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RATIONAL EXPECTATIONS AND LABOUR MARKET EQUILIBRIUM IN BRITAIN 1855-1913

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ABSTRACT*

This paper tests a two equation model of supply and demand for labour for 1857-1913, the period which was the focus of the original Phillips curve study. The basic structure is an equilibrium model of the labour market with "classical" characteristics arising from a surprise supply function and the assumption that expectations are formed rationally i.e. in a way consistent with the model itself. Tests of exclusion restrictions on a general reduced form tend to weakly reject these joint hypotheses. Tests on a structural model reject unanticipated wage change in favour of actual wage change as the appropriate variable in the supply function. This gives support to the original Phillips curve formulation.

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SUMMARY

In this paper the forces determining the wage rate and the rate of employment of labour in the British economy from 1857 to 1913 are analysed. This was the subject of the famous study which gave birth to the "Phillips Curve" relationship between wage changes and unemployment. Since Phillips wrote in 1958 the literature on labour market adjustment has increasingly focussed on models in which the labour market clears but in which there is imperfect information and slow adjustment.

The central idea of the paper is to set up a simple model which captures these elements but at the same time bears a resemblance to the Phillips curve. Thus the rate of employment depends positively on the deviation of actual from expected wages and on past values of the employment rate. Labour demand depends negatively on the wage and also on current and lagged values of other variables. In order to derive the expected wage from which to obtain the wage deviation, the rational expectations approach is applied. The expected wage is therefore assumed to be formed in a manner consistent with the model itself.

These assumptions give rise to different types of restrictions which provide tests of the model. One type is to take a fairly general model and test for the importance of variables which theory suggests should be excluded. The results of performing this test are not very decisive but suggest that the strict classical version of the model is not strongly supported. In order to determine the exact reason for these results a second type of test is used. This involves testing for specific values of parameters within and between supply and demand equations. The results suggest that the restrictions implied by the rational expectations postulate are not strongly rejected while those implied by the surprise supply function are. On testing another variant of the model it appears that it is the current change in the wage rather than the unanticipated change which important is

for labour supply. This supports the traditional interpretation of Phillips rather than the new classical interpretation of the labour market before the first World War.

I

In this paper we study the aggregate labour market in Britain for the half-century before 1914. It was for this period that Phillips (1958) estimated his famous relationship which gave rise to what is now a vast literature on the wage-unemployment trade-off[1]. Most of the subsequent literature on this important historical period has followed Phillip's approach in treating the relationship as measuring adjustment to Keynesian type labour market disequilibrium. In contrast, the model tested in this paper is one which assumes continuous market clearing and has strong classical properties. Such models estimated for the postwar period have produced mixed results and it might be expected that the model would be more strongly supported for the late nineteenth century.

The period before 1914 is often thought of as "classical" in the sense that the economy adjusted smoothly and rapidly, unimpeded by institutional rigidities. There were no periods of persistent disequilibrium comparable with that of 1921 - 1938 or periods of persistent and accelerating inflation such as have occurred since 1946. The government did not attempt systematically to manage the economy and the monetary authorities were constrained in their activities by the discipline of the international gold standard. In the labour market trade unions were relatively weak and there was no state system of unemployment insurance.[2] The limited dissemination of macroeconomic data and the costliness of acquiring available information suggests the appropriateness of a model based on the incomplete information paradigm .

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1 For an extensive survey of the literature produced by the first 20 years of debate, see Santomero and Seater (1978).

2 For a recent extensive survey of labour in the nineteenth century, see Hunt (1981), esp. pp. 286-341.

The model used is characterised by the supply function developed by Lucas (1973) adapted to the labour market so that variations in unemployment and therefore in economic activity as a whole, depend on the difference between the actual and expected wage rate. Expectations are formed rationally such that, given the available information, they are not systematically biased and are therefore formed in a way consistent with the underlying structure./ The Labour market is assumed to clear each period so that unlike the traditional Phillips curve literature, we specify a labour demand function to give a two-equation system. The course of the nominal wage and the unemployment rate are determined as the solution to this system.

In keeping with the recent literature we test the restrictions implied by the model. Sections II and III concentrate on the specification of a general model and the testing of exclusion restrictions on the general reduced form. In some respects these tests are unsatisfactory since the restrictions represent joint hypotheses and, if they are rejected, it is not clear why. Furthermore, they shed little light on labour market structure. In sections IV and V we turn to a more specific structural model which gives different types of restrictions: those implied by the assumption of rational expectations, and those arising from the particular form of supply function used. In the case where the latter are rejected, different variants of the model allow us to choose between well specified alternatives.

