DISCUSSION PAPER SERIES

No. 3013

IRREVERSIBLE INVESTMENT WITH STRATEGIC INTERACTIONS

Robin Mason and Helen Weeds

INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP3013.asp

ISSN 0265-8003

IRREVERSIBLE INVESTMENT WITH STRATEGIC INTERACTIONS

Robin Mason, Southampton University and CEPR
Helen Weeds, University of Warwick

Discussion Paper No. 3013
October 2001

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 3013

October 2001

ABSTRACT

Irreversible Investment with Strategic Interactions*

This Paper examines irreversible investment in a project with uncertain returns, when there is an advantage to being the first to invest, and externalities to investing when others also do so. Pre-emption decreases and may even eliminate the option values created by irreversibility and uncertainty. Externalities introduce inefficiencies in investment decisions. Pre-emption and externalities combined can actually hasten, rather than delay, investment, contrary to the usual outcome. These facts demonstrate the importance of extending 'real options' analysis to include strategic interactions.

JEL Classification: C73, D81, L13 and O31
Keywords: network effects, pre-emption and real options

Robin Mason
Department of Economics
University of Southampton
Highfield
SOUTHAMPTON
S017 1BJ
Tel: (44 23) 8059 3268
Fax: (44 23) 8059 3858
Email: robin.mason@soton.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=145252

Helen Weeds
Department of Economics
University of Warwick
COVENTRY
CV4 7AL
Tel: (44 24) 7652 3052
Fax: (44 24) 7652 3032
Email: helen.weeds@warwick.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=138689

* We are grateful to In Ho Lee, Davis Newbery, Patrick Rey and Jean Tirole for comments; we would like to thank Juuso Valimaki in particular. This is a work-in-progress; all other comments are welcome. The latest version of this paper can be found at http://www.soton.ac.uk/~ram2/

Submitted 17 July 2001

1. Introduction

The literature on irreversible investment under uncertainty teaches three major lessons. First, the net present value (NPV) rule for investment is generally incorrect, since it considers only a now-or-never decision and fails to appreciate that investment can be delayed. Secondly, an option value is created by the fact that the return is bounded below by the payoff from not investing; the effect of this option value is to delay investment, relative to the NPV rule. Finally, the greater the degree of uncertainty, the larger this delay: an increase in uncertainty increases the upside potential from investment, and so increases the value of the investment option.

Typically, the work on the 'real options' approach analyses investment decisions for a single agent in isolation. (Some exceptions to this are discussed below.) In many cases, however, investment takes place in a more competitive environment in which there are strategic interactions between investing agents. The purpose of this paper is to demonstrate that strategic interactions can have important consequences for irreversible, uncertain investments. Pre-emption significantly decreases option values. Externalities introduce, relative to the co-operative outcome, inefficiencies in investment decisions (which, of course, are absent in single-agent models). Pre-emption and externalities combined can actually hasten, rather than delay, investment.

We analyse irreversible investment in a project with uncertain returns in a dynamic two-player model. Two types of strategic interactions are considered. The first is pre-emption. When there is some advantage to being the first to undertake an investment, there will be competition to be the first. In this situation, any benefit from delaying investment due to real option effects has to be balanced against the loss from being pre-empted. The second interaction arises when the value of an investment depends on the number of agents who have also invested. In this case also, the timing of an agent's investment is influenced by the investment decisions of others. Two equilibrium patterns of investment are possible. Either the agents invest sequentially (i.e., the 'leader' invests early while the 'follower' invests late), or they invest simultaneously. The paper asks: is equilibrium investment efficient? There are two aspects to this. First, conditional on a particular pattern of investment (sequential or simultaneous), are equilibrium investment times efficient? Secondly, does the efficient pattern of investment occur in equilibrium? How do the various factors—externalities, uncertainty and pre-emption—influence any investment inefficiencies?

Three investment inefficiencies are identified. A 'leader inefficiency' arises because each agent ignores the effect of its investment on the other. As a result, in equilibrium sequential investment occurs too often, compared to the co-operative solution. A 'follower inefficiency' arises because, when investment occurs sequentially, the follower does not consider the effect of its investment on the leader. As a result, the follower's equilibrium investment point is inefficient. Finally, a 'pre-emption inefficiency' arises because, when investment occurs sequentially, the leader does not consider the effect of its investment on the follower. As a result, the leader invests too early in equilibrium. In short: the leader leads too frequently in equilibrium; the pre-emptor invests too early; and the follower invests inefficiently when it has been pre-empted.

The three factors—externalities, pre-emption and uncertainty—each separately affect these three inefficiencies. An increase in the degree of investment externalities exacerbates the leader and follower inefficiencies. The possibility of pre-emption gives rise to the pre-emption inefficiency. An increase in the degree of uncertainty affects the extent of the leader inefficiency. In addition, the three factors have effects in combination. The pre-emption inefficiency increases with the extent of positive investment externalities. The most unexpected result is that the pre-emption inefficiency can also increase with the degree of uncertainty. Put differently, uncertainty does not always delay investment: for a sufficiently positive externality, the introduction of a small amount of uncertainty into the model decreases the investment point of the pre-emptor in an equilibrium with sequential investment. Overall, therefore, strategic interactions give rise to significant qualitative and quantitative effects that are omitted from the standard real options analysis of investment.

There are many cases of technology investment in which uncertainty, externalities and pre-emption are important. Three examples are discussed here; clearly there are many others. While the focus of this paper is investment, a simple example concerns entry by firms into differentiated product markets; see Prescott and Visscher (1977), Lane (1980) and Neven (1987). Two entrepreneurs are considering opening shops on a street; they must decide when and where to open a shop. There are sunk costs (such as fitting) in opening a shop. If both entrepreneurs open a shop, they compete in prices against each other for custom. There may also be, however, positive externalities to being located on the same street; for example, because a common cost can be shared (such as a fixed cost of delivery of goods), or because aggregate demand is increased by lowering consumer search costs. Finally, total demand (the mass of consumers) is growing over time but is uncertain. The outcome of this model is analysed in the appendix to provide a micro-foundation for the reduced-form model of the next section.

A second example concerns two firms deciding whether to set up sites on the World Wide Web. There is some benefit to having a Web site; but the exact size of the benefit is uncertain.[1] Sunk costs are incurred in setting up a site: skilled labour is required to design and write the pages, a domain name must be purchased, marketing expenditures incurred etc..[2] An increasingly important reason to pre-empt is the ability of first-movers to buy their preferred domain names cheaply.[3] Generic Web addresses (such as `business.com` and `internet.com`), generally perceived to be the most valuable, are a limited resource. In 1997, `business.com` was sold for US$150,000, `consumers.com` and `internet.com` both sold for US$100,000.[4] In the words of one industry newspaper, the "Internet equivalent of an uptown address just got a little bit pricier" (see News.Com (1997)). A first-mover advantage may also arise because the firm that acts first to set up its Web site may face lower staff costs—site designers being relatively abundant—than later firms who have to hire when designers are more scarce. Finally, negative externalities arise through competition; positive externalities can also occur since a firm setting up a Web site benefits from the efforts of other firms, both directly (e.g., by being able to learn from the design of other sites) and indirectly (e.g., consumers already being accustomed to buying online).

As a third example, consider competing satellite systems for global communications. (The following discussion relies on Vu (1996).) Initially, there were two competing types of system: geosynchronous earth orbit (GEO) satellites and low earth orbit (LEO) satellites.[5]

[1] A recent study found that one-third of the small businesses that use the Internet increased their revenues by at least 10 per cent over the previous year. However, in the first nine months of 1999, consumer e-commerce in the U.S. initially fell and then plateaued; participation in online auctions has followed the same pattern. See InternetNews.Com (1999). Recent bankruptcies have emphasized the high degree of uncertainty facing internet-based businesses.

[2] Estimates of the cost of setting up the most basic web site range between US$225–1050, with an annual maintenance cost of between US$200–350; the most complex sites may cost several hundreds of thousands of dollars. See Magazine (1999). Since its inception, marketing expenditure has been 25% of `amazon.com`'s revenues.

[3] Before 1994, Internic, the primary international authority for registration of domain names, did not charge; after this date, registration fees were instituted (in September 1999, US$70 per address for first 2 years, with a renewal fee thereafter). See Radin and Wagner (1996) for details.

[4] It might be argued that the most famous web addresses, such as `amazon.com` and `yahoo.com`, are non-generic. The point is, however, that generic web addresses are advantageous in attracting uninformed consumers who are unaware of specific brands.

[5] The former orbit approximately 35,000 km above the equator, and require between three and fifteen satellites to deliver worldwide service. LEOs orbit at about 1,350 km above the earth's surface, and require a much larger network of satellites to cover the entire world. More recently, a third system—global stratospheric

There are large sunk costs to implementing either system: the GEO system was estimated to cost around US$4 billion, while the cheapest LEO proposal costs US$9 billion. Wireless communications systems such as satellites use frequencies within the radio spectrum. If two users employ the same frequency at the same time, interference is created. A broader range of lower frequencies is more desirable and a first-mover advantage arises through the allocation of these frequencies.[6] Externalities arise through competition (negative) and because the availability of satellite systems stimulates demand for global communications that all system operators benefit from (positive). Finally, the industry faced considerable uncertainty, about both the cost of satellite technology and the total demand for wireless communication services.

Three strands of literature are related to this paper. Real options models have been used to explain delay and hysteresis arising in a wide range of contexts. McDonald and Siegel (1986) and Pindyck (1988) consider irreversible investment opportunities available to a single agent. Dixit (1989) and Dixit (1991) consider product market entry and exit in monopolistic and perfectly competitive settings respectively. The second strand of literature concerns timing games of entry or exit in a deterministic setting. Papers analysing pre-emption games include Fudenberg, Gilbert, Stiglitz, and Tirole (1983) and Fudenberg and Tirole (1985), while wars of attrition have been modelled by e.g., Fudenberg and Tirole (1986). Finally, technology investment in the presence of network effects has been analysed by many papers, including Farrell and Saloner (1986) and Katz and Shapiro (1986). Existing real option models typically assume a monopolistic or perfectly competitive framework, and do not include externalities. Pre-emption models allow for incomplete information about the types of players, but not for common uncertainty about payoffs or externalities. Network papers have not (with the exception reviewed below) analysed explicitly the effect of 'option values'—created when there is exogenous uncertainty, investment is irreversible, and agents are able to choose the time of investment.

There are a number of papers related to this one. Choi (1994) examines a model in which there are positive network effects, uncertainty and the possibility of delay. Choi identifies two externalities (he calls them forward and backward externalities). In Choi's model, users

telecommunications system (GSTS)—has been proposed. GSTS involves floating communication platforms suspended 12 km above the earth by helium balloons.

