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ABSTRACT

Does Space Affect Search? A Theory of Local Unemployment*

The spatial dispersion of economic agents is an immediate determinant of informational imperfections. We investigate how this dispersion creates search frictions and, thus, rationing. For that we develop a model of local labour markets in which workers' search efficiency is negatively affected by distance to jobs. Workers' location in a city is endogenous and reflects a trade-off between commuting costs and the surplus associated with search. Different configurations emerge in equilibrium: notably, the unemployed workers may reside far away or close to the jobs. The labour market equilibrium itself depends crucially on these urban equilibria since the aggregate information about economic opportunities depends on the shape of the city. We show that there exists a unique and stable market equilibrium in which both land and labour markets are solved for simultaneously. We then decompose unemployment in two parts: the level reached if all agents were residing in the same location and an additional term due to the spatial dispersion.

JEL Classification: E24, J41, R14
Keywords: local labour markets, urban land use equilibrium, matching, equilibrium unemployment

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NON-TECHNICAL SUMMARY

It has been recognized for a long time that distance interacts with the diffusion of information. In his seminal contribution to search, Stigler (1961) puts geographical dispersion as one of the four immediate determinants of price ignorance. The reason is simply that distance affects various costs associated with search. In most search models, for example Diamond (1982), distance between agents or units implies a fixed cost of making another draw in the distribution. In other words, a spatial dispersion of agents creates frictions and, thus, unemployment.

We apply this methodological approach to an old and well known issue in the US, the spatial mismatch hypothesis. It states that, residing in segregated areas distant from and poorly connected to major centres of employment growth, black workers face strong geographic barriers to finding and keeping well-paid jobs. In this context, and perhaps because of discrimination and high prices in the housing market in the suburbs, black workers are forced to stay in the ghetto in the city-centre, far away from jobs located at the city outskirts. Indeed, in US metropolitan areas, unemployment rates tend to be larger in the city-centre than in the suburbs. One possible explanation of this empirical fact is that access to employment is not independent of residential location. This idea was first introduced by Kain (1968) and reviewed more than twenty years later by Holzer (1991).

Zax and Kain (1996) have recently illustrated this issue by studying a 'natural experiment' (the case of a large firm in the service industry which relocated from the centre of Detroit to the suburb Dearborn in 1974). Among workers whose commuting time was increased, black workers were over-represented, and not all could follow the firm. This had two consequences: first, segregation forced some blacks to quit their jobs; second, the share of black workers applying for jobs to the firm drastically decreased (53% to 25% in 5 years before and after the relocation), and the share of black workers in hires also fell from 39% to 27%. Zax and Kain concluding remarks are: '...with employment in the Detroit region shifting from locations at which black workers have higher probabilities of employment to locations at which they have lower probabilities, black workers must search more locations in order to prevent their aggregate probability of employment from declining. If search is costly, some decline is unavoidable.'

In Europe, there is a similar problem of access to employment centres. In France for instance, 500 'priority' districts (that need transfers) have been designed based on the percentage of foreigners, criminality figures or the unemployment rate, i.e. on non-spatial criteria. However, it turned out that the districts have a geographical specificity: they are closer to the periphery than the city-centre (0.9 km versus 2.3 km on average). In general, they are physically separated from the city-centre by ring roads, rivers or railroads, and poorly connected to it (for instance, 70% of the districts have a railroad but only 40% of these have a station (Castellan, Marpsat and Goldenberg, 1992). Since jobs (especially service jobs) in France tend to be concentrated in the city-centre, this supports the view that low-qualified workers in these areas have bad access to employment and poor information on jobs.

Both European and American experiences reflect a clear spatial dimension of the labour markets (see for example the recent survey of Crampton, 1999). For instance, workers living further away from jobs may have poorer labour market information than those living closer. This is particularly true for younger and/or less-skilled workers who rely heavily on informal search methods for obtaining employment (Holzer, 1987). In Holzer (1988), it is shown that among 16-23 years old workers who reported job acceptance, 66% used informal search channels (30% direct application without referral and 36% friends/relatives), while only 11% used state agencies and 10% newspapers. The reliance on these informal methods of job search suggests that information on available job opportunities may decay rapidly with the distance from home (Ihlanfeldt and Sjoquist, 1990) and that, for example, the disadvantage for black workers is entirely due to residential segregation (Ihlanfeldt 1997). Distance also implies higher search costs for the unemployed and this may be partly reflected by workers' choice of location.

Finally, differences in access to information on job opportunities can also be explained by the social network and the neighbourhood where workers live (Akerlof, 1997, Benabou, 1993, 1996, Montgomery, 1991, O'Regan and Quigley, 1993, Topa, 1997, Wilson, 1987, 1996). Geographical distance in the model may also reflect social distance.

The interaction between space and labour markets is in any case complex and our Paper aims at capturing some of the phenomena at work. In our model, the allocation of jobs and workers is a time-consuming process and the number of matches per unit of time between workers and open vacancies is represented by an aggregate matching function à la Diamond-Mortensen-Pissarides. In this line of search models, the spatial dimension is often implicit. Here, we explicitly introduce it to the matching representation of the labour market, by simply considering that the distance between workers' residential location and firms plays an adverse role in the formation of a match. In this respect, our model can be viewed as a natural extension of the standard matching model.

The housing market will be kept rather simple in order to provide closed-form solutions. The city is monocentric, i.e. firms are exogeneously located in the central business district (CBD) and workers consume inelastically one unit of space. In our analysis, local factors (rental price, distance to the CBD) and global factors (labour market tightness, wages) influence workers' location decisions, i.e. the urban equilibrium. Within this framework, we have different urban equilibria: the unemployed reside either close to the CBD or further from it. We then derive the labour market equilibrium in which urban unemployment is due to frictions in the labour market. Indeed, even if firms pay workers' reservation wage, there is still a level of durable unemployment in the city (due to stochastic rationing not being eliminated by price adjustment).

We explore different wage-setting mechanisms that yield similar results: on the one hand, the urban equilibrium depends on aggregate variables (such as wages and labour market tightness) since these variables affect the location choices of workers. On the other hand, the labour market equilibrium crucially depends on the urban equilibrium configurations. If, for example, unemployed workers reside far from the city-centre, aggregate matching is not efficient and firms are reluctant to make job offers.

We also provide a decomposition of unemployment into two parts, one depending on the labour market frictions if all agents were located in the same place, and a term where search frictions are augmented by the spatial dispersion of agents in the city.

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The paper is organized as follows. Section 1 presents the model and its notations. In section 2, we derive the different equilibrium urban configurations. The labor market equilibrium with exogeneous wages is then studied in section 3 whereas the case with endogeneous wage is analyzed in section 4. Section 5 shows the role of space in equilibrium unemployment while section 6 extends the model to endogenous search effort. Section 7 concludes.

1 The model and general notations

Firms and workers are all (ex ante) identical. A firm is a unit of production that can either be filled by a worker whose production is y units of output or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy that can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply that there is a number of contacts between the two sides of the market during a small time interval dt that we assume to be

[4] It is important to observe that search models have rarely been introduced in a spatial framework. Exceptions include Seater (1979), Jayet (1990a,b), and Simpson (1992). However, these authors focus on different issues and, in particular, they do not explicitly model the intra-urban equilibrium in which the location of unemployed and employed workers is endogeneously determined and influences the labor market equilibrium. Coulson, Laing and Wang (1997) is the closest paper to ours. They undertake to model the spatial mismatch hypothesis; their focus is however on firms' location whereas we investigate workers' location.

determined by the following matching function:

$$x(\bar{s}U, V)dt \tag{1}$$

where $\bar{s}$ is the average efficiency of search of the unemployed workers (a worker i has an efficiency of search equal to s_i). Since this terms represent the aggregate search frictions, it is also an index of aggregate information about economic opportunities.

We assume that $x(\cdot)$ is increasing both in its arguments, concave and homogeneous of degree 1 (or equivalently has constant return to scale).[5] In this context, the probability for a vacancy to be filled during a small time interval dt is $\frac{x(\bar{s}U, V)dt}{V}$. By constant return to scale, it can be rewritten as:

$$x(\frac{1}{\theta}, 1)dt \equiv q(\theta)dt \tag{2}$$

where $\theta = V/U\bar{s}$ is a measure of labor market tightness in efficiency units and $q(\theta)$ is a Poisson intensity. By using the properties of $x(\cdot)$, it is easily verified that $q'(\theta) \leq 0$: the greater the labor market tightness, the lower the probability for a firm of filling a vacancy . Observe that we assume that firms have no impact on their own search efficiency and consider $\bar{s}$, U and V as given. Similarly, for a worker i with efficiency s_i; the probability of obtaining a job during a small time interval dt is:

$$\frac{x(\bar{s}U, V)}{U}\frac{s_i}{\bar{s}}dt \equiv \theta q(\theta)s_i dt \equiv p_i dt \tag{3}$$

where p_i is defined as the intensity of the exit rate from unemployment. In contrast to the standard model of job matching as proposed by Pissarides (1990) where there is no spatial dimension, we make here the important assumption that s_i depends on the location of the unemployed workers in the city: the closer the location to the workplace is, the greater the efficiency and the more likely is a contact ($s_i = s(d_i)$ where d_i is the location of the worker with $s'(d_i) < 0$).[6] This is confirmed empirically by Barron and Gilley (1981) and Chirinko

[5] Here, the number of job contacts as described by (1) is equivalent to the number of job matches, i.e., all job contacts lead to job matches. We could have differentiated the two concepts (job contact versus job match) as in Pissarides (1990, ch.5) by multiplying $x(\bar{s}U, V)dt$ by a probability that a job contact is transformed into a job match but this would complicate the analysis without altering our main results.

