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OPTIMAL MONITORING IN HIERARCHICAL RELATIONSHIPS

Anke S Kessler

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Anke S Kessler, University of Bonn

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: http://www.cepr.org

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ABSTRACT

Optimal Monitoring in Hierarchical Relationships*

This Paper studies an agency framework in which a principal hires a supervisor to monitor the agent's productive effort. We consider several monitoring technologies which differ in the quantity (frequency) and the quality (accuracy) of the information they deliver. We show that the frequency of monitoring is irrelevant if the supervisor is honest or if the supervisor colludes with the agent but monitoring evidence can only be concealed and not forged. In either case, a first-best can be achieved if monitoring is sufficiently precise even though unbounded punishments are not feasible. Finally, if monitoring evidence can be falsified, the principal benefits both from the frequency and the accuracy of the supervisor's observations. This is the only case in which collusion imposes an additional cost on the relationship. The findings suggest that it is strictly better for the principal to monitor the agent's action rather than testing for an unknown ability or technology parameter.

JEL Classification: D82, L23
Keywords: internal organization of the firm, hierarchies, collusion

Anke S Kessler
Department of Economics,
University of Bonn
Adenauerallee 24-42, D-53113 Bonn
GERMANY
Tel. (49 228) 739246
Fax: (49 228) 739221
E-mail: kessler@wipol.uni-bonn.de

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NON-TECHNICAL SUMMARY

Consider an employment or procurement situation in which one individual (the agent) is hired by another individual (the principal) to perform a productive task on the latter's behalf. In many such principal-agent relationships, the parties pursue different objectives and the principal cannot determine directly whether the task is carried out in full compliance with her interests. The agent's inclination for opportunistic behaviour together with the prevailing informational asymmetry then give rise to an incentive problem. While the problem can sometimes be alleviated indirectly through performance based compensation, there is often an additional possibility of direct control through monitoring.

The Paper focuses on the possibility of monitoring the agent as a means for the principal to reduce the incentive problem and improve the profitability of the relationship. We examine a situation where the agent is privately informed both about the circumstances in which he carries out the task and about the effort he exerts to produce output. For instance, managers may be better informed than shareholders with regard to the firm's market conditions as well as the time they devote to their assignments. Similarly, a machine worker may have superior knowledge about the technological capabilities of his machine and about how much care he exerts in maintenance. In the model, the principal uses a performance-based pay system and in addition, may decide to monitor the agent's working effort. Based on the monitoring evidence, penalties may be inflicted on the agent. These cannot be arbitrarily high, however, because the agent is protected by limited liability (he may be wealth-constrained or there may be legal restrictions on the maximum punishment). The audits can either be conducted by the principal herself, or, if she lacks the time or the knowledge to do so, she must hire a third party for the job, a supervisor. In this latter situation, however, the agent has an incentive to bribe the supervisor (collude) in order avoid the penalty associated with poor work performance. Clearly, the possibility of tacit collusion between agent and supervisor could hamper the positive incentive effect of monitoring.

The analysis does not take the monitoring technology available to the principal as given. Instead, we allow the principal to have various information technologies with different characteristics at her disposal. The main attention of the analysis rests on two characteristics of the available monitoring technology. The first characteristic is the quantity of information generated, or, in other words, the frequency of making an observation (e.g. how often routine checks are conducted). The second property is the quality of information delivered, i.e. how precisely the observation reflects the true value of the agent's working effort. The framework thus allows us to study different aspects of information technologies and their virtues under various circumstances.

The findings indicate that it is only the quality of information which matters for the principal under quite general conditions. Specifically, we show that quantitative aspects of the information generated through monitoring are of no concern if either a) the principal undertakes the monitoring herself or, alternatively, if b) the agent bribes the supervisor but the latter can only conceal and not forge monitoring evidence. In either situation, monitoring increases the power of the optimal incentive scheme for the agent (his equilibrium effort). Moreover, the incentive problem can be fully eliminated whenever the signal generated through monitoring is sufficiently accurate, irrespective of the frequency with which it is obtained. Intuitively, the principal can always compensate for less frequent observations by increasing the agent's reward in the case where monitoring revealed that he has complied with the terms of the agreement. The principal strictly benefits from frequent observations only if the supervisor is corrupt and the supervisor-agent coalition can falsify monitoring evidence. Surprisingly, this is the only case in which the possibility of tacit side-arrangements imposes an additional cost on the principal.

These results are important in two respects. Ignoring cost differences, they first imply that organisations should generally opt for accurate rather than frequent evaluations of their employees.

Second, in comparison to some of the recent contributions to monitoring and collusion in hierarchies, they also indicate that the principal should *ceteris paribus* strictly prefer to monitor the agent's action (his effort) rather than to audit the reported information (the circumstances in which the task is carried out). This conclusion suggests for example that firms should focus more on evaluating a worker's performance on the job instead of relying on ability-tests.

1 Introduction

In the last two decades, agency problems with adverse selection have received much attention in the theoretical as well as the applied economic literature. While early contributions such as Baron and Myerson (1982) have focused on the inefficiencies generated by informational asymmetries between the principal and the agent, subsequent authors extended the standard framework by allowing for monitoring in order to reduce the prevailing incentive problems. Examples of these models include Nalebuff and Scharfstein (1987) who explore the role of testing in labor markets[1] and Baron and Besanko (1984) who consider auditing in a regulatory context. More recently, the possibility of monitoring has been investigated in three-layer hierarchies which include a third party, the supervisor. Tirole (1986, 1992), Laffont and Tirole (1991, 1992), and Kofman and Lawarrée (1993) point to an additional problem which arises in this context. Specifically, if the principal does not undertake the monitoring herself these authors show that the agent and the supervisor have an incentive to collude (form a coalition) which may reduce the welfare of the principal.[2] An assumption common to all these papers is that they take the information technology to be exogenously given.[3] In practice, however, the principal may have various information technologies with different characteristics at her disposal. As an example, suppose that the principal can by investing into a particular testing procedure improve the quality of a test on the agent's ability. Alternatively, the principal can conduct frequent inspection rounds to increase the probability that he observes the agent's work performance.

This paper presents a model which provides a framework for studying different aspects of information technologies and their virtues under various circumstances. The two distinct characteristics of information I will concentrate on have already been incorporated in the example above. The first characteristic is the *quantity* of information generated, or, in other words, the frequency of making an observation (e.g. how often routine checks are carried out). The second important property of an information technology is the *quality* of information it delivers, i.e. how precisely the observation

[1] See also Gale (1991) and Lockwood (1991).

[2] Demski and Sappington (1987) also consider a three-layer hierarchy but do not analyze the issue of collusion. Instead they focus on the problem of motivating the supervisor to collect relevant information. A different view on hierarchies is taken by Melumad and Mookherjee (1989) and Strausz (1997) who show that employing the supervisor can increase the principal's commitment to the contract, thereby increasing her expected return from the relationship.

[3] An exception is the paper by Nalebuff and Scharfstein (1987) which is briefly discussed below.

reflects the true value of the variable which is to be monitored. For illustrational purposes, the analysis will be confined to a simple agency framework with a three-layer hierarchy. The hierarchy consists of a principal, a supervisor, and an agent who carries out a productive task for the principal. The agent is privately informed about a technology parameter and the effort he exerts to produce output. In order to monitor the agent's working effort, the principal can employ the supervisor. I first consider the case where the supervisor cannot collude with the agent. In the present framework, this is equivalent to a situation in which the principal undertakes the monitoring herself. Next, the agent and the supervisor are allowed to collude, i.e. to write unobservable side contracts. Here, we will distinguish between the supervisor's signal being verifiable and non-verifiable, respectively. In the former case, the supervisor/agent coalition can only conceal but not forge the outcome of monitoring. In latter case, the supervisor's report may contain fake evidence.