[6] Two aspects of frequency are important. The first is amount: the bandwidth made available to a wireless operator determines the total demand that it can serve. Secondly, the range of the frequency has important consequences for transmission. Very high frequencies can be blocked by tree leaves, windows and even very heavy rain storms, causing loss of signal.

are exogenously asymmetric: user 1 is able to choose which of two technologies (with random returns) to invest in either of two periods, while user 2 is able to invest only in the second period. This paper departs from Choi's in several respects. Most importantly, it does not impose exogenously an asymmetry between players, but instead allows the first mover to be determined endogenously. To show the consequence of this, two versions of the model are presented. In the first, the roles of leader and follower are pre-assigned exogenously, so that (as in Choi) pre-emption is not an issue; see section 3.1. In the second version, the roles are determined endogenously: the leader invests at the point at which it is indifferent between leading and following; see section 3.2.[7] The fact that investment by the leader is determined by indifference, rather than optimally (for the leader), makes an important difference to investment behaviour.

In Farrell and Saloner (1986), a model of technology investment with uncertainty about the timing of (rather than return from) investment, positive network effects, and irreversibility is analysed (see section II). Unlike Farrell and Saloner, we allow agents to invest at any time, not just at random opportunities. If this assumption were used in the Farrell and Saloner model, then many of the features would disappear (although the basic co-ordination problem due to network effects would remain). Here, delay is endogenously determined through the optimization decisions of the agents, rather than imposed exogenously.

Smets (1991) examines irreversible market entry in a duopoly facing stochastic demand. Simultaneous investment may arise only when the leadership role is exogenously pre-assigned. Consequently, he does not consider fully the pre-emption externality. Weeds (1999) presents a model in which two firms may invest in competing research projects with uncertain returns. She does not impose an asymmetry between the firms, but allows the leader to emerge endogenously. She does not include, however, more general externalities. Finally, Hoppe (2000) analyses a timing game of new technology investment in an uncertain environment. She considers second, rather than first, mover advantages and models uncertainty in a different way to this paper.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 analyses the non-co-operative equilibria of two versions of the model—the first where the roles of leader and follower are exogenously pre-assigned, the second where they are endogenous. Section 4 determines the co-operative solution. Various inefficiencies in the

[7]This is the rent equalization principle identified in Fudenberg and Tirole (1985).

model are analysed in section 5, which also reviews the main results and comparative statics of the model. Section 6 concludes. The appendix details a micro-foundation for the reduced-form model of section 2 and contains lengthier proofs.

2. The Model

This section develops a model to capture the three effects that are the focus of this paper: (i) uncertainty, irreversibility and the possibility of delay in investment; (ii) investment externalities, where the return to investment depends on the number of investors; and (iii) pre-emption, where early investors have an advantage. The section deals with a reduced-form model; a specific model (of entry into a differentiated product market) that conforms to the reduced-form structure is given in the appendix.

Two risk neutral agents, labelled $i \in \{1, 2\}$ each can invest in a project. There is a cost $K > 0$ to doing so, which is the same for both agents. Investment is irreversible (the cost K is entirely sunk), and can be delayed indefinitely. Time is continuous and labelled by $t \in [0, \infty)$. The timing of investment is the main concern of the analysis. Investment by the two agents may occur sequentially—that is, the two agents invest at distinctly different times—or simultaneously.

Consider first the outcome when the agents invest sequentially. Call the first investor the 'leader' and the second investor the 'follower'. The leader's instantaneous payoff at time t from investment, before the follower has invested, is

$$\pi_L^I = \theta_t, \tag{1}$$

where θ_t is the stand-alone benefit from investment—the instantaneous payoff received by an agent that is the sole investor. After the follower has invested, the leader's instantaneous payoff becomes

$$\pi_L^{II} = \gamma_L(1 + \alpha)\theta_t. \tag{2}$$

The follower's instantaneous payoff at time t from investment is

$$\pi_2^{II} = \gamma_F(1 + \alpha)\theta_t. \tag{3}$$

Now suppose that the agents invest simultaneously. The instantaneous payoff at time t from investment is the same for both agents:

$$\pi^{III} = \gamma_S(1+\alpha)\theta_t. \tag{4}$$

The parameters $\gamma_L, \gamma_F, \gamma_S$ lie between 0 and 1; α is non-negative.

The instantaneous payoffs in equations (2)–(4) are parameterized to capture two separate effects. The parameters γ_L, γ_F and γ_S measure the payoffs to (and so externalities between) specific investors. If they are small (large), then the instantaneous return from investment when the other agent has invested is low (high), because e.g., competition is intense (mild). In addition, their relative sizes represent the extent of first-, second- or simultaneous-mover advantages (see below). The parameter α measures generally the extent of positive externalities between investors, e.g., network benefits. Since the γ parameters are positive, a higher α corresponds to higher payoffs i.e., greater positive externalities. The particular way in which this parameter appears in the instantaneous payoff functions—as a multiplicative factor—is chosen for analytical convenience only. The important feature is that the α and γ parameters are complements (i.e, the marginal effect of an increase in α is positively related to the level of γ).

For most of the calculations, it is convenient to re-define variables as follows:

$$\gamma_L(1+\alpha) \equiv 1+\delta_L, \qquad \gamma_F(1+\alpha) \equiv 1+\delta_F, \qquad \gamma_S(1+\alpha) \equiv 1+\delta_S.$$

We will not investigate all possible configurations of the model parameters. Instead, we restrict attention to cases described in the following assumption:

Assumption 1:

$$-\left(\frac{\beta}{\beta+1}\right) \leq \delta_F \leq 0, \tag{5}$$

$$\delta_F \leq \delta_S, \tag{6}$$

$$\frac{\delta_F}{\beta} \leq \delta_L \leq -\delta_F, \tag{7}$$

where $\beta \in (1, \infty)$ (and will be defined later).

This assumption ensures several things.[8] First, there is a first-mover advantage, since $\delta_L \geq \delta_F/\beta \geq \delta_F$ (from equations (5) and (7)). Secondly, the first-mover advantage is not too small nor too large (equation (7)). Thirdly, there is a second-mover disadvantage, in the sense that $\delta_S \geq \delta_F$ (equation (6)). Fourthly, positive externalities cannot be too large (see equations (5) and (7)). The role of particular aspects of assumption 1 will be pointed out as the analysis progresses.

θ_t is assumed to be exogenous and stochastic, evolving according to a geometric Brownian motion (GBM) with drift:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t \tag{8}$$

where $\mu \in [0, r)$ is the drift parameter, measuring the expected growth rate of θ, r is the continuous-time discount rate,[9] $\sigma > 0$ is the instantaneous standard deviation or volatility parameter, and dW is the increment of a standard Wiener process, $dW_t \sim N(0, dt)$. The parameters μ, σ and r are common knowledge and constant over time. The choice of continuous time and this representation of uncertainty is motivated by the analytical tractability of the value functions that result.

The strategies of the agents in the investment game are now defined. If agent i has not invested at any time $\tau < t$, its action set is $A_t^i = \{\text{invest, don't invest}\}$. If, on the other hand, agent i has invested at some $\tau < t$, then A_t^i is the null action 'don't move'. The agent therefore faces a control problem in which its only choice is when to choose the action 'invest'. After taking this action, the agent can make no further moves.

A strategy for agent i is a mapping from the history of the game H_t (the sample path of the stochastic variable θ and the actions of both agents up to time t) to the action set A_t^i. Agents are assumed to use stationary Markovian strategies: actions depend on only the

[8] Assumption 1 can be expressed in terms of the parameters $\gamma_L, \gamma_F, \gamma_S$ and α:

$$\frac{1}{(\beta+1)(1+\alpha)} \leq \gamma_F \leq \frac{1}{(1+\alpha)},$$
$$\gamma_F \leq \gamma_S,$$
$$\frac{1}{\beta}\left(\gamma_F + \frac{\beta-1}{1+\alpha}\right) \leq \gamma_L \leq \frac{2}{1+\alpha} - \gamma_F.$$

The first and third conditions together require that $\gamma_F \leq \gamma_L$.

[9] The restriction that $\mu < r$ ensures that there is a positive opportunity cost to holding the 'option' to invest, so that the option is not held indefinitely.

current state and the strategy formulation itself does not vary with time. Since θ follows a Markov process, Markovian strategies incorporate all payoff-relevant factors in this game. Furthermore, if one player uses a Markovian strategy, then its rival has a best response that is Markovian as well. Hence, a Markovian equilibrium remains an equilibrium when history-dependent strategies are also permitted, although other non-Markovian equilibria may then also exist. (For further explanation see Maskin and Tirole (1988) and Fudenberg and Tirole (1991).)

The formulation of the agents' strategies is complicated by the use of a continuous-time model. Fudenberg and Tirole (1985) point out that there is a loss of information inherent in representing continuous-time equilibria as the limits of discrete time mixed strategy equilibria. To correct for this, they extend the strategy space to specify not only the cumulative probability that player i has invested, but also the 'intensity' with which each player invests at times 'just after' the probability has jumped to one.[10] Although this formulation uses mixed strategies, the equilibrium outcomes are equivalent to those in which agents employ pure strategies. (See section 3 of Fudenberg and Tirole (1985).) Consequently, the analysis will proceed as if each agent uses a pure Markovian strategy i.e., a stopping rule specifying a critical value or 'trigger point' for the exogenous variable θ at which the agent invests. Note, however, that this is for convenience only: underlying the analysis is an extended space with mixed strategies.

The possible states of each agent are denoted $n_i \in \{0, 1\}$ when the agent has not invested and has invested, respectively. The following assumptions are made:

ASSUMPTION 2: *If* $n_i(\tau) = 1$, *then* $n_i(t) = 1$ *for all* $t \geq \tau$, $i \in \{1, 2\}$.

ASSUMPTION 3: $\mathbb{E}_0 \left[\int_0^\infty \exp(-rt)\theta_t dt \right] - K < 0$.

Assumption 2 formalizes the irreversibility of investment: if agent i has invested by date τ, it then remains active at all dates subsequent to τ. Assumption 3 states that the initial

[10] In Fudenberg and Tirole (1985), an agent's strategy is a *collection* of *simple strategies* satisfying an *intertemporal consistency condition.* A simple strategy for agent i in a game starting at a positive level θ of the state variable is a pair of real-valued functions $(G_i(\theta), \epsilon_i(\theta)) : (0, \infty) \times (0, \infty) \to [0, 1] \times [0, 1]$ satisfying certain conditions (see definition 1 in their paper) ensuring that G_i is a cumulative distribution function, and that when $\epsilon_i > 0$, $G_i = 1$ (so that if the intensity of atoms in the interval $[\theta, \theta + d\theta]$ is positive, the agent is sure to invest by θ). A collection of simple strategies for agent i, $(G_i^\theta(.), \epsilon_i^\theta(.))$, is the set of simple strategies that satisfy intertemporal consistency conditions.

value of the project is sufficiently low that the expected return from investment is negative, thus ensuring that immediate investment is not worthwhile. (The operator $\mathbb{E}_0$ denotes expectations conditional on information available at time $t = 0$.)

