DISCUSSION PAPER SERIES

No. 3012

DYNAMIC PROCESSES OF SOCIAL AND ECONOMIC INTERACTIONS: ON THE PERSISTENCE OF INEFFICIENCIES

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INDUSTRIAL ORGANIZATION

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ISSN 0265-8003

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Discussion Paper No. 3012
October 2001

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CEPR Discussion Paper No. 3012

October 2001

ABSTRACT

Dynamic Processes of Social and Economic Interactions: On the Persistence of Inefficiencies*

This Paper considers the efficiency and convergence properties of dynamic processes of social and economic interactions, such as exchange economies, multilateral negotiations, merger and divestiture transactions, or legislative bargaining. The key general feature of the economy is that agents can implement any move from one state to another as long as a pre-specified subset of agents approve. By means of examples, we show that inefficiencies may occur even in the long run. Persistent inefficiencies take the form of cycles between states or of convergence to an inefficient state. When agents are sufficiently patient, we show very generally that the initial state from which the process starts plays no role in the long-run properties of equilibria. Also, when there exists an efficient state that is externality free (in the sense that a move away from that state does not hurt the agents whose consent is not required for the move), then the system must converge to this efficient state in the long run. Conversely, long-run efficiency can only be attained in a robust way if there exists an efficient externality-free state. It is thus more important to design transitions guaranteeing the existence of an efficient externality-free state rather than to implement a fine initialization of the process.

JEL Classification: C70, D50 and D70
Keywords: dynamic games, efficiency, externalities, multilateral interactions

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* Gomes wishes thanks for the financial support provided by the Rodney L White Center for Financial Research.
Submitted 26 July 2001

1 Introduction

Most economic activities can be represented as ongoing processes with transitions from states to states in which agents can affect the course of transitions, and the approval of some agents is required for the transitions.

For example, in an exchange economy, the states stand for the allocations of assets, intermediaries may propose transactions between agents, and the old and new owners should approve of the transactions. In mergers and acquisitions or divestiture transactions, the states stand for the various market structure configurations, managers or boards of directors may propose mergers or dissolutions to their stock holder constituencies, and merger laws or regulations on industry concentration can be viewed as placing additional constraints on the possible transitions from one market structure to another. In the legislative context, the states stand for the current policies (or laws), some politicians (like the president or the head of the government) may have a lead on the agenda setting and the constitution places constraints on how new policies (or laws) can be implemented (whether by simple majority or any other procedure). In coalitional bargaining the states corresponds to the partitions of the agents into coalitions, and transitions from one partition to another (which may involve break-up or expansion of coalitions) require the consent of those agents whose coalition changed.

What are the efficiency properties of the equilibria of such dynamic processes? Do the agents monitoring the transitions eventually stabilize the system to some state or can the system cycle between states in the long run? If the system is to stabilize, are the stable states necessarily efficient? What is the effect (if any) of the initial state on the long run properties of the system?

These questions are of vital importance to assess and/or improve the economic performance of existing institutions. The answers to these questions depend on parameters such as the flows of payoffs at each possible state, the set of allowed transitions including the specification of whose consent is required to move from one state to another, the patience of the agents and the probabilities with which the various agents can make proposals in the various states.

Throughout the paper, we make the following assumptions: 1) Agents can always stay in the current state (i.e. choose to remain inactive if they wish); 2) If the transition from state a to state b is possible with the consent of agents in S, then the consent of extra agents (in addition to those in S) cannot make the move unfeasible; 3) A move from any state a to any state b is always feasible if everybody approves the change; and 4) Agents can make side-payments to facilitate the transitions from states to states, but no contracts involving future actions (or side-payments) are available.

Assumptions 1-2-3 are very natural ones, and are met in most applications of interest. The assumption that contracts involving future actions are not available is more restrictive. It is meant to capture simply some limitations in the contracting possibilities of the agents.[1] And it allows us to focus on dynamic issues (in particular credibility issues) that the complete contracting approach cannot address.[2]

With this structure in place, we obtain sharp characterizations of the efficiency and convergence properties of equilibria.[3] We first observe in a series of examples that in equilibrium sometimes the system cycles, sometimes it converges, and the limiting states may sometimes be efficient and sometimes inefficient. Whether or not the system converges to a stable state may also depend on the probabilities that the various agents are selected to make proposals. It is the absence of contracts involving future actions (say, the impossibility to commit not to move from the efficient state in the future) that is responsible for the potential inefficiency.[4]

An interesting robust conclusion that applies to any specification is that there is no effect of the initial state on the long run properties of the system, as long as agents are sufficiently patient. And this insight holds true whether the system cycles or converges and whatever the efficiency properties of the limit behavior of the system. The irrelevance of the initial state with respect to the long run properties indicates that there is no point in re-initializing the system to another state if one is to maintain the rest of the process unchanged.[5]

Since inefficiencies may sometimes occur, and since efficiency is affected by the specification of the allowed transitions, we may conclude that institutional reforms should mostly bear on the form of the allowed transitions (rather than on a fine initialization of the system).[6]

In this respect, we identify a necessary and sufficient condition that guarantees the convergence to an efficient state irrespective of the probabilities that the various agents are selected to make proposals (and irrespective of the initial state, as implied by the previous result). That condition combines properties of the allowed transitions and of the flows of

[1] Such limitations may reflect legal constraints or common business practices.

[2] An interpretation of the set of allowed transitions is that it fully describes the set of available contracts. For example, if contracts were allowed to involve several periods, then this could be accommodated by expanding the set of states to consist of these several periods.

[3] We restrict attention to Markov Perfect Equilibria throughout the paper.

[4] In the Coasian language, this lack of commitment possibilities is a transaction cost.

[5] This is in the spirit of the result obtained in Jehiel and Moldovanu (1999), and our result shows that such an insight carries over to a very broad class of situations.

[6] It may also bear on the contracting possibilities.

payoffs obtained by the agents in the various states. In short, the condition amounts to the existence of an *efficient* state that is *externality-free* in the sense that if a (possibly indirect) transition from that efficient state to another state is possible without the consent of some agent, then this agent is no worse off (in terms of immediate flows of payoffs) in the original (efficient) state than in the reachable state.

The existence of an efficient externality-free state is key for the following reason. Consider an efficient state a that is *not* externality-free. That is, there is a group S, a state b such that S can move from a to b (possibly in several steps), and an agent i outside S who derives higher immediate payoff in state a than in state b. There is always the temptation for group S to move from state a to state b in order to extract some surplus from agent i (in exchange for the equilibrium prospect of moving back to state a). This effect indeed destabilizes state a whenever the probability that agent i is the proposer at state b is sufficiently small.[7] In contrast, when the efficient state a is externality-free, no such move can destabilize a, and the system must converge to state a in a finite number of steps whatever the probabilities that the various agents are selected to make proposals in the various states and whatever the initial state.

We now review some implications of our general results and insights. Some implications echo results already present in the literature (although generally obtained in less general setups). Others shed new lights on strands of literature that used different (generally static) approaches.

In exchange economies, transactions can take place if sellers and buyers both agree. In absence of externalities (i.e. when agents care only about their allocations), such transitions guarantee the existence of efficient externality-free states, and thus convergence to an efficient state follows (this echoes results obtained by Gale (1986) in a setting with no discounting). When there are externalities (i.e., when agents care about the entire profile of allocations, see Jehiel and Moldovanu 1995ab), long run inefficiencies and cycles may arise (because there need not be an efficient externality-free state). In simple instances though, efficiency might obtain even with externalities (because externalities *per se* are not incompatible with the existence of an efficient externality-free state).[8]

[7]The equilibrium in this case may involve states other than a and b.

[8]For example, suppose there is only one good consisting of a cost-reducting technology and the agents are firms competing in an imperfectly competitive fashion. Suppose the efficient state (the one that maximizes the profit of all firms) requires that firm i gets the innovation and suppose that firm i is the toughest competitor (i.e., every firm k prefers that firm $j \neq i, k$ gets the innovation rather than firm i). Then our result shows that firm i will eventually own the cost-reducing innovation and will never resell it to another firm (because the efficient state -firm i owning the good - is externality-free).

In legislative bargaining, simple majority procedures do not in general guarantee the existence of an efficient externality-free state even in those contexts where one policy is preferred by a majority to any other policy (see Example 2 below). Received voting theory based on static approaches would predict in such a case that a policy winning against any other policy - a so-called Condorcet winner - should be stable. However, in our dynamic setup cycles between policies may arise at equilibrium due to the inexistence of an efficient externality-free state, and even a Condorcet winner may not be stable. It should be noted that unlike in static approaches (à la Condorcet, say), there is no problem in our dynamic setting to speak of cycles. And our insights (about the emergence of cycles in dynamic settings) may suggest a new reason for political instability in democracies. Besides, our theory suggests that in order to guarantee convergence to an efficient state irrespective of who sets the agenda, unanimity constraints may have to be imposed (at least to leave the efficient state).[9] Or, if unanimity constraints cannot be imposed, it is important to adjust finely the probabilities of who sets the agenda in every state of the world in order to improve the functioning of the legislative process.

Our setup can also be used to speak of the process of coalition formation in the presence of widespread externalities (see Ray and Vohra (1999) and Gomes (2000, 2001)). States correspond now to the partitions of the agents into coalitions, and transitions from one partition to another should require the consent of those agents whose coalition is affected (either because they changed coalition or because their coalition was re-organized). Our setup allows for coalitions to expand or break-up, while most non-cooperative approaches to coalition formation either assume that coalitions upon forming leave the game (see, for example, Chatterjee et al (1993), and Ray and Vohra (1999)) or that coalitions may only expand (see Gomes 2001). Our analysis sheds a new light on the issue of stability (or convergence) and efficiency by identifying a new concept (other than the core), i.e., the concept of efficient externality-free state (which appears to be the key stability concept with farsighted agents). In particular, we obtain that the coalition formation process (despite the existence of externalities) is asymptotically efficient if the grand coalition is efficient.

Our model can also be applied to understand how mergers, acquisitions, partnership dissolutions and other governance changes may affect the shape of the market structure. Similarly to Hart and Moore (1990), in this setting, agents are the owners (or controllers) of the physical assets that are necessary for productive purposes, and the states of the economy stand for the various control structures (i.e. mapping that defines which assets are controlled

[9] Such unanimity constraints guarantee the existence of an efficient externality-free state.

by which agents). The transition rules are defined by the allowed changes in control structure, and we can accommodate for the existence of regulations and laws, such as regulation on industry concentration, and laws that endow agents with inalienable rights. The approach developed in this paper thus allows us to explore the process of changes in market structure and how agents share the surplus in situations where, for example, off-the-shelf solution concepts (such as the Shapley value) may not even be available.

There are a few papers in the bargaining literature which share with the present paper the generality of the setup. These include Rosenthal's (1976) "effectiveness form", Greenberg's (1990) "inducement correspondence", and more recently Chwe (1994), Xue (1998), and Konishi and Ray (2001).[10] An important distinction between our approach and the approaches based on the "effectiveness form" or the "inducement correspondence" (see in particular Konishi and Ray (2001)) is that we adopt a purely non-cooperative view (in the sense that the only decision-makers are the agents themselves in our setup). Another distinction is that we allow for side-payments between agents and we impose some mild (and natural, see above) restrictions on the allowed transitions. That extra structure of our setup allows us to obtain sharp predictions in terms of the effect of the initial state and about the conditions ensuring convergence to an efficient state, which the previous literature did not obtain.

The rest of the paper is organized as follows. In Section 2 we describe the model. In Section 3 we explain how the model can be used in a variety of applications. In Section 4 we develop some general properties of equilibria and exhibit a few examples with interesting dynamic properties. We also note a positive role for intermediaries. In Section 5 we analyze the efficiency properties of the model. In Section 6 we analyze the convergence properties of the model.

2 The Model

Consider an economy with n agents, infinitely many periods of interactions, and m possible states. We let $N = \{1, ..., n\}$ denote the set of agents, and Z denote the set of states. Agents all have the same discount factor δ.[11] The flow of utility to agent i at state $a \in Z$ is $(1 - \delta)\, v_i(a)$ per period. Side payments between agents can also be made at any period

[10] The effectivity functions introduced by Moulin and Peleg (1982) also shares some common features with our approach.

[11] The analysis easily extends to the case where agents have different discount factors. We have chosen to present the model with equally patient agents to alleviate a bit the notation.

and in any state, and agents are endowed with enough wealth to afford any transitions from state to state (i.e., there is no budget constraint).[12] That is, let a^k and t_i^k be respectively the state in period k and the transfer received by agent i in period k. Let $\widetilde{\sigma}$ denote the stochastic process governing a^k and t_i^k.[13] Agent i's expected utility induced by this stochastic process is

$$E_{\widetilde{\sigma}}\left[\sum_{k=0}^{\infty}\delta^k[t_i^k+(1-\delta)v_i(a^k)]\right],$$

where E denotes the expectation operator.

The transition from states to states is determined by the agents themselves in every period. In state a, agent i is selected with probability $p_i(a)$ to make a proposal. The proposal consists of a transition proposal, from state a to state b say, and possibly side-payments.

We wish to cover situations in which the transition from state a to state b may require the consent of a subset $S \subseteq N$ of agents. Formally, let $a, b \in Z$ be two possible states, and let $S \subseteq N$ be a subset of agents. We write $a \rightarrow_S b$ to denote that the move from state a to state b is feasible with the consent of agents in S.

Throughout the paper, we make the following assumptions about the transition relation:
A1) For all $a \in Z$ and $S \subseteq N$, $a \rightarrow_S a$. (Staying in the same state is always possible.)
A2) For all $a, b \in Z$, if $a \rightarrow_S b$ then $a \rightarrow_T b$ whenever $S \subset T$. (If a subcoalition $S \subset T$ can move from state a to state b so does coalition T.)
A3) For all $a, b \in Z$, $a \rightarrow_N b$. (If everybody agrees, a transition from state a to state b is always possible.)

These three assumptions are - we believe - very natural in many of applications of interest. In words, A1 states that one may always (if one wishes) stay in the current state (the status quo is always available). A2 states that if the consent of agents in S is enough to move from state a to state b, then the extra consent of agents outside S cannot make the move unfeasible. In a related vein, A3 states that if everybody approves the change from a to b then it can be done.[14] In social and economic interaction contexts, Assumptions A2 and A3 seem extremely natural (we discuss in section 5 how our results are affected when assumptions A2 and A3 are relaxed).

Let a be the current state and let agent i be the agent selected to make a proposal at that state. Agent i's offer τ consists of a subset S of agents (with $i \in S$), a state b, such that

[12] If the payoffs are normalized so that all $v_i(a) \geq 0$, then a sufficient condition for agents not to be budget constrained is that they have wealth $w_i \geq \max_{a \in Z}\left(\sum_{j \in N} v_j(a)\right)$.

[13] Later on it will be made endogenous and result from the strategies employed by the agents.

[14] A2 implies A3 if we make the extra assumption A3' that for any two states a and b there is always a coalition S (not necessarily N) such that $a \rightarrow_S b$.

$a \to_S b$ and transfers $t = (t_j)_{j \in S}$ between members of S such that $\sum_{j \in S} t_j = 0$. In words, the offer $\tau = (S, b, t)$ stands for a proposal made by agent i to agents $j \in S$ to switch from state a to state b in exchange for side-payments $t = (t_j)_{j \in S}$ where t_j is the payment received by agent j.