For each of these three possibilities, the optimal contract for the principal given a particular monitoring technology is derived. We are then in a position to investigate how - ignoring relative costs - the principal's return from the relationship varies with the quantity and the quality of the information collected by the supervisor. The analysis indicates that it is only the quality of information which matters for the principal under quite general conditions. Quantitative aspects are of no concern if either the supervisor is honest or his report is hard information. The principal benefits from frequent observations only if the supervisor is collusive and his signal is non-verifiable. Moreover, this is the only case in which the possibility of collusion imposes an additional cost on the principal. These results are important in two respects. Ignoring cost differences, they first imply that organizations should generally opt for accurate rather than frequent evaluations of their employees. Second, in comparison to some of the recent contributions to monitoring in hierarchies the findings also indicate that the principal should *ceteris paribus* strictly prefer to monitor the agent's action (e.g. cost reducing effort) rather than an underlying technology parameter which is private information to the agent. This conclusion suggests for example that firms should focus more on evaluating a worker's performance on the job instead of relying on ability-tests.

Up to now, there have been few attempts to draw conclusions on endogenous information acquisition in agency problems. To my knowledge, there are only two papers who explicitly consider different information structures and their value for the principal.[4] The first of these is by Nalebuff and Scharfstein (1987) who investigate a competitive labor market framework in which firms can test for a worker's ability. Testing is costly and both the accuracy of the test and the percentage of workers tested are choice variables in the firms' decision problem. The authors demonstrate that workers are tested with strictly positive probability in equilibrium. Only if the testing technology is very accurate and unbounded punishments can be used, the full information equilibrium can be approximated. The second paper is by Khalil and Lawarrée (1995) who show that the choice between input and output monitoring is determined by the identity of the residual claimant. In their model, the principal prefers to monitor the inputs if she herself is the residual claimant whereas output monitoring is superior if the agent is the residual claimant.

The remainder of the paper is organized as follows. Section 2.1 introduces the basic framework. Section 2.2 examines collusion-free monitoring. In Section 2.3, the agent and the supervisor are allowed to collude. Optimal contracts are derived under the assumption that the supervisor's report to the principal is hard and soft information, respectively. A final section concludes.

2 The Model

2.1 The Basic Framework

There are three risk-neutral parties: a principal (P), a supervisor (S), and an agent or manager (A). The agent is a productive unit and produces output x for the principal. A is indispensable in that he is the only party in the hierarchy who possesses the time or the knowledge to perform the required task. Output is determined by a random productivity parameter θ and the agents effort e:

$$x = \theta + e.$$

The agent's productivity can either be high or low: $\theta \in \{\theta_h, \theta_l\}$ where $\theta_h > \theta_l$. The ex ante probability that productivity is high is common knowledge and denoted by $q > 0$. When exerting effort, the agent incurs a disutility. The monetary equivalent

[4] For models in which the agents' information structure is endogenously derived, see Crémer et al. (1992, 1998) and Kessler (1998).

of his disutility of effort is represented by an increasing and strictly convex function $\psi(e)$ which satisfies $\psi(0) = \psi'(0) = 0$ and $\psi'''(\cdot) \geq 0$.[5] In compensation for his work, the agent receives a transfer t from the principal. His utility is $u_A = t - \psi(e)$. While the output x is publicly observable and verifiable, both θ and e are private information of the agent. The principal can, however, monitor the agent's effort by employing the supervisor who receives a wage w for his services. The supervisor's utility is $u_S = w$. For simplicity, the respective reservation utilities of the supervisor and the agent are normalized to zero. Throughout the analysis, A and S are assumed to be wealth constrained.

To obtain information on the agent's effort, the supervisor has access to the following information technology: with probability $p \in (0, 1]$ he receives a signal s which is imperfectly correlated with the true effort exerted by A. Although effort is a continuous variable, I assume for analytical convenience that monitoring triggers only two possible signals which are labelled 'shirked' ($s = S$) and 'did not shirk' ($s = N$). One interpretation of this assumption is that the monitoring technology allows the principal to specify a 'benchmark' value of $\tilde{e}$ and generates a signal $s \in \{S, N\}$ whose realization depends on whether the agent's effort fell short of or exceeded the specified $\tilde{e}$, respectively.[6] Conditional upon receiving a signal, the probability that the signal is correct is denoted by $\alpha \in (\frac{1}{2}, 1]$ which is assumed to be independent of the true value of e. With probability $1 - p$, the supervisor observes 'nothing'. In what follows, I will refer to p as the *reception probability* or *frequency* and to α as the *precision* or *accuracy* of the signal. One of the main objectives of this paper is to analyze the effect of these two parameters on the optimal contract and the principal's utility. Suppose for concreteness that the principal has the choice between two different information technologies, one of which has a low reception probability but is very accurate whereas under the second technology, a signal is frequently observed but mistakes are relatively likely to occur. The question which can be addressed in this framework is which of the two technologies the principal will prefer.[7]

[5]The last assumption on $\psi(\cdot)$ ensures that the principal's problem is globally concave and that we can without loss of generality restrict contracts to be deterministic [see e.g. Laffont and Tirole (1986)].

[6]See also footnote 7 below for an alternative interpretation. Equivalently, one could assume that auditing is directed at the technology parameter θ which can assume only two values [see, however, the remark on effort versus productivity monitoring in Section 2.3.].

[7]The hypothesis that the principal can choose between different information technologies is not implausible. Consider a slightly modified framework in which S observes the signal $s = e + \epsilon$ where

The principal is the owner of the production technology (or, alternatively, the buyer of the good produced by the agent). Her utility is $u_P = \theta + e - t - w$. The principal designs contracts for the manager and the supervisor which can be conditioned on commonly observable variables such as output x and the supervisor's report r.[8] Before contracting takes place, nature chooses the state of productivity θ and the agent learns his type. Next, the principal proposes contracts to the agent and the supervisor which both can accept or reject. If contracts have been accepted, the manager exerts effort which the supervisor may or may not observe. In either case, the outcome of monitoring is revealed to the agent. Then, output is realized and the supervisor (if called for) reports the result of his investigation to the principal. Finally, the principal pays the transfers in accordance with the contracts. The sequence of events is summarized in Figure 1.

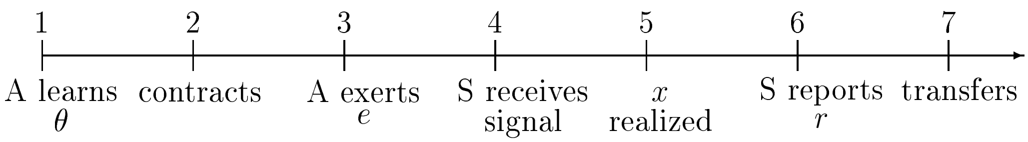

Figure 1

We can now go on to determine the optimal contracts for the principal and their characteristics. In Section 2.2, this is done under the assumption that side contracts between the agent and the supervisor are not feasible. Section 2.3 investigates collusion between A and S. Before proceeding, consider first the benchmark cases when information is symmetric and the supervisor is not available, respectively.

Benchmark Contracts

Suppose first θ is publicly observable so that there is no role for the supervisor. The principal maximizes her utility, $u_p = \theta + e - t$, subject to the manager's participation constraint $t - \psi(e) \geq 0$. Thus, the first-best contract is characterized by $\psi'(e^{FB}) = 1$ and $t^{FB} = \psi(e^{FB})$, independent of the manager's productivity θ.

