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ABSTRACT

Portfolio Choice, Liquidity Constraints and Stock Market Mean Reversion*

This Paper solves numerically for the optimal consumption and portfolio choice of an infinitely lived investor facing short sales and borrowing constraints, undiversifiable labour income risk and a predictable time varying equity premium. The investor aggressively times the market while positive correlation between permanent earnings shocks and stock return innovations generates a substantial hedging demand for the riskless asset. Moreover, a speculative increase in savings arises when stock returns are expected to be high and conversely when future returns are expected to be low. Small information/optimization costs can make it optimal for an investor to assume i.i.d excess stock returns, both because liquidity constraints can be frequently binding and because households can smooth idiosyncratic earnings shock using a small buffer stock of wealth.

JEL Classification: E21, G11
Keywords: buffer stock saving, liquidity constraints, portfolio choice, stock market mean reversion, stock market predictability

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NON-TECHNICAL SUMMARY

How does the presence of stock market predictability, undiversifiable labour income risk and liquidity constraints affect optimal consumption and portfolio choice? Based on the empirical evidence of stock market predictability at longer horizons, various recent papers have analysed the implications of stock market predictability for consumption and/or portfolio choice. These papers ignore non-tradable labour income risk and its effects on optimal consumption and portfolio choice. At the same time the effect of background labour income risk on portfolio choice while ignoring stock market predictability has also been analysed. This paper jointly models stock market predictability and non-diversifiable background labour income risk and analyses the implications for optimal consumption and portfolio choice.

With stock market predictability, the consumer/investor is shown to be an aggressive market timer. Relative to an i.i.d. returns model, high expected future returns generate a higher allocation of stocks in the portfolio for a given level of saving, while low expected future returns decrease the exposure in the stock market and can cause complete portfolio specialization in the riskless asset. Aggressive market timing behaviour is similar to behaviour predicted in Barberis (2000) and Brennan, Schwartz and Lagnado (1997) (in both papers there is no consumption choice and no undiversifiable labour income uncertainty). It seems, therefore, that labour income risk does not change qualitatively the effects of stock market predictability on portfolio choice.

The effects on the optimal consumption/savings policy rule are more interesting. Stock market predictability generates a speculative increase in savings when the excess return of stocks over the riskless asset is expected to be high and conversely when the excess stock return is expected to be low. Equivalently, the substitution effect from an increase in expected future returns on current savings outweighs the income effect for a wider range of parameters than is usually thought to be the case. In particular, the model predicts an increase in savings when future expected returns are high whereas for similar parameter values a model with no labour income and no constraints, but time varying investment opportunities predicts a decrease in savings (Campbell and Viceira, 1999). The difference in predictions arises from the presence of non-tradable labour income. The numerical results in this paper are shown to be consistent with the analytical results in Viceira (2001); utilizing the latter, it is shown that the substitution effect is stronger than the income effect because of labour income received in subsequent periods.

The model predicts that both bonds and stocks will be held on average over time, contrary to the complete portfolio specialization in stocks generated by the i.i.d. returns model. Nevertheless, for most factor realizations, the investor either allocates total savings completely in the stock market or allocates all savings in the riskless asset. The individual desires to short the stock market position for signals that predict very low future stock returns, leading to a complete portfolio specialization in the riskless asset regardless of the wealth level. Moreover, the model (counterfactually) predicts that the median share of wealth in stocks equals one.

The complete portfolio specialization in either the riskless asset or the stock market, conditional on the factor realization, would be inconsistent with a general equilibrium version of the model. This weakness can be remedied by considering the effects of fixed one-time stock market entry, or information processing costs and thereby performing two welfare comparisons. First, the welfare gain from participating in the stock market is assessed. The welfare gain is shown to be small for certain parameter values; the conflict between impatience and prudence generates this result. Impatience (or high future expected growth in earnings against which the agent cannot borrow) makes asset accumulation costly, while prudence gives rise to a precautionary saving motive to smooth consumption fluctuations. When prudence is weak, the impatient consumer accumulates low savings to smooth consumption and the household is liquidity constrained around one-third of the time. As a result, the benefit from entering the stock market (even in the presence of stock market predictability) is small. A positive demand for the riskless asset can therefore arise from agents who face high stock market entry costs.

Can demand for stocks always be generated as well? To answer this question, the welfare gain from using the predictability of stock market returns relative to using an i.i.d. model is quantified. Once more, for low (but plausible) degrees of prudence, the welfare gain is small. Campbell and Viceira (1999) instead argue that taking advantage of the information predicting future returns can lead to substantial welfare improvements. Three explanations can rationalize the lower welfare gain in the current set-up. First, the impatient household is liquidity constrained around one-third of the time, and second, it accumulates low wealth holdings to smooth consumption; both factors limit the benefit from stock market predictability. Third, the presence of borrowing and short sales constraints is very important. In the i.i.d. returns model, the household invests total savings in the stock market. For plausible levels of prudence, the same behaviour is predicted in the AR(1) model for most of the factor realizations. For the lowest factor realizations the household wants to short the risky asset but is prevented from doing so from the short sales constraint, thereby limiting the welfare loss that could have occurred in an economy without the short sales constraint.

1 Introduction

How does the presence of stock market predictability, undiversifiable labor income risk and liquidity constraints affect optimal consumption and portfolio choice? Various recent papers have analyzed the implications of stock market predictability[1] for consumption and/or portfolio choice while ignoring labor income risk; Brennan, Schwartz and Lagnado (1997), Campbell et. al. (1998), Campbell et. al. (1999), Campbell and Viceira (1999), Barberis (2000) and Balduzzi and Lynch (1999) show that stock market exposure varies substantially as a response to the predictive factor(s). The effect of background labor income risk on portfolio choice while ignoring stock market predictability has been analyzed numerically by Heaton and Lucas (1996, 1997, 2000(a), 2000(b)) and Haliassos and Michaelides (1999) and analytically by Viceira (2001). This paper jointly models stock market predictability and non-diversifiable background labor income risk and analyzes the implications for consumption and portfolio choice.

Liquidity constraints in both the risky and riskless asset markets are an important component of the current model for a number of reasons. First, in the absence of borrowing restrictions, households with long horizons facing nontradable labor income risk would borrow to invest in the stock market, given the equity premium (Viceira, 2001).[2] This theoretical prediction would not only contradict directly the observed zero stockholding puzzle (Mankiw and Zeldes, 1991, and Haliassos and Bertaut, 1995) but would make the equity premium puzzle[3] even harder to resolve as demand for the risky asset would rise relative to a model with explicit liquidity constraints. Indeed, Constantinides, Donaldson and Mehra (1998) and Storesletten, Telmer and Yaron (1998) argue that liquidity constraints faced by younger cohorts who expect higher earnings in the future can be one important component of a model that explains the equity premium. Second, an emerging literature on portfolio selection has stressed the importance of borrowing and short sales constraints in enhancing our understanding of observed portfolio choice patterns. Cocco, Gomes and Maenhout (1999) solve numerically a model with short sales and borrowing constraints over the life cycle in the presence of undiversifiable labor income risk and show that households should invest a larger proportion of their savings in the stock market when young because the future labor income they will receive (against which they cannot borrow) acts as a risk free asset that crowds out the accumulation of riskless assets[4]. This prediction resembles the advice given by financial planning consultants (Malkiel, 1999). Finally, the presence of both borrowing constraints and undiversifiable labor income risk is an important component of the buffer stock saving model (Deaton (1991) and Carroll (1992)) that has been proposed as the leading alternative to the classic Permanent Income Hypothesis (PIH) or Life Cycle model in an effort to explain the observed "excess smoothness"[5] and "excess sensitivity"[6] puzzles.[7] The buffer stock saving model outperforms the PIH in accounting for the microeconomic consumption data (see Carroll (1997) and Gourinchas and Parker (1999)) and the aggregate consumption data (see Ludvigson and Michaelides (forthcoming)).

The second important component of the model is undiversifiable labor income risk. In a related paper Barberis (2000) analyzes the portfolio choice implications of stock market predictability but in the absence of consumption choice and labor income risk. The buffer stock saving literature (Deaton, 1991 and Carroll, 1992, for instance) has shown, however, that nontradable labor income risk is an important factor that must be taken into account by households making optimal savings plans. The importance of undiversifiable labor income risk has also been stressed by Viceira (2001), who has shown that higher nontradable labor income risk can affect positively the level of savings through a precautionary savings channel and negatively the share of savings invested in the risky asset through a temperance channel (Gollier and Pratt (1996)). Integrating stock market predictability with labor income risk in a single model can potentially yield further insights on the effects of both labor income risk and predictability on optimal consumption and portfolio choice.

I rely on numerical techniques and calibration to draw out the implications of the model.[8] Heaton and Lucas (1997) find that positive demand for bonds is very difficult to generate even in the presence of sizeable transaction costs, habit formation in preferences or an equity premium as low as two percent. Heaton and Lucas (2000(a)), however, find that entrepreneurial risk (measured by small business/proprietary income) is positively correlated with stock market risk and show that such correlation can generate higher accumulation of the risk free asset in the optimal portfolio. Haliassos and Michaelides (1999) show that these results hold for a different labor income process but conclude that an unrealistically high correlation is needed (around 0.4)[9] for a potential explanation of the zero stock holding puzzle. These models generate, on average, a bias towards excessive stockholding, a demand side manifestation of the equity premium puzzle; stock market predictability could generate a more balanced portfolio.

With stock market predictability, the consumer/investor is shown to be an aggressive market timer. Relative to the i.i.d. returns model, high expected future returns generate a higher allocation of stocks in the portfolio for a given level of saving, while low expected future returns decrease the exposure in the stock market and can cause complete portfolio specialization in the riskless asset. Aggressive market timing behavior is similar to behavior predicted in Barberis (2000) and Brennan, Schwartz and Lagnado (1997) (in both papers there is no consumption choice and no undiversifiable labor income uncertainty). It seems, therefore, that labor income risk does not change qualitatively the effects of stock market predictability on portfolio choice.

The effects on the optimal consumption/savings policy rule are more interesting. Stock market predictability generates a speculative increase in savings when the excess return of stocks over the riskless asset is expected to be high and conversely when the excess stock return is expected to be low. Equivalently, the substitution effect from an increase in expected future returns on current savings outweighs the income effect for a wider range of parameters than is usually thought to be the case. In particular, the model predicts an increase in savings when future expected returns are high whereas for similar parameter values a model with no labor income and no constraints but time varying investment opportunities predicts a decrease in savings (Campbell and Viceira (1999)). The difference in predictions arises from the presence of labor income. The numerical results in this paper are shown to be consistent with the analytical results in Viceira (2001); utilizing the latter, it is shown that the substitution effect is stronger than the income effect because of labor income received in subsequent periods.

Liquidity constraints are also shown to be quite important because they are frequently binding. When future stock returns are expected to be high, the consumer wants to borrow to invest in the stock market and therefore the borrowing constraint becomes binding. When low future excess returns are predicted, on the other hand, the consumer wants to short the risky asset and the short sales constraint becomes binding. Positive correlation between labor income shocks and the stock market innovation is shown to decrease the stock market allocation (a hedging demand due to the possibility of low labor income when stock returns are low), while the effects of correlation between the factor innovation and the stock market return innovation are small in magnitude due to the presence of the constraints.

The model predicts that both bonds and stocks will be held on average over time, contrary to the complete portfolio specialization in stocks generated by the i.i.d. model. Nevertheless, conditional on the factor realization, the model generates a result similar in spirit to the Heaton and Lucas (1997) complete portfolio specialization in stocks prediction. Specifically, for most factor realizations, the investor either allocates total savings completely in the stock market or allocates all savings in the riskless asset. The individual desires to short the stock market position for signals that predict very low future stock returns, leading to a complete portfolio specialization in the riskless asset regardless of the wealth level. Moreover, the model (counterfactually) predicts that the median share of wealth in stocks equals one.

The complete portfolio specialization in either the riskless asset or the stock market, conditional on the factor realization, would be inconsistent with a general equilibrium version of the model. This weakness can be remedied by considering the effects of fixed one-time stock market entry, or information processing, costs[10] and thereby performing two welfare comparisons. First, the welfare gain from participating in the stock market is assessed. The welfare gain is shown to be small for certain parameter values; the conflict between impatience and prudence generates this result. Impatience (or high future expected growth in earnings against which the agent cannot borrow) makes asset accumulation costly, while prudence gives rise to a precautionary saving motive to smooth consumption fluctuations. When prudence is weak, the impatient consumer accumulates low savings to smooth consumption and the household is liquidity constrained around one third of the time. As a result, the benefit from entering the stock market (even in the presence of stock market predictability) is small. A positive demand for the riskless asset can therefore arise from agents who face high stock market entry costs.