Upon the offer $\tau = (S, b, t)$ being made, the agents in S respond to the offer by yes or no (the order is irrelevant as any agent can always reject the offer). If the offer is rejected by any agent then the economy moves to the next period and the state remains unchanged (i.e. it remains a) with no side-payments being made. Otherwise, if all agents in S accept the offer then the state of the economy moves to b and the (lump-sum) transfers t are made.

Given a transition relation, an economy is characterized by the payoff specification $v = \{v_i(a)\}_{i,a}$, the proposals' probabilities $p = \{p_i(a)\}_{i,a}$ and the discount factor δ. The dynamic game with the fixed transition relation and the above parameters is denoted as the *economy* $\mathcal{E}(v, p, \delta)$. Some properties will refer to the specification of the transition relation, v and p only (but not on δ); we will then speak of the (v,p)-*economy* and denote it by $\mathcal{E}(v, p)$. Some properties will refer to the transition relation and v (but not on p and δ); we will then speak of the v-*economy* and denote it by $\mathcal{E}(v)$.

We have assumed that agents derive flows of payoffs (and are impatient). An alternative interpretation of the model is one in which there is an exogenous risk of breakdown. In this variant, players are not impatient but, in each period, there is a probability $1 - \delta$ that the economy stops at the current state, say a, for ever, in which case players receive a lump-sum payoff equal to $v_i(a)$. The economy with exogenous risk of breakdown and lump-sum payoffs and the one with discounting and flows of payoffs are equivalent and thus have the same equilibria (see also Binmore, Rubinstein, and Wolinsky (1986) and Gomes (2001)). For concreteness, we refer to the economy with discounting.

Equilibrium, efficiency and externalities:

Throughout the paper, we restrict attention to Markov Perfect Equilibria (MPE), which we refer to as equilibria in the main text. In a MPE, the strategies used by the players may only depend on the current state of the economy (and also, for the proposer, on his identity, and, for the responders, on the proposal). A possible rationale for restricting attention to MPE is that players do not keep track of the entire history of play, and they can thereby only rely on the current state to condition their strategies.[15]

[15] Another rationale is that such equilibrium behaviors are presumably easier to learn, and equilibrium selections based on learnability may lead to that restriction (see Maskin and Tirole (1997)). Formal analysis of this statement as well as the analysis of other equilibria should be the subject of future research.

Formally, a Markovian strategy for player i specifies for every state a such that $p_i(a) > 0$ a probability distribution over all feasible offers (S, b, t) that agent i can possibly make at state a, and for every state a' and every offer (S', b', t') such that $j \in S'$, a probability of acceptance for player j.

A Markovian strategy for player i will be denoted by σ_i and $\sigma_i[a](S, b, t)$ will denote the probability that offer (S, b, t) is made by player i at state a when i is the proposer at that state. Similarly, $\sigma_i(a)(S, b)$ will denote the associated probability that player i at state a makes a proposal to coalition S to move to state b, and $supp\,(\sigma_i(a))$ will denote the support of $\sigma_i(a)$.

Definition 1 *(Markov Perfect Equilibrium) A strategy profile $\sigma = (\sigma_i)_{i=1}^n$ is a Markov Perfect Equilibrium if for each player i, σ_i is a Markovian strategy, and after every history of play, σ_i is a best-response for player i when other players $-i$ play according to σ_{-i}.*

Efficiency in our context boils down to welfare efficiency, since utilities are assumed to be transferable:

Definition 2 *(Pareto Efficiency) A state $a \in Z$ is efficient if $a \in \arg\max_{a' \in Z} \sum_{i=1}^{n} v_i(a')$. We let $ES \subset N$ denote the set of Pareto efficient states.*

The economies we consider in the paper may have widespread externalities (positive or negative externalities). The notion of externalities plays an important role in our analysis, and it is useful to formally define the concept:[16]

Definition 3 *(Externalities) An economy is said to be without negative externalities if for all $a, b \in Z$, if $a \rightarrow_S b$ and $i \notin S$ then $v_i(a) \leq v_i(b)$. Otherwise, the economy is said to be with (negative) externalities.*

Although, we allow for situations with both positive and negative externalities, the latter will prove to be more important in our analysis.[17] The no-negative-externality condition states that if the consent of player i is not required to move from state a to state b, then the flow of player i's payoff is no smaller in state a than in state b. Thus, the decision of the

[16] In the definition, we consider only direct links between states a and b. An alternative definition of absence of negative externalities is: for all players $i \in N$ and every pair of states a and b such that $a \rightarrow_{S_1} a_1 \rightarrow \cdots \rightarrow_{S_n} a_n = b$ with $S_k \subset N \backslash i$ (excluding player i) then $v_i(b) \geq v_i(a)$. However, it is readily verified that the two definitions are equivalent.

[17] See Ray and Vohra (2001) for an analysis of situations in which positive externalities play a more important role.

agents in S to move from state a to state b (where $a \to_S b$) creates no negative externality (at least in terms of instantaneous payoffs) to agents outside S.[18]

3 Applications

Our setting is very general in that it allows for any specification as to what set of agents should be consulted to move the economy from one state to another and it allows for any specification as to who has the lead in making proposals as a function of the state of the economy. It also allows for any specification of the flow of payoffs as a function of the state of the economy.

The aim of this Section is to show that as a result of the flexible nature of our setting, a large range of applications can be dealt with. Thus our results may receive many interpretations of interest for each possible application (we refer to Greenberg (1990), Mariotti (1997), and Konishi and Ray (2001) for applications not discussed below to games in strategic and extensive form).

3.1 Exchange Economies

Exchange economies without externalities:

Consider an economy with n agents $N = \{1, ..., n\}$ and m indivisible goods. Agents start with some endowment and they can exchange their commodities over the various time periods. In each period, the state space Z of the economy is represented by the profile ω of allocations $(\omega_i)_{i=1}^n$ where ω_i is agent i's allocation in the current period and $\omega_i \cap \omega_j = \emptyset$ for all $i, j \in N$ (joint ownership is not allowed). The flow of per period payoff of player i in state ω is $(1-\delta)u_i(\omega_i)$ where u_i stands for agent i's utility function.

As the notion of property right suggests, a move from state ω to state ω' requires the consent of agent i whenever agent i's allocation is modified, i.e. $\omega_i \neq \omega'_i$. Thus, the transition rule for the exchange economy is: for any ω, ω' and subset S of agents $\omega \to_S \omega'$ if $\{i \in N: \omega_i \neq \omega'_i\} \subseteq S$.

We do not make any restriction as to the probabilities $p_i(\omega)$ that agent i makes the offer in state ω. Thus, even someone whose allocation does not change may propose a trade from ω to ω'. This, in particular, allows us to analyze the role of intermediaries in exchange economies.

[18] A stronger property related to Definition 3 is that agent i is not affected whether he is in state a or in state b, i.e. $v_i(a) = v_i(b)$. Note that in those contexts where $a \to_S b$ implies $b \to_S a$ - with the interpretation that what a coalition can do it can also undo - then the two conditions are equivalent. For some applications though, $a \to_S b$ need not imply $b \to_S a$ due to some irreversibilities (see for instance the merger application).

Compared to Rubinstein-Wolinsky (1985) and Gale (1986), our setup allows for multilateral exchanges whereas Rubinstein-Wolinsky and Gale focus on bilateral exchanges. Note also that Gale and Rubinstein-Wolisnky assume that there is only one time of consumption whereas our setup uses a flow formulation for payoffs.[19]

Exchange economies with externalities:

The exchange economy described above assumes that agents' flows of utilities depend solely on their bundles, i.e. there are no externalities. A more general situation with respect to payoff specification is one in which the flow of per period payoff of player i for a given allocation profile ω does not solely depend on ω_i but on the entire allocation profile, i.e. it is of the form $(1-\delta)u_i(\omega)$.

This situation has first been studied in a one-object context in Jehiel-Moldovanu (1995ab-1999). Our setup is more general in that it allows for an arbitrary number of goods.

3.2 Coalitional Bargaining Games

Our approach is well suited to address the issues of coalition formation in dynamic settings either in the form of coalitional bargaining models with externalities (e.g., Ray and Vohra (1999), Bloch (1996), and Gomes (2001)) or in the traditional form without externalities (Gul (1989), Hart and Mas-Colell (1996)). Generally, the externalities of the coalitional game among $N = \{1, ..., n\}$ players are described by a partition function $v(S, \pi) \in R$ that stands for the value of coalition S given the partition $\pi = \{S_1, ..., S_k\}$ of the N players. When there are no externalities (or, in the language of cooperative game theory, when the game has a characteristic form representation) we impose the additional restriction that $v(S, \pi) = v(S, \pi')$ for all $S \in \pi \cap \pi'$. That is, the value of coalition S is $v(S)$ and does not depend on the whole architecture of coalitions, but solely on who is in S.

A natural way to embed the coalition formation problem among the set $N = \{1, ..., n\}$ of players into our setup is to view the states of the economy as the partitions π of the players ($Z = \Pi$, the set of partitions of the N players). The transition rule we specify for the coalitional bargaining game is applicable for situations where coalitions may expand or break-up at any point in time and players who change coalitions should approve the transition. Formally, the transition rule is defined as follows: for any two partitions (states) π and π', $\pi \rightarrow_S \pi'$ if and only if S contains those agents whose coalitions changed.

[19] In Gale (or Rubinstein-Wolinsky)'s setup the offer is made by either of the parties who exchange their goods, whereas in our model intermediaries may propose trades to whoever they wish. Also, our setup corresponds to the case of durable goods (but, see also the exogenous breakdown interpretation of our model).

For notational convenience, in the sequel (including the examples provided throughout the paper) we represent coalitions and partitions using brackets: for example [12] refers to the coalition of players 1 and 2, and [12][3] refers to the partition where player 3 is in a solo coalition whereas players 1 and 2 are partners in a coalition.

In order to make the transition rules defined above more tangible it is helpful to look at some concrete situations. For example, if the current state is [12][3] a move to [1][2][3] (which corresponds to a break-up of coalition [12]) can be done with the sole consent of players 1 and 2 (so neither player 1 nor 2 can leave the coalition [12] without the consent of the other player). On the other hand, a move from [12][3] to state [123] (expansion of coalition [12]) or a move to state [13][2] (reorganization of coalitions) requires the consent of all three agents. Also, a move from [123] to any other coalition structure (break-up of the grand coalition) requires the consent of all three agents.[20]

The coalitional bargaining game payoffs for each player i and partition (state) π are given by $v_i(\pi)$, where for every coalition S in π, $v(S,\pi) = \sum\limits_{i \in S} v_i(\pi)$. That is, the sum of what agents $i \in S$ achieve in partition π should be equal to what coalition S can achieve given the partition π. Alternatively, we could have chosen another division of the coalitional value $v_i'(\pi)$ also satisfying $v(S,\pi) = \sum\limits_{i \in S} v_i'(\pi)$, or even include the choice of division of the coalitional value as part of the state space. However, it can be shown that the equilibrium transitions generated by these variants are the same (although the payoffs and transfers depend on the payoff division), and therefore we can without loss of generality consider a specific choice of payoffs $v_i(\pi)$.[21]

Finally, to complete the specification of the coalitional bargaining model, the proposals' probabilities are defined, for all $i \in N$ and π, by $p_i(\pi) \geq 0$ such that $\sum_{i \in N} p_i(\pi) = 1$. The

[20] Formally, $\pi \rightarrow_S \pi'$ if and only if $N \setminus \bigcup\limits_{T \in \pi \cap \pi'} T \subset S$.

[21] **(The coalitional approach to bargaining)** We could have chosen instead another equivalent formulation of coalitional bargaining that uses only the partition function, and the players are the coalitions. Formally, let $\mathcal{N} = 2^N \backslash \{\emptyset\}$ be the set of players and $Z = \Pi$. Define the payoffs and proposals' probabilities as follows, for all $S \in \mathcal{N}$:

$$v_S(\pi) = \begin{cases} v(S,\pi) & \text{if } S \in \pi \\ 0 & \text{otherwise} \end{cases} \quad \text{and } p_S(\pi) = \begin{cases} p(S,\pi) & \text{if } S \in \pi \\ 0 & \text{otherwise} \end{cases}$$

which means that if a coalition does not belong to π then it is worth zero, and only coalitions that belong to π can make proposals. The transition rule is defined by $\pi \rightarrow_S \pi'$ if and only if $S \supset (\pi \cup \pi') / (\pi \cap \pi')$, which means that the consent of those coalitions which are reorganized is required. In this interpretation, once a coalition between various coalitions S_k forms it becomes a decision-maker $S = \cup_k S_k$ (whose objective is to maximize the payoff of the S player as defined above). Also, at the time of the coalition formation of S, the various S_k receive a transfer payment from S, but lose their decision making power for future moves as long as the coalition does not break apart.

economy with the transition rule described above and parameters (v, p, δ) is referred to as the coalitional bargaining model (note that it accommodates both situations with externalities and without externalities).

Alternative approaches to coalition formation:

Our setup allows for coalitions to expand or break-up in contrast to most previous non-cooperative models in the literature. For example, a large body of the literature assumes that coalitions upon forming leave the game (see Chatterjee et al. (1993), Okada (1996), Ray and Vohra (1999), and that coalitions may only expand - but may not break-up (see Gomes (2001).[22]

The constraint that coalitions once formed leave the game can easily be represented in our setup, simply by not allowing transitions from partition π to partition π' where coalition $S \in \pi$ but $S \notin \pi'$. However, note that Assumption 3 no longer holds with such an additional requirement, since a switch from π to π' is not allowed even if every agent is willing to do it. While it is of interest to analyze the effect imposed by the extra constraint that only a limited number of transitions is allowed, we believe in many applications Assumption 3 is a very natural one.

Hart and Kurz (1983), in a cooperative game theory setting, considers two interesting alternative formulations of coalition formation - the Γ and Δ games - and a similar approach was used by Konishi and Ray (2001) in a dynamic model. In both games coalitions are formed if and only if the consent of all players is obtained, but unlike our model, a coalition may break-up if a deviant subcoalition is willing to do so even without having to ask the permission of the remaining coalitional members (for example, player 1 may move from [12][3] to [1][2][3] without the consent of 2). The two games of Hart and Kurz differ with respect to the reaction of the remaining players after the break-up of a coalition: in the Γ game the remaining players break apart and form singleton coalitions, while in the Δ game the remaining players stay together. Thus, the main distinction between our coalition game formulation and the one of Hart and Kurz and Konishi and Ray is about the permission required for a coalition to break-up (see also the discussion about the core in Section 6).[23]

[22]The constraint that coalitions can only expand may fit well in some merger/acquisition contexts in which dissolving a partnership involves large transaction costs.

[23]Note, though, that the permission structures considered in Hart and Kurz and Konishi and Ray give rise to transition rules that can also be studied within our general framework.

3.3 Control Structures: Mergers, Acquisitions, and Joint Ventures

Another important application of our setup is the business world in which mergers, acquisitions, partnership dissolutions and other governance changes may affect the shape of the market structure. In this setting, similarly to that of Hart and Moore (1990), agents are the owners (or controllers) of the physical assets that are necessary for productive purposes, and states of the economy stand for the various market structure configurations, i.e. the various possible combinations of assets, including those that allow for joint ownership.

