DISTRIBUTIVE POLITICS AND THE COST OF DECENTRALIZATION

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ABSTRACT

Distributive Politics and the Cost of Decentralization*

This paper integrates the distributive politics literature with the literature on decentralization by incorporating inter-regional project externalities into a standard model of distributive policy. A key finding is that the degree of uniformity (or 'universalism') of the provision of regional projects is endogenous and depends on the strength of the externality. The efficiency of decentralization and the performance of 'constitutional rules' (such as majority voting) which may be used to choose between decentralization and centralization are then discussed in this framework. Stronger externalities and more heterogeneity between regions need not imply that decentralization becomes more efficient.

JEL Classification: H41, H70, H72
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NON-TECHNICAL SUMMARY

The different tax, expenditure and regulatory functions of government typically vary considerably in their degree of decentralization and moreover, decentralization also varies considerably by country, as well as by function. There is also an old and continuing debate over the desirable degree of decentralization. For example, the principle of subsidiarity in the Maastricht treaty is the subject of continuing controversy. This paper addresses both these issues.

The earlier literature on fiscal federalism, and in particular Oates' seminal work (Oates (1972)) gave the following account of the costs and benefits of decentralization. Sub-central governments may find it hard to coordinate to internalize inter-jurisdictional externalities or to exploit economies of scale in the provision of regional projects. On the other hand, the benefit of decentralization is greater responsiveness in the choice of project to the preferences of regions and localities. Specifically, in Oates' work, the cost of centralization was assumed to be policy uniformity e.g. if a regional public good is provided centrally, it must be provided at the same level in every region. This leads to the conclusion (Oates' 'decentralization theorem') that the efficient level of decentralization of the provision of a public good (or indeed any other government activity) is at the point where the benefits from less policy uniformity no longer exceed the costs of less internalization of externalities.

While providing important insights, Oates' account suffers from the problem that 'policy uniformity' is not derived from any explicit model of government behaviour and indeed, explicit public choice models tend to give a different account of what might happen with centralized provision of regional public goods. For example, the large literature on distributive politics (see Mueller (1997)) emphasizes the formation of minimum winning coalitions, rather than policy uniformity, in the provision of projects with region-specific benefits.

The distributive politics literature cannot be applied directly to refine Oates' argument, however, as it does not model the benefits of centralization that arise from the internalization of externalities. This paper attempts to integrate these two literatures, by formulating a model of distributive policy where: (i) legislative behaviour is rigorously modelled, with the primitives being legislative rules rather than outcomes; and (ii) spillovers between regions generated by distributive policies gives some rationale for centralization.

Absent externalities, the specific model we use is in many respects standard in the large theoretical literature with distributive politics. Specifically, every region has a discrete project that generates both intra-regional benefits and external benefits (or costs). All voters within a region are identical, but regions may vary both with respect to the costs and the benefit of the project. Central government then comprises a legislature of delegates, each delegate representing a region and elected from among the citizens of that region. The legislature then decides which projects are to be financed out of the proceeds of a uniform national tax.

Building on the important papers by Ferejohn, Fiorina, McKelvey (1987) and McKelvey (1986), we then propose some minimal legislative rules to ensure that behaviour in the legislature is determinate (i.e. that voting cycles are ruled out). First, legislators make proposals concerning subsets of regions whose projects are to be funded. These proposals are then ordered into an agenda, and are voted on sequentially and the winning motion is then paired with the status quo.

This procedure has a unique equilibrium outcome, where a proposal to fund projects in a particular set K of regions is proposed and approved independently of how items are ordered on the agenda. If externalities are negative or only weakly positive, this set comprises a simple majority of regions with the lowest costs as in the distributive politics literature (Ferejohn, Fiorina, McKelvey (1987)). If externalities are strongly positive, K comprises more than a simple majority of regions and may include all regions.

So, an important insight of this paper is that there is an interaction between project externalities and the legislative rules; the strength of the spillovers affects the degree of 'universalism' or uniformity in distributive policy. When spillovers are strong (and positive), outcome of legislative decision-making is closer to uniformity than it is when spillovers are small, or negative.

The second contribution of the paper is a thorough investigation of the constitutional choice between centralization and decentralization, using this model as a vehicle. We study first the benchmark case where unanimity is required for any change to the status quo, but side-payments between regions are possible. This case is a useful benchmark, in that the efficient alternative that maximizes aggregate welfare (the sum of utilities) will be chosen. We also consider the alternatives of unanimity rule without side-payments and majority rule. Generally, the picture confirms Oates' insights; centralization is chosen when externalities are strong and regions are relatively homogenous, and decentralization is chosen when the converse is true. But, there are some intriguing exceptions. For example, the relative benefit to centralization is not everywhere increasing in the size of the externality. These exceptions result from the fact that the legislative outcome is endogenously determined by the size of the externality.

1. Introduction

The different tax, expenditure and regulatory functions of government typically vary considerably in their degree of decentralization. For example, in the US, expenditure on education is highly decentralized, while expenditure on defense is almost entirely federal; property taxes are the main revenue-raising instrument at local level, whereas state and federal governments use income taxes. Moreover, countries differ in the degree to which functions are decentralized; for example, in contrast to the US, the only tax which is not centrally set in the UK is the local residential property tax.

Moreover, there is both an old and continuing debate over the desirable degree of decentralization. For example, there has been an ongoing debate about the appropriate sharing of tax and expenditure powers between Federal and State governments since the drafting of the US Constitution (Inman and Rubinfeld(1997)). In the European Union, the principle of subsidiarity, in introduced in the Maastricht Treaty, "remains vague and capable of conflicting interpretations" (Begg et. al. (1993)).

To understand this empirical diversity, and also to address the normative questions, we must understand both the underlying costs and benefits of (de)centralization, and the political processes that lead to the choice of a particular level of decentralization being chosen.

The earlier literature on fiscal federalism, and in particular Oates' seminal work (Oates(1972)) gave the following account of costs and benefits of decentralization. Sub-central governments may find it hard to coordinate to internalize inter-jurisdictional externalities, or to exploit economies of scale, in the provision of regional projects. On the other hand, the benefit of decentralization is greater responsiveness in the choice of project to the preferences of regions and localities. Specifically, in Oates' work, the cost of centralization was assumed to be *policy uniformity* i.e. it was assumed that if a regional public good was provided centrally, it must be provided at the same level in every region. This leads to the conclusion (Oates' "decentralization theorem"), that the efficient level of decentralization of the provision of a public good (or indeed any other government activity) is at the point where the benefits from less policy uniformity no longer exceed the costs of less internalization of externalities.

While providing important insights, Oates' account suffers from two problems. First, typically, spending by central governments is not uniform across regions in per capita terms. For example, the formulae used to allocate US federal block grants depends not only on population, but also on income per capita, tax raising effort, and several other factors (Boadway and Wildasin(1984)), and this is also true of other countries with formula-based intergovernmental grants (Costello(1993)).

Second, the hypothesis of "policy uniformity" is not derived from any explicit model of government behavior, and indeed, explicit public choice models tend to give a different account of what might happen with centralized provision of regional public goods. For example, the large literature on distributive politics (see e.g. Ferejohn, Fiorina and McKelvey (1987)) emphasizes the formation of minimum winning coalitions, rather than policy uniformity, in the provision of projects with region-specific benefits.

However, the distributive politics literature cannot be applied directly to refine Oates' argument, as it does not model the benefits of centralization that arise from the internalization of externalities. The objective of this paper is to integrate these two literatures, by formulating a model of distributive policy where (i) legislative behavior is rigorously modelled, with the primitives being legislative rules, rather than outcomes; (ii) spillovers between regions generated by distributive policies gives some rationale for centralization.

The main insight of this paper is that there is an *interaction* between these two features; the strength of the spillovers affects the degree of "universalism" or uniformity in distributive policy. When spillovers are strong (and positive), the *outcome* of legislative decision-making is closer to uniformity than it is when spillovers are small, or negative.

Absent externalities, the specific model we use is in many respects standard in the large theoretical literature with distributive politics. Specifically, every region has a discrete project which generates both intra-regional benefits and external benefits (or costs). All voters within a region are identical, but regions may vary both with respect to the costs and the benefit of the project. Central government then comprises a legislature of delegates, each delegate representing a region, and elected from amongst the citizens of that region[1]. The legislature then decides on which projects are to be financed out of the proceeds of a uniform national tax.

Building on the important papers by Ferejohn, Fiorina, McKelvey(1987), and McKelvey(1986), we then propose some minimal legislative rules to ensure that behavior in the legislature is determinate. First, legislators make proposals concerning subsets of regions whose projects are to be funded. These proposals are then ordered into an agenda, and are

[1]Another new feature is that of election of delegates to the national legislature. As all regions are homogenous, however, the delegate must have the preferences of any resident of that region. Besley and Coate(1998) consider the case where intra-regional preferences may differ.

voted on sequentially, and the winning motion is then paired with the *status quo*.

This procedure has a unique equilibrium outcome, where a proposal to fund projects in a particular set K of regions is proposed and approved, *independently* of how items are ordered on the agenda. The key finding is the following. If externalities are negative, or only weakly positive, this set comprises a bare majority of regions[2] with the lowest costs as in the distributive politics literature (Ferejohn, Fiorina, McKelvey(1987)). If externalities are strongly positive, K comprises more than a bare majority of regions, and may include all regions. So, the level of the externality effectively determines the degree of uniformity in project provision. A notable feature of the outcome is that the set of funded projects does not depend on the regional *benefits* of projects, but does depend on costs, ultimately because project costs are shared though national taxation.

The second contribution of the paper is a thorough investigation of the efficiency gains[3] from decentralization. In the earlier literature, where policy uniformity was assumed, the gains are higher when regions are heterogenous and/or inter-regional spillovers are small. It is not obvious that this should be so, as here the cost of centralization is not policy uniformity, but rather insensitivity of decision-making to project benefits and because the legislative outcome is endogenously determined by the size of the externality. We find that while conditions can be found under which both statements are true, there are some important qualifications, especially in the case of heterogeneity. For example, the efficiency gain to centralization is not everywhere increasing in the size of the externality, and the conditions under which increased heterogeneity increase the efficiency of decentralization are quite stringent. This is consistent with the results of Wallis and Oates(1988) and others, who do not find any strong evidence that linguistic and ethnic heterogeneity lead to greater decentralization[4].