Can demand for stocks always be generated as well? To answer this question, the welfare gain from using the predictability of stock market returns relative to using an i.i.d. model (when in fact returns are predictable) is quantified. Once more, for low (but plausible) degrees of prudence, the welfare gain is small. Campbell and Viceira (1999) instead argue that taking advantage of the information predicting future returns can lead to substantial welfare improvements. Three explanations can rationalize the lower welfare gain in the current setup. First, the impatient household is liquidity constrained around one third of the time and, second, it accumulates low wealth holdings to smooth consumption; both factors limit the benefit from stock market predictability. Third, the presence of borrowing and short sales constraints is very important. In the i.i.d. returns model, the household invests total savings in the stock market. For plausible levels of prudence, the same behavior is predicted in the AR(1) model for most of the factor realizations. For the lowest factor realizations the household wants to short the risky asset but is prevented from doing so from the short sales constraint, thereby limiting the welfare loss that could have occurred in an economy without the short sales constraint.

The paper is organized as follows. Section 2 describes the theoretical model and section 3 outlines the numerical algorithm and parameter choice. Section 4 discusses the policy functions for different parameter specifications and uses the invariant distribution associated with the model to compute time series averages for the variables of interest. Section 5 presents the welfare comparisons between abstaining or participating in the stock market and the welfare comparisons between using the i.i.d. model for excess returns when in fact returns are predictable. Section 6 concludes.

2 The Model[11]

Time is discrete, there is one non-durable good, one riskless financial asset and a risky time varying investment opportunity. The riskless asset yields a constant gross after tax real return, R_f, while the gross real return on the risky asset is denoted by $\widetilde{R}$. At time t, the agent enters the period with invested wealth in the stock market S_{t-1} and the bond market B_{t-1} and receives Y_t units of the non-durable good. Following Deaton (1991) cash on hand in period t is denoted by $X_t = S_{t-1}\widetilde{R_t} + B_{t-1}R_f + Y_t$. The investor then chooses savings in the bond (B_t) and stock (S_t) market to maximize welfare. The particular assumptions made about the economic environment are as follows:

2.1 Preferences

Preferences are of the constant relative risk aversion family; specifically, $U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$ when $\rho > 0$; if $\rho = 1$, $U(C_t) = \ln C_t$.

2.2 Liquidity Constraints

No borrowing and no short sales of stocks are allowed; $B_t \geqq 0$ and $S_t \geqq 0$.

2.3 Labor Income Process

The exogenous stochastic process for individual income is given by $Y_t = P_t U_t$ and $P_t = GP_{t-1}N_t$.[12] This process is decomposed into a permanent component, P_t, and a transitory component, U_t. $\ln U_t$, and $\ln N_t$ are i.i.d. normal with mean $\mu_u = -.5 * \sigma_u^2$ and $\mu_n = -.5 * \sigma_n^2$, and variances σ_u^2 and σ_n^2, respectively.[13] Given these assumptions, the growth in individual labor income follows

$$\Delta \ln Y_t = \ln G + \ln N_t + \ln U_t - \ln U_{t-1}, \tag{1}$$

where the unconditional mean growth for individual earnings is $\mu_g + \mu_n$, and the unconditional variance equals $(\sigma_n^2 + 2\sigma_u^2)$. Individual labor income growth in (1) follows a first order moving average process, consistent with microeconometric studies (MaCurdy [1982] and Abowd and Card [1989]).[14]

2.4 Mean Reversion

I follow Campbell (1999) in assuming that there is a single factor that can predict future excess returns. Letting $\{r_f, r_t\}$ denote the net risk free rate and the net stock market return respectively and f_t the factor that predicts future excess returns, we have

$$r_{t+1} - r_f = f_t + z_{t+1} \tag{2}$$

$$f_{t+1} = \mu + \phi(f_t - \mu) + \varepsilon_{t+1} \tag{3}$$

where the two innovations $\{z_{t+1}, \varepsilon_{t+1}\}$ could be contemporaneously correlated.

Mean reversion in the stock market is captured by the autoregressive nature of the factor (f_t) predicting stock market returns ($\phi > 0$). Negative correlation between the excess stock market return innovation (z_{t+1}) and the innovation to the factor (ε_{t+1}) is documented by Campbell and Viceira (1999). I will also be reporting results from a model with i.i.d. excess returns; in that case $r_{t+1} - r_f = \mu + z_{t+1}$.

3 The Euler Equations

In this economic environment, the individual maximizes

$$MAX_{\{B_t,S_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t), \quad (4)$$

where E_0 is the expectation conditional on information available at time 0, and $\beta = \frac{1}{1+\delta}$ is the constant discount factor.

The two Euler equations associated with the problem are:

$$U'(C_t) = MAX[U'(X_t - S_t), \frac{1+r}{1+\delta} E_t U'(C_{t+1})] \quad (5)$$

and

$$U'(C_t) = MAX[U'(X_t - B_t), \frac{1}{1+\delta} E_t \widetilde{R}_{t+1} U'(C_{t+1})] \quad (6)$$

where $C_t = X_t - S_t - B_t$.[15] Given the nonstationary process followed by labor income, I normalize by the permanent component of earnings P_t (see Carroll (1992)). Defining $Z_{t+1} = \frac{P_{t+1}}{P_t}$, taking advantage of the homogeneity of degree $(-\rho)$ of the marginal utility function and labelling the m factor states $i = 1, ..., m$, there are m bond and stock demand functions defined by the two Euler equations as the solutions to the functional equations

$$\begin{aligned} U'(x - s(x,i) - b(x,i)) &= MAX[U'(x - s(x,i)), \\ &\quad \frac{1+r}{1+\delta} E_t Z_{t+1}^{-\rho} U'(x' - s(x',j) - b(x',j))] \end{aligned} \quad (7)$$

and

$$(8) \qquad U'(x - s(x,i) - b(x,i)) = MAX[U'(x - b(x,i)), \frac{1}{1+\delta} E_t \widetilde{R}_{t+1} Z_{t+1}^{-\rho} U'(x' - s(x',j) - b(x',j))]$$

where primes are used to denote next period variables[16], j denotes the factor value expected for next period and lower case variables are normalized by P_t. The endogenous state variable X evolves according to $X' = S_t \widetilde{R}_{t+1} + B_t R_f + Y_{t+1}$ and its normalized equivalent therefore follows $x' = (s_t \widetilde{R}_{t+1} + b_t R_f) Z_{t+1}^{-1} + U_{t+1}$.

Conditional on the factor state (i), this is a system of two functional equations in two unknown functions $(s(x,i), b(x,i))$. Two questions arise: (a) Do solutions for $\{s(x_t,i), b(x_t,i)\}$ that satisfy (7) and (8) exist? (b) Are these solutions unique? The sufficient conditions for existence and uniqueness are given by Deaton and Laroque (1992) for a mathematically equivalent model of commodity prices with non-negative market inventories. The generalization of that framework in this setup gives as sufficient conditions the following inequalities

$$(9) \qquad \frac{1}{1+\delta} E_t \widetilde{R}_{t+1} Z_{t+1}^{-\rho} < 1$$

and

$$(10) \qquad \frac{1+r}{1+\delta} E_t Z_{t+1}^{-\rho} < 1$$

When stock returns $(\widetilde{R}_{t+1})$ are uncorrelated with permanent labor income shocks (Z_{t+1}) and with a positive mean equity premium, a sufficient condition is (9). Taking logs of (9) and using the approximation that for small $x, \log(1+x) \approx x$ the condition[17] becomes

$$(11) \qquad \frac{r_f + f_t - \delta}{\rho} + \frac{\rho}{2}\sigma_n^2 < \mu_g + \mu_n$$

When $f_t = \mu$ this simplifies to the convergence condition derived by Haliassos and Michaelides (1999) for the i.i.d. stock returns model. In the absence of a risky investment alternative, $f_t = 0$ and we have the Deaton (1991) and Carroll (1997) conditions. Appendix A details the numerical solution technique that involves solving simultaneously for the two policy functions by iterating over the two Euler equations of the problem.

3.1 Parameter Choice

The model is solved for a set of "baseline" parameters. The rate of time preference, δ, equals 0.12, and the net constant real interest rate, r, equals 0.01. Carroll (1992) estimates the variances of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and the benchmark simulations use values close to those: 0.1 percent per year for σ_u and 0.08 percent per year for σ_n.[18] Mean labor income growth (μ_g) equals 0.03. The coefficient of relative risk aversion is set either equal to 3 or 6. The high discount rate is chosen to accommodate the convergence conditions (11) for all factor realizations and chosen coefficients of risk aversion. The parameters describing the evolution of stock market returns are selected from Campbell (1999, Table 2C) who reports parameter estimates for a VAR model based on annual US data between 1891 and 1994. They are $\mu = .042, \phi = .798, \sigma_z^2 = .0319, \sigma_\varepsilon^2 = .9^2 * .001$, and $\sigma_{z,\varepsilon} = -.0039$.[19]

4 Results

4.1 Factor Follows an AR(1), $\rho_{z,\varepsilon} = 0, \rho_{n,z} = 0$

4.1.1 Optimal Consumption Choice

The consumption policy functions are plotted in figure 1 for $\rho = 3$ and in figure 3 for $\rho = 6$. A few observations can be made about the shape of the policy functions. First, the consumption policy rule has the familiar shape from the buffer stock saving literature without risky asset choice; below a cutoff point x^*, no saving takes place while the marginal propensity to consume falls quickly beyond x^* (see figures 1 and 3). Second, the total amount of precautionary saving is higher for $\rho = 6$ than for $\rho = 3$; saving becomes positive at lower levels of cash on hand, while the level of consumption beyond x^* is lower for higher ρ. Third, a high current factor realization signifying higher future stock returns induces an increase in saving to take advantage of more favorable future investment opportunities while a very low factor realization makes saving less desirable and induces an increase in consumption (figures 1 and 3). The increase in saving to take advantage of higher expected future returns can be thought of as a speculative demand for saving, and is particularly important for higher levels of intertemporal substitution; the consumer reacts to a greater extent when $\rho = 3$ than when $\rho = 6$ (compare figures 1 and 3). Equivalently, the substitution effect from a higher return on saving outweighs the income effect, even though in the standard two period single asset case with no labor income in the second period, the substitution effect is stronger only if $\rho < 1$ (see, for instance, Obstfeld and Rogoff (1998, p.30)).

The strength of the substitution effect is surprising. In a related model without labor income and with no liquidity constraints, Campbell and Viceira (1999) (figure 2) find the opposite result, namely that the consumption to wealth ratio is increasing in the expected excess return to stocks for a risk aversion of four (and an elasticity of substitution of 0.25 which must also be taken into account since they use a recursive utility formulation). The difference in results arises from the presence of labor income. To illustrate the argument I will use the analytical results in Viceira (2001) who studies optimal consumption and portfolio choice for long-horizon investors with nontradable labor income but no liquidity constraints. Viceira (2001) uses a CRRA utility function and computes the optimal consumption rule for both retired investors (who receive no labor income) and employed investors (who do receive labor income). For the retired investors, the substitution effect outweighs the income effect when $\rho < 1$ (the exact condition is $\frac{1}{\rho} > b_1^r = 1$ where b_1^r is the elasticity of consumption with respect to financial wealth for the retired investor). This is the Samuelson (1969) result. For the employed investors, on the other hand, the substitution effect dominates the income effect for a wider range of relative risk aversion coefficients. This occurs because the relevant condition now becomes $\frac{1}{\rho} > \overline{b}_1$ and $\overline{b}_1 < b_1^r$ ($\overline{b}_1$ is the expected elasticity of consumption with respect to financial wealth for the employed investor where the expectation is taken over the two possible states next period: employment and retirement). Viceira (2001) in fact notes that "since $\overline{b}_1 < b_1^r$, the substitution effect of an increase in expected return on wealth dominates the income effect at lower values of intertemporal substitution" (p. 14) [and therefore at higher values of risk aversion for the CRRA utility function being used].

But how much higher can ρ become before the substitution effect from a higher expected next period return is outweighed by the income effect? To answer this question, it is useful to use again the analytical results in Viceira (2001), Panel A ($\rho_{z,\varepsilon} = 0$), table I (for this calibration, the parameters are of similar magnitude as the ones used in the current study except for the discount rate). When the correlation between stock return and permanent earnings innovations is zero, $\bar{b}_1$ can be computed as $\frac{\alpha^r}{\alpha^e}$, namely the ratio of the share of wealth invested in stocks during retirement to the share of wealth in stocks during working life (compare equations (15) and (19) in Viceira(2001)). Using the longest retirement (35 years) for which results are available (to most closely approximate the infinite horizon used here) in Panel A, we find that $\bar{b}_1 = .31$ when $\rho = 2$. At this level, $\bar{b}_1 < \frac{1}{2}$ implying that the substitution effect dominates the income effect when $\rho = 2$. This does not hold for $\rho = 3$ as in figure 1 of the current paper but this can be rationalized by the change in horizon: the 35 years to retirement are quite far away from the infinite horizon assumed here. Viceira (2001, Panel A, Table 1) reports that the share of wealth in risky assets between 30 and 35 years to retirement rises from 136 to 148 percent when $\rho = 3$ implying that the share of wealth in the risky asset would continue to rise substantially enough to lower $\bar{b}_1$ even further making the substitution effect stronger than the income effect at higher levels of risk aversion. In this paper it is shown that the substitution effect could be mildly stronger than the income effect even for coefficients of relative risk aversion equal to 6 (see figure 3).

