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THE COSTS OF INFLATION: SOME THEORETICAL ISSUES

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ABSTRACT

Although for the most part this paper is concerned with others' work, it is not a survey of the literature. Notable omissions are: the shoe-leather effect, the Tobin effect, the real effects of nominal (government) institutions.

The first part looks at the relationship between inflation, relative prices and inefficiency. _Inter alia_, there is a critique of the way the price misperceptions model has been used in applied work, and a new model of the effects of consumers thinking in nominal terms.

The second part considers the consequences of, and possible reasons for, the non-indexing of loans. Finally, the relation between inflation and the stock market is discussed.

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SUMMARY

Conventional economic theory tells us that, since consumer and firm decisions depend on relative rather than absolute prices, a doubling of the money supply should leave the real side of an economy unchanged. It is therefore by no means clear why inflation, at least if it is fully anticipated, should be costly. However, there are a number of reasons for believing that this view is oversimplified. First, conventional economic theory tends to downplay the transactions role of money. Yet one cost of inflation may result from the rise of the (opportunity) cost of holding money for transactions purposes in inflationary times, which leads to people making an excessive number of trips to the bank (the "shoe leather effect"), and to a move from financial to real capital (the Tobin effect). Second, conventional theory tends to ignore taxes -- but if the tax system is not indexed, it is clear that inflation will have real effects as, for example, workers move into higher income tax brackets.

Such views about the cost of inflation are well known. The present paper is concerned with more recent contributions. I should point out that although it mainly looks at other economists' work, it is not meant to be a full survey.

I begin with the price misperceptions model of Lucas, Barro, and others, which is based on the idea that agents cannot distinguish between relative (real) shocks and absolute demand (nominal) shocks. This suggests that an increase in the variability of inflation due to an increase in the variability of the (unobserved) money supply can lead to inefficiencies in output and employment decisions -- with perhaps a concomitant increase in the variability of relative prices. An increase in the variability of relative shocks will also lead to inefficiencies. There have been a number of empirical studies which establish positive links between combinations of these variabilities. But, I contend, many of them correlate inappropriate pairs of

variables. Also, the theoretical predictions are less clear-cut than is often supposed. Certainly what is not true of the model is that an increase in the average level of inflation should have welfare costs, or that there should be a relationship between the average level of inflation and the variability of inflation or relative prices. Finally, it is not necessarily the case that greater price uncertainty leads to unambiguously lower welfare.

Also in Section A1, other lines of work concerning relative prices are touched upon. In particular, it may be that the act of changing nominal prices is itself costly, due, for example, to a firm having to update its sales catalogue. Sheshinski and Weiss have shown that under these circumstances, as the rate of inflation rise, when (monopolistic) firms adjust their prices they do so by greater percentages -- and out of phase from one another -- with the result that the variance of relative prices in the economy increases.

A new model is presented in Section A2 of the paper. (This is joint work with Oliver Hart.) Suppose consumers think in nominal, rather than real terms. There are two reasons why what they remember might be less informative when inflation is higher: (a) they may not have good recall of when they made their previous purchases; (b) they may have a form of "bounded rationality", in that they are poor at calculating percentage increases. The upshot of this fuzziness in consumers' knowledge of price distributions is that firms can exploit it in their pricing policies. All of the following are shown to rise with inflation: (i) the mean, variance, and width of the support of the distribution of prices; (ii) the amount of (costly) search which is done; and (iii) the total number of firms in the market (each incuring a fixed cost). From (ii) and (iii) it follows that the welfare effect is unambiguous: higher inflation imposes greater costs.

Section B1 of the paper considers the problems inflation can cause for firms (and also consumers) if loan sizes remain unchanged. The liquidity of firms may be severely reduced as nominal interest rates rise ("front-end loading"). This can cause firms to cut back on both output and employment. This effect seems potentially most important (see Wadhwani's empirical findings), but the economic and institutional puzzle is why loan sizes do not increase under inflation to keep real indebtedness constant.

A model by Gale may provide a clue; this is summarised in Section B2. Inflation may be related to business confidence in the following way. There is a conventional idea as to how much it is safe to lend on certain types of security. If this amount is expressed in terms of money, then the real value of it will be reduced by inflation. There is no money illusion at work here: each bank, say, is behaving rationally, but since it must take the behaviour of other banks as given, it must also take the convention as given. When the 'real' convention changes, the 'nominal' convention staying the same, everyone's real behaviour must adjust. Inflation may have real consequences (costs) simply because agents believe it will.

Finally, in Section B3, I briefly discuss Modigliani and Cohn's thesis that Stock Market values are adversely affected by inflation. Essentially, their idea is that investors commit two errors. First, they use nominal, rather than real, interest rates to capitalise equity earnings. Second, they use current cost accounting to measure firms' profits, instead of true economic profit.

Part A. Inflation and Relative Prices

This part of the paper is divided into two Sections. In Section A1 we review a number of the theories relating inflation to changes in relative prices. In the Section A2, a new model is presented based on an idea from Oliver Hart. Most of the technical discussion about this model has been put into an Appendix (1); one should therefore be able to read through the main text fairly easily.

A1. Inflation and relative prices: some current theories

It should be stated at the outset that this does not purport to be a review of the literature on inflation and relative prices. For comprehensive surveys of both theory and evidence, see the papers by Cukierman (1984) and Marquez and Vining (1983).

We will not consider (the rather poor) theories that seek to explain the strong empirical link between the level of inflation (anticipated or unanticipated) and the variance of (or uncertainty about) inflation. (On this, see the original paper by Okun (1971) and the subsequent work by, for example, Logue and Willet (1976), Foster (1978), Blejer (1979), Taylor (1981), and Pagan, Hall, and Trivedi (1983).) However, there is something we should bear in mind. In two of the sets of models given below -- the price misperceptions and the involuntary savings models -- it is uncertainty about inflation which is crucial. So if there is some other mechanism by which a higher level of inflation leads to greater uncertainty about inflation then these effects could be compounded -- thus making inflation *per se* the primary 'bad', which in a sense is the effect we seek to identify, should it exist.

Price Misperceptions

Perhaps the simplest of the price misperceptions models is Barro's (1976), which was based on a localized markets framework described by Phelps (1970) and employed by Lucas (1973). One of the main thrusts of this work is to show that anticipated monetary policy is neutral; only unanticipated shocks in the money supply can have real effects. In particular, an unanticipated increase in the money supply, with a concomitant increase in prices (inflation), can boost output and employment.

Barro's work was also concerned with how unanticipated changes in the money supply would affect relative prices. He found that, provided substitution effects dominate income effects in each of the markets, then an increase in the variance of unanticipated shocks to the money supply would increase the variance of relative prices.

There is more to all this than meets the eye, though. Barro's model assumes a perfectly symmetric market structure. Of crucial importance in analysing relative prices is his assumption that the price elasticities of demand and supply are the same in each market. To see why, and also to get a hold on some related matters, we now briefly look at a more general model based on that in Hercowitz (1981).

Let the (very large) set of markets be labelled z. At time t the (log) price in market z is $P_t(z)$. Participants in market z do not observe the (log) aggregate price level P_t, but form a rational expectation, EP_t based on $P_t(z)$ and their knowledge of the model. (Log) supply and demand in market z are respectively given by

$$y_t^s(z) = \alpha^s(z)[P_t(z) - EP_t] + e_t^s(z)$$

and $$y_t^d(z) = -\alpha^d[P_t(z) - EP_t] + (M_t - EP_t) + e_t^d(z)$$

where $\alpha^s(z)$ and α^d are positive. M_t is the (log) aggregate money supply, whose growth rate $M_t - M_{t-1}$ contains a predictable component, g_t, together with a white noise term, $m_t \sim N(0,\sigma_m^2)$. $e_t^s(z)$ and $e_t^d(z)$ are also white noise terms. All three noise terms are assumed to be independently distributed. Let $e_t(z) = e_t^d(z) - e_t^s(z)$, and suppose that $e_t(z) \sim N(0,\sigma_e^2)$. Notice that the market-specific shocks are assumed to be drawn from a common distribution: σ_e^2 does not depend on λ. But also notice that, unlike in Barro's model, the supply elasticities $\alpha^s(z)$ are not assumed to be the same across markets. (We could go further and have α^d depend on z too, but our point will be well made even without this.) Define $\Lambda(z) = 1/[\alpha^s(z) + \alpha^d]$, and let Λ be the average of the $\Lambda(z)$'s across all markets. Define $\lambda(z)$ be the deviation, $\Lambda(z) - \Lambda$, and let the 'variance' of the 'distribution' of $\lambda(z)$'s be σ_λ^2. (Our assumption that there is a large number of markets is useful because it enables us to treat the actual distribution of $\lambda(z)$ as though it were a true population distribution.) Incidentally, "knowledge of the model" includes knowledge of σ_m^2, σ_e^2, and σ_λ^2.

Using the method of undetermined coefficients, it can be shown that the rational expectations equilibrium satisfies

$$P_t = P_t^e + [\theta + \Lambda(1-\theta)]m_t \quad \text{(i)}$$

$$\text{and} \quad P_t(z) - P_t = (1-\theta)\lambda(z)m_t + [\theta + \Lambda(z)(1-\theta)]e_t(z), \quad \text{(ii)}$$

where $\theta = \sigma_m^2/[\sigma_m^2 + \sigma_e^2/\Lambda]$

and $P_t^e = M_{t-1} + g_t$ = aggregate price level at t, expected at t − 1.

We define the variance of (unanticipated) inflation to be Var $D[P_t - P_t^e]$, where $D[.]$ represents the first-difference operator. From (i), this equals $2[\theta + \Lambda(1-\theta)]\sigma_m^2$. If we assume, as does Barro, that substitution dominate wealth effects, then $\Lambda < 1$ and Var $D[P_t - P_t^e]$ is an increasing function of σ_m^2 and a decreasing function of σ_e^2.