[6] If one believes that within a city, the derivative of s to distance is small, one can reinterpret our urban model as a regional one in which employment in concentrated in the main city, and suburbs and the countryside are much poorer in jobs. Then, location clearly affects search (for empirical evidence, see e.g. Adnett, 1996, ch.5). Moreover, in section 6, the assumption about $s(d)$ is relaxed, and we show that with endogeneous search effort, we still retrieve $s_i'(d) < 0$, i.e. the negative dependence of search efficiency with distance.

(1982). Indeed, these papers show that there are diminishing returns to search when people live far away from jobs. More recently, Van Ommeren et al. (1997) show that people who expect to receive more job offers will generally not have to accept a long commute. Rogers (1997) also demonstrates that access to employment is a significant variable in explaining the probability of leaving unemployment. This may finally reflect the fact that some workers are less attractive to employers. For example Seater (1979) shows that workers searching further away from the residence are less productive than those who search closer to where they live.

Once the match is made, the wage is determined (we study first the case of exogeneous wages to give the intuition, and then the case of the generalized Nash bargaining solution). In each period, there is also a probability δdt that the match is destroyed. In order to determine the (general) equilibrium, we will proceed as follows. We first determine the partial urban equilibrium configurations. Then, depending on the location of workers and thus on the aggregate search efficiency $\bar{s}$, we determine the partial labor market equilibria. Hereafter, by labor (respectively urban) equilibrium, it has to be understood the partial equilibrium. The general equilibrium will be denominated a 'market equilibrium'.

It is important to observe that the few models in urban economics that incorporate a labor market and that involve expected lifetime utilities are able to solve the locational equilibrium prior to the labor market equilibrium because locational choices for both employed and unemployed workers involved only fixed transportation costs and consumption of composite goods which affect only short run utilities (see e.g. Zenou and Smith, 1995 and Smith and Zenou 1997). However, in the present model, locational choices involve an explicit trade-off between accessibility to the job market through search (affecting long run utilities) and consumption of composite goods (affecting short run utilities). This implies that expected lifetime utilities play a fundamental role so that two market equilibria (land and labor) have to be solved for simultaneously. To the best of our knowledge, this is the first model that has this complete interaction between land and labor markets.

In this context, by denoting by $R(d)$ the land market price at a distance d from the city-center and by w the wage earned by workers, we have the following definition.

Definition 1 A market equilibrium $(R(d); w; \mu; u)$ is such that the urban land use equilibrium and the labor market equilibrium are solved for simultaneously.

Thus, a market equilibrium requires solving simultaneously two problems:

(i) a location and rental price outcome (referred to as an urban land use equilibrium), and

(ii) a (steady state) matching equilibrium with determines w, θ and u (referred to as a labor market equilibrium).

We will give below more precise definitions of these two markets.

2 Equilibrium urban configurations

The city is monocentric, i.e., all firms are assumed to be exogeneously located in the central business district (CBD hereafter), linear, closed and landlords are absent.[7] There is a continuum of workers uniformly distributed along the linear city who endogeneously decide their optimal residence between the CBD and the city fringe. They all consume the same amount of land (normalized to 1) and the density of residential land parcels is taken to be unity so that there are exactly d units of housing within a distance d of the CBD.

Employed workers go to the CBD to work and to shop while unemployed workers go to the CBD to be interviewed and to shop. Let us denote by $\alpha_s d$, $\alpha_e d$ and $\alpha_u d$ the transportation cost at a distance d from the CBD for respectively shopping, working and unemployed specific activities (interviews, registration), with $\alpha_s > 0; \alpha_e > 0$, $\alpha_u > 0$. In this context, the transportation cost for the employed workers is equal to $(\alpha_s + \alpha_e)d$ and for the unemployed workers it is equal to $(\alpha_s + \alpha_u)d$. All workers bear land rent costs at the market price $R(d)$ and receive a wage w when employed and unemployment benefits b if unemployed. We denote by I_u and I_e the expected discounted lifetime net income of the unemployed and the employed respectively. We assume that location changes are costless. With the Poisson probabilities defined above, infinite lived workers have the following intertemporal utility functions:

$$rI_u(d) = b - (\alpha_s + \alpha_u)d - R(d) + p(d)\left[\left(\max_{d'} I_e(d')\right) - I_u(d)\right] \tag{4}$$

$$rI_e(d) = w - (\alpha_s + \alpha_e)d - R(d) + \delta\left[\left(\max_{d'} I_u(d')\right) - I_e(d)\right] \tag{5}$$

where r is the exogeneous discount rate. Let us comment on (4). When a worker is unemployed today, he resides in d and his net income is $b - (\alpha_s + \alpha_u)d - R(d)$. Then, he can get a job with a probability $p(d)$ and if so, he relocates optimally in d' and obtains an increase in income of $I_e(d') - I_u(d)$. The interpretation of (5) is similar.

[7] All these assumptions are very standard in urban economics (see e.g. Brueckner, 1987 or Fujita, 1989).

Due to the absence of relocation costs, the urban equilibrium is such that all the unemployed enjoy the same level of utility $rI_u = r\overline{I_u}$ as the employed $rI_e = r\overline{I_e}$. Indeed, any utility differential within the city would lead to the relocation of some workers which will rise bid rents up to the point where all differences in utility disappear. We now have to determine the optimal location of all workers in the city. The standard way of doing it is to use the concept of bid rents (Fujita, 1989) which are defined as the maximum land rent at a distance d that each type of workers is ready to pay in order to reach its respective equilibrium utility level. Therefore, the bid rents of the unemployed and employed are respectively equal to:

$$\Psi_u(d;\overline{I_u};\overline{I_e}) = b - (\alpha_s + \alpha_u)d + p(d)\overline{I_e} - (r + p(d))\overline{I_u} \tag{6}$$

$$\Psi_e(d;\overline{I_u};\overline{I_e}) = w - (\alpha_s + \alpha_e)d + \delta\overline{I_u} - (r + \delta)\overline{I_e} \tag{7}$$

We have now to determine the location of the unemployed and of the employed workers in the city. For this purpose, we need to calculate the bid rent slopes for each type of worker since a steeper bid rent corresponds to an equilibrium location closer to the CBD. They are respectively equal to:

$$\frac{\partial\Psi_u(d;\overline{I_u};\overline{I_e})}{\partial d} = -(\alpha_s + \alpha_u) + p'(d)(\overline{I_e} - \overline{I_u}) \leq 0 \tag{8}$$

$$\frac{\partial\Psi_e(d;\overline{I_u};\overline{I_e})}{\partial d} = -(\alpha_s + \alpha_e) \leq 0 \tag{9}$$

where $p'(d) = \theta q(\theta)s'(d) < 0$. These slopes (in absolute values) can be interpreted as the marginal cost that a worker is ready to pay in order to be marginally closer to the CBD.[8] This marginal cost for the unemployed is the sum of the marginal commuting cost, $(\alpha_s + \alpha_u)$, and the marginal probability of finding a job times the (intertemporal) surplus of being employed. On the other hand, the employed workers bear only marginal commuting cost equal to $(\alpha_s + \alpha_e)$ since the probability of losing a job δ is totally exogeneous and does not depend on the location of workers. If d_f denotes the city fringe, we have the following definition.

Definition 2 The urban land use equilibrium $R(d)$ is the upper envelop of all workers' bid rents and of the agricultural land rent R_A, i.e.,

$$R(d) = \max\left\{\Psi_u(d;\overline{I_u};\overline{I_e});\ \Psi_e(d;\overline{I_u};\overline{I_e});\ R_A\right\} \quad \text{at each } d \in [0; d_f]$$

[8] We assume that the intersection of the slopes of the two bid rents is of zero measure so that we exclude mixed configurations where both employed and unemployed are located in the same place. We only consider separated configurations where only one type of workers locate within the same segment.

With this general definition, a large number of urban equilibria can arise. However, for analytical simplicity, we here focus on linear bid rent slopes in which only two urban equilibria are possible. In the first (Equilibrium 1), the unemployed reside in the vicinity of the CBD and the employed at the outskirts of the city whereas in the second (Equilibrium 2), the unemployed locate at the outskirts of the city and the employed close to the city-center.[9] In order to have linear bid rents, we assume that $s(d) = s_0 - ad$ ($s_0 > 0$ and $a > 0$), which implies that $p''(d) = 0$ and thus:

$$\frac{\partial^2 \Psi_u(d;\overline{I_u};\overline{I_e})}{\partial d^2} = \frac{\partial^2 \Psi_e(d;\overline{I_u};\overline{I_e})}{\partial d^2} = 0$$

Thus, according to (8) and (9), the resulting urban equilibrium is determined by a trade-off between the difference in commuting costs, $(\alpha_e - \alpha_u)$; and the marginal probability of getting a job. This is due to the fact that both the employed and the unemployed want to be as close as possible to the CBD because the former want to save on commuting costs and the latter desire to increase their probability of getting a job.

