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A NORTH-SOUTH GROWTH MODEL ALONG KALDORIAN LINES

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ABSTRACT

A North-South growth model is presented which focusses on i) the complementarity of Southern output (consumption goods) and Northern output (capital goods) and ii) the terms of trade as a mechanism linking the growth rates of the two regions. This Kaldorian model is different from recent North-South models of Findlay and Taylor, because the Northern economy is neo-Keynesian and the Southern economy is Ricardian. In the long run the growth of both North and South is constrained in this model by the (effective) availability of agricultural land. Also the terms of trade linkage between the two regions is a loose one. As a consequence i) in the short run the terms of trade may overshoot following an exogenous disturbance, and ii) adjustment to full equilibrium may be cyclical, with phases of under- and over-investment in both North and South.

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SUMMARY

In the mid 1970's high prices of primary commodities were associated with a low rate of growth of advanced regions of the world (the "North"). In the early 1980's low prices of primary commodities have been associated with a low rate of growth of the less developed regions (the "South"). Does the first event lead to the second, or vice versa? Or is there an equilibrium growth process in which the North and South grow in step? How might the world economy converge to such an equilibrium? And what might determine such an equilibrium growth rate? Is it ultimately constrained by the availability or primary commodities?

This paper uses a formal model to suggest answers to these questions. Like other recent North-South models it focuses on the complementarity of Northern output (capital goods) and Southern output (consumption goods) and identifies the North-South terms of trade as a mechanism linking the growth rates of the two regions. What is new is that the paper formalizes ideas on these issues to be found in the writings of Lord Kaldor.

The model suggests that in the long run world growth is constrained by the availability of natural resources and land, and by technical progress in their use. In this it differs from the recent North-South analyses put forward by Ronald Findlay and Lance Taylor. They locate the ultimate determinants of world growth in labour force growth in the North and investment dynamism in the North respectively.

The paper also shows that the terms-of-trade linkage between North and South is a loose one. As a result we can show that, in response to events like energy crisis or agricultural crop failure, the North-South terms of trade may exhibit exaggerated overshooting behaviour. Furthermore a cyclical process of adjustment may occur in which there is under-investment, followed by over-investment in both the South and the North. The paper

thus formalizes Kaldor's conjectures that the terms-of-trade linkage operates slowly and wastefully and tends to set up perverse and unnecessary cycles in world economic activity.

1. INTRODUCTION

In the mid 1970's high prices of primary commodities were associated with a low rate of growth of output in the advanced regions of the world economy (the "North"). In the early 1980's low prices of primary commodities have been associated with a low rate of growth of less developed regions (the "South"). Does the first event lead to the second, and vice versa? Or is there an equilibrium growth process in which the North and South grow in step? How might the world economy converge to such an equilibrium? And what might determine such an equilibrium growth rate? Is it ultimately constrained by the availability of primary commodities?

Two important models have so far been produced (Findlay 1980, Taylor 1983) to answer some of these questions. One distinctive feature of these North-South models is that they focus on the complementarity of the outputs of the North and the South. The North produces manufactured capital goods and the South produces primary consumption goods. These goods are complements in production(1). Another distinctive feature of these models is that they focus on the terms of trade - the amount of capital goods which the South obtains for one unit of exports of consumption goods - as the mechanism linking the growth of the two regions together. Findlay's model is somewhat neoclassical: it combines a Solow growth model in the North with a Lewis-type model of the South. Taylor's is less so: he provides a more Kaleckian treatment of the North, but also in combination with a Lewis-type model of the South.

The purpose of this paper is to develop a further North-South model, due to Kaldor. This model combines a neo-Keynesian model of the North with a Ricardian treatment of the South, and its insights are strikingly different from those of the other models mentioned. For many years until 1975 Kaldor lectured on such a model, and there are hints of it in several of his writings, most notably in his essay in honour of Tibor Scitovsky (Kaldor, 1979) and in his Presidential Address to the Royal Economic Society (Kaldor, 1976)[(2)]. But Kaldor's model has never been properly formalized. The present paper takes up that challenge.

The answers given by this Kaldorian model to the questions posed in the first paragraph will be shown to depend upon the time horizon. In the short run in which the capital stocks in the North and South are fixed, the terms of trade must be such as to equate the level of supply and the level of demand for primary commodities. The growth rates of the two sectors may be out of step in such a short run, since that of the South is fast when the terms of trade stand in the South's favour but then the growth rate of the North will be slow. In the intermediate run growth rates in the two sectors are linked to each other; they are jointly determined with the terms of trade. Growth rates and the terms of trade then depend on a complex of factors including productivity and thrift in both of the sectors. In the long run, industry adapts itself to the availability and productivity of agricultural land and the growth rate of the North is constrained by the growth of primary commodity supply. The long-run properties of the model are Ricardian.

In the next section the fundamental assumptions of the model are explained, and the model's equations are set down in Section 3. Sections 4, 5 and 6 consider the growth of the North and the South in the short run, the intermediate run and the long run. In each section we describe equilibrium, and the adjustment back to equilibrium in the face of a shock to the world economy. And in each section the same shock is considered: a productivity reduction in the supply of primary commodities(3). Section 7 discusses possible extensions to the model and Section 8 concludes the paper.

2. FUNDAMENTAL ASSUMPTIONS

1. The neo-keynesian North (producing industrial goods)

Profits are invested and (given the capital to output ratio) this investment determines the Northern growth rate, without reference to further supply constraints. Three strong assumptions are made.

A.1. Industrial output is supply determined: fixed at any point in time by the productive capacity of the industrial capital stock. There is no possibility of "effective demand failure" leading to a change in the level of industrial output. In this sense the model is un-Keynesian[4]. The model shares this feature with Findlay (1980), but not with Taylor, (1983).

A.2. Surplus labour exists in the North (as well as the South - see below). The real wage is exogenous, fixed by custom. By contrast Findlay assumes for the North that there is a market-clearing real wage, determined so as to ensure full-employment Northern growth a-la-Solow. Taylor has a Kaleckian theory of real wages in the North: it is the outcome of markup pricing policies of firms. To assume with Kaldor that real wages are exogenous in the North (as well as the South - see below), makes the model somewhat Marxian[5].

A.3. There is no independent investment function for the North[6]. Investment equals the surplus of industrial output left over after sales to the South and after any consumption by Northern workers of investment goods. Without an investment function Northern animal spirits cannot influence the size of this surplus (by e.g. increasing demand for fixed Northern output and bidding up prices, profits and savings). The model is also un-Keynesian in this sense, which is different from the sense in which Assumption A.1 makes the model un-Keynesian.

These three assumptions are restrictive. Kaldor has indicated to me that he wished to "loosen them up". But I have been unable to close the Kaldorian model without them[7]. In Section 7 I discuss what further work would be needed to relax them.

2. The Ricardian South (producing agricultural goods)

The agricultural surplus is invested and the Southern growth rate depends upon the productivity of this investment, which may diminish as land becomes scarce. Four strong assumptions are made.

B.1. Agricultural output is supply determined.