In this stationary model, it would be natural to work with the variance of relative prices, Var $[P_t(z) - P_t]$. However, in most empirical studies, the variance of changes in relative prices is used: Var $D[P_t(z) - P_t]$. This has the advantage of staying constant whenever relative prices are adjusting steadily to changes in technology or taste.

From (ii) we can deduce that the variance of changes in relative prices is

$$\text{Var } D[P_t(z) - P_t] = 2(1-\theta)^2\sigma_\lambda^{\,2}\sigma_m^{\,2} + 2(1-\theta)^2\sigma_\lambda^{\,2}\sigma_e^{\,2} + 2[\theta + \Lambda(1-\theta)]^2\sigma_e^{\,2}.$$

Now Barro's finding was that Var $D[P_t(z) - P_t]$ increases with $\sigma_m^{\,2}$. His model is the special case where $\Lambda(z) \equiv \Lambda$; i.e. $\sigma_\lambda^{\,2} = 0$. Given $\Lambda < 1$, we can see that his result is indeed confirmed by our algebra, putting $\sigma_\lambda^{\,2} = 0$.

But the result is not true in general, because an increase in $\sigma_m^{\,2}$ decreases the second term in the expression for Var $D[P_t(z) - P_t]$, and decreases the first term if $\sigma_m^{\,2} > \sigma_e^{\,2}/\Lambda$. To understand this we must grasp the intuition behind all three terms on the RHS.

The first term is analogous to the Lucas hypothesis concerning the slope of the Phillips curve. The term captures how much variation in relative price is brought about not through genuine variations in relative excess demands, but rather through differential responses to changes in aggregate demand. (One can see why this must be an effect originating from differential elasticities $\alpha^s(z)$, because if there were no such differential then $\sigma_\lambda^{\,2}$ would be zero and the term would disappear.) Now for high starting values of $\sigma_m^{\,2}$, (i.e. more than $\sigma_e^{\,2}/\Lambda$), the effect of an increase in $\sigma_m^{\,2}$ on the variance of relative prices is negative because although the observed price $P_t(z)$ in market z may vary more, proportionately more of the variance is attributed to changes in the aggregate price P_t. On the other hand, for lower starting values of $\sigma_m^{\,2}$, not all the increase in the variance in $P_t(z)$ is attributed to shifts in P_t.

The second term comes from the positive interaction between the diversity in elasticities (the factor $\sigma_\lambda^{\,2}$) and the relative supply and demand shifts (the factor $\sigma_m^{\,2}$). (Note that a term of this sort would be present even if there was full information about prices.) With a more unpredictable money stock (higher $\sigma_m^2 \Rightarrow$ higher θ), this interaction is dampened as agents attribute a greater proportion of the fluctuation in $P_t(z)$ to changes in aggregate price P_t. So here the increase in σ_m^2 unambiguously dampens the variance in relative prices.

As we have already said, the third term is Barro's effect, and would be present even if all markets had identical price elasticities of supply (i.e. even if $\sigma_\lambda^2 = 0$). The point is that as σ_m^2 increases, the responsiveness of excess demand in market z, say, to shifts in the observed price $P_t(z)$ diminishes, because agents perceive these more as shifts in the aggregate price P_t. Accordingly, a given size of relative disturbance, $e_t(z)$, requires a larger movement in $P_t(z)$ in order clear the market. Thus increases in the variance of money supply amplify the effect on the variance of relative prices stemming from a given variance, σ_e^2, in relative disturbances.

Now the effect that a rise in the variance, σ_e^2, of relative disturbances has on the variation in relative price is not clear cut. The first two terms in the expression for Var $D[P_t(z) - P_t]$ do rise with σ_e^2, but we need conditions for the third term not to fall -- for example $\lambda > 1/3$, or $\sigma_e^2 > \sigma_m^2$, would suffice. [The specific condition is that $J \equiv \sigma_e^4/\Lambda + (3 - 1/\Lambda)\sigma_m^2\sigma_e^2 + \sigma_m^4$ should be non-negative.] The intuition underlying these three effects in many ways parallels that for the case of a rise in σ_m^2, so we won't work through it again. But notice how perverse it would be for an increase in the variability of relative disturbances not to increase the variability of relative prices. So, as a working assumption we might take $J > 0$, and so be assured that this perverse case does not arise.

There are two conclusions that can be drawn from all this which are less than reassuring, bearing in mind that (a) the model of price misperceptions supposedly provides a positive link between the variability of inflation and the variability of relative prices, (b) there is a strong empirical evidence (albeit with next to no theoretical underpinning) positively relating the level and the variability of inflation (see the references cited earlier: Okun et al), and (c) one is aiming to conclude that a cost of inflation is the inefficient resource allocation stemming from excessive variability in relative prices:

> First, if the cause of an increased variability in inflation is an increased variability in money stock, then although the theory does confirm a positive link between these two, it does not predict a necessarily higher variability in relative prices.
>
> Second, if there is an increase in the variability of relative disturbances (i.e. a rise in σ_e^2), then the theory predicts that although the variability of relative prices is most likely to rise, the variability in inflation will fall. This source of uncertainty, then, gives a negative relationship between the variabilities of inflation and of relative prices.

Notice that it must be assumed that the observed variability of inflation proxies unanticipated inflation. As Barro (and others) showed, all anticipated inflation is neutral in these models. If the actual economy was as modelled here then there would be nothing to choose between, say, a zero and a 20% inflation rate. The policy prescription is not to bring inflation down, but simply to keep it steady (or at least fully predictable).

In much of the applied work in this area, the chosen measure of relative price variability is VP_t, defined by

$$VP_t = \text{VARS D}[P_t(z) - P_t] = \frac{1}{|z|} \sum_z \{ [P_t(z) - P_t] - [P_{t-1}(z) - P_{t-1}] \}^2.$$

It follows from (ii) that

$$VP_t = (1-\theta)^2 (Dm_t)^2 + \{ (1-\theta)^2\sigma_\lambda^2 + [\Lambda + \theta(1-\Lambda)]^2 \} (\text{VARS } \varepsilon_t(z) + \text{VARS } \varepsilon_{t-1}(z))^2 \quad \text{(iii)}$$

where

$$Dm_t = m_t - m_{t-1}, \quad \text{VARS } \varepsilon_t(z) = \frac{1}{|z|} \sum_z \varepsilon_t(z), \quad \text{and} \quad \text{VARS } \varepsilon_{t-1}(z) = \frac{1}{|z|} \sum_z \varepsilon_{t-1}(z)$$

[Note that 'VARS' denotes the sample variance across markets.]

We can see from the RHS of (iii) what kinds of variables should (according to the theory) determine VP_t -- and what kinds should not. Clearly anticipated inflation (via P_t^e) should not. Nor should unanticipated inflation (via m_t); for consider a sequence of $\{m_t\}$ satisfying

$$m_t = \begin{cases} 0 & \text{for } t < \tau \\ m > 0 & \text{for } t > \tau. \end{cases}$$

This would lead to unanticipated inflation for all $t > \tau$; but VP_t would be constant except for $t = \tau$. (Unanticipated inflation <u>would</u> affect the variability of relative prices had the latter been measured as VARS $[P_t(z) - P_t]$ rather than as VP_t.) Finally, inflation <u>per se</u>, DP_t, should not affect VP_t. And yet all of these variables have been included as regressors in empirical work. For references, see Mizon and Thomas (1984).

The appropriate nominal shock variable is Dm_t. Since this is not observable, two alternatives are possible. The first is to estimate the m_t's using an unanticipated money equation. See Hercovitz (1981). The second possibility is to proxy it with $(D[P_t - P_t^e])^2$: recalling (i),

$$(D[P_t - P_t^e])^2 = [\Lambda + \theta(1-\Lambda)]^2 (Dm_t)^2.$$

Note that $D[P_t - P_t^e]$ can be written $D[(P_t - P_{t-1}) - (P_t^e - P_{t-1})]$, which is D[unanticipated inflation]. The point is that the more usual RHS inflation variables -- expected inflation, unanticipated inflation, actual inflation -- need to 'slip a derivative' if VP_t is on the LHS.

It could be strongly argued that not being able to see the economy-wide price level is too implausible an assumption to be taken seriously. There are, however, other, more convincing stories that one could tell. For example, in an economy with production lags, firms need to assess future relative prices. Now suppose that shocks are (imperfectly) serially correlated; the shocks are both to supply (technological) and demand (relative and aggregate). Cukierman (1982) has investigated this sort of economy. He assumed that all price information, past and current, is available to all agents. The agents, in particular the firms, still have a signal extraction problem: are the supply and demand shocks that they can see permanent or transitory?

This is clearly a considerably more complicated model to the one we analysed above. It is also driven by a very different effect; viz. confusion between permanent and temporary shocks, as opposed to local and aggregate. One should not necessarily expect the results to be the same. In fact they do turn out to be quite different. Cukierman's principal finding is that the variance of (unanticipated) inflation is positively related to the variance of relative price changes. [This former is the variance of the change in aggregate price between periods, and the latter is the variance of the change in any given relative price between periods. Now although we did not consider either of these two variances in our analysis above (we looked at variance of aggregate prices, P_t, and the variance of relative prices, $P_t(z) - P_t$), had we done so we would have found that our results would not have altered.] Cukierman finds reasonable conditions (see his Propositions 1 and 2) under which any source of increased uncertainty -- that is, an increase in the variance of any of the shocks itemised in the previous paragraph -- would lead to increases in both the variance of inflation and the variance of relative price changes.

In terms of providing theoretical support for empirical work, this alternative approach appears to be more fruitful, as well as being more realistic in its assumptions. Unfortunately it is also more complicated.

