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**FORWARD FOREIGN EXCHANGE  
RATES, EXPECTED SPOT  
RATES, AND PREMIA:  
A SIGNAL-EXTRACTION  
APPROACH**

**Christian Wolff**



**Centre for Economic Policy Research**

6 Duke of York Street, London SW1Y 6LA

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Christian C P Wolff

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Centre for Economic Policy Research  
6 Duke of York Street  
London SW1Y 6LA

Tel: 01 930 2963

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Forward Foreign Exchange Rates, Expected Spot Rates, and Premia:  
A Signal-Extraction Approach \*

ABSTRACT

In this paper, we implement a methodology to identify and measure premia in the pricing of forward foreign exchange. The methodology involves application of signal-extraction techniques from the engineering literature. Diagnostic tests indicate that these methods are quite successful in capturing the essence of the time-series properties of premium terms. The estimated premium models indicate that premia show a certain degree of persistence over time and that more than half the variance in the forecast error that results from the use of current forward rates as predictors of future spot rates is accounted for by variation in premium terms. The methodology can be applied straightforwardly to the measurement of unobservables in other financial markets.

JEL classification: 430

Keyword: exchange rates, premia, Kalman filtering

Christian C P Wolff  
London Business School  
Sussex Place  
Regent's Park  
London NW1 4SA

Tel: 01 262 5050

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## NON-TECHNICAL SUMMARY

Empirical research on the behaviour of forward exchange rates has focused on the discrepancy between the forward rate and the corresponding realization of the spot rate. This difference can be represented as the sum of errors in expectations concerning the future spot rate and a premium reflecting risk and inflation. If we assume that market forecasts of spot rates are made efficiently, using all the information available when the forecast is made, the errors in expectations will form a purely random sequence which is not correlated with its own past values. The remainder of the discrepancy, the premium, will generally vary over time in a partly systematic and partly stochastic fashion. Recent attempts to analyse these premia have used regression techniques in which the discrepancy (between the forward and realized spot rates) is regressed on a set of explanatory variables which represent the information available to the market. Conclusions concerning the behaviour of premia have been based on the nature of the estimated relationship, but the choice of explanatory variables is rather arbitrary and often dictated by the availability of data.

In this paper we examine the discrepancies between 30-day forward rates and the corresponding spot rates for the Dollar and the Deutschmark, the Dollar and the Yen and the Dollar and the Pound sterling over the period 1973-84. We attempt to separate the premia from the expectational errors. To separate the two we use the Kalman filter, a recursive algorithm based on Bayes's theorem, which divides the variation in the observed series into that due to "the signal" (the premia) and that due to the noise (the errors in expectations). The parameters of the filter are chosen by maximum likelihood methods. We assume that expectations are formed rationally, so that errors in these expectations are indeed "white noise". In order to apply the Kalman filter we must also specify the process governing the behaviour of the premium term. Here we assume that the premia are generated by an ARMA process, whose structure we determine by examining the autocorrelations and partial autocorrelations of

the data. The differences between the forward and realized spot rates for the \$/DM and \$/Yen both follow MA(1) processes, while the difference for the \$/£ rate requires an ARMA(1,1) representation.

Once an appropriate model for the premium term is identified, the Kalman filter is applied to the series in order to extract the "signal", i.e. the period by period estimates of the premium terms. The estimated premia are themselves of considerable interest; in addition, we can test whether the premium term contains a systematic element by testing whether the autoregressive or moving-average coefficient in the premium model is zero: we find that more than half the variation in the difference between the forward rate and the corresponding realization of the spot rate can be attributed to the premia.

The estimated premia reached magnitudes of between 4% and 6% for the three series studied: \$/£ premia show substantial volatility. The premium terms for different currencies have apparently moved together: correlations between the estimated premia lie in the range 0.40-0.55.

There exists a growing body of empirical research on premia in the pricing of forward foreign exchange. Conditional on the hypothesis that the foreign exchange market is efficient or rational, the existence of time-varying premia has been documented in the literature by Fama [6], Hansen and Hodrick [10],[11], Hodrick and Srivastava [15],[16], Hsieh [17], and Korajczyk [20]. Frankel [9] fails to identify such premia and Domowitz and Hakkio [5] obtain different results for different currencies.

