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ABSTRACT

Price Stability with Imperfect Financial Integration*

This Paper evaluates the welfare implications of policy rules when international financial markets are incomplete. Using a two-country dynamic general equilibrium model with incomplete markets, price stickiness and monopolistic competition, one finds that an allocation in which the producer inflation rates in both countries are stabilized to zero reproduces the flexible-price allocation. This allocation, however, is suboptimal with deadweight losses evaluated at around 0.05% of a permanent shift in steady-state consumption. The feasible first-best is a state-contingent producer inflation policy. The gains from pursuing this policy instead of price stability are, however, small in terms of reduction in the dead-weight losses. Therefore, under incomplete markets, price stability is a good approximation of the feasible first-best policy.

JEL Classification: E58, F41
Keywords: incomplete markets, optimal monetary policy, price stability

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NON-TECHNICAL SUMMARY

Recent advances in monetary theory advocate price stability as the primary objective of monetary policy. This conclusion stems from general equilibrium models embedded with some frictions, such as price stickiness and monopolistic competition, that allow monetary policy to have a role in explaining short-run fluctuations in the economy. These models evaluate the optimality of the various policy rules based on the direct maximization of the utilities of the consumers. The distortions existing in the economy are then measured for the deadweight losses they produce on the optimal decision path of the households. A key finding is that under the goal of price stability, these models reproduce the same fluctuations that would arise with perfectly flexible prices, i.e. the mark-up of prices over nominal marginal costs are stabilized. This allocation is also efficient. The case for price stability is robust across various specifications of the model, such as the price-setting mechanism or the type of shocks affecting the economy, be it from the demand or supply side. Studies supporting this conclusion are Aoki (1998), Goodfriend and King (1997, 2000), King and Wolman (1998), Obstfeld and Rogoff (2000a, 2000c), Woodford (1999b).

As outlined in Woodford (1999b), this argument should also consider numerous caveats. First, even if the flexible-price allocation is the efficient outcome, price stability is not always a feasible option. In a more complex model where different sectors are characterized by different degrees of rigidity, the simultaneous stabilization of the mark-ups cannot be achieved. This inconsistency also occurs in a model with sticky wages and prices, where under the flexible-price allocation, the real wage moves in response to real disturbances. Another case where price stability is not feasible is when shocks to the natural rate of interest can put the nominal interest rate below the zero lower bound, such as in Woodford (1999a, 1999b).

Second, even if price stability can achieve the flexible-price allocation, the latter does not coincide with the efficient equilibrium. This is the case if the monopolistic distortions are time-varying, assuming the form of cost-push shock, or if monetary frictions, of the kind that makes it desirable to avoid large fluctuations of the nominal interest rates, are considered. In an open economy context, the strategic interaction between different policy-makers is an important addition. But, abstracting from these issues, in a cooperative solution price stability reproduces the efficient allocation when there are complete markets and producer currency pricing, as shown in P Benigno (2001) and Obstfeld and Rogoff (2000a, 2000c). In fact, the exchange rate moves to accommodate asymmetric supply shocks. Instead the efficient equilibrium is not even feasible as a cooperative solution when the terms of trade cannot efficiently offset asymmetric shocks, because of an imperfect pass-through or pricing-to-market behaviour, as outlined in G Benigno (1999), Corsetti and Pesenti (2001b) and Devereux and Engel (2000).

In this Paper, we show an open-economy case in which price stability in each country is not the efficient equilibrium. The reason for the departure is the incomplete asset market structure. Here, in a two-country general equilibrium model, we compare the allocation that would arise if households can trade internationally in a set of securities that span all the states of nature with that in which they can only trade in a nominal risk-free bond. Under incomplete markets, it is still true that the stabilization of the producer price level can stabilize the mark-up and reproduce the flexible-price allocation. This allocation is inefficient, however, because an incomplete market structure cannot allow perfect stabilization of asymmetric demand and supply shocks. With our utility-based welfare criterion, we evaluate the magnitude of the deadweight losses to be around 0.05% of a permanent shift in steady-state consumption. This number is in the same order of magnitude as the gains that would be obtained by eliminating the uncertainty in the fluctuations of the economy, as in Lucas (1987).

In this model, we also show that stabilization of the mark-up is not the optimal feasible policy. Instead, producer inflation rates should be state contingent. We find that the gains under the state-contingent policy relative to that under price-stability are small in terms of the reduction in the deadweight losses. Considering that a policy of stabilization of producer prices in each country does not require any sort of international coordination between different monetary policy-makers and that the policy goals can be easily communicated to the private sector, it can be concluded that in our framework price stability is desirable even under incomplete markets.

1 Introduction

Recent advances in monetary theory advocate price stability as the primary objective of monetary policy. This conclusion stems from general equilibrium models embedded with some frictions, such as price stickiness and monopolistic competition, that allow monetary policy to have a role in explaining short-run fluctuations in the economy. These models evaluate the optimality of the various policy rules based on the direct maximization of the utilities of the consumers. The distortions existing in the economy are then measured for the deadweight losses they produce on the optimal decision path of the households. A key finding is that under the goal of price stability, these models reproduce the same fluctuations that would arise with perfectly flexible prices, i.e. the mark-up of prices over nominal marginal costs are stabilized. This allocation is also efficient. The case for price stability is robust across various specifications of the model, such as the price setting mechanism or the type of shocks affecting the economy, be it from the demand or supply side. Studies supporting this conclusion are Aoki (1998), Goodfriend and King (1997, 2000), King and Wolman (1998), Obstfeld and Rogoff (2000a, 2000c), Woodford (1999b).

As outlined in Woodford (1999b), this argument should also consider numerous caveats. First, even if the flexible-price allocation is the efficient outcome, price stability is not always a feasible option. In a more complex model where different sectors are characterized by different degrees of rigidity, the simultaneous stabilization of the mark-ups cannot be achieved. This inconsistency also occurs in a model with sticky wages and prices, where under the flexible-price allocation, real wage moves in response to real disturbances.[1] Another case where price stability is not feasible is when shocks to the natural rate of interest can put the nominal interest rate below the zero lower bound, such as in Woodford (1999a,b).

Second, even if price stability can achieve the flexible-price allocation, the latter does not coincide with the efficient equilibrium. This is the case if the monopolistic distortions are time-varying, assuming the form of cost-push shock, or if monetary frictions, of the kind that makes desirable to avoid large fluctuations of the nominal interest rates, are considered.[2]

[1] Benigno P. (2001) and Erceg et al. (2000) belong to this class of models.

[2] In the first case, as in Clarida et al. (1999), there is a trade-off between stabilizing the economy

In an open-economy context, the strategic interaction between different policymakers is an important addition. But, abstracting from these issues, in a cooperative solution price stability reproduces the efficient allocation when there are complete markets and producer currency pricing, as shown in Benigno P. (2001) and Obstfeld and Rogoff (2000a,c). In fact, the exchange rate moves to accommodate asymmetric supply shocks.[3] Instead the efficient equilibrium is not even feasible as a cooperative solution when the terms of trade cannot efficiently offset asymmetric shocks, because of an imperfect pass-through or pricing-to-market behavior, as outlined in Benigno G. (1999), Corsetti and Pesenti (2001b) and Devereux and Engel (2000).

In this paper, we show an open-economy case in which price stability in each country is not the efficient equilibrium.[4] The reason for the departure is the incomplete asset market structure.[5] Here, in a two-country general equilibrium model, we compare the allocation that would arise if households can trade internationally in a set of securities that span all the states of nature with that in which they can only trade in a nominal risk-free bond. Under incomplete markets, it is still true that the stabilization of the producer price level can stabilize the mark-up and reproduce the flexible-price allocation. However, this allocation is inefficient because an incomplete-market structure cannot allow perfect stabilization of asymmetric demand and supply shocks. With our utility-based welfare criterion, we evaluate the magnitude of the deadweight losses to be around 0.05 percent of a permanent shift in steady-state consumption. This number is in the same order of magnitude as the gains that would be obtained by eliminating the uncertainty in the fluctuations of the economy, as in Lucas (1987).

at the flexible-price allocation and offsetting optimally the cost-push shock. In the second case, as shown in Woodford (1999a,b), monetary policy should balance the objective of stabilizing the economy at the flexible-price allocation with that of reducing the volatility of the nominal interest rate.

[3] Obstfeld and Rogoff (2000a,c) shows a case in which the efficient equilibrium can be implemented also in a strategic context, with an ex-ante commitment. Benigno and Benigno (2001) discusses the discretionary equilibrium.

[4] This case applies also to a closed-economy model.

[5] Obstfeld and Rogoff (2001c) have shown that under incomplete markets the flexible-price allocation is not efficient.

In this model, we also show that stabilization of the mark-up is not the optimal feasible policy. Instead, producer inflation rates should be state contingent. We find that the gains under the state-contingent policy relative to that under price-stability are small in terms of the reduction in the deadweight losses. Considering that a policy of stabilization of producer prices in each country does not require any sort of international coordination between different monetary policymakers and that the policy goals can be easily communicated to the private sector, it can be concluded that in our framework price stability is desirable even under incomplete markets.

The structure of the work is the following. Section II presents the model. Section III discusses the incomplete-market equilibrium versus the complete one. Section IV evaluates the welfare costs of following a zero inflation-targeting policy in both countries, while section V computes the optimal policy under incomplete markets and evaluates the deadweight losses. Finally, section VI concludes.

2 The Model

The model belongs to the class of stochastic general equilibrium models that have been used for the evaluation of monetary policy both in the closed and open-economy literature.[6] The important addition is the treatment of an incomplete-market asset structure that can be directly compared to the complete-market one used in the literature. Our utility-based welfare criterion also allows for a direct evaluation of the welfare costs of imperfect risk-sharing, with particular emphasis on the different assumptions on the structural parameters of the model, the nature of the shocks – whether supply or demand– and the role of monetary policy.[7]

[6] Models in this class are Aoki (1998), Erceg et al. (2000), Goodfriend and King (1997, 2000), Henderson and Kim (1999), King and Wolman (1998), Woodford (1999a, 1999b, 2000) for the closed economy; Benigno P. (2001), Benigno and Benigno (2001), Corsetti and Pesenti (2001a, 2001b), Engel and Devereux (2000), Obstfeld and Rogoff (2000a, 2000c), Sutherland (2001) for the open economy.

[7] A non-exhaustive list of studies has analyzed the welfare costs of imperfect risk sharing in non-monetary models as in Backus et al. (1992), Cole and Obstfeld (1991), Kim et al. (2000), Lewis (1996), Mendoza (1995), Obstfeld (1994), Tesar (1995) and van Wincoop (1994, 1999).

We consider a world with two countries, H and F. The population on the segment $[0, n]$ belongs to country H while the population on the segment $(n, 1]$ belongs to country F. In each country, a continuum of differentiated goods is produced with measure equal to the population size. The utility of a generic consumer j belonging to country H is

$$U^j = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_t^j, \xi_{C,t}) + L\left(\frac{M_t^j}{P_t}, \xi_{M,t}\right) - \frac{1}{n}\int_0^n V(y_t(h), \xi_{Y,t}) dh \right] \right\}$$

where E_0 denotes the expectation conditional on the information set at date 0, while β is the intertemporal discount factor, with $0 < \beta < 1$.

Households obtain utility from consumption and the liquidity services of holding money, while they receive disutility from producing goods. The utility function is separable in these three factors. Moreover, each household contributes to the productions of all the goods produced in their own country with a separable disutility.[8] ξ_C, ξ_M and ξ_Y denote shocks to the preferences towards consumption, real money balances and production respectively. All these shocks are country specific. With starred variables we denote country's F variables.

The utility function U is an increasing concave function of the index C^j, defined as follows:

$$C^j \equiv \left[n^{\frac{1}{\theta}} (C_H^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} (C_F^j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where C_H^j and C_F^j are indexes of consumption across the continuum of differentiated goods produced respectively in country H and F,

$$C_H^j \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_o^n c^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

The elasticity of substitution across goods produced within a country is denoted by σ, which is assumed greater than one, while the elasticity of substitution between the bundles C_H and C_F is θ.

[8] This form of utility function can be seen as the outcome of a decentralized labor market in which households get disutility from supplying hours across all the firms within a country. This disutility is separable in the various efforts provided. On the other side, firms employ work, which is perfectly substitutable in production, from all the households belonging to their country.

L is an increasing concave function of the real money balances. Households derive utility from the liquidity service of holding money, where M_t^j is the agent j's money balance at the end of date t, while P_t is the appropriate country-specific price index used to deflate M_t^j. Indeed, P is defined as the minimum expenditure needed to buy one unit of the consumption index C^j.

We assume that all the goods are traded and that the law of one price holds. We further assume that the same composition of the consumption bundle C applies to country F. Given these assumptions, it follows that purchasing power parity holds, i.e. $P = SP^*$, $P_H = SP_H^*$ and $P_F = SP_F^*$, where S is the nominal exchange rate. Here we define the relative price T, the terms of trade, as $T \equiv P_F/P_H$.

The household j's demand of a generic good h, produced in country H, and of the generic good f, produced in country F, are

$$c^j(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\frac{P_H}{P}\right)^{-\theta} C^j, \qquad c^j(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} \left(\frac{P_F}{P}\right)^{-\theta} C^j. \quad (3)$$

Furthermore we assume that a country-specific level of public expenditure is allocated only among the goods produced in the country according to the following demands

$$g(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} G, \qquad g^*(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} G^*. \quad (4)$$

Combining (3) with (4) we can write total demand of good h and f as

$$y^d(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} [\left(\frac{P_H}{P}\right)^{-\theta} C^W + G], \qquad y^d(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} [\left(\frac{P_F}{P}\right)^{-\theta} C^W + G^*] \quad (5)$$

where the world consumption C^W is defined as

$$C^W \equiv \int_0^1 C^j dj.$$

We assume that households that belong to country H can allocate their wealth among domestic money and two bonds. Both bonds are risk-free with one-period maturity. One is denominated in domestic currency and the other in foreign currency. In contrast, households that belong to country F can allocate their wealth only in foreign money and in a risk-free nominal bond denominated in the foreign currency. Thus, the budget constraint of household j in country H (expressed in real terms with respect to the consumption-based price index) is

$$\frac{B^j_{H,t}}{P_t(1+i_t)}+\frac{S_tB^j_{F,t}}{P_t(1+i^*_t)}\frac{1}{\phi\left(\frac{S_tB_{F,t}}{P_t}\right)}+\frac{M^j_t}{P_t}\leq W^j_{t-1}+(1-\tau)\frac{1}{P_t}\frac{\int_0^n p_t(h)y_t(h)dh}{n}-C^j_t+\frac{TR^j_t}{P_t},$$

at each date t, with

$$W^j_{t-1}\equiv\frac{B^j_{H,t-1}+S_tB^j_{F,t-1}+M^j_{t-1}}{P_t}.$$

$B^j_{H,t}$ is household j's holding of the risk-free one-period nominal bond, denominated in units of currency H. The nominal interest rate on this bond is i_t. $B^j_{F,t}$ is household j's holding of the risk-free one-period nominal bond, denominated in units of currency F. The price of this bond is inversely proportional to its gross nominal interest rate $1+i^*_t$. The factor of proportionality is the function $\phi(.)$ which depends on the real holdings of the foreign assets in the entire home economy. This means that domestic households take the function $\phi(.)$ as given when deciding on the optimal holding of the foreign bond. We require some restrictions on $\phi(.)$: $\phi(0)=1$ and that $\phi(.)$ assumes the value 1 only if $B_{F,t}=0$; $\phi(.)$ is a, differentiable, (at least) decreasing function in the neighborhood of zero.

