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**THE TERMS OF TRADE, LABOUR SUPPLY, AND THE CURRENT ACCOUNT**

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The Terms of Trade, Labour Supply, and the Current Account

ABSTRACT\*

This paper extends Svensson and Razin's two period analysis of the Laursen-Harberger-Metzler effect to the important case where labour supply and output are variable. A terms of trade shift alters the relationship between the product and the consumption wage and so induces a change in output. This extension substantially enriches their analysis. A temporary current terms of trade deterioration has an ambiguous effect on the current account, but their finding that a future deterioration leads to an improvement in the current account is strengthened. The effect of a permanent terms of trade shift in a stationary state depends not only on the rate of time preference, but also on the strength of intertemporal substitution effects. In the canonical case of a constant rate of time preference the current account deteriorates in response to a permanent terms of trade deterioration if the degree of intertemporal substitution in consumption exceeds that in leisure, or equivalently whether the wealth effect on consumption exceeds that on labour supply.

The model is then extended to an infinite horizon by embedding it in an overlapping generations framework. Intergenerational linkages through the labour market are now the source of some complex dynamics. From an initial position of balance on current account an anticipated temporary terms of trade deterioration leads to a period of steadily increasing surplus, a deficit on impact, and a further period of declining surpluses. A permanent deterioration in the terms of trade leads to a similar period of anticipatory surpluses, followed by a period of declining deficits after impact.

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## SUMMARY

The object of this paper is to examine the effects of changes in the terms of trade on the current account of the balance of payments. This is a question which has been previously examined by a number of authors, but the large terms of trade shifts which have occurred in the last decade have led to a reappraisal of their work in the light of modern theories of consumer behaviour which emphasise the role of wealth in smoothing expenditure over time. It was previously argued that a terms of trade deterioration implied a fall in the real income of consumers, a reduction in savings and therefore, a worsening of the current account. However, matters are less clear cut if the terms of trade deterioration is permanent since although real income falls there is no reason for the pattern of savings to change. Recent analyses have demonstrated that the effect on the current account is complicated and depends on whether increments in wealth tend to lead to uniform increases in consumption over the consumer's lifetime or are concentrated in particular periods. In other words it depends on their degree of time preference or "impatience". If they tend to become less "impatient" as wealth rises then a permanent terms of trade deterioration will lead to a deterioration in the current account and vice versa.

We extend this recent work to allow output to vary as well. A terms of trade improvement makes production more profitable than before, leading to an expansion in output and an increase in employment and wages. The effect on the current account depends not only on the way the consumer's rate of time preference changes with the level of welfare, but also on the relative ease of substitution of consumption and leisure between different time periods. If consumption can be easily switched between different time periods then a terms of trade deterioration will also tend to lead to a deterioration in the current account. Conversely, if workers can be easily persuaded to work longer hours now in return for more leisure in the future this makes an improvement in the current account more likely.

(ii)

Finally, the paper goes on to enrich the existing analyses by replacing the simple two-period structure by one with a series of overlapping generations. Even though each individual is finitely-lived, the interaction between individuals of different generations in the labour market leads to very complicated behaviour in the current account. Temporary terms of trade shifts have effects of the current account long after those individuals who experienced it have died, and the adjustment to permanent changes is only gradual.

Although abstract the analysis confirms the view that the behaviour of the current account in response to terms of trade shifts is likely to be very complicated and depends critically on whether such changes are seen as temporary or permanent, and are anticipated or unanticipated. In general there seems to be little presumption that it will either improve or deteriorate.

## I. INTRODUCTION

The effect of terms of trade changes on savings and the current account was first pointed out by Harberger (1950) and Laursen and Metzler (1950). A terms of trade deterioration would reduce real income and savings; then if investment and the government deficit were to remain constant, the change in saving would be equal to the change in the current account surplus. Hence the Harberger-Laursen-Metzler effect implies that a deterioration in the current account would accompany a terms of trade deterioration.

Recent contributions by Obstfeld (1982), Sachs (1981) and Svensson and Razin (1983) have placed the analysis in an explicit intertemporal optimisation framework. The paper by Svensson and Razin is especially elegant and they decompose the effect of a terms of trade change into three components: an income effect through the direct revaluation of net exports; a wealth effect on spending; and an intertemporal substitution effect. They show that a temporary current (future) terms of trade deterioration implies a deterioration (improvement) of the trade balance, but that, in a stationary state, a permanent terms of trade change has an ambiguous effect whose sign depends critically on the rate of time preference. If the rate of time preference decreases with the level of wealth then the current trade balance will deteriorate and vice versa. In an extension from two periods to an infinite horizon the assumption of an increasing rate of time preference is shown to be crucial to the stability of the system.

One limitation with these analyses is that the effect of terms of trade changes on labour supply and production is ignored. A change in the terms of trade alters the relationship between the product

wage, which is relevant to the firm's labour demand decision, and the consumption wage, which is relevant to the worker's labour supply decision. It may also induce a wealth effect on labour supply.

The effect of changes in the price of imported intermediate inputs on activity has, of course, been of considerable interest in recent years and extensively investigated by a number of authors (see Sachs, 1981, and the references in Svensson and Razin for instance). Also the consequences of a change in export prices when money wages are fixed and employment is demand determined has been analysed by authors such as Helpman (1976, 1977) and McKinnon (1976). However the fact that changes in the terms of trade in final goods can have effects on output even when the labour market clears seems to have received little attention in the macroeconomic literature. Authors incorporating a model of the labour market similar in spirit to the one presented here include Branson and Rotemberg (1980), Buiter (1979), Riley (1982) and Sachs (1980) although both Branson and Rotemberg, and Sachs are primarily interested in examining the implications of nominal or real wage rigidity, and none of them consider the implications for the behaviour of the current account. The closest antecedent of this paper is Salop (1974) who notes that with flexible wages and a fixed money supply a devaluation worsens the terms of trade, reduces employment and output, but improves the trade balance. Neither she, nor any of the other authors mentioned above, carry out their analysis in an explicitly intertemporal model in which the distinction between temporary and permanent, and current and future, terms of trade changes can be properly investigated.

In the next section we extend Svensson and Razin's two-period analysis to incorporate the labour supply decision. This extension leads to a substantial modification of their results. In particular a temporary current terms of trade deterioration which leaves the real rate of interest unchanged has an ambiguous effect on the trade balance rather than leading to an increased deficit. On the one hand labour demand falls because of the reduced profitability of production, but at the same time labour supply increases as a consequence of the fall in wealth induced by the terms of trade change. Output may increase and the current account may actually improve. By contrast their result that a future terms of trade deterioration leads to an improvement in the current balance is strengthened. Labour demand in the current period is unchanged, but there is a wealth-induced increase in labour supply and a consequent increase in production.

