

# INSIDE VERSUS OUTSIDE FINANCING AND PRODUCT MARKET COMPETITION

Monika Schnitzer and Achim Wambach

Discussion Paper No. 2049  
December 1998

Centre for Economic Policy Research  
90–98 Goswell Rd  
London EC1V 7DB  
Tel: (44 171) 878 2900  
Fax: (44 171) 878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org)

This Discussion Paper is issued under the auspices of the Centre's research programme in **Industrial Organization**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Monika Schnitzer and Achim Wambach

## ABSTRACT

### Inside versus Outside Financing and Product Market Competition

This paper investigates the interaction of firms' financial structure and their competitive behaviour on oligopolistic product markets. We consider risk-averse entrepreneurs who produce with uncertain production costs. To reduce their exposure to risk they can sell stocks to risk-neutral outside-investors. We show that in equilibrium the entrepreneurs prefer not to fully transfer this risk to outside-financiers because it reduces the competitive pressure on the product market. Furthermore, we discuss how the optimal financial structure reacts to variations in entrepreneurs' risk aversion, the level of cost uncertainty and the number of competitors.

JEL Classification: D43, G32, L13, L22

Keywords: corporate finance, Bertrand competition, inside versus financing, impact of financial structure on competition, impact of product competition on financial structure

Monika Schnitzer  
Department of Economics  
University of Munich  
Akademiestr. 1/III  
D-80799 Munich  
GERMANY  
Tel: (49 89) 2180 2217  
Fax: (49 89) 2180 2767  
Email: schnitzer@lrz.uni-muenchen.de

Achim Wambach  
Department of Economics  
University of Munich  
Ludwigstrasse 28/VG  
D-80539 Munich  
GERMANY  
Tel: (49 89) 2180 2227  
Fax: (49 89) 2180 6282  
Email: wambach@lrz.uni-muenchen.de

Submitted 3 December 1998

## NON-TECHNICAL SUMMARY

In this paper we study the interaction of corporate finance and product market competition; i.e. we analyse the impact of firms' financial structure on price competition and conversely, how firms' financial decisions are affected if this impact is taken into account. For this purpose we consider a simple two-stage model with  $n$  firms. Each firm is run by a risk-averse entrepreneur who produces with uncertain production cost. He can either own the firm himself and bear all the risk resulting from the production cost uncertainty alone or he can sell non-voting stocks to risk-neutral outside investors in order to transfer some of his risk. At stage one, all firms simultaneously choose the ratio of inside and outside financing and at stage two firms engage in Bertrand competition on the product market.

*A priori* one should expect that entrepreneurs prefer to diversify as much as possible. But, as we will show, this intuition does not hold if price competition on the product market is modelled explicitly. To study price competition of risk-averse firms we use a model where firms sell homogeneous products and have symmetric uncertain marginal costs. But, in contrast to the standard Bertrand outcome risk-averse firms do not choose prices equal to expected marginal cost. If an entrepreneur undercuts the prices of his competitors in order to serve all consumers in the market he realizes an increase in profits if marginal costs turn out to be low, but he realizes an increase in losses if costs turn out to be high. Since risk-averse entrepreneurs suffer more from an increase in losses than they gain from an equivalent increase in profits they are less tempted to go for the whole market with aggressive price cuts.

When choosing the optimal ratio of inside and outside financing at stage one, each firm thus faces a trade-off between bearing more risk and benefiting from less aggressive price competition. We show that if firms can cooperatively agree on their financial structure they will never choose to fully diversify their risk because this leads to standard Bertrand competition with zero profits for all firms. If firms choose their financial ratio non-cooperatively, instead, each individual firm may have an incentive to deviate from this jointly optimal ratio to be in the position to compete more aggressively on the product market. In equilibrium, however, firms choose a positive ratio of inside and outside financing and hence competition will be less intense than under standard Bertrand competition.

From the point of view of social welfare, this financial structure implies too much risk and too little price competition. This analysis applies only if we can take the number of firms active in the market as exogenously given, however. Our welfare conclusions may have to be modified if we take the firms' market entry decision into account. We identify a potentially welfare enhancing effect

of the financing strategy: In a world with only risk-neutral participants no firm is able to cover its entry costs if more than one firm enters the market, so a monopoly arises. If firms carry some residual risk, however, Bertrand competition allows for positive profits and more than one firm can enter the market which will lead to prices below the monopoly price. As long as these entry costs are not too large, this will be welfare enhancing.

Finally, we point out another strategy by which a firm might commit to risk-averse behaviour: The employment of a risk-averse manager who is paid an uncertain wage. In this case the manager is given an incentive contract which is not intended to give an incentive to work harder, as in standard moral hazard problems. It is used as a means to raise prices in the market as risk-averse managers whose wage depend on the profit are more cautious in their decisions and behave less aggressively. In this sense incentive contracts can serve as an anti-competitive device which weakens the price competition.

Our analysis shows that entrepreneurs who decide on their financial structure non-cooperatively choose positive levels of inside-financing in equilibrium. In all our examples, however, this non-cooperative level of inside-financing falls short of the level that entrepreneurs would like to commit to if the financial decision could be taken cooperatively. This means price competition is weaker as compared to standard Bertrand competition with risk-neutral firms. But it is more intense than it would be if entrepreneurs could agree on their ratio of inside and outside financing. This is due to the fact that entrepreneurs have an incentive to free ride on the risk-aversion of their competitors.

This observation has interesting implications for the judgement of cooperation among firms. It points out that cooperation on some decisions can affect the outcome of other, seemingly unrelated decisions. In our case, cooperation on firms' financial decisions can reduce the competitive pressure on the product market and thus have an anti-competitive effect on price competition. This shows that even *prima facie* innocuous cooperative attempts can give rise to concern about potentially anti-competitive effects.

# 1 Introduction

For almost 40 years Modigliani and Miller's Irrelevance Theorem has challenged economists to explain why the financial structure of firms should matter. But although it stimulated a large and growing literature on corporate finance, surprisingly little attention has been paid to the relationship of firms' financial decisions and their competitive behavior on product markets.

In this paper we study the interaction of corporate finance and product market competition; i.e., we analyze the impact of firms' financial structure on price competition and conversely, how firms' financial decisions are affected if this impact is taken into account. For this purpose we consider a simple two-stage model with  $n$  firms. Each firm is run by a risk-averse entrepreneur who produces with uncertain production cost. He can either own the firm himself and bear all the risk resulting from the production cost uncertainty alone or he can sell non-voting stocks to risk-neutral outside investors in order to transfer some of his risk. At stage 1, all firms simultaneously choose the ratio of inside and outside financing and at stage 2 firms engage in Bertrand competition on the product market.

A priori one should expect that entrepreneurs prefer to diversify as much as possible. But, as we will show, this intuition does not hold if price competition on the product market is modelled explicitly. To study price competition of risk-averse firms we use a model of Bertrand competition based on Wambach (1998). In this model firms sell homogeneous products and have symmetric uncertain marginal costs. But in contrast to the standard Bertrand outcome risk-averse firms do not choose prices equal to expected marginal cost. If an entrepreneur undercuts the prices of his competitors in order to serve all consumers in the market he realizes an increase in profits if marginal costs turn out to be low, but he realizes an increase in losses if costs turn out to be high. Since risk-averse entrepreneurs suffer more from an increase in losses than they gain from an equivalent increase in profits they are less tempted to go for the whole market with aggressive price cuts.

When choosing the optimal ratio of inside and outside financing at stage 1, each firm thus faces a trade-off between bearing more risk and benefitting from less aggressive price

competition. We show that if firms can cooperatively agree on their financial structure they will never choose to fully diversify their risk because this leads to standard Bertrand competition with zero profits for all firms. If firms choose their financial ratio noncooperatively, instead, each individual firm may have an incentive to deviate from this jointly optimal ratio to be in the position to compete more aggressively on the product market. However, in equilibrium, firms choose a positive ratio of inside and outside financing and hence competition will be less intense than under standard Bertrand competition.