ϵ is normally distributed with mean zero and variance σ^2. Since the potential values of e are known, the principal can vary the supervisor's information technology by specifying 'confidence' intervals for the possible effort levels with the understanding that if s falls outside these intervals, S is to report 'nothing' and if s falls inside the intervals, S is to report 'shirked' and 'did not shirk', repectively. Narrowing these intervals increases the precision of the supervisor's report but decreases the frequency of a valuable observation.

[8]The assumption on r being commonly observable is imposed by most of the literature on monitoring but is not innocuous. As we will see below, it may be of interest for the principal to claim that S has observed 'nothing' rather than the correct value of e. Thus, it is necessary to assume that the principal cannot hide monitoring evidence.

Next, consider a situation where θ is private information of the agent but the supervisor is not available or, equivalently, $p = 0$. The principal's problem is a standard adverse selection problem. Supposing that the principal can commit herself not to renegotiate the contract, the Revelation Principle ensures that we can confine the analysis to a direct mechanism which guarantees truthful revelation and which prescribes an output $x(\hat{\theta})$ and a transfer $t(\hat{\theta})$ contingent upon the agent's announcement of his type, $\hat{\theta}$. For notational simplicity, let (t_l, x_l) and (t_h, x_h) be the contracts for an agent who claims to be of type θ_l and θ_h, respectively. Denoting $e_i = x_i - \theta_i, i \in \{l, h\}$, the participation constraints can be written as

$$t_i - \psi(e_i) \geq 0, \qquad i \in \{l, h\} \tag{PC$_i$}$$

In addition, the following incentive-compatibility constraints must hold

$$t_l - \psi(e_l) \geq t_h - \psi(e_h + \Delta\theta) \tag{IC$_l$}$$

$$t_h - \psi(e_h) \geq t_l - \psi(e_l - \Delta\theta), \tag{IC$_h$}$$

where $\Delta\theta = \theta_h - \theta_l > 0$. The principal maximizes her expected revenue, $u_P = q(\theta_h + e_h - t_h) + (1-q)(\theta_l + e_l - t_l)$, subject to the (PC) and (IC) constraints. As is easily seen, the incentive constraint for the low-productivity agent as well as the participation constraint for the high-productivity agent will not bind at the optimum and can be ignored. From the first-order conditions, the no-supervisor second best effort levels are determined by

$$\begin{aligned} 1 &= \psi'(e_h^{SB}) \qquad \Rightarrow \qquad e_h^{SB} = e^{FB} \qquad \text{and} \\ 1 &= \psi'(e_l^{SB}) + \frac{q}{1-q}\left[\psi'(e_l^{SB}) - \psi'(e_l^{SB} - \Delta\theta)\right] \qquad \Rightarrow \qquad e_l^{SB} < e^{FB}. \end{aligned}$$

The corresponding transfers are $t_l^{SB} = \bar{U} + \psi(e_l^{SB})$ and $t_h^{SB} = \bar{U} + \psi(e^{FB}) + \left[\psi(e_l^{SB}) - \psi(e_l^{SB} - \Delta\theta)\right]$. Under the optimal contract, the low-productivity manager exerts a level of effort which is distorted downward and obtains his reservation utility. In contrast, the effort of the high-productivity type is efficient and he earns an informational rent equal to $\Phi(e_l^{SB}) = \psi(e_l^{SB}) - \psi(e_l^{SB} - \Delta\theta) > 0$.[9]

[9]It is assumed here that the principal wants to employ the agent even if the supervisor is not available. Furthermore, the (IC$_h$) constraint implicitly requires $e_l \geq \Delta\theta$. If $e_l < \Delta$, the constraint either vanishes (if it is impossible for A to hide output) or reduces to $t_h - \psi(e_h) \geq t_l$. In order to ensure $e_l^{SB} \geq \Delta$, it is necessary to assume either $\psi'(\Delta) \leq 1 - q$ (if A can hide output) or that the principal's utility under the contract defined above strictly exceeds her utility from setting $e_l = \Delta$ (if A cannot hide output).

2.2 Collusion-Free Monitoring

Suppose now that the principal can employ the supervisor to obtain information on the effort exerted by the agent. In this section I assume that the supervisor always reports his signal truthfully. In particular, side contracts between the manager and the supervisor are ruled out. Contingent on the report of S, the principal may now punish the agent if she suspects that he shirked. Recall that the agent is wealth constrained and the worst punishment the principal can impose is therefore the retention of the transfer. Also observe that we can without loss of generality assume that the supervisor is not called for if the agent has truthfully revealed that he is of the high-productivity type. If output is low, though, a high-productivity agent may have misrepresented his true type and the principal will request the supervisor's report. Let t_l^r and w^r denote the compensations for the agent and supervisor, respectively, conditional on the report $r \in \{0, N, S\}$.[10]

Suppose the principal wants to induce an effort level e_l for the low-productivity agent. Then, a shirking high-productivity agent must exert the effort $e_l - \Delta\theta < e_l$. If the principal specifies a benchmark effort $\tilde{e} = e_l$, the manager's participation constraints if the supervisor reports truthfully in equilibrium can be written as[11]

$$p[\alpha t_l^N + (1-\alpha)t_l^S] + (1-p)t_l^0 - \psi(e_l) \geq \bar{U} \qquad (PC_l)$$

$$t_h - \psi(e_h) \geq 0 \qquad (PC_h)$$

In addition, incentive compatibility now requires

$$p[\alpha t_l^N + (1-\alpha)t_l^S] + (1-p)t_l^0 - \psi(e_l) \geq t_h - \psi(e_h + \Delta\theta) \qquad (\mathrm{IC}_l)$$

$$t_h - \psi(e_h) \geq p[(1-\alpha)t_l^N + \alpha t_l^S] + (1-p)t_l^0 - \psi(e_l - \Delta\theta) \qquad (\mathrm{IC}_h)$$

The assumption that $\alpha > \frac{1}{2}$ ensures that the signal is informative and - on average - correct. Thus, a high-productivity agent who shirked faces a different probability

[10] The fact that it is optimal to monitor with probability one after a low output has been observed hinges on the assumption that the supervisor is costless. This assumption may be justified if the supervisor is himself indispensable because he must be hired for reasons other than supervision [see Tirole (1986, 1990) and Laffont and Tirole (1991, 1992)]. If monitoring was costly, the results would remain valid on the condition that a supervisor is used.

[11] The assumption that $\alpha = Prob\{s = N | e \geq \tilde{e}\} = Prob\{s = S | e < \tilde{e}\}$ is not critical. If those probabilities differed in magnitude, the relevant measure of precision is given by the ratio of $Prob\{s = N | e < \tilde{e}\}$ to $Prob\{s = N | e \geq \tilde{e}\}$. Note, though, that this assumption is satisfied by the example given in footnote 7.

distribution over transfers than an agent with low productivity. To simplify matters, I will in the subsequent analysis directly invoke the Maximum Deterrence Principle [Baron and Besanko (1984)] which states that we can without loss of generality set $t_l^S = 0$. Furthermore, because the supervisor is honest the principal can pay him a flat wage equal to his reservation utility, i.e. the optimal contract specifies $w^N = w^S = w^0 = 0$ independent of his report r. The principal's problem is to

$$\begin{aligned} \text{maximize} \quad & q\left\{\theta_h + e_h - t_h\right\} + (1-q)\left\{\theta_l + e_l - p\alpha t_l^N - (1-p)t_l^0\right\} \qquad \text{(CF)} \\ \text{subject to} \quad & (\text{PC}_l),(\text{PC}_h),(\text{IC}_h) \quad \text{and} \quad t_h, t_l^N, t_l^0, \geq 0, \end{aligned}$$

where we have again omitted the (IC_l) constraint which will be satisfied by the solution to (CF). In order to guarantee that the above program is globally concave and has a unique solution, I impose the additional assumption that $q \leq \frac{1}{2}$. The results of the formal analysis in the Appendix are gathered in Proposition 1.