II

Our version of the labour supply function is cast in terms of the log of the employment rate $lnER = ln(100-U)$, where U is the percentage unemployed. The rate at which the labour force, in micro-market v supplies labour is composed of four components

$$lnER^s_t(v) = Y_t(v) + X_t(v) + \sum_{i=1}^{n} \beta_i \, lnER_{t-1}(v) + \varepsilon_{1t} \quad (1)$$

From the right they are a stochastic component which is common to all markets, a distributed lag in the employment rate, the surprise $X_t(v)$ and the systematic component $Y_t(v)$. Since the expectation of ε_{1t} and $X_t(v)$ is zero, the natural rate of unemployment is obtained from $lnER_t(v) = Y_t(v)/(1-\sum_{i=1}^{n} \beta_i)$. Thus the model is "classical" in the sense that there is no long run relationship between unemployment and other variables if we assume $Y_t(v)$ to be constant. The component $X_t(v)$ depends on the deviation between the currently observed wage in market v and that expected to prevail in the economy as a whole conditional on information currently available in market v .

$$X_t(v) = \omega\left[lnW_t(v) - E(v)lnW_t | I_t(v)\right] \quad (2)$$

Information set $I_t(v)$ consists of the currently observed wage in v and values of all relevant aggregates dated t-1 and earlier. It is assumed that consumption good prices do not vary cross sectionally so that nominal and real wage deviations are equivalent. The cross sectional wage variance, Var 1 and that of the time series forcast error, $lnW_t - ElnW_t$, Var 2 are known. Thus the expectation of lnW_t in v is:

$$E(v)(lnW_t | lnW_t(v), ElnW_t) = (1-\theta)lnW_t(v) + \theta ElnW_t \quad (3)$$

where θ = Var 1/(Var 1 + Var 2).

Substituting (3) into (2) and (2) into (1) gives

$$\ln ER_t^{s}(v) = Y_t(v) + \omega\theta\left|\ln W_t(v) - E(v)\ln W_t\right| + \sum_{i=1}^{n} \beta_i \ln ER(v)_{t-i} + \varepsilon_{1t} \tag{4}$$

Aggregating over micromarkets yields the economy wide (geometric) mean labour supply function as

$$\ln ER_t^S = \alpha_o + \alpha_1\left[\ln W_t - E\ln W_t\right] + \sum_{i=1}^{n} \beta_i \ln ER_{t-i} + \varepsilon_{1t} \tag{5}$$

where $\alpha_o = Y_t$ and $\alpha_1 = \omega\theta$.

The deviation from trend of labour demand is taken for the moment as a very general specification. It depends on the current and possibly lagged wage rate, a vector (Z) of other variables, such as prices or outputs dated t or earlier and a distributed lag of the employment rate as well as a random error

$$\ln ER_t^D = \phi_o + \sum_{i=1}^{m}\gamma_i \ln W_{t-i} + \sum_{i=0}^{k}\partial_i Z_{t-i} + \sum_{i=1}^{h}\lambda_i \ln ER_{t-i} + \varepsilon_{1t} \tag{6}$$

Equations (5) and (6) can be solved for the wage to give:

$$\ln W_t = \frac{\alpha_o-\phi_o}{\gamma_o-\alpha_1} - \frac{\alpha_1}{\gamma_o-\alpha_1} E\ln W_t + \frac{\sum_{i=1}^{n}\beta_i-\lambda_i}{\gamma_o-\alpha_1}\ln ER_{t-i} - \frac{\sum_{i=1}^{m}\gamma_i}{\gamma_o-\alpha_1}\ln W_{t-i}$$

$$- \frac{\partial_o}{\gamma_o-\alpha_1} Z_t - \frac{\sum_{i=1}^{k}\partial_L}{\gamma_o-\alpha_1} Z_{t-i} + \frac{\varepsilon_{it}-\varepsilon_{2t}}{\gamma_o-\alpha_1} \tag{7}$$

Taking the expectation at t conditioned on information at t-1 and earlier, the final term also drops out and Z_t is replaced by EZ_t. Thus the forecast error in logs is

$$\ln W_t - E\ln W_t = -\frac{\partial_o}{\gamma_o-\alpha_1}(Z_t - EZ_t) + \frac{\varepsilon_{1t}-\varepsilon_{2t}}{\gamma_o\ \alpha_1} \tag{8}$$