Turn now to the welfare costs of greater relative price variability. There are two obvious questions that arise. First, are welfare losses necessarily any greater if there is an increase, as opposed to a decrease, in the variance of changes in relative prices? We have been implicitly assuming that it is greater variability which is costly, and yet allocations are inefficient whenever prices deviate from their full-information levels. What is true of the models, though, is that the constrained-efficient policy is for the government to maintain a predictable money supply rule.

Second, as was first observed by Waugh (1944), and highlighted again by Fischer (1981) in the context of inflation, might not a reduction in uncertainty of price decrease consumer surplus? The point is that indirect utility is a convex function of price. However Samuelson (1972) showed that there is a crucial rider to this: it cannot be that both producers and consumers benefit from price destabilisation. But Waugh's argument does at least imply that the welfare effects of greater uncertainty about relative prices are not straightforward.

Costly Price Adjustment

The pioneering work was by Sheshinski and Weiss (1977). The argument is very simple. If the act of changing nominal prices is costly then during inflation a (monopolistic) firm will delay making a change until its relative price has dropped to some level s, say, which lies below that which would maximise profit in the absence of adjustment costs. Moreover, the new (relative) price S, say, will be above the profit maximising level.

There are quite a lot of reasons why changing the nominal price is costly. (Note that the assumption is of a fixed, real cost for each price adjustment -- the size of that adjustment is immaterial.) An obvious example is that of a firm having to update and recirculate its sales catalogue. Less obvious is the effect a price increase has on a firm's demand in a market with imperfect information; it may lose fewer customers on balance by changing price less frequently, because that way customers "know where they stand". (This last argument is speculative.)

If the inflation rate were to rise, then Sheshinski and Weiss show that s and S unambiguously fall and rise respectively. There is a corresponding greater variance (suitably measured) of relative prices in the economy. The welfare effects are not obvious, though. After all, their model starts with a monopoly, and so it cannot be said a priori whether consumer surplus is raised or lowered. Also, they found that, somewhat surprisingly, there can be circumstances where the rate of price adjustment -- and therefore the total adjustment cost incurred -- falls as inflation rises. Despite all this, the broad conclusion is pertinent: inflation is costly if nominal quantities cannot be changed costlessly.

A recent paper, Sheshinski and Weiss (1983), extends this work to the case of an economy where inflation varies in an unpredictable way -- specifically, the rate of inflation follows (a variant of) a 2-state continuous-time Markov Chain. (The (1977) paper was in effect a comparative statics exercise, looking at how price setting differs between two economies with different constant inflation rates.) Their interesting new finding is that now an increase in the variance of (expected) inflation causes s to fall and S to rise; this is reminiscent of results from the price misperceptions models, albeit via a totally different channel.

Involuntary Saving

Deaton (1977) proposed the following mechanism whereby consumption would fall if the rate of inflation unexpectedly rose. Consumers buy a basket of goods in sequence. That is, they make their first purchases before they have observed the prices of those goods they plan to buy last. If there has been a rise in the rate of inflation which they have not foreseen, then they might rationaly attribute the higher prices that they initially encounter to greater relative prices for those goods, and so buy less of them. Overall, they will be left holding more money (or wealth) than they would have chosen to had they known in advance about the general rise in prices. Thus unanticipated inflation induces a rise in the savings ratio, and, depending on one's macromodel, a fall in output and employment.

Once again it is unanticipated inflation which is bad. There are no long-run costs of high but steady inflation. In this respect Deaton's model is similar to, say, Barro's price misperceptions model. (Indeed, it is a special model of price misperceptions.) An interesting difference between Deaton's and Barro's models concerns the predictions about how an unanticipated rise in inflation would affect output and employment. In the former model, these would fall, whereas in the latter, they would rise.

Asymmetric Price Response

Suppose that prices adjust faster to excess demand than they do to excess supply (see Fischer (1982) and Pauls (1981). Then the distributions of price changes would be skewed to the right. An extreme version of this would be that prices never fall, or as Solow (1975) puts it: "prices have got out of the habit of falling". The idea of downwardly rigid wages was explored by Tobin (1972) in his model of the labour market.

If an economy whose prices respond asymmetrically is subject to a shock which, although on balance is in some sense 'neutral', affects markets differentially, then the average price level will increase. And if the money supply adjusts passively, this inflation will be accommodated. Moreover, the greater the variation in relative prices, the higher will be the rate of inflation.

Leaving aside whatever rationale there may be for why the prices respond asymmetrically, and ignoring the question of whether zero is the appropriate 'threshold', we can instead look at the welfare implications of these models. Suppose that x% is the threshold; price increases of less than x% are sluggish, whereas increases of more than x% are rapid. (In the previous paragraph, x was equal to zero.) Then surely the optimal rate of inflation would be above x%. Here (unless for some reason the threshold always catches up with the current rate of inflation), there are positive benefits to be had from higher inflation since it would prevent misallocations stemming from distorted relative prices.

A2. Consumers and nominal prices

Casual introspection leads one to realise that by and large we make our economic calculations in nominal rather than real terms. Perhaps, then, one of the costs of inflation is that the informational content of nominal values is lower. To be more specific: we carry around a whole variety of nominal prices in our minds, upon which we base our decisions. Suppose this information was gleaned at various past points in time and we would be hard pressed to say just when we observed what. Then, if there is inflation, our priors on what would today be a good price and what would be a bad price become more diffuse. The purpose of the analysis which follows is to see what ramifications -- in particular, welfare losses -- this might have in a well-specified model of consumer search.

The conclusions are all very clear. When, owing to inflation, consumers have what amounts to less information, firms can exploit this in their pricing policies. All of the following are shown to rise with inflation:

- the mean, variance, and width of the support, of the distribution of prices;
- the amount of (costly) search which is done;
- the total number of firms in the market (each incuring a fixed cost).

A formal model

Consider an economy in which the (continuous-time) inflation rate is a.

In the market for a certain good, all the firms have access to the same technology. B is the fixed (real) cost of setting up a plant which has capacity output of q units of good. Each of the q units costs b to produce. There is free entry, and suppose that in equilibrium there are M firms.

The only distinction between firms is their pricing policy. In equilibrium there are two (real) prices charged, p and λp, where $\lambda > 1$. A fraction μ of the firms charge the high price λp. The remaining fraction $1 - \mu$ charge p. Because of free entry, both kinds of firm make zero profit. The high price firms aim to sell to those "first arrival" customers who decide not to search further. Not all first arrivals will buy, and these firms' sales are accordingly less than capacity q. The low price firms sell to all their first arrival customers, and also to those consumers who initially visited a high price firm, but then decided to search. Each low price firm sells at full capacity q. The details of all this should become clearer once we have specified the consumers' behaviour.

There are N consumers in the economy, and periodically each buys one unit of the good. For each consumer, the interval T between purchases is uniformly distributed on $[t - \tau , t + \tau]$. The critical assumption we make is that, at the time of purchasing another unit, the consumer forgets when he made his last purchase. He does, however, remember the nominal price he paid last time, and also the posterior probability θ_o (possible equal to 1) he had then as to whether he was buying from a low price firm. He uses these two recollections, together with the distribution of $\tilde{T}$, to calculate the probability that the (nominal) price he observes at the first firm he visits is low or high. [We also need to assume that he randomly selects the first firm to visit, and does not (or cannot) revisit any firm that he knows from past experience charges the low price.]

These strong assumptions are made simply to capture the idea that consumers may think in nominal terms, with the consequence that in inflationary periods what they remember will have less informational value. In defence of the assumption that they forget when they made their last purchase, we might appeal to an alternative form of 'bounded rationality': even though people may have a good recall of the timing of purchases, nonetheless they are poor at calculating % increases. The effect of either poor memory or a low ability to calculate is much the same: there is scope for some firms to get away with charging a higher price.

It is actually simpler to carry out our analysis using real prices, even though inflation is at the heart of the model. Consider a "type" (p_o, θ_o) consumer who finds a price p_1 at the first firm he visits. p_o and p_1 may each be one of p or λp. (So for example if $p_o = p$ and $p_1 = \lambda p$, then he paid a low price last time, and faces the option of buying at the high price today.) Now he uses Bayes' Rule to calculate Prob $\{p_1$ is low$\}$.

At this point there are some details to sort out, but we can use a very convenient device. Define $\theta^* = (1 - 2\mu)/(1 - \mu)$, and assume that $0 < \mu < 1/2$ so that $0 < \theta^* < 1$. Suppose $\theta_o > \theta^*$. In Appendix 1 we show that if $\theta_o > \theta^*$ and $\lambda < e^{2a\tau}$, then

Prob $\{p_1$ is low$\}$ is <u>either</u> zero

<u>or</u> $> \theta^*$.

Search happens as follows. Visiting the first firm is costless. If the consumer decides to buy, then he will of course search no further. If on the other hand he does search, he incurs a (real) cost c, and learns <u>all</u> the other M − 1 firms' prices. (One might suppose that he bought a consumer magazine costing c.) The total costs from not searching are just p_1. He has to compare p_1 with the total costs from search:

$$c + p_1 \text{ Prob } \{p_1 \text{ is low}\} + p_1/\lambda\, [1 - \text{Prob } \{p_1 \text{ is low}\}].$$

Hence he decides no search / search according as to whether Prob $\{p_1$ is low$\}$ is greater than / less than $1 - \dfrac{c}{p_1(1 - 1/\lambda)}$.

Define $\theta_{crit} = 1 - \dfrac{c}{\lambda p(1 - 1/\lambda)}$.

Assume that μ and c are such that $0 < \theta_{crit} < \theta^*$. So, given $\theta_o > \theta^*$,

<u>either</u> the consumer (correctly) believes p_1 is high <u>with certainty</u>, and searches

<u>or</u> the consumer buys at price p_1.

