often the case in models under complete markets, debt is a time-invariant function only on those real state variables. Past shocks (and time), and the influence they possibly had on debt of any agent, bear no influence on today's equilibrium value of debt, over and above the effect that past shocks (and time) have on current fundamentals.

To show how the Theorem works we now introduce the two models that we analyze throughout the paper.

Model 1 - (No capital accumulation)

This example generalizes the model of Lucas and Stokey (1983) by introducing a productivity shock θ_t. Adding this productivity shock is quite simple and helps in matching the data better later in the paper.[7]

Output is given by $y_t = \theta_t(1 - l_t)$ where l_t denotes leisure. The resource constraint is $y_t = c_t + g_t$ where c denotes consumption and g government expenditure. The stochastic process $\{g_t, \theta_t\}$ is exogenous and Markov of order one. A consumer with utility function $E_0 \sum_{t=0}^{\infty} \beta^t\ u(c_t, l_t)$ has access to complete markets, behaves competitively, obtains a competitive wage (in equilibrium equal to θ_t) for each hour worked, and pays labor taxes to the government.

The government levies a proportional tax rate τ_t on the labor income of the consumer and faces the budget constraint

$$b_{t-1}^g(g_t, \theta_t) = g_t - \tau_t\theta_t(1 - l_t) + \int b_t^g(\overline{g}, \overline{\theta})\ p_t^b(\overline{g}, \overline{\theta})\ \ d(\overline{g}, \overline{\theta}) \tag{11}$$

The government has a fixed initial level of bonds and chooses tax rates and government bonds so as to maximize consumer welfare.

Following Lucas and Stokey, it can be shown that the Ramsey equilibrium satisfies

$$(\tau_{t,} c_t, l_t) = G(g_t, \theta_t). \tag{12}$$

for all $t > 0$, and a time-invariant function G.

This economy satisfies all the assumptions above[8]. Here, $x_t = (\tau_t, c_t, l_t)$, $s_t^1 = (g_t, \theta_t)$ and no variables play the role of s_t^2 or z_t. Proposition 1 therefore

[7] Gorostiaga (1999) also introduces a productivity shock in a similar model.

[8] Strictly speaking, the recursivity assumptions are only satisfied in this model for $t > 0$. This changes the results in a trivial way: Proposition 1 and Corollary 2 only hold for $t > 0$.

implies

$$b_t^g(\overline{g},\overline{\theta}) = \mathrm{D}^g(\overline{g},\overline{\theta})$$

for all $t > 0$ and all realizations. In other words, the government always buys the same amount of each security in equilibrium, regardless of the current state of the economy and the period. In this model, the debt portfolio does not respond to a shock in government spending or productivity, even though all other equilibrium variables respond to these shocks according to (12).[9]

Model 2 - Capital accumulation under uncertainty

As a key focus of our analysis is the relative persistence of debt compared to other variables we introduce in Model 2 an additional source of persistence - capital accumulation. Output is now given by

$$y_t = \theta_t k_{t-1}^{\alpha}(1 - l_t)^{1-\alpha}$$

and the resource constraint becomes

$$k_t - (1-\delta)k_{t-1} + c_t + g_t = y_t$$

where δ is the depreciation rate.

Chari, Christiano and Kehoe (1994) (CCK) show that if we introduce capital and labor taxes, then the model can be formulated recursively with $s_t^1 = (g_t, \theta_t)$, $x_t = (\tau_t^l, \tau_t^k, l_t, k_t, c_t)$, $z_t = k_t$ and the solution is a time-invariant function of (g_t, θ_t, k_{t-1}) for $t > 0$. Therefore, our theorem says that

$$b_t^g(\overline{g},\overline{\theta}) = \mathrm{D}^g(\overline{g},\overline{\theta}, k_t)$$

so that the porfolio of debt depends only on today's value of the capital stock. In this case, a shock to the economy has an effect on debt only to the extent that this shock affects capital. If capital has an ergodic distribution, then so does debt and will show fluctuations around a long run mean.

The examples above display an important feature of debt under complete markets: glancing at equation (1), where the payoff of last period's portfolio seems to determine this period's debt, one might guess that past shocks influence today's debt through $b_{t-1}^g(s_t)$. But the above result says that, after

[9] The portfolio would be no longer constant if we assumed that $\{g_t, \theta_t\}$ is Markov of order *two.* More precisely, if the conditional density $f_{g_{t+1},\theta_{t+1}|g_t,\theta_t,g_{t-1},\theta_{t-1},\ldots}(\cdot) = \mu(\cdot; g_t, \theta_t, g_{t-1}, \theta_{t-1})$ then, Proposition 1 implies $b_t^g(\cdot) = \mathrm{D}^g(\cdot, g_t, \theta_t)$ for $t > 0$, so that the portfolio depends on current shocks.

taking into account the state variables z_t, s_t^2, and since these do not include debt, the type and number of bonds issued in a given period does not depend on the level or structure of debt in previous periods. Under complete markets, the equilibrium debt portfolio is structured in such a way so as to guarantee that the payoff of the portfolio is equal to the present value of future deficits in all periods and for all realizations. This feat is performed in such a way that shocks to the economy do not influence debt over and above the effect of these shocks on z and s_t^2.

Instead of just looking at the equilibrium portfolio of Arrow-Debreu securities, it is useful to characterize the behaviour of the market value of debt, both because this does not depend on the structure of the bond market (as long as markets remain complete) and because the value of debt has an obvious empirical counterpart. The following Corollary explains the determinants of the value of debt[10]:

Corollary 2 *The value of the equilibrium portfolio of contingent bonds* $vb_t^i \equiv \int_{S^1} b_t^i(\overline{s}^1)\ p_t^b(\overline{s}^1)\ d\overline{s}^1$ *is uniquely given by a (time-invariant) function* $\mathrm{V}^i : Z \times S^1 \times S^2 \rightarrow R$ *such that*

$$vb_t^i = \mathrm{V}^i(z_t, s_t) \equiv E_t \left[\mathrm{D}^i(z_t, s_{t+1}^1, s_{t+1}^2)\ \mathrm{p}(s_{t+1}^1, z_t, s_t) \right]$$

for all t, i, *for all realizations.*

Proof

The Markov and recursive assumptions imply that V^i is time invariant. Proposition 1 and (4) imply

$$\int_{S^1} b_t^i(\overline{s}^1)\ p_t^b(\overline{s}^1)\ d\overline{s}^1 = \int_{S^1} \mathrm{D}^i(\overline{s}^1, s_{t+1}^2, z_t)\ \mathrm{p}(\overline{s}^1, z_t, s_t)\ \mu(\overline{s}^1, s_t)\ d\overline{s}^1 = \mathrm{V}^i(z_t, s_t)$$

■

Corollary 2 implies that, in Model 1,

$$vb_t^g = \mathrm{V}^g(g_t, \theta_t) \tag{13}$$

while in Model 2

$$vb_t^g = \mathrm{V}^g(g_t, \theta_t, k_t). \tag{14}$$

[10] Chari, Christiano and Kehoe (1994) show a similar result for their model.

In other words, for Model 1 we now have the less extreme result that the market *value* of debt is influenced by current shocks to the economy. But, again, contrary to casual inspection of (1) the inherited value of government debt does not affect its current value directly. Again, past shocks, and the influence they possibly had on government debt, bear no influence on today's equilibrium value of government debt, over and above the effect that past shocks have on current fundamentals.

These examples illustrate our key proposition and corollary. We now turn to simulations to add additional insight into these results, enabling us to gauge the extent to which debt displays less persistence than other variables. These simulation results show that debt under complete markets has two properties: *i)* it has the same (or less) persistence as real variables in the model and *ii)* debt *decreases* in response to *higher* primary deficits. However, before outlining our simulations it is necessary to define our measures of persistence.

2.1 Measures of Persistence

We provide two measures of persistence. The first are detailed impulse response functions of endogenous variables to the fundamental shocks in the economy (g_t, θ_t). For this purpose, we use the law of motion from numerical solutions to compute how each variable would evolve if the innovation to each shock had a realization equal to $(\sigma_{\varepsilon}^{j}, 0, 0, ...)$, $j = g, \theta$ and σ_{ε}^{j} denotes the standard deviation of the innovation of variable j. We call this the 'fundamental' impulse response function (FIRF), as it is based on the model. When the FIRF coefficients at low frequencies are large relative to the coefficients at high frequencies we will say that the variable has high persistence.

While FIRFs are useful in understanding how the model works, it is difficult to study a direct counterpart to the FIRF in the data, since fundamental shocks cannot be observed directly. To compare the model with the data we therefore identify innovations in *both* the data and our simulations using the Cholesky decomposition. The FIRFs help us understand the properties of our model under complete and incomplete markets while the estimated IRFs enable us to compare directly the model with the data.

Since IRFs are very high-dimensional objects we also find it useful to consider an overall measure of persistence. For this purpose we utilize the k-variance ratio defined as[11]

$$P_y^k = \frac{Var(y_t - y_{t-k})}{k\ Var(y_t - y_{t-1})}$$

In the case of a stationary and ergodic variable we have $Var(y_t - y_{t-k}) \to 2\ var(y_t)$ as $k \to \infty$. Therefore, for a stationary case P_y^k goes to zero as $k \to \infty$. For instance, in the case of an i.i.d. process $P_y^k = 1/k$. By contrast in the case of a pure unit root $P_y^k = 1$ for all k. If the variable shows more than unit root persistence $P_y^k > 1$.

2.2 Persistence of Complete Market debt

For the rest of the paper, and in all models, we assume the utility function

$$u(c_t, l_t) = \frac{c_t^{1-\gamma_1}}{1-\gamma_1} + \frac{l_t^{1-\gamma_2}}{1-\gamma_2}$$

and set β =0.98 and γ_1=1, γ_2=2 which sets the share of leisure in the time endowment equal to 30%.

We assume g follows a truncated AR(1), and θ_t a log AR(1) process

$$g_t = \begin{cases} \overline{g} & \text{if } (1-\rho^g)g^* + \rho^g\ g_{t-1} + \varepsilon_t^g > \overline{g} \\ \underline{g} & \text{if } (1-\rho^g)g^* + \rho^g\ g_{t-1} + \varepsilon_t^g < \underline{g} \\ (1-\rho^g)g^* + \rho^g\ g_{t-1} + \varepsilon_t^g & \text{otherwise} \end{cases}$$

$$\log\theta_t = \rho^\theta\ \log\theta_{t-1} + \varepsilon_t^\theta$$

for $\varepsilon_t^g, \varepsilon_t^\theta$ i.i.d., mean zero and mutually independent. To facilitate comparisons across examples, we assume both shocks have the same persistence, i.e. $\rho^g = \rho^\theta$. We assume $\varepsilon_t^\theta \sim N(0, 0.007^2)$, $\varepsilon_t^g \sim N(0, 1.46^2)$, $g^* = 17.5$, with an upper bound of $\overline{g} = 24.5$, and a lower bound $\underline{g} = 10.5$. These values are chosen so that in the non-stochastic steady state of Model 1, government expenditure amounts to 25 percent of GDP and fluctuates within the range 15 and 35% of output. We consider two different assumptions regarding the persistence of the shocks a) both sequences are i.i.d ($\rho^g = \rho^\theta = 0$) and, b) strongly positively serially correlated shocks $\rho^g = \rho^\theta = 0.95$.[12]. For Model 2

[11] Cochrane (1988) uses this statistic in the macroeconomics literature to measure persistence in US GDP data.

we set $\alpha = 0.4$ and the depreciation rate $\delta = 0.05$.

