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RATIONAL EXPECTATIONS AND EXCHANGE RATE DYNAMICS

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ABSTRACT*

Dornbusch's overshooting model of the exchange rate has proved a very influential alternative to the monetary model. The original Dornbusch model was specified in continuous time and assumed perfect foresight. It also imposed the restriction of a sticky price level which does not respond instantaneously to new information. While convenient for analytic purposes, this particular model is less suitable for empirical analysis in which the data are aggregated over time and expectations are not formed perfectly. This paper presents a discrete time, rational expectations version of the Dornbusch model in which the price level is permitted to respond immediately, but not necessarily fully, to new information. The resulting dynamic behaviour of the exchange rate is analysed and interpreted. The conditions under which exchange rate overshooting occurs are derived and the effect of pre-announced policy changes are studied. Although the main purpose of the paper is expositional, an interesting feature of the results is that price stickyness is shown to be neither a necessary nor a sufficient condition for a change in monetary policy to bring about exchange rate overshooting.

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SUMMARY

Dornbusch's model of the exchange rate has proved a very influential alternative to the monetary model, primarily because it demonstrates that by reformulating the monetary model to have a sticky price level the exchange rate will respond differently to monetary shocks. Instead of the exchange rate jumping instantaneously to its new long run level, initially it will overshoot this value and then converge back to it through time along the stable manifold. Dornbusch's analysis is carried out in continuous time and with perfect foresight, which is not a convenient framework for empirical work which uses, for example, quarterly data. With this in mind, in the present paper Dornbusch's model is respecified in discrete time with rational expectations and its properties are examined.

Using a different method of analysis from that customarily employed in solving rational expectations a number of interesting results can be obtained which throw further light on

a) the conditions under which the exchange rate will overshoot
b) the nature of the dynamic adjustment path of the exchange rate, and
c) the effect on the exchange rate of announcements.

For example, it is shown that price stickyness is neither a necessary nor a sufficient condition for monetary policy to bring about exchange rate overshooting. The jump onto the stable manifold and the subsequent movement along it that is characteristic of continuous time models is given a more natural re-interpretation as a partial adjustment mechanism. Greater understanding of the reasons is obtained on why and by how much the exchange rate moves in anticipation of future changes in exogenous variables such as monetary and fiscal policy instruments.

1. Introduction

The object of this paper is to examine the dynamic behaviour of discrete time overshooting models of the exchange rate when expectations are formed rationally. Usually the dynamic analysis of overshooting models is carried out in continuous time with the assumption of perfect foresight e.g. Dornbusch (1976) and Buiter and Miller (1981). These models also assume a sticky price level which does not respond instantaneously to new information or shocks. Although convenient for analytic purposes, such models are less suitable for empirical analysis in which data are aggregated over time, expectations are not formed perfectly and any delays in the response of the price level are not sustained for a whole observation period. Hitherto there have been few attempts to use rational expectations in discrete time overshooting models. Perhaps the best known is that by Mussa (1982). The present method of analysis is however somewhat different as is the model to be analysed.

The difference between rational expectations and perfect foresight is often blurred. Dornbusch (1976, fn 10) describes the perfect foresight path that is characteristic of the dynamic adjustment of overshooting models as "obviously the deterministic equivalent of rational expectations". He also states that the perfect foresight path is more than a mere curiosum, "it is the only expectational assumption that is not arbitrary (given the model) and that does not involve persistent prediction errors". Rational expectations, whilst sharing the property of being model consistent, does not make such stringent informational demands as perfect foresight. This has a number of important implications, especially for policy. Moreover, the value of rational expectations in empirical work is now well established.

A characteristic of many asset market models of the exchange rate is that the market for foreign exchange is fully efficient with the result that following any disturbance or new information the exchange rate jumps instantaneously to maintain asset market equilibrium. The well known models of Mundell (1963), Fleming (1962), Frenkel (1976), Dornbusch, and Buiter and Miller all have this property. Where these models differ most is in their assumption about the speed of adjustment of goods prices; these range from zero (Mundell-Fleming model) to infinite (the monetary model of Frenkel). In between lie the sticky price models of Dornbusch, and Buiter and Miller, in which, immediately following a disturbance, the price level remains unchanged but the exchange rate jumps to clear the asset market. Subsequently prices begin to adjust enabling the exchange rate to proceed to its equilibrium value along the perfect foresight path. The new equilibrium will depend on the type and permanence of the disturbance. For example, a permanent increase in the money stock would cause both the equilibrium price level and the exchange rate - the domestic price of foreign exchange - to increase. However, the initial jump in the exchange rate exceeds the new equilibrium level, hence the general description of these models as overshooting models.

In continuous time perfect foresight models, the initial jump in the exchange rate following the disturbance can be interpreted as the result of the economy placing itself on a stable manifold, knife-edge adjustment, or perfect foresight path, along which it then moves to the new equilibrium once the price level begins to change. This solution is the result of the saddle-point property of the dynamic model that emerges. It describes the only way that equilibrium can be internally restored in these models and is in contrast with globally stable models in which there are an infinite number of ways of reaching a new equilibrium, Begg (1982, pp36-41). The explanation of how the economy manages to place itself on this unique adjustment path is that the exchange rate adjusts to maintain asset market equilibrium by arbitraging net yields. Further understanding of this process and, in particular, the role played by future expectations, can be gained by replacing the assumptions of perfect foresight and continuous time by rational expectations and discrete time. It can then be seen that this result does not depend on perfect foresight; a wide class of expectations will produce a similar result. Moreover, the dynamic path of exchange rates will differ according to the expectations held.