B.2. Surplus labour exists available to agriculture at an exogenous real wage which is fixed by custom or subsistence[8]

B.3. There is no independent investment function. All of the surplus is sold to the North in exchange for investment goods.

These three assumptions seem much less problematic for the South than the North.

B.4. In the "short run" and the "intermediate run" the productivity of investment in agriculture is constant (the idea being that diminishing returns set in only "slowly"). But in the longer run this productivity continues to fall if the growth in the agricultural capital stock is faster than that of the supply of effective land.

Thus in the long run the availability (and productivity) of agricultural land will exert a rigid constraint on agricultural output growth. This "dismal" aspect of the vision of Ricardo and Malthus (which influences all of Kaldor's later work) is very different from the cheerful optimism of his early one sector growth models (e.g. Kaldor, 1956a). The standpoint is also quite different from that of Findlay where (quite extraordinarily) diminishing returns to land exerts no constraint.

Findlay has, as it were, an endless prairie supplying agricultural goods. Ultimately his model produces a growth rate for both regions which is determined by the growth in the (effective) Northern labour force and which is unconstrained by the availability of Southern land. Kaldor's land constraint seems a very important one, although perhaps the present model makes it bind in too rigid a manner.

3. Relations between the sectors

C.1. There is no foreign lending or "foreign investment": trade is balanced between North and South. Each sector invests only its own surplus.

The terms of trade do not play this trade-balancing role. As we shall see, the burden of adjustment is thrown upon investment in the South.

3. EQUATIONS OF THE MODEL

We assume a closed economy with two activities, industry and agriculture. The 'economy' represents the world as a system. Industry produces "steel", which is a capital good and which is also used as a consumption good in the industrial sector. Steel is produced by means of inputs of labour and capital goods, the latter being steel retailed from previous production periods. Agriculture produces the main consumption good (food or "corn"). It does this by means of inputs of labour, land and capital goods (steel). Industry sells steel to agriculture in exchange for corn.

1. Definitions

a) Variables in steel units (8)

Q_i: output of industry

K_i, K_a: capital stock in industry, agriculture

D_i: demand for steel by industrial workers

I_i, I_a: investment in industry, agriculture

W_i, Π_i: wages, profits in industry

b) Variables in corn units (6)

Q_a: output of agriculture

D_a: demand for corn by industrial workers

C_a: agricultural consumption (which is entirely corn)

W_a, Π_a, Z_a: wages, profits, rent in agriculture

c) Other variables (5)

R: effective stock of agricultural land

L: labour employed in industry

p: price of corn in terms of steel

σ: the (average) productivity of capital in agriculture

s: the agricultural savings ratio

d) Derived variables used to simplify the model

$x = (K_i/K_a)$ the ratio of the capital stocks in the two sectors

$y = (R_a/K_a)$; the ratio of land to capital in agriculture

e) Constants

μ: average productivity of capital in industry

l: labour per unit output in industry

ρ: labour per unit capital in agriculture

w: the industrial real wage in cost of living units

v: the agricultural real wage in corn units

α: the share of industrial workers' wages spent on steel

$\theta = wl$

$\emptyset = \mu\theta(1-\alpha)/\sigma s$

The superscript "." denotes the time derivative and the superscript "~" denotes the time rate of change (e.g. $\tilde{x} = \dot{x}/x$).

2. Accounting relationships

Steel is used for investment in industry and agriculture (I_i, I_a) and for consumption by industrial workers, (D_i). Corn is consumed by the agricultural sector, C_a, and by industrial workers, D_a. Thus:

$$Q_i = I_i + D_i + I_a \qquad (1)$$

$$Q_a = D_a + C_a \qquad (2)$$

Steel output is distributed as wages and profits (W_i, Π_i, both valued in steel units) and corn output is distributed as wages, profits, and rent (W_a, Π_a, Z_a; all valued in corn units)

$$Q_i = W_i + \Pi_i \qquad (3)$$

$$Q_a = W_a + \Pi_a + Z_a \qquad (4)$$

3. Production

In the agricultural sector output is supply determined (Assumption B.1) given by

$$Q_a = \sigma K_a \qquad (5)$$

The value of σ is found as follows. Let the production function be

$$Q_a = R_a^{\beta} K_a^{1-\beta}$$

This function is Cobb-Douglas for convenience and effective land is defined in units such that the constant is unity. Agricultural labour is absent from this equation on account of a simplifying assumption of fixed coefficients between agricultural labour and agricultural capital stock; by Assumption B.2 the necessary labour is always available.(8) This equation implies that

$$\sigma = Q_a/K_a = (R_a/K_a)^{\beta} \qquad (6)$$

In the short and intermediate run it is supposed (Assumption B.4) that diminishing returns to land set in so slowly that σ is constant; this was the assumption made by Kaldor in his lectures. In such circumstances we replace (6) by

$$\sigma \text{ is exogenous} \qquad (6^*)$$

In the industrial sector output is supply determined (Assumption A.1), given by

$$Q_i = \mu K_i \qquad (7)$$

Also

$$L_i = l Q_i \qquad (8)$$

The parameters μ and l are fixed. (By assumption A.2. the necessary labour is always available). The model shares the Leontief fixed coefficients assumption with single sector neo-Keynesian growth models, for reasons of simplicity. That assumption was adopted by Kaldor in his lectures. The consequences of relaxing it are not crucial for our main argument and are discussed in Section 7.

4. Income, Expenditure and Market Clearing in Agriculture.

In agriculture income accrues to rentiers, agricultural capitalists, and agricultural workers. Assume that the land rental equals the marginal product of land. Then from the production function beneath equation (5)

$$Z_a = \beta Q_a \tag{9}$$

Agricultural wages are equal to output multiplied by i) the real wage, v (constant by Assumption B.2), ii) the agricultural labour to capital ratio, ρ (also assumed constant (9)), and iii) the reciprocal of the productivity of capital, $1/\sigma$. Thus

$$W_a = v\rho Q_a/\sigma \tag{10}$$

Equation 4 implies that agricultural profits Π_a are the residual surplus.

There is no independent investment function for the South (Assumption B.3). All of the agricultural surplus, $Q_a - C_a$ is simply offered to the North in exchange for investment goods.

This surplus is found as follows.

All agricultural wages and rent are consumed and no profits are consumed. All agricultural consumption is on corn(10). Thus

$$C_a = W_a + Z_a = [\beta - v\rho/\sigma]\ Q_a \tag{11}$$

Hence the ratio, s, of the agricultural surplus to agricultural output is derived from equations (9), (10), (11) and (4) as

$$s = (Q_a - C_a)/Q_a = [1 - \beta - v\rho/\sigma] \tag{12}$$

In the short run and medium run σ is constant (see above) and so equation (12) is replaced by

s is exogenous. (12*)

Equations (11) and (4) imply that $\Pi_a = Q_a - C_a$ and that the surplus available to purchase investment goods exactly equals profits. But with a more general consumption function than (11) this will not be true.

The corn market clears as follows. Consider equation (2) which states that demand for corn, $C_a + D_a$, equals supply Q_a. At any point in time the surplus, $Q_a - C_a$, is fixed.[11] Demand for corn by industrial workers D_a must adapt to equal this surplus $Q_a - C_a$. As we shall see in equation (20), changes in the terms of trade are the only mechanism by which this can be brought about.