Finally, we study constitutional choice (via unanimity and majority rule) between centralization and decentralization, using this model as a vehicle. Strikingly, even if there are no spillovers, *some* region will strictly gain from centralization, so the choice of decentralization can never be unanimous (this is because the gain though cost-pooling will always

[2]That is, $m = (n+1)/2$ regions, where n is the (odd) number of regions.

[3]As the model is a transferable utility one, the appropriate efficiency criterion is the sum of utilities, or aggregate surplus, and that is the one we use.

[4]Wallis and Oates study fiscal decentralization in the US, as measured by local government share in state-local government revenues and expenditures. The find that the percentage of the state population that are white, and the percentage that live on farms (two proxies for heterogeneity) have no stable effect on decentralization for different specifications of their regression equation.

benefit some high-cost region). If costs are sufficiently heterogenous, however, a majority will always prefer decentralization when there are no spillovers. Second, with sufficient homogeneity in both costs and benefits across regions, and strongly positive externalities, centralization is unanimously preferred, but only homogeneity in costs is required for a majority to prefer centralization when externalities are strongly positive.

There is already a body of work[5] which addresses (explicitly or implicitly[6]) the choice between centralization and decentralization, while taking a political economy approach to the modelling of government behavior (Alesina and Spolare(1997), Bolton and Roland(1997), Cremer and Palfrey(1996), Ellingsen(1997)). However, with the exception of Ellingsen, this literature follows Oates in assuming that centralized provision of a public good is uniform.

Finally, there is independent contribution of Besley and Coate(1998). Their paper also reexamines Oates' decentralization theorem from a political economy perspective. The focus of Besley and Coate's paper, however, is really quite different; they explicitly model the election of delegates to the national legislature in a citizen-candidate setting, and how this process interacts with the behavior of the legislature. By contrast to this paper, theirs does *not* model all the rules of operation of the legislature explicitly. Rather, in the setting of a "one-shot" version of Baron and Ferejohn's model of legislative bargaining, they capture the degree of "universalism" in an *ad hoc* way by supposing that the agenda-setter places some (exogenous) weight on the utility of the other delegate when formulating his agenda.

The rest of the paper is laid out as follows. Section 2 exposits the model. Sections 3 and 4 analyse political equilibrium under centralization. Section 5 derives conditions under which centralization or decentralization is the more efficient. Section 6 considers issues of constitutional design. Section 7 considers the robustness of the results to various extensions

[5] One should also note the work of Edwards and Keen(1996), and Seabright(1996)), where government is modelled as a Leviathan. The problem with such models of government behaviour, however, is that they are not based explicitly on the primitives of voters, legislative rules and the principal-agent relationship between voters and bureaucrats. There are also a number of papers which model government as welfare-maximizing (see e.g. Caillaud, Gilbert and Picard(1996), Gilbert and Picard(1996), Klibanoff and Poitevin(1996), Seabright(1996)). The challenge for these papers is to explain why decentralization might ever be welfare-superior to centralization; if central government can precommit, it can always replicate the decentralised outcome.

[6] Bolton and Roland focus on the closely related issue of when regions might choose to secede from a federation. One of the main themes of Bolton and Roland's work is how policy might be designed by the federation (assuming policy uniformity), subject to the constraint that it is not in either region's interest to secede. In our paper, we abstract from these issues by (implicitly) assuming that secession is infinitely costly.

of the model. Section 8 discusses some related literature in more detail than above, and concludes.

2. The Model

2.1. Preliminaries

There are an odd number $i = 1,..n$ of regions or districts each populated by a number of identical individuals with a population size normalized to unity. In each district there is a discrete project $x_i \in \{0,1\}$. Each project has a resource cost c_i, and generates benefit b_i for residents of i, and also external benefits e for residents of all regions $j \neq i$. There are two ways of interpreting this externality. The first is if there are three contiguous regions located in two-dimensional space, in which case the externality is "local" i.e. a project only impacts on neighboring regions, as shown in Figure 1.

Figure 1 in here

The second is that the externality is "global", that is, the project affects all regions, whether neighboring or not. Also, the externality e may be positive or negative, and may be interpreted as technological or pecuniary. This is a very stylized way of modelling externalities, but is analytically convenient. Some of the results of this paper extend to the case where externalities are depend both on the source and destination region i.e. where the project in region i inflicts externality e_{ij} on region j (Lockwood(1998)).

The following notation will be useful. Let $x = (x_i)_{i \in N}$ be any vector of projects, and $X = \{0,1\}^n$ be the feasible set of project vectors. If $F = \{i \in N | x_i = 1\}$ is the set of regions that have funded projects, let

$$x_i^F = \begin{cases} 1 & if\ i \in F \\ 0 & otherwise \end{cases}$$

and let $x^F = (x_i^F)_{i \in N}$. Also, let $f = \#F$.

All residents of region i have identical preferences over x^F and a numeraire good of the form

$$u_i = b_i x_i^F + y_i + (f - x_i^F)e \tag{1}$$

where b_i is the benefit from the project for those in region i, and y_i the level of consumption of a numeraire good. The term $(f - x_i^F)e$ indicates that region i gets external benefit of

fe from x^F if it does not have a project funded, and benefit of $(f-1)e$ from x^F if it has a project funded.

A resident of region i has initial endowment of the numeraire of unity, and pays a lump-sum of t_i either to regional or central government. So, the budget constraint for residents of region i is $y_i = 1 - t_i$. Substituting this constraint into (1), and suppressing the constant of unity, we get

$$u_i = b_i x_i^F - t_i + (f - x_i^F)e \tag{2}$$

2.2. Decentralization

With decentralization, the cost of the project is funded by a lump-sum regional tax[7], so the regional budget constraint is $t_i = x_i c_i$. Consequently, the net benefit of the project to any resident is $b_i - c_i$.

We make the natural assumption that a decision about the project is made by majority voting over the alternatives $x_i \in \{0, 1\}$. So, as all agents in a region are identical, the outcome under decentralization is simply that the project in i is funded if $b_i \geq c_i$. For future reference, note that the payoff to a resident of i with decentralization can be written

$$u_i^d = \max\{b_i - c_i,\ 0\} + (d - x_i^D)e \tag{3}$$

where $D = \{i \,|b_i \geq c_i\}$ is the set of projects funded under decentralization, and $d = \#D$. Obviously, in the presence of externalities, the outcome with decentralization is not efficient.

2.3. Centralization

We assume that in this case, both the decision about which projects to fund, and the setting of a tax to fund them, are made by a legislature that comprised of delegates from all regions. This is the way that centralization is often defined, but there are of course, two alternative kinds of *partial* centralization; the first is *centralized expenditure*, where projects are decided upon by central government, but are funded by regions as in Section 2.2 above, and the second *centralized funding*, where projects are decided upon regionally, but funded though a national tax (these alternatives are discussed in Section 6.2 below).

[7]This tax could easily be made distortionary, by introducing a factor of production in elastic supply (e.g. labour), and supposing that the tax is levied on this factor.

Revenue is raised by a national lump-sum tax, t i.e. a tax rate that is uniform across regions[8] So, the national government budget constraint is

$$nt = \sum_{j \in C} c_j \tag{4}$$

where C is the set of projects funded with centralization.

We make the reasonable assumption that the delegate from region i must be drawn[9] from the (homogenous) population in that region, consistently with the citizen-candidate model (Besley and Coate(1997)). Combining this with (1) and (2), we see that the payoff to both any resident of region i and its delegate from any x^C is:

$$u_i^c = x_i^C b_i - \frac{1}{n}\sum_{j \in C} c_j + (c - x_i^C)e \tag{5}$$

where $c = \#C$. This indicates that with centralization, there are *two* spillovers at work; the first is the project spillover, captured by the term $(c - x_i^C)e$, and the second is the *cost-sharing* spillover, captured by the term $\frac{1}{n}\sum_{j \in C} c_j$. Thus a project in region j benefits i by the *net* spillover

$$e - c_j/n \tag{6}$$

Net spillovers play a crucial role in what follows.

The set C of projects is determined by voting in a legislature, as described in Section 4 below. There, our modelling strategy is to take as given not the outcome, but the *rules of operation* of the legislature governing agenda-setting and voting. A key prior question is whether there exist alternatives $x \in X$ which are Condorcet winners, and it is to this issue that we now turn.

3. When Do Condorcet Winners Exist?

An alternative[10] $x \in Y$ is a *Condorcet winner in* $Y \subseteq X$ if x cannot be defeated by any $y \in Y$ in a majority vote, if voters who are indifferent between x, y abstain. Our

[8] This is obviously in contrast to expenditure decisions, which are allowed to be non-uniform. Empirically, taxes levied by central government are uniform in the sense that rates do not vary by region; one reason for this convention may be to protect minority regions from expropriation.

[9] Of course, if voters in a region had differing preferences over projects, then the choice of delegate would be non-trivial, and some explicit modelling of the procedure for the selection of a delegate would be appropriate. This issue is pursued in Besley and Coate(1998).

[10] Formally, let the majority voting preference relation R over pairs (x, y) in X be defined by

$$xRy \Longleftrightarrow \#\{i \,|u_i^c(x) > u_i^c(y)\} \geq \#\{i\,|u_i^c(y) > u_i^c(x)\} \tag{7}$$

space of alternatives is multi-dimensional, and so one might conjecture that in general, no Condorcet winner (CW) will exist in X. In fact, in the special case of our model without externalities, it is well-known that under weak conditions, there is no[11] Condorcet winner in X (Ferejohn, Fiorina, and McKelvey(1987)). Our main finding in this section is that the Ferejohn-Fiorina-McKelvey result generalizes to the case of negative or weakly positive externalities, but that the case of strongly positive externalities is quite different, with a unique CW.