The best known overshooting model is that of Dornbusch and it was decided to use this to explore the implications of rational expectations in discrete time. A suitable alternative choice would be the model of Buiter and Miller but as this generates more complicated dynamic paths for the exchange rate it seemed better to analyse this model separately. As the Dornbusch model is specified in continuous time it is necessary to respecify it in discrete time. This creates a problem because in the Dornbusch model the price level and aggregate demand are assumed not to respond instantly to new information. Although it would be possible to introduce corresponding delays into the discrete time version it was decided not to do so for two reasons: there would be an element of arbitrariness in the choice of lag but, more important, as the period of time increases in length, the assumptions of an instantaneously fixed price level and aggregate demand become less tenable. As part of the interest of this analysis is to apply it in empirical studies of the exchange rate when quarterly or annual data are used, and hence where the exchange rate measure would be the average over the time period, it seemed preferable not to impose such delays in the discrete model. The analysis of the behaviour of the exchange rate with a lag in the response of the price level is therefore confined to the appendix.

Since the feature of the Dornbusch model which most distinguishes it from the monetary model and which produces exchange rate overshooting is the imposition of sticky prices, it might be epxected that in permitting the price level to respond immediately overshooting would no longer occur. One of the interesting aspects of the results derived in this paper, therefore, is the finding that overshooting may still take place, though admittedly in more restrictive circumstances that depend on the interest elasticity of the money demand equation. It is also shown that even if the price level response with a lag, overshooting does not necessarily take place. Thus price stickyness is neither a necessary nor a sufficient condition for monetary policy to bring about exchange rate overshooting.

The paper is set out as follows. In Section 2 the fundamental dynamic equation of the exchange rate is derived for our rational expectations discrete time version of the Dornbusch model. This equation is given a partial adjustment interpretation and it is compared with that derived from the rational expectations monetary model. The dynamic behaviour of the exchange rate consequent upon exogenous shocks or policy changes is examined in the next three Sections. It is shown in Sections 2 and 3 that overshooting only occurs as a result of domestic or foreign monetary policy changes and even then this is not guaranteed. Fiscal policy is examined in Section 4 and is found not to produce overshooting. In Section 5 the effects of pre-announced fiscal and monetary policy changes are examined. Some conclusions are drawn briefly in Section 6 and finally, in the appendix, the effect of introducing a delay in the response of the price level is analysed.

2. The Dornbusch Model

The small country model of Dornbusch can be re-specified in discrete time as follows:

$$d_t = \delta(e_t + p_t^* - p_t) + \gamma y_t - \sigma r_t + z_t \tag{1}$$

$$p_t - p_{t-1} = \alpha(d_t - y_t) + v_t \tag{2}$$

$$m_t - p_t = \phi y_t - \lambda r_t + u_t \tag{3}$$

$$E_t e_{t+1} - e_t = r_t - r_t^* + w_t \tag{4}$$

where d = aggregate demand, e = the exchange rate (the domestic price of foreign exchange), p = the price level, y = output, m = the money stock, r = the nominal interest rate, z, v, u, w = stochastic i.i.d. variables with, for convenience, zero means, m and y are exogenous stochastic variables, all variables except r are natural logarithms, * = the equivalent foreign variable all of which are exogenous and E_t is the conditional expectation operator based on information available at the time t. Equation (1) determines the aggregate demand for goods. Equation (2) describes how the price level adjusts to excess demand in the goods market. In the limit as α tends to infinity the price level adjusts instantaneously thereby continuously maintaining goods market equilibrium. If, in addition, δ is infinite, purchasing power parity prevails continuously. The model would then reduce to the rational expectations monetary model of Mussa (1976). Equation (4) is the uncovered interest parity condition required for asset market equilibrium and implies that asset arbitrage equates the rate of return on domestic and foreign assets once allowance is made for expected changes in the exchange rate.

In Dornbusch's original continuous time specification it was assumed that while e was a jump variable capable of responding instantly to new information, p could only respond with a delay. Such a distinction is vital in producing the overshooting property characteristic of Dornbusch's model. In discrete time it is less clear that this distinction is justified. Although it would be possible to introduce time lags into the model so that p responds with a delay, the analysis here will be conducted using the time structure of equations (1) - (4). Corresponding results for a one period lag in the response of the price level to excess demand are reported in the appendix. It might be expected therefore that one consequence of this respecification of Dornbusch's model from continuous to discrete time is likely to be the absence of exchange rate overshooting.