In both models we only allow for labor taxes, even though Model 2 features capital accumulation. The focus of our analysis is how debt behaves under complete and incomplete markets and introducing capital taxation causes problems for this issue. CCK show that if the government has access to a full set of ex post contingent capital taxes, these can be used to complete the markets; in which case it does not matter whether bond markets are complete or incomplete. Our analysis in the rest of the paper is therefore predicated on the assumption that contingent capital taxes are unavailable to the government. In Model 2 we formulate this in an extreme way by setting the capital tax to zero. However, our general results would still hold so long as capital taxes react with a delay to contingencies.[13]

Figures 1 to 3 show the FIRFs of a number of endogenous variables, and Figure 4 shows the k-variance ratio for various models under both complete and incomplete markets. Figure 1 shows Model 1 under i.i.d. shocks[14]. Not surprisingly, under complete markets, given the i.i.d nature of the shocks, there is little in the way of a persistent response in any of the variables and the persistence of how the market value of debt responds to these shocks is no greater than in any other variable, confirming the intuition of Proposition 1. Figure 4 shows the k-variance ratio for the same variables and shows an identical degree of persistence across them all.

Figure 2 shows the FIRFs from the same model but now assuming persistent shocks. As expected from Proposition 1, under complete markets the persistence in the shocks is passed through all the endogenous variables and the response of each variable is now much more distinctive. In all cases the response declines roughly geometrically, at a rate of decay approximately equal to $\rho^g = .95$. The middle panel of Figure 4 shows that all variables have approximately equal persistence.

Figure 3 shows the FIRFs from Model 2 with capital accumulation. The additional persistence provided by capital accumulation is obvious, and the

[12] All models are solved using the Parameterised Expectations Algorithm described in den Haan and Marcet (1990). To solve the incomplete markets models we use the approach in Marcet, Sargent and Sepälä (2000). Also, to solve the models with capital accumulation we use the adaptations described in Abraham, Marcet and Scott (2001).

[13] This equivalence between complete markets and a complete set of contingent capital taxes mean that our later empirical findings can be interpreted as rejecting *both* assumptions.

[14] The vertical axis of all the FIRF graphs is in units of the variable under consideration.

deficit now shows a long and protracted decline after an adverse expenditure shock. With the exception of the response of output, which puts almost all weight at low frequencies, all of the complete market responses yet again display persistence declining close to a geometric rate. The final panel of Figure 4 shows that, according to the k-variance ratio, output is the most persistent variable with debt sharing the persistence of the deficit. So, in this example, debt is even less persistent than output under CM. All of these results are consistent with our proposition and corollary - under complete markets debt has less than or the same persistence as other variables in the model[15].

2.3 Response of debt to deficit shocks

Inspection of Figures 2 and 3 reveal another distinguishing feature of the complete market model: the market value of debt *falls* in response to persistent shocks that increase the deficit. The reason for this negative response is as follows. Consider the standard case when bond prices are given as in (6). Letting $MV_t(\equiv -vb_t^g)$ denote the market value of debt we have

$$MV_t = \omega_t^g - b_{t-1}^g(s_t^1) = -E_t \sum_{j=1}^{\infty} \beta^j \, \frac{u_{c,t+j}}{u_{c,t}} \, \omega_{t+j}^g \tag{15}$$

where the first equality comes from (1), the second from (9). Therefore when ω_{t+j}^g is positively serially correlated, a shock to the deficit today (say, higher than expected g) forecasts higher than average ω_{t+j}^g in the future, hence the right side of equation (15) goes down and so does the value of debt.[16] To be more precise, note that in the case that the deficit follows an AR(1) process $\omega_t^g = \rho^\omega \, \omega_{t-1}^g + \varepsilon_t^\omega$ and that $u_{c,t}$ is a constant then $MV_t = -\frac{\rho^\omega \beta}{1-\rho^\omega \beta}\omega_t^g$, so that higher primary deficits lead to a *fall* in government debt.

Under complete markets the government's bond portfolio is such that, if a bad shock occurs, the payoff should be sufficiently high to cover today's larger deficit, but also for the expected larger future deficits.

[15] FIRFs for all other endogenous variables are available on request.

[16] One can find complete market examples where government spending and debt increase in the same period. One case is Example 4 in LS. This occurs due to the fact that a high level of government spending (a 'war') occurs in only one period in that example and is not followed by similar high levels of spending.

3 Characterizing Post-War US data

The previous section outlined two specific features of government debt behaviour when bond markets are complete : *(i)* debt is no more persistent than any other variable in the model and *(ii)* in response to an adverse but persistent government expenditure shock, government debt should fall. In this section we examine whether either of these CM features are present in the behavior of US government debt.

We start our analysis by estimating a trivariate VAR using annual data on the primary deficit, GDP and debt. More precisely, and in order to work with data that is roughly stationary, we use the primary deficit/GDP ratio, the change in the logarithm of GDP and the debt/GDP ratio. We do not think of this VAR as a direct way of testing a particular structural model nor as providing a definitive characterization of how fiscal policy impacts on the economy[17]. We merely use the VAR as a convenient way to summarize the data. Also, by estimating the same VAR for US data as for the models we study, we have a systematic way of comparing models to data.

Since we are not aiming to identify structural VAR's we take a straightforward approach to orthogonalization and use triangular leading matrices (i.e., a Cholesky decomposition), and then check that if this particular decomposition is applied to the model it brings out the same insights. Since all the fundamental shocks in the model (e.g. ε^g and ε^θ) are, in the first instance, a shock to the deficit, we order the variables by placing the deficit first, debt last, and output growth in second place. We follow the common practice of calling the first orthogonalized shock the 'deficit shock', the second 'the output shock' and so on. However, we note that this nomenclature may mislead - for instance, the first shock in our estimated VAR contains a large part of the innovation to productivity, since an innovation to output will influence tax revenues and, therefore, the deficit.[18]

The data used throughout our study is from the 1999 version of the OECD Fiscal Position and Business Cycles CD-ROM. The deficit refers to the primary deficit of the general government and debt refers to the gross general government debt.The data is annual from 1970-1999.

Figure 5 shows the estimated impulse response functions from our basic

[17] See Blanchard and Perotti (1999) or Burnside, Eichenbaum and Fisher (2000) for such an analysis.

[18] We will also see in Appendix B that this orthogonalization brings out most of the properties of the incomplete markets model that we are exploring.

trivariate VAR specification[19] from which we glean the following results:

1. the shocks to both deficit and GDP have a large and very persistent effect on debt which increases for many periods and which is more persistent than for all other variables.

2. the effect of deficit shocks on debt and deficit is *positive*.

3. the effect of GDP shocks on debt and deficit is *negative*.

4. the effect of deficit shocks and GDP on the deficit disappears in the medium run (after around three years for this particular VAR specification).

5. the responses to the debt shock are only significantly different from zero for the first few lags of the Debt/GDP equation[20].

Facts 1 and 2 run directly counter to the features *i)* and *ii)* of our complete market analysis. Deficit and GDP are affected by fiscal or GDP shocks but the impact dies out quite quickly; however, the impact on debt is much more persistent and takes a very long time (if ever) to fade away. Therefore, Fact 1 contradicts feature *i)* of complete markets. Furthermore, deficit shocks have a positive effect on debt, so that facts 2 and 3 contradict feature *ii)* of complete markets. These results suggest that governments use variations in the market value of debt as a buffer stock for fiscal shocks rather than using

[19]Results are robust to a wide range of different VAR specifications and estimators.

[20]A puzzle in this VAR is what is meant by debt shocks. Since debt today is an exact function of past debt, interest rates and deficit, the debt innovation would be zero in a non-linear VAR and if interest rates were constant. Figure 5 suggests debt shocks have a minimal role to play in explaining variations in the VAR, but that the role is not zero at short lags. It first occured to us that this innovation may be capturing interest rate effects, but extending our VAR to include interest rates made no substantive changes to our results and neither did it appreciable reduce the role of the debt shock. A number of potential explanations exist. The first is that official debt and deficit statistics do not satisfy the simple accounting identity (1) because of definitional changes, revaluation effects and asset sales which are excluded from measurements of the fiscal deficit. Measurement error is therefore a possible explanation. Another explanation is that the linear approximation implied by a VAR is sufficiently far away from the true non-linear relationship for the linear innovation to be non-zero; this explanation receives some support from the fact that a similar effect of the debt innovation is found in the EIRFs from the simulated data in Appendix B.

the insurance role that bond interest payments play in the case of complete markets.[21]

Further evidence against the complete market model is given in Figure 6 which displays the k-variance ratios for GDP, taxes, expenditure and the debt/GDP ratio in the data. Comparing this with the charts of Figure 4 confirms strikingly the result that government debt contains far more persistence than any other variable in the economy. Overall our analysis suggests overwhelmingly that US data is not consistent with optimal fiscal policy under complete markets.

4 Debt under Incomplete Markets

In this section we argue that incomplete markets perform much better in accounting for the data. Because market incompleteness covers a wide range of possibilities we cannot show a completely general result. However, by using a variety of examples combined with intuition we suggest that under incomplete markets the persistence of government debt is *greater* than that in other variables in a large variety of models, and that the response of debt to a deficit shock is of the observed sign.

4.1 An Intuition

Under complete markets if the government experiences, for example, unexpectedly low revenue or high spending, it would have previously insured against this kind of shock via issuing contingent debt, and the payoff from the portfolio would help offset the shock. But under incomplete markets it is the size of debt that helps to partly insulate tax rates from these shocks. Consider a government that runs a balanced budget. Such a policy causes taxes to be very volatile and under standard assumptions on preferences leads to a loss of welfare. Therefore, a better policy is to use government debt as a buffer stock: by increasing debt in bad periods and decreasing it in good times the government could achieve smoother taxes. The problem with

[21] To confirm that this difference with the data is not an artifact of the orthogonalization chosen, we can see in Appendix A that the estimated IRF using data simulated under Model 1 and CM with the same orthogonalisation as in the data confirms our interpretation of the CM behavior and how it differs from the data.

financing the deficit this way is that the higher debt will, via the intertemporal budget constraint, generate higher interest payments in the future and so higher future tax rates. Optimal fiscal policy involves reducing the volatility of tax rates in the very short run, and shifting this volatility to the very far future, using debt for this purpose, much in the way that precautionary savings are used in the consumption literature.