Formally, consider an economy consisting of $N = \{1, ..., n\}$ agents and of $\mathcal{A} = \{a_1, ..., a_m\}$ assets. Hart and Moore (1990) describe an ownership and control structure using the control structure mapping $\alpha : 2^N \rightarrow 2^{\mathcal{A}}$. In this representation $\alpha(S) \subset \mathcal{A}$ stands for the assets that are controlled by coalition S.[24] Hart and Moore (1990) assume that each coalition S with assets A can generate a total value worth $v(S, A)$, and thus, for a given control structure α, the coalition S is worth $v(\alpha)(S) = v(S, \alpha(S))$, where $v(\alpha)$ is a standard characteristic function form.

Our approach allow us to talk about two types of situations. First, for a given control structure α the coalitional bargaining game among the N agents can be described by the characteristic function form $v(\alpha)$.[25] Interestingly, the solution that arises from our bargaining model is likely to be different from the Shapley value, which is the solution used by Hart and Moore (1990) (and more recently by Segal 2001).[26] The difference in the solution concept may be important for example in the study of the optimal control structures in incomplete contracting situations, which is the main focus of Hart and Moore (1990). As De Meza and Lockwood (1998) and Chiu (1998) have shown, alternative solutions of the (ex-post) bargaining game may well lead to very different implications about the optimal control structures, and the endogenous solution that arises from our bargaining game may well be another source for different predictions about the optimal control structure.

Second, we can talk about the process of selection of the control structure. In this situation, the players are N, the states of the economy are the control structures α, and the payoff functions are the equilibrium values that arise in the ex-post bargaining. Joint

[24] Two natural assumptions that the control structure mapping satisfy are (i) $\alpha(S') \subset \alpha(S)$ for all $S' \subset S$ - all the assets controlled by a subcoalition of S are also controlled by S; and (ii) $\alpha(S) \cap \alpha(N \backslash S) = \emptyset$ - assets cannot be controlled by two disjoint coalitions.

[25] We remark that the interpretation that we adopted for the transition rules for coalitional bargaining game is particularly suited to describe this contracting situation (see discussion about alternative representations of the bargaining game in Section 3.2.).

[26] For some values of the parameters it may well coincide with the Shapley value (see Gomes (2000)).

ventures, mergers, and acquisitions describe the transitions or changes of the control (or market) structure of the economy. For example, firms 1 and 2 each controlling assets $\alpha(1)$ and $\alpha(2)$ may form a joint venture in which some of their assets are now jointly controlled (the new control structure could be represented by β, where $\beta(i) \subset \alpha(i)$ and $\beta(12)$ represent the jointly controlled assets so that $\beta(1) \cup \beta(1) \cup \beta(12) = \alpha(1) \cup \alpha(2)$). In addition, the merger of firms 1 and 2 could be represented by a control structure β in which $\beta(1) = \beta(2) = \emptyset$ and $\beta(12) = \alpha(1) \cup \alpha(2)$. Moreover, the acquisition of firm 1 by firm 2 (or divestiture of some of firm 1's assets) can be described by a new control structure β where $\beta(1) = \emptyset$ and $\beta(2) = \alpha(1) \cup \alpha(2)$. These changes in control structure $\alpha \to_S \beta$ - including joint ventures, mergers, and acquisitions - define the transition rule.

In describing the transition rule we can also naturally accommodate further restrictions that may be imposed by regulations and laws, such as regulation on industry concentration, and other constraints on contracting such as inalienable rights of agents. The approach developed in this paper thus allows us to explore the dynamics of changes in contracts and how agents share the surplus in environments with rich contracting possibilities, situations where, for example, off-the-shelf solution concepts (such as the Shapley value) may not even be available.

3.4 Legislative bargaining

Baron and Ferejohn (1989) have proposed a model of legislative voting with endogenous agenda setting. A key feature is that a proposal is implemented whenever a majority vote in favor of the proposal.

Majority procedures a la Baron-Ferejohn can easily be captured in our model through appropriate specifications of the transition relation. Formally, let Z be the set of policies, which are the states of the economy. Consider a situation in which there is an ongoing process of policy choice by n legislators $i = 1, ..., n$.[27] The current policy $a \in Z$ may (or may not) affect the probabilities with which the legislators have control over the agenda setting. Accordingly, we let $p_i(a)$ be the probability that legislator i has the control in state a. The flow of payoff of legislator i is assumed to depend only on the current policy a, and we let $v_i(a)$ be that payoff.[28]

Majority rules are represented as follows. Suppose a simple majority is required to switch

[27] Baron and Ferejohn consider the case in which once a policy a is implemented this is the end (see also Banks and Duggan (2001)).

[28] Extensions to the case where legislators care about the last m policies raise no conceptual difficulty. It would just require defining the states of the economy as streams of policies of length m.

from policy a to policy b. Then legislators will be able to move from policy a to policy b whenever the consent of a majority is obtained. That is, $a \to_S b$ whenever the number of members of S (i.e. $\mid S \mid$) represents a (strict) majority of N (i.e. $\mid S \mid > n/2$). Other voting rules, such as for example supermajority voting, can be easily incorporated in our framework.

4 Cycles, Inefficiencies and Other Effects

We first provide some general characterization results and then illustrate a number of interesting dynamics that may arise in our setup.

4.1 Characterization and Existence of Equilibria

Consider a Markov Perfect Equilibrium σ of our economy $\mathcal{E}(v, p, \delta)$. For every state a and player i, we let $\phi_i(a)$ represent the associated *expected equilibrium outcome of player* i when the system is in state a. If the system moves to state a, player i's expected payoff (gross of transfers) is given by $x_i(a)$ where

$$x_i\left(a\right) = \delta\phi_i(a) + \left(1 - \delta\right) v_i\left(a\right). \tag{1}$$

That is, player i receives the flow of payoff $(1 - \delta)v_i(a)$ for the current period, and at the start of next period the system is in state a, resulting in a payoff of $\delta\phi_i(a)$.

Consider a Markov Perfect Equilibrium σ such that the strategy $\sigma_i\left(a\right)$ satisfies $\sigma_i[a]\left(S, b, t\right) > 0$. Then the (equilibrium) transfer that player i proposes to j is $t_j = x_j(a) - x_j(b)$. This is indeed the minimum transfer required by j to accept the transition from a to b, and it is such that player j is indifferent between rejecting and accepting the offer (since $t_j + x_j(b) = x_j(a)$). Thus, when player i at state a approaches S to move to state b, transfers are uniquely determined, and the equilibrium strategy of player i is characterized by $\sigma_i(a)\left(S, b\right)$. The above argument also shows that in equilibrium, player i at state a will approach coalition S and propose a transition to b whenever $\sum_{j\in S}(x_j(b) - x_j(a))$ is maximal over feasible transitions. This in turn yields the following characterization result:

Proposition 1 *A strategy profile σ is a MPE of an economy $\mathcal{E}(v, p, \delta)$ whenever the following conditions hold:*
i) The support of $\sigma_i\left(a\right)$ satisfies:

$$supp\left(\sigma_i\left(a\right)\right) \subset \underset{(S,b)}{\arg\max} \left\{ \sum_{j\in S} \left(x_j(b) - x_j(a)\right) : a \to_S b \text{ and } i \in S \right\}; \tag{2}$$

ii) For all i and a,

$$\phi_i(a) = \left(\sum_{j \in N} p_j(a) \phi_i^j(a) \right) \tag{3}$$

where

$$\phi_i^j(a) = \begin{cases} x_i(a) + \max\limits_{(b,S)} \left\{ \sum_{j \in S} (x_j(b) - x_j(a)) : a \to_S b \text{ and } i \in S \right\} & j = i \\ \sum_{(S,b)} \sigma_j(a)(S,b)(I(i \in S)x_i(a) + I(i \notin S)x_i(b)) & j \neq i \end{cases} \tag{4}$$

is player i's equilibrium payoff at state a when player j is the proposer at that state.

Concatenating (3) and (4), the expected payoffs $x_i(a)$ must satisfy the following system of equations:

$$x_i(a) = \begin{array}{l} (1-\delta) v_i(a) + \delta \sum_{(S,b)} \sigma_j(a)(S,b) \sum_{j \in S} (x_j(b) - x_j(a)) \\ +\delta \sum_{j \in N} p_j(a) \sum_{(S,b)} \sigma_j(a)(S,b)(I(i \in S)x_i(a) + I(i \notin S)x_i(b)) \end{array} \tag{5}$$

Given a Markov Perfect Equilibrium, it is useful to define the excess of player i at state a as

$$e_i(a) = \max_{(b,S)} \left\{ \sum_{j \in S} (x_j(b) - x_j(a)) : a \to_S b \text{ and } i \in S \right\} \tag{6}$$

This is the gain that agent i realizes at state a when he is the proposer at that state. Note that as player i can always decide to stay in the same state (see Assumption A1), we always have that $e_i(a) \geq 0$ for all i and a. Note also that Assumption A2 implies that in equilibrium it must be that if player i finds it optimal at state a to approach coalition S and propose a move to state b, i.e. if $\sigma_i(a)(S,b) > 0$, then agents outside S should be no worse off in state a than in state b, i.e. $x_j(b) \leq x_j(a)$ for all $j \notin S$. Indeed, if that were not the case, say $x_j(b) > x_j(a)$ for some $j \notin S$ and if i were to propose a move from a to b to coalition S, then player i could extract an extra transfer from agent j, i.e., $x_j(b) - x_j(a) > 0$, by including agent j in the coalition (which Assumption A2 permits, thus showing that the original proposal of agent i was not optimal).[29]

Our first general result establishes the existence of equilibrium.

Proposition 2 *(Existence) There exists at least one Markov perfect equilibrium for all economies $\mathcal{E}(v, p, \delta)$.*

The proof of the proposition relies on a standard use of the Kakutani fixed point theorem, and is provided in the appendix.[30]

[29] See also section 5 for more on the role of assumption A2.

[30] Note that both propositions 1 and 2 hold even if we drop assumptions A2 and A3. Moreover, it can be shown that the equilibrium correspondence is upper-hemi-continuous on the parameters (v, p, δ).

4.2 Stable States and Sets of the Economy

We are primarily interested in understanding the long-run properties of the economy and in particular their relation to efficiency and stability. To this end, we introduce the concepts of stable sets (or ergodic classes) and stable states (or absorbing states). The stable sets/states are the states to which the process converges after a long period of time.

The equilibrium σ of an economy $\mathcal{E}(v,p,\delta)$ induces a transition probability in the state space

$$\mu(a,b) = \sum_{j\in N}\sum_{S} p_j(a)\,\sigma_j(a)(S,b).$$

The Markov chain with transition probability μ captures the dynamics of the economy, and for this reason the transition probability μ is of central interest.

The long-run behavior of the economy is described by the stable sets (or ergodic classes).

Definition 4 *The stable sets (or ergodic classes) of the economy (associated with the equilibrium σ with transition probability μ) are the sets $E\subset Z$ such that:*
(i) (Closedness) For any $a\in E$ there exists no $b\in Z\backslash E$ such that $\mu(a,b)>0$;
(ii) (Irreducibility) For any $a,b\in E$ there exists a sequence $a=a_0,...a_k,...a_m=b$ with $a_k\in E$ and $\mu(a_{k-1},a_k)>0$.

The definition of stability captures the idea that starting from any state that belongs to a stable set the process remains at the stable set forever, and that no (non-trivial) subset of a stable set is stable.[31] A well-known result of the theory of Markov chains (see for example Doob (1953)) yields that starting from any state the process converges, in a finite number of steps, to a stable set. Therefore, the stable sets describe the long-run behavior of the economy.

When the stable set has a unique state, we refer to it as a stable state or absorbing state. Note that when the stable set contains several states, the system cycles between these states.

We now investigate a few examples showing how equilibrium cycles might emerge in our setup. We first consider a coalitional bargaining game with externalities.

[31] States that do not belong to a stable set are also referred to as a transient states.

Example 1 *Consider the following coalitional bargaining game with partition function:*

$v_i(\cdot)$	*[1][2][3]*	*[12][3]*	*[13][2]*	*[23][1]*
1	*1*	*2*	*2*	*-2*
2	*1*	*2*	*-2*	*2*
3	*1*	*-2*	*2*	*2*

where coalitions may expand and break-up (with the permission of the involved players, see section 3.2),[32] *and all proposers are chosen with equal probability.*

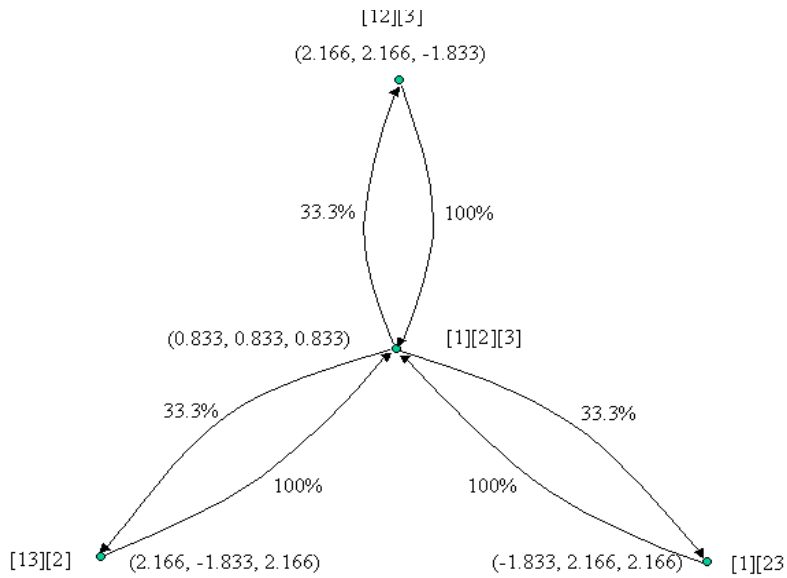

Figure 1: Limit equilibrium payoffs and transition probabilities for example 1 (an arrow leaving state a and pointing to state b indicates that the process can move from state a to b, and the percentage in the arrow is probability $\mu(a, b)$; the equilibrium payoffs at each state are given by the vector (ϕ_1, ϕ_2, ϕ_3)).

For every δ sufficiently close to 1, this economy has a unique equilibrium. The limit as δ goes to 1 of this equilibrium is such that equilibrium payoffs and equilibrium transition probabilities, for each state, converge to the values indicated in Figure 1.

[32]The grand coalition [123] is not considered because it yields low payoffs to all players, say.

The unique stable set is the set of all states $E = Z$, and the efficient state of this economy is [1][2][3], which is not a stable state. If it were, each player would only get a payoff of 1. But, player 1 say could achieve a higher payoff by proposing to player 2 to form a coalition (i.e. move to [12][3]). Hence, the system must cycle and the only equilibrium cycle is the one shown above in which all states may occur in equilibrium.

The next example illustrates the possibility of cycles in voting games. The example also illustrates that even if a Condorcet winner (a policy that cannot be defeated by any other policy by majority vote) exists it need not correspond to a stable state.