The important condition for having one equilibrium or the other is given by an inequality comparing the slopes of the bid rents of employed and unemployed workers. Equilibrium 1 occurs if and only if:

Condition 1:

$$(\alpha_e - \alpha_u) < \theta_1 q(\theta_1) a(\overline{I_{e_1}} - \overline{I_{u_1}}) \tag{10}$$

Equation (10) means that the differential in commuting costs between the employed and the unemployed is lower than the unemployed expected return of being more efficient in search. Thus if (10) holds, the unemployed bid away the employed towards the periphery of the city. In the other equilibrium (Equilibrium 2), we have the opposite inequality:

Condition 2:

$$(\alpha_e - \alpha_u) > \theta_2 q(\theta_2) a(\overline{I_{e_2}} - \overline{I_{u_2}}) \tag{11}$$

Observe that everything that may improve the return to search for unemployed workers ($\theta q(\theta)$ and $\overline{I_e} - \overline{I_u}$) can help to switch from equilibrium 2 to equilibrium 1. Observe also that when the employed have lower commuting costs than the unemployed, the only possible equilibrium is the first one since the unemployed have all the incentives to reside closer to the CBD.

[9] All variables determined at the equilibrium $k(= 1;2)$ are indexed by k. At each urban equilibrium k corresponds a labor market equilibrium k.

By using Definition 2 and the two conditions, (10) and (11), we are now able to define the two urban equilibria (remember that the city is closed, linear and that landlords are absent). For analytical simplicity, we normalize the labor force L to 1, i.e. $L = U + N = 1$ (N is the number of employed workers) so that the unemployment rate u is equal to the unemployment level U. If we denote the city border between the unemployed and the employed by d_b^k $(k = 1; 2)$, we have the following definitions:

Definition 3 The urban equilibrium 1 $(\overline{I_{u_1}}; \overline{I_{e_1}}; d_b^1; d_f^1)$ is such that:

$$d_b^1 = u_1 \tag{12}$$

$$d_f^1 = 1 \tag{13}$$

$$\Psi_e(d_b^1; \overline{I_{u_1}}; \overline{I_{e_1}}) = \Psi_u(d_b^1; \overline{I_{u_1}}; \overline{I_{e_1}}) \tag{14}$$

$$\Psi_e(d_f^1; \overline{I_{u_1}}; \overline{I_{e_1}}) = R_A \tag{15}$$

This equilibrium is illustrated in Figure 1. By using (6), (7), (12) and replacing them in (14) and (15), we obtain:

$$\overline{I_{e_1}} - \overline{I_{u_1}} = \frac{w_1 - b - (\alpha_e - \alpha_u)u_1}{r + \delta + p(u_1)} \tag{16}$$

where $w_1; u_1; \theta_1$ will be determined at the labor market equilibrium 1. The average efficiency intensity in equilibrium 1 is equal to:

$$\overline{s_1} = s_0 - a\overline{d_1} = s_0 - a\frac{u_1}{2} \tag{17}$$

[Insert Figure 1 here]

We can now define the second urban equilibrium.

Definition 4 The urban equilibrium 2 $(\overline{I_{u_2}}; \overline{I_{e_2}}; d_b^2; d_f^2)$ is such that:

$$d_b^2 = 1 - u_2 \tag{18}$$

$$d_f^2 = 1 \tag{19}$$

$$\Psi_e(d_b^2; \overline{I_{u_2}}; \overline{I_{e_2}}) = \Psi_u(d_b^2; \overline{I_{u_2}}; \overline{I_{e_2}}) \tag{20}$$

$$\Psi_u(d_f^2; \overline{I_{u_2}}; \overline{I_{e_2}}) = R_A \tag{21}$$

This equilibrium is illustrated in Figure 2. By using (6), (7), (18) and replacing them in (20) and (21), we obtain:

$$\overline{I_{e_2}} - \overline{I_{u_2}} = \frac{w_2 - b - (\alpha_e - \alpha_u)(1 - u_2)}{r + \delta + p(1 - u_2)} \tag{22}$$

where $w_2; u_2; \theta_2$ will be determined at the labor market equilibrium 2. Similarly, the average efficiency intensity in equilibrium 2 is equal to:

$$\overline{s_2} = s_0 - a\overline{d_2} = s_0 - a(1 - \frac{u_2}{2}) \tag{23}$$

[Insert Figure 2 here]

Proposition 1: Whatever the labor market equilibrium, the average search efficiency in urban equilibrium 1 is greater than in urban equilibrium 2, i.e., $\overline{s_1} > \overline{s_2}$.

Proof. $\overline{s_1} > \overline{s_2} \Leftrightarrow -u_1/2 > -(1 - u_2/2) \Leftrightarrow u_1 + u_2 < 2$ which is always true. ■

This proposition is quite intuitive. Since the unemployed reside in the vicinity of the CBD in equilibrium 1, their probability of getting a job is higher than in equilibrium 2 and thus the duration of unemployment is shorter and their search efficiency is higher ($\overline{s_1} > \overline{s_2}$). However, the result is not as obvious as it might seem, since the average distance between unemployed workers and jobs depends on u_1 or u_2 which are still unknown.

Observe that we have assumed a perfect access to the credit market; this is necessary since, for instance in equilibrium 1, the unemployed who pay high land rents to maximize their lifetime utility have to borrow.[10] This assumption of perfect access to credit could be relaxed by assuming for example that the unemployed face credit constraints. This would obviously complicate the model without changing our main results.

In order to better understand our results, we first study the labor market equilibrium under exogeneous wages and then under bargain wages. The following definition is valid in both cases.

Definition 5 A (steady-state) labor market equilibrium $(w; \theta; u)$ is such that, given the matching technology defined by (1), all agents (workers and firms) maximize their respective objective function, i.e. this triple is determined by a steady-state condition, a free-entry condition for firms and a wage-setting mechanism.

[10] In this first equilibrium, unemployed may have exploding debt, since they receive b and pay at most $R(0)$. A sufficient condition for avoiding the problem is that $R(0) < b$.

3 The market equilibrium under exogeneous wages

In this section, we assume that wages are exogeneous and equal to $w_1 = w_2 = \overline{w}$.

3.1 Free-entry condition and labor demand

Let us denote by I_J and I_V the intertemporal profit of a job and of a vacancy, respectively. If γ is the search cost for the firm per unit of time and y is the product of the match, then I_J and I_V can be written as:

$$rI_J = y - \overline{w} + \delta(I_V - I_J) \tag{24}$$

$$rI_V = -\gamma + q(\theta)(I_J - I_V) \tag{25}$$

Following Pissarides (1990) and Mortensen and Pissarides (1994), we assume that firms post vacancies up to a point where:

$$I_V = 0 \tag{26}$$

which is a free entry condition. From this free entry condition, we have the following decreasing relation between labor market tightness and wages:

$$q(\theta) = \gamma.\frac{r + \delta}{y - \overline{w}} \tag{27}$$

In other words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm. Notice that this value is increasing in $\overline{s}$ (which is the average search efficiency determined at the urban equilibrium) since $\theta = V/(u.\overline{s})$ and $q'(\theta) < 0$.

Equation (27) defines in the space (θ, w) a curve (see Figure 3) representing the supply of vacancies (what we shall also call a labor demand curve L^d). This curve is independent of the type of urban equilibrium since, independently of workers' location, firms post vacancies up to $I_V = 0$. The L^d curve (27) is independent of d_b and is downward sloping in the plane (θ, w). Since the wage is independent of the urban equilibrium, we have a unique $\theta^* = \theta_1 = \theta_2$. This means that, depending on the values of θ^*, only one urban equilibrium prevails. Using Lemma A5,[11] one can now show that there is a critical value $\overline{m}$, obtained from conditions (10) and (11), and defined by:

$$\overline{m} = \frac{(\alpha_e - \alpha_u)(r + \delta)}{a(\overline{w} - b) - (\alpha_e - \alpha_u)s_0}$$

[11] All Lemmas are stated and demonstrated in the Appendix.

determining which urban equilibrium prevails. The intuition of Lemma A5 is straightforward: when the probability of obtaining a job is very high, the marginal return to be closer to the center is higher for the unemployed, and they tend to locate close to the employment center.

[Insert Figure 3 here]

3.2 Steady-state labor market equilibrium and market equilibrium

As stated above, equation (27) determines a unique $\theta^* = V/(u\overline{s_k})$ which gives a relation between V and u. This is an upward sloping curve in the $u - V$ space called the VS curve (see Lemma A4). Moreover, even though there is a unique θ, we have two VS curves (VS_k for urban equilibrium $k = 1, 2$) since $\overline{s}_1 \neq \overline{s}_2$ (see Figure 4). We show in Lemma A4, that VS_1 is steeper than VS_2. We can now close the model by the following steady-state condition on flows:

$$\theta q(\theta)\overline{s}_k u_k = (1 - u_k)\delta \quad , \quad k = 1, 2 \tag{28}$$

which is equivalent to:

$$u_k = \frac{\delta}{\delta + \theta q(\theta)\overline{s}_k} \quad , \quad k = 1, 2 \tag{29}$$

and can be rewritten as:

$$\delta(1 - u_k) - V_k q(\theta_k) = 0 \tag{30}$$

This equation can be mapped in figure 4 in the plane (u_k, v_k): it is the so-called Beveridge curve UV_k. Its properties are given in Lemma A6. We have the following result.[12]

Theorem 1: When wages are exogeneous, there exists a unique and stable market equilibrium $(R(d), \theta^*, u_k^*)$, $k = 1, 2$ and only the two following cases are possible:

- If $\theta^* q(\theta^*) > \overline{m}$, urban equilibrium 1 prevails;
- If $\theta q(\theta^*) < \overline{m}$, urban equilibrium 2 prevails.