We begin by making three quite weak assumptions. The first is very weak; it simply says that each region derives a greater benefit from its project than the benefit it generates for any other region;

A0: $b_i > e$, $i \in N$

Now w.l.o.g, order the regions by increasing cost. The second assumption is that no two regions have the same cost i.e.

A1: $c_1 < c_2 < ... < c_n$.

Next, define $M = \{1, ..m\}$, with $m = (n+1)/2$, so M is the "minimum winning coalition" of regions with lowest costs. So, x^M is the policy that funds projects in these regions only. Define the status quo to be a situation with no project in any region, described by $0 \in X$. Our final assumption says that all $i \in M$ strictly prefer x^M to the *status quo* 0. Formally;

A2: $b_i - \frac{1}{n}\sum_{j \in M} c_j + (m-1)e > 0$, $i \in M$

Also, for neater statement of results, define a number $c_{n+1} = \infty$. Our fist result[12] on the existence of CWs is then the following.

Proposition 1. *Assume that A0-A2 hold. (i) If $e < c_1/n$, then there exists no Condorcet winner in X. However, x^M is the unique Condorcet winner in the set of those alternatives that are not beaten by the* status quo, $Y = \{x \in X \,|xR0\}$. *(ii) If $c_{k+1}/n > e \geq c_k/n$ for some $n \geq k \geq m$, then x^K, $K = \{1, 2, ..k\}$ is the unique Condorcet winner in X.*

Then $x \in Y$ is a *Condorcet winner in* $Y \subseteq X$ if xRy, all $y \in Y$. Also, define $xPy \iff xRy$ & $\neg yRx$. Note that if the Condorcet winner x is unique, then we must have xPy for all $y \in Y$.

[11] Ferejohn, Fiorina, and McKelvey(1987) also prove a positive result, namely that there is an $x^* \in X^n$ which beats all $y \in X^n$ that beat the status quo, and moreover, that this CW is the proposal that funds project in a bare majority of regions with the lowest costs. This result carries over to our model - see Proposition 1(i).

[12] This and all subsequent results are proved in the Appendix, when proof is required.

So, if externalities are negative or weakly positive ($e < c_1/n$), our result is a simple extension of Ferejohn, Fiorina, and McKelvey(1987). By contrast, however, if externalities are strongly positive and large enough ($e \geq c_m/n$), a CW exists. Moreover, this CW will typically involve funding projects in *more* than a bare majority of regions; indeed, if $e \geq c_n/n$, the CW funds projects in *all* regions (universalistic provision).

The intuition for these results is as follows. First, when externalities are negative or only weakly positive ($e < c_1/n$), then net spillovers from *all* projects are negative, as in the case without externalities, and the Ferejohn-Fiorina-McKelvey argument applies. That is, the proposal x^M that gives projects to the minimum winning coalition with lowest costs beats any proposal that gives projects either to more regions, or to a different set of m regions. But, nevertheless, x^M cannot be a CW, as it is beaten - for example - by a proposal that only gives a project to the $k < m-1$ lowest-cost regions. But, this last proposal imposes a negative net externality on a majority of regions, and so is then beaten by the *status quo*.

When externalities are strongly positive ($e \geq c_m/n$), then net spillovers from a majority of projects are positive. Specifically, if $c_{k+1}/n > e \geq c_k/n$ for some $n \geq k \geq m$, *every* region prefers x^K to some proposal that gives projects to fewer regions. Also, a majority of regions (i.e. all $i \in K$) prefer x^K to a proposal that gives projects to more regions, as the net spillover from any project in $j \notin K$ is negative. Consequently, x^K beats every other alternative.

We do not yet have a result in the intermediate case ($c_m/n > e \geq c_1/n$). Indeed, in this case, we may generally have no Condorcet winner, even relative to those alternatives that beat the *status quo*.

Example 1.

Assume $n = 3$, and $c_2/3 > e > c_1/3$. Not counting the status quo, there are seven subsets of N and so seven possible alternatives in X. However, any proposal that is not "least-cost" (in the sense that it provides a given number of projects at smallest total cost) cannot be a Condorcet winner. Specifically, $\{1,2\}$ is strictly preferred by 1 and 2 to $\{1,3\}$ and $\{2,3\}$. Also, $\{1\}$ is strictly preferred by 1 and 3 to $\{2\}$, and $\{1\}$ is strictly preferred by 1 and 2 to $\{3\}$. So, we only need consider $K = \{1\}, M = \{1,2\}$ and $N = \{1,2,3\}$.

As project 1 has a positive net spillover ($e > c_1/3$), and as from A0, $b_1 > e$, the payoffs from x^K are

$$u_1^K = b_1 - \frac{c_1}{3} > 0,\ u_i^K = e - \frac{c_1}{3} > 0,\ i = 2,3$$

So, $x^K \in Y$ i.e. it beats the *status quo*. Also, by A2, $i = 1, 2$ get a positive payoff from x^M:

$$u_i^M = b_i + e - \frac{(c_1 + c_2)}{3} > 0, \ i = 1, 2$$

so again, $x^M \in Y$ Also, note that $u_1^K - u_1^M = u_3^K - u_3^M = c_2/3 - e > 0$, so $x^K P x^M$ where "P" denotes strict preference under majority voting (see footnote 9). Finally, payoffs from X^N are

$$u_i^N = b_i + 2e - \frac{(c_1 + c_2 + c_3)}{3}, \ i = 1, 2, 3$$

Also, note that $u_1^M - u_1^N = u_2^M - u_2^N = c_3/3 - e > 0$, so $x^M P x^N$. Now assume that

$$b_2, b_3 > \frac{(c_1 + c_2)}{3} - e \tag{8}$$

Then, from (8), we see that

$$u_i^N = b_i + 2e - \frac{(c_1 + c_2 + c_3)}{3} > e - \frac{c_1}{3} = u_i^K > 0, \ i = 2, 3$$

So, we conclude that $x^N \in Y$ and that $x^N P x^K$ So, we have a cycle $x^K P x^M P x^N P x^K$, where each alternative in the cycle beats the *status quo*. We conclude that there exists no CW in Y, as claimed. ‖

Intuitively, a voting cycle arises as there is a conflict of preferences; only project 1 has positive net spillover, so on externality grounds, delegates all prefer just this one project to be funded, but projects 2 and 3 have high benefits for the regions concerned, so a majority prefer all projects to be funded.

The example also makes clear however, that the only way that this cycle can be avoided is by making *either* b_2 or b_3 less than $(c_1 + c_2)/3 - e$. For then, two out of three delegates would then prefer x^K to x^N, and the cycle would be broken, making x^K the CW. The following assumption extends this reasoning to the general case;

A3:(Cycle-Breaker) Suppose that $c_{k+1}/n > e \geq c_k/n$ for some $k < m$. Let $K = \{1, ..k\}$ and $L \subset N$ with $l = \#L$. If $l > n + k - m$, then for some $S \subset L/K$ with $\#S = m - k$, all $i \in S$ prefer x^K to x^L i.e. $b_i + (l-1)e - \frac{1}{n}\sum_{i \in L} c_j < ke - \frac{1}{n}\sum_{i \in K} c_j$, $i \in S$.

Assumption A3 ensures[13] that benefits are not so high so that a majority of regions prefer (say) all projects to be funded in preference to the set K. It is easily checked[14]

[13]It remains to check that A3 is consistent with A2. It is easy to check that if (9) holds for $i = 2$, it would violate A2. But A2 does not place any restriction on b_3, so we can always choose b_3 so that (9) holds.

[14]Note that in the example, $k = 1, m = 2$, and $n = 3$, so the only relevant set L is $L = N$ ($l > n + k - m$

that in the Example above, this cycle-breaking assumption reduces to the requirement that either b_2 or b_3 less than $(c_1 + c_2)/3 - e$.

Given A3, we can now show that a Condorcet winner emerges even in for intermediate-value externalities;

Proposition 2. *If $c_{k+1}/n > e \geq c_k/n$ for some $1 \leq k < m$, and in addition A0-A3 holds, then x^K, $K = \{1, 2, ..k\}$ is the unique Condorcet winner in X.*

So, given an additional assumption A3, we have a unique CW, but now projects are only funded in a minority of regions (and possibly only one!).

4. Legislative Rules and Endogenous Agenda Equilibrium

Propositions 1 and 2 above make it clear that even with assumptions A0-A3 imposed, unrestricted majority voting over alternatives in X in the legislature will lead to voting cycles unless externalities are positive and large enough. So, in order to ensure a determinate outcome in general, we need to specify some minimal rules of procedure for the legislature. Rules of procedure specify how proposals get on the agenda, what amendments (if any) may be put against them, and when voting takes place.

From Proposition 1, it is clear that voting cycles can be avoided if the rules eliminate alternatives that do not defeat the status quo. As shown by Ferejohn, Fiorina, and McKelvey(1987), it turns out that some quite unrestrictive rules will do this: the key is that the *status quo* must be privileged, in the sense that any motion is only passed if it defeats the *status quo* in a final round of voting. This rule is one that is used in the US Congress (Ordeshook(1986)). In this section, we study a three-stage legislative procedure that privileges the status quo in this way.

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implies $l > 3 - 1 = 2$, implying $l = 3$). Also, if $S \subset L/K$ and $\#S = m - k$, then we must have $S \subset \{2, 3\}$ and $\#S = 1$ implying $S = \{2\}$ or $S = \{3\}$. So, A3 requires that

$$b_i + 2e - \frac{(c_1 + c_2 + c_3)}{3} < e - \frac{c_1}{3}, \; i = 2 \text{ or } 3 \tag{9}$$

which of course is equivalent to the converse of (8) for $i = 2$ or $i = 3$.

1. **Proposals**

Any delegate i can propose any motion $a^i \in X$ as an alternative to the status quo.