This effect of debt on future taxes under incomplete markets is at the heart of the unit-root/persistence properties of taxes and debt that we outline in this section. As shown in Barro (1979), Marcet, Sargent and Seppälä (2000) (henceforth MSS) and Scott (2000) an increase in government spending has a permanent effect on taxes (in the sense that taxes are a risk-adjusted martingale) precisely because under incomplete markets the government uses debt to buffer the impact of shocks on tax rates. As a consequence, under incomplete markets debt displays substantial persistence compared to other variables as well as a positive response to deficit shocks.

4.2 An Example

We provide a special case of Model 1 where the only shock to the deficit occurs in period $t = 1$. The evolution of all variables after this period can be interpreted as the response to the shock in period 1. The analytic solution we provide to an infinitely lived agent model under incomplete markets is of independent interest. We will see that under incomplete markets the deficit responds strongly in period 1 and weakly thereafter, while the response of debt is permanent This is very different from what would occur under a similar probabilistic structure in a standard growth model, where the effect on capital stock of this one-time shock would die out as time passed.

Assume, in Model 1, that g_t is only random for $t = 1$. In particular $P(g_1 = g^H) = P(g_1 = g^L) = .5$ and $g^H = \overline{g} + \eta, \quad g^L = \overline{g} - \eta$, for some $\eta, \overline{g} > 0$. Government spending is constant in all other periods: $g_0 = g_t = \overline{g}$ for all $t \geq 2$. Assume that $u(c, l) = c + H(l)$ and that initial debt is zero: $b^g_{-1} = 0$.

We have that in this model, the first order conditions of the consumer and of the Ramsey optimizer under both complete and incomplete markets imply[22]

$$H'(l_t) = 1 - \tau_t \tag{16}$$

[22] See, for example, MSS.

$$H'(l_t) + \lambda_t \ [1 - H'(l_t) + H''(l_t)(1 - l_t)] = 0 \tag{17}$$

where λ_t is the Lagrange multiplier of the government budget constraint. Denote by $\Lambda(\cdot)$ the function mapping feasible revenue values into the corresponding multiplier λ guaranteeing that (17) is satisfied.[23] We assume that Λ is monotone.[24] We also need to assume

$$\sup_{\widetilde{R},\widehat{R}\in(\overline{g}-2\eta,\overline{g}+2\eta)} \left|\frac{\Lambda'(\widetilde{R})}{\Lambda'(\widehat{R})}\right| < \frac{1}{1-\beta} \tag{18}$$

Since $\sup\left|\frac{\Lambda'(\widetilde{R})}{\Lambda'(\widehat{R})}\right| > 1$, the above condition says that the sup can not be too far from 1, meaning that the derivative can not change too much over the specified interval for revenues. One could interpret this condition as requiring that "Λ is sufficiently linear around $\overline{g}$".[25] In the usual case that β is close to 1 this is not a very restrictive assumption.

We assume that there is an interior solution for both the consumer and the government.

As in all the incomplete market models in this paper we assume that the only financial asset in the economy is a risk-less one-period bond and that government levies only labor taxes. Therefore, the budget constraint of the government is

$$g_t + p_t^b \ b_t^g = w_t \tau_t (1 - l_t) + b_{t-1}^g \tag{19}$$

Denote with superscripts H and L the values of all variables under each realization of g_1. The following result states how a government reacts to this shock under complete or incomplete markets.

[23] Formally, for any feasible $\overline{R}$, and the corresponding labor $\overline{l}$ satisfying $\overline{R} \equiv (1 - H'(\overline{l}))(1-\overline{l})$, Λ is defined as $\Lambda(\overline{R}) \equiv -\frac{1-H'(\overline{l})+H''(\overline{l})(1-\overline{l})}{H'(\overline{l})}$

[24] This is satisfied under standard assumptions on H. See footnote 14 in MSS.

[25] This holds, in particular, if H is linear-quadratic, as in many of the examples of Lucas and Stokey (83). Another sufficient condition is that $\sup_{\widetilde{R},\widehat{R}\in(\overline{g}-2\eta,\overline{g}+2\eta)} \left|\frac{\Lambda''(\widetilde{R})}{\Lambda'(\widehat{R})}\right| < \frac{-\log(1-\beta)}{4\eta}$ This is because, under this condition

$$\log\left|\Lambda(\widetilde{R})\right| - \log\left|\Lambda(\widehat{R})\right| = \left|\frac{\Lambda''(\widetilde{\widetilde{R}})}{\Lambda'(\widetilde{\widetilde{R}})}\right| 4\eta < -\log(1-\beta)$$

This will be satisfied if the second derivative of Λ is sufficiently small, which in turn is guaranteed by a sufficiently small third derivative of H.

Result

- *Under* ***complete markets***

  only the deficit in period 1 is influenced by the shock:

$$\omega_0^g = \omega_2^g = \omega_3^g = ... = 0 \qquad \textit{for all realizations}$$
$$\omega_0^g < \omega_1^{g,H} > \omega_2^g$$

  Furthermore, debt does not respond to a deficit shock:[26]

$$MV_t = 0 \;\text{ for all } t = 0, 1, ... \text{ and all realizations}$$

- *Under* ***incomplete markets***

  the deficit has an immediate positive response to a positive shock in g^1 *and future deficits display a permanent negative response:*

$$\omega_0^g < \omega_1^{g,H} > \omega_2^{g,H} = \omega_3^{g,H} = ... \quad \textit{and} \quad \omega_0^g > \omega_2^{g,H} \tag{20}$$

  Furthermore, debt has a positive and permanent response to a deficit shock:

$$MV_0 < MV_1^H = MV_2^H = ... \tag{21}$$

All inequalities reversed (and equalities still hold) in the event $g_1 = g^L$*.*

Proof

For the government spending process above we have

$$E_t\left(g_{t+j}\right) = \overline{g} \text{ for all } t \geq 0 \text{ and for all } j > 0. \tag{22}$$

Following Lucas and Stokey (1983), under complete markets (17) is satisfied with $\lambda_t = \lambda$ for all t. This, together with (16) implies that leisure and taxes are constant, say $l^{cm} = l_t,\ \tau^{cm} = \tau_t$.

From the budget constraint of the government at period zero we have

$$0 = E_0 \sum_{t=0}^{\infty} \beta^t\ \omega_t^g = \sum_{t=0}^{\infty} \beta^t\ \left(E_0\left(g_t\right) - \tau^{cm}(1 - l^{cm})\right) = \frac{\overline{g} - \tau^{cm}(1 - l^{cm})}{1 - \beta}$$

[26] This example does not display the negative correlation of debt and deficit under CM which we have discussed at the end of section 2. This is not surprising since that feature of CM is present under positively serially correlated deficits a feature absent from the present example.

where the first equality uses $u_c' = 1$, the second equality uses constancy of taxes and leisure, and the third equality uses (22). Therefore,

$$\begin{array}{rcll} \omega_t^g &=& g_t - \tau^{cm}(1-l^{cm}) = \overline{g} - \tau^{cm}(1-l^{cm}) = 0 & \quad t = 0, 2, 3, ... \\ \omega_t^{g,H} &=& g^H - \tau^{cm}(1-l^{cm}) = g^H - \overline{g} = \eta > \omega_0^g & \quad t = 1 \end{array}$$

This proves the statements about the deficit under complete markets.

The value of debt satisfies

$$MV_t = -\sum_{j=1}^{\infty} \beta^j \ \left(E_t\left(g_{t+j}\right) - \tau^{cm}(1-l^{cm})\right) = 0$$

for all t, where the first equality follows from (15) and constancy of taxes and hours, and the second equality follows from (22) and our analysis of deficit. This shows our statements about debt under complete markets.

Under incomplete markets, following MSS we have that the first order conditions of the planner are (16), (17) and

$$\lambda_t = E_t(\lambda_{t+1}) \tag{23}$$

for all t. Since there is no uncertainty after period 1, (23) implies that $E_t(\lambda_{t+1}) = \lambda_{t+1}$ for all $t \geq 1$, so that $\lambda_t = \lambda_1$ for all $t \geq 1$. Then (17) implies $l_1 = l_t$ and (16) $\tau_1 = \tau_t$ for all $t \geq 1$ and all realizations, and since g is also constant after period 2 we have $\omega_2^g = \omega_t^g$ for all $t > 2$ and all realizations, proving the equalities of (20).

From proposition 1 in MSS, and since in this example $p_t^b = \beta$, we have

$$b_{t-1}^g = E_t \sum_{j=0}^{\infty} \beta^j \ \omega_{t+j}^g \qquad \text{for all } t. \tag{24}$$

This and the fact that deficit is constant for $t \geq 2$ imply

$$MV_t = -\beta b_t^g = -\beta \frac{\overline{g} - \tau_1(1-l_1)}{1-\beta} \qquad \text{for all } t \geq 1$$

which proves the equalities in (21).