Substituting the solution for the expected wage into (7) we can rewrite the reduced form wage as

$$\ln W_t = \frac{\alpha_o - \phi_o}{\gamma_o} + \frac{\sum_{i=1}^{n} \beta_i - \lambda_i}{\gamma_o} \ln ER_{t-i} - \sum_{i=1}^{m} \ln W_{t-i} - \frac{\partial_o}{\gamma_o} EZ_t - \frac{\sum_{i=1}^{k}}{\gamma_o} Z_{t-i}$$

$$- \frac{\partial_o}{\gamma_o - \alpha_1} (Z_t - EZ_t) + \frac{\varepsilon_{1t} - \varepsilon_{2t}}{\gamma_o - \alpha_1} \qquad (9)$$

Using (8), the reduced form employment rate can be written as:

$$\ln ER_t = \alpha_o + \sum_{i=1}^{n} \beta_i \ln ER - \frac{\alpha_1 \partial_o}{\gamma_o - \alpha_1} (Z_t - EZ_t) + \frac{\gamma_o}{\gamma_o - \alpha_1} \varepsilon_{1t} - \frac{\alpha_1}{\gamma_o - \alpha_1} \varepsilon_{2t}$$

(10)

It follows from (10) that the employment rate is a function only of its own lagged values and a random serially uncorrelated term which is independent of all data at t-1 or earlier. From (9) the wage depends on lags of its own value and lags of the employment rate as well as the lagged values of Z entering directly and also through the expectation EZ_t. Thus for the estimating equation:

$$\ln ER_t = a + \sum_{i=1}^{n} \ln ER_{t-i} + \sum_{i=1}^{m} c_i \ln W_{t-1} + \sum_{i=1}^{k} d_i Z_{t-i} + U_{1t} \qquad (11)$$

it should not be possible to reject the null hypothesis that the c's and d's are jointly zero.

In the estimating equation for the wage:

$$\ln W_t = a' + \sum_{i=1}^{n} b'_i \ln ER_{t-i} + \sum_{i=1}^{m} c'_i \ln W_{t-i} + \sum_{i=1}^{k} d'_j Z_{t-j} + U_{2t}$$

(12)

the null hypothesis that the c''s and d''s are jointly zero should be rejected.

These tests have been employed by Sargent (1973, 1976) who pointed out that this is equivalent to using Granger's criterion for causality. Focusing on the jointly dependent variables above, the model predicts that, in Granger's sense, the employment rate causes the wage but the wage does not cause the employment rate. These predictions arise from three characteristics of the model. First, apart from the autoregressive term, only the wage surprise enters the supply function; the natural rate is not a function of other economic variables. Second, lags of the wage and other relevant variables enter the demand function (though, even if these were eliminated, they might still operate through the expectation EZ_t). Third, the structure used to generate the expected wage is rational in the sense that it is formed in a manner consistent with the model itself.

III

Annual observations on the wage and unemployment rates for 1855 - 1913 were taken from Feinstein (1972, T124-6, T140). These are essentially the same series as used by Phillips though not identical. The results of estimating a pure autoregressive model for the employment rate and wage rate for different lag lenths are given in Table I. These show significant coefficients for up to second order lags in the employment rate and fourth order in the wage. Though the autoregressive model explains a much larger proportion of the annual variation in the wage than in unemployment, the latter exhibits serial correlation, judged by the LM4 test.

Table I

	Autoregressive Variable					
	$\ln(ER)_t$	$\ln(ER)_t$	$\ln(ER)_t$	$\ln W_t$	$\ln W_t$	$\ln W_t$
Constant	2.9657 (0.4984)	3.1109 (0.6311)	3.0839 (0.7666)	0.1265 (0.1016)	0.1737 (0.0971)	0.1530 (0.1003)
Lag 1	0.8751 (0.1212)	0.8040 (0.1387)	0.7964 (0.1399)	1.5083 (0.1157)	1.5614 (0.1230)	1.5196 (0.1365)
Lag 2	-0.4663 (0.1213)	-0.3318 (0.1718)	-0.3150 (0.1785)	-0.5362 (0.1157)	-0.7383 (0.2184)	-0.5446 (0.2469)
Lag 3		-0.1545 (0.1389)	-0.1569 (0.1788)		0.1387 (0.1273)	-0.2558 (0.2388)
Lag 4			-0.0007 (0.1403)			0.2474 (0.1270)
$\bar{R}^2$	0.4741	0.4764	0.4749	0.9710	0.9737	0.9726
R.S.S.	0.0176	0.0172	0.0165	0.0168	0.0138	0.0129
D.W.	2.1104	1.9527	1.7000	1.7427	2.0837	1.9918
BP4	1.8793	1.9676	0.6850	3.9485	5.1791	1.0465
LM4	2.2287	2.3632	3.1735	11.5767*	16.3072**	17.8640**

BP4 and LM4 are test statistics for Box Pierce and Lagrange Multiplier tests for 4th order serial correlation. ** denotes significance at the 1% level, * at the 5% level.