2. **Agenda Formation**

All the motions made by the delegates are incorporated into an *agenda*. Motions proposed by delegates $i = 1,..n$ are put on the agenda in a random order, with the final item on the agenda being the status quo. Formally, a permutation function $\pi : N \rightarrow N$ is selected randomly from Π, the set of all such functions, with probability[15] $p_\pi > 0$. Given π, an agenda is an $n+1-$tuple $y = (y^1, y^2, y^3..y^n, 0)$, where $y^i = a^{\pi(i)}$

3. **Voting**

Voting on the agenda is as follows. The first and second motions y^1, y^2 are voted on, the winner is paired with y^3, and so on, until finally the winner after $n-1$ rounds of pairwise voting (the amended motion) is paired with the *status quo*, 0, and there is a final vote for the amended motion against the *status quo*. [If the motion on the floor and the newest amendment get equal numbers of votes, the tie-breaking rule selects the motion on the floor.]

This procedure is rather general in two senses. First, we allow for *endogenous* formation of agendas. Second, the structure of the agenda is very general; the only restriction is that the items on the agenda are compared pairwise (the agenda is binary[16]), and the last item is the *status quo*.

Steps 1-3 above describe an extensive-form game played by the delegates. We suppose that delegates have Von-Neumann-Morgenstern preferences over risky outcomes, and we place the following weak restrictions on strategies: (i) indifferent voters abstain at all decision nodes in the voting subgame; (ii) weakly dominated strategies are not played in the voting subgame. Call any subgame-perfect equilibrium of the above game that satisfies (i) and (ii) an *endogenous agenda equilibrium*[17].

Building on results by Fiorina, Ferejohn, and McKelvey, we can show that given assumptions A0-A3, although the endogenous agenda equilibrium is not unique, there is a unique equilibrium outcome, *independent* of the ordering of the proposals π. Specifically, let

$$C = \begin{cases} M & \text{if } e < c_1/n \\ K = \{1,..k\} & \text{if } c_{k+1}/n > e \geq c_k/n,\ k \in N \end{cases}$$

[15] These probabilities need not be equal.

[16] An agenda is binary if at every stage, voters vote between two alternatives, alternatives being subsets of the space of alternatives.

[17] Both the equilibrium and the equilibrium outcome are defined formally in the Appendix.

Proposition 3. *If A0-A3 hold, in any endogenous agenda equilibrium, at least one $i \in C$ proposes the motion x^C. Consequently, whatever $\pi \in \Pi$, the unique endogenous agenda equilibrium outcome is x^C*

This result is essentially a generalization of Ferejohn, Fiorina, and McKelvey(1987), in a setting which allows for endogenous agenda formation, as in McKelvey(1986).

Proposition 3 has the following striking implications. First, the set of projects undertaken in equilibrium is *independent* of the local benefits b_i of the projects (subject to A2 and A3 being satisfied). This makes precise the idea, expressed in Oates(1972), that centralization means that decisions are less responsive to regional preferences.

Second, the proportion of regions obtaining projects, $\lambda = c/n$ depends on the size of the spillover e, as shown in Figure 2.

Figure 2 in here.

When e is positive and large enough, we clearly have universal provision of projects, whereas e is small or negative, we have only provision to a majority. So, although formally, voting in the legislature is by majority vote (what Inman and Rubinfeld(1997a) call the "minimum winning coalition legislature"), the *outcome* may be similar to a legislature where there is implicit agreement to provide universal provision, as in Weingast(1979) and Niou and Ordeshook(1985). However, in our setting, this arises not though implicit cooperation, but through the fact that legislative rules allow for (partial) internalization of externalities.

Note finally that the proportion of projects funded, λ, is *not* monotonic in the size of the externality; when the spillover is of intermediate size, (i.e. in the range $[c_1/n, c_m/n)$), λ actually falls. As remarked above, the intuition is that with intermediate externalities, *all* regions may prefer the funding of projects in a few very low-cost regions to the *status quo*, whereas when externalities are very low (or zero) the *status quo* can only be defeated by a "minimum winning coalition"

5. When is Decentralisation Efficient?

Now that we have characterized the outcome of the political process with centralization, we are in a position to assess the relative efficiency of decentralization. As utility is linear in income, the model is one of transferable utility, and so the natural measure of efficiency is the aggregate surplus, or sum of utilities. If the aggregate surplus is greater under decentralization, then decentralization is unambiguously potentially Pareto-preferred[18] The informal literature of the 1970s, and more formal models based on this literature, suggest that decentralization is more efficient in this sense if (i) inter-regional externalities are small; (ii) regions are relatively heterogenous. For example, on (ii), Oates(1970, p37) says: "the welfare gain from the decentralized provision of particular local public good becomes greater as the diversity of individual demands within the country as a whole increases."

In this section, we investigate whether these results carry over to our model. It is not obvious that this should be so, as here the cost of centralization is not policy uniformity, but rather insensitivity of decision-making to project benefits. We find that while conditions can be found under which both statements are true, there are some important qualifications, especially in the case of heterogeneity.

A useful first step is to calculate the gain from decentralization for a single region. From (3),(4), this can be written;

$$\begin{aligned} & u_i^d - u_i^c \\ = & [\max\{b_i - c_i, 0\} - x_i^C(b_i - c_i)] + [\frac{1}{n}\sum_{j\in C} c_j - x_i^C c_i] + [(d - x_i^D) - (c - x_i^C)] \end{aligned} \tag{10}$$

The three terms in (10) illustrate the gains from decentralization for each region in an illuminating way. The term in the first square bracket reflects the efficiency gain, due to additional responsiveness to regional project benefits, that comes with decentralized provision and is always non-negative. Second, the term in the second square bracket is the share of aggregate cost borne by i, minus the true economic cost of i's project, under centralization. This term captures the *distributional* impact of moving to decentralized funding taking as given the set of projects that are funded. The term in the third square bracket measures additional spillovers accruing to i that arise with decentralization. Decentralization is inefficient here in the sense that project externalities are not internalized at all (d does not vary with e). Centralization may be more efficient as project externalities are *partially internalized* through the legislative process (from Proposition 3, c is increasing in e, except over a range).

Now, from (10), summing over all regions, we see[19] that the efficiency gain from decen-

[18] Of course, the Kaldor-Hicks criterion is only of interest here if lump-sum transfers between regions are possible at the point where the choice between centralization and decentralization is made. In the next section, we investigate under what conditions (de)centralization is Pareto-preferred without lump-sum transfers i.e. unanimously preferred.

[19] Note that in the aggregate, the distributional gains and losses in (10) from cost-pooling net out.

tralization is

$$W^d - W^c = \sum_{i \in N} [\max\{b_i - c_i, 0\} - x_i^C (b_i - c_i)] + (n-1)(d-c)e \tag{11}$$

The first term in (11) captures the fact that decentralization is always more responsive to regional preferences, and is always non-negative. The second term may be positive or negative. Note that the distributional gains and losses from decentralization sum to zero; that is, what determines the sign of $W^d - W^c$ is simply the set of projects that are funded in each case, and not how they are funded.

We first turn to the question of the whether decentralization is more efficient when spillovers are small. We can prove the following;

Proposition 4. *Assume that A0-A3 hold. If there are no spillovers ($e = 0$), then decentralization is more efficient ($W^d \geq W^c$) and strictly so unless $d = m$. If spillovers are large enough ($e \geq c_n/n$), then centralization is more efficient ($W^d \leq W^c$) and strictly so unless $d = n$.*

One might conjecture from this result that the gain to centralization would be *everywhere* non-decreasing in e. In fact, this is not the case, and is related to the non-monotonicity of the number of projects in e discussed above. The following example makes this point.

Example 2.

The example has three regions. Assumptions A0-A3 are assumed to hold, and it is assumed that $D = \{1\}$. Also, suppose initially $e < c_1/3$, so from Proposition 1, $C = \{1, 2\}$. Then

$$W^c - W^d = b_2 - c_2 + 2e$$

As $D = \{1\}$, $b_2 - c_2 = -\varepsilon < 0$. Let $2e > \varepsilon$; then $W^c > W^d$ i.e. centralization is strictly more efficient. Now let e increase to e', with $c_1/3 \leq e < c_2/3$. Then, if A3 is satisfied, $C = \{1\}$, so now $W^c = W^d$ But b_3 can always be chosen to satisfy A3, as the discussion following Example 1 makes clear. So, in this example, $W^c - W^d$ is *not* everywhere non-decreasing in e.‖

We now turn to investigate whether decentralization becomes more desirable as regional characteristics become more heterogenous. The first issue is how to measure heterogeneity. As regions differ in cost and benefit characteristics, a first sight a natural definition of increased heterogeneity might be a mean-preserving spread (MPS) in *either* the distribution of benefits, *or* costs, or both, across regions. However, a moment's reflection indicates that it is heterogeneity of the *net* project benefits, $\nu_i = b_i - c_i$ that is important in Oates's argument cited above; for if all regions have the same net benefit, there is no efficiency loss from policy uniformity, no matter how the gross benefits, or the costs, or projects vary across regions. Indeed, if we measure heterogeneity in net benefits, we can obtain a result, albeit under some stringent conditions. We will assume:

A4: $\{v_k\}_{k \in N}$ is symmetrically distributed around zero.

Also, define a *symmetric* MPS of $\{v_k\}_{k \in N}$ to be an MPS of this distribution that results in a symmetric distribution with mean zero. We will of course, only consider the class of symmetric MPSs such that assumptions A0-A3 are satisfied both before and after the change[20] Then we have;

Proposition 5. *Assume that A0-A4 hold, and that either (i) $e < 0$ or (ii) costs c_i remain fixed. Then the efficiency gain from decentralization, $W^d - W^c$, does not fall following a symmetric MPS in the distribution of the net project benefits $\{v_k\}_{k \in N}$.*

The intuition behind this result is as follows. Assumption A4, plus the construction of the MPS, implies that net benefits do not change sign following the MPS; they rise (fall) only in regions where they were initially positive (negative). So, the set of projects funded under decentralization, D, is unchanged following the MPS. Also, (i) or (ii) implies that set of projects funded under centralization, C, is unchanged. Finally, the fact that net benefits rise (fall) only in regions where they were initially positive (negative) implies that the gain in "responsiveness" i.e. the first term in (11) cannot fall - and will usually rise.

Perhaps the most restrictive condition in Proposition 5 above is that net benefits are symmetrically distributed with mean zero. However, both these assumptions are necessary, in that it is possible to find counter-examples to Proposition 5 when either assumption is relaxed.

