No. 2344

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FINANCIAL ECONOMICS

Centre for Economic Policy Research

ISSN 0265-8003

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Discussion Paper No. 2344
December 1999

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CEPR Discussion Paper No. 2344

December 1999

ABSTRACT

Optimal Capital Allocation Using RAROC And EVA*

This paper analyzes financial institutions' capital allocation decisions when their required equity capital depends on the risk of their projects chosen. We discuss the relevance of strict position limits against discretionary trading through the use of an optimal compensation function. We show that (under full information) the first-best investment decision can be delegated through an economic value added (EVA) compensation contract and solve for the optimal capital allocation rules. We demonstrate how the concept of incremental value at risk (VaR) must be used to deal with the multidivisional firm. The results are extended to deal with asymmetric information on the part of the trading division(s). The analysis defines precisely the notion of risk-adjusted return on capital (RAROC) and how it can be used as a performance measure.

JEL Classification: G21, G28, G31, G32
Keywords: RAROC, EVA, capital, allocation , budgeting, banking, institution, divisions

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* RAROC is a trademark of Bankers Trust and EVA is a registered trademark of Stern Stewart & Co. This paper has been presented at seminars at the European Finance Association, Amsterdam, Frankfurt, Hong Kong University, Houston, Konstanz, Mainz, NYU, UC Irvine, the Stockholm School of Economics and Wisconsin. We appreciate the comments of participants at the seminars. We especially acknowledge the helpful comments of Wolfgang Bühler and Bruce Grundy.

Submitted 3 November 1999

NON-TECHNICAL SUMMARY

Financial institutions, like their counterparts throughout the economy, are facing increasing pressure to enhance shareholder value. This has led to the widespread adoption of concepts such as Economic Value Added (EVA) and Risk Adjusted Return on Capital (RAROC). When implemented by financial institutions, both these concepts involve capital charges that depend on risk measures. This intricate link between bank performance and risk management is the focus of this paper.

Regulation provides an immediate justification for the link between risk and bank capital. The Bank of International Settlements (BIS) has played the leading role in formalizing the regulatory relationship between credit risk, various forms of market risk and equity capital requirements. The BIS Accords of 1988 and 1996 have both been translated into national law by a large number of industrialized countries, including the EU-countries and the US.

However, an even more important link between risk and equity capital is the fact that banks will generally find it in their own interest to provide equity in accordance with the risks they are exposed to. A bank will only be able to generate new transactions if their customers do not doubt its ability to remain solvent. Furthermore, even if customers do not require equity capital to reflect the bank's risk exposure, the bank's equity-holders will generally do so as long as the bank charter has positive value. Since this value would be lost in the event of default, equity-holders have an incentive to accord the bank's risk exposure with the amount of equity capital.

To capture the link between capital requirements and risk, we model a bank whose equity must exceed an amount defined by its value at risk (VaR), that is, by a maximum acceptable bankruptcy probability. In such a setting, our paper addresses several central issues related to the risk management of financial institutions. The main purpose is to derive an optimal capital allocation mechanism so that the utilization of EVA and RAROC concepts is appropriate to achieving overall value maximization from the perspective of shareholders.

Although in such an environment equity capital requirements will be determined at the firm level, it is important to allocate capital at the divisional or project level. The key question involved in capital allocation is how much equity capital to charge each unit with so that the effect of individual investment decisions on the overall level of risk is internalized. In this way the capital charge to the division may be different from the actual amount of capital required by this division's investments. The most dramatic example is a division entirely engaged in derivatives trading for which no literal cash investment may be required.

The problem of the correct implementation of EVA and RAROC is most challenging in a multidivisional setting where risk management activities need to be coordinated. How can the institution decentralize the use of EVA compensation at the divisional level? If divisional VaR is utilized for economic capital, then the sum of divisional economic capital exceeds firm-wide economic capital due to the correlation structure of investments. Ad hoc procedures, such as proportional scaling may create significant distortions in the decisions taken.

The method this paper derives involves the central authority of the institution specifying a mechanism under which divisions are allocated economic capital equal to a division-specific price of risk multiplied by their own VaR. The price is chosen appropriately so that the economic capital is equal in realization to the *incremental value at risk* (IVaR), which takes into account the externality that one division's risk imposes on that of the other divisions within the institution. Moreover, this mechanism has the virtue that the sum of divisional economic capital is exactly equal to the overall VaR of the firm. An important feature is that the 'price' must be personalized for the division's own investment opportunities. The use of this risk pricing mechanism 'separates' the investment decisions in a way that allows each division to act independently of the others.

This paper demonstrates that the above capital allocation mechanism allows the bank's central authority to delegate investment decisions, whereby the divisions take decisions which are consistent with overall shareholder value maximization and which are consistent with the capital structure constraint. Initially the capital allocation mechanism is derived for a situation in which there is full information within the firm about investment opportunities. Subsequently the analysis is extended to deal with the case of asymmetric information. It is shown that asymmetric information makes the capital allocation more sensitive to risk-taking. Thus, in the presence of asymmetric information, the bank allocates more capital for a given increase in volatility than in the presence of symmetric information.

It is found that a rigorous definition of RAROC involves, in the denominator, an economic capital amount equal to the unit's incremental VaR; and in the numerator, the business unit's expected or realized return minus an adjustment factor less a risk adjustment comprised of economic capital. This adjustment reflects both the economic value added at the optimal risk level and risk externalities imposed by one business unit on the incremental VaR of other business units. The mechanism derived has the property that RAROC is larger for divisions that make optimal investment decisions and for those with more favourable investment opportunities. Furthermore, a business unit's economic value added equals RAROC times the unit's incremental VaR.

1 Introduction

Governmental regulatory authorities as well as international financial organizations focus significant attention on the risk exposures, capital adequacy and the risk management practices of banking and financial service industries. This contrasts with the traditional corporate finance paradigm, based on frictionless markets, which leaves little scope for risk management considerations.

The banking sector does not fit well in this paradigm, however. The choice of capital structure of financial institutions merits special consideration. There is widespread agreement that banks favor debt-financing. Banks, by their very nature, are in the business of accepting deposits, which implies a bias in favor of debt financing. The provision of services such as liquidity through demand deposits, letters of credit etc. imply that depositors are willing to loan their funds at rates below those prevailing in money markets. By contrast, in order to raise equity capital, banks must compete with other non-financial firms in the security markets. Thus, in the absence of governmental or self-regulation, banks will prefer the maximum use of debt finance.

Therefore, regulating bank's capital structure and/or risk-management may be desirable for a number of reasons. First, banks' role as financial intermediaries requires that their debt is held by a large number of small depositors. This in turn would make it very costly for each depositor to monitor the risk of a particular bank. It may therefore be optimal to regulate banks' risk exposures. Second, the presence of deposit insurance may create incentives for excessive risk taking of the bank. Third, negative externalities imposed on the rest of the economy by a risky and highly levered banking sector may require regulation. Many countries have therefore implemented the recommendations of the Basle Banking Committee and define capital requirements on the basis of various risk measures. In addition to monitoring risks to comply with regulation, many banks and financial institutions have also adopted their own internal risk management systems designed to measure and limit their risks in accordance with their equity capital.

Although in such an environment equity capital requirements will be determined at the firm level, it is important to allocate capital at the divisional or project level. The key question involved in capital allocation is how much equity capital to charge each unit with so that the effect of individual investment decisions on the overall level of risk is internalized. In this way the capital charge to the division may be different from the actual amount of capital required by this division's investments. The most dramatic example is a division entirely engaged in derivatives trading for which no literal cash investment may be required.

In standard capital budgeting contexts the need to assess project-specific systematic risks for the purpose of deciding on an appropriate cost of capital is well-documented. Less well-understood, however, is the need to incorporate project-specific capital structure weights. Stulz (1998) discusses these issues in the context of risk management and demonstrates that the capital budgeting process must incorporate an adjustment to net present value to represent the impact of project-specific unsystematic risk on the capital of the firm.