The solution of the model is best obtained by first deriving the reduced form exchange rate equation. From (1) - (3) we obtain

$$p_t = \mu p_{t-1} + \theta e_t + q_t \qquad (5)$$

where $\mu = (1 + \alpha(\delta + \sigma/\lambda))^{-1}$, $\theta = \mu\alpha\delta$ and

$$q_t = \mu\alpha\ (\gamma - 1 - \sigma\phi/\lambda)y_t + (\mu\alpha\sigma/\lambda)\ m_t$$

$$+\mu\alpha z_t + \mu\alpha\delta p_t^* - (\mu\alpha\sigma/\lambda)u_t + \mu v_t \qquad (6)$$

From (3) and (4) we have

$$e_t = E_t e_{t+1} - (1/\lambda)p_t + s_t \qquad (7)$$

where

$$s_t = (-\phi/\lambda)y_t + (1/\lambda)m_t + r_t^* - (1/\lambda)u_t - w_t \qquad (8)$$

Combining (5) and (7) to eliminate p_t gives the reduced form exchange rate equation

$$\lambda E_t e_{t+1} - \mu\lambda E_{t-1} e_t - (\lambda + \theta) e_t + \mu\lambda e_{t-1} = x_t \qquad (9)$$

where

$$x_t = q_t - \lambda s_t + \mu\lambda s_{t-1} \qquad (10)$$

In order to determine the spot exchange rate e_t it is necessary to take expectations of equation (9) conditional on information available in period t-1. Using the algebra of operators we obtain

$$\lambda E_{t-1} e_{t+1} - (\mu\lambda + \lambda + \theta) E_{t-1} e_t + \mu\lambda e_{t-1} = E_{t-1} x_t \qquad (11)$$

Noting that $E_{t-1} e_{t-1} = e_{t-1}$ and using the forward operator L where $L^i e_t = E_t e_{t+i}$, we can write the characteristic equation of (11) as

$$\lambda L^2 - (\mu\lambda + \lambda + \theta) L + \mu\lambda = 0$$

which has the solution

$$L = \left[\mu\lambda + \lambda + \theta \pm \sqrt{(\mu\lambda + \lambda + \theta)^2 - 4\mu\lambda^2}\right] / 2\lambda$$

Denoting the two roots η_1 and η_2, it can be seen that they satisfy $\eta_1 < 1$ and $\eta_2 > 1$. In other words, the solution is a saddlepoint with one root (η_1) stable and the other (η_2) unstable.

Equation (11) can now be re-written as:

$$\lambda(L - \eta_1)(L - \eta_2)\, e_{t-1} = E_{t-1}x_t \ ,$$

$$-\lambda\eta_2 L(1 - \eta_1 L^{-1})(1 - \eta_2^{-1} L)\, e_{t-1} = L\, x_{t-1} \ ,$$

$$(1 - \eta_1 L^{-1})\, e_t = -x_t / [\lambda\eta_2(1 - \eta_2^{-1} L)]$$

Eliminating the operators we obtain the required equation for the spot exchange:

$$e_t = \eta_1 e_{t-1} - (\lambda\eta_2)^{-1} \sum_{i=0}^{\infty} \eta_2^{-i} E_t x_{t+i} \qquad (12)$$

This solution thus decomposes the dynamics of the exchange rate into a forward looking and a backward looking component. A similar split solution was proposed by Tinsley (1971) in the entirely different context of multi-period optimisation leading to labour demand functions. A more recent use of this solution procedure in the context of rational expectations is by Hansen and Sargent (1980).

The interpretation of equation (12) and the implied dynamic behaviour of the exchange rate is straightforward. Following the availability of new information about any of the current and expected future values of the variables of which x_t is comprised, the last term of equation (12) - the forward looking component - will immediately change. This will cause a corresponding jump in the spot exchange rate. Adjustment to the new equilibrium exchange rate will take place through the lagged exchange rate. Because $\eta_1 < 1$, equation (12) is a stable autoregressive process in e_t which ensures convergence to equilibrium.

Equation (12) can also be given a partial adjustment interpretation because it can be re-written:

$$\Delta e_t = (1 - \eta_1)\ (\bar{e}_t - e_{t-1}) \tag{13}$$

where

$$\bar{e}_t = -\ (\lambda \eta_2)^{-1} \sum_{i=0}^{\infty} \eta_2^{-i}\ E_t x_{t+i}/(1-\eta_1) \tag{14}$$

is the long-run solution and $1>(1-\eta_1)>0$ is the coefficient of adjustment.

The solution can be compared with that of the perfect foresight path. The jump in the spot exchange rate required to place the economy on the perfect foresight path has a natural interpretation here. This interpretation dispels any mystery there may have been over how the economy knows by how much to jump in order to place itself on the unique perfect foresight path. The size of the jump is determined simply by the amount of the change in the forward looking term in equation (12). Whatever the size of the jump, convergence to equilibrium is assured. In this sense there is no more uniqueness about the path of the exchange rate than there is in the behaviour of any other variable generated by a stochastic autoregressive process.

Several further remarks can be made about the solution given by equation (12).

1. It is consistent with a wide class of expectation processes. All that is required is that expectations are consistent with the use of iterated operators in the solution; in particular, full rationality is not required.

2. The solution, although not necessarily rational, does, however, make use of the structure of the model and in this sense the expectations of the exchange rate can be said to be consistent.