How robust are these conclusions to changes in the parameters governing the labor income process? When we eliminate the permanent earnings shocks ($\sigma_n = 0, \sigma_u = .1$) the substitution effect still outweighs the income effect while the magnitude of the speculative increase in savings becomes higher; the investor operates in a less uncertain economic environment and can therefore afford to take on more stock market risk. Similar conclusions are gleaned from setting ($\sigma_n = 0.08, \sigma_u = 0.05$) and ($\sigma_n = 0.02, \sigma_u = 0.1$); a reduction in background labor income risk makes the consumer/investor more eager to take advantage of market timing opportunities.

4.1.2 Optimal Portfolio Choice

The optimal portfolio allocation is substantially changed conditional on the factor realization. A high factor realization this period signifies higher future returns, and therefore generates additional demand for stocks compared to the i.i.d. case. With current realizations above the mean[20] (five cases in total in the discretization scheme chosen), the stock market allocation is higher than in the i.i.d. model due to the increase in total saving. Nevertheless, the borrowing constraint provides an upper bound on the ability of market timing to generate additional demand for stocks; the maximum amount that can be invested in the stock market is total savings on account of the borrowing constraint. Since the borrowing constraint is already binding in the i.i.d. model, the additional demand for stocks comes only from the increase in saving. The share of wealth invested in the stock market stays the same as in the i.i.d. model, therefore, and equals one; the consumer would like to borrow to invest in the stock market but is unable to do so (figures 2 and 4).

On the other hand, for the five cases where the current factor realization is below its mean, the demand for stocks (relative to the i.i.d. model) falls, since the factor is signalling lower returns in the future. There are now substantial portfolio allocation effects since the borrowing constraint does not prevent the individual from lowering the proportion of stocks in the portfolio and indeed the individual aggressively lowers the stock market exposure (figures 2 and 4). Moreover, market timing becomes so important that for the two lowest realizations of the factor (signalling very low future stock market returns) the investor allocates savings completely in the riskless asset market. On account of the low expected stock returns, the investor is now even willing to short the stock market position, but is prevented from doing so from the short sales constraint.

To see why this is happening, we must go back to the Euler equations. For the two lowest realizations of the factor, the consumer saves everything in the riskless asset market. For this to be the case, the normalized versions of (5) and (6) imply that[21]

$$\frac{1+r}{1+\delta}E_t\{Z_{t+1}^{-\rho}U'(c_{t+1})\} > \frac{1}{1+\delta}E_t\{\widetilde{R}_{t+1}Z_{t+1}^{-\rho}U'(c_{t+1})\} \tag{12}$$

with equality holding when neither constraint is binding.[22] When $s_t = 0$ and stock returns are uncorrelated with labor income shocks, (12) implies[23]

$$\frac{1+r}{1+\delta}E_t\{Z_{t+1}^{-\rho}U'(c_{t+1})\} > \frac{1}{1+\delta}E_t\{\widetilde{R}_{t+1}\}E_t\{Z_{t+1}^{-\rho}U'(c_{t+1})\} \tag{13}$$

The constraint ($s_t = 0$) will therefore continue to bind for as long as $(1+r) > E_t\widetilde{R}_{t+1} = 1 + r + f_t$ (see (2)). This is indeed the case for the two lowest realizations of the factor state ($f_t = -.04, -.01$ respectively). Intuitively, the expected next period stock return conditional on time t information is lower than the risk free rate and therefore the riskless asset dominates the stock market as a saving vehicle for all levels of cash on hand.

These results are broadly similar to the findings of Brennan, Schwartz and Lagnado (1997) with a three factor model, liquidity constraints and no labor income risk and Barberis (2000), figure 5, in a model with liquidity constraints but no labor income and no consumption choice. Barberis (2000) finds that for a risk aversion equal to 5 the household invests all its financial wealth in stocks when the factor predicting the mean excess return to stock is at or above its average. On the contrary, when the factor is such that the expected excess return to stocks is negative, the household invests all its financial wealth in the riskless asset. It seems that adding labor income risk, therefore, does not affect the portfolio choice implications of the model.

Are these results robust to changes in the parameters governing the labor income process? Changing the variances of the permanent and transitory shocks shows a broadly similar pattern; the investor becomes more aggressive in his market timing activities when faced with lower background labor income risk and vice versa when the background risk is increased. Given that these comparative statics exercises hold for the cases considered in the next few subsections as well, we conclude that adding labor income does not affect qualitatively the market timing properties of the portfolio decision.

It is also worth noting that for factor realizations generating positive stock holdings, total savings is first completely allocated in the stock market and stock market exposure is a non-increasing function of cash on hand (see figures 2 and 4). It is important to recall that even though labor income is uncertain, it is always positive with probability one. Labor income acts as a risk free asset crowding out the accumulation of a riskless asset. More puzzling is the reduction in the share of wealth in stocks that is observed for some states beyond a certain cash on hand level.[24] This finding is, however, consistent with Viceira (2001) who argues that non-tradable labor income is a forced investment in the riskless asset making investors with labor income allocate to stocks a larger share of their wealth than investors with no labor income. In the current setup, as cash on hand ($X_t = S_{t-1}\widetilde{R_t} + B_{t-1}R_f + Y_t$) rises, the share of labor income in cash on hand falls and therefore the investor's implicit holdings of the riskless asset in the form of non-tradeable labor income are reduced (as a fraction of total liquid wealth). The investor makes up for this reduction in riskless assets by allocating explicitly some savings in the bond market beyond a certain level of cash on hand.

4.1.3 Time Series Implications

Individual policy functions are informative about microeconomic behavior; nevertheless, we are very often interested in either the aggregate or the time series implications of a microeconomic model. One usual way of investigating the time series implications of non-linear microeconomic models is to simulate individual life histories over time by generating the random shocks from the exogenous distributions of the model and then using the computed policy functions to derive time series statistics. In the current model, however, normalized cash on hand follows a renewal process[25] and therefore the aggregate or individual time series implications of the model can be derived by computing the time invariant distribution of cash on hand.[26] The numerical computation details of the invariant distribution are left for appendix B (i.i.d. model) and appendix C (factor model).

Time series moments reported in Table 1 ($\rho = 3$ and $\rho = 6$) support the conclusions gleaned by comparing policy functions. The third column of table 1 reports the results for the i.i.d. model ($\phi = 0, \rho_{z,\varepsilon} = \rho_{z,\eta} = 0$); for either coefficient of relative risk aversion there is complete specialization of the portfolio in stocks (Heaton and Lucas, 1997). Stronger prudence ($\rho = 6$ relative to $\rho = 3$) generates higher precautionary saving and therefore a higher stock market allocation (mean normalized stocks rise from .03 to .07) and greater consumption smoothing (the standard deviation of normalized consumption falls from .07 to .06).

In the presence of a factor predicting excess returns (first column; $\{\phi = .798, \rho_{z,\varepsilon} = \rho_{z,\eta} = 0\}$), complete portfolio specialization in stocks does not occur. Relative to the i.i.d. model, normalized bonds rise from .00 to .01 when $\rho = 3$ or $\rho = 6$, while normalized stock holdings remain unchanged. Consistent with active market timing activity, the standard deviation of total savings rises by 25 percent when $\rho = 3$ relative to the i.i.d. model. When $\rho = 6$, on the other hand, the volatility of savings stays the same as in the i.i.d. model, consistent with the policy function results where the income and substitution effect outweighed each other. The prediction that, on average, the investor holds positive bond and stock holdings simultaneously is more consistent with empirical evidence than the implication of the i.i.d. model that the investor holds all savings in the stock market. On the other hand, the median share of wealth in stocks remains equal to one (in direct conflict with the zero stock holding puzzle).

4.2 Factor Follows AR(1), $\rho_{z,\varepsilon} = 0, \rho_{n,z} = 0.3$[27]

4.2.1 Optimal Consumption Choice

Positive correlation between labor income shocks and the stock market innovation does not change the consumption choice relative to the case where $\rho_{n,z} = 0$; the shape of the consumption policy rule retains the shape from the buffer stock saving model and a signal for higher future returns generates an increase in saving to take advantage of the improvement in future investment opportunities (compare figure 5 to figure 1 and figure 7 to figure 3).

4.2.2 Optimal Portfolio Choice

Figures 6 and 8 show that the hedging demand for the riskless asset induced by positive correlation between the stock market return and permanent labor income innovations can be substantial. There is now zero stock market exposure for three factor state realizations, as opposed to two in the zero correlation case. To see why this is happening, we must return to the Euler equations. For the consumer to be saving everything in the riskless asset market, (12) must be satisfied, with equality holding when neither constraint is binding. When $s_t = 0$, (12) is equivalent to

$$\begin{aligned}\frac{1+r}{1+\delta}E_t Z_{t+1}^{-\rho}U'(c_{t+1}) \quad > \quad & \frac{1}{1+\delta}E_t\widetilde{R}_{t+1}E_t Z_{t+1}^{-\rho}U'(c_{t+1}) \\ & + cov_t(\widetilde{R}_{t+1}, Z_{t+1}^{-\rho}U'(c_{t+1}))\end{aligned}$$

Relative to the zero correlation case, this condition includes a covariance term that turns out to be negative; the constraint ($s_t = 0$) therefore binds for some cases even if $(1+r) < E_t\widetilde{R}_{t+1} = 1 + r + f_t$. This situation occurs for the three lowest realizations of the factor state. Intuitively, the conditional expected next period return on the risky asset at time t must be higher than in the zero correlation case to induce demand for stocks because now stocks have the undesirable attribute of offering low returns when labor income is low.

4.2.3 Time Series Implications

A positive correlation between the stock market innovation and the permanent labor income innovation generally increases mean bond holdings, reduces exposure in the stock market and thereby reduces the mean share of wealth invested in stocks, with the effects being more important for higher risk aversion parameters. For $\rho = 3$ the differences are very small relative to the case of no correlation (first and second column of table 1) but when $\rho = 6$, mean normalized bond holdings rise from .01 to .02 and mean normalized stock holdings fall from .07 to .05. Nevertheless, the median share of wealth in stocks remains equal to 1.00. Once more, the volatility of savings is around 25 percent higher than in the i.i.d. model when $\rho = 3$.

4.3 Factor Follows AR(1), $\rho_{z,\varepsilon} = -.69, \rho_{n,z} = 0$

Introducing negative correlation between the stock market innovation and the factor innovation gives rise to a different type of hedging demand (see Merton, 1973) arising from a deterioration of future investment opportunities when current stock market returns are high. Hedging demand differs from market timing since the former arises as protection from unfavorable shifts in the investment opportunity set, reflecting an attempt to minimize (unanticipated) consumption variability. On the other hand, market timing demand arises from the desire to take advantage of current information regarding future returns. This subsection assesses the differential demand for stock investment when $\rho_{z,\varepsilon} = -.69$ relative to the case where $\rho_{z,\varepsilon} = 0$.

4.3.1 Optimal Consumption Choice

The consumption rules are depicted in figures 9 and 11. As before, the consumption policy function retains its shape from the buffer stock saving literature. Moreover, conditioning on the same factor realization, consumption is not significantly changed from the case when

$\rho_{z,\varepsilon} = 0$ (see figures 9 and 11 for $\rho = 3$ and $\rho = 6$ respectively). We conclude that total precautionary saving is not affected by this correlation.

4.3.2 Optimal Portfolio Choice

Perhaps surprisingly, the portfolio allocation decision is not significantly affected, either. The differences in the share of wealth allocated in the stock market (relative to the case when $\rho_{z,\varepsilon} = 0$) are relatively small in magnitude due to the constraints. For the highest realizations of the factor, the policy functions are the same since the borrowing constraint is binding in both cases, while total saving is approximately the same due to the consumption response. For the two lowest realizations of the factor, virtually identical behavior is generated on account of the short sales constraint (see figures 10 and 12). There is a small increase in the stock market allocation when $\rho_{z,\varepsilon} = -.69$ (a hedging demand for stocks) for one or two factors but this difference in the policy functions is very small in magnitude.