Our paper addresses several central issues related to the risk management of financial institutions. There is considerable interest from the financial community in utilizing Economic Value Added (EVA) as a compensation mechanism.[1] A related measure, risk adjusted return on capital (RAROC) is also applied to assign performance rankings across divisions. The main purpose of our paper is to derive an optimal capital allocation mechanism so that the utilization of these techniques is appropriate to achieving overall value maximization from the perspective of shareholders. Essentially the optimal capital allocation mechanism we derive requires the institution to compute an *economic capital*, rather than a book capital for use in the EVA and RAROC computations. Moreover, we show that the economic capital is related to the widespread use of value-at-risk (VaR).

Despite these important developments, a puzzle has remained unresolved. In a multidivisional setting, where risk management activities need to be coordinated, how can the institution decentralize the use of EVA compensation at the divisional level? If divisional VaR is utilized for economic capital, then the sum of divisional economic capital exceeds firm-wide economic capital due to the correlation structure of investments. Ad hoc procedures, such as those in Kimball (1997) have been proposed, but these may create significant distortions in the decisions taken.

Froot and Stein (1998) discuss this problem of divisional interdependence in a model in which risk management arises endogenously from the need to avoid an adverse selection problem with respect to external finance. They indicate the set of conditions a RAROC measure would have to satisfy in order to

[1] See, for example, Uyemura, Kantor and Pettit (1996), in the banking context.

produce optimal decisions using a bank-wide hurdle rate criterion. We extend this research by analyzing specific forms of capital allocation and RAROC and their effects on delegated investment decisions. We also model explicitly the influence of asymmetric information on optimal risk adjusted performance measures in an agency framework. If the institution uses an EVA-based incentive compensation system with an appropriately designed economic capital allocation, then we demonstrate that the firm-wide optimal risk decision can be implemented through local control. RAROC is more appropriately used in our view as an ex post performance measure, as opposed to a bank-wide hurdle rate determination.

The method we propose involves the central authority of the institution specifying a mechanism under which divisions are allocated economic capital equal to a division-specific price multiplied by their own value-at-risk. The price is chosen appropriately so that the economic capital is equal in realization to the *incremental value at risk* (IVaR), which takes into account the externality that one division's risk imposes on that of the other divisions within the institution. Moreover, this mechanism has the virtue that the sum of divisional economic capital is exactly equal to the overall VaR of the firm. In some ways our results are therefore reminiscent of the literature on internal capital markets (Stein, 1997). However an important distinction is that the "price" must be personalized for the division's own investment opportunities. The use of this risk pricing mechanism "separates" the investment decisions in a way that allows each division to act independently of the others.

We model a bank whose equity capital must exceed an amount defined by its VaR, that is, by a maximum acceptable bankruptcy probability. Each of the bank's business units must decide among various investment projects with different expected returns and standard deviations. Since investment decisions are delegated, the bank's center must induce an optimal investment decision consistent with the capital structure constraint. Initially we consider a situation in which there is full information within the firm about investment opportunities. This, of course, is not the realistic case. However it allows for a relatively simple exposition of the main ideas, which are then subsequently extended to deal with asymmetric information.

In two relevant papers, James (1996) and Zaik, Walter, Kelling and James (1996) analyze the capital budgeting process at Bank of America where equity capital is defined by a value at risk measure. Milbourn and Thakor (1996) consider an asymmetric information model of capital allocation and compensation. They focus on the moral hazard problem of managerial effort as well as enhanced bargaining opportunities for the central authority.

One of the original papers to look at capital budgeting under asymmetric information and solve for the amount of inefficiency induced was Harris, Kriebel and Raviv (1982). Harris and Raviv (1996) and Harris and Raviv (1998) analyze capital budgeting decisions in the presence of asymmetric information about project quality and empire building preferences by divisional managers. At a cost, headquarters can obtain information about a division's investment opportunity set. The paper demonstrates under which circumstances headquarters will delegate the decision how to allocate capital across projects and what form this delegation may take. Our paper focuses on capital budgeting decisions of banks which are facing capital adequacy restrictions based on the risk of the projects chosen by the different business units. In particular we consider capital allocations for investment decisions which do not require any initial investment.

The remainder of the paper is structured as follows. Section 2 discusses the economic and institutional environment and develops the model. Section 3 provides the analysis for the case of full information; whereas section 4 extends the model to the case of asymmetric information. The most general setup appears in section 5. Section 6 concludes.

2 Model Development

Financial institutions and banks, in particular, face market imperfections such as costs of financial distress costs, transactions costs in accessing capital markets, or simply regulatory constraints. These frictions imply that risk management, capital structure and capital budgeting are interdependent. We begin by discussing the nature of investment opportunities faced by the financial institution.

2.1 Investment Opportunities

We first specify how a business unit's investment opportunities are modeled. A financial institution consists of n divisions, each of which may choose investment projects, defined by their standard deviations of cash flows to equity holders. Expected cash flows at the end of the period are given by

$$\mu_i = \mu_i(\sigma_i, \theta_i) \tag{1}$$

where σ_i is the standard deviation of cash flows of division i and θ_i defines the investment technology specified below.[2]

We assume that, *ceteris paribus*, more "aggressive" risk taking by a division translates into higher expected returns, i.e.

$$\frac{\partial \mu_i}{\partial \sigma_i} > 0. \tag{2}$$

To ensure interior solutions we also assume that the investment technology is concave, $\partial^2 \mu_i / \partial \sigma_i^2 < 0$, and that $\lim_{(\sigma_i \to 0)} \; \partial \mu_i / \partial \sigma_i = \infty$ and $\lim_{(\sigma_i \to \infty)} \; \partial \mu_i / \partial \sigma_i = 0$.

The parameter θ_i defines the functional relationship between risk and expected return. Specifically, we assume that the following cross-partial derivative condition applies:

$$\frac{\partial^2 \mu_i}{\partial \sigma_i \partial \theta_i} \geq 0. \tag{3}$$

This is the well-known Spence (1973) - Mirrlees (1971) *sorting condition*, that is critical in the analysis under asymmetric information. Condition (3) implies that a higher θ_i makes "risk" more productive and makes a higher σ_i more desirable.

2.1.1 Examples

Equation (1) is a reduced form, representing the composition of a number of concurrent operating activities of financial institutions. We provide two specific descriptions of activities that underlie the specification in equation (1). The first example is that of a bank which engages in deposit-financed lending activity. We hereby assume that deposit rates are below the expected return on loans. This may be due to imperfect competition between banks or due to fixed costs in setting up a bank. We normalize the interest rate paid to depositors to zero.

[2] In order to generate the functional representation of investment opportunities embodied in (1) it may be necessary to apply an efficiency criterion to a more general investment opportunity set.

Let L denote the amount of lending undertaken. The marginal loan quality is decreasing in the aggregate amount of lending and so we can represent the expected net cash flows from lending activities by $\mu = f(L)$ with f being an increasing concave function. Suppose, for instance, that the aggregate risk is increasing linearly in aggregate lending, so that $\sigma = sL$ with $s > 0$. We can therefore rewrite the expected cash flows in the following way

$$\mu = f(\frac{\sigma}{s}) \equiv \mu(\sigma).$$

Therefore equation (1) characterizes the deposit-financed lending activity described above.

The next situation we consider is that of a trading division engaged in self-financing derivatives trading. That is zero net investment positions are taken in financial instruments such as forwards, futures, swaps etc. These activities may either be thought of as arising from offering structured products to corporate customers or from trading in the capital markets based on proprietary valuation models. Let σ represent the risk of the cash flows generated by the derivatives exposure. Similar to the previous example, suppose that σ is linear in the face value and that the expected cash flow is an increasing and concave function of the face value of the derivative position. This concavity may be due to a number of sources, such as price impact, or estimation risk in the valuation model used to derive expected cash flows.

Although we have given two examples of activities that financial institutions are involved in, our model can be interpreted to reflect a variety of other investment activities. These investment activities may be interpreted in our model, without loss of generality, as requiring zero initial cash outlay. Positive initial cash investments can be modeled by assuming that they are partly debt financed. Cash flows are always defined after appropriate interest costs.