3. The dynamic behaviour of the exchange rate in this model can be compared with that of the rational expectations monetary model. It was explained earlier that the assumption of instantaneous price adjustment requires that α equals infinity, and if, in addition, $\delta = \infty$ then purchasing power parity holds. It can be shown that as $\alpha \to \infty$ and $\delta \to \infty$ then $\lim \mu = 0$, $\lim \theta = 1$, $\lim \eta_1 = 0$, and $\lim \eta_2 = 1 + 1/\lambda$. Hence (12) converges to

$$e_t = -\sum_{i=0}^{\infty} (1 + 1/\lambda)^{-i} E_t x_{t+i}/(1+\lambda) \qquad (15)$$

Equation (15) corresponds to that derived by Mussa (1976) and Smith and Wickens (1984). It shows that in the monetary model new information causes the spot exchange rate to jump immediately to its new long-run equilibrium value with no overshooting, nor any other tendency to move due to an internal dynamic.

3. Domestic Monetary Policy

Further understanding of the dynamic behaviour of the exchange rate under rational expectations, and an examination of whether the model displays the overshooting property, can be gained by considering the effect of changes in the exogenous variables. We examine first the case of domestic monetary policy in which the domestic money supply is assumed to be generated by

$$m_t = \bar{m} + \varepsilon_t \qquad (16)$$

where ε_t is an i.i.d. process with zero mean and constant variance and the mean of the money supply is increased from $\bar{m}_0$ to $\bar{m}_1$. For convenience the other exogenous and random variables are assumed to be fixed at their means. Thus x_t, defined by equation (10), can be written

$$x_t = -\mu(1 + \alpha\delta)m_t + \mu m_{t-1} + \psi \qquad (17)$$

where ψ is a constant. If the increase in the mean of m_t occurs in period t then

$$\left.\begin{aligned} E_t x_{t+i} &= -\mu(1 + \alpha\delta)(\bar{m}_1 + \varepsilon_t) + \mu\, m_{t-1} + \psi && i = 0 \\ &= -\mu\,\alpha\delta\bar{m}_1 + \mu\,\varepsilon_t + \psi && i = 1 \\ &= -\mu\alpha\delta\bar{m}_1 + \psi && i > 1 \end{aligned}\right\} \qquad (18)$$

Substituting (17) into (12) and simplifying it can be shown that

$$e_t = \eta_1 e_{t-1} + \nu_1\bar{m}_1 + \nu_2\, m_{t-1} + \nu_3\varepsilon_t + \nu_4 \qquad (19)$$

where $\nu_1 = \mu[\eta_2(1 + \alpha\delta)-1]/\lambda\eta_2(\eta_2-1)$, $\quad \nu_2 = -\mu/\lambda\eta_2$,

$\nu_3 = \mu[\eta_2(1+ \alpha\delta)-1]/\lambda\eta_2^2$, and $\nu_4 = -\psi/\lambda(\eta_2-1)$

The increase in e_t in the first period arising from a change in $\bar{m}$ is therefore $\nu_1 > 0$, and from a temporary monetary shock ε_t is $\nu_3 > 0$, which is less than ν_1 since $\eta_2 > 1$. The long-run effect of a change in $\bar{m}$ is unity. This can be obtained either directly from the structural model or from (19) by simplifying $(\nu_1 + \nu_2)/(1 - \eta_1)$.

For the exchange rate to overshoot its long-run value it is necessary (and sufficient) that $de_t/d\bar{m} = \nu_1 > 1$. In general this condition will not hold. In fact, it seems more likely that $0 < \nu_1 < 1$. To see this note that

$$\nu_1 - 1 = \frac{(\eta_2 - 1)(\mu - \lambda\eta_2) + \mu\alpha\delta\eta_2}{\lambda\eta_2(\eta_2 - 1)} \qquad (20)$$

As $0 < \mu < 1$, $\eta_2 > 1$ and λ is likely to be greater than unity, ν_1 will not necessarily be greater than unity.

Further understanding of the dynamic behaviour of the exchange rate and an alternative but more informative condition than (20) can be obtained by making use of the partial adjustment interpretation of the dynamics of the exchange rate given by equations (13) and (14). For a partial adjustment mechanism convergence to the 'desired' value following a single disturbance is monotonic. Therefore overshooting, if it occurs, must be due to more than one change in the 'desired' value. More precisely, a necessary (but not sufficient) condition for the exchange rate to overshoot is that $d\bar{e}_t/d\bar{m} > d\bar{e}/d\bar{m}$ i.e. starting from a position of equilibrium the change in the 'desired' value in period t, $\bar{e}_t$, must exceed the change in the long-run equilibrium value, $\bar{e}$. To show that $d\bar{e}_t/d\bar{m} > 1$ we need only focus on the term involving m_{t-1} in equation (19). In calculating $\bar{e}_t$ the mean value of m_{t-1} is $\bar{m}_o$, while to obtain $\bar{e}$ the mean of m_{t-1} is set equal to its new value $\bar{m}_1$. Hence we can prove that

$$d\bar{e}_t/d\bar{m} - d\bar{e}/d\bar{m} = -\nu_2/(1 - \eta_1) > 0 \qquad (21)$$

from which it follows that there is a reduction in the 'desired' value after the first period. To see why this is not sufficient to produce overshooting we note from (13) that

$$de_t/d\bar{m} = (1 - \eta_1)d\bar{e}_t/d\bar{m}$$

and hence that

$$\begin{aligned} de_t/d\bar{m} - d\bar{e}/d\bar{m} &= (1 - \eta_1)(d\bar{e}_t/d\bar{m} - d\bar{e}/d\bar{m}) - \eta_1\, d\bar{e}/d\bar{m} \\ &= -\nu_2 - \eta_1 \\ &= \mu\,(1-\lambda)/\lambda\eta_2 \qquad (22) \end{aligned}$$

Thus overshooting occurs if $\lambda < 1$. This result can be confirmed from equation (20) by setting $\lambda = 1$ in (20) and noting that the right-hand side is then zero, etc.