4.3.3 Time Series Implications

Time series results corroborate that there is a very small difference in the time series moments for the variables of interest when $\rho_{z,\varepsilon} = -.69$ relative to the case when $\rho_{z,\varepsilon} = 0$ (compare first two columns of table 2 for $\rho = 3$ and $\rho = 6$ respectively). Moreover, the median share of wealth in stocks remains equal to one. Note that the small change in the results from varying $\rho_{z,\varepsilon}$ suggests that the hedging demand arising from correlation between labor income shocks and stock market return innovations is a substantially more important component of total hedging demand for stocks than the correlation between the stock market and factor innovations.

4.4 AR(1), $\rho_{z,\varepsilon} = -.69, \rho_{z,n} = .3$

Positive correlation between labor income shocks and the stock market return innovation in the presence of negative correlation between the stock market and the factor innovation does not affect total precautionary savings (see figures 13 and 15), a result identical to the case when $\rho_{z,\varepsilon} = 0$. Moreover, positive correlation between stock return innovations and shocks to permanent labor income continues to crowd out stock holdings; figures 14 and 16 show the share of wealth invested in stocks falls for all factor realizations for which the borrowing and short sales constraints are not binding (compared to the case when $\rho_{z,n} = 0$).

4.4.1 Time Series Implications

The time series implications are consistent with the discussion for the policy functions; the second and fourth columns in table 2 compare the results when $\rho_{z,\varepsilon} = -.69$ and $\rho_{z,n}$ is increased from 0 to 0.3. The moments of the variables of interest remain virtually unchanged for $\rho = 3$ (reflecting the frequency with which the borrowing and short sales constraints are binding, in which case the policy rules are identical), while for $\rho = 6$ mean normalized stock holdings fall from .07 to .06 and mean normalized bond holdings rise from .01 to .02. Comparing the first to the second and the third to the fourth column of table 2 allows us to quantify the effect of varying $\rho_{z,\varepsilon}$. The effect depends on the presence or absence of correlation between the labor income shocks and stock market returns; in either case the magnitude of the changes is very small. Moreover, the zero stockholding puzzle persists throughout, since the median share of wealth in stocks remains equal to one.

4.5 Conclusions

We conclude this section with a brief summary of the main findings. First, the consumption policy rule has a familiar shape with functions derived in the buffer stock saving literature. Second, stock return predictability gives rise to a speculative demand for saving; equivalently, the substitution effect from a change in the expected rate of return on investment opportunities dominates the income effect for a wider range of relative risk aversion coefficients than the model with no labor income risk. Relatedly, the volatility of total saving rises by around 25 percent for low ρ, a reflection of saving patterns that take into account future investment opportunities. Third, for a given amount of saving, the share of wealth allocated in the stock market rises, when future investment opportunities are perceived to be improving. Fourth, complete portfolio specialization in stocks arises for factors predicting high future stock returns. Stock market predictability can also generate a complete portfolio specialization in the riskless asset, however, for factors predicting very low future returns. Time series simulations therefore lead to a portfolio allocation that includes both the riskless and the risky asset. Fifth, the correlation between permanent labor income shocks and stock market return innovations appears to be a more important determinant of optimal portfolio allocation than correlation between factor innovations and stock market return innovations due to the presence of both short sales and borrowing constraints. Sixth, the zero stock holding puzzle cannot be explained by the model since the portfolio is still heavily skewed towards the stock market. In particular, the median share of wealth in stocks equals one.

5 Welfare Costs from Failing to Time the Market

Two predictions of the current theoretical model are problematic. First, the prediction that the median share of wealth in stocks is one is counterfactual. Second, the prediction that conditional on the factor realization, every consumer either fully invests in the stock market or in the riskless asset market, would be inconsistent with a general equilibrium version of the model. I undertake two welfare analyses in this section to investigate whether fixed, one-time, transaction costs can mitigate these problems. First, I evaluate the transaction cost that is needed to make investors indifferent between participating in the stock market or not. Haliassos and Michaelides (1999) argue using the i.i.d. model that this entry cost is small; impatient consumers are liquidity constrained around one third of the time and accumulate a small buffer stock of assets to smooth consumption; a small stock market entry cost is therefore sufficient to prevent stock market participation. The question then arises whether the size of this transaction cost remains small in the presence of stock market predictability. Second, I quantify the welfare loss that results from using the i.i.d. policy rules when in fact stock market returns are predictable. Campbell and Viceira (1999) analyze an unconstrained model without labor income risk and argue that failure to time the stock market in the presence of a factor predicting future returns can lead to substantial welfare losses. Nevertheless, the policy function results have shown that the borrowing (short sales) constraints interfere heavily with the desired stock market allocation for high (low) realizations of the factor and prevent the investor from increasing (decreasing) the share of wealth in the stock market to the desired level. This result raises the possibility that welfare relative to the i.i.d. model, where saving is held completely in the stock market, might not be significantly higher. I address this question by evaluating the cost an investor would be willing to pay to be informed about the factor realization; the investor is now participating in the stock market and has to decide whether to behave optimally or make decisions according to the i.i.d. model policy rules.

5.1 Welfare Differences between Bonds Only vs Factor Predictability Models: A Certainty Equivalent Approach

The first comparison assumes that the individual begins with having access only to the riskless asset market. The consumer has to make a decision whether to enter the stock market given that there is a one-time entry fee (the fee could be in the form of brokerage commissions, information costs, the opportunity cost of time or simply inertia).

Welfare comparisons are performed using the value functions associated with the optimal policy rules[28]. Let V_S denote the value function associated with stock market entry and V_B the bonds only value function[29]. One measure of welfare is the unconditional expectation of the value functions in the two regimes[30]. Alternatively, we can compute the certainty equivalent level of cash on hand that would make the consumer indifferent between entering, or abstaining from, the stock market. Letting this function be denoted by $k(x,i)$, it is defined as the solution to $V_S([x-k(x,i)],i) = V_B(x,i)$ where i is the factor state variable. Given that the value function is concave, its inverse exists and a numerical interpolation procedure can be used to invert V_B and derive $k(x,i)$ as $k(x,i) = x - V_S^{-1}(V_B(x,i))$. If the one time entry cost is higher than the benefit for all the possible realizations of x, then the investor will optimally never choose to enter the stock market.

Figures 17-20 plot the functions $k(x,i)$ for the highest and lowest realizations of the factor when $\rho = 3$ and $\rho = 6$ vis-a-vis the bonds only model for different parameter specifications.[31] For all comparisons, the certainty equivalent function is increasing in normalized cash on hand; the cost must be higher for wealthier individuals for them to stay out of the stock market. Moreover, the cost must be higher for the high factor states since these states predict higher future stock returns and therefore carry a higher benefit from entering the stock market. Furthermore, the magnitude of the certainty equivalent is increasing in ρ for a given level of normalized cash on hand; stronger prudence requires the accumulation of a higher buffer stock of assets and the stock market provides a superior saving vehicle than the bond market.

Table 3 finds the largest normalized certainty equivalent that will induce stock market non-participation when $\rho = 3$. To compute this value, I first compute the maximum possible realization of cash on hand (call it $\widehat{x}$) when the economy is without a stock market. Using the invariant distribution for normalized cash on hand in the bonds only model, $\widehat{x}$ is computed as that value of x such that $\Pr(x \geqslant \widehat{x}) = 0$.[32] The maximum possible certainty equivalent that will induce non-participation after a stock market begins operation in this economy is then given by $\{MAX_i k(\widehat{x}, i)\}$. From figures 17-20, we know that this value is $k(\widehat{x}, 10)$ where 10 denotes the highest realization of the state variable f. $k(\widehat{x}, 10)$ varies between .09 and .12 (table 3, panel A) for $\rho = 3$ and between .15 and .26 (table 3, panel A) when $\rho = 6$.

Two conclusions arise from this analysis. First, the certainty equivalent is higher with stronger prudence because the stock market is a superior vehicle in generating the precautionary wealth buffer than the riskless asset market. Second, for low levels of ρ, the certainty equivalent tends to be small, ranging between .09 and .12 units of normalized cash on hand. Since mean normalized earnings equal one, the results imply that a cost between nine and twelve percent of mean labor income is sufficient to generate stock market non-participation.

Why is the cost small for $\rho = 3$? The answer lies in the fundamental conflict between impatience and prudence. Impatient households want to consume earlier rather than later, while prudence requires some savings to smooth consumption in the face of substantial undiversifiable earnings uncertainty. The chosen impatience parameter is high ($\delta = .12$) to ensure that the contraction mapping condition is satisfied and it generates two results that help explain the low cost. Firstly, the liquidity constraints are binding very often. Panel C of Table 3 uses the time invariant distributions associated with each model to compute the percentage of time an individual will be with zero savings. The range varies between 39 and 44 percent for $\rho = 3$, thereby reducing the benefit from incurring a cost to have access to the stock market. Furthermore, impatience makes asset accumulation very expensive and results in a very low stock market investment; normalized stock holdings range around .03 for the different specifications of the economic environment when $\rho = 3$ (table 1). The much higher entry costs for the case when $\rho = 6$ can also be explained in this light; Panel C of

Table 3 shows that the percentage of time that total saving is zero for the higher level of risk aversion/prudence ranges between 19 and 21 percent. Moreover, normalized stock holdings now rise to around .07 (from .03 when $\rho = 3$) generating a higher benefit from the equity premium. For both reasons the benefit from entering the stock market is higher, and leads to a higher level of entry costs required to explain stock market non-participation.

5.2 Welfare Differences between the i.i.d. and Factor Predictability Model

The previous subsection argued that demand for the riskless asset can be generated for all factor realizations in the presence of one time stock market entry costs. Can demand for stocks also be generated for all factor realizations? To answer this question, welfare analysis can be performed for the case when the consumer/investor behaves as if returns were i.i.d.. The value functions when agents are using the i.i.d. policy functions are now computed by replacing the optimal policy functions in the different factor model specifications with their i.i.d. counterparts; expectations are taken as before using the probabilities associated with the correct specification of the economic environment. Let $V_F(x,i)$ denote the value functions when the i.i.d. policy rules are used in place of the optimal policy rules.[33] Conditional on the factor, these value functions are everywhere below the optimal value functions; the certainty equivalent that would make agents indifferent between the two is then given by $k(x,i) = x - V_S^{-1}(V_F(x,i),i)$ where V_S is the optimal value function using the correct specification of the economic environment. The certainty equivalent can now be interpreted as the cost of acquiring the information about the true nature of returns in the economy.

The certainty equivalents have the same shape and offer conclusions broadly similar to those from the previous subsection. For low risk aversion/prudence ($\rho = 3$) the information cost can be very small ranging between .04 and .05 units of normalized cash on hand (table 4).[34] There are three reasons for this result. First, we know from table 3 that even with optimal behavior, high impatience means that the investor has zero savings around forty percent of the time (Panel C, table 3). In these cases, acting in the non-optimal way does not hurt the investor since there are no savings carried over in the next period. Second, the power of the borrowing constraint can now be seen in full force; we have seen that when high future returns are predicted, the investor wants to invest everything in the stock market whether in the i.i.d. world or in the factor model. The binding borrowing constraint prevents any desired differences in behavior from materializing and therefore the end result in terms of returns is the same in both models. Third, the small buffer of assets that is accumulated over time (especially when $\rho = 3$) further reduces the benefit from acting optimally (see table 1 for instance, where mean normalized stocks is .03).

The welfare cost is much higher for $\rho = 6$ (table 4). Prudence now becomes much stronger relative to impatience with the following results. First, the agent has zero savings around twenty percent of the time (Panel C, table 3) and therefore deviation from the optimal rule carries a heavier penalty in terms of foregone returns. Second, there are more deviations between the optimal policy rules and the i.i.d. model for $\rho = 6$ (see the discussion of the policy functions). Third, there is a larger buffer of assets being accumulated for higher degrees of risk aversion/prudence leading to an enhanced loss from deviating from optimality (mean normalized stock holdings range around .07 (compare to .03 for $\rho = 3$)).

6 Conclusion

Undiversifiable labor income risk does not seem to affect the implications of stock market predictability for optimal portfolio choice. Consistent with Barberis (2000) and Brennan, Schwartz and Lagnado (1997), stock market predictability implies that portfolio holdings will very often be either completely allocated in the stock market or in the riskless asset market. Moreover, consistent with previous findings in the literature (Heaton and Lucas (2000(b)) for instance) stock market return correlation with permanent shocks to labor income generates a wider range of cases where bonds and stocks co-exist in the portfolio and gives rise to a quantifiably important hedging demand for the riskless asset. Stock market return correlation with the innovation in the factor predicting future returns does not generate as large (in magnitude) changes in hedging demands as those generated by the correlation of stock market return innovations with permanent labor income shocks largely due to the borrowing and short sales constraints. Moreover, median stock holding (counterfactually) equals one.

Undiversifiable labor income risk affects substantially the optimal consumption policy rule, however. Specifically, high future expected stock returns generate an increase in the speculative demand for saving for a wider range of risk aversion parameters than is usually thought to be the case and vice versa when future expected returns are expected to be low. Equivalently, the substitution effect from a higher expected return on saving dominates the income effect for a wider range of coefficients of relative risk aversion than what is usually thought to be the case.