In the case of a permanent terms of trade deterioration which leaves the real rate of interest constant we find in Section III that, in a stationary state, the effect on the current balance is in general ambiguous. If labour supply or demand is inelastic we obtain Svensson and Razin's result that a necessary and sufficient condition for an improvement in the trade balance is that the rate of time preference increase with the level of welfare. For the general case of elastic labour supply and labour demand we find that in the canonical case of a constant rate of time preference the current account improves if the intertemporal substitutability of leisure with respect to the real wage exceeds that of consumption or equivalently if the wealth effect on labour supply is stronger than that on consumption.

A two period analysis is rather limited and not entirely satisfactory since it does not accommodate a very rich pattern of dynamics in the current account with a deficit in the first period implying a surplus in the second and vice versa. However, rather than adopt the infinite horizon approach of Obstfeld, and Svensson and Razin we embed, in Section IV, the two period model in an overlapping generations framework. The labour market provides an intergenerational linkage which is both forward and backward looking. This linkage, which is not present in the conventional overlapping generations model, proves to be a source of complex dynamics. An anticipated temporary terms of trade deterioration leads, from an initial position of balance, to a period of anticipatory surplus, a deficit on impact, and a period of declining surpluses thereafter. In the case of an anticipated permanent terms of trade deterioration, there is again a period of anticipatory surplus, which is followed by a period of declining deficits when the terms of trade shift occurs. Section V contains a few concluding remarks.



## II SPENDING, OUTPUT AND THE TRADE BALANCE

There are two periods, indexed by the superscript  $t = 1, 2$  and  $m$  goods, indexed  $i = 1, \dots, m$ . The country is small and can trade freely at the given world price vector  $\underline{p}^t$ , and has free access to the world credit market with nominal discount factor  $D$  (measured in terms of an arbitrary numeraire). Borrowing and lending only takes place on this integrated capital market and is the only source of assets and liabilities there being no real domestic capital. A representative consumer possesses a well-behaved utility function defined over  $\underline{c}^t$ , the  $m$ -vector of consumption in period  $t$ , and leisure  $\bar{l} - l^t$ , where  $\bar{l}$  is the leisure endowment and  $l^t$  is labour supply in period  $t$ . To enable us to define real variables we assume the utility function is weakly separable in each periods consumption and leisure:

$$u = u(z^1, z^2, \bar{l} - l^1, \bar{l} - l^2) \quad \dots (1)$$

where  $z^t = g^t(\underline{c}^t)$

Without loss of generality the functions  $g^t(\underline{c}^t)$  can be chosen to be linearly homogenous. Then the expenditure function corresponding to  $u$  can be written:

$$E[\phi^1(\underline{p}^1), D\phi^2(\underline{p}^2), w^1, DW^2, u] = \min\{P.\underline{c}^1 + D\underline{p}^2.\underline{c}^2 + w^1(\bar{l}-l^1) + DW^2(\bar{l}-l^2) : u(z^1, z^2, \bar{l}-l^1, \bar{l}-l^2) \geq u\} \dots (2)$$

where  $\phi^t(\underline{p}^t)$  is the expenditure function corresponding to the sub-utility function  $g^t(\underline{c}^t)$  and  $w^t$  is the wage. Since  $\underline{p}^t.\underline{c}^t = \phi^t(\underline{p}^t)z^t$ ,  $z^t$  may be

interpreted as real spending and  $\phi^t(\underline{p}^t)$  the associated price index. Defining the real price vector  $\underline{p}^t = \underline{p}^t / \phi^t(\underline{p}^t)$  the intertemporal budget constraint may be written:

$$E(1, \delta, w^1, \delta w^2, u) = \bar{l}(w^1 + \delta w^2) + \pi = a \quad \dots (3)$$

where  $\delta = D\phi^2(\underline{p}^2) / \phi^1(\underline{p}^1) =$  real discount factor

$w^t = W^t / \phi^t(\underline{p}^t) =$  real consumption wage

$\pi =$  real profits (measured in terms of period 1 consumption)

$a =$  real wealth (present value of human and non-human wealth in terms of period 1 consumption)

and we have used the property that the expenditure function is linearly homogenous in prices. Thus real spending ( $z^t$ ) and labour supplies are given by:

$$z^1 = E_1(1, \delta, w^1, \delta w^2, u) = z^1(w^1, \delta w^2, \delta, u) \quad \dots (4a)$$

$$z^2 = E_2(1, \delta, w^1, \delta w^2, u) = z^2(w^1, \delta w^2, \delta, u) \quad \dots (4b)$$

$$l^1 = \bar{l} - E_3(1, \delta, w^1, \delta w^2, u) = l^1(w^1, \delta w^2, \delta, u) \quad \dots (4c)$$

$$l^2 = \bar{l} - E_4(1, \delta, w^1, \delta w^2, u) = l^2(w^1, \delta w^2, \delta, u) \quad \dots (4d)$$

On the production side we assume that the technology is well-behaved, separable between periods and takes the form  $f^t(v^t, n^t) = 0$  where  $v^t = d(\underline{x}^t)$  is a linearly homogenous value added function,  $\underline{x}^t$  is an  $m$ -vector of outputs/inputs, and  $n^t$  is labour input. The assumption that the technology can be written in this form is not entirely innocuous since it implies that all inputs are co-operant with labour and that a rise in the price of a non-labour input leads, ceteris paribus, to a fall in labour demand. Non-labour inputs may, however, be non-co-operant. Then the profit function may be written as:

$$\Pi^t[\psi^t(\underline{p}^t), w^t] = \max\{\underline{p}^t \cdot \underline{x}^t - W^t n^t : f^t(v^t, n^t) = 0\} \quad \dots (5)$$

where  $\psi^t(\underline{p}^t) = \underline{p}^t \cdot \underline{x}^t / v^t$  is the price of a unit of value added. Using the linear homogeneity of the profit function the real present value of profits is:

$$\pi = \Pi^1(\lambda^1, w^1) + \delta \Pi^2(\lambda^2, w^2) \quad \dots (6)$$

where  $\lambda^t = \psi^t(\underline{p}^t) / \phi^t(\underline{p}^t) =$  terms of trade.<sup>1</sup>