In addition, we have: $u_1^* < u_2^*$ and $\overline{s}_1^* > \overline{s}_2^*$.

Proof.

Existence and uniqueness: In the plane (u, V), for each urban equilibrium, we have an upward sloping curve given by θ in (27). This is the VS curve, that cuts the axes and a downward sloping curve (30), the Beveridge curve (UV), that does not cut the axes. We have therefore a unique labor equilibrium.

[12] A variable with a star as a superscript refers to the market equilibrium.

Stability: We use the same argument as in Pissarides (1990), chapter 3, p47, see graph 3.2 for instance.

Next, from Lemma A7, the Beveridge curve 2 (the one corresponding to the urban equilibrium 2) is always above the Beveridge curve 1 while VS_1 is steeper than VS_2 (see Lemma A4). We have therefore that: $u_2^* > u_1^*$. Moreover, by Proposition 1, we have $\bar{s}_1^* > \bar{s}_2^*$. ■

Since when the wage is exogeneous $\theta^* = \theta_1 = \theta_2$, the main difference in terms of unemployment rate between the two urban equilibria is on search efficiency. It follows that in equilibrium 1, the unemployed have good access to jobs since they reside at the vicinity of the CBD. In this case, the unemployment rate is given by u_1 and defined by (29) and the average efficiency by $\bar{s}_1$. In equilibrium 2, the unemployed are far away from jobs and their search efficiency is quite low $\bar{s}_1 > \bar{s}_2$. An intuitive interpretation is that "segregated" cities i.e., cities where unemployed workers are far away from the employment-center, are less "efficient" than "integrated" cities. In the case of endogeneous wages, the urban configuration matter not only for $\bar{s}$, but also for θ, since wages and thus job creations must reflects the costs associated with distance.

[Insert Figure 4 here]

4 Wage bargaining

With endogeneous wages, new effects of distance on the equilibrium unemployment will appear, notably through commuting costs which affect wages. The usual assumption about wage determination is that, at each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between firms and workers. The total surplus is the sum of the surplus of the workers, $\overline{I_e} - \overline{I_u}$; and the surplus of the firms. At each period, the wage is determined by :

$$w_k = \operatorname{Arg\,max}(\overline{I_{e_k}} - \overline{I_{u_k}})^{\beta}(I_{J_k} - I_{V_k})^{1-\beta} \qquad k = 1;2 \tag{31}$$

where k denotes the urban equilibrium. Observe that $\overline{I_{u_k}}$, the threat point for the worker in the urban equilibrium k, does not depend on the current location of the worker, who will relocate if there is a transition in his employment status. First order condition yields:

$$\frac{\beta}{1-\beta}\left(\frac{\partial \overline{I_{e_k}}}{\partial w_k} - \frac{\partial \overline{I_{u_k}}}{\partial w_k}\right) I_{J_k} + \left(\overline{I_{e_k}} - \overline{I_{u_k}}\right)\frac{\partial I_{J_k}}{\partial w_k} = 0 \qquad k = 1;2 \tag{32}$$

Since the wage is negotiated at each period, $\overline{I_{u_k}}$ does not depend on current w_k and so $\frac{\partial \overline{I_{u_k}}}{\partial w_k} = 0$. By using (5) and (26), we can rewrite (32) as:

$$(\overline{I_{e_k}} - \overline{I_{u_k}})(w_k) = \frac{\beta}{1-\beta}\frac{\gamma}{q(\theta_k)} \qquad k = 1, 2 \tag{33}$$

Equation (33) defines a positive relationship between wages and labor market tightness which is sometimes interpreted as a wage setting function or a labor supply function. In urban equilibrium 1, we obtain:

$$w_1 = (1-\beta)\left[b + (\alpha_e - \alpha_u)u_1\right] + \beta\left[y + s(u_1)\theta_1\gamma\right] \tag{34}$$

The wage setting curve described by (34) is a positively sloped curve in the plan (w, θ). In urban equilibrium 2, we would instead obtain :

$$w_2 = (1-\beta)\left[b + (\alpha_e - \alpha_u)(1-u_2)\right] + \beta\left[y + s(1-u_2)\theta_2\gamma\right] \tag{35}$$

From the free entry condition (26), we still have a decreasing relation between labor market tightness and wages (27).

Let us interpret (34) with a focus on how wages depend on unemployment u_1.[13] The first part of the RHS of (34), $(1-\beta)\left[b + (\alpha_e - \alpha_u)u_1\right]$, is what firms must pay to induce workers to accept the job offer: firms must exactly compensate the transportation cost difference (between the employed and the unemployed) of the unemployed worker who is the further away from the CBD, i.e. located at $d_b^1 = u_1$ (remember that here unemployment is also a distance). Therefore, when u_1 increases, this marginal unemployed worker is further away from the CBD and wages must compensate the transportation cost difference. The second part of the RHS of (34), $[\beta y + s(u_1)\theta_1\gamma]$ implies that distance (u_1) has the opposite effect. When u_1 rises, the average unemployed (i.e. the one located in $\bar{d} = u_1/2$) is further away from the city-center and thus his outside option decreases. This is what we call the "outside option effect". The first effect is a pure spatial cost since $(\alpha_e - \alpha_u)u_1$ represents the space cost differential between employed and unemployed workers paying the same bid rent whereas the second effect is a mixed labor-spatial cost one.[14]

[13] For empirical evidence of links between local wages and unemployment, see Blanchflower and Oswald (1994) and Topel (1986).

[14] In addition to the thin market externality of the matching technology, already present in Pissarides, one can notice that these effects are two new external effects introduced by the spatial dimension. The exact status

The equation (35) is exactly similar to (34) if one replaces u_1 by $1 - u_2$. Also observe that while the two wages are upward sloping in the plane $(w_k; \theta_k)$, WS_1 is steeper than WS_2 but the intercept of WS_1 is lower (see lemma A8). This is illustrated in Figure 5: for small values of θ; $w_2 > w_1$ whereas for large values of θ; $w_2 < w_1$: In other words, if the labor market is not very tense (there are few vacancies compared to efficient units of unemployed workers), the probability to get a job is low and thus the compensation effect dominates the outside option effect. In the other case when the labor market is very tense, we obtain the opposite result.

[Insert Figure 5 here]

The two equations (27) and (34) (resp. (35) in the second equilibrium) determine wages and labor tightness parametrized by u_k. The model is closed by the steady-state condition on flows defined as in the previous section by (29). Therefore, we have three equations (34) (or (35) for equilibrium 2), (27) and (28) and three unknowns $w_k; \theta_k$ and u_k $(k = 1; 2)$. By Lemma A10, we have again a critical value

$$\bar{\theta} = \frac{1}{\gamma}\frac{(\alpha_e - \alpha_u)}{a}\left(\frac{1-\beta}{\beta}\right)$$

obtained from the conditions (10) and (11), which depends only on exogeneous parameters. Since $\bar{\theta}$ determines which urban equilibrium prevails, we have the following result.

Theorem 2: When wages are endogeneous, there exists a unique and stable market equilibrium $(R(d); w_k^*; \theta_k^*; u_k^*)$, $k = 1; 2$ and only the two following cases are possible:

- If $\bar{\theta} < \theta_1^* < \theta_2^*$; urban equilibrium 1 prevails;
- If $\theta_2^* < \theta_1^* < \bar{\theta}$; urban equilibrium 2 prevails.

Proof. Since θ_k is determined by the intersection of the labor demand curve and the wage setting line (see Figure 5), two situations are possible: either the labor demand curve cuts the wage setting before the intersection point $(\hat{w}; \hat{\theta})$ (which defined in lemma A8) or after it. In the first case, we have: $\theta_2^* < \theta_1^* < \bar{\theta} = \hat{\theta}$; and then according to Lemma A10, there is only one unique urban equilibrium which is equilibrium 2. In the second case, we have: $\bar{\theta} = \hat{\theta} < \theta_1^* < \theta_2^*$ and according to Lemma A12 only Equilibrium 1 exists. Moreover, since there exists a unique

of these externalities and their systematic studies are beyond the scope of the paper. They seem to be close to the "distributional externalities", similar in nature to the pecuniary externalities, but creating here some inefficiency.

labor market equilibrium in each urban equilibrium (see Lemma A11), then there exists a unique market equilibrium. Finally, for the stability of the market equilibrium, we use the same argument as in Theorem 1. ■

Observe that multiple urban equilibria can never happen here because the condition for multiple equilibria is $\theta_2^* < \bar{l} < \theta_1^*$ and it can never be satisfied since the intersection of the labor demand with the wage setting line never satisfies this condition. This is due to the fact that $\bar{l} = \hat{\theta}$. Observe also that we exclude the case when the labor demand curve intersects the two wage setting curves at exactly $(\hat{w}; \hat{\theta})$.