Proposition 1. *The optimal contract under collusion-free monitoring is characterized by a first-best effort of the high-productivity type, $e_h^{CF} = e^{FB}$, and an effort of the low-productivity type which strictly exceeds the no-supervisor second-best effort, $e_l^{CF} > e_l^{SB}$. Moreover, for* any $p \in (0,1]$ *the utility of the principal is independent of the frequency p and strictly increasing in the precision α of the signal. A first best is achieved for values of $\alpha \geq \alpha^* = \psi(e^{FB})/[\psi(e^{FB}) + \psi(e^{FB} - \Delta\theta)]$.*

The first part of the proposition simply states that the possibility of benevolent monitoring increases the power of the incentive scheme for the low-productivity manager.[12] While this conclusions is standard in the literature, the second part of Proposition 1 is more surprising. Under the optimal contract, the utility of P is not affected by the probability that S actually observes a signal on e. It solely depends on the accuracy of the supervisor's information, even though the *ex ante* probability that S detects shirking by A increases with p. Besides, the principal can implement a first-best solution if α is above some critical level, regardless of p. This implies that the principal, if

[12]For similar results, see Laffont and Tirole (1992). In contrast, Baron and Besanko (1984) and Kofman and Lawarrée (1993) derive a separation property. In these models, the principal uses monitoring to extract informational rents, leaving the output of a low-productivity agent largely unaffected. The output decision is thus independent of the monitoring decision. The crucial difference is that these authors interpret limited liability as an exogenous upper bound on the maximum punishment while in this model limited liablity is interpreted as the inability of the principal to extract money from the agent. As a concequence, the maximal punishment varies with the transfer to the low-productivity agent and hence, with his effort. A brief discussion of this issue is provided below.

given the choice, prefers to minimize the supervisor's probability of making mistakes relative to increasing the frequency of the signal. In order to develop some intuition for this result, let us examine the informational rent of a high-productivity agent in more detail. Using $t_l^S = 0$, and substituting for t_l^0 from (PC$_l$) into (IC$_h$), we can write this rent as

$$\Phi(e_l) = \psi(e_l) - \psi(e_l - \Delta\theta) - p(1 - 2\alpha)t_l^N$$

which is strictly decreasing in p as expected. The last term on the right hand side reflects the expected punishment: if the manager has not truthfully reported his type, the supervisor will detect that he shirked with probability $p\alpha$ whereas with probability $p(1-\alpha)$, the manager's shirking will remain undetected. Since the signal is informative, the rent which the agent can secure himself with this strategy is reduced relative to the situation without monitoring. This reasoning, however, does not take into account the optimal level of t_l^N. Since the punishment increases in t_l^N,[13] it is optimal for the principal to set the manager's compensation in case the supervisor observes that he did not shirk as large as possible. By inspection of (PC$_l$), the highest value of t_l^N can be attained if the value of t_l^0 is as small as possible. Setting $t_l^0 = 0$ and combining (PC$_l$) and (IC$_h$) reveals that the agent's rent is independent of p. As I formally show in the Appendix, the optimal contract indeed specifies $t_l^0 = 0$ as long as the incentive-compatibility constraint of the high-productivity agent is binding. Under this contract, the agent is rewarded *only if* the monitoring outcome confirms that he did not shirk. In all other cases (in particular, if the supervisor's investigation was unfruitful) he is punished.[14] If α is sufficiently high, the expected payoff of an agent who shirked eventually becomes negative, even if t_l^N is relatively large. In this case, the principal can implement the first-best effort level at no additional cost of inducing truthful revelation.

Notice, too, that the issue of commitment to a certain monitoring probability does not arise in this context. Throughout the analysis, I have assumed that monitoring

[13] This must be the case because the highest possible punishment is the retention of the transfer, given that the manager is wealth constrained.

[14] It is important to note that this result does not depend on the normalization of the agent's wealth and reservation utility to zero. Fruthermore, while it may seem odd that the agent is 'punished' if no signal on his performance has been received, such an incentive mechanism is not uncommon. In the internal labor markets of firms, employees often are promoted only if their employer has obtained favorable information concerning their performance on a particular job. Individuals who have not succeded in generating a signal are often excluded from promotion rounds. This is in particular true for 'up-or-out' schemes.

is costless and that the principal will send the supervisor with probability one. If the supervisor is costly, the qualitative results of the above proposition are unaffected on the condition that the supervisor is used. In this case, however, it is conceivable that the principal may *ex post* prefer not to monitor since the agent in equilibrium never shirks (always announces his type truthfully) and monitoring costs can thus be saved. Some of the recent literature on monitoring has pointed to this problem of commitment.[15] In the present model, this issue is irrelevant as long as monitoring costs are not too large. To see this, observe that the optimal contract is such that the agent is punished with a positive probability if he is monitored. The agent's transfer in this situation accrues directly to the principal which makes it ex post attractive to monitor. Thus, even if P could not commit to monitor, it would be sequentially rational for her to send the supervisor.[16]

2.3 Collusive Monitoring

To incorporate the possibility of collusion between supervisor and agent, suppose that after monitoring has taken place but before the supervisor makes his report r, the two parties can sign a side-contract which is unobservable to the principal. This contract is assumed to be fully enforceable[17] and consists of transfers from the agent to the supervisor (or vice versa) which may be contingent on realized output and the supervisor's report. Instead of imposing a particular bargaining technology between the supervisor and the agent, it suffices to assume that the outcome of the negotiations is Pareto optimal for the coalition (recall that the agent observes the outcome of S's

[15] See e.g. Melumad and Mookherjee (1989), Khalil (1997), and Strausz (1997).

[16] Although the problem of commitment to a certain audit probability is solved, it should be pointed out here that yet another problem emerges if punishments are used: the principal has ex post an incentive to manipulate the supervisor's report in order to retain the agent's transfer. While one could argue that it is difficult for her to falsify evidence without the agent's consent, it is less clear why the monitoring outcome cannot be concealed. Laffont and Tirole (1992) provide a model of effort monitoring in which $\alpha = 1$ but $p < 1$. Contrary to the last result of Proposition 1, they conclude that a first best in not achieved unless $p = 1$. The reason for this result is that the authors implicitly assume that the agent cannot be punished if the supervisor observed nothing. The presumption that the principal can only partially condition the agent's compensation on r may be appropriate if P is able to hide monitoring evidence.

[17] The assumption that side contracts are perfectly enforceable is standard in the literature on collusion in hierarchies. In practice, of course, it may be impossible for the parties to rely on a court for enforcement, not least because side transfers are often illegal. An alternative mechanism to judicial enforcement could be that the parties are concerned with their reputation of keeping promises. For an extensive discussion of this point see Tirole (1992).

investigation and the bargaining therefore takes place under symmetric information) and that each party can guarantee itself the utility under no collusion. The latter assumption eliminates potential blackmailing of the manager by the supervisor and may be justified on the grounds that the supervisor needs the manager to falsify or conceal evidence. Since e is already sunk at this stage, the optimal report from the supervisor/agent coalition's point of view is one which maximizes the total wage bill $t(x,r)+w(x,r)$. The fact that side contracts can be written between S and A gives rise to a new set of constraints which any final allocation must satisfy. The principal now designs a contract subject to the incentive-compatibility and participation constraints, plus the new coalition incentive constraints. As we will see, the optimal contract is collusion-proof, i.e. it is such that coalitions do not form and no side transfers are made in equilibrium.

