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ABSTRACT

An Exploration into Pigou's Theory of Cycles*

This Paper proposes a model of business cycles in which recessions and booms arise as the result of difficulties encountered by agents in properly forecasting the economy's future needs in terms of capital. The idea has a long history in the macroeconomic literature, as reflected by the work of Pigou (1926). The contribution of this Paper is twofold. First, we illustrate the type of general equilibrium structure that can give rise to such phenomena. Second, we examine the extent to which such a model can explain the observed pattern of US recessions (frequency, depth) without relying on technological regress. We argue that such a model may offer an explanation as to why recession appear to be driven by declines in aggregate demand even in the absence of any significant price rigidities, and may also help understand elements of the recent downturns in Asia.

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Keywords: equilibrium business cycles, expectations, recessions and technological progress

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1 Introduction

Equilibrium business cycle theory is often criticized on the ground that it does not provide a convincing theory of recession. In particular, it is well known that standard real business cycles models have difficulties explaining recessions[1]– at least of the size observed in Post War US data– without invoking technological regress[2]. This observation has lead many macroeconomists to regard equilibrium business cycle theory as incomplete for understanding recessions[3].

An alternative view, one that we favor, is that the failure of modern equilibrium business cycle theory to provide a consensus explanation of recessions may simply reflect its failure to incorporate the elements considered most important by early equilibrium macro-theorists. In particular, prior to Keynes and Keynesianism, Pigou [1926] and many others argued that recessions and booms likely arise as the result of difficulties encountered by agents

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[1] Similarly, nominal-real confusion models (see Lucas [1972]) generate persistent downturns only if agents' ability to access price and money supply information is severely limited.

[2] See Kydland and Prescott [1982] or King, Plosser, and Rebelo [1988]. A notable exception is King and Rebelo [1998], where it is shown that a one sector business cycle model can explain business cycles with "a low probability of technological regress", provided that we are in a "'high substitution economy", i.e. with large elasticity of labor supply and elastic capacity utilization.

[3] Note that over the post WWII period, almost 20% of the semesters experience strictly negative per capita output growth (see section 3 for a more detailed empirical characterization of U.S. postwar recessions).

in properly forecasting the economy's future capital needs[4]. This difficulty was seen by Pigou as being an inherent feature of an economy with technological progress. For example, if agents are optimistic about the future and decide to build up capital in expectation of future demand then, in the case were their expectations are not met, there will be a period of retrenched investment which is likely to cause a recession. The object of this paper is to offer a formalization of this idea and to explore its quantitative plausibility as a theory of recessions. A key aspect of this paper is to explore the extent to which such a mechanism can explain the depth and frequency of recessions within an equilibrium framework where technological regress never occurs.

At first glance, the idea of a business cycle model where optimism and pessimism play a dominant role may appear counter to the notion of rational expectations. However, this will not be the case in our model. In effect, we consider an environment where agents get imperfect signals about future productivity growth and use these signals to make decisions about investment; knowing that the received signals are imperfect. The notion of optimism simply refers to a state where agents receive an above average signal. In this environment, periodic recessions are most likely to arise when agents signals

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[4] This view have been recently surveyed by De Long [1991], and advocated by Black [1995] and Greenwood and Yorukoglu [1997].

about the future are precise. In effect, occasional recessions can be viewed as a sign of a well functioning economy since they reflect the availability of good quality information upon which people act.

The analysis conducted in this paper can be viewed as being complementary to the literature emphasizing how rational herding and information cascades may be important for understanding macroeconomic phenomena (see for example Banerjee [1992], Bickhchandani, Hirshleifer, and Welch [1992], Chamley and Gale [1994], Caplin and Leahy [1993] and Zeira [1994]). In particular, this strand of literature has emphasized how information may occasionally be aggregated improperly thereby leading to significant forecast errors that are shared by a large fraction of the population. The current paper adds to this research program by examining whether (rational/non-systematic) aggregate forecast errors can explain the observed pattern of recessions within a fully-specified dynamic general equilibrium model. It should also be noted that the mechanisms at work in this paper are very close to those discussed in Phelps [1999]. In this sense, this paper can be see as offering a particular formalization to Phelps idea of structural booms and structural slumps.

The remaining sections of the paper are structured as follows. In Section

2 we illustrate how booms and recessions can arise in a dynamic general equilibrium model as the result of forecast errors. The focus of this section is on qualitative properties. In particular, we want to highlight the economic structure that can generate such behavior and discuss the differences between this structure and standard models used in the macroeconomic literature. To this end, the model is first presented in a non-stochastic continuous-time framework so as to present the mechanisms at work using phase diagrams. In Section 3 we reformulate the model in a stochastic discrete-time setting and evaluate its quantitative properties using standard numerical techniques. The main question addressed here is whether such a model can explain the observed depth and frequency of recession without invoking technological regress. Section 4 offers concluding comments.

2 A model of booms and recessions driven by forecast error

The object of this section is to present a simple dynamic general equilibrium model in which : *(i)* a forecast of future technological improvement first leads to a boom, that is, an increase in aggregate output, employment, investment and consumption, and *(ii)* the realization that a forecast is too optimistic leads to a recession, that is, a fall in all the same aggregate quantities. We will refer to such paths as Pigou Cycles. Since the focus of this section is on illustrating a general equilibrium structure which can produce Pigou cycles, we adopt a continuous-time non-stochastic framework. This approach has the advantage that the dynamics of the model can be illustrated using phase diagrams and thereby can be easily compared to the dynamics associated with more standard macro models. However, this framework has the disadvantage that forecast errors must be modeled as complete surprises. This drawback will be remedied in the following section where we embed the model in a stochastic setting where rational agents receive signals, make forecasts, and take decisions, knowing that received signals may be wrong.

Before setting out the structure of our model, it is worth emphasizing that the standard equilibrium models used in the macroeconomic literature do not produce boom and bust cycles of the type suggested by Pigou. For example, in the standard one sector model, an anticipated increase in technology generally leads to a fall in aggregate output (and investment) with an increase in aggregate consumption, while the realization of a forecast error would lead to a rise in output and a fall in consumption. This pattern does not appear to capture the type of dynamics suggested by Pigou, and certainly does not reproduce the pattern observed in recessions. In effect, the real constraint with the standard one sector model[5] is that in the absence of any change in technology, consumption and investment must always move in opposite direction. This is a constraint on the set of temporary equilibria since it is implied by the labor market equilibrium condition. This is not a property that is tied to either rational expectations or forward looking behavior. Since this property of one sector models is extremely restrictive, and since it is not a ubiquitous property of general equilibrium models, we believe that it is desirable to identify the type of economic structure which can produce Pigou cycles. In particular, if we can identify a type of structure that can produce an expectation driven boom and bust cycle, we can then ask whether such a structure may be a reasonable framework for modelling the short-run behavior of the macroeconomy.