The inability of the model to generate positive demand for the riskless asset or the risky investment for some factor state realizations can be problematic for recasting the model in general equilibrium. This weakness can be remedied, however, by considering the effects of fixed one time entry costs. For plausible levels of prudence, impatient households will be constrained often enough to optimally choose not to incur a small one time stock market entry cost, generating stock market non-participation. Impatience and the frequency with which the borrowing and short sales constraints bind also imply that a small cost associated with acquiring information about the economy can generate an equilibrium where the agent makes decisions based on the belief that stock returns are i.i.d., when in fact they are predictable.

Two broad directions for future research arise. First, the importance of impatience in generating the predictions of the model leads one to seek sensitivity analysis with respect to this parameter. Unfortunately, given the equity premium, the convergence conditions limit the sensitivity analysis that can be performed with respect to impatience. An alternative approach that avoids this problem, involves investigating the implications of the model over the life cycle (see Cocco, Gomes and Maenhout, 1999). Solving the model over the life cycle need not satisfy any restrictions on the parameter space while the model can then address issues of wealth accumulation and be more amenable to estimation using household level data. Second, taking estimation risk for the factor predictability model into account will mitigate the extreme portfolio re-allocations and generate co-existence of bonds and stocks in the portfolio (Barberis, 2000). Both extensions could potentially have interesting implications for portfolio advice in a multifactor world.

A Numerical Procedures

A.1 Numerical Dynamic Programming

The factor that predicts future stock returns, f_{t+1}, follows the AR(1) process (3) where (μ) is the unconditional mean of the factor. This continuous autoregressive process is discretized around (μ) with a standard deviation equal to $\sigma_f = \sigma_\varepsilon/\sqrt{1-\phi^2}$. Letting $\{z_i\}_{i=1}^{n=10}$ denote the ten point discrete approximation to the standard normal, $f = \mu + \sigma_f * z_i$.[35] The transition probabilities $\pi_{ij}$of moving from interval i to interval j can be computed numerically. From the properties of the normal distribution of the error term z_i, we have

$$
\begin{aligned}
(14) \qquad \pi_{ij} &= \Pr(\sigma_f z_j \geq f_t - \mu \geq \sigma_f z_{j-1} | \sigma_f z_i \geq f_t - \mu \geq \sigma_f z_{i-1}) = \\
&\frac{1}{\sigma_f\sqrt{2\pi}} \int_{\sigma_f z_{i-1}}^{\sigma_f z_i} \exp(\frac{-x^2}{2\sigma_f^2})\{\Phi(\frac{\sigma_f z_j - \phi x}{\sigma}) - \Phi(\frac{\sigma_f z_{j-1} - \phi x}{\sigma})\}dx
\end{aligned}
$$

For any given $\{\sigma, \phi\}$, this integral can be calculated directly using GAUSS routines that approximate the cumulative normal (Φ).

Ten policy functions (one for each f) are defined by solving simultaneously the two Euler equations (7) and (8). The expectation operator E_t is conditional on all information known at time t (x_t, f_i). Conditional on this information and discretizing $\widetilde{R}_{t+1}$ by $\widetilde{R}_{t+1} = 1 + f_i + \{\sigma_z * z_i\}_{i=1}^{n=10}$,

$$
\begin{aligned}
& \frac{1+r}{1+\delta} E_t U'(x_{t+1} - s(x_{t+1}, j) - b(x_{t+1}, j)) Z_{t+1}^{-\rho} \\
= \; & \frac{1+r}{1+\delta} \sum_j \sum_m \sum_k \sum_l \pi_{ij}\pi_m\pi_k\pi_l (GN_l)^{-\rho} * U'\{ \\
& (GN_l)^{-1} * [s(x_t, i) * (1 + f_i + z_k) + b(x_t, i) * R_f] + U_m - \\
& s((GN_l)^{-1} * [s(x_t, i) * (1 + f_i + z_k) + b(x_t, i) * R_f] + U_m, j) - \\
& b((GN_l)^{-1} * [s(x_t, i) * (1 + f_i + z_k) + b(x_t, i) * R_f] + U_m, j)\}
\end{aligned}
$$

where π_m, π_k, and π_l are transition probabilities associated with $U_{t+1}, z_{t+1}, N_{t+1}$, while π_{ij} is the probability that the factor takes the value of f_j next period when the current period's realization is f_i. All the probabilities except the one associated with the autoregressive process (π_{ij}) are equal to .1 when the stock market return innovation, the permanent shock to labor income and the factor innovation are uncorrelated with each other. We discretize the state variable x by dividing it into 100 equidistant grid points.

A.2 Allowing Correlation Between Innovations

To compute the joint distribution of two correlated random variables, we use a discrete approximation of

$$F(y_1 \leq Y \leq y_2, z_1 \leq Z \leq z_2) = F(y_2, z_2) - F(y_2, z_1) - F(y_1, z_2) + F(y_1, z_1)$$

where F is the bivariate standard normal of the two random variables (Y, Z). This probability is evaluated using the $CDFBVN$ command in GAUSS. When the stock market innovation is correlated with the factor innovation, we follow these steps. The numerical algorithm outlined in the previous subsection uses a ten point discretization of the process followed by the factor predicting stock returns: $f = \mu + \sigma_f * z_i\}_{i=1}^{i=10}$. Moreover, the functional equations are solved for conditional on the current state $f_t = f_i$ and on next period's state being $f_{t+1} = f_j$. Conditional on this information, $\varepsilon_{t+1} = f_j - \mu - \phi(f_i - \mu) = \overline{\varepsilon}$. To find the probability of observing a particular realization of z_{t+1}[36] conditional on $\{f_t, f_{t+1}\}$ use

$$\Pr(z_{t+1}|f_i, f_j) = \Pr(z_{t+1}|\varepsilon_{t+1} = \overline{\varepsilon}) = \frac{\Pr(z_{t+1} \cap \varepsilon_{t+1} = \overline{\varepsilon})}{\Pr(\varepsilon_{t+1} = \overline{\varepsilon})} \tag{15}$$

The numerator can be computed for any correlation coefficient between the two innovations using the properties of the bivariate normal, while the denominator equals (1/10) since 10 points have been used to approximate this distribution.

In the most general case where both $\rho_{z,n}$ and $\rho_{z,\varepsilon}$ are not zero we use the fact that $\rho_{n,\varepsilon} = \rho_{z,\varepsilon} * \rho_{z,n}$ and apply the logic of (15) to find the probabilities of observing permanent labor income innovations conditional on f_i, f_j.

B Computing The Time Invariant Distribution in the I.i.d. Model

To find the time invariant distribution of cash on hand in the i.i.d. stock returns model, we first compute the bond and stock policy functions; $b(x)$ and $s(x)$ respectively. Note that the normalized cash on hand evolution equation is $x_{t+1} = [b(x_t)R_f + s(x_t)\widetilde{R}_{t+1}]\frac{P_t}{P_{t+1}} + U_{t+1} = w(x_t|\widetilde{R}_{t+1}, \frac{P_t}{P_{t+1}}) + U_{t+1}$, where $w(x)$ is defined by the last equality and is conditional on $\{\widetilde{R}_{t+1}, \frac{P_t}{P_{t+1}}\}$. Denote the transition matrix of moving from x_j to x_k,[37] as T_{kj}. Let Δ denote the distance between the equally spaced discrete points of cash on hand on the grid. The risky asset return $\widetilde{R}$ and $\frac{P_t}{P_{t+1}}$ are discretized using 10 grid points respectively: $R = \{R_l\}_{l=1}^{l=10}$ and $\frac{P_t}{P_{t+1}} = \{GN_m\}_{m=1}^{m=10}$. $T_{kj} = \Pr(x_{t+1=k}|x_{t=j})$ is found using

$$(16) \quad \sum_{l=1}^{l=10}\sum_{m=1}^{m=10} \Pr(x_{t+1}|x_t, \widetilde{R}_{t+1} = R_l, \frac{P_t}{P_{t+1}} = N_m) * \Pr(\widetilde{R}_{t+1} = R_l) * \Pr(\frac{P_t}{P_{t+1}} = N_m)$$

where both the independence of $(\widetilde{R}_{t+1}, \frac{P_t}{P_{t+1}})$ from x_t and the independence of $\frac{P_t}{P_{t+1}}$ from $\widetilde{R}_{t+1}$ were used. Numerically, this probability is calculated using

$$T_{kjlm} = \Pr(x_k + \frac{\Delta}{2} \geqslant x_{t+1} \geqslant x_k - \frac{\Delta}{2}|x_t = x_j, \frac{P_{it}}{P_{it+1}} = N_m, R_{t+1} = R_l)$$

Making use the approximation that for small values of σ_u^2, $U \sim N(\exp(\mu_u + .5 * \sigma_u^2), (\exp(2 * \mu_u + (\sigma_u^2)) * (\exp(\sigma_u^2) - 1)))$, and denoting the mean of U by $\overline{U}$ and its standard deviation by σ, the transition probability conditional on N_m and R_l then equals

$$(17) \quad \begin{aligned} T_{kjlm} &= \Phi(\frac{x_k + \frac{\Delta}{2} - w(x_t|N_m, R_l) - \overline{U}}{\sigma} \geqslant x_{t+1} \geqslant \frac{x_k - \frac{\Delta}{2} - w(x_t|N_m, R_l) - \overline{U}}{\sigma} \\ |x_t &= x_j, \frac{P_{it}}{P_{it+1}} = N_m, R_{t+1} = R_l) \end{aligned}$$

The unconditional probability from x_j to x_k is then given by

$$(18) \quad T_{kj} = \sum_{l=1}^{l=10}\sum_{m=1}^{m=10} T_{kjlm} \Pr(N_m) \Pr(R_l)$$

Given the matrix T, the probabilities of each of the states can be updated by $\pi_{kt+1} = \sum_j T_{kj} * \pi_{jt}$ so that the invariant distribution can be found by repeatedly multiplying the transition matrix by itself until all its columns stop changing. The invariant distribution π is instead calculated (faster) as the normalized eigenvector of T corresponding to the unit eigenvalue by solving the linear equations

$$\begin{pmatrix} T-I & e \\ e' & 0 \end{pmatrix}\begin{pmatrix} \pi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{19}$$

where e is an M-vector of ones.

C Time Invariant Distribution with Stock Market Mean Reversion

There are two state variables in the presence of a factor predicting future excess returns; cash on hand (x_t) and the current factor ($f_t = f_k$, $k = 1, ..., 10$). Given individual policy functions for bonds and stocks as $b(x,k)$ and $s(x,k)$ respectively, the endogenous state variable evolves as $x_{t+1} = [b(x_t,k)R_f + s(x_t,k)(1+f_k+z_{t+1})]\frac{P_t}{P_{t+1}} + U_{t+1} = w(x_t,k|z_{t+1},\frac{P_t}{P_{t+1}}) + U_{t+1}$ where $\{w(x,k|z_{t+1},\frac{P_t}{P_{t+1}})\}$ is defined by the last equality. We need to solve for the joint invariant distribution of cash on hand (x_t) and factor realizations (f_t). Define the conditional transition probability from $\{x_i, f_k\}$ to $\{x_j, f_l\}$ as T_{ijkl}. This probability can be found by conditioning on f_l and multiplying by the probability that f_l occurs. Then,

$$\begin{aligned} T_{ijkl} &= \Pr(x_{t+1} = x_j, f_{t+1} = f_l | x_t = x_i, f_t = f_k) \\ &= \Pr(x_{t+1} = x_j | x_t = x_i, f_t = f_k, f_{t+1} = f_l) * \Pr(f_{t+1} = f_l | f_t = f_k) \end{aligned}$$

The first probability is evaluated using the same method as in the i.i.d. model; conditioning on the permanent shock and the stock return innovation and integrating out as in (17) and (18), while the second probability is the probability associated with the discretization of the Markov process followed by f_t (see (14)).

The time invariant joint distribution between cash on hand and the factor is then calculated using a version of equation (19) which now has dimension MN by MN, where M is the number of grid points on the cash on hand space and N is the number of grid point used to discretize f_t. Once the invariant distribution has been derived, the moments of interest for the different variables can be easily computed. Mean consumption, for instance, equals

$$\sum_l \sum_j \pi_{lj} * c(x_j, f_l) \tag{20}$$

References

[1] Abowd, John and David Card. 1989. "On the Covariance Structure of Earnings and Hours Changes" *Econometrica* 57: 411-45.

[2] Attanasio, Orazio and Guglielmo Weber. 1993. "Consumption Growth, the Interest Rate and Aggregation." *Review of Economic Studies* 60: 631-49.

[3] Balduzzi, P. and Anthony W. Lynch. 1999. "Transaction Costs and Predictability: Some Utility Cost Calculations." *Journal of Financial Economics* 52: 47-78.