5. Trade balance between North and South.

The amount of investment goods which the South receives equals the amount of the corn surplus offered to the North, sQ_a, times the value of this in terms of steel, p. Thus

$$I_a = psQ_a \tag{13}$$

There is no "lending" to the South which would permit deliveries of steel in excess of psQ_a and conversely the South does not "lend" to the North. Investment in the South is thus a residual, determined in order to ensure balanced trade. If we were to introduce an independent investment function for the South then balanced trade would not in general emerge unless the Southern real wage was also endogenous.[12]

6. Income, Expenditure, and Market Clearing in Industry.

The industrial labour force (L) receives a fixed real wage per worker, w (Assumption A.2) fixed in cost of living units (see below). Industrial workers do not save so that all of this wage is spent either on corn or steel. We can allow for the substitutability in consumption between corn and steel so that as the relative price of steel falls the (physical) demand for steel by industrial workers rises and the (physical) demand for corn falls.(13) The simplest way of doing this is to postulate a Cobb-Douglas demand system (see also Cardoso, 1981) in which a fixed proportion of workers' money expenditure is assumed to be spent on steel (α) and corn $(1 - \alpha)$, respectively. We obtain the following demands for corn and steel (both in physical units) by industrial workers

$$D_a = (1-\alpha)wLp^{-\alpha} \tag{14}$$

$$D_i = \alpha wLp^{1-\alpha} \tag{15}$$

These equations can be derived as follows. If there are L workers each of whom receives a money wage, m, and if p_i and p_a are the money prices of steel and corn, then the money demand for corn is $(1-\alpha)mL$ and the real demand for corn is $D_a = (1-\alpha)mL/p_a$. Also the money demand for steel is αmL and the real demand for steel is $D_i = \alpha mL/p_i$. But for this Cobb-Douglas demand system there is 'true' cost of living index, $p_i^{\alpha} p_a^{1-\alpha}$ and so for there to be a fixed real wage of w, we must replace m by $m = wp_i^{\alpha} p_a^{1-\alpha}$. Since $p = p_a/p_i$ equations (14) and (15) follow.

These equations imply that the total of industrial wages, valued in steel units, is given by

$$W_i = wLp^{1-\alpha} \tag{16}$$

Equation (3) shows that steel profits, Π_i, are the surplus, Q_i-W_i.

There is no independent investment function for the North (Assumption A.3). Investment is simply obtained from equation (1) as the steel residual, $Q_i-I_a-D_i$, left over after consumption by industrial workers and after sales to the South. We may derive the result that $\Pi_i = I_i$ (14); i.e. that the residual available for investment exactly equals profits. But with an independent investment function, or a more general consumption function, or if there is unbalanced trade with the South, this will not be true.

The steel market clears by means of this residual determination industrial investment. If we introduced an independent investment function then the clearance of the steel market would require either deviations of output from capacity or changes in the real wage (15). This would either violate assumption A.1 or assumption A.2.

7. Dynamic equations

Capital stocks are supposed not to depreciate, so that

$$\dot{K}_a = I_a \tag{17}$$

$$\dot{K}_i = I_i \tag{18}$$

The rate of growth of effective land is exogenous, so that

$$\tilde{R} = n \tag{19}$$

8. The full system

In Section 3.1 we defined 19 variables and 19 equations have been presented. The model may thus be solved. It is straightforward to show that the values of all of these 19 variables at any point in time are fully determined once K_a and K_i are given. We thus confine our attention to solving for these variables, and for the terms of trade since this is of intrinsic interest. Solutions for outputs Q_i and Q_a can be obtained in a trivial manner from what follows, using equations (5) and (7).

4. THE SHORT RUN.

1. Determinants of the terms of trade and growth rates.

In the short run the capital stocks K_i and K_a are predetermined and the terms of trade are endogenous. Growth rates of the two sectors are determined by the resulting terms of trade. Also, σ and s are fixed.

The terms of trade are determined as follows. At any time, since K_a and K_i are predetermined at their values inherited from the past, Q_i and Q_a are fixed. If Q_a is low, there will be a small agricultural surplus, $Q_a - C_a$, and if Q_i is high the demand for corn, D_a, will be high. In these circumstances the relative price of corn must be high to discourage consumption of it. From equation (2) after substituting for C_a from equation (11) and for D_a from equation (14) we obtain

$$(1-\alpha)\ wlQ_i p^{-\alpha} = sQ_a \tag{20}$$

After using equations (5) and (7), and making use of the definitions for x, $\emptyset$, and Θ :

$$x = K_i/K_a$$

$$\emptyset = \mu\Theta(1-\alpha)/\sigma s$$

$$\Theta = wl$$

we obtain from equation (20)

$$p = (\emptyset x)^{1/\alpha} \tag{21}$$

Equation (21) is drawn as the CC curve in figure 1a. Any point on this curve shows for a given x the value of p for which the corn market is in equilibrium. In what follows it will often be convenient to use a linear approximation to equation (21). We note that

$$dp/dx = (1/\alpha)(\phi x)^{1/\alpha \;\; -1}$$

so that in the region of p=1

$$p = 1 + (\phi x - 1)/\alpha \qquad (22)$$

The growth of the agricultural capital stock is obtained from equations (17) and (18) as

$$\tilde{K}_a = psQ_a/K_a$$

Using equation (5) this gives

$$\tilde{K}_a = \sigma sp \qquad (23)$$

This relationship is shown as the g_a curve in figure 1b. The higher is the price of corn the more investment goods can be purchased by the agricultural sector and the faster the growth of agricultural capital (and therefore of agricultural output). An increase in agricultural productivity will shift out the line as will a rise in the agricultural savings ratio.

The growth of the industrial capital stock is of obtained from equations (18) and (1) as

$$\tilde{K}_i = (Q_i - D_i - I_a)/K_i$$

Use equations (15) and (7) to substitute for D_i. Also use equations (13) and (5) to substitute for I_a, and then use equation (21). The result is

$$\tilde{K}_i = \mu\ [1 - (1-\alpha)\ \theta p^{1-\alpha} - \alpha\theta\ p^{1-\alpha}]$$

$$= \mu\ [1 - \theta p^{1-\alpha}\ ] \qquad (24)$$

This equation is shown as the g_i curve in figure 1b. The higher is the price of corn the more steel is consumed by industrial workers and the more is purchased by agriculture for investment there. Thus the slower must be the growth of industrial capital (and also of industrial output). When corn is free the growth rate of industrial output is at a maximum equal to the productivity of investment in industry. The point where the g_i curve cuts the vertical axis gives the price of corn in terms of steel at which no steel can be retained by industry for investment; from equation (24) this price is $\theta^{1/(\alpha-1)}$. An increase in the productivity of capital in industry will shift the maximum growth rate outwards and an improvement in labour productivity unmatched by an increase in the real wage will shift the vertical intercept upwards. In what follows it will often be convenient to use a linear approximation to equation (24). We note that

$$\frac{\partial \tilde{K}_i}{\partial p} = \mu\ [1 - (1-\alpha)\ \theta p^{-\alpha}\ ]$$

so that in the region of $p = 1$,

$$\tilde{K}_i = \mu[(1-\alpha\theta) - \theta(1-\alpha)p] \qquad (25)$$

2. <u>Comparative Dynamics: a permanent reduction in σ.</u>

In response to a reduction in the agricultural output to capital ratio, σ, the CC curve shifts up and the price of corn rises. Using the approximation in equation (22) and noting that $\emptyset = \mu\theta(1-\alpha)/\sigma s$, we have the following expression for this change in price

$$dp/d\sigma = -\emptyset x/\sigma\alpha < 0 \qquad (26)$$

The industrial growth rate slows since following the rise in the price of corn the economy moves to the left along the g_i curve:

$$\frac{d\tilde{K}_i}{d\sigma} = \frac{\partial K_i}{\partial p} \frac{dp}{d\sigma}$$