Example 2 *Consider the voting game*

$v_i(\cdot)$	a	b	c	d
1	*1.5*	*1*	*0*	*1.6*
2	*1*	*0*	*1.5*	*1.6*
3	*0*	*1.5*	*1*	*1.6*

,

where any policy can be passed by majority voting, i.e. any two players can move from the current policy (status quo) to any other policy, all players are the proposers with equal probability in all states, and players are arbitrarily patient).

For every δ sufficiently close to 1, this economy has a unique equilibrium where the payoffs and transition probabilities are as shown in Figure 2.

Note that policy d is a Condorcet winner (a core state), since no majority prefers a, b or c over d. In fact, policy d is even unanimously preferred over a, b, c. Despite the fact that d is a Condorcet winner (and is even unanimously preferred to other states) state d is not stable!

At first sight it might seem strange that state d - which is unanimously preferred to other states - happen to be unstable (it does not even belong to the ergodic set). The problem is that two agents, say agents 1 and 2, are enough to move away from the efficient policy d, to say policy a, and the left aside agent (i.e. 3) suffers a lot from the move to a. In the language of our paper (see below), we will say that the efficient state d is not externality-free. And whenever there is no efficient externality-free state, we will show that the efficient state need not be stable, at least for some proposers' probabilities (see below Sections 5 and 6).

In words, agents 1 and 2 move away from state d (despite the fact that they incur immediate losses) because they can improve their bargaining position by moving to a state where agent 3 is in a weak bargaining position.

It should be noted that if in the same example we had required that to move away from state d *unanimous* consent is required, then the economy would have converged to the efficient

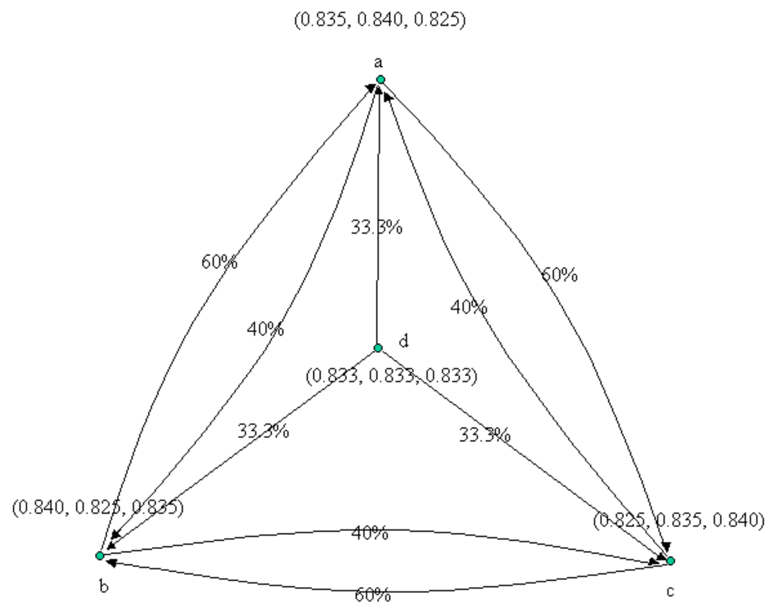

Figure 2: Equilibrium payoffs and transition probabilities for the voting game of example 2 (an arrow leaving state a and pointing to state b indicates that the process can move from state a to b, and the percentage in the arrow is probability $\mu(a, b)$; the equilibrium payoffs at each state are given by the vector (ϕ_1, ϕ_2, ϕ_3)).

state and no cycle would have occurred.[33]

The next example illustrates in the context of coalitional bargaining games that the economy may sometimes converge to an *inefficient* state.

[33]Thus, our analysis allows us to disentangle the effects of payoffs and of the allowed transition on the asymptotic efficiency, which static frameworks (say the core concept) do not allow.

Example 3 *Consider the coalitional bargaining game with partition function* [34]

$v_i(\cdot)$	[123]	[12][3]	[13][2]	[23][1]	[1][2][3]
1	0	3	2.5	-10	1
2	0	2	-10	2	1
3	0	-10	2	2	1

where all proposers are chosen with equal probability.

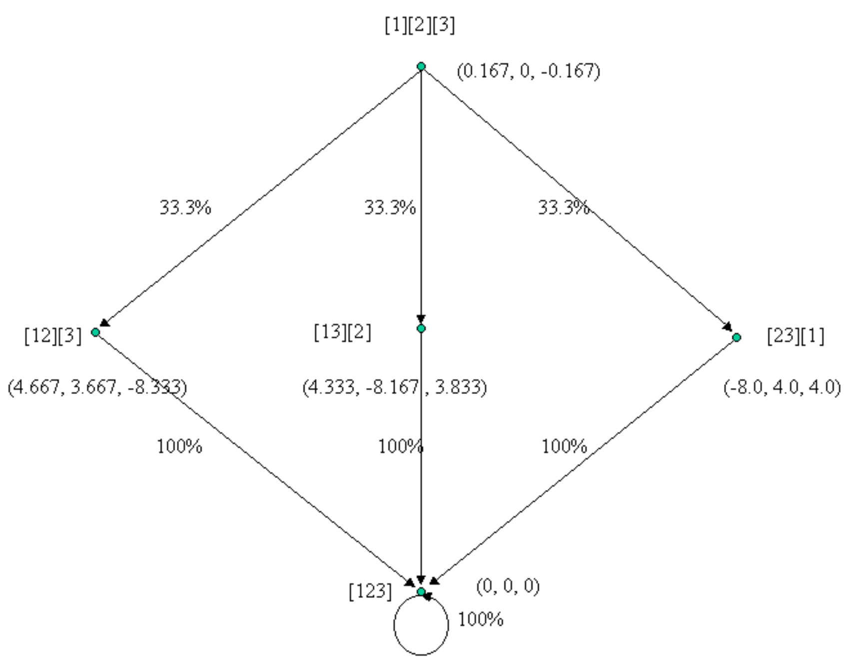

Figure 3: Equilibrium payoffs and transition probabilities in the coalitional bargaining game of example 3.

In Figure 3 we show the unique equilibrium payoff and transition probabilities for δ very close to 1.

[34]The payoffs above could describe for example the values of firms for different market structures. Suppose that the merger of the three firms is inefficient, and that the merger of any two firms has the effect of making aggressive policies toward firms outside the coalition less costly. In that context, the efficient coalition structure (for the set of firms) may well having the three firms operating independently (because of the high cost of mergers). But, the merger of two firms may induce high profit to the merged firms (and low ones to the firm left aside) because of the weakening of the firm left aside. A payoff structure with similar qualititative features would then arise.

Even though state [123] is inefficient -it yields a welfare of 0 whereas state [1][2][3] yields a welfare of 3-, it is stable!!! Player i does not move from state [123] to state [1][2][3] (which is more efficient) because player i is afraid that whenever he has no opportunity to make an offer in state [1][2][3], one of the other players decide to form a coalition without i (i.e., move to state $[jk][i]$), which is very unfavorable to player i.

As we will see, for convergence to an inefficient state to occur, players have to be sufficiently patient. When players are impatient or myopic inefficient states cannot be stable (see subsection 6.2.).

4.3 Intermediaries and the Excess Function

The goal of this subsection is to develop some basic properties about the excess function (see expression (6)). When the excess of player i is positive at state a, it means that player i can extract some surplus (equal to the excess) when he is the proposer at state a. The following proposition shows that in the limit of very patient players, there is always a state in a stable set such that the limit excess of player i at that state is zero.

Proposition 3 *Consider an equilibrium strategy profile $\sigma^{(\delta)}$ of $\mathcal{E}(v,p,\delta)$ such that the equilibrium transition probabilities and payoffs $\mu^{(\delta)}$ and $\phi^{(\delta)}$ are such that $\mu^{(\delta)} \to \mu$ and $\phi^{(\delta)} \to \phi$, as $\delta \to 1$, and let $E \subset Z$ be a stable set of μ. Consider a player i that can be the proposer with positive probability at some state in E. Then there exists a state $a^* \in E$ such that $p_i(a^*) > 0$ and i's excess at a^* converges to zero, i.e., $\lim_{\delta \to 1} e_i^{(\delta)}(a^*) = 0$.*

The intuition for Proposition 4 is that if the limit excess of say player i were strictly positive in all states, then it would not be possible that the expected equilibrium payoffs of this player be finite in all states, which is clearly absurd. (The formal argument relies on a reasoning about the state where the player gets its minimal equilibrium payoff.) It should be noted though that the limit excess of players need not be zero in *all* states of a stable set, as Example 1 shows (the excesses of all players at state [1][2][3] are strictly positive).

The following Proposition shows that when players are very patient the path of players' equilibrium value departing from a certain state is markedly different depending on whether the excesses are zero or positive:

Proposition 4 *Consider an equilibrium strategy profile $\sigma^{(\delta)}$ of $\mathcal{E}(v,p,\delta)$ that converges as $\delta \to 1$ (and such that the payoffs also converge $\phi^{(\delta)} \to \phi$). Then, given that the process is at state a either the excess at state a for player i is:*

(i) $p_i(a)e_i(a) = 0$, in which case the equilibrium value of player i at state a is independent of

who makes the proposal at that state;
(ii) $p_i(a)e_i(a) > 0$, *in which case the equilibrium value of player i at state a varies with the identity of the proposer, and the system may move to states b and c such that player i's equilibrium value is higher (resp. lower) in state b (resp. c) than in state a.*

In specific applications, such as the mergers and acquisitions or the coalitional bargaining games we discussed in the previous section, the result implies that if the excess of firm or coalition is positive given a current market or coalition structure, then the firm's stock price or the coalition's value is uncertain and can either go up or down from its current value. The examples previously considered also serve to illustrate the proposition: In examples 1 and 3 the excesses of all players were positive at state [1][2][3] and the players' values went up when they happened to participate to a pairwise coalition and went down otherwise; and in example 2 the excesses were zero at all states and, accordingly, players' values remained (approximately) constant.

The positiveness of the limit excess value has also implications for the role of intermediaries (where an intermediary (or a dummy player) is an agent who serves no fundamental role in allowing for transitions and gets a constant payoff in all states). We will show that an intermediary may make positive profits despite his non-essentiality in situations where the excess is positive and he has some initiative to make proposals. Notice that this property of our solution is markedly distinct from cooperative solution concepts such as the Shapley value, where an intermediary (as just defined) makes no profits. We first define intermediaries formally:

Definition 5 *A player i is considered an intermediary (or a dummy player) if* $v_i(a) = 0$ *(or any constant) for all* $a \in Z$ *and for all feasible transitions* $a \to_S b$ *then* $a \to_{S \setminus \{i\}} b$ *is also feasible.*

Consider the following variant of example 1 with the addition of player 4 as an intermediary.

Example 4 *Consider the coalitional bargaining game with partition function similar to example 1,*

$v_i(\cdot)$	*[1][2][3][4]*	*[12][3][4]*	*[13][2][4]*	*[23][1][4]*
1	*1*	*2*	*2*	*-2*
2	*1*	*2*	*-2*	*2*
3	*1*	*-2*	*2*	*2*
4	*0*	*0*	*0*	*0*

where all player are proposers with equal probability and the discount rate are infinitesimally close to 1. The equilibrium payoff is

$\phi_i(\cdot)$	*[1][2][3][4]*	*[12][3][4]*	*[13][2][4]*	*[23][1][4]*
1	*0.525*	*2.125*	*2.125*	*-1.875*
2	*0.525*	*2.125*	*-1.875*	*2.125*
3	*0.525*	*-1.875*	*2.125*	*2.125*
4	*0.925*	*0.125*	*0.125*	*0.125*

,

and the equilibrium transition probability is as in example 1.

In this example, the excess of players $i \in \{1,2,3\}$ at state $a = [1][2][3][4]$ are equal to $e_i(a) = 3.2$ and the excess of player 4 is $e_4(a) = 2.4$. Interestingly, despite the fact that player 4 is an intermediary, he makes strictly positive profits.

In general, we have that if the economy is at a certain state where at least one player has positive excess (in the limit) an intermediary with initiative (i.e., positive probability of being the proposer) is able to make strictly positive profits (and these profits are monotonically increasing in the excess).[35] The intuition is that whenever the intermediary is able to seize the initiative to move, he can extract positive rents from a player with positive excess. This is so because, as we have seem in proposition 4, the value of a player with positive excess can either go up or down, and thus if the intermediary proposes to move to a state where the players' value go up then he can obtain some positive rents (which are increasing in the excess).[36]

5 Efficiency Analysis

What are the efficiency properties of equilibria? The Coasian view would be that in absence of transaction costs and in the transferable utility case individualistic bargainers should make

[35]This monotonic relationship can be seen in example 4: at state [1][2][3][4] the maximum excess is 3.2 and the intermediary's profits is 0.925, while at all other states the maximum excess is zero and the intermediary's profit only 0.125 (note that the intermediary's proposer probability were kept fixed at 0.25 in all states).

[36]More formally, the equilibrium value of an intermediary is given by

$$x_i(a) = p_i(a)e_i(a) + \sum_{(S,b)} \mu\left(a,b,S\right)\left(I(i \in S)x_i(a) + I(i \notin S)x_i(b)\right),$$

where $x_i(b) \geq 0$ (because $v_i(a) \geq 0$ for all states and $\sum_{(S,b)} \mu\left(a,b,S\right) I(i \in S) \leq p_i(a)$). Thus equation above implies that

$$x_i(a) \geq \frac{p_i(a)}{1-p_i(a)} e_i(a),$$

(notice that the lower bound for the value is increasing in $e_i(a)$).

efficient decisions (and share the surplus as a function of their relative bargaining power). This view would suggest that whoever has to move and whatever the state of the economy there should be an immediate move to an efficient state (and no further move or moves only within the set of efficient states).

However, we have already seen examples in Section 4 in which efficiency did not obtain (either due to long run cycles or to convergence to inefficient states), thus suggesting that the inability to commit of the players is a form of transaction costs.

In our analysis, we distinguish two forms of efficiency: the strong efficiency which requires that starting at any given state there is an immediate move to an efficient state (i.e., any state that yields $\max_{a \in Z} v_N(a)$) and the asymptotic efficiency which only requires that in the long run (after a number of possible transitory moves) the system stabilizes to states that are efficient.

Clearly, strong efficiency implies asymptotic efficiency. And a strongly efficient equilibrium pattern induces the highest possible welfare. However, in those contexts where players are very patient, the potential loss induced by asymptotic efficiency relative to strong efficiency is very small (it vanishes to 0 as players get infinitely patient). Asymptotic efficiency is thus the economically relevant measure of efficiency when considering arbitrarily patient players.

In the next subsection, we characterize sufficient conditions for asymptotic efficiency. In short, the condition is that there exists an efficient-externality-free state (EFS), that is, an efficient state a^* such that if a group S can move from a^* to b (possibly in several steps), then players outside S derive no smaller flows of payoffs in state b than in state a^*. When we require robustness with respect to the (possibly state-dependent) probabilities that the various players are selected to make proposals, the existence of an efficient externality-free state appears to be necessary. Finally, in subsection 5.3, we characterize the very restrictive conditions under which strong efficiency holds.

