DISCUSSION PAPER SERIES

No. 2897

EDUCATION, DISTRIBUTIVE JUSTICE AND ADVERSE SELECTION

Marc Fleurbaey, Robert Gary-Bobo and Denis Maguain

PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP2897.asp**

ISSN 0265-8003

EDUCATION, DISTRIBUTIVE JUSTICE AND ADVERSE SELECTION

Marc Fleurbaey, Universite de Cergy-Pontoise
Robert Gary-Bobo, Universite de Cergy-Pontoise and CEPR
Denis Maguain, THEMA, Université de Cergy-Pontoise and Université Catholique de Leuven

Discussion Paper No. 2897
April 2002

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 2897

April 2002

ABSTRACT

Education, Distributive Justice and Adverse Selection*

We consider a model of education planning in an economy in which agents differ in their costs of acquiring education. The agents' cost parameter, called "talent," is not observed. The Principal is endowed with a fixed sum of money, with which two types of transfer can be made: in cash and in kind. The Principal can finance transfers in kind, called "help," by means of schooling expenditures, which reduce the agent's education cost. The Principal seeks to maximize a social welfare function which is a CES index of utility levels. We study the optimal allocation of individual education effort, schooling expenditures (help), and cash, under self-selection and budget constraints. Assuming first that the set of types is finite, and that help and effort are sufficiently substitutable, we find that individual education investment levels are an increasing function, and help is a decreasing function of talent. Utility levels cannot be equalized because of self-selection constraints. More aversion for inequality unequivocally leads to more inequality of educational achievements, and to more assistance through redistribution. This remains true in the limit, under strictly egalitarian preferences of the Principal. The same qualitative properties hold in the general case of a continuum of types. Bunching at the lower end of the talent scale is a feature of the solution for sufficiently high degrees of inequality aversion.

JEL Classification: D82 and H41
Keywords: effort, help, inequality aversion, redistribution and talent

Marc Fleurbaey
Département des Sciences Economiques
Université de Cergy Pontoise 33 Boulevard du Port
95011 Cergy-Pontoise Cédex
FRANCE

Robert Gary-Bobo
Université de Cergy-Pontoise
THEMA
33, boulevard du Port
95011 Cergy-Pontoise Cedex
FRANCE
Tel: 34 25 6062
Fax: 34 25 6233
Email: robert.gary-bobo@eco.u-cergy.fr

Denis Maguain
Center for Economic Studies
Katholieke Universiteit Leuven
Naamsestraat 69
3000 Leuven
BELGIUM
Tel: (33 1) 6326 826
Fax: (33 1) 6326 796
Email: denis.maguain@econ.kuleuven.ac.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=154807

* A preliminary version of this paper was presented during the 4th international meeting of the Social Choice and Welfare Society, held in Vancouver, University of British Columbia, June-July 1998. D. Maguain gratefully acknowledges financial support from the EU, through the TMR network "Living Standards, Inequality and Taxation", contract No A-3733-FMRX-CT98-0248. We also thank seminar audiences in Namur, Leuven, Paris, Copenhagen, and the Free University of Brussels, Pierre Cahuc, Patrick Legros, Pierre Pestieau, Jean-Charles Rochet, Thomas Piketty for their remarks and comments. Special thanks are due to François Maniquet for his help. The usual caveat applies.

Submitted 18 March 2002

1. Introduction

The role of education in the realization of economic justice is more than ever a hotly debated topic. Access to education is seen as the crucial element in the process of equalizing opportunities, and education itself — the quality of teachers, the efficiency of schools — is at the center of the politician's concerns, in many countries. The debate in the US is on inequalities of school funding, on affirmative action as a means of promoting minorities, and on the merits of competition in fostering quality, while experiences are being conducted with voucher systems. In the UK, school performance and the teacher's incentives lie at the heart of the debate, but the underlying hopes are in fact that public education could play a more powerful role in the reduction of social inequalities.[1] The debate is very similar, although phrased differently, in the highly centralized French *Education Nationale*, with the development of classroom violence, strikes and quasi-riots in the poor urban areas, demanding a form of affirmative action in favor of problem schools. Spontaneous conceptions of Justice and Democracy, in their relation with Education, often clash with vested interests, and make every attempt at reforming the rights of access to, and the tuition fees of, higher education in public universities a very acute political question. This problem is again capable of triggering massive demonstrations, and of leading the career of the (usually front-row) politician in charge of the Ministry of Education to a premature end.[2]

To sum up, it seems that higher demands are placed on education by citizens in most social groups, and as a consequence, that both the forms of public intervention, and the organization of public teaching institutions need more than superficial changes in many countries. The desirable direction of change however, is far from gathering unanimous consent. The past decades have witnessed an impressive development of secondary and higher education in all industrialized countries, but the speed and the form of the growth process exhibits striking differences from one country to another. For instance, the rate of enrolment of 17-year-olds in secondary education has been consistently and markedly lower in the UK than in the US during the entire twentieth century (see Reid (1991)). In Sweden — probably one of the most egalitarian societies —, the expansion of schooling has been largely based on vocational training, which is then responsible for the bulk of the increase in the rates of access to secondary education of children from lower socioeconomic backgrounds (see Erikson and Jonsson (1996)). But in spite of qualitative similarities in the general tendency towards expansion of the education sector, the changes in inequality, as measured by the impact of social origin and parental income on the number of school years completed, are significantly different from one country to another (see Shavit and Blossfeld (1993)). The determinants of these differences are not easy to understand, and even less easily amenable to quantitative analysis in an economic framework.

---

[1] See *The Economist,* (1999a), (1999b).

[2] It was the case in the winter of 1986-87, in France, when the Minister for Research and Higher Education, Alain Devaquet, was forced to resign and his plans for university reform subsequently abandoned. Important student demonstrations were also triggered, in major German cities, in november 1997, by insufficient capacities in universities and plans to increase the admission fees.

Recent contributions to the theoretical literature have analyzed the industrial organization of the education sector and its screening properties, such as Epple and Romano (1998). Bénabou (1996a), (1996b) pays attention to links between urban stratification and human capital investment: he shows how school funding centralization could help reducing inequalities. Fernandez and Rogerson (1996) show how education spending redistribution can improve overall efficiency, while at the same time reducing heterogeneity in a multi-community model. In contrast, Hoxby (1994), (1999) emphasizes the improvements that can be expected from increased school competition. Yet, given the apparent importance of the subject, the economic theory of education in its relation with social justice and income redistribution is still under-developed. To be more precise, the modern tools of Mechanism Design have not been systematically applied to explore this question from a normative point of view; the present contribution is an attempt in this direction.

We consider a stylized model of education planning in an economy in which agents differ in their costs of acquiring a given education level. The agents' cost parameter, called "talent," is not observed by the Planner, or Principal. Individual education effort yields a level of human capital. The Principal is endowed with a fixed amount of a resource, called money. There are two types of transfer, in cash and in kind, both playing a role in redistribution. On top of usual money transfers, the Principal can use transfers in kind, by producing, and then allocating, a good called "help," which reduces the agent's education cost. Help is an abstract representation of public expenditure on schools. The model is consistent with a number of recent empirical studies having demonstrated the positive effects of an increase of school resources per student on earnings.[3] According to Card and Krueger (1996), empirical results converge to suggest that a ten percent increase in schooling outlays per student is associated with something like a 1.5 percent increase in earnings for each year of a student's working life. The data also shows that each additional year of education raises workers' wages by 9.5 percent in the United States.

In our model, the Principal seeks to maximize a Social Welfare Function, which is a CES index of utility levels.[4] This welfare function is parameterized by a degree of *social aversion for inequality*. In the limit, an infinite degree of aversion for inequality corresponds to the Rawlsian egalitarian social preferences.

In this framework, we study the optimal allocation of individual education investment, schooling help, and cash, under self-selection and budget constraints. This allocation is a second-best because of informational asymmetries: the optimal solution can be viewed as a *menu of contracts*, each type of agent choosing a given combination of education level, cash transfer and help.

Assuming first that there are only 2 types, when help and individual effort are sufficiently close substitutes as inputs of the education process, we find that the optimal education level of an agent is an increasing function, and optimal help is a decreasing function of talent. In spite of the presence of direct redistribution by means of money transfers, utility levels cannot be equalized, because of self-selection constraints. We obtain a very clear characterization of the equity-efficiency trade-off, in relation with the

[3] Card (1998) provides an up-to-date survey of the literature.

[4] The CES welfare function is associated with the well-known Kolm-Atkinson inequality index, e.g. Atkinson (1970), Kolm (1968).

importance of informational asymmetries. The qualitative property that the less-talented will be helped more and exert less educational effort than the talented is also a feature of the first-best allocation, but is accentuated by distortions under second-best conditions, leading to inefficiently low effort and achievement on the part of the less-talented. If help and effort are complementary inputs, optimal education and help levels are both increasing functions of talent, the other results remaining the same.

We then turn to a comparative statics exercise on the level of aversion for inequality. While differences in utilities tend to decrease when aversion for inequality increases, the downward distortion of the less-talented's education levels somewhat paradoxically increases with the aversion index. More aversion for inequality unequivocally leads to more inequality of educational achievements, and thus to more assistance through redistribution. This remains true in the limit, under strictly egalitarian preferences of the Principal.

We finally explore the general case of a continuum of types with the help of Optimal Control techniques. The same qualitative properties hold as in the discrete case. Bunching at the lower end of the talent scale is a robust feature of the solution for sufficiently high degrees of inequality aversion.

In our model, the qualitative property of the optimal allocation of public spending on education can be called *input progressivity*, in the sense of Arrow (1971).[5] Arrow's 1971 note and the subsequent work have popularized the idea that public expenditures on education are regressive by their very nature, because an utilitarian planner will always allocate education to those who are the most able to benefit from it. Our analysis shows that this conclusion depends on the fact that spending on education and individual education level are constrained to be equal. When the two notions are unbound, and more precisely, if, in addition, individual effort and schooling expenditures are sufficiently close substitutes, one finds the more natural conclusion that help is progressive, while individual education levels are regressive.

Relationships with the literature

Arrow's utilitarian analysis has been pushed further by Bruno (1976), who considers varying degrees of aversion for inequality and their impact on the progressivity or regressivity of optimal public expenditures. His analysis is conducted under complete information assumptions. Ulph (1977), and Hare and Ulph (1979) have taken the subject over to examine situations in which both education and income redistribution policies are simultaneously performed. These authors are forerunners in the study of the problems addressed below; they have sketched the analysis of the incomplete information case, using the optimal taxation techniques of Mirrlees (1971). Tuomala (1986) studies the interaction of individual educational and labour supply choices with the design of the optimal income tax policy, and shows that this changes some qualitative features of the optimal solution, in particular, the properties of the marginal tax rate at the boundary of the set of types. More recently, in a very interesting contribution, De Fraja (1998) has addressed the problem

[5] When the benefit derived by an individual from a given volume of government expenditures depends upon a characteristic of the individual called "ability," an allocation of public expenditures is input progressive if it is decreasing with ability.

of optimal education policies with the help of mechanism design techniques. He assumes that households differ in their income and in the unobservable ability of their children, and introduces the classic justifications for public education: externalities created by the average education level, and capital market imperfections. De Fraja (1998) does not consider varying degrees of aversion for inequality as we do here, and solves his model in a simple quasi-linear utilitarian framework, but he analyzes the constraints imposed on optimal policy implementation by the existence of a private education sector. Finally, Kranich (1997) addresses the equalization-of-opportunities problem in a simple model with education and unobservable abilities, assuming that the planner's preferences are represented by the leximin extension of the Rawlsian criterion; he also finds that informational asymmetries are an obstacle to full compensation of the handicaps.

The problem studied below, and more particularly the continuum-of-types version of the model, is formally close to an optimal taxation problem à la Mirrlees (1971). The results of our last section are reminiscent of the qualitative properties of a standard optimal taxation model, as highlighted in the contributions of Seade (1977), Lollivier and Rochet (1983), and Ebert (1992). But on the other hand, our problem can be viewed as a Principal-Agent problem under pure adverse selection. Due to the nonlinearity of the social welfare function, the classic "tricks," as exposed in Guesnerie and Laffont (1984), cannot be employed to solve the general case, in which bunching typically occurs. The technical problems posed by the Principal's risk aversion are known to be very hard, and few contributions have tackled the case. Salanié (1990) is a good example; he studies a Principal-Agent problem under pure adverse selection, in which a risk-averse agent must contract with the Principal without knowing the value of his or her hidden characteristic. The problem is non-linear because of risk-aversion. Salanié characterizes the solution in the particular case of a uniform distribution of the agent's type. This solution, which exhibits bunching, has a striking formal similarity with ours.