2.1 The Production Sector

As we noted above, standard one and two sector macro models are not capable of producing Pigou cycles. For this reason, let us consider a stylized economy composed of three sectors: a final consumption goods sector, a non-durable goods (or intermediate good) sector and a durable goods sector.

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[5] This is also true of the standard two sector model.

The durable good sector is best thought as the construction industry with the stock of the durable good representing plant and housing infrastructure. The final good, denoted C_t, is produced as CES composite of the nondurable good (or service) X_t and the stock of infrastructure K_t:

$$C_t = (aX_t^{\nu} + (1-a)K_t^{\nu})^{\frac{1}{\nu}}, \quad \nu \leq 0$$

The final good C_t is a flow of consumption services, which could model as being either produced inside the household (by households purchasing X_t and K_t) or in the market. For the sake of concreteness, we choose to treat C_t as being produced in the market.

The non-durable good X_t is produced using labor according to:

$$X_t = \theta_{x,t} l_{x,t}^{\alpha_x} \ \ \widetilde{l}_x^{(1-\alpha_x)} \quad , \quad 0 < \alpha_x \leq 1$$

where $\theta_{x,t}$ is the state of technology in the non-durable goods sector and $l_{x,t}$ is the level of employment in this sector. $\widetilde{l}_x$ represents a fixed factor that is required in production. The introduction of the fixed factor assures that overall returns to scale are constant, but forces returns to scale in the variable factor to be decreasing.

The capital good accumulates according to:

$$\frac{\partial K_t}{\partial t} = I_t - \delta K_t \quad , \quad 0 < \delta < 1$$

where δ is the rate of depreciation and I_t is investment which is provided by the construction sector. Production in the construction sector depends on the state of technology in this sector, $\theta_{k,t}$, the levels of employment $l_{k,t}$ and a fixed factor $\widetilde{l}_k$.

$$I_t = \theta_{k,t} l_{k,t}^{\alpha_k} \ \widetilde{l}_k^{(1-\alpha_k)} \quad , \quad 0 < \alpha_k \leq 1$$

We will restrict attention to cases where the elasticity of substitution between K_t and X_t in the final goods sector is no greater that one (which seems reasonable given our interpretation of K_t as infrastructure). Obviously, both the intermediate good sector and the construction sector should have production technologies which use both physical capital (machines) and labor. However, in order to make our model concise we exclude this possibility and instead introduce fixed factors. This simplifies exposition greatly since it allows us to remain in the family of models with only one capital stock.

2.2 The Household Sector

The representative household has preferences defined over consumption of the final good and over the labor supplied in each of the two sectors. In the continuous time formulation, the household's objective is to maximize:

$$\int_0^\infty \{log(C_t) + v^x(\bar{l}_x - l_{x,t})^{\gamma_x} + v^k(\bar{l}_k - l_{k,t})^{\gamma_k}\}e^{-\rho t}dt$$

where C_t is the level of consumption of the final good , $\bar{l}_x$ and $\bar{l}_k$ are the endowments of labor available in each of the two sectors, γ_x and γ_k belong to the unit interval, v^x and v^k are fixed constants, and ρ is the discount rate. Note that household preferences are assumed to be separable in consumption and in the two types of labor. The household's within period budget constraint is:

$$C_t + p_t I_t = w_{x,t}\,(l_{x,t}) + w_{k,t}\,(l_{k,t}) + r_t K_t + \Pi_{x,t} + \Pi_{k,t}$$

where the final good C_t is the numéraire, p_t is the price of capital, r_t is the rental rate of capital, $w_{x,t}$ and $\Pi_{x,t}$ are respectively the wage rate and returns to the fixed factor in the intermediate goods sector, and finally $w_{k,t}$ and $\Pi_{k,t}$ are the wage rate and returns to the fixed factor in the construction sector.

2.3 Equilibrium Dynamics

A Walrasian Equilibrium for this economy is a set of time paths for k, l_x, l_k, C, r, p, w_x and w_l such that (1) allocations are optimal given prices (that is, consumers maximize utility and firms maximize profits) and (2) markets clear. Given an initial capital stock K_0 and time paths for $\theta_{x,t}$ and $\theta_{k,t}$, equilibrium allocations for this economy can be found by solving the following social planner's problem:

$$\max_{C_t, l_{x,t}, l_{k,t}, K_t} \int_0^\infty \{log(C_t) + v^x(\bar{l}_x - l_{x,t})^{\gamma_x} + v^k(\bar{l}_k - l_{k,t})^{\gamma_k}\} e^{-\rho t} dt$$

subject to

$$\begin{aligned}
C_t &= (a(\theta_{x,t} l_{x,t}^{\alpha_x} \widetilde{l}_x^{(1-\alpha_x)})^\nu + (1-a)K_t^\nu)^{\frac{1}{\nu}} \\
\frac{\partial K_t}{\partial t} &= -\delta K_t + \theta_{k,t} l_{k,t}^{\alpha_k} \widetilde{l}_k^{1-\alpha_k} \\
l_{x,t} &\leq \bar{l}_x \\
l_{k,t} &\leq \bar{l}_k
\end{aligned}$$

Assuming an interior solution for employment and assuming that technology parameters $\theta_{x,t}$ and $\theta_{k,t}$ are fixed, the equilibrium paths (transitional dynamics) for $\{l_{x,t}, l_{k,t}, K_t\}$ are defined by the following set of equations (plus the transversality condition):

$$\begin{aligned}
\frac{\partial K_t}{\partial t} &= \theta_k l_{k,t}^{\alpha_k} \widetilde{l}_k^{1-\alpha_k} - \delta K_t \\
\frac{\partial l_{k,t}}{\partial t} \frac{1}{l_{k,t}} &= \frac{(\bar{l}_k - l_{k,t})}{(1-\alpha_k)(\bar{l}_k - l_{k,t}) + (1-\gamma_k) l_{k,t}} \times \\
&\quad \left[(\delta + \rho) - \frac{(1-a)K_t^{\nu-1} \alpha_k \theta_k l_{k,t}^{(\alpha_k - 1)} \widetilde{l}_k^{(1-\alpha_k)}}{(a\theta_x^\nu l_{x,t}^{\alpha_x \nu} \widetilde{l}_x^{\nu(1-\alpha_x)} + (1-a)K_t^\nu) v^k \gamma_k (\bar{l}_k - l_{k,t})^{\gamma_k - 1}} \right] \\
0 &= \frac{\alpha_x a \theta_x^\nu l_{x,t}^{(\alpha_x \nu - 1)} \widetilde{l}_x^{\nu(1-\alpha_x)}}{(a\theta_x^\nu l_{x,t}^{\alpha_x \nu} \widetilde{l}_x^{\nu(1-\alpha_x)} + (1-a)K_t^\nu)} - v^x \gamma_x (\bar{l}_x - l_{x,t})^{\gamma_x - 1}
\end{aligned}$$

Since the last equation is a static condition, it can be solved for $l_{x,t}$ as a function of K_t and θ_x (it can be verified that $l_{x,t}$ is increasing in K_t and decreasing in θ_x) and used to reduce the problem to a pair of differential equations in the capital stock and the level of employment in the construction sector. In order to describe the properties of the transitional dynamics associated with this pair of equations, it is best to examine the phase diagram given in Figure 1.