With this paper we intend to make two contributions. First of all, as emphasized above, the paper establishes a relationship between a firm's financial decisions and its competitive behavior on product markets. Although the interaction between the financial market and the output market is seen to be a potentially important determinant in the financial contracting decision of a firm (see e.g. Harris and Raviv, 1992), it has not received much attention in the literature. The first work in this direction was Brander and Lewis (1986) who studied the impact of a firm's debt/equity ratio on product market competition. In their model, a risk neutral firm that is engaged in Cournot competition has an incentive to take on a large debt to commit itself to a more aggressive output stance due to the limited liability. However, when both competitors use this strategy they both end up being worse off. In a companion paper, Brander and Lewis (1988) introduce additional bankruptcy cost and show under what conditions highly-leveraged firms engage in more aggressive output decisions than firms with no debt. Showalter (1995) extends the Brander and Lewis (1986) analysis to Bertrand competition. He shows that the incentive to use strategic debt depends on the type of competition and on the type of uncertainty in the output market. If demand is uncertain, debt is advantageous for risk neutral firms because it induces higher prices of both competitors. In case of cost uncertainty, however, firms prefer not to take on debt because this would induce too low prices. Another extension of Brander and Lewis' (1986) analysis is Hughes et al. (1998). They investigate whether or not firms can eliminate the strategic incentives for taking on debt under Cournot competition if they can acquire and share information before the financial structure is chosen. It is shown that firms prefer to resolve this uncertainty only if its level is sufficiently high. In the context of repeated oligopoly under tacit collusion, Maksimovic (1988) shows that high levels of debt weakens the ability to collude, since

future debt repayments decrease the collusive profit relative to the one-period deviation gain.

Our paper considers a different financial structure decision, the ratio of inside versus outside financing. However, there are interesting parallels to be seen. In the case of the debt/equity decision firms become more aggressive if they take on more debt due to the limited liability. Similarly, in our model entrepreneurs become more aggressive if they take on more outside financing because this makes them less risk-averse. Like Showalter who shows that risk-neutral firms avoid debt in order to commit to less aggressive price competition we find that risk-averse firms choose less outside financing than is optimal from the point of view of risk diversification for the same strategic reason, to reduce price competition.

Another paper that studies the impact of outside financing on product market competition is Aghion et al. (1998). The authors find a non-monotonic (U-shaped) relationship between outside finance and the manager's effort. If outside finance is low a further rise reduces the manager's incentive to work hard. If outside finance is high, however, the manager has to commit to working hard to attract more funds. The nature of competition and strategic interaction (strategic substitution or strategic complementarity) depends on the regime under which the manager operates.

Empirically it is striking that most investment projects are financed internally. Mayer (1988) reports that around 70 percent of all investments in Anglo-American or European firms are financed through retained earnings. Myers and Majluf (1984) provide a theoretical explanation for this phenomenon based on the idea that internal financing reduces agency costs due to asymmetric information. Hellwig (1998) argues instead that managers prefer inside financing because this implies less interference from outside investors. Our paper points to a complementary explanation which is that inside financing is used as a strategic commitment device to reduce price competition.

The second contribution of this paper is related to the literature on Bertrand competition and on alternative ways to overcome the Bertrand Paradox. Formally, the anticompetitive impact of risk aversion in our model is similar to the effects of capacity constraints

or convex costs on Bertrand competition (Allen and Hellwig 1986, 1989; Benassy 1989; Dastidar 1995; Dixon 1990, 1992; Edgeworth, 1897; Kreps and Scheinkman 1983; Maskin 1986). The crucial question is, however, whether or not firms can influence their degree of risk aversion in the same way as they can choose for example their capacity constraint (Kreps and Scheinkman 1983) and if so, what degree of risk aversion they will choose. Our paper shows that entrepreneurs can indeed affect their degree of risk aversion by choosing their ratio of inside and outside financing. Furthermore, in an interesting parallel to the result of Kreps and Scheinkman we find that in equilibrium firms will indeed choose a financial structure that reduces price competition at stage 2.

The paper is organized as follows. In section 2, we introduce the basic set up of Bertrand competition with uncertain cost and risk averse entrepreneurs. In section 3, we discuss as a benchmark the case of perfect competition, where firms face given prices. Section 4 shows the main effect of product competition on financial ratio in a simple duopoly model. In section 5 we carry out some comparative static analysis with respect to the number of firms and the degree of risk aversion. In section 6, we introduce entry costs and derive welfare implications of the financial structure. Section 7 concludes with a discussion of possible extensions and some implications for judging cooperation on financial decisions.

## 2 The basic set up

Consider a market where  $n$  firms sell homogeneous products with symmetric production technologies. Production costs are linear in quantities but uncertain. The cause of this uncertainty is not modelled explicitly. What we have in mind is that production costs could be uncertain because of uncertain factor prices, resulting from uncertain supplies or uncertain exchange rates. Similarly, firms could be uncertain about the amount of inputs required for production. An example is the construction industry where firms typically compete for contracts without knowing precisely their construction costs. To make ideas precise we assume that all firms produce with identical, but uncertain unit production



costs  $c$ , where  $c$  is drawn from  $[\underline{c}, \bar{c}]$ ,  $\underline{c} < \bar{c}$ , according to the probability distribution  $F(c)$ .<sup>1</sup>

Firms are run by risk-averse entrepreneurs. As the potential profit stream is uncertain the entrepreneurs may want to sell some of their cash flow rights to outside financiers, who are assumed to be risk neutral. Outside investors do not obtain control rights if they buy cash flow rights. This is for example the case if they buy non-voting stock rights. We assume that there exists a large number of potential outside investors and that each entrepreneur addresses one single investor with a take-it-or-leave-it offer when seeking outside-financing.<sup>2</sup>

The time structure of the game is the following:

1. At stage 1, each risk averse entrepreneur  $i$ ,  $i = 1, \dots, n$ , simultaneously makes a take-it-or-leave-it offer to a risk neutral outside financier, which specifies  $1 - \alpha_i$  and  $G_i$ , where  $(1 - \alpha_i)$  is the percentage of the cash flow rights sold and  $G_i$  is the price the investor has to pay. Then each outside investor either accepts or rejects his offer, without seeing the offers made by other entrepreneurs.<sup>3</sup>
2. At stage 2, firms compete in prices on the output market. One problem that arises with production cost uncertainty is that once unit production costs are realized firms may not want to serve the demand they face.<sup>4</sup> However, we assume that output is demand determined, i.e. firms are required to serve the residual demand they face. This assumption is commonly made (Dixon 1990; Dastidar 1995; Wambach 1998) because it guarantees existence of pure strategy equilibria. One possible justification could be that it is extremely costly to turn customers away.

Total market demand is denoted by  $x(p)$ . Let  $x_i(p_i, p_{-i})$  denote the individual market

---

<sup>1</sup>Results do not change if costs are i.i.d. distributed between the firms.

<sup>2</sup>Both assumptions are made to keep the model as simple as possible. It is straightforward to see how outside-financing becomes less attractive if there is less competition among outside-investors and thus entrepreneurs have less bargaining power when seeking risk-neutral capital.

<sup>3</sup>This assumption greatly simplifies our analysis of out of equilibrium behavior, as will be shown below.

<sup>4</sup>Note that similar problems arise in models with capacity constraints or convex costs and Bertrand competition (Allen and Hellwig 1986, 1989; Benassy 1989; Dastidar 1995; Dixon 1990, 1992; Edgeworth 1897; Kreps and Scheinkman 1983; Maskin 1986). In this literature, firms are not willing to take the whole market as this is either not possible due to capacity constraints or very costly due to the convex cost function. In general, under these conditions there exist only equilibria in mixed strategies (Maskin 1986; Allen and Hellwig 1986, 1989).

demand faced by firm  $i$ , where  $p_i$  is the price charged by firm  $i$  and  $p_{-i}$  the vector of prices charged by all other firms. Note that  $x_i(p_i, p_{-i}) = x(p_i)$  if firm  $i$  charges the lowest price and it is zero if other firms charge the lowest price. If several firms charge the same lowest price, they share total market demand at this price symmetrically.

Entrepreneurs maximize their respective utilities which depend on their degree of risk aversion. We specify this risk aversion more explicitly below.

### 3 Financing Decision under Perfect Competition

Before we start with our analysis of an oligopolistic product market we consider first the case of perfect competition as a point of reference. For this purpose, we assume that once the entrepreneurs have taken their decision about their financial ratio, each firm faces a given price and market demand, so that there is no longer a decision to take. In particular, we assume that the firms face a given price  $p$  and an individual demand  $x_i(p) = x(p)/n$ .<sup>5</sup> Note that  $p$  is determined before cost uncertainty is resolved which exposes producers to some risk.

In this case, the decision problem of each firm reduces to its decision about the financial structure at stage 1 and (omitting the subscript  $i$ ) it is formally given by:

$$\max_{\alpha} \int_{\underline{c}}^{\bar{c}} U\left(\alpha \frac{x(p)}{n}(p - c) + G\right) dF(c) \quad (1)$$

where  $G$  is the price which can be charged to the risk neutral outside financiers. The outside investors accept any  $G$  that guarantees a non-negative expected profit. As the price and the demand is given at stage 2 for a given number of firms, this implies that the maximal  $G$  which can and will be demanded satisfies:

$$G = (1 - \alpha) \frac{x(p)}{n} (p - \hat{c}) \quad (2)$$

where  $\hat{c}$  is the expected unit cost:  $\hat{c} = \int_{\underline{c}}^{\bar{c}} c dF(c)$ .