Example 3.

Suppose that there are three regions with $\nu_1 = \nu - \delta$, $\nu_2 = \nu$, $\nu_3 = \nu + \delta$, (so that net benefits are symmetrically distributed, but with positive mean) and $\nu - \delta > 0$ initially, and that $e > c_3/3$. So, it is efficient to fund all three projects. That is also initially the outcome under centralization; $C = \{1, 2, 3\}$ as $e > c_3/3$, from Proposition 3. It is also the outcome under decentralization, as $\nu_i > 0$, $i = 1, 2, 3$. Now increase δ (this is a symmetric MPS), so

[20] The main requirement is from A2 that $\nu_i > \frac{m}{n} \sum_{j \in M} c_j - c_i - (m-1)e = \underline{\nu}_i$, but as long as $\underline{\nu}_i < 0$, $i = 1, ..m-1$, A2 is consistent with A4.

that $-2e < v - \delta < 0$, and suppose that this change takes place though changes in benefits. Then, project 1 is no longer funded under decentralization, although it is still efficient (as $v_1 + 2e > 0$). As neither costs, nor the size of the spillover, e, have changed, centralization is just as efficient as before. So, now decentralization is less efficient than centralization.

Now modify the example so that assumption so that $\nu_1 < \nu_2 = 0 < \nu_3$, so that net benefits have mean zero, but are no longer necessarily symmetrically distributed. Suppose that $v_1 + 2e > 0$ so it is efficient to fund all projects. Initially, the set of projects funded under decentralization is $D = \{2, 3\}$. Now consider a (non-symmetric) MPS with ν_2 changing to $-\delta$, with $\delta < 2e$ and ν_3 changing to $\nu_3 + \delta$, with the change taking place through changes in benefits. Then following the MPS, only the project in region 3 is funded, but it is still efficient to fund project 2 (as $-\delta + 2e > 0$). So decentralization becomes less efficient. But by the previous argument, centralization is just as efficient as before. ||

These examples indicate that Proposition 5 is unlikely to generalize significantly. So, the belief that "increased heterogeneity" leads to increased relative efficiency of decentralization is not generally confirmed by this model. The underlying reason is that in our model, the cost of centralization is not policy uniformity, but lack of responsiveness of decision-making to project benefits.

6. Constitutional Design

At some initial constitutional design stage, regions choose between centralization and decentralization. In practice, constitutional (re)design occurs through the political process, via what Buchanan calls constitutional rules. Depending on the nature of the constitution, reallocation of tax and spending powers may be decided upon by ordinary legislation in a national parliament, or may[21] require formal constitutional amendment, which may in turn, require referenda. In unitary states, such referenda may be only national, such as the referendum in the UK to decide on membership of the EU. However, in truly federal states, constitutional amendment always requires, in some way or other, approval of a (super)majority the constituent states or regions[22]

In this model, as all voters in a given region are identical, and all regions have identical

[21] Constitutional amendments are used routinely in Switzerland, and less frequently in the US, Canada and Australia, to reallocate tax and spending powers (Wheare(1963)).

[22] Constitutional amendments in Australia and Switzerland require majority approval of the population as a whole, and also majorities is all the regions (cantons), but in the US, approval of a supermajority (3/4) of the states is required (Wheare(1963)).

populations, constitutional rules of this type reduce to a simple regional referendum: regions (or their delegates) vote on the *status quo* versus the alternative, and the *status quo* is selected unless a proportion[23] of at least α of regions prefer the alternative. We focus on two special cases; ordinary majority rule ($\alpha = 0.5$), and unanimity rule ($\alpha = 1$). To avoid tedious discussion of "non-generic" cases, we assume that;

A5: $d \neq m \neq n$, $b_i \neq c_i$, $i \in N$

i.e. that the set of projects funded under decentralization is never m or n, and that no region is indifferent about their project. We can now move to an analysis of the two cases. We focus on the extent to which Proposition 4 above extends to these two alternative decision rules[24]

6.1. Unanimity Rule

Proposition 4 above shows that when the spillover is zero, decentralization is strictly more efficient than centralization, but when it is large and positive, the reverse is the case. One might conjecture that there must be some way of choosing the remaining parameters (the b_i and c_i) so that *all* agents can share in the relevant efficiency gain i.e. so that decentralization is *unanimously* preferred when the spillover is zero, and centralization is *unanimously* preferred when it is large and positive. Surprisingly, it turns out that only half of this conjecture is true.

Say that the regions are ε*-homogenous* if there exists a number ε such that

$$\left|b_i - \overline{b}\right| < \varepsilon,\ \left|c_i - \overline{c}\right| < \varepsilon, \text{ all } i \in N.$$

where $\overline{b} = \frac{1}{n}\sum_{i \in N} b_i$, and $\overline{c} = \frac{1}{n}\sum_{i \in N} c_i$ are average benefits and costs. We assume that $\overline{b} \neq \overline{c}$ i.e. average net benefit from the project is not zero. Note that this definition of homogeneity is consistent with A1 above. We then have;

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[23]In the event of a tie, we assume that the status quo is selected, which we take w.l.o.g. to be decentralization.

[24]Of course, to the extent that constitutional revision is costly or infrequent, regions will take an *ex ante* view of project costs and benefits, and so from this perspective, regions will be more homogenous than at the stage when projects are actually chosen. In the extreme case, one can imagine all regions are ex ante identical, in which case (assuming that behind the veil of ignorance, agents evaluate lotteries according to expected utility criterion, Harsanyi(1953)), agents will simply choose the alternative that maximises the expected value, or equivalently the sum, of utilities. In this case, every region would choose decentralization iff $W^d \geq W^c$ under both unanimity and majority rules, in which case Proposition 4 would continue to apply unchanged.

Proposition 6. *Assume A0-A3 and A5 hold. If externalities are strongly positive ($e > c_n/n$), then, there exists an $\delta > 0$ such that if the regions are $\varepsilon-$homogenous, with $\delta > \varepsilon$, then $u_i^c > u_i^d$, $i \in N$. But, even if $e = 0$, then $u_i^c > u_i^d$, some i.*

Note first the striking result that even if there are no spillovers, *some* region will strictly gain from centralization, so the choice of decentralization can never be unanimous. This is because the gain though cost-pooling will always benefit some high-cost region. Second, we see that with sufficient homogeneity across regions, and strongly positive externalities, centralization is Pareto-preferred. In fact the combination of strongly positive externalities, plus homogeneity, means that centralization chooses the efficient set of projects (i.e. N projects in all regions).

6.2. Majority Rule

With majority rule, (de)centralization is selected if (of the regions that are not indifferent) a majority strictly prefer (de)centralization. In this case, it is possible to find conditions, on the distribution of costs only[25], sufficient for decentralization to be chosen when project externalities are zero, and for centralization to be chosen when externalities are large. Say that the costs are *$\varepsilon-$homogenous* if there exists a number ε such that

$$|c_i - \overline{c}| < \varepsilon, \text{ all } i \in N.$$

where $\overline{c} = \frac{1}{n}\sum_{i \in N} c$. Also, let β_m be the median benefit in the distribution of benefits across regions. We have;

Proposition 7. *Assume A0-A4 hold. If $e = 0$, and costs are sufficiently heterogenous ($c_1 < \frac{1}{n}\sum_{j=1}^{m} c_j$) then majority rule selects decentralization. If $e \geq c_n/n$, $\beta_m > \overline{c} - (m-1)e$, and there is a $\delta > 0$ such if costs are $\varepsilon-$homogenous, with $\delta > \varepsilon$, then majority rule selects centralization.*

For the case of large positive externalities, this result can be contrasted with Proposition 6: whereas we needed homogeneity in *both* costs and benefits to get a result about unanimous preference, we need only homogeneity in costs and a weak condition on the median benefit to get a result about majority preference.

[25] Plus a weak lower bound on the median benefit.

7. Some Extensions

7.1. Vote Trading

It is often asserted that legislators have an opportunity for "vote trading"; that is, an agreement between two or more legislators for mutual support, even though it requires each to vote contrary to his real preferences on some legislation (Ordeshook(1986)). A standard way of modelling vote-trading is to suppose that legislators can form coalitions to coordinate their strategies. Associated with any coalition S is a characteristic function i.e. a set of feasible utility vectors for that coalition. In our model (given the agenda-setting and voting procedure 1-3 described in Section 4 above), the set of feasible utility vectors for S is defined as the set that S can guarantee themselves by coordinating their agenda-setting and voting behavior. Then, given the characteristic function, the core of the voting game can be defined, and a point in the core (if the core is non-empty) is an equilibrium with vote-trading.

Here, the characteristic function $v(.)$ takes a very simple form. If some set S of voters has $\#S \geq m$, then this coalition S can propose and vote though *any* $x \in X$. So, in this case, the members of S can guarantee themselves any feasible payoff;

$$v(S) = \{(v_i)_{i \in S} | v_i \leq u_i = u_i^c(x),\ some\ x \in X,\ all\ i \in S\}$$

If on the other hand, $\#S < m$, then member i of S can guarantee only $\underline{u}_i = \min_{x \in X} u_i^c(x)$, so in this case

$$v(S) = \{(v_i)_{i \in S} | v_i \leq \underline{u}_i,\ all\ i \in S\}$$

Say that x^* is an *equilibrium with vote-trading* if no coalition of delegates can form, and by co-ordinating their votes, and vote in a better alternative i.e. there does not exist a coalition S and a $w \in v(S)$ such that $w_i > u_i^c(x^*)$, $i \in S$. Note that the set of equilibria with vote-trading comprises the set of core allocations..