Table II

Dependent Variable: $\ln ER_t$

Order of lag	Constant	lnER	lnW	lnP	lnQ
	3.5828 (0.7534)				
t-1		0.7230 (0.2274)	0.4867 (0.1840)	-0.0641 (0.1539)	-0.0046 (0.1903)
t-2		-0.8176 (0.2649)	-0.8386 (0.2916)	0.4794 (0.2062)	0.3698 (0.2140)
t-3		0.2133 (0.2208)	0.4152 (0.1923)	-0.3655 (0.1470)	-0.3867 (0.1954)

$\bar{R}^2$ = 0.5439 R.S.S. = 0.0123, D.W. = 2.1409, BP4 = 1.0896, LM4 = 4.452

Dependent Variable: $\ln W_t$

Order of lag	Constant	lnER	lnW	lnP	lnQ
	1.0013 (0.6526)				
t-1		0.2605 (0.1970)	1.3655 (0.1594)	-0.0843 (0.1333)	-0.0329 (0.1648)
t-2		-0.4879 (0.2295)	-0.4051 (0.2525)	0.0046 (0.1786)	0.3551 (0.1854)
t-3		0.0843 (0.1333)	-0.1440 (0.1665)	0.1300 (0.1273)	-0.2653 (0.1693)

$\bar{R}^2$ = 0.9786, R.S.S. = 0.0093, D.W. = 2.0462, BP4 = 1.0830, LM4 = 4.396

BP4 and LM4 are test statistics for Box-Pierce and Lagrange Multiplier tests for 4th order serial correlation.

Table III

R.H.S. Variable	L.H.S. Variable					
	$lnER_t$	$lnER_t$	$lnER_t$	lnW_t	lnW_t	lnW_t
Constant	3.3866 (0.5672)	3.3950 (0.7571)	3.5606 (0.9162)	0.7534 (0.5392)	1.1145 (0.6447)	1.5815 (0.7678)
$lnER_{t-1}$	0.7029 (0.1392)	0.6776 (0.1525)	0.7124 (0.1568)	0.1850 (0.1323)	0.1541 (0.1298)	0.1500 (0.1314)
$lnER_{t-2}$	−0.4330 (0.1186)	−0.3157 (0.1749)	−0.3154 (0.1846)	−0.3268 (0.1128)	−0.1856 (0.1489)	−0.2402 (0.1547)
$lnER_{t-3}$		−0.0947 (0.1395)	−0.1758 (0.1853)		−0.1751 (0.1878)	−0.0071 (0.1553)
$lnER_{t-4}$			0.0194 (0.1445)			−0.0209 (0.1211)
lnW_{t-1}	0.3125 (0.1049)	0.3287 (0.1593)	0.2526 (0.1835)	1.4891 (0.1339)	1.3998 (0.1356)	1.3333 (0.1538)
lnW_{t-2}	−0.3259 (0.1413)	−0.4467 (0.2671)	−0.4169 (0.2915)	−0.5126 (0.1343)	−0.4073 (0.2275)	−0.3396 (0.2443)
lnW_{t-3}		0.1056 (0.1649)	0.3002 (0.2850)		−0.0304 (0.1404)	−0.0718 (0.2388)
lnW_{t-4}			−0.1582 (0.1691)			0.0370 (0.1417)
$\bar{R}^2$	0.5056	0.4948	0.4771	0.9741	0.9771	0.9754
R.S.S.	0.0160	0.0156	0.0151	0.0144	0.0113	0.0106
D.W.	2.0168	1.9482	1.8380	2.0276	2.1299	2.0014
BP4	1.2320	0.0380	0.6097	2.6967	2.5739	1.9661
LM4	2.3541	3.8192	5.5605	8.4873	6.1824	4.3285

BP4 and LM4 are test statistics for Box-Pierce and Lagrange Multiplier tests for 4th order serial correlation.

Under the maintained hypothesis that the model previously specified is appropriate, its essential properties can be obtained from the autoregression in the employment rate. These indicate that the process is stable with the long-run solution or natural rate emerging at values very close to the mean of the series - equivalent to an unemployment rate of 3.8%.

Unrestricted models were estimated for each of the restricted versions in Table I by including additional variables with lags up to the same order. Besides lags of the two jointly dependent variables, two other variables which might be expected to influence the wage and employment are included i.e. the variables which are represented by Z in equation (2). These are the GDP deflator and the index of real GDP (Feinstein, 1972, T18-19, T132)[1]. For illustration, the equations for third order lags are given in Table II. In addition, a series of bivariate equations were estimated for the wage and employment rates and each of the other variables. The results for the employment-wage equations are given in Table III.