---

<sup>5</sup>Usually, models of perfect competition assume only that the price is given. However, as the model is linear in expected costs, a risk-neutral firm would choose to produce an infinite amount if  $p > E(c)$ . To avoid this problem, we impose here and in the following that for a given price every firm faces a demand which it has to satisfy, i.e.  $x_i(p)$ , the output of firm  $i$ , is not a choice variable but is given by the residual demand.

Proposition 1 describes the individually optimal ratio of inside and outside financing under these circumstances.

**Proposition 1** *If the product market is perfectly competitive, each firm chooses to fully diversify its risk, i.e. the optimal ratio of inside and outside financing  $\frac{\alpha}{1-\alpha}$  is zero.*

Proof:

Substituting  $G$  from equation 2 in expression 1 and taking the derivative with respect to  $\alpha$  leads to:

$$E \left[ U' \left( \frac{x(p)}{n}(p - \hat{c}) - \alpha \frac{x(p)}{n}(c - \hat{c}) \right) \cdot \left( -\frac{x(p)}{n}(c - \hat{c}) \right) \right] = 0 \quad (3)$$

It is easy to verify that the second derivative w.r.t.  $\alpha$  is negative. Note that this first order condition is solved at  $\alpha = 0$ . In that case the marginal utility is a constant, and the expectation of the second term is zero. Q.E.D.

This result is quite intuitive - a risk-averse entrepreneur would like to insure himself, and as the outside financiers are assumed to be risk-neutral, all risks are shifted towards them.

## 4 Financing Decision under Bertrand Competition

To investigate the situation under Bertrand competition we study first a simple example with only two firms. Both entrepreneurs have symmetric utility functions which are assumed to be logarithmic, and symmetric initial wealth which is denoted by  $w$ . Costs per unit of production can take one of two values, 0 or 1, with equal probability.

We first analyse how price competition at stage 2 is affected by the firms' ratio of inside and outside financing which in turn determines their risk aversion. To illustrate the impact of the financial structure on price competition it is useful to consider the symmetric case where  $\alpha = \alpha_1 = \alpha_2$  and  $G = G_1 = G_2$ . Note first that for positive  $\alpha$

it cannot be an equilibrium that both firms charge a price equal to expected marginal cost.<sup>6</sup> In our example this would imply charging a price of  $\frac{1}{2}$ . However, due to the cost uncertainty and the firms' risk aversion firms would be better off to forego sales at this price since

$$\begin{aligned} \frac{1}{2} \ln \left( w + \alpha \left( \frac{1}{2} - 0 \right) \frac{\pi(\frac{1}{2})}{2} + G \right) + \frac{1}{2} \ln \left( w + \alpha \left( \frac{1}{2} - 1 \right) \frac{\pi(\frac{1}{2})}{2} + G \right) = \\ \frac{1}{2} \ln \left( w + \frac{\alpha \pi(\frac{1}{2})}{4} + G \right) + \frac{1}{2} \ln \left( w - \frac{\alpha \pi(\frac{1}{2})}{4} + G \right) < \ln(w + G) \end{aligned} \quad (4)$$

Suppose next that firms think of charging a price  $p$  above expected marginal cost. Then this can be an equilibrium price only if no firm has an incentive to unilaterally undercut this price to capture the whole market. Thus, a necessary condition for this price to be an equilibrium price is that

$$\begin{aligned} \frac{1}{2} \ln(w + \alpha \frac{\pi(p)}{2}(p - 0) + G) + \frac{1}{2} \ln(w + \alpha \frac{\pi(p)}{2}(p - 1) + G) \geq \\ \frac{1}{2} \ln(w + \alpha x(p)(p - 0) + G) + \frac{1}{2} \ln(w + \alpha x(p)(p - 1) + G) \end{aligned} \quad (5)$$

that is no firm has an incentive to lower the price, which would increase the residual demand from  $\frac{\pi(p)}{2}$  to  $x(p)$  but comes with the risk of higher losses.<sup>7</sup> To determine the maximum equilibrium price  $p^*$ , condition (5) has to be satisfied with equality. This leads to

$$p^*(\alpha) = \frac{1}{2}(1 - u + \sqrt{1 + u^2}) \quad (6)$$

where  $u = \frac{4(w+G)}{3\alpha x(p)}$ . Note that for  $\alpha \rightarrow 0$ ,  $p \rightarrow 1/2$ ; i.e., if firms become more and more risk neutral, the Bertrand price converges to expected marginal costs. This is the standard result from the industrial organisation literature: under Bertrand competition two firms are sufficient to restore perfect competition. As our example shows this result holds even if costs are uncertain as long as firms are risk neutral. If firms are risk averse, however, this result no longer holds. If  $\alpha$  is positive the price is above  $1/2$ . In the extreme case, where  $w$  is very small (approaching  $-G$ ) so that firms are very risk averse due to the DARA utility function, the equilibrium price approaches 1, the high cost level.

We still have to show that the maximum equilibrium price described by condition (6) is higher than the price necessary to induce risk averse firms to make positive sales at all;

<sup>6</sup>Sandmo (1971) shows that risk averse firms require a price above expected cost to enter the market at all.

<sup>7</sup>A detailed derivation of the strategies and the according equilibria can be found in Wambach (1998).

i.e. that  $p^*$  guarantees a positive utility to risk averse firms. To derive the entrepreneurs' utility for a given level of  $\alpha$ , we need to solve for the equilibrium price  $G$  paid by the outside investors for their cash flow rights. Note that given  $\alpha_1 = \alpha_2 = \alpha$ , each investor anticipates the equilibrium price  $p$  at stage 2 derived above. Firm  $i$ 's take-it-or-leave-it offer  $G_i$  will be accepted if  $(1 - \alpha)\frac{1}{2}x(p^*)(p^* - \hat{c}) - G_i \geq 0$ . If this inequality is not satisfied, then the financier rejects the offer. Solving this condition for the maximum  $G_i$  and plugging this back into equation (6) we can solve for  $p^*$

**Proposition 2** *Suppose the two firms engage in Bertrand competition on the product market. If  $\alpha_1 = \alpha_2 = \alpha > 0$  and hence both firms are risk averse, then the maximum equilibrium price lies above the competitive price and the two firms receive a strictly positive level of utility.*

Proof: See Wambach (1998)

This result establishes the driving effect of our model. If firms sell all their cash flow rights in order to fully diversify their risk, Bertrand competition will drive down prices to expected marginal cost, leading to zero utility. If firms choose  $\alpha > 0$  instead, they have to carry some risk but this in turn enables them to sustain prices above marginal costs in the output market. Proposition 2 shows that firms benefit from this reduction in price competition even though they have to bear more risk.

If firms can cooperate on their choice of  $\frac{\alpha}{1-\alpha}$ , the ratio of inside and outside financing, they will choose the  $\alpha^*$  that maximizes joint profits.

To illustrate our results we have plotted the function  $p^*(\alpha)$  in Figure 1 for the following parameters:  $w = 30$ ,  $x(p) = 100$ . As expected,  $p^*$  increases with  $\alpha$ , more risk averse capital leads to an increase in price.

*Figure 1 about here*

In Figure 2 the expected utility of the firms is shown (the solid curve). Note that

for  $\alpha = 0$ , the expected utility is  $3.4 = \ln(30)$ ; i.e., firms receive just their outside option utility. At this point, both firms are risk neutral and the resulting price is 0.5. The maximum utility is reached at  $\alpha^* = 0.8$ , i.e. we get an interior solution for the optimal  $\alpha$ . This reflects the trade-off between risk aversion and reduced price competition.

*Figure 2 about here*

So far we have treated  $\alpha$  as exogenously given or assumed that the two firms cooperate when choosing their ratio of inside and outside financing. The crucial question is, however, whether there exists a symmetric equilibrium where both firms choose positive ratios noncooperatively. This is the subject of the following proposition.

**Proposition 3** *Suppose the two firms engage in Bertrand competition on the product market at stage 2 and suppose further that they choose  $\alpha_1$  and  $\alpha_2$  noncooperatively at stage 1. In this case there exist symmetric equilibria, where  $\alpha_1 = \alpha_2 = \alpha > 0$ ; i.e., both firms choose the same level of inside financing, and enjoy positive utility levels.*

Proof: To prove this proposition consider the decision of entrepreneur 1 at stage 1. If the strategy of firm 2 is to choose  $\alpha_2 = \alpha$ , what is the best response of firm 1 to that? Note first, that it cannot improve its payoff by choosing  $\alpha_1 > \alpha$ . In that case the market price at stage 2 is solely determined by  $\alpha$ , as it must hold that no firm has an incentive to capture the whole market. But for any price, the incentive to deviate is larger for the firm with more outside financing, because it is less risk averse. Thus, if  $\alpha_1 > \alpha_2 = \alpha$  the resulting equilibrium price is independent of  $\alpha_1$ . However, if the price is not affected by firm  $i$ 's choice of  $\alpha_i$  then, as we know from the last section, it prefers to choose its own residual risk as low as possible. Hence,  $\alpha_1 > \alpha$  cannot be a profitable deviation.