5.1 Asymptotic Efficiency

In Section 4, we showed examples in which the equilibrium cycled between several states. The flows of aggregate payoffs in such cyclical equilibria also cycle as the various states between which the economy cycles need not have the same aggregate flows of payoffs. However, the aggregate equilibrium value attached to a state incorporates not only the instantaneous aggregate flow of payoffs attached to that state but also the subsequent flows of aggregate payoffs as derived from the equilibrium transitions from states to states.

The next proposition shows that the aggregate equilibrium value cannot cycle and must converge to a well defined limit as players get very patient. That is, the aggregate equilibrium value is approximately the same at all states if players are very patient. Therefore, social welfare does not depend on the particular initial state from which the process starts. Even though the long run pattern may display cycles or inefficiencies, the initial state from which the process starts has no effect on the long run welfare efficiency properties of the system. We use from now on the following simplifying notation to denote aggregate value:

$$v_S(a) = \sum_{i \in S} v_i(a),\ \phi_S(a) = \sum_{i \in S} \phi_i(a) \text{ etc., where } S \subset N.$$

Proposition 5 *The aggregate equilibrium values are approximately the same at all states for all economies $\mathcal{E}(v, p, \delta)$ if players are patient enough (δ close to 1). More precisely,* $\lim_{\delta \to 1} \max \left\{ |\phi_N^{(\delta)}(a) - \phi_N^{(\delta)}(b)| : a, b \in Z \right\} = 0$*, where $\phi_N^{(\delta)}(\cdot)$ is the aggregate equilibrium payoffs associated with any equilibrium $\sigma^{(\delta)}$ of the economy $\mathcal{E}(v, p, \delta)$.*

To illustrate Proposition 5 we refer to Examples 1, 2, and 3. It is no coincidence that the aggregate welfare was the same in all states (the aggregate welfare was equal to 2.5, 2.5, and 0, respectively, in examples 1, 2, and 3).

The intuition for Proposition 5 is as follows. When players are very patient, whatever the state one starts from, the aggregate value attached to that state must correspond to the average aggregate value obtained in the ergodic set reached from that state. However, it might be *a priori* that two different states lead to two different ergodic sets (see however Section 6), and thus this argument alone does not permit to conclude. But, if there were two ergodic sets corresponding to two different aggregate welfares, any player would be willing to move (even with unanimous consent) from the less efficient ergodic set to the more efficient ergodic set extracting for himself the generated surplus. Thus, the two ergodic sets would have to be in fact a single ergodic set (since players would move from one to the other), and it is thus impossible to have two ergodic sets with different aggregate values in the limit.[37]

Proposition 5 guarantees that there is a well defined notion of limit aggregate welfare. But, this limit aggregate welfare may well be suboptimal. An equilibrium is said to be asymptotically efficient if the aggregate equilibrium value, at any given state, is approximately equal to the aggregate value at an efficient state. Our focus is on economies where *all* equilibria are approximately efficient when players are very patient. Formally,

[37] Note that the argument does not *a priori* rule out the possibility of having two ergodic sets with the same aggregate values in the limit as δ goes to 1.

Definition 6 *A (v,p)-economy $\mathcal{E}(v,p)$ is asymptotically efficient if and only if* $\lim_{\delta\to 1} \phi_N^{(\delta)}(a) = \max_{b\in Z} v_N(b)$ *for any equilibrium $\sigma^{(\delta)}$ of the economy $\mathcal{E}(v,p,\delta)$ and any state $a \in Z$.*

Again, the examples in Section 4 may be used to show that economies need not be asymptotically efficient. However, assume there is an efficient state that is a stable state in a given equilibrium $\sigma^{(\delta)}$ of $\mathcal{E}(v,p,\delta)$ for all δ sufficiently close to 1. According to proposition 5, in such an equilibrium and for sufficiently patient players, the aggregate equilibrium value must be close to the efficient level at all states. In economies without negative externalities it can be shown that efficient states are stable, and thus economies without negative externalities are asymptotically efficient.

The same line of argument can also be applied to economies with externalities. Suppose that there is an efficient state a that can be left only with unanimous consent. Then such an efficient state must be stable whatever the equilibrium, which in turn implies asymptotic efficiency.

More generally, we develop the concept of an efficient-externality-free state (EFS):[38,39]

Definition 7 *A state a is an efficient-externality-free-state (EFS) of the v-economy $\mathcal{E}(v)$ if and only if for all players $i \in N$ and moves $a \to_{S_1} a_1 \to \cdots \to_{S_n} a_n = b$ by subcoalitions $S_k \subset N\backslash i$ (excluding player i) then $v_i(b) \geq v_i(a)$. We let $EFS \subset Z$ (of $\mathcal{E}(v)$) be the set of states that satisfies the EFS property.*

Observe that the absence of negative externalities and/or the requirement of unanimous consent to move away from one efficient state implies the existence of an efficient externality-free state.

The next result (and its corollary) establishes asymptotic efficiency under any of the conditions discussed above:

Proposition 6 *All (v,p)-economies $\mathcal{E}(v,p)$ are asymptotically efficient if there exists at least one efficient externality free state of $\mathcal{E}(v)$ ($EFS \neq \emptyset$).*

Corollary 1 *An economy $\mathcal{E}(v,p)$ is asymptotically efficient (i) if the economy is without negative externality, or (ii) if there exists at least one efficient state where unanimity is required to move to any other state.*

[38] Strictly speaking, the efficient state a should be prone to *negative* (and not necessarily *positive*) externalities, as $v_i(b) > v_i(a)$ is allowed in the definition.

[39] If one adds a transitivity axiom (i.e. for any three states b, c, d and coalition S, $b \to_S c$ and $c \to_S d$ imply that $b \to_S d$), then the definition can be simplified to consider only direct transitions from a to b.

In the definition of an efficient externality-free state we allow for positive externalities. Thus, Proposition 6 shows that efficiency obtains even if leaving the efficient state a to some sub-optimal state b can be done without the consent of player i, as long as player i is not worse off (in terms of flow of payoffs) in the new state b relative to the original state a. Our analysis will show that negative externalities and the possibility of exclusion are the essential features that are responsible for potential inefficiencies (see more on the effect of positive externalities below).

It is worth pointing out that we cannot weaken the definition of EFS to consider only states that are reachable in a one step deviation instead of multiple steps, if we wish to guarantee asymptotic efficiency.[40]

Proposition 6 and Corollary 1 have several practical implications.

Corollary 1 (i) can be used to establish asymptotic efficiency in exchange economies without externalities (see subsection 3.1), which is reminiscent of Gale (1986). It can also be used to establish asymptotic efficiency in coalitional bargaining games (see subsection 3.2) with characteristic function forms (see also Seidmann and Winter (1998)).

Corollary 1 (ii) can be used to establish asymptotic efficiency in coalitional bargaining games with externalities where the grand coalition is efficient (see also Gomes (2001)).

The next Example illustrates the scope of Proposition 6 when Corollary 1 does not apply (because there are negative externalities and one can move away from the efficient state without unanimous consent):

Example 5 *Consider the coalitional bargaining game with partition function*

$v_i(a)$	*[123]*	*[12][3]*	*[13][2]*	*[23][1]*	*[1][2][3]*
1	*1*	*3*	*2*	*2*	*0*
2	*1*	*2*	*2*	*0*	*0*
3	*1*	*-1*	*-2*	*1*	*0*

.

It is readily verified that the efficient state [12][3] is EFS, and therefore Proposition 6 guarantees asymptotic efficiency. Note that the economy above is not without negative externality (since 1 and 3 can move from [1][2][3] to [13][2] and 2 is worse off when 1 and 3 form a coalition), and moreover, unanimity is not required to move away form state [12][3].

Applying Proposition 6 to general coalitional bargaining games, we get:

[40] Say, that a is an efficient state satisfying $a \rightarrow_S b$ implies $v_i(b) \geq v_i(a)$ for all $i \notin S$. The following example illustrates that this restriction would not imply efficiency. Example: $N = \{1, 2, 3\}$, $Z = \{a, b, c\}$, and $v(a) = (0, 0, 3)$, $v(b) = (-1, -1, 4)$, $v(b) = (1, 1, 0)$ and $a \rightarrow_{12} b$, $b \rightarrow_{12} c$, and $c \rightarrow_{123} a$.

Corollary 2 *Consider a coalitional bargaining game with partition function where there exists an efficient coalition structure* π^* *satisfying* $v_S(\pi) \leq v_S(\pi^*)$ *for all* $S \in \pi^*$ *and partitions* π *with* $S \in \pi$. *Then* π^* *is an efficient externality-free state, and the coalitional bargaining game is asymptotically efficient.*

In Examples 1, 2 and 3 above (see Section 4), we identified circumstances under which asymptotic inefficiencies occurred in equilibrium (either due to cycles in Examples 1 and 2 or to convergence to an inefficient state in Example 3). In all three examples, the reason for the inefficiency is the non-existence of an efficient externality-free state. That is, asymptotic inefficiencies occur because by deviating from the efficient state a subgroup may impose negative externalities on excluded players.

Discussion of Assumptions:

Assumption A3 which states that one can always move in one single step to another state with unanimous consent cannot be dispensed with in Propositions 5 and 6. Suppose that one weakens the assumption allowing for a move in multiple steps from any state to any other state.[41] The next example illustrates that even if there is an EFS state the economy may not be asymptotically efficient.

Counter-example 1 *(Necessity of assumption A3) Consider a coalitional bargaining game with payoffs*

$v_i\,(a)$	[1][2][3]	[12][3]	[13][2]	[23][1]	[123]
1	0	4	4	0	3
2	0	4	0	4	3
3	0	0	4	4	3

where the only change with respect to the transition rules specified in subsection 3.2 is that it is no longer possible to move directly from state [ij][k] to state [123] (although it is possible to move in two steps: [ij][k]$\rightarrow_{ij}$*[1][2][3]*$\rightarrow_{123}$*[123]). Consider that each player has an equal probability of being the proposer and that they are very patient.*

Note that assumption A3 does not hold (but all other assumptions are maintained) and the efficient state [123] is an EFS state. The equilibrium transitions in this economy can be described as follows. From state [1][2][3], player i will move to a state of the form $[ij][k]$ (i.e. he will form a coalition with one of the two other players) with no further move afterwards;

[41] Formally, this corresponds to: For all $a, b \in Z$ there exists sequences $x_k \in Z$ and $S_k \subset N$ such that: $a \rightarrow_{S_1} x_1 \rightarrow \cdots \rightarrow_{S_k} x_k = b$. (It is always possible to move from state a to state b in a finite number of steps.)

and from the grand coalition state [123], there is no further move. So there are four stable sets in this equilibrium corresponding to the three pairwise coalitions and the grand coalition. We do not have asymptotic efficiency as the system may stabilize to one of the pairwise coalition states, which are suboptimal.

The reason for the inefficiency here is that it is not possible to move from an inefficient pairwise coalition to the grand coalition directly, and the transition to the disaggregated state [1][2][3] would involve a loss to the original pairwise coalition (as it would be too favorable to the third player).

We have already noted that if we assume A3' that any two states can communicate in one step (i.e., for all a, b there exists S such that $a \to_S b$) then A2 implies A3 (so A2 and A3 are equivalent to A2 and A3'). We now show that relaxing A2 while maintaining A3' invalidates the results in Propositions 5 and 6. The main reason why assumption A2 is important in order to achieve efficiency is that it facilitates a move to efficient states in that proposers are able to extract from other players any gains that such move entails. The following war of attrition example illustrates this point.

Counter-example 2 *(Necessity of assumption A2) Consider the following war of attrition game with payoffs*

$v_i(\cdot)$	a	b	c
1	*0*	*1*	*2*
2	*0*	*2*	*1*

where the transition rule is as follows: $a \to_1 b$, $a \to_2 c$, *and* $b \to_{12} x$, $c \to_{12} x$ *and* $x \to_i x$ *for* $x = a, b, c$ *and* $i = 1, 2$. *Consider that players have an equal probability of being proposers and have discount rate* δ.

Note that assumption A2 does not hold since the moves $a \to_{12} b$ and $a \to_{12} c$ are not feasible (assumptions A1 and A3' are maintained though). We now show that despite the fact that states b and c are EFS states, the economy above have equilibria that are not asymptotically efficient. It is clear that in any equilibrium, starting from states b or c the payoffs of players are given by $v(b)$ and $v(c)$, and thus we only need to analyze the strategy profiles at state a. Consider a symmetric strategy profile where both players randomize whether to give in or not whenever they are proposing (i.e. $\sigma_1(a)(b) = \sigma_2(a)(c) = p$ and $\sigma_i(a)(a) = 1 - p$). A necessary and sufficient condition for the strategy profile above to be an MPE is that

$$\begin{aligned} x_i(a) &= \delta\left((1-p)\,x_i(a) + \frac{p}{2}1 + \frac{p}{2}2\right), \\ x_i(a) &= 1, \end{aligned}$$

which implies that $p = \frac{2(1-\delta)}{\delta}$, and thus for $\delta \geq \frac{2}{3}$ (probability $p \leq 1$) there is an equilibrium with the strategy profile above. Note that the aggregate welfare $\phi_N(a)$ at state a is given by $x_N(a) = \delta\phi_N(a) + (1-\delta)\, v_N(a)$, and thus $\phi_N(a) = \frac{2}{\delta}$. So when $\delta \to 1$ the aggregate welfare converges to $\phi_N(a) = 2 < 3$, and thus the economy is not asymptotically efficient.[42] Note that the equilibrium above has two ergodic classes: stable states b and c (and these ergodic classes are robust to parameter perturbations).

5.2 The Role of Proposer's Probabilities

In this part, we discuss the role of proposer's probabilities. We start with an illustrative example.

Example 6 *Consider the game where the only possible moves are $a \to_1 b$, $a \to_{12} b$, and $b \to_{12} a$ and the payoffs are*

$v_i(\cdot)$	a	b
1	*1*	x
2	*1*	*0*

We assume that both players may propose with positive probability in state a and we let p be the probability that player 1 is the proposer at state b, and let δ be the discount rate.

What are the conditions under which the equilibrium outcome is asymptotically efficient?

It might seem that as long as $x < 1$, player 1 will not be willing to move from state a to state b (even though he may) because such a move would hurt him (in addition to hurting player 2), and thus such a move might seem non-credible. However, this argument ignores the possibility that player 1 may have a strong bargaining position in state b, thus providing player 1 with a payoff greater than 1 in state b. This turns out to be the case when the probability that player 1 is the proposer at state b is sufficiently large.

Formally, let

$$\begin{aligned} \phi_1(b) &= x + p(2-x) \\ \phi_2(b) &= (1-p)(2-x) \end{aligned}$$

be the payoffs that players 1 and 2 would obtain in expectation at state b if player 1 (resp. 2) were to make the proposal with probability p (resp. $1-p$) and the proposal could only

[42] The war of attrition game though have two other asymmetric equilibrium in which one player immediately gives in resulting in an efficient outcome (see also Gomes (2001)).

consist of a move to state a (with no further moves allowed). In such a setup, the condition for the stability of state a is

$$v_1(a) \geq \delta\phi_1(b) + (1-\delta)\, v_1(b)\,,$$

and thus a is stable if and only if

$$p \leq \frac{1-x}{(2-x)\delta},$$

(the ergodic classes are $\{a\}$ and $\{a,b\}$ if p is, respectively, smaller or greater than $\frac{1-x}{(2-x)\delta}$).