Note that this definition of the terms of trade as the price of value added relative to the price of consumption is rather different from the usual one, but is both convenient and the natural one to use when supply is variable since it gives the rate at which production can be traded for consumption. Real net output in terms of consumption goods  $y^t$  and labour demands are then given by:

$$y^1 = \lambda^1 v^1 = \lambda^1 \Pi_{\lambda}^1(\lambda^1, w^1) = \lambda^1 v^1 (w^1 / \lambda^1) \quad \dots (7a)$$

$$y^2 = \lambda^2 v^2 = \lambda^2 \Pi_{\lambda}^2(\lambda^2, w^2) = \lambda^2 v^2 (w^2 / \lambda^2) \quad \dots (7b)$$

$$n^1 = - \Pi_w^1(\lambda^1, w^1) = n^1(w^1/\lambda^1) \quad \dots (7c)$$

$$n^2 = - \Pi_w^2(\lambda^2, w^2) = n^2(w^2/\lambda^2) \quad \dots (7d)$$

And labour market equilibrium by:

$$l^1(w^1, \delta w^2, \varepsilon, u) = n^1(w^1/\lambda^1) \quad \dots (8a)$$

$$l^2(w^1, \delta w^2, \delta, u) = n^2(w^2/\lambda^2) \quad \dots (8b)$$

Differentiating (3) we obtain:

$$E_u du = l^1 \bar{d}w^1 + l^2 \bar{d}(\delta w^2) - z^2 \bar{d}\delta + d\pi \quad \dots (9)$$

And from (6) using the linear homogeneity of  $\Pi^t(\lambda^t, w^t)$ :

$$d\pi = v^1 \bar{d}\lambda^1 - l^1 \bar{d}w^1 + v^2 \bar{d}(\delta \lambda^2) - l^2 \bar{d}(\delta w^2) \quad \dots (10)$$

Hence:

$$E_u du = v^1 \bar{d}\lambda^1 + \delta v^2 \bar{d}\lambda^2 + b^2 \bar{d}\delta \quad \dots (11)$$

where  $b^t = y^t - z^t =$  real trade balance in period  $t$  in terms of that periods consumption goods.

The change in real spending is obtained by differentiating (4a) and using (11):

$$dz^1 = z_1^1 \bar{d}w^1 + z_2^1 \bar{d}(\delta w^2) + z_0^1 \bar{d}\delta + z_a^1 (v^1 \bar{d}\lambda^1 + \delta v^2 \bar{d}\lambda^2 + b^2 \bar{d}\delta) \quad \dots (12)$$

where  $z_a^1$  is the marginal propensity to spend out of total wealth  $a$ .

The change in real output is obtained from (7a):

$$dy^1 = (v^1 - v_1^1 w^1 / \lambda^1) d\lambda^1 + v_1^1 dw^1 \quad \dots (13)$$

To obtain the change in wages we differentiate the system (8)

and use (11) and the fact that  $v_1^t = n_1^t w^t / \lambda^t$  to obtain:

$$\begin{aligned} & \begin{bmatrix} \epsilon_a^1 v^1 + v_1^1 / \lambda^1 & \epsilon_a^1 \delta v^2 & \epsilon_\epsilon^1 + \epsilon_a^1 b^2 \\ \epsilon_a^2 v^1 & \epsilon_a^2 \delta v^2 + v_1^2 / \lambda^2 & \epsilon_\epsilon^2 + \epsilon_a^2 b^2 + v_1^2 / \delta \end{bmatrix} \begin{bmatrix} d\lambda^1 \\ d\lambda^2 \\ d\delta \end{bmatrix} \\ &= \begin{bmatrix} v_1^1 / w^1 - \epsilon_1^1 & -\epsilon_2^1 \\ -\epsilon_1^2 & v_1^2 / \delta w^2 - \epsilon_2^2 \end{bmatrix} \begin{bmatrix} dw^1 \\ d(\delta w^2) \end{bmatrix} \quad \dots (14) \end{aligned}$$

whence:

$$\begin{bmatrix} dw^1 \\ d(\delta w^2) \end{bmatrix} = [d_{ij}] \begin{bmatrix} d\lambda^1 \\ d\lambda^2 \\ d\delta \end{bmatrix} \quad \dots (15)$$

where  $d_{11} = [(v_1^2 / \delta w^2 - \epsilon_2^2) (\epsilon_a^1 v^1 + v_1^1 / \lambda^1) + \epsilon_2^1 \epsilon_a^2 v^1] / \Delta$

$$d_{12} = [(v_1^2 / \delta w^2 - \epsilon_2^2) \epsilon_a^1 \delta v^2 + \epsilon_2^1 (\epsilon_a^2 \delta v^2 + v_1^2 / \lambda^2)] / \Delta$$

In analysing the effects of terms of trade shifts it is necessary to make an assumption about how changes in the terms of trade affect the discount rate  $\delta$ . This requires modelling monetary policy and the determination of real interest rates in the rest of the world. As a benchmark we assume, by the principle of insufficient reason, that  $\delta$  is unchanged in the face of terms of trade shifts. There is something of an asymmetry involved in assuming away intertemporal relative price effects, but admitting intertemporal relative wage effects in this manner. However, it does seem the most natural assumption to make, at least for pedagogical purposes. It is perhaps more convincing for permanent rather than temporary shifts. However, it should be borne in mind throughout the following discussion that if the discount rate also changes then intertemporal relative price effects may modify the conclusions. In particular  $b^2 > 0$  is sufficient to ensure the coefficient on  $d\delta$  in (16) is negative so that in this case an increase in the discount rate will tend to lead to a deterioration in the current account.

Svensson and Razin find that a temporary deterioration in the terms of trade which leaves the discount rate unchanged ( $d\lambda^1 < 0$ ,  $d\lambda^2 = d\delta = 0$ ) unambiguously worsens the trade balance, there being a direct income effect which is only partially offset by a wealth induced fall in spending in the current period. This effect is captured by the term  $(1 - z_a^1)v^1$  in (16). However when production is variable there will also be a wealth induced rise in labour supply in both periods as well as a direct fall in labour demand in the current period as the product wage rises. Consequently the consumption wage falls in both periods inducing substitution away from consumption towards leisure. Thus the overall effect is ambiguous and the trade