Consequently, in both cases his posterior probability, θ_1, that he paid a low price this time, satisfies $\theta_1 > \theta^*$. This is true for all consumers, for all of their purchases. The assumption $\theta_o > \theta^*$ is therefore a justified one.

We deduce that the low price firms will <u>always</u> sell to "first arrival" customers. And a high price firm will manage to sell to <u>all</u> its first arrival customers who paid a high price last time, but will only manage to sell to those of its first arrival customers who paid a low price last time and whose Prob $\{ \lambda p \text{ is low} \} \neq 0$,

i.e. whose p_o satisfies $\quad 1/a \ln (\lambda p / p_o) < t + \tau$

i.e. whose T satisfies $\quad 1/a \ln (\lambda p / pe^{-aT}) < t + \tau$

or $T < t + \tau - 1/a \ln \lambda$.

Assuming the N consumers first arrive evenly distributed across the M firms, a high price firm will sell to

$$\phi N/M \quad + \quad (1 - \phi) N/M \left[1 - \frac{\ln \lambda}{2a\tau} \right]$$

consumers, where ϕ is the fraction of the N consumers who paid a high price for their last purchase. (In long run equilibrium ϕ also equals the fraction who pay high this time.)

A high price firm will choose λ to maximise profit:

$$(\lambda p - b) N/M \left[1 - \frac{(1 - \phi) \ln \lambda}{2a\tau} \right] - B$$

for which the first order condition is

$$2a\tau = (1 - \phi) \left[\ln \lambda + (1 - b/\lambda p) \right]. \qquad \text{(i)}$$

[Actually, matters are not quite this simple, because this is a left-hand derivative of profit. But this can be rationalised by appealing to a smoothing argument applied to the top of the support of the (uniform) distribution of T.]

The high price firm's maximised profit equals

$$(\lambda p - b)\ N/M\ \frac{(1 - b/\lambda p)(1 - \phi)}{2a\tau} \quad - \quad B \quad = \quad 0, \text{ by free entry.} \tag{ii}$$

There are μM high price firms. Therefore the number of consumers who pay the search cost equals

$$\frac{\mu N(1 - \phi)\ \ln \lambda}{2a\tau} \quad = \quad n, \text{ say.} \tag{iii}$$

Now ϕ is defined as the proportion of consumers who pay the high price,

$$\text{i.e.} \quad \phi N \quad = \quad \mu N \quad - \quad n. \tag{iv}$$

The $(1 - \mu)M$ low price firms sell to all their $(1 - \mu)N$ first arrival consumers, together with the n consumers who search. Each firm has a capacity q, so equating demand to supply,

$$(1 - \mu)N \quad + \quad n \quad = \quad (1 - \mu)Mq \tag{v}$$

Finally, with free entry, each (low price) firm makes zero profit:

$$(p - b)q \quad = \quad B. \tag{vi}$$

There are 6 equations, (i) - (vi), and 6 endogenous variables: M, μ, p, λ, ϕ, and n.

In Appendix 1 we show that provided c is not too large, there is a non-empty range $(\underline{a}\ , \overline{a})$ of values of the inflation rate, where $\underline{a} > 0$, such that, at the solution to (i) - (vi), the assumptions we have made are in fact satisfied: namely,

$$1 \quad < \quad \lambda \quad < \quad e^{2a\tau}$$

$$\text{and} \qquad 0 \quad < \quad \phi_{crit} \quad < \quad \phi^* \quad < \quad 1.$$

Suppose, then, a satifies $\underline{a} < a < \overline{a}$ and we are at an equilibrium. From (vi) we see that the price p is independent of the rate of inflation a. In Appendix 1 we also show that λ, μ, ϕ, M, and n are all strictly increasing functions of the inflation rate a. So too is the variance of prices:

$$V \quad = \quad p^2 \; (\lambda - 1)^2 \; \mu(1 - \mu).$$

Summary

The low price p is unchanged by inflation. But the high price λp rises with inflation. So too does the proportion μ of firms who charge the high price. Inflation also increases the variance V of the distribution of prices.

Now to the welfare effects. The number of consumers, n, who search and incur the cost c rises with inflation. So too does the total number of firms, M, who each have to pay the fixed cost B of setting up a plant. There is thus an unambiguous welfare cost to higher inflation.

Part B. Inflation, the Liquidity of Firms, and the Stock Market

Perhaps the most striking potential cost of inflation arises when loans are not indexed. This has ramifications for both households and firms, but in this part of the paper we consider only firms. [Jackman and Sutton (1982) present an interesting model in which inflation affects households' (real) borrowing, and hence consumption paths, because of the front-end loading associated with non-indexed loans.] The thrust of the discussion is that inflation reduces the liquidity of companies, which in turn increases the number of bankruptcies or at least causes firms to cut back their levels of output so as to avoid bankruptcy.

To see why inflation can have a dramatic effect on the cash flow of firms, imagine a firm which has borrowed £200 to purchase a machine. The loan is indefinite (the firm sold a perpetuity), and it is not indexed. By 'not indexed' we mean that the nominal value of the principal is fixed and in each period interest is paid at the prevailing nominal rate (which will of course vary with the rate of inflation -- so here 'not indexed' does not mean fixed interest). The real interest rate remains throughout 5%, and there is initially no inflation. Each period the revenue from sales of output produced by the machine is £100, and the cost of all inputs amount to £90. The firm is thus just breaking even, once the interest of £10 has been paid on debt. Suppose now that in some period, inflation rises to 20%. Revenue and input costs in the following period will then be £120 and £108. But since the (nominal) interest rate will be 25%, the firm's interest charges in that period will rise by 500% to £50 -- and the firm will make an accounting loss of £38. [For more about inflation accounting, see Section B3].

The point is that inflation causes front-end loading on non-indexed loans, and it is this which hurts cash flow. In the example, the firm is in effect paying back £38 / £240 = 15.3% of the outstanding real principal when it makes the interest payment £50. Of course, if inflation continued at 20%, then ultimately the real value of the nominal debt of £200 would drop to zero.

On the other hand, with an indexed loan the nominal value of debt each period rises in line with inflation, and interest would be paid at the real rate of 5%. Banks might argue that there is an informal equivalent to this: the firm services the original loan of £200 at the nominal interest rate of 25%, but meantime a second loan of £38 is made in the second period so as to maintain real indebtedness. The need to write formal indexed loan agreements is thereby obviated. The question is then, do banks actually behave this way? In a fascinating pair of papers, Wadhwani (1983 a and b) presents empirical and anecdotal evidence for the UK to suggest otherwise. Here, we look at some purely theoretical issues which arise in this context. The reader is urged to look at Wadhwani's work to be convinced of the practical importance of front-end loading induced by inflation.

First, there are a number of papers which ask why loans (or bonds) are not indexed. For example, see Levhari and Liviatan (1977) and, especially, Fischer (1977). However, there really are very few coherent reasons offered. One thing does emerge, however, which seems surprising. In practice it is the suppliers of indexed bonds (typically companies) who are unwilling to innovate, rather than the lending institutions. In what follows I will not take up this point any further, although this does not reflect the fact that I believe it to be unimportant; it is just that at present there is no theory to survey.

To reach a deeper theoretical understanding of the effects of inflation-induced front-end loading, we need to know precisely what is meant by the liquidity constraint of a firm. This is a vexed question, for it opens up a number of related issues, for example: what is the objective function of a firm, in which ways are capital markets imperfect, and which party or parties has the final control over when bankruptcy is declared? On this last, see Bulow and Shoven (1978). It is not surprising, then, that there is next to no theoretical work on how and why inflation might affect firms' liquidities. Gale (1982, Chapter 4, Part 2) has, however, provided a stimulating first attempt, and we will come to this later on.

B1. Owner-managed firms

The liquidity of corporations whose shares are publicly quoted on the Stock Exchange will of course be affected by the share price. However we defer discussing this until Section B3. In this Section, for simplicity we start by considering firms that are entirely funded by loans. That is, we consider owner-managed firms, where the entrepreneur has no personal resources. Although we will often refer to 'banks', this should be taken to mean any institution (or individual) that gives credit — for example, pension funds or insurance companies.

To focus our thinking, we need a model of the firm in which liquidity constraints are unambiguous, and can be shown to have real effects. A useful starting point is the model in Section 4.4 of Gale (1983) where there is a no bankruptcy condition. The enormous simplification which this condition brings is that there can be a single riskless rate of interest ρ — the credit market is therefore anonymous.

Incidentally, the subject of inflation will not crop up again for another 4 or 5 pages. The reader is asked to bear with the diversion, because it will help to clarify just why firms are liquidity-constrained, what is meant by a firm in 'financial distress', and why inflation may have real consequences on such a firm. In case you wish to skip the details, the intuition is rather simple. Suppose a bank has made the firm a long term (non-indexed) loan, on the basis of a probable healthy future stream of profit. If for some reason the long run prognosis changes for the worse (after the loan has been agreed), then the bank might decide not to extend any further credit. And other banks might also reach the same decision. When this happens, the firm is relying on current revenue to pay meet all costs, in particular interest payments. As long as this interest is being paid, the bank cannot exercise control over the firm and call in the receiver as it might wish. However, should inflation rise then the induced-front end loading could create difficulty with cash flow and, at worst, enable the bank to precipitate bankruptcy. Or at best the firm might have to cut back on output and employment in order to try to avoid bankruptcy; these real effects occur in the following model.)

Time is in discrete periods t indexed 0, $\pm$ 1, $\pm$ 2, ... Consider a firm which produces a single output which is sold at the end of each period at relative price $\tilde{p}$ -- a random variable with support $[\underline{p}, \overline{p}]$. $\underline{p}$ is strictly positive. The wage is chosen as the numeraire. During each period t, production takes place using labour ℓ_t to make $f(\ell_t)$ units of output, and the workers are hired at the beginning of the period.