2.2 Capital Structure Regulation

Financial institutions frequently find it advantageous to minimize their use of equity capital. This occurs for a number of reasons. First, due to liquidity and security offered to depositors, bank deposits are considered to be a cheap source of capital. Second, as in other industries, debt offers a tax subsidy due to deductibility of interest. Third, as emphasized by Merton and Perold (1993), a bank's customers are frequently also its debtholders so that the nature of banking requires leverage. The more "customers" a bank wants to attract, the more debt it must be prepared to accept in its capital structure. In our model, we capture this effect by assuming that the after-tax cost of deposits is zero.

On the other hand, depositors and other debtholders are only willing to lend their capital to a bank if it has a sufficient amount of equity capital to ensure its solvency. In addition regulators specify minimum equity standards which are based on various risk measures. Thus, a bank's capital structure is determined on the one hand by several advantages of debt, and on the other hand either by the bank's own desire to limit costs of financial distress or by regulatory constraints. If the equity capital required is greater than the investment outlay, it is invested in the riskless asset. This is assumed to incur deadweight costs which, as in Froot and Stein (1998) may be interpreted as a tax. To simplify the exposition, we normalize the bank's after-tax return on the riskless asset to zero.

The cost of equity capital raised by the institution, $r > 0$, will generally depend on the exposure of cash flows to various risk factors. While the question of the pricing of risk and the determination of the cost of capital is an interesting one, we abstract from this issue in this paper and assume that r is constant.

To enable the bank to realize the maximum benefits of leverage without violating regulatory constraints or incurring excessive transactions costs, most financial institutions have adopted sophisticated risk management systems to monitor and limit their risks in accordance to their equity capital. The basic concept is to specify a maximum loss, which may only be exceeded with a specified probability. This loss is referred to as *Value at Risk* (VaR). The value at risk is a function of the entire cash flow distribution and is measured by looking at the point on the lower tail where the probability is equal to a specified threshold value. When cash flows are normally distributed, the Value at Risk can be expressed as

$$\text{VaR} = \alpha\sigma, \tag{4}$$

where α is a factor defined by the probability with which the actual loss may not exceed the VaR;

σ is the standard deviation of cash flows on the institutions's portfolio over a specified time period.[3] Note that the definition of VaR in equation (4) does not take the expected return on investments into account.[4] There are several reasons why VaR is generally defined ignoring the mean return. First, since VaR calculations are usually made for short periods of time, i.e., for one to ten day periods, the first moments are of second order. Second, most VaR models used in practice do not take the mean return into account when calculating risk and, finally, regulators are conservative with respect to allowing banks' capital standards to be reduced by modeling the mean returns.

Bank management as well as regulators increasingly define capital requirements in terms of its VaR. More precisely, the equity capital of the bank, C, is given by the constraint[5]

$$C \geq \text{VaR} = \alpha\sigma. \tag{5}$$

According to equation (5) a bank is limited in its risk taking such that the resulting VaR does not exceed the amount of equity capital. This capital structure constraint ultimately determines the required capital allocation to investment projects or divisions.[6]

2.3 The Institution's Objective and Managerial Compensation

Based on the investment and regulatory environment described above, the net present value of the financial institution is

$$NPV = \frac{\sum_i \mu_i + C}{1+r} - C, \tag{6}$$

where r is the cost of capital. Maximizing NPV is equivalent to maximizing the *economic value added* (EVA), where

$$EVA = \sum_i \mu_i - rC. \tag{7}$$

[3]For a detailed analysis of the concept of Value at Risk, see Jorion (1997). Stulz (1998) demonstrates it's applicability in risk management contexts.

[4]Alternatively, VaR can be defined as $\text{VaR} = \alpha\sigma - \mu$, where μ denotes the expected cash flow on the bank's investments.

[5]Our formulation can be easily extended to the case where a non-regulated firm wishes to satisfy a constraint on its debt rating; here one may wish to redefine the VaR as mean-adjusted.

[6]The Basle Accord has been amended as of January 1, 1998 to allow banks to use their own internal models to assess risk. In this case a standard of 99% for ten days times a multiplier of between 3 and 4 is used for the bank's trading activities. That is, α is between 6.9 and 9.2, and σ is defined as the ten day standard deviation.

EVA represents the contribution to shareholder value.[7]

In our model the central authority of the financial institution wishes to maximize EVA after compensating the various divisions. Hence the institution's objective function is

$$I = EVA - \sum_i U_i, \tag{8}$$

where U_i denotes the transfer or compensation of business unit i. The transfer to the divisional managers must achieve two objectives. First, it must create incentives for the manager to expend some minimum effort to ensure a strictly positive θ_i. Second, the transfer to the division must induce optimal investment decisions for given θ_i.

Consistent with the overall objective of the firm, we consider the divisional compensation to be determined by divisional EVA as in

$$U_i = \gamma(\mu_i(\sigma_i, \theta_i) - rT_i), \tag{9}$$

where T_i is a function of certain variables that can be contracted upon. We refer to T_i as the *capital allocation* function. Thus, the manager receives a fraction of the economic value added by his division given the specified capital allocation rule. It is important to recognize that the sum of allocated capital, $\sum_i T_i$, does not necessarily add up to the bank's total capital, C. The optimal functional form of T_i is determined in the subsequent sections.

In the next section we solve for the optimal investment decision under full information from the standpoint of the institution. Then we show how this decision can be delegated at the divisional level.

3 The Full Information Optimization

In this section we develop the model under the conditions of full information, where the central authority of the financial institution and the trading division have the ability to contract fully on outcomes. The

[7] The correspondence between EVA and net present value (NPV) was perhaps first discussed by Preinreich (1937). The precise specifications of EVA and the corresponding depreciation schedule are discussed in Rogerson (1997) and Reichelstein (1997a), who showed that EVA-type measures represent the unique optimum among the class of linear measures. Unfortunately this uniqueness result depends critically on an assumption that cash flows are certain. The results are extended to the case of moral hazard in Reichelstein (1997b).

main purpose of this section is to derive the first-best outcome in order to facilitate comparison with the second-best mechanisms considered in the next section.

3.1 The First-best Problem

In the first-best situation, the institution makes all the decisions and compensates the trading division according to some minimum utility level. Given the information θ, the institution solves:

$$\max_{\sigma,C} \quad E[\mu(\sigma,\theta) - rC] \tag{10}$$

subject to

$$C \geq \alpha\sigma. \tag{11}$$

Equation (11) represents the regulatory constraint on total capital such that the VaR is limited at the appropriate level. The cost of capital, r, is applied to this amount to yield the EVA objective (10). Note that this objective function assumes that the institution is risk-neutral with respect to θ.

At this point we distinguish between two forms of capital constraints. We call *flexible capital* the situation in which the institution can adjust the amount of capital in response to information. That is C becomes a function of θ in this case. The second regime, which we call *inflexible capital* occurs when C must be chosen ex ante, before θ is known.

First, consider the inflexible capital case. C is raised and then may be used to satisfy the VaR constraint. If there are no alternative uses of capital other than for the set of investments represented by μ, then once C is fixed, the optimal decision is to select the highest possible σ subject to (11). This implies that the decision on risk will be independent of the information embodied in θ. It is straightforward to see that the optimal decision is given (implicitly) by the following condition:

$$E[\mu_\sigma(\sigma,\theta)] = r\alpha, \tag{12}$$

where μ_σ is the partial derivative with respect to the level of risk.

Second, consider the flexible capital case. Now the amount of total capital can be tailored to reflect the expected returns on the investment decision. Substituting $C = \alpha\sigma$ from equation (11) into (10) and maximizing pointwise over σ gives the following implicit expression for the optimal investment decision:

$$\mu_\sigma(\sigma, \theta) = r\alpha. \tag{13}$$

For instance, suppose $\mu(\sigma, \theta) = \theta \log \sigma$. Then the first order condition given by (13) yields $\sigma = \theta/(r\alpha)$. Thus, the optimal allocation of risk is increasing in the information variable and decreasing with respect to the cost of capital and the severity of the VaR requirement.

3.2 Delegation

We now consider what happens when the trading decision is delegated to the manager of a single division. The manager of the division begins with an expenditure of effort. If a minimum level of effort, e^*, is provided then the manager learns the information parameter θ. If this minimum level of effort is not expended, then $\theta = 0$.