Until the end of this proof we show that $\omega_1^{g,H} > \omega_0^g$. Since (24) and the equalities in (20) imply that $b_1^g = \frac{\overline{g}-\tau_1(1-l_1)}{1-\beta}$, and by assumption $b_{-1}^g = 0$, the budget constraint of the government at periods 0 and 1 imply

$$\beta^{-1}\left(\tau_0(1-l_0) - \overline{g}\right) = b_0^g = g_1 - \tau_1(1-l_1) + \beta \frac{\overline{g} - \tau_1(1-l_1)}{1-\beta} \tag{25}$$

Let us denote revenue by $R_t \equiv \tau_t(1-l_t)$. Since the last equation holds for both realizations of g_1 and b_0^g is not random, we have

$$b_0^g = g^H - R_1^H + \beta\frac{\overline{g} - R_1^H}{1-\beta} = g^L - R_1^L + \beta\frac{\overline{g} - R_1^L}{1-\beta} \quad . \tag{26}$$

implying

$$R_1^H - R_1^L = (1-\beta)2\eta > 0. \tag{27}$$

Given the process for g, (23) at $t = 0$ implies

$$\Lambda(R_0) = \frac{\Lambda(R_1^H) + \Lambda(R_1^L)}{2} \tag{28}$$

thus, since $R_1^H > R_1^L$ and Λ is monotone we have

$$R_1^H \geq R_0 \geq R_1^L \tag{29}$$

Assume towards a contradiction that $R_1^H < \overline{g}$. Then equation (26) implies that $b_0^g > 0$. But equation (29) implies $R_0 < \overline{g}$ and together with the first equality in (25) implies that $b_0^g < 0$, which is a contradiction. Therefore, we have that $R_1^H > \overline{g}$. Similarly, we could prove $R_1^L < \overline{g}$ which, together with (27) implies

$$R_1^H, R_1^L \in (\overline{g} - 2\eta, \overline{g} + 2\eta) \tag{30}$$

The following is true:

$$\begin{aligned} (R_1^H - R_0)\Lambda'(\overline{R}) &= \Lambda(R_1^H) - \Lambda(R_0) = \\ \frac{\Lambda(R_1^H) - \Lambda(R_1^L)}{2} &= \frac{\Lambda'\left(\overline{\overline{R}}\right)(R_1^H - R_1^L)}{2} = \Lambda'\left(\overline{\overline{R}}\right)(1-\beta)\eta \end{aligned} \tag{31}$$

where the first and third equalities follow from the mean-value theorem for some $\overline{R}$, $\overline{\overline{R}} \in (R_1^L, R_1^H)$, the second equality from (28) and the last equality from (27). This equation implies the first inequality in

$$R_1^H - R_0 \leq \left|\frac{\Lambda'(\overline{\overline{R}})}{\Lambda'(\overline{R})}\right|(1-\beta)\eta < \eta$$

and the last inequality follows because (30) implies $\left|\overline{R} - \overline{\overline{R}}\right| \leq 4\eta$ and from the bound in equation (18)

Thus we have $R_1^H - R_0 < \eta$ and since $g_1^H - g_0 = \eta$, we have that $\omega_1^{g,H} > \omega_0^g$.

All the remaining inequalities stated about incomplete markets follow from this fact and analogous arguments.■

Therefore in this stylized example debt shows a permanent response to a fiscal shock under incomplete markets: if government spending is high (low) in period 1, debt increases (decreases) and it remains at this higher (lower) level forever. Even if the shock dies out after one period, its effect on the deficit is much larger in the period when the shock occurs than in the future, and debt stays at the same level implied by the shock forever, thus displaying a persistence greater than any other variable. This is very different from the effects of a shock in a standard RBC model, where under the same stochastic structure the effects of the shock would die out as time passes.[27]

4.3 Simulation Results for Incomplete Markets

To compare more closely the case of incomplete markets we turn once again to Models 1 and 2 but simulate them under the assumption of incomplete markets. There are of course many ways in which bond markets may be incomplete but we consider the polar case where the government can only issue one period risk free bonds. So, the models analyzed in this section are the same as in section 2, except that the budget constraint of the government is now (19) plus some debt limits to rule out Ponzi schemes. The results are shown in Figures 1 to 4, together with the complete market results.

Figure 1 shows the FIRFs for the case of i.i.d shocks. In response to government expenditure shocks the initial response of debt is muted compared to other variables but it continues to accumulate so that it eventually becomes the most affected variable. The response of debt to productivity shocks is also shifted to long lags, indicating that productivity shocks have a highly persistent effect on debt. The k-variance ratio in Figure 4 (top right panel) confirms that the market value of debt contains more persistence than any of the other variables.

Figure 2 shows the case of Model 1 with persistent shocks and the FIRF for incomplete markets once again reveals a more complex picture. The immediate response of debt to the adverse expenditure shock is again small compared to other variables but once more it accumulates so that the effect

[27] For example, in a classical growth model à la Brock and Mirman, the effect of g_1 on the capital stock would be stronger in period 1 and it would die out as time passed.

of the shock is shifted to the low frequencies. The same profile appears in response to the persistent productivity shocks, although here the accumulation in the debt response takes a lot longer. Unlike the case of complete markets, the response of debt to both shocks is hump-shaped and then declines at a slow rate. The combination of the humped response and the slower rate of decline mean that the k-variance ratios reveal substantially more persistence in the case of incomplete markets - compare the middle panels of Figure 4. Debt is not only more persistent than any other variable under incomplete markets but more than ten times as persistent as under complete markets. Figure 2 also shows that incomplete markets reverses the response of debt to persistent government expenditure shocks. Now the market value of debt rises in response to an adverse but persistent government expenditure shock, just like in the data. Under incomplete markets, deficits in the distant future have to decrease in order to prevent debt from exploding.

Figure 3 confirms all of these results for our model with capital accumulation. It is interesting to note that both output and debt have a roughly equally persistent response to an innovation in g, while the response of debt to an innovation in θ is much more persistent than the response of ouptut to the same shock. Overall, debt displays the greatest persistence amongst the endogenous variables, and it increases in response to shocks that cause the primary deficit to grow.

The degree of persistence in our incomplete market simulations far exceeds that found in the data (see in particular the k-variance ratio). This suggests that the nature of incomplete markets that we have assumed - the government only being able to issue one period risk free bonds - is too extreme. However, as mentioned earlier, it is also apparent that introducing some degree of market incompleteness improves dramatically the fit of the model with the data on US government debt.[28,29]

[28] We have checked that this sort of behavior of debt under incomplete markets is present under most parameter values and under various models that we have explored. For example, the same sort of behavior occurs in the model of Gorostiaga (1999), which introduces frictions in the labor market and endogenous government spending.

[29] Appendix B shows the estimated IRFs from simulations of the incomplete market model 1 with persistent shocks using the same orthogonalization as in the data. The impulse-responses based on the VAR are able to tell the same story as the FIRFs from our simulations: in the model debt is more persistent than any other variable under incomplete markets, and it falls in response to a lower deficit. Although it is clear that Model 1 falls short of matching the VAR from the data, it is clear that it does much better than CM.

5 Sustainability, Debt Limits and Debt Management

A large literature has discussed the desirability of imposing limits on government debt, both from an academic and a policy-oriented point of view. It is worthwhile to reconsider this issue in the light of the above results.

We will show that under incomplete markets it is very difficult to distinguish between optimizing and irresponsible governments. In this environment, the virtue of debt limits is that they are an effective way to ensure sustainability of fiscal policy, and they can be monitored with very little information. On the other hand, debt limits constrain the choices of government and as a result will be very costly under some circumstances. We argue that improvements in debt management are desirable not only because it provides better insurance against shocks to the primary deficit but also because it enables debt limits to target unsustainable policies more effectively.

5.1 Sustainability

We will argue here that the large swings in government debt that are a feature of incomplete markets also cause problems for efforts to assess sustainability. A large literature has developed tests of sustainability of fiscal policy.[30] Since many of these tests have been criticized because of their low power[31] we will argue that a monitoring agency processing information optimally, would very often mistake the optimal (and sustainable) policy with an unsustainable policy.

Consider a monitoring agency that, given T observations on the debt/GDP ratio, has to assess whether a government is pursuing a sustainable fiscal policy when, in fact, the data are generated by Model 1 with persistent shocks. The agency does not know the true model, and assumes that the debt/output ratio ($\equiv D_t/Y_t$) follows an AR(n) process. The agency is likely to choose the

[30] See inter alia, Flavin and Hamilton (1986), Trehan and Walsh (1991), and Bohn (1998). Hansen, Roberts and Sargent (1991) discuss difficulties in testing sustainability if observations on debt are not available.

[31] For example, Bohn (1998) argues that the lack of power of unit root tests plagues most sustainability tests. The test proposed by Bohn, allows for strong increases in debt in the face of large increases in government spending, so it would not detect unsustainable policies that did not repay debt incurred in the face of large deficits.

simplest AR(1) model

$$\frac{D_t}{Y_t} = \alpha + b\frac{D_{t-1}}{Y_{t-1}} + e_t \tag{32}$$

since this would fit the data very well (in fact, the R^2 coefficient in this regression is always very close to one). We say the fiscal policy is unsustainable if $b > 1$. If $0<b < 1$ we say fiscal policy is sustainable and for low b we say debt is strongly mean-reverting.

The best summary of the available information is given by Bayesian posterior probabilities for b. We say that the agency perceives that fiscal policy is likely to be unsustainable after T periods if it finds

$$Post_T(b \geq 1) > Post_T(b < 1) \tag{33}$$

where $Post_T$ are the posterior probabilities with a sample of T observations. If (33) holds, the agency would conclude that fiscal policy is more likely to be unsustainable than otherwise; this would be a wrong conclusion since in our simulations the policy is, in fact, sustainable. The question we want to study is: what is the probability that (33) holds?. Under standard assumptions and assuming the agency has no prior beliefs on b, the Bayesian posterior is normally distributed with mean equal to the OLS estimator (b_T^{OLS}). Therefore, (33) holds if and only if $b_T^{OLS} \geq 1$, and the above question can be answered by studying the probability that the $b_T^{OLS} \geq 1$.

Figure 7 shows the Monte-Carlo distribution of b_T^{OLS} under complete and incomplete markets from 500 simulations. The data are simulations of Model 1 with persistent shocks and where T=40. Under complete markets we have $E(b_T^{OLS})$ =0.814, but under incomplete markets $E(b_T^{OLS})$ =0.988. Hence, under incomplete markets, the policy is on average perceived to be borderline unsustainable. More striking is the fact that in only 0.6% of cases did b_T^{OLS} exceed 1 under complete markets, compared to 48% under incomplete markets.

In other words, since government policy is sustainable in Model 1 (in fact, it is *optimal* within all sustainable fiscal policies) the agency would almost never have 'the wrong perception' under complete markets, but it would have 'the wrong perception' with probability 48% if markets were incomplete. The monitoring agency might just as well decide about sustainability by tossing a coin! The reason for this substantial error is that the large and sustained swings in the debt/output ratio that are optimal under incomplete markets and persistent shocks will often be mistaken as explosive and unsustainable path for debt unless allowance is made for the incompleteness of bond markets. Another way to say this is that the debt/output ratio is weakly (strongly) mean reverting under incomplete (complete) markets. So under complete markets a test of mean reversion is likely to give a correct answer, but not so under incomplete markets where it has very little power.

5.2 Debt Limits

The fact that it is so difficult to test for sustainability under incomplete markets can be used to make a case for imposing debt limits as a way to ensure sustainability. Debt limits are costly, but if they help to ensure sustainability, it may be worthwhile imposing them. We now turn to consider the cost of debt limits for an optimising government.

Under incomplete markets, an optimizing government will experience large swings in debt - especially if shocks are highly persistent. In the face of adverse expenditure shocks the debt/GDP ratio will rise sharply whereas in response to a string of lower government expenditure outcomes the government will find it optimal to repay debt and purchase assets.