The entrepreneur running the firm can borrow from a bank at the interest rate ρ. At the beginning of period t, the firm's total indebtedness to the bank is b_t, say (which includes the interest payable; interest accrues between periods). The firm is set up in period 1, so $b_1 = 0$, reflecting the fact the entrepreneur has no personal resources. Now if $y_t \geq 0$ are the earnings retained by the entrepreneur once the price p_t has been observed, then the firm's debt next period is given by

$$b_{t+1} = (1 + \rho)\{b_t - p_t f(\ell_t) + \ell_t + y_t\}. \qquad (1a)$$

The firm's objective is to choose $\{\ell_t, y_t, b_{t+1} | t \geq 1\}$ so as to maximise expected discounted value of the earnings stream:

$$E \sum_{t \geq 1} \delta^{t-1} y_t \qquad (2)$$

where δ is the firm's discount factor. But what is the firm's budget constraint?

Gale shows that with a no bankruptcy condition the firm's liquidity constraint takes the form

$$b_t \leq b^* \quad \text{for all } t \tag{1b}$$

for some constant b^*. A sketch of his argument, together with the definition of b^*, is given in Appendix 2.

The crucial no bankruptcy condition might be rationalized as follows. First, suppose that the costs to the entrepreneur -- in terms of loss of future creditworthiness, for example -- of declaring himself bankrupt are so high that he will always avoid it with certainty. (For this to be feasible, the worst price $\underline{p}$ must not be too low.) And second, suppose that banks cannot observe the outcome of a risky venture, and so cannot make the terms of interest and repayment of a loan contingent. If they could, then from a firm's perspective at least, there is a set of contingent markets — which we know leads to full efficiency (and the model would be robbed of its interest).

Notice that the fact that there is no bankruptcy does not mean that liquidity is not an issue -- quite the opposite: firms have to take every precaution to ensure that they can never become bankrupt.

The implication of (1b) is that the firm cannot always choose ℓ_t to equal the ℓ^* which attains the maximum of the expected present value of the stream of gross trading profits:

$$\pi^* = \underset{\{\ell_t\}}{\text{maximum}} \; E \sum_{t \geq 1} \beta^t \{ p_t F(\ell_t) - \ell_t \}$$

(ℓ^* is independent of t because the price distribution $\tilde{p}$ is stationary.) The reason is that with probability 1 there will eventually be a sequence of sufficiently poor prices that the firm's debt b_t will rise near to the upper limit b^*. When b_t rises near to (or reaches) b^*, then the firm has to cut back its labour demand to at least an $\ell_t < \ell^*$ satisfying

$$(1 + \rho) \left\{ b_t - \underline{p}f(\ell_t) + \ell_t \right\} \leq b^*.$$

(This is putting $y_t = 0$.) Thus the liquidity constraint <u>will</u> have real consequences. See all of Chapter 4 of Gale (1983).

Before leaving the model, it should be noted that had it been possible to sell the firm <u>as a going concern</u>, then the owner could realize the maximum expected present value of the firm, π^*, without ever going bankrupt. The strategy is to set $\ell = \ell^*$ provided $b_t - \underline{p}f(\ell^*) + \ell^* \leq \pi^*$, otherwise sell the firm for π^*. The crucial part of this is that we are implicitly assuming an unlimited supply of capital to pay for the firm's losses, no matter how large they may be.

This useful piece of analysis clears the ground for a more general model in which bankruptcy is admitted. First, recall that in having a no-bankruptcy condition we are implicitly assuming that creditors are unable to issue <u>contingent</u> loans — a possible justification for which is that the p_t's are only observable to the firm. (A less extreme assumption is that profitability is not observed by creditors, and profitability depends on other things besides price.) Now if bankruptcies can occur, then the loan might be conditional on the publicly observed signal: bankrupt/not bankrupt. In the event of bankruptcy, the capital assets of the firm (in the above model, there was only the entrepreneur's technology f(.)) may well be turned over to the creditor(s), although they may not be able to derive as much profit from them as could the entrepreneur. And if bankruptcy is not declared, then interest is paid at a rate $\rho + \xi$, say, where ξ is a risk premium. Gale and Hellwig (1983) have analysed a <u>one-period</u> loan model with bankruptcy. They have shown that even this seemingly straightforward problem is surprisingly tough. In particular, the size of the entrepreneur's initial wealth can have very subtle effects on the terms of the loan — i.e. on its magnitude and the value of ξ. These authors are currently examining a two-period model and report that the optimal structure there is yet more complicated. (The relevance to the many period model of the fact that entrepreneur's wealth has ambiguous effects in the one period model is that the wealth carried forward between future periods will presumably have equally ambiguous effects -- even though we might start by considering the benchmark model in which his initial wealth is zero, as above.)

The message from this is that it will be extremely difficult to understand a firm's real intertemporal liquidity constraint when bankruptcy is admitted, let alone take it one stage further to incorporate the effects of inflation. And all the while we are ignoring the possibility of a stock market.

What follows, then, is necessarily suggestive rather than rigorous.

Return to the model, but now, as well as allowing for bankruptcy, suppose that the firm's relative price of output no longer has a stationary distribution. We want to study not just short-run fluctuations in relative price, but also the longer term prospects for the product, over which there is also uncertainty. Suppose that as time evolves, so does knowledge about the distribution of future relative prices. For example, at time t information might become publicly available that the distribution of the relative price for all periods beyond some time $\tau > t$ has shifted downwards. In effect, the long term prospects for the firm are gloomier than before. This may mean that the firm is now not viable in the long run on average -- although of course there is always the possibility that later on prospects improve and the distribution of relative price is revised upwards.

Put formally, in each period t two random variables are sampled: the current (relative) price of output, and a signal s_t which adds to an evolving "core" of information concerning the distribution of future output prices. The current price is observed by the firm alone -- that is why loans cannot be contingent. And the sequence of publicly observed signals $\{s_t\}$ is consistent in that for any three periods $t < T < \tau$, the support of the distribution of τ-prices in s_T is a subset of the corresponding support in s_t.

Now even though all this makes a full analysis of lending still more daunting, consider a firm at time t for which s_t heralds (on average) lower future relative output prices. (The word 'relative' is of course important for when we come to inflation.) In fact suppose that s_t signals such a poor long run prognosis that no bank would make any further loans. But suppose also that the firm's present cash reserves (or more generally, liquid assets) are sufficient to meet the interest payments on outstanding debt, which is long term. Then provided the loans do not mature, creditors are unable to force bankruptcy, even though they might wish to in order to prevent the firm making any further losses.

The front-end loading induced by inflation could however make a substantial difference. Now at time t, the firm's cash reserves may be not be large enough to cover both the interest and the capital repayments concomitant with a nominal loan. Creditors can seize the opportunity to force bankruptcy.

If this is correct, and in inflationary periods bankruptcies do increase for liquidity reasons, then there is going to be an associated fall in employment ℓ and rise in risk premia ξ. The latter will exacerbate the firm's liquidity problems. Wadwhani (1983 a and b) discusses and estimates these effects of inflation on firms which are in what he terms "financial distress".

Unfortunately, the above arguments do not really stand up to rigorous examination. If creditors can use the onset of inflation to shut down firms in financial distress, then why are they unable to achieve the same end when there is no inflation? The point is to choose the duration of loans so as to maintain the same control over the firm's liquidity. Consider again a firm that has borrowed £200 from a bank; the loan is long term and not indexed. If the real interest rate is 5% and inflation is running at 20%, then in the first period £38, or 15.8% of the principal is repaid. We have seen why the bank will find it useful to have the option of forcing bankruptcy, in the event that the firm cannot meet its (nominal interest) repayment of £50. If there were no inflation the bank could exercise the same control by scheduling 15.8%, or £31.66, of the principal to be repaid after one period. Of course, this is a robust feature of any analysis: any nominal argument can be mimicked in real terms.

However, there are many codes of practice which are currently in use whose effect is not independent of inflation. Some of these will be discussed in the Section B3, where we introduce the stock market into our discussion. In this Section we mention one which is relevant to an owner-managed firm which only has access to funds borrowed from a bank, say.

Loans typically specify a minimum interest cover (say equal to 4), which is the ratio of gross trading profits to nominal interest payments. The agreement is that if this ratio falls below the specified minimum, then the creditor has the option of declaring the firm bankrupt. By this simple rule, the bank does retain a form of control over firms with long dated loans, even when the firm's cash flow is enough to meet the interest payments. Unlike inflation-induced front-end loading, this interest cover arrangement could very easily be made independent of inflation. In practice, however, the denominator of the ratio is nominal interest, which includes a capital repayment which rises with inflation. Hence the ratio falls with inflation, which presumably means that firms are more exposed to the possibility of the bank calling in a receiver in inflationary periods. (Broadly speaking, the minimum threshold appears not to fall with inflation.) Part of the CBI's evidence to the Wilson Committee (1980) was that the average level if interest cover for industrial and commercial companies had steadily fallen from 10.5 in 1960 to less than half that value by the mid-70's. See Wadhwani (1983 b).

Can this kind of 'collective irrationality' be given a sounder theoretical footing? Gale (1982, Chapter 4.2) has provided a stimulating model, which may well point to a fruitful line of research. His idea is that, owing to market imperfections, there can be an entire sequence (more specifically, a continuum) of "bootstraps" equilibria in the market for loanable funds. When there are multiple equilibria, some convention is needed to select one. A good example would be the minimum interest cover arrangement described in the previous paragraph. Now by using this particular convention, it happens that inflation has real costs, because with higher inflation the economy slips from one equilibrium on the continuum to an inferior one: higher inflation means the minimum interest cover constraint will bind more tightly. Another, slightly less transparent, example of a convention which is not independent of inflation is the use of non-indexed loans. As inflation increases from zero, the code of practice concerning the speed of repayment of principal shifts from one equilibrium in which none is paid until the loan matures to another equilibrium in which there is front end loading.