Optimal taxation problems have also been studied under the assumption of a finite number of types by Guesnerie and Seade (1982), and Weymark (1986), (1987). The results obtained by these authors cannot be transposed directly to our education model because of (i) nonlinearity of the welfare function with respect to "utilities", and (ii), multidimensionality of the screening problem. A distinctive feature of our approach is the coexistence of in-kind and cash transfers at the optimal solution.[6] The introduction of in-kind transfers as an additional policy tool creates a multidimensional screening problem, with inevitable technical difficulties. A small number of contributions have tackled these difficulties; in the field of Public Economics, Besley and Coate (1995) propose a very elegant normative analysis of income maintenance programmes in a finite economy; their policy design problem becomes multi-dimensional with the addition of "workfare," that is, when hours of work in the public sector can be required from recipients in exchange for welfare payments. Besley and Coate's contribution illustrates the type of problems created by a three-dimensional menu of contracts when analyzing incentive constraints. Any attempt at generalizing our

[6] The superiority of transfers of purchasing power over transfers of goods and services is known to disappear if information about preferences is not available to the government. In a context different from ours, Blackorby and Donaldson (1988) have demonstrated the usefulness of transfers in kind when the government is constrained by adverse selection.

approach to the case of an arbitrary finite number of unobservable preference types would possess the same degree of complexity. In Besley and Coate (1995), the planner's objective is a simple linear function of the variables, whereas a general version of the problem studied below would involve a nonlinear objective, which is substantially more difficult.

In the following, Section 2 presents the model and its basic assumptions; section 3 presents the case of two types; section 4 is devoted to the study and interpretation of the second-best optimum. Section 5 exposes the comparative statics of inequality aversion. Finally, our results on the case of a continuum of types are exposed in Section 6.

2. The model: basic assumptions

We consider an economy composed of agents with differing privately observed characteristics. The agents can invest in education by choosing a level of effort denoted x. Each agent is characterized by a hidden type, a real number denoted θ, which is the agent's marginal disutility of effort x while investing in education, which is assumed constant. Effort is an input used to "produce" an individual education level denoted e. Schooling expenditure, denoted s, and called "help" in the following, is another input, which can be viewed as an in-kind transfer, provided by the public sector. We assume that education technological possibilities are described by a production function g, giving the educational achievement of an individual as an increasing function of both effort and help,

$$e = g(x, s).$$

The private returns to education, or human capital, are given by a real-valued function, $B(e)$. To simplify the exposition, we assume that $B(e) = e$ identically, which is tantamount to assuming that education levels are directly denominated in units of human capital. By definition, the effort expenditure of agent θ is θx. The minimal amount of effort needed to achieve the education level e while receiving help s is denoted $x = C(e, s)$, where by definition

$$e \equiv g[C(e, s), s]. \tag{0}$$

The total cost of education to agent θ is therefore $\theta C(e, s)$. It follows that θ can be interpreted as the unit price of effort for an agent of type θ, while C is denominated in units of effort.

On top of her private investment decisions, agent θ is subject to a direct money tranfer imposed by the State, and denoted t. We assume that agent θ's utility, denoted $u(\theta)$, has the following quasi-linear form,

$$u(\theta) = t + e - \theta C(e, s). \tag{1}$$

This model is consistent with Becker's classic theory of education as investment (*e.g.*, Becker (1967)); yet, there are several possible interpretations of this formulation. First, the type can be viewed, if such a thing exists, as "pure talent": a level of aptitude that is given at birth, or more realistically, that is inherited from personal history, and marks individuals without any manipulation possibility, at the moment at which education investments are chosen, and the policy problem is posed. Alternatively, the type can be understood as reflecting the characteristics of family background, as opposed to pure gifts, and $(1-\theta)$ can then be interpreted as the share of the education burden which is taken care of by the agent's family, in the broadest sense [7]. According to this alternative view, a high-θ agent is handicapped by a weak family background, whatever the effort C associated with the chosen investment. We will come back to these interpretations after the statement of our results. To sum up, we have assumed that schooling expenditures can reduce the costs with which a given level of human capital can be produced by means of individual effort.

In the following, we show that a number of interesting features of our model are implied by the property that effort and help are substitutes or complements, when viewed as inputs of human capital production.

In this economy, investment in education is planned by a benevolent, but imperfectly informed Planner, or Principal, whith some equity concerns. The Principal has the power to tax and redistribute a scarce resource, called money, the total quantity of which is denoted M. Money can also be used to finance the in-kind schooling help decisions, one unit of help being produced by means of one unit of money, with some constant returns-to-scale technology. To fix ideas and simplify the exposition, we assume that there is no market for the good called help, that is, no private schools.[8]

The Principal cannot observe the types of individuals, but is assumed to know the probability distribution F of these types in the consumer population. The Principal's objective is to choose e, s and t, as a function of θ, so as to maximize the following social welfare functional, denoted W_σ,

$$W_\sigma(u) = \left(\int u(\theta)^\sigma dF(\theta) \right)^{\frac{1}{\sigma}}, \tag{2}$$

where σ is a parameter smaller than 1. Parameter $\rho = 1-\sigma$, is an index of *social aversion for inequality.*

A possible interpretation of (2) is to consider $u(\theta)$ as agent θ's utility, while σ reflects the ethical views of the Principal, who maximizes a CES index of the utility profile, trying to minimize utility differences. There are other possible interpretations.[9] We remain agnostic in the following and use the word u-value, instead of utility, when referring to expression (1).

[7] See Bénabou (1996a) for a discussion of the impact of social networks, community and neighborhood quality on the costs of attaining a certain level of human capital.

[8] If there was a competitive market for help, it would be traded at a money price p=1, because of constant returns to scale. Our first-best solution would be exactly the same. Our second-best solution would also be the same, provided that individual consumptions s are observable by the Principal.

[9] Another possible, and formally equivalent interpretation is utilitarian, or Harsanyian (on these questions, see d'Aspremont and Mongin (1998), Blackorby, Bossert and Donaldson (1999)). According to this view, the utility of agent θ is $v(\theta)=(1/\sigma)u(\theta)^\sigma$, where $\rho=1-\sigma$ is a parameter of utility functions, and $u(\theta)$, as defined by (1) above, is the agent's human capital, net of investment costs. The Principal is then utilitarian, trying to maximize $\int v(\theta)dF(\theta)$, and the social aversion for inequality is directly linked with the individual's degree of risk aversion.

When $\rho = 1 - \sigma$ goes from 0 to $+\infty$, the Principal exhibits more and more aversion for inequality of u-values. In the limit, an infinite degree of aversion corresponds to the Rawlsian maximin, *i.e.*, maximization of the least-favored type, $\min\{u(\theta)\}$.

In the language of the Theory of Justice, an agent's type θ is viewed as a *handicap* that should be compensated to equalize the realized u-levels.[10]

In the following, our goal will be to study second-best optimal educational and redistribution policies under adverse selection (non-observable types), and to perform comparative statics exercises on σ.

For the sake of simplicity, we will make assumptions directly on the cost function C and its derivatives, but, clearly, each of our assumptions has a counterpart in terms of the production function g and its derivatives, using identity (0) above. The partial derivatives of any function with respect to one or several variables will be indicated by subscripts, as usual. For the needs of the analysis, we assume the following,

Assumption 1

The mapping C is twice continuously differentiable, with partial derivatives satisfying,
(i) $C_e > 0$; $C_s < 0$;
(ii) $C_e \to 0$ as $e \to 0$ *for all* s; $C_s \to 0$ as $s \to +\infty$, *and* $C_s \to -\infty$ as $s \to 0$ *for all* e;
(iii) C *is strictly convex;*
(iv) *if* $C_e + C_s = 0$, *then,* $C_{ee} > |C_{es}|$ *and* $C_{ss} > |C_{es}|$.

Education is strictly costly, while help decreases the total cost. Assumption 1(ii) will simplify the analysis by ensuring the interiority of education and help; it is an expression of the classic Inada conditions, in terms of the cost function derivatives. Assumption 1(iii) says that the returns to scale in the production of an *individual* level of education are strictly decreasing. Assumption 1(iv) is more restrictive; it says that when $C_e + C_s = 0$, that is, when help is efficiently allocated[11], the cross second-order derivative C_{es} is sufficiently small in absolute value. It is relatively straightforward to show that condition 1(iv) will be satisfied if effort x and help s are sufficiently close substitutes in the production function g. But effort and help need not be perfect substitutes. For instance, if g has the form $g(x,s) = \phi[h(x) + k(s)]$, it can be shown that assumption 1(iv) is satisfied if the three functions ϕ, h and k are twice continuously differentiable, strictly increasing and concave.[12]In the following, we consider variations on the technical assumption 1(iv), and for the sake of completeness, we also treat the case of strict complementarity of effort and help.

[10] Although the quantity x is dubbed effort, and the parameter θ represents a personal talent, or handicap, our model bears little relation to the theory of equity with responsibility (*e.g.,* Fleurbaey (1998), Roemer (1998)), because the agents all have the same structural preferences, and differ only in the parameter θ, for which they are not responsible. Their choice of effort is entirely determined by this parameter and the shape of their option set; this is why our Planner is interested in equality of final utilities, and not in some intermediate outcome.

[11] Remark that $C_e + C_s = 0$ is a necessary condition for maximization of the net surplus $t + e - s - \theta C(e,s)$ with respect to e and s.

[12] A classic example would be the CES form $g(x,s) = (x^a + s^a)^{b/a}$ with $0 < b < a < 1$. The elasticity of substitution $1/(1-a)$ must therefore be higher than 1 here.

3. The two-types case with schooling help

3.1. The Principal's optimal education contract problem

We start with a simple framework in which the set of types is discrete. Section 6 below addresses the more technical case of a continuum of types.

Assumption 2

The set of types is $\{\theta_1, \theta_2\}$*, with* $\theta_1 < \theta_2$*, and the probability of type* θ_i *is* $p_i > 0$*,* $i = 1, 2$*.*

Under Assumption 2, type 2 has a higher cost, as compared to type 1. In the following, we will call the low-cost types the "talented" and the high-cost ones the "less-talented".

Now, by definition, a contract is a mapping $\theta_i \to (e_i, s_i, t_i)$, specifying an education level, a quantity of help, and a money transfer for each type $i = 1, 2$.

We define the first-best situation as the one in which both θ_i and e_i are observed by the Principal. By contrast, in the second-best world, θ_i is not observed. By the Revelation Principle, there is no loss of generality in restricting the Principal to use *direct revelation mechanisms* in which each agent declares a type θ_i, while the Principal decides which level e_i should be achieved by, and the amounts s_i and t_i that will be given to, agent θ_i. This is equivalent to saying that the Principal uses self-selection by means of a *menu of contracts* satisfying incentive compatibility (IC) constraints, to ensure differential treatment of types.

The second-best education planning problem of the Principal is as follows,

$$\max_{(e_i, s_i, t_i)} (1/\sigma)(p_1 u_1^\sigma + p_2 u_2^\sigma)$$

subject to the constraints

$$\begin{aligned} u_i &= t_i + e_i - \theta_i C(e_i, s_i), \quad (i = 1, 2) && (U0) \\ M &= p_1(t_1 + s_1) + p_2(t_2 + s_2), && (BB) \\ u_1 &\geq t_2 + e_2 - \theta_1 C(e_2, s_2), && (IC_1) \\ u_2 &\geq t_1 + e_1 - \theta_2 C(e_1, s_1). && (IC_2) \end{aligned}$$

3.2. Computation of the first best

To compute the first-best allocation (e_i^*, s_i^*, t_i^*), we simply solve the above optimization problem while ignoring the constraints IC_1 and IC_2.

The necessary (and sufficient) first-order conditions for optimality are obtained by standard computations, and read as follows.

$$1 = \theta_i C_e(e_i^*, s_i^*), \tag{$FB1$}$$

$$1 = -\theta_i C_s(e_i^*, s_i^*), \tag{$FB2$}$$

$$\text{if } \sigma < 1, \quad u_1^* = u_2^*, \tag{$FB3$}$$

$$M = p_1(t_1^* + s_1^*) + p_2(t_2^* + s_2^*). \tag{BB}$$

In the above expression, we use the notation $u_i^* = e_i^* + t_i^* - \theta_i C(e_i^*, s_i^*)$. Expression FB1 says that the optimal allocation equalizes the marginal cost of investment in education for type i with its marginal value, equal to 1 here. FB2 says that help should be chosen so as to equalize the marginal cost reduction for type i with the resource cost of a unit of help, which is equal to 1 by assumption. Note that substracting FB1 and FB2 yields the relation $C_e^* + C_s^* = 0$. Expression FB3 shows that utilities are perfectly equalized at the first best, while BB is a simple restatement of the Principal's budget constraint ($FB3$ and BB together define the optimal money transfers (t_1^*, t_2^*)).

Consider now the first-best education and help functions $\theta \to (e^*(\theta), s^*(\theta))$, obtained as a solution to $FB1$ and $FB2$.

Proposition 1

Under Assumption 1, e^* is decreasing and s^* is increasing as a function of θ.