The above game is simple majority-rule voting game. In such games it is well-known (Ordeshook(1986)) that any Condorcet winner is in the set of core allocations. In fact, we can go further, using the special structure of our model, and prove that if the Condorcet winner x^* is unique, then so is the equilibrium with vote-trading. Formally;

Proposition 8. *Assume that A0-A3 hold. If $c_{k+1}/n > e > c_k/n$ for some $k \in N$, then x^K; $K = \{1, 2, ..k\}$ is the unique equilibrium outcome with vote-trading.*

So, in the event that externalities are sufficiently positive, there is a unique equilibrium with vote-trading, which coincides with the outcome of the voting game studied above. So, this proposition has a striking implication that if $c_{k+1}/n > e > c_k/n$ for some $k \in N$, the outcome with vote-trading is *exactly the same* as with no coordination between legislators. Specifically, coordination does not allow legislators to incorporate the benefits of projects into the political decision-making process. So, Propositions 4-7 of the previous section, concerning the relative efficiency of (de)centralization, continue to hold.

7.2. Alternative Models of Legislative Behavior

We have focussed on the legislative model of Fiorina, Ferejohn and McKelvey(1987), which can be characterized as a two-stage process; first, a (binary) agenda is formed, and then voting takes place. The other leading model of legislative behavior is the Baron and Ferejohn(1989) model of legislative bargaining, which has been applied to public finance issues by Baron(1989), Besley and Coate(1998), and Persson (1998). There are two problems with using the Baron/Ferejohn model in this context. First, the infinite-horizon model is analytically complex when regions are heterogenous[26], and perhaps for this reason, Besley and Coate(1998) and Persson (1998) both use a "one-shot" version of the model, where each legislator is chosen with probability $1/n$ to make a proposal which is then voted on in a pairwise comparison with the *status quo*, after which the game ends. This is both restrictive and unrealistic, as it does not allow other legislators to make amendments to the initial proposal.

A second problem with the Baron/Ferejohn model is that it is possible that even when a Condorcet winner exists, alternatives other than the CW alternative will be chosen in equilibrium. The reason is that (in the "one-shot" closed-rule version of the Baron/Ferejohn model) the legislator who is selected to make a proposal then chooses her proposal to maximise her payoff, subject to the constraint that at least $m-1$ other legislators also prefer that proposal to the *status quo*, and the solution to this constrained maximization problem need not be a CW. In particular, the proposer may wish to grant herself a project, even though a majority of other delegates may prefer the proposer *not* to have a project. The following example illustrates this point.

Example 4.

The example has three regions. Assumptions A0-A2 are assumed to hold. Suppose

[26] Baron and Ferejohn(1989) make heavy use of the assumption of identical agents in characterising the (subgame-perfect) equilibrium of the model.

that $c_2/3 \leq e < c_3/3$, so that the CW is $x^C = (1,1,0)$. Now suppose that 3 is chosen as proposer. Let F be the set of projects he decides to fund[27] Let $b_3 > c_3/3$; then, he will always prefer to fund his own project than not, even though this makes the other two regions worse off, as $c_3/3 > e$. So, 3 chooses between $F = \{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}$, subject to the constraint that *one* other delegate must prefer F to the *status quo*. It is easily checked that $F = \{1,2,3\}$ is the solution to this problem as long as $b_1, b_2 > c_3/3$ also.||

In general, however it is possible to show that this divergence between the CW outcome and the Baron/Ferejohn equilibrium outcome is negligible when n is large. The reason is that a region not in C can only enforce its will on the others when that region's delegate is proposer, which occurs with probability $1/n$. In fact, we have:

Proposition 9. *Assume that $e \geq 0$ and $b_i \geq c_i/n$, $i \in N$. Then, in the equilibrium of the Baron/Ferejohn model, any $i \in C$ receives a project with probability 1. Moreover, if $e \geq c_m/n$ or $e < c_1/n$, and any $i \notin C$ receives a project with probability $1/n$.*

This result says that for large n, the "one-shot" version of the Baron/Ferejohn model gives us an outcome that approximates (in terms of expected payoffs) the outcome of the model presented in Section 4 above, except for the parameter range $c_m/n > e \geq c_1/n$ - which itself becomes negligible as n becomes large. So, with this qualification, Propositions 4,5,6 will carry over to this alternative model.

7.3. Partial Decentralization

We have compared two polar cases of the possible allocation of powers, full decentralization and full centralization. However, as mentioned above, there are two intermediate alternatives which are worthy of mention. The first, and the empirically more common case, is where expenditure decisions are decentralized, but are financed by a national tax. In this case, the perceived cost of a project for region i is c_i/n, so the project will be selected if $b_i \geq c_i/n$. So, in this case, the cost spillover, or "common pool" problem leads to overprovision of projects, and the outcome is always less efficient than with full decentralization.

The other case is where expenditure decisions are centralized, but are financed by regional taxes. In this case, there is no cost-sharing. Without externalities, all regions

[27] In the original Baron/Ferejohn model, proposers can also make side-payments to regions. Howoever, Besley and Coate(1998) use a variant of the Baron/Ferejohn model similar to this one, where side-payments cannot be made.

$j \neq i$ will be indifferent about i's project, and so the outcome under full decentralization, x^D, will be a Condorcet winner. Consequently, when $e = 0$, the outcome is equivalent to full decentralization. If $e > 0$, on the other hand, all $j \neq i$ strictly prefer $x_i = 1$, so the alternative where all projects are funded ($x = (1, ..1)$) is the unique Condorcet winner. This is of course the uniform outcome that some have associated with decentralization studied by Oates(1972). Under some conditions, this outcome may be more efficient than full centralization. However, in general, the outcome is insensitive not only to regional benefits (as is full centralization), but also to regional costs (unlike full centralization). Consequently, there can be no presumption that this form of partial centralization is generally more efficient than full centralization.

8. Conclusions and Related Literature

This paper has presented a model where the relative merits of centralization and decentralization, and the performance of various constitutional rules for choosing between the two, can be evaluated. One key feature of the paper is that (in the centralized case), we present a fully explicit model of a national legislature, where legislative rules, rather than behavior, are taken as primitive. This model is a generalization of the well-known model of distributive policy to the case of inter-regional externalities. An important finding is that the uniformity of provision is *endogenously* determined by the strength of the externality. When externalities are large and positive, an outcome closer to universalistic provision, rather than just a bare majority of funded projects, will occur. Second, there is likely to be greater consensus on the merits of the equilibrium set of projects when externalities are large i.e. a Condorcet winner may emerge. Moreover, this characterization of the behavior of the legislature is robust to the introduction of logrolling, and of different specifications of the legislative rules.

This model allows to investigate in detail both the relative efficiency of centralization and decentralization, and of the performance of various constitutional rules for choosing between them. To some extent, our analysis confirms Oates' insights that decentralization is more efficient when externalities are small and/or regions are heterogenous, and centralization to be preferred under the reverse conditions. However, the conditions required for increased heterogeneity to imply increased efficiency of decentralization are strong, essentially because the cost of centralization is not policy uniformity, but inefficient choice of projects due to cost-sharing and lack of responsiveness of the legislative process to benefits.

There are also some intriguing findings, which emerge due to the *interaction* of the strength of the externality and legislative behavior. For example, while centralization may be welfare-superior to decentralization when externalities are very large, over some range an increase in the strength of the externality may make *decentralization* more attractive. Second, sufficient conditions for a majority of the population to prefer centralization (or decentralization) can be formulated only with reference to the heterogeneity of costs, not benefits.

Some related literature has already been mentioned in the introduction. Here, we discuss in more detail the two papers that are most closely related to this one. Ellingsen's paper does provide an explicit model of political decision-making with centralization. However, his model has only two types of agent, one of which is more numerous than the other, and direct, rather than representative, democracy. So, with centralization, the more numerous type is effectively a dictator. Moreover, expenditure is on a pure (national) public good, so the strength of inter-regional externalities cannot be varied. (Ellingsen does discuss informally an extension to the case where goods produced by the two jurisdictions are not perfect substitutes, but does not present any results.) However, his results in Section 3.2 of his paper (which are comparable to this paper as they assume homogenous regions) have some of the flavor of Propositions 4-6 above.

The work much the closest to this one is the independent work of Besley and Coate(1998), which addresses the same issue - the choice between centralized and decentralized provision of regional public goods - in a political economy model. However, this paper and theirs are really complementary in the way that they view centralization. First, Besley and Coate(1998) focus on the role of strategic voting for delegates to the legislature. Specifically, in their model, populations in regions are heterogenous, and any citizen may stand a candidate for election. So, voting in a delegate with a strong preference for public spending is a precommitment mechanism that allows that region to capture more of the available tax revenue for its own projects. This is a source of inefficiency with centralized provision. We abstract from this important issue in our model, by assuming that the population within any region is homogenous.

The second key difference is that Besley and Coate do not model all the rules of operation of the legislature explicitly. Specifically, they assume that each of the two delegates to the legislature (there are only two regions in their model) is selected with equal probability to be agenda-setter, and then the agenda-setter maximises the sum of his own payoff and the weighted payoff of the other delegate, where the weight μ is exogenously fixed at some value between zero and one[28] By contrast, in this paper, we study a model where all the rules of operation of the legislature are explicit (and quite general). This really makes a difference; one of the key insights of our model is that the degree to which policy is universalistic rather than majoritarian (i.e. the proportion of regions that get projects) depends crucially on the level of the project externality; the higher this is, the closer provision is to universalistic. This suggests that the comparative static exercises of Besley and Coate, where the size of the externality and the weight μ are varied independently. may not be consistent with a "micro-founded" model of the legislature[29]

Perhaps because we do not model the possibility of strategic voting or delegates, (and because projects are discrete rather than continuous), our model is also more general in some other important respects, while remaining analytically tractable. We have an arbitrary number of regions (where Besley and Coate have two), and can obtain analytical results for the case where regions differ in both project benefits and costs (in Besley and Coate, the two regions have the same costs, and most analytical results are obtained only for the case where the two regions also have the same benefits). So, they are not really ale to look at issues of heterogeneity.

[28]This weight is a proxy for the outcome of a dynamic model of legislative bargaining, where implicit cooperation is possible.

[29]This key difference is reflected also in the results. For example, Besley and Coate find that the gain from centralisation is monotonically increasing in the size of the project externality (Proposition 2(i)), whereas from Example 2 above, we do not.