[4] Barberis, Nicholas. 2000. "Investing for the Long Run when Returns are Predictable." *Journal of Finance*, Vol. 55 (1).

[5] Bodie, Zvi, Robert C. Merton and William F. Samuelson. 1992. "Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model." *Journal of Economic Dynamics and Control*, 16, 427-449.

[6] Brennan, M. E. Schwartz and R. Lagnado. 1997. "Strategic Asset Allocation." *Journal of Economic Dynamics and Control*, 21, 1377-1403.

[7] Campbell, John. 1999. Clarendon Lectures in Economics.

[8] —— 2000. "Asset Pricing at the Millenium." Forthcoming in *Journal of Finance*, August 2000.

[9] Campbell, John Y., and Deaton Angus. 1989. "Why is consumption so smooth?" *Review of Economic Studies* 56, 357-74.

[10] Campbell, John Y., and N. Gregory Mankiw, 1989."Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence', in Olivier J. Blanchard and Stanley

Fischer (Eds..), NBER *Macroeconomics Annual 1989*, Cambridge, Mass, MIT Press, 185-216.

[11] Campbell, John Y., A. Lo, and C. MacKinlay. 1997. “The Econometrics of Financial Markets.” *Princeton University Press.*

[12] Campbell, J. Y., J. Cocco, F. Gomes, P. Maenhout. 1998 “Stock Market Mean Reversion and the Optimal Allocation of a Long Lived Investor.” mimeo, Harvard University.

[13] Campbell, John Y. and Luis Viceira. 1999. “Consumption and Portfolio Decisions When Expected Returns are Time Varying.” *Quarterly Journal of Economics*, 114, 433-495.

[14] Campbell, J. Y., Y. L. Chan and L. M. Viceira. 1999. “A Multivariate Model of Strategic Asset Allocation.” Harvard University Working Paper.

[15] Carroll, Christopher D., 1992. “The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence.” *Brookings Papers on Economic Activity* no. 2: 61-156.

[16] ————1997. “Buffer Stock Saving and the Life Cycle / Permanent Income Hypothesis’. *Quarterly Journal of Economics* CXII no. 1: 3-55.

[17] Carroll, Christopher and Andrew Samwick. 1997. “The Nature of Precautionary Wealth.” *Journal of Monetary Economics*, 40(1).

[18] —— 1998. “How important is precautionary saving?” *Review of Economics and Statistics* 80 no. 3: 410-19.

[19] Cocco, J., F. Gomes and P. Maenhout. 1999. “Consumption and Portfolio Choice over the Life-Cycle.” Harvard University, mimeo.

[20] Cochrane, John. 1999. “New Facts in Finance.” *Economic Perspectives*, Federal Reserve Bank of Chicago, pp. 36-58.

[21] Constantinides, George, Donaldson John and Mehra Rajnish. 1998. “Junior Can’t Borrow: A New Perspective on the Equity Premium Puzzle.” Working paper, University of Chicago.

[22] Davis, Steven and Willen, Paul. 1999. "Using Financial Assets to Hedge Labor Income Risks: Estimating the Benefits." University of Chicago, mimeo.

[23] Deaton, Angus.1991. "Saving and Liquidity Constraints." *Econometrica* 59 no.5: 1221-48.

[24] Deaton, Angus, and Guy Laroque. 1992. "On the Behavior of Commodity Prices" *Review of Economic Studies* 59: 1-23.

[25] Flavin, Marjorie. 1981."The Adjustment of Consumption to Changing Expectations About Future Income." *Journal of Political Economy* 89: 974-1009.

[26] Gollier C. and J.W. Pratt. 1996. "Risk Vulnerability and the Tempering Effect of Background Risk." *Econometrica*, 64: 1109-1124.

[27] Gourinchas, Pierre-Olivier and Jonathan Parker. 1999. "Consumption over the Life Cycle." Princeton University working paper.

[28] Haliassos, Michael and Carol C. Bertaut. 1995. "Why do so few hold stocks?" *The Economic Journal*, 105, 1110-1129.

[29] Haliassos, Michael and Alexander Michaelides. 1999. "Portfolio Choice and Liquidity Constraints." Mimeo, University of Cyprus.

[30] Hall, Robert. 1978."Stochastic Implications of the life cycle-permanent income hypothesis: theory and evidence." *Journal of Political Economy* 96: 971-87.

[31] Heaton John, and Deborah Lucas. 1996. "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing." *Journal of Political Economy* 104: 443-487.

[32] —— 1997. "Market Frictions, Savings Behavior, and Portfolio Choice." *Macroeconomic Dynamics* 1: 76-101.

[33] —— 2000(a). "Asset Pricing and Portfolio Choice: The Importance of Entrepreneurial Risk." *Journal of Finance*, Vol. 55, Number 3, 1163-1198.

[34] Heaton John, and Deborah Lucas. 2000(b). "Portfolio Choice in the Presence of Background Risk." *The Economic Journal*, 110 (January), 1-26.

[35] Hubbard Glenn, Jonathan Skinner, and Stephen Zeldes. 1994."The importance of precautionary motives for explaining individual and aggregate saving." in Allan Meltzer and Charles I Plosser, eds., *The Carnegie Rochester Conference Series on Public Policy*, XL (Amsterdam, North Holland).

[36] Ludvigson, Sydney and Alexander Michaelides. "Does Buffer Stock Saving Explain the Smoothness and Excess Sensitivity of Consumption?" Forthcoming in *American Economic Review.*

[37] Malkiel, G., Burton. 1999. "A Random Walk Down Wall Street." 7th edition, *New York: Norton.*

[38] Mankiw, N. Gregory and Stephen P. Zeldes. 1991. "The Consumption of Stockholders and Non-Stockholders." *Journal of Financial Economics,* 29, 97-112.

[39] Merton, R.C. 1969. "Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case." *Review of Economics and Statistics*, 51, 247-57.

[40] Merton, R.C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous Time Model." *Journal of Economic Theory*, 3, 373-413.

[41] Merton, R.C. 1973. "An Intertemporal Capital Asset Pricing Model." *Econometrica*, 41, 867-87.

[42] Obstfeld, Maurice and Kenneth Rogoff. 1996. *Foundations of International Macroeconomics*, MIT Press.

[43] Samuelson, Paul. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics*, 51, 239-46.

[44] Storesletten, Kjetil, Telmer Chris and Yaron Amir. "Persistent Idiosyncratic Shocks and Incomplete Markets." Working paper, Carnegie Mellon University, 1998.

[45] Tauchen, George. 1986. "Finite State Markov chain approximations to univariate and vector autoregressions." *Economic Letters* 20: 177-81.

[46] Viceira, Luis. April 2001. "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income." *The Journal of Finance.*

[47] Vissing-Jorgensen, A. 1999. "Limited Stock Market Participation." University of Chicago, mimeo.

Notes

[1]Most financial economists nowadays consider the predictability of stock returns over longer horizons as a stylized fact in finance (for an exhaustive list of references, see Campbell, Lo and MacKinlay (1997), Cochrane (1999) and Campbell (2000)).

[2]Viceira (2001) rigorously verifies the popular advice (Malkiel (1999)) in the financial management industry that higher exposure in the stock market be taken during working life with a shift towards safe assets after retirement. It is perhaps useful to recall that the infinite horizon models of portfolio choice (Merton 1969, 1971 and Samuelson 1969) that assume fully tradable human capital and a constant investment opportunity set, predict that the share of wealth invested in the risky asset should be constant.

[3]Mehra and Prescott (1985) show how the coefficient of relative risk aversion must be unrealistically high to reconcile the smoothness of aggregate consumption with the observed equity premium.

[4]Viceira (2001) finds similar results in a stylized life-cycle model with no explicitly imposed constraints. Bodie, Merton and Samuelson (1992) also show that it is optimal for employed investors to hold proportionately more stocks in their portfolios than retired investors in the presence of nontradable certain future labor income.

[5]The "excess smoothness" puzzle arises in the context of the PIH; given the observed positive serial correlation of labor income growth in aggregate data, the representative agent PIH predicts that consumption growth should be more volatile than aggregate income growth (Campbell and Deaton (1989)). In actual data, non-durables consumption growth is around half as volatile as labor income growth; see Ludvigson and Michaelides (forthcoming) for an updated look at post war US data.

[6]The PIH predicts that consumption changes should be orthogonal to predictable, or lagged, income changes; see Hall (1978). The correlation between consumption growth and predictable or lagged labor income changes has become one of the most robust features of aggregate data (see Flavin (1981), Campbell and Mankiw (1989) and Attanasio and Weber (1993)).

[7]The importance of buffer stock saving has been stressed by Carroll (1997), Carroll and Samwick (1997, 1998), and Hubbard, Skinner and Zeldes (1995).

[8] A numerical technique to solve the portfolio choice problem with i.i.d. stock market returns has been used by Heaton and Lucas (1996, 1997, 2000(a), 2000(b)) and Haliassos and Michaelides (1999).

[9] Davis and Willen (1999) obtain estimates ranging between .1 and .3 over most of the working life for college educated males and around $-.25$ at all ages for male high school dropouts. Such estimates would imply, according to the theoretical model, that less educated individuals should have a higher exposure to the stock market than individuals with college education to take advantage of the hedging opportunity that stocks offer; this is empirically counterfactual. Vissing (1999) estimates a correlation around 0.2.

[10] These costs could either come from the high opportunity cost of time or from actual transaction costs like brokerage commissions or from high costs in accessing and analyzing information; see Haliassos and Michaelides (1999).

[11] The framework extends the Heaton and Lucas (1997) model to study the optimal saving and portfolio choice problem in the presence of a mean reverting equity premium.

[12] See Carroll (1992).

[13] In this way the mean level equals 1; for instance, $EU_t = \exp(.5 * \sigma_u^2 - .5 * \sigma_u^2\ ) = 1$.

[14] Although these studies generally suggest that individual income changes follow a MA(2), the MA(1) is found to be a close approximation.

[15] At this point, there are three state variables associated with this problem; the current level of cash on hand (X_t), the current level of the permanent component of labor income (P_t) and the current realization of the factor predicting future returns (f_t).

[16] U' denotes marginal utility, all other variables with primes denote next period variables.

[17] This requires no correlation between the stock market innovation and the innovation to the permanent income component. Positive correlation between the two random variables $\{\widetilde{R}_{t+1}, Z_{t+1}\}$ makes the constraint less stringent, since the extra covariance term $\{cov(\widetilde{R}_{t+1}, Z_{t+1}^{-\rho})\}$ is then negative and further reduces the right hand side of (11).

[18] These values generate an autocovariance structure for the growth rate of labor income that is almost identical to the one used by Deaton (1991), who in turn deflates the MaCurdy (1982) estimates to take into account the effects of measurement error.

[19] Campbell (1999, Table 2C) estimates r_f to be .0199 and $\sigma_\varepsilon = .001$. I decrease both quantities so that the convergence condition (11) can be satisfied for all factor state realizations and for all coefficients of relative risk aversion ($\rho = 3$ and $\rho = 6$). Calibrating the model over the life cycle need not satisfy the condition and can therefore allow richer experimentation with different parameter values. This is the subject of current research.

[20] The i.i.d. model policy functions have been computed by setting the mean equity premium equal to 4.2 percent; the factor also has the same unconditional mean in the AR(1) model ($\mu = .042$). In the AR(1) model, the conditional expectation of next period excess returns (conditional at time t information) equals f_t (see (2)). Similar conclusions hold when setting the equity premium equal to 6 percent.

[21] For notational convenience I suppress the dependence of consumption on the current factor (f_t).

[22] $c_{t+1} = g(Z_{t+1}^{-1} b_t R_f + U_{t+1})$ when $s_t = 0$, where g is a non-decreasing, continuous function.

[23] This step utilizes the fact that $s_t = 0$ and that $\widetilde{R}_{t+1}$ is uncorrelated with $Z_{t+1}^{-\rho}$.

[24] Heaton and Lucas (1997) and Haliassos and Michaelides (1999) offer similar numerical results in the i.i.d. returns infinite horizons model and Cocco, Gomes and Maenhout (1999) report similar results in a life cycle model.

[25] See Deaton and Laroque (1992) for the proof for a mathematically equivalent model of commodity prices.

[26] Computation of the invariant distribution offers high numerical accuracy at a low computational cost; time invariant probabilities are equivalent to simulating an infinite number of invividuals (or a single individual over an infinite number of periods).

[27] Haliassos and Michaelides (1999) and Viceira (2001) find that correlation between stock market returns and permanent shocks to labor income is much more important for the portfolio allocation decision than correlation between transitory innovations to labor income and stock returns. I therefore limit my attention to correlation between the innovation to the permanent component of labor income (N_{t+1}) and the stock market return innovation (z_{t+1}).