Consider next  $\alpha_1 < \alpha$ . If both firms still share the market at stage 2, the market price is given by solving equation (5) for the lowest  $\alpha$ . Thus, if  $\alpha \leq \alpha^*$ ; a marginal decrease of

$\alpha_1$ , such that  $\alpha_1 < \alpha$ , does not pay if  $\alpha \leq \alpha^*$  and the two firms share the market because the combination of  $\alpha^*$  and  $p(\alpha^*)$  maximizes the expected utility.

However, firm 1 may consider a drastic decrease of  $\alpha_1$  and a price reduction at stage 2 such that firm 2 will not even serve half the market. In that case, the condition for the equilibrium price  $\bar{p}$  at stage 2 is:

$$\frac{1}{2} \ln(w + \alpha \frac{x(\bar{p})}{2}(\bar{p} - 0) + G) + \frac{1}{2} \ln(w + \alpha \frac{x(\bar{p})}{2}(\bar{p} - 1) + G) \leq \ln(w + G) \quad (7)$$

i.e. firm 2 prefers to stay out of the market rather than to charge  $\bar{p}$  and serve half the market. Note that also here  $G$  is given by  $G = (1 - \alpha)\frac{1}{2}x(p^*)(p^* - \hat{c})$ , which is the fee the outside investors have paid to firm 2.

This leads to the following equation for  $\bar{p}$ :

$$\bar{p}(\alpha) = \frac{1}{2}(1 - v + \sqrt{1 + v^2}) \quad (8)$$

where  $v = \frac{4(w+G)}{\alpha x(\bar{p})}$ . Comparing this with equation (6) it follows that  $\bar{p} < p^*$

As this price does not depend on  $\alpha_1$  we know that the best response of firm 1 is to set  $\alpha_1 = 0$ . In that case, the income of firm 1 is certain. The outside financiers anticipate the price  $\bar{p}$  so that they are willing to pay  $\bar{G} = x(\bar{p})(\bar{p} - \hat{c})$ . The utility of firm 1 is then:

$$\ln(w + x(\bar{p})(\bar{p} - \hat{c})) \quad (9)$$

Next we show that there exist values for  $\alpha$  where the utility of serving half the market at price  $p^*(\alpha)$  is larger than the utility of serving the whole market at  $\bar{p}(\alpha)$ . As  $p^* > \bar{p}$  the utility of serving half the market at price  $p^*$  is larger than the utility of serving half the market at  $\bar{p}$  which is equal to  $\ln(w + G)$ . The proof will be done if we show that for small  $\alpha$ ,  $\ln(w + G)$  is larger than the utility of the deviator given in equation (9). Thus we have to show that

$$w + G > w + x(\bar{p})(\bar{p} - \hat{c}) \quad (10)$$

where  $G$  is given above and  $\hat{c} = 0.5$ . By using equations (6) and (8) this transforms into:

$$(1 - \alpha)\frac{1}{2}x(p^*)(-u + \sqrt{1 + u^2}) > x(\bar{p})(-v + \sqrt{1 + v^2}) \quad (11)$$

Multiplying both sides by  $\alpha$  we get:

$$(1 - \alpha) \frac{1}{2} x(p^*) (-\bar{u} + \sqrt{\alpha^2 + \bar{u}^2}) > x(\bar{p}) (-\bar{v} + \sqrt{\alpha^2 + \bar{v}^2}) \quad (12)$$

with  $\bar{u} = \alpha * u$ ,  $\bar{v} = \alpha * v$ . If  $\alpha$  tends to zero, both sides are zero. Taking the derivative with respect to  $\alpha^2$  at  $\alpha = 0$  we get for the left hand side:

$$\frac{1}{2} x(1/2) \frac{1}{2\sqrt{\bar{u}^2}} = \frac{1}{4} x(1/2) \frac{3}{4} \frac{x(1/2)}{w} \quad (13)$$

while the derivative of the right hand side is:

$$x(1/2) \frac{1}{2\sqrt{\bar{v}^2}} = \frac{1}{2} x(1/2) \frac{1}{4} \frac{x(1/2)}{w} \quad (14)$$

which is smaller. So for small  $\alpha$  a drastic deviation does not pay.

*Q.E.D.*

Proposition 3 confirms that there exist noncooperative equilibria where both firms choose positive levels of inside financing. But it is not clear that the joint utility maximizing  $\alpha^*$  can be sustained as a noncooperative equilibrium outcome. This is illustrated in the numerical example presented above. Figure 1 shows both price functions,  $p^*(\alpha)$  and  $\bar{p}(\alpha)$ , under the assumptions on  $x$ ,  $w$  and  $K$  as given above, for any  $\alpha$ . In Figure 2 the expected utility of firm 1 is shown as a function of  $\alpha$  under the two scenarios: Either  $\alpha_1 = \alpha_2 = \alpha$  and both firms share the market with  $p = p^*(\alpha)$  or  $\alpha_1 = 0$ ,  $\alpha_2 = \alpha$ , firm 1 has the whole market and  $p = \bar{p}(\alpha)$ . For the symmetric solution to hold as the outcome of a noncooperative equilibrium  $\alpha$  must not be larger than 0.53. And in fact, this is the constrained optimum from the firms point of view.

Thus, in our numerical example entrepreneurs do have an incentive to free ride on the risk aversion of their competitors and choose a smaller level of inside financing than is optimal from the point of view of joint profit maximization. Both firms would increase their utility if they could write a binding agreement on choosing the same  $\alpha$ .

## 5 Comparative statics

In this section we want to do some comparative static analysis with respect to the number of competitors, the degree of the entrepreneurs' risk aversion and the level of cost



uncertainty. For this purpose it is useful to study a slightly different set up with  $n$  entrepreneurs whose utility functions exhibit constant absolute risk aversion (CARA) with a Pratt-Arrow risk coefficient  $\gamma$ . Furthermore, costs are normally distributed with mean  $\hat{c}$  and variance  $\sigma^2$ .<sup>8</sup> One nice feature about normally distributed costs is that  $E[\exp(-\phi c)] = \exp(-\phi \hat{c} + \frac{\phi^2}{2} \sigma^2)$ . Thus in the following it suffices to argue with the certainty equivalent of the uncertain income. In our case, the utility function of each entrepreneur is  $E[\exp(-\alpha \gamma \frac{\pi}{n}(p - c) - \gamma G)]$  and the certainty equivalent is therefore  $\exp(-\alpha \gamma \frac{\pi}{n}(p - \hat{c}) + \frac{\alpha^2}{2} \alpha^2 \gamma^2 \frac{\sigma^2}{n^2} - \gamma G)$ .

Consider again the maximum equilibrium price if all firms have the same financial structure  $\alpha/(1 - \alpha)$ . This is found by looking at the analogue of condition (5) which is here

$$\alpha \gamma \frac{x(p)}{n}(p - \hat{c}) - \frac{\sigma^2}{2} \alpha^2 \gamma^2 \frac{x(p)^2}{n^2} + \gamma G \geq \alpha \gamma x(p)(p - \hat{c}) - \frac{\sigma^2}{2} \alpha^2 x(p)^2 \gamma^2 + \gamma G \quad (15)$$

To determine the maximum equilibrium price again this expression has to hold with equality. From this we get:

$$p^* = \hat{c} + \gamma \frac{\sigma^2}{2} \alpha x(p^*) \frac{n+1}{n} \quad (16)$$

The following proposition summarizes the comparative static results for this equilibrium price:

**Proposition 4** *Suppose  $n$  firms engage in Bertrand competition on the product market. If  $\alpha_1 = \alpha_2 = \dots = \alpha_n = \alpha > 0$  then the maximum equilibrium price lies above the competitive price and it is higher*

- *the higher the firms' risk aversion due to the level of inside financing ( $\alpha$ ),*
- *the higher the firms' risk aversion arising from their utility function, as captured by the risk coefficient  $\gamma$ ,*
- *the higher the level of cost uncertainty, as measured by the cost variance  $\sigma^2$ , and*

---

<sup>8</sup>This assumption is unattractive on economic grounds, as it allows for negative costs. However, it simplifies the calculation to a great extent and is therefore very commonly used in the literature.

- the smaller the number of firms  $n$ .