3. Non-Co-operative Equilibrium

Two models are studied. In the first, there are no pre-emption effects: one agent is assigned exogenously the role of investing first. This model is well-suited to cases in which, for example, one agent has a clear advantage in adoption of a new technology—it may be technically more literate, have a more flexible organization (if it is a firm), or be less dependent on an existing technology. More importantly, by ruling out the possibility of pre-emption (the first mover always moves first), it isolates the option and externality effects (and so allows a comparison with Choi (1994)). In the second model, agents are *ex ante* symmetric, but may be *ex post* asymmetric; which agent invests first and which second is determined endogenously.

3.1. Without Pre-emption

Start by assuming that the pre-assigned leader and follower invest at different points. The possibility of simultaneous investment is considered below. As usual in dynamic games, the stopping time game is solved backwards. Thus the first step is to consider the optimization problem of the follower who invests strictly later than the leader. Given that the leader has invested irreversibly, the follower's payoff on investing has two components: the flow payoff from the project, $(1 + \delta_F)\theta_t$; and the cost of investment, $-K$. The follower's value function $F(\theta_t)$ at time t given a level θ_t of the state variable is therefore

$$F(\theta_t) = \max_{T_F} \mathbb{E}_t \left[\int_{T_F}^{\infty} \exp\left(-r(\tau - t)\right)(1 + \delta_F)\theta_\tau d\tau - K \exp\left(-r(T_F - t)\right) \right] \tag{9}$$

where T_F is the random investment time for the follower, and the operator $\mathbb{E}_t$ denotes expectations conditional on information available at time t. The value function F has two components, holding over different ranges of θ: one relating to the value of investment before the follower has invested, the other to after investment. Let these value functions be denoted F_0 and F_1, respectively.

Prior to investment, the follower holds an option to invest but receives no flow payoff.

In this 'continuation' region, in any short time interval dt starting at time t the follower experiences a capital gain or loss dF_0. The Bellman equation for the value of the investment opportunity is therefore

$$F_0 = \exp(-rdt)\mathbb{E}_t\left[F_0 + dF_0\right]. \tag{10}$$

Itô's lemma and the GBM equation (8) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 F_0''(\theta) + \mu\theta F_0'(\theta) - rF_0(\theta) = 0. \tag{11}$$

From equation (8), it can be seen that if θ ever goes to zero, then it stays there forever. Therefore the option to invest has no value when $\theta = 0$, and must satisfy the boundary condition $F_0 = 0$. Solution of the differential equation subject to this boundary condition gives $F_0 = b_F\theta^\beta$, where b_F is a positive constant and $\beta > 1$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$; i.e., $\beta = \frac{1}{2}\left(1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right)$.

Now consider the value of the agent in the 'stopping' region, in which the value of θ is such that it is optimal to invest at once. Since investment is irreversible, the value of the agent in the stopping region is given by the expected value alone with no option value terms. When the level at time t of the state variable is θ_t, this is

$$F_1(\theta_t) = \mathbb{E}_t\left[\int_t^\infty \exp(-r(\tau - t))(1+\delta_F)\theta_\tau d\tau - K\right].$$

θ is expected to grow at rate μ, so that

$$F_1(\theta) = \frac{(1+\delta_F)\theta}{r-\mu} - K. \tag{12}$$

The boundary between the continuation region and the stopping region is given by a trigger point θ_F of the stochastic process such that continued delay is optimal for $\theta < \theta_F$ and immediate investment is optimal for $\theta \geq \theta_F$. The optimal stopping time T_F is then defined as the first time that the stochastic process θ hits the interval $[\theta_F, \infty)$ from below. Putting together the two regions gives the follower's value function:

$$F(\theta) = \begin{cases} b_F\theta^\beta & \theta < \theta_F, \\ \frac{(1+\delta_F)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \tag{13}$$

given that the leader invests at $\theta_L < \theta_F$.

By arbitrage, the critical value θ_F must satisfy a value-matching condition; optimality requires a second condition, known as 'smooth-pasting', to be satisfied. (See Dixit and Pindyck (1994) for an explanation.) This condition requires the two components of the follower's value function to meet smoothly at θ_F with equal first derivatives, which together with the value matching condition implies that

$$\theta_F = \frac{\beta}{\beta - 1}\left(\frac{K}{1+\delta_F}\right)(r-\mu), \tag{14}$$

$$b_F = \frac{(1+\delta_F)\theta_F^{-(\beta-1)}}{\beta(r-\mu)}. \tag{15}$$

Equation (14) for the follower's trigger point can be interpreted as the effective flow cost of investment with an adjustment for uncertainty. The sunk investment cost is K, but this yields a flow payoff of $(1+\delta_F)\theta$; hence the effective sunk cost is $\frac{K}{1+\delta_F}$. With an effective interest rate of $r-\mu$ (i.e., the actual interest rate r minus the expected proportional growth in the flow payoff μ), this gives an instantaneous cost of $\left(\frac{K}{1+\delta_F}\right)(r-\mu)$. If a Marshallian rule were used for the investment decision, the trigger point would be simply this cost. But with uncertainty, irreversibility and the option to delay investment, the Marshallian trigger point must be adjusted upwards by the factor $\frac{\beta}{\beta-1} > 1$.

There are three components to the leader's value function holding over different ranges of θ. The first L_0 describes the value of investment before the leader (and so the follower) has invested; the second L_1 after the leader has invested, but before the follower has done so; and the third L_2, after the follower has invested. The first and third components are equivalent to those of the follower, determined previously. The second component is new, and so is derived first.

After the leader has invested, it has no further decision to take and its payoff is given by the expected value of its investment. This payoff is affected, however, by the action of the follower investing later at θ_F. Taking account of subsequent investment by the follower, the leader's post-investment payoff is given by

$$L_1(\theta_t) = \mathbb{E}_t\left[\int_t^{T_F} \exp\left(-r(\tau-t)\right)\theta_\tau d\tau + \int_{T_F}^{\infty} \exp\left(-r(\tau-t)\right)(1+\delta_L)\theta_\tau d\tau - K\right]. \tag{16}$$

The Bellman equation for the leader is

$$L_1 = \theta dt + \exp(-rdt)\mathbb{E}_t\left[L_1 + dL_1\right]. \tag{17}$$

Using Itô's lemma and equation (8) gives

$$\frac{1}{2}\sigma^2\theta^2 L_1''(\theta) + \mu\theta L_1'(\theta) - rL_1(\theta) + \theta = 0. \tag{18}$$

As before, investment has no value when $\theta = 0$, and so $L_1 = \frac{\theta}{r-\mu} + b_{L1}\theta^\beta$, where b_{L1} is a constant. The first part of the value function L_1 gives the expected value of investment before the follower invests, while the second is an option-like term reflecting the value (due to externalities) to the leader of future investment by the follower.

The other components of the leader's value function follow immediately from the calculations of the previous section:

$$L(\theta) = \begin{cases} b_{L0}\theta^\beta & \theta < \theta_L, \\ \frac{\theta}{r-\mu} + b_{L1}\theta^\beta - K & \theta \in [\theta_L, \theta_F), \\ \frac{(1+\delta_L)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \tag{19}$$

given the leader's trigger point θ_L and investment by the follower at the higher θ_F.

The value of the unknown constant b_{L1} is found by considering the impact of the follower's investment on the payoff to the leader. When θ_F is first reached, the follower invests and the leader's expected flow payoff is altered. Since value functions are forward-looking, L_1 anticipates the effect of the follower's action and must therefore meet L_2 at θ_F. Hence, a value-matching condition holds at this point (for further explanation see Harrison (1985)); however, there is no optimality on the part of the leader, and so no corresponding smooth-pasting condition. This implies that

$$b_{L1} = \frac{\delta_L \theta_F^{-(\beta-1)}}{r-\mu}. \tag{20}$$

The usual value matching and smooth pasting conditions at the optimally-chosen θ_L determine the other unknown variables:

$$\theta_L = \frac{\beta}{\beta - 1} K(r - \mu), \tag{21}$$

$$b_{L0} = \frac{\theta_L^{-(\beta-1)}}{\beta(r-\mu)} + b_{L1}. \tag{22}$$

Assumption 1 (specifically $\delta_F \leq 0$) ensures that $\theta_L \leq \theta_F$, so that the leader does indeed invest before the follower. If $\delta_F > 0$, then investment would occur as a cascade: the 'leader' would invest at θ_L, and the 'follower' would invest immediately afterwards.

So, in the model without pre-emption when equilibrium investment is sequential, the leader invests at $\theta_L = \frac{\beta}{\beta-1} K(r-\mu)$ and the follower at $\theta_F = \frac{\beta}{\beta-1}\left(\frac{K}{1+\delta_F}\right)(r-\mu)$. Uncertainty and externalities have very simple impacts. (Of course, there is no pre-emption incentive, since the roles are pre-assigned.) Uncertainty leads to delay (higher trigger points for both agents), since $\frac{\beta}{\beta-1}$ is increasing in σ. Externalities affect the follower—when they are negative ($\delta_F < 0$), they cause the follower to invest later (and the converse). They have no effect, however, on the leader's investment point. These findings are discussed further in section 5.

Now consider the alternative case, in which investment is simultaneous at the trigger point θ_S. The previous analysis indicates that the value function of each agent is then

$$S(\theta) = \begin{cases} b_S \theta^\beta & \theta < \theta_S, \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_S. \end{cases} \tag{23}$$

(This value function can be derived from the appropriate Bellman equation, following the steps shown above.) There is a continuum of simultaneous solutions; it is straightforward to show that they can be Pareto ranked, with higher trigger points yielding higher value functions. In this case, it seems reasonable that the agents invest at the Pareto optimal point, given by both value matching and smooth pasting. So

$$\theta_S = \frac{\beta}{\beta-1}\left(\frac{K}{1+\delta_S}\right)(r-\mu), \tag{24}$$

$$b_S = \frac{(1+\delta_S)\theta_S^{-(\beta-1)}}{\beta(r-\mu)}. \tag{25}$$

Note that $\theta_S \leq \theta_F$, since $\delta_S \geq \delta_F$; and $\theta_S \geq (\leq)\theta_L$ if $\delta_S \leq (\geq)0$.

The following lemma describes when simultaneous investment is an equilibrium.