Concerning the supervisor's information, we will distinguish two cases. Following Tirole (1986, 1992) and others, the supervisor's information is first assumed to be 'hard' in the sense that the outcome of his investigation could be concealed but not forged. Accordingly, if the supervisor has observed a signal $s \in \{N, S\}$, he can either present the true evidence or else claim that he has observed nothing. In the second scenario, the agent and the supervisor are allowed to falsify evidence, i.e. the supervisor's signal is non-verifiable. This is the case of 'soft' information which has been investigated for example by Kofman and Lawarrée (1993). As will become clear shortly, outcomes differ drastically depending on which assumption is made.

The Supervisor's Report is Hard Information

Clearly, the only case in which collusion can potentially occur is when the supervisor has observed that the manager shirked. Notice that because the optimal contract is incentive compatible, the supervisor has necessarily made a mistake in this case. Because the manager can bribe the supervisor to change his report to having observed 'nothing', the following coalition incentive constraint must hold

$$t_l^S + w^S \geq t_l^0 + w^0. \tag{CIC}$$

Given our previous analysis, however, it is immediate that the optimal contract as characterized in Proposition 1 and the subsequent discussion satisfies this constraint. Under this contract, the supervisor obtains his reservation utility and the agent is punished when S has observed that A shirked *and* when S has observed 'nothing'. Since the only possible deviation is to claim that S has not received a signal, a report other than $r = S$ does not increase the utility of the A-S coalition. This result is stated in Proposition 2.

Proposition 2. *If the supervisor's information is hard (the monitoring signal verifiable), the optimal effort levels as well as the principal's return from the relationship are unaffected by the possibility of collusion. Expressed differently, the optimal contract under collusion-free monitoring is already collusion-proof and collusion can be fended off costlessly.*

Several remarks on the robustness of this result are in order. First, I have interpreted limited liability as the inability of the principal to extract money from the agent [see also Laffont and Tirole (1990), Border and Sobel (1987) and Melumad and Mookherjee (1989)]. Penalties are therefore transfer dependent. The higher the agent's remuneration, the more he can be held liable. In contrast, Baron and Besanko (1984) and Kofman and Lawarrée (1993) interpret limited liability as an exogenous upper bound on the penalty that can be inflicted on the agent. To see why the nature of punishments does not matter for the result, let $\bar{t} > 0$ be such an upper bound on the penalty. Again, the principal will inflict the maximal punishment on the agent if the supervisor detects shirking and the transfer for a report $r = S$ will be $t_l^S = t_l^N - \bar{t}$. As before, t_l^N denotes the agent's remuneration when the signal was favorable. Inserting this value into (IC_h) and using (PC_l), we see that the agent's informational rent is now given by $\psi(e_l) - \psi(e_l - \Delta\theta) - p(2\alpha - 1)\bar{t}$ which is independent of t_l^N and t_l^0. Both transfers can therefore be chosen arbitrarily to ensure that (PC_l) holds. Consequently, the principal can again adjust t_l^0 to satisfy the coalition incentive constraint (CIC) at no additional cost. Second, observe that the result does not depend on the assumption that the agent and the supervisor can sign side-contracts only after the effort has been sunk. The argument above is still valid if the two parties can collude ex ante, e.g. prior to the contract offer by the principal.

This finding stands in sharp contrast to the results obtained in Tirole (1986, 1992) and Laffont-Tirole (1991). These authors generally conclude that collusion hurts the principal and reduces the power of the incentive scheme, even though the supervisor's information is hard. The remark below briefly addresses the rationale underlying this difference.

A Remark on Monitoring of Effort versus Monitoring of Productivity

The present paper can be seen as an attempt to integrate and compare two different approaches in the literature on monitoring and collusion in hierarchical relationships. In the first branch of the literature [Kofman and Lawarrée (1993)], the supervisor observes a relevant variable (productivity, cost parameter) *ex post*, i.e. after the agent has revealed his type. Furthermore, the supervisor *always* observes a signal which may or may not correspond to the true value of the variable in question and this signal is non-verifiable. In the second branch [Tirole (1986, 1992), Laffont and Tirole (1991)], the supervisor observes the agent's type *ex ante*, i.e. before the agent makes his announcement. These papers presume that the supervisor either observes the true value of the parameter or else observes 'nothing'. The signal is verifiable and he can when colluding with the agent only claim that his investigation was unfruitful. A general result of this line of research is that the first best cannot be achieved unless $p = 1$ and that collusion reduces the value of the relationship for the principal. These conclusions seemingly contradict the results of Propositions 1 and 2 where I demonstrate that the first best is possible independently of p and that the possibility of collusion is not harmful for the principal as long as the supervisor's report is hard information. The crucial difference between these models and this paper is the timing of the observation of the supervisor. If S monitors ex ante and the agent simultaneously observes the signal,[18] the agent's participation constraint must hold for all possible signals separately. Punishments are therefore infeasible. In contrast, the present model follows the first branch of the literature in that the supervisor receives his signal ex post. This implies that the agent is at stage 1 still uncertain about the outcome of subsequent monitoring. The agent's uncertainty facilitates punishments when he is detected shirking which in turn greatly improve his incentives to provide the correct effort.[19] Moreover, optimal punishments are such that they cannot be avoided by claiming that S has observed nothing so that collusion can be deterred at no additional cost.

The stark difference in conclusions raises the question which of the assumptions on the exact timing of the supervisor's signal is more plausible. One possible answer to this question is based on the characteristics of the relevant variable which is to be monitored: naturally, if the supervisor is to audit a parameter whose value has already been realized

[18]A common feature of most of the literature on collusion is that the supervisor's signal is simultaneously revealed to himself and the agent.

[19]This point can also be made by comparing the results presented here with those of Nalebuff and Scharfstein (1987) who consider competitive firms conducting pre-employment tests on workers' abilities. In their paper, the the probability of testing matters and the first best can only be achieved if unbounded punishments are feasible.

at stage 0, there is no *a priori* reason to restrict his observation to later stages. If, however, the supervisor is to monitor the agent's effort, a different picture emerges. Clearly, it is quite inconceivable that a signal which is triggered by an action could be observed before this action has been completed.[20] This line of reasoning suggests a strong bias in favor of effort monitoring irrespective of how production functions and monitoring technologies are specified: a principal who faces an agency problem which consists of a moral-hazard and a hidden-information component should rationally prefer to monitor the agent's action rather than to audit his reported information.