These tables illustrate that, for both dependent variables, additional lagged variables were found to give some individually significant coefficients. The relevant computed values for the likelihood ratio test are given in Table IV for both F and χ^2 distributions. The first panel gives some evidence that the three additional variables improve the prediction of the wage consistent with the theory but also some weak evidence that they improve the prediction of the employment rate. The middle panel shows that, in the bivariate relationship between the wage and employment rates, lags of the latter explain the former but not the reverse. The third panel indicates that adding the two additional variables gives a marginal improvement in the prediction of each variable and from the other bivarate regressions, it is clear that this predictive power comes from output rather than the price level.

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[1] The GDP deflator series begins only in 1870 so for 1855 to 1869 a series was generated using a regression of the GDP deflator on four other price series, a constant and a time trend for 1870-1913. The four series were the cost of living index (Feinstein, 1972, T140), import and export prices (Imlah, 1959, pp. 94-98) and the Rousseaux index for the prices of principal industrial products (Mitchell and Deane, 1962, p. 471).

Table IV

Test Statistics for Restrictions

	L.H.S. Variable: $\ln ER_t$			L.H.S. Variable : $\ln W_t$		
	2 lags	3 lags	4 lags	2 lags	3 lags	4 lags
	Restricted Table I vs Unrestricted Table II					
F	1.2026	1.8495	1.6269	3.2000*	2.3118*	1.4754
χ^2	7.8426	18.3239*	23.2175*	18.8424**	22.1006**	21.4186*
	Restricted Table I vs Unrestricted Table III					
F	2.6000	1.6750	1.0662	4.3333*	3.6136*	2.4953
χ^2	5.4327	5.4678	4.8766	8.7866*	11.1925*	10.8000*
	Restricted Table III vs Unrestricted Table II					
F	0.5490	1.8495	1.8303	2.4000	1.5412	0.9716
χ^2	2.5052	12.8562*	18.2522*	10.2100*	10.9081	10.4217

* indicates significance at the 5% level, ** at the 1% level.

Most of the test statistics are on the borderline of significance at the 5% level and, hence, the data does not discriminate sharply between hypotheses and provides only weak support for the model. These results give little insight into exactly why the model is only weakly supported - whether due to some element of structural misspecification or because expectations are non rational. In order to pursue these issues, we turn to examining a structural specification.

IV

In order to perform tests on structural equations, we retain the supply function of equation (5) but derive an equation for labour demand as the short run marginal productivity condition. The underlying production function is taken to be C.E.S. of the form

$$Q_t = Ae^{\tau t}\left[\psi E_t^{-\eta} + (1-\psi)K_t^{-\eta}\right]^{\frac{\mu}{\eta}} \tag{13}$$

where Q is output, E employment and K the capital stock. τ is the parameter determining the rate of disembodied technical progress, $\frac{1}{1+\eta}$ gives the elasticity of factor substitution and μ the degree of returns to scale. Taking the capital stock as pre-determined, the first order condition for profit maximisation gives the labour demand function

$$E_t^D = A^{\frac{\eta}{(1+\eta)\mu}} (\mu\ \psi)^{\frac{1}{1+\eta}} e^{\frac{\eta\tau}{(1+\eta)\mu}t} \left(\frac{W}{P}\right)_t^{\frac{-1}{1+\eta}} Q_t^{\frac{1+\eta/\mu}{1+\eta}} \tag{14}$$

where the maximised value of output has been substituted back into the expression solving out for K_t and giving an expression linear in logs. To convert this to an employment rate, the trend in the growth of the labour force is represented as

$$L_t = L_o e^{\phi t} \tag{15}$$

Dividing (15) into (14) taking logs and adding a stochastic error term gives the employment rate demand equation for ln(E/L) = lnER

$$\ln ER_t^D = \sigma_o + \sigma_1 \ln\left(\frac{W}{P}\right)_t + \sigma_2 \ln Q_t + \sigma_3 t + \varepsilon_{2t} \tag{16}$$

where

$$\sigma_1 = \frac{-1}{1+\eta},\ \sigma_2 = \frac{1+\eta/\mu}{1+\eta},\quad \sigma_3 = \frac{\eta\tau}{(1+\eta)\mu} - \phi$$

This simple demand side structure is adhered to as a maintained hypothesis and only variants of the supply side are tested. This is justified largely on the pragmatic ground that this particular function appeared to be empirically robust. It should be noted that there are no lags involved so that firms are always on their marginal product schedule. By itself, this would tend to undermine the tests of section III though if the expectation of the current values of product price and output are related to past values, they may remain valid. Our basic system then, consists of equations (5) and (16).