[28] The computation of the value function is straightforward once the optimal policy rules are available; the description of the algorithm is omitted due to space considerations.

[29] V_S is a function of normalized cash on hand and the factor state, while V_B depends only on normalized cash on hand; moreover, we know that $V_S \geqslant V_B$ because there are more options available when the stock market exists.

[30] Let E denote the unconditional expectation of the value function using the time invariant probability distributions (π_i in the single riskless asset model and π_{ij} in the two asset model with stock market predictability). Then $EV_B(x) = \sum_i \pi_i V_B(x_i)$ and $EV_S(x) = \sum_i \sum_j \pi_{ij} V_S(x_i, j)$.

[31] REV denotes the model with $\{\phi = .798, \rho_{z,\varepsilon} = 0, \rho_{n,z} = 0\}$, REV1 the model with $\{\phi = .798, \rho_{z,\varepsilon} = -.69, \rho_{n,z} = 0\}$, REV2 the model with $\{\phi = .798, \rho_{z,\varepsilon} = -.69, \rho_{n,z} = .3\}$, and REV3 the model with $\{\phi = .798, \rho_{z,\varepsilon} = 0, \rho_{n,z} = 0.3\}$.

[32] $\widehat{x} = 1.73$ for $\rho = 3$ and $\widehat{x} = 1.88$ when $\rho = 6$.

[33] Subscript F stands for false since the non-optimal policy rules are used in the computation of the value function.

[34] The joint distribution of cash on hand and the factor when agents are using the i.i.d. model are derived by replacing the optimal policy functions with their i.i.d. counterparts in the algorithm computing the invariant distribution; everything else remains the same as when computing the distributions in the AR(1) models (this leads to the computation of a "false" distribution). The certainty equivalents are derived by first finding $\widehat{x}$ such that $\Pr(x > \widehat{x}) = 0$, where the "false" distribution was used ($\widehat{x}$ also depends on i, the factor state). The certainty equivalent reported in the text is then given by $MAX_i k(\widehat{x}, i)$.

[35] The actual values used to discretize f were {-0.04087909, -0.00733293, 0.01001418, 0.02374757, 0.03604977,

0.04795023, 0.06025243, 0.07398582, 0.09133293, 0.12487909 } generated by setting $\mu = .042$, and $\sigma_f = .0472$.

[36] z_{t+1} refers to the stock market innovation and is different from $z_i\}_{i=1}^{i=10}$.

[37] The normalized grid is discretized between $(x\min, x\max)$ where $x\min$ denotes the minimum point on the equally spaced grid and $x\max$ the maximum point.

Table 1: $\rho = 3$ and $\rho = 6$

Effects on consumption, bond and stock holdings from varying the parameters determining market timing (ϕ) and hedging demand due to correlation between the stock market return innovation and the innovation to the permanent component of labor income ($\rho_{z,\eta}$)

	$\phi = .798$	$\phi = .798$	$\phi = 0$	$\phi = 0$
	$\rho_{z,\varepsilon} = 0$	$\rho_{z,\varepsilon} = 0$	$\rho_{z,\varepsilon} = 0$	$\rho_{z,\varepsilon} = 0$
	$\rho_{z,\eta} = 0$	$\rho_{z,\eta} = 0.3$	$\rho_{z,\eta} = 0$	$\rho_{z,\eta} = 0.3$
Mean Normalized Bond Holdings	.01,.01,	.01,.02	.00,.00	.00,.00
Mean Normalized Stock Holdings	.03,.07	.03,.05	.03,.07	.03,.07
Mean Normalized Savings	.04,.08	.04,.07	.03,.07	.03,.07
Median Share of Wealth in Stocks	1.0,1.0	1.0,1.0	1.0,1.0	1.0,1.0
σ(Normalized Bond Holdings)	.02,.04	.02,.05	.00,.00	.00,.00
σ(Normalized Stock Holdings)	.05,.08	.05,.07	.04,.07	.04,.07
σ(Normalized Consumption)	.07,.06	.07,.06	.07,.06	.07,.06
σ(Normalized Savings)	.05,.07	.05,.07	.04,.07	.04,.07
σ(Normalized Earnings)	.10,.10	.10,.10	.10,.10	.10,.10

Notes to Table 1: Normalized variables are with respect to the permanent component of labor income (P_{it}). The first number in each cell reports results for $\rho = 3$ and the second for $\rho = 6$. The reported numbers are generated using the invariant distribution of cash on hand associated with the relevant model. Benchmark parameter values are: $\mu = .042, \phi = .798, \sigma_z^2 = .0319, \sigma_\varepsilon^2 = .9^2 * .001, r_f = .01, \delta = .12, \mu_g = .03, \sigma_u = .1, \sigma_n = .08.$

Table 2: $\rho = 3$ and $\rho = 6$

Effects on consumption, bond and stock holdings from varying the parameters determining hedging demand due to correlation between the stock market return innovation and the innovation to the permanent component of labor income ($\rho_{z,\eta}$) *and due to correlation between the stock market return innovation and the factor innovation* ($\rho_{z,\varepsilon}$)

	$\phi = .798$	$\phi = .798$	$\phi = .798$	$\phi = .798$
	$\rho_{z,\varepsilon} = 0$	$\rho_{z,\varepsilon} = -.69$	$\rho_{z,\varepsilon} = 0$	$\rho_{z,\varepsilon} = -.69$
	$\rho_{z,\eta} = 0$	$\rho_{z,\eta} = 0$	$\rho_{z,\eta} = 0.3$	$\rho_{z,\eta} = 0.3$
Mean Normalized Bond Holdings	.01,.01	.01,.01	.01,.02	.01,.02
Mean Normalized Stock Holdings	.03,.07	.03,.07	.03,.05	.03,.06
Mean Normalized Savings	.04,.08	.04,.08	.04,.07	.04,.08
Median Share of Wealth in Stocks	1.0,1.0	1.0,1.0	1.0,1.0	1.0,1.0
σ(Normalized Bond Holdings)	.02,.04	.02,.04	.02,.05	.02,.04
σ(Normalized Stock Holdings)	.05,.08	.05,.08	.05,.07	.05,.07
σ(Normalized Consumption)	.07,.06	.07,.06	.07,.06	.07,.06
σ(Normalized Savings)	.05,.07	.05,.08	.05,.07	.05,.07
σ(Normalized Earnings)	.10,.10	.10,.10	.10,.10	.10,.10

Notes to Table 2: See Table 1.

Table 3

Certainty Equivalent Costs for Stock Market Non-Participation (Normalized by permanent component of labor income) $\rho = 3$ and $\rho = 6$

Panel A:	$\phi = .798$	$\phi = .798$	$\phi = .798$	$\phi = .798$
	$\rho_{\varepsilon,z} = 0$	$\rho_{\varepsilon,z} = -.69$	$\rho_{\varepsilon,z} = -.69$	$\rho_{\varepsilon,z} = 0$
	$\rho_{\eta,z} = 0$	$\rho_{\eta,z} = 0$	$\rho_{\eta,z} = .3$	$\rho_{\eta,z} = .3$
highest	.11,.19	.12,.15	.09,.26	.11,.15
Panel B: x^*				
Highest state	.97,.94	.97,.94	.97,.94	.97,.94
Lowest State	1.01,.97	1.01,.97	1.01,.97	1.01,.97
Panel C: Percentage of time total saving is zero				
	39,19	39,19	40,20	44,21

Notes to Table 3: The first number in each cell reports results for $\rho = 3$ and the second for $\rho = 6$. Panel A reports the highest normalized certainty equivalent that induces stock market non-participation. Panel B reports the minimum normalized level of cash on hand where no saving takes place. Panel C reports the percentage of time that an individual is located below x^* and therefore goes into the next period without any savings. By comparison, in the bonds only model, $x^* = 1.01$ (.97) and probability of being liquidity constrained is 41 (25) percent when $\rho = 3$ ($\rho = 6$).

Table 4

Certainty Equivalent Costs for Being Indifferent Between Acquiring Information about the Factor or Not

(Normalized by permanent component of labor income) $\rho = 3$ and $\rho = 6$

$\phi = .798$	$\phi = .798$	$\phi = .798$	$\phi = .798$
$\rho_{\varepsilon,z} = 0$	$\rho_{\varepsilon,z} = -.69$	$\rho_{\varepsilon,z} = -.69$	$\rho_{\varepsilon,z} = 0$
$\rho_{\eta,z} = 0$	$\rho_{\eta,z} = 0$	$\rho_{\eta,z} = .3$	$\rho_{\eta,z} = .3$
.04,.09	.05,.10	.05,.09	.04,.11

Notes to Table 4: The first number in each cell reports results for $\rho = 3$ and the second for $\rho = 6$.

Market Timing vs No Market Timing, rho=3 or rho=6, $\rho_{\varepsilon,z}=0$ and $\rho_{n,z}=0$

Fig.1: Normalized Consumption (rho=3)

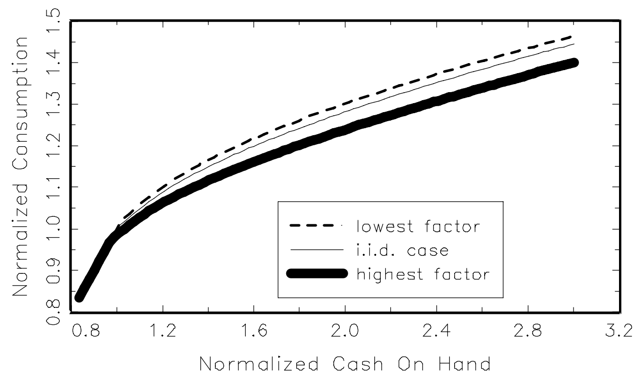


Fig.2: Share of Wealth in Stocks (rho=3)

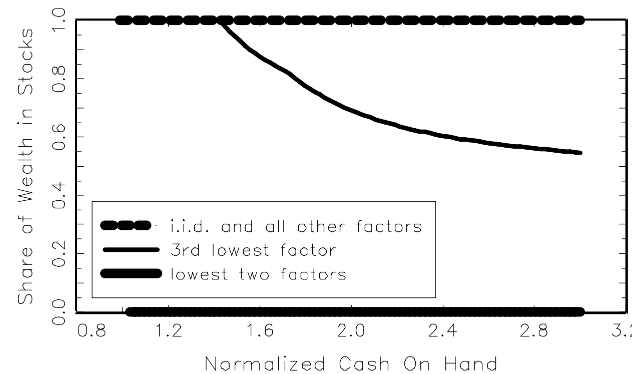


Fig.3: Normalized Consumption (rho=6)

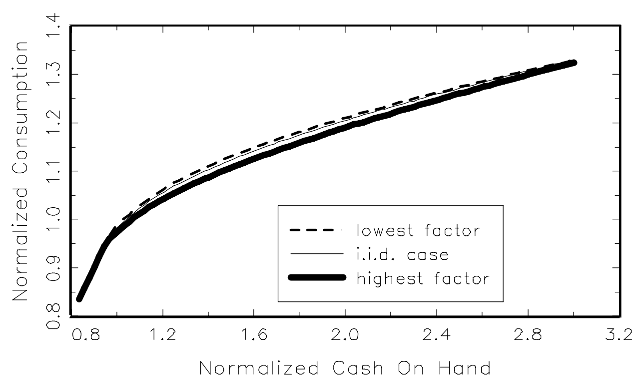
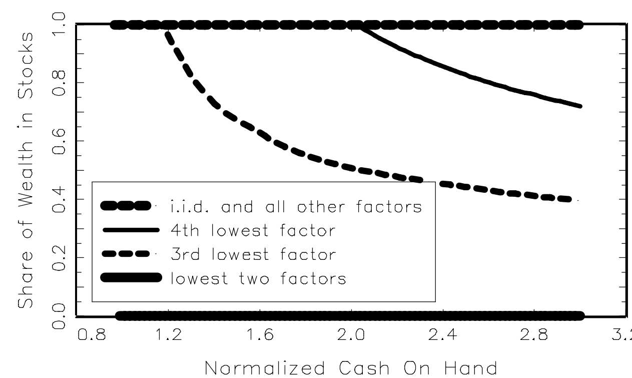


Fig.4: Share of Wealth in Stocks (rho=6)



Market Timing vs No Market Timing, rho=3 or rho=6, $\rho_{\varepsilon,z}=0$ and $\rho_{n,z}=0$

Fig.1: Normalized Consumption (rho=3)

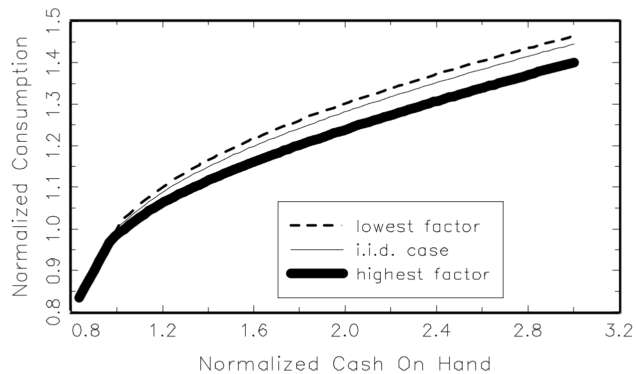


Fig.2: Share of Wealth in Stocks (rho=3)