Proof: Using the implicit function theorem, the proof is straightforward:

$$\begin{aligned}\frac{dp^*}{d\alpha} &= -\frac{-\gamma\frac{\sigma^2}{2}x\frac{n+1}{n}}{1-\gamma\frac{\sigma^2}{2}\alpha x'} > 0 & \frac{dp^*}{d\gamma} &= -\frac{-\frac{\sigma^2}{2}\alpha x\frac{n+1}{n}}{1-\gamma\frac{\sigma^2}{2}\alpha x'} > 0 \\ \frac{dp^*}{d\sigma} &= -\frac{-\gamma\sigma\alpha x\frac{n+1}{n}}{1-\gamma\frac{\sigma^2}{2}\alpha x'} > 0 & \frac{dp^*}{dn} &= -\frac{-\gamma\frac{\sigma^2}{2}\alpha x\frac{n-(n+1)}{n^2}}{1-\gamma\frac{\sigma^2}{2}\alpha x'} < 0\end{aligned}$$

*Q.E.D.*

It is easy to see why the equilibrium price should increase in  $\alpha$ ,  $\gamma$  and  $\sigma$ , as all these factors make the firm more risk averse. The comparative statics result with respect to the number of competitors is less straightforward. As the number of competitors grows the increase in market share when undercutting the competitors gets larger because it implies moving from  $\frac{x(p)}{n}$  to  $x(p)$ . This means that the potential for gains increases if costs are low, but also the potential for losses if costs are high. As our proposition shows, the positive effect dominates the negative effect which makes it more difficult to sustain a price above the competitive price. Finally, note that in contrast to the previous section,  $p^*$  does not depend on  $w$  or  $G$  as the degree of risk aversion is independent of the wealth level of the firm due to the CARA assumption.

Figure 3 illustrates with a numerical example how the maximum equilibrium price reacts to different values of  $\alpha$ ,  $\gamma$ ,  $\sigma$  and  $n$ . Take curve 1 as a starting point. As  $n$  increases, the maximum equilibrium price decreases for a given value of  $\alpha$  which implies a downward shift of curve 1 (curve 2). An increase in  $\gamma$  or  $\sigma$  instead leads to an upward shift of curve 1 (curve 3).

*Figure 3 about here*

Let us suppose next that the firms can collude on their financing strategy, but as before not on their prices. Call the resulting symmetric level of inside financing  $\alpha^c$ , and

the resulting equilibrium prices  $p^c$ . Then the optimal collusive outcome is given by the following maximization problem:

$$\max_{\alpha, p} \quad \frac{x(p)}{n}(p - \hat{c}) - \gamma \frac{\sigma^2}{2} \alpha^2 \frac{x(p)^2}{n^2} \quad (17)$$

$$\text{s.t.} \quad p = \hat{c} + \gamma \frac{\sigma^2}{2} \alpha x(p) \frac{n+1}{n} \quad (18)$$

where condition (18) denotes the maximum equilibrium price specified above in equation (16). The outcome of this maximization problem is characterized in the following Proposition.

**Proposition 5** *Suppose  $n$  firms engage in Bertrand competition on the product market but can agree on a jointly optimal level of inside financing before price competition starts. Then the optimal level of inside financing is given by*

$$\alpha^c = \frac{p^c - \hat{c}}{x(p^c)} \frac{2}{\gamma \sigma^2} \frac{n}{n+1} \quad (19)$$

The reaction of  $\alpha^c$  to variations of the firms' risk aversion, cost uncertainty or the number of firms is ambiguous and depends on the elasticity of market demand. However, if demand is differentiable with finite first and second derivative, then for large values of  $\gamma$ ,  $\sigma$  and  $n$   $\alpha^c$  decreases in  $\gamma$  and  $\sigma$  but increases in  $n$ .

The corresponding equilibrium price is always lower than the monopoly price, but tends towards the monopoly price

- if the firms' risk aversion is large, as measured by  $\gamma$ ,
- if the level of cost uncertainty is large, as measured by  $\sigma$  and
- if the number of firms,  $n$ , is large.

Proof: The Lagrange-Function of the maximization problem above is given by

$$L = \frac{x(p)}{n}(p - \hat{c}) - \gamma \frac{\sigma^2}{2} \alpha^2 \frac{x(p)^2}{n^2} - \lambda [p - \hat{c} - \gamma \frac{\sigma^2}{2} \alpha x(p) \frac{n+1}{n}] \quad (20)$$

This leads to the following first order conditions:

$$\frac{x(p)}{n} + \frac{x'(p)}{n}(p - \hat{c}) - \gamma \frac{\sigma^2}{2} \alpha^2 \frac{2x(p)x'(p)}{n^2} - \lambda \left[ 1 - \gamma \frac{\sigma^2}{2} \alpha x'(p) \frac{n+1}{n} \right] = 0 \quad (21)$$

$$-\gamma \frac{\sigma^2}{2} 2\alpha \frac{x(p)^2}{n^2} + \lambda \left[ \gamma \frac{\sigma^2}{2} x(p) \frac{n+1}{n} \right] = 0 \quad (22)$$

$$p - \hat{c} - \gamma \frac{\sigma^2}{2} \alpha x(p) \frac{n+1}{n} = 0 \quad (23)$$

Rewriting equation (22) gives:

$$\lambda = \frac{2}{n+1} \alpha \frac{x(p)}{n} \quad (24)$$

Multiplying equation (22) by  $\alpha x'/x$ , subtracting equation (22) from equation (21), substituting  $\alpha$  from equation (23) and using the expression for  $\lambda$  leads to:

$$x'(p^c)(p^c - \hat{c}) + x(p^c) = (p^c - \hat{c}) \frac{4}{\gamma \sigma^2} \frac{n}{(n+1)^2} \quad (25)$$

The interpretation of this expression is straightforward. On the left hand side we have marginal profit and on the right hand side we have an expression that is related to the firms' risk aversion. So the optimal price will always be below the monopoly price of a risk neutral monopoly as long as the right hand side of equation (25) is positive. If the firms are very risk averse, if the uncertainty is large or if there are many firms in the market, this price tends towards the monopoly price.

To see this, consider once more equation (25) and define  $A = \frac{4}{\gamma \sigma^2} \frac{n}{(n+1)^2}$ . Then we can use the implicit function theorem to show that

$$\frac{dp^c}{dA} = - \frac{-(p^c - \hat{c})}{x''(p^c)(p^c - \hat{c}) + 2x'(p^c) - A} < 0 \quad (26)$$

The denominator is negative as the first part of the expression is the second order derivative of the profit maximizing problem a risk neutral monopolist faces, which is negative. By subtracting  $A$  this term remains negative. Thus the overall expression is negative given that  $p^c > c$ . As  $A$  depends negatively on  $\gamma$ ,  $\sigma$  and  $n$  we get the desired results.

To see how the optimal  $\alpha$  reacts to variations of risk aversion and the number of competitors, we use equations (21) and (22) to get the following expression:

$$\alpha^c = \frac{n+1}{2x} (x + x'(p^c)(p^c - \hat{c})) \quad (27)$$

From this expression we get the following derivatives:

$$\frac{d\alpha^c}{d\gamma} = - \underbrace{\frac{(n+1)2x'(p)\frac{dp^c}{d\gamma}}{4x(p)^2}}_{+} \underbrace{(x(p) + x'(p)(p^c - \hat{c}))}_{(i)} + \frac{n+1}{2x(p)} \underbrace{(2x'(p) + x''(p)(p^c - \hat{c}))}_{(ii)} \frac{dp^c}{d\gamma} \quad (28)$$

$$\frac{d\alpha^c}{d\sigma} = - \underbrace{\frac{(n+1)2x'(p)\frac{dp^c}{d\sigma}}{4x(p)^2}}_{+} \underbrace{(x(p) + x'(p)(p^c - \hat{c}))}_{(i)} + \frac{n+1}{2x(p)} \underbrace{(2x'(p) + x''(p)(p^c - \hat{c}))}_{(ii)} \frac{dp^c}{d\sigma} \quad (29)$$

$$\frac{d\alpha^c}{dn} = \underbrace{\frac{2x(p) - (n+1)2x'(p)\frac{dp^c}{dn}}{4x(p)^2}}_{+} \underbrace{(x(p) + x'(p)(p^c - \hat{c}))}_{(i)} + \frac{n+1}{2x(p)} \underbrace{(2x'(p) + x''(p)(p^c - \hat{c}))}_{(ii)} \frac{dp^c}{dn} \quad (30)$$