Lemma 1: *The necessary and sufficient condition for simultaneous investment to occur in equilibrium in the model without pre-emption is*

$$(1 + \delta_S)^\beta \geq 1 + \beta\delta_L(1 + \delta_F)^{\beta-1}. \tag{26}$$

A sufficient condition is $\delta_S \geq 0 \geq \delta_L$*.*

Proof: For equilibrium simultaneous investment, it must be that $S(\theta) \geq L(\theta)$ for $\theta \in [\theta_L, \theta_S]$. Due to the convexity of the value functions, this requires that $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_L]$, and so that $b_S \geq b_{L0}$. Therefore from equations (22) and (25), the necessary and sufficient condition is

$$\frac{(1 + \delta_S)\theta_S^{-(\beta-1)}}{r - \mu} \geq \frac{\theta_L^{-(\beta-1)}}{\beta(r - \mu)} + \frac{\delta_L \theta_F^{-(\beta-1)}}{r - \mu}.$$

When the expressions for θ_S and θ_L are substituted, this reduces to equation (26). The sufficient condition follows directly from equation (26). □

Whether simultaneous investment occurs in equilibrium is determined by whether the leader wishes to invest before the follower, or at the same time (i.e., by the comparison of $L(\theta)$ and $S(\theta)$). The lemma shows the reasonable condition that, in order for simultaneous investment to occur in equilibrium, it must be the case that δ_S is sufficiently large and/or δ_L and δ_F sufficiently small. (This is clearest in the sufficient condition.) Note that the simultaneous investment equilibrium, when it exists, Pareto dominates the sequential outcome; this is an immediate consequence of the condition for existence of the simultaneous investment equilibrium: $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_S]$.

3.2. With Pre-emption

Instead of pre-assigning roles to the two agents, suppose that the leader is determined endogenously. This supplements the option and externality effects with a pre-emption incentive (c.f. Fudenberg and Tirole (1985) and Weeds (1999)). The applied motivation is that this set-up reflects agents' concerns when adopting network technologies: they want to wait until the market is developed, but are concerned that they will be at some disadvantage at that stage relative to agents that have adopted earlier. Nevertheless, agents are symmetric before moving—no agent has an intrinsic (dis-)advantage from the start. Analytically, this allows the pre-emption incentive to be studied.

As before, start by supposing that one agent (the pre-emptor) invests strictly before the other. The follower's value function and trigger point is the same as for the model without pre-emption; so

$$\theta_F = \frac{\beta}{\beta - 1}\left(\frac{K}{1+\delta_F}\right)(r-\mu).$$

The pre-emptor's value function is as described in the previous section. As before, value matching at θ_F determines the unknown variable $b_{L1} = \frac{\delta_L}{r-\mu}\theta_F^{-(\beta-1)}$. But the pre-emptor can no longer choose its investment point optimally, as it could when roles were pre-assigned. Instead, the first agent to invest does so at the point at which it prefers to lead rather than follow, not the point at which the benefits from leading are largest. Clearly, it cannot be that the first agent invests when the value from following is greater than the value from leading—if this were the case, the agent would do better by waiting. Likewise, it cannot be that the first agent invests when the value from leading is strictly greater than the value from following , since in this case without pre-assigned roles, the other agent could pre-empt it and still gain. Hence the investment point is defined by indifference between leading and following. Whereas in the model without pre-emption, θ_L was determined by value matching and smooth pasting, the trigger point θ_P in the pre-emption model is given by indifference: $L(\theta_P) = F(\theta_P)$.

The first step is to show that there is such a trigger point.

Lemma 2: *There exists a unique $\theta_P < \theta_L$ such that $L(\theta_P) = F(\theta_P)$ and $L(\theta) < F(\theta)$ for $\theta < \theta_P$, $L(\theta) > F(\theta)$ for $\theta > \theta_P$.*

Proof: See the appendix.

The indifference relation $L(\theta_P) = F(\theta_P)$ gives a non-linear equation for θ_P:

$$\frac{\theta_P}{r-\mu} - K - \frac{K}{\beta-1}\left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\left(\frac{\theta_P}{\theta_F}\right)^{\beta} = 0. \quad (27)$$

The solution for simultaneous investment in the pre-emption model is the same as in the model without pre-emption: the trigger point is the same, θ_S, and the necessary and sufficient condition for simultaneous investment to occur in equilibrium is given by equation (26). The conditions of simultaneous investment are unaltered because the value (function) from being the first to invest is the same regardless of whether the roles are pre-assigned or determined endogenously. This feature of the model arises because the follower's trigger point θ_F is independent of trigger point of the other agent.

4. Co-operative Solution

This section analyses the co-operative solution, in which the agents' investment trigger points are chosen to maximize the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies in the next section. Notice that there is only one co-operative solution—the previous distinction between pre-assigned and endogenous leader/follower roles is not relevant.

Consider first the co-operative solution when investment is sequential. Two trigger points, $\theta_1 < \theta_2$, are chosen to maximize the sum of the leader's and follower's value functions. Call the co-operative value function in this case C_{L+F}; using the same steps as before,

$$C_{L+F}(\theta) = \begin{cases} b_0\theta^\beta + b_1\theta^\beta & \theta < \theta_1, \\ \frac{\theta}{r-\mu} + b_2\theta^\beta - K + b_3\theta^\beta & \theta \in [\theta_1, \theta_2), \\ \frac{(2+\delta_L+\delta_F)\theta}{r-\mu} - 2K & \theta \geq \theta_2, \end{cases} \tag{28}$$

where $b_i,\ i = 0, 1, 2, 3$ are constants. The co-operative trigger points are determined by value matching and smooth pasting conditions at both points. Therefore

$$\theta_1 = \left(\frac{\beta}{\beta-1}\right) K(r-\mu) = \theta_L, \tag{29}$$

$$\theta_2 = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_L+\delta_F}\right)(r-\mu). \tag{30}$$

Assumption 1 ensures that $\theta_2 > \theta_1$, since $\delta_L \leq -\delta_F$.

Now consider the co-operative solution with simultaneous investment at the trigger point

θ_3. The co-operative value function in this case is

$$C_S(\theta) = \begin{cases} b_4\theta^\beta & \theta < \theta_3, \\ \frac{2(1+\delta_S)\theta}{r-\mu} - 2K & \theta \geq \theta_3. \end{cases} \tag{31}$$

Again, value matching and smooth pasting determine θ_3:

$$\theta_3 = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_S}\right)(r-\mu) = \theta_S. \tag{32}$$

A similar analysis to that undertaken with the non-co-operative equilibria shows when co-operation involves simultaneous investment.

LEMMA 3: *The necessary and sufficient condition for simultaneous investment to be a co-operative solution is*

$$2(1+\delta_S)^\beta \geq 1 + (1+\delta_L+\delta_F)^\beta. \tag{33}$$

A sufficient condition is $\delta_S \geq 0$.

PROOF: The necessary and sufficient condition is that the value function for simultaneous investment $C_S(\theta) \geq C_{L+F}(\theta)$, for all $\theta \in [\theta_1, \theta_3]$. The strict convexity of the value functions means, however, that this requires that $C_S(\theta) \geq C_{L+F}(\theta)$ for all $\theta \in [0, \theta_1]$ i.e., $b_4 \geq b_0 + b_1$. From above,

$$\begin{aligned} b_0 + b_1 &= \left(\frac{1+(1+\delta_L+\delta_F)^\beta}{\beta-1}\right)\left(\left(\frac{\beta-1}{\beta}\right)\frac{1}{K(r-\mu)}\right)^\beta K, \\ b_4 &= \left(\frac{2}{\beta-1}\right)\left(\left(\frac{\beta-1}{\beta}\right)\frac{1+\delta_S}{K(r-\mu)}\right)^\beta K. \end{aligned}$$

It is immediate that $b_4 \geq b_0 + b_1$ iff condition (33) holds. The sufficient condition follows directly from equation (33), noting that assumption 1 implies that $\delta_L + \delta_F \leq 1$. □

Equation (33) is very similar to equation (26) and the intuition behind the conditions is the same.

5. Inefficiencies and Comparative Statics

The previous two sections have established conditions under which investment is sequential or simultaneous; and the trigger points for investment, for equilibrium with and without pre-emption, and for the co-operative solution. This section first analyses inefficiencies that arise in the non-co-operative equilibria, and then assesses how the conditions for simultaneous investment and the trigger points vary as the parameters of the model change.

5.1. Inefficiencies

The next proposition compares equilibrium outcomes with the co-operative solution. In order to identify the investment inefficiencies precisely, the following definitions are useful:

$$\begin{aligned} \lambda_{26} &\equiv (1+\delta_S)^\beta - \left(1+\beta\delta_L(1+\delta_F)^{\beta-1}\right), \\ \lambda_{33} &\equiv 2(1+\delta_S)^\beta - \left(1+(1+\delta_L+\delta_F)^\beta\right), \\ \lambda &\equiv \lambda_{33}-\lambda_{26}. \end{aligned}$$

The necessary and sufficient condition (26), which ensures the equilibrium investment is simultaneous, therefore requires that $\lambda_{26} \geq 0$, while condition (33), for co-operative simultaneous investment, requires that $\lambda_{33} \geq 0$.

Proposition 1: *There are three investment inefficiencies:*
Follower*: $\theta_L = \theta_1$ and $\theta_F > (<)\ \theta_2$ when $\delta_L > (<)\ 0$. In words, conditional on both equilibrium and the co-operative solution involving sequential investment, the non-co-operative leader invests at the co-operative point when there is no pre-emption. The non-co-operative follower invests too late (early) when δ_L is greater (less) than zero.*
Pre-emption*: $\theta_P < \theta_L = \theta_1$: conditional on both equilibrium and the co-operative solution involving sequential investment, the non-co-operative leader invests too early when there is pre-emption.*
Leader*: $\lambda \geq 0$: there is insufficient simultaneous investment in equilibrium.*

Proof: See the appendix.

The follower inefficiency arises when investment is sequential. In the model without pre-emption, the leader invests at the correct (i.e., co-operative) point, but the follower invests at the wrong point. For the leader, externalities cause a constant proportional change (an increase if $\delta_L > 0$, a decrease otherwise) to its value function; this change has no marginal effect on the leader, and so its trigger point is unaffected.[11] The follower does not consider the effect on the leader of its investment, and consequently invests either too soon (when $\delta_L < 0$) or too late (when $\delta_L > 0$). The pre-emption inefficiency arises when leadership is endogenous and investment is sequential: the leader invests too early, relative to the co-operative solution. The leader inefficiency arises through inefficient simultaneous investment. In equilibrium, whether investment is sequential or simultaneous is determined by the leader's incentive to invest. The proposition shows that the leader wishes to invest before the follower too often, compared to the co-operative solution. In short: the leader leads too much in equilibrium; the pre-emptor invests too early; and the follower invests inefficiently when it has been pre-empted.