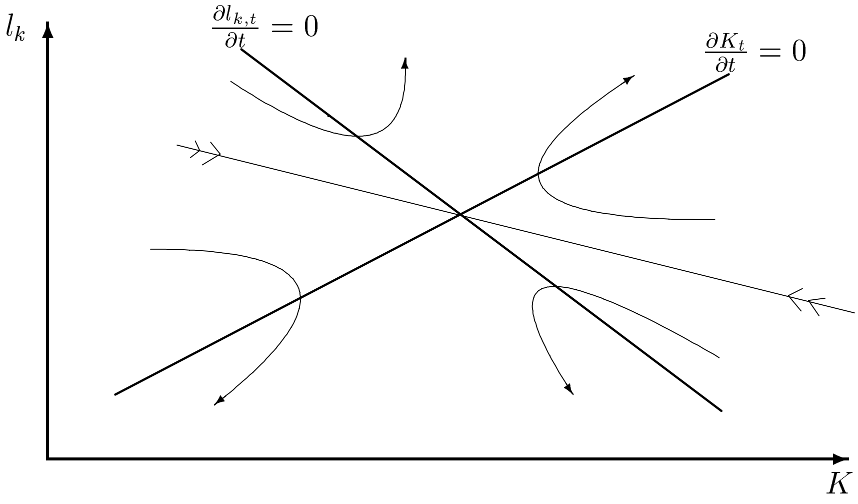

Figure 1: Phase diagram of the economy

In Figure 1, we have graphed the $\frac{\partial K_t}{\partial t} = 0$ line, which is always upward sloping, and the $\frac{\partial l_{k,t}}{\partial t} = 0$, which is always downward sloping. The $\frac{\partial K_t}{\partial t} = 0$ line corresponds to the set of points where increased employment in the construction sector is balanced off with the increased depreciation associated with a higher capital stock. The $\frac{\partial l_{k,t}}{\partial t} = 0$ line corresponds to the set of points where the marginal product of capital is equal to the marginal cost of producing one more unit of capital. This latter locus is downward sloping since a lower level of capital is associated with a high marginal product and hence it must be balanced off by a high marginal cost of producing capital. This occurs precisely when l_k is high.

In Figure 1 we have also represented the family of dynamic trajectories consistent with the two dynamic equations, and we have plotted the saddle path which is the trajectory consistent with the transversality condition. As can be seen, the saddle path is downward sloping and the transitional dynamics are simple. In effect, if K_0 is below its steady state, employment in the construction sector begins above its steady state and gradually converges to it, which allows capital to be built up. During this transition, employment in the non-durable goods sector is below its steady state level. This pattern of dynamics is represented in Figure 2.

There are two points to take from Figure 2. First, along the transition path, the aggregate level of employment (as defined by $l_x + l_k$) can be either above or below its steady state level. Second, these dynamics are qualitatively similar to those derived for the one sector model generally used in

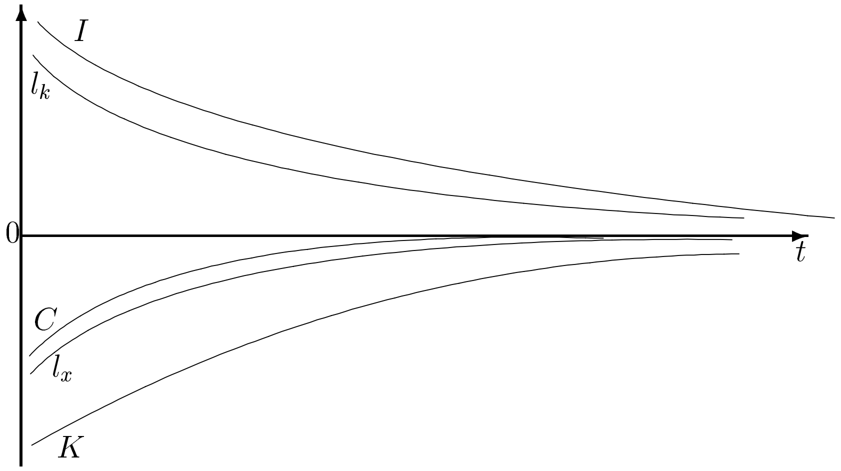

Figure 2: Transitional dynamics with K_0 below steady state (all variables are measured in relative deviations from their respective steady-state level)

real business cycle models. However, as we will see, the dynamics of this model will differ from those of the more standard model when anticipated technological change is introduced.

Let us now turn to examining how such an economy would respond to an anticipated increase in technology. The first case we consider is a balanced improvement in technology where agents anticipate increases in θ_x and θ_k of identical magnitude. This technological improvement is expected to arise at time T', that is, at time T it is learned that technology will improve at time $T' > T$. A typical depiction of the dynamics associated with such a change are given in Figure 3.

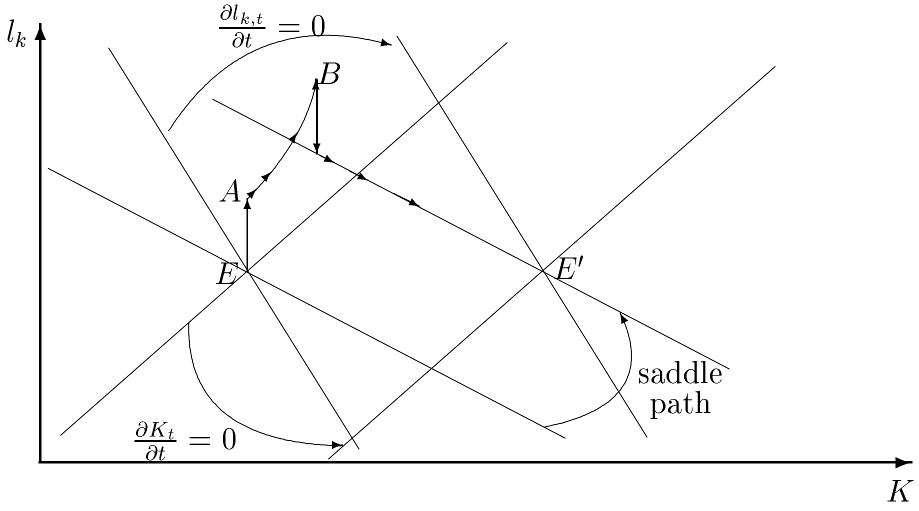

Figure 3: Response of the economy to an announcement at time 0 of future positive shock on the technology and a realization of that shock at time T