Term (i) is the marginal profit of a risk neutral monopolist and hence positive for all prices below the monopoly price. Term (ii) is the second derivative of this monopoly profit and hence negative. Note that  $\frac{dp^c}{d\gamma}$  and  $\frac{dp^c}{d\sigma}$  are positive, as shown above. Thus, the sign of the derivative of  $\alpha^c$  with respect to  $\gamma$ , and  $\sigma$  depends on which of the two terms (i) and (ii) dominates. However, as we have seen above, for large values of  $\gamma$ ,  $\sigma$  and  $n$  term (i) tends to zero and hence the negative impact of term (ii) dominates. This implies that for large values of  $\gamma$ ,  $\sigma$  and  $n$  the expressions in equations (28) and (29) are negative. A similar argument does not hold for equation (30), as term (i) is not just multiplied by  $\frac{dp^c}{dn}$  but also by  $2x(p)$ . For large values of  $\gamma$ ,  $\sigma$  and  $n$  both (i) as well as  $\frac{dp^c}{dn}$  goes to zero. Expanding this expression in  $A$  one gets:

$$\frac{d\alpha^c}{dn} \rightarrow \frac{(p^c - \hat{c})A}{2x(p)} \left| 1 - \frac{n-1}{n} \right| > 0$$

*Q.E.D.*

It is intuitively clear that  $p^c$  increases in  $\gamma$  and  $\sigma$  as large risk aversion makes it easier to stabilize larger prices. The result for large  $n$  is more surprising. Although we know that for a given  $\alpha$ , prices decrease in  $n$ , Proposition 5 implies that the equilibrium prices resulting after optimally choosing  $\alpha$  converge to the monopoly price for large  $n$ . The reason is that for larger  $n$ 's firms have to provide a smaller market share which makes

them more, but not fully risk neutral. Larger  $\alpha$ 's can therefore be optimal which in turn increases the price.

The crucial question of course is again, what levels of symmetric inside financing the firms can sustain in a non-cooperative equilibrium. This is described in the following Proposition.

**Proposition 6** *Suppose the  $n$  firms engage in Bertrand competition on the product market at stage 2 and suppose further that they choose  $\alpha_i$  noncooperatively at stage 1. In this case there exist symmetric equilibria in which all firms choose the same positive ratio of inside and outside financing. However, in the optimum firms will acquire a positive level of outside financing if demand is elastic.*

Proof:

To see whether a firm has an incentive to deviate from the symmetric choice of ( $\alpha_1 = \alpha_2 = \dots = \alpha_2 = \alpha > 0$ ) we have to formulate the analogue to equation (7). This has the following form:

$$\alpha \frac{x(\bar{p})}{n} (\bar{p} - \hat{c}) - \gamma \frac{\sigma^2}{2} \alpha^2 \frac{x(\bar{p})^2}{n^2} + G \leq G \quad (31)$$

To determine the maximum equilibrium price with which a deviating firm can capture the whole market this condition has to be satisfied with equality which leads to

$$\bar{p} = \hat{c} + \gamma \frac{\sigma^2}{2} \alpha x(\bar{p}) \frac{1}{n} \quad (32)$$

It is clear that  $\bar{p} < p^*$

The symmetric solution for the level of inside financing only holds in equilibrium if the expected utility of serving  $1/n$ th of the market at price  $p^*$  and  $\alpha > 0$  is larger than the utility of serving the whole market at price  $\bar{p}$  with  $\alpha = 0$ . The outside financiers anticipate the price  $p^*$  or  $\bar{p}$  respectively, so we get by using the corresponding expressions for  $G$  and  $\bar{G}$ .

$$\frac{x(p^*)}{n} (p^* - \hat{c}) - \gamma \frac{\sigma^2}{2} \alpha^2 \frac{x(p^*)^2}{n^2} \geq x(\bar{p}) (\bar{p} - \hat{c}) \quad (33)$$

Using equations (16) and (32) it follows:

$$x(p^*)^2 \frac{n+1-\alpha}{n} \geq x(\bar{p})^2 \quad (34)$$

If the demand is somewhat elastic, the output is larger on the right hand side since  $\bar{p}$  is smaller than  $p^*$ . This implies that  $\alpha$  must be strictly smaller than one for the symmetric solution to hold. Furthermore, it is clear that for small  $\alpha$  the condition in equation (34) is not violated, as in that case  $x(p^*) = x(\bar{p})$  and  $n+1-\alpha > n$ . This proves the last part of the Proposition. Q.E.D.

To compare the noncooperative financial structure with the cooperative solution we consider two examples of different demand structures:

(i) *completely inelastic demand:  $x(p) = x$  if  $p \leq \bar{v}$  and  $x(p) = 0$  for  $p > \bar{v}$*

Consider a market with  $m$  consumers who have a valuation of  $\bar{v}$  of the good and desire

to buy  $\frac{x}{m}$  units each. From equation (25) we can derive the optimal equilibrium price  $p^c$  if the level of inside financing is chosen cooperatively.

$$p^c = \min\left\{\hat{c} + \frac{(n+1)^2 \gamma \sigma^2}{n} x, \bar{v}\right\} \quad (35)$$

Furthermore,  $\alpha^c$  is given by:

$$\alpha^c = \min\left\{\frac{1}{2}(n+1), \frac{\bar{v} - \hat{c}}{x} \frac{2}{\gamma \sigma^2} \frac{n}{n+1}\right\} \quad (36)$$

However, from Proposition 6 we know that  $\alpha$  has to be smaller than 1, otherwise one of the firms would have an incentive to deviate by choosing  $\alpha = 0$  and capturing the whole market. Thus we conclude that in this case the optimal noncooperative  $\alpha^*$  is given at either  $\alpha^* = 1$ , i.e. all entrepreneurs own their firm completely, and the market price is  $p^* = \hat{c} + \gamma \sigma^2 (n+1)/(2n)$ , or  $\alpha^* = \frac{\bar{v} - \hat{c}}{x} \frac{2}{\gamma \sigma^2} \frac{n}{n+1}$  if the latter expression is smaller than one, and  $p^* = \bar{v}$ .

(ii) *linear demand*:  $x(p) = A - Bp$

So far an increase in  $\alpha$  had two consequences - first it increased the degree of risk exposure, which lead ceteris paribus to lower expected utility. Second, it could be used as a commitment device to sustain higher prices, which in turn increased expected profits. Now, in addition to these two effects, an increase in  $\alpha$  increases the price which leads to a decrease in demand. This in turn decreases the degree of risk exposure as the risk premium scales with  $x^2$

Suppose for simplicity that  $\hat{c} = 0$ . Then, using equation (25), the optimal price if all  $n$  firms choose cooperatively the same  $\alpha$  is given by

$$p^c = \frac{A}{2B + \theta} \quad (37)$$

where  $\theta = \frac{in}{\gamma \sigma^2 (n+1)^2}$ . Note that the monopoly price is  $p^{mon} = \frac{A}{2B} > p^c$ . From equation (19) it follows that:

$$\alpha^c = \frac{\theta(n+1)}{2(B + \theta)} \quad (38)$$

Finally,  $\bar{p}$  for this  $\alpha^c$  is given by equation (32):

$$\bar{p} = \frac{A}{B(n+2) + \theta(n+1)} \quad (39)$$



The verification of condition (34) however shows that for any value of  $B$ ,  $\theta$  and  $n$  this inequality is not satisfied. Thus if firms cannot collude both constraints will be binding and the equilibrium is given by the solution of equations (16, 32, 34). Reformulating these expressions we get one single equation for  $\alpha^*$ , namely:

$$(\theta(n+1)^2 + 2\alpha^*B)^2(n+1-\alpha^*) = (n+1)^2n(\theta(n+1) + 2\alpha^*B)^2 \quad (40)$$

No analytical solution exists. We therefore discuss two extreme cases. First consider  $\theta/B$  small. This may be the case, because  $\gamma\sigma^2$  is large, i.e. the risk is very large or the firms are very risk averse, or because  $B$  is large. After some calculation we get that  $\alpha^* = \frac{n+1}{2} \frac{\theta}{B} \phi$  with  $\phi = (\sqrt{(n^2-1)^2 + (n+1)(n^2+n-1)} - (n^2-1))/(n^2+n-1)$ .  $\phi$  is smaller than 1 and tends to zero for large  $n$ . This has to be compared with equation (38), which is the solution if firms can collude on their financing strategy:  $\alpha^c = \frac{n+1}{2} \frac{\theta}{B}$ . Firms would always like to agree on a common financial strategy, and the value of such a precommitment increases in the number of firms.

Second, consider the case with a large  $\theta/B$ . This may be large either because firms are not risk averse, or because  $B$  is small, that is the demand is very inelastic. In that case,  $\alpha^* = 1$  while the  $\alpha^c$  from equation (38) satisfies  $\alpha^c = \frac{n+1}{2}$ . This does not come as a surprise as  $B = 0$  is the case discussed in the last paragraph on inelastic demand, and the same solution was obtained there.