The Supervisor's Report is Soft Information

If the supervisor's information is soft, the agent can bribe S to report that he did not shirk. As already indicated, I do not allow for blackmailing.[21] The preceding analysis suggests that bribing the supervisor may be optimal for the manager if S has observed that he shirked *and* if S has observed nothing. Since both parties can agree in those states to jointly claim that the supervisor has observed the agent worked correctly, we must have

$$t_l^0 + w^0 \geq t_l^N + w^N \qquad (\text{CIC}^0)$$

$$t_l^S + w^S \geq t_l^N + w^N \qquad (\text{CIC}^S)$$

It is straightforward to check that other possible coalition-incentive constraints as well as the incentive constraint for the low-productivity agent do not bind and can again be ignored. Clearly, there is also no gain for the principal from rewarding the supervisor if his report was favorable for the agent. We can therefore set $w^N = 0$ without loss of generality. The optimal contract for the principal taking into account the (CIC) constraints then solves

$$\begin{aligned} \max_{e,t,w} \quad & q\left\{\theta_h + e_h - t_h\right\} \qquad\qquad (\text{C}) \\ & + (1-q)\left\{\theta_l + e_l - p[\alpha t_l^N + (1-\alpha)(t_l^S + w^S)] - (1-p)(t_l^0 + w^0)\right\} \\ \text{subject to} \quad & (\text{PC}_l),(\text{PC}_h),(\text{IC}_h),(\text{CIC}^0),(\text{CIC}^S) \text{ and } t_h, t_l^N, t_l^0, w^S, w^0 \geq 0, \end{aligned}$$

As before, the optimal contract will be such that no side transfers are made in equilibrium. Yet, in contrast to the previous analysis the principal is now strictly worse off as

[20] Notice that Propositions 1 and 2 remains valid if the signal is received simultaneously with e.

[21] If the supervisor could blackmail the agent by falsely claiming that he shirked, it is never optimal for the principal to request S's report. This no-supervisor outcome will also prevail in case the supervisor learns a non-verifiable signal on θ ex ante (instead of ex post).

compared to a situation where side contracts between A and S are not feasible. As is formally shown in the Appendix, the possibility of collusion now prompts the principal not to rely on the supervisor's report if the signal is too noisy. More specifically, if α is below some critical level $\hat{\alpha}$, the manager's compensation does not depend on r and the optimal contract is equal to the no-supervisor second-best scheme as defined above. If the signal is sufficiently accurate the effort of the low-productivity agent is higher than e_l^{SB}. In this case, the principal again optimally punishes the agent if the supervisor has observed that he shirked. If S has observed nothing, however, the agent will be paid a positive amount, i.e. it is no longer optimal to retain the transfer in this case. Furthermore, the principal now benefits from both a higher frequency and a higher precision of the signal and a first-best solution can only be attained if $\alpha = 1$ and p is sufficiently large. Proposition 3 summarizes these findings.

Proposition 3. *Define $\hat{\alpha} = \frac{1}{1+q}$. The solution to program (C) is characterized as follows:*

a) if $\alpha \leq \hat{\alpha}$, it is optimal for the principal not to request the supervisor's report. The optimal contract is equal to the no-supervisor second-best scheme with $e_h^C = e^{FB}$ and $e_l^C = e_l^{SB}$. The principal's expected payoff is independent of p and α.

b) if $\alpha > \hat{\alpha}$, the supervisor is used with probability one. The agent is punished when detected shirking but obtains a positive transfer if the supervisor's investigation was unfruitful. The effort levels satisfy $e_h^C = e^{FB}$ and $e_l^C \in (e_l^{SB}, e_l^{CF})$. The principal's expected return from the relationship is increasing in p and α.

c) A first best is achieved if only if $p \geq \hat{p} = [\psi(e^{FB}) - \psi(e^{FB} - \Delta)]/\psi(e_l^{FB})$ and $\alpha = 1$.

The intuition for part a) of the proposition is as follows. Since the optimal contract is collusion-proof, the principal must pay the supervisor an amount equal to the punishment of the agent if the former has detected shirking. Using the supervisor is therefore relatively costly if mistakes are likely to occur. To see why $\hat{\alpha}$ is inversely related to q, recall that S is only used if output is low which occurs with probability $1 - q$ because in equilibrium the agent never shirks. If this probability is high (q low) and the supervisor's information rather imprecise, the expected wage of the supervisor exceeds the principal's benefit from monitoring.[22] Part b) states that if α is sufficiently large, the supervisor is used with probability one. In this case, it is still optimal for her to punish the agent if S has observed that he shirked. The rationale behind this result is that P has to pay at least t_l^N, either to the agent or to the supervisor (this is directly implied by the two CIC constraints). From the point of view of the principal, this transfer serves two purposes. First, it can be retained in order to give incentives to a high-productivity agent not to shirk. Second, it can be used to compensate a low-productivity agent for his efforts. If it is efficient to monitor, the former effect dominates the latter since incentives can be given at relatively low cost for P.[23] Concerning the agent's compensation in case S observed nothing, however, the latter effect dominates the former. Incentives cannot be given through t_l^0 because both types of agents face the same probability that the supervisor does not receive a signal on their effort. Raising t_l^0 now puts the principal into a position to lower t_l^N which is beneficial for her (recall that she has to pay t_l^N in any case). This line of reasoning also explains why contrary to the two previous cases, the principal's payoff now increases with the frequency p of the signal.

Finally, to see why a first best can only be achieved if $\alpha = 1$ and p sufficiently large consider first a situation where the supervisor's signal is perfectly accurate ($\alpha = 1$) but $p < \hat{p}$. If the supervisor's information is soft, he can claim that A did not shirk even though he has observed nothing. This prevents the principal from imposing the punishment $t_l^0 = 0$ that is needed to relax the incentive constraint of the high-productivity agent (see Proposition 1). Thus, we must have $t_l^0 > 0$ and $(IC)_h$ continues to bind if the probability of this transfer being paid is large enough, i.e. p sufficiently low.

Next, consider $p \geq \hat{p}$ but $\alpha < 1$. Clearly, if S has observed that A shirked he must be given a wage equal to the punishment of A in order to tell the truth. Hence, only if the principal knows that the signal is perfectly accurate, she can costlessly use a compensation scheme in which S is paid a wage equal to the punishment of A in case he detected shirking. Since shirking never occurs in equilibrium, the probability of this wage being paid is zero and shirking can be prevented at no loss for the principal.[24]

[22] For the case of an exogenously given punishment, a similar result has been obtained by Kofman and Lawarée (1993).

[23] The principal has to compensate S only if he observed that A shirked which occurs with a small probability given that the signal is very informative.

[24] It is important to recognize that this conclusion (in contrast to Proposition 2) crucially depends on

3 Concluding Remarks

This paper has reconsidered the issue of monitoring and collusive behavior in hierarchies. In contrast to previous contributions to the subject, the analysis has focused on how different information technologies affect the optimal contract and the principal's expected return from the relationship. To this end, I have distinguished between quantitative and qualitative aspects of the information technology: first, the probability that a signal on effort is received (the frequency of the observation) and second, the probability that this signal is correct (the accuracy of the observation). It was demonstrated that if the supervisor and the agent cannot collude or, alternatively, if the supervisor's signal is hard information, the principal's utility under the optimal contract is independent of the frequency with which a signal on the effort level is actually observed. In those cases, the principal's sole interest lies in improving the precision of monitoring. A first-best allocation from the principal's point of view can be implemented as long as the monitoring outcome is sufficiently accurate. Furthermore, any collusion between the supervisor and the agent can be fended off costlessly. These findings hold even though both the supervisor and the agent are wealth constrained and unbounded punishments cannot be used. If the signal of a collusive supervisor is non-verifiable, however, the possibility of collusion imposes an additional cost on the principal. These costs induce her not to rely on the supervisor's report provided that mistakes are likely to occur. Again, this decision is independent of the signal's reception probability. If the signal is relatively accurate, the supervisor will be used in spite of collusion. This is the only case in which the frequency of making a valuable observation has a positive impact on the principal's return from the relationship.