Recent methodologies to measure time-varying premia are usually centered around regression equations. Fama [6] applied regression analysis to estimate empirically the degree of variation of the premium over time and to investigate the degree of covariation of the premium with the expected future spot exchange rate. His findings indicate that most of the variation in forward rates is due to variation in premia and that the premium and expected future spot rate components of forward rates are negatively correlated. Hodrick and Srivastava [16] confirmed Fama's results on the basis of Generalised Method of Moments (GMM) estimation and other techniques. Hansen and Hodrick [11] relied on the first-order conditions of optimality for a rational representative investor (and some auxiliary assumptions) to construct a single-beta latent variable model to measure risk. Hodrick and Srivastava [15] re-examined the Hansen and Hodrick model and statistically reject the restrictions imposed by the model. They point at parameter instability as a factor responsible for the rejection. The methodologies used in these papers usually involve estimation of equations in regression format. Conclusions about the behaviour of premia in the pricing of forward foreign exchange are drawn on the basis of the observed relationship between a regressand and a set of regressors. Since the choice of a set of regressors to be used in this context is fairly arbitrary, we propose an alternative, complementary approach to identifying and measuring premia. The

approach implements procedures for extracting signals from noisy environments that are widely used in the engineering literature in a variety of contexts.

At the conceptual level, one can divide the forward foreign exchange rate observed at time  $t$  for currency to be delivered at  $t+1$  into an expected future spot rate component and a premium component:

$$F(t,t+1) = E[S(t+1)|t] + P(t), \quad (1)$$

where  $F(t,t+1)$  is the natural logarithm of the forward rate at time  $t$  for an exchange at  $t+1$ ,  $E[S(t+1)|t]$  is the rational or efficient forecast of the log of the spot exchange rate at  $t+1$ , conditional on all information available at  $t$ , and  $P(t)$  is a premium term. Throughout the paper we will use logarithms of exchange rates rather than their simple levels in order to circumvent any problems arising from Jensen's inequality.<sup>1</sup> The premium term  $P(t)$ , as it is defined in equation (1), does not yet have any economic content. In order to give it content, we need a model of international asset pricing which describes the determination of  $P(t)$ . Equilibrium models of international asset pricing that provide us with such descriptions are presented, for instance, in Adler and Dumas [1], Fama and Farber [7], Hodrick [14], Hodrick and Srivastava [15], Roll and Solnik [22], Solnik [23] and Stulz [24]. Subtracting  $S(t+1)$  from both sides of equation (1), and defining  $v(t+1) \equiv E[S(t+1)|t] - S(t+1)$ , we obtain

$$F(t,t+1) - S(t+1) = P(t) + v(t+1), \quad (2)$$

where  $v(t)$  is an uncorrelated, zero mean sequence. Equation (2) states that the forecast error resulting from the forward rate as a predictor of the future spot rate consists of a premium component and a white noise error term due to the arrival of new information between  $t$  and  $t+1$  concerning the spot rate at  $t+1$ . It is convenient to refer to the premium component  $P(t)$  as the signal that we would like to characterise and to  $v(t+1)$  as noise that is added to the signal. The problem that we face thus involves extracting a signal from a noisy environment.

The presentation of the paper is as follows. In section I we start with a description of our dataset and a preliminary look at the time series properties of  $F(t,t+1) - S(t+1)$ . The latter is necessary because the methodology that is described in section II requires a prespecified time series process for the premium term. In section III the empirical results are presented and discussed and section IV offers some concluding comments.

### I. Identifying Premium Models

In this section of the paper we will discuss our model identification procedure. We study premia for three important bilateral exchange rates involving the U.S. Dollar: the Dollar/Pound, Dollar/Mark and Dollar/Yen exchange rates. Thirty-day forward rates and subsequently observed spot rates are taken from the Harris Bank Data Base supported by the Center for Studies in International Finance at the University of Chicago. The rates are Friday closes, sampled at four-week intervals. There are 148 observations covering the period April 6, 1973 to July 13, 1984.<sup>2</sup>

From equation (2) we see that autocorrelations of  $F(t,t+1)-S(t+1)$  must at the same time be the autocorrelations of the combined premium plus noise process  $P(t)+v(t+1)$ . Our modelling strategy in this paper is based on precisely this observation. We will study the autocorrelations of  $F(t,t+1)-S(t+1)$  in order to infer an appropriate time series model for  $P(t)$ . This identification process is based on a summation theorem for moving average processes from the literature. Once an appropriate premium model is identified, a Kalman filter model is estimated and used for recursive signal extraction in order to obtain period-by-period estimates of premium terms. In Table I we present autocorrelations and partial autocorrelations of  $F(t,t+1)-S(t+1)$ .