## 6 Entry Costs

So far, our analysis draws a negative picture of the welfare consequences of the firms' financial decision. Entrepreneurs choose inside financing in order to reduce price competition. Thus, from the point of view of social welfare, this financial structure implies too much risk and too little price competition. However, this analysis applies only if we can take the number of firms active in the market as exogenously given. In this section we show that our welfare conclusions may have to be modified if we take the firms' market entry decision into account.

Consider a stage zero at which the firms can decide whether they want to enter

the market or not. Suppose furthermore that market entry is costly, e.g. because the entrepreneur has to investigate the market, set up his company, carry out R&D or invest in marketing its products. In the standard Bertrand model such entry costs induce a monopolistic market structure: If more than one firm enters the market at stage 0, profits will be competed down in the subsequent price competition, so that the entry costs will not be recovered. The only subgame perfect equilibrium in pure strategies is where only one firm enters the market and recovers its entry costs by charging the monopoly price.

This is different in our model. Even if more than one firm enters the market the expected utility is positive. Thus, as long as the entry costs are not too high, more than one firm will enter the market in equilibrium. The following proposition shows that this can have positive consequences for social welfare.

**Proposition 7** *Consider a three-stage game with  $n$  risk-averse entrepreneurs who face positive market entry costs  $k$  at stage 0, choose their financial structure at stage 1 and compete in prices at stage 2. Then there exist positive values for  $k$  such that the welfare is higher in case of risk-averse entrepreneurs who choose their financial structure than in case of risk-neutral firms.*

Proof: To prove this proposition we construct an example for which this statement is true. Consider the scenario discussed in section 4 with a slight modification. Costs are as before (with probability 1/2 costs are zero, with probability 1/2 they are 1), firms have logarithmic utility functions, initial wealth is 39. If the price is not greater than 1, demand is 100, for prices between 1 and 2 the demand is 50 and zero for prices above 2. At stage zero firms decide whether to enter the market or not. In case of entering entry costs of 9 have to be paid. If risk neutral firms were to compete in prices, in equilibrium only one firm would enter the market. This firm would charge the monopoly price which is 2 for the parameter values given above. This changes if firms are risk-averse. For two firms, we have seen in section 4 that if the firms do not cooperate in their financial decision, the optimal  $\alpha$  is given at 0.53 which leads to a price of .725.<sup>9</sup> In that case,

---

<sup>9</sup>Note that for a wealth of 39 and entering costs of 5, the sure wealth at stage 2 is 30, as in the numerical example in section 4.

expected utility is 3.67 which is larger than the utility firms have by not entering the market, which is  $\ln(39) = 3.66$ . The parameter values are chosen in such a way that no third firm would enter the market. To determine the net welfare effect note first that social surplus in case of a risk-neutral monopolist is equal to the expected profit of the monopolist and is given by  $50 \cdot (2 - 0.5) - 9 = 66$  since consumer surplus is zero. In case of two risk-averse duopolists instead total social welfare is equal to the consumer surplus, given by  $50 \cdot (2 - 0.725) + 50 \cdot (1 - 0.725) = 77.5$ , and the change in utility of both entrepreneurs,  $\Delta u$ , which is positive. Note that  $77.5 + 2\Delta u > 66$ . Thus, the overall social surplus is higher in case of risk-averse entrepreneurs because it induces more market entry and hence lower prices than in case of risk-neutral firms. *Q.E.D.*

This proposition identifies a potentially welfare enhancing effect of the financing strategy: In a world with only risk-neutral participants no firm is able to cover its entry costs if more than one firm enters the market, so a monopoly arises. However, if firms carry some residual risk Bertrand competition allows for positive profits and more than one firm can enter the market which will lead to prices below the monopoly price. As long as these entry costs are not too large, this will be welfare enhancing.

## 7 Extensions and Conclusions

In this paper we have shown that risk-averse oligopolists charge prices above the competitive price and enjoy positive utilities. We have demonstrated further that firms can commit to being risk-averse by choosing a particular financing strategy, i.e. inside financing. In this final section we want to point out another strategy by which a firm might commit to risk-averse behavior: the employment of a risk-averse manager who is paid an uncertain wage.

Consider e.g. a risk-neutral entrepreneur who employs a risk-averse manager and pays a linear wage:  $w(x) = w_0 + \alpha\pi(p)$  where  $\pi$  is the profit of the firm which is  $x(p)(p - c)$ . In this case the optimisation problem is given by:

$$\max_x (1 - \alpha)x(p)(p - c) - w_0 \tag{41}$$

subject to the constraint

$$E[U(\alpha x(p)(p - \hat{c}) + w_0)] \geq U(0) \quad (42)$$

which is the dual problem to the one considered in the previous sections. If firms run by risk-averse managers compete in prices the equilibrium price  $p$  and demand  $x(p)$  are again functions of  $\alpha$  as in the previous sections.

Note that the 'incentive contract' designed for the manager is not intended to give an incentive to work harder, as in standard moral hazard problems. It is used as a means to raise prices in the market as risk-averse managers whose wage depend on the profit are more cautious in their decisions and behave less aggressively. In this sense incentive contracts can serve as an anticompetitive device which weakens the price competition.

Our analysis has also shown that entrepreneurs who decide on their financial structure non-cooperatively choose positive levels of inside-financing in equilibrium. However, in all our examples this non-cooperative level of inside-financing falls short of the level that entrepreneurs would like to commit to if the financial decision could be taken cooperatively. This means price competition is weaker as compared to standard Bertrand competition with risk-neutral firms. But it is more intense than it would be if entrepreneurs could agree on their ratio of inside and outside-financing. This is due to the fact that entrepreneurs have an incentive to free-ride on the risk-aversion of their competitors.

This observation has interesting implications for the judgment of cooperation among firms. It points out that cooperation on some decisions can affect the outcome of other, seemingly unrelated decisions. In our case, cooperation on firms' financial decisions can reduce the competitive pressure on the product market and thus have an anticompetitive effect on price competition. This shows that even prima facie innocuous cooperative attempts can give rise to concerns about potentially anticompetitive effects.

## References

- Aghion, P., M. Dewatripont and P. Rey 1998, "Agency Costs, Firm Behavior and the Nature of Competition", *mimeo*, University College London.
- Allen, B. and M. Hellwig 1986, "Bertrand-Edgeworth Oligopoly in Large Markets", *Review of Economic Studies* 8, 175-204
- Allen, B. and M. Hellwig 1989, "The Approximation of Competitive Equilibria by Bertrand-Edgeworth Equilibria in Large Markets", *Journal of Mathematical Economics* 18, 103-127
- Benassy, J.-P. 1989, "Market Size and Substitutability in Imperfect Competition: A Bertrand-Edgeworth-Chamberlin Model", *Review of Economic Studies* 56, 217-234
- Brander, J.D. and T.R. Lewis 1986, "Oligopoly and Financial Structure: The Limited Liability Effect", *American Economic Review* 76, 956-970
- Brander, J.D. and T.R. Lewis 1988, "Bankruptcy Costs and the Theory of Oligopoly", *Canadian Journal of Economics*, 221-243
- Dastidar, K. G. 1995, "On the Existence of Pure Strategy Bertrand Equilibrium", *Economic Theory* 5, 19-32
- Dixon, H. 1990, "Bertrand-Edgeworth Equilibria when Firms Avoid Turning Customers Away", *The Journal of Industrial Economics* 39, 131-146
- Dixon, H. 1992, "The Competitive Outcome as the Equilibrium in an Edgeworthian Price-Quantity Model", *The Economic Journal* 102, 301-309
- Edgeworth, F. 1897, "The Pure Theory of Monopoly", in *Papers Relating to Political Economy*, London:Macmillan
- Harris, M. and A. Raviv 1992, "Financial Contracting Theory", in: Laffont, J.J. (ed.), "Advances in economic theory: sixth World Congress", Cambridge University Press (New York), 64-150
- Hellwig, M. 1998, "Unternehmensfinanzierung, Unternehmenskontrolle und Ressourcenallokation: Was leistet das Finanzsystem?", *mimeo*, University of Mannheim.