### III PERMANENT TERMS OF TRADE CHANGES IN A STATIONARY STATE

To shed further light on the conditions under which (17) holds we consider the special case of a stationary state with initial trade balance equilibrium. Specifically we assume the utility function can be written in the form:

$$u = \bar{u}(e^1, e^2) \quad \dots (18)$$

where  $e^t = h(z^t, \bar{x} - l^t)$

$$z^t = g(\underline{c}^t)$$

Then the expenditure function can be written  $\bar{E}[\chi(1, w^1), \chi(\delta, \delta w^2), u]$  where  $\chi(\cdot)$  is a linearly homogenous expenditure function, per unit of utility, corresponding to the (linearly homogenous) sub-utility function  $h(\cdot)$ . Thus  $e^t$  may be interpreted as an index of real "welfare" in period  $t$  i.e. incorporating leisure as well as consumption, and  $\chi^t$  is the associated "cost-of-living" index. Then we may write the demand functions for "welfare" in each period as the derivative of the expenditure function:

$$e^1 = \bar{E}_1 = e^1[\chi(1, w^1), \delta\chi(1, w^2), u] \quad \dots (19a)$$

$$e^2 = \bar{E}_2 = e^2[\chi(1, w^1), \delta\chi(1, w^2), u] \quad \dots (19b)$$

We also assume the production technology, and hence the profit function, is the same in both periods.

Consider a stationary state in which world prices  $\underline{p}^t$ , the wage  $w^t$  and the terms of trade  $\lambda^t$  are the same in each period. This implies output  $y^t$ , expenditure  $z^t$ , and employment  $l^t$  are also the same in each period and the trade balance  $b^t$  is identically zero.

Since the Hicksian demands are homogenous of degree zero we have, deleting time superscripts when redundant:

$$e_1^1 + \delta e_2^1 = e_2^1 + \delta e_2^2 = 0 \quad \dots (20a)$$

$$x_{11} + w x_{12} = x_{12} + w x_{22} = 0 \quad \dots (20b)$$

Using these results we have from the properties of the expenditure function:

$$z_1^1 = -w z x_{22}/x_1 + l_2^1 x_1/x_2 \quad \dots (21a)$$

$$z_2^1 = -l_2^1 x_1/x_2 \quad \dots (21b)$$

$$l_1^1 = l_2^2 = -z x_{22}/x_1 - l_2^1 \quad \dots (21c)$$

$$l_2^1 = e_1^1 (x_2)^2 \quad \dots (21d)$$

$$z_a^1 = e_a^1 x_1 \quad \dots (21e)$$

$$z_a^2 = e_a^2 x_1 \quad \dots (21f)$$

$$l_a^1 = -e_a^1 x_2 \quad \dots (21g)$$

$$l_a^2 = -e_a^2 x_2 \quad \dots (21h)$$



Finally the budget constraint implies:

$$1 = (e_a^1 + \delta e_a^2)(x_1 + wx_2) = (e_a^1 + \delta e_a^2)x \quad \dots (22)$$

We proceed to consider three special cases:

Case 1:  $x_2 (= x_{22}) = 0$ .

This case of inelastic labour supply is implicitly that considered by Svensson and Razin. Substituting (21) and (22) into (6), setting appropriate terms to zero, we obtain:

$$db^1/d\lambda = \delta v_x (e_a^2 - e_a^1) = \delta v (z_a^2 - z_a^1) \quad \dots (23)$$

$$\gtrsim 0 \text{ as } z_a^2 \gtrsim z_a^1$$

Not surprisingly we also obtain their result that a permanent terms of trade deterioration leads to a deterioration in the current trade balance if and only if the slope of the wealth expansion path in  $(z^1, z^2)$  space is greater than unity. This result is quite intuitive and is portrayed graphically in Figure 1. A is the initial equilibrium, and a permanent terms of trade deterioration shifts the endowment point along the  $45^\circ$  line OA to B. The line AC is the wealth expansion path formed by the locus of points where the marginal rate of substitution is  $(1/\delta)$  and drawn with  $z_a^2 > z_a^1$ . The new equilibrium is point C where the indifference curve is tangent to the new budget line and there is a trade deficit in period 1. As drawn the slope of the indifference curves along the  $45^\circ$  line OBA must be decreasing in absolute magnitude in the region of A as welfare increases. Hence the condition  $z_a^2 > z_a^1$  may be equivalently expressed as the requirement that the rate of time

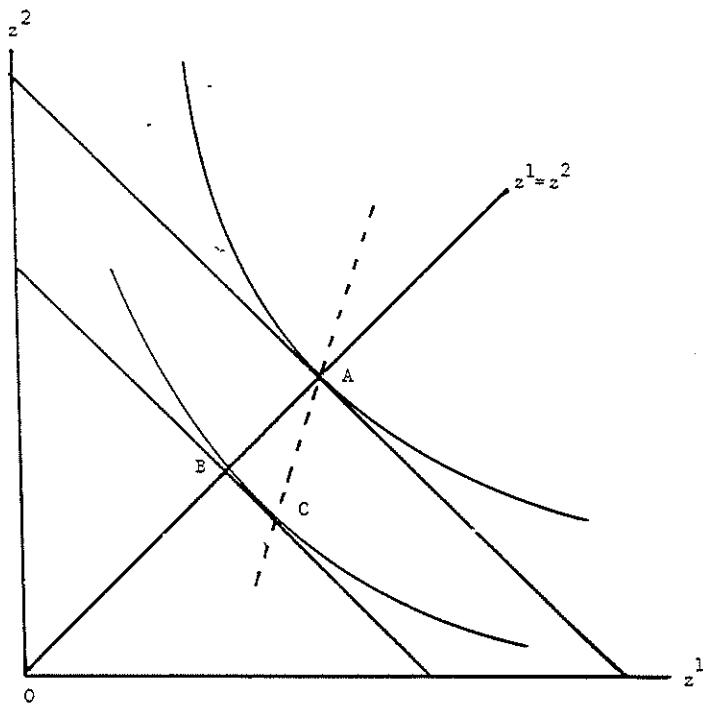


FIGURE 1

preference  $\rho(u)$ , defined as

$$\rho(u) = [\tilde{u}_1(e^1, e^2)/\tilde{u}_2(e^1, e^2)]_{e^1=e^2}^{-1}, \quad \dots (24)$$

decrease with the level of utility.

Case 2:  $v_1 = 0$ .

The case of inelastic labour demand is superficially very similar to Case 1. Substituting (20) and (21) into (16) after setting appropriate terms to zero yields, after a little algebra, the effect of a permanent change in the terms of trade on the current trade balance as:

$$db^1/d\lambda = v_X(e_a^2 - e_a^1)(\delta k_1^1 + k_2^1)/(k_1^1 - k_2^1) \quad \dots (25)$$

$$\geq 0 \text{ as } (e_a^2 - e_a^1)(\delta k_1^1 + k_2^1) \geq 0$$

If  $(\delta k_1^1 + k_2^1) > 0$  we obtain the same result as Case 1 that a necessary and sufficient condition is  $e_a^2 > e_a^1$  (or equivalently  $z_a^2 > z_a^1$ ).