In interpreting this result a comparison may be made with Dornbusch's continuous time model. It will be recalled that the domestic price level and aggregate demand are fixed temporarily so that in the first instance an increase in the supply of money will affect only the domestic interest rate and the exchange rate. The interest rate will fall and create a negative interest rate differential. In order to satisfy the interest parity condition, equation (4), this will generate an expected future appreciation (fall) of the exchange rate and hence convergence back to equilibrium from above. In the long run the exchange rate will depreciate (rise) and therefore in order to achieve convergence to equilibrium from above, the spot rate must jump above, or overshoot, its new long-run value. In contrast, in the discrete time model considered here, the long-run effect of a change in the money supply will be the same but the short-run dynamics will be different. All four endogenous variables p, d, r and e are now permitted to respond in the first period to the change in m. There is therefore no necessity for the interest rate to bear the full impact of the change in the money supply. Moreover, because the price level will increase in the first period, the direction of change in the interest rate will now depend on the behaviour of the real and not the nominal money supply. Exchange rate overshooting will only occur if the real money supply rises. Attention is thus focused on the response of the price level and this will depend in part on the size of λ. To see how, we can envisage the following sequence of events: with the price level initially fixed, the increase in the money supply will reduce the interest rate; this in turn raises aggregate demand, which in its turn creates excess demand and causes a rise in the price level. The amount of the increase in the price level will depend on λ. The greater λ, the larger the rise in the price level and hence the less likely is it that the exchange rate will overshoot. This is consistent with the earlier result of equation (21) that overshooting occurs for small λ.

The practical implication of this result is primarily for empirical work which is based on infrequently observed data such as quarterly or annual data. In the very short run it seems highly implausible to expect that the price level would respond sufficiently fast to prevent exchange rate overshooting, but over a period of say a quarter, when quarterly averages are being used to measure the exchange rate, matters are not as clear cut and overshooting may not be observed. In these cases the discrete time model may be more appropriate.

The dynamic response of the exchange rate to an increase in the money is depicted in Figures 1a and 1b.

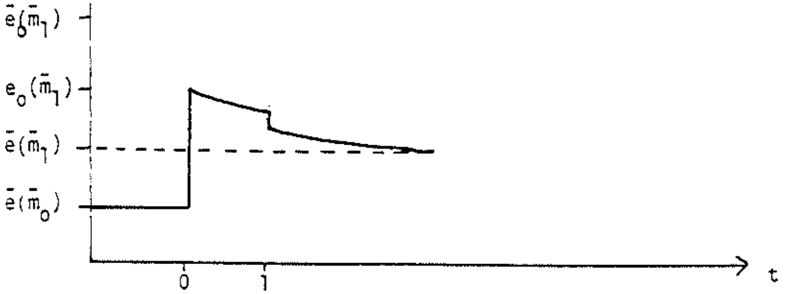

Figure 1a: Monetary policy with $\lambda < 1$

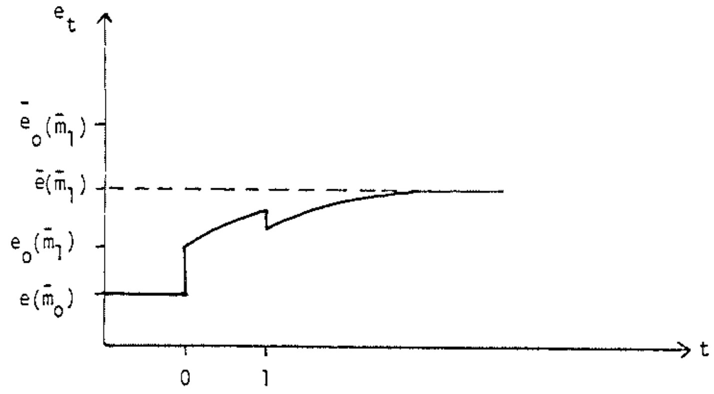

Figure 1b: Monetary policy with $\lambda>1$

$\bar{e}(\bar{m}_o)$ denotes the initial equilibrium exchange rate based on $\bar{m}_o$ - this presupposes it was previously in equilibrium - and $\bar{e}(\bar{m}_1)$ the new equilibrium rate; $\bar{e}_o(\bar{m}_1)$ is the 'desired' value in the first period and $e_o(\bar{m}_1)$ the actual value. In Figure 1a the exchange rate overshoots but in Figure 1b it does not. In period 1 there is a step down due to the response of the 'desired' value following the change to the lagged money supply. This step only occurs once.