A second and related result of this paper is that if the agent has private information *and* takes an unobservable action, the principal should prefer to monitor the action rather than testing for the unknown characteristics of the agent. The rationale behind this result is straightforward: in the former case the agent is definitely uncertain about the outcome of the supervisor's investigation at the time of contracting and at the time he has to take the prescribed action. The principal benefits from the agent's

the assumption that side contracts between A and S can only be written *after* the agent has announced his type. If the supervisor and the agent could collude ex ante, S would have an incentive to bribe a high-productivity agent to shirk in order to obtain his reward with probability one. However, it is still possible to show that the principal is strictly better off as compared to a situation where S observes a signal on θ ex ante.

uncertainty since it relaxes his participation and incentive constraints and punishments can be invoked. This conclusion is unaffected by the exact timing of side contracting. Specifically, it remains valid if the two parties can agree to a side contract before the agent announces his type.

An important extension of the present framework would be to allow for coalitions other than the supervisor/agent coalition. As we have seen, the principal has an incentive to falsify or at least to hide monitoring evidence. Clearly, the fact that she can collude with the supervisor alters the optimal contract in a non-trivial way which may give rise to additional considerations concerning her preferred monitoring technology.

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Appendix

Proof of Proposition 1

Setting $t_l^S = w^r = 0$, the Lagrangian of the principal's problem is

$$\begin{aligned}\mathcal{L} &= q\{\theta_h + e_h - t_h\} + (1-q)\{\theta_l + e_l - p\alpha t_l^N + (1-p)t_l^0\} \\ &\quad +\lambda_1\{p\alpha t_l^N + (1-p)t_l^0 - \psi(e_l)\} + \lambda_2\{t_h + \underline{t} - \psi(e_h)\} \\ &\quad +\lambda_3\{t_h - \psi(e_h) - p(1-\alpha)t_l^N - (1-p)t_l^0 + \psi(e_h - \Delta\theta)\},\end{aligned}$$

with the non-negativity constraints. The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial e_h} = q - (\lambda_2 + \lambda_3)\psi'(e_h) \leq 0, \quad e_h \geq 0 \quad \text{and} \quad e_h \frac{\partial L}{\partial e_h} = 0 \tag{1}$$

$$\frac{\partial L}{\partial t_h} = -q + (\lambda_2 + \lambda_3) \leq 0, \quad \tilde{t}_h \geq 0 \quad \text{and} \quad t_h \frac{\partial L}{\partial t_h} = 0 \tag{2}$$

$$\frac{\partial L}{\partial e_l} = 1 - q - \lambda_1\psi'(e_l) + \lambda_3\psi'(e_l - \Delta\theta) \leq 0, \quad e_l \geq 0 \quad \text{and} \quad e_l \frac{\partial L}{\partial e_l} = 0 \tag{3}$$

$$\frac{\partial L}{\partial t_l^N} = -(1-q)p\alpha + \lambda_1 p\alpha - \lambda_3 p(1-\alpha) \leq 0, \ t_l^N \geq 0 \quad \text{and} \quad t_l^N \frac{\partial L}{\partial t_l^N} = 0 \tag{4}$$

$$\frac{\partial L}{\partial t_l^0} = -(1-q)(1-p) + \lambda_1(1-p) - \lambda_3(1-p) \leq 0, \ t_l^0 \geq 0 \quad \text{and} \quad t_l^0 \frac{\partial L}{\partial t_l^0} = 0, \tag{5}$$

plus the constraints and their complementary slackness conditions. From (1) and $\psi'(0) = 0$, we have $e_h > 0$ which implies $t_h > 0$ by (PC$_h$). Hence,

$$\begin{aligned} q &= \lambda_2 + \lambda_3. \\ 1 &= \psi'(e_h) \ \Rightarrow \ e_h = e^{FB} \end{aligned} \tag{6}$$

Next, we also have $e_l > 0$ so that (3) holds with equality (see footnote 9). Similar to the argument above, (PC$_l$) then requires either $t_l^N > 0$ or $t_l^0 > 0$ or both. We first show $t_l^N > 0$. Suppose by way of contradiction that $t_l^N = 0$. Then, $t_l^0 > 0$ so (5) holds with equality and $\lambda_1 = (1-q) + \lambda_3 > 0$. Substituting for λ_1 in (4) yields $\lambda_3 \leq 0$. Thus, we can have $t_l^N = 0$ only if $\lambda_3 = 0$. But then (3) implies $e_l = e^{FB}$ which together with (PC$_l$) contradicts (IC$_h$). Therefore, we must have $t_l^N > 0$.

Using (4), $t_l^N > 0$ implies $\lambda_1 = (1-q) + \frac{1-\alpha}{\alpha}\lambda_3 > 0$. Substituting for λ_1 in (3) yields

$$(1-q) = (1-q)\psi'(e_l) + \lambda_3\left[\frac{1-\alpha}{\alpha}\psi(e_l) - \psi'(e_l - \Delta\theta)\right]. \tag{7}$$

Consider first $\lambda_3 = 0$. Then, $e_l = e^{FB}$ by (7) and $\lambda_2 = q > 0$ from (7), so (PC$_h$) is binding. In order for (IC$_h$) to hold in this case, we must have $\frac{1-\alpha}{\alpha}\psi(e^{FB}) - \psi(e^{FB} - \Delta\theta) \leq 0$ or

$$\alpha \geq \alpha^* = \frac{\psi(e^{FB})}{\psi(e^{FB}) + \psi(e^{FB} - \Delta\theta)}.$$

Note that α^* is independent of p. Conversely, if $\alpha < \alpha^*$, $\lambda_3 = 0$ yields a contradiction and we must have $\lambda_3 > 0$ so (IC$_h$) is binding. Substituting $\lambda_1 = (1-q) + \frac{1-\alpha}{\alpha}\lambda_3$ into (5), we obtain

$\lambda_3(1-2\alpha) \leq 0$. Hence, $\lambda_3 > 0$ implies $t_l^0 = 0$. There are two cases to distinguish: a) $\lambda_2 = 0$ implies $\lambda_3 = q$ and the optimal level of e_l can recovered from equation (7)

$$1 = \psi'(e_l) + \frac{q}{1-q}\left[\frac{1-\alpha}{\alpha}\psi(e_l) - \psi'(e_l - \Delta\theta)\right];$$

b) $\lambda_2 > 0$ implies that both (PC$_h$) and (IC$_h$) are binding at the optimum and e_l is given by

$$\frac{1-\alpha}{\alpha}\psi(e_l) - \psi(e_l - \Delta\theta) = 0.$$

In either case, e_l is increasing in α and does not depend on p. Furthermore, $\lambda_3 > 0$ together with (5) implies $t_l^0 = 0$. The level of t_l^N is chosen so as to satisfy (PC$_l$). That is, $\tilde{t}_l^N = [\psi(e_l)]/(\alpha p)$. Therefore, the informational rent of the high-productivity type is independent of p [if $\lambda_3 = 0$ and (IC$_h$) is not binding, the values of t_l^N and t_l^0 can be chosen arbitrarily so as to satisfy (PC$_l$)]. Applying the envelope theorem, we have after substituting for t_l^N and λ_1,

$$\frac{\partial L}{\partial \alpha} = \frac{\lambda_3}{\alpha^2}\psi(e_l) > 0 \quad \Leftrightarrow \quad \lambda_3 > 0,$$

which completes the proof. □

Proof of Proposition 3

Note first that both coalition constraints must be binding. Since it can never be optimal for P to set $w_N > 0$, we can without loss of generality set $w^N = 0$, $w^0 = t_l^N - t_l^0$ and $w^S = t_l^N - t_l^S$. The coalition-proofness constraints can now be written as $w^0 = t_l^N - t_l^0 \geq 0$ and $w^S = t_l^N - t_l^S \geq 0$. The following lemma simplifies the analysis:

Lemma. *Without loss of generality, we can restrict attention to contracts where the supervisor is used with probability one.*

Proof: Let γ be the probability that the supervisor's report is requested. Denote the transfers depending on this report as before and let t_l be the transfer to A if no monitoring occurs. Now consider an alternative contract where monitoring occurs with probability one and $\hat{t}_l^N = (1-\gamma)t_l + \gamma t_l^N$, $\hat{t}_l^0 = (1-\gamma)t_l + \gamma t_l^0$ and $\hat{t}_l^S = (1-\gamma)t_l + \gamma t_l^S$. This contract satisfies all constraints and leaves the principal's payoff unchanged. □.