The function $\phi(.)$ captures the costs, for the households belonging to country H, of undertaking positions in the international asset market.[9] As borrowers, they will be charged a premium on the foreign interest rate; as lenders, they will receive a remuneration lower than the foreign interest rate. Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by the Foreign households) who can borrow from and lend to households of country F at the rate $1+i^*$, but can borrow from and lend to households of country H at the rate $(1+i^*)\cdot\phi(.)$. The profits from this activity are given by

$$K=\frac{B_{F,t}}{P^*_t}\frac{1}{(1+i^*_t)}\left[\frac{1}{\phi\left(\frac{B_{F,t}}{P^*_t}\right)}-1\right]>0$$

[9] For characterizing the incomplete financial structure, we do not really need to introduce this additional cost. However, this will be useful in pinning down a well-defined steady-state for consumption and assets. For related use in small open-economy models, see Senhadji (1997) and Schmitt-Grohé and Uribe (2000). Ghironi (2000) discusses how an overlapping generation structure can succeed for this purpose.

which are positive, given the shape of the function $\phi(.)$.

In characterizing the budget constraint, we also assume that all the households belonging to a country share the revenues from running the firms in equal proportion. Finally, τ denotes a country-specific proportional tax on nominal income, while TR_t^j denotes the government transfers to household j. The budget constraint at date t of the fiscal authority in country H is

$$\tau \int_0^n p_t(h) y_t(h) dh = \int_0^n M_t^j - \int_0^n M_{t-1}^j + G_t + \int_0^n TR_t^j.$$

Households of country F can allocate their wealth only between money and the risk-free nominal bond denominated in units of their own currency. However, they do not face any cost of intermediation and they can lend and borrow at the risk-free nominal interest rate i^*. The characterization of the budget constraint is

$$\frac{B_{F,t}^{*j}}{P_t^*(1+i_t^*)} + \frac{M_t^{*j}}{P_t^*} \leq W_{t-1}^{*j} + (1-\tau^*)\frac{1}{P_t^*}\frac{\int_{1-n}^1 p_t^*(f) y_t^*(f) df}{1-n} + \frac{K}{1-n} - C_t^{*j} + \frac{TR_t^{*j}}{P_t^*},$$

with

$$W_{t-1}^{*j} \equiv \frac{B_{F,t-1}^{*j} + M_{t-1}^{*j}}{P_t^*}.$$

We further assume that the initial level of wealth is the same across all the households belonging to the same country. This assumption combined with the fact that all the households within a country work for all the firms sharing the profits in equal proportion, implies that within a country all the households face the same budget constraint. In their consumption decisions, they will choose the same path of consumption. We can then drop the index j and consider a representative household for each country. However, consumption will not necessarily be risk shared at an international level. Given the sequences of $\{P_t, P_t^*, T_t, i_t, i_t^*,\}$ the optimal allocation of $\{C_t, C_t^*, B_{F,t}\}$ will be characterized by the following conditions:

$$U_C(C_t, \xi_{C,t}) = (1+i_t)\beta \mathrm{E}_t \left\{ U_C(C_{t+1}, \xi_{C,t+1}) \frac{P_t}{P_{t+1}} \right\}, \tag{6}$$

$$U_C(C_t^*, \xi_{C,t}^*) = (1+i_t^*)\beta \mathrm{E}_t \left\{ U_C(C_{t+1}^*, \xi_{C,t+1}^*) \frac{P_t^*}{P_{t+1}^*} \right\}, \tag{7}$$

$$U_C(C_t, \xi_{C,t}) = (1+i_t^*)\phi\left(\frac{B_{F,t}}{P_t^*}\right)\beta \mathrm{E}_t \left\{ U_C(C_{t+1}, \xi_{C,t+1}) \frac{P_t^*}{P_{t+1}^*} \right\}, \tag{8}$$

and

$$\frac{B_{F,t}}{P_t^*(1+i_t^*)}\frac{1}{\phi\left(\frac{B_{F,t}}{P_t^*}\right)}=\frac{B_{F,t-1}}{P_t^*}+\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta}C_t^W-C_t, \tag{9}$$

plus the appropriate no-Ponzi games and transversality conditions. Equations (6) and (7) represent the home and foreign Euler equations obtained by optimally choosing the holdings of the nominal bonds denominated in their respective currencies. Equation (8) represents household H's Euler equation derived by maximizing the holdings of the nominal bond denominated in currency F. Equation (9) represents the resource constraint of country H, which is obtained by aggregating the equilibrium budget constraint of the households with that of the government. We further impose the condition that bonds denominated in currency H are in zero-net supply within that country. The resource constraint of country F is redundant by Walras's law, having assumed that profits of intermediation are added up to the resources of the foreign economy.

2.1 Price-setting decisions

In this model suppliers behave as monopolists in selling their products. They do not take demand for the goods as given and can affect the quantity demanded through their pricing decisions as shown in equation (5). However, they are small with respect to the overall market and take as given the indexes P, P_H, P_F and C, C^*. Prices are subject to changes at random intervals as in the Calvo-Yun model. In each period, a seller faces a fixed probability $1-\alpha$ of adjusting the price, irrespective on how long it has been since the last change had occured. In this event the price is chosen to maximize the expected discounted profits under the circumstance that the decision on the price is still maintained; in fact, the seller also assumes that the price chosen at a certain date t will apply in the future at date $t+k$ with probability α^k. It is important to note that all the sellers that belong to the same country and that can modify their price at a certain time will face the same discounted future demands and marginal costs under the hypothesis that the new price is maintained. Hence they will set the same price. We denote with $\widetilde{p}_t(h)$ the price of the good h, in country H, chosen at date t and with $\widetilde{y}_{t,t+k}(h)$ the total demand of good h at time $t+k$ under the circumstances that the price $\widetilde{p}_t(h)$ still applies. From (5), $\widetilde{y}_{t,t+k}(h)$ is

$$\widetilde{y}_{t,t+k}(h) = \left(\frac{\widetilde{p}_t(h)}{P_{H,t+k}}\right)^{-\sigma} \left[\left(\frac{P_{H,t+k}}{P_t}\right)^{-\theta} C^W_{t+k} + G_{t+k}\right].$$

The optimal choice of $\widetilde{p}_t(h)$ is:

$$\widetilde{p}_t(h) = \frac{\sigma}{(\sigma-1)(1-\tau)} \frac{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k V_y(\widetilde{y}_{t,t+k}(h), \xi_{Y,t+k}) \widetilde{y}_{t,t+k}(h)}{\mathrm{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \lambda_{t+k} \widetilde{y}_{t,t+k}(h)}, \tag{10}$$

where λ is the marginal utility of nominal income which is common across agents within a country. Under the Calvo-style price-setting behavior, a fraction $(1-\alpha)$ of sellers, that can chose to adjust the price, sets the same price. Thus, we obtain the following state equation for $P_{H,t}$

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\widetilde{p}_t(h)^{1-\sigma}. \tag{11}$$

A similar optimal price-setting decision also holds in country F, with the appropriate starred variables.

2.2 Complete asset structure

The incomplete asset market model of the previous section is compared to a complete-market one. In this case, both domestic and foreign households can trade in a set of state-contingent securities that deliver one unit of the home and/or foreign currency in each state of nature. Under this market structure, the marginal utilities of consumption will be equated across countries at all dates and states of nature

$$U_C(C_t, \xi_{C,t}) = U_C(C_t^*, \xi_{C,t}^*). \tag{12}$$

as in Chari et al. (2000). In characterizing the allocation of consumption, conditions (6) and (7) still hold. Instead, conditions (8) and (9) are not anymore relevant. In fact, under complete markets, there is no need to outline the path of the current account for characterizing the consumption allocations in both countries. No-Ponzi games and transversality conditions should be appropriately modified.

3 Complete versus incomplete asset market equilibria

A good benchmark for the comparison of the equilibrium allocation under sticky prices is the flexible-price complete-market equilibrium. With flexible prices, the real marginal costs are constant across time and states of nature. In fact, from the price-setting decisions, it follows that

$$U_C(C_t, \xi_{C,t}) \frac{P_{H,t}}{P_t} = \frac{1}{1-\tau} \frac{\sigma}{\sigma-1} V_y \left(\left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t^W + G_t, \xi_{Y,t} \right), \tag{13}$$

$$U_C(C_t^*, \xi_{C,t}^*) \frac{P_{F,t}}{P_t} = \frac{1}{1-\tau^*} \frac{\sigma}{\sigma-1} V_y \left(\left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t^W + G_t^*, \xi_{Y,t}^* \right). \tag{14}$$

Combining equations (12), (13) and (14), one can characterize the allocation of $\{C, C^*, T\}$. Using a log-linear approximation around the steady-state (defined in appendix A), we obtain[10]

$$\begin{aligned} \widetilde{C}_t^R &= v_t^R, \\ \widetilde{C}_t^W &= \frac{\rho}{\rho+\eta} v_t^W + \frac{\eta}{\rho+\eta} (\overline{Y}_t^W - g_t^W), \\ \widetilde{T}_t &= \frac{\eta}{1+\theta\eta} (\overline{Y}_t^R - g_t^R), \end{aligned}$$

where a variable with an upper index W denotes a weighted average of the home and foreign variables with weights n and $1-n$ respectively, while a variable with an upper index R denotes the difference between the home and foreign variables. Thus, C^W denotes world consumption and C^R denotes the difference between the home and foreign consumptions. Furthermore, v^i and $\overline{Y}_t^i$ are reparametrization of the preference shock toward consumption and leisure, respectively (with $i = H, F$); g_t^i is a reparametrization of the public expenditure shock. We can interpret v^i as a country-specific demand shock. Since $\overline{Y}_t^i$ and g_t^i enter with diametrically opposite effects, we can interpret their difference as a supply disturbance. Finally, η and ρ are the inverses of the elasticity of labor supply and the intertemporal elasticity of

[10] Note that the steady-state around which we log-linearize is the same independent of the market structure.

Table 1: The sticky-price complete-market model

$\rho \mathrm{E}_t(c^W_{t+1} - c^W_t) = ni_t + (1-n)i^*_t - \widetilde{R}^W_t - \mathrm{E}_t\pi^W_{t+1}$	IS_W
$c^R_t = 0$	IS_R
$\mathrm{E}_t\Delta S_{t+1} = i_t - i^*_t$	UIP
$\pi_{H,t} = k[(\rho+\eta)c^W_t + (1-n)(1+\eta\theta)(\widehat{T}_t - \widetilde{T}_t)] + \beta \mathrm{E}_t\pi_{H,t+1}$	AS_H
$\pi^*_{F,t} = k^*[(\rho+\eta)c^W_t - n(1+\eta\theta)(\widehat{T}_t - \widetilde{T}_t)] + \beta \mathrm{E}_t\pi^*_{F,t+1}$	AS_F
$\widehat{T}_t = \widehat{T}_{t-1} + \Delta S_t + \pi^*_{F,t} - \pi_{H,t}$	TT

Notes : The index R refers to the difference between Home and Foreign variables. The index W refers to a weighted average of Home and Foreign variables with weights n and $1-n$. We have defined the consumption gap as the difference between the consumption that arises under sticky prices and the one that arises under the flexible-price complete-market allocation, i.e. $c_t \equiv \widehat{C}_t - \widetilde{C}_t$. While the world natural rate of interest is defined as $\widetilde{R}^W_t \equiv \rho \mathrm{E}_t\{(\widetilde{C}^W_{t+1} - \widetilde{C}^W_t) - (v^W_{t+1} - v^W_t)\}$.Other definitions are $k \equiv [(1-\alpha\beta)(1-\alpha)/\alpha] \cdot [1/(1+\sigma\eta)]$ and $k^* \equiv [(1-\alpha^*\beta)(1-\alpha^*)/\alpha^*] \cdot [1/(1+\sigma\eta)]$.Note that $\pi^W_t \equiv n\pi_t + (1-n)\pi^*_t = n\pi_{H,t} + (1-n)\pi^*_{F,t}$.

substitution in consumption, respectively.[11]

The presence of complete markets, as specified, do not necessarily equalize consumption across countries. Only the marginal utilities of nominal and real income are equal across countries.[12] Asymmetric demand shocks, that originate from shifts in consumption preferences, create a departure from complete equalization of consumption across countries. World consumption depends only on world shocks. Favorable supply and demand shocks both increase world consumption. On contrast, the terms of trade are not affected by demand shocks, since relative consumption adjusts to absorb these shocks. Terms of trade depend only on asymmetric supply shocks. Whenever there are asymmetric disturbances that induce the households in a country to work more, changes in the terms of trade optimally shift part of the burden to the households in the other country.

Tables 1 and 2 summarize the first-order approximation of the sticky-price model

[11] We have defined $\rho \equiv -U_{CC}\overline{C}/U_C$ and $\eta \equiv V_{yy}\overline{C}/V_y$, while v_t, $\overline{Y}_t$ are defined as $U_{CC}\overline{C}v_t = -U_{C\xi_C}\xi_{C,t}$ and $V_{yy}\overline{CY}_t = -V_{Y\xi_Y}\xi_{Y,t}$ while $g_t = G_t/\overline{C}$. v^*_t, $\overline{Y}^*_t, g^*_t$ are defined accordingly.

[12] With imperfect pass-through or pricing-to-market, it is the case that only the marginal utilities of nominal income are equated.

Table 2: The sticky-price incomplete-market model

$\rho \mathrm{E}_t(c_{t+1}^W - c_t^W) = ni_t + (1-n)i_t^* - \widetilde{R}_t^W - \mathrm{E}_t\pi_{t+1}^W$	IS$_W$
$\rho \mathrm{E}_t(c_{t+1}^R - c_t^R) = i_t - i_t^* - \mathrm{E}_t\Delta S_{t+1},$	IS$_R^I$
$\mathrm{E}_t\Delta S_{t+1} = i_t - i_t^* + \delta b_t$	UIP^I
$\beta b_t = b_{t-1} - (1-n)c_t^R - (1-n)v_t^R + (1-\theta)(n-1)(\widehat{T}_t - \widetilde{T}_t) + (1-\theta)(n-1)\widetilde{T}_t$	CA^I
$\pi_{H,t} = k[(\rho+\eta)c_t^W + \rho(1-n)c_t^R + (1-n)(1+\eta\theta)(\widehat{T}_t - \widetilde{T}_t)] + \beta \mathrm{E}_t\pi_{H,t+1}$	AS$_H^I$
$\pi_{F,t}^* = k^*[(\rho+\eta)c_t^W - \rho n c_t^R - n(1+\eta\theta)(\widehat{T}_t - \widetilde{T}_t)] + \beta \mathrm{E}_t\pi_{F,t+1}^*$	AS$_F^I$
$\widehat{T}_t = \widehat{T}_{t-1} + \Delta S_t + \pi_{F,t}^* - \pi_{H,t}$	TT

Notes : The index R refers to the difference between Home and Foreign variables. The index W refers to a weighted average of Home and Foreign variables with weights n and $1-n$. We have defined the consumption gap as the difference between the consumption that arises under sticky prices and the one that arises under the flexible-price complete-market allocation, i.e. $c_t \equiv \widehat{C}_t - \widetilde{C}_t$. While the world natural rate of interest is defined as $\widetilde{R}_t^W \equiv \rho \mathrm{E}_t\{(\widetilde{C}_{t+1}^W - \widetilde{C}_t^W) - (v_{t+1}^W - v_t^W)\}$. Other definitions are $k \equiv [(1-\alpha\beta)(1-\alpha)/\alpha] \cdot [1/(1+\sigma\eta)]$ and $k^* \equiv [(1-\alpha^*\beta)(1-\alpha^*)/\alpha^*] \cdot [1/(1+\sigma\eta)]$, while $\delta \equiv -\phi'(0)\overline{C}$.Note that $\pi_t^W \equiv n\pi_t + (1-n)\pi_t^* = n\pi_{H,t} + (1-n)\pi_{F,t}^*$.

under complete and incomplete markets, respectively.[13] The world IS equation, IS_W, is derived from a weighted average, with weights n and $1-n$, of the log-linear approximation of equations (6) and (7), which hold under both specifications of the asset structure. As a first step, we define the consumption gap as the difference between the sticky-price allocation, no matter what the structure of the market is, and the flexible-price complete-market allocation, i.e. $c \equiv \widehat{C} - \widetilde{C}$. Then, from IS_W, we find that the world consumption gap depends on the present and expected future values of the difference between the world real interest rate and the world natural interest rate, $\widetilde{R}^W$. The relevant deflator of the world nominal interest rate is the weighted average of the producer inflation rates, π^W.