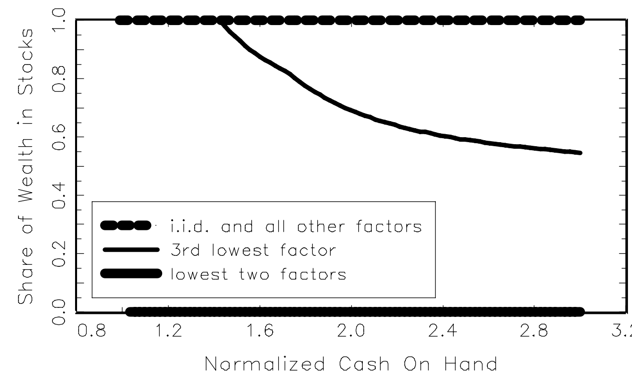


Fig.3: Normalized Consumption (rho=6)

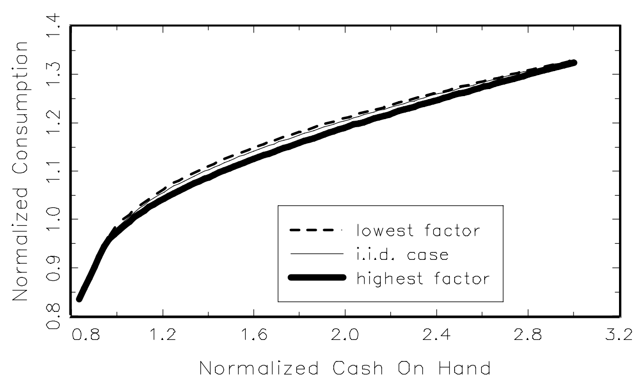
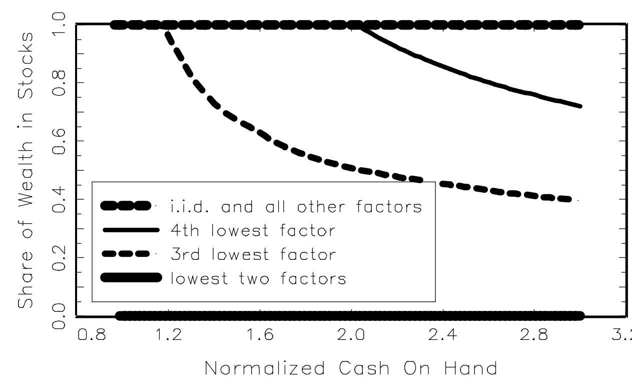


Fig.4: Share of Wealth in Stocks (rho=6)



Market Timing, rho=3 or rho=6; $\rho_{\varepsilon,z}$ is either zero or $-.69$ and $\rho_{n,z}=0$

Fig.9: Normalized Consumption (rho=3)

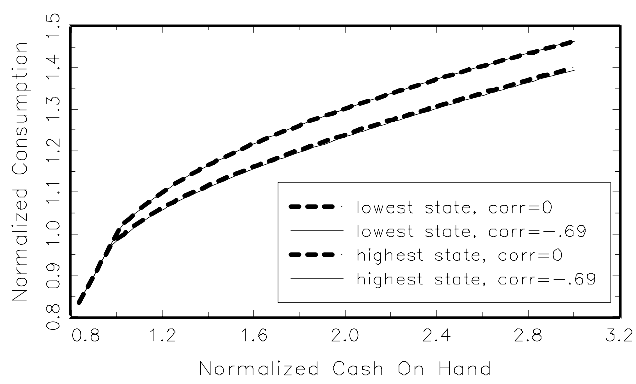


Fig.10: Share of Wealth in Stocks (rho=3)

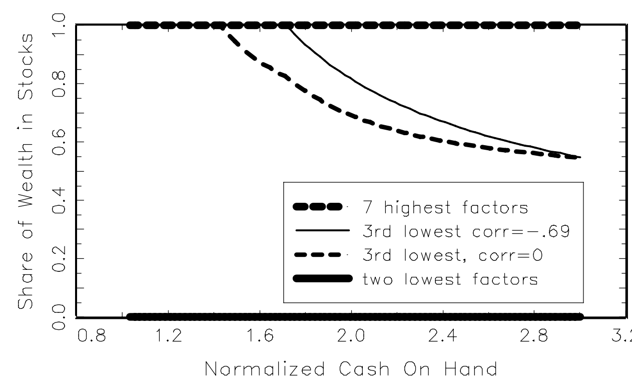


Fig.11: Normalized Consumption (rho=6)

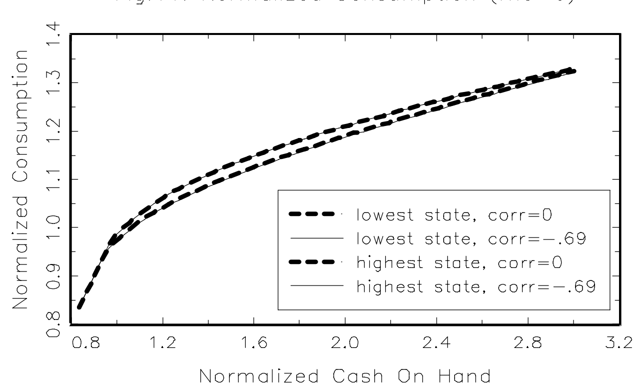
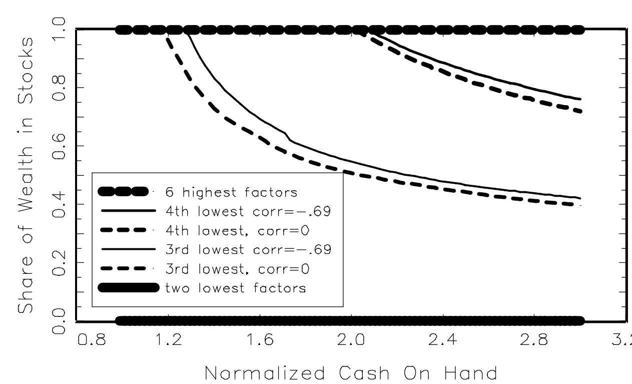


Fig.12 : Share of Wealth in Stocks (rho=6)



Market Timing; $\rho=3$ or 6, $\rho_{\varepsilon,z}=-.69$ and $\rho_{n,z}$ is either 0 or 0.3

Fig.13: Normalized Consumption (rho=3)

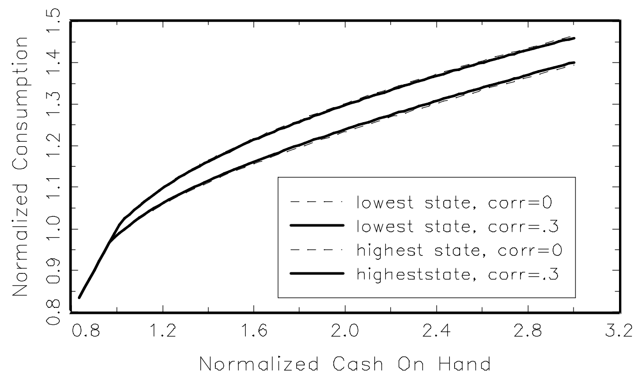


Fig.14: Share of Wealth in Stocks (rho=3)

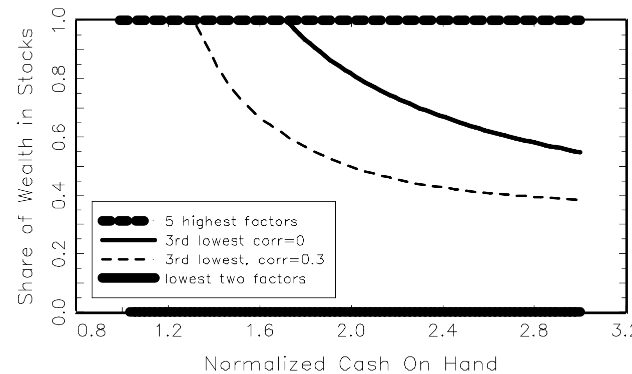


Fig.15: Normalized Consumption (rho=6)

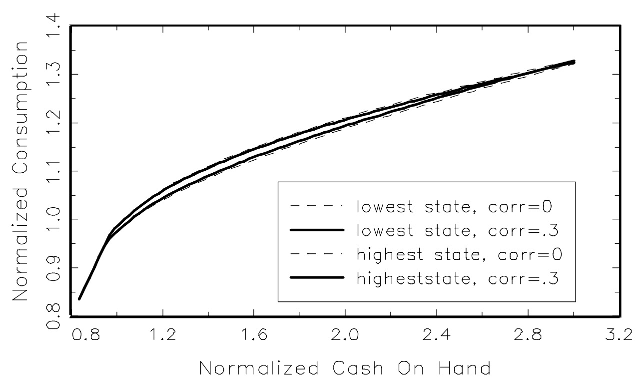
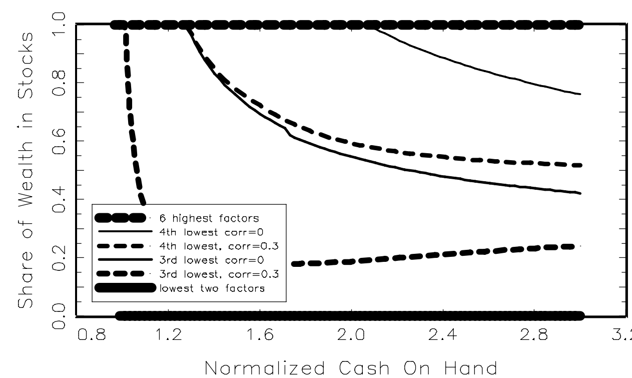


Fig.16: Share of Wealth in Stocks (rho=6)



Certainty equivalents for Stock Market Non−Participation; $\rho=3$, equity premium=4.2%

Fig.17: Certainty Equivalent to stay out of Stock Market, REV

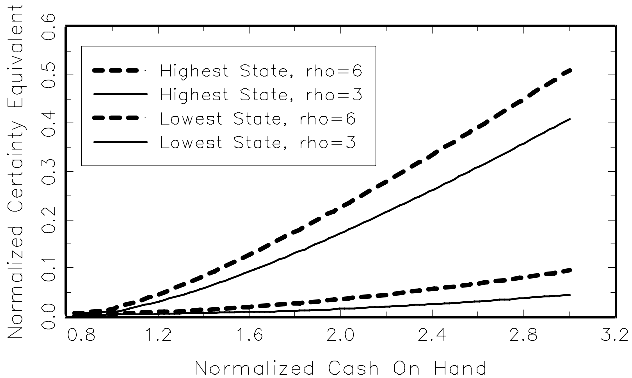


Fig.18: Certainty Equivalent to stay out of Stock Market, REV1

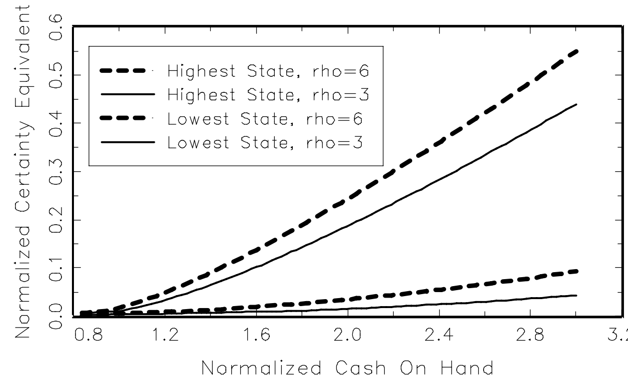


Fig.19: Certainty Equivalent to stay out of Stock Market, REV2

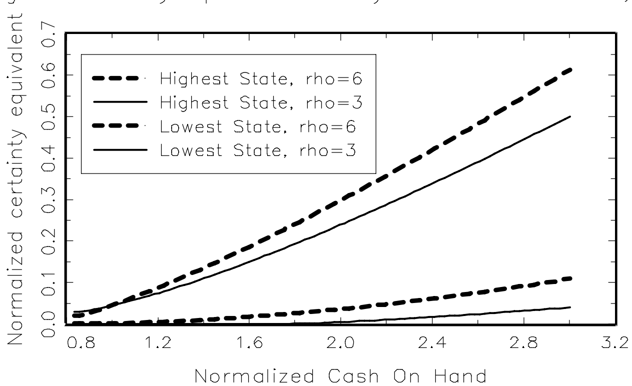


Fig.20: Certainty Equivalent to stay out of Stock Market, REV3