5.2. Comparative Statics I: Factors in Isolation

Pre-emption on its own has one effect in this model: it causes the leader to invest earlier when investment is sequential. It does not alter the follower's investment behaviour, nor the prevalence of sequential versus simultaneous investment in equilibrium. These observations can be seen by comparing the trigger points of the non-co-operative equilibria in the models with and without pre-emption when investment is sequential, and the necessary and sufficient conditions for equilibrium simultaneous investment. Pre-emption does not affect the follower's investment behaviour, since the follower's (single agent) optimization decision is independent of the leader's investment point. This is also the reason why pre-emption does not alter the leader inefficiency: since the follower's trigger point is independent of the leader's investment decision, the leader's value function is the same whether pre-emption can occur or not. The comparison of the leader's value function with the value function from simultaneous investment determines the equilibrium pattern of investment; and so pre-emption makes no difference to that pattern.

Recall that two sets parameters represent the degree of investment externalities: the triple

[11] In terms of the calculation, any term in δ_L or θ_F drops out of the value matching and smooth pasting conditions that determine θ_L.

γ_L, γ_F and γ_S, and α. The effects of the γ parameters are straightforward to determine, and are stated without proof in the next proposition.

PROPOSITION 2: *An increase in*

- γ_L *has no effect on* $\theta_L, \theta_F, \theta_S, \theta_1$ *and* θ_2*; decreases* θ_P*; decreases* λ_{26} *and* λ_{33}*, and increases (decreases)* λ *if* $\delta_L < (>)0$.
- γ_F *has no effect on* θ_L, θ_S *and* θ_1*; decreases* θ_F *and* θ_2*; increases* θ_P*; increases (decreases)* λ_{26} *if* $\delta_L < (>)0$ *and decreases* λ_{33}*, and, if* $\delta_L \leq 0$*, increases* λ.
- γ_S *has no effect on* $\theta_L, \theta_F, \theta_1$ *and* θ_2*; decreases* θ_S*; and increases* λ_{26}*,* λ_{33} *and* λ.

Most of these effects—particularly the comparative statics of the trigger points—are quite obvious. For example, θ_P is decreasing in γ_L and increasing in γ_F because this trigger point is determined by indifference. If the gain to being the leader increases or to being the follower decreases, then indifference requires that the leader invests earlier. In both equilibrium and the co-operative solution, simultaneous investment is favoured by an increase in the flow payoff from simultaneous investment (i.e., higher γ_S) and a decrease in the flow payoff to the leader (i.e., lower γ_L). A change in γ_S has a larger effect on the co-operative solution, and so λ increases in this parameter. The least obvious effect comes from an increase in γ_F. If the flow payoff to being the follower increases, then simultaneous investment is less favoured in the co-operative solution. The same is true in equilibrium only if δ_L is negative. In equilibrium, the payoff to being the leader relative to a simultaneous investor determines whether investment is simultaneous. If the follower's payoff increases, then it invests earlier (θ_F decreases). If δ_L is negative, earlier investment by the follower decreases the payoff to being the leader, and so encourages simultaneous investment. Therefore, when δ_L is weakly less than zero, an increase in γ_F favours simultaneous investment in equilibrium but not in the co-operative solution, and hence increases λ. (When δ_L is sufficiently positive, the effect of γ_F on λ is ambiguous.)

The general level of positive externalities, measured by α, affect the trigger points and the sequential/simultaneous investment decisions. When investment is sequential, α affects the follower's equilibrium θ_F and co-operative θ_2 trigger points, but not the leader's without pre-emption. Differentiation of the conditions (26) and (33) with respect to α shows that simultaneous investment is more likely to occur in equilibrium and the co-operative solution when positive externalities are larger. In addition, positive externalities exacerbate the leader inefficiency: although a greater positive externality causes less sequential investment in equilibrium, it causes even less in the co-operative solution. These observations are stated more formally in the next proposition.

PROPOSITION 3: *(i) θ_F and θ_2 are decreasing in α. If $\delta_L < (>)0$, then the gap between θ_F and θ_2 decreases (increases) as α increases.*

(ii) If $\lambda_i \geq 0$ for $\alpha = \alpha'$, then $\lambda_i \geq 0$ $\forall \alpha'' \geq \alpha'$, for $i \in \{26, 33\}$.

(iii) λ is increasing in α.

PROOF: See the appendix.

Turning now to the volatility parameter σ, greater uncertainty usually leads to investment delay (in the sense of higher trigger points). The intuition is that delay allows for the possibility that the random process (8) will go up; if it goes down, then the agent need not invest. The greater the variance of the process, the more valuable is the option created by this asymmetric situation, and so the more delay occurs. All but one of the trigger points in the paper have this general feature: $\theta_L, \theta_F, \theta_1, \theta_2$ and θ_3 are all increasing in σ. The exception, θ_P, is analysed below. Despite causing investment delay, uncertainty on its own does not affect the follower inefficiency: both θ_F and θ_2 increase equally as σ rises. This feature is a consequence of the way in which uncertainty has been modelled: as the variance parameter of a geometric Brownian motion. This leads to a common term, $\beta/(\beta-1)$, relating to uncertainty, appearing in the expressions for θ_F and θ_2.

The degree of uncertainty affects the leader inefficiency, but unfortunately there is no counterpart to proposition 3. To see the problem, consider λ_{26}. Recall that two terms in θ appear in the two parts of the leader's value function before the follower's investment: a direct option value associated with the leader's own investment, and an option-like term relating to the follower's investment.[12] Consider the effect of an increase in σ when $\delta_L < 0$. The leader's value increases due to the first, direct option term—this is the standard comparative static of an option value. But the leader's value decreases due to the second term: the magnitude

[12]Refer to equation (19). Notice that both terms are important for $\theta \leq \theta_F$. This is explicit over the range $\theta \in [\theta_L, \theta_F)$, and implicit for $\theta < \theta_L$: for the latter, the two factors show up in the expression for b_{L0}—see equation (22).

of the option-like value increases, but it is a negative value, since $\delta_L < 0$. Hence there are two conflicting effects when σ increases, and consequently the comparative static with respect to σ may be (and in fact is) non-monotonic.

In some cases, however, the comparative statics are unambiguous. Consider the effects of an increase in uncertainty for the non-co-operative equilibrium without pre-emption. The value from simultaneous investment increases, in line with the standard option value comparative static. The marginal effect on the simultaneous investment value function of an increase in σ is therefore positive; but it is decreasing in δ_S. This is because as δ_S increases, for any given level of σ, simultaneous investment occurs sooner (θ_S decreases). Hence an increase in δ_S acts in the opposite direction to an increase in σ, which increases θ_S. The first term in leader's value function increases with σ; and the marginal effect of an increase in uncertainty is independent of δ_L and δ_F. The second term increases with uncertainty if $\delta_L \geq 0$. In this case, the marginal effect of an increase in uncertainty is decreasing in δ_F: as δ_F increases, for any given level of σ, the follower invests sooner (θ_F decreases). Hence an increase in δ_F acts in the opposite direction to an increase in σ, which increases θ_F. This argument establishes that the value of the leader increases with uncertainty by more than the value of a simultaneous investor if (i) δ_S is sufficiently large; (ii) δ_L is sufficiently large; and (iii) δ_F is sufficiently small.

The second case to consider is when $\delta_L \leq 0$: then, the second term in the leader's value function increases with uncertainty. In this case, the marginal effect of an increase in uncertainty is increasing in δ_F: as δ_F increases, for any given level of σ, the follower invests later (θ_F decreases). Hence an increase in δ_F acts in the same direction to an increase in σ. This argument establishes that the value of the leader increases with uncertainty by more than the value of a simultaneous investor if (i) δ_S is sufficiently large; (ii) δ_L is sufficiently small; and (iii) δ_F is sufficiently large.

A similar argument can be used to analyse the non-co-operative equilibrium with pre-emption and the co-operative solution. The results of the argument are summarized in the following proposition.

Proposition 4: *(i) Joint sufficient conditions for λ_{26} to be a decreasing function of σ are: $\delta_S \geq 0$ and either (i) $\delta_L \geq 0$ and $\delta_F \leq e^{-1} - 1$ or (ii) $\delta_L \leq 0$ and $\delta_F \geq e^{-1} - 1$. A sufficient condition for λ_{33} to be a decreasing function of σ is $\delta_S \geq 0$.*

(ii) Joint sufficient conditions for λ to be a decreasing function of σ are: $\delta_S \geq 0$ and either (i) $\delta_L \geq 0$ and $\delta_F \geq e^{-1} - 1$ or (ii) $\delta_L \leq 0$ and $\delta_F \leq e^{-1} - 1$.

PROOF: See the appendix.

Part (ii) of the proposition therefore gives sufficient conditions for the leader inefficiency to be exacerbated by an increase in the degree of uncertainty.

5.3. Comparative Statics II: Interactions

There are two interactions between positive externalities and pre-emption effects. First, proposition 5 shows that (conditional on sequential investment) the pre-emptor's trigger point θ_P, which is below the co-operative level due to pre-emption, is decreasing in the size of positive externalities measured by the parameter α.

PROPOSITION 5: *θ_P is decreasing in α.*

PROOF: See the appendix.

Proposition 5 shows that the pre-emptor invests earlier when positive externalities are larger. This result is not immediately obvious: at its trigger point, the pre-emptor is indifferent between leading and following; but both the leader's and follower's returns increase as α increases; and so it must be shown that the increase in the leader's return is the stronger effect. To gain an intuition for the result, rewrite the pre-emptor's indifference condition as $L(\theta_P; \alpha) - F(\theta_P; \alpha) = 0$. Then

$$\frac{\partial \theta_P}{\partial \alpha} = -\left(\frac{\partial L}{\partial \alpha} - \frac{\partial F}{\partial \alpha}\right) / \left(\frac{\partial L}{\partial \theta_P} - \frac{\partial F}{\partial \theta_P}\right).$$

Since $L < (>) F$ for $\theta < (>) \theta_P$, it must be that $\frac{\partial L}{\partial \theta_P} > \frac{\partial F}{\partial \theta_P}$. Hence the sign of $\frac{\partial \theta_P}{\partial \alpha}$ is determined by whether $\frac{\partial L}{\partial \alpha}$ is greater or less than $\frac{\partial F}{\partial \alpha}$. (The partial derivatives here hold θ_P constant, but allow θ_F to vary.)

There are two effects as α increases. First, the agents' flow returns once both agents have invested increase. Secondly, the follower invests earlier (θ_F decreases); when $\delta_L > 0$, this benefits both agents, while when $\delta_L < 0$, it benefits the follower but not the leader. For the leader, both effects are important. For the follower, only the first effect is of first-order significance: since the follower chooses θ_F optimally, any variation in the trigger point due to a small change in α induces only a second-order variation in returns. When $\delta_L > 0$, this suggests qualitatively that the leader's return increases more when α rises. The comparison is more complicated when $\delta_L < 0$; note, however, that the second effect for the leader is limited by assumption 1, which requires that $\delta_L \geq -\frac{1}{\beta+1}$. In fact, proposition 5 shows that the first effect dominates for the leader, and to such an extent that the leader's value function increases with α by more than does the follower's.