5 The role of space in the theory of local unemployment

The aim of this section is to develop further the role and the importance of space in the formation of unemployment.

5.1 Interaction between land and labor markets: the dependence of search efficiency on distance

One of the key assumption of our model is that the search efficiency s_i of each worker i depends on the distance between residence and the job-center, i.e., $s_i = s_i(d)$ with $s_i'(d) < 0$. To evaluate the importance of this relation, let us assume that s_i is independent of d but workers still locate in the city and thus bear land rents and commuting costs. For that we have $a = 0$ so that $\bar{s} = s_0$. In this context, the inequalities (10) and (11) collapse to $\alpha_e - \alpha_u \gtreqless 0$. In other words, the land market equilibrium is independent of the labor market equilibrium. The location choices of the employed and the unemployed, which depend on the slopes of the bid rents, involve only transportation costs so that: when $\alpha_u > \alpha_e$, the unemployed locate at the vicinity of the CBD whereas when $\alpha_u < \alpha_e$, the unemployed reside at the outskirts of the city, irrespective of the labor market equilibrium outcome. So, when wages are exogenous, the equilibrium unemployment and vacancy rates would be exactly the same as in the standard non-spatial matching models (see e.g. Pissarides, 1990); there would be only one θ^*, $\bar{s} = s_0$ being independent of the urban land use equilibrium. When the wage is a result of a bargaining between workers and firms, commuting costs affect wages so that each urban equilibrium determines an equilibrium θ_k^* that depends on the urban configuration k. This is because wages and thus job creations rely on commuting costs, which are different in each equilibrium k. Thus, the labor market equilibrium will depend on the land market equilibrium through θ_k while the reverse is not true. We can summarize our discussion by the following table.

Table 1: Interaction between land and labor markets

	Exogeneous wages	Exogeneous wages
$a = 0$	No Interaction	Partial Interaction (UE $\rightarrow$ LE)
$a > 0$	Complete Interaction	Complete Interaction

(UE $\rightarrow$ LE means that the interaction is from the Urban Equilibrium to the Labor Equilibrium)

This table highlights the role of the relation between search efficiency and distance to the CBD. When there is no such relation ($a = 0$), the land market analysis is either totally independent of the labor market one (exogeneous wages) or relies only on commuting costs (endogeneous wages). When it exists ($a > 0$), by contrast, there is a complete interaction between land and labor markets. This allows us to shed some new lights on the working on the urban labor markets and especially on the understanding of urban unemployment: space is a major determinant of the process of search for both workers and firms.

5.2 Decomposition of unemployment

We pursue our analysis of the importance of space in equilibrium unemployment by determining the part of unemployment only due to spatial frictions.

Let us start with exogeneous wages. Since θ_k is independent of $\bar{s}$ (see (27)), we have $\theta_1 = \theta_2 = \theta$. Then by using (29), the unemployment rate in Equilibrium 1 is given by:

$$u_1 = \frac{\delta}{\delta + \theta q(\theta)(s_0 - a u_1/2)} \tag{36}$$

Let us further define by:

$$u_0 = \frac{\delta}{\delta + \theta q(\theta) s_0} \tag{37}$$

the part of unemployment that is independent of spatial frictions, i.e. when $a = 0$ so that $\bar{s}_0 = s_0$. Observe that here θ is independent of $\bar{s}$ so that u_0 is a function of θ (this will no longer be true when wages are endogeneous). Moreover, by a Taylor first-order expansion for small a/s_0, we easily obtain:

$$u_1^* = u_0 \left(1 + \frac{a}{2s_0} u_0 (1 - u_0)\right) = u_0 + u_1^s \tag{38}$$

where $u_1^s \equiv u_0 \left[a u_0 (1 - u_0)/2s_0\right]$ is the unemployment that is only due to spatial frictions in Equilibrium 1 and u_0 is defined by (37). Observe that u_1^s is increasing in a/s_0, the parameter representing the loss in information through distance and null when $a = 0$. Observe also that the pure frictional unemployment u_0 affects u_1^s in the following way:

$$\text{If} \quad u_0 < \frac{2}{3}, \quad \text{then} \quad \frac{\partial u_1^s}{\partial u_0} > 0$$

In general $u_0 < 2/3$ so that u_0 affects positively u_1^s; showing the complete interaction between land and labor markets. This is quite natural: higher 'spaceless' unemployment u_0 affects positively frictions due to spatial heterogeneity (this is a side-effect of the dispersion of space on the unemployed themselves). In Equilibrium 2, by using the same technique, we obtain:

$$u_2^* = u_0 \left[1 + \frac{a}{s_0}(1 - u_0)\left(1 - \frac{u_0}{2}\right)\right] = u_0 + u_2^s \tag{39}$$

where $u_2^s \equiv u_0 \left[a\,(1 - u_0)(1 - u_0/2)/s_0\right]$ is the unemployment that is only due to spatial frictions in Equilibrium 2. The comments are the same as in Equilibrium 1. We also have:

$$\text{If} \quad u_0 < 1 - \frac{\sqrt{3}}{3} \simeq 0.42, \quad \text{then} \quad \frac{\partial u_2^s}{\partial u_0} > 0$$

Proposition 2: The part of unemployment due to spatial frictions is lower in Equilibrium 1 than in Equilibrium 2, i.e.,

$$u_1^s < u_2^s \tag{40}$$

In fact, the demonstration of this proposition is the same as for Proposition 1. Indeed, we have:

$$u_2^s/u_1^s = \frac{2 - u_0}{u_0} > 1 \qquad \Longleftrightarrow \qquad u_0 < 1$$

so that spatial frictions are higher in Equilibrium 2 because the unemployed are in average further away from jobs so that the aggregate matching is less efficient. Moreover, the exact position of the unemployed is determined by the level of unemployment and thus, in a first order approximation, on u_0.[15] This highlights the debate on the spatial mimatch hypothesis discussed in the introduction. Indeed, what is crucial here is the access to employment so that when the unemployed reside far away from jobs, their efficiency is low because of high spatial frictions.

Under endogenous wage setting, a larger set of parameters determines the spatial component of unemployment since θ_K depends now on wages, which in turn are affected by

[15] Recall that, in a first order approximation, the location of the average unemployed worker is $u_0/2$ in Equilibrium 1 and $1 - u_0/2$ in Equilibrium 2.

transportation costs and the prevailing urban equilibrium. To give the main intuitions, we only describe the decomposition of unemployment in Equilibrium 2 and thus we can drop the superscript k.

First, the endogeneous wage w defined by (35) can be decomposed into three parts:

$$w = w_0 + w_\alpha - w_a \tag{41}$$

where $w_0 \equiv (1-\beta)b + \beta(y + s_0\theta\gamma)$ is the wage that would receive workers if all agents were located in the same point (it is the standard Pissarides and Mortensen-Pissarides wage), $w_\alpha \equiv (1-\beta)(\alpha_e - \alpha_u)(1-u)$ reflects the impact of distance on transportation costs and thus on wages, and $w_a \equiv \beta\, a\,(1-u)\theta\gamma$ the fact that search efficiency varies with distance to jobs (this was called the 'outside option effect' of distance in the previous section). By using a Taylor expansion, assuming both that a/s_0 and $(\alpha_e - \alpha_u)/b$ are small compared to 1, we have from (27):

$$q(\theta) = \gamma \frac{r+\delta}{y - w_0}\left(1 + \frac{w_\alpha - w_a}{y - w_0}\right) \tag{42}$$

This implies that:

$$\theta = \theta_0\left(1 + \frac{1}{\eta}\frac{w_\alpha - w_a}{y - w_0}\right)$$

where θ_0 is obtained by setting $w_a = w_\alpha = 0$ and where η is the elasticity of $q(\theta)$ with respect to θ (see Lemma A1). Second, by using a Taylor expansion, it follows from (29) that:

$$\begin{aligned} u_2^* &\simeq u_0\left[1 + \frac{a}{s_0}(1-u_0)\left(1 - \frac{u_0}{2}\right)\right] + \frac{s_0}{\delta}\frac{1-\eta}{\eta}\frac{w_\alpha - w_a}{y - w_0} \\ &\simeq u_0 + u^s(w_0) + u^s(w_\alpha) - u^s(w_a) \end{aligned} \tag{43}$$

where $u^s(w_0) = u_0\, a/s_0(1-u_0)(1-u_0/2)$ is the standard non-spatial unemployment rate (when $w = w_0$), $u^s(w_\alpha)$, the spatial unemployment due only to commuting costs (when $w = w_\alpha$) and $u^s(w_a)$; the unemployment resulting from the outside option of workers (when $w = w_\alpha$). A rise in the loss of information with distance, a, has the same positive effect has a negative impact through w_a since the outside option of the unemployed workers is reduced. Therefore $u^s(w_a)$ contributes to a reduction in unemployment. On the other hand, a rise in a does increase $u^s(w_0)$ through an increase in u_0[16] because of lower search efficiency. Finally, and contrary to the exogeneous wage case (see (38)), the spatial part of unemployment is now directly affected by transportation costs. Indeed, when $\alpha_e - \alpha_u$ increases, w_α and thus wages

[16] again if $u_0 < 1 - \frac{\sqrt{3}}{3}$

are higher to compensate the employed workers. It follows that there are fewer job creations which increase the spatial part of unemployment $u^s(w_\alpha)$.

