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SPURIOUS CORRELATION IN EXCHANGE RATE TARGET ZONE MODELLING: TESTING THE DRIFT-ADJUSTMENT METHOD ON THE US DOLLAR, RANDOM WALK AND CHAOS

Zsolt M Darvas

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Centre for Economic Policy Research
90–98 Goswell Rd
London EC1V 7DB
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

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ABSTRACT

Spurious Correlation in Exchange Rate Target Zone Modelling: Testing the Drift-Adjustment Method on the US Dollar, Random Walk and Chaos*

The drift-adjustment method estimates the expected rate of depreciation within an exchange rate band by simple equations. Papers applying this method claim that, while forecasting a freely floating currency is hopeless, predicting an exchange rate within the future band is successful. This paper shows that the results achieved by applications to EMS and Nordic currencies are not specific to data of target zone currencies. For example, application to US dollar leads qualitatively to the same result as application to EMS currencies. Simulation evidence suggests that the closer the dominant inverted autoregressive root to unity the higher the chance of reproducing the empirical target zone results, since the finite sample biases of the parameters of interest in the unit root case are such that the random walk seemingly significantly fits the model in the vast majority of experiments. HAC standard errors do not help much in hypothesis testing either. The paper develops a simple model coinciding with stylized facts of target zones that demonstrates the unpredictability of the expected rate of depreciation within the band. Surprisingly, application of drift-adjustment method to a process switching between stationary periods and chaotic periods, the fit is similar to reported target zone results.

JEL Classification: C22, F31
Keywords: exchange rate traget zone, unit root, Monte Carlo simulation

Zsolt Darvas
Economics & Research Department
Modelling Division
National Bank of Hungary
Szabadság tér 8–9
H–1850 Budapest
HUNGARY
Tel: (36 1) 312 3059
Fax: (36 1) 331 7906
Email: darvaszs@mnbpost2.mnblan.mnb.hu

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Submitted 19 February 1998

NON-TECHNICAL SUMMARY

The drift-adjustment method is a widely used device for estimating realignment expectations in an exchange rate target zone. The method applies simple equations for estimating expected depreciation within the band. Estimated expected depreciation within the band is subtracted from the interest differential to derive expected devaluation of the zone. This step is based on the assumption of uncovered interest parity, which states that the yield differential between home and foreign bonds free of default risks equals the expected rate of depreciation of the home currency. Therefore, the method concentrates solely on estimating expected future exchange rates within the future band.

This method was proposed by the working paper version of Bertola and Svensson (1993) and has been applied by several authors to a wide range of currencies. Besides EMS and Nordic currencies' application the method had been recently applied to developing countries, such as Chile, Columbia, Czech Republic and Israel. Authors consider empirical results good if: (1) some summary statistics of the estimated regressions are adequate; (2) estimated equations imply that exchange rates within the band display mean reversion; (3) the estimates for expected devaluation predict actual devaluation reasonably well or coincide with the prior conception of the authors; and (4) the estimates fit the curves implied by a theoretical model. Fitted data turn out to coincide with the predictions of the Bertola-Svensson model. Estimated expected devaluation has usually a high value before devaluation and a low value afterwards and shows some correlation to the inflation differential. See, for example, the third volume of the *Handbook of International Economics* (1995) suggesting the drift-adjustment method to be 'the solution' for empirical target zone modelling.

This paper shows that the results achieved by applications to EMS and Nordic currencies are not specific to data of target zone currencies. Using other data having a close to unity autoregressive root and evaluating the model solely by criteria applied to EMS lead qualitatively to the same fit. For example, the US dollar against the Deutsche mark, Japanese yen and sterling fit requirements (1), (2), and (4) above. At least requirement (2) (i.e. mean reversion of the exchange rate level) is in sharp contrast to stylized facts of floating rates. Even more interestingly, those fitted curves that are independent of the interest rate show the same shape for most unit root processes. Results are independent of whether a limiting zone for the generated processes exists or not. This suggests the possibility of spurious regression and that the empirical results

Noting that $E_t[v_{t+i}] = v$ for $i = 1, \dots, K$, and K denote the last period before the switch to the next expectation process. Iterating (33) for $\tau < K+1$ yields:

$$E_t[x_{t+\tau}] = \sum_{i=1}^{\tau}\prod_{j=0}^{i-1}\left(1 - P(x_{t+j-1}|NR^{j-1})\right)P(x_{t+i-1}|NR^{i-1})(-B + (\tau - i)v) + \prod_{j=1}^{\tau}\left(1 - P(x_{t+j-1}|NR^{j-1})\right)E_t\left(x_{t+\tau}|NR^{\tau}\right) \quad (34)$$

where $P(x_{t-1}|NR^{-1}) = 0$ and $P(x_t|NR^0) = P(x_t)$. Although the algebra seems a bit notation demanding the intuition is simple as can be inferred from Figure 34 and Equation (34) might be simulated. Since all conditional probabilities are functions of x_t and the conditional expected value of the last term in (34) is also a function of x_t, the expected exchange rate within the band will be a non-linear deterministic function of x_t.

To show that Equation (34) might imply chaos let write out the formulas for $E_t[x_{t+1}]$ and $E_t[x_{t+2}]$:

(35) $E_t[x_{t+1}] = P(x_t)(-B) + [1 - P(x_t)]\,[x_t(1-\delta) + B\delta]$

(36) $E_t[x_{t+2}] = P(x_t)\,[-B+v] + [1 - P(x_t)]\,\{P[x_t(1-\delta) + B\delta]\,(-B) +$

$+ (1 - P[x_t(1-\delta)+B\delta])([x_t(1-\delta) + B\delta]\,(1-\delta) + B\delta)\,\} =$

$= E_t[x_{t+1}]\,(1 - P[x_t(1-\delta) + B\delta])(1-\delta) + \Big[- P[x_t(1-\delta) + B\delta])(1-\delta)(-B) +$

$+ P(x_t)\,[v] + [1 - P(x_t)]\,\{P[x_t(1-\delta) + B\delta]\,(-B) + (1 - P[x_t(1-\delta)+B\delta])\,B\delta\,\}\Big] =$

$$= E_t[x_{t+1}]\left\{1 - P\left(\frac{E_t[x_{t+1}] - P(x_t)(-B)}{1 - P(x_t)}\right)(1-\delta)\right\} + \left[\ \dots\ \right]$$

Iterating further the expected value of the exchange rate within the band for period $s+1$ at time t will be a non-linear function of the expected value for period s at time t. Thus, the expected exchange rate within the band will be a deterministic logistic type function of its previous value and a constant.

are not necessarily the outcome of a theoretical model of exchange rate target zones.

Simulation examines the asymptotic properties of OLS parameter estimators in the unit root case. Estimated equations differ from the Dickey-Fuller case by: (1) overlapping observations; (2) time-varying intercepts; (3) other explanatory variables; and (4) the *ad hoc* non-linear (in variables) specification where applied. All four factors modify Dickey-Fuller distributions. Asymptotic distributions in cases (1), (2), and (4) are possible to characterize and evidence suggests that critical values change substantially, even if the standard errors are estimated by the Newey-West heteroskedasticity and autocorrelation consistent (HAC) estimator. Therefore, using critical values from the simulated distributions the US dollar (and also the FF/DM rate) does not seem to be stationary anymore.

The non-linear specification yields interesting insights into the OLS polynomial regressions. First, the US dollar seems to have significant parameter values and fits all curves implied by the Bertola-Svensson model. Second, simulation evidence suggest that: (1) the closer the dominant inverted autoregressive root to unity the higher the chance of reproducing the non-linear estimates of Rose-Svensson (1995) for the FF/DM rate and the dollar estimates of this paper; (2) the estimated parameters are consistent in unit root cases; but (3) in the unit root case the larger the sample size the higher the chance of reproducing the results (i.e. the t-value of the critical parameter estimate does not go to zero); (4) the asymptotic distribution of the estimator of the critical parameter is highly asymmetric and skewed to the left in unit root cases; (5) HAC standard errors do not validate standard statistical tables. Using critical values from the simulated distributions the US dollar (and also the FF/DM rate) does not have significant parameter estimates anymore.

It is also easy to show that the drift-adjustment method infers higher devaluation expectations when the exchange rate is closer to the weak edge and the higher the interest differential. Since usually both precede a devaluation and their converses follow, the specification secures nice estimates for devaluation, especially when the estimate of the autoregressive parameter is downward biased.

One of the main messages of this paper is that the real question is not whether there is a unit root in an exchange rate within the band or not. The band is devalued when the central bank can not resist market pressure anymore so the exchange rate within the band is always within the band. Therefore, it might not have a unit root. The real questions are whether one can model the exchange rate within the band independently of other factors (e.g. expectations for devaluation of central parity); how to model the exchange rate within the band; and, especially, whether one can assume the same data generating process (DGP) for the whole sample. For example, the Dutch guilder might have had the same DGP since March 1983. For currencies that were devalued, however, the process might have different characteristics at least before and after a devaluation.

A simple model is offered for fundamentally weak currencies in which the exchange rate within the band follows a switching process. After a period of low devaluation probability and stationary exchange rates within the band process, at a threshold level of devaluation probability the process is switched to an 'expectation' process. A devaluation switches the process back to the stationary one. The paper argues that application of the drift adjustment method for the first period might be misleading. For the second period it is shown that for certain parameter values the exchange rate within the band process will exhibit unpredictability. Surprisingly, application of the method to a process switching between trend-stationarity and chaos results are qualitatively the same as the EMS application.

The conditional expectation of $x_{t+\tau}$, conditional on that a devaluation happened in $t+\tau-1$ is $-B$.

Using the above notation, the following figure shows expectation formation for $t+3$: the four possible outcomes with the four conditional expectations and the associated probabilities.

Figure 34. Expectation formation for x_{t+3} at time t.

t	$t+1$	$t+2$	$t+3$	P
x_t	R — $-B$	$-B+v_1$	$-B+v_1+v_2$	$P(x_t)$
NR	$E_t(x_{t+1}, NR^1)$ R — $-B$		$-B+v_1$	$[1-P(x_t)]P(x_{t+1}, NR^1)$
	NR	$E_t(x_{t+2}, NR^2)$ R — $-B$		$[1-P(x_t)][1-P(x_{t+1}, NR^1)]P(x_{t+2}, NR^2)$
		NR	$E_t(x_{t+3}, NR^3)$	$[1-P(x_t)][1-P(x_{t+1}, NR^1)][1-P(x_{t+2}, NR^2)]$

The expected value of $x_{t+\tau}$ at time t conditional on no realignment is an iteration of Equation (21):

(30) $E_t[x_{t+2} \mid NR^2] = \{x_t(1-\delta) + \delta B\}(1-\delta) + \delta B = x_t(1-\delta)^2 + \delta B\,[1 + (1-\delta)]$

(31) $E_t[x_{t+3} \mid NR^3] = \{x_t(1-\delta)^2 + \delta B\,[1+(1-\delta)]\}(1-\delta) + \delta B =$

$$= x_t(1-\delta)^3 + \delta B\,[1+(1-\delta)+(1-\delta)^2]$$

(32) $E_t[x_{t+\tau} \mid NR^\tau] = x_t(1-\delta)^\tau + \delta B\,[1+(1-\delta)+(1-\delta)^2 + \ldots + (1-\delta)^{\tau-1}]$

Since for $0 < a < 1$

$$1 + a + a^2 + a^3 + \cdots + a^t = \frac{1}{1-a} - (a^{t+1} + a^{t+2} + \cdots) = \frac{1}{1-a} - \frac{a^{t-1}}{a^{t-1}}(a^{t+1} + a^{t+2} + \cdots) =$$

$$= \frac{1}{1-a} - \frac{1}{a^{-t-1}}\frac{1}{1-a} = \frac{1-a^{t+1}}{1-a} \text{, then}$$

$$\delta B\{1 + (1-\delta) + (1-\delta)^2 + \cdots + (1-\delta)^{\tau-1}\} = \delta B\frac{1-(1-\delta)^\tau}{\delta} = B\{1-(1-\delta)^\tau\} \text{, therefore}$$

(32) $E_t[x_{t+\tau} \mid NR^\tau] = x_t(1-\delta)^\tau + B\,\{1 - (1-\delta)^\tau\}$.