Using the approximations in equations (25) and (26) we have

$$\frac{d\tilde{K}_i}{d\sigma} = \mu\theta(1-\alpha)\emptyset x/\sigma\alpha$$

$$= sx\emptyset^2/\alpha > 0 \qquad (27)$$

The agricultural growth rate could in principle rise or fall. This is because although the agricultural sector's purchasing power over investment goods is higher (and so a higher point along the g_a curve is obtained) the agricultural sector's productivity falls (the g_a curve rotates anti-clockwise). However (given the precise specifications of the model) the first effect necessarily predominates and K_a rises when falls. This can be shown as follows.

$$\frac{d\bar{K}_a}{d\sigma} = \frac{\partial\bar{K}_a}{\partial\sigma} + \frac{\partial\tilde{K}_a}{\partial p}\frac{dp}{d\sigma}$$

Using equation (23) and the approximation in equation (26) we have

$$\frac{d\tilde{K}_a}{d\sigma} = sp - \sigma s\,\emptyset x/\sigma\alpha$$

$$= s(p - \emptyset x/\alpha\ )$$

But using equation (22) this gives

$$\frac{d\tilde{K}_a}{d\sigma} = -s(1-\alpha)/\alpha < 0 \qquad (28)$$

Thus when σ falls, $\tilde{K}_a$ increases.

5. THE INTERMEDIATE RUN.

In the medium run capital stocks and the terms of trade are jointly endogenous. But the productivity of agricultural investment, σ is assumed to remain constant, and so is the agricultural surplus ratio, s.

1. Equilibrium growth in the intermediate run.

We define intermediate run "equilibrium growth" as a stationary growth process in which all variables in groups (a) and (b) in Section 3.1 grow at the same rate. Because the values of all these variables at any point in time are completely determined by K_a and K_i we continue to confine our attention to those variables. In equilibrium growth, the terms of trade, p, and the ratio of the capital stocks in the two sectors, $x = K_i/K_a$, are constant.

The equilibrium growth rate K* is obtained from the intersection of the g_a and g_i curves in figure 1b. Using equation (23) and the approximation in equation (25) we obtain, when $\tilde{K}_i = \tilde{K}_a = \tilde{K}^*$,

$$\tilde{K}^* = \frac{(1 - \alpha\theta)}{\frac{1}{\mu} + \frac{\theta(1-\alpha)}{\sigma s}} \tag{29}$$

This intersection also specifies an equilibrium terms of trade, p*; again from equations (23) and (25)[16]

$$p^* = \frac{\mu(1 - \alpha\theta)}{\sigma s + \mu\theta(1-\alpha)} \tag{30}$$

The equilibrium growth rate will be faster, the higher is the productivity of investment in industry and agriculture, μ and σ; the higher is the agricultural savings ratio, s and the lower are industrial wage costs per unit of output, θ. The terms of trade move in favour of agriculture the lower are θ, σ and s, and the higher is μ. The effect of α, the proportion of industrial wages spent on steel, on $\tilde{K}^*$ and p^* is ambiguous.(16)

The existence of equilibrium growth also fixes the ratio, x, of the capital stocks in the two sectors. This is obtained as x* in figure 1a. Using the approximation in equation (22) and the definition of $\emptyset$, we have

$$x^* = \alpha p^* + (1-\alpha)/\emptyset$$

$$= \frac{\sigma s}{\mu\theta} \left[\frac{\mu(1-\alpha\theta)}{\{\alpha s+\mu\theta(1-\alpha)\}\ (1-\alpha)} + 1\right] \qquad (31)$$

This complex expression is easily understood if we realise that in intermediate run equilibrium it is the role of x to equate the level of the surplus of corn from agriculture to the level of demand for it by industry. The terms of trade, p, cannot fulfil this function since that variable equilibriates the growth rate of the two sectors. In the simple case in which $\alpha \to 0$ the surplus of corn sold to industry is $s\sigma K_a$ and the demand for it by the industrial sector is $\mu\theta K_i$, and so $x = \sigma s/\mu\theta$.(17) But as α increases from zero, x must take a higher value because workers demand less corn and more steel.

2. Transition to equilibrium growth.

From figure 1 we may deduce a phase diagram for $x = K_i/K_a$. The economy is always on the CC line in Figure 1a. When $x < x^*$ and so $p > p^*$ then from figure 1b, $\tilde{K}_i > \tilde{K}_a$ and so x is rising. The opposite applies when $x > x^*$. Thus the phase diagram is shown in figure 2 and convergence to equilibrium growth is stable.

To a linear approximation the rate of convergence may be found from equations (22), (23), and (25):

$$\dot{x} = x_o\ (\tilde{K}_i - \tilde{K}_a)$$
$$= x_o\ \{ - [\sigma s + \mu\theta(1 - \alpha)]p + \mu\ (1 - \alpha\theta)\}$$

Thus

$$\frac{d\dot{x}}{dx} = -x_o\ \{\frac{\mu\theta(1 - \alpha)}{\alpha}\ [1 + \frac{\mu\theta(1 - \alpha)}{s}]\ \} \qquad (32)$$

Convergence is faster the larger are σ, s, μ and θ and the smaller is α.

3. Comparative dynamics and "overshooting": a permanent reduction in σ.

In response to a reduction in the productivity of agricultural capital the g_a curve rotates anti-clockwise in Figure 1b. Thus the equilibrium growth rate K^* falls and the equilibrium terms of trade, p^*, rises. (See equations 29 and 30.)

What happens to the terms of trade in the transition? Does the short term movement in the terms of trade (which we discussed in Section

4) overshoot the movement required for long run equilibrium? This can be immediately determined from equations (26) and (30). To a linear approximation there is overshooting in response to a fall in σ if

$$\frac{\emptyset x}{\sigma\alpha} > \frac{\partial p^*}{\partial \sigma} = \frac{\mu(1-\alpha\theta)s}{[\sigma s+\mu\theta(1-\alpha)]^2} = \frac{sp^*}{\sigma s+\mu\theta(1-\alpha)}$$

But in the region of an initial equilibrium $p^* \simeq \emptyset\, x^* \simeq 1$. Thus there is overshooting if

$$\frac{s\ \sigma\alpha}{\sigma s+\mu\theta(1-\alpha)} < 1 \qquad (33)$$

This inequality always holds, and so overshooting always occurs.