The manager of the division is compensated according to some share, γ, of the divisional EVA, $\mu(\sigma, \theta) - rT(\sigma)$, where $T(\sigma)$ represents the capital allocated to the division. We assume throughout that it is optimal for the minimum level of effort to be provided by the manager. When effort is unobservable, the center must provide incentives through γ so that effort is undertaken. That is, γ must be set equal to the value such that

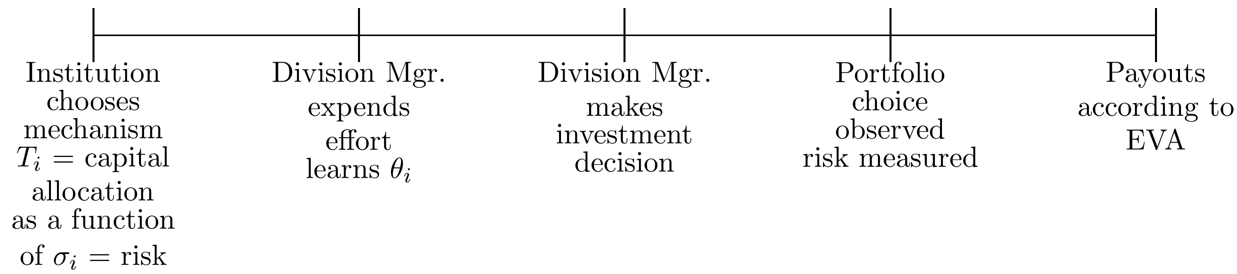
$$E[\gamma(\mu(\sigma, \theta) - rT(\sigma)) - e^*] \geq E[\gamma(\mu(\sigma, 0) - rT(\sigma))]. \tag{14}$$

Throughout the analysis, we assume that the headquarters sets $\gamma > 0$ to the value that solves (14). Therefore we focus on the subproblem that ensues once θ has been revealed. Figure 1 illustrates the sequence of decisions in the delegation problem.

3.2.1 The Institution's Problem

The financial institution maximizes its share, $(1 - \gamma)$, of the economic value added subject to the regulatory requirement that the total capital, C is at least equal to the amount prescribed by the value

Figure 1: Sequence of Events



at risk, VaR. Given that the financial institution delegates the decision on the risk level to the trading division, the overall problem to be solved is as follows:

$$\max_{\sigma,\bar{\sigma},T(\sigma),C} \quad I = E[\mu(\sigma,\theta) - rC - U] \tag{15}$$

subject to

$$\sigma \in \arg\max_{\sigma \leq \bar{\sigma}} \quad \gamma(\mu(\sigma,\theta) - rT(\sigma)), \tag{16}$$

$$U = \gamma(\mu(\sigma,\theta) - rT(\sigma)) \geq \underline{U} \tag{17}$$

$$C \geq \alpha\sigma. \tag{18}$$

Equation (18) is the regulatory constraint that assures that the financial institution must raise sufficient capital from the external market so that the VaR condition is satisfied. This capital is evaluated at the cost r in the objective function (15), in which the institution must subtract the amount of EVA allocated to the division. The delegation constraint is represented by (16) implying that the optimal investment decision is optimal from the standpoint of the divisional manager relative to EVA given the capital allocated to him, T. This formulation of the delegation problem allows the institution to set a *position limit*, $\bar{\sigma}$. Further the manager must be given some minimum (reservation) utility in order to be willing to commit the effort required to make the observation. This is embodied in (17).

Suppose that capital is flexible so that it can be chosen after information is known. Further, suppose that $T(\sigma)$ is differentiable and weakly convex and consider the first-order condition of (16). The solution to this problem then gives:

$$\mu_\sigma(\sigma,\theta) = rT_\sigma.$$

Notice that this is independent of γ. By contrasting this expression with the first-best solution (13), it is clear that if $T_\sigma = \alpha$ then the first-best risk allocation will be achieved in the delegation problem. Integrating this over σ then yields the following capital allocation function:

$$T(\sigma) = \alpha\sigma + \nu, \tag{19}$$

where ν is some constant. If σ^* denotes the first-best risk decision, then the constant is determined by the reservation utility constraint. Substituting into (17) yields

$$r\nu = \mu(\sigma^*,\theta) - r\alpha\sigma^* - (\underline{U}/\gamma). \tag{20}$$

We have shown that with flexible capital, the optimal capital allocation scheme (19) consists of two parts. The first part, which we term *economic capital* is VaR and is linear in the risk of the division. The second part is a constant *adjustment factor*, ν. In the full information problem this adjustment is negatively related to the amount of reservation utility required to keep the manager. Holding the reservation utility constant and varying the productivity of investment, this adjustment factor is positively related to the EVA ($\mu - r\alpha\sigma$) evaluated at the *optimal* risk level.

3.3 RAROC

We next explore the implications of the RAROC performance measure. As discussed in the introduction, many banks use the concept of RAROC for capital allocation and performance evaluation.[8] While there are numerous different definitions of RAROC used in practice, it is most commonly used in the following way:

$$RAROC = \frac{\text{(expected) return minus risk adjustment}}{\text{economic capital}}.$$

[8]See Matten (1996) for a very complete discussion of the various procedures for risk adjusted performance measurement.

One of the major contributions of the paper is in precisely characterizing the nature of this RAROC measure. Specifically we shall illustrate exactly how the risk adjustment should be determined as well as how economic capital should be computed.

Consider the following implementation of RAROC:

$$RAROC = \frac{\mu(\sigma, \theta) - r\nu - r\alpha\sigma}{\alpha\sigma^*}. \tag{21}$$

In this case, it is clear that net income is adjusted for realized risk in the numerator of this RAROC measure. The denominator is exactly equal to the economic capital term from equation (19) evaluated at the optimal risk level. Using this definition, it is easy to see that the institution's EVA is equal to

$$EVA = (RAROC)\alpha\sigma^*.$$

That is, EVA is the risk adjusted return on capital as applied to the allocated economic capital.

To study how this performance measure is related to the amount of optimal trading risk, we consider the expected value of RAROC. Under full information, using the above capital allocation schedule, the optimal risk level, σ^* will always be attained. To study the behavior of RAROC in this section, we will consider the possibility that somehow the wrong investment decision was made. Substituting (20) into (21) yields the following equivalent RAROC measure:

$$RAROC = \frac{\mu(\sigma, \theta) - r\alpha\sigma - (\mu(\sigma^*, \theta) - r\alpha\sigma^*) + (\underline{U}/\gamma)}{\alpha\sigma^*}. \tag{22}$$

It is clear from equation (22) and the fact that σ^* is optimal that this measure of RAROC attains its maximum when the optimal decision is taken. At this point,

$$RAROC = \frac{\underline{U}/\gamma}{\alpha\sigma^*}.$$

We summarize our result for the case of flexible capital in Proposition 1.

Proposition 1 *The optimal flexible capital allocation scheme, (19) consists of value at risk plus an* *adjustment factor related to the potential economic value added. Using the optimal flexible capital allocation scheme, the expected risk adjusted return on capital (RAROC) is maximized at the optimal level of risk,* σ^*.

Proposition 1 shows that RAROC is a perfect performance measure under the assumptions of full information. In general it will be positive at the optimal decision, with the magnitude of this value being related to the compensation necessary to induce the manager to become informed. For suboptimal levels of risk it is entirely possible that RAROC becomes negative.

Now consider what happens with inflexible capital. Again, supposing that capital cannot be put to alternative uses once it is raised, the first-best analysis has shown that the institution desires to implement some (constant) risk decision, σ_0, independent of θ. It is easy to see that if the institution uses a position limit $\bar{\sigma} = \sigma_0$ coupled with a constant capital allocation $T(\sigma) = \nu$, then the institution can implement the first-best in the delegation case with inflexible capital. This follows because the divisional manager will have the incentive to increase σ up to its maximum limit given in the constraint to (16).

We thus obtain the following proposition.

Proposition 2 *In the case of inflexible capital, the financial institution optimally delegates the investment decision to the division through a pure position limit coupled with a capital allocation that is independent of the risk attained. Using the optimal inflexible capital allocation scheme, the expected risk adjusted return on capital (RAROC) is maximized at the optimal level of risk,* σ_0.

3.4 Multidivisional Capital Allocation

We now consider the problem of a multidivisional firm with a central authority under complete information. As before we derive the first-best solution and show how it can be implemented in a delegation environment. The basic aspects of the single divisional model are preserved in this environment.