The expected value of the exchange rate within the band for $\tau = 2$ is

$$(33)\quad \begin{aligned} E_t[x_{t+2}] &= P(x_t)(-B + E_t[v_{t+2}]) + \\ &+ (1-P(x_t))\left[P(x_{t+1}|NR^1)(-B) + (1-P(x_{t+1}|NR^1))(x_t(1-\delta)^2 + B\{1-(1-\delta)^2\})\right] \end{aligned}$$

Table 12. Unit root simulation for the Koedijk *et al.* specification

	Random Walk
R^2	0.0071
adj. R^2	0.0031
$\hat{\beta}_0$	-0.0022
$\hat{\delta} = 1 - \hat{\beta}_1$	0.0089
$\hat{\beta}_2^+$	-2.9184
$\hat{\beta}_2^-$	2.7438
$\hat{\beta}_3$	-0.00005
$t(\hat{\beta}_0)$	-0.004
$t(\hat{\delta})$	0.285
$t(\hat{\beta}_2^+)$	-0.021
$t(\hat{\beta}_2^-)$	0.011
$t(\hat{\beta}_3)$	-0.647
% of $\lvert t(\hat{\beta}_0)\rvert > 2$	5,0 %
% of $\lvert t(\hat{\delta})\rvert > 2$	5.0 %
% of $\lvert t(\hat{\beta}_2^+)\rvert > 2$	4,9 %
% of $\lvert t(\hat{\beta}_2^-)\rvert > 2$	6,2 %
% of $\lvert t(\hat{\beta}_3)\rvert > 2$	7,2 %

Note: Except for the percentage statistics (denoted by %) all other statistics in the table are average of the 2307 estimates.

Random walks generated as $s_t^{(i)} = s_{t-1}^{(i)} + \varepsilon_t^{(i)}$, $\varepsilon_t^{(i)} \sim N(0,1)\ \forall t, i$, $s_0 = 0$, $t = 1, \ldots, 1000$, $i = 1, \ldots, 2400$, where (i) stands for the i-th experiment, and estimated equation is (29) with $c_t = 0$. In 93 of the 2400 cases the random walk did not change sign so regression was impossible[13]. Therefore, statistics refer to the 2307 estimations.

2. The second regime of the model

Let NR^t denote the conditioning information that no realignment takes place in t periods to come, and x_t the sequence for the first (stationary) regime of the process. Probabilities are denoted as follows:

$P(x_t)$: probability of devaluation in $t+1$

$P(x_t \mid NR^1)$: probability of devaluation in $t+2$, conditional that there was not a devaluation in $t+1$

$P(x_t \mid NR^\tau)$: probability of devaluation in $t+\tau+1$, conditional that there was not a devaluation up to and including $t+\tau$

[13] Since the square terms in equation (29) are dummied up by the sign of $s_t - c_t$, if there is no change in sign, then there is a regressor equals zero for all t.

I. Introduction

The *drift-adjustment method* is a widely used device for estimating realignment expectations in an exchange rate target zone. The method applies simple equations for estimating expected depreciation within the band. Estimated expected depreciation within the band is subtracted from the interest differential to derive expected devaluation of the zone. This step is based on the assumption of uncovered interest parity, which state that the yield differential between home and foreign bonds free of default risks equals the expected rate of depreciation of the home currency. Therefore, the method concentrates solely on estimating expected future exchange rate within the future band.

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Simulation examines the asymptotic properties of OLS parameter estimators in unit root case. Estimated equations differs from the Dickey-Fuller case by (1) overlapping observations, (2) time-varying intercepts, (3) other explanatory variables, and (4) by the *ad hoc* nonlinear (in variables) specification where applied. All four factors modify Dickey-Fuller distributions. Asymptotic distributions in cases (1), (2), and (4) are possible to characterize and evidence suggests that critical values change substantially, even if the standard errors are estimated by the Newey-West heteroskedasticity and autocorrelation consistent (HAC) estimator. Therefore, using critical values from the simulated distributions the US dollar (and also the FFR/DM rate) do not seem to be stationary anymore.

The non-linear specification yielded interesting insights into the OLS polynomial regressions. First, the US dollar seems to have significant parameter values and fits all curves implied by the Bertola-Svensson model. Secondly, simulation evidence suggest that (1) the closer the dominant inverted autoregressive root to unity the higher the chance of reproducing the non-linear estimates of Rose-Svensson [1995] for the FF/DM rate and the dollar estimates of this paper, (2) the estimated parameters are consistent in unit root cases, but (3) in the unit root case the larger the sample size the higher the chance of reproducing the results (i.e. t-value of the critical parameter estimate do not goes to zero), (4) the asymptotic distribution of the estimator of the critical parameter is highly asymmetric and skewed to the left in unit root cases, (5) HAC standard errors do not validate standard statistical tables. Using critical values from the simulated distributions the US dollar (and also the FFR/DM rate) do not have significant parameter estimates anymore.

It is also easy to show that the drift-adjustment method infers higher devaluation expectations when the exchange rate is closer to the weak edge and the higher the interest differential. Since usually both precede a devaluation and their converses follow, the specification secures nice estimates for devaluation, especially when the estimate of the autoregressive parameter is downward biased.

One of the main message of this paper is that the real question is not whether there is a unit root in exchange rate within the band or not. The band is devalued when the central

Number of experiments: 10,000

test equation: $y_{t-1} = \sum_{1}^{k} \hat{\alpha}_i + \hat{\rho} y_t + \hat{u}_{t-1}$, $t = 1, \ldots, 3000$

α_i: dummies having value 1 for $1/k$ of the sample in succession

Figure 32. A pure switching process

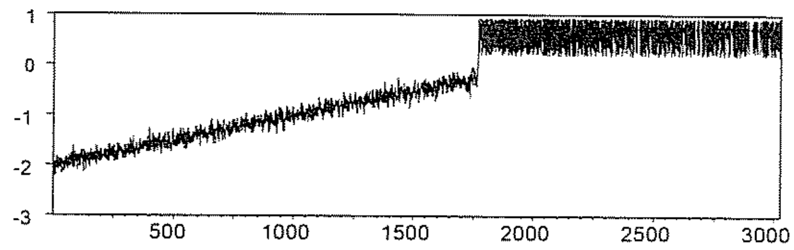
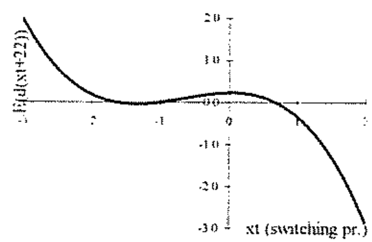


Figure 33. Estimation of equation (6) to a pure switching process



Notes for Tables 7 - 9.
Number of experiments: 5,000
Random walks generated as $y_{t+1} = y_t + u_{t+1}$, $u_t \sim$ i.i.d. N(0,1), t = 1, 2, ..., 3064, y_1=0
(1) Dickey-Fuller critical values for T = 500, source: Hamilton [1994], Table B.6. and B.5.case 2, p.762-3.
(2) test equation: $y_{t+1} = \hat{\alpha} + \hat{\rho} y_t + \hat{u}_{t+1}$ (= Dickey-Fuller equation), t = 1, ..., 3000
(3) test equation: $y_{t+1} = \sum_1^6 \hat{\alpha}_i + \hat{\rho} y_t + \hat{u}_{t+1}$. α_i : six dummies, t = 1, ..., 3000
(4) test equation: $y_{t+65} = \hat{\alpha} + \hat{\rho} y_t + \hat{u}_{t+65}$, t = 1, ..., 3000
(5) test equation: $y_{t+65} = \sum_1^6 \hat{\alpha}_i + \hat{\rho} y_t + \hat{u}_{t+65}$. α_i : six dummies, t = 1, ..., 3000
(6) test equation: $y_{t+65} = \hat{\alpha} + \hat{\rho} y_t + \hat{u}_{t+65}$, with excluded observations*, t = 1, ..., 3000
(7) test equation: $y_{t+65} = \sum_1^6 \hat{\alpha}_i + \hat{\rho} y_t + \hat{u}_{t+65}$. α_i six dummies, with excluded observations*, t = 1, ..., 3000

* excluded observations: 436-500, 936-1000, 1436-1500, 1936-2000, 2436-2500, therefore, included observations for columns (6) and (7) are not 3000 but 2675
α_i: dummies having value i for 1/6 of the sample in succession

Table 10. Critical values for unit root test for different number of intercepts based on OLS t-statistic $(\hat{\rho}-1)/\hat{\sigma}_{\hat{\rho}}$

No. of intercepts	1	2	3	4	5	6	7	8	9	10
1.0%	-3.47	-3.89	-4.28	-4.56	-4.91	-5.18	-5.39	-5.64	-5.85	-6.04
2.5%	-3.10	-3.58	-3.99	-4.30	-4.59	-4.87	-5.08	-5.29	-5.50	-5.69
5.0%	-2.86	-3.34	-3.72	-4.03	-4.32	-4.58	-4.81	-5.02	-5.24	-5.42
10.0%	-2.54	-3.04	-3.42	-3.72	-4.01	-4.25	-4.51	-4.69	-4.94	-5.11
90.0%	-0.44	-0.86	-1.19	-1.50	-1.78	-2.00	-2.20	-2.43	-2.64	-2.85
95.0%	-0.11	-0.52	-0.84	-1.17	-1.47	-1.69	-1.85	-2.09	-2.33	-2.50
97.5%	0.23	-0.24	-0.52	-0.90	-1.16	-1.37	-1.58	-1.81	-2.06	-2.22
99.0%	0.62	0.04	-0.16	-0.56	-0.80	-1.09	-1.25	-1.49	-1.75	-1.91

See notes below Table 11.

Table 11. Critical values for unit root test for different number of intercepts based on OLS autoregressive coefficient $T(\hat{\rho}-1)$

No. of intercepts	1	2	3	4	5	6	7	8	9	10
1.0%	-20.82	-26.70	-31.38	-35.65	-41.14	-45.37	-49.65	-53.77	-57.92	-61.66
2.5%	-16.97	-21.88	-26.98	-31.62	-35.76	-39.77	-44.33	-48.05	-51.89	-56.22
5.0%	-13.89	-18.75	-23.37	-27.61	-31.59	-35.58	-40.47	-43.47	-47.84	-51.37
10.0%	-10.98	-15.57	-19.71	-23.75	-27.74	-31.36	-35.54	-38.85	-42.94	-46.19
90.0%	-0.86	-2.40	-4.23	-6.30	-8.50	-10.49	-12.65	-14.93	-17.27	-19.62
95.0%	-0.20	-1.42	-2.79	-4.73	-6.64	-8.36	-10.37	-12.51	-14.95	-16.86
97.5%	0.39	-0.66	-1.66	-3.50	-5.13	-6.79	-8.52	-10.37	-12.81	-14.44
99.0%	1.03	0.10	-0.52	-2.13	-3.49	-5.18	-6.43	-8.34	-10.35	-12.27

Notes for Tables 10 and 11.
Random walks generated as $y_{t+1} = y_t + u_{t+1}$, $u_t \sim$ i.i.d. N(0,1), t = 1, 2, ..., 3000, y_1=0

bank can not resist market pressure anymore so the exchange rate within the band is always within the band. Therefore, it might not have a unit root. The real questions are whether one can model the exchange rate within the band independently of other factors (e.g. expectations for devaluation of central parity), how to model the exchange rate within the band, and especially, whether one can assume the same data generating process (DGP) for the whole sample. For example, the Dutch guilder might have the same DGP since March 1983. However, for currencies that were devalued the process might have different characteristic at least before and after a devaluation.

A simple model is offered for fundamentally weak currencies in which the exchange rate within the band follows a switching process. After a period of low devaluation probability and stationary exchange rate within the band process, at a threshold level of devaluation probability the process is switched to an "expectation" process. A devaluation switches the process back to the stationary one. The paper argues that application of the drift adjustment method for the first period might be misleading. For the second period it is shown that for certain parameter values the exchange rate within the band process will exhibit unpredictability. Surprisingly, application of the method to a process switching between trend-stationarity and chaos results are qualitatively the same as the EMS application.