The restrictions implied by fully rational expectations can be obtained as before by solving for the reduced form wage, taking expectations and substituting back into (5) to give the supply equation as

$$\ln ER_t^S = \alpha_o + \frac{\alpha_1(\sigma_o - \alpha_o)}{\sigma_1} + \alpha_1(\ln W_t - E\ln P_t) + (1 - \frac{\alpha_1}{\sigma_1})\sum_{i=1}^{n}\beta_i \ln ER_{t-i}$$

$$+ \frac{\alpha_1\sigma_2}{\sigma_1} E_{t-1}\ln Q_t + \frac{\alpha_1\sigma_3}{\sigma_1}t + \varepsilon_{1t} \qquad (17)$$

In the system formed by (16) and (17) there are two non-linear cross equation restrictions implied by the parameters $\frac{\alpha_1\sigma_2}{\sigma_1}$ and $\frac{\alpha_1\sigma_3}{\sigma_1}$

There are now two types of restriction which can be tested following the approach used by Mishkin (1982) and Liederman (1981). The first allows the expected wage to enter the structural equation in addition to the surprise but the expectation to be formed consistently with this new structure. Adding the term $\rho E\ln W_t$ into equation (5) and solving through as before gives

$$\ln ER_t^S = \alpha_o + \frac{(\alpha_1-\rho)(\alpha_1-\sigma_o)}{\sigma_1-\rho} + \alpha_1 \ln W_t - \frac{(\alpha_1-\rho)\sigma_1}{\sigma_1-\rho} E\ln P_t$$

$$+ \left(1 - \frac{-(\alpha_1-\rho)}{\sigma_1-\rho}\right)\sum_{i=1}^{n}\beta_i \ln ER_{t-i} + \frac{(\alpha_1-\rho)\sigma_2}{\sigma_1-\rho} E\ln Q_t + \frac{(\alpha_1-\rho)\sigma_3}{\sigma_1-\rho}t + \varepsilon_{1t}$$

(18)

The test of the systemmatic effect of expected wage change is that $\rho \neq$ with the cross equation restrictions imposed. This may be regarded as a test of "expected wage neutrality" under the maintained hypothesis of expectations formed rationally. Similarly testing the cross equation restrictions in (17) can be regarded as a test of "rationality" under the maintained hypothesis of "neutrality". In addition, each of these restrictions can be tested without the other imposed. If these restrictions are rejected, particularly the within equation restriction, this raises the question of exactly what the alternative model is.

For a labour supply function it may be more appealing to pose an alternative model which is more consistent with the theory of labour supply. One obvious suggestion would be to make Y_t in equation (5) depend on the real wage facing workers: $Y_t = \alpha_o + \alpha_2(\ln W_t - \ln C_t)$ where C is the cost of living index. α_2 could be positive or negative but in either case the supply function becomes non-neutral in the wage. Following the same procedure as before gives the labour supply equation as

$$\begin{aligned} \ln ER^s = \alpha_o - \frac{\alpha_1(\alpha_o - \sigma_o)}{\sigma_1 - \alpha_2} + (\alpha_1 + \alpha_2) \ln W_t - \left(1 - \frac{\alpha_1}{\alpha_1 - \alpha_2}\right)\alpha_2 \ln C_t \\ - \frac{\alpha_1 \sigma_1}{\alpha_1 - \alpha_2} E\ln P_t + \frac{\alpha_1 \sigma_2}{\sigma_1 - \alpha_2} E\ln Q_t + \frac{\alpha_1 \sigma_3 t}{\sigma_1 - \alpha_2} + \\ + \left(1 - \frac{\alpha_1}{\sigma_1 - \alpha_2}\right) \sum_{i=1}^{n} \beta_i \ln ER_{t-i} + \varepsilon_{1t} \end{aligned} \tag{19}$$

In this new system formed by (16) and (19) the cross-equation and within equations are now no longer independent and hence one cannot readily test for neutrality and rationality independently unless $\alpha_2 = 0$ in which case the system collapses back into the previous model.