4. Foreign Monetary Policy

This may be represented in the model most easily by considering the effect of a change in r^*. it could be argued, however, that changes in the foreign money supply would affect both r^* and p^* and not r^* alone. In this case it would be easier to drop all foreign variables from the model and, with the exception of the exchange rate, to redefine the domestic variables as the difference between the domestic and the corresponding foreign variable. The model would then express the difference between the domestic and the foreign economies. This interpretation presupposes that all domestic and foreign equations are identical. The analysis of a decrease in the foreign money supply would then be the same as for an increase in the domestic money supply,which we have already considered. One way of justifying our proposed analysis based on changes in r^* is to assume that the domestic economy is small relative to the rest of the world and so there are no feedback effects from abroad.

It is assumed that r^* is generated by

$$r^*_t = \bar{r}^* + \eta_t$$

where η_t is an i.i.d. variable with zero mean and constant variance.

Keeping all other exogenous variables constant, and noting that x_t depends on r^*_t only through s_t, enables x_t to be written

$$x_t = -\lambda r^*_t + \mu\lambda r^*_{t-1} + \omega \qquad (23)$$

Following an increase in $\bar{r}^*_0$ to $\bar{r}^*_1$ in period t the exchange rate equation becomes

$$e_t = \eta_1 e_{t-1} + \pi_1 \bar{r}^*_1 + \pi_2 r^*_{t-1} + \pi_3 \eta_t + \pi_4 \qquad (24)$$

where $\pi_1 = (\eta_2 - \mu)/\eta_2(\eta_2 - 1)$, $\pi_2 = -\mu/\eta_2$,

$\pi_3 = (\eta_2 - \mu)/\eta_2^2$ and $\pi_4 = -\omega/\lambda(\eta_2 - 1)$. Because $\pi_1 > 0$, an increase in $\bar{r}^*$ causes a jump depreciation in the exchange rate. The long-run effect of the change in $\bar{r}^*$ is

$$d\bar{e}/d\bar{r}^* = \lambda + \sigma/\delta > 0 \qquad (25)$$

Hence overshooting occurs if $\pi_1 > \lambda + \sigma/\delta$. By a similar argument to that used to derive (21) it can be shown that

$$de_t/d\bar{r}^* - d\bar{e}/d\bar{r}^* = \mu(1 - \lambda - \sigma/\delta)/\eta_2 \qquad (26)$$

implying that overshooting occurs if $\lambda + \sigma/\delta < 1$. The dynamic behaviour of the exchange rate after an increase in $\bar{r}^*$ can therefore be represented by a similar diagram to Figure 1,once the condition for overshooting is amended appropriately.

4. Fiscal Policy

The effects of fiscal policy can be analysed by considering the dynamic consequences of a change in z_t, the exogenous variable in the aggregate demand equation. The results obtained will also apply to foreign demand shocks such as changes in world trade or the foreign price level. Once again interest centres on x_t which, from equation (10), can now be written

$$x_t = \mu\alpha z_t + \rho \qquad (27)$$

because x_t depends on z_t only through q_t. Assuming that

$$z_t = \bar{z} + \xi_t \qquad (28)$$

where ξ_t is an i.i.d variable with mean zero and constant variance, it can be shown that the exchange rate equation becomes

$$e_t = \eta_1 e_{t-1} - (\mu\alpha\bar{z} + \rho)/\lambda(\eta_2-1) - \mu\alpha\xi_t/\lambda\eta_2 \qquad (29)$$

It follows that an increase in $\bar{z}$, or a positive shock ξ_t, will cause a jump appreciation of the exchange rate. The long-run effect on the exchange rate of a change in $\bar{z}$ is given by

$$d\bar{e}/d\bar{z} = -1/\delta < 0 \qquad (30)$$

and of a change in p^*, by -1.

Moreover,

$$de_t/d\bar{z} - d\bar{e}/d\bar{z} = \eta_1/\delta > 0 \qquad (31)$$

which implies that the exchange rate does not overshoot. The adjustment path is depicted in Figure 2 for an increase in $\bar{z}_0$ to $\bar{z}_1$.

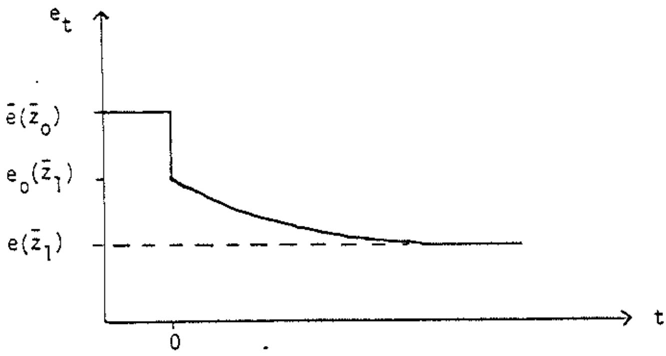

Figure 2: Fiscal Policy.

Thus, in the first period the exchange rate jumps from $\bar{e}(z_o)$, its equilibrium value prior to the policy change, to $e_o(\bar{z}_1)$. Subsequently, it follows a partial adjustment path to the new long-run equilibrium value $\bar{e}(\bar{z}_1)$. Since in x_t there is no lag in z_t there is now no step in period 1.