The rest of the paper is organized as follows. Section II sketches the drift adjustment method and surveys its reported empirical performance. Section III applies the method to the DEM/USD, JPY/USD and USD/GBP exchange rates as well as to several generated processes with and without limits. Section IV tries to explain the puzzle. Section V offers possible solutions: a model demonstrating the unpredictability of the exchange rate within the band, and the analysis of the theory based non-liner specification of Koedijk *et al.* [1997]. Section VI concludes.

II. The drift adjustment method

By definition, the logarithm of the exchange rate in an exchange rate target zone can be written as the sum of the logarithm of the central parity and the logarithm of the deviation from the central party:

(1) $s_t = c_t + x_t$.

where s_t denotes the exchange rate, c_t the central parity and x_t the deviation from the central parity that is usually called "the exchange rate within the band". Under the assumption of uncovered interest parity, the interest rate differential at time t equals the expectations of currency depreciation conditional on information available at time t:

(2) $r_t - r^*_t = E_t[\Delta s_{t+a}] / \tau \quad \tau = a$ / (number of observation a year)

where r_t and r^*_t are annualized domestic and foreign interest rates maturing in τ years. Taking expectations of (1) and combining with (2), the expected rate of realignment can be measured as the difference between the interest differential and the expected rate of depreciation within the band:

(3) $E_t[\Delta c_{t+a}] / \tau = r_t - r^*_t - E_t[\Delta x_{t+a}] / \tau$

After some pages of algebraic manipulation developers of the drift adjustment method conclude that it is sufficient to find an estimate for the expected depreciation within the band in order to give an estimate for the expected rate of realignment.

The estimate is given by the following equation (e.g. Rose and Svensson 1995):

(4) $x_{t+a} = \beta_0 + \beta_1 x_t + \alpha_2 X_t + u_t$,

or in a difference form (e.g. Svensson 1993):

(5) $x_{t+a} - x_t = \beta_0 + \beta'_1 x_t + \beta_2 X_t + u_t$,

where X_t might contain the square and cube of the exchange rate within the band, domestic and foreign interest rates, other EMS rates within the band, effective limits[1], or nothing. The basic equation used by Rose and Svensson is

(6) $x_{t+22} = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \beta_3 x_t^3 + u_t$.

The empirical results summarized by Bertola and Svensson (1993, p. 706.) are the following.

[1] The EMS requires all bilateral rates to fall within the bilateral band. Therefore, the bilateral bands jointly limit each currency's deviation from each other. For example, if the FF/DEM rate was 2,25 percent lower and the IP/DEM rate were 2,25 percent higher than the corresponding central rates then the IP/FF rate would be 4.6 percent higher than the IP/FF central rate. See Rose and Svensson, 1995.

Table 7. Critical values for unit root test of Dickey-Fuller and non-Dickey-Fuller test equation specification based on OLS t-statistic $(\hat{\rho}-1)/\hat{\sigma}_{\hat{\rho}}$

T = 3000	(1) Dickey-Fuller	(2) period: 1 interc.: 1	(3) period: 1 interc.: 6	(4) period: 65 interc.: 1	(5) period: 65 interc.: 6	(6) period: 65 interc.: 1 with exl.	(7) period: 65 interc.: 6 with exl.
1.0%	**-3.44**	-3.45	-5.22	-28.69	-44.70	-27.92	-45.53
2.5%	**-3.13**	-3.14	-4.91	-26.19	-41.80	-25.62	-42.54
5.0%	**-2.87**	-2.87	-4.64	-23.99	-39.67	-23.42	-39.79
10.0%	**-2.57**	-2.59	-4.29	-21.27	-36.92	-20.60	-37.01
90.0%	**-0.43**	-0.48	-2.00	-3.58	-16.24	-3.05	-16.10
95.0%	**-0.07**	-0.11	-1.65	-0.38	-13.25	0.39	-13.43
97.5%	**0.24**	0.21	-1.36	2.33	-10.76	2.96	-10.94
99.0%	**0.61**	0.61	-1.01	5.06	-7.55	5.74	-8.42

See notes below Table 9.

Table 8. Critical values for unit root test of Dickey-Fuller and non-Dickey-Fuller test equation specification based on OLS t-statistic $(\hat{\rho}-1)/\hat{\sigma}_{\hat{\rho}}$

T = 3000	(1) Dickey-Fuller	(2) period: 1 interc.: 1	(3) period: 1 interc.: 6	(4) period: 65 interc.: 1	(5) period: 65 interc.: 6
1.0%	**-3.44**	-3.52	-5.33	-10.55	-16.85
2.5%	**-3.13**	-3.16	-4.98	-9.56	-15.51
5.0%	**-2.87**	-2.91	-4.68	-8.72	-14.32
10.0%	**-2.57**	-2.60	-4.33	-7.62	-13.12
90.0%	**-0.43**	-0.48	-2.01	-1.27	-5.50
95.0%	**-0.07**	-0.11	-1.62	-0.14	-4.44
97.5%	**0.24**	0.22	-1.34	0.86	-3.75
99.0%	**0.61**	0.60	-1.02	1.81	-2.59

See notes below Table 9.

Table 9. Critical values for unit root test of Dickey-Fuller and non-Dickey-Fuller test equation specification based on OLS autoregressive coefficient $T(\hat{\rho}-1)$

T = 3000	(1) Dickey-Fuller	(2) period: 1 interc.: 1	(3) period: 1 interc.: 6	(4) period: 65 interc.: 1	(5) period: 65 interc.: 6	(6) period: 65 interc.: 1 with exl.*	(7) period: 65 interc.: 6 with exl.*
1.0%	**-20.5**	-20.9	-46.3	-1143.8	-2140.2	-1035.0	-2086.6
2.5%	**-16.8**	-16.7	-41.0	-986.4	-1971.8	-896.8	-1928.1
5.0%	**-14.0**	-14.4	-35.8	-835.9	-1821.8	-757.1	-1799.4
10.0%	**-11.2**	-11.3	-31.7	-670.1	-1657.8	-611.2	-1630.0
90.0%	**-0.84**	-0.91	-10.48	-55.34	-628.07	-45.27	-637.43
95.0%	**-0.13**	-0.20	-8.40	-7.55	-501.32	6.24	-522.65
97.5%	**0.42**	0.39	-6.71	31.85	-419.86	39.48	-422.80
99.0%	**1.06**	1.06	-4.91	71.43	-285.87	80.35	-319.04

Table 5. Sampling distribution of the t-values of equation (6) parameter estimates to random walks for different sample sizes

	$\hat{\beta}_1 - 1$ / σ_{OLS}							$\hat{\beta}_2$ / σ_{OLS}							$\hat{\beta}_3$ / σ_{OLS}						
	0.01	0.05	0.10	0.50	0.90	0.95	0.99	0.01	0.05	0.10	0.50	0.90	0.95	0.99	0.01	0.05	0.10	0.50	0.90	0.95	0.99
100	-9 99	-7 24	-6 03	-1 90	1 61	2 67	5 01	-6 49	-4 19	-3 26	-0 08	3 08	4 11	5 95	-6 57	-4 83	-3 84	-0 72	1 97	2 87	4 38
200	-9 88	-7 66	-6 48	-2 25	2 19	3 48	5 81	-8 35	-5 77	-4 36	0 02	4 28	5 62	8 22	-9 70	-7 43	-6 15	-1 78	1 74	2 60	4 39
500	-11 39	-8 92	-7 58	-2 75	2 64	4 35	8 34	-11 23	-7 83	-6 08	0 04	6 21	7.91	11 08	-13 14	-10 29	-8 73	-3 79	0 56	1 77	4 25
1000	-11 93	-9 47	-8 08	-2 90	3 34	5 52	9 23	-11 77	-8 49	-6 83	0 30	7 25	8 85	12 06	-14 35	-11 35	-9 81	-4 76	-0 16	1 18	4 09
3000	-12 93	-10 46	-9.00	-3 25	3 73	5 91	9 66	-12 80	-9 81	-7 93	-0 19	8 07	9 87	13 10	-14 95	-12 29	-10 74	-5 70	-0 70	0 72	3 77
5000	-13 54	-10 64	-9 06	-3 36	3 88	6 10	9 80	-13 29	-10 17	-8 29	0 06	8 17	10 01	13 23	-15 17	-12 62	-11.09	-5 90	-1 00	0 35	3.04
10000	-13 96	-10 63	-9 15	-3 57	3 95	5 97	9 56	-13 28	-10 06	-8 23	-0 02	8 40	10 01	13.61	-15 18	-12 52	-11 08	-6 01	-1.04	0 48	3 29
30000	-14.05	-11.05	-9.42	-3.46	3.76	6.01	10.05	-13.37	-10.26	-8.49	0.07	8.61	10.45	13.66	-15.68	-12.83	-11.28	-6.25	-1.27	0.37	2.76

Table 6. Sampling distribution of the Newey-West t-values of equation (6) parameter estimates to random walks for different sample sizes

	$\hat{\beta}_1 - 1$ / σ_{NW}							$\hat{\beta}_2$ / σ_{NW}							$\hat{\beta}_3$ / σ_{NW}						
	0.01	0.05	0.10	0.50	0.90	0.95	0.99	0.01	0.05	0.10	0.50	0.90	0.95	0.99	0.01	0.05	0.10	0.50	0.90	0.95	0.99
100	-8 60	-5 35	-4 24	-1 21	1 02	1 71	3 31	-4 51	-2 77	-2 10	-0 05	1 98	2 64	4 19	-4 67	-3 13	-2 50	-0 48	1 38	1 97	3 35
200	-6 08	-4 07	-3 23	-1 00	0 95	1 54	2 79	-4 10	-2 70	-2 03	0 01	2 01	2 68	3 84	-4 89	-3 57	-2 90	-0 82	0 88	1 31	2 37
500	-4 94	-3 68	-2 97	-0 97	0 85	1 37	2 66	-4 17	-2 81	-2 19	0 01	2 21	2 83	4 15	-5 22	-3 77	-3 18	-1 34	0 21	0 68	1 66
1000	-4 37	-3 29	-2 73	-0 91	0 93	1 52	2 67	-3 83	-2 83	-2 22	0 09	2 32	2 92	4 05	-4 82	-3 76	-3 23	-1 53	-0 05	0 39	1 41
3000	-4 02	-3 12	-2 63	-0 91	1 01	1 57	2 69	-3 66	-2 82	-2 29	-0 06	2 31	2 86	3 81	-4 36	-3 61	-3 14	-1 65	-0 20	0 20	1 10
5000	-3 93	-3 05	-2 55	-0 92	1 03	1 62	2 58	-3 75	-2 82	-2 32	0 02	2 27	2 79	3 80	-4 32	-3 52	-3 11	-1 64	-0 28	0 10	0 86
10000	-3 77	-2 90	-2 49	-0 95	1 02	1 57	2 50	-3 61	-2 70	-2 19	-0 01	2 25	2 71	3 66	-4 11	-3 39	-2 98	-1 62	-0 28	0 13	0 87
30000	-3.76	-2.94	-2.46	-0.90	0.98	1.57	2.61	-3.54	-2.69	-2.23	0.02	2.26	2.71	3.59	-4.16	-3.35	-2.94	-1.64	-0.33	0.10	0.75

Notes: $x_t = x_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$, t = -20, -19 . T . $x_{-21} = 0$. Equation (6): $x_t = \beta_0 + \beta_1 x_{t-22} + \beta_2 x_{t-22}^2 + \beta_3 x_{t-22}^3$. t = 1, . T

Newey-West HAC errors calculated with 22 lags . Number of estimations: 5000 for each T

"(1) Exchange rates within the band display strong mean reversion, and expected rates of depreciation within the band are often the same magnitude as interest rate differentials. ... (2) The current exchange rate within the band, x_t, is the dominant determinant of the expected future depreciation within the band. ... (3) For the maturities examined (from one up to twelve months) a linear specification .. consistently delivers reasonable results for the expected rate of depreciation within the band, although for the FF/DM case Rose and Svensson also find some support for a cubic specification. (4) Estimated expected rates of devaluation vary considerably over time ... (5) Estimated expected rates of devaluation to some extent predict actual realignments. (6) Estimated expected rates of devaluation are correlated with some macro-variables."