We could use this decomposition to analyze the potential policy implications of the model, even though the welfare function of the local planner is not explicitly introduced. Notably, the role of the policy maker would be to affect the parameters of conditions (10) and (11) in order to switch from equilibrium 2 to equilibrium 1, at least as long as the employed workers do not pay too high commuting costs.[17]

6 Endogeneous search effort

So far, the interaction of space and unemployment is present because of the negative dependence between s and d; the interpretation being that distance results in a loss of information. In this section, we show a complementary channel through which spatial dispersion affects the labor market: the fact that search costs depend on distance.

Each individual's search efficiency s_i depends now on his effort e. We assume decreasing returns to scale in the effort, i.e., $s'(e) > 0$ and $s''(e) \leq 0$. As before, each interview is made in the CBD and thus involves transport costs. We denote by $C_u(e; d)$ the search costs associated with a level of effort e for resident in d. For instance, using the previous linear costs in distance, one would have $C_u(e; d) = \alpha_u(e).d$. We assume that the search commuting cost is an increasing and convex function of the effort level e devoted to job search, i.e., $\partial C_u/\partial e > 0$ and $\partial^2 C_u/\partial^2 e \geq 0$, and that, quite naturally, $\partial^2 C_u/\partial e \partial d > 0$: the search effort marginally costs more further away from job.[18] There is therefore a trade-off between search costs and returns associated with a higher probability to exit from unemployment. The value

[17] In this logic, subsidizing all the commuting costs tends to reduce the spatial component of unemployment and thus global unemployment. This is in accordance to the well known empirical fact that in most American cities one major problem of access to employment is the bad transportation network. In the same spirit, the (local) government can also reduce fixed search cost s_0, increase variable search cost a and/or decrease the level of unemployment benefits b (observe that b don't affect directly unemployment by has an indirect effect through (11)). Indeed, inspection of conditions (10) and (11) shows that when the unemployment benefits are quite low, Equilibrium 1 is more likely to hold so that the unemployment is reduced compared to Equilibrium 2.

[18] Note that, in theory, one could perfectly imagine that s, at a given e, also depends on distance (through the informational imperfections). However, in this section, we only focus on the impact of search effort on search efficiency.

of unemployment can be written as:

$$rI_u(d) = b - (\alpha_s d + C_u(e; d)) - R(d) + \theta.q(\theta).s(e)\left[\mathrm{Max}_{d'} I_e(d') - I_u(d)\right] \tag{44}$$

while the value of employment is still given by (5). The unemployed worker located at a distance d from the CBD chooses e^* that maximizes his intertemporal utility (44). We have therefore the following proposition.

Proposition 3. When workers choose endogeneously their effort, search efficiency and thus the probability of obtaining a job decreases with distance to the city-center.

Proof: The first order condition on effort yields[19]:

$$\theta.q(\theta).s'(e^*)(\overline{I_e} - \overline{I_u}) = \partial C_u(e^*; d)/\partial e \tag{45}$$

By totally differentiating (45), we obtain:

$$\frac{\partial e^*}{\partial d} = \frac{\partial^2 C_u(e^*; d)/\partial e \partial d}{\theta q(\theta)(\overline{I_e} - \overline{I_u})s''(e^*) - \partial^2 C_u(e^*; d)/\partial^2 e} < 0$$

and thus

$$\frac{\partial s}{\partial d} = s'(e)\frac{\partial e}{\partial d} < 0$$

■

It is readily verified that the unemployed's bid rents are not anymore linear but convex since s is now a non-linear function of distance. This implies that three different urban configurations can emerge. As before, urban equilibria 1 and 2 are possible but also a third one (equilibrium 3) in which the unemployed reside both at the vicinity of the CBD and at the outskirts of the city and the employed in between the unemployed (see Figure 6). In the latter case, even though in equilibrium all unemployed workers have the same utility level, one can distinguish the ones located close to the CBD from the others residing further away. The latter can be considered as long term unemployed (whose level is given by u_{lr}) since their probability of obtaining of job is lower and their unemployment duration is longer than the former who are seen as short term unemployed (whose level is given by u_{sr}). Actually, the trade off is quite clear. Either workers are long run unemployed, live on welfare but pay very low land rents since they reside far away from jobs or they are short run unemployed but pay

[19] The interpretation of (45) is straightforward. The LHS is the marginal probability generated by one more interview times the surplus of a worker when he leaves unemployment whereas the RHS is the marginal commuting cost of searching for a job.

a very high rent since they live close to the employment center. Thus, space (or location) makes workers heterogeneous in terms of access to employment: those who are further away from jobs experience longer unemployment spells (see e.g. Rogers, 1997 for empirical results).

[Insert Figure 6 here]

We can also analyze the interaction between space and equilibrium unemployment. By assuming for simplicity that wages are exogeneous (so that $\theta_1 = \theta_2 = \theta_3 = \theta$), the unemployment rate is given by:

$$u_k = \frac{\delta}{\delta + \theta.q(\theta)\bar{s}_k} \qquad k = 1; 2; 3$$

Thus, the only spatial interaction in unemployment is reflected through:

$$\bar{s}_k = \int_{\text{Unemployed}} s(e^*(z))dz \qquad k = 1; 2; 3$$

where z replaces distance in the integral for notational convenience. Assume $s(e) = e^{\sigma}$, with $\sigma < 1/2$ for reasons that will become clear below, and that $C(e; d) = \alpha_u(e).d = (\alpha_{0u} + \alpha_u.e)d$. By using (45), we easily obtain:

$$s(e_k^*) = \left[\frac{\theta q(\theta)(\overline{I_{e_k}} - \overline{I_{u_k}})}{2\alpha_u d}\right]^{\frac{\sigma}{1-\sigma}} \qquad k = 1; 2; 3 \tag{46}$$

or

$$e_k^* = \left[\frac{\theta q(\theta)(\overline{I_{e_k}} - \overline{I_{u_k}})}{2\alpha_u d}\right]^{\frac{1}{1-\sigma}} \qquad k = 1; 2; 3 \tag{47}$$

One can see that $\sigma < 1/2$ allows us to integrate $s(e)$ between 0 and any number. Focusing again on the second urban equilibrium, we obtain:

$$\bar{s}_2 = \left(\frac{1-2\sigma}{1-\sigma}\right)\left[\frac{\theta q(\theta)(\overline{I_{e_2}} - \overline{I_{u_2}})}{2\alpha_u}\right]^{\frac{\sigma}{1-\sigma}}\left[1 - (1-u)^{\frac{1-2\sigma}{1-\sigma}}\right] \tag{48}$$

Space has an impact on $\bar{s}$ through the parameter α_u which is the cross-derivative of the cost function for the unemployed, i.e. $\frac{\partial^2 C(e;d)}{\partial e \partial d}$. Subsidizing search costs for the unemployed would decrease α_u and increase aggregate search efficiency. It is also reflected in the term $(1-u)^{\frac{1-2\sigma}{1-\sigma}}$ which, as before, reflects the average distance between unemployed workers and jobs. Equilibrium 3 is also interesting since we have:

$$\bar{s}_3 = \left(\frac{1-2\sigma}{1-\sigma}\right)\left[\frac{\theta q(\theta)(\overline{I_{e_3}} - \overline{I_{u_3}})}{2\alpha_u}\right]^{\frac{\sigma}{1-\sigma}}\left[(u_{sr})^{\frac{1-2\sigma}{1-\sigma}} + 1 - (1-u_{lr})^{\frac{1-2\sigma}{1-\sigma}}\right] \tag{49}$$

where u_{sr} and u_{lr} are the short run and long run unemployment rates respectively. In $\bar{s}_3$, the last term represents the spatial dispersion of short-term and long-term unemployed, both contributing much more to higher aggregate search efficiency.

7 Conclusion

In this paper, we have modelled the general interaction between the spatial dispersion of economic agents and the imperfection in information about economic opportunities. Notably, we have built an index of aggregate information about these opportunities, $\bar{s}$, and showed how the location of agents relative to the economic opportunities determines $\bar{s}$. We have also introduced, in the last section, the cost function of agents to acquire information, and showed again how the index $\bar{s}$ depends on that cost function and its parameters.

Although these mechanisms are extremely general, we have applied them to a context of local labor markets, i.e., a context in which search frictions and locations of agents interact in a very natural way. In this context, we have decomposed unemployment in a part due to the usual labor market parameters, which would be obtained if all agents were located in the same point - the usual assumption, and a new part due to the dispersion of agents.

We have also shown that the shape of a city has a strong impact on the spatial part of unemployment. According to the di¤erent incentives for workers to be closer from the CBD (transport costs and search e¢ ciency), a category of workers bid away the other to the periphery. When the unemployed are far from the city-center, the outcome is likely to be quite negative since search e¢ ciency will be very low in average and the unemployment rate high.

We believe that the role of space is however not limited to those two mechanisms: for example, the role of social networks, neighborhood peer group e¤ects, education externalities are also very relevant in the context of search models... We leave this for future research.