However if the real interest rate is sufficiently high ( $\delta$  small) and/or the intertemporal substitutability of leisure is relatively high ( $k_2^1/k_1^1$  near unity in absolute value) then these conclusions are precisely reversed.

At first glance this seems to be in contradiction to Figure 1, since the new endowment B and equilibrium C are as before. The source of the paradox lies in the fact that AC no longer represents the wealth expansion path because along AC changes in wages are taking place. Figures 2(a) and 2(b) portray events in  $(e^1, e^2)$  space.  $A_1AA_2$  is the "welfare possibility frontier" representing the combination of  $e^1$  and  $e^2$  the consumer can pick given the fact that his consumption of leisure and his wealth are fixed. It is easy to demonstrate that that it is concave and that it has a slope of  $(-1/\delta)$  along the 45° ray OA. A terms of trade deterioration shifts this frontier into  $B_1BB_2$ . The wealth expansion path AC corresponds, as before, to the locus of points where the marginal rate of substitution is  $(1/\delta)$  and is drawn with  $e_a^2 > e_a^1$ . However quasi-concavity of  $u(z^1, z^2, \bar{l}-l^1, \bar{l}-l^2)$  does not necessitate the quasi-concavity of  $\tilde{u}(e^1, e^2)$ , the only requirement being that the indifference curves corresponding to  $\tilde{u}(e^1, e^2)$  are less concave than the welfare possibility frontier. If they are convex, as in Figure 2(a), then a point on BC such as D must be the equilibrium where there is a deficit in the first period. However if they are concave, as in Figure 2(b), then the new equilibrium must be on  $B_1B$  such as D where there is trade surplus in the first period. Note, however, that in this second case the slope of the indifference curves increase in absolute magnitude along OBA as the index of utility increases. Hence the conclusion that there is a surplus only if the rate of time preference increases with the level of welfare is still valid.

Intuitively the following sequence of events occurs. A permanent terms of trade deterioration leads to not only a wealth-induced fall in expenditure on goods but also a wealth-induced rise in labour supply.

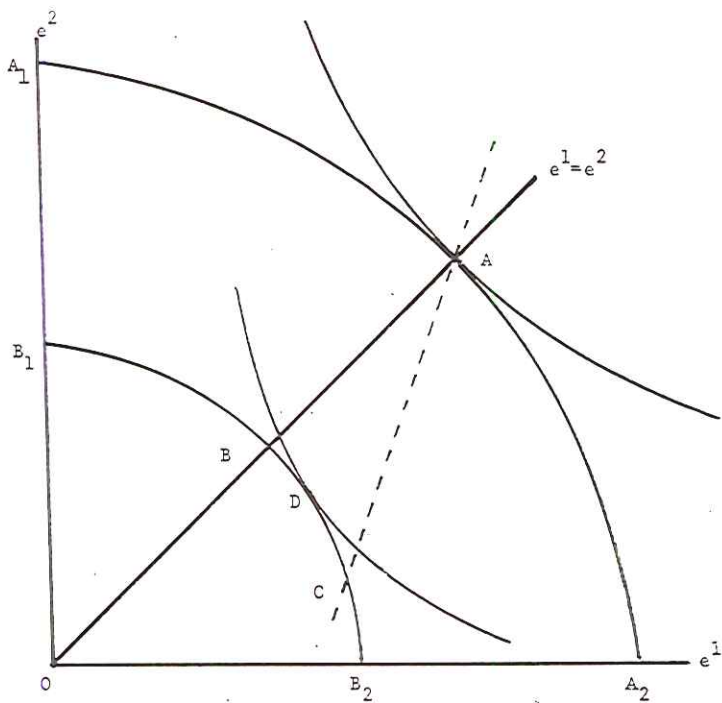


FIGURE 2(a)

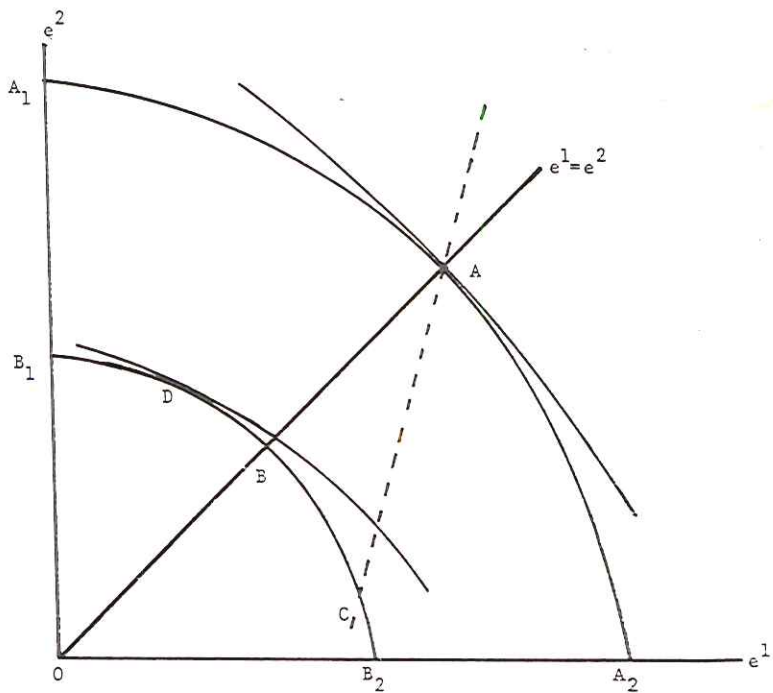


FIGURE 2(b)

With inelastic labour demand the (discounted) wage will fall in both periods and if  $e_a^1 > e_a^2$  then the fall will be greater in period 1. This in turn encourages intertemporal substitution of spending which may be sufficiently large to offset the initial bias in favour of relatively greater spending in period 2. Alternatively one could view the consumer as being "forced" to consume relatively too much leisure in period 1, this raising the marginal utility of period 1 consumption relative to that in period 2 and inducing a redistribution of spending in favour of period 1. If this is sufficiently profitable ( $\delta$  small) and intertemporal substitutability is sufficiently high a deficit rather than a surplus in period 1 will result.