Since this model exhibits balanced growth, the new steady state corresponds to a situation with increased capital (and consumption) but no change in employment. However, during the transition (as illustrated by points A, B and E' on the graph) the employment level in the construction sector immediately jumps to point A, then continues to increase until time T' where it reaches B. At this point technology improves and employment in the construction sector jumps on to the new saddle path[6], and then grad-

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[6] At first pass, it may be surprizing to see the the level of employment in the construction sector jump at point B in response to an anticipated change in technology. However, this jump is l_k is necessary to assure that the underlying price of capital does not jump. In effect, if we depicted the dynamics in the $k - P$ space (that is, the space of capital and the price of capital), it would be the case that the price of capital would be exactly on the the new saddle path at time T'.

ually decreases to its steady state level while the capital stock continuously increases. These dynamics are such that anticipated technological improvement can be said to cause an expectation lead boom, that is, from time T to T', employment in both sectors, total output (defined as $C_t + p_t I_t$), investment and consumption are all increasing even though technology has not yet improved.

Let us now consider what happens if, at time T', instead of technology improving as anticipated, individuals learn that their forecast is incorrect and that technology does not actually change (it remained at its initial level). In this case, there would be a fall in output and employment in the construction sector at T', as individuals realized that they previously over-accumulated. Following this drop, employment would gradually return to its previous steady state as the capital stock returned to its initial level. These dynamics are reproduced on a phase diagram in Figure 4.

In Figure 4, the economy is first pictured at time T' at point B, then jumps to C and finally converges to the original steady state E. To illustrate the dynamics further, Figure 5 graphs the time paths of all the main variables through this entire sequence of anticipation and realization.

Note from Figure 5 that the economy first experiences a boom and then a

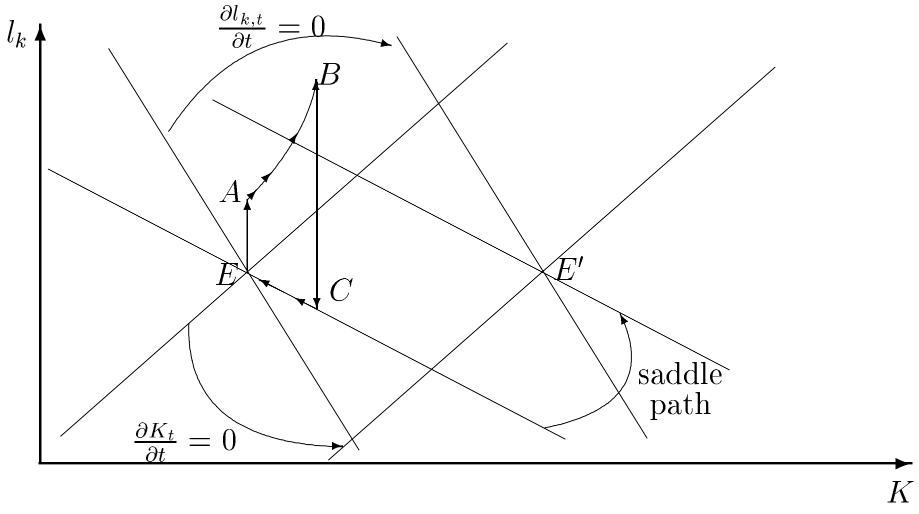

Figure 4: Response of the economy to an announcement at time T of future positive shock on the technology and no realization of that shock at time T'

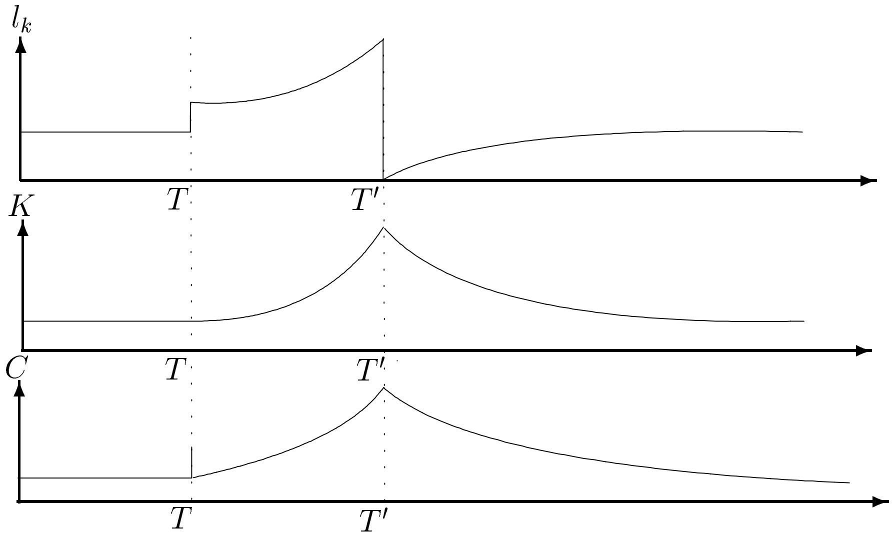

Figure 5: Response of l_k, K and C to an announcement at time T of future positive shock on the technology and no realization of that shock at time T'

recession without ever having experienced an actual change in technology. In particular, at time T' aggregate output, investment and employment all fall, while consumption (of the final good) falls with a lag. It is interesting to note the pattern of prices that decentralizes this behavior. During the first phase, from T to T', individuals invest in infrastructure in anticipation of realizing capital gains. Throughout this phase, the price of infrastructure increases thereby fulfilling these expectations of capital gains. At time T', however, the price of infrastructure falls drastically in recognition of an over-supply. The new low price for infrastructure makes investment unprofitable and therefore employment in the construction industry collapses. As the oversupply of capital slowly diminishes as the result of depreciation, incentives for new investment reemerge and thereby pulling the economy out of the recession.

In our view, these dynamics capture the idea, suggested by Pigou and others, that forecast errors may be key in understanding recessions. In effect, in this model, a boom and a recession can arise as the result of overly optimistic expectations about future technological growth. Two questions arise immediately. Which property of this model (in comparison to the standard one sector model) allows it to generate Pigou cycles, and is this property reasonable?