Svensson (1992, p.133) {and Garber-Svensson, 1995, p. 1883} adds that:

"This way of estimating the expected realignment has the great advantage is that it does not depend on any specific theory of exchange rates; nor does it matter whether expected rates of realignments are exogenous or endogenous (for instance, whether or not they influenced by the exchange rate s position within the band)."

*** Figures 1, 2, 3, see page 28 ***

Theoretical predictions of the Bertola-Svensson model are illustrated on Figures 1-3. Figure 1 shows the expected depreciation within the band against the exchange rate within the band. The slope of the locus is negative and its curvature is likely to be convex towards the strong edge and concave towards the weak edge. Figure 2 shows the expected exchange rate within the band against the exchange rate within the band and the 45° line. It is clear at first sight that Figure 1 implies Figure 2. Figure 3 shows the exchange rate within the band against the "aggregate fundamental" and the 45° line. The aggregate fundamental is defined as the linear combination of the fundamentals of the monetary model of exchange rates and the expected devaluation (see equations (7) through (10) later on page 10). The theoretical model suggests *exchange rate honeymoon effect* in terms of the aggregate fundamental, that is, a unit shock for the aggregate fundamental cause less than a unit shock to the exchange rate within the band.

Rose and Svensson [1995] estimate some specifications containing different variables in X_t of equation (4). x^2 and x^3 are included in every specification. They examine the FF/DM rate for the period 1979-1993 with daily data and α equals 22 (number of working days in a month). One month before each devaluation is excluded and the intercept in equation (4) is replaced with seven dummies for the seven periods around the six cases of devaluation. Their base results {equation (6) and (6) augmented with effective limits}are shown in Table 1, and they claim that their estimates display both statistically and economically significant mean reversion.

*** Table 1, see page 28 ***

The coefficients of the non-linear terms and one of the effective limits are significant according to the standard statistical tables. Calculating and plotting $x_{t-22} - x_t$ against x_t, x_{t-22} against x_t, and x_t against the aggregate fundamental Rose and Svensson report how striking the similarities of these estimated figures to Figures 1, 2 and 3 are. In addition, they found some evidence that the estimated expected rate of devaluation predicts actual devaluation.

Svensson [1993] estimates the linear equation (5) in which X_t includes domestic and foreign interest only, a equals 65 and 261 (number of working days in three month and in a year), and the intercept is allowed to change following a devaluation. The t-values for the autoregressive parameters for the three months estimates ranged between -3,63 and -7,47 which were compared to the -2,86 critical value from the Dickey-Fuller tables[2].

Rose and Svensson [1994] examine the relation of macroeconomic variables to realignment expectations and the effects of realignment expectations on the macroeconomic variables when the estimated equations do not include powered terms of the regressor. They found that except for the inflation differentials it is difficult to find economically meaningful relationships. They also found that for estimating exchange rate changes within the band their "... equations predict the actual change in the exchange rate within the band relatively poorly." (Rose-Svensson, 1994, p.1195) They overcome this fact by claiming that their goal is to estimate the expected rather than the actual future exchange rate within the band. They also claim that their estimated "... equation should be interpreted as a statistical projection, rather than as anything structural." (p. 1189) In contrast to the note of poor predicting power for future exchange rate within the band, the Handbook paper claims that

"For floating exchange rates, predicting future exchange rate is usually considered a futile exercise ... However, what is stake here is predicting the expected future

[2] In a footnote Svensson mentions that overlapping observations and variable intercepts increase the absolute value of the critical values but regard the estimated values large enough to soundly reject the hypothesis of unit root (taking into account that standard errors are estimated consistently by the Newey-West HAC estimator).

6

Table 4. Estimation of equation (6) to random walks for different sample sizes

sample (T)	mean $\hat\beta_1$	mean $\hat\beta_2$	mean $\hat\beta_3$	mean $\lvert\hat\beta_1 - 1\rvert$	mean $\lvert\hat\beta_2 - 0\rvert$	mean $\lvert\hat\beta_3 - 0\rvert$	% of $\hat\beta_1 < 1$	% of $\hat\beta_2 < 0$	% of $\hat\beta_3 < 0$	% of $\hat\beta_3$'s t-value "significant" at 5%	% of $\hat\beta_3$'s Newey-West t-value "significant" at 5%	mean R^2	mean DW
100	0.4053	-0.003062	-0.003159	0.926527	0.149904	0.013082	74.9%	51.2%	62.3%	39.9%	21.3%	0.3270	0.2352
200	0.5084	0.000634	-0.003182	0.697787	0.086628	0.005723	74.5%	49.7%	74.0%	56.1%	24.6%	0.4353	0.1389
500	0.6446	0.000551	-0.001447	0.448714	0.039214	0.001721	75.5%	49.7%	86.9%	74.4%	32.0%	0.6712	0.0995
1000	0.7821	0.000854	-0.000554	0.268679	0.018199	0.000610	73.6%	48.0%	90.6%	81.9%	36.7%	0.8074	0.0919
3000	0.9073	-0.000032	-0.000086	0.112066	0.004612	0.000090	74.1%	50.8%	92.7%	86.6%	39.7%	0.9278	0.0901
5000	0.9396	0.000033	-0.000035	0.071678	0.002327	0.000036	74.7%	49.6%	94.0%	86.9%	38.8%	0.9550	0.0903
10000	0.9689	-0.000010	-0.000009	0.036905	0.000868	0.000010	74.0%	50.0%	93.7%	87.7%	37.2%	0.9761	0.0905
30000	0.9892	0.000002	-0.000001	0.012583	0.000171	0.000001	74.4%	49.7%	94.1%	88.4%	37.7%	0.9922	0.0907

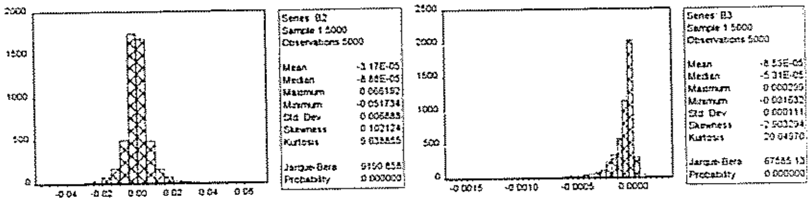
Notes: $x_t = x_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$, $t = -20, -19, \ldots, T$, $x_{-21} = 0$

Equation (6): $x_t = \beta_0 + \beta_1 x_{t-22} + \beta_2 x_{t-22}^2 + \beta_3 x_{t-22}^3$, $t = 1, \ldots, T$

Newey-West HAC errors calculated with 22 lags

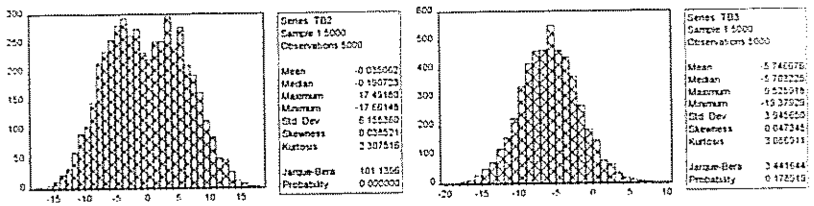
Number of estimations: 5000 for each T

Figures 26-27. The sampling distribution of $\hat{\beta}_2$ and $\hat{\beta}_3$ by estimating equation (6) 5000 times for sample size 3000



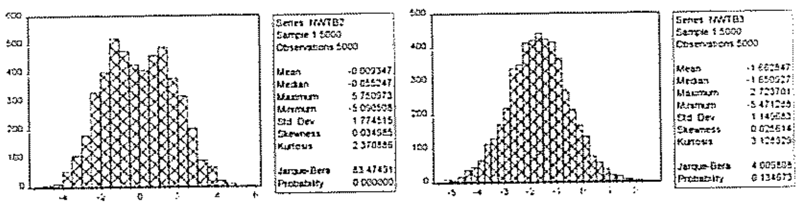
Note: shaded regions show values higher than 0.

Figures 28-29. The sampling distribution of the t-ratio of $\hat{\beta}_2$ and $\hat{\beta}_3$ by estimating equation (6) 5000 times for sample size 3000



Note: shaded regions show values higher than 0.

Figures 30-31. The sampling distribution of the t-ratio $\hat{\beta}_2$ of $\hat{\beta}_3$ and with Newey-West HAC errors (22 lags) by estimating equation (6) 5000 times for sample size 3000



Note: shaded regions show values higher than 0.

exchange rate within the band: that is, the future exchange rate's expected deviation from the future central parity. Predicting this has turned out to be much more fruitful than predicting (total) future exchange rates, since — unlike floating exchange rates — exchange rates within the band both theoretically (see above) and empirically, display strong mean reversion." (Svensson, 1992, p.132 and Garber-Svensson, 1995, p.1883)

The question that naturally arises is how one can judge whether the equation for future exchange rate within the band approximates reasonably well agents' expectations in the whole sample? There is no clear answer to this question but one might infer from the literature that the authors consider empirical results good if (1) some summary statistics of the estimated regression are adequate especially the significance of the autoregressive parameter, (2) the equation implies that exchange rate within the band displays mean reversion, that is, the autoregressive parameter is less than one for the level estimates or less than zero for the difference estimates, (3) the estimates for expected devaluation predict actual devaluation reasonably well or coincide with the prior conception of the authors, and (4) the estimates fit the curves implied by the theoretical model of Bertola-Svensson.

The next section of this paper applies this projection technique to the US dollar and the random walk and evaluate the results by criteria applied to EMS.

III. The US dollar and the random walk

This section demonstrates that the results achieved by applications to EMS and Nordic currencies that reported to be favorable are not specific to data of target zone currencies. However, the exercise reported below is not intended to demonstrate the adequate forecastability of floating exchange rates.

Throughout this paper the terms *non-linearity* and *linearity* refer to the regressor

1. Linearity

Table 2 shows the results of the same linear specification as Svensson [1993] for the logarithm of US dollar against the mark, the pound and the yen (equation (5) when X_t

includes interest rates only). The sample period is 01/02/88 - 03/13/97 with daily data. t-statistics corresponding both the standard OLS and Newey-West corrected standard errors are reported. The results are very similar to EMS: coefficients of x_t were seemingly significantly negative in all cases "implying strong mean reversion" in the DEM/USD and JPY/USD cases the coefficients of the domestic interest rate (that is, German and Japanese) were negative and the coefficients on the US interest rate were positive, and most of the were significant even with Newey-West HAC errors. R-squared ranged 0.13-0.25 for the three-month estimates and 0.41-0.69 for the twelve-month estimates. The former is below the EMS results (0.26-0.62) while the later is in line with that (0.45-0.70). The mean reversion coefficients range {-0.15, -0.47} for three months and {-0.49, -1.23} for one year, while these ranges for the EMS were {-0.31, -0.68}[4] and {-0.83, -1.12}.

*** Table 2, see page 29 ***

Since the model is linear, estimated parameters (expect the intercept) are the same if we transform the US dollar rates to be percentage deviations from 1.60 DEM/USD, 120 JPY/USD, and 1.6 USD/GBP. The resulting series, call them "exchange rates within an infinite wide band" were stayed within the ranges of (-15.2%, 27.6%), (-32.6%, 33.3%), (-16.9%, 17.8%), respectively.

2. Non-linearity

a. The dollar

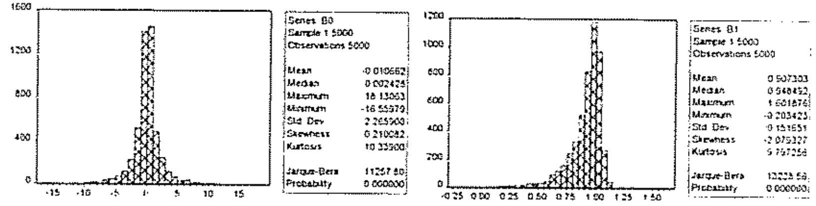
Suppose that the dollar was "fixed" in infinitely wide band around the values specified above, with occasional intramarginal intervention. Let estimate equation (6) for the mark, the pound and the yen. Figures 4 through 15 show fitted lines of Figures 1 and 2.

*** Figures 4 through 15, see page 30 ***

[3] The data source is the database of the National Bank of Hungary. Data are stored as Hungarian Forint per one unit of foreign exchange and reflect the 11 o'clock cross rates in Frankfurt am Mainz.

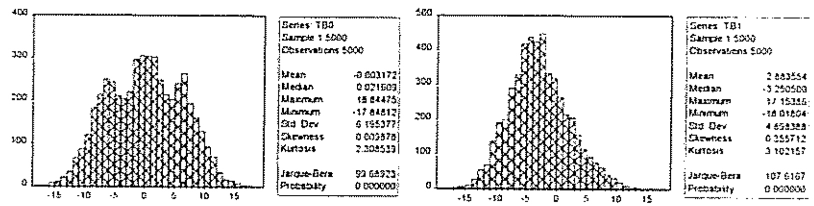
[4] Reported statistics in Table 2 of Svensson [1993] are transformed back to the level of the autoregressive coefficient since the regressand $x_{t+n} - x_t$ were divided by (65/261) to yield estimates at annual percentage level.

Figures 20-21. The sampling distribution of $\hat{\beta}_0$ and $\hat{\beta}_1$ by estimating equation (6) 5000 times for sample size 3000



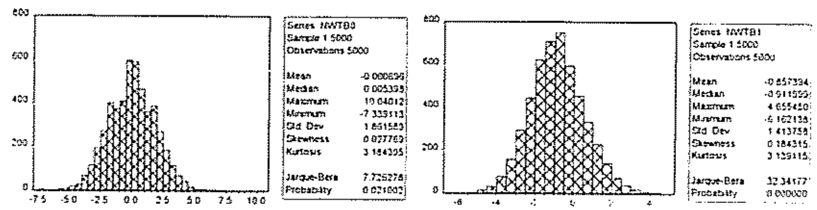
Note: shaded regions show values higher than 0 and 1 respectively.

Figures 22-23. The sampling distribution of the t-ratio of $\hat{\beta}_0$ and ($\hat{\beta}_1$ -1) by estimating equation (6) 5000 times for sample size 3000



Note: shaded regions show values higher than 0.

Figures 24-25. The sampling distribution of the t-ratio of $\hat{\beta}_0$ and ($\hat{\beta}_1$ -1) with Newey-West HAC errors (22 lags) by estimating equation (6) 5000 times for sample size 3000



Note: shaded regions show values higher than 0.

Figures 16-17. Estimation "expected change of the random walk" based on equation (6): at the average value of estimation and a case when the constant is positive

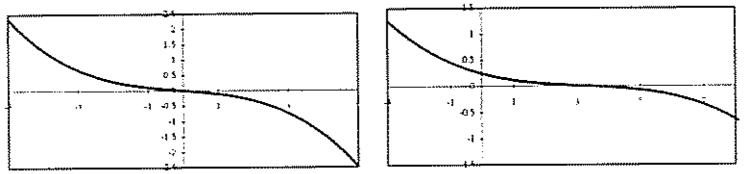


Figure 18. The exchange rate function: DEM/USD

$\alpha = 0.5$ year
Thick curve is x plotted against h
Dots are x plotted against $f - c$

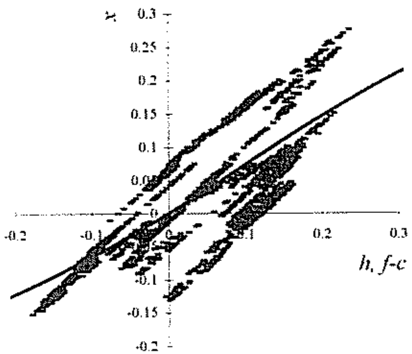
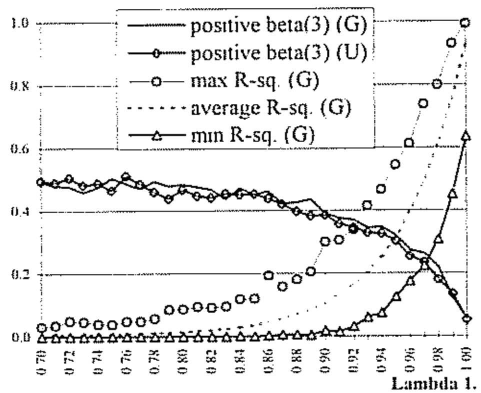


Figure 19. Equation (6) for AR(2) processes with different dominant inverted AR roots



The one month and three months ahead "expectations" look very similar to Figure 1 and 2 and the empirical estimates for real target zone data (see, for example, Figures 1 and 2 of Rose-Svensson 1995, pp. 179, 181.)

The corresponding statistics to Table 1 are shown in Table 3. According to standard statistical tables the coefficients of the non-linear regressors are significantly different from zero if OLS standard errors are used. With Newey-West HAC errors both nonlinear terms are significantly different from zero in the case of the pound, the cube term seem significant in the case of the mark, and the marginal significance level of the cube term is 6.3 percent in the case of the yen. R-squared estimates are in line with the Rose-Svensson estimates for the FF/DM.

*** Table 3, see page 31 ***

b. Random walk

Performing a Monte-Carlo experiment with random walks the mean statistics were also very similar to the FF/DM estimates. The vast majority of the powered terms' parameters are seemingly significantly different from zero (see the tabulation later). For illustration, Figures 16 and 17 plot two series of estimated $x_{t+22} - x_t$ against x_t.

*** Figures 16, 17, see page 32 ***

The slope and curvature of Figure 16 and 17 seems very similar to Figure 1, the FF/DM and the dollar estimates cited and showed above.

c. The exchange rate function for the dollar

The Bertola-Svensson model derives an exchange rate function for the exchange rate within the band as a function of a aggregate fundamental denoted as $x_t = X(h_t)$. This fundamental consists of the fundamental of the monetary model of exchange rate (denoted by f_t) and the expected rate of devaluation. Rose and Svensson [1995] implement an estimate for this fundamental (see equations (7) to (10) below) and plot the exchange rate function ($x_t = X(\hat{h}_t)$) in the same charts with dots of x_t against $\hat{f}_t - c_t$. They interpret the horizontal difference between the dots and the exchange rate function as α times expected devaluation. The figures of their paper (p. 194) are very similar to the plots and dots of the same exercise for the DEM/USD showed in Figure 18. The US dollar's exchange rate function shows a clear honeymoon effect in terms of h.

*** Figure 18, see page 32 ***

Up to now I have showed similarity between estimates for this dollar rates and EMS. I have also "defined" central rates for the dollar, seven cases of devaluation when the dollar jumped a big, and ±11 percent wide bands. After calculating the exchange rate "within the band" series I excluded one month before each devaluation and replaced the constant in Equation (6) with eight dummies for the eight "regime" around the seven larger jumps. Estimation led qualitatively the same fit as the application to the infinitely wide dollar band and the FF/DM, with R-squared equals 0,67 and t-statistics for the non-linear terms equal -6,7 and -4,7.

The similarity between the EMS estimation and simulation and free floating evidence presented in this section allows to formulate two hypotheses. (1: Linear specification) A unit root process estimated in a model with overlapping observations and other explanatory variables spuriously seems stationary evaluated by the Dickey-Fuller table, even with HAC standard errors. (2: Nonlinear specification) Several time series having both positive and negative values perform similar outputs under the drift adjustment method considering criteria applied for the EMS: plots and dots, "significance" of parameters, R-squared and standard error of regression. The next section tries to resolve this puzzle.

IV. Possible explanations

1. Exploring non-linearity

Let start with the third type of plot, the aggregate fundamental.

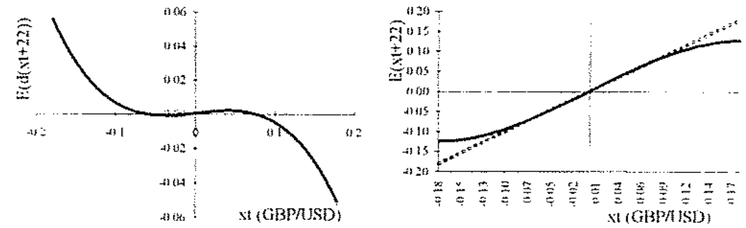
a. The aggregate fundamental

The simple monetary model derives the exchange rate as

(7) $s_t = f_t + \alpha E_t[\Delta s_{t+1}]$, $\alpha > 0$,

where f_t is referred as fundamental (the linear combination of domestic and foreign real incomes and money supplies) and α is the interest rate semi-elasticity of money demand

Figures 12-13. The Pound for one month



Figures 14-15. The Pound for three months

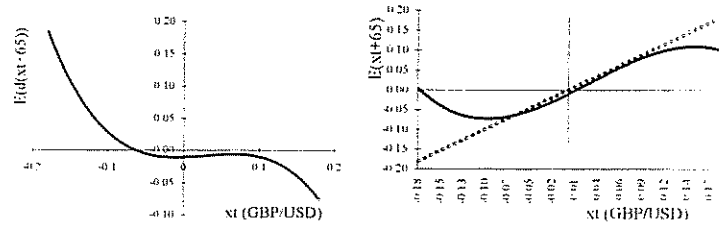
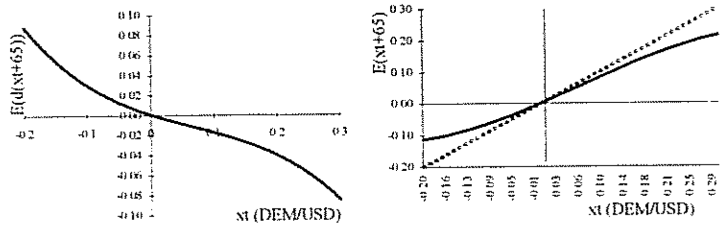


Table 3. Estimation of equation (6) to the dollar rates and random walks

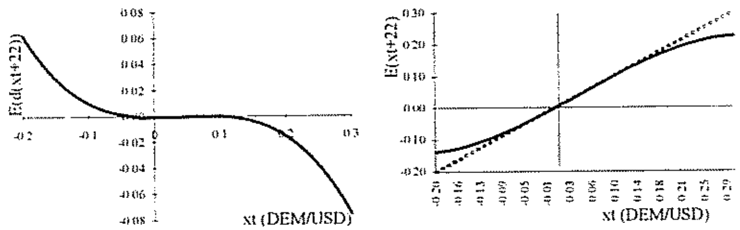
x_{t+22}	DEM / USD			JPY / USD			GBP / USD		
	coef.	std. er.	NW s.e.	coef.	std. er.	NW s.e.	coef.	std. er.	NW s.e.
c	-0.0011	0.0009	0.0034	0.0020	0.0009	0.0034	-0.0003	0.0011	0.0035
x_t	0.9955	0.0134	0.0529	1.0169	0.0085	0.0293	1.0079	0.0149	0.0563
x_t^2	0.6130	0.1173	0.3949	-0.0741	0.0298	0.1245	1.5902	0.2426	0.7793
x_t^3	-4.7535	0.6921	2.2603	-0.9301	0.1557	0.4998	-10.9395	1.0844	4.4091
N	2359			2359			2359		
R^2	0.8629			0.9489			0.8434		
σ	0.0330			0.0324			0.0344		

Notes: NW s.e. Newey-West standard errors (22 lags)

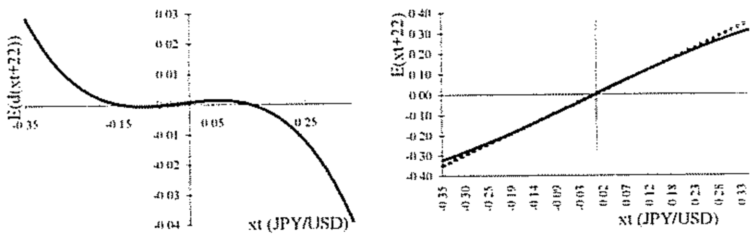
Figures 4-5. Deriving expected exchange rate depreciation "within the band" based on equation (6) to the dollar: The Mark for three months



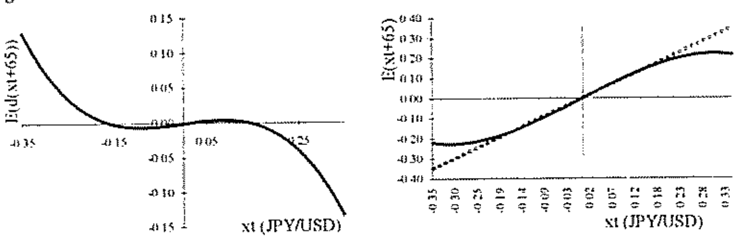
Figures 6-7. The Mark for one month



Figures 8-9. The Yen for one month



Figures 10-11. The Yen for three months



assumed to be the same for both countries. Under the assumption of uncovered interest parity, an estimate for the fundamental is given by

(8) $\hat{f}_t = s_t - \hat{\alpha}\delta_t$.

where δ_t is the interest differential and $\hat{\alpha}$ is a guess or estimate of α. Earlier empirical works showed that plots of s_t against $\hat{f}_t$ did not display the theoretical prediction of the underlying basic target zone model. The Bertola-Svensson model offered the explanation that there are two fundamentals: besides the "old" one (f) the expected rate of devaluation (g) should be also take into account. This model derives the exchange rate function as

(9) $x_t = h_t + \alpha E_t[\Delta x_{t+1}]$ $\alpha > 0$,

where x_t is the exchange rate within the band and h_t is the aggregate fundamental containing the both the "old" fundamental and the expected rate of devaluation. This is defined as

(10) $\hat{h}_t \equiv \hat{f}_t - c_t + \hat{\alpha}\hat{g}_t$.

where $\hat{f}_t$ is given by equation (8) and $\hat{g}_t$ is the estimate of the expected rate of devaluation by the drift adjustment method.

Equations (7) to (10) were taken from Rose-Svensson (1995, p.192-193). However, a last step is not written down there: inserting (8) into (10) and noting that $\delta_t = \hat{g}_t + E_t[\Delta x_{t+1}]$ yields:

(11) $\hat{h}_t \equiv \hat{f}_t - c_t + \hat{\alpha}\hat{g}_t = s_t - \hat{\alpha}\delta_t - c_t + \hat{\alpha}(\delta_t - E_t[\Delta x_{t+1}]) = x_t - \hat{\alpha}E_t[\Delta x_{t+1}]$

Thus, the empirical definition of the aggregate fundamental is the difference between the exchange rate within the band and the estimated expected rate of depreciation within the band $\hat{\alpha}$ times. Having unmasked the aggregate fundamental it does not resemble to a fundamental process. Since $\hat{\alpha}$ is positive and provided that the empirical counterpart of Figure 1 has the same shape, plotting ($x_t - \hat{\alpha}E_t[\Delta x_{t+1}]$, x_t) points ensure the exchange rate honeymoon effect. For positive x the abscissa is likely to be larger than the ordinate and the converse for negative x.

It was stated earlier that Figure 1 implies Figure 2. The preceding paragraphs revealed that the empirical shape of Figure 1 is sufficient to produce the empirical shape of Figure 3 as well. Therefore, one should examine why estimation of equation (6) likely produces significant looking empirical counterpart of Figure 1.

b. The curvature of $E[\Delta x_{t+a}]$ against x_t

The empirical fit of Figures 1 is a plot of third order polynomial where x steps from the lowest possible value the to the highest, for example, from -2.25 percent to +2.25 percent. Every third order polynomial goes to plus and minus infinity at the different ends of the axis and the signs are determined by the parameter of the cube term. It is also clear that there might or might not be a hump somewhere in the "middle" range of the polynomial. There is a hump only if the first derivative changes sign. It is in turn depend on the roots of the polynomial, whether there are three distinct real roots or not.

Suppose that the scaling of x_t provides that its mean is around zero. In this case the "middle range" of the polynomial is around zero.

Define $Dx_{t+22} \equiv (x_{t+22} - x_t) = y_t$ and subtract x_t from each side of equation (6) and leave the time index. The resulting polynomial is the function plotted on Figure 1 in the range $(-B, B)$, where B denotes the one sided width of the band,

(12) $y = \beta_0 + (\beta_1 - 1)x + \beta_2 x^2 + \beta_3 x^3$

Let differentiate equation (12) once and twice,

(13) $\dfrac{\partial y}{\partial x} = (\beta_1 - 1) + 2\beta_2 x + 3\beta_3 x^2$

(14) $\dfrac{\partial^2 y}{\partial x\, \partial x} = 2\beta_2 + 6\beta_3 x$

Regardless of β_2, the second derivative will change sign at $x = \beta_2/(3\beta_3)$ (provided, of course, that the parameter of the cube term is nonzero). This means that the curvature changes from concavity to convexity or inversely along the axis. In the case of a negative β_3 the plot of the polynomial (12) goes to plus infinity moving left on the axis and to minus infinity at right direction. A continuously negative first derivative ensures the absence of

Table 2. Estimation of equation (5) to the dollar, X_t includes interest rates only

	DEM USD					
dep. var.	$x_{t-65} - x_t$			$x_{t+261} - x_t$		
	coef.	t	NW t	coef.	t	NW t
c	0.118	16.81	6.91	0.341	30.09	12.57
x_t	-0.251	-16.28	-5.83	-0.817	-36.47	-15.71
r_t	-0.185	-3.30	-1.02	-0.106	-1.20	-0.39
$r_{US,t}$	0.249	3.77	1.36	0.894	9.39	4.12
N	2317			2121		
R^2	0.130			0.405		
σ	0.056			0.080		
	JPY USD					
dep. var.	$x_{t+65} - x_t$			$x_{t-261} - x_t$		
	coef.	t	NW t	coef.	t	NW t
c	0.965	9.35	2.51	2.099	25.19	10.89
x_t	-0.151	-9.78	-2.65	-0.494	-26.74	-11.85
r_t	-0.119	-1.29	-0.51	-0.686	-5.62	-2.68
$r_{US,t}$	1.186	17.38	6.25	4.675	61.40	29.30
N	2317			2121		
R^2	0.156			0.688		
σ	0.057			0.063		
	USD GBP					
dep. var.	$x_{t-65} - x_t$			$x_{t+261} - x_t$		
	coef.	t	NW t	coef.	t	NW t
c	0.161	21.81	8.08	0.420	50.31	20.39
x_t	-0.461	-26.93	-7.92	-1.232	-64.13	-23.36
r_t	0.609	10.73	4.03	1.424	21.36	7.64
$r_{US,t}$	0.177	2.12	0.55	0.871	9.12	3.33
N	2317			2121		
R^2	0.249			0.680		
σ	0.053			0.059		

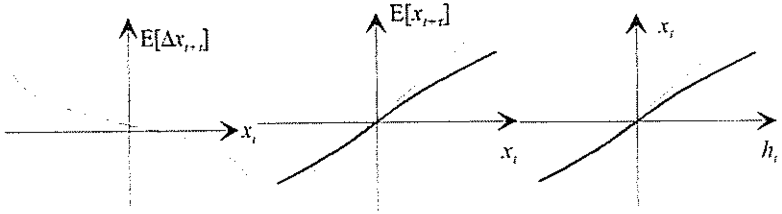
Notes: 3-month interest rates were used for all estimates (source: OECD Main Economic Indicators, available at monthly frequency). Daily frequencies for the interest rates were created by smoothing.

NW t : t-ratios calculated with Newey-West HAC errors, 8 and 7 lags for a = 65 and 261, respectively.

VIII. Appendix

i. Tables and figures

Figures 1-2-3. Theoretical predictions of the Bertola-Svensson model



Sources: Bertola-Svensson 1993, pp. 693-7, 708-9 and Rose-Svensson 1995, pp.179, 181, 194.

Table 1. Estimation of expected future exchange rates within the band, Rose and Svensson (1995)

dep.var.	x_{t+22}	x_{t+22}
	7 dummies	7 dummies
x_t	0.98 (0.08)	0.89 (0.10)
x_t^2	-0,05 (0,02)	-0.03 (0,02)
x_t^3	-0.05 (0.02)	-0.06 (0.02)
p-value	0.02	0.03
a		-0.02 (0.08)
b		0.23 (0.08)
p-value		0.001
N	3316	3316
R^2	0.81	0.81
σ	0.48	0,47

Notes: OLS: Newey-West Standard errors (22 lags); a and b are the effective limits: p-values for the joint hypothesizes of coef.(x_t^2) = coef.(x_t^3) = 0 and coef.(a) = coef.(b) = 0, respectively with χ^2 hypothesis test.

hump at the middle range. Therefore, one should find out why first derivative and β_3 are likely to be negative.

Since equation (6) is a difference equation, β_1 is like to be positive and less than one. Therefore, the first element of the first derivative is likely to be negative. The Rose-Svensson estimates for β_1 ranged 0,71-0,98 and average of the 5000 estimates for random walks resulted in 0,907 for β_1 in a 3000 sample. If β_3 was negative than the third element of the first derivative would be negative as well. Thus, provided

$$(15) \quad (\beta_1 - 1) + 3\beta_3 x^2 < -2\beta_2 x$$

the slope of estimated locus of Figures 1 will be negative without any hump in the whole range. Condition (15) more likely to be fulfilled if β_2 was close to zero.

Therefore, the key question reduces to whether one can say anything about β_2 and β_3. All estimates of the Rose-Svensson paper and the dollar estimates of this paper yielded a negative estimate for β_3 and the average estimate for random walks were negative as well. Most of the negative estimates seemed significantly different from zero according to standard statistical tables with Newey-West errors as well. At the other hand, estimates of β_2 were close to zero for random walks, but OLS standard errors were even closer to zero showing seemingly significant estimates. However, we know that at least for a random walk these two true population parameters are zero.

c. Simulation to random walks

It is well known since the works of Dickey and Fuller that in a simple first order autoregression the OLS estimate of the autoregressive parameter is downward biased in small samples both for the stationary and the unit root cases. The limiting distribution of the deviation of the estimated coefficient from the true value multiplied by the convergence rate is symmetric in the stationary case [that is, $\sqrt{T}(\hat{\rho} - \rho)$] but asymmetric in the unit root case [$T(\hat{\rho} - 1)$] with negative values much more likely.

Figures 20-31 show the sampling distribution of $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ and that of their OLS t-values, and their Newey-West t-values for sample size 3000. It seems that all distributions differ from each other and from the Dickey-Fuller distribution.

*** Figures 20-31, see page 33 ***

Table 4 and 5 explore the effect of the sample size on the estimates. There are several interesting features. (1) The estimate for β_1 and β_3 is downward biased (β_3 more severely), while β_2 is not; (2) the R^2 increase with the sample size; (3) all three parameter estimates seems consistent; but (4) the higher the sample size, the more likely to have an negative estimate for β_3 which is seemingly significant at 5 percent; (5) the probability of $\hat{\beta}_3 < 0$ is above 90 percent; (6) the 5% critical t-values values for $\hat{\beta}_1$-1, $\hat{\beta}_2$, $\hat{\beta}_3$ are about -10.5, -9.8, and -12.3 with OLS errors and -3.05, -2.82, -3.52 with HAC errors (22 lags), respectively.

*** Table 4 and 5, see page 35 and 36 ***

Using these critical values neither the Rose-Svensson estimates for the franc, nor the dollar estimates of this paper differ from the random walk.[5]

Considering the evidence reported above, I hypothesize that in unit root case the asymptotic distribution of $(\hat{\beta}_1 - 1)$ and $(\hat{\beta}_3 - 0)$ multiplied by the corresponding convergence rates are highly asymmetric and skewed to left, while that of $(\hat{\beta}_2 - 0)$ is approximately symmetric. These would imply that the expected values of the parameter estimates in finite samples fulfill condition (15).

Suppose that the hypotheses are true. Then for $x > \beta_2/(3\beta_3)$ the curve is convex and the converse applies to the concave part. Therefore, if the average of x is close to zero, then for positive x values the curve is likely to be negatively sloped and convex, and for negative x values the curve is likely to be negatively sloped and concave. A unit root in the examined series likely yields an empirical fit of Figure 1. Since it was shown earlier that Figure 1 implies Figures 2 and 3, a unit root likely yields all the fitted curves examined by the non-linear estimates of the drift-adjustment method without reference to any theoretical model of exchange rate target zone. The next important question is whether anything can be said for stationary processes having a root close to unity.

[5] Rose-Svensson recognize that their autoregressive parameter estimate differs from zero, but do not recognize that it does not differ from 1 even based on the t-distribution. However, since the model is nonlinear, even if the autoregressive parameter was one the process might be stationary if the squared and the cube terms are in certain ranges.

exchange rate's position within the band, a postponement of devaluation causes the exchange rate to depreciate further in the band, and a devaluation leads to a jump into the strong region of the realigned band. Surprisingly, applying the drift-adjustment method to a process switching between trend-stationary and chaotic periods, the fit is similar to the EMS result.

A careful examination of the theoretical and empirical validity of the switching model might be the scope of future research. However, deriving expectations from a simple forecast for an exchange rate within a band by the drift-adjustment method seems as hopeless as forecasting a freely floating currency, at least in periods when macroeconomic factors call for a devaluation.

VII. References

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exchange rate's position within the band. The method infers higher estimated expected devaluation when the higher the interest differential and the weaker the exchange rate within the band. Since both interest differential and the exchange rate within the band usually increase before devaluation and decrease afterwards, the specification produces nice estimates for expectations, especially if the autoregressive parameter is downward biased.

The non-linear version of the drift adjustment method is also complicated by the presence of unit root or a root close to unity in the whole sample autoregressive representation. A third-order polynomial OLS approximation of the random walk yields different asymptotic distributions for all estimated parameters. These are different from the Dickey-Fuller distribution as well. The sampling distributions of parameters of the lagged variable and the cube of the lagged variable are downward biased, the magnitude of the bias is higher than that of the Dickey-Fuller distribution, and the cube term has the largest bias. The parameter of the squared lagged variable seems to have a symmetric asymptotic distribution. Although estimates are consistent, t-values of the squared and cube term do not converge to zero even in the limit. This is also true if the standard errors are estimated consistently by the Newey-West estimator. Therefore, in the unit root case the mean of the parameter estimates is less than one, zero, and less than zero, respectively, in finite samples, and all are seemingly significant. This paper showed that if the three parameters of interest are less than one, zero, and less than zero, respectively, then the first type of figure of the Bertola-Svensson model materializes. It is very easy to show that the first type of figure implies the second type of figure. This paper unmasked the third type of figure, the exchange rate function in terms of the aggregate fundamental. It was shown that the aggregate fundamental do not seem to be a fundamental process and that the first type of figure implies this third type of figure as well. Therefore, the mean of parameter estimates of a third order-polynomial for a unit root process leads to all three figures of the Bertola-Svensson theory. Using critical values from the simulated distributions neither the FF/DM rate, nor the US dollar rates seems to have significant nonlinear terms in their autoregressive representations anymore.

One might sensibly presume that the exchange rate within the band process has different characteristic in periods of "low" and "high" devaluation risk. This paper offered a switching model showing that chaotic dynamics and hence unpredictability of the exchange rate within the band can arise when the probability of devaluation is positively related to the

d. Simulation to stationary processes

A reasonable way to carry out a simulation would be the application of equation (6) to some AR(2) processes with different roots. A zero mean AR(2) might be written in the form of

$$(16) \quad (1 - \lambda_1 L)(1 - \lambda_2 L)\, x_t = \varepsilon_t$$

where λ_i denotes the inverted roots (which must be inside the unit circle for covariance stationarity), L the lag operator and let ε_t be a white noise process. I have set $\lambda_2 = 0.5$ and stepped λ_1 from 0.70 to 1.00 by 0.01. For each pair of roots I have generated one thousand AR(2) processes from Gaussian white noises and another thousand from uniformly distributed white noises for $t = 1, \ldots$ 3023 and estimated equation (6). Figure 19 shows the percentage of estimates with positive β_3 for both distributions and the average, minimum and maximum R-squared for the Gaussian case.

*** Figure 19, see page 32 ***

Figure 19 suggests that a root close to unity in the autoregressive process is likely to produce a negative estimate for β_3. More than 90 percent of the two thousands unit root processes yielded negative estimate for β_3, but even for a dominant root of 0.98 approximately 80 percent of the estimates yield a negative estimate for β_3 with an average R-squared of 0.6. The estimated "significance" of all three right hand side variables increase to 0.00 in the unit root case. When the dominant inverted root is far from unity the estimated "goodness of fit" decreases dramatically and the chance of a negative β_3 estimate drops to one half.

A second result is that in the case of a unit root, the "significance" of β_2 and β_3 and the R-squared increases with the sample size. For example, at sample size 30 thousands the R-squared do not differ much from one.

Carrying out the simulation for regulated random walks and autoregressive processes, i.e. any times the series would move outside the preset range are bounded to the limits, the results are unchanged.

Therefore, simulation evidence confirms the hypothesis that a root close to unity likely produces the curvature of Figure 1. This implies that statistical inference for the estimated coefficients would require tables from nonstandard distributions.

2. Linear specification

Let examine the case when the estimated equation is linear in the regressor. For example, Svensson [1993] estimates equation (5) which is in a differenced form and X_t includes only domestic and foreign interest rates or nothing:

$$(17) \quad Dx_{t+a} = \beta_0 + (\beta_1 - 1)x_t + \beta_4 r_t + \beta_5 r^*_t \qquad Dx_{t+a} \equiv x_{t+a} - x_t$$

Combining equations (3) and (17) the result is equation (18) for the expected rate of devaluation:

$$(18) \quad E_t(Dc_{t+a}) = -\beta_0 + (1-\beta_4)(r_t - r^*_t) - (\beta_1 - 1)\, x_t - (\beta_5 + \beta_4)\, r^*_t$$

One should assume that the true population value of β_1 is less than unity, otherwise it is not take much sense to apply the drift adjustment method. In this case the cited claim of Svensson on page 5 does not hold since the estimated expected devaluation does depend on the exchange rate's position within the band.[6] The more positive x_t (the weaker the currency) the higher the estimated expected devaluation.

All regressions of Svensson for the EMS and of this paper for floating rates yielded negative estimates for (β_1-1). Both for the floating and EMS estimates the longer the time horizon the smaller the estimated $(\hat{\beta}_1-1)$. For example, for three and twelve months horizons the average estimates were -0.28 and -0.83 for the dollar (with 1 intercept) and -0.46 and -1.00 for EMS (with varying intercepts). At least the floating evidence suggests that the overlapping observations problem is quite serious for the autoregressive parameter estimate. One explanation might be that the longer the overlapping (i.e. higher a), the more severely downward biased the autoregressive parameter. Another explanation might be the non-complete randomness of floating rates. For example, if either there are speculative bubbles that burst out the end or short and medium-run movements occur around the stable fundamental equilibrium exchange rate, than for longer horizon one is expected to see more "reversion" for floating rates.

If equation (17) do not include interest rates (i.e. $\beta_4 = 0$) then the coefficient of interest differential in equation (18) is positive. Moreover, β_4 had negative sign in eleven of the

*** Table 12, see page 40 ***

R-squared is close to zero, average t-values are far below even 1 and only about 5 percent of the experiments resulted in significant parameter estimates for the random walk according to standard statistical tables. Interestingly, exactly 5 percent of the experiments rejected the hypothesis of unit root with the with the critical value -2. Therefore, the significant parameter estimates for the Netherlands and Ireland is not the spurious result of a unit autoregressive root.

VI. Conclusions

Empirical applications of the drift-adjustment method reported to describe successfully realignment expectations of real world target zones and to coincide with the theoretical predictions of the Bertola-Svensson model. This paper showed that several time series having autoregressive root close to unity perform similar outputs under the drift adjustment method considering criteria applied for the EMS: plots and dots, significance of parameters according to standard statistical distributions, R-squared and standard error of regression. These time series include the US dollar against the mark, yen, and pound and the vast majority of random walks.

Although several conclusions of this paper are based on unit root Monte-Carlo simulations, one of the key message is that the relevant question is not whether there is a unit root in exchange rate within the band or not. The crucial questions are whether one can model the exchange rate within the band independently of other factors (i.e. expectations for devaluation of central parity), how to model the exchange rate within the band, and especially whether one can assume the same data generating process (DGP) for the whole sample. The paper suggest that for those currencies that are to be devalued, the process might have different characteristic at least before and after a devaluation.

In analyzing the linear version of the drift adjustment method this paper pointed out that overlapping observations and variable intercepts switches the Dickey-Fuller critical values substantially, even with Newey-West HAC errors. That's why considering the specification that was used to judge that EMS currencies display strong mean reversion, the US dollar seems to display strong mean reversion as well. However, using critical values from the simulated distributions, both the dollar and most EMS rate do not seem to be stationary anymore. It was also shown that estimated expected devaluation does depend on the

2. A theory based non-linear estimate

After deriving the theoretical model in discrete time, Koedijk-Stork-de Vries [1997] present an empirical method to (1) examine the empirical validity of the model and to (2) estimate devaluation probabilities. Here I only consider the empirical specification of the first task. They are able to derive the exchange rate function as

(27) $s_{t+1} = a(s_t) + \phi_{t+1}$.

and show that the $a(.)$ function is convex toward the weak edge and concave toward the strong edge. They approximate the exchange rate function by a Taylor series expansion around the central parity,

(28) $s_{t+1} = c_t + \beta_1(s_t - c_t) + \beta_2(s_t - c_t)^2 + \beta_3(s_t - c_t)^3 + \psi_{t+1}$.

where ψ_{t+1} denotes the sum of the innovations and the remainder of the Taylor-series. An important result is that they recognize that the second derivative of the exchange rate function is zero evaluated at the middle of the band, and negative when the exchange rate is in the upper part of the zone and positive in the lower part. This is the consequence of the fact that the function is convex to the left of the central parity and concave to the right. That's why they estimate the following equation:

(29) $s_{t+1} - s_t = \beta_0 - \delta x_t + \beta_2^+ x_t^2 + \beta_2^- x_t^2 + \beta_3 x_t^3 + \psi_{t+1}$.

where $x_t = s_t - c_t$ as before, and they dummy the squared deviation from the central parity by whether it is above or bellow the central parity.

Authors raise doubts on the drift adjustment method as well. These are twofold: (1) Authors claim that when $(1-\beta_1)$, β_2^+, β_2^-, and β_3 are collapsed into a single coefficient (= a first order approximation as in several other papers, like in Svensson [1993]), then the estimate of β_1 is likely be less than one in contrast to the case when these effects are separated out. (2) Without the distinction made for the squared term (β_2^+, β_2^-) the single estimate of β_2 likely collapses to zero and eliminates the non-linear effect when there are.

I have also carried out the simulation for the empirical method of the Koedijk *et al.* paper. Since the dependent variable in these cases are a white noise (change in the random walk) one might expect no explanatory power and insignificant parameter estimates.

twelve[7] estimates of Svensson [1993] and four of six estimates of this paper[8], therefore, the coefficient of interest differential in equation (18) is positive. Thus, the higher the interest differential and the weaker the exchange rate within the band the higher the estimated expected devaluation. Since both interest differential and the exchange rate within the band usually increase before devaluation and decrease afterwards, the specification (17) will produce nice estimates for expectations, especially when the estimate of β_1 is downward biased.

3. Unit root

First of all, I would like to emphasize that the relevant question is not whether there is a unit root in exchange rate within the band or not. A unit root implies an ever increasing variance and loosely speaking the possibility of being anywhere as the sample size increases. These are definitely not the characteristics of an exchange rate within the band, since it is *always* within the band. The real questions are whether the exchange rate within the band can be modeled independently of everything else, how to model the exchange rate within the band, and especially, whether one can assume the same data generating process (DGP) for the whole sample.

Nonetheless Bertola and Svensson [1993] claim that the hypotheses of unit roots in the exchange rates within the band processes were rejected. However, Rose and Svensson [1995] do not mention any unit root tests. Svensson [1993] regard his estimate of equation (5) as suitable unit root test as well, however, there are severe methodological problems. Problems arise from the fact the Dickey-Fuller tables refer to the case when (1) the constant is the same through the whole sample, (2) a equals 1, and (3) there are no other explanatory variables in the regression. Here nothing is said about the statistical properties of these variables. Theoretically, each of the three factors invalidates the Dickey-Fuller tables and a convincing empirical demonstration is the floating evidences: t-values ranged between -9.8 and -26.9 for three months and -26.7 to -64.1 for one year estimates of this

[6] This is true, of course, for every specification that assumes or implies (whole sample) mean reversion.

[7] The only positive coefficient was 0,17, thus the coefficient of the interest differential $(1 - \beta_1)$ in equation (18) is positive as well.

paper showing an even "more significantly rejection" of the unit root hypothesis for the floating rates.

It is easy to find critical values for modifications in (1) and (2). Tables 7 - 9 show critical values when (a) a equals 65, (b) there are six intercepts in succession, (c) some observations excluded, and the combination of these for sample size 3000. Table 7 contains values for OLS t-statistic $(\hat{\rho}-1)/\sigma_{\hat{\rho}}$, Table 8 shows the same with standar errors estimated by Newey-West HAC, and Table 9 contains vaies for OLS autoregressive coefficient $T(\hat{\rho}-1)$.

*** Tables 7 - 9, see page 37 ***

The table also contains the true critical values from the Dickey-Fuller distribution (first data column) and the Dickey-Fuller values implied by the generated random walks (second data column). These two columns are virtually identical so we can regard the approximation error to be very small. The Dickey-Fuller critical value is -2.87. This is switched by the six intercepts to -4.64 (with HAC errors to -4.68), and by the overlapping observations for 65 periods to -23.99 (HAC. -8.72). They jointly switches the value to -39.67 (HAC. -14.32). These seems to be really huge changes. Comparing the -23.99 and -8.72 values to the dollar estimates, they do not seemed to be stationary anymore[9]. This is also true for the EMS estimates. Simulations for $T(\hat{\rho}-1)$ reveals the same conclusion expect that the Dutch guilder is stationary.

Tables 10 and 11 show critical values when a equals 1 as in the Dickey-Fuller case but there are different number of (non-overlapping) dummies, from 1 to 10. It is clear that the more intercept terms are included the more the critical values change.

*** Tables 10 and 11, see page 38 ***

Lindberg and Söderlind [1994] report unit root tests for the Swedish Krona. Interestingly, they could reject the hypothesis of unit root for a nine year period (01/82-11/90) but could not reject in two subperiods covering three and a half years each (01/82-

[8] Of course, there does not seem to be any hidden reason why this estimate is likely to be negative.

[9] One should be care because the interest differential is included in the test equation.

rarely emerge in real world data (e.g. CUSUM plots). The R-squared did not differ much from zero.

However, the model sketched above consists of two regimes. Let the stationary regime written in the trend stationary form

(25) $x_t = -B + \mu t + \varepsilon_t$.

where μ represents, for example, the inflation differential and for simplicity let ε_t be a Gaussian white noise with "small" variance. (This does not exclude the possibility of x_t moving outside the lower bound of the band for "small" t.). For simplicity I also assumed that $-B = -2$, the upper end of the zone is 1, and the threshold level is zero. These implies that trend-stationarity characterizes the exchange rate in the lower two/third of the band. When x_t turns to be positive the process switches to:

(26) $x_{t+1} = b(1-x_t)\,x_t$.

where I set b = 3.9. Simulation of these switching process was carried out in sample sizes 3023 with μ set to 2/2000, 2/1500, and 2/1000. These values imply that the switch will take place at a bit less than two thirds, one-half, and one-third of the sample sizes, respectively. Initial values for chaos are random by construction. A series for the μ = 2/2000 case is shown in Figure 32.

*** Figure 32, see page 39 ***

I have estimated Equation (6) two hundred times each μ. The interesting fact is that 600 of the 600 estimates produced negative β_2 and β_3. The average R-squared was 0.949 for the two-third stationary and even 0.924 for the one-third stationary processes. The average values of the t-statistics were 31-40 and 27-29 for β_2 and β_3 . Plotting the counterpart of Figure 1 shows the same shape as the theoretical the FF/DEM, and USD estimates. However, there are some hump within the range of [-2,1], see Figure 33.

*** (Figure 33), see page 39 ***

This model does not prove anything. It only shows that it is possible to construct a model based on assumption coinciding with some stylized facts that generates excess sensitivity to initial conditions and unpredictability of the exchange rates within the band.

where k is the highest power of x_t in the determination of P_t and θ_i (i = 0, 1, ... , k) are parameters. For the non-linear cases it seems logical to require that:

(i) not all $\theta_i = 0$,

(ii) $P(B) = 1$,

(iii) $\frac{\partial P}{\partial x} > 0$ for $-B < x < B$ and $\frac{\partial P}{\partial x} \to 0$ as $x \to B$.

and let postpone the requirements for the lower bound for a moment. Condition (ii) assumes that the central bank never let the exchange rate reach the weak edge of the zone or when it does then the devaluation is certain at the next discrete point in time. For the later case no-arbitrage condition requires the domestic interest rate to be "high"

b. The stationary regime and the switch

A devaluation switches the process to a stationary one till the devaluation probability reaches the threshold level again. This requires to model both the stationary exchange rate process and the devaluation probability as well. A possibility would be to assume trend-stationarity with the trend equal the inflation differential and some noise around it. When x_t reaches a certain value agents switch the process to the expectation period described above.

Now it is clear that the counterpart of requirements (ii) and (iii), i.e. $P(-B) = 0$ and $\frac{\partial P}{\partial x} \to 0$ as $x \to -B$ has undesirable implications. These would imply that both x_t and P_t might take low values as well in the expectation period. A possibility might be to require that in the expectations regime x_t does not fall below the threshold level x_{TH} .

The conditions for θ_i are not sufficient for creating chaos but do not exclude its possibility. The second part of the appendix derives that chaos might emerge in this setup.

c. A simulation

This model might have relevance for this paper only if it produces an estimate for equation (6) similar to observed target zone data. Therefore, I have generated several hundreds of chaotic processes with initial values drawn from the uniform distribution on (0,1) for sample sizes 5023 and run regression (6) from 2001 to 5023. Results were interesting in the sense that every parameter seemed chaotic and showed such pictures that

06/85 and 02/86-10/89). Vlaar and Plam [1993] reject unit root for the six EMS exchange rates within the band for weekly observations.[10]

As many FF/DM data were available for me (since January 1988) unit root tests produced conflicting results. When the estimated equation did not include a constant the hypothesis of unit root could not be rejected at every usually applied significance levels. In the case of constant the hypothesis could be reject at 5%.[11] One might refer, for example, to Hamilton (1994, pp. 444-447) noting that how difficult to distinguish a unit root process from a stationary one in finite samples.

Recall the beginning of this subsection the important point is not whether there is a unit root. However, an estimated whole sample autoregressive representation will provide an estimate for the dominant inverted autoregressive root. Recall also that Rose-Svensson found non-linear effects only in case of FF/DM rate. It is possible that the FF/DM rate's (whole sample) autoregressive representation had the highest inverted autoregressive root among the EMS countries that lead to the significant looking empirical fit of the non-linear estimation in the form of equation (6).

4. Time dependent exchange rates within the band process

One of the most important objection to the drift adjustment method is the implicit assumption of time-independent exchange rate within the band process between two cases of devaluation. Rose and Svensson (1995, p.184) states that "there is no theoretical reason to presume that the stochastic process of the exchange rate within the band remains the same through all regimes"[12], but they implicitly presume that the process is the same within a regime. Clearly, there is no theoretical reason for this assumption. One might sensibly presume that the process has different characteristic in periods of "low" and "high" devaluation risk.

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[10] It seems an interesting problem why the Dickey-Fuller test reject unit root for weekly observations, while the overlapping version of the test for daily observations not.

[11] Both the Phillips-Perron and the Augmented Dickey-Fuller produced this result, with lag truncation for Bartlett kernel for the first was selected by the Newey-West method and the lag length for the second by four methods.

[12] A regime is defined as the period between two cases of devaluation.

V. Possible solutions

1. A simple model of non-predictability

Let assume a currency that is fundamentally weak, i.e. inflation is inherently higher than in the anchoring country and trade balance deteriorates in a period with prolonged exchange rate adjustment. Suppose that it is possible to divide the period between realignments into two regimes: (1) a devaluation is followed by a stationary exchange rate within the band process first and (2) at some point in time the process switches to an "expectation" period. This means that at a threshold level of devaluation risk agents base their exchange rate decisions primarily on the consideration of the devaluation risk. In the first period there might be mean reversion but not necessarily in the second.

Applying the drift-adjustment method to the first regime is misleading since any change in the intramarginal intervention policy of the central bank or any other short term shock result in variability of the expected rate of realignment. For the second time period a simple model described below shows that under its assumptions and certain parameter values the exchange rates within the band process will be non-predictable. It is convenient to describe the second regime first.

a. The "expectation" regime

The model for the second period is based on he following stylized facts: (1) the position of the exchange rate within the band reflects devaluation expectations, i.e. the closer the rate to the weak edge the higher the agents' expected rate of devaluation; (2) upon postponement of devaluation agents expect the exchange rate moving further to the weak edge of the original zone; (3) upon devaluation the exchange rate jumps to the strong region of the realigned band.

The drift-adjustment method assumes mean reversion in every situation, so Fact 1 and the opposite of Fact 2 are implicit assumption of the drift adjustment method, while Fact 3 is neglected.

Let the exchange rate be somewhere within the band. The market assumes probability P_t to a devaluation on or before the discrete point of time $t+1$. For simplicity, in the case of a devaluation let the exchange rate within the band jump to the strong edge of the realigned band. However, if a devaluation will not occur, the assumed probability of realignment will increase and the exchange rate within the band will move toward the weak edge of the original band.

The expected exchange rate within the band for period $t+1$ is:

(19) $E_t[x_{t+1}] = P_t\, E_t[x_{t+1} \mid R] + (1 - P_t)\, E_t[x_{t+1} \mid NR]$

where R and NR denote realignment and no realignment. Considering the simplifying assumptions and the stylized facts it follows that

(20) $E_t[x_{t+1} \mid R] = -B$

(21) $E_t[x_{t+1} \mid NR] = x_t + \delta(B - x_t) = x_t(1 - \delta) + \delta B \qquad 0 < \delta \leq 1$

where B denotes the one sided width of the band, that is, the maximum possible deviation from the central parity measured in percent. Suppose that δ is constant in this regime, so $0 < \delta < 1$.

Suppose further that in this regime the probability of realignment relates to the exchange rate within the band by the linear equation

(22) $P(x_t) = \theta_0 + \theta_1 x_t$

and θ_0 and θ_1 are such that $\theta_1 > 0$ and $0 \leq \theta_0 + \theta_1 x \leq 1$ for all $-B \leq x \leq B$.

In this case the expected exchange rate within the band for period $t+1$ is:

(23) $E_t[x_{t+1}] = (\theta_0 + \theta_1 x_t)(-B) + (1 - \theta_0 - \theta_1 x_t)\{x_t(1 - \delta) + \delta B\} =$

$= \Phi_0 + \Phi_1 x_t + \Phi_2 (1 - \Phi_3 x_t) x_t$

Equation (23) is a mutation of the well-known logistic function. <u>If we were able to show that the expected value of x_{t+1} is a logistic difference function of x_t as well than it is possible that for certain parameter values the exchange rates within the band process might follow chaotic dynamics.</u> Chaos might remain even if we replace equation (22) with other functional form, e.g. with a higher order polynomial:

(24) $P_t = \sum_{i=0}^{j} \theta_i x_t^i$