The 'relative' IS equations, respectively IS_R and IS_R^I for the complete and incomplete-market model, depend instead on the structure of the asset market. Under complete asset markets, even in the case where prices are sticky, equation (12) holds,

[13] We denote with $\widehat{C}_t \equiv \ln C_t/\overline{C}$, $\widehat{C}_t^* \equiv \ln C_t^*/\overline{C}$ $\widehat{T}_t \equiv \ln T_t/\overline{T}$, $\pi_{H,t} \equiv \ln P_{H,t}/P_{H,t-1}$, $\pi_{F,t}^* \equiv \ln P_{F,t}^*/P_{F,t-1}^*$, $\Delta S_t = \ln S_t/S_{t-1}$ and $b_t = (B_{F,t}/P_t^*) \cdot \overline{C}^{-1}$.

and the consumption differential purely reflects asymmetric demand shocks. Relative consumption gap will be always zero. Moreover, taking the difference between the log-linear approximation of equations (6) and (7), one observes that uncovered interest parity holds and that the nominal interest-rate differential reflects expected depreciation of the exchange rate (UIP equation). However, under incomplete asset markets, uncovered interest parity does not hold and the spread in the nominal interest rates reflects a premium on top of the expected exchange rate depreciation (UIP^I equation)

$$i_t - i_t^* = \mathrm{E}_t \Delta S_{t+1} - \delta b_t. \tag{15}$$

The premium can be positive or negative depending on the Home country being a borrower or a lender in the market of the international asset. Equation (15) is derived by taking the difference between the log-linear approximation of equations (6) and (8). An additional characterization of the incomplete market model is that the consumption gaps are not necessarily equalized across countries. In fact, combining equations (6) and (7), it can be shown that

$$\rho E_t (c_{t+1}^R - c_t^R) = i_t - i_t^* - E_t \Delta S_{t+1},$$

which is the IS_R^I. Equivalently, by using equation (15) one finds that the relative consumption gap can be related to the foreign assets holdings

$$\rho E_t (c_{t+1}^R - c_t^R) = -\delta b_t. \tag{16}$$

With incomplete markets, the dynamic of the current account (equation CA^I) is needed in order to determine the path of the relative consumption gap and the nominal interest rate differential.

The aggregate supply block does not differ across asset structures. As it is common with a Calvo-style price-setting model, the producer inflation rates depend on the current and expected discounted future deviations of the real marginal costs from the steady-state value. However, the specification of the real marginal costs depends on the market structure. An important difference, under incomplete markets, is that an increase in the consumption gap of the home country relative to that of the foreign country pushes home inflation up. In fact, the marginal utility of nominal income decreases in the home economy and producer prices tend to rise so as to protect revenues. Aside from this channel, all other elements in the decomposition of the real marginal costs are similar across asset structures. A positive world consumption gap increases both home and foreign inflation, while the difference between the terms of trade under sticky prices and under the complete-market flexible-price allocation creates a dispersion of inflation across countries.

Finally TT denotes the terms of trade identity which shows that the terms of trade growth depends on the exchange rate depreciation and the inflation rate differential. The model is then closed with the policy rules chosen by the two central banks.

Although the complete and incomplete market allocations look different, there is one particular case in which the two allocations coincide independent of the policy followed by the monetary policymakers.

Proposition 1 *When the home and foreign demand shocks are symmetric such that* $v_t = v_t^*$ *at all dates* t, *and the intratemporal elasticity of substitution between Home and Foreign goods* θ *is unitary, given the initial condition* $b_{-1} = 0$*, then the complete-market and incomplete-market allocations coincide.*

In fact, under such conditions, the current account equation boils down to

$$b_t = \frac{b_{t-1}}{\beta} - (1-n)c_t^R,$$

which can be combined with equation (16) in a dynamic system of the form,

$$\mathrm{E}_t x_{t+1} = A x_t,$$

where $x_t' = [c_t \; b_{t-1}]$. Given that b is a predetermined variable, there exists a unique and bounded rational expectation solution if and only if there is one eigenvalue inside the unit circle. For this to be the case, δ should be positive. This happens to be true given the assumption that the function $\phi(.)$ is decreasing in a neighborhood of zero. It can be further shown that in this unique solution

$$b_t = \mu_1 b_{t-1}$$

where μ_1 is the stable eigenvalue of the matrix A. It then follows that given the initial condition, $b_{-1} = 0$, b is always zero at all dates t.

It is a well-established result that, with Cobb-Douglas preferences, the terms of trade provides a risk-sharing role. Indeed this finding has been well emphasized by Cole and Obstfeld (1991), Corsetti and Pesenti (2001a) and Helpman and Razin (1978). In this paper, we have shown that this result applies only to asymmetric supply shocks, and not to asymmetric demand shocks that originate instead from perturbations to the marginal utility of consumption.

4 Welfare and the costs of imperfect risk sharing

A natural criterion for evaluation of different monetary policies under the two asset market structures is the sum of the utilities of the consumers

$$\begin{aligned} W &\equiv \int_0^1 U^j dj \\ &= \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t w_t \right\}, \end{aligned}$$

where

$$w_t \equiv nU(C_t, \xi_{Y,t}) + (1-n)U(C_t^*, \xi_{Y,t}^*) - \int_0^n V(y_t(h), \xi_{Y,t})dh - \int_n^1 V(y_t^*(f), \xi_{Y,t}^*)df$$

In this specification, the utility derived from the liquidity services of holding money has been neglected.[14] By taking a second-order approximation of W around the steady state in which a taxation subsidy completely offsets the monopolistic distortions in both countries, we obtain that

$$W = -\frac{U_C\overline{C}}{2}\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}, \tag{17}$$

where

$$\begin{aligned} L_t &= (\rho+\eta)\cdot[c_t^W]^2 + n(1-n)\rho[c_t^R]^2 + n(1-n)(1+\eta\theta)\theta\cdot[\widehat{T}_t - \widetilde{T}_t]^2 \\ &\quad + n\frac{\sigma}{k}(\pi_{H,t})^2 + (1-n)\frac{\sigma}{k^*}(\pi_{F,t}^*)^2 + \text{t.i.p.} + o(\|\xi\|^3), \end{aligned}$$

[14] This is a common assumption in the literature. In a closed-economy model, Woodford (1999a,b) addresses the case in which the utility derived from the liquidity preference is not negligible.

and $t.i.p.$ denotes elements that are independent of the policy while $o(\|\xi\|^3)$ measures residuals of third-order in the maximum amplitude of the shocks.

Using equation (17), one can evaluate the deadweight losses implied by the distortions existing in the model. This idea goes back to King and Wolman (1998), Rotemberg and Woodford (1997) and Woodford (1999b), where a standard public finance approach is used to evaluate the magnitude of the distortions existing in the economy.[15]

From equation (17), we can see that when the monopolistic distortions are offset by appropriate taxation subsidies, the flexible-price complete-market allocation is the efficient allocation for the whole economy. Any departure from this allocation produces losses for society. Price stickiness is a source of distortions when, combined with staggered prices, creates dispersion of demand across goods that are produced according to the same technology. The squares of the producer inflation rates in each country capture these distortionary costs. On the other side, relative prices should move when there are asymmetric productivity shocks. In fact, the terms of trade should offset asymmetric supply shocks. In the welfare function, this is captured by the square of the terms of trade with respect to their efficient level. Finally, the world and relative consumption gap should be completely stabilized. In particular, a departure from the complete risk-sharing of the marginal utilities of nominal incomes creates welfare costs. And the microfounded welfare criterion delivers appropriate weights for each of these distortions.

In this work, we limit the analysis to cooperative equilibria in which the central planner maximizes (17) under the structural equilibrium conditions. In particular, the optimal plan can be described as a commitment to a certain path of producer inflation rates $\{\pi_{H,t},\, \pi^*_{F,t}\}$. To the end of analysing the optimal plan, a useful result is the following.

Remark 2 *A combined policy of zero producer inflation rates in both countries, i.e.*

[15] In particular, in a closed-economy model, Rotemberg and Woodford (1998) and Woodford (1999) use a quadratic approximation of the welfare that can be correctly evaluated by a log-linear approximation of the equilibrium conditions. The application of this technique to an open-economy model requires appropriate qualifications, as discussed in Benigno and Benigno (2000) and Kim and Kim (1999). See Appendix B for details.

$\pi_{H,t} = \pi^*_{F,t} = 0$ *at all dates* t*, reproduces the same allocation as under flexible prices.*

When producer inflation rates are stabilized to zero in both countries, the real marginal costs are constant in all periods and the fluctuations of the economy reproduce the allocation that would arise if prices were perfectly flexible. But, among the flexible-price allocations, the structure of the asset market matters for the equilibrium path of various variables. The distinction between a complete versus incomplete-market structure is still crucial.

Proposition 3 *When asset markets are complete, stabilizing the producer inflation rates to zero in each country achieves the first-best; when asset markets are incomplete, stabilizing the producer inflation rates to zero does not achieve the first-best, except for the cases considered in proposition 1.*

The first part of the proposition with respect to complete markets is self-explanatory. However when asset markets are incomplete, policy intervention cannot achieve the first-best because in the economy there are too many distortions than instruments. By stabilizing producer inflation rates to zero, the two monetary policymakers can eliminate the distortions induced by the fact that prices are sticky, in a staggered way, and can thus close the world consumption gap to zero. The response to world shocks is still the efficient one, as also outlined in Obstfeld and Rogoff (2000c). However, the adjustment to asymmetric demand and supply shocks is inefficient. In fact, under a combined path of zero producer inflation rates in both countries, the terms of trade gap will move in the opposite direction of the consumption gap as shown by the following equation

$$\widehat{T}_t - \widetilde{T}_t = -\frac{\rho}{1+\eta\theta} c^R_t. \tag{18}$$

If consumption increases in one country relative to the other, producer prices tend to increase in order to offset the fall in the marginal utility of revenue. As a result, the terms of trade of the country with the higher consumption worsens in order to reduce the pressure to its producer inflation rate. By using condition (18) into CA^I and equation (16), we can reduce the equations describing the flexible-price incomplete market allocation into a dynamic system as follows

$$\mathrm{E}_t x_{t+1} = D x_t + M \varepsilon_t$$

where $x_t' = [c_t^R \; b_{t-1}]$, $\varepsilon_t = [\widetilde{T}_t \; v_t^R]$, while D and M are 2×2 matrices. We assume AR(1) processes for the shocks $\widetilde{T}_t$ and v_t^R. Under the conditions of determinacy, the unique and stable solution can be written as[16]

$$\begin{aligned} c_t^R &= -e_1 b_{t-1} - z_1(1-\theta)\widetilde{T}_t - z_2 v_t^R, \\ b_t &= \mu_1 b_{t-1} - z_3(1-\theta)\widetilde{T}_t - z_4 v_t^R, \end{aligned}$$

where μ_1 (with $0 < \mu_1 < 1$) is the stable eigenvalue of the matrix D, and e_1 (with $e_1 < 0$) is the second element of the left eigenvector associated with the unstable eigenvalue of the matrix D; z_1, z_2, z_3 and z_4 are all positive coefficients which are combinations of the structural parameters of the model. Asymmetric demand shocks have an unambiguous effect on the relative consumption gap and the accumulation of assets. With complete markets, the marginal utilities of consumption are equated across countries. Asymmetric demand shocks are balanced by offsetting movements in relative consumption. As asset markets depart from perfect completeness, it is no longer possible to perfectly insure internationally the marginal utilities of nominal income. When a demand shock happens in country H, consumption rises relative to the other country through an increase in international borrowing; but consumption cannot rise enough to completely match the shock. Hence, the consumption gap of country H falls relative to the other country.

Instead, an asymmetric supply shock, such as a terms of trade shock, has an ambiguous effect on the relative consumption gap and the accumulation of assets. The effect depends on whether the value of the intratemporal elasticity of substitution between Home and Foreign goods is above or below one. When the value of θ is below unity, a terms of trade shock can result in what is known in the trade literature as 'immiserising growth'. As shown in Bhagwati (1956), the deterioration of the terms of trade can offset the beneficial effect of an expansion by reducing real income. Indeed, when θ is less than 1, a temporary positive productivity shock in country H produces a temporary negative effect on the real income of country H, consumption decreases less than proportionally because of consumption-smoothing behavior; the

[16] For determinacy of the equilibrium, it is required that $\theta \geq \max(\underline{\theta}, 0)$ where $\underline{\theta} = (\rho - 1)/(\eta + \rho)$. Note that if ρ is less than 1, then $\underline{\theta}$ is less than 0, while as ρ increases above 1, $\underline{\theta}$ becomes positive but always less than 1.

level of borrowing increases in order to finance a level of consumption above the fall in the real income. The actual level of the terms of trade overshoots the efficient level. A totally opposite pattern arises in the case when θ is bigger than 1. A temporary positive productivity shock produces a temporary positive effect on real income, consumption increases, but less than proportionally, and assets accumulate.

With complete markets, the terms of trade gap and consumption gap are stabilized to zero. When markets are incomplete, consumption and the terms of trade deviate from the efficient equilibrium. This inefficient path creates deadweight losses that can be evaluated using the microfounded welfare criterion. There is an extensive literature on the welfare costs of market incompleteness and the results are controversial. Some of the papers report very small gains from international risk-sharing (less than 0.5% of units of steady-state consumption) while others report much higher values (sometimes of the order of 20%).[17]

The model presented here resembles that of Cole and Obstfeld (1991), but with some important qualifications. First, we characterize an economy with an endogenous labor supply; second, we stress the role of demand shocks in characterizing the departure of the incomplete market allocation from the complete-market one. We have seen that the role of the terms of trade as an insurance device does not apply to these relative demand shocks. Third, the criterion (17) allows for an analytical evaluation of the elements that contribute to the welfare costs by using only a log-linear approximation to the structural equilibrium conditions. Finally, as will be shown in the next section, our framework allows also for an analysis of the contribution of monetary policy in improving or worsening the welfare costs of market incompleteness.

Consistent with the literature and following Lucas (1987), we reparametrize the welfare costs in terms of a permanent percentage shift in steady-state consumption. We denote with W^C, the welfare under the flexible-price complete-market allocation and with W^I, the welfare under the flexible-price incomplete-market allocation. Both allocations can be obtained by stabilizing the producer price index in both countries.

[17] A non-exaustive list is Backus et al. (1992), Cole and Obstfeld (1991), Lewis (1996), Mendoza (1995), Obstfeld (1994), Tesar (1995) and van Wincoop (1994, 1996).