The key property that allows this model to generate Pigou cycles is the fact that current consumption decisions in the model are decoupled from current investment decisions. That is, agents in the economy can determine how much investment in infrastructure to undertake without this decision having any direct feedback on how much the economy can currently consume. In other words, the shadow price of investment in the above three sector model is not directly related to foregone consumption but only relates to the cost of reduced leisure for workers in the construction industry. The fact that increased investment is not directly reducing consumption possibilities is a property that may be a sensible description of short-term substitutability constraints in a modern economy. For example, if an economy has an oversupply of buildings, it seems a reasonable simplification to exclude– at least in the short run –the possibility of immediately transforming the output of construction sector workers into others goods. It is this type of constraint which differentiates the above three sector model from more standard macro models and thereby allows for Pigou cycles.

It is interesting to conjecture how individuals may perceive a downturn generated by a Pigou cycle and how this may lead them to choose inappropriate policies. For example, at the onset of a recession, individuals in our model are likely to perceive the cause of the recession as being a fall in aggregate demand. In response, they may be tempted to favor policies, that would stimulate investment demand; such as temporary tax breaks or investment tax credits. However, such policies would be misplaced in this model since it is precisely an excess of investment that caused the recession. Policies which stimulate investment may even appear to individuals as a cure to downturns —since they would temporarily increase employment and output— when in fact such policies would at best be a postponement of needed adjustment.

Our objective now is to go a step further and examine whether the above model, once embedded in a stochastic setting with imperfect signals and rational expectations, can mimic some of the quantitative features of recessions; in particular, their frequency and depth. However, it seems unreasonable to examine the properties of this model under the scenario where shocks to technology are perfectly correlated between sectors. Instead, we will examine the case where technology grows stochastically only in the non-durable goods sector. This choice appears reasonable given that expectations about technological improvements in the construction sector do not stand out as an important driving force behind business cycles. Moreover, technological improvements in the non-durable goods sector can be interpreted (and formalized) as the arrival of new differentiated goods in an economy with tastes for variety. In this case, it would be the expected arrival of new goods and the associated infrastructure requirements implied requirements in terms of infrastructure which would lead a Pigou cycle. As can be easily verified, anticipated growth in θ_x gives rises to dynamics identical to that derived above under the assumption of equal increase in θ_x and θ_k.

3 A Quantitative Evaluation of the Model

3.1 Reformulation the Model in a Discrete-Time Stochastic Setting

The model presented in Section 2 can be easily extended to a discrete time setting by making two simple changes. First, the consumers preferences can be changed to:

$$E_O\left[\sum_{t=0}^{\infty}\beta^t\{log(C_t)+v_0\ (\bar{l}_x+\bar{l}_k-l_{x,t}-l_{k,t})\}\right]$$

where β is the discount factor and v_0 a positive constant. Note that, following Hansen [1985] and Rogerson [1988], we now assume that preferences are linear with respect to labor at the representative agent level.

The law of motion for capital is modified to:

$$K_{t+1}=(1-\delta)K_t+I_t$$

where now δ represents the per-period depreciation rate. All other equations remain the same with the obvious change in interpretation from instantaneous flows to per-periods flows.

3.2 Processes for Technology

As far as technology is concerned, we want to examine an economy where (*i*) technology only improves (never regresses), (*ii*) the economy exhibits balanced growth in the long-run and (*iii*) technological progress is stochastic only in the non-durable good sector. To this end, we assume technology in the construction sector grows deterministically according to

$$\log\theta_{k,t} = g_{o,k} + g_1 t,$$

while technology in the intermediate goods sector evolves stochastically according to:

$$\begin{aligned} \log\theta_{x,t} &= g_{o,x} + g_1 t + \log\widehat{\theta}_{x,t} \\ \log\widehat{\theta}_{x,t} &= \lambda\log\widehat{\theta}_{x,t-1} + \varepsilon_t, \qquad 0 < \lambda < 1, \end{aligned}$$

where ε is a zero mean i.i.d random variable. To exclude the possibility of technological regress, we impose that the minimum value for ε be greater than -g_1. With $\lambda = 1$, the restriction $\min\varepsilon \geq -g_1$ guarantees that θ_x never regresses. When λ is very close but smaller than 1, which will be the case in our simulations ($\lambda = .99$), this restriction on the support of ε guarantees that technological regressions almost never regresses. In effect, with this support restriction, technology never regresses in our simulations.

3.3 Information Structure

We specify the information structure as follows. In every period, the representative households observes a signal, S_t, about a technological innovation (ε_{t+n}) that will arise in n periods. A key issue is how best to specify the joint distribution of S_t and ε_{t+n} . In order to capture the idea that forecast errors can sometimes be substantially wrong, we assume that with probability q the signal is perfect ($S_t = \varepsilon_{t+n}$) and, with probability $1 - q$ the signal is wrong ($S_t = \mu_t$, where μ_t is drawn from the same distribution as ε_t but is independent from ε_t). The idea here is to have signals which are sometimes entirely void of information as is suggested by the herding literature (e.g. Banerjee [1992]).

To limit the number of parameters to be calibrated in the quantitative exercise, we adopt the simplest possible specification for the random variables S_t and ε_t. We assume that ε_t can either take on a high level, which implies growth, or a low level, which implies no growth. The respective probabilities of these different states are $1-p$ and p. With the restriction that technology grows at factor g_1 on average, and that bad times are periods with zero productivity growth, p fully characterizes the distribution of technological innovations. In effect, these restrictions imply that ε_t takes on the value $g_1 \times p/(1-p)$ in the growth state. The signal is also assumed to only take on two values: one indicating a future high growth state and one indicating a future no growth state. We denote by q the probability that the signal indicates the correct state. The economy can therefore go through one of the following four realizations of signal and subsequent growth: a growth signal at time t which is validated by technological growth at time $t+n$ (probability $(1-p)q$); a growth signal at t but no realized growth at time $t+n$ (probability $(1-q)p$); a no-growth signal at time t but a growth realization at $t+n$ (probability $(1-p)(1-q)$); a no-growth signal at time t and a no-growth realization at time $t+n$ (probability pq).

3.4 Calibration

In our calibration exercise, our goal is not to suggest that a three sector model is a fully adequate description of the economy. In effect, we believe that the above model is an extreme simplification of reality and that it omits many important elements (for example: adjustment costs, variable rates of factor utilization, inventories, additional capital stocks). Nonetheless, we believe that a calibration exercise is useful for evaluating whether the theoretical mechanism by which this model produces booms and recessions (in the absence of technological regress) can be considered quantitatively relevant. To this end, we will examine whether a reasonably calibrated version of the model can reproduce the observed pattern of recessions (frequency and depth) while simultaneously capturing the variances and co-movements emphasized in much of the modern business cycle literature. Throughout this exercise, we will interpret a time period as representing six-months. The advantage of adopting a semester as our unit of time is that it allows a decline in output in the model to be referred to as a recession. Moreover, we believe that a semester is a more reasonable notion for a period than a quarter in model without any adjustment costs.