The nature of the market imperfections might be as follows. (From here on, the discussion is couched in <u>real</u> terms.) A bank will lend to a firm an amount based on, among other things, the future stream of profits. In general this amount will exceed the second-hand price that the capital stock would fetch, because there is more to a firm than this; there are also the specific skills of the workforce, the goodwill in the product market, and so on. Assume that the firm cannot be sold <u>intact</u> -- i.e. there is no stock market. Then there are a number of reasons why the bank might not take solely the future stream of profit (and associated risks of bankruptcy) into account when making the loan, but would also be concerned with the firm's access to funds from <u>other</u> banks. For example the bank may want to diversify, and so would not want be the sole lender. Or the firm may at some future date need to increase its indebtedness, at which time the bank may not itself be in a position to make a second loan. Or finally, for flexibility, the bank may only want to lend short, so that it can have ready access to its own capital. (It is this last which Gale models formally, and which we summarise below in Section B2.)

Notice that if a bank's lending policy is conditional on (future) lending by other banks, then there is scope for many Nash equilibria. In our context, if inflation is generally thought to be damaging to industrial profits, and banks generally cut back their supply of credit, then it is irrational for any particular bank to step out of line -- given that it is not willing to provide a firm with credit indefinitely.

This informal discussion of Gale's ideas may be helpful, but it is important to examine critically just what assumptions are required to make such a model hang together. For this reason, we now look at the bare essentials of his formal analysis. Note that this model will also be very germane when we come in Section B3 to discuss similar 'collective irrationalities' -- that is, codes of practice that are not independent of inflation -- in the stock market.

B2. "Bootstraps" Equilibria in the credit market.

The following is a simplified presentation of the model in Chapter 4.2 of Gale (1982). It should be pointed out that the definition of equilibrium has to be somewhat more precise than that given here, where we examine only stationary equilibria.

The economy comprises consumers and firms, with time divided into discrete intervals. In addition to labour, there is a single good, which can be both consumed and invested as capital. Consumers live for just two periods, and there are overlapping generations. Equal numbers of young consumers are born each period. Firms last for ever.

Only the young consumers work. They each supply their labour inelastically, and are paid a wage of ω units of the good. Some of this is consumed, and the rest saved at an interest rate ρ. If $s(\omega,\rho)$ is the amount saved, then their (optimal) consumption pattern is $[\ \omega - s(\omega,\rho)\ ,\ (1+\rho)s(\omega,\rho)\ ]$. A "bank" is a syndicate of young consumers who each have deposited $s(\omega,\rho)$.

Banks compete to lend to firms, but because all banks' "depositors" lend for one period only, a firm is only able to borrow short. Every period each firm has to find another bank from which to borrow, so it can repay the loans it took out in the previous period.

This is the central feature of the model, and it is on these grounds that the model is most open to criticism. Notice however that the two period assumption is not crucial; the results would essentially carry through were one instead to model consumers with longer (but finite) lives.

Another crucial assumption is that firms cannot be bought and sold; there is no stock market. Each firm is run by an entrepreneur who has no initial wealth but borrows from banks to invest in capital. There is free entry (an unlimited supply) of entrepreneurs. Once capital is installed by an entrepreneur it cannot be used by someone else; each entrepreneur has specific skills.

The upshot of these rather extreme assumptions is that, in equilibrium, the only way a bank can recover a loan made to some firm is to find another bank willing to make a new loan. No doubt the assumptions can be weakened, but the basic feature has to remain: the capital market is imperfect in so far as the extent to which lenders can recoup their money depends on what others are willing to lend.

If in period t the capital stock of a typical firm is k_t, then it can produce $f(k_t,\ell)$ units of good by employing ℓ workers. Define $\ell(k_t,\omega)$ to be the labour demand which attains the maximum gross profit

$$\pi(k_t,\ell) = \underset{\ell}{\text{maximum}} \quad f(k_t,\ell) - \omega\ell.$$

Should the firm borrow b_t in period t in order to finance a (non-negative) investment $k_{t+1} - k_t$, then the net income of the entrepreneur is

$$y_t = \pi(k_t,\omega) + b_t - (1 + r)b_{t-1} - (k_{t+1} - k_t).$$

Notice that, owing to the banks' two-period horizons, the firm has to repay all of last period's debt, b_{t-1}, with interest.

The firm maximises the discounted sum

$$\sum_{t \geq 0} \frac{1}{(1+\rho)^t} y_t \qquad \text{(M)}$$

subject to, for all t,

$$y_t \geq 0 \qquad \text{(Bankruptcy constraint)}$$

$$k_{t+1} - k_t \geq 0 \qquad \text{(No capital decumulation)}$$

and

$$b_t \leq v^* \qquad \text{(Borrowing constraint)}$$

The interest of the model lies with the borrowing constraints. We are going to demonstrate that there does exist indeed a stationary equilibrium in which v^* is a constant. But this by no means exhausts the possibilities. Broadly, the borrowing constraint depends on the entrepreneur's collateral, which in this case is not k_t (since this cannot be sold), but the value of the firm as a going concern. Now this value will in general depend on today's capital stock k_t and outstanding debt b_{t-1}. Because no bank lending today is interested in future lending the value of the firm will depend on the borrowing constraints imposed by <u>other</u> banks tomorrow, which in turn will depend on other banks' behaviour in the next period, and so on. We suppose that today's banks take these other constraints as given (the Nash hypothesis). To repeat, in general the value of the firm will also depend on its current capital stock k_t and its outstanding debt b_{t-1}, but we are concentrating on an equilibrium in which v^* is a constant.

An immediate observation is that in equilibrium a bank should never <u>impose</u> v^*, even though it makes the assumption that other banks will do so in the future. If a firm would wish to borrow $b_t > v^*$ at interest rate ρ, then the bank could offer more than v^* at a rate higher than ρ. Both parties would benefit. In equilibrium, then, the firm must <u>choose</u> b_t no higher than v^*.

Given this, it might appear that firms are not at all constrained in their borrowing. But consider a vector (ρ^*, ω^*, k^*, α^*; v^*), where α^* is the number of firms divided by the number of workers in each generation (recall that there is free entry of entrepreneurs), such that

(i) for each firm:

in the first period: $k_o = 0$ (no initial capital)

and $b_o = k_1 = k^* = v^*$ (maximum borrowing to invest in capital)

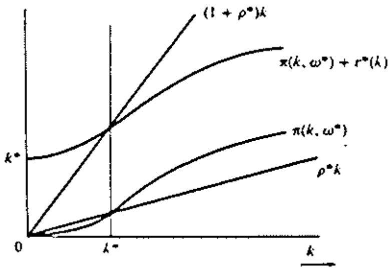
thereafter: $b_t = k_{t+1} = k^* = v^*$ and $\pi(k^*,\omega^*) - \rho^*k^* = 0$

(gross profit equals interest; new borrowing pays off the old debt)

(ii) $s(\rho^*,\omega^*) = \alpha^*k^*$ (goods market equilibrium; savings = investment)

(iii) $1 = \alpha^*\ell(k^*,\omega^*)$ (labour market equilibrium)

We will show that this is an equilibrium by making an assumption about $\pi(k,\omega^*)$ as a function of k, which is best seen graphically:



The critical features are that

(a) $k < (=) k^* \Rightarrow \pi(k,\omega^*) < (=) \rho^*k$

and (b) $k > k^* \Rightarrow (1 + \rho^*)k > \pi(k,\omega^*) + v^*.$

(a) amounts to assuming that returns to capital are initially increasing. (b) will be satisfied if profits never rise disproportionately with capital (i.e. if $\frac{\partial\pi}{\partial k}$ is never > 1.)

To confirm that, under these assumptions, we are at an equilibrium, we need to see if the sequence as defined in (i) maximises (M), and in each period the constraint v^* does not bind, taking the future constraints as given.

Notice that, because of free entry, in the purported equilibrium the net income y^*, say, of any entrepreneur is zero in every period.

Consider some other sequence $\{ k_t, b_t \}_{t>0}$ yielding net incomes $\{ y_t \}_{t>0}$ For $T > 0$, define Y_T as the present value of the first T period's net incomes:

$$Y_T = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} y_t$$

$$= \sum_{t=1}^{T} \frac{1}{(1+\rho)^t} \{\pi(k_t,\omega) - \rho k_t\} + \frac{1}{(1+\rho)^T}(b_T - k_{T+1}).$$

If Y_∞ is to be strictly positive, then appealing to (a) we can see that k_t must exceed k^* for at least one $t \geq 1$. Let τ be the minimum such t. Since $Y_{\tau-1} \geq 0$, from (a) we have that $b_{\tau-1} \geq k_\tau$. Hence $b_{\tau-1} > v^*$ (because $k^* = v^*$). That is, the borrowing constraint was exceeded in period $\tau - 1$. But because $b_{\tau-1} \geq k_\tau$ and $k_{\tau+1} - k_\tau \geq 0$,

$$0 \leq y_\tau \leq \pi(k_\tau,\omega^*) + b_\tau - (1+\rho)k_\tau, \quad \text{which from (b) is}$$

$$< b_\tau - v^* \qquad (\text{since } k_\tau > k^*)$$

-- which means that in period $\tau - 1$, when the borrowing constraint was exceeded, the bank which made the loan $b_{\tau-1}$ would know that borrowing the following period, b_τ, would have to also exceed v^*. But this runs counter to the Nash hypothesis that banks take other banks' lending policies as given. Therefore $\{ k_t, b_t \}_{t>0}$ cannot be admissible.

So $\{ \rho^*, \omega^*, k^*, \alpha^*; v^* \}$ is an equilibrium vector. Notice that this equilibrium is parameterised by our initial choice of v^*. Now it is not difficult to show that for any v^{**} in a neighbourhood of v^*, a corresponding equilibrium vector can be found. That is, there exists a continuum of equilibria.