To summarize, the example illustrates that the inefficiencies are driven by the asymmetry of payoffs and/or bargaining power at states that can be reached from the efficient state. It also shows that the search for stronger bargaining positions may cause distortions in the allocation of resources. However, as we have demonstrated in proposition 6 the bargaining power of players create no distortions if there exists at least one efficient externality-free state (which is not the case in Example 6).

The following proposition shows that the existence of an EFS state is a necessary condition for asymptotic efficiency to hold irrespective of the proposers' probabilities.

Proposition 7 *Any v-economy $\mathcal{E}(v)$ that has no efficient-externality-free-state ($EFS = \emptyset$) is such that there exist (an open set of) proposers' probabilities p such that $\mathcal{E}(v', \delta, p)$ is not asymptotically efficient, for almost all v' in a neighborhood of v.*

Consider an economy with no efficient externality-free state. That is, coalition S may move (possibly in several steps) from the efficient state a to a suboptimal state b such that player $i \notin S$'s flow of payoff is lower at state b than at state a. Proposition 7 shows that for some specifications of p, asymptotic inefficiencies may occur. In fact, inefficiencies will occur when the probabilities that player i is the proposer at state b is sufficiently small. Under such circumstances, state a will not be stable because coalition S would rather leave state a to move to state b in order to exploit the weak bargaining position of player i at state b.

Remark: In proposition 7 it is important to consider perturbations of the payoff. For example, if all states of the economy are efficient, then even though there may be no EFS, the economy is certainly asymptotically efficient.

5.3 Strongly Efficient Equilibria

Strong efficiency requires that any player who has a chance to make an offer at an inefficient state proposes to move to the efficient state right away (with no delay). That is the pattern that an omniscient benevolent planner would implement, and we characterize the conditions under which the players voluntarily conform to it in equilibrium.

Proposition 8 *Suppose the proposers' probabilities are state independent (i.e. $p_i(a) = p_i$ for all a). If an economy $\mathcal{E}(v, p, \delta)$ satisfies*

$$v_S(b) - v_S(a) \leq p_S(v_N(b) - v_N(a)), \tag{7}$$

for all $a \to_S b$ then it has an equilibrium σ where, starting from any state, there is an immediate move to an efficient state with unanimous agreement, and the equilibrium payoff corresponds to the Nash Bargaining solution, i.e.

$$\sigma_i(a)(N, a^*) = 1, \tag{8}$$

$$\phi_i(a) = v_i(a) + p_i(v_N(a^*) - v_N(a)), \tag{9}$$

for all $a \in Z$, $i \in N$, and a^* is an efficient state. Reciprocally, if an economy $\mathcal{E}(v, p, \delta)$ with patient players (δ arbitrarily close to 1) has an equilibrium satisfying (8) then the payoffs satisfy inequalities (7).

Strong efficiency is, of course, a much stronger requirement than asymptotic efficiency. In Example 5, where all players have an equal probability of being proposer in all states, players 1 and 3 would move from [1][2][3] to [13][2] (instead of [23][1]) in order to improve their bargaining position vis a vis player 2.

Notice that for the case where all players have an equal probability of being the proposer the condition (7) is equivalent to

$$\frac{v_S(b) - v_S(a)}{|S|} \leq \frac{v_N(b) - v_N(a)}{|N|},$$

which resembles the condition given in Chatterjee et al. (1993) and Okada (1996) for existence of a no-delay stationary equilibrium.

6 Convergence and Stability Analysis

In this Section, we analyze the effect of the initial state on the long run properties of the system (convergence). We establish in Proposition 9 that in generic economies there is no effect of the initial state on the long run properties of the system whatever the equilibrium under study. We also analyze the conditions under which a state (not necessarily efficient) can be stable and relate the condition to the core condition and the existence of an efficient externality-free state.

6.1 Convergence

In Section 5 we have shown in economies with arbitrarily patient players that the aggregate welfare must converge and cannot cycle. We now show that generically[43], any equilibrium has a unique stable set or state whenever players are sufficiently patient. Thus, the long-run behavior of the dynamic process (not only the aggregate welfare and not only in the limit of arbitrarily patient players) is (generically) the same regardless of the initial state from which the system starts. And this conclusion holds true whether the system converges to a single state or cycles between several states.

Proposition 9 *For generic economies $\mathcal{E}(v, p, \delta)$ there exists a $\overline{\delta} > 0$ such that if $\delta \geq \overline{\delta}$ all equilibria of $\mathcal{E}(v, p, \delta)$ have only one stable set (ergodic class). Therefore, the long-run properties of economies are not dependent of the initial state, if players are patient enough.*

In the context of exchange economies (with or without externalities) or the merger/dissolution application, this result means that there is no long run effect of the initial allocation of property rights whenever players are sufficiently patient, which is reminiscent of Jehiel and Moldovanu (1999) (even though applied to a broader context).

When players are not patient enough, the following example shows that there may be equilibria with multiple stable sets. The example considers the case of myopic or totally impatient players.

Example 7 *Consider a voting problem where three legislators are myopic (each have discount rate $\delta = 0$) and derive the following utility with respect to four policy choices (players have equal probability of being proposers):*

$v_i(\cdot)$	a	b	c	d
1	1.5	1	0	1
2	1	0	1.5	1
3	0	1.5	1	1

Any of the policies a, b, or c can be approved and changed by a simple majority (at least two legislators), and policy d requires unanimity to be approved and changed.

The equilibrium for this game, for $\delta = 0$, is as depicted in Figure 4.

In this example there are two stable sets: $\{a, b, c\}$ in which the system cycles between a, b, c, and the stable state d. This structure of ergodic classes prevail until the discount rate reaches $\overline{\delta} \approx 0.75$, and are robust to perturbations of payoffs.

[43]Except in a subset of the set of payoffs $v \in R^{n \cdot |Z|}$ of Lesbegue measure zero .

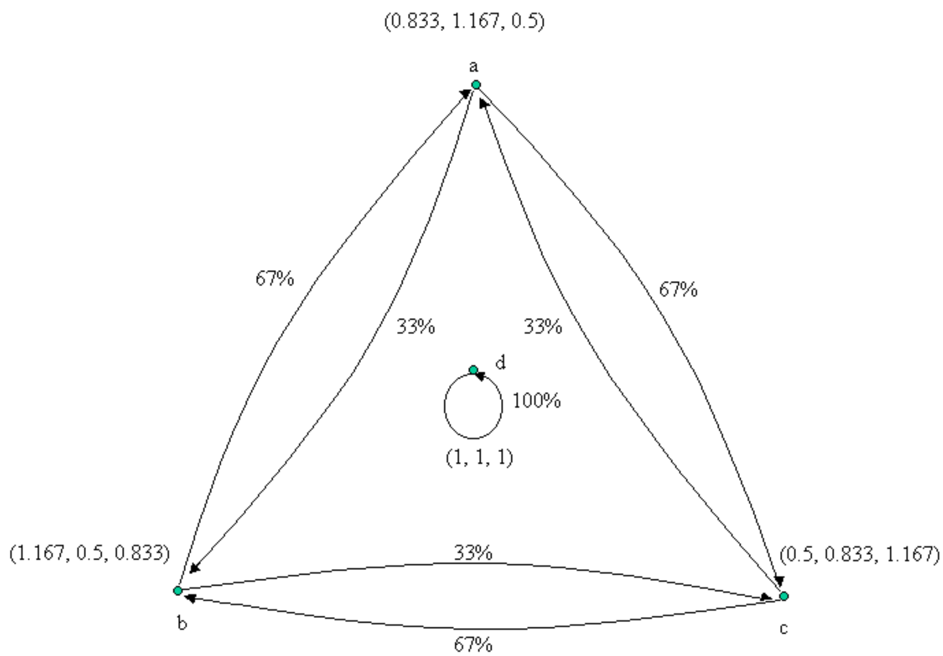

Figure 4: Equilibrium payoffs and transition probabilities for the voting game in example 7.

Notice that the process does not converge to the efficient state d, despite the fact that it satisfies the unanimity property of Corollary 1 (ii). However, if players are patient enough $(\delta > \overline{\delta})$ the stable state d is the unique limit state.

We conclude this subsection by noting that the dynamics in economies without negative externalities is particularly simple irrespective of the patience of the players. We have already noted in Corollary 1 (i) that an economy without negative externalities is asymptotically efficient (i.e. in the limit of infinitely patient players). We now extend this result to show that no matter what the patience of the players is, there is convergence to an efficient state in economies without negative externalities.

Proposition 10 *Any v-economy $\mathcal{E}(v)$ without negative externalities is such that any equilibrium of $\mathcal{E}(v, p, \delta)$ converges, in a finite number of steps, to the set of efficient states irrespective of δ and p.*

The convergence result of proposition 10 can in fact be extended for economies with externalities satisfying the unanimity property or for economies admitting efficient externality-free-states (EFS) at least when players are sufficiently patient.[44] Such convergencies are illustrated by Examples 5 and 7, which satisfy, respectively, the EFS and unanimity properties.

6.2 Stability

In this subsection we analyze the conditions under which the economy converges to a (stable) state a^* (whether or not efficient). To this end, for each economy $\mathcal{E}(v,p,\delta)$ and state a^*, we define the a^*-*restricted* economy $\mathcal{E}_{a^*}(v,\delta,p)$, obtained from $\mathcal{E}(v,p,\delta)$ by deleting all moves $a^* \to_S b$ with $S \neq N$ allowed in $\mathcal{E}(v,p,\delta)$ (while keeping $a^* \to_N b$ by A3). We have:

Proposition 11 *Let σ be an equilibrium of $\mathcal{E}(v,p,\delta)$, and assume that state a^* is a stable state of σ. Then*

$$v_S(a^*) \geq x_S(b) = \delta\phi_S(b) + (1-\delta)\, v_S(b). \tag{10}$$

*Moreover, if a^* is a stable state of some equilibrium σ of the a^*-*restricted *economy $\mathcal{E}_{a^*}(v,\delta,p)$ and if inequalities (10) hold then σ is an equilibrium of $\mathcal{E}(v,p,\delta)$ with stable state a^*.*

Remember that, as Example 3 shows, a stable state may well be inefficient. We now investigate the link between the stable states and the states lying in the core, which is formally defined by:[45]

Definition 8 (Core) *The core of v-economy $\mathcal{E}(v)$ is the set of states $C \subset Z$, where $a \in C$ if and only if for all coalitions $S \subset N$ and states $b \in Z$ with $a \to_S b$ then $v_S(a) \geq v_S(b)$.*

The relation of our concept of core to the classical definition deserves a few comments. Even tough the above definition of core corresponds to the classical concept, the application of the concept to, for example, coalitional bargaining games leads to some notable differences. For example, the characteristic function game $v(i) = 0$, $v(ij) = 8$, $v(123) = 9$ is typically referred to, in the context of cooperative game theory, as a game with empty core (no imputation $x \in R^3$ with $\sum_{i=1}^{3} x_i = 9$ satisfies $x_S \geq v(S)$ for all $S \subset N$). In our formulation of

[44] To see this, consider an efficient state $a \in Z$ that satisfies the unanimity property or the externality-free (EFS) property as defined in section 5.1. Consider now a slight perturbation of the payoffs of the economy so that the condition of proposition 9 holds. As we have seen in the proof of proposition 6, efficient states satisfying the unanimity or the externality-free (EFS) property can be sustained as stable states of any equilibrium, and thus, according to proposition 9, are unique limit states when $\delta > \overline{\delta}$.

[45] The core of the games we considered so far are: examples 1, 3, and 4 have empty core; example 2 core={d}, or equivalently, state d is a Condorcet winner; example 5 core={[12][3]}; example 6 core={a}; and example 7 core={d}.

the coalitional bargaining game (see subsection 3.2) the core of the corresponding economy is non-empty and consists of the state where the grand coalition [123] forms. The distinction comes not from the concept of core *per se* but from the transition relation that specifies the possible moves by coalitions. Implicit in the cooperative interpretation, is the view that a subcoalition can break-up from a coalition without the consent of the players left behind (see also discussion in Section 3.2). But this view is inconsistent with the idea that once players have agreed to form a coalition it cannot be re-organized without the consent of all players in the coalition. We believe that in an explicit dynamic setup in which players have agreed to form the grand coalition [123] it makes more sense (at least in some contexts) to assume that breaking that grand coalition apart requires the consent of all three players.

Clearly, the conditions expressed in inequalities (10) bear some resemblance with the conditions for the Core (as just defined) whenever $\delta = 0$. The following Proposition formalizes the intimate relationship between absorbing states and the core when players are myopic. It also shows that whenever the core is empty there must be cycles in economies with myopic players.

Proposition 12 *Any economy $\mathcal{E}(v, p, 0)$ where players are myopic ($\delta = 0$) satisfies:*
(i) the stable states (absorbing states) are contained in the core[46]*;*
(ii) if the core is empty all equilibria of $\mathcal{E}(v, p, 0)$ have cycles.

Non-myopic or farsighted players though, when considering deviations from a state, put weight δ on the expected equilibrium value (which incorporates the expected value of all future payoffs) of the deviant coalition in the new state (see inequality 10), as opposed to putting all the weight on the coalitional value in the new state (which is the case in the definition of the core).

As our examples illustrate the relationship between stability and the core, and the existence of cycles and emptiness of the core break-down in farsighted situations. In example 3, [123] is an (inefficient) stable state and there are no cycles despite the fact that the core is empty, and in examples 2 and 6 the core is non-empty but there are no stable states and the economy cycles.

We use the following example to provide some further intuition for why the notion of core is not useful to capture stability in farsighted economies, as well as to illustrate that an economy may have an empty core and still have an efficient state that is absorbing.

[46]Moreover, if there is a core state a such that $v_S(b) < v_S(a)$ for any state b and $S \subset N$, then the core is the unique stable state.

Example 8 *Consider a situation with two patient players with the following payoffs*

$v_i(\cdot)$	a	b	c
1	*1*	*1.5*	*0*
2	*1*	*0*	*1.5*

where $a \to_1 b$, $b \to_2 c$, *and all* $x \to_{12} y$ *for all* $x, y \in Z$ *(both players are proposers with equal probability in all states).*

While state a (which is efficient) is not in the core (because 1 can move to state b, which he prefers), it is a stable state. This is so because player 1 knows that if he moves from state a to state b player 2 will then move to state c which would be very unfavorable to player 1.

So when players are farsighted, the core concept does not capture the long run stability property of our dynamic processes of social and economic interactions.

When stability is required *irrespective* of the proposers' probabilities, it turns out that the notion of efficient externality-free state (as introduced earlier) is key to the analysis of stability:

Proposition 13 *If state* a^* *is not an externality-free-state of* $\mathcal{E}(v, \delta, p)$ *then there exists proposers probability* p *and* $\overline{\delta} > 0$ *such that if* $\delta \geq \overline{\delta}$ *state* a^* *is not a stable state of any equilibrium of* $\mathcal{E}(v, \delta, p)$*. Reciprocally, if state* a^* *is an efficient externality-free-state of* $\mathcal{E}(v)$ *then* a^* *is a stable state of any equilibrium of* $\mathcal{E}(v, \delta, p)$ *for all* p *and* δ*.*

The intuition for this result is as follows. For a state to be stable *irrespective* of the proposer's probabilities it has to be efficient. (Otherwise, there would be a player who is better off in the efficient state than in the candidate stable state, and by making this player the proposer with a probability close to 1 in all states, we would get a contradiction.) Also, an efficient state can be stable irrespective of the proposer's probability only if it is externality-free. (Otherwise, choose the proposer's probabilities such that a player who is suffering the negative externality from the move away from the candidate efficient stable state is the proposer at the deviating state with a very small probability).