Thus when σ falls p initially rises by an exaggerated amount. We know from Section 4 that initially $\tilde{K}_i$ falls and $\tilde{K}_a$ rises. Gradually therefore $x = K_i/K_a$ falls, p falls towards its intermediate run equilibrium, and $\tilde{K}_i$ and $\tilde{K}_a$ converge towards $\tilde{K}^{*\prime}$. This process is depicted in Figure 3.

6. THE LONG RUN

In the long run the productivity of agricultural investment, σ, and the agricultural savings ratio, s, both join the list of endogenous variables. This is significant because if the growth of the capital stock employed in agriculture has any tendency to run ahead of the effective growth of agricultural land, then diminishing returns will set in and σ will fall. Furthermore, as this happens the distribution of income in agriculture will shift away from profits and s will fall. (A rising capital to output ratio implies rising wage costs per unit output since the capital to labour ratio is assumed fixed). Both of these effects will tend to bring the growth of agricultural output into line with the effective growth of agricultural land. They work in reverse if for any reason the growth of capital formation in agriculture becomes very low.

The formal analysis of this proceeds as follows. We have

$$\tilde{K}_a = \sigma sp \qquad \text{(from equation 23)}$$

$$\tilde{K}_i = \mu[(1 - \alpha\theta) - \theta(1 - \alpha)p] \qquad \text{(from equation 25)}$$

$$\tilde{R}_a = n \tag{34}$$

Making use of the definitions $x = (K_i/K_a)$, $y = (R_a/K_a)$ we have from these equations

$$\tilde{x} = \tilde{K}_i - \tilde{K}_a = \mu(1 - \alpha\theta) - [\sigma s + \theta(1 - \alpha)]p \tag{35}$$

$$\tilde{y} = n - \sigma sp \tag{36}$$

Equations (35) and (36) contain the variables x and y, and a further three endogenous variables, p, σ, and s, which must be eliminated, using equations (22), (6) and (12). Furthermore the parameter $\emptyset$ is also now endogenous since it depends on σs: $\emptyset = \mu\theta(1-\alpha)/\sigma s$. After substitution we obtain

$$\tilde{x} = \mu(1-\alpha\theta) - [y^{\beta}(1-\beta) - v\rho + \theta(1-\alpha)]\left[\frac{\mu\theta(1-\alpha)}{y^{\beta}(1-\beta)-v\rho}\right]x^{1/\alpha} \tag{37}$$

$$\tilde{y} = n - \mu\theta(1 - \alpha)\, x^{1/\alpha} \tag{38}$$

These are the two basic equations which we shall use.

Because the values of all the variables in the model at any point in time are entirely determined by x, y, and R, and because R is exogenous, we confine our attention to equations (37) and (38).

1. Equilibrium growth in the long run.

We define long run equilibrium growth as a stationary growth process in which (i) all variables in groups (a) and (b) of Section 3.1 grow at the same rate and (ii) this growth rate is the same as that of the effective stock of agricultural land, R. Long run equilibrium implies that both x and y are constant.

From equation (38), the constancy of y implies that

$$x = \left[\frac{n}{\mu\theta(1-\alpha)} \right]^{\alpha} \tag{39}$$

The interpretation of this is simple. If the growth rate of land increases, a higher p is required in order for agriculture to keep the growth rate of its capital stock in line with the increased growth rate of land. But from equation (22) a higher p requires a higher value of x. Equation (38) is plotted as the vertical line, $\tilde{y} = 0$, in figure 4.

From equation (27) the constancy of x implies the following tradeoff between y and x

$$\mu\,(1 - \alpha\theta) = \left[1 + \frac{\theta(1-\alpha)}{y^{\beta}(1-\beta) - v\rho} \right] x^{1/\alpha} \tag{40}$$

Clearly as $y \to \infty$, $x^{1/\alpha} \to \mu(1-\alpha\theta)$, and as $y \to 0$, $x^{1/\alpha} \to 0$. This equation is plotted as the upward sloping line, $\tilde{x} = 0$ in figure 4. This equation depicts the fact that if R_a is high relative to K_a, and so if y is high, then corn supply will be plentiful and p low. Industrial growth will then be low unless K_i is high enough to keep x high. For this will keep demand for corn high and so make p high.

Long run equilibrium growth occurs when $\tilde{x} = \tilde{y} = 0$, as depicted by the point X in figure 4.

2. Transition to long-run equilibrium growth.

When x is to the left of the $\dot{y} = 0$ line, equation (38) shows that y is positive, and vice versa. When y is above the $\dot{x} = 0$ line, equation (37) shows that x is increasing, and vice-versa. Movement out of long run equilibrium is thus as depicted by the arrows, and is stable.

3. Comparative dynamics and cycles: a reduction in y

A reduction in y is the equivalent in the present formulation to the reduction in σ considered in earlier sections. The positions of the curves in figure 4 are not altered by such a change. Instead an economy which was initially at point X moves downwards to point Y.

As figure 4 shows the economy converges back to the same equilibrium values for x and y. In this the outcome differs from that discussed in section 5 (which corresponds only to a gradual horizontal movement leftwards from Y towards the $\dot{x} = 0$ curve). Of course if the fall in y has been brought about by a fall in the level of R (see footnote 3), then the new equilibrium growth process will have levels of K_i and K_a lower than if the disturbance had not happened.

Figure 4 also shows that the convergence to equilibrium is certainly not monotonic in x, and that both x and y may quite possibly cycle. Such a case of cyclical convergence is depicted in the figure. The interpretation of the cycle is as follows. Following the initial drop in y the price of corn rises, as discussed in Section 5. As a result $\dot{K}_a > \dot{K}_i$ and x gradually falls. The fall in x makes the price level fall and, as a result, agricultural capital accumulation lags behind agricultural land growth. The agricultural land-to-capital ratio may well overshoot as shown. At the point where the spiral crosses the $\tilde{y} = 0$ line, there has been under-investment in agriculture, even although initially at Y agricultural capital was over-abundant compared with agricultural land. As drawn there is a further phase of over-investment in agriculture before convergence to X is finally obtained. This agricultural cycle is associated with a sympathetic oscillation in industrial capital accumulation, as the figure shows.

7. EXTENSIONS

1. Further comparative dynamic experiments.

The model could be used, for example, to examine the Prebisch effect. This would show the consequence of industrial real wages rising as fast as or faster than improvements in industrial output per man, and would contrast this with the effects of agricultural productivity increases unmatched (as Prebisch argued) by wage increases in the South. The model would present a very different picture of this effect than that given by Findlay (1980). Findlay's model found a basis for the Prebisch effect in the market-clearing determination of wages in the North, whereas the present model suggests an underpinning for Prebisch's effect in wage bargaining institutions and political structures (since θ is exogenous in the model). This is more in line with Prebisch's original insights. But what the model also points out is that the Ricardian spectre fights against the Prebisch effect so as to turn the terms of trade against industry. If both diminishing returns and the Prebisch effect are operating then there can be no "iron law of the terms of trade".