9. References

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Appendix

A.1. Endogenous Agenda Equilibrium

Here, we define formally and characterise the endogenous agenda equilibrium. First, it is possible to obtain the following characterization of the voting subgame:

Lemma A1 *Under restrictions (i) and (ii) in the text, the subgame-perfect equilibrium outcome of the voting subgame is unique, and equal to y_1^*, where y_i^* is recursively defined*[30] *as follows, with $y_{n+1}^* = 0$:*

$$y_i^* = \begin{cases} y_i & \text{if } y_i R y_j^*, \forall j > i \\ y_{i+1}^* & \text{otherwise} \end{cases} \quad \text{for } i = 1, ..n-1 \tag{A.1}$$

Proof of Lemma A1

Given the restrictions on strategies stated in the Lemma, and the fact that at any node, there only two alternatives, it is easy to check that (given unique continuation payoffs), the unique equilibrium strategy for a non-indifferent voter is to vote sincerely i.e. for his most preferred alternative. Also, in the event of a tie, the tie-breaking rule gives a unique outcome. It now follows by a backward induction argument that there exists a unique SPE in this voting subgame. Moreover, the outcome must be described as in the Lemma, by backward induction. ■

Thus, conditional on y, this subgame generates a unique outcome $y_1^* = z(y)$. Let $y(a, \pi)$ be the unique map from a vector of motions $a = (a_1, .a_n)$ to an agenda y given a permutation π. So, define the composition

$$y_1^* = z(y(a, \pi)) = \sigma(a, \pi)$$

Then, conditional on $(a_1, .a_n)$ we can write the expected utility of delegate i over possible agendas as

$$v_i(a_1, ..a_n) = \sum_{\pi \in \Pi} p_\pi u_i^c(\sigma(a_1, ..a_n, \pi)) \tag{A.2}$$

We can now formally define:

[30] In the voting literature, y_i^* is known as the *sophisticated equivalent* of y_i. So, (A.1) says that if y_i cannot be beaten by all the sophisticated equivalents of proposals further down the agenda (including the status quo), y_i is its own sophisticated equivalent.

Definition. An *endogenous agenda equilibrium* is an n-tuple $(a_1^*, ...a_n^*)$, such that $v_i(a_i^*, ..a_{-i}^*) \geq v_i(a_i', ..a_{-i}^*)$, all $a_i' \in A$.

Definition. An $x^* \in X$ is an *outcome of an endogenous agenda equilibrium conditional on* π if $x^* = \sigma(a_1^*, ...a_n^*, \pi)$, where $(a_1^*, ...a_n^*)$ is an endogenous agenda equilibrium.

A.2. Proofs of Propositions

Proof of Proposition 1

(i) For any $K \subset N$, define $v(K) = ke - \frac{1}{n}\sum_{j\in K} c_j$, and $w(j) = ej - \frac{1}{n}\sum_{i=1}^{j} c_i$. So, $v(K) \leq w(k)$, with equality iff $K = \{1, ..k\}$. Consequently, if

$$\frac{c_{j+1}}{n} > e \geq \frac{c_j}{n} \Longleftrightarrow w(j) \geq w(l),\ l \neq j$$

with $w(j) > w(l)$ unless $e = c_j/n$, in which case $w(j) = w(j-1)$. Also, note that if $l > k > j$, or $l < k < j$, then $w(k) > w(l)$. These two properties say that $w(l)$ is quasi-concave in l with a maximizer of j (which is unique unless $e = c_j/n$, in which case $j - 1$ is also a maximizer). Finally, note that if $e < c_1/n$, $w(j) < 0$, all $j \in N$.

Now let $K, L \subset N$ be two sets, with $K = \{1, ..k\}$ so it comprises the k lowest-cost regions, and L arbitrary. Let $A = K \cap L$, $B = K \cup L$. Using the above results, we see that following a switch from x^L to x^K, we have the following gains for all $i \in (N/L) \cup K = S$;

$$\begin{aligned} u_i^c(x^K) - u_i^c(x^L) &= v(K) - v(L) \geq w(k) - w(l),\ i \in N/B \\ u_i^c(x^K) - u_i^c(x^L) &= [b_i - e + v(K)] - v(L) > w(k) - w(l),\ i \in K/A \\ u_i^c(x^K) - u_i^c(x^L) &= [b_i - e + v(K)] - [b_i - e + v(L)] \geq w(k) - w(l),\ i \in A \end{aligned} \tag{A.3}$$

(ii) Now let $c_{k+1}/n > e \geq c_k/n$, $k \geq m$. We will show that $x^K P x^L$, implying that x^K is the unique CW. Note first that as $k \geq m$, then $\#S = s \geq m$. Then, we see from (A.3) that

$$u_i^c(x^K) - u_i^c(x^L) \geq w(k) - w(l),\ i \in S \tag{A.4}$$

Now from the properties of $w(.)$, if $e > c_k/n$, and/or $l \neq j - 1$, then $w(k) > w(l)$. Consequently, from (A.9), $u_i^c(x^K) > u_i^c(x^L), i \in S$ and consequently $x^K Py$, all $y \in X^n$

If $e > c_k/n$, and $l = j - 1$, then there are two cases. First, if $L \neq \{1, ..j-1\}$, $v(L) > w(l)$, implying that

$$v(K) - v(L) > w(k) - w(l)$$

Consequently, all the inequalities in (A.3) hold strictly, and so again $u_i^c(x^K) > u_i^c(x^L), i \in S$ and consequently $x^K Py$, all $y \in X$. Finally, if $L = \{1, ..j - 1\}$, then it is easy to check that all delegates are indifferent between x^K and x^L except for j, who strictly prefers x^K Again, $x^K Py$, all $y \in X$.

(iii) Now consider the case with $e < c_1/n$. We first show that x^M is a Condorcet winner in $Y = \{x \in X \,|xR0\}$. First, $0 \in Y$ by definition, and by assumption A1, $x^M R0$.

Next, assuming $x^L \neq 0$, if $x^L R0$, it must be the case that $\#L = l \geq m$. First we show that delegates $i \in N/L$ always prefer 0 to x^L To see this, note that following a switch from 0 to x^L, regions $i \in N/L$ have a net gain of at most $w(l) < 0$ in external benefit. So, regions $i \in N/L$ always lose from the switch. Now if $\#L < m$, delegates $i \in N/L$ are in the majority, implying $0Px^L$

So, let $L \subseteq N$ be such that $\#L = l \geq m$. It is then sufficient to show that x^M is preferred to any x^L But, from the argument in (ii),

$$u_i^c(x^M) - u_i^c(x^L) \geq w(m) - w(l),\ i \in S$$

Now, from the properties of $w(.), w(m) > w(l)$. So, all $i \in S$ prefer x^M to x^L, and as $\#S \geq m$, it follows that $x^M Rx^L$.

Finally, we need to show that there does not exist a Condorcet winner overall. To do this, in view of (ii), we only need show that (a) x^M is not a CW in X; (b) no $z \in X/Y$ is a CW in X.

The proof of (a) is simple. Let $x^{\{i\}}$, $i \in M$, be the proposal which only funds the project in i. Then obviously, the delegate from region i prefers $x^{\{i\}}$ Moreover, as $w(1) > w(m)$, all $i \in N/M$ also prefer $x^{\{i\}}$ As these delegates constitute a majority, so $x^{\{i\}} Rx^M$, implying that x^M is not a CW in X.

Also, (b) follows immediately from the fact that if $z \in X/Y$, z is beaten by the status quo 0. □

Proof of Proposition 2

In this case, $c_{k+1}/n > e \geq c_k/n$, for some $k < m$. We will show that $x^K Px^L$ for any $L \subset N$, $L \neq K$. Define the set a S exactly as in (i) of the proof of Proposition 1 above. If $s \geq m$, then the argument is as in (i) and (ii) of the proof of Proposition 1 above. However, as $k < m$, it is now possible that $s < m$. This can occur iff $l > n + k - m$.

So, it is sufficient to show that $x^K Px^L$ for all $L \subset N$ with $l > n + k - m$. In turn, to show that $x^K Px^L$ in this case, it is certainly sufficient to show that $m - k$ of delegates $i \in N/S$ strictly prefer x^K to x^L; for then, $m - k + s \geq m$ delegates overall strictly prefer x^K to x^L Now,

$$u_i^c(x^K) - u_i^c(x^L) = -(b_i - e) + v(K) - v(L),\ i \in N/S$$

So, A4 implies directly that $u_i^c(x^K) > u_i^c(x^L)$ for $m-k$ delegates in N/S, as required. □

Proof of Proposition 3

Note from (10) that if an agenda y contains x^C, then the sophisticated equivalent of y_1 must be x^C - this is because from Proposition 1, x^C beats both the *status quo* and anything that beats the *status quo* (See Ferejohn, Fiorina, and McKelvey(1987)). So, $x^C = z(y)$, and the map from a vector of proposals $a = (a_1, .a_n)$ to an outcome is $x_1^* = \sigma(a, \pi) = x^C$ iff a contains x^C

(iii) We now claim that in any endogenous agenda equilibrium, $(a_1^*, ...a_n^*)$ must contain x^C For suppose not: then the outcome must be some $x' \in Y = \{x \in X\,|xR0\}$. But for some $i \in M$, $u_i(x^C) > u_i(x')$ [otherwise, $i \in C$, $u_i(x^C) < u_i(x')$, all $i \in M$, which contradicts the definition of x^C as a CW in Y]. So, by proposing x^C, some $i \in C$ can do strictly better than $u_i(x')$. □

Proof of Proposition 4

(i) When $e = 0$, $c = m$ so from A4, $c \neq d$. Then, as $c \neq d$, we have

$$\sum_{i \in N}[\max\{b_i - c_i, 0\} - x_i^C(b_i - c_i)] > 0$$

so (i) follows immediately from (11).