Interestingly, if we consider a v-economy $\mathcal{E}(v)$ that does not have any efficient externality-free-state ($EFS = \emptyset$), then it is easy to see that there exist a proposer's probability p and $\overline{\delta} > 0$ such that if $\delta \geq \overline{\delta}$ no equilibrium of $\mathcal{E}(v, \delta, p)$ has stable states, and thus all solutions are cyclical (note the similarity of this result and proposition 12).

Finally, it should be noted that whenever there exists an efficient externality-free state, the core (as defined above) is non-empty and contains that efficient state.[47] However, the non-emptiness of the core does not guarantee the existence of an EFS. Thus, when stability is to be obtained *irrespective* of the proposer's probability a more stringent condition than the non-emptiness of the core is required; that is,–the existence of an externality-free state.

7 Conclusion

This paper has made two important points regarding the efficiency and convergence analysis of dynamic processes of social and economic interactions. First, there is no effect of the initial state on the long run properties of the economy if players are sufficiently patient. Second, if efficiency and stability (of a single state) are to be obtained irrespective of the proposers' probabilities (with the idea that there is little control over these probabilities), then it is indispensable to have an efficient state that is externality-free (in the sense that if a coalition can move away from that state to some alternative state without the consent of some agent then this agent should get a flow of payoff at least as large in the new state than in the original (efficient) state). Imposing that one can move from the efficient state only with unanimous consent is a simple way to guarantee the existence of an EFS, but there are many other possible transition mappings that allow for such a property. Applications of these insights to contract theory should be the subject of future research.

[47]Suppose that $a \in EFS$ and consider any $a \to_S b$. Then $v_N(a) \geq v_N(b)$ and $v_{N \setminus S}(b) \geq v_{N \setminus S}(a)$ (from EFS property). Concatenating the two inequalities we have that $v_S(a) \geq v_S(b)$, which shows that $a \in core$.

A Appendix

Proof of Proposition 1: The necessary part follows directly from the discussion before the statement of the result and the definition of *MPE* solution. Let us prove the sufficient part of the theorem. Suppose that the strategy profile σ satisfies all the conditions of the lemma. We use the one-stage deviation principle for infinite-horizon games. This result states that in any infinite-horizon game with observed actions that is continuous at infinity, a strategy profile σ is subgame perfect if and only if there is no player i and strategy σ_i' that agrees with σ_i except at a single stage t of the game and history h^t, such that σ_i' is a better response to σ_{-i} than σ_i conditional on history h^t being reached (see Fudenberg and Tirole (1991)).Q.E.D.

Proof of Proposition 2: Let the map $f : R^d \times \Sigma \to R^d$ be defined as

$$\begin{aligned} f_i(a)(x,\sigma) &= (1-\delta)v_i(a) + \\ &+\delta \left(\begin{array}{c} p_i(a) \sum_{(b,S)} \sigma_i(a)(S,b) \left(\sum_{j\in S}(x_j(b) - x_j(a))\right) + \\ + \sum_{j\in N} p_j(a) \sum_{(b,S)} \sigma_j(a)(S,b)\left(I(i\in S)x_i(a) + I(i\notin S)x_i(b)\right) \end{array} \right), \end{aligned}$$

for $(x,\sigma) \in R^d \times \Sigma$. Consider the set

$$\Sigma_i(a) = \{(S,b) : \text{ where } a \to_S b \text{ and } i \in S\}$$

and $\sigma_i(a) \in \Delta^{\Sigma_i(a)}$ be the set of probability distributions over $\Sigma_i(a)$. Let

$$\Sigma_i = \underset{a\in Z}{\times} \Sigma_i(a),$$

and Σ be the set of offering strategies for all player.

Let the correspondence $F : R^d \to\to R^d$ be defined as

$$F(x) = \{f(x,\sigma) : \sigma \in \Sigma(x)\},$$

where

$$\Sigma(x) = \{\sigma \in \Sigma : \ supp(\sigma(i,a)) \subset \underset{(b,S)}{\arg\max} \{e(a)(S,b)(x) : a \to_S b \text{ and } i \in S\}.$$

According to proposition 1 a payoff $x \in R^d$ is a MPE if and only if x is a fixed point of F.

(1) Let $X \subset R^m$ be a compact and convex set defined where the coordinate

$$\min_{a\in Z} v_i(a) \le x_i(a) \le \max_{a\in Z} v_N(a) - \sum_{j\in N} \min_{a\in Z} v_j(a)$$

and $X = \times_{i\in N} I_i$. It is immediate that $F(X) \subset X$.

(2) $F(x)$ is a convex (and non-empty) set for all $x \in X$: Say that $z, z' \in F(x)$ with $z = f(x,\sigma)$ and $z' = f(x,\sigma')$ where $\sigma, \sigma' \in \Sigma(x)$. Then, for any $\lambda \in [0,1]$, $\lambda z + (1-\lambda) z' = f(x, \lambda\sigma + (1-\lambda)\sigma') \in F(x)$ because $\lambda\sigma + (1-\lambda)\sigma' \in \Sigma(x)$ ($\Sigma(x)$ is convex).

(3) F is u.h.c., that is, for any sequence $(x^n, f(x^n, \sigma^n)) \to (x, z)$ with $\sigma^n \in \Sigma(x^n)$ then $z \in F(x)$ (i.e., there exists an $\sigma \in \Sigma(x)$ such that $f(x,\sigma) = z$). The sequence (σ^n) belongs to Σ a compact subset of a finite-dimension Euclidean space. Therefore, there exists a subsequence of (σ^{n_k}) that converges to $\sigma \in \Sigma$. Rename this subsequence as (σ^n) for notational simplicity. We have that $\sigma_i^n(a)(b,S) \to \sigma(i,a)(b,S)$, and that $f(x^n, \sigma^n) \to f(x,\sigma)$, due to the continuity of y, and thus $z = f(x,\sigma)$.

It is sufficient to show that $\sigma \in \Sigma(x)$. By the definition of $\Sigma(x)$, $\sigma \in \Sigma(x)$ if and only if $\sigma \in \Sigma$ and $\sigma_i(a)(b,S) = 0$ for all (b,S) such that

$$x_S(b) - x_S(a) < \max_{(b,S)} \{e(a)(S,b)(x) : a \to_S b \text{ and } i \in S\}.$$

Consider any $S \subset \pi$ for which the inequality above holds. By continuity, we have that there exists a large enough n_0 such that for all $n \geq n_0$,

$$x_S^n(b) - x_S^n(a) < \max_{(b,S)} \{e(a)(S,b)(x^n) : a \to_S b \text{ and } i \in S\}.$$

But since $\sigma^n \in \Sigma(x^n)$, this implies that $\sigma_i^n(a)(b,S) = 0$, and $\sigma_i(a)(b,S) = 0$.

Since all the conditions for the Kakutani fixed point theorem holds, the correspondence F has a fixed point, which yields an MPE. Q.E.D.

Proof of Proposition 3 and 4: Let $e_i(a) = \lim_{\delta \to 1} e_i^{(\delta)}(a)$. Taking the limit of the expressions in equation (5) we have that

$$x_i(a) = p_i(a)e_i(a) + \sum_{(S,b)} \mu(a,b,S)\left(I(i \in S)x_i(a) + I(i \notin S)x_i(b)\right) \tag{11}$$

where $\mu(a,b,S) = \sum_{j \in N} p_j(a)\sigma_j(a)(S,b)$. We have already seen that $x_i(b) \leq x_i(a)$ for all $b \in Z$ such that $\mu(a,b,S) > 0$ and $i \notin S$ (see remarks after the statement of proposition 1).

Let player i be a proposer with positive probability in at least one state in E (i.e, $p_i(a') > 0$ for some $a' \in E$). Let x be the limit solution and μ the limit transition probability. For any player i let a^* be a state where the $\min_{a' \in E}\{x_i(a')\}$ is attained ($x_i(a^*) = \min_{a' \in E}\{x_i(a')\}$) and thus $\mu(a^*, b, S) > 0$ implies that $x_i(b) \geq x_i(a^*)$ (because E is a closed class, and $\mu(a,b,S) > 0$ and $a \in E$ implies that $b \in E$).

Suppose that $p_i(a^*) > 0$. Applying equation (11) to state a^*, and taking into account that $\sum_{(S,b)} \mu(a^*, b, S) = 1$, we get that

$$x_i(a^*) = p_i(a^*)e_i(a^*) + x_i(a^*), \tag{12}$$

which implies $e_i(a^*) = 0$, since $p_i(a^*) > 0$.

Now if $p_i(a^*) = 0$, then any state b such that $\mu(a^*, b, S) > 0$ is also such that $x_i(b) = \min_{a' \in E}\{ x_i(a')\}$ (this is so because $x_i(b) \leq x_i(a)$ for all $b \in Z$ such that $\mu(a, b, S) > 0$). Thus, one can find a state a^{**} such that $x_i(a^{**}) = \min_{a' \in E}\{ x_i(a')\}$ and $p_i(a^{**}) > 0$, and use the same argument above to this state.

Consider now proposition 4.

(i) If $p_i(a)e_i(a) = 0$ then if player i is the proposer ($e_i(a) = 0$) his payoff is $\phi_i^j(a) = \phi_i(a) + e_i(a) = \phi_i(a)$ (note that in the limit $\phi_i(a) = x_i(a)$); if player j is the proposer then i's payoff is $\phi_i^j(a) = \sum_{(S,b)} \sigma_j(a)(S, b)(I(i \in S)x_i(a) + I(i \notin S)x_i(b))$. But since equation (11) corresponds to $x_i(a) = \sum_{(S,b)} \mu(a, b, S)(I(i \in S)x_i(a) + I(i \notin S)x_i(b))$ and $x_i(b) \leq x_i(a)$ then $x_i(b) = x_i(a)$, and thus $\phi_i^j(a) = \phi_i(a)$.

(ii) If $p_i(a)e_i(a) > 0$ then in the event that i is the proposer (which happens with positive probability) he gets $\phi_i^j(a) = \phi_i(a) + e_i(a) > \phi_i(a)$. But equation (11), $p_i(a)e_i(a) > 0$, and $x_i(b) \leq x_i(a)$ imply that there must exists $\mu(a, b, S)$ with $i \notin S$ such that $x_i(b) < x_i(a)$. Therefore there exists a proposer j that proposes an acceptable move to go b where the payoff of player i is $\phi_i^j(a) = \phi_i(b) < \phi_i(a)$. Q.E.D.

Proof of Proposition 5: Suppose by contradiction that there exists a subsequence $\delta_n \to 1$ such that $\lim_{\delta_n \to 1} \max |\phi_N^{(\delta_n)}(a) - \phi_N^{(\delta_n)}(b)| : a, b \in Z\} > 0$, where $\mu^{(\delta_n)}$ and $\phi^{(\delta_n)}$ are the equilibrium transition probabilities and payoffs. Now, consider a convergent subsequence of δ_n such that $\mu^{(\delta_n)} \to \mu$ and $\phi^{(\delta_n)} \to \phi$ (of course, $\max_{a,b \in Z} |\phi_N(a) - \phi_N(b)| > 0$).

The aggregate value $\phi_N^{(\delta)}(\cdot)$ satisfies

$$\phi_N^{(\delta)}(a) = \sum_b \mu^{(\delta)}(a, b)\left[(1 - \delta)v_N(b) + \delta\phi_N^{(\delta)}(b)\right], \text{ for all } a \in Z,$$

which is equivalent to

$$\left[I - \delta\mu^{(\delta)}\right]\phi_N^{(\delta)} = (1 - \delta)\mu^{(\delta)}v_N. \tag{13}$$

Taking the limit when $\delta \to 1$ we have,

$$[I - \mu]\phi_N = 0. \tag{14}$$

Let $E_1, \cdots, E_m$ be the ergodic classes and $T \subset Z$ the class of transient states of the limit transition probability μ ($Z = E_1 \cup \cdots \cup E_m \cup T$). Equation (14) is equivalent to $\phi_N = \mu\phi_N$ and thus ϕ_N is an (right) eigenvector of μ corresponding to the eigenvalue 1. A well-known result from the theory of Markov chains (see Doob (1953)) implies that $\phi_N(\cdot)$ is a constant within each ergodic class E_j and that the value of $\phi_N(\cdot)$ at any transient state is a linear combination of the values of $\phi_N(\cdot)$ at the ergodic states.

By Proposition 3 there a exists state a_j in each ergodic class E_j such that the excess $e_i^{(\delta_n)}(a_j)$ of player i at state a_j converges to zero: $e_i^{(\delta_n)}(a_j) \to 0$, for all $j = 1, ..., m$.

We now show that the values of $\phi_N(\cdot)$ across ergodic classes are equal: say that there are two ergodic classes E_j and E_k such that $\phi_N(a_j) < \phi_N(a_k)$. But since it is feasible for player i to move from state a_j to state a_k with the agreement of all players N then $\limsup_{\delta_n \to 1} e_i^{(\delta_n)}(a_j) \geq \phi_N(a_k) - \phi_N(a_j) > 0$ (contradiction). Also, because the value of $\phi_N(\cdot)$ at transient states is a linear combination of the values at ergodic states, then we conclude that $\phi_N(\cdot)$ is constant across all states in Z. Finally, this leads to a contradiction with $\max_{a,b \in Z} |\phi_N(a) - \phi_N(b)| > 0$, completing the proof. Q.E.D.

Proof of Proposition 6: We first show that economies is without negative externality (i), the unanimity property (ii), and the EFS property (iii), imply that there exists a state a^* such that $\phi_N(a^*) = \max_{a \in Z} v_N(a)$.

(i) If the economy is without negative externality then

$$\phi_i(a) \geq v_i(a) \text{ for all } a \in Z \text{ and } i \in N : \tag{15}$$

an utility level at least equal to $v_i(a)$ can be achieved by player i if he does not make any proposals and if he does not accept any proposals due to the no-negative-externality assumption (whenever $a \to_{S_1} \cdots \to_{S_k} a_k \to \cdots \to_{S_n} a_n$ and $i \notin S_k$ then $v_i(a_n) \geq \cdots \geq v_i(a_k) \geq \cdots \geq v_i(a)$). Therefore, if a^* is an efficient state then $\phi_N(a^*) \geq v_N(a^*)$ and thus $\phi_N(a^*) = \max_{a \in Z} v_N(a)$.

(ii) If a^* is an efficient state where unanimous agreement is needed to move to any other state then $a^* \in EFS$. We thus have (i)$\Longrightarrow$(ii)$\Longrightarrow$(iii).