5. Announcement Effects

In a very elegant analysis of the perfect foresight version of the Dornbusch model, Wilson (1979) demonstrated that a pre-announced (and credible) change in the money supply would cause the spot exchange rate to jump on the instant the announcement was made and to continue to alter each period so that when the change in the money supply actually takes place the spot exchange rate will be exactly on the perfect foresight path. Thereafter convergence to equilibrium takes place once more along this path. The dynamic behaviour of both the spot and expected future exchange rates following a policy announcement can be analysed more easily using the rational expectations version of the Dornbusch model.

Fiscal Policy

The simplest case to examine first is that of a pre-announced change in fiscal policy. Assume that $\bar{z}$ is expected to increase in period $t + n$ $(n > 0)$ from $\bar{z}_0$ to $\bar{z}_1$. It follows that there will be an immediate change in the forward looking term in the exchange rate equation (12). From (27) - (28) the spot exchange rate in period t is now given by

$$e_t = \eta_1 e_{t-1} - \left[\sum_{i=0}^{n-1} \eta_2^{-i} (\mu\alpha\bar{z}_0 + \rho) + \sum_{i=n}^{\infty} \eta_2^{-i} / \mu\alpha\bar{z}_1 + \rho)/\lambda\eta_2 \right] - \mu\alpha\xi_t/\lambda\eta_2$$

$$= \eta_1 e_{t-1} - (\mu\alpha\bar{z}_0 + \rho)/\lambda(\eta_2 - 1) - \mu\alpha(\bar{z}_1 - \bar{z}_0)/\lambda\eta_2^n - \mu\alpha\xi_t/\lambda\eta_2. \qquad (32)$$

Thus when the announcement is made there is an immediate appreciation of the spot exchange rate of $\mu\alpha(\bar{z}_1 - \bar{z}_0)/\lambda\eta_2^n$. The further ahead is the proposed implementation of the fiscal expansion the smaller is the jump appreciation because $\eta_2 > 1$. In each period until $t + n$ the exchange rate adjusts dynamically both to the changing lagged exchange rate and to the increasing size of the announcement term. At period t + n, equation (31) reverts to equation (28). This is analagous to Wilson's result that when the policy is implemented the exchange rate is once more on the perfect foresight path. Figure 3 illustrates the dynamic behaviour of the exchange in this case from t = 0. As we get closer

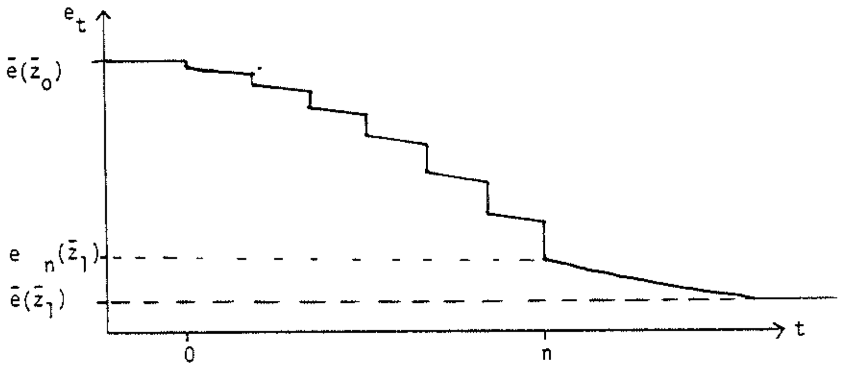

Figure 3: n period ahead fiscal policy announcement.

to period n the step appreciations get larger. After period n the dynamic behaviour of the exchange rate is determined entirely by the lagged exchange rate.

<u>Domestic Money Supply</u>

Pre-announced changes in the money supply are complicated by the presence of the lagged money supply. Suppose that $\bar{m}$ is expected to increase from $\bar{m}_0$ to $\bar{m}_1$ in period n (n > 1), then from (16) - (18) the spot exchange rate in period t is determined by

$$e_t = \eta_1 e_{t-1} + \Big\{ \mu(1+\alpha\delta)\left[\sum_{i=o}^{n-1} \eta_2^{-i}\bar{m}_o + \sum_{i=n}^{\infty} \eta_2^{-i}\bar{m}_1\right] - \mu\left[\sum_{i=o}^{n} \eta_2^{-i}\bar{m}_o + \sum_{i=n+1}^{\infty} \eta_2^{-i}\bar{m}_1\right]$$

$$- \mu\left[\eta_2(1+\alpha\delta)-1\right]\varepsilon_t/\eta_2 + \mu\,\varepsilon_{t-1} - \sum_{i=o}^{\infty} \eta_2^{-i}\psi \Big\} / \lambda\eta_2$$

$$= \eta_1 e_{t-1} + \mu\alpha\delta\bar{m}_o/\lambda(\eta_2-1) + \mu\left[\eta_2(1+\alpha\delta)-1\right](\bar{m}_1 - \bar{m}_o)/\lambda\eta_2^{n+1}(\eta_2-1)$$

$$- \mu\left[\eta_2(1+\alpha\delta)-1\right]\varepsilon_t/\lambda\eta_2^2 + \mu\,\varepsilon_{t-1}/\lambda\eta_2 - \psi/\lambda(\eta_2-1) \qquad (33)$$

There is therefore an instant depreciation of the exchange rate when the announcement is made of $\mu\left[\eta_2(1+\alpha\delta)-1\right](\bar{m}_1 - \bar{m}_o)/\lambda\eta_2^{n+1}(\eta_2-1)$. Again the jump depreciation declines the further ahead is the implementation date, while at t + n equation (33) reverts to (19).