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#### INSERT TABLE I

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Significant autocorrelations and partial autocorrelations are reported in a number of cases, in particular at lag one. Traditional identification procedures in the spirit of Box and Jenkins [4] can be applied to identify time series models for  $F(t,t+1)-S(t+1)$ . The autocorrelations in Table I are consistent with an ARMA(1,1) model for the Dollar/Pound case and MA(1) models for the Dollar/Mark and Dollar/Yen cases.<sup>3</sup> These ARMA (1,1) and MA(1) processes are also representations for the combined premium plus noise processes  $P(t)+v(t+1)$ . Using a summation theorem for moving average processes, we can infer appropriate models for the premium processes alone. An ARMA(1,1) model for  $P(t)+v(t+1)$  is consistent with an AR(1) model for the premium, to which a white noise error term,  $v(t+1)$ , is added. This can be seen as follows. Let us specify an AR(1) model for the premium:



$$P(t) = \phi P(t-1) + a(t), \quad (3)$$

where  $a(t)$  is normally distributed with mean zero and variance  $\sigma_a^2$ . Using equations (2) and (3), we can then derive:

$$[F(t,t+1)-S(t+1)] - \phi[F(t-1,t)-S(t)] = v(t+1) - \phi v(t) + a(t). \quad (4)$$

The right hand side of (4) consists of a first-order moving average process and a white noise error term. The summation theorem for moving averages in Ansley, Spivey and Wroblewski [3] states that the summation of two uncorrelated moving average processes of orders  $q_1$  and  $q_2$ , respectively, has a  $MA(q^*)$  representation, where  $q^* \leq \max[q_1, q_2]$ . Making use of this theorem, we can conclude that  $F(t,t+1)-S(t+1)$  in equation (4) has an ARMA(1,1) representation.<sup>4</sup> Using the same theorem, we can also conclude directly that a MA(1) model for  $F(t,t+1)-S(t+1)$  is consistent with a MA(1) model for  $P(t)$ . In the remainder of the paper we will employ the AR(1) specification for the premium in the Dollar/Pound case and the MA(1) specification for the Dollar/Mark and Dollar/Yen premia.<sup>5</sup> These specifications are parsimonious and fully logically consistent with the autocorrelation patterns that are observed.

## II. Methodology

The methodology of this paper involves signal extraction by means of recursive application of the Kalman filter. Kalman filtering is described in Kalman [19], Anderson and Moore [2] and Harvey [13]. Kalman filtering models have appeared in the finance literature in different contexts. For instance, Fama and Gibbons [8] apply the Kalman filter to study the relationship between expected real rates of return and expected inflation and Hsieh and Kulatilaka [18] use a Kalman filter model in a paper on primary metals markets.

In order to be able to apply the Kalman filter, the models for various premia are formulated in so-called state space form. Consider the following general model:

$$y(t) = Hb(t) + e(t) \quad (5)$$

$$b(t) = Tb(t-1) + u(t). \quad (6)$$

Here  $y(t)$  is a  $k \times 1$  vector of observed variables,  $H$  is a  $k \times m$  matrix of known constants,  $b(t)$  is a  $m \times 1$  vector of unobservable state variables,  $T$  is a  $m \times m$  state transition matrix and  $e(t)$  and  $u(t)$  are vectors of disturbance terms. In this context it is usual to refer to (5) as the

observation equation and to (6) as the state transition equation. In addition the following properties are assumed:

- (a)  $E[e(t)] = 0$ ,  $\text{var}[e(t)] = R$
- (b)  $E[u(t)] = 0$ ,  $\text{var}[u(t)] = Q$
- (c)  $[e(t)]$  is an independent sequence
- (d)  $[u(t)]$  is an independent sequence
- (e)  $e(t)$  and  $u(r)$  are independently distributed for all  $r, t$
- (f)  $[e(t), u(t+1)]$  and  $b(r)$  are independent for all  $r \leq t$ .

Together with these assumptions, the system (5)-(6) forms a state space model. Models of this type can be recursively estimated by means of the Kalman filter.

The Kalman filter is essentially an algorithm that allows us to compute the mean and covariance matrix of the vector  $b(t)$  on a period-by-period basis. We assume that the error terms  $e(t)$  and  $u(t)$  are normally distributed and that  $b(t)$  has a normal prior distribution with mean  $b(0|0)$  and covariance matrix  $V(0|0)$ .<sup>6</sup> At every point in time  $t$ , after the history of the process  $Y(t) = \{y(r); r=1, 2, \dots, t\}$  has been observed, we want to revise our prior distribution of the unknown state vector  $b(t)$ . The Kalman filter enables us, given knowledge of  $b(0|0)$ ,  $V(0|0)$ ,  $Q$ ,  $R$  and  $T$ , to compute recursively the mean and covariance matrix of  $b$  for each period in time. Denote the conditional distribution of  $b(t)$  given  $Y(t)$  by  $p[b(t)|Y(t)]$ . Given the normality assumptions above,  $p[b(t)|Y(t)]$  and  $p[b(t+1)|Y(t+1)]$  are also normal and completely characterised by their first two moments. If we denote the mean and covariance matrix of  $p[b(t)|Y(t)]$  by  $b(t|t)$  and  $V(t|t)$ , respectively, and those of  $p[b(t+1)|Y(t+1)]$  by  $b(t+1|t)$  and  $V(t+1|t)$ , then the Kalman filter recursions for  $t=0, 1, 2, \dots, N$ , where  $N$  is the total number of observations, are given by equations (7)-(12):