First consider the institution's problem when all decisions are centralized. With two divisions, let $\mu_i(\sigma_i, \theta_i)$ represent the expected cash flows from taking risk level σ_i for division $i; i = 1, 2$ given information θ_i. The overall risk of the portfolio of investments from the two divisions is defined to be

σ_p where

$$\sigma_p(\sigma_1, \sigma_2)^2 = \sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2. \tag{23}$$

For simplicity, we assume that information θ_i pertains only to trading division i and that it is independent across divisions.

The first-best problem therefore becomes:

$$\max_{\sigma_1, \sigma_2} \quad E[\mu_1(\sigma_1, \theta_1) + \mu_2(\sigma_2, \theta_2)] - rC, \tag{24}$$

where

$$C \geq \alpha\sigma_p(\sigma_1, \sigma_2). \tag{25}$$

We focus in this subsection on the flexible capital case; the inflexible capital situation would be handled analogously. As before, constraint (25) obviously holds with equality; substituting into (24) gives

$$\max_{\sigma_1, \sigma_2} \quad \mu_1(\sigma_1, \theta_1) + \mu_2(\sigma_1, \theta_2) - r\alpha\sigma_p(\sigma_1, \sigma_2).$$

The necessary condition for optimality then becomes:

$$\frac{\partial \mu_i(\sigma_i, \theta_i)}{\partial \sigma_i} = r\alpha \frac{\partial \sigma_p}{\partial \sigma_i}.$$

Simplifying this expression using (23) gives the following result:

$$\frac{\partial \mu_i(\sigma_i, \theta_i)}{\partial \sigma_i} = r\alpha \left[\frac{\sigma_i + \rho\sigma_j}{\sigma_p} \right], \tag{26}$$

for $i = 1, 2; j \neq i$.

To interpret this result we use the concept of *incremental value at risk* (IVaR), as defined here:

Definition The incremental value at risk, $v_i(\sigma_i, \sigma_j)$ for division i is defined as

$$v_i(\sigma_i, \sigma_j) = \alpha \frac{\sigma_i^2 + \rho\sigma_i\sigma_j}{\sigma_p}. \tag{27}$$

It can be shown that the incremental value at risk for each division is $[\partial(\alpha\sigma_p)/\partial\sigma_i]\sigma_i$. The incremental value at risk is proportional to the regression coefficient from a regression of the cash flows of division i on the institution's overall portfolio. Specifically, if β_{ip} is the regression coefficient, then $v_i = \alpha\beta_{ip}\sigma_p$. Not surprisingly therefore, the incremental value at risk has the property that $\alpha\sigma_p = v_1 + v_2$, i.e., the sum of the IVaRs equals the institution's overall VaR.

Substituting this definition into the first-order condition above gives the following representation for the optimal investment decision of each division:

$$\frac{\partial\mu_i(\sigma_i,\theta_i)}{\partial\sigma_i} = r\frac{v_i}{\sigma_i}. \tag{28}$$

That is, investment occurs at the point where the marginal increment in expected returns is balanced by the cost of the ratio of the incremental value at risk to the risk level chosen.

3.4.1 Delegation in the Multidivisional Problem

Given the perfect observability of the center, it is straightforward to find a delegation contract that implements the first-best. Indeed, suppose that the central authority establishes capital allocation rules, $T_i(\sigma_1,\sigma_2)$ for each division. Notice that this assumes that the capital allocation rules are interdependent. The need for this derives from the possible correlation of returns across trading divisions.

In the case of flexible capital, the formal delegation problem solves:

$$\max_{\sigma_i,\bar{\sigma}_i,T_i(\sigma_1,\sigma_2),C} \quad I = \mu_1(\sigma_1,\theta_1) + \mu_2(\sigma_2,\theta_2) - rC - U_1 - U_2 \tag{29}$$

subject to

$$\sigma_i \in \arg\max_{\sigma_i\le\bar{\sigma}_i} \quad \gamma(\mu_i(\sigma_i,\theta_i) - rT_i(\sigma_1,\sigma_2)), \tag{30}$$

$$U_i = \gamma(\mu_i(\sigma_i,\theta_i) - rT_i(\sigma_1,\sigma_2)) \ge \underline{U}_i \tag{31}$$

$$C \ge \alpha\sigma_p. \tag{32}$$

As in the single division case, the key constraint governing the incentives provided to the division is present through (30). As long as the position limit constraint $\bar{\sigma}_i$ is not operative, the first order condition for optimal incentives on the part of the division is

$$\frac{\partial \mu_i}{\partial \sigma_i} - r\frac{\partial T_i}{\partial \sigma_i} = 0. \tag{33}$$

Therefore (from (28) and (33)) the implementation condition requires that at the first-best risk levels, σ_i^* and σ_j^*

$$\frac{\partial T_i}{\partial \sigma_i} = \alpha \frac{\sigma_i^* + \rho \sigma_j^*}{\sigma_p^*}. \tag{34}$$

It therefore follows by integrating (34) that the optimal capital allocation functions are of the following form:

$$T_i(\sigma_1, \sigma_2) = \alpha \left[\frac{\sigma_i^* + \rho \sigma_j^*}{\sigma_p^*} \right] \sigma_i \;+\; \nu_i(\sigma_j),$$

where $\nu_i(\sigma_j)$ denotes a function that depends only on the risk taken by the other division.

Expressing this result in terms of the IVaR, we obtain the following characterization result:

Proposition 3 *In the multidivisional capital allocation problem under complete information, the optimal capital allocation scheme to each division satisfies*

$$T_i(\sigma_1, \sigma_2) = \nu_i(\sigma_j) + \lambda_i(\sigma_i^*, \sigma_j^*)\sigma_i, \tag{35}$$

where

$$\lambda_i(\sigma_i^*, \sigma_j^*) = \frac{v_i(\sigma_i^*, \sigma_j^*)}{\sigma_i^*} \equiv \frac{v_i^*}{\sigma_i^*} \tag{36}$$

and

$$r\nu_i(\sigma_j) = \mu_i(\sigma_i^*, \theta_i) - r\alpha v_i(\sigma_i^*, \sigma_j) - \frac{U_i}{\gamma}. \tag{37}$$

Notice that according to the the mechanism derived above, a division's capital allocation is a constant adjustment factor plus an amount of economic capital that is proportional only to its own standard deviation, σ_i. The proportionality factor, λ, however, depends on its marginal contribution to portfolio risk, $\partial \sigma_p / \partial \sigma_i$, evaluated at the optimal risk levels. In the multidivisional model, λ, can be interpreted as the "price" of division-specific risk. In solving the delegation problem, the divisional manager sees the impact of the divisional risk on other divisions only through the price itself. There is no need for one division to know the investment opportunities of other divisions.

Note that in the mechanism (35) above the amount of capital allocated to division i at the optimum risk level is equal to $v_i + \nu_i$. Since the IVaRs of the two division add up to total institution-wide VaR, the total capital allocation at the optimal risk level is given by

$$T_1(\sigma_i^*, \sigma_2^*) + T_2(\sigma_i^*, \sigma_j^*) = \alpha\sigma_p(\sigma_1^*, \sigma_2^*) + \nu_1(\sigma_2^*) + \nu_2(\sigma_1^*). \tag{38}$$

As in the single-division case the capital allocation levels are clearly related to the reservation utilities of the trading divisions. However, the key difference is that the variable capital allocation depends both on the standard deviation chosen by the division and on the covariance with the other division, evaluated at the optimal risk levels.

Once again, it is possible to characterize the RAROC performance measure. The definition consistent with the single division analysis is:

$$RAROC = \frac{\mu_i(\sigma_i) - r\nu_i(\sigma_j) - r\lambda_i\sigma_i}{\lambda_i\sigma_i^*}. \tag{39}$$

This definition provides the result that the EVA of division i at the optimal risk level is equal to RAROC times the amount of economic capital as given by the optimal IVaR:

$$EVA = (RAROC)v_i^*, \tag{40}$$

where v_i^* is the optimal incremental value at risk of division i. While the characterization of RAROC in (39) is similar to the single trading division, here we notice that the denominator weights the standard deviation of the division by its marginal contribution to portfolio risk. The performance measurement properties of RAROC are as in the single trading division case, as indicated in the following proposition.