6. Conclusions

This paper has suggested a method of analysing discrete time models of the exchange rate in which expectations are formed rationally. This method is likely to prove useful for studying analytically the properties of empirical models. It also provides a convenient alternative way of expositing the dynamic behaviour of exchange rate models.

Dornbusch's overshooting model was chosen as a suitable vehicle for demonstrating this method. An interesting feature of the results obtained is that price stickyness is shown to be neither a necessary nor a sufficient condition for a change in monetary policy to bring about exchange rate overshooting.

Appendix

The effect of a delay in the response of the price level

We examine briefly the effect of introducing a delay in the response of the price level to the excess demand for goods. It may be thought that this more closely approximates Dornbusch's original sticky price model. Thus we replace equation (2) with

$$p_t - p_{t-1} = \alpha(d_{t-1} - y_{t-1}) + v_t \qquad (2')$$

Equations (5) and (6) now become

$$p_t = \mu p_{t-1} + \theta e_{t-1} + q_t \qquad (5')$$

$$q_t = \alpha(\gamma - 1 - \sigma\phi/\lambda)y_{t-1} + (\alpha\sigma/\lambda)(m_{t-1} - u_{t-1}) + \alpha z_{t-1} + \alpha\delta p^*_{t-1} + v_t \qquad (6')$$

where now $\mu = 1 - \alpha(\delta + \sigma/\lambda) < 1$ and $\theta = \alpha\delta$. Thus the price level no longer responds immediately to a change in the money supply. The reduced form exchange rate equation can be written

$$E_t e_{t+1} - e_t - \mu E_{t-1} e_t + (\mu - \theta/\lambda)e_{t-1} = x_t \qquad (9')$$

where

$$x_t = -s_t + \mu s_{t-1} + (1/\lambda)q_t \qquad (10')$$

Taking expectations of (9') based on E_{t-1} gives

$$E_{t-1}e_{t+1} - (1 + \mu)E_{t-1}e_t + (\mu - \theta/\lambda)e_{t-1} = E_{t-1}x_t \qquad (11')$$

and hence a characteristic equation

$$L^2 - (1 + \mu)L + (\mu - \theta/\lambda) = 0$$

which has the solution

$$L = \tfrac{1}{2}(1 + \mu) \pm \sqrt{[\tfrac{1}{2}(1+ \mu)]^2 - (\mu - \theta/\lambda)}$$

Denoting the roots η_1 (< 1) and η_2 (> 1) enables us to write the exchange rate equation as

$$e_t = \eta_1 e_{t-1} - \sum_{i=0}^{\infty} \eta_2^{-i-1} E_t x_{t+i} \qquad (12')$$

Thus the equation describing the dynamic behaviour of the exchange rate when there is a fixed delay in the response of the price level has the same structure and hence interpretation as before.

We shall confine further analysis of (12') to the case of a permanent increase in the domestic money supply from a mean of $\bar{m}_0$ to $\bar{m}_1$. It can be s[illegible] that

$$x_t = [-m_t + (\mu + \alpha\sigma/\lambda)m_{t-1} + \psi]/\lambda \qquad (16')$$

and hence

$$\left.\begin{aligned} \lambda E_t x_{t+i} &= -(\bar{m}_1 + \varepsilon_t) + (\mu + \alpha\sigma/\lambda)m_{t-1} + \psi & i = 0 \\ &= -\bar{m}_1 + (\mu + \alpha\sigma/\lambda)(\bar{m}_1 + \varepsilon_t) + \psi & i = 1 \\ &= -\bar{m}_1 + (\mu + \alpha\sigma/\lambda)\bar{m}_1 + \psi & i > 1 \end{aligned}\right\} \qquad (17')$$

from which we can derive e_t as

$$e_t = \eta_1 e_{t-1} + \nu_1 \bar{m}_1 + \nu_2 m_{t-1} + \nu_3 \varepsilon_t + \nu_4 \qquad (18')$$

where $\nu_1 = (\eta_2 - \mu - \alpha\sigma/\lambda)\bar{m}_1/\lambda\eta_2(\eta_2-1)$, $\quad \nu_2 = -(\mu + \alpha\sigma/\lambda)m_{t-1}/\lambda\eta_2$,

$\nu_3 = (1 - \mu - \alpha\sigma/\lambda)\varepsilon_t/\lambda\eta_2$, $\quad$ and $\quad \nu_4 = -\psi/\lambda(\eta_2-1)$.

The condition under which overshooting occurs can now be derived. Once again $d\bar{e}/d\bar{m} = 1$ and

$$de_t/d\bar{m} - d\bar{e}/d\bar{m} = -\nu_2 - \eta_1 = \frac{1 - \mu\lambda}{\lambda\eta_2} \qquad (21')$$

Hence overshooting occurs if $\lambda < 1/\mu$. As $0 < \mu < 1$, this condition is less stringent than the earlier result that overshooting occurs if $\lambda < 1$. Thus adding a fixed delay in the response of the price level to excess demand, whilst making overshooting more likely, does not guarantee it.

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