(iii) Let $a^* \in EFS$. We claim that $\phi_i(a^*) \geq v_i(a^*)$ for all $i \in N$. Indeed any player i can get an utility level at least equal to $v_i(a^*)$ if he does not make any proposals and if he does not accept any proposals (whenever $a^* \to_{S_1} \cdots \to_{S_k} a_k \to \cdots \to_{S_n} a_n$ and $i \notin S_k$ then $v_i(a_n) \geq v_i(a^*)$). Therefore, if $a^* \in EFS$ then $\phi_N(a^*) \geq v_N(a^*)$ and thus $\phi_N(a^*) = \max_{a \in Z} v_N(a)$. Moreover, $\phi_i(a^*) = v_i(a^*)$.

Now suppose there exists a state a^* such that $\phi_N(a^*) = \max_{a \in Z} v_N(a)$. By proposition 5 if players are patient enough ($\delta \geq \overline{\delta}$) then $\min_{a \in Z} \phi_N(a) \geq \phi_N(a^*) - \varepsilon$ for any given $\varepsilon > 0$, which implies that $\mathcal{E}$ is asymptotically efficient. Q.E.D.

Proof of Corollary 1: Let a^* be an absorbing state. Then $e_i(a^*) = 0$ for all i, and because $\mu(a^*, a^*) = 1$ equation (5) correspond to

$$x_i(a^*) = \delta x_i(a^*) + (1 - \delta)\, v_i\,(a^*)\,.$$

Thus $x_i(a^*) = v_i\,(a^*)\,$, and $\phi_N(a^*) = x_N(a^*) = v_N(a^*)$. Since a^* is an efficient state then $\phi_N(a^*) = \max_{a \in Z} v_N(a)$. The result now follows from proposition 6. Q.E.D.

Proof of Proposition 7: Consider any small perturbation v' of the payoff v such that the new payoff v' has only one efficient state, say a. Of course, for any small enough perturbation, a is also an efficient state of $E(v)$, and since $a \notin EFS(v)$ then there must exist a player i and a state b such that $a \rightarrow_{S=N \backslash i} b$ and $v_i(b) < v_i(a)$. Note that any small perturbation also satisfies $v_i'(b) < v_i'(a)$ and $v_N'(b) < v_N'(a)$ (a is the only efficient state of $E(v')$). Consider now any proposer probability p such that $p_i(b)$, the probability that i is proposer at b, is close enough to zero (for simplicity, say that $p_i(b) = 0$).

We now show that the economy $E(v', p)$ is not asymptotically efficient. Suppose to the contrary that it is asymptotically efficient, so that any sequence $\left(\phi^{(\delta)}, \sigma^{(\delta)}\right)$ with $\phi^{(\delta)} \rightarrow \phi$ and $\mu^{(\delta)} \rightarrow \mu$ the payoffs satisfy $\phi_N(\cdot) = \phi_N(a) = v_N'(a)$. Since a is the only efficient state of $E(v')$ and $\phi_N(a) = v_N'(a)$ then $\mu\,(a, a) = 1$, which implies that $v_{N \backslash i}'(a) \geq \phi_{N \backslash i}(b)$ (otherwise the excess from moving away from state a would be positive). But $v_N'(a) = \phi_N(b)$ and thus $v_i'(a) \leq \phi_i(b)$. Also, because player i is not proposer at b then i's excess at b is zero, and equation 5 implies that $\phi_i(b) \leq v_i'(b)$. Concatenating the last two inequalities yields, $v_i'(a) \leq v_i'(b)$, a contradiction. Q.E.D.

Proof of Proposition 8: Let us first show that if conditions (7) hold then the pair (ϕ, σ) proposed is a MPE. Note that $\phi_N\,(a) = v_N\,(a^*)$ and the excesses are,

$$e_i(a) = x_N\,(a^*) - x_N\,(a) = \delta\,(\phi_N\,(a^*) - \phi_N\,(a)) + (1 - \delta)\,(v_N\,(a^*) - v_N\,(a))\,,$$

which simplifies to

$$e_i(a) = (1 - \delta)\,(v_N\,(a^*) - v_N\,(a))\,. \tag{16}$$

We now show that both equations (5) and inequalities (2) hold, and thus, by Proposition 1, (ϕ, σ) is a MPE:

(i) Equations (5) correspond to

$$x_i(a) = \delta p_i e_i(a) + \delta x_i(a) + (1 - \delta)\, v_i\,(a)\,,$$

which hold given the expressions for the excesses and the payoffs $x_i(a)$,

$$x_i(a) = v_i(a) + \delta p_i (v_N(a^*) - v_N(a)); \tag{17}$$

(ii) Inequalities (2) are equivalent to

$$e(a)(S,b) = x_S(b) - x_S(a) \leq e_i(a) = (1-\delta)(v_N(a^*) - v_N(a))$$

for all $a \to_S b$. But from equation (17) these inequalities are equivalent to

$$x_S(b) - x_S(a) = v_S(b) - v_S(a) - \delta p_S (v_N(b) - v_N(a)) \leq (1-\delta)(v_N(a^*) - v_N(a)). \tag{18}$$

But inequality (7) is equivalent to

$$v_S(b) - v_S(a) - \delta p_S (v_N(b) - v_N(a)) \leq (1-\delta) p_S (v_N(b) - v_N(a)), \tag{19}$$

and, because $a^* \in ES$ and $p_S \in [0,1]$, then

$$p_S (v_N(b) - v_N(a)) \leq v_N(a^*) - v_N(a) \tag{20}$$

always hold. These two inequalities jointly imply that inequalities (18) hold, which completes the proof of the first part of the proposition.

Reciprocally, consider an economy with MPE σ satisfying (8). By a similar argument as above, inequalities (18) must hold. Suppose now, by contradiction, that there is a $a \to_S b$ with

$$(v_S(b) - v_S(a)) > p_S (v_N(b) - v_N(a)).$$

It is easy to verify that there exists a $\overline{\delta}$ close enough to one such that inequality (18) is violated for all $\delta \geq \overline{\delta}$ (contradiction). Q.E.D.

Proof of Proposition 9: Suppose by contradiction that there is a sequence of δ_n converging to one ($\delta_n \to 1$) with equilibrium transition probability $\mu^{(\delta)}$ and payoff $\phi^{(\delta)}$ having two ergodic classes $E_1^{(\delta)}$ and $E_2^{(\delta)}$ with invariant probabilities $\lambda_1^{(\delta)}$ and $\lambda_2^{(\delta)}$ (given by $\lambda_i^{(\delta)} \mu^{(\delta)} = \lambda_i^{(\delta)}$ and $\sum_{a' \in E_i^{(\delta)}} \lambda_i^{(\delta)}(a') = 1$). Consider a convergent subsequence of δ_n (also named δ_n) such that $\mu^{(\delta_n)} \to \mu$, $\phi^{(\delta_n)} \to \phi$, $\lambda_i^{(\delta_n)} \to \lambda_i$, and $E_i = E_i^{(\delta_n)}$.

Multiplying equation (13) to the left by $\lambda_i^{(\delta)}$ yields,

$$\lambda_i^{(\delta)} \left[I - \delta \mu^{(\delta)} \right] \phi_N^{(\delta)} = \lambda_i^{(\delta)} (1-\delta) \mu^{(\delta)} v_N, \tag{21}$$

which is equivalent to $(1-\delta)\lambda_i^{(\delta)}\phi_N^{(\delta)} = \lambda_i^{(\delta)}(1-\delta)\mu^{(\delta)}v_N$, (after taking in to account that $\lambda_i^{(\delta)}\mu^{(\delta)} = \lambda_i^{(\delta)}$), and thus,

$$\lambda_i^{(\delta)}\phi_N^{(\delta)} = \lambda_i^{(\delta)}\mu^{(\delta)}v_N.$$

Taking the limit of the above expression (using $\lambda_i\mu = \lambda_i$), we have

$$\lambda_i\phi_N = \lambda_i\mu v_N = \lambda_i v_N.$$

But proposition 5 implies that $\phi_N(\cdot)$ is constant over Z, and thus

$$\lambda_1 v_N = \sum_{a'\in E_2} \lambda_i(a') v_N(a') = \sum_{a'\in E_2} \lambda_i(a') v_N(a') = \lambda_2 v_N. \qquad (22)$$

Finally, the equality $\lambda_1 v_N = \lambda_2 v_N$ cannot be satisfied generically: The two ergodic classes are disjoint $E_1 \cap E_2 = \emptyset$ and the invariant measures λ_i only depend on the payoffs $v(a)$ for $a \in E_i$. If equality (22) happened to be satisfied for some choice of parameters, changing slightly the payoffs in one of the classes (say by adding an ε to the payoff of a player in one of the classes) would lead to violation of the equality. Q.E.D.

Proof of Proposition 10: We first show that the set of efficient states is a closed set. According to item (i) in the proof of proposition 6 for any state a, $\phi_i(a) \geq v_i(a)$ for all $i \in N$. Therefore, for any $a \in ES$, $\phi_i(a) = v_i(a)$. Suppose by contradiction that in equilibrium $a \to_S b$, where b is not an efficient state. Since the move from a to b is in the equilibrium path then the excess is non-negative, i.e. $x_S(b) - x_S(a) \geq 0$, and $v_i(b) \geq v_i(a)$ for all $i \notin S$ (no-externality). Therefore, $\phi_i(b) \geq v_i(b) \geq v_i(a)$ and $x_i(b) = \delta\phi_i(b) + (1-\delta)v_i(b) \geq v_i(a)$ for all $i \notin S$, and thus, $x_{N\backslash S}(b) \geq v_{N\backslash S}(a)$. Because a is an efficient state and b is an inefficient state we have that

$$x_S(b) + x_{N\backslash S}(b) < v_N(a),$$

and, using that $x_S(a) + x_{N\backslash S}(a) = v_N(a)$,

$$x_S(b) - x_S(a) < x_{N\backslash S}(a) - x_{N\backslash S}(b) \leq 0,$$

which is a contradiction. Thus set of efficient states (core) is a closed class and thus the union of all stable sets (ergodic states).

A well-known result of the theory of Markov chains (Doob (1953)) yields that the equilibrium converges, in a finite number of steps, to ES if and only if there are no ergodic classes E containing inefficient states (i.e, $E \subset Z\backslash ES$).

Suppose now that there is an equilibrium with and ergodic class $E \subset Z \backslash ES$. We claim that $v_N(a) = v_N(b)$ for all $a, b \in E$: let $\overline{a} \in \underset{a' \in E}{\arg\max}\, v_N(a')$ and suppose that $v_N(\overline{a}) > v_N(b)$ for some $b \in E$. Inequalities (15) imply that $\phi_N(\overline{a}) \geq v_N(\overline{a})$, and since E is an ergodic class, then $\phi_N(\overline{a})$ is an average (with strictly positive weights) of $v_N(a')$ for $a' \in E$ (contradiction).

We thus have that $\phi_N(a) = v_N(\overline{a})$ for all $a \in E$, and inequalities (15) imply that $\phi_i(a) = v_i(a)$ for all $a \in E$. We now show that the excess is zero for all players at all states in E. Suppose that there is a move $a \to_S b$ with $a, b \in E$ with positive excess:

$$x_S(b) - x_S(a) = \delta\left(\phi_S(b) - \phi_S(a)\right) + (1-\delta)\left(v_S(b) - v_S(a)\right) = v_S(b) - v_S(a) > 0.$$

By the no-externality assumption $v_{N\backslash S}(b) \geq v_{N\backslash S}(a)$. But then $v_N(b) > v_N(a)$ with $a, b \in E$ (contradiction).

But if the excess is zero for all players at all states in E then E cannot be an ergodic class, because any player can move to an efficient state b from any state $a \in E$ and get a positive excess ($a \to_N b$): $x_N(b) - x_N(a) = v_N(b) - v_N(a) > 0$ (contradiction). Q.E.D.

Proof of Proposition 11: Suppose that (ϕ, σ) is equilibria of the restricted economy $\mathcal{E}(a^*)$, with a^* absorbing state. The same argument used in the proof of item (iv) of proposition 6 implies that $\phi_i(a^*) = v_i(a^*)$. By Proposition 1, in order to prove that (ϕ, σ) is equilibria of economy $\mathcal{E}$, it is sufficient to verify that equations (5) and inequalities (2) hold. But all equations and inequalities are already satisfied, with the exception of the inequalities $x_S(b) - x_S(a^*) \leq 0$ for all $a^* \to_S b$, which also hold because $v_S(a^*) \geq x_S(b)$ and $\phi_i(a^*) = x_i(a^*) = v_i(a^*)$. The reciprocal result is immediate. Q.E.D.

Proof of Proposition 12: Note that any equilibrium (ϕ, σ) of $\mathcal{E}^{(0)}$ is such that $x_i(a) = \delta\phi_i(a) + (1-\delta)\, v_i(a) = v_i(a)$.

(i) Let a^* be an absorbing state of an MPE. Then all moves $a^* \to_S b$ for all S and b must have non-positive excess $v_S(b) - v_S(a^*) \leq 0$, which implies that a^* belongs to the core.

(ii) If the core is empty then there are no absorbing states (item i). But the associated equilibrium Markov chain in the state space must have an ergodic class, and since it cannot be an absorbing ergodic class, it yields a cycle.

(iii) State a is an absorbing state because the excess from moving $a^* \to_S b$ is $v_S(b) - v_S(a^*) < 0$, and thus all players propose to remain at a, where the excess is zero. Q.E.D.

Proof of Proposition 13: Suppose that state a is an stable state (thus $v_i(a) = \phi_i(a)$) and is not an EFS state. Since $a \notin EFS$ then there exists $a \to_{S=N\backslash i} b$ such that $v_i(b) < v_i(a)$.

Note that the stability of state a implies that $v_{N\backslash i}(a) \geq \phi_{N\backslash i}(b)$ (otherwise the excess from moving away from state a would be positive). Consider now any proposer probability p such that $p_i(b)$, the probability that i is proposer at b, is close enough to zero.

Proposition 5 implies that $v_N(a) = \phi_N(b)$ and thus $v_i(a) \leq \phi_i(b)$. Also, because player i is not proposer at b then i's excess at b is zero, and equation 5 implies that $\phi_i(b) \leq v_i(b)$. Concatenating the last two inequalities yields, $v_i(a) \leq v_i(b)$, a contradiction.

Consider now the reciprocal. Suppose that $a \in EFS$. We now show that any move $a \to_S b$ by coalition S yields non-positive excess, where the excess associated with the move is $e = x_S(b) - x_S(a)$, which is sufficient to prove that a is an stable state.

We have show in Proposition 6 (item iii) that if $a \in EFS$ then $\phi_i(a) = v_i(a)$ for all $i \in N$ and the efficiency of a implies that $v_N(a) \geq v_N(b)$ and $v_N(a) \geq \phi_N(b)$ for all $b \in Z$. We also have, due to the fact that $a \in EFS$ and $a \to_S b$, that $\phi_{N\backslash S}(b) \geq v_{N\backslash S}(a)$ (this comes from the definition of EFS). Concatenating the inequalities yields that $e = x_S(b) - x_S(a) = \delta(\phi_S(b) - \phi_S(a)) + (1-\delta)(v_S(b) - v_S(a)) \leq 0$: $\phi_S(b) - \phi_S(a) \leq (\phi_S(b) - \phi_S(a)) + \left(\phi_{N\backslash S}(b) - v_{N\backslash S}(a)\right) = \phi_N(b) - v_N(a) \leq 0$ and also it is easy to see that $v_S(b) - v_S(a) \leq 0$. Q.E.D.

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