Proposition 4 *In the multiple trading divisions case the expected risk adjusted return on capital for each division as defined in (39) is maximized at the joint optimal set of investment decisions.*

These results show that the appropriate use of capital allocation can be accomplished via a form of internal capital markets where each division faces its own price of risk and multiplies its own divisional risk (VaR) by this factor to determine its economic capital. The price internalizes the externality across divisions. This procedure stands in strong contrast to the case where a division is given some allocation of firm-wide VaR as economic capital and therefore is impacted by the decisions of other divisions.

It is quite possible that for some (but not all) division(s) within the institution, the IVaR will be negative, as it is derived as a regression coefficient on overall firm cash flows. What does the optimal capital allocation scheme prescribe in such as case? Here, rather than using up capital, the division is essentially freeing up capital for use by other divisions. This means that if we preserve our definition (39) of RAROC above, then divisional EVA and therefore divisional compensation should be redefined as

$$EVA = (-RAROC)v_i^*.$$

This is analogous to what happens in translating between internal rate of return and net present value in corporate finance. The usual case is that internal rates of return above costs of capital indicate value-creating projects. But it is also possible to have situations where projects that contribute capital create value when their internal rates of return are less than the cost of capital.

4 Asymmetric Information

Although the previous section derived an explicit functional form for the optimal capital allocation process, the major deficiency is that the mechanism depends on information that would likely be private to the trading division(s). In the case of equation (19) the adjustment to returns, $r\nu$ is a function of the expected return at the optimal risk level. The situation is even more sensitive in the multidivisional case. Equations (35) and (36) show that the dependence on information is critical. The IVaR of division i must be evaluated at the optimal portfolio decision of division j as well as that of the overall institution, σ_p. Further the deviation from IVaR is also a function of the overall portfolio risk at the optimum. Lastly, the adjustment function, ν_i depends on expected returns as in the single divisional case. Therefore it is of considerable interest to explore the optimal mechanism in the presence of private information of the divisions. The case studied in this section features a single trading division.

As in the previous section, we assume that the trading division must expend some unobservable effort to learn the information parameter, θ. This requires, as before that the division manager is allocated a fraction, γ of the divisional EVA. As before, we assume that it is optimal for the institution to encourage informational collection. Information is modeled as a single-dimensional draw, $\theta \in [\underline{\theta}, \bar{\theta}]$ given distribution function $F(\theta)$.

As is well-known in the theory of private information and agency (Fudenberg and Tirole, 1992), the optimal mechanism may be derived by using the revelation principle (Myerson, 1979). The direct revelation mechanism is $\sigma(\hat{\theta})$ and $T(\hat{\theta})$, where $\hat{\theta}$ represents the division's *report* of information or "type". The Bayesian Nash equilibria of the asymmetric information game are only those that can be supported by the *incentive compatibility* condition that $\hat{\theta} = \theta$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

Formally under asymmetric information, the problem to be solved by the financial institution is as follows:

$$\max_{\sigma(\theta), T(\theta), TC(\theta)} \quad I = E[\mu(\sigma(\theta), \theta) - rTC(\theta) - U(\theta)] \tag{41}$$

subject to

$$\theta \in \arg\max_{\hat{\theta}} \quad \gamma(\mu(\sigma(\hat{\theta}), \theta) - rT(\hat{\theta})), \tag{42}$$

$$U(\theta) = \gamma(\mu(\sigma(\theta), \theta) - rT(\theta)) \geq \underline{U} \tag{43}$$

$$TC(\theta) \geq \alpha\sigma(\theta). \tag{44}$$

The basic structure of this problem is analogous to the full information problem. The institution's objective is still to maximize total EVA of the institution less the compensation awarded to the trading division. The institutions's objective reflects the cost of the total capital raised as embodied in the regulatory restriction (44). Clearly this constraint will be met with equality. In designing the mechanism, the central authority must satisfy the *incentive compatibility* condition on truth-telling by the trading division (42) and must also provide a minimum level of utility in order to get the manager to participate (equation (43)).

4.1 Optimal Delegated Capital

As before, there are two possibilities where capital may be flexible or inflexible. As the flexible allocation scheme is more interesting, we focus entirely on that scenario here.

Consider the general problem in which the institution can select capital to respond to private information. The optimal mechanism involves a trade-off between investment efficiency and informational rent given to the trading division. This is a familiar situation given that this is a screening problem. The capital allocation used in the first-best case (19) is a increasing function of the private information of the division. As a result divisions with better information (higher θ) would intentionally misreport their information as lower than it really is in order to have a higher EVA. This creates information rent for the higher types that the institution would like to extract in the optimal second-best mechanism.

The major step in solving problem (41) is to first-convert the global incentive-compatibility condition (42) into a local representation. Following Fudenberg and Tirole (1992, p. 264) it can be shown that a necessary and sufficient condition for (42) to hold is that

$$U(\theta) = \underline{U} + \int_{\underline{\theta}}^{\theta} \gamma \mu_\theta(\sigma(\hat{\theta}), \hat{\theta}) d\hat{\theta}, \tag{45}$$

and $\sigma(\theta)$ non-decreasing.[9] Using this representation, the optimal risk of the institution solves the following problem:

$$\max_{\sigma(\theta)} \quad \int_{\underline{\theta}}^{\bar{\theta}} [\mu(\sigma(\theta), \theta) - r\alpha\sigma(\theta) - U(\theta)] dF(\theta). \tag{46}$$

subject to (45)

The solution to this problem is straightforward and is relegated to the appendix. Proposition 5 gives the solution to the problem.

Proposition 5 *The optimal risk taken by the division when capital is flexible is given by*

$$\mu_\sigma(\sigma(\theta), \theta) - r\alpha = \frac{\gamma(1 - F(\theta))}{F'(\theta)} \mu_{\theta\sigma}(\sigma(\theta), \theta). \tag{47}$$

Proposition 5 shows that when there is asymmetric information, optimal risk is reduced relative to the

[9] The sorting condition (3) is the critical assumption that is necessary to obtain this representation of the incentive compatibility conditions.

first-best level. This can be seen from (47) by noting that the right-hand side is positive. Therefore, the marginal benefit of taking on increased risk at the second-best optimum is greater than the marginal cost of increased VaR. The extent of this deviation is greater for lower types. This is intentional on the part of the institution as it strives to make it more costly for the better trading divisions to misreport. As an illustration of the optimal mechanism, we find that when $\mu(\sigma, \theta) = \theta \log \sigma$ and F is uniformly distributed,

$$\sigma(\theta) = \frac{(1+\gamma)\theta - \gamma\bar{\theta}}{r\alpha}.$$

Since our interest is primarily in deriving the optimal capital allocation rules, we now consider a transformation of the direct revelation mechanism. Consider the implementation of the optimal $\sigma(\theta)$ via the following incentive schedule:

$$\hat{T}(\hat{\theta}, \sigma) = \nu(\hat{\theta}) + \lambda(\hat{\theta})\sigma. \tag{48}$$

Suppose now that the division has "reported" $\hat{\theta}$, thereby determining the functional form of (48). Consider the sub-problem where the institution now allows the division to select the risk level according to:

$$\max_{\sigma} \quad \gamma(\mu(\sigma, \theta) - r\hat{T}(\hat{\theta}, \sigma)). \tag{49}$$

Definition The indirect mechanism $\hat{T}(\hat{\theta}, \sigma)$ *implements* the direct mechanism $\langle \sigma(\hat{\theta}), T(\hat{\theta}) \rangle$ whenever the solution to (49), $\hat{\sigma}(\hat{\theta}) = \sigma(\hat{\theta})$ and $\hat{T}(\hat{\theta}, \hat{\sigma}(\hat{\theta})) = T(\hat{\theta})$ for all $\hat{\theta}$.

Using this definition, we now see that a necessary condition for implementation of the optimal second-best mechanism is that

$$\mu_\sigma - r\hat{T}_\sigma = 0$$

yields the optimal decision according to (47). Substituting for the second-best optimality condition and the definition of $\hat{T}$ from (48) we arrive at

$$\lambda(\theta) = \alpha + \frac{\gamma(1 - F(\theta))}{rF'(\theta)} \mu_{\theta\sigma}(\sigma, \theta). \tag{50}$$

This result leads to the next proposition.