There are several parameters in our model, some of which do not have immediate counterparts in the literature. Therefore we approach this calibration exercise by first setting parameters (as is most commonly done) based on known estimates or based on matching certain steady state properties. We estimate the remaining parameters using a simulated method of moments technique. In particular, the discount factor β is set equal to .98, the depreciation rate δ is set to .05. Total disposable time $\bar{l}$ is normalized to 2, and the disutility of labor scale parameter v_0 is set to 1, so that one third of total time is devoted to work in the steady state. The average growth factor of productivity is set to its observed level in our sample period (see below for a description of the data set). The ratio $\theta_{o,x}/\theta_{o,k}$ and the relative weight of K and X in the CES production function, that is the parameter a, are set so that, in conjunction with the other parameters, the labor share is 66% and consumption's share in total output is 75%.[7] We also need to set values for the short-run returns-to-labor parameters α_x and α_k. The literature on scale parameters suggest that the short run returns to labor are close to the labor share in output.[8] However, the literature on the construction industry arrives at a somewhat different conclusion. Allen [1985], for example, estimates the short-run return to labor in construction to be very close to one. In order to reflect these two considerations, we set $\alpha_x = .6$ and $\alpha_k = .97$.[9]

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[7] It should be noted that total output (GDP) in our model is calculated as the sum of the flow of consumption services C_t plus the value of investment $p_t * I_t$. The production of the non-durable good is treated as an intermediate input.

[8] See for example Burnside, Eichenbaum, and Rebelo [1995]

[9] These values for α_x and α_k can also be justified based on some of the results found in Burnside, Eichenbaum, and Rebelo [1995]. In particular, when focusing on industries for which there are good direct measures of output, Burnside, Eichenbaum and Rebelo estimate the short run return to labor in durable manufacturing to be .98, while the counterpart for non-durable manufacturing is estimated to be .61 (see their last columns of Table 10). Clearly, these are estimates offer an alternative justification for the returns to scale parameters that we use in our calibration.

There remain four parameters that we cannot infer from previous studies, namely the two parameters governing the technology and information processes (p, q), the technological parameter ν[10], and the number of periods n between the arrival of a signal and the related realization of ε. Therefore we choose to estimate these four parameters by Simulated Method of Moments.[11] We implement this procedure by finding, for different values of n, the vector $\pi = (p, q, \nu)$ that provides the best match for the following six moments: the volatilities of output, consumption and investment calculated for both the Hodrick-Prescott (HP filter) cyclical components[12] and growth rate ($1 - L$ filter). We denote this set of six moment by $M^o = (\sigma_y, \sigma_c, \sigma_i)_{HP,1-L}$. The HP moments are chosen for reasons of comparability with previous studies, while our interest in "classical cycles" (cycles in terms of growth rate) suggests the use of the $1 - L$ filter. Let us denote by Ω the variance-covariance

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[10] Note that the parameter representing the elasticity of substitution between capital and non-durable goods is for the final goods production function. Given that this production function describes the process of aggregating goods and services into a final flow of consumption goods, it does not seem appropriate to set it based on estimates derived from industry studies.

[11] Roughly speaking, simulated method of moments consists in choosing those model parameters values that produce the best match between a set of empirical and simulated moments, where the distance between those moments is evaluated using the inverse of a consistent estimate of the moments estimators asymptotic variance matrix. See Duffie and Singleton [1993] for an exposition and Hairault, Langot, and Portier [1997] for an application.

[12] In calculating HP filtered moments we set $\lambda = 800$ since this appeared to give reasonable cyclical components to other semestrial data.

matrix of these estimators. For a given vector of parameters π and n, we simulate the model N times for T periods ($N = 20$ and T=77[13]) and compute a vector of simulated moments $M^s(\pi)$. We performed the simulations using a log-linearized approximation of the model (around its (locally) unique steady state). The estimate of π is then

$$\widehat{\pi} = \text{Arg Min}_{\pi} \quad J = \frac{NT}{NT+1}(M^s(\pi) - M^o)\Omega(M^s(\pi) - M^o)'$$

We estimated a π vector for each of the ten cases where n was allowed to vary between 1 and 10. We then chose n and the corresponding π vector based on the lowest value for the J statistic.

The data we use are US National Income and Product Account data covering the period 1959 to the end of 1997. We build the relevant empirical counterparts to our theoretical constructs in the following manner. Durable goods and inventories are considered investment, and net exports are split into consumption and investment according to the relative share of consumption and investment. More precisely, the three series are constructed as follows: Investment (I) = Fixed investment + Durable goods + Change in business inventories + Net export of good and service $\times(i/y)$, Consumption (C) = Nondurable goods + Services + Net export of good and service

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[13] T =77 corresponds to the length of our sample.

$\times(c/y)$, Output (Y) = Consumption + Investment. Variables are then expressed in per capita terms. Estimation results for the case where $n = 2$ (which corresponds to a minimum for the J statistic)[14] are given in Table 1, and the model's predictions relative to the targeted moments are given in Table 2.

Table 1: SMM Estimators of p, q and ν (standard-deviations in parenthesis)

p	.71 (.04)
q	.82 (.31)
ν	-3.78 (1.21)
n	2
J	3.30
$\chi^2(2)$ at 95%	5.99

Table 2: Targeted and Simulated Moments

	U.S. Data	model simulation
σ_c (HP)	1.055 (.087)	1.060
σ_c $(1-L)$	0.687 (.059)	0.714
σ_y (HP)	2.162 (.222)	1.825
σ_y $(1-L)$	1.438 (.125)	1.477
σ_i (HP)	6.872 (.669)	5.742
σ_i $(1-L)$	4.996 (.488)	5.100

The results from the estimation using simulated method of moments implies an economy where (*i*) infrastructure K and other goods X are strong complements (elasticity of substitution close to .2), (*ii*) agents receive rather

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[14] We did not find any significant differences for J calculated using $n = 1$ or $n = 2$. However, the value of J does increase substantially for $n > 2$.

informative signals, that is, signals are right 82% of the time, (*iii*) technological growth is quite sporadic with 71% of semesters registering no technological progress and 29% percent of semesters registering growth of 4.17%, and (*iv*) the delay between signals and realizations is one year (2 periods or semesters). It is interesting to note that, under the null that the model is the Data Generating Process and that $n = 2$, the J statistics (which conditional on n would follow a $\chi^2(3)$) cannot be rejected at a 5% level ($\chi^2(3)$ at 95% is 7.8). Although this is not an appropriate test since we are choosing n to minimize J (in which case it is more appropriate to compare J with a $\chi^2(2)$ distribution), it nevertheless suggests that this simple three sector model can fit these data surprisingly well.