$$b(t+1|t) = Tb(t|t) \tag{7}$$

$$y(t+1|t) = Hb(t+1|t) \tag{8}$$

$$V(t+1|t) = TV(t|t)T' + Q \tag{9}$$

$$D(t+1) = HV(t+1|t)H' + R \tag{10}$$

$$b(t+1|t+1) = b(t+1|t) + V(t+1|t)H'[D(t+1)]^{-1}[y(t+1) - Hb(t+1|t)] \tag{11}$$

$$V(t+1|t+1) = V(t+1|t) - V(t+1|t)H'[D(t+1)]^{-1}HV(t+1|t) \tag{12}$$

The Kalman filter gives us  $b(t|t)$  and  $V(t|t)$  for  $t=1, \dots, N$ . These quantities are conditioned on the information that is available at time  $t$ . More efficient estimates can be obtained on the

basis of the entire sample of y's. The fixed interval smoothing algorithm allows us to compute such improved estimates  $b(t|N)$  and  $V(t|N)$ . After the data are passed through the Kalman filter, the fixed interval smoothing algorithm involves a backward "sweep" over the data from  $t=N$  back to  $t=1$ . The smoothing equations for  $t=N-1, N-2, \dots, 1$  are given by equations (13) and (14):

$$b(t|N) = b(t|t) + V(t|t)T'[V(t+1|t)]^{-1}[b(t+1|N) - b(t+1|t)] \quad (13)$$

$$V(t|N) = V(t|t) + V(t|t)T'[V(t+1|t)]^{-1}[V(t+1|N) - V(t+1|t)][V(t+1|t)]^{-1}TV(t|t). \quad (14)$$

In the backward pass through the data the Kalman filter estimates of  $b$  and  $V$  are revised on the basis of all available information on the  $y$ -vector. In the above discussion it was assumed that the matrices  $Q$ ,  $R$  and  $T$  are known. In practice, this will almost never be the case. Maximum likelihood techniques are available to estimate the model parameters, as will be seen below.

In order to be able to apply the Kalman filter in the context of our premium models, the premium models have to be arranged in state space format. From the discussion in Section I, we have the following two-equation model for the premium in the Dollar/Sterling case:

$$F(t,t+1) - S(t+1) = P(t) + v(t+1) \quad (15)$$

$$P(t) = \phi P(t-1) + a(t). \quad (16)$$

These equations correspond precisely to the format that was outlined above. This can be easily seen if we employ following definitions:  $y(t) \equiv F(t,t+1) - S(t+1)$ ,  $H \equiv 1$ ,  $b(t) \equiv P(t)$ ,  $e(t) \equiv v(t+1)$ ,  $T \equiv \phi$  and  $u(t) \equiv a(t)$ . Equations (15) and (16), together with appropriately modified versions of assumptions (a)-(f) above, form a state space model. Harvey [12],[13] shows that the concentrated<sup>7</sup> log-likelihood function for the problem at hand can be expressed as:

$$\log L = -(N/2) \log(2\pi) - (N/2) \log\left\{\frac{1}{N} \sum_{t=1}^N [w(t)]^2\right\} - 0.5 \sum_{t=1}^N \log[f(t)] \quad (17)$$

where  $f(t) \equiv V(t|t-1)/\sigma_v^2 + 1$  and  $w(t) \equiv [y(t) - y(t|t-1)]/[f(t)]^{0.5}$ . Note that  $f(t)$  and  $w(t)$  are quantities which can be computed directly by the Kalman filter.<sup>8</sup> Thus, the Kalman filter can be used to evaluate the log-likelihood function (17). For the Dollar/Mark and Dollar/Yen cases, we have the following two-equation systems:

$$F(t,t+1) - S(t+1) = P(t) + v(t+1) \quad (18)$$

$$P(t) = a(t) + \theta a(t-1) \quad (19)$$

where  $a(t)$  is again white noise. Equations (18) and (19) are not in state space form, but can be rearranged in order to obtain a state space model. Define the variable  $z(t) \equiv \Theta a(t)$ . Then (18) and (19) can be rearranged as (20) and (21):

$$F(t,t+1) - S(t+1) = [1 \ 0] \begin{bmatrix} P(t) \\ z(t) \end{bmatrix} + v(t+1) \quad (20)$$

$$\begin{bmatrix} P(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P(t-1) \\ z(t-1) \end{bmatrix} + \begin{bmatrix} a(t) \\ \Theta a(t) \end{bmatrix}. \quad (21)$$