To recap: v* might be viewed as an index of business confidence, perhaps related to the rate of inflation. The link might be that "there is a conventional idea as to how much it is safe to lend on certain types of security. This amount will be expressed in terms of money. Inflation will reduce the real value of this nominal amount. The economy slides along the continuum of Nash equilibria as a result, but there is no money illusion at work here. Each agent is behaving rationally but since he must take the behaviour of other agents as given, he must also take the convention as given. When the 'real' convention changes, the 'nominal' convention staying the same, everyone's real behaviour must adjust. Thus inflation may have real consequences simply because agents believe it will have real consequences." (I.e. with higher inflation, confidence is lower, and the borrowing limits of firms are tighter, which has deleterious real effects on output and employment.) "The fault lies not with inflation, but with the nature of our beliefs." (Gale (1982) pp 178-179.)

Finally, it should be noted that the formal model is actually a comparative statics exercise -- there are no dynamics specified as to how an economy might slide from one equilibrium to another. This contrasts with the earlier informal discussion which was largely about the effects of a rise in inflation. Also, there is no close analogy between the 'rules of thumb' that we discussed (e.g. nominal interest cover, or the use of non-indexed loans) and the model's v*. Nonetheless, Gale's formal analysis is extremely useful because it not only clarifies the kinds of assumptions that are likely to be needed to sustain multiple "bootstraps" equlibria, but, as said earlier, it also points towards a potentially fruitful line of research.

B3. Corporations and the Stock Market

Modigliani and Cohn (1979) argue that in inflationary periods, private corporations are severely under-valued by the Stock Market. Specifically, in the US in 1977 the Standard and Poor's 500 stock index was 100 -- when, they say, it should have been 200. The source of this under-valuation is twofold.

First, investors and managers capitalize equity earnings using the nominal interest rate, even though the earnings themselves have been deflated.

Second, they use current cost accounting (CCA) to measure firms' profits, rather than true economic profit. The former is an 'entity' based profit measure; the latter is 'equity' based. The distinction between these two is that CCA deducts the whole of nominal interest payment -- which, as we have seen, will contain some repayment of real principal when there is inflation -- whereas true equity profit treats debt as if it were indexed. [For an illuminating helpful reference on inflation accounting, see Whittington (1983). Also refer to the article on "Indexing Company Accounts" by A.J. Merrett in Liesner and King (1975).]

We will here be mainly concerned with the second source of under-valuation, for which the evidence is particularly strong. See Wadhwani (1983 b) as well as Modigliani and Cohn.

Consider the simplest possible case of a firm which at the beginning of period 0 has an outstanding non-indexed bank loan of £ $D(0)$. The real interest rate is constant at ρ. Inflation is running at λ each period, and the nominal interest rate is $R = \rho + \lambda$. Modigliani and Cohn's point is seen most starkly by assuming that the firm does not change its leverage policy. To this end, the firm must take out an additional loan each period so as to maintain real indebtedness. The nominal debt outstanding at the beginning of period t, then, is £ $D(t)$, where $D(t) = (1 + \lambda)^t D(0)$. Notice that in terms of both nominal and real quantities, this scenario is almost identical to another in which the original period 0 loan is instead indexed. The only difference is that in our case, as the firm pays nominal interest to the bank with one hand, it gets a new loan from the bank with the other hand; and the former amount exceeds the latter by $\rho\, D(t)$, which is of course the interest the firm would have paid had the initial loan been indexed. Given the real economic equivalence of these two scenarios, it seems extraordinary that, as a result of an accounting convention, the firm's share price should in practice be different between the two.

The firm's gross trading profit (i.e. before any interest is deducted) equals £ $X(t)$ in period $t > 0$. We assume that this is constant in real terms; i.e. $X(t) = (1 + \lambda)^t X(0)$. Then the CCA profit in period t is given by

$$\Pi(t) = X(t) - R\, D(t)$$

And the equity profit in period t is

$$\Pi^*(t) = X(t) - \rho \left[(1 + \lambda)^t D(0) \right]$$
$$= \Pi(t) + \lambda D(t).$$

If investors use $\Pi(t)$ as their profit figure, then the value of equity will be

$$S(0) = \sum_{t>0} \frac{1}{(1 + \rho)^t} \left[X(t) - R\, D(t)\right] (1 + \lambda)^{-t}$$

$$= \frac{(1 + \rho)}{\rho} \left[X(0) - \lambda D(0)\right] - (1 + \rho)\, D(0).$$

If, however, they instead used $\Pi^*(t)$ then the value of equity would be

$$S^*(0) = \sum_{t>0} \frac{1}{(1 + \rho)^t} \left[X(t) - \rho\, (1 + \lambda)^t D(0)\right] (1 + \lambda)^{-t}$$

$$= \frac{(1 + \rho)}{\rho} X(0) - (1 + \rho)\, D(0),$$

which exceeds $S(0)$ by $\lambda D(0)(1 + \rho)/\rho$.

One central tenet of Modigliani and Cohn's work is that the stock market values the firm at $S(0)$, rather than $S^*(0)$. It is clear that $S^*(0)$ is the correct figure to an economist; $S(0)$ varies with the inflation rate λ, and therefore cannot be right because nothing real changes with inflation.

The other under-valuation comes about because instead of using $1/(1 + \rho)$ as the discount factor, many financiers use $1/(1 + R)$.

The Modigliani-Cohn theory has of course met with criticisms. For a selection, see Boeckh and Coghlan (1982). It seems most implausible that this kind of irrationality should persist. Return to the choice between CCA and equity profit; both in the UK and US some loose equivalent to CCA is currently used. An obvious opportunity for a kind of arbitrage would be to buy a controlling influence in a company with a high debt-equity ratio, and then issue new shares to pay off the debt. If there is inflation, CCA profit will rise, and so will the value of one's equity. (Of course this is ignoring, inter alia the costs of issuing new shares.) What is interesting is to know what non-economists think. See Carsberg and Day (1983). Particularly illuminating is the response of investors and managers alike when it is pointed out that the reduction in real debt caused by inflation should be added to CCA in order to determine true economic profit. They say that what matters is not the value of true economic profit, but rather the value that other people put on a company. This is very reminiscent of Gale's "bootstraps" argument -- albeit that he had to explicitly rule out a Stock Market in his model. It would be nice to have a coherent theoretical model which rationalises the kind of behaviour which Modigliani and Cohn describe.

Finally, return to the question of liquidity -- this time of corporations which can raise money by selling shares. As we saw earlier, specifying the liquidity constraint is extremely difficult unless one makes strong assumptions -- like no bankruptcy and a stationary distribution of relative output price. In broad terms, though, it is likely that if inflation does strain a firm's liquidity, then this will create uncertainty about its future prospects, which will in turn be reflected in the price of its shares. Conversely, any drop in Stock Market value brought on by inflation (e.g. via a 'Modigliani-Cohn' effect) will cause banks to lower their borrowing limits. A well-specified model must of course make clear the interplay beween these two effects.

Appendix 1.

Facing price p_1, a "type" (p_o, θ_o) consumer uses Bayes Rule to calculate Prob $\{ p_1 \text{ is low} \}$ =

$$\frac{(1-\mu)\left[\theta_o f\left(\frac{1}{a}\ln\frac{p_1}{p_o}\right) + (1-\theta_o) f\left(\frac{1}{a}\ln\frac{\lambda p_1}{p_o}\right)\right]}{(1-\mu)\left[\theta_o f\left(\frac{1}{a}\ln\frac{p_1}{p_o}\right) + (1-\theta_o) f\left(\frac{1}{a}\ln\frac{\lambda p_1}{p_o}\right)\right] + \mu\left[\theta_o f\left(\frac{1}{a}\ln\frac{p_1}{\lambda p_o}\right) + (1-\theta_o) f\left(\frac{1}{a}\ln\frac{p_1}{p_o}\right)\right]\}}$$

$$\text{where} \quad f(x) = \begin{cases} 1/2\tau & \text{if} \quad t-\tau \leq x \leq t+\tau \\ 0 & \text{otherwise.} \end{cases}$$

Hence, for this consumer, Prob $\{ p_1 \text{ is low} \}$ equals

$$\begin{cases} 0 & \text{if} \quad t-\tau \leq \frac{1}{a}\ln\frac{p_1}{\lambda p_o} \leq t+\tau < \frac{1}{a}\ln\frac{p_1}{p_o} < \frac{1}{a}\ln\frac{\lambda p_1}{p_o} \\ \frac{\theta_o(1-\mu)}{\theta_o(1-\mu)+\mu} & \text{if} \quad t-\tau \leq \frac{1}{a}\ln\frac{p_1}{\lambda p_o} < \frac{1}{a}\ln\frac{p_1}{p_o} \leq t+\tau < \frac{1}{a}\ln\frac{\lambda p_1}{p_o} \\ 1-\mu & \text{if} \quad t-\tau \leq \frac{1}{a}\ln\frac{p_1}{\lambda p_o} < \frac{1}{a}\ln\frac{p_1}{p_o} < \frac{1}{a}\ln\frac{\lambda p_1}{p_o} \leq t+\tau \\ \frac{1-\mu}{1-\mu+(1-\theta_o)\mu} & \text{if} \quad \frac{1}{a}\ln\frac{p_1}{\lambda p_o} < t-\tau \leq \frac{1}{a}\ln\frac{p_1}{p_o} < \frac{1}{a}\ln\frac{\lambda p_1}{p_o} \leq t+\tau \\ 1 & \text{if} \quad \frac{1}{a}\ln\frac{p_1}{\lambda p_o} < \frac{1}{a}\ln\frac{p_1}{p_o} < t-\tau \leq \frac{1}{a}\ln\frac{\lambda p_1}{p_o} \leq t+\tau \end{cases}$$

Note: The sixth possibility,

$$\frac{1}{a}\ln\frac{p_1}{\lambda p_o} < t-\tau \leq \frac{1}{a}\ln\frac{p_1}{p_o} \leq t+\tau < \frac{1}{a}\ln\frac{\lambda p_1}{p_o}$$

is ruled out as it is shown that, in equilibrium, $\lambda < e^{2a\tau}$.