The Lagrangian of the principal's problem is now

$$\begin{aligned} L &= q\{\theta_h + e_h - t_h\} + (1-q)\{\theta_l + e_l - t_l^N\} \\ &\quad + \lambda_1\{p\alpha t_l^N + (1-p)t_l^0 + p(1-\alpha)t_l^S - \psi(e_l)\} + \lambda_2\{t_h - \psi(e_h)\} \\ &\quad + \lambda_3\{t_h - \psi(e_h) - p(1-\alpha)t_l^N - (1-p)t_l^0 - p\alpha t_l^S + \psi(e_l - \Delta\theta)\} \\ &\quad + \lambda_4\{t_l^N - t_l^0\} + \lambda_5\{t_l^N - t_l^S\}, \end{aligned}$$

with the non-negativity constraints. The Kuhn-Tucker conditions are

$$\begin{aligned}
\frac{\partial L}{\partial e_h} &= q-(\lambda_2+\lambda_3)\psi'(e_h)\leq 0, \quad e_h\geq 0 \quad \text{and} \quad e_h\frac{\partial L}{\partial e_h}=0 && (8)\\
\frac{\partial L}{\partial t_h} &= -q+(\lambda_2+\lambda_3)\leq 0, \quad t_h\geq 0 \quad \text{and} \quad t_h\frac{\partial L}{\partial t_h}=0 && (9)\\
\frac{\partial L}{\partial e_l} &= 1-q-\lambda_1\psi'(e_l)+\lambda_3\psi'(e_l-\Delta\theta)\leq 0, \quad e_l\geq 0 \quad \text{and} \quad e_l\frac{\partial L}{\partial e_l}=0 && (10)\\
\frac{\partial L}{\partial t_l^N} &= -(1-q)-\lambda_1 p\alpha-\lambda_3 p(1-\alpha)+\lambda_4+\lambda_5\leq 0, \; t_l^N\geq 0, t_l^N\frac{\partial L}{\partial t_l^N}=0 && (11)\\
\frac{\partial L}{\partial t_l^0} &= \lambda_1(1-p)-\lambda_3(1-p)-\lambda_4\leq 0, \quad t_l^0\geq 0 \quad \text{and} \quad t_l^0\frac{\partial L}{\partial t_l^0}=0 && (12)\\
\frac{\partial L}{\partial t_l^S} &= \lambda_1 p(1-\alpha)-\lambda_3 p\alpha-\lambda_5\leq 0, \quad t_l^S\geq 0 \quad \text{and} \quad t_l^S\frac{\partial L}{\partial t_l^S}=0. && (13)
\end{aligned}$$

As in the proof of Proposition 1, we have $e_h, t_h, e_l, t_l^N > 0$ with $\psi'(e_h)=1$,

$$\begin{aligned}
q &= \lambda_2+\lambda_3 && (14)\\
(1-q) &= \lambda_1\psi'(e_l)-\lambda_3\psi'(e_l-\Delta\theta) && (15)\\
(1-q) &= \lambda_1 p\alpha-\lambda_3 p(1-\alpha)+\lambda_4+\lambda_5. && (16)
\end{aligned}$$

Suppose first $\lambda_4, \lambda_5 > 0$ so that $t_l^N = t_l^0 = t_l^S > 0$. From (12) and (13), $\lambda_4 = (1-p)(\lambda_1-\lambda_3)$ and $\lambda_5 = p(1-\alpha)\lambda_1 - p\alpha\lambda_3$. Substituting these equations into (16) and using (15), we obtain

$$1=\psi'(e_l)-\frac{\lambda_3}{1-q}[\psi'(e_l)-\psi'(e_l-\Delta\theta)]. \quad (17)$$

Note that since the agent's compensation does not depend on the supervisor's report in this case, the solution must be identical to the no-supervisor second-best scheme. From (17), we have $e_l = e_l^{SB}$ if $\lambda_3 = q$ ($\lambda_2 = 0$). But $\lambda_3 = q$ implies $\lambda_1 = 1$ by (15) and (17) and we can have $\lambda_5 > 0$ only if $(1-\alpha)-\alpha q > 0$ or $\alpha < \hat{\alpha} = \frac{1}{1+q}$. Conversely, $\alpha \geq \hat{\alpha}$ contradicts the assumption that $\lambda_4, \lambda_5 > 0$ and we must have either $\lambda_4 = 0$ or $\lambda_5 = 0$ or both. $\lambda_4 = 0, \lambda_5 > 0$ immediately yields a contradiction from (12), (13) and (16). Likewise, $\lambda_4 = \lambda_5 = 0$ again contradicts (12), (13) and (16) under the assumption that $q < \frac{1}{2}$. Hence, we must have $\lambda_4 > 0$ and $\lambda_5 = 0$. Then, $\lambda_1(1-\alpha)-\lambda_3\alpha \leq 0$ from (13) as long as $\lambda_3 \leq q$ with strict inequality if $\alpha > \hat{\alpha}$. Hence, $t_l^S = 0$. There are three possibilities. First, $\lambda_2 = 0$ implies $\lambda_3 = q$. Substituting for λ_4 in (16), we get $\lambda_1 = [(1-q)+q(1-p\alpha)]/(1-p+p\alpha) > 0$. Note that $\lambda_1 < 1$ as long as $\alpha > \hat{\alpha}$. Thus, $e_l > e_l^{SB}$. Using the expression for λ_1, the optimal level of e_l can be recovered from (15) which, after some rearrangements yields

$$1=\frac{1}{1-p+p\alpha}\psi'(e_l)+\frac{q}{1-q}\left\{\frac{1-p\alpha}{1-p+p\alpha}\psi'(e_l)-\psi'(e_l-\Delta\theta)\right\}.$$

Since $\alpha > \frac{1}{2}$, e_l is strictly lower than the optimal e_l under collusion-free monitoring. Second, $\lambda_2 > 0$ implies that both (PC$_h$) and (IC$_h$) are binding at the optimum. e_l can then be obtained from

$$\frac{1-p\alpha}{1-p+p\alpha}\psi(e_l)-\psi(e_l-\Delta\theta)=0. \quad (18)$$

Finally, if $\lambda_2 > 0$ and $\lambda_3 = 0$, we have $e_l = e^{FB}$ and (15), (16) and (18) hold only if $\alpha = 1$ and $p \geq \hat{p} = [\psi(e_l^{FB}) - \psi(e_l^{FB} - \Delta)]/\psi(e_l^{FB})$. Again, we can apply the envelope theorem which, after substituting $t_l^S = 0$ and $t_l^0 = t_l^N$ yields

$$\frac{\partial L}{\partial p} = \lambda_3 \alpha - \lambda_1 (1 - \alpha) \geq 0,$$

by (13), with strict inequality if $\alpha > \hat{\alpha} = \frac{1}{1+q}$ and $\lambda_3 > 0 \Leftrightarrow p < \hat{p}$ and $\alpha < 1$ which completes the proof. □