A second alternative model is suggested by the original Phillips curve literature, namely that labour supply depends on wage change instead of, or in addition to, the wage surprise. In equation (5) in place of the aggregate surprise X_t, we now have: $X_t = \alpha_1 \left[\ln W_t - E\ln W_t \right] + \alpha_3 \left[E\ln W_t - \ln W_{t-1} \right]$. In the case where $\alpha_1 = \alpha_3$ we have a pure Phillips curve in the sense that only the current change in the wage matters. Solving this model through for the expected wage and substituting back into the structural equation gives:

$$
\begin{aligned}
\ln ER^S = {} & \alpha_o - \frac{(\alpha_1-\alpha_3)(\alpha_o-\sigma_o)}{\sigma_1-\alpha_3} + \alpha_1 \ln W_t - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_3)} \sigma_1 \ln P_t \\
& - \alpha_3 \left(1 - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_3)}\right) \ln W_{t-1} + \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_3)} \sigma_2 \ln Q_t + \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_3)} \sigma_3 t \\
& + \sum_{i=1}^{n} \beta_i \left(1 - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_3)}\right) \ln ER_{t-i} + \varepsilon_{1t}
\end{aligned} \tag{20}
$$

As before the system formed by (20) and (16) has relatively complex restrictions but if $\alpha_1 = \alpha_3$, the terms involving the σ's drop out. Applying this restriction gives the simple Phillips curve:

$$
\ln ER^S = \alpha_o + \alpha_1(\ln W_t - \ln W_{t-1}) + \sum_{i=1}^{n} \beta_i \ln ER_{t-i} + \varepsilon_{it} \tag{21}
$$

Finally one may wish to test the two variants given by (19) and (20) against a more general model involving both a real wage term and a Phillips curve term. Solving and substituting as before gives:

$$
\begin{aligned}
\ln ER^S = {} & \alpha_o - \frac{(\alpha_1-\alpha_2)(\alpha_o-\sigma_o)}{(\sigma_1 - \alpha_3)} + (\alpha_1 + \alpha_2)\ln W_t - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_2-\alpha_3)} \sigma_2 E\ln P_t \\
& - \alpha_2 \quad 1 - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_2-\alpha_3)} \ln C_t - \alpha_3 \quad 1 - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_2-\alpha_3)} \ln W_{t-1} \\
& + \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_2-\alpha_3)} \sigma_2 E\ln Q_t + \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_2-\alpha_3)} \sigma_3 t \\
& + \left(1 - \frac{(\alpha_1-\alpha_3)}{(\sigma_1-\alpha_2-\alpha_3)}\right) \sum_{i=1}^{n} \beta_i \ln ER_{t-i} + \varepsilon_{1t}
\end{aligned} \tag{22}
$$

V.

In this section we estimate the models set out previously and test the relevant restrictions. In order to obtain unbiased estimates of the structural parameters, appropriate steps must be taken to form the expectations of lnP_t and lnQ_t which appear in the supply equation, and to purge their observed counterparts of their obvious endogeneity when they appear in the demand equation. A simple approach is to estimate equations for these using appropriate independent variables, placing no restriction on the first stage estimates, and use the fitted values at the second stage. Given that we do not have a model of the rest of the economy which would be required to obtain restrictions, this is an appropriate method.

The expectations of lnP_t and lnQ_t are formed on information at time t-1 or earlier, hence the expectations must exclude current information. Thus the variables $ElnP_t$ and $WlnQ_t$ used in estimation are formed by a first stage regression on lagged values of the logs of all the variables in the model, namely lnW, lnER, lnP and lnQ with the constant and time trend. In order to help in identification, the instrumental variables for lnP_t and lnQ_t must include some current information to distinguish them from their expected values. In addition to the variables entering the expectations, the current value of four price series used previously to generate missing observations of lnP_t are included as well as the volume indices for export of goods and services and gross domestic fixed capital formation (Imlah, 1959, pp. 94-98, Feinstein, 1972, T85). The lag length in the supply function was set at n = 3 which was also the highest order used in first stage regressions.

Estimation of the structural model was undertaken, using Full Information Maximum Likelihood on RSMI. The most restricted model was estimated using 2SLS estimates as starting values and then using the resulting estimates as starting values in turn, for the less restricted models. The parameter estimates for the most restricted model (16), (17) are given in the first column of Table V. Testing the two sets of restrictions with and without the other imposed gave the following results:

Test Statistics for Restrictions (X^2)

Test of "neutrality"		Test of "rationality"	
restricted	unrestricted	restricted	unrestricted
59.416**	54.600**	6.888*	2.072

** significant at 1%   * significant at 5%

The test statistics clearly reject the within equation restriction whether or not the cross equation restriction is imposed. The cross equation restrictions are only rejected marginally when the other restriction is applied.

Table V gives the results for three other systems with all the restrictions imposed and the test statistic below each equation is the X^2 for all the restrictions together. For the model including the real wage (16), (19), the index for the cost of living was again taken from Feinstein (1972, p. T 140)[1]. The additional term α_2 takes a very small and insignificant coefficient and does not change the other parameter estimates very

[1] The set of regressors used in the first stage to obtain the instrumental variables and forecasts for $\ln P_t$ and $\ln Q_t$ were not changed since the current value of the cost of living index is one of the regressors for the former. The current value is assumed to be exogenous and known by workers and is used directly in the estimation.

much. In the joint test, all the restrictions together are still massively rejected. But when the model formed by (16) and (20) is estimated the results change considerably and, taken together, the restrictions cannot be rejected. The point estimates of α_1 and α_3 are very nearly equal indicating that the other restrictions are virtually redundant. Thus the labour supply function looks more like a pure Phillips curve than a surprise supply function. It is interesting also to note the reduced size of the coefficients on the lagged dependent variables which are now all insignificant. Not surprisingly, when (16), (19) and (16), (20) are nested in the more general model represented by (16), ((22), (the results of which are not reported), the restriction implied by the former is rejected but not the latter[1].

Given our findings, we take the most parsimonious specification by estimating (16), (21) but with only one lagged dependent variable (n = 1) These restrictions cannot be rejected against the full model represented by (16), (20) even at the 1% level[2]. Thus the final specification looks remarkably like the original Phillips curve except that there are very simple first order dynamics represented by the lagged dependent variable and the equation is estimated jointly with a labour demand function. The zero wage change unemployment rate implied by this supply function is 5.56% which is almost identical to that estimated by Phillips for 1861 to 1913.

Additional support for the model is given by the parameter estimates of the labour demand curve. In each case it is downward sloping with a significantly negative coefficient, though if this value is interpreted as the elasticity of factor substitution, it is rather low. The coefficient

---

[1] The χ^2 values are, for one restriction, 144.2** and 0.784 respectively.

[2] The χ^2 value for 3 restrictions is 3.242.

Table V

FIML Parameter Estimates

	Equation System			
	16, 17	16, 19	16, 20	16,21
α_o	2.8590 (0.4406)	2.9420 (0.4394)	2.5954 (0.7091)	3.2272 (0.5141)
α_1	0.9411 (0.6087)	0.9047 (0.5740)	1.3940 (0.3639)	1.2234 (0.2102)
α_2		-0.0065 (0.0115)		
α_3			1.3814 (0.4284)	
β_1	0.7192 (0.1050)	0.7122 (0.1042)	0.2113 (0.2043)	0.2904 (0.1129)
β_2	0.1759 (0.1102)	0.1734 (0.1143)	0.0525 (0.1936)	
β_3	0.1782 (0.0943)	-0.1842 (0.0935)	0.1649 (0.1875)	
σ_o	1.0307 (0.3969)	1.0458 (0.3988)	1.1619 (0.3111)	1.1456 (0.3116)
σ_1	-0.3083 (0.0643)	-0.3130 (0.0638)	-0.1843 (0.0348)	-0.1869 (0.0344)
σ_2	0.9746 (0.1098)	0.9703 (0.1103)	0.9496 (0.0875)	0.9538 (0.0876)
σ_3	-0.0156 (0.0018)	-0.0155 (0.0018)	-0.0161 (0.0015)	-0.0161 (0.0015)
χ^2 Statistic for restrictions	61.488**	62.384**	5.320	1.473
No. of restrictions	3	3	3	1

on output is strongly significant and numerically close to the value 1 which, from the production function, implies constant returns to scale. Finally the coefficient on time indicates that the combined effect of technical progress and labour force growth is significant and negative but small as would have been expected.

VI

Conclusions from the results must be drawn with the utmost caution. For one thing, the data for the period before 1914 is subject to a wide margin of error. Even so, it has been widely used in quantitative work particularly that following in the tradition of Phillips (1958). Given the influential nature of this work, it is legitimate to ask whether the historical data in which the distinctive Phillips curve was first identified is consistent with a model of labour market equilibrium with rational expectations. The tests on general reduced form equations do not support the extremely classical version of the model very strongly and, in this respect, they are similar to the results of Sargent (1973, 1976).

Such findings do not take one very far because there are several reasons why such hypotheses might be rejected. Further insight can only be gained from estimating a structural model using a framework within which alternative hypotheses can be tested. The evidence from this exercise indicates that one can clearly identify aggregate supply and demand curves for labour. Furthermore, within this model the process of expectations formation and the structure of the model are not inconsistent. However, there appears to have been some long term trade off between the nominal wage and the rate of employment which cannot be accounted for by labour supply being a positive function of the real wage.

On further investigation, the current period wage change appears to dominate the wage surprise derived from rational expectations. This gives strong support to the original Phillips formulation and suggests that Phillips' results cannot be automatically translated into contemporary models.

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