Now $\theta^* = (1 - 2\mu)/(1 - \mu)$ satisfies

$$\theta^* = \frac{\theta^*(1-\mu)}{\theta^*(1-\mu) + \mu} .$$

So if $\theta_o < \theta^*$, if $\lambda < e^{2a\tau}$, and if $\mu < 1/2$ (=> $\theta^* > 0$), then

Prob $\{ p_1$ is low $\}$ is <u>either</u> = 0

<u>or</u> > θ^*.

There is one further consideration. If a firm charged a (real) price either higher than λp, or lower than p, then there would be <u>some</u> consumers who would know with certainty that the firm was "off the equilibrium path" -- to use the language of games with incomplete information. (We are talking about a type (p_o, θ_o) consumer who observes a price p_1, where

either $1/a \ln (p_1/\lambda p_o) > t + \tau$ or $1/a \ln (\lambda p_1/p_o) < t - \tau$.)

In such games, the specification of beliefs off the equilibrium path -- that is, what such consumers would think if faced with a deviant firm's price -- can critically affect the nature of the (sequential) equilibrium. Moreover, it can be fairly arbitrary as to what constitutes 'reasonable' beliefs; Bayes' Rule cannot be applied at events whose probability is zero. Here, however, we are fortunate because there is really only one sensible specification of beliefs:

if $1/a \ln (p_1/\lambda p_o) > t + \tau$ then Prob $\{p_1$ is low $\}$ = 0,

and if $1/a \ln (\lambda p_1/p_o) < t - \tau$ then Prob $\{ p_1$ is low $\}$ = 1.

Equation (ii) can be written

$$M = \frac{\lambda pN (1 - \phi) (1 - b/\lambda p)^2}{B\ 2a\tau} \qquad \text{(vii)}$$

Substituting for M and n from (iii) and (vii) into (v) gives

$$\frac{(1-\mu)\, q\lambda p\, (1-\phi)\, (1-b/\lambda p)^2}{B\ 2a\tau} = 1 - \mu + \frac{\mu\,(1-\phi)\ln\lambda}{2a\tau}$$

Substituting $qp/B = 1/(1 - b/p)$ from (vi), and for $2a\tau$ from (i), gives, on cancelling $(1 - \phi)$:

$$\frac{(1-\mu)\,\lambda\,(1-b/\lambda p)^2}{(1-b/p)} = (1-\mu)\left[\ln\lambda + (1-b/\lambda p)\right] + \mu\ln\lambda$$

$$\text{or} \qquad 1-\mu = \frac{(1-b/p)\ln\lambda}{(1-b/\lambda p)\,(\lambda-1)}, \qquad \text{(viii)}$$

which means that μ does lie in (0,1).

From (i), (iii), and (iv),

$$\mu - \phi = \frac{\mu\ln\lambda}{\ln\lambda + (1-b/\lambda p)}$$

$$\text{or} \qquad \phi = \frac{(1-b/\lambda p)}{\ln\lambda + (1-b/\lambda p)} - \frac{(1-\mu)\,(1-b/\lambda p)}{\ln\lambda + (1-b/\lambda p)}$$

$$\text{Using (viii),} \quad \phi = \frac{(1-b/\lambda p)}{\ln\lambda + (1-b/\lambda p)} - \frac{(1-b/p)\ln\lambda}{(\lambda-1)\left[\ln\lambda + (1-b/\lambda p)\right]}$$

$$\Rightarrow \qquad 1-\phi = \frac{(\lambda-b/p)\ln\lambda}{(\lambda-1)\left[\ln\lambda + (1-b/\lambda p)\right]}. \qquad \text{(ix)}$$

Substituting (ix) into (i) gives

$$2a\tau = \frac{(\lambda-b/p)\ln\lambda}{(\lambda-1)}. \qquad \text{(x)}$$

The variance in prices V, say, is given by

$$V = p^2\,(\lambda-1)^2\,\mu(1-\mu)$$

$$\text{or from (viii)} \quad = \frac{p^2\,\mu\lambda\,(1-b/p)\,(1-1/\lambda)\ln\lambda}{(1-b/\lambda p)}. \qquad \text{(xi)}$$

Routine differentiation of (x), and then (viii), shows that $\lambda(a)$ and $\mu(a)$ are strictly increasing functions of the inflation rate a. [We use the fact that if $\lambda > 1$, then $\ln\lambda < \lambda - 1 < \lambda \ln\lambda$.] From (ix) and (xi) it follows that $\phi(a)$ and $V(a)$ are also strictly increasing functions of a.

Substituting for $(1 - \phi)/2a\tau$ from (i) into (vii):

$$M = \frac{\lambda p N (1 - b/\lambda p)^2}{B\left[\ln\lambda + (1 - b/\lambda p)\right]}$$

which, on differentiating, gives that $M(a)$ is strictly increasing in a.

Finally, substituting for $(1 - \phi)/2a\tau$ from (i) into (iii), and using (viii) to eliminate μ:

$$n = \frac{N \ln\lambda}{\ln\lambda + (1 - b/\lambda p)}\left\{1 - \frac{(1 - b/p)\ln\lambda}{(1 - b/\lambda p)(\lambda - 1)}\right\}$$

which, on differentiating, gives that $n(a)$ is strictly increasing in a.

It remains to demonstrate that there is a non-empty interval $(\underline{a}, \overline{a})$ of inflation rates such that, at a solution to (i) - (vi), the assumptions we have so far made are in fact satisfied. Namely,

$$1 < \lambda < e^{2a\tau}$$

$$\text{and} \quad 0 < \theta_{crit} < \theta^* < 1.$$

Let $a_{min} = (1 - b/p)/2\tau$. From (vi), a_{min} is strictly positive, and from (x) we have that if $a > a_{min}$ then $1 < \lambda < e^{2a\tau}$, as required. Moreover from (x), $\lim_{a \downarrow a_{min}} \lambda(a) = 1$ and $\lim_{a \uparrow \infty} \lambda(a) = \infty$.

Therefore from (viii) it follows that $\lim_{a \downarrow a_{min}} \mu(a) = 0$ and $\lim_{a \uparrow \infty} \mu(a) = 1$.

For $a > a_{min}$, $\theta^*(a)$ [defined as $(1 - 2\mu)/(1 - \mu)$] is a strictly decreasing function of a because it decreases with μ and $\partial\mu/\partial a > 0$. Also

$$\lim_{a \downarrow a_{min}} \theta^*(a) = 1 \quad \text{and} \quad \lim_{a \uparrow \infty} \theta^*(a) = -\infty.$$

$\theta_{crit}(a)$ [defined as $1 - c/(\lambda p - p)$] is a strictly increasing function of a because it increases with λ and $\partial\lambda/\partial a > 0$. Also

$$\lim_{a \downarrow a_{min}} \theta_{crit}(a) = -\infty \quad \text{and} \quad \lim_{a \downarrow \infty} \theta_{crit}(a) = 1.$$

Let $a = \bar{a}$ be the (well defined) solution of $\theta^*(a) = \theta_{crit}(a)$, where $\bar{a} > a_{min}$. Now the value of $\bar{a}$ decreases as the cost of search, c, decreases. For sufficiently low c, then, $\bar{a}$ will satisfy $\theta^*(\bar{a}) > 0$. Moreover, there will be some $\underline{a}$ contained in $(a_{min}, \bar{a})$ such that $\theta_{crit}(\underline{a}) = 0$.

By construction, for any a satisfying $\underline{a} < a < \bar{a}$, we have, as required:

$$0 < \theta_{crit} < \theta^* < 1.$$

Appendix 2

One possibility for the firm's budget constraint is that the present value of any profit stream can never be lower than present value of any earnings stream, plus initial wealth b_1 (= 0):

$$\sum_{t \geq 1} \frac{1}{(1+\rho)^{t-1}} \left\{ p_t f(\ell_t) - \ell_t - y_t \right\} \geq 0 \text{ with probability 1.} \qquad (3)$$

The difficulty with this is that there is nothing that resembles a liquidity constraint; the b_t's, $t \geq 2$, do not enter. That is, we cannot use dynamic programming to analyse the firm's decisions through time.

Of course replacing (3) by (1a) as the constraints will not do, because there is no terminal debt constraint to limit the growth of $\{b_t\}$. (If there was such a transversality condition, then summing (1a) yields (3).) However, Proposition 4.4.1 of Gale (1983) shows that if (1a) is augmented by

$$b_t \leq b^* \quad \text{for all } t \qquad (1b)$$

where b^* is defined as $\sup \left\{ b \mid \underline{p} f(\ell) - \ell \geq \frac{\rho b}{1+\rho} \text{ for some } \ell \right\}$,

then (1a) and (1b) are together equivalent to (3). The intuition behind this is straightforward: if debt is allowed to grow higher than b^* in some period t then, from (1a) and by construction of b^*, there is no way the firm can avoid a positive probability of b_{t+1} exceeding b_t. By repeating this argument, it follows that the firm inevitably faces the possibility of its debt growing without limit.

In sum, if there is a no-bankruptcy condition, then firms' liquidity constraints take the form (1b).

It is worth mentioning an observation made by Gale. Any constant b^{**} such that $b^* < b^{**} < \infty$ would suffice on the RHS of (1b). The firm voluntarily limits its indebtedness to b^*, even though the constraint $b_t \leq b^{**}$ is thereby always slack, because it knows that once b^* is overstepped there is a chance that debt will later grow beyond b^{**} regardless of the choices of ℓ's.

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