Equations (20) and (21) are in state space form. In terms of the model that was presented in equations (5) and (6) this can be seen if we define:  $y(t) \equiv F(t,t+1) - S(t+1)$ ,  $H \equiv [1, 0]$ ,  $b'(t) \equiv [P(t), z(t)]$ ,

$e(t) \equiv v(t+1)$ ,  $T \equiv \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $u(t) \equiv [a(t), \Theta a(t)]$ . Thus, the Kalman filter can also be applied in these

cases. The models' parameters can again be estimated by maximisation of the likelihood function in (17). Note that in this context  $V(t|t-1)$  in the definition of  $f(t)$  needs to be replaced with its first diagonal element, since  $V(t|t-1)$  is now a matrix. In order to start off the Kalman filter in the estimation procedure the starting values  $b(0|0)$  and  $V(0|0)$  need to be specified. Since the premium models are stationary,<sup>9</sup> such initial values are automatically available in the unconditional means and variances of the  $b$ -vectors. (See Harvey [13].)

In Section III of the paper maximum likelihood estimates of the premium models will be presented. Standard likelihood ratio tests will be employed to test the null hypothesis that the autoregressive or moving average parameter in a particular premium model is zero against the alternative hypothesis that it is not. This amounts to testing for the presence of systematic (nonrandom) components in the premia. An interesting diagnostic check of the models is possible, based on autocorrelations. In Table I we studied the autocorrelations of  $F(t,t+1) - S(t+1)$  or, equivalently, of  $P(t) + v(t+1)$ . In that table the systematic component of  $P(t)$  gives rise to significant autocorrelations and partial autocorrelations. If our signal-extraction models have successfully captured the essence of the premium terms, then it should be the case that  $F(t,t+1) - S(t+1) - P^S(t)$ , where  $P^S(t)$  is the systematic component of  $P(t)$ , is not autocorrelated. To see this, consider equations (22) and (23), which are equivalent to (15) and (16) or (18) and (19):

$$F(t,t+1) - S(t+1) = P(t) + v(t+1) \quad (22)$$

$$P(t) = P^S(t) + a(t). \quad (23)$$

The systematic component of  $P(t)$  is defined as  $P^S(t) = \phi P(t-1)$  for the AR(1) premium model and as  $P^S(t) = \theta a(t-1)$  for the MA(1) models. If we substitute (23) into (22) and bring  $P^S(t)$  to the left hand side, we obtain:

$$F(t,t+1) - S(t+1) - P^S(t) = a(t) + v(t+1). \quad (24)$$

Since  $a(t)$  and  $v(t+1)$  were assumed to be uncorrelated white noise sequences, it should be the case that  $F(t,t+1) - S(t+1) - P^S(t)$  is an uncorrelated sequence. This implication of the model is tested below on the basis of estimated autocorrelation functions and the Box and Ljung [21] portmanteau test of randomness.

A brief discussion of the relative merits of the methodology employed in this paper and more traditional regression-based approaches is in order. The regression-based approaches depend heavily on the econometrician's choice of the set of variables that are in traders' information sets at time  $t$ , onto which the regressand [usually  $F(t,t+1) - S(t+1)$ ] is to be regressed, which is "arbitrary and dictated in most cases by availability of data" (Hansen and Hodrick [11], p.120). For instance, Fama [6] studied a pair of complementary regression equations, one of which was the following:

$$F(t,t+1) - S(t+1) = \alpha + \beta [F(t,t+1) - S(t)] + \epsilon(t). \quad (25)$$

Here  $F(t,t+1) - S(t)$ , the current forward-spot differential, is a variable that was known at time  $t$ ,  $\alpha$  and  $\beta$  are parameters to be estimated and  $\epsilon(t)$  is a disturbance term. The particular specification of equation (25) allowed Fama to make a number of interesting inferences about the behavior of premia in the pricing of forward foreign exchange. In general, however, many other potentially useful regressors could be included in equation (25). Hansen and Hodrick [11], for instance, employed up to ten right-hand-side variables in a similar context. The signal-extraction approach used in this paper circumvents this problem of arbitrariness of regressors, at the expense of having to specify an exact time series model for the premium. Given these different limitations of the two approaches, we think it is useful to regard them as complementary.

### III. The Empirical Results

Maximum likelihood estimates of the state space models for premia in the pricing of forward foreign exchange are presented in Table II.

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INSERT TABLE II

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The estimated parameters imply a stationary premium model for the Dollar/Pound rate and invertible premium models for the Dollar/Mark and Dollar/Yen rates.<sup>10</sup> Marginal significance levels (p-values) associated with the likelihood ratio test statistics which test the significance of the autoregressive or moving average parameter indicate the parameters are highly significant. Both the autoregressive and moving average parameter estimates indicate that premia show a certain degree of persistence over time. That is, if today's premium is positive the probability that next month's premium will also be positive is greater than fifty percent. The variance estimates that are reported in Table II are interesting. In all three cases  $s_p^2$  (the variance of the premium term) is greater than  $s_v^2$  (the variance of the noise term  $v$ ). Variation in the premium terms accounts for more than half of the variation in the forecast errors  $F(t,t+1) - S(t+1)$ . Comparison of  $s_p^2$  and  $s_a^2$  (the estimated variance of the noise term  $a$ ) indicates that most of the variation in the premia is of a random (unsystematic) nature.

In Table III autocorrelations and partial autocorrelations of  $F(t,t+1) - S(t+1) - P^S(t)$  are presented. These were calculated on the basis of estimates of  $P^S(t)$  that result from the Kalman filter.

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INSERT TABLE III

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None of the individual autocorrelations and partial autocorrelations in Table III are significant. The marginal significance levels associated with the Box and Ljung [21] portmanteau test of randomness also indicate that no significant residual autocorrelation is present. Thus the estimation of the premium models is successful in the sense that the autocorrelation patterns that were reported in Table I are fully accounted for. The signal-extraction models appear to have captured the essence of the time series properties of the premium terms. The estimates reported in Table III imply time series of estimated premium terms. As an example the Dollar/Sterling premia are presented in Figure 1. The premium

estimates that are presented in this figure are smoothed estimates, obtained via the fixed interval smoothing algorithm.

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INSERT FIGURE 1

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The figure indicates substantial volatility in the Dollar/Sterling premium. For the three exchange rates that we study premia have reached magnitudes of 4 to 6 percent in absolute value. As is to be expected on the basis of theoretical models of international asset pricing, the premium terms can be positive as well as negative. (See, e.g., the formulas in Adler and Dumas [1].) Cross-currency correlations of premia are in the range 0.40-0.55, indicating that premium terms for different currencies, relative to the U.S. Dollar, have moved together to a considerable extent.

#### IV. Concluding Observations

In this paper we implemented a previously unexplored methodology to identify and measure premia in the pricing of forward foreign exchange. The methodology involves application of signal-extraction techniques from the engineering literature. While the approach is in principle limited by the particular time series representations that we have to adopt for premium terms, diagnostic tests indicate that the methodology was quite successful in capturing the essence of the time series properties of premia.

The estimated premium models indicate that premia show a certain degree of persistence over time. Fama [6] and Hodrick and Srivastava [16] found that most of the variation in forward rates is due to variation in premia. The variance estimates presented in this paper also suggest substantial variation in premia: more than half of the variance in the forecast error that results from the use of current forward rates as predictors of future spot rates, is accounted for by variation in premia.

Finally, the methodology can be applied straightforwardly to similar problems in the context of different financial markets. Identification and estimation of premia in the term structure of interest rates, for instance, is a problem that can be attacked within the framework presented in this paper.

#### Footnotes

1. Because of Jensen's inequality the best predictor of the level of the spot exchange rate expressed as units of currency  $i$  per unit of currency  $j$  is not generally the best predictor of the level of the same expressed as units of currency  $j$  per unit of currency  $i$ .
2. The same data source was used in Fama's [6] paper on forward and spot exchange rates.
3. Alternative possibilities will be entertained in footnote 9.
4. Note that a white noise sequence can be interpreted as a MA(0) process.
5. Alternative possibilities will be considered in footnote 9.
6. Without the normality assumptions, the Kalman-filtering results hold for best linear unbiased predictions rather than conditional expectations.
7. The likelihood function is concentrated over  $\sigma_v^2$ , the variance of the noise term  $v$ . The particular form of the likelihood function that is shown in equation (17) exploits the fact that only the ratio of the variance of  $v(t+1)$  to the variance of  $a(t)$  matters for the calculation of premium terms, not the individual variances. (See Harvey [12].)
8. See Harvey [12], [13].
9. The MA(1) premium models are obviously stationary. Preliminary maximum likelihood estimation of the model for the Dollar/Pound rate, on the basis of starting values that were constructed from a subset of the observations, indicated that the autoregressive parameter was well within the region of stationarity.
10. In order to try to assess how sensitive measured premium terms are to different model specifications, we also estimated state space models with AR(1) premium processes for the Dollar/Mark and Dollar/Yen rates and a state space model with a MA(1) premium process for the Dollar/Pound rate. The likelihood functions associated with the AR(1) premium processes for the Dollar/Mark and Dollar/Yen exchange rates did not have well-defined maxima. This is probably the result of misspecification of the premium models. The likelihood function associated with the MA(1) premium for the Dollar/Pound rate did have a maximum. The maximum likelihood estimate of the moving average parameter was 0.402. The marginal significance level associated with the likelihood ratio statistic which tests the significance of this parameter was 0.008. The time series of smoothed premium estimates that were computed from the AR(1) and MA(1) premium models for the



Dollar/Pound rate are highly correlated: the coefficient of correlation for these two series is 0.996.

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Table I

Autocorrelations and Partial Autocorrelations of  $F(t,t+1)-S(t+1)$   
6 April 1973-13 July 1984.

Lag	Dollar/Pound	Dollar/Mark	Dollar/Yen
<u>Autocorrelations<sup>a</sup></u>			
1	0.236**	0.151*	0.192**
2	0.144*	-0.016	0.038
3	-0.017	0.036	0.077
4	-0.042	-0.111	0.031
5	0.060	-0.090	0.090
6	0.092	-0.078	-0.019
7	0.086	0.024	-0.096
8	0.030	0.118	0.037
<u>Partial Autocorrelations<sup>a</sup></u>			
1	0.236**	0.151*	0.192**
2	0.094	-0.040	0.001
3	-0.075	0.045	0.073
4	-0.039	-0.127	0.003
5	0.097	-0.052	0.086
6	0.075	-0.069	-0.060
7	0.028	0.055	-0.088
8	-0.015	0.100	0.064

<sup>a</sup> Under the hypothesis that the true autocorrelations and partial autocorrelations are zero, the standard error of sample autocorrelations and partial autocorrelations is approximately 0.08.

\*\* Indicates statistical significance at the 5 percent level.

\* Indicates statistical significance at the 10 percent level.

Maximum Likelihood Estimates of State Space Models for Premia  
in the Pricing of Forward Foreign Exchange

Table II

	$\hat{\phi}$	$\hat{\theta}$	LR <sup>a</sup> (p-value)	$s_V^2$ ( $\times 10^3$ )	$s_a^2$ ( $\times 10^3$ )	$s_D^2$ ( $\times 10^3$ )	log L
Dollar/Pound	0.462***	-	9.857 (0.002)	0.300	0.285	0.362	395.19
Dollar/Mark	-	0.340**	4.009 (0.045)	0.442	0.486	0.542	373.46
Dollar/Yen	-	0.404**	5.621 (0.018)	0.392	0.404	0.470	363.12

<sup>a</sup>LR denotes the value of the likelihood ratio test statistics which test the significance of the autoregressive or moving average parameter. Marginal significance levels (p-values) associated with these test statistics are presented in parentheses.

\*\*\* Indicates statistical significance at the 1 percent level.

\*\* Indicates statistical significance at the 5 percent level.

Table III

Autocorrelations and Partial Autocorrelations of  
 $F(t,t+1)-S(t+1)-P^S(t)$ , 6 April 1973 - 13 July 1984