The permanent percentage shift in steady-state consumption, λ, can be written as

$$\lambda \equiv \frac{W^C - W^I}{U_C \overline{C}} = \frac{n(1-n)\rho}{2} \cdot \left[\frac{1+\theta(\eta+\rho)}{1+\theta\eta}\right] \cdot \mathrm{E}_0 \left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t (c_t^R)^2\right\}. \qquad (19)$$

It follows that the costs of following a zero-producer-inflation-rate policies with incomplete markets are proportional to a proxy of the variance of the consumption-gap differential. The measure λ is highly sensitive to the parametrization used.[18] In our case, the relative consumption gap and its variance depend crucially on the kind of shocks, their persistence and the other structural parameters. As a particular case, when the intratemporal elasticity of substitution between Home and Foreign goods is unitary and the demand shocks are perfectly symmetric across countries, then the costs of incomplete markets are zero. It is then interesting to evaluate how the variance of the relative consumption gap and the costs of market incompleteness vary as the model departs from these assumptions. Moreover, in equation (19), the coefficient of proportionality that multiplies the variance is a function of the parameters of the model. In particular, it is increasing in the degree of risk aversion of the households. In the next section, we will perform comparisons of the welfare costs across different parametrization.

4.1 A calibrated example

In this section, we compare the flexible-price incomplete-market allocation to the efficient allocation using a calibrated example, both from a quantitative and a normative perspective. We set $\beta = 0.99$ which implies that the steady-state real interest rate is around 4% (in a quarterly model). We assume that countries are of equal size, $n = 0.5$. The inverse of the elasticity of labor supply is calibrated according to Rotemberg and Woodford (1998), $\eta = 0.47$. They also estimate the risk aversion of the consumer, $\rho = 0.16$. Later, we perform an sensitivity analysis around the values suggested by Eichenbaum et al. (1988), varying from 0.5 to 3. The parameter δ, that measures the cost of intermediation in the foreign market, is set to 10^{-3}. Note that,

[18] This is common in the literature on the costs of imperfect risk sharing. A detailed discussion is in Kim et al. (2000) and Van Wincoop (1999).

for these comparisons, we do not need to calibrate the parameters indicating the degrees of monopolistic competition and price rigidities, since a zero inflation-targeting policy in both countries makes the allocation independent of these parameters. Instead, we let the intratemporal elasticity of substitution between home and foreign goods in consumption vary across reasonable values. We assume that $\widetilde{T}$ and ν are processes of the form

$$\begin{aligned}\widetilde{T}_t &= \rho_1 \widetilde{T}_{t-1} + u_{1,t} \\ \nu_t^R &= \rho_2 v_{t-1}^R + u_{2,t}\end{aligned}$$

where u_1 and u_2 are white-noise processes. The international real business cycle literature has in general assumed autoregressive productivity shocks with degree of persistency around 0.9. We adopt this assumption both for the supply and demand shocks.

Figure 1 shows the impulse response functions of the terms of trade gap, the consumption gap and foreign assets to a terms of trade shock. As outlined in the discussion above, any departure of the intratemporal elasticity of substitution from a unitary value creates inefficient fluctuations of consumption and terms of trade. The greater the departure from the unitary value, the larger the size of the fluctuations. The sign of the response has already been discussed in the previous section. Note that the time of the adjustment towards the initial steady-state depends on the parameter δ and the degree of persistency of the shocks.

Figure 2 shows the impulse responses of the terms of trade gap, the consumption gap and foreign assets to a relative demand shock, v^R. An interesting result is that an increase in the intratemporal elasticity of substitution between home and foreign goods dampens the departure of the terms of trade and consumption from their efficient levels. When there is a demand shock in country H, households of country H tend to borrow in order to sustain a higher level of consumption. The higher consumption increases the demands for home and foreign goods. However the overall consumption of home households does not rise enough to match the demand shock. The home marginal utility of nominal income increases. The exchange rate of country H depreciates to sustain the demand and a stable level of prices in country H. The improvement in the home terms of trade produces a wealth effect for households in country H that pushes up home consumption. This effect is stronger the higher is the intratemporal elasticity of substitution in consumption. In this case, weaker movements in the terms of trade and assets are required to sustain a higher level of home consumption.

For the evaluation of welfare costs, we proceed in two steps. First, we evaluate only the costs induced by supply shocks. We analyze two cases: one characterized by relatively low persistence of the shock ($\rho_1 = 0.9$), and the other by high persistence ($\rho_2 = 0.95$).[19] In these two scenarios, we allow the intratemporal elasticity of substitution (θ) to vary from the lower bound required for determinacy, $\underline{\theta}$, to 10 and the coefficient of risk aversion (ρ) to vary from 1/6 to 3. Our measure of the welfare costs is the fraction λ over the variance of the relative supply shocks. Figures 3 and 4 plot $\lambda/\sigma_{u_1}^2$ with respect to the values of the intratemporal elasticity of substitution θ. Consistent with the theoretical analysis, when the intratemporal elasticity of substitution is equal to 1, the movement in the terms of trade offsets supply shocks completely. An increase in the intratemporal elasticity of substitution amplifies the costs of market incompleteness. This also happens with an increase in the risk aversion and in the persistence of the shocks. The picture shows that the magnitude of the costs is highly sensitive to the parametrization used. Hence we try to fix some of the parameters to values that have been estimated and used frequently in the literature. As outlined in Obstfeld and Rogoff (2000b) the estimation of the intratemporal elasticity of substitution is a crucial parameter. According to some recent studies, such as Harrigan (1993) and Trefler and Lai (1999), a sensible assumption for this parameter values is 6. As mentioned above, the degree of risk aversion can be assumed to be around 1 or 2. Other crucial parameters are the degree of persistence and the variance of the relative supply shocks. For this purpose, we exploit the calibration used for the technology shock in the international real business cycle literature. Following Kehoe and Perri (2000), Backus et al. (1992) and Baxter and Crucini (1995), we assume that the persistence of the supply shocks is 0.9 and the variance of each country productivity shock is 0.007^2 with a correlation of 0.25.

[19] Comparisons of this kind are used in the international business cycle literature, see Baxter and Crucini (1995) and Kehoe and Perri (2000).

Thus, we obtain that the variance of the relative supply shocks, $\sigma^2_{u_1}$, to be 0.0086^2.

For these parameter values, we can see from figure 3 and 4 that the measure $\lambda/\sigma^2_{u_1}$ assumes values between 2 to 8 depending on whether the persistence of the shocks is low or high. For the assumed value of $\sigma^2_{u_1}$, we obtain that the gains from having complete markets ranges in the order of 0.02 % to 0.07% of a permanent shift in steady-state consumption. These results are of the same order of magnitude of the estimate of the costs of the business cycle given by Lucas (1987).

Next, we consider the costs implied by demand shocks. We repeat the same experiment by varying some of the structural parameters of the model. As shown in figures 5 and 6, we now find that in contrast to the previous case with supply shocks an increase in the intratemporal elasticity of substitution reduces the costs implied by the imperfect offsetting of the relative demand shocks. As outlined above, the wealth effect induced by the movement in the terms of trade is higher and so consumption rises more in response to the demand shock. But, for values of θ around 6, the costs of the incomplete markets are of the order of 1/5 of the variance of relative demand shocks. Assuming that relative demand shocks have the same variance as relative supply shocks, the costs implied by relative demand shocks are small with respect to the costs implied by relative supply shocks.

5 Optimal Monetary Policy under Incomplete Markets

In the previous section, we showed that the optimal policy under a complete asset structure is to stabilize the producer inflation rates and real marginal costs. All distortions are simultaneously eliminated by replicating the flexible-price allocation. This is not generally the case under incomplete markets.

Proposition 4 *In the optimal plan, with incomplete asset markets, unless $\theta = 1$ and $\upsilon_t = \upsilon^*_t$ at all dates t, the producer inflation rates are state contingent.*

The proof is related to an application of the Ramsey's principle of optimal taxation in a monetary policy environment. In a model with incomplete markets and sticky prices, there are several distortions as shown in equation (17): imperfect risk-sharing, a non-efficient adjustment of the terms of trade and consumption, a non-zero producer inflation rates. Given that there are several trade-offs among these deadweight losses, it is optimal to distribute the losses across different uses. Hence, if the producer inflation rates are stabilized to zero, it would be optimal to marginally increase the inflation rates – thereby creating a distortion of a second order – and reducing the magnitude of the other distortions – by adding a first-order benefit. An outcome in which each country pursues a policy of price stability is then sub-optimal.

This result has been also suggested, but in a static context, by Obstfeld and Rogoff (2000c). However, all the previous studies, using open-economy models, have focused on a flexible-price allocation that coincides with the efficient equilibrium. And, with producer currency pricing and complete markets, stabilization of the real marginal costs in each country results in such an efficient outcome, as shown in Benigno P. (2001), Corsetti and Pesenti (2001b), Devereux and Engel (2000) and Obstfeld and Rogoff (2000a,c). This is because the exchange rate adjusts for asymmetric shocks while monetary policymakers pursue their stabilization policy.[20]

Here, we have an interesting case in which even in a cooperative solution the flexible-price allocation is not efficient because of the frictions induced by incomplete asset markets. The flexible-price allocation is still a feasible solution, even when

[20] With imperfect pass-through or pricing-to-market, the terms of trade is partially insulated from movements of the exchange rate and cannot allocate efficiently asymmetric shocks. The efficient allocation is not feasible, as shown in Benigno G. (1999), Corsetti and Pesenti (2001b) and Devereux and Engel (2000). In a non-cooperative solution, the strategic interaction between policymakers may exacerbate the inefficiency. However, as shown in Obstfeld and Rogoff (2000a,c), when each monetary policymaker can commit to not systematically fool the price setters, then the non-cooperative solution can achieve efficiency. This is because the strategic interaction between the terms of trade and the monopolistic distortions, stressed by Corsetti and Pesenti (2001a) and Tille (2000), is eliminated by the ex-ante commitment. In a discretionary equilibrium, the tension between monopolistic distortions and terms of trade arises again and then the non-cooperative solution leads to a sub-optimal allocation, as in Benigno and Benigno (2000). With imperfect pass-through, cooperative and non-cooperative solution differs since already the cooperative solution is affected by inefficiencies. Instead, Clarida et al. (2001) discusses a small open-economy case which is isomorphic to the standard closed economy model.

markets are incomplete. But, with incomplete markets and sticky prices, monetary policy can do better than replicate the flexible-price allocation. The frictions imposed by sticky prices should be used to reduce the distortions due to market incompleteness. In the next section we examine whether the reduction in the deadweight losses by pursuing the state-contingent producer inflation rate policy is of a significant amount.

5.1 A calibrated example

In this example, we use the calibration values used in the previous section for the parameters β, δ, ρ, η, n, ρ_1, ρ_2, setting them equals to 0.99, 0.001, 0.16, 0.47, 0.5, 0.9, 0.9 respectively. In addition, we now need to calibrate the parameters indicating the degrees of rigidity and monopoly power. We assume that country H is the U.S. while country F is Europe. Following recent empirical works on the estimation of forward-looking aggregate supply equations, such as Galí and Gertler (1998), Galí et al. (2001) and Sbordone (1998, 2000), we assume that α and α^* are 0.66 and 0.75, which implies that the durations of the contracts are 3 and 4 quarters, respectively. The degree of monopolistic competition is taken from Rotemberg and Woodford (1998), where they set $\sigma = 7.66$ which implies an average mark-up of 15%.

Figures 7 and 8 show the impulse response functions of the terms of trade gap, consumption gap, foreign asset holdings, home and foreign inflation rates in response to asymmetric supply and demand shocks, respectively. Comparing these figures with figures 1 and 2, it appears that both the optimal policy and the zero inflation-targeting policy imply similar paths for the consumption gap and foreign assets. The only remarkable difference is in the terms of trade gap. Indeed, inflation rates move to coordinate changes in the exchange rate so as to stabilize the terms of trade to the efficient level. If, under the flexible-price allocation, the terms of trade is below the efficient level, inflation in country H should increase while inflation in country F should decrease in order to produce a depreciation of the home currency which then stabilizes the terms of trade. This exchange rate adjustment has only a minor impact on asset accumulation and on the relative consumption gap. Note that the movements in the inflation rates are small in magnitude. All these results suggest that the optimal policy does not differ that much from the zero inflation-targeting policy.

This can also be seen in figures 9 and 10, where the welfare loss obtained under the optimal policy is compared with the zero inflation-targeting policy. In these figures, we assume that the coefficient of risk aversion in the consumers' preferences ρ is 2 and that θ varies between $\underline{\theta}$ and 20. But regardless of the value of the intratemporal elasticity of substitution, the gains obtained by pursuing the optimal policy instead of the zero inflation-targeting policy are small when measured in terms of the reduction of the deadweight losses.

The important issue to observe is that our results can be sensitive to the degree of rigidity assumed. Indeed, as we showed earlier, under the zero inflation-targeting policy the welfare is independent of the degree of rigidity, while this is not the case under the optimal policy. In other simulations, we have also explored other values for the degree of nominal rigidity, but without any significant difference from the conclusion reached above.

There is a natural intuition for this result. When the degrees of rigidity are small in both countries or at least in one country, then there is not much room for improving welfare by exploiting price-stickiness. However, as the degrees of rigidity increase, we can see from equation (17) that the weights given to the costs of producing inflation become larger (σ/k and σ/k^* increases) and thus it is optimal to stabilize producer inflation rates and reproduce the flexible-price allocation.

Even with incomplete markets, price stability – which entails the stabilization of the mark-up– is a quasi-optimal policy. There are two desirable aspects of this policy: first, it is transparent and easy to communicate so that private sectors can adjust expectations appropriately; second, as stressed by Obstfeld and Rogoff (2000c), it does not require any sort of coordination across countries. Following their intuition, cases in which the optimal cooperative solution can improve the flexible-price allocation are also cases in which the non-cooperative allocation differs from the cooperative one. However, given that the gains from pursuing the optimal cooperative solution instead of the flexible-price allocation are small, our result points toward minimizing the need of international coordination, as in Obstfeld and Rogoff (2000c). However, differently from their analysis, this is not to say that the costs of imperfect financial integration are small. It is instead true that the costs of imperfect financial integrations can be neglected when designing the conduction of monetary policy, and that policies that aim at stabilizing the economy in the single countries can be quasi-optimal. However, integration of the financial market is an important achievement for modern economies, not a job of monetary policy, with gains that are not necessarily of second order.

6 Conclusion

In this paper, we have shown that the argument for price stability holds even if markets are incomplete. However, there are some caveats that needs to be addressed. For instance, we have only considered rigidities in the market for the final goods. Introducing rigidities in the intermediate sector, as in McCallum and Nelson (2001), or in the labor market, as in Erceg et al. (2000), would alter our conclusion as to the appropriate price index to target. This would also be the case in a model with non-traded goods and sticky-prices in those goods. These results would also need to be qualified, if one considers differences in price-setting across various markets as in the case of an exchange rate pass-through. All this qualifications would certainly have altered the finding on the optimal policy to pursue. But, they would not affect the main result that monetary policy should not be conducted in a way to eliminate the frictions induced by imperfect financial markets.

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Appendix A

This appendix solves for the steady-state allocation.