2. Tinkering with the model structure.

One example of this would be to allow for choice of technique in industry. In the model above we assumed that for the industrial sector both labour inputs per unit output, l , and the productivity of capital, μ, are independent of the relative factor rewards to labour and capital. The consequences of relaxing this assumption are interesting. Let us for simplicity assume the Cobb-Douglas production function

$$Q_i = AK_i^{\gamma} L_i^{1-\gamma} \tag{41}$$

We can show that, if industrial producers equate the marginal product of each factor to its rewards, then[18]

$$l = (1-\gamma)p^{(\alpha-1)}/w \tag{42}$$

$$\mu = A^{1/\gamma}[(1-\gamma)p^{(\alpha-1)}/w]^{(1/\gamma-1)} \tag{43}$$

These results add an additional mechanism to the way in which the industrial sector responds to an increase in the relative price of corn, p. Even although the real cost-of-living-wage is constant in industry, a rise in the relative price of corn increases the cost of this wage in terms of steel. This encourages steel producers to adopt techniques of production which have less labour per unit of output, $\partial l/\partial p > 0$, and less output per unit of capital, $\partial \mu/\partial p < 0$. Both of these features of the new methods of production mean that industrial employment, L, falls. The fall in employment will reduce the demand for corn since this demand depends on L.

Thus with flexible choice of technique the short run demand curve for corn of the industrial sector will be more elastic than that shown in figure 1a. This will assist in moderating short run fluctuations in the terms of trade, at the expense of induced fluctuations in industrial employment. It will also mean that the g_i curve in figure 1b is less strongly downward sloping, since producers by changing technique can partly defend themselves from higher corn prices. This will moderate the negative effect on the intermediate run growth rate of a reduction in agricultural productivity.

3. Radical surgery: can the model be made Keynesian ?

The strongly un-Keynesian features of the model have already been noted in detail. In particular, there are two important limitations which need to be investigated. First, at least in the industrial sector, output should be allowed to diverge from full capacity in the downward direction. Kanbur and Vines (1984) produce a short run model of this kind, but not the required analysis of growth. Second, and perhaps more important, the fixed-real-wage assumption should be abandoned. This would allow a Keynesian treatment of investment to be introduced. Increases in investment in the industrial sector caused by improved animal spirits could then "force the pace of growth" by raising prices and pre-empting resources from the predetermined supply. One could then analyse an attempt by wage earners to defend themselves against the resulting reduction of their real wage, by means of money wage increases. This would then produce a North-South growth model which had as an <u>intrinsic</u> part of its working a price-wage-price inflationary process of the Latin American structuralist kind, working at the world level. Such a model might be valuable. The tools necessary for this exercise are at hand in Cardoso (1981) and Marglin (1984).

8. CONCLUSIONS

This paper has presented a Kaldorian model of the North-South growth process. As Kaldor wrote in 1976 (op. cit, p. 704)

> "Continued and stable economic progress requires that the growth of output in [the primary and secondary] sectors should be at the required relationship with each other [sic] - that is to say, the growth of the saleable output of agriculture and mining should be in line with the growth of demand, which in turn reflects the growth of the [secondary sector].
> However from a technical standpoint there can be no guarantee that the rate of growth of primary production, propelled by land-saving innovations, proceeds at the precise rate warranted by growth of production and incomes in the [secondary sector]. To ensure that it does is the function of the price mechanism, more particularly of relative prices, or the 'terms of trade' between primary commodities and manufactured goods".

The formal model of this paper suggests that the price mechanism may be rather bad at performing this function. In particular we have shown that in the face of changes in primary sector productivity, the terms of trade may exhibit exaggerated overshooting behaviour. Furthermore, a cyclical process of adjustment may occur, in which there is under-investment followed by over-investment in both agriculture and industry. The paper thus formalizes Kaldor's conjectures that the price mechanism "operates slowly and wastefully, and tends to set up perverse and unnecessary cycles in world industrial activity" (op. cit. p. 713).

Kaldor went on to argue from this conclusion that there is a case for the stabilization of primary commodity prices. To establish a formal case for primary commodity price stabilization using the present growth model is a task for future work.

FOOTNOTES

(1) Lewis (1972, p. 92) has suggested that there are "ready answers available in the corpus of international trade theory" to deal with the questions posed above. But this focus on complementarity in the new North-South models differs from the focus in Johnson's classic article (Johnson, 1957), recommended for the purpose of Lewis. Johnson's two sectors produce substitutes in consumption rather than complements in production. For greater generality in what follows we will allow for some substitution in consumption. Industrial workers will be supposed to consume capital goods as well as consumption goods, in proportions which depend upon relative prices.

(2) Aspects of the ideas are also discussed in Kaldor (1972, 1975).

(3) This comparative static experiment most closely corresponds to a sinking into the sea of a chunk of agricultural land. It is only remotely related to an "energy crisis" (which is to do with non-renewable resources) or with a harvest failure (which is temporary).

(4) But note that this is not the full-employment-of-labour assumption of Kaldor's 1960's growth models. Rather it is full employment of machines. This distinction has been clarified in a forthcoming paper on Malthus by Costabile and Rowthorn (1984).

(5) See the typology of models offered by Marglin (1984) and Kaldor (1956a).

(6) In a one-sector model this feature would be a logical consequence of the "Marxian" theory of wages embodied in Assumption A.2. coupled with the exogenous supply-side determination of output embodied in Assumption A.1. (see Marglin, 1984). This is not so in a two-sector model since, even given these assumptions, changes in the profit share can accommodate changes in investment by means of changes in the North-South terms of trade.

(7) Assumptions 1.1 and 1.3 are easily relaxed to give a short run static Keynesian model (Kanbur and Vines, 1984). Such a model lurks behind some of the discussions in Kaldor (1976). But I have been unable to obtain the kind of model which underlies the analysis of growth in Kaldor (1976) without keeping these assumptions.

(8) In his lectures Kaldor ignored changes in agricultural employment by making the assumption of a disguised unemployment. This enabled him to treat the marginal product of labour as zero, even if the marginal product of labour time was not zero, on the argument that the total number of hours worked remained the same when a unit of labour was removed from agriculture. I do not find this argument entirely convincing, and instead adopt the opposite simplifying assumption of a fixed labour to fixed capital ratio in agriculture.

(9) See the previous footnote.

(10) This is a strong limitation of the model: the South does not import any consumption goods. Neither Findlay nor Taylor deal with this problem.

(11) Q_a and σ are predetermined. Thus using equation (11) $Q_a - C_a$ is fixed.

(12) Changes in the terms of trade cannot be the mechanism by which balanced trade is brought about because as we shall see in equation (20) changes in the terms of trade are the only way in which the corn market can clear. This feature would disappear in a different model with flexible real wages and an independent investment function in the South. Given Northern demands for corn, the Southern real wage and the terms of trade could then jointly clear the corn market and ensure balanced North South trade. Note that a truly Ricardian model would allow foreign lending and would allocate investment between the North and the South so as to equalize the profit rate throughout the world.

(13) Kaldor assumed in his lectures that all industrial wages are spent on corn. The present assumption is more general and incorporates Kaldor's assumption as the special case $\alpha \to 0$.