To investigate the extent to which debt limits influence optimal debt and taxes, we simulated Model 1 under three different assumptions about debt limits: tight debt limits which bind at 10% of steady state GDP, moderate limits of 40% of steady state GDP and finally loose constraints of 105% of steady state GDP. The results are shown in Table 2. Under the assumption of i.i.d. shocks and complete markets the variations in the market value of debt are minimal (a result consistent with Proposition 1 and Corollary 2). Under incomplete markets debt fluctuations are more noticeable but constraints rarely bind.[32] In the case of persistent shocks, even under complete markets the model generates enormous variation in the debt/GDP ratio.As the range of variation from the loose debt limits show, in the face of incomplete markets the debt/GDP ratio needs to show enormous variation.

Viewed from the perspective of these simulations the problem with debt limits is clear: they fail to discriminate between governments which have been unlucky and require large variations in debt to pursue an optimal (incomplete

[32]Notice the asymmetry in the behaviour of debt - the minimum values are always greater in absolute value than the peaks. This is a similar finding to Chari, Kehoe and McGrattan (1994) who find that because of precautionary motives governments try and accumulate assets to avoid hitting the upper bound on debt limits.

markets-) fiscal policy and governments who are pursuing insolvent policies which also produce a large increase in debt. Debt limits insure sustainability at the cost of further constraining those governments that are experiencing bad times. Unless debt management helps insulate tax rates from adverse shocks then an optimizing government will often experience a binding debt limit in bad periods.

5.3 Debt management and Fiscal Policy

Our analysis provides a clear role for debt management - to buy/issue securities which offer a negative covariance between interest payments and the primary deficit and so reduce fluctuations in debt and tax rates. The better debt management is, the more appropriate the complete market model is in explaining the data. Getting closer to complete markets is important for two reasons: it helps reduce the volatility of taxes in a direct way, and it makes sustainability easy to test by requiring debt to have a strong reversion to the mean (a low b in equation (32)), therefore eliminating the need for harsh measures such as debt limits to ensure sustainability.

Table 1 shows some details from our simulations of Models 1 and 2. As to be expected in all the complete market models the negative correlation between interest payments and the primary deficit is larger than under incomplete markets and as a consequence tax rates show very little volatility. As emphasized in Scott (2000) tax rates change only because of variations in the elasticity of labor supply and not because of shocks to the government's intertemporal budget constraint. Because the elasticity of labor supply changes very little there is little variation in taxes under complete markets. But when governments issue only risk free debt, interest payments can no longer ensure the intertemporal budget constraint holds: debt goes up in response to a higher deficit, and taxes have to increase in the distant future in order to pay for the higher interest payments. Hence, taxes need to be more variable and, in particular, they need to have a large low-frequency variability, as found in our k-variance ratios of the previous section. In Model 1 with persistent shocks taxes are ten times more volatile in the incomplete market model, whereas in Model 2 they are 17 times more volatile. It is interesting to note that the negative correlation between interest payments and the primary deficit is larger for the case of persistent shocks than for i.i.d shocks. Because in our model we assume the government can only issue one-period risk-free debt, interest payments are unable to adjust to these i.i.d shocks.

However, when shocks are persistent future interest rates are also affected and this provides some insurance to the government.

Another interesting feature of our simulations is how under complete markets the primary deficit is more volatile compared to the incomplete markets case. Under complete markets taxes respond less to government expenditure shocks so that shocks feed through mostly into the primary deficit. However, swings in the primary deficit are offset by fluctuations in interest payments to produce very limited fluctuations in the total deficit. By contrast, under incomplete markets taxes shift in the same direction as government expenditure shocks reducing the volatility of the primary deficit. Therefore incomplete markets lead to a more muted automatic fiscal stabilizer.

6 Conclusion

We have characterized the optimal behaviour of government debt under both complete and incomplete financial markets. When markets are complete then government debt shows the same or less persistence than other variables in the economy and it falls in response to a lower deficit. By contrast, under incomplete markets debt shows a much larger persistence than other variables and debt increases in response to adverse shocks to the fiscal deficit.

Examining US data reveals that the incomplete market model is a far better explanation of the data. While our results suggest that through issuing a variety of different types of debt the government achieves partial contingent insurance, the behavior of government debt in the data is a far cry from complete markets.

Given the presence of incomplete markets, optimal debt will show large and persistent fluctuations, and it will be very hard to discriminate between an optimizing but unlucky government from an insolvent government. In our leading model, a monitoring agency using information on debt optimally to assess sustainability of fiscal policy would do as good a job as a monitoring agency that did the assessment by flipping a coin. The fact that optimal behavior is hard to distinguish from insolvency may be justify the use of debt limits in order to insure sustainability. But debt limits are costly, and they are particularly likely to hurt optimizing but unlucky governments.

Our analysis suggests that there are significant shocks to the fiscal deficit which are not hedged by current US debt management. Discovering these shocks and how to hedge against them promises to reduce fluctuations in the level of US debt and volatility in tax rates[33]. Also, it provides a different way to assess sustainability: being closer to complete markets, we can enforce sustainability by requiring a strong reversion to the mean of the debt/GDP ratio, and in this way avoid the costs of debt limits.

We have discussed these issues in the context of a particular set of models (Ramsey equilibrium, perfectly competitive markets, etc.). Our conjecture (to be verified by future research) is that these features of complete vs. incomplete markets will be present in most models where the government cares about tax volatility, whenever debt is used as buffer stock to smooth shocks.

[33] See Marcet and Scott (2001) for an empirical attempt at doing so for OECD economies.

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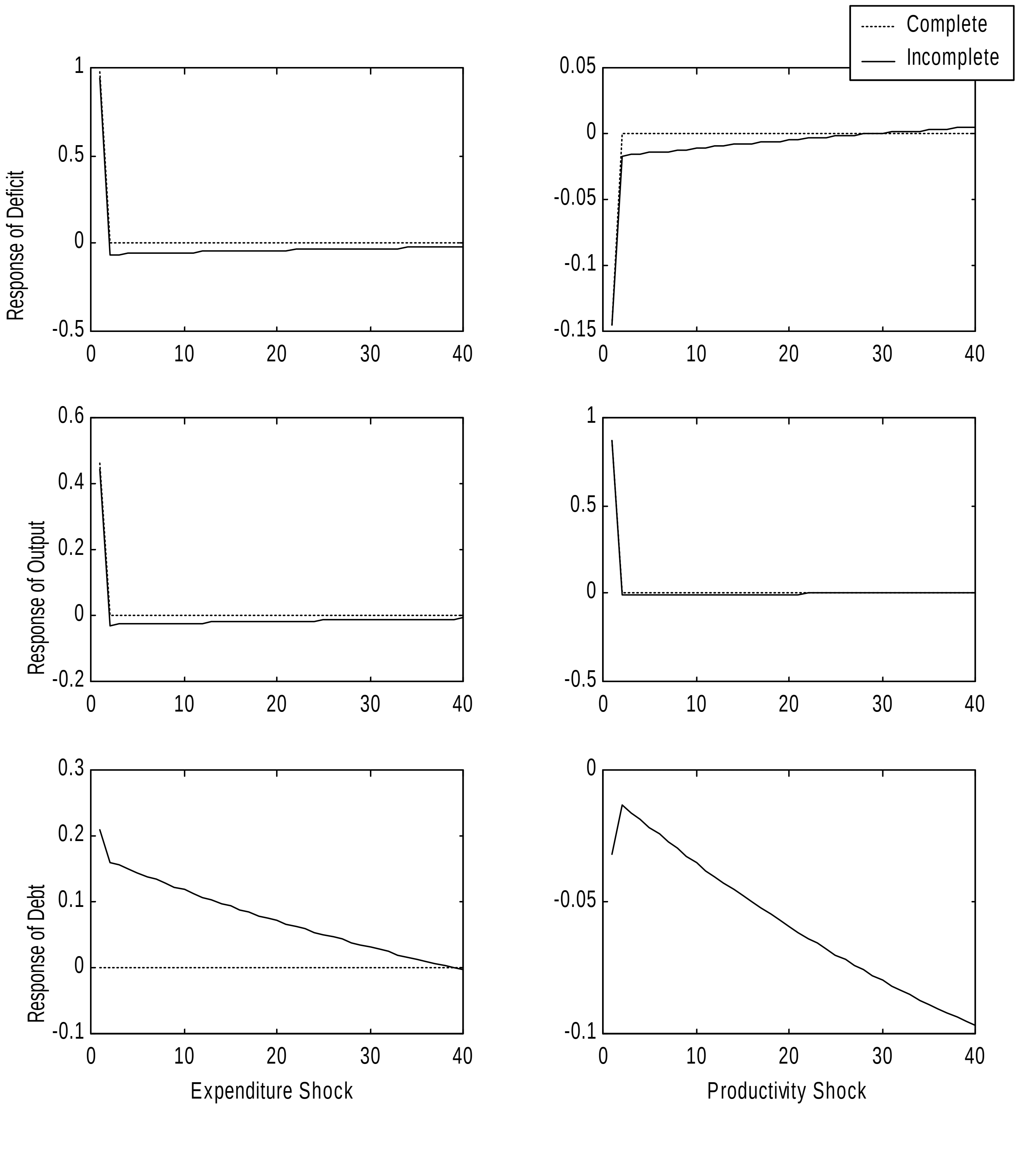