Proposition 6 *Under asymmetric information, the optimal second-best mechanism may be implemented via a modified VaR capital allocation schedule such that*

$$\hat{T}(\theta, \sigma) = \nu(\theta) + \left[\alpha + \frac{\gamma(1 - F(\theta))}{rF'(\theta)}\mu_{\theta\sigma}\right]\sigma, \tag{51}$$

where

$$r\nu(\theta) = -\int_{\underline{\theta}}^{\theta} r\sigma(\hat{\theta})d\lambda(\hat{\theta}) + \mu(\underline{\theta}) - r\lambda(\underline{\theta})\sigma(\underline{\theta}) - (\underline{U}/\gamma). \tag{52}$$

The proof of proposition 6 is in the appendix.

Proposition 6 therefore shows that the division is made marginally more sensitive to risk than at the first-best optimum according to the degree of information asymmetry and the share of divisional EVA going to the manager. That is, the institution uses a more conservative capital allocation rule than that under full information. Given the monotone hazard rate property, $(1 - F(\theta))/F'(\theta)$, decreasing in θ, it follows that generally λ is decreasing in the type of the trading division. This implies that trading divisions with better investment opportunities face less of a deviation from the VaR factor as faced by the institution itself.

The second part of the proposition illustrates the dependency on the fixed part of the capital allocation function, ν, on the VaR factor λ. As $\lambda(\theta)$ is a decreasing function, the integral is negative. Apart from the reservation utility, $\underline{U}$, this shows that ν is positive as long as $\mu - r\lambda\sigma$ is positive at the lower endpoint. Further, the above relation shows that ν is increasing in θ; therefore there is an inverse relationship between this fixed part of the capital allocation function and the VaR factor.

RAROC may be implemented under asymmetric information in a manner consistent with the full information analyses. Define expected RAROC as follows:

$$RAROC(\theta) = \frac{\mu(\sigma(\theta), \theta) - r\nu(\theta) - r\lambda(\theta)\sigma(\theta)}{\lambda(\theta)\sigma(\theta)}. \tag{53}$$

Now consider how RAROC behaves. At the lower endpoint, $\underline{\theta}$, substituting (52) into (53) shows that

$$RAROC = \frac{\underline{U}/\gamma}{\lambda(\underline{\theta})\sigma(\underline{\theta})}.$$

analogous to the full information case.

The performance measure is monotonically increasing in the information of the trading division so that better divisions are correctly identified *ex post*. Of course, given that the capital allocation schedule has already identified them *ex ante*, there is actually no reason in this single-period model to utilize performance measurement.[10]

5 Multidivisional Capital Allocation under Asymmetric Information

This section features the most general set of conditions in the paper. We consider a multidivisional financial institution where the divisions have private information about their expected cash flows from trading activity. The problem is first formulated as a direct revelation game in which divisions each report the value of their private information subject to a Bayesian Nash incentive compatibility condition.[11]

The direct revelation mechanism with two divisions is defined by the functions $\sigma_i(\theta_1, \theta_2); i = 1, 2$, denoting the risk level as a function of the information of the two divisions, and $T_i(\theta_1, \theta_2); i = 1, 2$, the capital allocation function. Information is represented by the joint distribution function, $F(\theta_1, \theta_2) \in [\underline{\theta}_1, \bar{\theta}_1] \times [\underline{\theta}_2, \bar{\theta}_2]$, exhibits independence with respect to (θ_1, θ_2). Given the Bayesian Nash structure of the problem, we use the following notation: a bar over a variable (e.g., $\bar{\mu}_i$) indicates that expectations are taken by division i with respect to the private information of division j. With these features, the

[10]For a multiperiod model of portfolio management in which performance measurement plays an important role see Heinkel and Stoughton (1994).

[11]Darrough and Stoughton (1989) discuss a joint venture mechanism design problem in which there is mutivariate private information. They derive the functional form of optimal screening mechanisms. Mookherjee and Reichelstein (1992) discuss the formulation of a multi-agent screening problem and the conditions under which implementation via *dominant* strategies is possible. Unfortunately due to the joint dependence through the portfolio effects their "condensation" condition does not hold. As a result we consider Nash implementation.

multidivisional problem for two divisions under joint asymmetric information is:

$$\max_{\sigma_i(\theta_1,\theta_2),T_i(\theta_1,\theta_2),TC} \quad I = E[\bar{\mu}_1(\sigma_1(\theta_1,\theta_2),\theta_1) + \bar{\mu}_2(\sigma_2(\theta_1,\theta_2),\theta_2) - rTC - \bar{U}_1 - \bar{U}_2] \tag{54}$$

subject to

$$\theta_i \in \arg\max_{\hat{\theta}_i} \quad \gamma(\bar{\mu}_i(\sigma_i(\hat{\theta}_i,\theta_j),\theta_i) - r\bar{T}_i(\hat{\theta}_i,\theta_j)), \tag{55}$$

$$\bar{U}_i = \gamma(\bar{\mu}_i(\sigma_i(\theta_1,\theta_2),\theta_i) - r\bar{T}_i(\theta_1,\theta_2)) \geq \underline{U}_i \tag{56}$$

$$TC \geq \alpha\sigma_p(\sigma_1(\theta_1,\theta_2),\sigma_2(\theta_1,\theta_2)). \tag{57}$$

The objective function of the institution, (54) reflects the need to take expectations over the joint distribution of divisional types, $F(\theta_1,\theta_2)$. The set of equations embodied in (55) indicates that each division reports the "true" value of private information while taking the truthful report of the other division as given. Similarly in (56) each division must take expectations over the other division's information parameter. The total capital regulatory constraint, (57) is as before.

Given the structure of the problem, it is fairly straightforward to apply analysis similar to that of section 4, except with respect to the two divisions. The incentive compatible representation condition for equations (55) is

$$\bar{U}_i(\theta_i) = \underline{U} + \int_{\underline{\theta}_i}^{\theta_i} \gamma\bar{\mu}_\theta(\sigma_i(\hat{\theta}_i,\theta_j),\hat{\theta}_i)d\hat{\theta}_i, \tag{58}$$

and the expectation (over θ_j) of the designated risk level for division i, $\bar{\sigma}_i(\theta_i)$, is non-decreasing in θ_i. By analogy with equation (47), we therefore find that the optimal joint levels of risk obtained in the multidivisional mechanism satisfies:

$$\mu_{i\sigma}(\sigma_i,\theta_i) - r\frac{v_i}{\sigma_i} - \gamma\mu_{i\theta\sigma}\left[\frac{1-F_i(\theta_i)}{F_i'(\theta_i)}\right] = 0, \tag{59}$$

where $v_i(\sigma_i,\sigma_j)$ is defined as before in (27), F_i denotes the marginal distribution of θ_i and $\sigma_i(\theta_i,\theta_j)$ depends jointly on the two divisions' private information.

The optimal solution to the multidivisional problem (59) is similar to the single divisional case, and exhibits less risk-taking than first-best. The degree of risk-taking is complicated by the interactive effect of both divisions in each of the three terms of (59). However the qualitative behavior of σ_i as a function of θ_i as θ_j is fixed is essentially the same as the single divisional analysis.

5.1 Implementation

In order to operationalize the above mechanism through an indirect mechanism where the risk level is delegated to each divisional manager, we propose the following multivariate mechanism: delegate the decision, σ_i to each division i so as to solve

$$\max_{\sigma_i} \quad \gamma(\mu_i(\sigma_i, \theta_i) - r\hat{T}_i(\hat{\theta}_i, \theta_j, \sigma_i)), \tag{60}$$

where

$$\hat{T}_i(\hat{\theta}_i, \theta_j, \sigma_i) = \bar{\nu}_i(\hat{\theta}_i) + \lambda_i(\hat{\theta}_i, \theta_j)\sigma_i. \tag{61}$$

That is, each division is presented with a menu of capital allocation schedules with a fixed component, $\bar{\nu}$ that does not depend on risk taken or the action of the other division. The risk factor, λ, however is impacted by the choice of the other division. Generally, with better private information of division j, the risk factor for division i will be greater. This induces a kind of internal capital market within the financial institution.