We consider a steady-state in which all the shocks are zero and in which the home and foreign monetary policymakers set their respective CPI inflation rates to zero:

$$P_t/P_{t-1} = P_t^*/P_{t-1}^* = 1.$$

Equations (6) and (7) then imply that the nominal interest rates, $1+i$ and $1+i^*$ equal to

$$1+i = 1+i^* = \frac{1}{\beta},$$

which along with equation (8) implies that $B_F = 0$ in the steady state. From equation (9) we obtain that

$$\overline{C} = \left(\frac{\overline{P_H}}{P}\right)^{1-\theta} \overline{C}^W, \tag{A.1}$$

and from the resource constraint of the foreign country

$$\overline{C}^* = \left(\frac{\overline{P_F}}{P}\right)^{1-\theta} \overline{C}^W, \tag{A.2}$$

where we recall that

$$n\left(\frac{\overline{P_H}}{P}\right)^{1-\theta} + (1-n)\left(\frac{\overline{P_F}}{P}\right)^{1-\theta} = 1. \tag{A.3}$$

From the price-setting condition we get

$$U_C(\overline{C})\frac{\overline{P_H}}{P} = \frac{1}{1-\tau}\frac{\sigma}{\sigma-1}V_y\left(\left(\frac{\overline{P_H}}{P}\right)^{-\theta}\overline{C}^W\right), \tag{A.4}$$

$$U_C(\overline{C}^*)\frac{\overline{P_F}}{P} = \frac{1}{1-\tau^*}\frac{\sigma}{\sigma-1}V_y\left(\left(\frac{\overline{P_F}}{P}\right)^{-\theta}\overline{C}^W\right). \tag{A.5}$$

Assuming $\tau = \tau^*$ and combining conditions (A.1), (A.2), (A.3), (A.4), (A.5), we obtain that $\overline{C} = \overline{C}^*$ and $\overline{\frac{P_H}{P_F}} = 1$.

Appendix B

In this appendix we derive the second-order approximation of equation (17) in the text, following Woodford (1999b). The welfare criterion is

$$W = \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t w_t\right\},$$

where the utility flow is defined as a weighted average of the utility of both countries, disregarding the liquidity effects of holding real money balance

$$w_t \equiv \left[nU(C_t, \xi_{C,t}) + (1-n)U(C_t^*, \xi_{C,t}^*) - \int_0^n V(y_t(h), \xi_{Y,t})dh - \int_n^1 V(y_t(f), \xi_{Y,t}^*)df \right]. \tag{B.6}$$

First, we take a second-order expansion of the term

$$nU(C_t, \xi_{C,t}) + (1-n)U(C_t^*, \xi_{C,t}^*)$$

around the steady state where $\overline{C} = \overline{C}^*$ and where $\xi_{C,t} = 0$ at each date t obtaining

$$nU(C_t, \xi_{C,t}) + (1-n)U(C_t^*, \xi_{C,t}^*) = U(\overline{C}) + nU_C(C_t - \overline{C}) + (1-n)U_C(C_t^* - \overline{C}) + \tag{B.7}$$

$$\begin{aligned} &+\frac{n}{2}U_{CC}(C_t - \overline{C})^2 + \frac{1-n}{2}U_{CC}(C_t^* - \overline{C})^2 + nU_{C\xi_C}(C_t - \overline{C})\xi_{C,t} \\ &+(1-n)U_{C\xi_C}(C_t^* - \overline{C})\xi_{C,t}^* + o(\|\xi\|^3) \end{aligned} \tag{B.8}$$

where $o(\|\xi\|^3)$ represents all the terms that are of third order or higher in the deviations of the various variables from their steady-state values. Furthermore, expanding C_t, C_t^*, C_t^W with a second-order Taylor approximation we obtain

$$C_t = \overline{C}(1 + \widehat{C}_t + \frac{1}{2}\widehat{C}_t^2) + o(\|\xi\|^3), \tag{B.9}$$

$$C_t^* = \overline{C}(1 + \widehat{C}_t^* + \frac{1}{2}\widehat{C}_t^{*2}) + o(\|\xi\|^3), \tag{B.10}$$

$$C_t^W = \overline{C}(1 + \widehat{C}_t^W + \frac{1}{2}(\widehat{C}_t^W)^2) + o(\|\xi\|^3), \tag{B.11}$$

where $\widehat{C}_t = \ln(C_t/\overline{C})$, $\widehat{C}_t^* = \ln(C_t^*/\overline{C})$ and $\widehat{C}_t^W = \ln(C_t^W/\overline{C})$. Substituting (B.9), (B.10) and (B.11) into (B.8) we obtain that

$$\begin{aligned} nU(C_t, \xi_{C,t}) + (1-n)U(C_t^*, \xi_{C,t}^*) = U_C\overline{C}[\widehat{C}_t^W + \frac{1}{2}(\widehat{C}_t^W)^2 - n\frac{\rho}{2}\widehat{C}_t^2 \\ -(1-n)\frac{\rho}{2}(\widehat{C}_t^*)^2 + n\rho\widehat{C}_t v_t + (1-n)\rho\widehat{C}_t^* v_t^*] + \text{t.i.p.} + o(\|\xi\|^3) \end{aligned} \tag{B.12}$$

where we have defined $\rho \equiv -U_{CC}\overline{C}/U_C$, $U_{CC}\overline{C}v_t = -U_{C\xi_C}\xi_{C,t}$, $U_{CC}\overline{C}v_t^* = -U_{C\xi_C}\xi_{C,t}^*$ and where in t.i.p. we include all the terms that are independent of monetary policy.

Similarly, we take a second-order Taylor expansion of $V(y_t(h), \xi_{Y,t})$ around a steady state where $y_t(h) = \overline{Y} = \overline{C}$ for each h, at each date t, and where $\xi_{Y,t} = 0$ at each date t, obtaining

$$\begin{aligned} V(y_t(h), \xi_{Y,t}) &= V(\overline{Y}, 0) + v_y(y_t(h) - \overline{Y}) + v_{\xi_Y}\xi_{Y,t} + \frac{1}{2}v_{yy}(y_t(h) - \overline{Y})^2 \\ &\quad + v_{y\xi_Y}(y_t(h) - \overline{Y})\xi_{Y,t} + \frac{1}{2}v_{\xi_Y\xi_Y}(\xi_{Y,t})^2 + o(\|\xi\|^3), \end{aligned} \tag{B.13}$$

where $\widehat{y}_t(h) = \ln(y_t(h)/\overline{Y})$. In the same way, we can take an expansion of $v(y_t(f), \xi^*_{Y,t})$ obtaining

$$\begin{aligned} V(y_t(f), \xi^*_{Y,t}) &= V(\overline{Y}, 0) + v_y(y_t(f) - \overline{Y}) + v_{\xi^*_Y}\xi^*_{Y,t} + \frac{1}{2}v_{yy}(y_t(f) - \overline{Y})^2 \\ &\quad + v_{y\xi^*_Y}(y_t(f) - \overline{Y})\xi^*_{Y,t} + \frac{1}{2}v_{\xi^*_Y\xi^*_Y}(\xi^*_{Y,t})^2 + o(\|\xi\|^3). \end{aligned} \tag{B.14}$$

Combining (B.13) and (B.14), we obtain

$$\int_0^n V(y_t(h), \xi_{Y,t})dh + \int_n^1 V(y_t(f), \xi^*_{Y,t})df = \int_0^n v_y(y_t(h) - \overline{Y})dh + \int_n^1 v_y(y_t(f) - \overline{Y})df$$

$$\begin{aligned} &+\frac{1}{2}\int_0^n v_{yy}(y_t(h) - \overline{Y})^2 dh + \frac{1}{2}\int_n^1 v_{yy}(y_t(f) - \overline{Y})^2 df + \\ &+\int_0^n v_{y\xi_Y}(y_t(h) - \overline{Y})\xi_{Y,t}dh + \int_n^1 v_{y\xi^*_Y}(y_t(f) - \overline{Y})\xi^*_{Y,t}df + o(\|\xi\|^3). \end{aligned} \tag{B.15}$$

Note that we can write

$$\int_0^n v_y(y_t(h)-\overline{Y})dh+\int_n^1 v_y(y_t(f)-\overline{Y})df = v_y\left[y_t(w) + \int_0^n y^g(h)dh + \int_n^1 y^g(f)df - \overline{Y}\right], \tag{B.16}$$

where we have defined

$$\begin{aligned} y^g(h) &\equiv \left(\frac{p(h)}{P_H}\right)^{-\sigma} G, \\ y^g(f) &\equiv \left(\frac{p(f)}{P_F}\right)^{-\sigma} G^*. \end{aligned}$$

Moreover, we have defined

$$y_t(w) \equiv \left(\frac{P'_t}{P_t}\right)^{-\theta} C^W_t,$$

and

$$(P_t^{'})^{-\theta} \equiv n \left(\frac{P_H^{'}}{P_H}\right)^{-\sigma} P_H^{-\theta} + (1-n)\left(\frac{P_F^{'}}{P_F}\right)^{-\sigma} P_F^{-\theta},$$

$$\begin{aligned} P_H^{'} &\equiv \left[\frac{1}{n}\int_0^n p(h)^{-\sigma} dh\right]^{-\frac{1}{\sigma}}, \\ P_F^{'} &\equiv \left[\frac{1}{1-n}\int_n^1 p(f)^{-\sigma} df\right]^{-\frac{1}{\sigma}}. \end{aligned}$$

First, we observe that by taking a second-order expansion of $y_t^g(h)$ and $y_t^g(f)$ as follows,

$$\begin{aligned} y_t^g(h) &= \overline{Y} \cdot (\widehat{y}_t^g(h) + \frac{1}{2} \cdot [\widehat{y}_t^g(h)]^2) + o(\|\xi\|^3), \\ y_t^g(f) &= \overline{Y} \cdot (\widehat{y}_t^g(f) + \frac{1}{2} \cdot [\widehat{y}_t^g(f)]^2) + o(\|\xi\|^3), \end{aligned}$$

all the terms of order less than $o(\|\xi\|^3)$ are independent of monetary policy, since the shocks G and G^* equal to zero in the steady state. We can neglect these terms. We need to consider only the term $y(w)$. After some algebra, it can be shown that $y(w)$ can be approximated in a second-order expansion as

$$\begin{aligned} y_t(w) &= \overline{Y} \cdot \{1 + \widehat{C}_t^W + n\mathrm{E}_h[\widehat{y}_t^p(h)] + (1-n)\mathrm{E}_f[\widehat{y}_t^p(f)] \\ &\quad + \frac{1}{2}(\widehat{C}_t^W)^2 + n(1-n)\theta(\widehat{T}_t)^2 + n\mathrm{E}_h[\widehat{y}_t^p(h)]^2 + \\ &\quad + (1-n)\mathrm{E}_f[\widehat{y}_t^p(f)]^2\} + o(\|\xi\|^3), \end{aligned} \tag{B.17}$$

where we have made use of the restrictions on P_H and P_F implied by P. We have also defined $\widehat{y}_t^p(h) = \ln y_t^p(h)$, $\widehat{y}_t^p(f) = \ln y_t^p(f)$ and $\widehat{T}_t = \ln T_t$ where

$$\begin{aligned} y^p(h) &\equiv \left(\frac{p(h)}{P_H}\right)^{-\sigma}, \\ y^p(f) &\equiv \left(\frac{p(f)}{P_F}\right)^{-\sigma}. \end{aligned}$$

We also note that we can expand $y_t(h)$ and $y_t(f)$ as

$$y_t(h) = \overline{Y} \cdot (\widehat{y}_t(h) + \frac{1}{2} \cdot [\widehat{y}_t(h)]^2) + o(\|\xi\|^3), \tag{B.18}$$

$$y_t(f) = \overline{Y} \cdot (\widehat{y}_t(f) + \frac{1}{2} \cdot [\widehat{y}_t(f)]^2) + o(\|\xi\|^3). \tag{B.19}$$

Finally, plugging (B.17), (B.18), (B.19) into (B.15), we obtain

$$\int_0^n V(y_t(h),\xi_{Y,t})dh+\int_n^1 V(y_t(f),\xi^*_{Y,t})df = v_y\overline{Y}\cdot[\widehat{C}^W_t+\frac{1}{2}(\widehat{C}^W_t)^2+n\mathrm{E}_h[\widehat{y}^p_t(h)]+(1-n)\mathrm{E}_f[\widehat{y}^p_t(f)]$$

$$\begin{aligned}
&+n(1-n)\theta(\widehat{T}_t)^2+\frac{n}{2}\mathrm{E}_h[\widehat{y}^p_t(h)]^2+\frac{1-n}{2}\mathrm{E}_f[\widehat{y}^p_t(f)]^2+n\frac{\eta}{2}\cdot\mathrm{E}_h[\widehat{y}_t(h)]^2\\
&+(1-n)\frac{\eta}{2}\cdot\mathrm{E}_f[\widehat{y}_t(f)]^2-n\eta\cdot\overline{Y}_t\mathrm{E}_h\widehat{y}_t(h)-(1-n)\eta\cdot\overline{Y}^*_t\mathrm{E}_f\widehat{y}_t(f)\\
&+o(\|\xi\|^3).
\end{aligned}\tag{B.20}$$

where $\overline{Y}_t$, $\overline{Y}^*_t$ are defined as $v_{y\xi_Y}\xi_{Y,t}\equiv -v_{yy}\overline{Y}\overline{Y}_t$, $v_{y\xi^*_Y}\xi^*_{Y,t}\equiv -v_{yy}\overline{Y}\overline{Y}^*_t$ and $\eta\equiv V_{yy}(\overline{Y},0)\overline{Y}/V_y(\overline{Y},0)$.

Next, we analyze a steady-state in which a subsidy completely offset the monopolistic distortions. From conditions (A.4), one can show that

$$U_C(\overline{C},0)=V_y\left(\overline{C},0\right),\tag{B.21}$$

Here we note that

$$\begin{aligned}
\mathrm{E}_h[\widehat{y}^p_t(h)]^2 &= \mathrm{var}_h\widehat{y}^p_t(h)+[E_h\widehat{y}^p_t(h)]^2, &\text{(B.22)}\\
\mathrm{E}_h[\widehat{y}^p_t(f)]^2 &= \mathrm{var}_h\widehat{y}^p_t(f)+[E_h\widehat{y}^p_t(f)]^2, &\text{(B.23)}\\
\mathrm{E}_h[\widehat{y}_t(h)]^2 &= \mathrm{var}_h\widehat{y}_t(h)+[E_h\widehat{y}_t(h)]^2, &\text{(B.24)}\\
\mathrm{E}_h[\widehat{y}_t(f)]^2 &= \mathrm{var}_h\widehat{y}_t(f)+[E_h\widehat{y}_t(f)]^2, &\text{(B.25)}
\end{aligned}$$

We can use the aggregators

$$\begin{aligned}
1 &= \left[\left(\frac{1}{n}\right)\int_o^n y^p(h)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}},\\
1 &= \left[\left(\frac{1}{n}\right)\int_o^n y^p(f)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}},\\
Y_{H,t} &= \left[\left(\frac{1}{n}\right)\int_o^n y(h)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}}=\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}C^W_t+G_t,\\
Y_{F,t} &= \left[\left(\frac{1}{1-n}\right)\int_n^1 y(f)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}}=\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}C^W_t+G^*_t.
\end{aligned}$$

We can further take a second-order approximation of the aggregators

$$\begin{aligned}
0 &= E_h\widehat{y}^p_t(h)+\frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\mathrm{var}_h\widehat{y}^p_t(h)+o(\|\xi\|^3) &\text{(B.26)}\\
0 &= E_f\widehat{y}^p_t(h)+\frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\mathrm{var}_f\widehat{y}^p_t(f)+o(\|\xi\|^3) &\text{(B.27)}
\end{aligned}$$

$$\widehat{Y}_{H,t} = E_h\widehat{y}_t(h) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\mathrm{var}_h\widehat{y}_t(h) + o(\|\xi\|^3), \quad \text{(B.28)}$$
$$\widehat{Y}_{F,t} = E_f\widehat{y}_t(f) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\mathrm{var}_f\widehat{y}_t(f) + o(\|\xi\|^3). \quad \text{(B.29)}$$

Finally substituting (B.22), (B.23), (B.24), (B.25), (B.26), (B.27), (B.28) and (B.29) into (B.20) we obtain,

$$\int_0^n V(y_t(h),\xi_{Y,t})dh + \int_n^1 V(y_t(f),\xi^*_{Y,t})df = U_C\overline{C}\cdot[\widehat{C}_t^W + \frac{1}{2}(\widehat{C}_t^W)^2 + n(1-n)\theta\widehat{T}_t^2$$

$$+\frac{1}{2}\eta\cdot[n\widehat{Y}^2_{H,t} + (1-n)\widehat{Y}^2_{F,t}] - \eta\cdot[n\widehat{Y}_{H,t}\overline{Y}_t + (1-n)\widehat{Y}_{F,t}\overline{Y}^*_t]$$
$$+\frac{1}{2}(\sigma^{-1}+\eta)[n\mathrm{var}_h\widehat{y}_t(h) + (1-n)\mathrm{var}_f\widehat{y}_t(f)] + o(\|\xi\|^3). \quad \text{(B.30)}$$

where we have used the fact that $\mathrm{var}_h\widehat{y}_t(h) = \mathrm{var}_h\widehat{y}^p_t(h)$ and $\mathrm{var}_f\widehat{y}_t(f) = \mathrm{var}_f\widehat{y}^p_t(f)$.