3.5 A First Look at the Models Quantitative Properties

In order to clarify the quantitative properties of the model in the sharpest manner, we begin by studying the response of the economy to the following experiment. Initially the economy is at the steady state, then in period 1 agents receive a signal indicating that technological growth will be 4.17% in one year. Agents know the signal has a 18% probability of being false. In period 2 nothing is announced and technology grows at its trend rate.

Finally, in period 4 is it discovered that the signal was false and the anticipated growth in technology does not occur. Note that such an experiment is never observed in our simulated economy because a new signal arrives every period. Nevertheless, such an experiment provides a good way to illustrate the functioning of the model economy.

As can be seen in Figure 6, the responses to this experiment are qualitatively identical to the ones we derived from the analytical continuous time model. Investment, consumption and output start expanding at the signal of the future growth. When the information is revealed to be false, investment falls dramatically and output falls by over 3%. It is interesting to observed that consumption responds by falling below trend but does not decrease enough to generate a negative growth rate. Finally, recall that technology does not deviate from its trend during this whole experiment, so that all movements are explained by forecasts and revisions.

3.6 Comparing the Model's Cyclical Properties with Those of the Data

Business cycle and recession statistics for the U.S. economy are given in Tables 3 and 4. Our construction and reporting of recession statistics, in addition to standard business cycle statistics, reflects our desire to evaluate

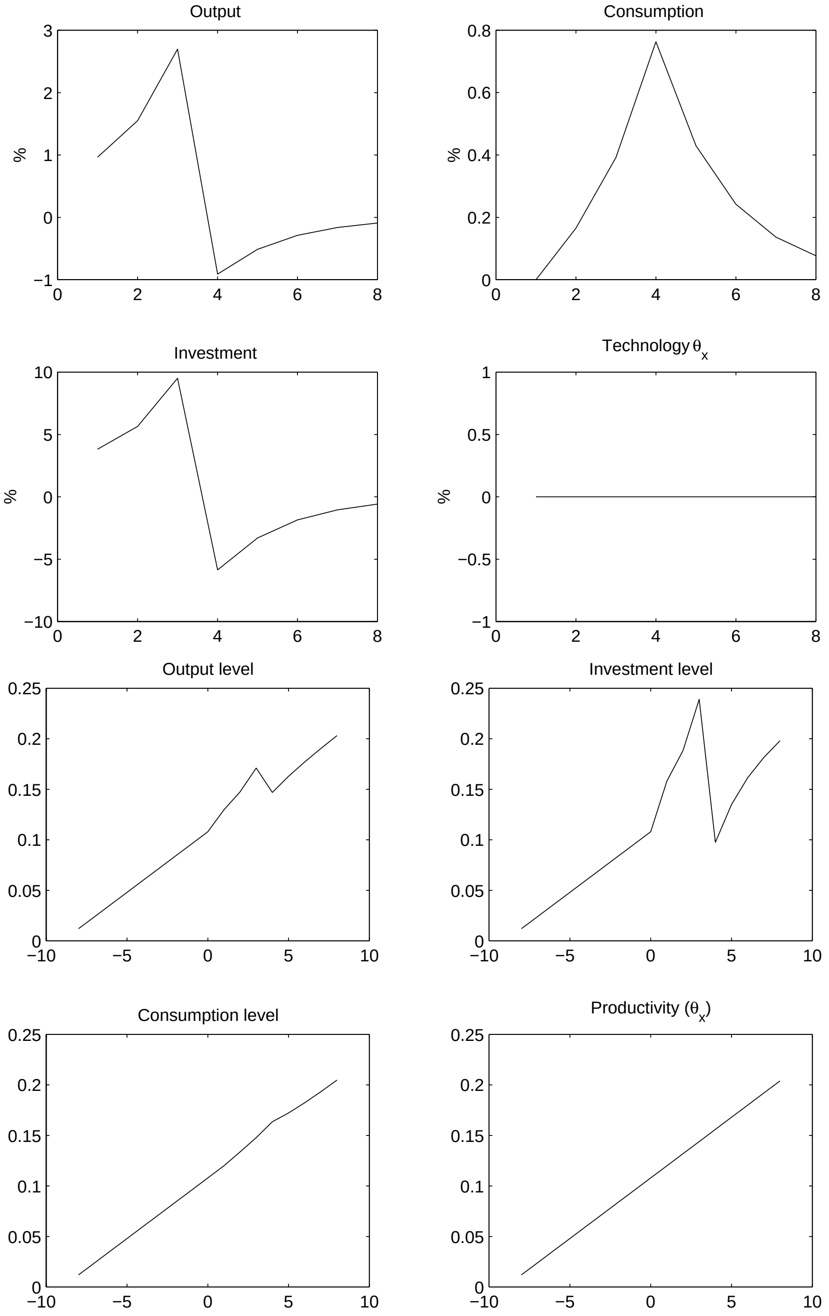

Figure 6: Impulse Response (relative deviations from trend and levels)

the capacity of our model to explain this particular phase of the cycle.

Table 3: Statistics on Recessions (U.S. NIPA, 59:1–97:2, semi-annual)

$\overline{\Delta^- y}$	$F(\Delta^- y)$	$\overline{\Delta^- c}$	$F(\Delta^- c)$	$\overline{\Delta^- i}$	$F(\Delta^- i)$
-1.1	19.5	-.15	7.8	-3.87	33.8
(.2)	(5.3)	(.03)	(3.7)	(.69)	(6.5)
$\min(\Delta^- y)$	$\min(\Delta^- i)$	$\min(\Delta^- c)$	$\overline{\Delta c}\vert_{\Delta y<0}$	$\overline{\Delta i}\vert_{\Delta y<0}$	
-2.56	-13.56	-.29	.51	-5.8	
–	–	–	(.17)	(.79)	

Table 4: Statistics on Business Cycle (U.S. NIPA, 59:1–97:2, semi-annual)

HP Filter Data		
σ_y	σ_c	σ_i
2.16	1.06	6.85
(.22)	(.09)	(.67)
ρ_y	ρ_c	ρ_i
.79	.84	.74
(.08)	(.06)	(.09)
$\mathrm{cor}(y,c)$	$\mathrm{cor}(y,i)$	
.64	.95	
(.08)	(.01)	

In these two tables, the figures in parenthesis are standard deviations of estimators. In Table 3, the variables of the form $\overline{\Delta^- x}$ represents the average growth rate of x conditional on Δx being negative, $F(\Delta^- x)$ represents the percentage of semesters for which Δx is negative, $\min(\Delta^- x)$ represents the largest recession (percentage decrease) of x, and finally $\overline{\Delta x}|_{\Delta y<0}$ is the average growth rate of x conditional on Δy (growth in aggregate output) being negative. Note that recessions are not rare events: almost one fifth of semesters experienced output drops and on average the falls are 1.1%. For investment, recessions happen one third of the time, and the average fall in a semester is almost 4%. In contrast, recession for consumptions happen rarely and when they do happen they are on average very shallow at .1%.