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Appendix : Proof of Lemmas

Lemma A1: We have the following properties of the function $q(\theta)$ and $\theta q(\theta)$:

$$q'(\theta) < 0 \quad ; \quad q''(\theta) > 0$$

$$-1 < \eta = \frac{\partial q(\theta)}{\partial \theta}\frac{\theta}{q(\theta)} < 0$$

Proof. The results stem directly from the properties of the matching function $x(.)$. ■

Lemma A2: For the two urban equilibria, the following relations always hold along the (VS) curve:

$$\frac{\partial \theta_k}{\partial V_k} > 0 \quad ; \quad \frac{\partial \theta_k}{\partial u_k} < 0 \qquad k = 1,2$$

Proof. Since $\theta_k = V_k/(\bar{s}_k u_k)$, we have:

$$\frac{\partial \theta_k}{\partial V_k} = \frac{1}{u_k \bar{s}_k} > 0 \qquad \forall k = 1,2$$

Next,

$$\frac{\partial \theta_k}{\partial u_k} = -\frac{V_k}{(u_k \bar{s}_k)^2}\left(\bar{s}_k + u_k \frac{\partial \bar{s}_k}{\partial u_k}\right)$$

Let us start with urban equilibrium 2 where $\bar{s}_2 = s_0 - a(1 - u_2/2)$ and $\partial \bar{s}_2/\partial u_2 = a/2 > 0$. This implies that:

$$\frac{\partial \theta_2}{\partial u_2} < 0$$

Consider now urban equilibrium 1 where $\bar{s}_1 = s_0 - a u_1/2$ and $\partial \bar{s}_1/\partial u_1 = -a/2 < 0$. We have therefore:

$$\frac{\partial \theta_1}{\partial u_1} = -\frac{V_1}{(u_1 \bar{s}_1)^2}\left(\bar{s}_1 - u_1 \frac{a}{2}\right)$$

This can easily be rewritten as:

$$\frac{\partial \theta_1}{\partial u_1} = -\theta_1 \left(\frac{1}{u_1} - \frac{a}{2\bar{s}_1}\right)$$

But

$$\frac{1}{u_1} > \frac{a}{2\bar{s}_1} \quad \Leftrightarrow \quad s_0 - a u_1 = s(u_1) > 0$$

which is always true. Therefore,

$$\frac{\partial \theta_1}{\partial u_1} < 0$$

■

Lemma A3: For both the exogeneous and endogeneous wage, in the plane (θ, w), the labor demand curve (27) is downward sloping and convex. It cuts the axes at $w(\theta = 0) = y$ and $\theta(w = 0) = \bar{\theta} > 0$ where $\bar{\theta}$ is defined by the following equation: $q(\bar{\theta}) = (r + \delta)\gamma/y$.

Proof. By differentiating (27), we obtain:

$$\frac{\partial w}{\partial \theta} = (r + \delta)\gamma \frac{q'(\theta)}{q(\theta)^2} < 0$$

$$\frac{\partial^2 w}{\partial \theta^2} = -2.(r+\delta)\gamma \left[\frac{q''(\theta)q(\theta) - 2q'(\theta)^2}{q(\theta)^3}\right]$$

By assuming that $\eta_{q'} > 2\eta$, we have:

$$\frac{\partial^2 w}{\partial \theta^2} > 0$$

where $\eta_{q'} = \frac{\partial q'(\theta)}{\partial \theta}\frac{\theta}{q'(\theta)}$ and $\eta = \frac{\partial q(\theta)}{\partial \theta}\frac{\theta}{q(\theta)}$.

Then by using (27), we easily obtain:

$$w(\theta = 0) = y \;\text{ and }\; \theta(w = 0) = \bar{\theta} > 0$$

■

Lemma A4: When wages are exogeneous, VS_1 is steeper than VS_2.

Proof. By totally differentiating (27), it is easily checked by using Lemma A2 that:

$$\frac{dV}{du} = -\frac{\partial\theta/\partial u}{\partial\theta/\partial V} > 0$$

This upward sloping curve in the $u - V$ space is called the VS curve. There is a unique θ but since $\bar{s}_1 \neq \bar{s}_2$, there are two VS curves labelled VS_1 and VS_2: Moreover, since equation (27) has only one unknown, θ, we can rewrite it in the $u - V$ space as:

$$V_k = \theta^* u_k \bar{s}_k \qquad k = 1;2$$

whose slopes are:

$$\frac{\partial V_1}{\partial u_1} = \theta^*(\bar{s}_1 + u_1.\frac{\partial \bar{s}_1}{\partial u_1}) = \theta^*(s_0 - a.u_1)$$

$$\frac{\partial V_2}{\partial u_2} = \theta^*(\bar{s}_2 + u_2.\frac{\partial \bar{s}_2}{\partial u_2}) = \theta^*(s_0 - a.(1 - u_2))$$

and with:

$$\frac{\partial V_1}{\partial u_1} > \frac{\partial V_2}{\partial u_2} \;\Leftrightarrow\; u_1 + u_2 < 1$$

Since the latter inequality is satisfied with usual values of unemployment rates, this implies that the VS curve in equilibrium 1 will be steeper than in equilibrium 2. ■

Lemma A5: When wages are exogeneous, there is a unique urban equilibrium. If $\theta^* q(\theta^*) > \bar{m}$ (resp. $< \bar{m}$), Equilibrium 1 (resp. Equilibrium 2) prevails, where

$$\bar{m} = \frac{(\alpha_e - \alpha_u)(r+\delta)}{a(\bar{w} - b) - (\alpha_e - \alpha_u)s_0} > 0 \qquad \text{(A1)}$$

Proof. By using (27) and Lemma A3, we have: $\bar{w}(\theta = 0) = y$. Since by Lemma A3, the labor demand (27) is downward sloping and since $y > \bar{w}$, there exists a unique θ^* and thus a unique urban equilibrium. Moreover, by replacing (16) and (22) in (10) when equality holds, the result of equation (A1) follows. ■

Lemma A6: For both the exogeneous and endogeneous wage, in the plane $(V_k; u_k)$, $k = 1;2$, the Beveridge curve (28) is decreasing. It cuts the axes at $u_k(V_k = 0) = +\infty$ and $V_k(u_k = 0) = +\infty$.

Proof. By totally differentiating (30):

$$\frac{dV_k}{du_k} q(\theta_k) + V_k q'(\theta_k)(\partial\theta_k/\partial u_k) = -\delta.\overline{s_k}.[1 + (1-u_k)/\overline{s_k}.\partial\overline{s_k}/\partial u_k]$$

Then, since $q'(\theta_k)(\partial\theta_k/\partial u_k) > 0$ from Lemma A2, in equilibrium 1, we always have $dV_1/du_1 < 0$. In equilibrium 2, $1 + (1-u)/\overline{s}.\partial\overline{s}/\partial u_k$ can be rewritten as $1/\overline{s_k} : [s(1/2)] > 0$. Then, in all cases,

$$\frac{dV_k}{du_k} < 0 \quad \forall k = 1,2$$

Next, by using (28), we have:

$$V_k(u_k = 0) = +\infty \qquad \text{and} \qquad u_k(V_k = 0) = +\infty$$

■

Lemma A7: For both the exogeneous and endogeneous wage, in the plane (θ, w), the Beveridge curve in urban equilibrium 2 is always above the Beveridge curve in urban equilibrium 1.

Proof. Since the two Beveridge curves have the same slopes everywhere and do not cut the axes, we must show that if $V_1 = V_2 = V$ then $u_1 < u_2$. By using (28) and by observing that $\delta(1-u_k) = x(\overline{s_k}u_k, V)$, we have:

$$x(\overline{s_k}u_k, V) + \delta.u_k = \delta \tag{A2}$$

By the implicit functions theorem, there exists locally a function $u = F(\overline{s_k}, V)$. Given the monotonicity of the arguments in (A2), the function exists everywhere, and it is easy to check that $F_1 < 0$. Therefore, since $s_1 > s_2$ from Proposition 1, we get the result.

■

Lemma A8: For the endogeneous wage only, in the plane (θ_k, w_k), $k = 1,2$, the two lines (34) and (35) are upward sloping. The line (34) is steeper than (35) but the intercept of (34) is lower than the one of (35). The intersection point between these two lines is equal to:

$$\hat{w} = (1-\beta)\left[b + \frac{1}{a}(\alpha_e - \alpha_u)s_0\right] + \beta y \tag{A3}$$

$$\hat{\theta} = \frac{1-\beta}{\beta}\frac{(\alpha_e - \alpha_u)}{a\gamma} \tag{A4}$$

Proof. Along (34) we have $\partial w_1/\partial\theta_1 = \beta(s_0 - au_1)\gamma$. Along (35) we have $\partial w_2/\partial\theta_2 = \beta(s_0 - a(1-u_2))\gamma$. Then we have $\partial w_1/\partial\theta_1 > \partial w_2/\partial\theta_2 \Leftrightarrow u_1 + u_2 < 1$. This is always true for reasonable values of u_1 and u_2. The intercepts of (34) and (35) are calculated as:

$w_1(\theta_1 = 0) = (1-\beta)[b + (\alpha_e - \alpha_u)u_1] + \beta y$

$< w_2(\theta_2 = 0) = (1-\beta)[b + (\alpha_e - \alpha_u)(1-u_2)] + \beta y \Leftrightarrow u_1 + u_2 < 1$.