Combining (B.12), (B.30) and (B.6) we obtain,

$$\begin{aligned} w_t = {} & U_C\overline{C}\{-n\frac{\rho}{2}\widehat{C}_t^2 - (1-n)\frac{\rho}{2}(\widehat{C}_t^*)^2 + \\ & +n\rho\widehat{C}_t v_t + (1-n)\rho\widehat{C}_t^* v_t^* - n(1-n)\theta\widehat{T}_t^2 - \\ & -\frac{1}{2}\eta\cdot[n\widehat{Y}^2_{H,t} + (1-n)\widehat{Y}^{*2}_{F,t}] + \eta\cdot[n\widehat{Y}_{H,t}\overline{Y}_t + (1-n)\widehat{Y}^*_{F,t}\overline{Y}_t] + \\ & -\frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\mathrm{var}_h\widehat{y}_t(h) + (1-n)\mathrm{var}_f\widehat{y}_t(f)]\} \\ & +\text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \quad \text{(B.31)}$$

and after substituting the expressions for $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$ with some simplification we get

$$\begin{aligned} w_t = {} & U_C\overline{C}\{-\rho(\widehat{C}_t^W)^2 - n(1-n)\frac{\rho}{2}(\widehat{C}_t^R)^2 + \rho\widehat{C}_t^W v_t^W + n(1-n)\rho\widehat{C}_t^R v_t^R \\ & -n(1-n)\theta\widehat{T}_t^2 + \eta[\widehat{C}_t^W\overline{Y}_t^W + n(1-n)\theta\widehat{T}_t\overline{Y}_t^R] \\ & -\frac{1}{2}\eta\cdot[(\widehat{C}_t^W)^2 + n(1-n)\theta^2\widehat{T}_t^2 + 2\widehat{C}_t^W g_t^W + 2n(1-n)\theta\widehat{T}_t g_t^R] \\ & -\frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\mathrm{var}_h\widehat{y}_t(h) + (1-n)\mathrm{var}_f\widehat{y}_t(f)]\} \\ & +\text{t.i.p.} + o(\|\xi\|^3), \end{aligned} \quad \text{(B.32)}$$

which can be written as

$$
\begin{aligned}
w_t &= -U_C\overline{C}\{\frac{1}{2}(\rho+\eta)[\widehat{C}_t^W - \widetilde{C}_t^W]^2 + \frac{1}{2}n(1-n)\rho[\widehat{C}_t^R - \widetilde{C}_t^R]^2 \\
&\quad + \frac{1}{2}n(1-n)(1+\eta\theta)\theta[\widehat{T}_t - \widetilde{T}_t]^2 \\
&\quad + \frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\text{var}_h\widehat{y}_t(h) + (1-n)\text{var}_f\widehat{y}_t(f)]\} \\
&\quad + \text{t.i.p.} + o(\|\xi\|^3). \qquad \text{(B.33)}
\end{aligned}
$$

after having used the definitions of $\widetilde{C}_t^W$, $\widetilde{C}_t^R$, $\widetilde{T}_t$. Using the definition of consumption gap, we obtain

$$
\begin{aligned}
w_t &= -U_C\overline{C}\{\frac{1}{2}(\rho+\eta)[c_t^W]^2 + \frac{1}{2}n(1-n)\rho[c_t^R]^2 + \frac{1}{2}n(1-n)(1+\eta\theta)\theta[\widehat{T}_t - \widetilde{T}_t]^2 \\
&\quad + \frac{1}{2}(\sigma^{-1}+\eta)\cdot[n\text{var}_h\widehat{y}_t(h) + (1-n)\text{var}_f\widehat{y}_t(f)]\} + \text{t.i.p.} + o(\|\xi\|^3), \qquad \text{(B.34)}
\end{aligned}
$$

Following Woodford (1999), we derive $\text{var}_h\widehat{y}_t(h)$ and $\text{var}_f\widehat{y}_t(f)$ to get

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t\text{var}_h\{\text{log}y_t(h)\} &= \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sigma^2\sum_{t=0}^{\infty}\beta^t(\pi_{H,t})^2 + \text{t.i.p.}+o(\|\xi\|^3), \\
\sum_{t=0}^{\infty}\beta^t\text{var}_f\{\text{log}y_t(f)\} &= \frac{\alpha^*}{(1-\alpha^*)(1-\alpha^*\beta)}\sigma^2\sum_{t=0}^{\infty}\beta^t(\pi_{F,t}^*)^2 + \text{t.i.p.}+o(\|\xi\|^3)
\end{aligned}
$$

We finally obtain

$$
W = -\frac{1}{2}U_C\overline{C}\text{E}_0\left\{\sum_{t=0}^{\infty}\beta^t L_t\right\} \qquad \text{(B.35)}
$$

where

$$
\begin{aligned}
L_t &= (\rho+\eta)\cdot[c_t^W]^2 + n(1-n)\rho[c_t^R]^2 + n(1-n)(1+\eta\theta)\theta\cdot[\widehat{T}_t - \widetilde{T}_t]^2 \\
&\quad + n\frac{\sigma}{k}(\pi_{H,t})^2 + (1-n)\frac{\sigma}{k^*}(\pi_{F,t}^*)^2 + \text{t.i.p.} + o(\|\xi\|^3),
\end{aligned}
$$

which corresponds to equation (14) in the text.

Figure 1. Flexible-Price Equilibrium, Impulse Response Function, Asymmetric Supply Shocks

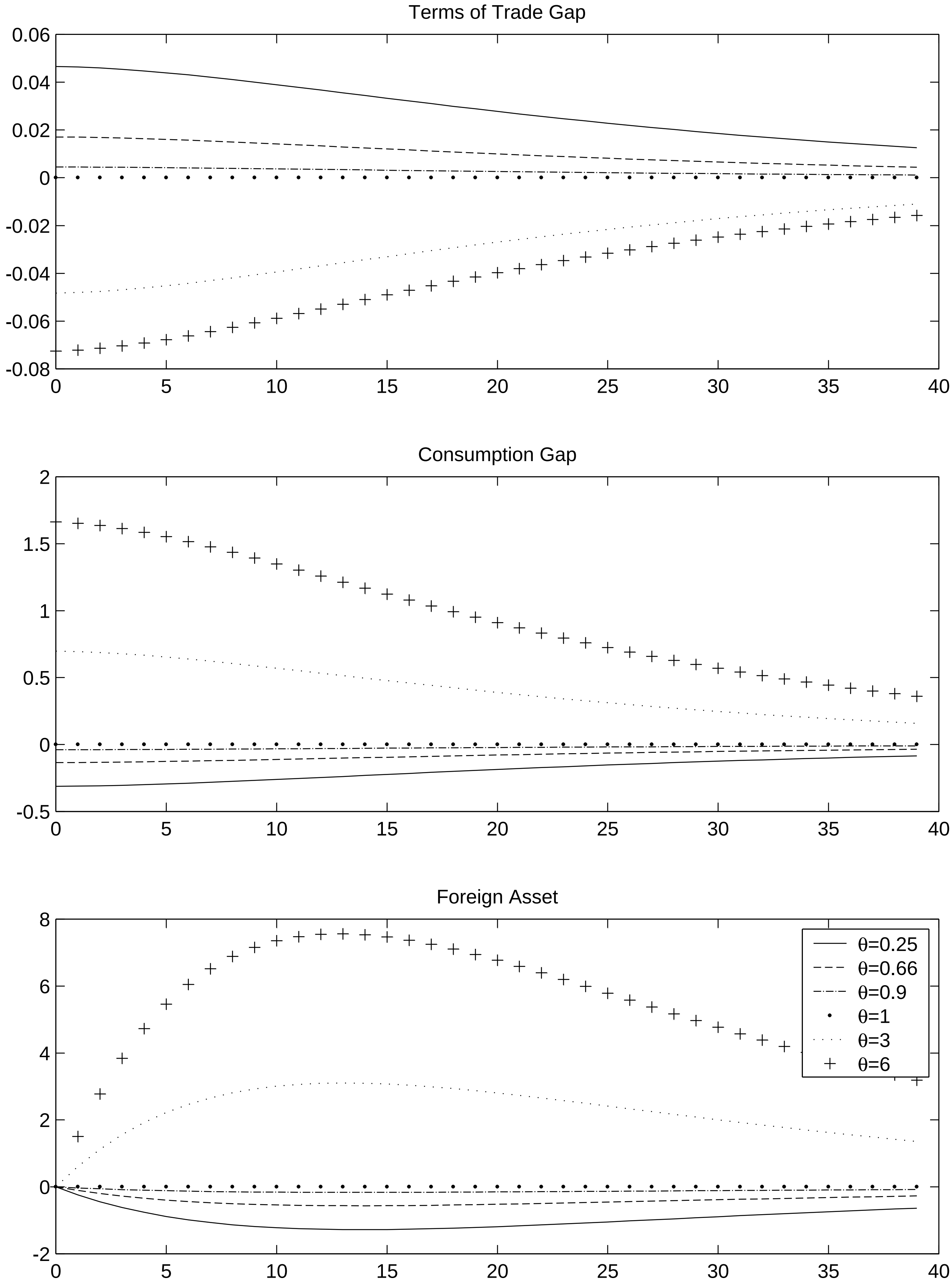

Figure 2. Flexible-Price Equilibrium, Impulse Response Function, Asymmetric Demand Shocks

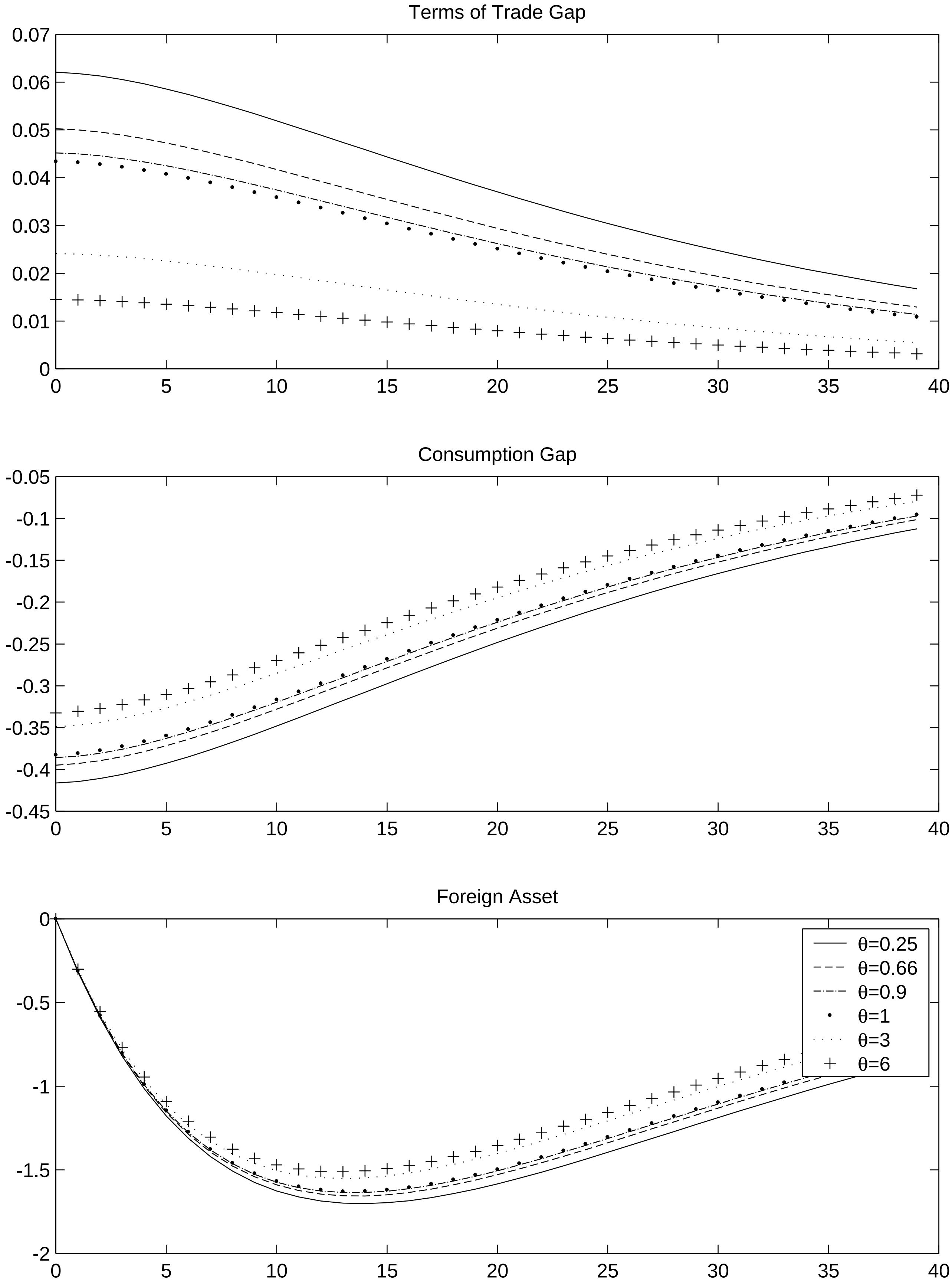

Figure 3: Flexible-Price Allocation, Welfare Costs of Market Incompleteness, Low Persistent Relative Supply Shocks