Let us now turn to the statistics generated by the model. We evaluate the model's ability to match the data in the following way. We generate 1000 simulations of length 77 (the number of observations in our sample), and compute the mean and the standard deviations of the moments of interest. We then ask the question: "Is the data at odds with the statistics generated by the model?". We ask this question for several different moments, focusing on one moment at a time. Tables 5 and 6 report statistics generated by the model, with standard deviations given in parenthesis. A $\star$ on a statistic indicates that the empirical moment lies within a interval of ± 2 s.d. around the mean of the model simulations (2 $\star$ for ± 3 s.d.). We interpret this as follows: as far as this particular moment is concerned, we cannot reject that the data could have been generated by our model.

We first comment on the ability of the model to reproduce standard business cycle statistics as reported in Table 6. Recall that the model has been

Table 5: Statistics on Recessions (Model)

$\overline{\Delta^- y}$	$F(\Delta^- y)$	$\overline{\Delta^- c}$	$F(\Delta^- c)$	$\overline{\Delta^- i}$	$F(\Delta^- i)$
-.95*	13.99*	-.00	.03	-3.17*	40.27*
(.35)	(4.2)	(.02)	(.39)	(.99)	(6.3)
$\min(\Delta^- y)$	$\min(\Delta^- i)$	$\min(\Delta^- c)$	$\overline{\Delta c}\vert_{\Delta y<0}$	$\overline{\Delta i}\vert_{\Delta y<0}$	
-3.22*	-16.25*	.27	.97	-6.61*	
(.84)	(3.42)	(.14)	(.10)	(1.78)	

Table 6: Statistics on Business Cycle (Model)

HP filtered Data		
σ_y	σ_c	σ_i
1.85*	1.07*	5.84*
(.27)	(.18)	(.80)
ρ_y	ρ_c	ρ_i
.67*	.81*	.58*
(.09)	(.05)	(.10)
$\mathrm{cor}(y,c)$	$\mathrm{cor}(y,i)$	
.63*	.91**	
(.06)	(.01)	

calibrated to give a good fit for the standard deviations of HP-filtered output, consumption and investment. However, the model was not calibrated to match the other statistics in Table 6. In particular, it is interesting to note that the serial correlations and cross correlation of the HP filtered data are well reproduced by the model. We interpret these results as suggesting that the model does a good job at matching the moments most often discussed in the RBC literature. Let us now look at recession statistics (Table 5). Again recall that the calibration has been done without targeting these statistics. As far as output and investment are concerned, the model does a very good job at reproducing the recession statistics. Average and maximum depth of recessions, as well as frequency of recessions, are all matched by the model even though there is never technological regress. The only major failure of the model is that consumption is too smooth: in effect, the model does not produce significant recessions in per capita consumption it only produces significant slowdowns. However, we do not interpret this failure of the model to be a fatal drawback given that drops in consumption are also rare and small in the data (see Table 3).

In order to get an additional view of the model's ability to reproduce observed output growth, Figure 7 plots the histogram of output growth. As can

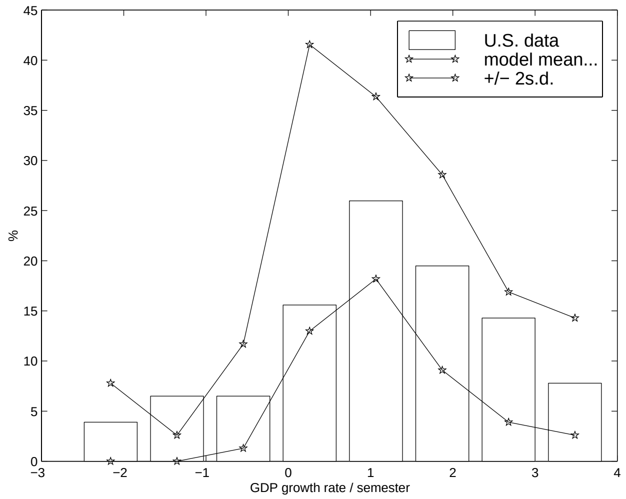

Figure 7: Output Growth Histogram

be seen from the Figure, the empirical histogram lies almost entirely within the 2 s.e. bands associated with the histogram generated by the model. This further illustrates how a simple three sector equilibrium model without any technological regress can reproduce the observed pattern of output growth.

3.7 Explaining the Success of RBC Models

Balke and Wynne [1995] have shown, among others, that a simple *RBC* model can generate business cycles of plausible duration and depth. If one does accept the possibility of technological regress, this indicates that an RBC models may provide a good theory of cycles. However, if one questions the plausibility of high frequency technological regresses, then the observed declines and variability of the Solow residual need to be explained. In this section, we want to argue that our model can account for the success of the standard RBC model by explaining observed movements in the Solow residual (as an improper measure of technological shocks).

To this end, let us compute from our model an implied Solow residual as if the series were generated with a one sector model. That is, let us use our simulated data to compute a Solow residual series as follows:

$$\Delta SR_t = \Delta \log Y_t - s_l \Delta \log(l_{k,t} + l_{x,t}) - s_k \Delta \log K_t$$

where s_l and s_k are the share of labor and capital in total income. In performing this exercise on our simulated data, we obtained a standard deviation of ΔSR of 1.58 based on 1000 simulations (with a standard deviation of .09). In comparison, the standard deviation of the innovation in the Solow residual used by Hansen [1997] to calibrate a one sector RBC model is 1.59. Hence, if a macroeconomist was given data generated by our model and he used this data to calibrate a one sector growth model, he would find that the resulting RBC model would fit the data rather well. The point we want to emphasize here is that, our three sector model can not only reproduce business cycle facts, it can also explain why RBC models (and, as previously discussed, Keynesian aggregate demand models) might appear as reasonable theories of the cycle even if they were incorrect.

4 Conclusion

In this paper we have illustrated an equilibrium business cycle model where anticipations and realizations of technological growth were qualitatively and quantitatively able to explain several patterns associated with business cycles and recessions. We think that the mechanism of this model —the importance of forecasts and forecast errors in explaining aggregate movements of activity— may help understand certain episodes of cyclical downturns in industrialized economies. In particular, this type of model may provide a useful framework for understanding the recent downturns in South-East Asia since it has been argued that revisions of expected growth were central in generating the crises observed in these economies. However, we leave a detailed exploration of this last issue for future research.

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