Proof. By the Implicit Function Theorem, we get by differentiation of $FB1$ and $FB2$,

$$\begin{pmatrix} C_{ee} & C_{es} \\ C_{se} & C_{ss} \end{pmatrix} \begin{pmatrix} e_\theta^* \\ s_\theta^* \end{pmatrix} = \begin{pmatrix} -1/\theta^2 \\ +1/\theta^2 \end{pmatrix}. \tag{3}$$

Inverting (3) yields

$$\begin{pmatrix} e_\theta^* \\ s_\theta^* \end{pmatrix} = \frac{1}{det(C'')} \begin{pmatrix} C_{ss} & -C_{se} \\ -C_{es} & C_{ee} \end{pmatrix} \begin{pmatrix} -1/\theta^2 \\ +1/\theta^2 \end{pmatrix}. \tag{4}$$

Under Assumption 1, $det(C'') = C_{ee}C_{ss} - (C_{se})^2 > 0$, $C_{ee} + C_{es} > 0$ and $C_{ss} + C_{se} > 0$, and the result easily follows from (4). *Q.E.D.*

First-best optimality thus says that, when help and effort are substitutes, the less-talented should reach a smaller education level ($e_1^* > e_2^*$), but should be helped more ($s_1^* < s_2^*$) than the talented, while full equality of u-levels is achieved by means of direct money redistribution.

The question is then to determine which, among this first set of results, is robust to a variation of Assumption 1(iv). In the statement of Assumption 1(iv), the inequality $C_{ss} \geq |C_{es}|$ is typically satisfied if returns to scale are strictly decreasing and inputs exhibit enough complementarity; only the inequality $C_{ee} \geq |C_{es}|$ is really restrictive, for it can be shown to be equivalent to $g_{sx} \leq 0$. With enough complementarity of factors x and s, standard production functions g, such as the CES, would satisfy the property $C_{ss} \geq |C_{es}| \geq C_{ee}$ and, of course, $C_{es} < 0$. A glance at the proof of Proposition 1 above then yields the following Corollary of Proposition 1.

Corollary 1

If Assumption 1(i)-(iii) is satisfied, and if $C_e + C_s = 0$ implies $C_{ss} \geq |C_{es}| \geq C_{ee}$ and $C_{es} < 0$, then $e^*(\theta)$ and $s^*(\theta)$ are both decreasing with respect to θ.

Corollary 1 shows that under enough complementarity of effort and help, the more talented should receive more help in the first-best optimal allocation. The result will be the same in the strict complements case, studied in Appendix A.

To sum up, under very general conditions, the optimal education level is an increasing function of talent (or decreasing function of the handicap); help will compensate the handicaps only if individual effort and help are good substitutes. This conclusion might seem "ethically repugnant" to the reader: we propose an interpretation of why it is so in the general discussion of our results below.

4. Incentive constraints and second-best optimality

4.1. Analysis of incentive constraints

Consider now the complete optimization program including the IC constraints. We will transform this program before a solution can be found. To simplify notation, from now on, let us denote $C_i = C(e_i, s_i)$.

Lemma 1

IC_1 holds as an equality at the optimum.

Proof. Assume that Lemma 1 is false. Then, $u_1 > t_2 + e_2 - \theta_1 C_2 > u_2$ because $\theta_2 > \theta_1$. The u-value inequality between type 1 and type 2 can then be reduced. To this end, diminish t_1 by $\epsilon > 0$ and increase t_2 by $\epsilon p_1/p_2$, which preserves BB and IC_2. The impact of this change on the social objective is approximately $(-p_1\epsilon)u_1^{\sigma-1} + p_2(\epsilon p_1/p_2)u_2^{\sigma-1} = p_1\epsilon(u_2^{\sigma-1} - u_1^{\sigma-1}) > 0$, for ϵ sufficiently small, because $\sigma < 1$ and $u_1 > u_2$. It follows that social welfare can be increased without violating BB and IC constraints, a contradiction. *Q.E.D.*

Lemma 2

If IC_1 holds as an equality then, IC_2 is equivalent to $C(e_1, s_1) \geq C(e_2, s_2)$.

Proof. Remark that if IC_1 holds as an equality, $t_1+e_1-\theta_1 C_1 = t_2+e_2-\theta_2 C_2+(\theta_2-\theta_1)C_2$. Then, using IC_2 and the above equation, one gets $t_1+e_1-\theta_1 C_1-(\theta_2-\theta_1)C_2 \geq t_1+e_1-\theta_2 C_1$, which is equivalent, after simplification, to $C_1 \geq C_2$, given that $\theta_2 > \theta_1$. *Q.E.D.*

Recalling that $x_i = C(e_i, s_i)$, it follows from Lemma 2 that in any incentive-compatible optimal solution, the talented exert more effort than the less-talented. Let now u_i^{**} denote the second-best optimal levels of u.

Lemma 3

$u_1^{**} > u_2^{**}$ if and only if $C(e_2^{**}, s_2^{**}) \neq 0$.

Proof. By IC_1, we immediately get $u_1^{**} = u_2^{**} + (\theta_2 - \theta_1)C_2^{**}$. *Q.E.D.*

Using the above Lemmata, the Principal's optimization problem can be rewritten in a convenient form. Transfers can first be eliminated from BB, using $t_i = u_i - e_i + \theta_i C_i$. Condition BB then becomes BB',

$$M = \sum_{i=1,2} p_i(-e_i + \theta_i C_i + s_i + u_i). \qquad (BB')$$

Condition IC_1, holding as an equality, can also be rewritten as a relation between u-levels, $u_1 = u_2 + (\theta_2 - \theta_1)C_2$. To clarify notations, define now the function,

$$M(e, s) = M + \sum_{i=1,2} p_i(e_i - \theta_i C_i - s_i). \qquad (5)$$

Using (5), it is easy to see that BB' and IC_1 together form a linear system in two unknowns, u_1 and u_2, for any fixed vector (e, s), that is,

$$p_1 u_1 + p_2 u_2 = M(e, s), \qquad (BB')$$

$$u_1 - u_2 = (\theta_2 - \theta_1)C_2. \qquad (IC_1')$$

Solving this system yields u_1 and u_2 as a function, of (e, s), a solution that we shall denote $\hat{u}_1$ and $\hat{u}_2$ (see expressions $U1$ and $U2$ below). With the help of these results, the Principal's optimization program can be rewritten as follows.

$$\max_{(e,s)} \sigma^{-1}(p_1 \hat{u}_1^\sigma + p_2 \hat{u}_2^\sigma)$$

subject to the constraints,

$$\hat{u}_1 = M(e, s) + p_2(\theta_2 - \theta_1)C(e_2, s_2), \qquad (U1)$$

$$\hat{u}_2 = M(e, s) - p_1(\theta_2 - \theta_1)C(e_2, s_2), \qquad (U2)$$

$$C(e_1, s_1) \geq C(e_2, s_2). \qquad (IC_2')$$

Remark that subtracting $U2$ from $U1$ yields IC_1', and $(\theta_2 - \theta_1)C_2$ can be interpreted as the informational rent of type 1. It is then easy to provide a graphical representation of the constraints in the (u_1, u_2)-plane, for any fixed value of (e, s) such that $C_1 \geq C_2$. By the above Lemmata, the optimal solution is necessarily at the intersection of BB' and IC_1'. See Figure 1. In utility space, constraint IC_2 writes,

$$u_2 \geq u_1 - (\theta_2 - \theta_1)C_1,$$

and it is clear from Figure 1 that it does not bind.

[Insert Figure 1 around here]

4.2. Second-best optimum: first-order conditions

In the above reformulation, once conditions $U1$ and $U2$ have been substituted in the Principal's objective, the first-order necessary conditions for second-best optimality can be obtained by unconstrained optimization of the objective with respect to e_i and s_i, $i = 1, 2$, provided that IC_2' holds. These necessary conditions can also be shown to be sufficient, because C is convex and $(1/\sigma)u^\sigma$ is a concave function of u for $\sigma \leq 1$. After standard computations, this yields the following equations.

$$1/\theta_1 = C_e(e_1, s_1), \tag{$SB1$}$$

$$-1/\theta_1 = C_s(e_1, s_1), \tag{$SB2$}$$

$$\frac{1}{\theta_2 + (\theta_2 - \theta_1)\delta(\hat{u})} = C_e(e_2, s_2), \tag{$SB3$}$$

$$\frac{-1}{\theta_2 + (\theta_2 - \theta_1)\delta(\hat{u})} = C_s(e_2, s_2), \tag{$SB4$}$$

where,

$$\delta(\hat{u}) = \frac{p_1(\hat{u}_2^{\sigma-1} - \hat{u}_1^{\sigma-1})}{p_1\hat{u}_1^{\sigma-1} + p_2\hat{u}_2^{\sigma-1}}, \tag{6}$$

is a distortion term.

Conditions $SB1$ and $SB2$ are merely a restatement of the first-best conditions $FB1$ and $FB2$, showing that the most talented types θ_1 have the same education conditions than in the first best. By constrast, the education and help levels of the type 2 are distorted; they are treated as if they were more handicapped than they actually are, their type parameter θ_2 being increased by the distortion term $(\theta_2 - \theta_1)\delta(\hat{u})$.

Let the mapping $\theta_2 \to (e^{**}(\theta_2), s^{**}(\theta_2))$ denote the solution of equations $SB3$ and $SB4$, given that $e^{**}(\theta_1)$ and $s^{**}(\theta_1)$ are fully determined by equations $SB1$ and $SB2$. A double star ** will indicate that the marked variable, or function, is evaluated at its second-best optimal value. We summarize some of the results that can be deduced from the above conditions in the following Proposition.

Proposition 2. *Under Assumptions 1 and 2,*

(i) there is no distortion at the top,

$$e^*(\theta_1) = e^{**}(\theta_1); \qquad s^*(\theta_1) = s^{**}(\theta_1);$$

(ii) the less-talented are less educated and helped more as compared to first-best optimality,

$$e^{**}(\theta_2) < e^*(\theta_2) < e^{**}(\theta_1); \qquad s^{**}(\theta_2) > s^*(\theta_2) > s^{**}(\theta_1);$$

(iii) the effort of the talented is strictly greater than that of the less-talented (constraint IC_2' is satisfied as a strict inequality).

$$C_1^{**} > C_2^{**};$$

(iv) finally, types are unequal, because of the informational rent,

$$u_1^{**} - u_2^{**} = (\theta_2 - \theta_1)C_2^{**}.$$

Proof. We use e_i^{**}, s_i^{**} as a shorthand notation for the solution $e^{**}(\theta_i)$, $s^{**}(\theta_i)$, $i = 1, 2$. Point (i) is trivially true since $SB1$ and $SB2$ are equivalent to $FB1$ and $FB2$. To prove point (ii), remark that e_2^{**} and s_2^{**} are determined just like e_2^* and s_2^*, but as if a virtual type $\theta_2 + (\theta_2 - \theta_1)\delta(\hat{u})$ had been substituted for the real type. The result then follows from Proposition 1 and the fact that $\delta(\hat{u}) > 0$. Point (iii) is a consequence of Assumption 1 and of point (ii), because $C(e_1^{**}, s_1^{**}) > C(e_2^{**}, s_1^{**}) > C(e_2^{**}, s_2^{**})$. Finally, (iv) is simply a restatement of IC_1. *Q.E.D.*

Some remarks are useful at this stage. Note first that if $\sigma = 1$, there is no social aversion for inequality, and $\delta(\hat{u}) = 0$. It then follows that the first-best and second-best allocations coincide, $(e_2^{**}, s_2^{**}) = (e_2^*, s_2^*)$.

By contrast, if $\rho = 1 - \sigma$ goes to infinity, then, $\delta \to p_1/p_2$ pointwise. (To check this, write $\delta(\hat{u}) = p_1[p_1(\hat{u}_1/\hat{u}_2)^{\sigma-1} + p_2]^{-1} - p_1[p_1 + p_2(\hat{u}_2/\hat{u}_1)^{\sigma-1}]^{-1}$. Since $\hat{u}_1 > \hat{u}_2$, we get that δ goes to $(p_1/p_2) - 0$ when $1 - \sigma$ goes to infinity.)

An important consequence of this is that second-best optimality continues to entail downward distortions of effort for type 2 in the limit, because the limiting value of the distortion appearing in $SB3$ and $SB4$ is $(\theta_2 - \theta_1)(p_1/p_2) > 0$. And, provided that non-pathologically, $C(e_2^{**}, s_2^{**}) \neq 0$, the limit continues to exhibit inequality of u-levels. This solution is obtained under the Rawlsian maximin procedure. To see this more rigorously, note that the Rawlsian egalitarian allocation solves the program,

$$\max_{(e,s)} \hat{u}_2, \quad \text{s. t.} \quad (U1) \,\text{and}\, (U2),$$

given that under IC_1, $\hat{u}_1 \geq \hat{u}_2$. It is then easy to see that the first-order necessary conditions for this problem are exactly SB1-4 above, with $\delta = p_1/p_2$. It follows that point (iii) and (iv) in Proposition 2 typically continue to hold in the egalitarian limit. Figure 2 represents the solution in the Rawlsian case.

[Insert Figure 2 around here]

Insofar as the less-talented choose smaller effort and education levels, equality is more efficiently achieved through direct redistribution than by means of education in the first best situation. Now, under the second-best conditions of unobservable talents and handicaps, there will be more help for the less-talented than in the first-best solution, but at the cost of more education level regressivity. The less-talented will choose inefficiently low levels of effort as a result of the second-best logic.

The Principal faces a trade-off between equity and efficiency, which is created by revelation incentives. From the Revelation Principle, we know that the Principal cannot do better than to rely on revealing mechanisms, and is thus limited by incentive constraints. To induce truthful revelation by the talented types, who can by assumption always pose as less-talented, the Principal must pay an informational rent, the exact value of which is $(\theta_2 - \theta_1)C(e_2, s_2)$. This informational rent forces a minimal level of inequality between types, that can only be reduced by decreasing the education costs of the less-talented, more precisely, by a decrease in the effort e_2, and (or) by an increase in the amount of help s_2.

In-kind transfers of help are used for the purpose of reducing C_2, and thus to reduce the informational rent, but at some point, it is less costly to use money tranfers to redistribute directly, and the incentive constraint of the talented is binding. The educational effort of the less-talented will typically not be distorted downwards to zero, because this would be very inefficient, that is, very costly in terms of social surplus, and particularly costly if the less-talented are numerous. As a result, inequality is incompressible, even if the aversion for inequality is very high. In the next section, we prove the striking result that a more egalitarian Principal will distort the educational allocation of the less-talented more than a less inequality-averse Principal.

But before we turn to this, and for the sake of completeness, we should explore how Proposition 2 would change under the variant of Assumption 1(iv) studied above. It happens that results do not change, except that e^{**} and s^{**} would both be decreasing functions of θ (which parallels Corollary 1 above).

Corollary 2

If the production function g is strictly concave, Assumption 1(i)-(iii) are satisfied, and if $C_e + C_s = 0$ implies $C_{ss} \geq |C_{es}| \geq C_{ee}$ and $C_{es} < 0$, then, points (i), (iii) and (iv), in Proposition 2 remain true, but e^{**} and s^{**} are both decreasing functions of θ, so that,

$$e^{**}(\theta_2) < e^*(\theta_2) < e^{**}(\theta_1); \qquad s^{**}(\theta_2) < s^*(\theta_2) < s^{**}(\theta_1).$$

For proof, see Appendix B.

In Appendix A below, we study the case of strict complementarity of effort and help, assuming that $g(x, s) = \min\{x, s\}$, the results are essentially the same as in Corollary 2.

4.3. Discussion and interpretation of the results

To sum up, we have shown that the main features of our second-best optimal allocation are reasonably robust: inequality is essentially imposed by informational asymmetries, which also cause distortions of the allocation in the form of inefficiently low effort on the part of the more handicapped. The fact that educational achievements and help levels vary in opposite directions with the handicap parameter depends on a property of the cost function's second-order derivatives. This property is in turn satisfied when individual effort and help are sufficiently good substitutes in the production of education. It is of course an empirical question to determine whether indeed help and effort are substitutes or complements. According to specialists of the field,[13] and to the best of our knowledge, this question has not been studied by econometricians. But some difficulties will certainly appear while trying to decide in favor of the hypothesis. There are more than two inputs in the real-life process of educating a person, some of which might be complements, while others are substitutes. Our "help" and "effort" variables can be seen

[13] D. Card (personal communication).

as aggregates of several sub-inputs. The existence of self-made men (and women) might be seen by some as evidence of the high substitutability of help and effort, while others will emphasize that individuals completely deprived of some essential type of "educational food" or initiation into crucial knowledge cannot succeed in life. The present contribution shows the importance of these ideological positions, and ideally, empirical work is needed to choose among alternative assumptions.

Now, the robust conclusion that the optimal education level is a decreasing function of the handicap might seem ethically repugnant to the reader, but the result's meaning varies with the interpretation given for the type parameter θ, as well as with other implicit assumptions of our model. If the handicap parameter θ is viewed as measuring pure talent or gifts, then, the conclusion is more or less inescapable: the regressivity of education levels is entirely driven by efficiency considerations, *given that* the Planner has degrees of freedom in the use of direct redistribution to equalize utilities. If agents differed not only by their talents, but also by an observable "endowment" (their parents' income for instance), then, our menu of contracts could easily be generalized to become conditional on the observed parental characteristics. Then, within each income class, the qualitative features of the self-selective menu would be the same as described above. With this first interpretation in mind, our model describes an idealized meritocracy, in which possibilities of direct, means-tested redistribution can be used: poor talented children will end up being treated exactly as the talented children of rich families, in terms of education levels and effort demands (for optimal education levels are independent of bequests).

If in contrast, the handicap parameter more or less reflects the family background of the agent, then, several questions can be raised. First of all, the efficiency argument is no longer entirely convincing, at least from an ethical point of view. Families being very important in the process of human capital investment, the model could then be extended to describe the allocation of expenditures aimed at narrowing the gap $\theta_2 - \theta_1$ between weak and strong families directly, that is, to describe a form of assistance to families in combination with expenditure on schools. This would mitigate the troubling character of the conclusion, by adding a policy instrument to the analysis,[14] but the use of this policy instrument would only be pushed to the point at which its marginal social value and cost are equated, and presumably, this would not completely "level the playing field" by fully equalizing the types, and it follows that our analysis remains valid. If parental income and handicap parameters are statistically correlated, a special analysis is required. These questions could become the topic of some future work.

In addition to these considerations, our way of modelling direct redistribution by money transfers deserves a comment. Once informational rents are taken into account, redistribution imposes no additional cost in our setting, and there is no tendency towards second-best over-use of the alternative in-kind transfers of help to equalize utilities. The burden of equalization essentially falls on taxes, while that of investing an efficient level of resources in human capital falls on education, explaining why the latter remains hopelessly regressive. Introducing other social costs of taxation would impose a limit to direct redistribution in the optimal allocation, and would mitigate the regressivity of education. More generally, from the point of view of Political Economy, our normative solution im-

[14] See S. E. Mayer (1997) for a discussion of the empirical aspects of this question in the U.S.

plicitly rests on blind confidence in the redistribution system: the handicapped people are made completely dependent on a commitment of the public sector, whereas the willingness of future legislatures to continue paying the money transfers is uncertain. In contrast, a high level of education is an inalienable (albeit not perfect) protection against poverty, and against long run changes in redistribution policies. This is probably the real reason why the Egalitarian — and to a certain extent, the Utilitarian — instinctively want to reduce the inequality of education levels: this would happen in our model if tranfers were discounted to take commitment credibility problems into account. From the point of view of modern Political Economy, the present contribution is thus a possible benchmark, and more work is needed to explore these ideas.

We finally devote a few comments to possible relationships of the present contribution with the *affirmative action* debate. Our work has nothing to do with affirmative action in the usual North-American sense of the term, and first of all because the policies described above are strictly *color-blind*. If color-blind policies are viewed as desirable (as they obviously would in a country like France, for instance), then, our results describe a possible direction of reform, based on self-selection in a menu of contracts. Some formal principles of equality or non-discrimination can be, in some sense, reconciled with a form of "positive discrimination" through self-selection: replacing color-based affirmative action with color-blind self-selection is like changing from first-degree to second-degree discrimination. These remarks are of course very preliminary and incident, and we will not push the discussion further here.

5. The comparative statics of inequality aversion

We now turn to the comparative statics of parameter ρ. Define the mapping $\sigma \rightarrow (u_1^{**}(\sigma), u_2^{**}(\sigma))$ which gives the second-best optimum u-levels of the two types as a function of parameter σ. Assume that the index of inequality aversion, $\rho = 1 - \sigma$ increases, or equivalently, that σ decreases toward $\sigma' < \sigma$. Will this yield a reduction of inequality? The answer is yes. A look at Figure 3 will show why.

[Insert Figure 3 around here]

The indifference curves of the CES welfare function W_σ are convex in the (u_1, u_2)-plane. It is well-known that the curvature of these indifference curves increases when σ diminishes, and that they tend to look like the rectangular indifference curves of $min\{u_1, u_2\}$. Remark in addition that the set of feasible pairs (u_1, u_2) is the mapping of the set of feasible vectors (e, s) in the (u_1, u_2)-plane by expressions $U1$ and $U2$, and that this set is independent of σ. The feasible set of u-level pairs must lie entirely below the diagonal $u_1 = u_2$, and below the indifference curve of W_σ passing through the solution $u^{**}(\sigma) = (u_1^{**}(\sigma), u_2^{**}(\sigma))$. Consider now the indifference curve of $W_{\sigma'}$ passing through the same point $u^{**}(\sigma)$. With $\sigma' < \sigma$, the slope of the latter indifference curve is smaller in absolute value at point $u^{**}(\sigma)$. A simple revealed-preference argument shows that the new solution, that is, $u^{**}(\sigma')$ must be located, *(i)*, above the indifference curve of $W_{\sigma'}$ passing through $u^{**}(\sigma)$, *(ii)*, below the diagonal, and *(iii)*, below the indifference curve of W_σ passing through $u^{**}(\sigma)$; to sum up, the new solution is in the shaded region on Figure 3. This shows that $u_1^{**}(\sigma) - u_2^{**}(\sigma)$ must diminish with σ. We have proved the following result.

Lemma 4

The difference of second-best optimal u-levels $u_1^{**}(\sigma) - u_2^{**}(\sigma)$ is a nonincreasing function of the aversion for inequality $\rho = 1 - \sigma$.

From this result, we deduce the next Proposition.

Proposition 3

Under Assumptions 1 and 2, the second-best effort e_2^{**} is a decreasing function of the index of inequality aversion ρ; the second-best help level s_2^{**} is an increasing function of ρ; and the optimal value of the distortion δ^{**} increases with ρ.

Proof. To prove the result, recall first that e_1^{**} and s_1^{**}, are determined by equations $SB1$ and $SB2$ and do not depend on σ. Remark then that by IC_1, $u_1^{**} - u_2^{**} = (\theta_2 - \theta_1)C(e_2^{**}, s_2^{**})$, and therefore, when σ decreases, by Lemma 4, it must be that $C(e_2^{**}, s_2^{**})$, also decreases. It remains to prove that e_2^{**} and s_2^{**} must vary in opposite directions with σ. Differentiation of $SB3$ and $SB4$ yields the following system, after some computations.

$$\begin{pmatrix} C_{ee} & C_{es} \\ C_{se} & C_{ss} \end{pmatrix} \begin{pmatrix} e_{2\sigma}^{**} \\ s_{2\sigma}^{**} \end{pmatrix} = \begin{pmatrix} -a_\sigma \\ +a_\sigma \end{pmatrix}, \tag{7}$$

where,

$$a_\sigma = \frac{(\theta_2 - \theta_1)\delta_\sigma}{(\theta_2 + (\theta_2 - \theta_1)\delta)^2}, \tag{8}$$

and δ_σ is the total derivative of the optimal distortion (6) with respect to σ. Standard inversion of (7) then yields,

$$\begin{pmatrix} e_{2\sigma}^{**} \\ s_{2\sigma}^{**} \end{pmatrix} = \frac{1}{det(C'')} \begin{pmatrix} C_{ss} & -C_{se} \\ -C_{es} & C_{ee} \end{pmatrix} \begin{pmatrix} -a_\sigma \\ +a_\sigma \end{pmatrix}. \tag{9}$$

Under Assumption 1, $det(C'') > 0$, $C_{ss} + C_{es} > 0$, $C_{ee} + C_{es} > 0$, and it is easy to check that

$$sign(e_{2\sigma}^{**}) = sign(-a_\sigma) = sign(-\delta_\sigma) = sign(-s_{2\sigma}^{**}). \tag{10}$$

Therefore, e_2^{**} and s_2^{**} vary in opposite directions when σ varies. Since C_2^{**} decreases, under Assumption 1, it must be that the effort decreases and the help increases when σ decreases. As a corollary of this result, we deduce from (10) that $\delta_\sigma < 0$, *i.e.*, the optimal distortion increases with inequality aversion. *Q.E.D.*

Given the interpretation provided for Proposition 2 above, Proposition 3 becomes easy to understand. When inequality aversion increases, the Principal will sacrifice more and more surplus, by means of downward distortions of the educational efforts, to reduce the informational rent of the talented types, thereby reducing the inequality of u-values accross types. This is why distortions increase with ρ. The Principal is all the more keen on helping and reducing the education level of the less-talented, since inequality aversion is high. Under informational asymmetries, the optimal policy is to promote de luxe, subsidized vocational training with a lot of teaching personnel for the less talented, and materially austere, yet intellectually demanding graduate programs for the brilliant ones.

The essential results do not change under the alternative assumption on the derivatives of C. Apart from the fact that help and education would vary in the same direction, the distortions still increase with inequality aversion. We get the following result.

Corollary 3

If the production function g is strictly concave, if Assumptions 1(i)-(iii) are satisfied, and if $C_e + C_s = 0$ implies $C_{ss} \geq |C_{es}| \geq C_{ee}$ and $C_{es} < 0$, then, second-best effort e_2^{**} and second-best help s_2^{**} are decreasing functions of aversion for inequality ρ; the optimal value of the distortion δ^{**} increases with ρ.

For proof, see Appendix B

6. The case of a continuum of types

We consider now the case of a continuum of types θ. We replace Assumption 2 with the following.

Assumption 3

The probability distribution of types θ has a compact support $[\underline{\theta}, \bar{\theta}]$, a continuous cumulative distribution denoted F, with a strictly positive density $f = F'$.

To simplify the analysis, it is assumed here that the amount of "help" is fixed and uniformly distributed among agents. The cost of effort is simply $C(e)$, and is assumed strictly increasing and convex with respect to e. We will study the direct revelation mechanisms $\theta \rightarrow (e(\theta), t(\theta))$, where e specifies the education levels, and t is a transfer.

6.1. Necessary and sufficient conditions for second-best optimality

If agent θ declares type $\hat{\theta}$ to the Principal, she gets the utility level $t(\hat{\theta}) + e(\hat{\theta}) - \theta C[e(\hat{\theta})]$. If $\hat{\theta} = \theta$ is the best report of agent θ, then necessarily, the following fist-order condition holds,

$$t'(\theta) = -e'(\theta)[1 - \theta C'(e(\theta))], \tag{11}$$

for all θ, together with the second-order condition,

$$t''(\theta) + e''(\theta)[1 - \theta C'(e(\theta))] - e'(\theta)^2 \theta C''(e(\theta)) \leq 0. \tag{12}$$

Using the fact that (11) holds identically and taking the second derivative of t in (11) yields, $t''(\theta) = -e''(\theta)[1 - \theta C'(e(\theta))] + e'(\theta)[\theta C''(e(\theta))e'(\theta) + C'(e(\theta))]$. Substituting this relation in (12) yields the monotonicity condition, for all θ,

$$e'(\theta) \leq 0, \tag{13}$$

given that $C' > 0$. To sum up, a direct revealing mechanism satisfies (11) and (13). It is also possible to check that (11) and (13) are sufficient for truthful revelation (this is a well-known result, see Guesnerie and Laffont (1984)).

We redefine,

$$u(\theta) = t(\theta) + e(\theta) - \theta C(e(\theta)). \tag{1'}$$

In addition let M^s be the amount of available resources, net of the fixed in-kind transfers. The optimization program of the Principal now becomes,

$$\max_{e(.),t(.)} \sigma^{-1} \int_{\underline{\theta}}^{\bar{\theta}} u(\theta)^\sigma f(\theta) d\theta,$$

subject to the constraints,

$$\int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta) d\theta = M^s, \tag{BB}$$

$$t'(\theta) = -e'(\theta)[1 - \theta C'(e(\theta))], \tag{IC^1}$$

$$e'(\theta) \leq 0, \tag{IC^2}$$

and $e(\theta) \geq 0$. This problem is typically difficult to solve, except if $\sigma = 1$. In particular, known methods, such as those used in the classic contribution of Guesnerie and Laffont (1984) cannot be employed here, because of the nonlinear structure of the objective function.

We transform the problem before writing the necessary conditions for optimality. First integrate IC^1 to obtain,

$$t(\bar{\theta}) - t(\theta) = -\int_{\theta}^{\bar{\theta}} e'(\tau)[1 - \tau C'(e(\tau))] d\tau. \tag{14}$$

Remark that $(e - \theta C)' = e'(1 - \theta C') - C$, so that, integrating by parts the right-hand side of (14), we get $t(\theta) - t(\bar{\theta}) = \int_{\theta}^{\bar{\theta}} C d\tau + [e - \tau C]_{\theta}^{\bar{\theta}}$, which can again easily be transformed, using $(1')$, into the following expression.

$$u(\theta) = u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} C(e(\tau)) d\tau. \tag{15}$$

This relation holds for all θ. The inequality of u-values is again a consequence of IC constraints. The utility of type θ is the sum of the utility of the least-favored type $u(\bar{\theta})$ to which an informational rent is added. This rent is denoted r and defined as,

$$r(\theta) = \int_{\theta}^{\bar{\theta}} C(e(\tau))d\tau. \tag{16}$$

The term $u(\bar{\theta})$ is a constant of integration, to be determined. We will denote $\beta = u(\bar{\theta})$ to simplify notation. Substituting (15) into (BB) yields $M^s = \int_{\underline{\theta}}^{\bar{\theta}}(u - e + \theta C)fd\theta = \beta - \int_{\underline{\theta}}^{\bar{\theta}}(e - \theta C)fd\theta + \int_{\underline{\theta}}^{\bar{\theta}} rd\theta$. Computing the double integral at the right-hand side of this expression finally yields,

$$M^s = \beta + \int_{\underline{\theta}}^{\bar{\theta}} \Big[\Big(\theta + \frac{F(\theta)}{f(\theta)}\Big)C(e(\theta)) - e(\theta)\Big] f(\theta)d\theta, \tag{17}$$

which expresses the budget constraint, once the total cost of informational rents has been taken into account. This constraint in integral form can be transformed into a differential equation, introducing the function $k(\theta)$, with $k' = [e - (\theta + F/f)C]f$, and the initial and terminal conditions, $k(\underline{\theta}) = 0$ and $k(\bar{\theta}) = \beta - M^s$. With this reformulation, the Principal's optimization program can be rewritten as a standard optimal control problem, as follows.

$$\max_{\beta, y(.)} \; \sigma^{-1}\int_{\underline{\theta}}^{\bar{\theta}} (\beta + r(\theta))^{\sigma} f(\theta)d\theta, \tag{18}$$

subject to the constaints,

$$r'(\theta) = -C(e(\theta)), \tag{19}$$

$$k'(\theta) = \Big[e(\theta) - \Big(\theta + \frac{F(\theta)}{f(\theta)}\Big)C(e(\theta))\Big] f(\theta), \tag{20}$$

$$e'(\theta) = y(\theta), \tag{21}$$

$$y(\theta) \leq 0, \tag{22}$$

$$k(\underline{\theta}) = 0, \qquad k(\bar{\theta}) = \beta - M^s. \tag{23}$$

$$r(\bar{\theta}) = 0 \tag{24}$$

This problem has three state variables, $e(.)$, $r(.)$, and $k(.)$, a control variable $y(.)$ and a control parameter β. In this formulation, (19) and (24) take care of the informational-rent expression of u-values, given above by (15) and (16); conditions (20) and (23) are equivalent to the budget constraint (17), as explained above; and (21)-(22) represent the monotonicity or second-order incentive constraint IC^2.

We apply Pontryagin's Maximum Principle to this Optimal Control problem. The case is a bit more complicated than in standard applications, because of the control parameter β; see Theorems 6.5.1. and 7.11.1 in Leonard and Van Long (1995). We introduce three costate variables, or multipliers, denoted $\pi_1(.)$, $\pi_2(.)$ and $\pi_3(.)$, associated with the state variables r, k, and e respectively, for the differential constraints, (19), (20) and (21), and a Lagrange multiplier $\lambda(.)$ associated with $y(.)$, for the inequality constraint (22).

The necessary conditions for an optimum can be stated as follows.

Proposition 4

If (β^*, y^*) is an optimal solution of the above problem (18)-(24), and e^*, k^*, r^* are the corresponding optimal trajectories of the state variables, then, there exist continuous and piecewise differentiable costate variables $\pi_1(.)$, $\pi_2(.)$ and $\pi_3(.)$, and a Lagrange multiplier $\lambda(.) \geq 0$ such that the following relations hold. For all θ,

$$\pi_1(\theta) = -\int_{\underline{\theta}}^{\theta} (\beta^* + r^*(\tau))^{\sigma-1} f(\tau) d\tau; \tag{25}$$

π_2 is a constant and

$$\pi_2 = -\pi_1(\bar{\theta}); \tag{26}$$

the optimal effort function satisfies,

$$\frac{1}{C'(e^*(\theta))} = \theta + \delta(\theta), \tag{27}$$

where δ is a distortion function defined as

$$\delta(\theta) = \frac{F(\theta)}{f(\theta)} + \frac{\pi_1(\theta)}{\pi_2 f(\theta)} - \frac{\pi_3'(\theta)}{\pi_2 f(\theta) C'(e^*(\theta))}. \tag{28}$$

In addition, the following bunching condition holds, for all θ,

$$\pi_3(\theta) y^*(\theta) = 0; \qquad \pi_3(\theta) = \lambda(\theta) \geq 0; \tag{29}$$

and finally, the solution satisfies (16), (17) and (22). The above necessary conditions are also sufficient.

Expressions (25) and (26) define π_2 as the marginal social value of increasing β, the utility of the high-cost agents; it is also the multiplier of the budget constraint in integral form, as given by (17). Condition (27) is the core of the analysis, for it determines education levels $e(\theta)$ by equating the marginal cost of effort with a distorted marginal value. The distortion has a complex expression, given by (28). We shall learn more about δ below.

First-best optimality would have required maximization of $\frac{1}{\sigma} \int u(\theta)^\sigma f(\theta) d\theta$ under constraint (BB) only, which would have given the marginal condition $1/C'(e^*(\theta)) = \theta$ for all θ. The second-best education function is thus obtained by replacing the individual's type with a distorted, or virtual type $\theta + \delta(\theta)$, which is typically higher than θ, as shown below.

Remark also that in the particular case in which $\sigma = 1$, the solution will be undistorted and first-best optimal. To see this, remark that $\sigma = 1$ implies $\pi_1(\theta) = -F(\theta)$, and $\pi_2 = 1$. Assuming that $e' < 0$ everywhere yields $\pi_3 \equiv \pi_3' \equiv 0$ by (29), and finally $\delta(\theta) \equiv 0$. Thus, (27) determines the first-best solution, that is, $e^*(\theta) = [C']^{-1}(1/\theta)$. It is easy to check that e^* is decreasing, which confirms the assumption just made and $\pi_3' = 0$. The solution is interior for $\sigma = 1$, and will continue to be interior if σ is smaller than but close to 1.

But, the solution will typically involve *bunching* for high values of $\rho = 1 - \sigma$, that is, an interval of types over which the solution e^* is constant will appear. Bunching is entirely caused by constraint IC^2, insofar as it imposes a non-increasing solution e^*. It happens that, by the complementarity relation (29), we get $\pi_3 = 0$, each time that $y = e' < 0$.

It would be easy to check that the Hamiltonian of our optimization problem is a concave function, implying that the necessary conditions (25)-(29) are also sufficient. We now turn to detailed analysis of these conditions.

6.2. The interior case

The system of necessary conditions (25)-(29) cannot be solved explicitly, but more can be learned about the solution if we assume that $y = e' < 0$ for all θ (the interior case). We then get $\pi_3 = 0$, and a simplified expression for the distortion term,

$$\delta_0(\theta) = \frac{\pi_2 F(\theta) + \pi_1(\theta)}{\pi_2 f(\theta)}. \tag{30}$$

Result 1. *The monotonicity constraint IC^2 holds as a strict inequality at the optimum, i.e., $e^{*\prime} < 0$, if and only if $\delta_0'(\theta) > -1$ for all θ.*

Proof. From (27), we get by differentiation, $e^{*\prime} = -((C')^2/C'')(1 + \delta_0')$, and the result follows, because C is assumed strictly increasing and convex. *Q.E.D.*

Assume now, for the rest of the present sub-section, that the interiority condition of Result 1 holds; we then find that there is no distortion "at the top" and "at the bottom".

Result 2. *The distortion is zero at the boundary of the set of types, i.e., $\delta_0(\underline{\theta}) = 0 = \delta_0(\bar{\theta})$.*

Proof. The result easily follows from computation of δ_0 at $\underline{\theta}$ and $\bar{\theta}$, taking into account that $F(\bar{\theta}) = 1$ and $-\pi_1(\bar{\theta}) = \pi_2$, $F(\underline{\theta}) = 0$ and $\pi_1(\underline{\theta}) = 0$. *Q.E.D.*

To understand Result 2, recall that increasing the distortion on some interval allows a reduction of the informational rents paid to more talented types. Economizing on rents will create more equality, at the expense of productive efficiency. When θ is close to the "bottom" $\bar{\theta}$, the less talented students who could benefit from redistribution through a reduction of the rent are only a negligible number: efficiency considerations then dominate and dictate almost undistorted education. When θ is equal to the most efficient type $\underline{\theta}$ (*i.e.*, the "top"), there is no informational rent that can be economized on the lower types by distorting the education level. We can learn a little more about δ in the interior case.

Result 3. *The distortion δ_0 is strictly positive for all type in the interior of $[\underline{\theta}, \bar{\theta}]$. As a consequence, the second-best optimal education level of all interior types will be distorted downwards, as compared to the first-best allocation.*

For proof: see Appendix B

Result 4. *The distortion δ_0 is increasing in the neighborhood of $\underline{\theta}$ and decreasing in the neighborhood of $\bar{\theta}$.*

For proof: see Appendix B

Result 5. *The distortion δ_0 is concave if the probability distribution of types is uniform on $[\underline{\theta}, \bar{\theta}]$.*

Proof. Direct computation yields the result. Using $f = 1/(\bar{\theta} - \underline{\theta})$, one finds

$$\delta_0'(\theta) = 1 + (\bar{\theta} - \underline{\theta})\frac{\pi_1'(\theta)}{\pi_2}, \tag{31}$$

and

$$\delta_0''(\theta) = +(\bar{\theta} - \underline{\theta})\frac{\pi_1''(\theta)}{\pi_2}. \tag{32}$$

Given that

$$\pi_1''(\theta) = -(1-\sigma)(\bar{\theta} - \underline{\theta})^{-1}(\beta + r(\theta))^{\sigma-2}C(e(\theta)) < 0, \tag{33}$$

we conclude that $\delta_0'' < 0$. *Q.E.D.*

With a general distribution F, it is difficult to find further qualitative properties of the distortion function, and thus of the education allocation. Before we turn to the case in which bunching occurs at the optimum, a detour is needed to study the Rawlsian egalitarian optimum.

6.3. The egalitarian solution, and solutions with bunching

The Rawlsian solution will be found by letting ρ go to $+\infty$, or directly, by maximizing the smallest u-level, that is, $\beta = u(\bar{\theta})$, under IC^2 and the modified budget constraint (17). A glance at (17) shows that this problem is equivalent to that of maximizing the *virtual surplus*,

$$\int_{\underline{\theta}}^{\bar{\theta}} \Big[e(\theta) - \Big(\theta + \frac{F(\theta)}{f(\theta)}\Big)C(e(\theta))\Big]f(\theta)d\theta,$$

if we relax the problem by neglecting IC^2. The necessary condition for a minimum of the above expression is simply $1 = (\theta + F/f)C'$, for all θ, or,

$$\frac{1}{C'(e^*(\theta))} = \theta + \frac{F(\theta)}{f(\theta)}, \tag{34}$$

and we easily check that the education allocation e^* which satisfies (34) is strictly decreasing under the classic Monotonic Hazard Rate condition that f/F is a decreasing function. The immediate consequence of this goes as follows.

Result 6. *If F satisfies the Monotonic Hazard Rate property, the Rawlsian solution is determined by (34) and the distortion at the "bottom" is positive, equal to $1/f(\bar{\theta})$.*

The problem is now to reconcile Result 6, which holds when $\rho = +\infty$, with Result 2, which is known to hold for $\rho = 0$: bunching will bridge this gap for sufficiently large values of ρ. We define the *bunching set* $\mathcal{B}$ as the union of all subsets of $[\underline{\theta}, \bar{\theta}]$, with a nonempty interior, such that the optimal education solution e^* is constant (or such that the control $y = 0$).

Consider the mapping $\sigma \to e^*_\sigma(.)$ which gives the optimal education trajectories as a function of parameter σ. When σ goes to $-\infty$, these trajectories should converge to the Rawlsian solution given by (34). This can only be done continuously if an interval over which e^* is constant appears to the left of $\bar{\theta}$ for values of σ between 1 and $-\infty$. We therefore state the following result.

Result 7. *If F satisfies the Monotonic Hazard Rate property, and if the optimal education levels $e^*_\sigma(\theta)$ vary continuously with σ for all θ, then, for sufficiently high ρ, the highest handicap $\bar{\theta}$ will be included in a bunching interval with a nonempty interior.*

This general property is illustrated on Figure 4, which is obtained by numerical simulations under the assumption of a uniform distribution F, and a quadratic cost function C. A bunching interval of the form $[\theta_0(\sigma), \bar{\theta}]$ appears, where $\theta_0(\sigma)$ first decreases, and then increases, when σ goes to $-\infty$.

[insert Figure 4 around here]

It is impossible to prove that the bunching set $\mathcal{B}$ is convex in general, although it is the most reasonable conjecture for well-behaved distributions F. It is however easy to prove that bunching cannot take place in the neighborhood of the lowest handicap $\underline{\theta}$, using Result 4. The analysis in the general case is made difficult by the fact that, due to nonlinearities, the solution under bunching typically does not coincide with the interior solution out of the bunching set, in contrast with the problem studied by Guesnerie and Laffont (1984). To obtain a better understanding of a solution with bunching, let us assume from now on that F is the uniform distribution.

The bunching set will be nonempty if the mapping $\theta \to \theta + \delta_0(\theta)$ is decreasing on some interval, when evaluated at the optimal solution, due to (27) and Result 1 above, for then without bunching, e^* would be increasing on this interval, which is excluded by IC^2. A glance at the proof of Result 5 above, and particularly at (31), shows that $\theta + \delta_0(\theta)$ decreases if and only if

$$-\pi_1'(\theta) > \frac{2\pi_2}{(\bar{\theta} - \underline{\theta})}. \tag{35}$$

Given that $-\pi_1'$ is monotonic increasing (as shown by (33)), and that π_2 is a constant for any given solution e^*, the concavity of δ_0 imposes that if it holds, (35) holds on a single interval $(\hat{\theta}, \bar{\theta}]$, including the highest handicap. From this, we can deduce the following result.

Result 8. *If F is the uniform distribution on $[\underline{\theta}, \bar{\theta}]$, the optimal bunching set $\mathcal{B}^*$ is an interval including the highest handicap.*

For proof, see Appendix B.

This result is reassuring, and proves that optimal solutions look like the trajectories depicted on Figure 4, when F is sufficiently close to the uniform distributions. Result 8 also establishes what should be viewed as the typical form of the solution: with enough aversion for inequality, the presence of bunching at the lower end of the talent scale is second-best optimal, and all types, except the most talented, suffer a downward distortion with respect to first best. The bunching interval restores a form of equality of educational achievement among the mediocre. An interesting question is of course to determine the empirical extent of the bunching interval, for the vast majority of the population could be bunched, except for a few highly talented individuals.[15] Clearly, the practical meaning of the solution depends on the shape of the distribution of talents in the population.

7. Conclusion

To sum up, we have studied the optimal allocation of public expenditures on education when individual talents are unobservable and when direct money transfers can be imposed by the State. Education levels are the result of a combination of individual effort and publicly funded help. Expenditure on schools (help) reduces the individual cost of acquiring a given level of education (and thus of human capital). When effort and help are good substitutes, the second-best optimum distributes more educational resources to the less-talented than to the talented, and uses money transfers to reduce inequality, at the expense of a lower individual effort of the less-talented. When effort and help are complements, the more talented should be helped more. In any case, education levels and individual efforts increase with the talent parameter. Full equality of utilities can never be realized because of incentive compatibility constraints faced by the Principal. When the degree of social aversion for inequality increases, second-best optimality requires a reduction of utility differences, which in turn implies more unequal individual education levels. If in addition, effort and help are sufficiently good substitutes, the reduction of utility differences requires at the same time more help for the less-talented than for the talented, in the form of public expenditures on schools. We get the following paradoxical result: the inequality of education levels is maximal under "pure egalitarianism", that is, when the degree of aversion for inequality is infinite. This stems from the fact that under asymmetric information, when direct money transfers can be used by the State, equalization of individual educational achievements conflicts with that of utility levels. The conclusion will be somewhat repugnant to those who think, as we do, that a greater equality of educational attainments, and thus a greater equality of the distribution of human capital is desirable: the conclusion is a consequence of the Planner's efforts to equalize the after-transfer present values of the agents' incomes, and not the education levels themselves.

More work is needed to understand fully the mechanics of education policies under informational asymmetries, and their interplay with redistribution and ethical concerns. Several extensions of the above model could be interesting. Introducing a private market for education, together with imperfect observation of the consumption of education (otherwise the conclusions would be the same as above), would enrich the analysis. Generalizing the analysis with the introduction of a two-dimensional distribution of family (or parental) income and of student ability would also be of interest, particularly if the income distribution is not assumed independent of the ability distribution.

---

[15] This form of solution, by a stretch of the imagination, could be seen as a reconciliation of the French ideal of equality with the (also French) *Elitisme Républicain*, insofar as this system sends its very best, mercilessly screened students to a few *Grandes Ecoles*, while the vast majority, the less-brilliant subjects, packs "downwardly distorted" State University programs. On the French case, see Neave (1993).

Appendix A

We study here the case in which effort and help are strict complements. Let us assume that the production function is $e = \min\{x, s\}$. The associated cost (or effort) function C thus writes $C(e,s) = e$ if $e \leq s$ and $C(e,s) = +\infty$ if $e > s$. It follows that the entire discussion will be conducted under the assumption $e_i \leq s_i$. The benefits of education are assumed to be a strictly increasing, strictly concave and continuously differentiable function of e, denoted $B(e)$. This assumption is reasonable, and will ensure the existence of nice interior solutions.

The second-best education planning problem of the Principal can be written as follows,

$$\max_{(e_i, s_i, t_i)} (1/\sigma)(p_1 u_1^\sigma + p_2 u_2^\sigma)$$

subject to the constraints

$$e_i \leq s_i, \quad (i = 1, 2)$$

$$u_i = t_i + B(e_i) - \theta_i e_i, \quad (i = 1, 2) \tag{$U0$}$$

$$M = p_1(t_1 + s_1) + p_2(t_2 + s_2), \tag{BB}$$

$$u_1 \geq t_2 + e_2 - \theta_1 e_2, \tag{IC_1}$$

$$u_2 \geq t_1 + e_1 - \theta_2 e_1. \tag{IC_2}$$

It is easy to check that at the optimum, $e_i = s_i$ will always hold, because schooling expenditures are costly and should never be wasted under the budget constraint BB. This remark being made, the equivalent of Lemmata 1, 2 and 3 can be proved, along the same line of reasoning as above. The incentive constraint IC_1 will hold as an equality at the optimum. Given this result, IC_2 is simply equivalent to $e_1 > e_2$. Finally the inequality between types can be expressed as $u_1 = u_2 + (\theta_2 - \theta_1)e_2$, which is equivalent to IC_1.

To solve the above program, first eliminate transfers t_i using $U0$. The constraint BB can be rewritten

$$M(e) \equiv M + \sum_i p_i[(1 + \theta_i)e_i - B(e_i)] = \sum_i p_i u_i.$$

This and the expression of IC_1 above can be solved for the utilities $(\hat{u}_1, \hat{u}_2)$. We get, $\hat{u}_1 = M(e) + p_2 e_2(\theta_2 - \theta_1)$ and $\hat{u}_2 = M(e) - p_1 e_2(\theta_2 - \theta_1)$. Substituting these expressions in the Planner's objective function and writing the first-order conditions yields the equations defining e_1^{**} and e_2^{**}. The first-best and the second-best optimal allocation of the talented are given by

$$(1 + \theta_1) = B'(e_1^{**}),$$

(there is no distortion at the top), and the second-best allocation of the less talented is given by,

$$(1 + \theta_2) + (\theta_2 - \theta_1)\delta(\hat{u}) = B'(e_2^{**}),$$

where δ is given by expression (6) above, showing that there is a downward distortion of the less-talented individual's effort and education levels, with respect to first-best. It follows that the results are essentially the same as those of Corollary 2 above.

Appendix B

Proof of Corollary 2.

Define the distortive term $\alpha = (\theta_2 - \theta_1)(1 + \delta(\hat{u}))$. The first-order conditions SB3 and SB4 are equivalent to $C_e^{**} + C_s^{**} = 0$ and $C_e^{**} = 1/(\theta_1 + \alpha)$. Differentiating the first relation with respect to α yields,

$$[C_{ee}^{**} + C_{es}^{**}]e_{2\alpha}^{**} + [C_{ss}^{**} + C_{es}^{**}]s_{2\alpha}^{**} = 0,$$

which, given the assumptions made on second-order derivatives, shows that $e_{2\alpha}^{**}$ and $s_{2\alpha}^{**}$ have the same sign. Everything is then as before, except that we must make sure that C^{**} is decreasing with the distortive term α, to ensure that the neglected constraint IC_2 holds at the solution of the system SB1-4. Total differentiation of C^{**} with respect to α yields

$$\frac{d}{d\alpha}C^{**} = C_e^{**}e_{2\alpha}^{**} + C_s^{**}s_{2\alpha}^{**}.$$

Using the above relation between $e_{2\alpha}^{**}$ and $s_{2\alpha}^{**}$ and the fact that $C_e^{**} = -C_s^{**}$ holds for all types, then gives

$$\frac{d}{d\alpha}C^{**} = C_e^{**}e_{2\alpha}^{**}\left(1 + \frac{(C_{ee}^{**} + C_{es}^{**})}{(C_{ss}^{**} + C_{es}^{**})}\right).$$

Now, multiplying the expression of $\frac{d}{d\alpha}C^{**}$ by $C_{ss}^{**} + C_{es}^{**}$, which is positive by assumption, we get,

$$sign[\frac{d}{d\alpha}C^{**}] = sign[e_{2\alpha}^{**}(C_{ee}^{**} + 2C_{es}^{**} + C_{ss}^{**})].$$

It is then easy to check that $C_{ee}^{**} + 2C_{es}^{**} + C_{ss}^{**} > 0$ is equivalent to $g_{ss} < 0$, which happens to be true under the assumed concavity of g. (To perform the computation, differentiate definition (0), which gives $C_e = 1/g_x$, $C_s = -g_s/g_x$, $C_{ee} = (1/g_x)^3(-g_{xx})$, $C_{es} = -(1/g_x)^3(g_{sx}g_x - g_{xx}g_s)$, and $C_{ss} = (1/g_x)^3(-g_{ss}g_x^2 + 2g_{sx}g_sg_x - g_{xx}g_s^2)$. Use the fact that $C_e = -C_s$ is equivalent to $g_s = 1$). We conclude that the second-best effort C^{**} is decreasing with α, showing that point (iii) in Proposition 2 holds under the assumptions of Corollary 2. *Q.E.D.*

Proof of Corollary 3.

Consider, as in the proof of Proposition 3, the derivatives of help and education with respect to σ. The assumption made on the second-order derivatives of C yields

$$sign(e_{2\sigma}^{**}) = sign(s_{2\sigma}^{**}) = sign(-\delta_\sigma).$$

We know from Lemma 4 that C_2^{**} increases with σ. We thus get, using the first-order condition $C_e^{**} = -C_s^{**}$,

$$\frac{d}{d\sigma}C_2^{**} = C_e^{**}e_{2\sigma}^{**} + C_s^{**}s_{2\sigma}^{**} = (C_e^{**})(e_{2\sigma}^{**} - s_{2\sigma}^{**}) \geq 0.$$

We can then replace $e^{**}_{2\sigma}$ and $s^{**}_{2\sigma}$ by their expressions in (9). Taking account of the fact that $det(C'') > 0$, $C^{**}_e > 0$ and $sign(\delta_\sigma) = sign(a_\sigma)$, the above inequality implies,

$$[C^{**}_{ss} + 2C^{**}_{es} + C^{**}_{ee}](-\delta_\sigma) \geq 0.$$

To end the proof, it is sufficient to check that $C^{**}_{ee} + 2C^{**}_{es} + C^{**}_{ss} > 0$ if g is strictly concave, as in the proof of Corollary 2 above. *Q.E.D.*

Proof of Result 3.

It will be sufficient to establish that the distortion δ_0 is strictly positive in $(\underline{\theta}, \bar{\theta})$, for then, the statement on the downwards distortion easily follows from (27) and the convexity of C.

Clearly, $\delta_0 > 0$ if and only if $\pi_2 > -\pi_1(\theta)/F(\theta)$. Using (25) and (26), this is equivalent to $Z(\theta) < Z(\bar{\theta})$, where by definition,

$$Z(\theta) = \int_{\underline{\theta}}^{\theta} (\beta + r(\tau))^{\sigma-1} \frac{f(\tau)}{F(\theta)} d\tau.$$

It is not difficult to establish that $Z'(\theta) > 0$ if and only if

$$(\beta + r(\theta))^{\sigma-1} > \int_{\underline{\theta}}^{\theta} (\beta + r(\tau))^{\sigma-1} (f(\tau)/F(\theta)) d\tau,$$

which is true because $\sigma < 1$ and r is decreasing.
Q.E.D.

Proof of Result 4.

Remark first that $\delta_0(\theta) = (F(\theta) - G(\theta))/f(\theta)$, where by definition, $G(\theta) = -\pi_1(\theta)/\pi_2$, and $0 \leq G \leq 1$. Define $\Delta = F - G$. Then, $sign\,(\delta_0') = sign\,(\Delta' - (f'/f)\Delta)$, and given that $\Delta(\underline{\theta}) = \Delta(\bar{\theta}) = 0$, we get, $sign\,(\delta_0'(\underline{\theta})) = sign\,(\Delta'(\underline{\theta}))$ and $sign\,(\delta_0'(\bar{\theta})) = sign\,(\Delta'(\bar{\theta}))$. It is then easy to check that $\Delta' > 0$ if and only if $-\pi_1'/f < \pi_2$. Using (25) and (26), we find that $\Delta'(\underline{\theta}) > 0$ can be equivalently rewritten

$$\int_{\underline{\theta}}^{\bar{\theta}} (\beta + r(\theta))^{\sigma-1} f(\theta) d\theta > (\beta + r(\underline{\theta}))^{\sigma-1},$$

and the latter inequality is always true, given that r is decreasing and $\sigma < 1$. We also find that $\Delta'(\bar{\theta}) < 0$ can be equivalently rewritten

$$\int_{\underline{\theta}}^{\bar{\theta}} (\beta + r(\theta))^{\sigma-1} f(\theta) d\theta < \beta^{\sigma-1},$$

which holds true for the same reasons as above, given that $r(\bar{\theta}) = 0$.
Q.E.D.

Proof of Result 8.

Let e^* be the optimal trajectory and $\mathcal{B}^*$ be the optimal bunching set, with a nonempty interior. Remark that the solution $e^*(\theta)$ coincides with the function

$$e^0(\theta) = (C')^{-1}\left(\frac{1}{\theta + \delta_0(\theta)}\right), \tag{36}$$

on the complement of $\mathcal{B}^*$ in $[\underline{\theta}, \bar{\theta}]$, and that e^0 is increasing at θ if and only if (35) holds at θ. It must be that $\mathcal{B}^*$ includes the sub-interval of $[\underline{\theta}, \bar{\theta}]$ over which e^0 is increasing, for otherwise, the solution would violate IC^2.

Assume now that $\mathcal{B}^*$ contains two disjoint intervals, $[\theta_0, \theta_1]$ and $[\theta_2, \bar{\theta}]$, with $\theta_1 < \theta_2$. It must be that the solution is strictly decreasing and equal to e^0 over (θ_1, θ_2). In addition, by continuity of state variables, e^* is constant and equal to $e^0(\theta_1)$ over $[\theta_1, \theta_2]$. Therefore, the optimal trajectory e^* must jump downwards at $\theta = \theta_1$, contradicting the continuity of e^*. It follows that $\mathcal{B}^*$ is an interval of the form $[\theta_0, \bar{\theta}]$.
Q.E.D.

References

Arrow, K.J. (1971), "The Utilitarian Approach to the Concept of Equality in Public Expenditure," *Quarterly Journal of Economics*, 85, 409-415.

Atkinson, A.B. (1970), "On the Measurement of Inequality", *Journal of Economic Theory*,2, 244-263.

d'Aspremont, C. and P. Mongin (1998), "Utility Theory and Ethics", Chapter 10 in S. Barbera, P.J. Hammond and C. Seidl eds., *Handbook of Utility Theory, Volume 1: Principles*, Kluwer Academic Publishers, Dordrecht.

Becker, G. (1967), *Human Capital and the Personal Distribution of Income.*, University of Michigan Press, Ann Arbor.

Bénabou, R. (1996a), "Equity and Efficiency in Human Capital Investment: The Local Connection", *Review of Economic Studies*, 63, 584-609.

Bénabou, R. (1996b), "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance", *American Economic Review*, 86, 237-264.

Besley, T. ans S. Coate (1995), "The Design of Income Maintenance Programmes", *Review of Economic Studies*, 62, 187-221.

Blackorby, C. Bossert, W, and D. Donaldson, (1999), "Utilitarianism and the Theory of Justice", forthcoming as Chapter 20 in Arrow, Sen and Suzumura eds.,*Handbook of Social Choice and Welfare*, Elsevier, Amsterdam.

Blackorby, C. and D. Donaldson (1988), "Cash versus Kind, Self-Selection, and Efficient Transfers", *American Economic Review*, 78, 691-700.

Bruno, M. (1976), "Equality, Complementarity and the Incidence of Public Expenditures", *Journal of Public Economics*, 6, 395-407.

Card, D. (1998), "The Causal Effect of Education on Earnings", *Center for Labor Economics, University of California Berkeley, Working Paper* No 2, forthcoming in O. Ashenfelter and D. Card eds., *Handbook of Labor Economics, vol. 3*, North Holland, Amsterdam.

Card, D. and A. B. Krueger (1996), "The Economic Return to School Quality", Chapter 6 in W. E. Becker and W. J. Baumol eds., *Assessing Educational Practices. The Contribution of Economics*, MIT Press, Cambridge, Massachusetts.

De Fraja, G. (1998), "The Design of Optimal Education Policies", *University of York, manuscript.*

Ebert, U. (1992), "A Reexamination of the Optimal Nonlinear Income Tax", *Journal of Public Economics*, 49, 47-73.

Epple, D. and R. E. Romano (1998), "Competition between Private and Public Schools, Vouchers, and Peer-Group Effects", *American Economic Review*, 88, 33-62.

Erikson, R. and J. O. Jonsson (1996), *Can Education be Equalized? The Swedish Case in Perspective*, Westview Press, Oxford.

Fernandez, R. and R. Rogerson (1996), "Income Distribution, Communities, and the Quality of Public Education", *Quarterly Journal of Economics*, 111, 135-164.

Fleurbaey, M. (1998), "Equality among Responsible Individuals", in Laslier, Fleurbaey, Gravel and Trannoy eds., *Freedom in Economics*, Routledge, London.

Guesnerie, R. and J-J. Laffont (1984), "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm," *Journal of Public Economics*, 25, 329-369.

Guesnerie, R. and J. Seade (1982), "Nonlinear Pricing in a Finite Economy," *Journal of Public Economics*, 17, 157-179.

Hare, P. G. and D. Ulph (1979), "On Education and Distribution", *Journal of Political Economy*, 87, 193-212.

Hoxby, C. M. (1994), "Do Private Schools provide Competition for Public Schools?", *NBER Working Paper*, no 4978.

Hoxby, C. M. (1999), "The Productivity of Schools and other Local Public Goods Producers", *Journal of Public Economics*, 74, 1-30.

Kolm, S. C. (1968), "The Optimal Production of Social Justice", in H. Guitton and J. Margolis eds., Economie Publique, CNRS, Paris.

Kranich, L. (1997), "Equalizing Opportunities through Public Education when Innate Abilities are Unobservable", *manuscript, Universidad Carlos III, dept. de economia*, Madrid, Spain.

Leonard D. and N. Van Long (1995), *Optimal Control Theory and Static Optimization in Economics*, Cambridge University Press, Cambridge.

Lollivier, S. and J-C. Rochet (1983), "Bunching and Second-Order Conditions: A Note on Optimal Tax Theory", *Journal of Economic Theory*, 31, 392-400.

Mayer, S. E. (1997), *What Money Can't Buy: Family Income and Children's Life Chances*, Harvard University Press, Cambridge , Massachusetts.

Mirrlees, J. A. (1971), "An Exploration in the Theory of Optimum Income Taxation", *Review of Economic Studies*, 38, 175-208.

Neave, G. (1993), "Séparation de Corps: The Training of Advanced Students and the Organization of Research in France", in Clark B.R. ed., *The Research Foundations of Graduate Education: Germany, Britain, France, United States, Japan*, University of California Press, Berkeley, California.

Reid, W. A. (1991), "The Ideology of Access in Comparative Perspective", in C. Chitty ed., *Post-16 Education: Studies in Access and Achievement*, London Education Studies, Kogan Page, London.

Roemer, J. E. (1998), *Equality of Opportunity*, Harvard University Press, Cambridge.

Salanié, B. (1990), "Sélection adverse et aversion pour le risque", *Annales d'Economie et de Statistique*, 18, 131-149.

Seade, J. K. (1977), "On the Shape of Optimal Tax Schedules", *Journal of Public Economics*, 7, 203-236.

Shavit, Y. and H-P. Blossfeld (1993), *Persistent Inequalities: Changing Educational Attainment in 13 Countries*, Westview Press, Oxford.

The Economist, (1999a), "Caning the Teachers," october 23rd, 49-50.

The Economist, (1999b), "Teachers: Good Scores, or Else," october 23rd, 68-69.

Tuomala, M. (1986), "On the Optimal Income Taxation and Education Decisions", *Journal of Public Economics*, 30, 183-198.

Ulph, D. (1977), "On the Optimal Distribution of Income and Educational Expenditure", *Journal of Public Economics*, 8, 341-356.

Weymark, J. A. (1986), "A Reduced-Form Optimal Nonlinear Income Tax Problem," *Journal of Public Economics*, 30, 199-217.

Weymark, J. A. (1987), "Comparative Static Properties of Optimal Nonlinear Income Taxes," *Econometrica*, 55, 1165-1185.

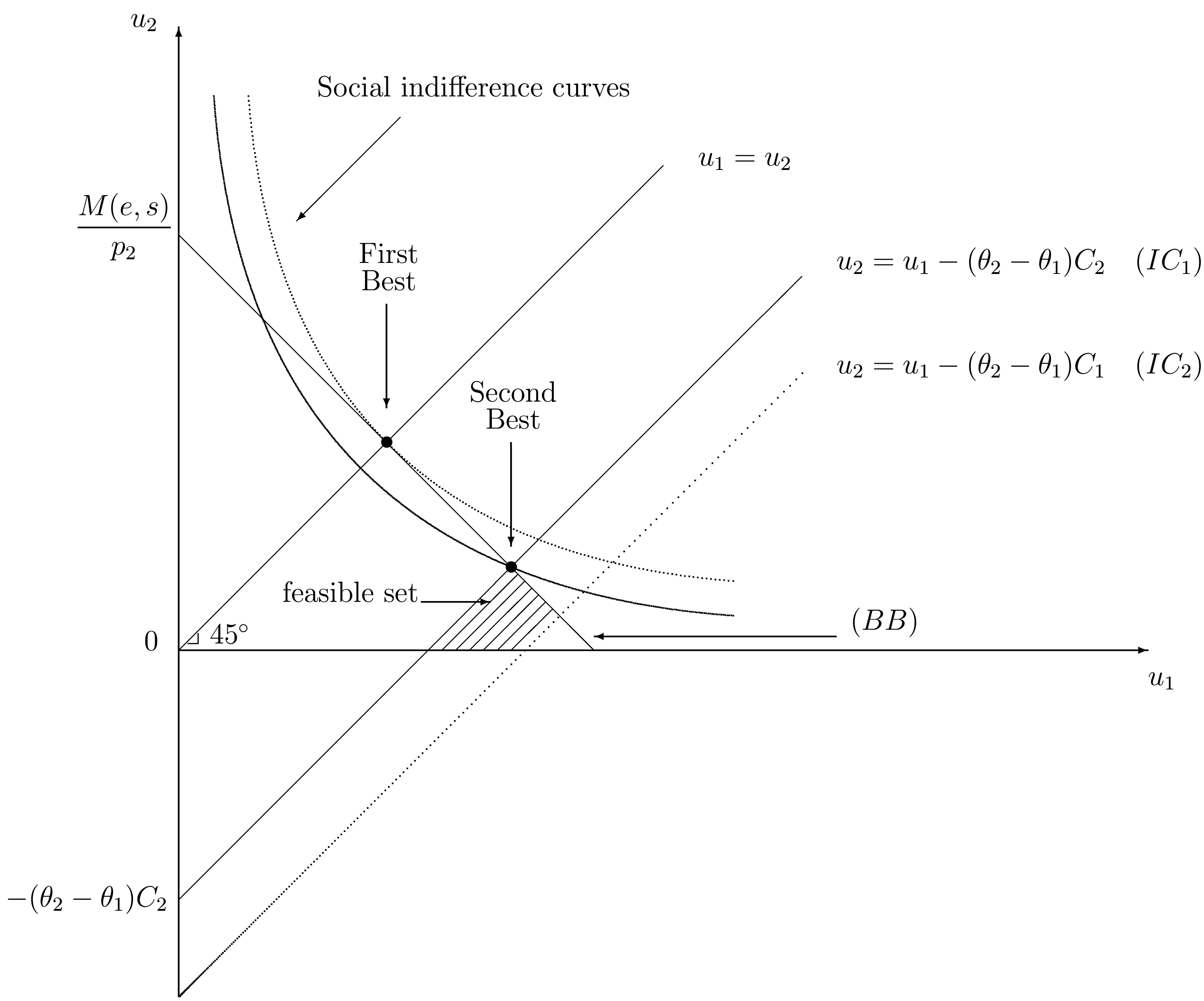

Figure 1: *First Best and Second Best solutions in utility space ; $C_1 \geq C_2$, (e, s) fixed.*

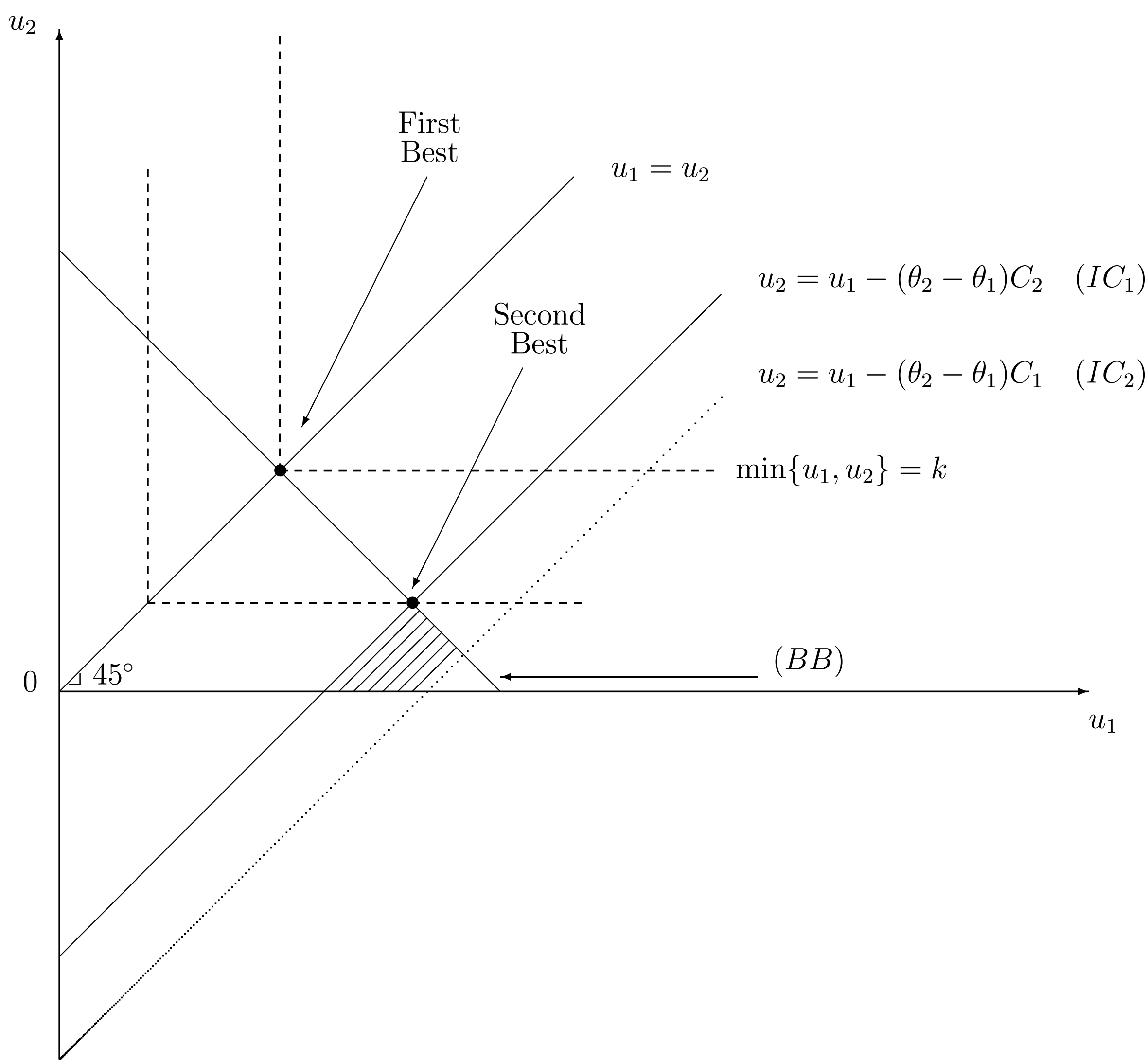

Figure 2: *The Rawlsian Case.*

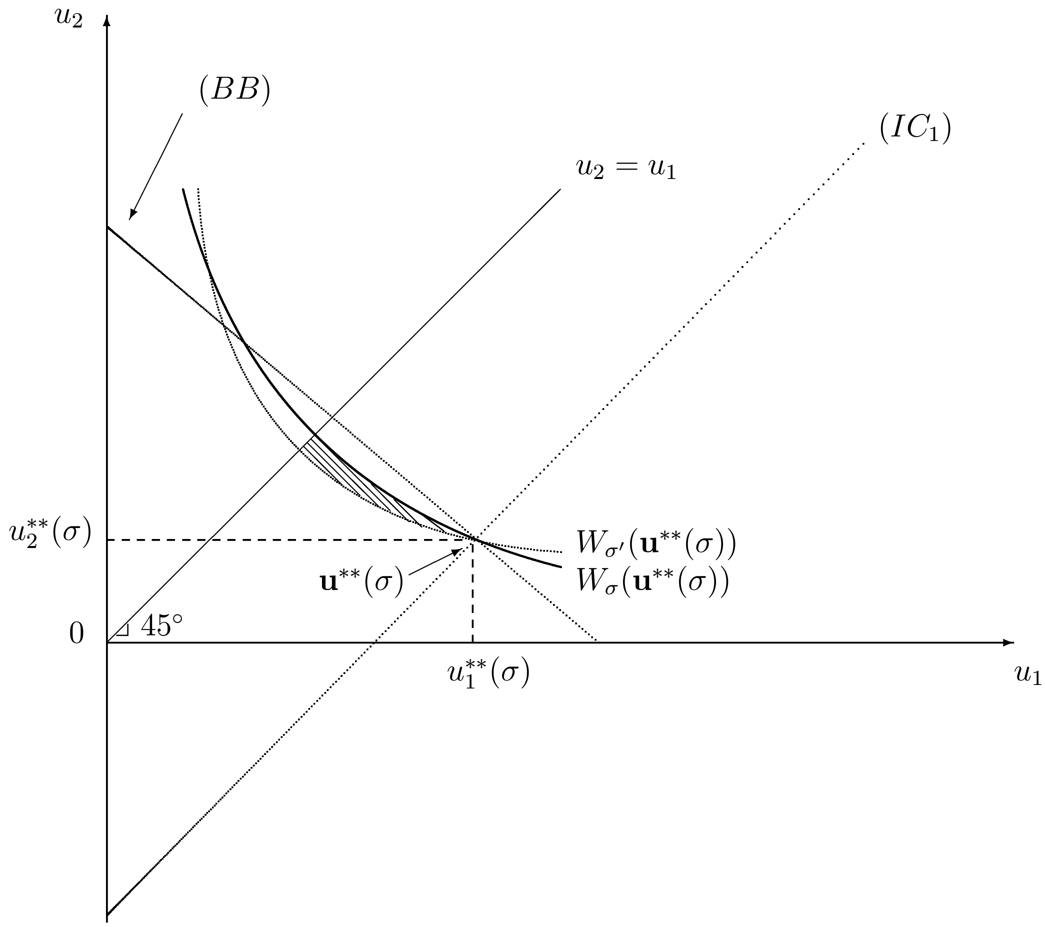

Figure 3: *Comparative Statics with respect to σ.*

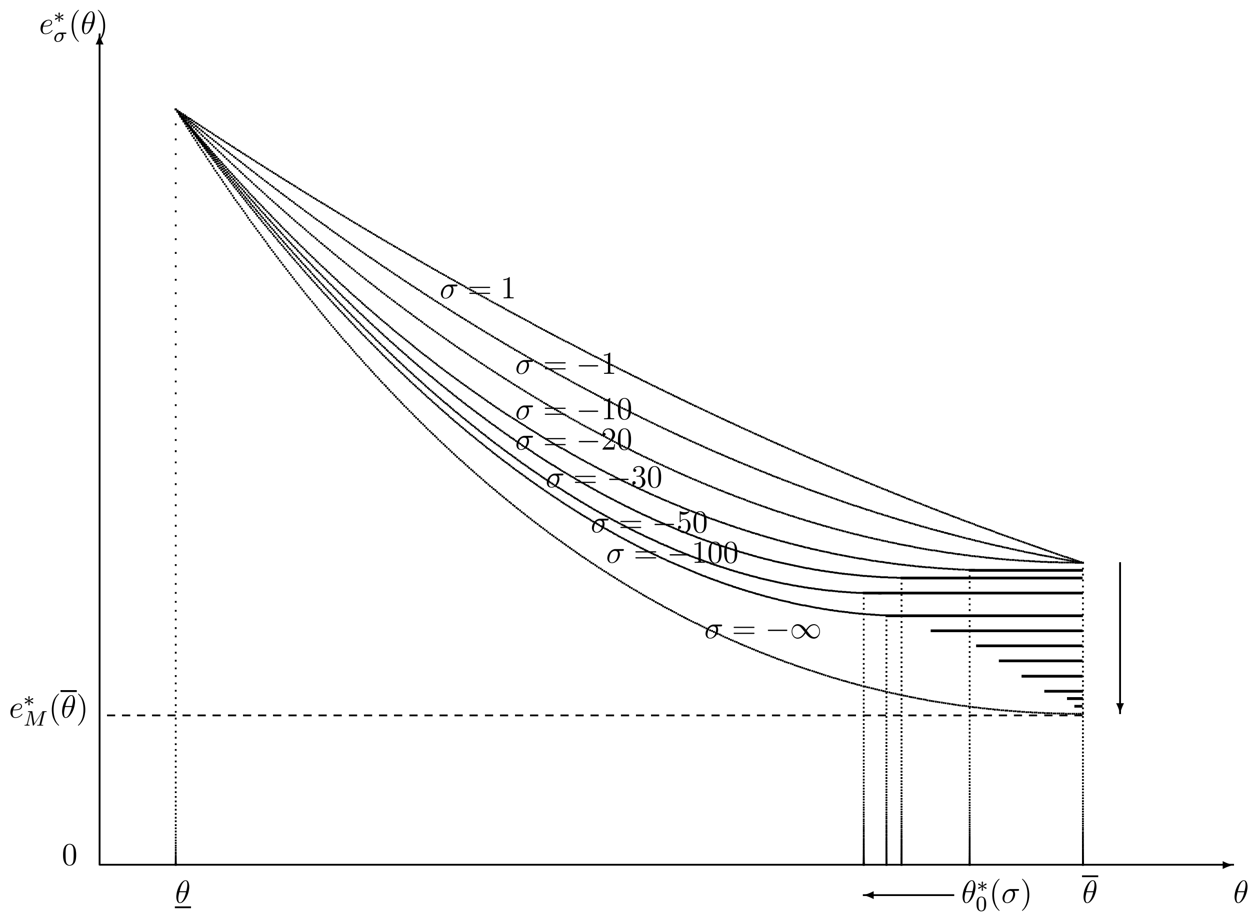

Figure 4: *The Bunching.*