- Hughes, J.S., Kao, J.L. and A. Mukherji 1998, "Oligopoly, Financial Structure, and Resolution of Uncertainty", *Journal of Economics and Managements Strategy* 7, 67-88
- Kreps, D. and J. Scheinkman 1983, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", *Bell Journal of Economics* 14, 326-337
- Maksimovic, V. 1988, "Capital Structure in Repeated Oligopolies", *RAND Journal of Economics*, 19, 389-407
- Maskin, E. 1986, "The Existence of Equilibrium with Price-Setting Firms", *American Economic Review*, papers and proceedings 57, 1243-1276
- Mayer, C. 1988, "New Issues in Corporate Finance", *European Economic Review* 32, 1167-1189
- Myers, S.C. and N.S. Majluf 1984, "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have", *Journal of Financial Economics*, 13, 187-221
- Sandmo, A. 1971, "On the Theory of the Competitive Firm Under Price Uncertainty", *American Economic Review* 61, 65-73
- Showalter, D.M. 1995, "Oligopoly and Financial Structure: Comment", *American Economic Review* 85, 647-653
- Wambach, A. 1998, "Bertrand Competition under Cost Uncertainty", *The International Journal of Industrial Organisation*, forthcoming

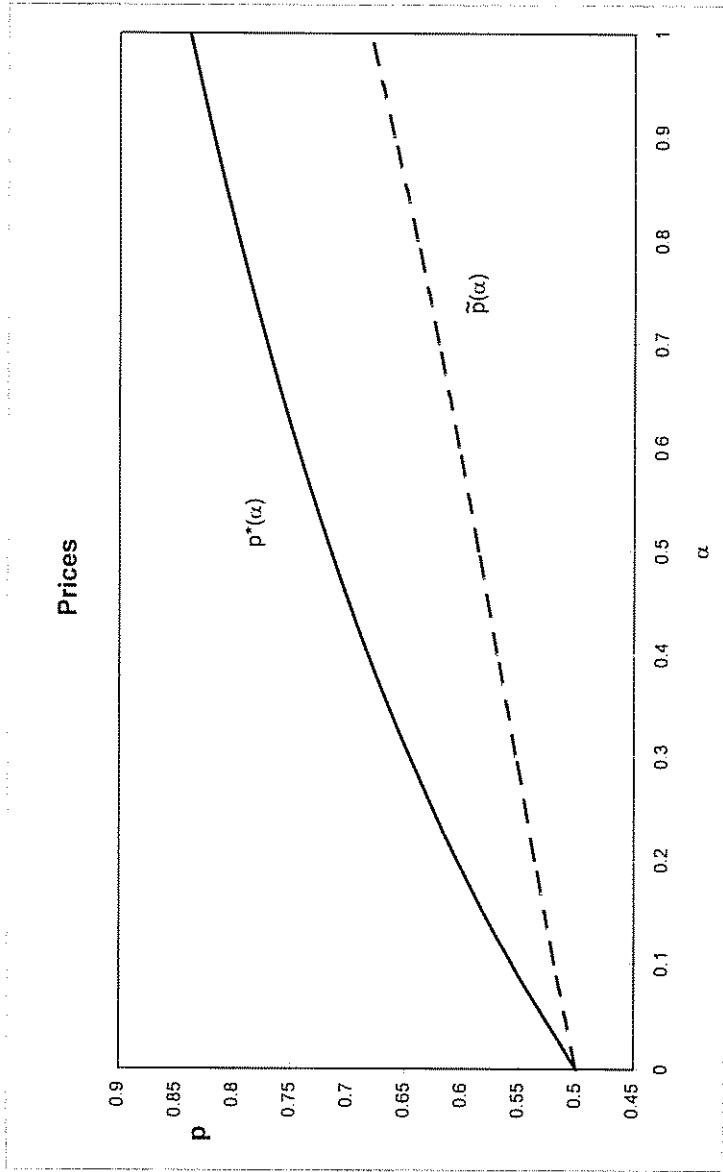


Diagram 1

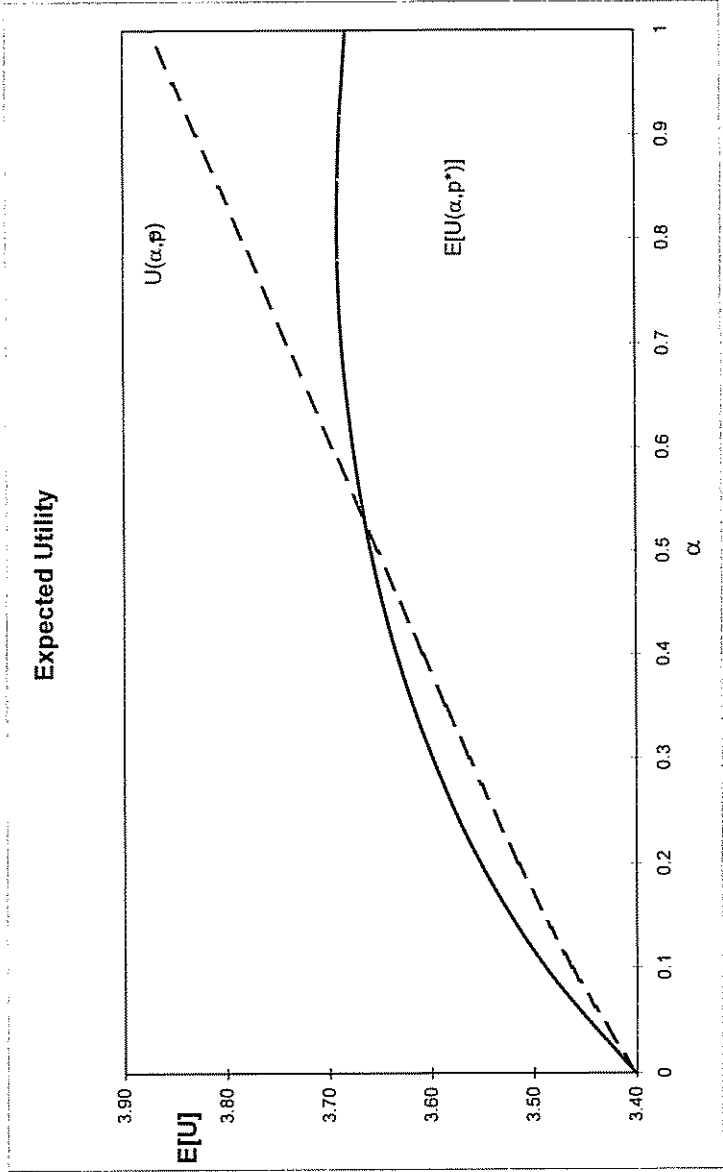


Diagram 2



# Prices

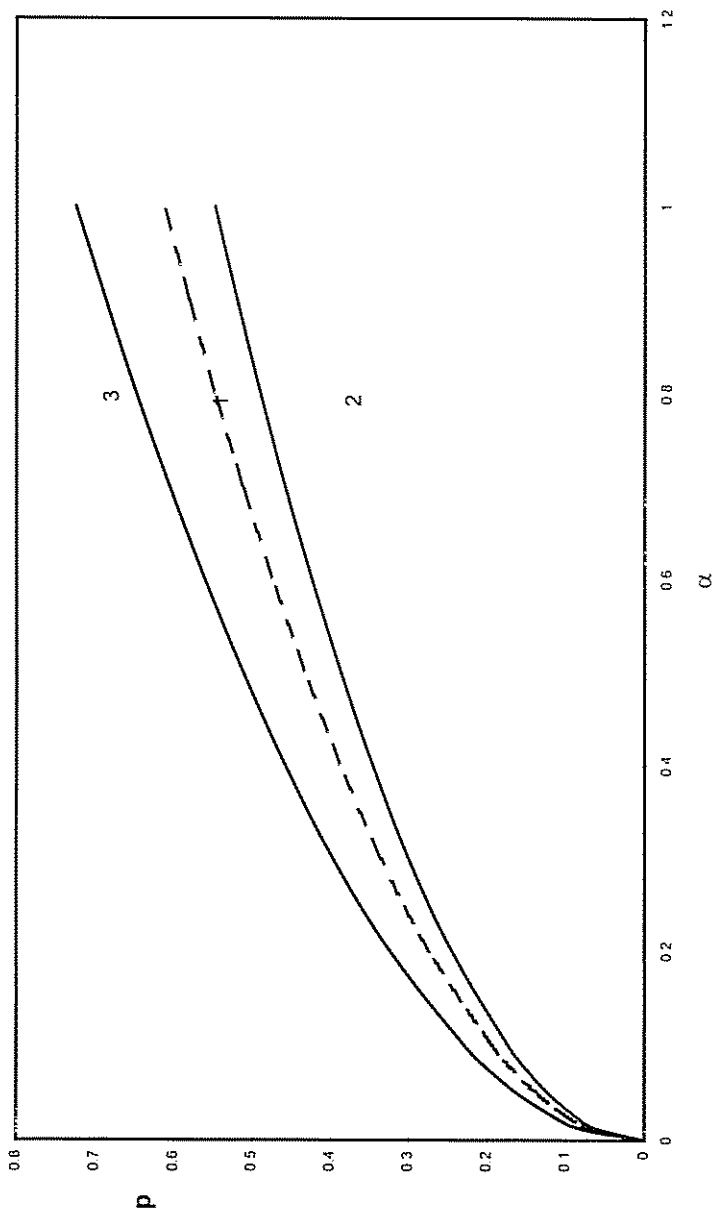


Diagram 3