Secondly, positive externalities and pre-emption interact in the way in which uncertainty affects the pre-emptor's trigger point θ_P, raising the possibility that an increase in uncertainty lowers the trigger point.

PROPOSITION 6: *Sufficient conditions for* $\frac{\partial \theta_P}{\partial \sigma} > 0$ *are: either*

1. $1 - \beta\delta_L + \delta_F > 0$ *and* $\max[\phi_1, \phi_2] < \epsilon$*; or*
2. $1 - \beta\delta_L + \delta_F < 0$ *and* $\min[\phi_1, \phi_2] > \epsilon$*.*

Sufficient conditions for $\frac{\partial \theta_P}{\partial \sigma} < 0$ *are: either*

1. $1 - \beta\delta_L + \delta_F > 0$ *and* $\min[\phi_1, \phi_2] > \epsilon$*; or*
2. $1 - \beta\delta_L + \delta_F < 0$ *and* $\max[\phi_1, \phi_2] < \epsilon$*.*

$\phi_1 \equiv -\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right)$, $\phi_2 \equiv \frac{\theta_P}{\theta_F}$, *and* ϵ *is the solution to the equation* $\epsilon + \ln \epsilon = 0$ *(i.e.,* $\epsilon \approx 0.57$*).*

PROOF: See the appendix.

The result therefore raises the striking possibility that greater uncertainty lowers the pre-emptor's trigger point. This is counter to the usual comparative static. The difference arises from the lack of optimality in the choice of the pre-emption trigger point. An optimal trigger point is such that the marginal benefit from delaying investment for a period equals the marginal cost. The marginal benefit is the interest saved on the investment cost plus the expected gain from the possibility that the flow payoff increases. The marginal cost is the flow payoff foregone by not investing. In this marginal calculation, the agent does not consider the effect of its delay on the investment decision of the other agent, since in the models considered in this paper, each agent's trigger point (with the exception of θ_P) does not depend on the other's. Increased uncertainty raises the expected gain from delay, causing the (optimally chosen) trigger point to increase. This reasoning does not apply in the case of θ_P, however: it is not chosen according to a marginal equality, but an absolute equality between the value from leading and the value from following. The proposition shows that this difference in the determination of the trigger point can lead to θ_P decreasing as uncertainty increases.

There are two cases in which this can occur. In the first, $1-\beta\delta_L+\delta_F > 0$ and $\min[\phi_1, \phi_2] > \epsilon$. This sufficient condition requires that $\delta_L < 0$ and that $\beta < \frac{1}{\epsilon}$. In the second case, the sufficient conditions $1-\beta\delta_L+\delta_F < 0$ and $\max[\phi_1, \phi_2] < \epsilon$ require that $\beta < \frac{1+\delta_F}{\delta_L} + \frac{1}{\epsilon} < \frac{1}{\epsilon}$ if $\delta_L < 0$, and $\beta > \frac{1+\delta_F}{\delta_L} + \frac{1}{\epsilon} > \frac{1}{\epsilon}$ if $\delta_L > 0$. In summary, therefore, there are two situations that are conducive to a rise in uncertainty increasing the pre-emptor's trigger point (i.e., must obtain for the sufficient conditions to hold): either positive externalities are sufficiently small and uncertainty is sufficiently large; or the converse.

In order for this unusual comparative static to hold, it must be that the leader's value function increases by more than the follower's when uncertainty rises, holding constant the pre-emptor's trigger point θ_P. (This statement follows directly from using the implicit function theorem on the non-linear equation (27) defining θ_P.) The leader's value function depends on uncertainty due to the option-like term that anticipates investment by the follower: $b_{L1}\theta^\beta$, where $b_{L1} \equiv \frac{\delta_L \theta_F^{-(\beta-1)}}{r-\mu}$. Hence this option-like term is positive if δ_L is positive, and negative otherwise. As σ increases, two factors are important. First, holding the trigger point of the follower constant, there is a change in the value of the option-like term: if $\delta_L < 0$, it becomes more negative; if $\delta_L > 0$, it becomes more positive. Secondly, the follower's trigger point θ_F increases. When $\delta_L < 0$, this increases the option-like term (makes it less negative), since investment by the follower reduces the pre-emptor's instantaneous return; when $\delta_L > 0$, the option-like term decreases. Hence the first, direct effect always works in the opposite direction to the second, indirect effect. It is straightforward to show that the direct effect dominates the indirect effect at low levels of uncertainty, but at higher levels of σ, the indirect effect dominates. The follower's value function increases with uncertainty since the agent holds a standard (call) option relating to its future irreversible investment.

The same two factors are relevant, but the effect of a change in the follower's trigger point is of second-order, by the envelope theorem, so that only the direct effect is of first-order significance.

The favourable conditions identified above ($\delta_L < 0$ and σ large, or $\delta_L > 0$ and σ small) arise immediately from these observations. When $\delta_L < 0$, the direct effect is negative and the indirect effect positive. When σ is small, the former dominates; and so it is only when uncertainty is large that the leader's value function is increasing in σ. In the other case ($\delta_L > 0$), the direct effect is positive and the indirect effect negative. When σ is large, the latter dominates; and so it is only when uncertainty is small that the leader's value function is increasing in σ.

The case $\delta_L > 0$ and σ small is of particular interest, since it implies that for sufficiently positive externalities (such that $\delta_L > 0$), the introduction of a small amount of uncertainty into the model increases all trigger points except the pre-emptor's, which decreases. More precisely, large positive externalities mean that $\frac{\partial \theta_P}{\partial \sigma}\Big|_{\sigma=0} < 0$.

6. Conclusions

This paper has analysed irreversible investment in a project with uncertain returns, when there is an advantage to being the first investor, but externalities to investing when others also invest. It therefore extends standard 'real options' analysis to a case where there are general strategic interactions between investing agents. We believe that this is an important area of research. The real options literature has taught us that an option value is created by irreversibility and uncertainty; this option value typically leads to delayed investment. Strategic interactions, omitted from the standard real options analysis, can change and may even eliminate this option value. This has significant qualitative and quantitative effects on investment. For example, we have shown that the interaction of pre-emption with both externalities and uncertainty can actually hasten, rather than delay, investment.

APPENDIX

A.1. A Sequential Location Micro-foundation of the Reduced-form Model

The reduced-form model of section 2 can be supported by many micro-models of investment. In this section, the investment of a new technology by two agents is modelled as Hotelling-style entry into a horizontally differentiated market. In the spatial model, a first-mover advantage arises because the first entrant can locate so as to attract more demand than the later entrant. The entry game is treated quite informally in this section; see section 2 for a more formal statement of the agents' strategies, equilibrium etc..

Consumers are uniformly distributed on the unit interval. A consumer located at $x \in [0,1]$ gains a utility from purchasing a unit of the good located at $y \in [0,1]$ given by $U(x,y) = V - l(|x-y|)^2 - p$, where V is a constant that is the same for all consumers, $l > 0$ is the transport cost, or measure of horizontal differentiation, and p is the price that is charged for the good. Each consumer buys one or zero units of any good. It is assumed that V is 'sufficiently large'; exactly how large and the role of this assumption is explained below. Time is continuous and labelled by $t \in [0,\infty)$. The mass of consumers is time-varying and is described further in section 2.

Two risk neutral agents, labelled $i = 1,2$ can each enter the industry. There is a cost K to doing so, which is the same for both agents. Entry is irreversible (the cost K is entirely sunk), and can be delayed indefinitely. Once an agent has entered, it can sell its product at zero marginal cost. There are three possible locations at which the agents can enter: at $x = 0$, $x = \frac{1}{2}$ and $x = 1$. The restriction on locations is made to keep the analysis clear.

Three factors affect the location decision of an entrepreneur who enters before its rival (the leader): profit before the entry of the rival; profit after the entry of the rival; and the expected time to entry of the rival. Suppose that the leader decides to locate in the middle of the street. In doing so, while it has the only shop on the street its profit is higher than if it had located at one end of the street. Once its rival (the follower) enters, its profit is lower than if it had located at one end of the street, since price competition is more intense. Its rival, given standard assumptions, locates as far away as possible to minimize price competition. The furthest away it can locate from a shop in the middle of the street is half the street length, rather than the whole street length if the leading entrepreneur locates at one end. But this fact also means that the follower delays opening its shop until the level of demand is higher, and so the leader enjoys monopoly profits for longer.

The timing of entry is considered in the main paper. In this section, the analysis concentrates on the location of entry and the prices set by the firms. The firms will not enter at the same location on the line, since Bertrand competition would drive flow profits to zero; with the sunk cost of entry K, entry would not be profitable in this case. Two configurations are possible: one of the firms locates at $x = \frac{1}{2}$ while the other locates at $x = 0$ (or, equivalently, $x = 1$); or both firms locate at the ends of the line, at $x = 0$ and $x = 1$. In the static Hotelling model with endogenous locations and a quadratic transport cost function, there is maximum differentiation. Therefore, when (a) entry is simultaneous, the firms locate at the ends of the line; (b) entry is sequential, the second entrant, or follower, locates as far away from the first entrant, or leader, as possible. In the latter case, there are two possibilities: (i) the leader locates at $x = \frac{1}{2}$, or (ii) the leader locates at one end. In either case, the follower locates at the (other) end. Only the first possibility is considered here; the conditions required for this to be the optimal choice of the leader are derived below. The purpose of this restriction is to focus attention on the choice of interest: whether a firm considering investment before its rival should be aggressive or accommodating. If it is aggressive, by locating in the middle, then the conditions ensure that the other firm delays investing. If it is accommodating, by locating at one end, then the other firm locates at the other end simultaneously.