Figure 1: Fundamental Impulse Response Functions,
Model 1, i.i.d. Shocks

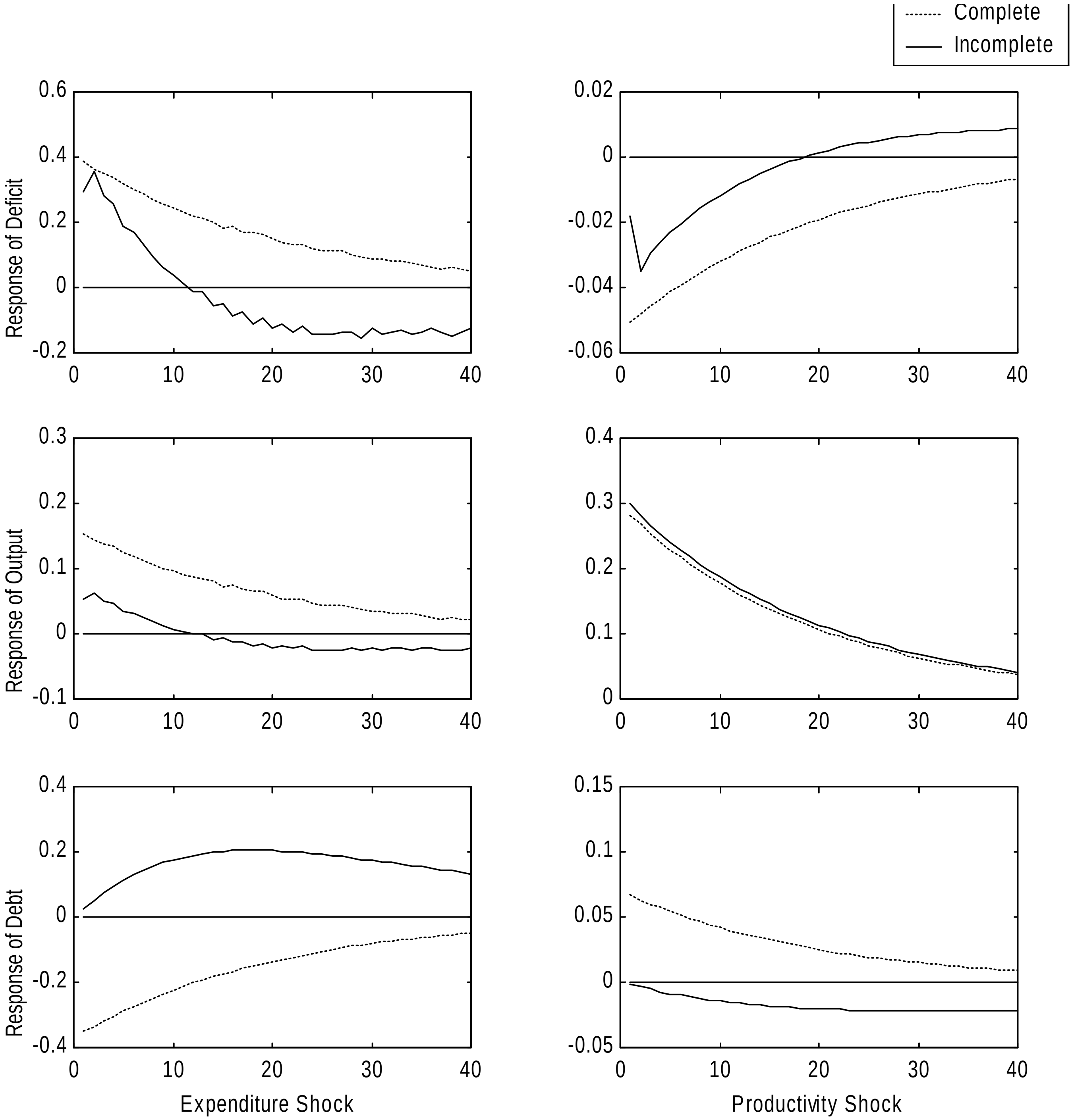


Figure 2: Fundamental Impulse Response Functions,
Model 1, Persistent Shocks

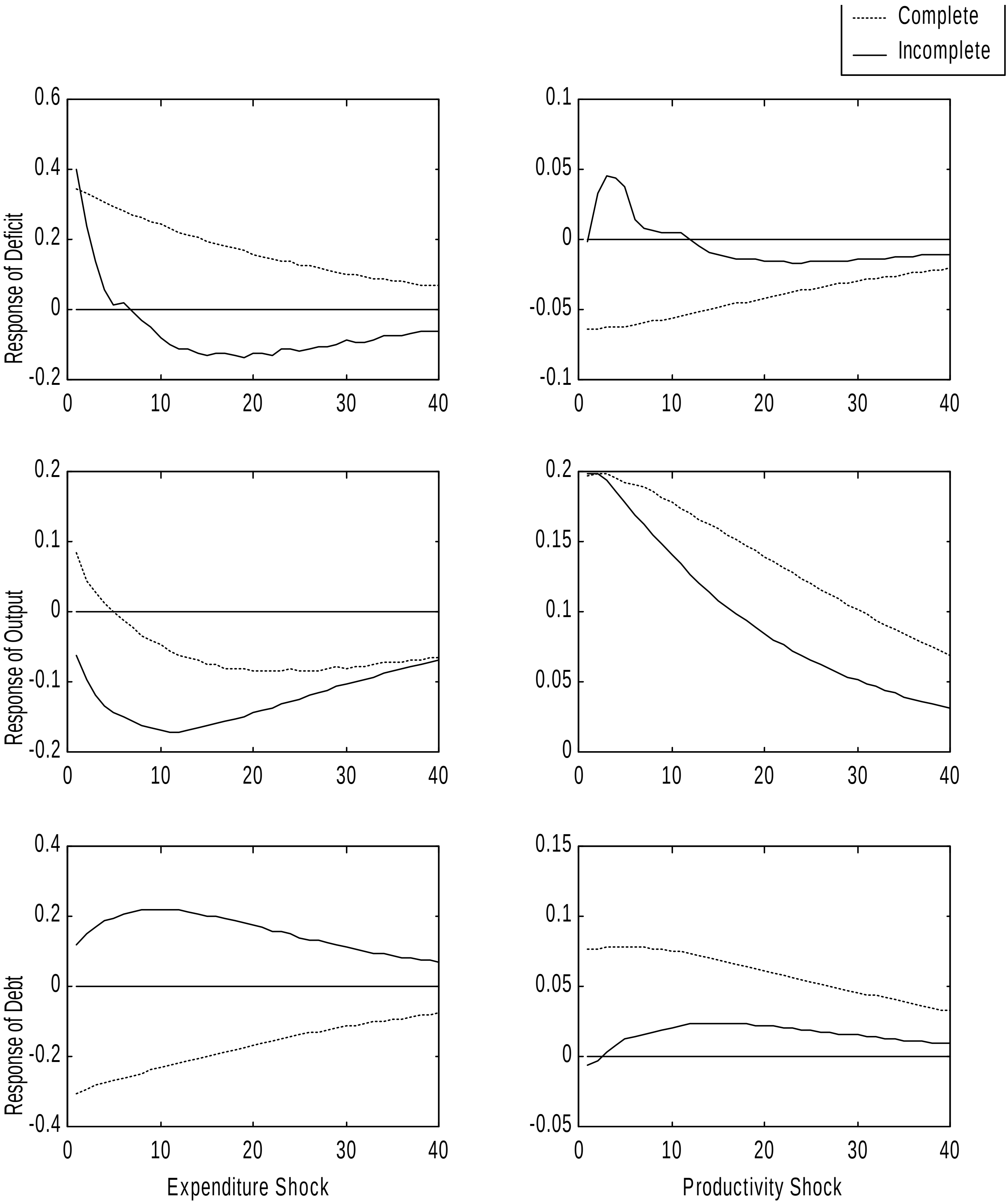


Figure 3: Fundamental Impulse Response Functions,
Model 2 – Capital Accumulation

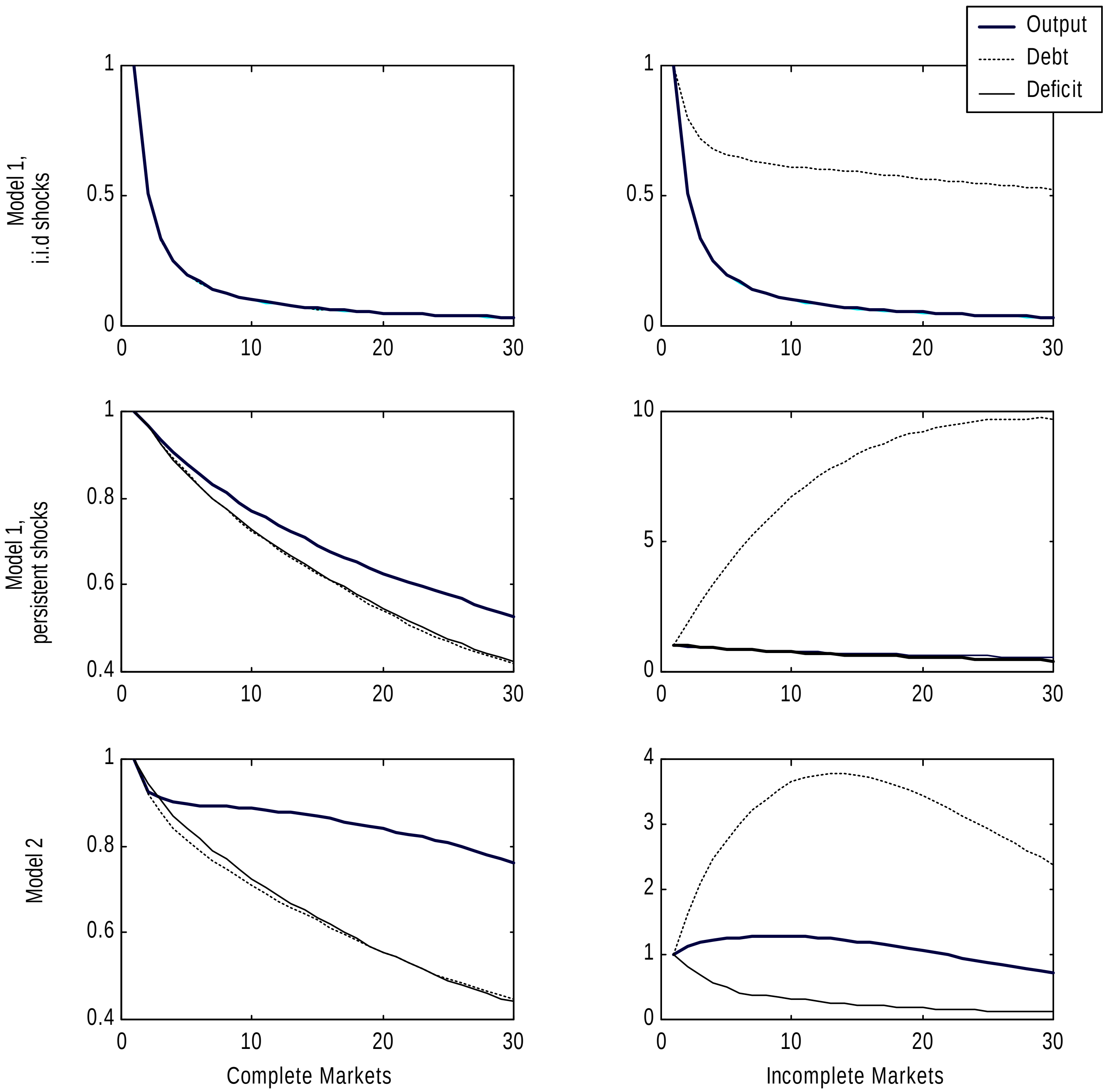


Figure 4: k-variance (Persistence) Measures Across Model

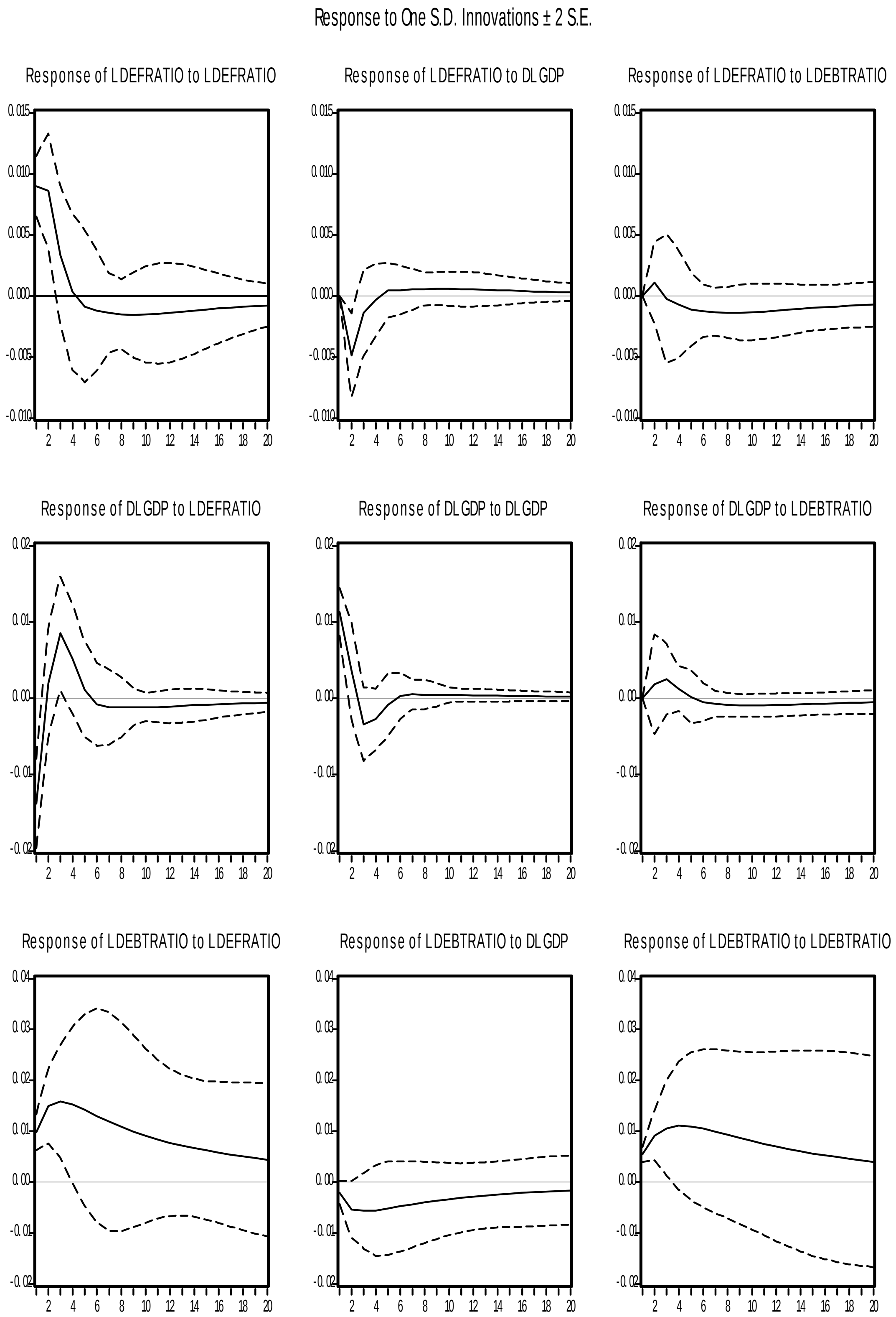


Figure 5 : Estimated Impulse Response Functions on US Data 1970-99

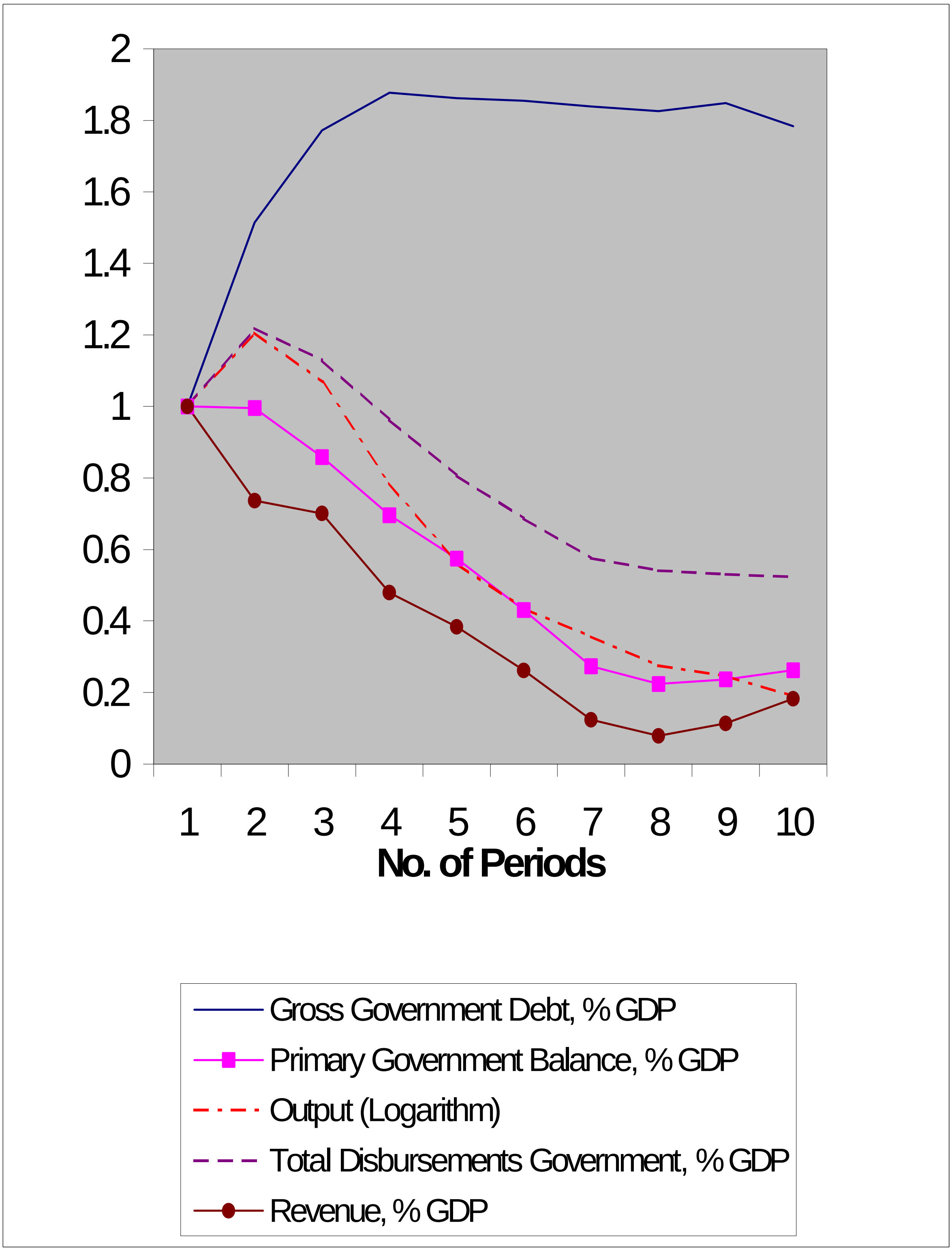
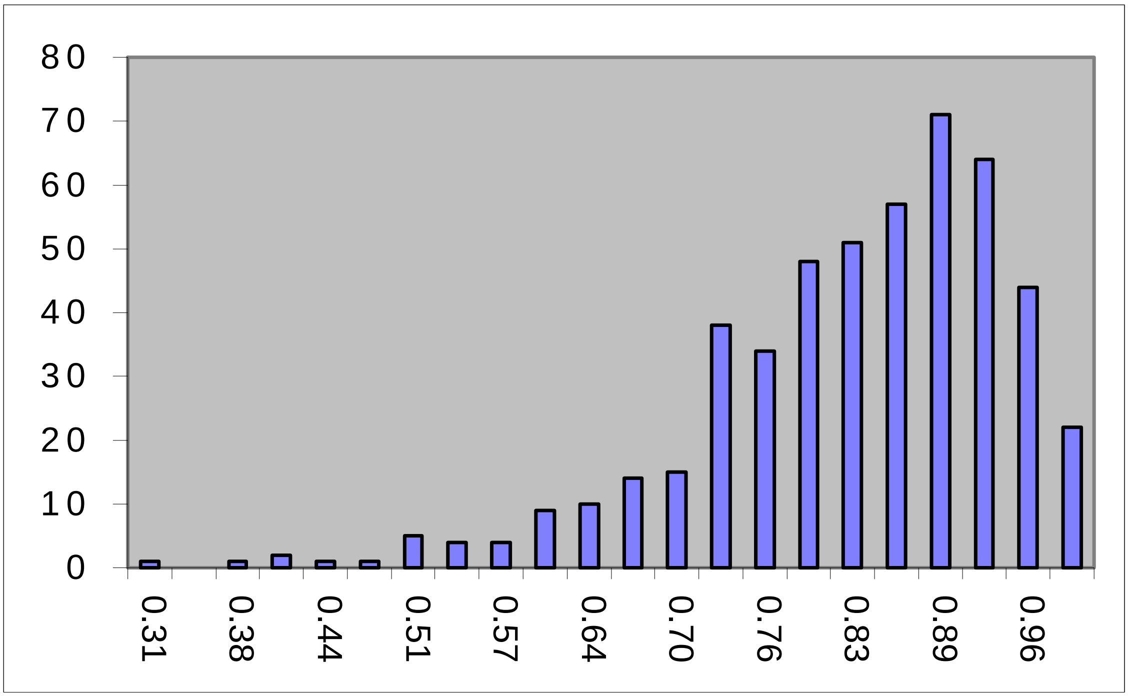
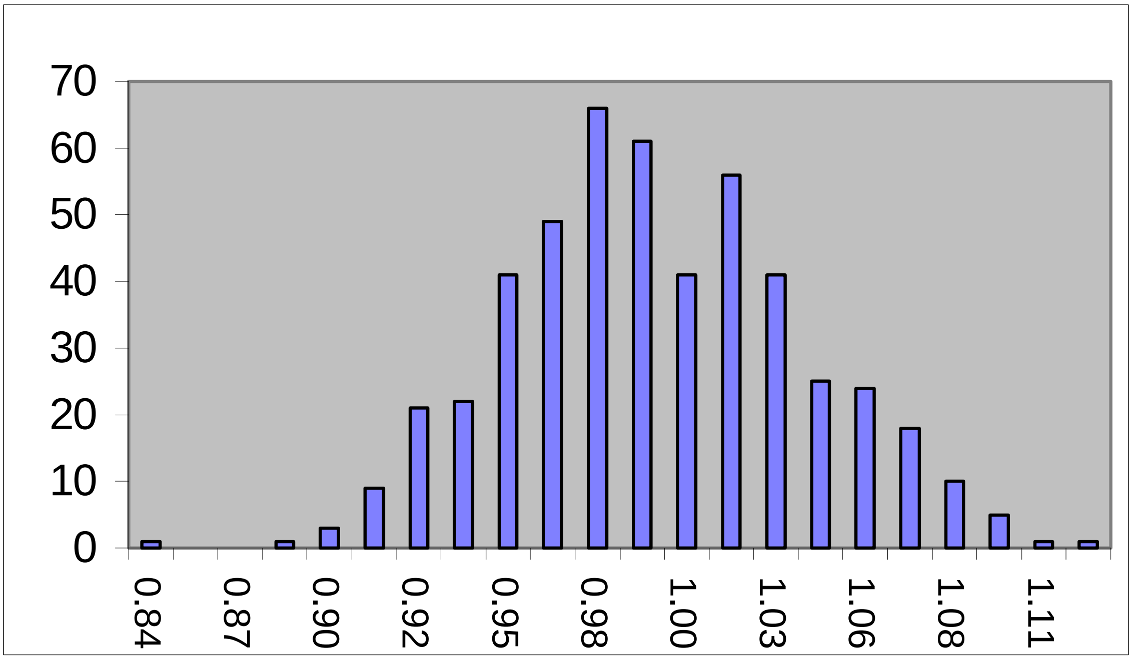


Figure 6 – k-variance (persistence) statistics for US data.

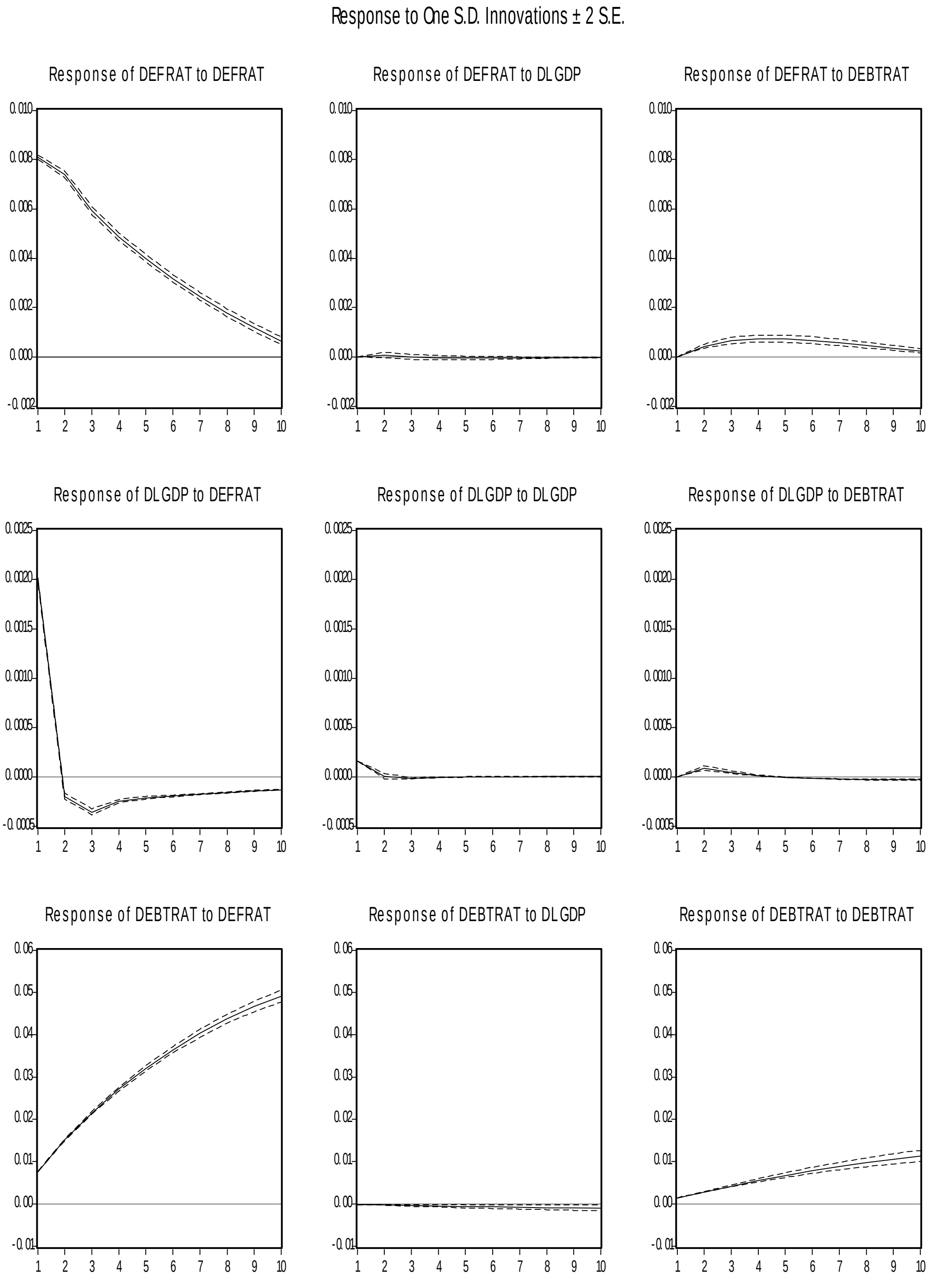


Complete Markets

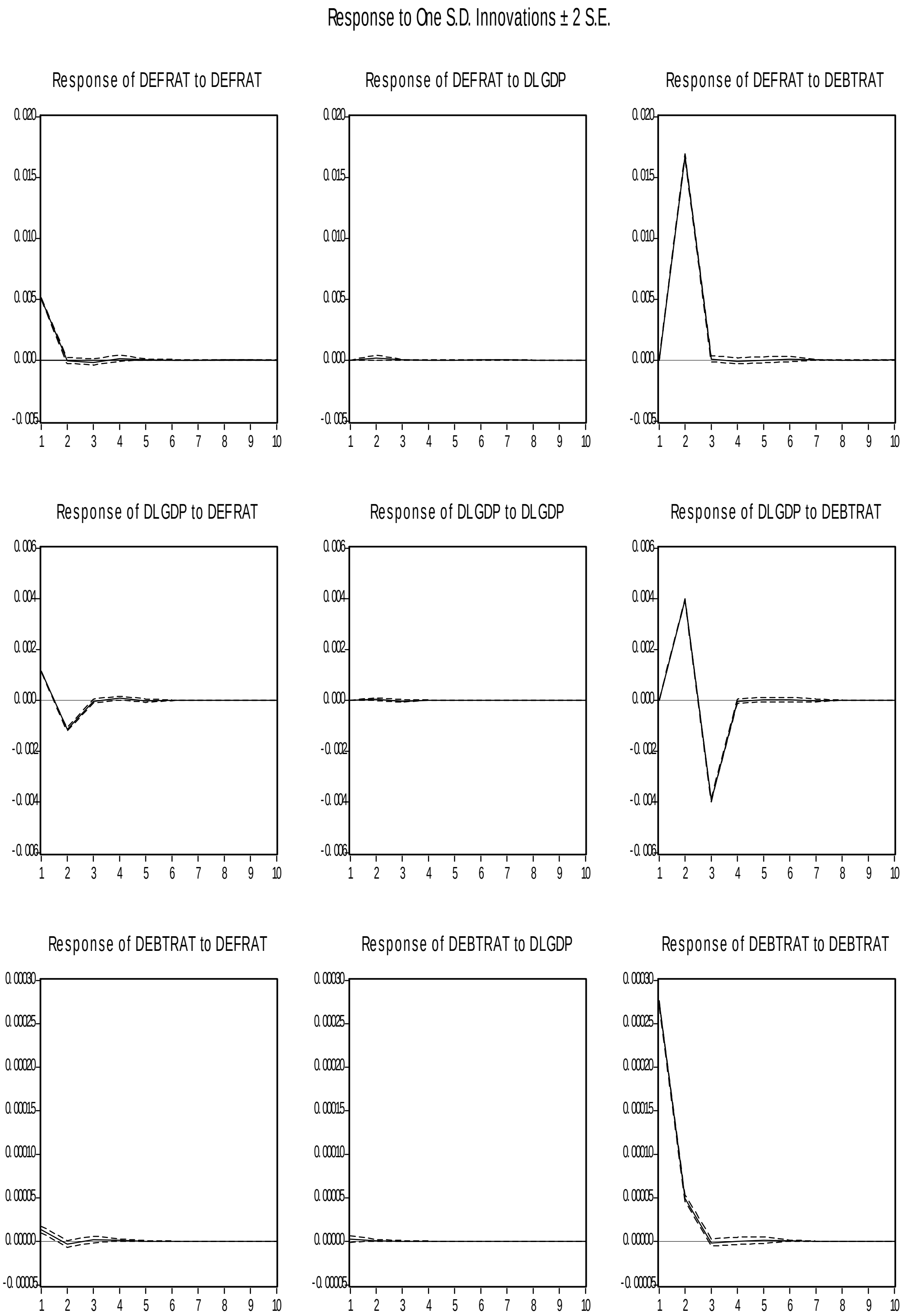


Incomplete Markets

Figure 7. Distribution of OLS estimator of *b*,
Model 1, persistent shocks.
Sample *T*=40.



Appendix A: Estimated Impulse Response Functions using Simulations of Model 1 with Persistent Shocks, Incomplete Markets



Appendix B: Estimated Impulse Response Functions using Simulations of Model 1 with Persistent Shocks, Complete Markets

Table 1 : Debt Management Issues

	Standard Deviations			
	Primary Deficit/ GDP	Total Deficit/GDP	Tax Rate	Correlation Interest payments and Primary Deficit
Model 1.a				
Complete Markets	0.018	0.017	0.002	-0.837
Incomplete Markets	0.017	0.017	0.006	-0.139
Model 1.b				
Complete Markets	0.046	0.029	0.006	-0.992
Incomplete Markets	0.020	0.017	0.052	-0.574
Model 2				
Complete Markets	0.047	0.040	0.005	-0.593
Incomplete Markets	0.026	0.023	0.083	-0.380

Table 2 : Debt Limit Issues

	Min. Mark. Value of Debt	Max. Mark. Value of Debt	% Times Lower Limit Binds	% Times Upper Limit Binds
Model 1.a				
Complete Markets	-1.4	-0.5	-	-
Incomplete Markets – Tight Limits	-14.7	13.3	0.7	0.1
Incomplete Markets – Moderate Limits	-25.7	20.1	0	0
Model 1.b				
Complete Markets	-59.3	74.6	-	-
Incomplete Markets (Tight Limits)	-13.6	13.4	14.1	2.2
Incomplete Markets (Moderate Limits)	-36.0	40.0	0	0.3
Model 2				
Complete Markets	-141.8	151.8	-	-
Incomplete Markets (Moderate Limits)	-37.9	55.1	0	0.4