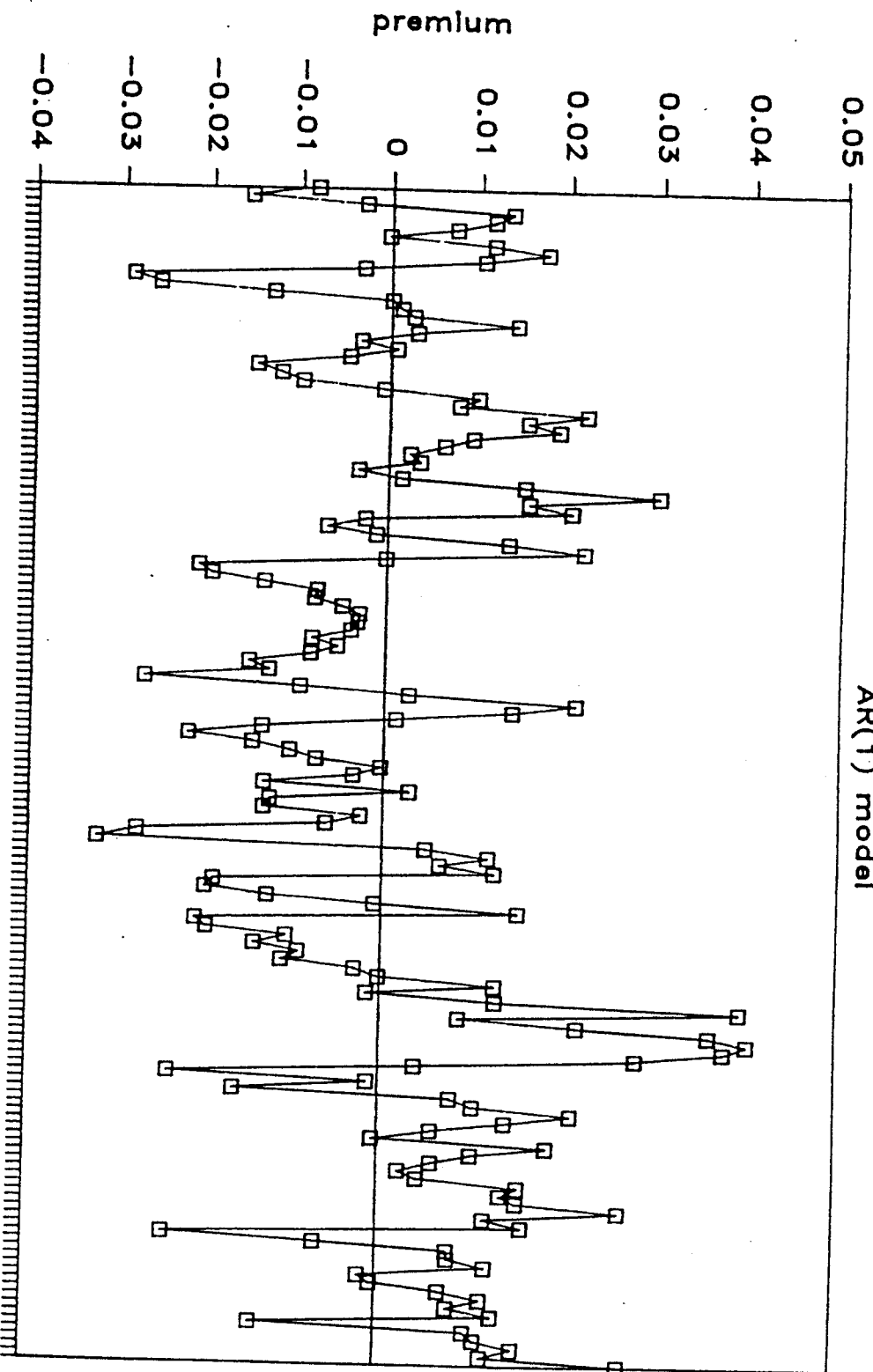
	Dollar/Pound	Dollar/Mark	Dollar/Yen
<u>Autocorrelations<sup>a</sup></u>			
<u>Lag</u>			
1	-0.015	-0.011	0.002
2	0.056	-0.025	0.023
3	-0.071	0.060	0.076
4	-0.076	-0.110	0.000
5	0.053	-0.059	0.092
6	0.063	-0.075	-0.017
7	0.065	0.017	-0.106
8	-0.004	0.123	0.074
<u>Partial Autocorrelations<sup>a</sup></u>			
1	-0.015	-0.011	0.002
2	0.056	-0.025	0.023
3	-0.070	0.060	0.076
4	-0.081	-0.109	0.001
5	0.060	-0.058	0.086
6	0.070	-0.087	-0.023
7	0.050	0.026	0.111
8	-0.009	0.117	0.063
<u>Box-Ljung Tests</u>			
B-L(8) <sup>b</sup>	3.892	6.007	4.915
p-value	(0.867)	(0.646)	(0.767)

<sup>a</sup>Under the hypothesis that the true autocorrelations are zero, the standard error of sample autocorrelations and partial autocorrelations is approximately 0.08.

<sup>b</sup>B-L(8) refers to values of the Box and Ljung [21] portmanteau test of randomness based on the first 8 autocorrelations. Marginal significance levels are reported in parentheses.

# PREMIA FOR DOLLAR/POUND RATE

AR(1) model



6 April 1973 - 15 June 1984

Figure 1. Estimated premia for the Dollar/Pound exchange rate.