Case 3:  $e_a^1 = e_a^2$

Cases 1 and 2, where output is fixed, focus on the role of differential marginal propensities to consume out of wealth in each period. In examining the general case it is helpful to abstract from such effects by assuming they are identical, or equivalently that the rate of time preference is constant. Once again using equations (16), (20) and (21):

$$db^1/d\lambda = v_1(1-\delta)[v_1/\lambda + v(1+\delta)l_a^1](w\lambda_2^1 + z_2^1)/w\delta\Delta \quad \dots (26)$$

$$\geq 0 \quad \text{as} \quad z_2^1 \geq -w\lambda_2^1$$

$$\text{or} \quad z_a^1 \geq -w\lambda_a^1$$

$$\text{or} \quad z \geq w(\bar{l}-l)$$

where  $\Delta = v_1[v_1/w - \lambda_1^1(1+\delta)]/w\delta + (\lambda_1^1 + \lambda_2^1)(\lambda_1^1 - \lambda_2^1)$ .

Thus a permanent terms of trade deterioration leads to a current deficit if the degree of intertemporal substitution of spending in response to a wage change in any period exceeds that of leisure, or equivalently that wealth effects are stronger on consumption than on labour supply, or equivalently if the share of wealth devoted to consumption exceeds that devoted to leisure. Unfortunately a complete diagrammatical analysis is difficult because there are now four choice variables in the problem. In terms of Figures 2(a) and 2(b) the wealth expansion path AC is now coincident with OBA. However the chosen levels of labour supply are now parametric to the welfare possibility frontier. If  $\epsilon^1 > \epsilon^2$  then the absolute value of its slope along OBA is greater than  $(1/\delta)$  and the equilibrium point C is to the right of OBA and vice versa if  $\epsilon^1 < \epsilon^2$ . The optimal combination obviously involves choosing  $\epsilon^1$  and  $\epsilon^2$  so as to reach the highest utility level, but there does not seem to be a simple way to interpret the conditions associated with (26) graphically. However, it is clear that even in a stationary state a decreasing rate of time preference is neither necessary nor sufficient for a permanent terms of trade deterioration to lead to a current deficit.

## IV THE MULTIPERIOD CASE

In the simple two period model developed above a deficit in the first period implies a surplus in the second and vice versa (since  $b^1 + \delta b^2 = 0$ ). Whether a terms of trade deterioration leads to a deficit or a surplus is therefore not of great concern for the stability of the stationary state. However, as noted by Obstfeld, and Svensson and Razin, if the planning horizon of the model is extended from two periods to infinity then, in the simple case of inelastic labour supply, a decreasing rate of time preference implies instability of the stationary state. The reason this problem arises is as follows: a permanent terms of trade deterioration also lowers real wealth, but if the terms of trade deterioration also leads to a deficit on the current account there will be further decumulation of assets moving the economy further away from the equilibrium level of wealth. In principle this process could continue until all production is used to service the resulting debt and consumption ceases completely. However, long before this state is reached the small country assumption of a fixed interest rate is likely to become inappropriate. The same limitations apply, *mutatis mutandis*, for a permanent terms of trade improvement which leads to a current account surplus.

The results derived in the preceding section suggest that an increasing rate of time preference is neither necessary nor sufficient to guarantee stability of the stationary state in the general case where labour supply and output are variable. In the special case considered above of a constant rate of time preference stability will obtain if the degree of intertemporal labour substitution is high relative to that of consumption or equivalently that wealth effects on labour supply are more powerful than those on spending.



Equations (4) still describe the spending and labour supply decisions of households, while equations (7) still describe the supply of output and labour demand. The profit function for each period is assumed to be the same so that the output supply and labour demand functions are time invariant, as is the discount rate  $\delta$  which is omitted as an argument in the household decision rules. Then labour market equilibrium in period  $t$  is given by:

$$\begin{aligned} n(w^t/\lambda^t) &= \ell^{1,t} + \ell^{2,t} \\ &= \ell^1(w^t, \delta w^{t+1}, u^t) + \ell^2(w^{t-1}, \delta w^t, u^{t-1}) \quad \dots (27) \end{aligned}$$

Whence:

$$\begin{aligned} \ell_1^{2,t} \, d w^{t-1} + (\ell_1^{1,t} + \delta \ell_2^{2,t} - v_1^t/w^t) \, d w^t + \delta \ell_2^{1,t} \, d w^{t+1} + \ell_u^{1,t} \, d u^t + \ell_u^{2,t} \, d u^{t-1} \\ = -v_1^t \, d \lambda^t / \lambda^t \quad \dots (28) \end{aligned}$$

Let  $\theta^t$  denote the share of profits paid to the young in period  $t$ . Then the analogue of equation (11) is:

$$\begin{aligned} E_u^t \, d u^t &= (\ell^{1,t} - \theta^t \ell^{2,t}) \, d w^t + \delta [\ell^{2,t+1} - (1-\theta^{t+1}) \ell^{1,t+1}] \, d w^{t+1} \\ &\quad + \theta^t v^t \, d \lambda^t + \delta (1-\theta^{t+1}) v^{t+1} \, d \lambda^{t+1} \quad \dots (29) \end{aligned}$$

Suppose profits are distributed in proportion to labour supply as in a profit-sharing scheme so that  $\theta^t = \ell^{1,t}/\ell^t$ . Then the first two terms on the right-hand side of (29) are identically zero and (28) becomes:

$$\begin{aligned}
& -k_1^{2,t} \frac{dw^{t-1}}{d\lambda} + (v_1^t/w^t - k_1^{1,t} - \delta k_2^{2,t}) \frac{dw^t}{d\lambda} - \delta k_2^{1,t} \frac{dw^{t+1}}{d\lambda} = \\
& k_a^{2,t} \theta^{t-1} v^{t-1} \frac{d\lambda^{t-1}}{d\lambda} + \{ [\delta(1-\theta^t) k_a^{2,t} + \theta^t k_a^{1,t}] v^t + v_1^t/\lambda^t \} \frac{d\lambda^t}{d\lambda} \dots (30) \\
& + \delta k_a^{1,t} (1-\theta^{t+1}) v^{t+1} \frac{d\lambda^{t+1}}{d\lambda}
\end{aligned}$$

This is a second-order difference equation describing the evolution of the wage rate in terms of past, current and future shocks to the terms of trade. More will be said about its properties below. The current account is given by:

$$\begin{aligned}
b^t &= y^t - (z^1{}^t + z^2{}^t) \\
&= \lambda^t v (w^t/\lambda^t) - z^1 (w^t, \delta w^{t+1}, u^t) - z^2 (w^{t-1}, \delta w^t, u^{t-1}) \dots (31)
\end{aligned}$$