To find the intersection point, one has to solve (34) = (35). ■

Lemma A9: For the endogeneous wage only and whatever the urban equilibrium, we have the following relation:

$$\frac{\partial\theta_k}{\partial u_k} < 0 \quad ; \quad \forall k = 1,2 \tag{A5}$$

Proof. Observe from (34) or (35) that:

$$\frac{\partial w_k}{\partial d_{bk}} = (1 - \beta)(\alpha_e - \alpha_u) - \beta a \gamma \theta_1$$

along the wage setting curves. From (A6) $\partial w_k / \partial d_{bk}$ is positive in Equilibrium 1, and negative in Equilibrium 2. Therefore, since $d_{b1} = u_1$ and $d_{b2} = 1 - u_2$, we have in all cases $\partial w_k / \partial d_{bk} > 0$, then by intersection with (27), (A5) is true in all urban equilibria. ■

Lemma A10: At the labor market equilibrium with endogeneous wage, urban equilibrium 1 prevails iff $\theta_1^* > \bar{I}$ and urban equilibrium 2 prevails iff $\theta_2^* < \bar{I}$, where

$$\bar{I} = \frac{(\alpha_e - \alpha_u)}{a\gamma} \left(\frac{1 - \beta}{\beta} \right) > 0 \tag{A6}$$

Proof. If we replace $\overline{I_e} - \overline{I_u}$ in (10) with (33), the result follows. ■

Lemma A11: When the wage is endogeneous, in each urban equilibrium, there exists a unique labor market equilibrium.

Proof. Since for each urban equilibrium the Beveridge curve is decreasing and convex and does not cut the axes, and since the curve $\theta_k(u_k)$ is increasing and concave for the urban equilibria, the labor equilibrium exists and is unique. ■

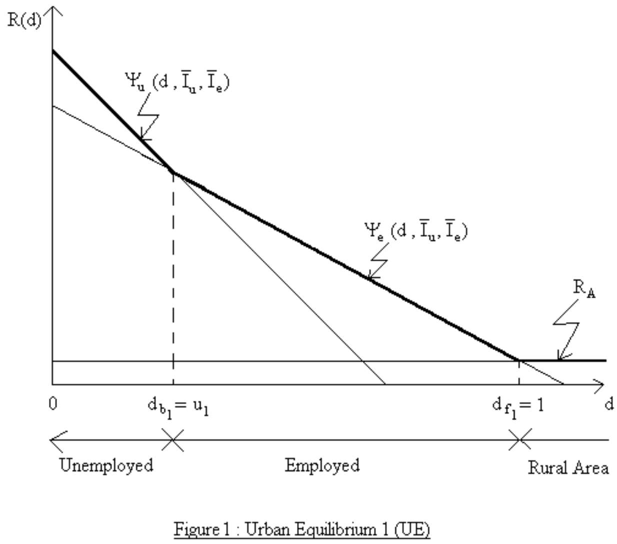
Lemma A12: When wages are endogeneous, we have the following possible situations:
When $\theta_1^* < \theta_2^*$:

- if $\theta_1^* < \theta_2^* < \bar{I}$; urban equilibrium 2 exists and is unique;
- if $\bar{I} < \theta_1^* < \theta_2^*$; urban equilibrium 1 exists and is unique;
- if $\theta_1^* < \bar{I} < \theta_2^*$; there is no urban equilibrium.

When $\theta_2^* < \theta_1^*$:

- if $\theta_2^* < \theta_1^* < \bar{I}$; urban equilibrium 2 exists and is unique;
- if $\bar{I} < \theta_2^* < \theta_1^*$; urban equilibrium 1 exists and is unique;
- if $\theta_2^* < \bar{I} < \theta_1^*$; both urban equilibria exist.

Proof. By using Lemma A10, it is straightforward. ■



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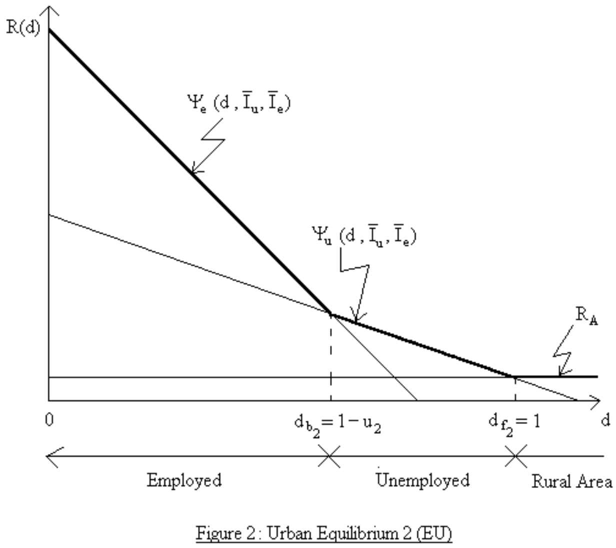
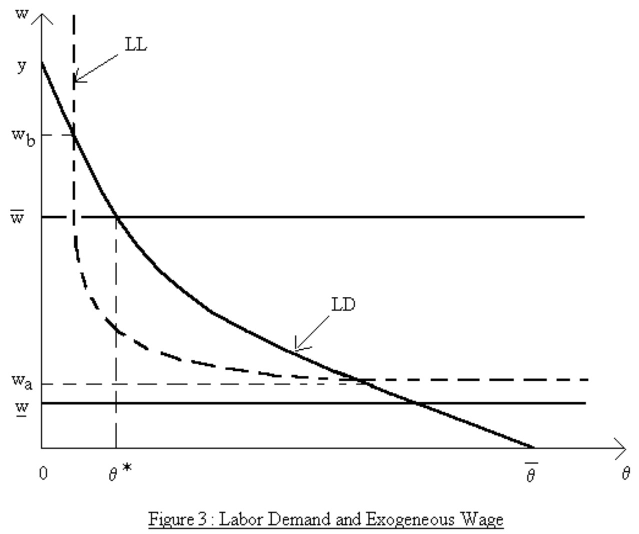


Figure 2:



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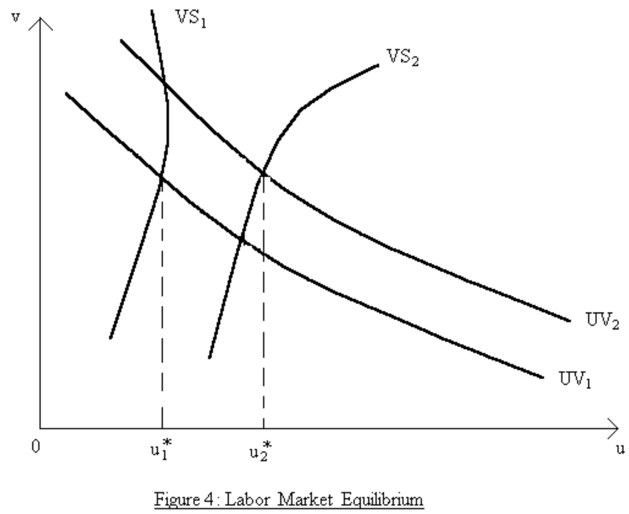

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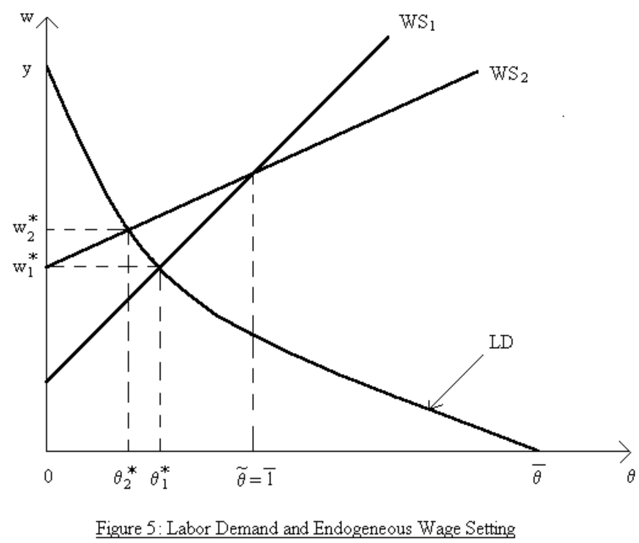

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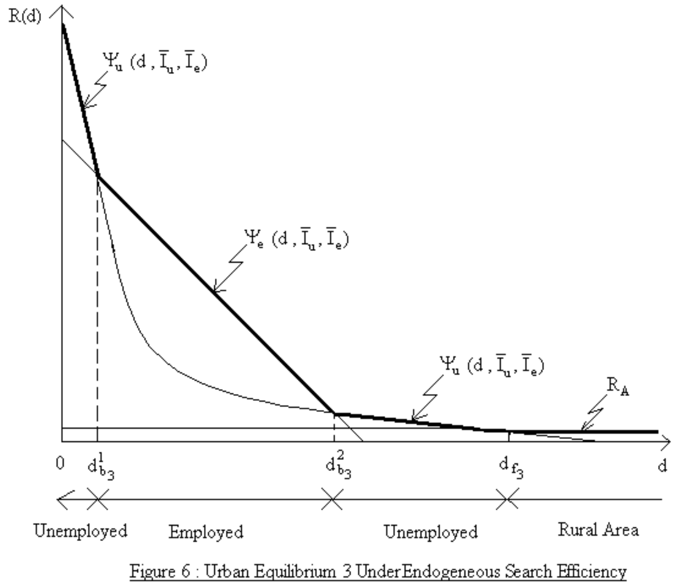

Figure 6: