No. 2102

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INTERNATIONAL TRADE

Centre for Economic Policy Research

ISSN 0265-8003

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Discussion Paper No.2102
March 1999

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CEPR Discussion Paper No. 2102

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ABSTRACT

Agglomeration with Human and Physical Capital: An Analytically Solvable Case*

This paper suggests a simple modification of the core-periphery model by Krugman (1991), which makes the model easy to solve analytically. We use the modified model to analyse the tendencies for geographical agglomeration of manufacturing industry as regions integrate economically. Two cases of human capital mobility and physical or knowledge capital moblity are treated separately. In the human capital case the model behaves qualitatively just as the core-periphery model, where manufacturing tends to agglomerate for low trade costs. In the physical capital case, on the contrary, agglomeration will not occur as a consequence of economic integration.

JEL Classification: F12, F15, F21, R12
Keywords:economic geography, agglomeration, capital mobility

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* This paper is produced as part of a CEPR research network on *Foreign Direct Investment and the Multinational Corporation: New Theories and Evidence*, funded by the European Commission under the Training and Mobility of Researchers Programme (Contract no. ERBFMRX–CT98–0215).

This paper was written during the author’s stay at the Norwegian School of Economics and Business Administration in Bergen. Financial support from the Research Council of Norway (grant no. 124559/510) and from the European Commission (TMR grant) is gratefully acknowledged. Thanks to Fredrik Andersson and Richard Baldwin for valuable comments.

Submitted 15 February 1999

NON-TECHNICAL SUMMARY

The effect of economic integration on industrial location has become an issue of considerable political interest following regional integration agreements such as the EU and NAFTA. At the same time trade and location, or 'new economic geography', has become a mainstream research area of trade theory, starting with the seminal papers by Krugman (1991), Krugman and Venables (1995) and Venables (1996). These papers show that economic integration may lead to increased concentration of industrial production.

One weakness of most of the trade and location literature is, however, that the models have to be solved by numerical simulation rather than analytically due to their inherent non-linearity.

This paper suggests a simple modification of the Dixit-Stiglitz framework in the model by Krugman (1991), which makes the model easy to solve analytically. The novelty lies in the assumption that the fixed cost in the manufacturing sector consists of a separate factor, which may migrate between regions. We interpret this factor as either human capital or physical/knowledge capital. For production (the variable cost) firms use geographically immobile unskilled labour. It may also be noted that human capital and particularly physical or knowledge capital is fairly mobile between countries – much more so than unskilled workers. While the original core-periphery model is sometimes interpreted as an American model, since it relies on geographical mobility of workers, the modified model may also be applicable in a European context.

We use this model to analyse the tendencies for agglomeration of manufacturing production when countries integrate economically. Two cases are analysed. First, we treat the case of human capital mobility. In this case the model behaves qualitatively just as the core-periphery model, where manufacturing tends to agglomerate for low trade costs. We then analyse the case where capital is interpreted as physical or knowledge capital. In this case, on the contrary, agglomeration will not occur as a consequence of capital mobility. This result hinges on two effects. First, we assume that this type of capital moves in response to nominal returns rather than real returns. This nullifies the usual supply-link whereby a larger region is more attractive due to its larger number of locally supplied varieties, which may be consumed without incurring trade costs. Second, owners of physical or knowledge capital spend the return to capital in their home region. This implies that there will be less expenditure shifting as a result of capital mobility than in the human capital case.

Much recent public debate centres on the effects of rapid cross-border movements of financial/physical capital. The results of this paper suggest that

the effects of physical capital outflow may be transitory, however. Countries that worry over a potential long-term loss of manufacturing production as a consequence of ongoing economic integration should concentrate on keeping its human capital.

1 Introduction

The effect of economic integration on industrial location has become an issue of considerable political interest following regional integration agreements such as the EU and NAFTA. At the same time trade and location, or 'new economic geography', has become a mainstream research area of trade theory, starting with the seminal papers by Krugman (1991), Krugman and Venables (1995) and Venables (1996). These papers show how economic integration may lead to increased concentration of industrial production via self-reinforcing demand- and supply- linkages.

One weakness of the trade and location literature, however, is that the models generally have to be solved by numerical simulation rather than analytically due to their inherent non-linearity. Two exceptions, however, exist. Baldwin (1997) develops an analytically solvable model where factor accumulation and decumulation, rather than factor migration, causes agglomeration. While labour is geographically immobile in this model, capital mobility is allowed. Interestingly capital mobility is a stabilising force in this model. Ottaviano and Thiesse (1998) develop a model of agglomeration and trade that departs from the usual Dixit-Stiglitz framework by using a quadratic utility model *a lá* Dixit (1979) and Vives (1990). This gives linear demand functions, which makes the model analytically tractable. The model allows for the analysis of a rich set of issues including segmented market pricing. The model, however, shares one property with the Dixit-Stiglitz framework – namely that it suffers from a set of rather restrictive assumptions.

This paper suggests a simple modification of the Dixit-Stiglitz framework in the core-periphery model by Krugman (1991), which makes the model easy to solve analytically. The novelty lies in the assumption that the fixed cost in the manufacturing sector consists of a separate factor, which may migrate between regions. This factor will be intrepreted as either human capital or physical/knowledge capital. The analysis shows that the agglomeration effects are quite different in these two cases. In the human capital case the model behaves qualitatively just as the core-periphery model, where manufacturing tends to agglomerate for low trade costs. In the physical capital case, on the contrary, agglomeration will not occur as a consequence of economic integration.

It may also be noted that human capital and particularly physical or knowledge capital is fairly mobile between countries - much more so than unskilled workers. While the original core-periphery model is sometimes interpreted as an American model, since it relies on geographical mobility of workers, the modified model of this paper may also be applicable in a European context.

2 The Model

The world is composed of two symmetric regions, 1 and 2. Each region has two factors of production (skilled workers or human capital L^S and unskilled workers L^U) and two sectors (industry X and agriculture A). The agricultural product is homogeneous and freely traded with identical production technology in the two regions. The agricultural sector employs only unskilled workers that are internationally immobile. The monopolistically competitive X sector employs both L^S and L^U to produce output and faces increasing returns with a linear cost function. L^S is internationally mobile and only enters the fixed cost, which may be interpreted as cost for R&D or headquarter services. L^U only enters the variable cost of manufacturing production. Both A and X are traded; A trade is costless, but trade in X is inhibited by frictional trade costs.

Preferences are given by:

$$U = C_X^{\mu} C_A^{1-\mu}, \qquad C_X = \left(\int_{i=0}^{n_1+n_2} x_i^{\,1-1/\sigma} \right)^{\frac{1}{1-1/\sigma}}, \quad \sigma > 1 \tag{1}$$

where C_X is the manufactures aggregate, C_A the consumed quantity of agricultural products, x_i the consumption of variety *i*, n_J is the number of produced varieties in region j, and $\sigma > 1$ the elasticity of substitution between products.

Each region has a fixed amount of unskilled labour, and for skilled labour the world supply is fixed:

$$L_1^U = L_2^U = L^U; \quad L_1^S + L_2^S = L_W^S \tag{2}$$

The agricultural good is produced under constant returns to scale and units are normalised so that one unit of unskilled labour is required to produce one unit of this good. We also choose this good as numeraire. Free trade in this good implies that for both regions:

$$p_A = w_U = 1 \tag{3}$$

The cost function for an X-sector firm in region j is given by:

$$\chi_j = w_j^S \alpha + \beta x_j \tag{4}$$

where α is a fixed cost of skilled labour and β the amount of unskilled labour per unit *x*. We will choose units of skilled labour so that α=1. A single firm produces each variety. Firms cannot discriminate between consumers in different regions and therefore charge a common (producer) price for their good. Trade between regions is costly. We assume these costs are of the "iceberg" type where $\tau > 1$ units must be shipped for one unit to arrive. With the Chamberlinian large group assumption each firm sets its price as a constant mark-up on variable cost, $p = (1-1/\sigma)^{-1}\beta$. Choosing units of x so that $\beta = 1-1/\sigma$ gives:

$$p = 1 \tag{5}$$

When exported, the consumer price of a differentiated good is τp. The fact that manufacturing prices can be normalised to one in this model is the key property that makes the model analytically solvable, since it takes away most of the non-linearity of the demand functions.

We assume free entry in the X-sector. New firms will enter until operating profit x/σ equals the fixed cost w^S. This gives the equilibrium quantity of each firm:

$$x = w^S \sigma \tag{6}$$

Finally the income in a region j is:

$$Y_j = L^U + w_j^S L_j^S \tag{7}$$

Short-run equilibrium

In the short run skilled labour is immobile between regions, and the equilibrium in the market for skilled labour is given by:

$$n_1 = L_1^S; \quad n_2 = L_2^S \tag{8}$$

The X-sector market clearing condition closes the model:

$$\begin{aligned} \sigma w_1^S &= \frac{\mu Y_1}{n_1 + \phi n_2} + \frac{\phi \mu Y_2}{\phi n_1 + n_2} \\ \sigma w_2^S &= \frac{\phi \mu Y_1}{n_1 + \phi n_2} + \frac{\mu Y_2}{\phi n_1 + n_2} \end{aligned} \tag{9}$$

where $\phi = \tau^{1-\sigma}$, which ranges between 0 and 1, stands for “freeness” of trade (0 is autarchy and 1 is zero trade costs). Substituting the expressions (7) and (8) in (9) determines nominal wages in each region for given factor stocks.

The endowment of skilled labour determines the size of the manufacturing sector and therefore also the fraction of unskilled labour that is allocated to manufacturing. The amount of unskilled labour in manufacturing equals $n\beta x$. Substituting x and n from (6) and (8) gives $n\beta x = (\sigma - 1) w^s L^s$. We rule out corner solutions by assuming that $(\sigma - 1) w^s L^s < L^u$, which ensures that the agricultural sector is active in both regions.

Long-run equilibrium

In the long run skilled labour is mobile between regions. Agents maximise their real return to labour by moving to the region with the higher real wage. A long-run equilibrium with both regions producing manufactured goods will arise when

$$\frac{w_1^s}{P_1^{\mu}} = \frac{w_2^s}{P_2^{\mu}} \qquad (10)$$

where $P_1 = \left(n_1 + \phi n_2\right)^{\frac{1}{1-\sigma}}, \quad P_2 = \left(\phi n_1 + n_2\right)^{\frac{1}{1-\sigma}}$ are the perfect CES-price indices in respective region.

Stability

Given the symmetry of the model, a symmetric allocation of resources is always an equilibrium. The question is, however, if this equilibrium is stable. It is well known that the symmetric equilibrium in the original core-periphery model is only stable for sufficiently high trade costs. This assertion is established using the following approach: Symmetry is perturbed by exogenously moving a small mass of skilled labour from one region to the other. Nominal wages adjust to restore equilibrium in the X-sector and the resulting relative real wage is checked. If the post-shock real wage is lower in the ‘receiving’ region than in the ‘sending’ region, workers would wish to move back and thereby restore the symmetric equilibrium. The symmetric equilibrium in this case is said to be stable. However, if the receiving region’s post-shock real wage is higher, the equilibrium is unstable since additional workers would be attracted to the receiving region. We will apply this analysis below.

The cases of free trade and prohibitive trade costs

Consider first the case of free trade (ϕ=1). It is easy to check that real wages are always equal in this case. This is no surprise since distance disappears from the model when trade costs are zero. Consequently the equilibrium allocation of workers is undetermined.

Second consider the case of infinite trade costs, ϕ=0. Solving the model in a straight forward way gives the relative real wage:

$$\frac{w_1^S P_1^{-\mu}}{w_2^S P_2^{-\mu}} = \left(\frac{L_1^S}{L_2^S}\right)^{\frac{\mu}{\sigma-1}-1} \tag{11}$$

The symmetric equilibrium is stable if $\mu < \sigma - 1$, which is assumed to hold. This condition corresponds to the "no-black-hole" condition in the original core-periphery model.[1]

The two special cases of free trade and prohibitive trade costs (ϕ=1 and 0) can be analysed analytically in most location models. The more interesting case of intermediate trade costs is, however, not analytically tractable in standard models. Here due to the linearity of the demand functions in (9), we can solve the model analytically for general trade costs.

General Trade Costs

Solving the model for general trade costs gives the relative real wage according to:

$$\frac{w_1^s P_2^{\mu}}{w_2^s P_1^{\mu}} = \frac{-s_L\mu(\phi^2-1) - s_L\sigma(\phi-1)^2 + (\mu+\sigma)\phi^2 + (\sigma-\mu)}{s_L\mu(\phi^2-1) + s_L\sigma(\phi-1)^2 + 2\sigma\phi}\left(\frac{\phi s_L + 1 - s_L}{s_L + \phi(1-s_L)}\right)^{\frac{\mu}{1-\sigma}} \tag{12}$$

where $s_L \equiv L_1^S / (L_1^S + L_2^S)$. Equation (12) expresses the relative real wage as a function of the state variable s_L and parameters of the model. This is not possible in the original core-periphery model, which cannot be solved analytically. Two agglomeration forces are at work. On the demand-side, more skilled workers and firms in a region imply a larger market and therefore an even more attractive place for additional firms and skilled workers to locate in. Second a larger region produces more varieties locally and therefore has a lower price index. This implies a higher real wage, which attracts even more skilled labour and firms. In equation (12) this supply link can be seen as the last right hand side parenthesis. Against these

[1] This condition is in fact somewhat less restrictive than the condition, $\mu<(\sigma-1)/\sigma$ in the original core-periphery model (see Fujita, Krugman and Venables 1998).

agglomeration forces stand trade costs as a dispersion force. Higher trade costs make it less attractive to serve markets via exports, which tends to disperse production.

Figure 1 plots (12) for different levels of trade costs. For high trade costs the symmetric equilibrium is stable, as the cost of supplying a market by exports is too large. For lower trade costs agglomerative forces come to dominate, which leads to a core-periphery outcome. [2]

{ Figure 1 about here}

For intermediate trade cost the possibility of asymmetric equilibria appears, as shown in Figure 2. These equilibria are, however, unstable. This model, thus, behaves qualitatively exactly as the Krugman (1991) model.

{ Figure 2 about here}

It is also straightforward to evaluate the stability of different equilibria by differentiating (12) with respect to s_L and evaluating the derivative in the equilibrium of interest. The symmetric equilibrium (s_L =0.5) becomes unstable for trade costs lower than the "breakpoint", which is given by:

$$\phi^{break} = \frac{(\sigma-\mu)(\sigma-1-\mu)}{(\sigma+\mu)(\sigma+\mu-1)} \qquad (13)$$

The breakpoint is decreasing in μ and increasing in σ by inspection. A larger expenditure share on manufacturing, μ, increases agglomeration forces, which implies that the symmetric

[2] The slope of the line in Figure 1 changes sign (the breakpoint of the symmetric equilibrium) for very high trade costs in the original core-periphery model by Krugman (1991) (e.g. t^{break}=1.66 for σ=4 and μ=0.3). It is probably an attractive feature of the modified model in this paper that the breakpoint occurs for more relevant levels of trade costs.

equilibrium is stable for a smaller range of trade costs. A larger σ works in the opposite direction since it implies a lower mark-up in manufacturing and therefore lowers agglomeration forces.

The core-periphery equilibrium in turn cannot be sustained for trade costs above the "sustainpoint", which is given by:

$$\phi^{sust} = \sqrt{\frac{(\sigma^2 - \sigma - \mu(3\sigma - 1))(\sigma - \mu)}{(\sigma^2 - \sigma + \mu(3\sigma - 1))(\sigma + \mu)}} \tag{14}$$

As in the Krugman (1991) model $\phi^{break} - \phi^{sust} > 0$, which implies hysteresis in location. Once the core-periphery equilibrium is reached, trade costs have to rise above the breakpoint (which preserves the symmetric equilibrium) before the agglomerated equilibrium becomes unstable. As σ goes towards infinity the manufacturing sector approaches perfect competition, and the breakpoint and sustainpoint converge.

3 Physical and Knowledge Capital Mobility

In this section we treat the case when the fixed cost in the manufacturing sector consists of physical or knowledge capital rather than human capital. The difference being that the return to physical capital is repatriated to the region of the capital owners as opposed to the return to human capital, which is spent where capital is located. As a consequence physical capital moves in response to differences in nominal returns rather than real returns. We will assume that each region is endowed with a fixed quantity of capital $\overline{K}$, which may be allocated in either region. K_{ij} denotes the quantity of region i capital that is located in region j. We choose units of capital so that α=1. Installed capital in region j, n_j, is given by:

$$n_j = K_{jj} + K_{ij} \quad i, j \in 1, 2\,. \tag{15}$$

The income equation (7) is modified according to:

$$Y_j = L^U + r_j K_{jj} + r_i K_{ji} \tag{16}$$

where r_j is the return to capital in region j. The X-sector market clearing equation (9), (15) and (16) determine the short-run equilibrium values for factor returns, incomes and installed capital given the configuration of K. We will again assume that the stock of unskilled labour is large enough to rule out corner solutions where the agricultural sector becomes inactive in one region. This is assured by assuming that $(\sigma - 1)rn < L^u$.

Stability of the symmetric equilibrium

The stability experiment is identical to the case of human capital: move one unit of capital between the regions, let nominal returns adjust and compare the relative returns. However, for physical capital the decision to move will be based on relative *nominal* returns. Starting from the symmetric equilibrium ($K_{11} = K_{22} = \overline{K}$) and moving δ units of capital from region 1 to region 2 gives the following configuration of capital:[3]

$$K_{11} = \overline{K} - \delta, \quad K_{12} = \delta, \quad K_{22} = \overline{K}, \quad K_{21} = 0$$

Income in the two regions is now given by:

$$Y_1 = L^u + r_1(\overline{K} - \delta) + r_2\delta, \quad Y_2 = L^u + r_2\overline{K} \tag{17}$$

and the X-sector market-clearing condition is:

$$\begin{aligned} \sigma r_1 &= \frac{\mu Y_1}{\overline{K} - \delta + \phi(\overline{K} + \delta)} + \frac{\phi\mu Y_2}{\phi(\overline{K} - \delta) + \overline{K} + \delta} \\ \sigma r_2 &= \frac{\phi\mu Y_1}{\overline{K} - \delta + \phi(\overline{K} + \delta)} + \frac{\mu Y_2}{\phi(\overline{K} - \delta) + \overline{K} + \delta} \end{aligned} \tag{18}$$

Using (17) and (18) gives:

[3]The location of capital in the symmetric steady-state may be determined by assuming a small cost epsilon of moving capital geographically.

$$\frac{d\left(\frac{r_1}{r_2}\right)}{d\delta}=\frac{2K\sigma(\phi+1)(\phi-1)^2(\phi(\sigma+\mu)+\sigma-\mu)}{\left(((\sigma+\mu)\phi^2+\sigma-\mu)(\overline{K}-\delta)+2\sigma\phi(\overline{K}+\delta)\right)^2}>0 \tag{19}$$

(19) implies that the symmetric equilibrium is stable since capital will move back to region 1 for all trade costs. Physical capital mobility will, thus, contrary to human capital mobility, not lead to agglomeration of production. There are two forces that produce this result. First, physical capital moves according to the nominal return. This implies that the supply link, whereby a larger region is more attractive since it has a lower price index due to a larger number of locally produced varieties, does not operate. There is also a second "expenditure shifting" effect in that local demand, which is determined by GNP rather than GDP, stays unchanged or increases as capital is employed abroad. Both these forces are necessary for the result that physical capital mobility does not lead to agglomeration, as can be shown by deriving the equivalent of (19) when only one of the stabilising forces is operational.

4 CONCLUSIONS

This paper develops an analytically solvable version of the core-periphery trade and location model by Krugman (1991). The key assumption being that the fixed cost in manufacturing production consists of a separate factor, which is geographically mobile. Two cases are analysed: when the mobile factor consists of human capital and when this factor consists of physical or knowledge capital.

While the human capital case gives qualitatively the same results as in the original core-periphery model, where manufacturing production may agglomerate geographically for low trade costs, the physical capital case does not lead to agglomeration as regions integrate. This may be of importance for regions that worry about the potential loss of their industrial base. Much policy debate concerns the large and rapid cross-border flows of capital in today's increasingly integrated world economy. This paper suggests that loss of manufacturing associated with physical capital mobility may be transitory rather than long-run. For human capital, however, this conclusion does not hold. Human capital mobility may cause agglomeration, and therefore the loss of industry in peripheral regions. It may therefore be more important, for a small peripheral region, to keep the human capital stock in the region than trying to dampen physical capital mobility.

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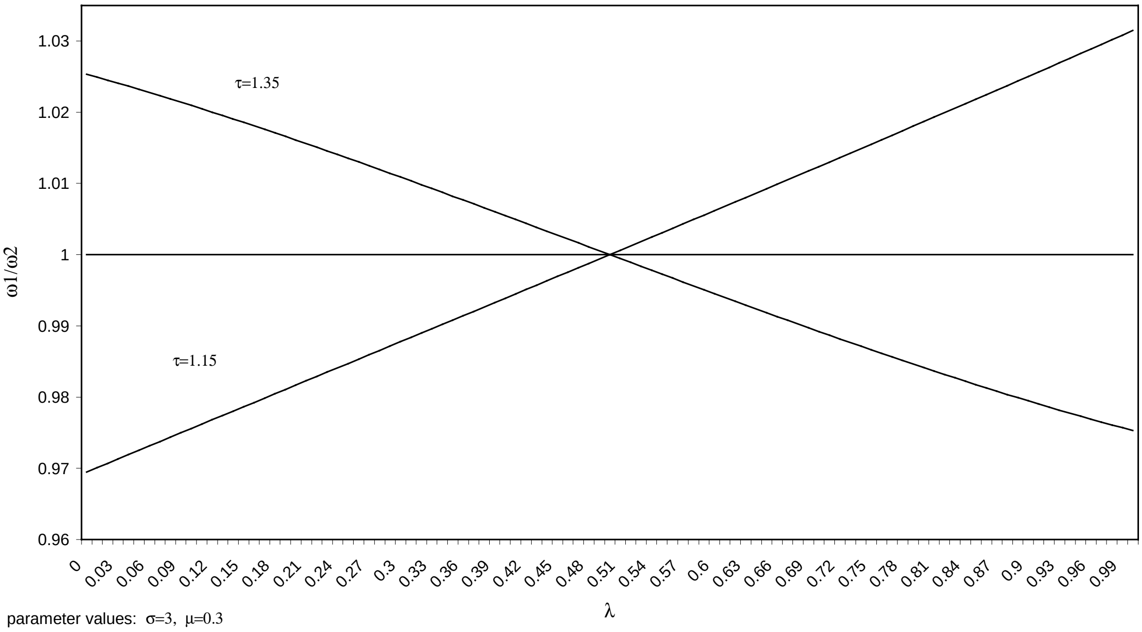
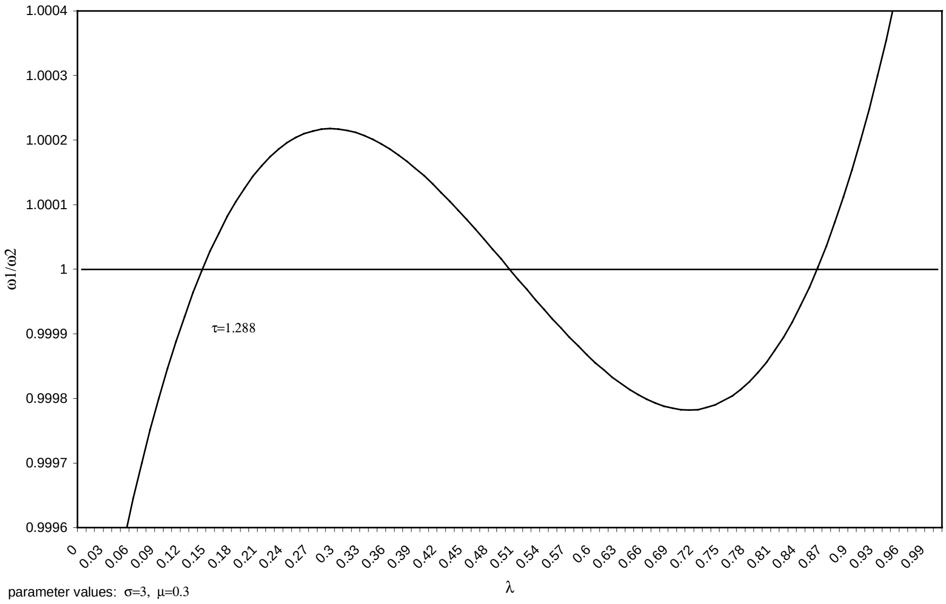
Figure 1 Stability of the Symmetric Equilibrium



parameter values: σ=3, μ=0.3

Figure 2   Stability of the Symmetric Equilibrium for Intermediate Trade Costs



parameter values: σ=3, μ=0.3