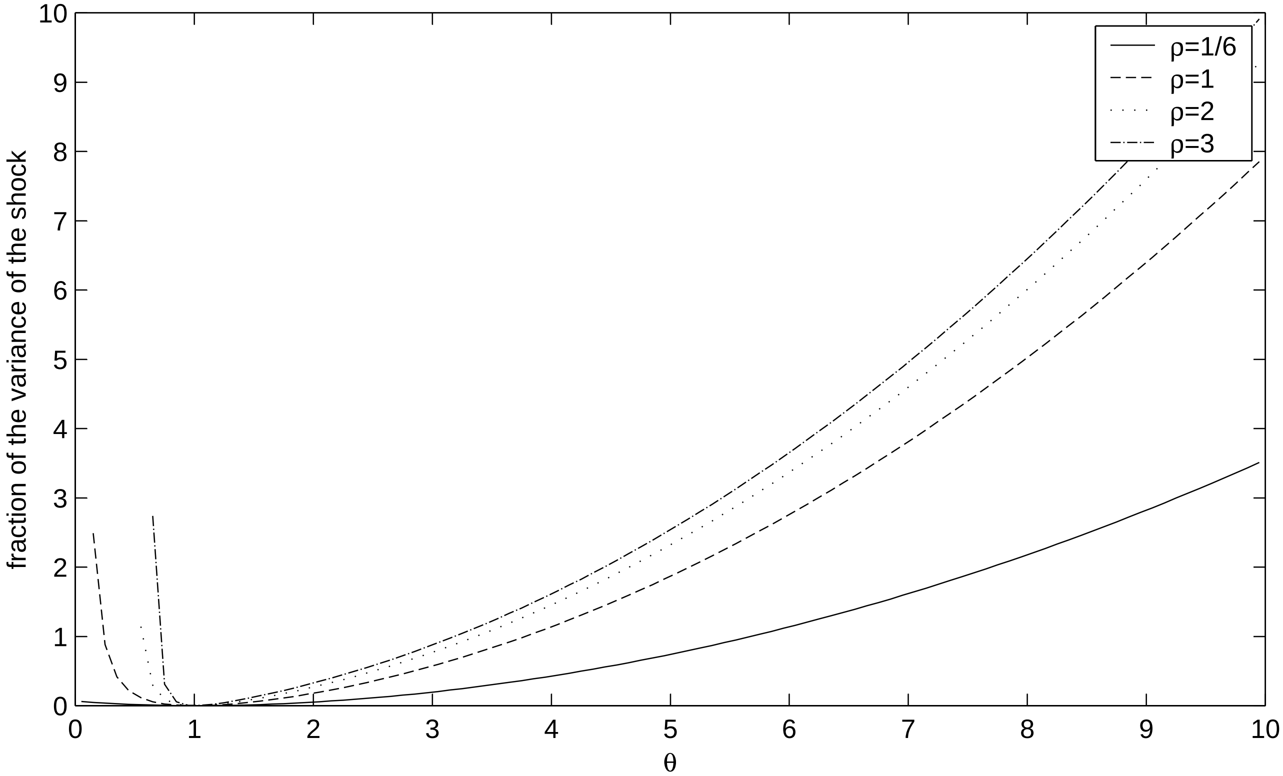

Figure 4: Welfare Costs of Market Incompleteness, High Persistent Relative Supply Shocks

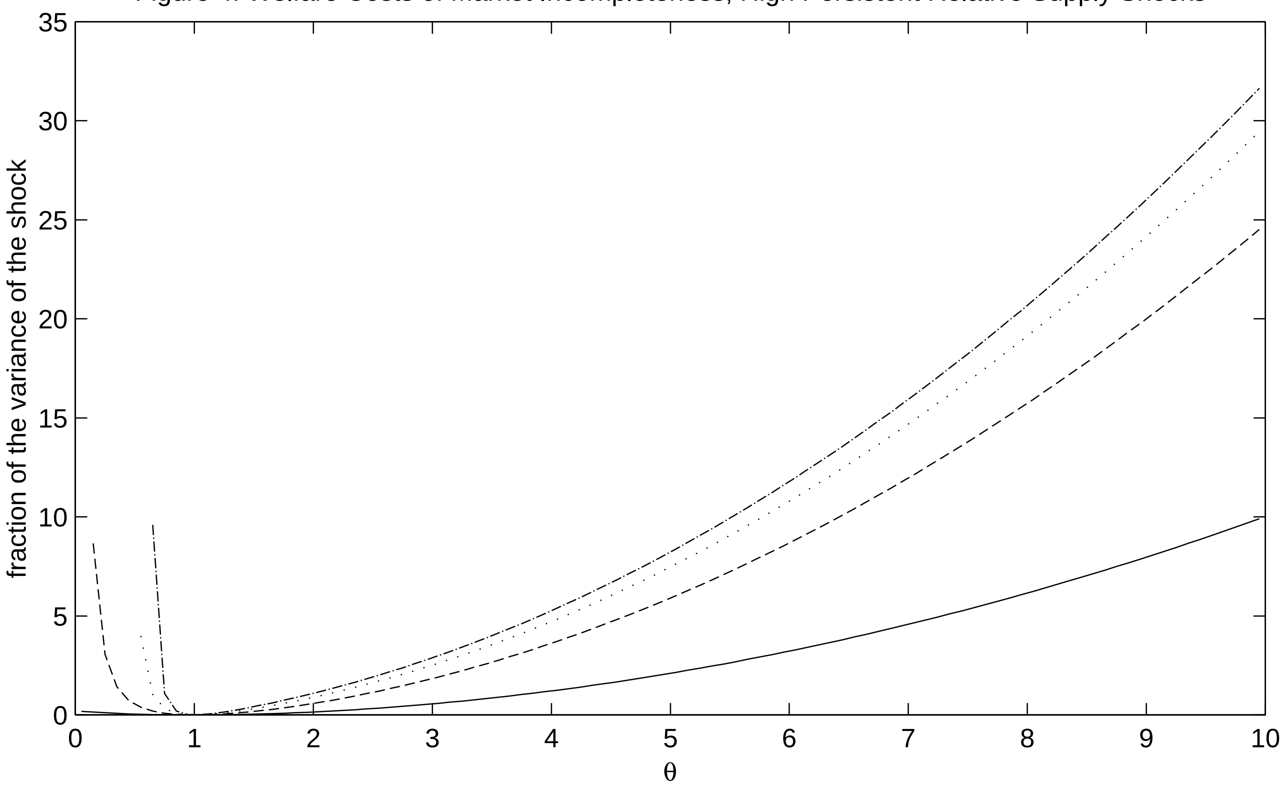

Figure 5. Flexible-Price Equilibrium, Welfare Costs of Market Incompleteness, Low Persistent Relative Demand Shocks

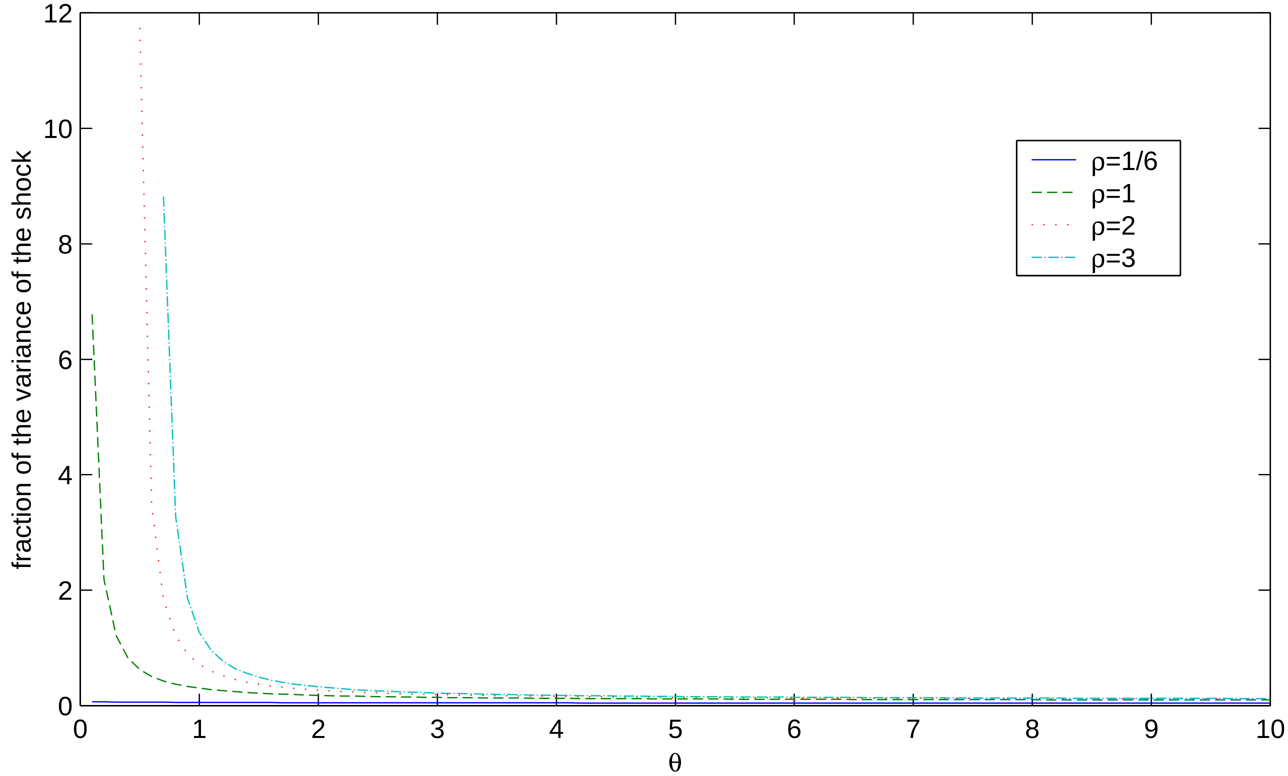

Figure 6. Flexible-Price Equilibrium, Welfare Costs of Market Incompleteness, High Persistent Relative Demand Shocks

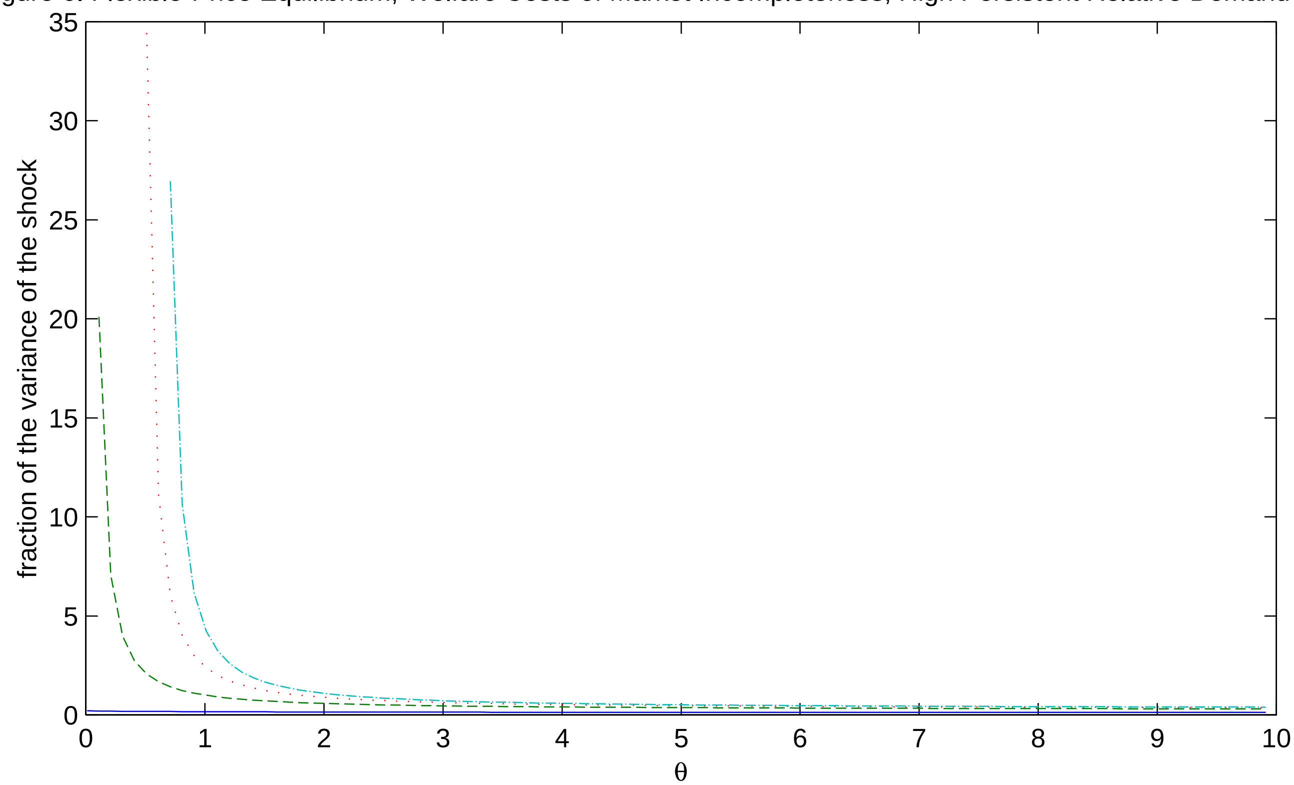

Figure 7. Optimal Policy, Impulse Response Function, Asymmetric Supply Shocks

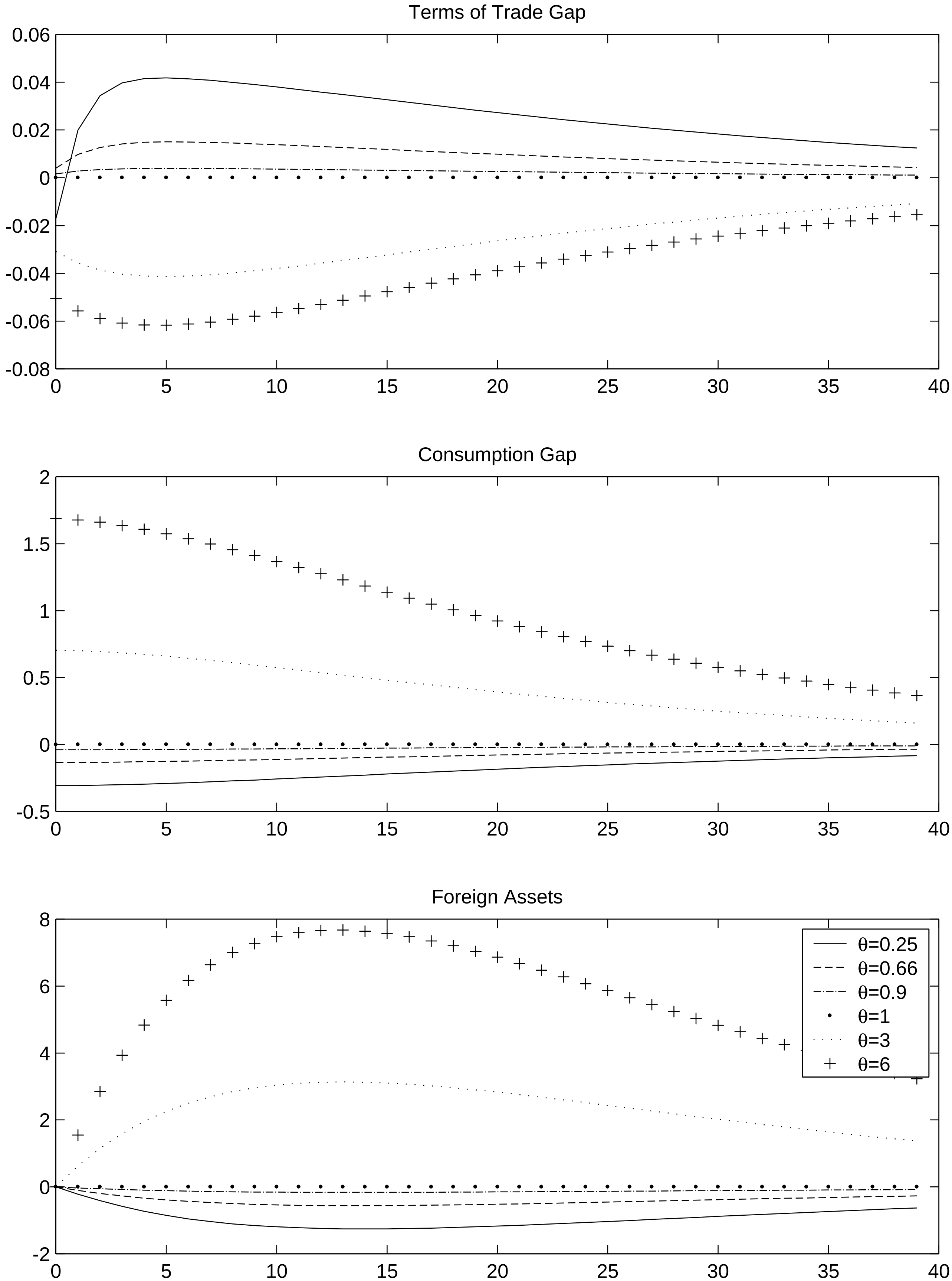

Figure 7. Optimal Policy, Impulse Response Function, Asymmetric Supply Shocks

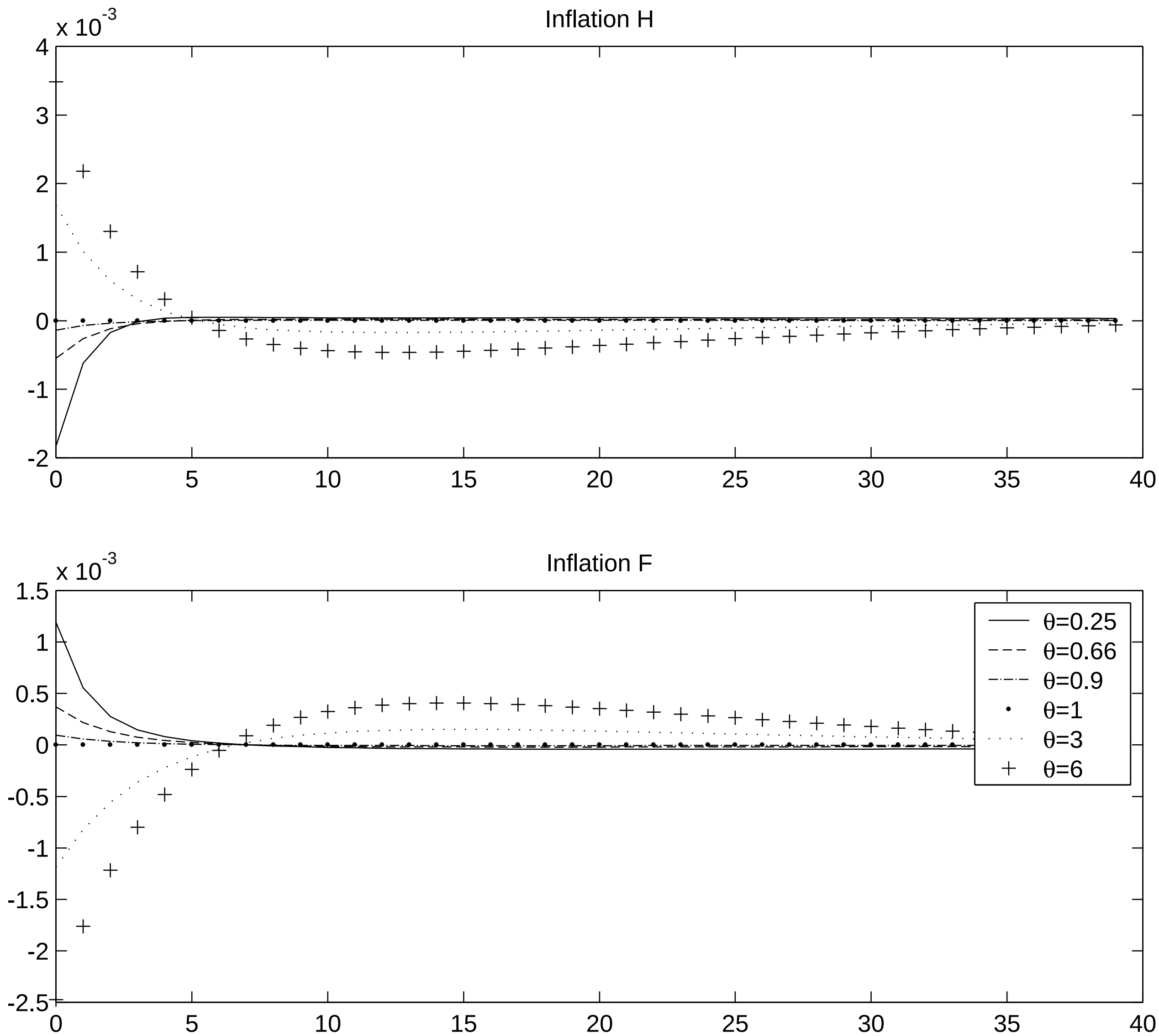

Figure 8. Optimal Policy, Impulse Response Function, Asymmetric Demand Shocks

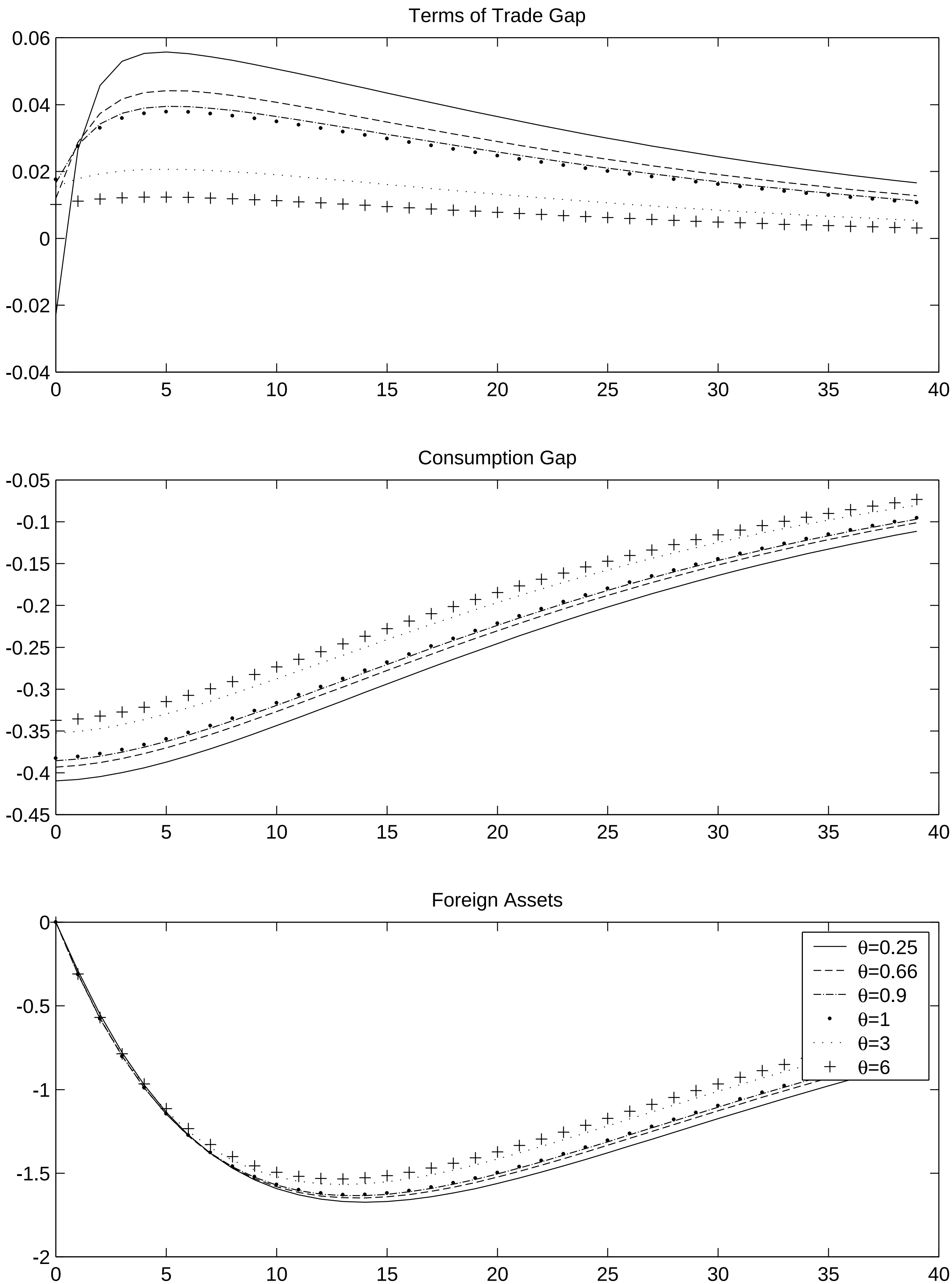

Figure 8. Optimal Policy, Impulse Response Function, Asymmetric Demand Shocks

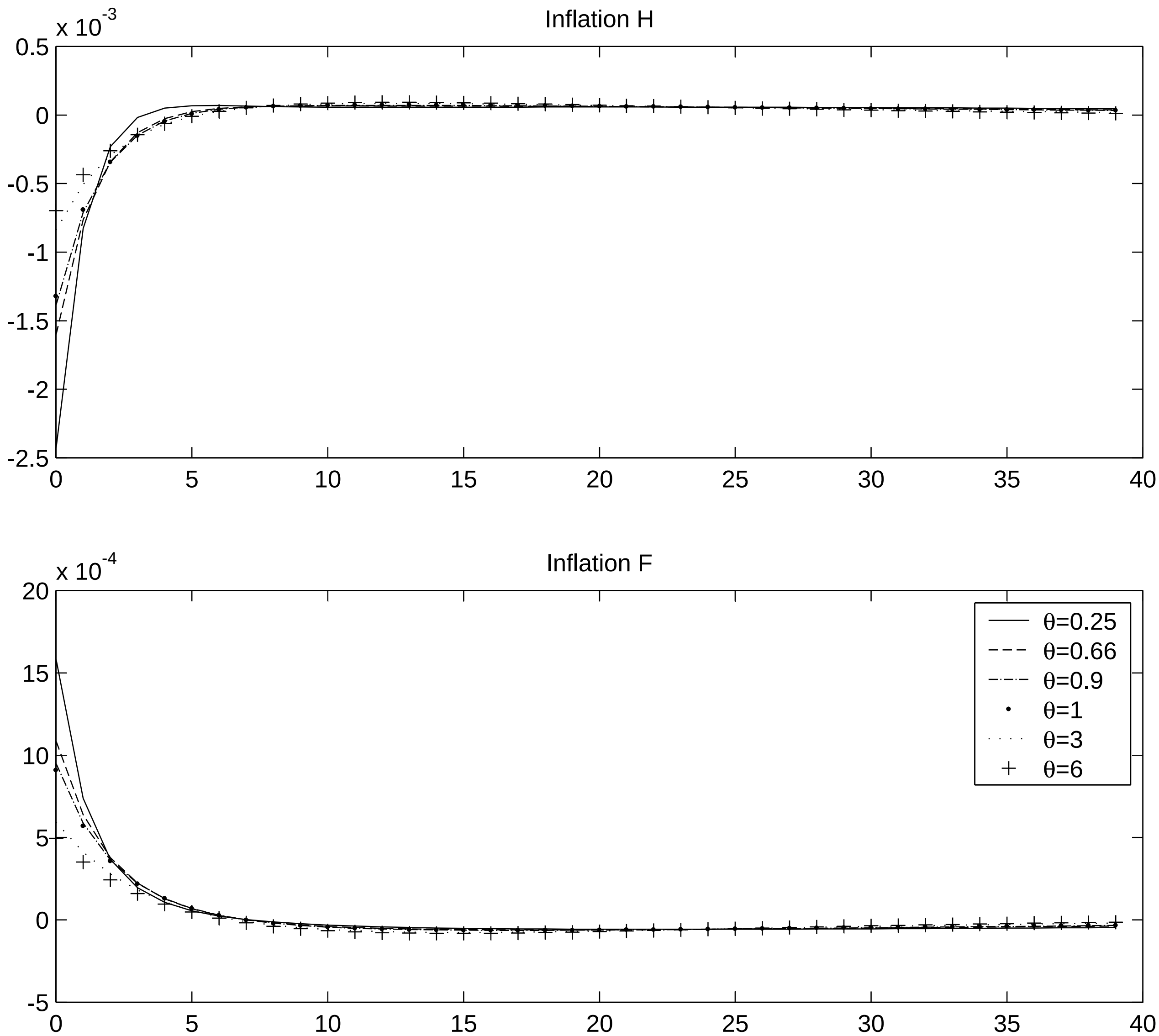

Figure 9. Welfare Comparisons, Relative Supply Shocks

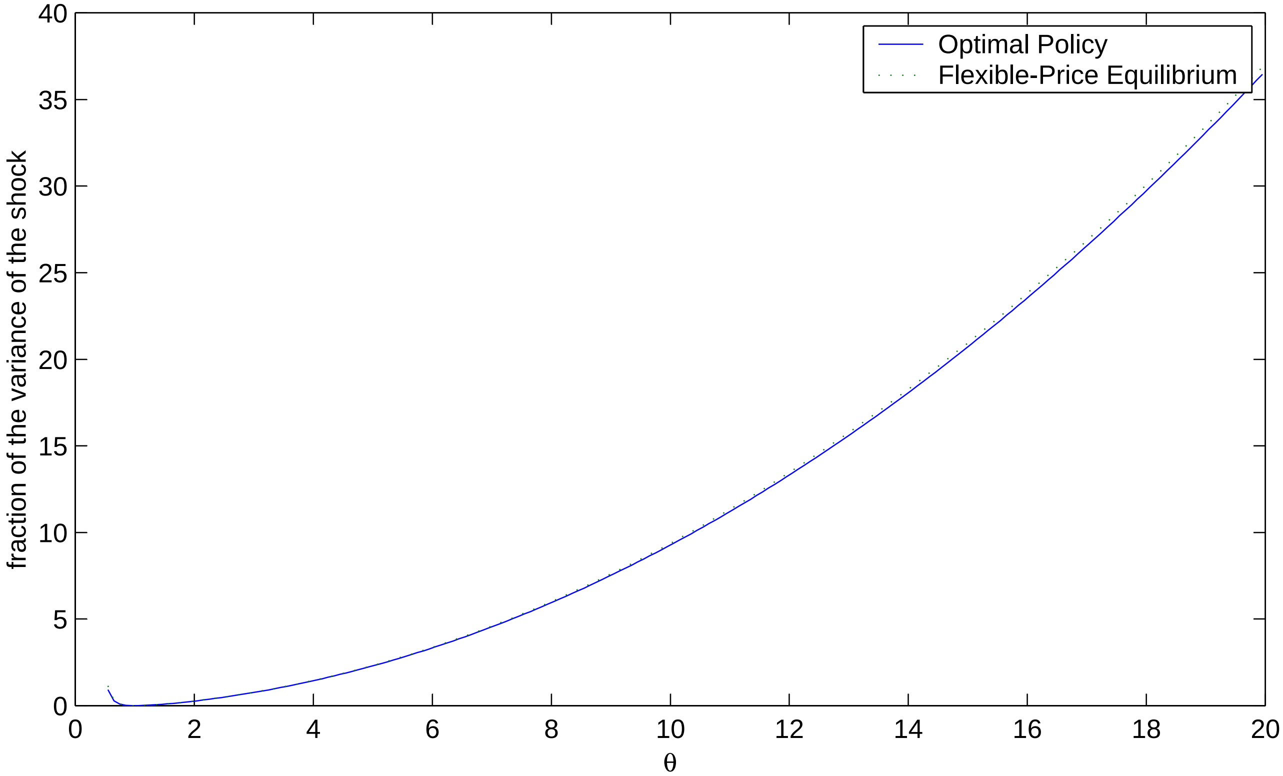

Figure 10. Welfare Comparisons, Relative Demand Shocks