Consider first the outcome when the firms enter the market sequentially. The extensive form of the game is as follows: one firm enters and, having entered at its preferred location, sets its price to maximize profit. The second firm then enters at its preferred location; once it has entered, the firms compete in prices. Without loss of generality (wlog), let firm 1 be the firm that enters first. Let the mass of consumers in the market at time t in this case be θ_t. Once the firm has entered (at location $x = \frac{1}{2}$) and before firm 2 has entered, it chooses its price to maximize its profit $\pi_1 = \theta p_1(1-2x^*)$, where $(x^*, 1-x^*)$ are the locations of the marginal consumers who are indifferent between buying and not: i.e., for whom $V - l(\frac{1}{2} - x^*)^2 - p_1 = 0$, when the firm sets a price p_1. A straightforward calculation shows that, when V is sufficiently large, the firm chooses to sell to all consumers, setting its price so that the consumers located at $x = 0$ and 1 are indifferent between buying and not.[13] Therefore the firm's profit maximizing price and maximum profit are

$$p_1^I = V - \frac{l}{4}, \qquad \pi_1^I = \left(V - \frac{l}{4}\right)\theta.$$

Now consider the outcome once the second firm has entered, wlog at $x = 0$. The assumption that V is sufficiently large (greater than $\frac{3}{2}l$) ensures that in equilibrium all consumers buy from

[13]The alternative is that $x^* > 0$. In this case, the firm's profit maximizing price would be $p = \frac{2V}{3}$ and $x^* = \frac{1}{2} - \sqrt{\frac{V}{3l}}$. In order for $x^* > 0$, therefore, it must be that $V < \frac{3}{4}l$. It is assumed below, however, that $V \geq \frac{3}{2}l$, so this case does not occur.

one of the firms. The usual calculations show that the Nash equilibrium prices are

$$p_1^{II} = \frac{7}{12}l, \qquad p_2^{II} = \frac{5}{12}l.$$

$V \geq \frac{3}{2}l$ means that these prices are lower than p_1^I, and so certainly all consumers receive a greater net surplus in this case.

Positive externalities are introduced by letting the market size at time t when two firms have invested be $(1+\alpha)\theta_t$, where $\alpha \geq 0$. There are several ways in which this feature can be justified. First, with two firms in the industry, competition is more intense and consumer surplus is greater. Consequently, more consumers will be willing to buy the firms' goods. Alternatively, there may be a positive externality arising from the presence of search costs for consumers that ensures an increase in aggregate demand when two firms are located on the line.[14] Finally, it could be that the firms must cover jointly a fixed cost for delivery of inputs from a perfectly competitive supplier. When there is one firm in the industry, it pays the entire delivery cost; when there are two firms in the industry, the fixed cost can be shared. This decrease in fixed cost is represented as a multiplicative increase in the profit function of the firms. The firms' instantaneous profits are therefore

$$\pi_1^{II} = \left(\frac{49l}{144}\right)(1+\alpha)\theta_t, \qquad \pi_2^{II} = \left(\frac{25l}{144}\right)(1+\alpha)\theta_t.$$

Finally, consider the outcome when the firms enter simultaneously at either end of the line. The standard calculation gives the Nash equilibrium prices as $p_1^{III} = p_2^{III} = l$. Since $V \geq \frac{3}{2}l$, these prices are below p_1^I, but they are above both p_1^{II} and p_2^{II}. Again, there are positive externalities, but smaller than previously: the payoff is $\gamma(1+\alpha)\theta_t$ where $\gamma \in (0,1)$. Depending on which justification is used, this is because price competition is less fierce in this case, and so consumers are less willing to buy the firms' goods; or because consumers' expected search costs are higher because the firms on the line are farther apart; or because the fixed cost of delivery to two firms that are further apart is bigger. The firms' symmetric profits are then (each)

$$\pi^{III} = \frac{l}{2}\gamma(1+\alpha)\theta_t.$$

Hence instantaneous profits at time t of the 'leader' and 'follower' when entry—or investment

[14] In this story, there is some 'outside' good that consumers can buy when they do not find their preferred good at a firm on the line. When there is only one firm on the line, expected search costs are higher and so fewer consumers are willing to buy than when there are two firms on the line.

of the technology—is sequential, normalized by the leader's profit before the follower invests:

$$\pi_1^I = \theta_t, \tag{A1}$$

$$\pi_1^{II} = \left(\frac{49l}{36(4V-l)}(1+\alpha) \right) \theta_t \;\equiv\; (1+\delta_L)\theta_t, \tag{A2}$$

$$\pi_2^{II} = \left(\frac{25l}{36(4V-l)}(1+\alpha) \right) \theta_t \;\equiv\; (1+\delta_F)\theta_t. \tag{A3}$$

The instantaneous profits of the agents at time t when investment is simultaneous is

$$\pi^{III} = \left(\frac{8l}{(4V-l)}\gamma(1+\alpha) \right) \theta_t \equiv (1+\delta_S)\theta_t, \tag{A4}$$

again normalized by π_1^I.

Now conditions are derived to support the assumption that when entry is sequential, the leader locates at $x = \frac{1}{2}$ and the follower locates at the end. Suppose that the leader locates at the end, $x = 0$ say. Then before the follower enters, a straightforward calculation shows that the leader maximizes profit by setting a price $p_0 = \frac{2V}{3}$, earning a profit of $\pi_0 = \frac{2V}{3}\sqrt{\frac{V}{3l}}\theta_t$. Note two things. First, this solution holds while $V < 3l$; the modification if $V \geq 3l$ is straightforward and so omitted. Secondly, the mass of consumers is taken to be θ_t i.e., the same as when the leader locates at $x = \frac{1}{2}$. The calculations can easily be revised to alter this aspect; little would change. Normalizing by π_1^I, this means that

$$\pi_0 = (1+\delta_0)\theta_t \equiv \left(\frac{8V\sqrt{\frac{V}{3l}}}{3(4V-l)} \right) \theta_t.$$

If $\frac{V}{l} \in [0.3196, 6.2180]$, then $\delta_0 < 0$. This assumption is maintained in the paper.

Therefore the value function of the leader when it locates at $x = 0$ and investment is sequential is

$$L(\theta) = \begin{cases} b_{L0}^0 \theta^\beta & \theta < \theta_0, \\ \frac{(1+\delta_0)\theta}{r-\mu} + b_{L1}^0 \theta^\beta - K & \theta \in [\theta_0, \theta_F^0), \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_F^0, \end{cases}$$

given the leader's trigger point θ_0 and investment by the follower at the later point θ_F^0. The value function of the follower in this case is

$$F(\theta) = \begin{cases} b_F^0 \theta^\beta & \theta < \theta_F^0, \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_F^0. \end{cases}$$

Using the techniques of section 3.1, the trigger points when the agents' roles are pre-assigned are

$$\begin{aligned}\theta_0 &= \frac{\beta}{\beta-1}\left(\frac{K}{1+\delta_0}\right)(r-\mu) > \theta_L, \\ \theta_F^0 &= \theta_S < \theta_F.\end{aligned}$$

It is clear that the leader will prefer to locate at $x=\frac{1}{2}$ (and hence invest at θ_L) rather than at $x=0$ (θ_0) iff $b_{L0} \geq b_{L0}^0$ i.e., iff $\delta_0 \leq \overline{\delta}_0$, where

$$(1+\overline{\delta}_0)^\beta - \beta\overline{\delta}_0(1+\delta_S)^{\beta-1} = 1 + \beta\delta_L(1+\delta_F)^\beta - \beta\delta_S(1+\delta_S)^{\beta-1}.$$

Given that the pre-emptor's value function is strictly below the pre-assigned leader's, $\delta_0 \leq \overline{\delta}_0$ is also sufficient to ensure that the pre-emptor will prefer to locate at $x=\frac{1}{2}$.

A.2. Proof of Lemma 2

Define

$$\Delta(\theta) \equiv \frac{\theta}{r-\mu} - K - \left(\frac{\theta}{\theta_F}\right)^\beta \left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\frac{K}{\beta-1}$$

i.e., $L(\theta)-F(\theta)$, where $L(\theta)$ is conditional on the pre-emptor having invested, and $F(\theta)$ is conditional on the pre-emptor having invested but not the follower. Then the following facts can be shown by straightforward manipulations, using assumption 1 throughout: (i) $\Delta(0) = -K < 0$; (ii) $\Delta(\theta_L) = \frac{K}{(\beta-1)(1+\delta_F)}\left(\left(\frac{\theta_L}{\theta_F}\right)^\beta \beta\delta_L + (1-\left(\frac{\theta_L}{\theta_F}\right)^\beta)(1+\delta_F)\right) > 0$; (iii) $\Delta(\theta_F) = \frac{\beta(\delta_L-\delta_F)K}{(\beta-1)(1+\delta_F)} > 0$; (iv) $\Delta'(0) = \frac{1}{r-\mu} > 0$; (v) $\Delta'(\theta_F) = \frac{\beta\delta_L-\delta_F}{r-\mu} > 0$; (vi) hence there exists a $\tilde{\theta} < \theta_L$ such that $\Delta(\tilde{\theta}) = 0$; (vii) for any $\hat{\theta}$ such that $\Delta(\hat{\theta}) = 0$ and $\Delta'(\hat{\theta}) < 0$, then it must be that for another $\overline{\theta} > \hat{\theta}$ such that $\Delta(\overline{\theta}) = 0$, $\Delta'(\overline{\theta}) < 0$; (viii) since $\Delta(\theta_F) > 0$ (point (iii)), it must be therefore that there is no $\hat{\theta}$ such that $\Delta(\hat{\theta}) = 0$ and $\Delta'(\hat{\theta}) < 0$. Hence there is a unique $\tilde{\theta} = \theta_P < \theta_L$ such that $\Delta(\theta_P) = 0$, and $\Delta(\theta) \gtreqless 0$ as $\theta \gtreqless \theta_P$.

A.3. Proof of Proposition 1

The first two parts of the proposition (relating to the follower and pre-emptor inefficiencies) follow from equations (14), (21), (30) and (29) and lemma 2. The proof of the third part of the proposition (relating to the leader inefficiency) requires a comparison of the necessary and sufficient conditions

(26) and (33). Rewrite the conditions as

$$\begin{aligned} 2(1+\delta_S)^\beta - 1 &\geq 1 + 2\beta\delta_L(1+\delta_F)^{\beta-1}, \\ 2(1+\delta_S)^\beta - 1 &\geq (1+\delta_L+\delta_F)^\beta. \end{aligned}$$