And using (29) the change in the current balance is:

$$\begin{aligned}
\frac{db^t}{d\lambda} &= -z_a^{2,t} \theta^{t-1} v^{t-1} \frac{d\lambda^{t-1}}{d\lambda} + \{ v^t [1-\theta^t z_a^{1,t} - \delta(1-\theta^t) z_a^{2,t}] - w^t v_1^t/\lambda^t \} \frac{d\lambda^t}{d\lambda} \\
& - z_a^{1,t} \delta (1-\theta^{t+1}) v^{t+1} \frac{d\lambda^{t+1}}{d\lambda} - z_1^{2,t} \frac{dw^{t-1}}{d\lambda} + (v_1^t - z_1^{1,t} - \delta z_2^{2,t}) \frac{dw^t}{d\lambda} \\
& - \delta z_2^{1,t} \frac{dw^{t+1}}{d\lambda} \dots (32)
\end{aligned}$$

Equation (30) may be solved for  $\frac{dw^t}{d\lambda}$  and then substituted into (32) to give the path of the current account which is clearly fairly complicated. To achieve some further simplification return to the stationary state of Section III, in which not only aggregate variables are constant over time, but also  $z^1{}^t = z^2{}^{t+1}$  and  $k^1{}^t = k^2{}^{t+1}$  and the rate of time preference is constant. In that case  $\theta = \frac{1}{2}$  and equation (30) becomes, where L is

the lag operator:

$$(L + \alpha_1 + \delta L^{-1})d\omega^t = (-\lambda_a v/2\lambda_2^1)(L + \alpha_2 + \delta L^{-1})d\lambda^t \quad \dots (33)$$

$$\text{where } \alpha_1 = [\lambda_1^1(1+\delta) - v_1/w_1/\lambda_2^1] < 0$$

$$\alpha_2 = 1 + \delta - 2\varepsilon/w\lambda_a > 0$$

$$\varepsilon = -wv_1/\lambda v = \text{elasticity of supply of output}$$

Now the lag polynomial on the left hand side can be rewritten:

$$L + \alpha_1 + \delta L^{-1} = (1-\rho_1 L^{-1})(L-\rho_2) = (-\rho_2)(1-\rho_1 L^{-1})(1-\rho_2^{-1}L) \quad \dots (34)$$

where  $\rho_1 + \rho_2 + \alpha_1 = 0$  and  $\rho_1 \rho_2 = \delta$ . Now  $\alpha_1 < -(1+\delta)$

So both  $\rho_1$  and  $\rho_2$  are positive with at least one root less than unity.

Let  $\rho_1$  be the smallest root. Then  $\rho_1 \leq \delta^{\frac{1}{2}}$  and  $\rho_2 \geq \delta^{\frac{1}{2}}$ . Suppose  $\rho_2 < 1$ ; it follows that  $\rho_1 + \rho_2 < 1 + \delta^{\frac{1}{2}} < -\alpha_1$  which is a contradiction. Hence  $\rho_2 > 1$ ,  $\rho_1 < \delta$  and the lag polynomial (34) is invertible.

The current account (32) may now be written as:

$$d\omega^t = (-z_a v/2)(L + \alpha_3 + \delta L^{-1})d\lambda^t - z_2^1(L + \alpha_4 + \delta L^{-1})d\omega^t \quad \dots (35)$$

$$\text{where } \alpha_3 = 1 + \delta - 2(1 + \varepsilon)/z_a \geq 0$$

$$\alpha_4 = [z_1^1(1+\delta) - v_1]/z_2^1 > 0$$

From (21)  $\lambda_a z_2^1 = \lambda_2^1 z_a$  so (33) and (35) may be combined to yield:

$$db^t = (z_a v/2) [(a_2 - a_3) + (a_4 - a_1) (L + a_2 + \delta L^{-1}) (L + a_1 + \delta L^{-1})^{-1}] d\lambda^t \dots (36)$$

Rearranging and factorising using (34):

$$\begin{aligned} db^t &= (z_a v/2) [(a_2 + a_4 - a_1 - a_3) + (a_2 - a_1) (a_1 - a_4) \rho_2^{-1} (1 - \rho_1 L^{-1})^{-1} (1 - \rho_2^{-1} L)^{-1}] d\lambda^t \\ &= \frac{z_a v}{2(\rho_2 - \rho_1)} \{[(a_1 a_3 - a_2 a_4) + 2\rho_1 (a_1 + a_3 - a_2 - a_4)] + \\ &\quad \rho_1 (a_2 - a_1) (a_1 - a_4) [L(\delta - \rho_1 L)^{-1} + L^{-1} (1 - \rho_1 L^{-1})^{-1}]\} d\lambda^t \end{aligned} \dots (37)$$

Hence the response of the current account to an anticipated shift in the terms of trade consists of a contemporaneous impact effect given by the first-term on the right-hand side of (37) and a two-sided geometric lag. Further past changes in the terms of trade have a somewhat larger effect and decay slower than changes equivalently far into future. To sign the impact effect note that using equations (21) and (22), after a little algebra:

$$\begin{aligned} (a_1 a_3 - a_2 a_4) + (1 + \delta) (a_1 + a_3 - a_2 - a_4) &\dots (38) \\ &= (a_1 + 1 + \delta) (a_3 + 1 + \delta) - (a_2 + 1 + \delta) (a_4 + 1 + \delta) \\ &= 0 \end{aligned}$$

Hence equation (37) can be rewritten:

$$\begin{aligned} db^t &= \frac{z_a v}{2(\rho_2 - \rho_1)} \{[(1 + \delta - 2\rho_1) (a_2 + a_4 - a_1 - a_3)] + \\ &\quad \rho_1 (a_2 - a_1) (a_1 - a_4) [L(\delta - \rho_1 L)^{-1} + L^{-1} (1 - \rho_1 L^{-1})^{-1}]\} d\lambda^t \end{aligned} \dots (39)$$

Thus, since  $\rho_1 < \delta$ , the impact effect is positive if and only if:

$$(\alpha_2 + \alpha_4) > (\alpha_1 + \alpha_3) \quad \dots (40)$$

Substituting for the  $\alpha_i$  and using equations (21) and (22):

$$\begin{aligned} (\alpha_2 + \alpha_4) - (\alpha_1 + \alpha_3) &= [(\ell_2^1 - \ell_1^1)(1 + \delta)w\ell_a + v_1\ell_a - 2\varepsilon\ell_2^1] / (1 + \delta)w\ell_a z_a \ell_2^1 \\ &> 0 \quad \dots (41) \end{aligned}$$

Hence the impact effect, for this special case of a stationary state and constant rate of time preference, is positive. Thus an anticipated temporary terms of trade deterioration leads to an exponentially improving current account until the moment the terms of trade deterioration occurs when there is a deficit. Thereafter there is again a period of surplus which decays exponentially. This is illustrated in Figure 3.

Now let us consider an anticipated permanent terms of trade deterioration occurring at  $t = 0$ . As before there will be growing surpluses as the terms of trade shift draws near (they will be larger than in the case of a temporary shift, of course). From the moment the shift occurs there is a constant tendency to deficit from the impact term coupled with a growing tendency to surplus as the date of the terms of trade shift recedes into the past. The long-run impact is found by setting  $L = 1$  in equation (36):

$$\begin{aligned} \lim_{t \rightarrow \infty} db^t / d\lambda^t &= z_a v [(\alpha_2 \alpha_4 - \alpha_1 \alpha_3) + (1 + \delta)(\alpha_2 + \alpha_4 - \alpha_1 - \alpha_3)] / 2(1 + \delta + \alpha_1) \quad \dots (42) \\ &= 0 \quad \text{from (38)} \end{aligned}$$

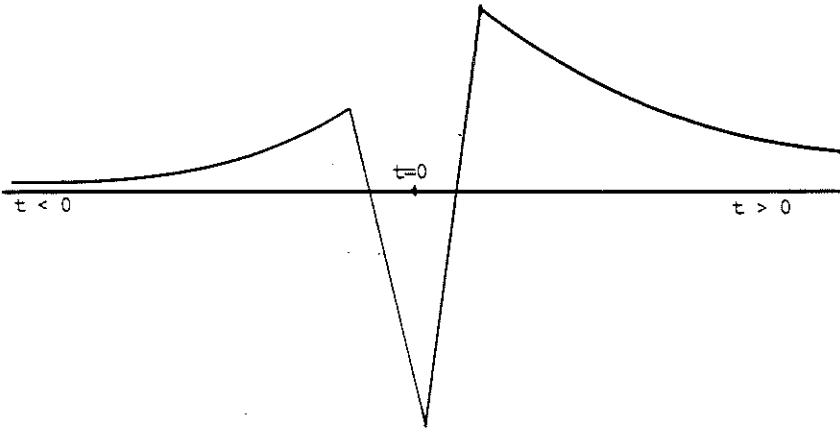


Figure 3: Time path of current account in response to an anticipated temporary terms of trade deterioration at  $t = 0$ .

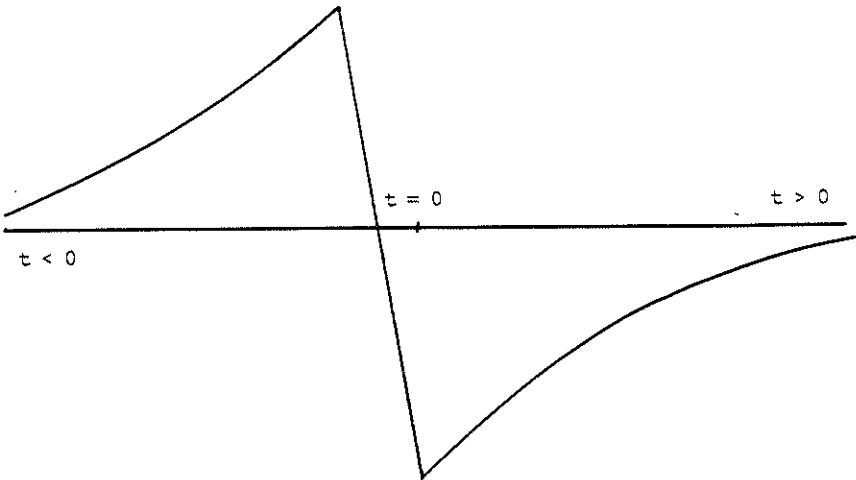


Figure 4: Time path of current account in response to an anticipated permanent terms of trade deterioration at  $t = 0$ .

This is not a trivial result since a non-zero long-run value of the current balance which is just sufficient to service the new steady-state value of debt is perfectly feasible. It follows that there must be a deficit throughout the period after the terms of trade deterioration. This is portrayed in Figure 4.

The details of behaviour under unanticipated shifts will, of course, be slightly different. Suppose it is announced at  $t = \tau \leq 0$  that a previously unanticipated terms of trade shift will occur at  $t = 0$ . Then equations (30) and (31) must be modified for  $t = \tau$  to take account of the fact that the old are constrained by the decisions they made in period  $(\tau-1)$  on the basis of faulty expectations of  $w^T$ . However, the behaviour of the current account in ensuing periods is still qualitatively similar to that for  $t \geq \tau$  in figures 3 and 4. The details are left to the reader.

## V CONCLUDING REMARKS

In the previous section we have shown how the introduction of variable labour supply and output into an open economy overlapping generations model can produce a complex pattern of dynamics in the current account through intergenerational linkages in the labour market. Particularly noteworthy is the two-sided nature of the distributed lag which leads to a current account improvement in anticipation of a future terms of trade deterioration. Obviously this is not the only source of dynamics. Persson and Svensson (1983), for instance, investigate the implications of introducing domestic capital formation into an open economy overlapping generations model with inelastic labour supply. A change in the discount factor changes the rate of capital formation by the young and has implications for output and the incomes of the young in the following period. Alternating periods of deficit and surplus may result. Integrating these two propagation mechanisms in a tractable manner is not trivial, but it would appear that such a combination would have the potential to produce yet more complex dynamics including the possibility of extended cycles.

Finally, the assumption of an initial stationary state is also important since it rules out the possibility of revaluation effects on financial wealth from terms of trade changes. Since many countries are substantial net debtors or creditors such effects are likely to play an important role in the real world.



## FOOTNOTES

- 1 It follows that a change in nominal world market prices  $\underline{P} = (P_1, \dots, P_m)$  therefore affects the terms of trade in each period according to:

$$d\lambda/\lambda = \sum_{i=1}^m (P_i \psi_i / \psi - P_i \phi_i / \phi) (dP_i / P_i)$$

where  $\psi_i = \partial\psi/\partial P_i$  and  $(P_i \psi_i / \psi)$  and  $(P_i \phi_i / \phi)$  are the value added and expenditure shares of good  $i$  respectively.

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