(ii) To prove (ii), note that we can write

$$W^c = \sum_{i \in C}(b_i - e + ne - c_i),\ W^d = \sum_{i \in D}(b_i - e + ne - c_i)$$

Now, for $e \geq c_n/n$, $C = N$, so

$$W^c - W^d = \sum_{i \in N/D}(b_i - e + ne - c_i)$$

where N/D is non-empty from A4. As $e \geq c_n/n$, and from A0, $b_i - e + ne - c_i > 0$ all $i \in N$, so $W^c > W^d$ as claimed. □

Proof of Proposition 5

Note that as only projects with positive v_i are undertaken with centralization, we can write

$$W^d - W^c = \sum_{j \in D/C}\nu_i - \sum_{j \in C/D}\nu_i + (n-1)(d-c)e \qquad \text{(A.5)}$$

Now, any symmetric MPS can be decomposed into a sequence of simple symmetric MPSs (Rothschild and Stiglitz(1970)), so it is sufficient to show that the result is true for a single simple symmetric MPS. First, recall that we have ordered the regions by increasing cost. Reorder them by increasing net benefit i.e.

$$\nu_1 \leq \nu_2 ... \leq \nu_n$$

With this ordering, a simple symmetric MPS of $\{\nu_k\}_{k\in N}$, $\{\nu'_k\}_{k\in N}$, is a transformation such that $\nu'_{m-i} = \nu_{m-i} - \delta$, $\nu'_{m+i} = \nu_{m+i} + \delta$, for some $1 \leq i \leq m-1$, and $\nu'_j = \nu_j$ all other j. But it is clear that this transformation leaves D and d unchanged (as no ν_i changes sign), and (weakly) raises $\sum_{j\in D/C} \nu_i$, and (weakly) lowers $\sum_{j\in C/D} \nu_i$. The proof is completed by noting that if (i) $e < 0$, or (ii) costs are left unchanged in the MPS, then from Proposition 3, c is left unchanged. So, from (A.5), $W^d - W^c$ cannot fall following the MPS. □

Proof of Proposition 6

(i) First, if $e = 0$, all i not in C strictly prefer decentralization, as they no longer pay a share of other regions' costs, and only undertake their own project if the benefit is non-negative.

So, we focus on $i \in C$. From Proposition 1, $C = M$ as $e = 0$, so $i \in C/D = M/D$ only get a project with centralization. So, by A2, all $i \in M/D$ strictly prefer centralization. So, the only way in which decentralization could be Pareto-preferred is if $M/D = \emptyset$, i.e. if $M \subset D$. But then

$$\begin{aligned} u_m^d &= b_m - c_m \\ &< b_m - \frac{1}{m}\sum_{j=1}^{m} c_j \\ &< b_m - \frac{1}{n}\sum_{j=i}^{m} c_j \\ &= u_m^c \end{aligned}$$

i.e. the agent with the median cost strictly prefers centralization.

(ii) As $D = \{i \in N \,|b_i \geq c_i\}$, then for ε small enough, recalling $\overline{b} \neq \overline{c}$ we see

$$D = \begin{cases} N & if\, \overline{b} > \overline{c} \\ \emptyset & if\, \overline{b} < \overline{c} \end{cases}$$

So, for ε small enough,

$$u_i^d = \begin{cases} b_i - c_i + (n-1)e & if\, \overline{b} > \overline{c} \\ 0 & if\, \overline{b} < \overline{c} \end{cases}$$

Also, as $c_i \to \bar{c}$, $e > \bar{c}/n$ implies $e > c_n/n$ for ε small enough. So, from Proposition 1, $e > \bar{c}/n$ implies $C = N$. So, for ε small enough

$$u_i^c = b_i - c_i + (n-1)e$$

Now, by A4, $d \neq n$ so we are in the case where $\bar{b} < \bar{c}$. So, to show $u_i^c > u_i^d$, $i \in N$, we only need show that $b_i - c_i + (n-1)e > 0$. Now note that for ε small enough,

$$\begin{aligned} A_i &= b_i - \frac{1}{n}\sum_{j \in M} c_j + (m-1)e & \text{(A.6)} \\ &< b_i - \frac{m}{n}c_i + (m-1)e + \varepsilon & \text{(A.7)} \\ &= b_i - c_i + (n-1)e - (n-m)(e - \frac{c_i}{n}) + \varepsilon & \text{(A.8)} \end{aligned}$$

Also, from A2, we must have $A_i > 0$. So, from (A.6), for $\varepsilon < (n-m)(e - \frac{c_i}{n})$, we have

$$b_i - c_i + (n-1)e > 0$$

as required. □

Proof of Proposition 7

(i) When $e = 0$, clearly all i not in C strictly prefer decentralization, as $\max\{b_i - c_i, 0\} > -\frac{1}{n}\sum_{j \in C} c_j$. As $\#C = m$, it suffices to find only one $i \in C$ who strictly prefers decentralization also, and we are done. Now note that by definition, $1 \in C$. So, combining this fact with $c_1 < \frac{1}{n}\sum_{j=1}^{m} c_j$ we see

$$u_1^d \geq b_1 - c_1 > u_1^c = b_i - \frac{1}{n}\sum_{j=1}^{m} c_j$$

So, 1 is the required region.

(ii) If $e \geq c_n/n$, then

$$\begin{aligned} u_i^c &= b_i - \bar{c} + (n-1)e \\ u_i^d &= \max\{b_i - c_i, 0\} + (d - x_i^D)e \end{aligned}$$

By A4, $d \neq n \neq m$. Assume first that $n > dm$. Now, as $|c_i - \bar{c}| < \varepsilon$, if we choose $\varepsilon < e(n-d)$, then

$$\begin{aligned} u_i^c &> b_i - c_i + (n-1)e - \varepsilon \\ &> b_i - c_i + (d-1)e \\ &= u_i^d \end{aligned}$$

for all $i \in D$. So, a majority strictly prefer C.

Now suppose that $d < m$. Then for all $i \in D$, we can show that $u_i^c > u_i^d$ as before. Also, by definition of β_m we can find $m-d$ members of N/D with $b_i \geq \beta_m$. Let the set of such members be S.

$$\begin{aligned} u_i^c &> b_i - \overline{c} + (n-1)e - \varepsilon \\ &= b_i - \overline{c} + (n-1-d)e + de - \varepsilon \\ &\geq b_i - \overline{c} + (m-1)e + de - \varepsilon \\ &\geq \beta_m - \overline{c} + (m-1)e + de - \varepsilon,\ i \in S \end{aligned}$$

But by assumption, $\beta_m - \overline{c} + (m-1)e > 0$. So, for ε small enough, $u_i^c > u_i^d = de$, $i \in S$. But then overall, a strict majority of regions prefer centralization. □

Proof of Proposition 8

We know from Proposition 1 that if $c_{k+1}/n > e > c_k/n$, x^K is the unique CW. We also know that as the inequality $e > c_k/n$ is strict, a majority of agents strictly prefer x^K to any alternative (see the proof of Proposition 1). So, $y \neq x^K$ cannot be a core allocation. Suppose to the contrary that this were true. But, as $x^K P y$, we can find a coalition S and $w = (u_i(x^K))_{i \in S} \in v(S)$ such that $w_i > u_i^c(y)$, $i \in S$, a contradiction. □

Proof of Proposition 9

By assumptions $b_i \geq c_i/n$ and $e \geq 0$, if any i is agenda-setter, he will always prefer to give a project to his own region. Let A_i be the set of "coalition members" that i chooses when he is selected as proposer i.e. every $j \in A_i$ prefers i's proposal to the *status quo*. Let $S = A_i \cup \{i\}$. Then A_i must solve problem P, which is

$$\max_{A_i \subset N} b_i - \frac{1}{n}\sum_{j \in S} c_j + (\#S - 1)e$$

$$s.t.\ x_j^S b_j - \frac{1}{n}\sum_{j \in S} c_j + (\#S - x_j^S)e \geq 0,\ j \in A_i \tag{A.9}$$

$$\#S \geq m - 1 \tag{A.10}$$

There are then three cases.

(i) $e < c_1/n$. Here, i can induce any j to vote for x^S only by offering j a project, as without a project j always prefers the *status quo* ($-\frac{1}{n}\sum_{j \in S} c_j + \#Se < 0$, all $S \subset N$). So, i will offer exactly $m-1$ other regions projects, and clearly these will be the ones with the lowest cost i.e. $A_i = \{1, ..m-1\}$. By A2, $\{1, ..m-1\}$ is feasible in P, and by the above argument, it clearly solves P.

(ii)$c_k/n \leq e < c_{k+1}/n$, $k < m$. In this case, ignoring the constraints (A.9),(A.10), i would prefer to set $A_i = K = \{1, ...k\}$ (or $K/\{i\}$ if $i \in K$). Let $h > k$ be the largest integer such that

$$-\frac{1}{n} \sum_{j \in K \cup \{h\}} c_j + (k+1)e \geq 0$$

If $i \in H = \{1, ..h\}$, then if i offers projects to regions in K, as well as a project in its own region, then every region gets a non-negative payoff from x^S, $S = K \cup \{i\}$, and thus $A_i = K$ is feasible in P. If $i > h$, $A_i = K$ is not feasible in P (i.e. externalities are not strong enough to induce regions who do not get projects to vote for $x^S, S = K \cup \{i\}$) and so i must offer projects to the minimum winning coalition i.e. set $A_i = \{1, ..m-1\}$.
(iii) $c_k/n \leq e < c_{k+1}/n$, $k \geq m$. Here, $S = K \cup \{i\}$, by the previous argument.

By the above arguments, it is clear that whatever e, projects in C are funded with probability one. Moreover, if $c_1/n \leq e$ or $c_m/n \leq e$, projects not in C are funded with probability $1/n$ only. □

Figure 1

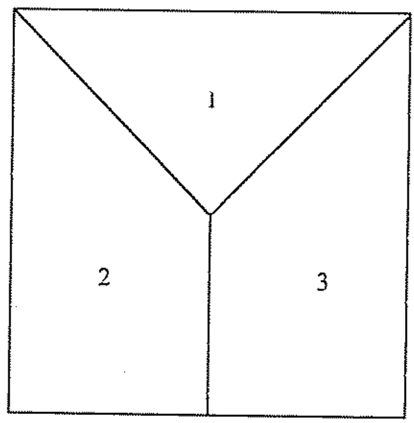

Figure 2