(14) From (1) and (3)

$$W_i + \Pi_i = I_i + D_i + I_a$$

Now

$$W_i = D_i + pD_a \qquad \text{(from equations 14-16)}$$

$$pD_a = p(Q_a - C_a) \qquad \text{(from equation 2),}$$

and

$$p(Q_a - C_a) = I_a \qquad \text{(from equations 12 and 13)}$$

Substitution for W_i gives $\Pi_i = I_i$.

(15) The steel market cannot clear by means of changes in the terms of trade because (as noted in the previous footnote) changes in the terms of trade are the only way in which the corn market can clear. This peculiar feature would disappear in a different model with,

for example, independent investment equations in each sector and flexible real wages in each sector. The two real wages and the terms of trade could then jointly clear both markets and ensure balanced North-South trade.

(16) If $\alpha = 0$ (i.e. all industrial wages are spent on corn), the equilibrium solutions of the model are:

$$\tilde{K}^* = 1/[\frac{1}{\mu} + \frac{\theta}{\sigma s}]$$

and $p^* = 1/[\sigma s/\mu + \theta]$

These are the equations underlying the diagram in Kaldor (1979). They were explicitly presented by Kaldor in his lectures.

(17) Notice that in this case instead of the CC curve in figure 1a, there would be a vertical line because there is no substitution between steel and corn. The terms of trade would be powerless to reconcile the supply of and demand for corn in the short-run. Some other mechanism is then required in the short run to ensure that relative outputs are in the necessary ratio fixed by the parameters s and θ. Kaldor did not notice this feature of his model.

(18) We have

$$Q_i = A\, K_i^{\gamma}\, L_i^{1-\gamma} \qquad \text{(i)}$$

The marginal produce of labour in steel production is

$$\frac{\partial Q_i}{\partial L} = (1-\gamma)Q_i/L = (1-\gamma)/l \qquad \text{(ii)}$$

The real wage per worker as it costs capitalists (i.e. in terms of steel) may be obtained from equation (16) as

$$wp^{(1-\alpha)} \qquad \text{(iii)}$$

Equating (ii) and (iii) we obtain

$$l = (1-\gamma)\ p^{\alpha-1}/w \qquad \text{(iv)}$$

From equation (i)

$$\mu = [A\, l^{\,1-\gamma}]^{1/\gamma} \qquad \text{(v)}$$

and using (iv) gives

$$\mu = A^{1/\gamma}\ [(1-\gamma)\ p^{\alpha-1}/w]^{(1-\gamma)/\gamma} \qquad \text{(vi)}$$

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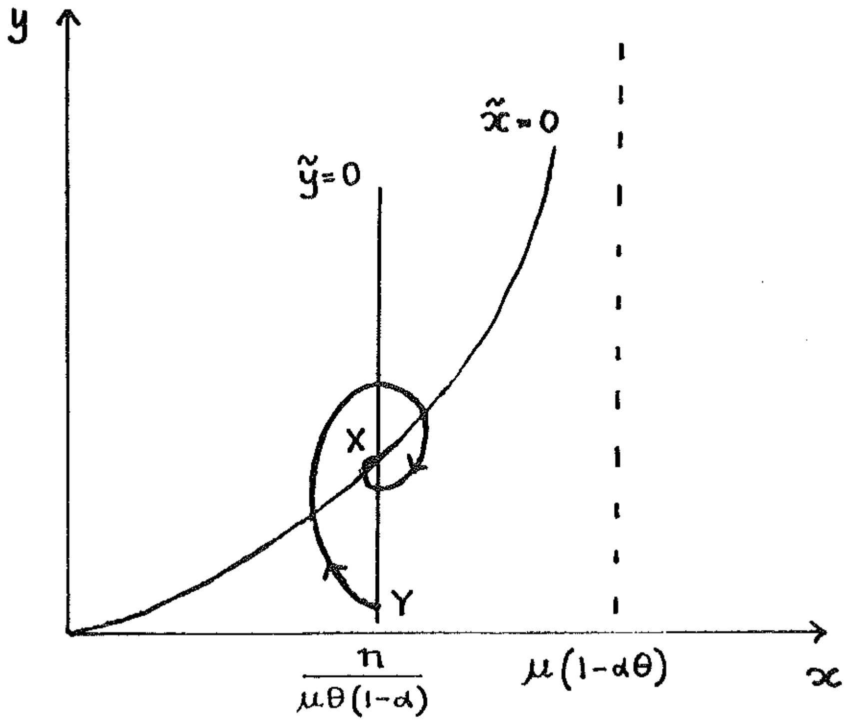
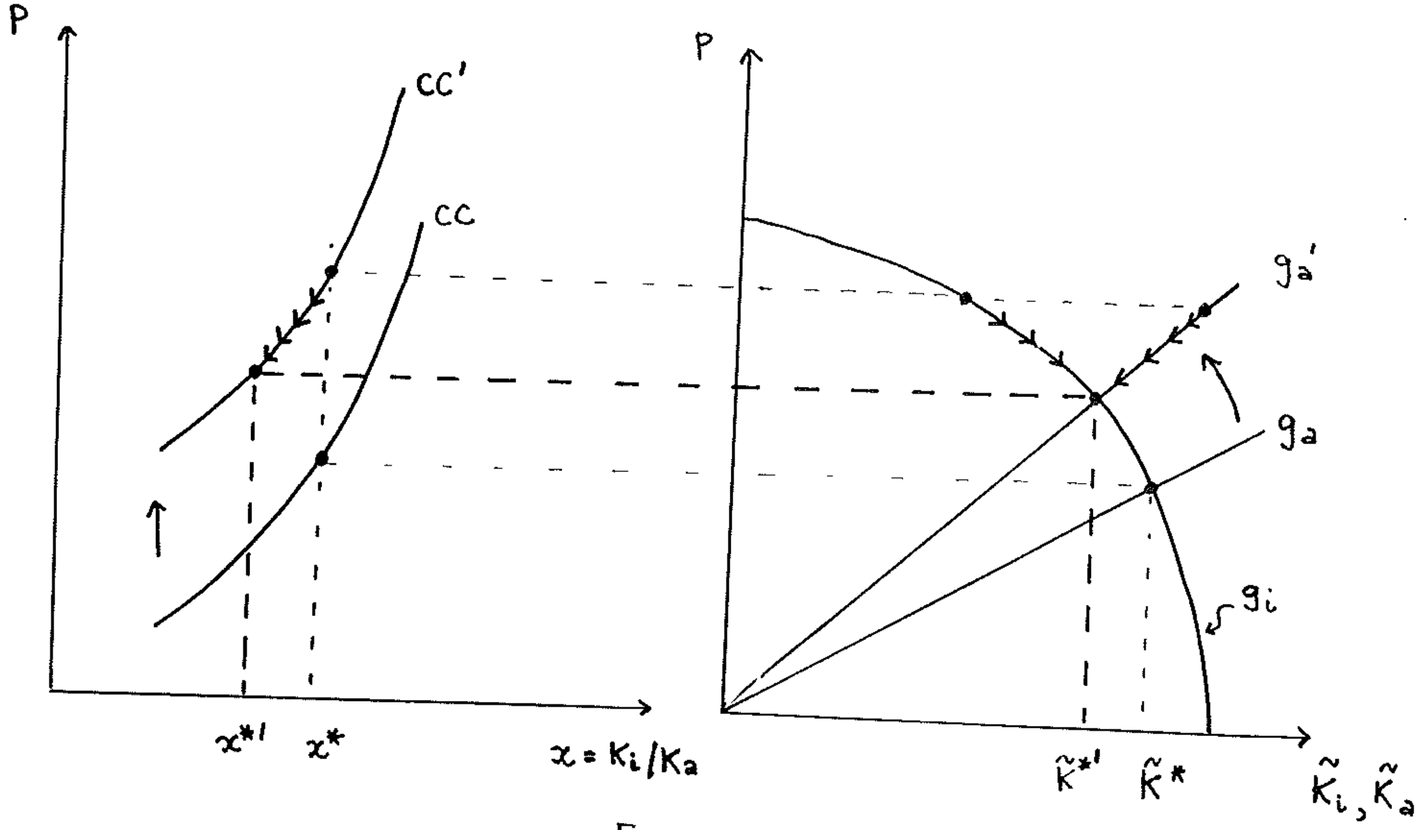