Proposition 7 establishes the functional form of this optimal indirect mechanism. It's proof is essentially the same as that of proposition 6.

Proposition 7 *The optimal multidivisional capital allocation mechanism can be implemented by a modified IVaR schedule such that*

$$\lambda(\theta_i, \theta_j) = \frac{v_i}{\sigma_i} + \frac{\gamma\mu_{i\theta\sigma}}{r}\frac{(1 - F_i(\theta_i))}{F_i'(\theta_i)}, \tag{62}$$

and

$$r\bar{\nu}_i(\theta_i) = -\int_{\underline{\theta}_i}^{\theta_i} r\bar{\sigma}_i(\hat{\theta}_i)d\bar{\lambda}_i(\hat{\theta}_i) + \bar{\mu}_i(\underline{\theta}_i) - r\bar{\lambda}_i(\underline{\theta}_i)\bar{\sigma}(\underline{\theta}_i) - (\underline{U}/\gamma), \tag{63}$$

where $\bar{\lambda}_i$ denotes expectations of λ_i with respect to θ_j.

Proposition 7 is the key to the representation of RAROC in the multidivisional firm under asymmetric information. We define

$$RAROC = \frac{\mu_i - r\bar{\nu}_i - r\lambda_i\sigma_i}{\lambda_i\sigma_i}. \tag{64}$$

This risk-adjusted return on capital exhibits the same properties with respect to each division as in the single division case: it perfectly indicates the divisions with more favorable investment opportunities.

6 Conclusion

Discussions with practitioners support the view that corporate CFOs as well as their counterparts at financial institutions view capital budgeting, risk management and capital structure as being inextricably linked. They seem less concerned with the determination and maintenance of an optimal debt to equity ratio in response to capital budgeting decisions. Rather, the main issue is to optimally allocate capital to various business units or investments and manage the resulting risks for a given financial structure. Although firms focus considerable attention on this internal capital allocation process, very little research has been done to provide management with robust normative rules. This paper attempts to make a contribution in this direction.

This paper analyzed decentralized capital budgeting decisions of financial institutions in the presence of capital adequacy restrictions. The results provide the capital allocation mechanism which induces optimal investment behavior by various business units. It is shown that optimal equity capital allocation is based on a business unit's contribution to the institution's total capital requirement, and can be decomposed into two parts. The economic capital term is measured by the division's incremental VaR. In the case of a single-division firm, this is the same as the firm's overall VaR. However, in the case of multiple business units, the economic capital is equal to a price of risk multiplied by the division's own standard deviation. The risk adjustment term from the capital allocation function is a constant related to the economic value added at the optimal investment level.

When there is asymmetric information about the business units' investment opportunity set, then the optimal capital allocation is changed. It is shown that asymmetric information makes the capital allocation more sensitive to risk taking. Thus, in the presence of asymmetric information, the bank allocates more capital for a given increase in volatility than in the presence of symmetric information.

The analysis also demonstrates how banks should calculate the risk adjusted return on capital, or RAROC to induce optimal investment decisions. It is found that a rigorous definition of RAROC involves in the denominator an economic capital amount equal to the unit's incremental VaR and in the numerator the business unit's expected or realized return minus an adjustment factor less a risk adjustment comprised of economic capital . This adjustment reflects both the economic value added at the optimal risk level and risk externalities imposed by one business unit on the incremental VaR of other business units. The mechanism derived has the property that RAROC is larger for division's that make optimal investment decisions and for those with more favorable investment opportunities. Furthermore, a business unit's economic value added equals RAROC times the unit's incremental VaR.

We believe that optimal capital allocation in the presence of capital adequacy regulation is a fruitful area for future research. For instance it would be interesting to consider multiperiod settings in which RAROC can be applied as a true performance measure. This extension would also more adequately address the situation where capital cannot respond immediately to changes to the investment opportunity set. Although tremendous advances have been made recently in the area of measuring and analyzing risk of financial institutions, questions have arisen about application of these modern methodologies. This paper represents a first step in developing a normative theory of the specialized nature of risk management in financial institutions.

A Appendix

A.1 Proof of Proposition 5

The solution to this problem is standard within the screening literature (Guesnerie and Laffont, 1984).

With flexible capital, there is no need to exceed the capital regulatory constraint given by (44). Using the representation of the incentive compatibility constraints (45), it is also clear that the reservation utility constraint is only binding at the lower endpoint $\underline{\theta}$. The institution's overall objective function can be written as the total EVA minus the utility of the division, $\mu - r\alpha\sigma - U(\theta)$.

Therefore the problem of (41) can be represented equivalently as:

$$\max_{\sigma(\theta)} \quad I = \int_{\underline{\theta}}^{\bar{\theta}} [\mu(\sigma(\theta), \theta) - r\alpha\sigma(\theta) - U(\theta)]dF(\theta), \tag{65}$$

subject to (45). Using integration-by-parts, we find that

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} U(\theta)dF(\theta) &= U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} F(\theta)dU(\theta) \\ &= U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu_\theta(\sigma, \theta)F(\theta)d\theta \\ &= U(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu_\theta(\sigma, \theta)(1 - F(\theta))d\theta, \end{aligned}$$

using (45). Substituting this expression into (65), gives the following maximization problem:

$$\begin{aligned} \max_{\sigma(\theta)} \quad I &= \int_{\underline{\theta}}^{\bar{\theta}} [\mu(\sigma, \theta) - r\alpha\sigma]dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} \gamma\mu_\theta(\sigma, \theta)(1 - F(\theta))d\theta - U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\mu(\sigma, \theta) - r\alpha\sigma - \gamma\mu_\theta(\sigma, \theta)\frac{(1 - F(\theta))}{F'(\theta)}\right] dF(\theta) - U(\underline{\theta}). \end{aligned}$$

The determination of the optimal $\sigma(\theta)$ can now be accomplished in a pointwise manner. This gives the following first order condition:

$$\mu_\sigma(\sigma, \theta) - r\alpha - \gamma\mu_{\sigma\theta}(\sigma, \theta)\frac{(1 - F(\theta))}{F'(\theta)} = 0. \qquad \square$$

A.2 Proof of Proposition 6

The first statement of the proposition (equation (51)) follows from the definition of $\hat{T}$ and equation (50).

To derive the optimal ν, substitute into the definition of the objective of the division to get

$$U(\theta) = \gamma(\mu(\sigma(\theta), \theta) - r\nu(\theta) - r\lambda(\theta)\sigma(\theta)).$$

Substituting the representation of utility under incentive compatibility, (45),

$$\underline{U} + \int_{\underline{\theta}}^{\theta} \gamma\mu_\theta(\sigma(\hat{\theta}), \hat{\theta})d\hat{\theta} = \gamma\mu(\theta) - \gamma r\nu(\theta) - \gamma r\lambda\sigma(\theta).$$

Rearranging provides the following sequence of expressions:

$$\begin{aligned}
\gamma r\nu(\theta) &= \gamma \int_{\mu(\underline{\theta})}^{\mu} d\mu' - \gamma r\lambda\sigma(\theta) - \int_{\underline{\theta}}^{\theta} \gamma\mu_\theta d\hat{\theta} + \gamma\mu(\underline{\theta}) - \underline{U} \\
&= \int_{\sigma(\underline{\theta})}^{\sigma(\theta)} \gamma\mu_\sigma d\sigma' - \gamma r\lambda\sigma + \gamma\mu(\underline{\theta}) - \underline{U} \\
&= \int_{\sigma(\underline{\theta})}^{\sigma(\theta)} \gamma r\lambda(\sigma^{-1}(\sigma'))d\sigma' - \gamma r\lambda(\theta)\sigma(\theta) + \gamma\mu(\underline{\theta}) - \underline{U} \\
&= -\gamma r\lambda(\underline{\theta})\sigma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \gamma r\sigma(\hat{\theta})d\lambda(\hat{\theta}) + \gamma\mu(\underline{\theta}) - \underline{U}.
\end{aligned}$$

Equation (52) is derived from the above equation. $\square$

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