If $1 + 2\beta\delta_L(1+\delta_F)^{\beta-1} \geq (1+\delta_L+\delta_F)^\beta$, then the proposition is proved. Let $\Delta \equiv (1+\delta_L+\delta_F)^\beta - 2\beta\delta_L(1+\delta_F)^{\beta-1}$; the object of the proof is to show that $\Delta \leq 1$. Suppose not i.e., suppose that $\Delta > 1$. A necessary condition for this to be the case is $\delta_L < 0$, since $\delta_L + \delta_F \leq 0$, from assumption 1. Consider the values δ_L^* and δ_F^* that maximize Δ (giving a value Δ^*), for any given β. (i) Suppose first that these choices are interior. The first-order conditions for maximization imply that $\delta_L^* = (1+\delta_F^*)/(\beta-1)$. But from assumption 1, $\delta_F \geq -\beta/(\beta+1) \geq -1$; this implies that $\delta_L^* \geq 0$, which is inconsistent with the necessary condition for $\Delta > 1$. Hence the maximization problem must involve non-interior solutions. (ii) Suppose that δ_L^* is not interior. Then either $\delta_L^* = \delta_F^*/\beta$ or $\delta_L^* = -\delta_F^*$, from assumption 1. If the former, then $\Delta = (1+(\beta+1)\delta_F/\beta)^\beta - 2\delta_F(1+\delta_F)^{\beta-1}$. It is straightforward to show that this is maximized at $\delta_F^* = 0$, giving $\Delta^* = 1$. If the latter, then $\Delta = 2\beta\delta_F(1+\delta_F)^{\beta-1} \leq 0$, where the inequality follows from assumption 1 ($\delta_F \leq 0$). (iii) Suppose that δ_F^* is not interior. Then either $\delta_F^* = -\beta/(\beta+1)$ or $\delta_F^* = 0$, from assumption 1. If the former, then $\Delta = (1/(\beta+1)+\delta_L)^\beta - 2\beta\delta_L(1/(\beta+1))^{\beta-1}$. It is straightforward to show that is a decreasing function of δ_L for $\delta_L \leq 0$ (which is required for $\Delta > 1$). Δ is therefore maximized by choosing the lowest value of δ_L, which is δ_F^*/β by assumption 1 i.e., $-1/(\beta+1)$. Hence $\Delta^* = 2\beta/(\beta+1)^\beta$, which is less than or equal to 1 for all values of β. If the latter, then $\Delta = (1+\delta_L)^\beta - 2\beta\delta_L$. This is a decreasing function of δ_L, and so is maximized by choosing the lowest value of δ_L, which is 0 since $\delta_L \geq \delta_F/\beta$ by assumption 1. Hence $\Delta^* = 1$. Since if either δ_L or δ_F is non-interior, the other is non-interior, all relevant cases have been analysed. Therefore $\Delta \leq 1$ for all values of δ_L, δ_F and β that satisfy assumption 1. This implies that $\lambda \geq 0$ i.e., the necessary and sufficient condition for simultaneous investment is stricter in the equilibrium of the model without pre-emption than in the co-operative solution.

A.4. Proof of Proposition 3

The proof of part (i) comes directly from differentiation of the expressions for θ_F and θ_2. The proof of parts (ii) and (iii) involves a comparison of the necessary and sufficient conditions (26) and (33) as α increases. Differentiation of λ_{26} and λ_{33} gives

$$\begin{aligned}\frac{\partial \lambda_{26}}{\partial \alpha} &= \frac{\beta\lambda_{26}}{1+\alpha} + \frac{\beta(1-(1+\delta_F)^{\beta-1})}{1+\alpha} \geq \frac{\beta\lambda_{26}}{1+\alpha},\\ \frac{\partial \lambda_{33}}{\partial \alpha} &= \frac{\beta(1+\lambda_{33})}{1+\alpha}.\end{aligned}$$

Hence if $\lambda_i \geq 0$, then $\partial\lambda_i/\partial\alpha \geq 0$, for $i \in \{26, 33\}$. This proves part (ii) of the proposition.

To prove part (iii) of the proposition, note that

$$\frac{\partial \lambda}{\partial \alpha} = \frac{\beta\lambda}{1+\alpha} + \frac{\beta(1+\delta_F)^{\beta-1}}{1+\alpha}.$$

Since $\lambda \geq 0$, by proposition 1, this means that $\partial\lambda/\partial\alpha \geq 0$.

A.5. Proof of Proposition 4

In order to prove the first part of the proposition, differentiate λ_{26} and λ_{33} with respect to β:

$$\frac{\partial \lambda_{26}}{\partial \beta} = (1+\delta_S)^{\beta}\ln(1+\delta_S) - \delta_L(1+\delta_F)^{\beta-1}(1+\ln(1+\delta_F)), \tag{A5}$$

$$\frac{\partial \lambda_{33}}{\partial \beta} = 2(1+\delta_S)^{\beta}\ln(1+\delta_S) - (1+\delta_L+\delta_F)^{\beta}\ln(1+\delta_L+\delta_F). \tag{A6}$$

It is sufficient for λ_{26} to be an increasing function of β that all terms in equation (A5) be positive. Hence joint sufficient conditions are: (i) $\delta_S \geq 0$, so that $\ln(1+\delta_S \geq 0$; (ii) $-\delta_L(1+\ln(1+\delta_F)) \geq 0$, which in turn requires that either (a) $\delta_L \geq 0$ and $1+\ln(1+\delta_F) \leq 0$ i.e., $\delta_F \leq e^{-1}-1$, or (b) the converse. It is sufficient for λ_{33} to be an increasing function of β that all terms in equation (A6) be positive. Hence a sufficient condition is that $\delta_S \geq 0$, since $\delta_L + \delta_F \leq 0$ by assumption 1.

The second part of the proposition follows directly from the first, given the definition $\lambda \equiv \lambda_{33} - \lambda_{26}$. Finally, note that if a function is an increasing (decreasing) function of β, it is a decreasing (increasing) function of σ.

A.6. Proof of Proposition 5

Rewrite equation (27) as

$$-\psi\theta_P^{\beta} + \frac{\theta_P}{r-\mu} - K = 0, \tag{A7}$$

$$\psi \equiv \frac{K}{\beta-1}\left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\theta_F^{-\beta}. \tag{A8}$$

Total differentiation gives

$$\frac{\partial\theta_P}{\partial\alpha} = \left(\frac{\theta_P^{\beta}}{\frac{1}{r-\mu} - \beta\psi\theta_P^{\beta-1}}\right)\frac{\partial\psi}{\partial\alpha}.$$

The denominator is positive from equation (A7). Hence Sign$\frac{\partial\theta_P}{\partial\alpha}$ = Sign$\frac{\partial\psi}{\partial\alpha}$. Differentiation gives

$$\frac{\partial\psi}{\partial\alpha} = -\left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\alpha}\right)\left(\frac{\beta\delta_L-\delta_F}{1+\delta_F}\right)\theta_F^{-\beta} < 0,$$

where the inequality follows from assumption 1.

A.7. Proof of Proposition 6

Differentiation of equation (27) gives

$$\frac{\partial\theta_P}{\partial\beta} = \frac{\psi\theta_P^{\beta}\left(-\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right)+\ln\left(\frac{\theta_P}{\theta_F}\right)\right)}{\frac{1}{r-\mu} - \beta\psi\theta_P^{\beta-1}}, \tag{A9}$$

where ψ was defined earlier. Hence Sign$\frac{\partial\theta_P}{\partial\sigma}$ = $-$Sign$\left[\psi\left(-\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right)+\ln\left(\frac{\theta_P}{\theta_F}\right)\right)\right]$. (This statement uses the fact that $\frac{\partial\beta}{\partial\sigma} < 0$.) Let $\phi_1 \equiv -\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right)$ and $\phi_2 \equiv \frac{\theta_P}{\theta_F}$. When $1-\beta\delta_L+\delta_F > 0$, $\psi > 0$. Hence Sign$\frac{\partial\theta_P}{\partial\sigma}$ = $-$Sign$[\phi_1 + \ln\phi_2]$. A sufficient condition for $\phi_1 + \ln\phi_2 > 0$ is that $\min[\phi_1, \phi_2] > \epsilon$. Conversely, a sufficient condition for $\phi_1 + \ln\phi_2 < 0$ is that $\max[\phi_1, \phi_2] < \epsilon$. An equivalent argument holds when $1-\beta\delta_L+\delta_F < 0$.

REFERENCES

Choi, J. P. (1994): "Irreversible Choice of Uncertain Technologies with Network Externalities," *RAND Journal of Economics*, 25(3), 382–401.

CNET News.Com (1997): "Domain Name Fetches Record Price," Internet magazine, Available at `http://news.cnet.com/news/0-1005-200-319447.html`.

Dixit, A. K. (1989): "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, 97, 620–638.

——— (1991): "Irreversible Investment with Price Ceilings," *Journal of Political Economy*, 99, 541–557.

Dixit, A. K., and R. S. Pindyck (1994): *Investment under Uncertainty*. Princeton University Press, Princeton.

Farrell, J., and G. Saloner (1986): "Installed Base and Compatibility—Innovation, Product Preannouncements, and Predation," *American Economic Review*, 76(5), 940–55.

Fudenberg, D., R. Gilbert, J. Stiglitz, and J. Tirole (1983): "Preemption, Leapfrogging, and Competition in Patent Races," *European Economic Review*, 22, 3–31.

Fudenberg, D., and J. Tirole (1985): "Preemption and Rent Equalization in the Adoption of New Technology," *Review of Economic Studies*, 52, 383–401.

——— (1986): "A Theory of Exit in Duopoly," *Econometrica*, 54, 943–960.

——— (1991): *Game Theory*. MIT Press, Cambridge.

Harrison, J. M. (1985): *Brownian Motion and Stochastic Flow Systems*. John Wiley & Sons.

Hoppe, H. (2000): "Second-mover Advantages in the Strategic Adoption of New Technology under Uncertainty," *International Journal of Industrial Organization*, 18, 315–338.

InternetNews.Com (1999): "Slow Downloads Costing Billions," Available at `http://www.internetnews.com/ec-news/article/0,1087,4_199731,00.html`.

Katz, M. L., and C. Shapiro (1986): "Technology Adoption in the Presence of Network Externalities," *Journal of Political Economy*, 94(4), 822–41.

Lane, W. J. (1980): "Product Differentiation in a Market with Endogenous Sequential Entry," *Bell Journal of Economics*, 11(1), 237–60.

Maskin, E., and J. Tirole (1988): "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," *Econometrica*, 56(3), 549–69.

McDonald, R., and D. Siegel (1986): "The Value of Waiting to Invest," *Quarterly Journal of Economics*, 101, 707–728.

Neven, D. (1987): "Endogenous Sequential Entry in a Spatial Model," *International Journal of Industrial Organization*, 5, 419–34.

P.C. World Magazine (1999): "Got Web?", January.

Pindyck, R. S. (1988): "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review*, 79, 969–985.

Prescott, E. C., and M. Visscher (1977): "Sequential Location among Firms with Foresight," *Bell Journal of Economics*, 8, 378–93.

Radin, M. J., and R. P. Wagner (1996): "Reflections on Exclusion and Coordination in Cyberspace: The Case Of Domain Names," Available at `http://www.stanford.edu/~mradin/downloads/dom_name.txt`.

Smets, F. (1991): "Exporting versus Foreign Direct Investment: The Effect of Uncertainty, Irreversibilities and Strategic Interactions," Mimeo.

Vu, B. (1996): "Will Satellite Broadband Services Fulfill Their Promise?," Conference Presentation, Available at `http://ksgwww.harvard.edu/iip/doeconf/vu.html`.

Weeds, H. (1999): "Strategic Delay in a Real Options Model of R&D Competition," Mimeo.