Figure 4


P
CC'
CC
$x^{*\prime}$
x^*
$x = K_i/K_a$
P
g_a'
g_a
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$\tilde{K}^*$
$\tilde{K}_i, \tilde{K}_a$

Figure 3

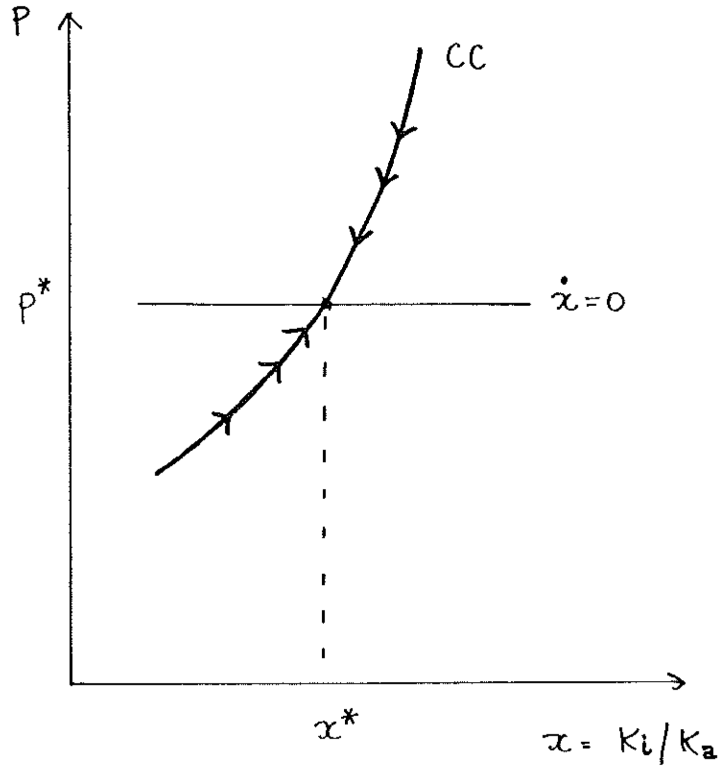

Figure 2

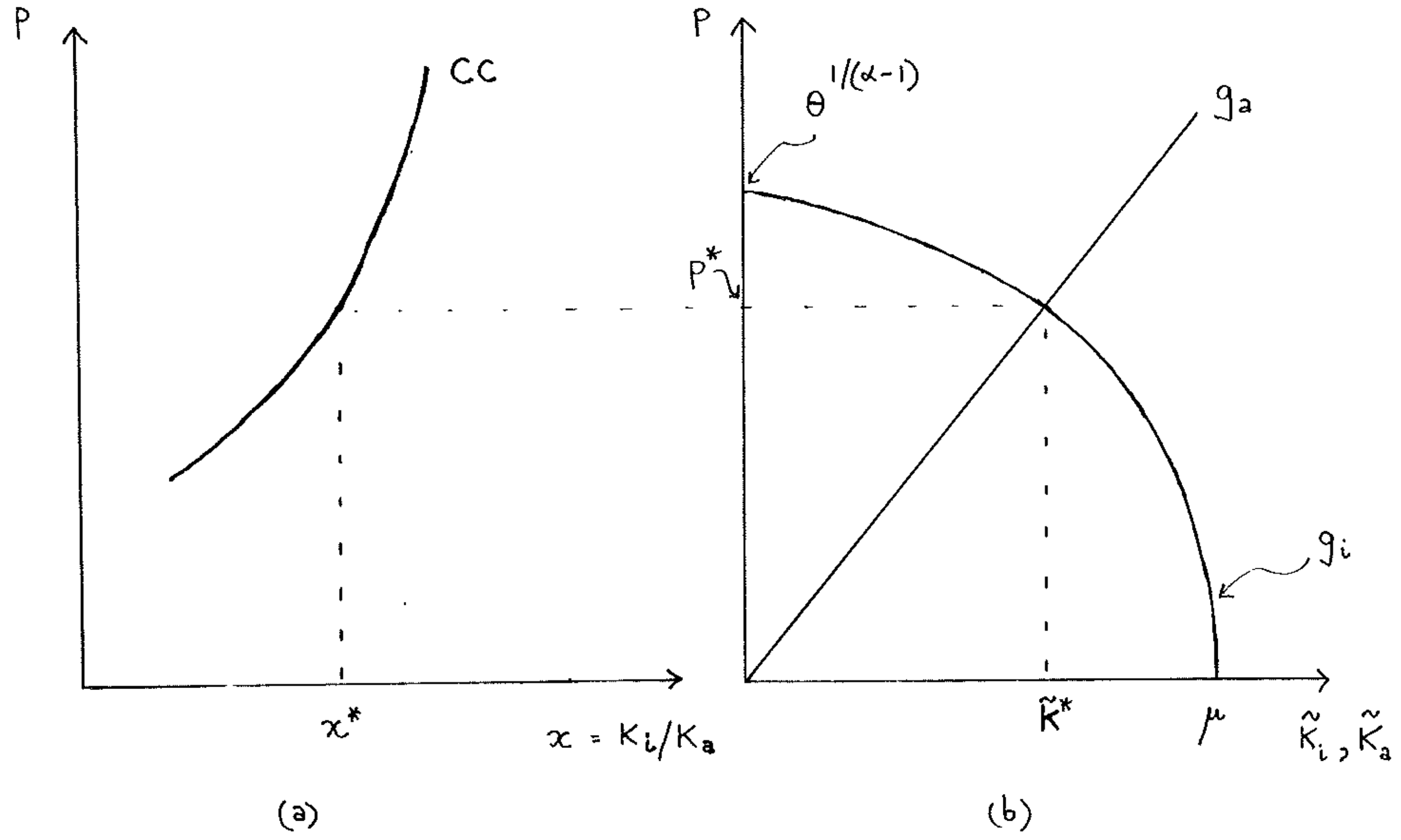

Figure 1