

**KEEPING UP WITH THE JONESES:
COMPETITION AND THE EVOLUTION OF
COLLUSION IN AN OLIGOPOLISTIC
ECONOMY**

Huw David Dixon

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Centre for Economic Policy Research
90-98 Goswell Rd
London EC1V 7DB
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

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ABSTRACT

Keeping Up With the Joneses: Competition and the Evolution of Collusion in an Oligopolistic Economy*

An economy consists of many duopolistic markets. Firms must earn normal profits in the long run if they are to survive. Normal profits are interpreted as the long-run limit of average profits in the whole economy. We adopt the aspiration based model of firm behaviour, and link it to the economy with the requirement that in the long run the profit aspiration must be at least as great as normal profits. We assume that the joint profits can be maximized with symmetric payoffs, and with very few other assumptions are able to show that the (almost) global attractor is the cooperative outcome.

JEL Classification: C7, D4, L13

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Huw David Dixon
Department of Economics
University of York
York YO1 5DD
UK
Tel: (44 1904) 433 788
Fax: (44 1904) 433 759
Email: hdd1@york.ac.uk

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NON-TECHNICAL SUMMARY

Is there a natural tendency for markets to become collusive over time? In this paper I argue that indeed there is. Most studies of competition have tended to focus on the interaction between oligopolists within a particular market. This paper focuses on the economy-wide interaction of oligopolists competing in different markets. The key linkage is that the level of normal profits in the economy is determined by the average level of profits in the economy. This link occurs because of capital markets.

What this paper shows is that this link leads to collusion, since excessively competitive markets are unsustainable, since one or more firms must then be earning low profits. In the long run, the only sustainable outcome is for complete collusion in all markets. While many studies have emphasized the *possibility* of collusive outcomes, this study argues that collusion is almost the only and the inevitable outcome.

The implications of the framework of this paper for policy are that we should expect that collusion is the norm in the economy. The market mechanism will tend to lead to competition being a transitory phenomenon.

It has long been argued that firms must earn at least normal profits to survive in the long run. Failure to achieve this will activate some market mechanism such as bankruptcy, the possible replacement of managers by shareholders, or take-over. The level of normal profits is taken to be the average level of profits in the economy. This paper explores the implications of this hypothesis in the context of an economy consisting of many identical oligopolistic markets: what conclusions can we reach about the level of competition that will emerge in such an economy? We model the behaviour of firms using an aspiration based model of bounded rationality. Firms at any time adopt a pure strategy. If they are achieving their aspiration level, then they are likely to continue with the same strategy. If they are below their aspiration level, however, then they are likely to experiment and try something new. Furthermore, we adopt a formulation which allows the aspiration level to be endogenous reflecting the past experience of the firm and the current profitability of the economy.

The key feature of this paper is to link together the aspirations of firms with the level of normal profit by requiring that in the long run the aspiration level of all firms is to have at least normal profits.

Making only a few very weak assumptions about the nature of the markets and the behaviour of firms, the paper shows that the proportion of markets that are collusive will eventually grow and converge to unity. The two key assumptions are:

- First, the duopoly joint profits are maximized at a payoff-symmetric outcome. This means that the maximisation of industry profits does not require an asymmetric outcome where one firm earns more than the other.
- Second, the probability that a firm will experiment needs to depend on the relation between actual profits and aspirations in the following manner: when firms are below aspiration and decide to experiment, there is a strictly positive lower bound on the probability of switching to any particular strategy (i.e. any of the possible strategies may be chosen); when firms are above aspiration, the probability of experimenting tends to zero rapidly over time (i.e. in the long run, if firms achieve their aspiration level they tend not to experiment).

As in conventional approaches, there is an incentive for an individual firm to deviate from the collusive outcome. This will tend not to occur, however, because the colluding firms will be above aspiration, and hence unlikely to experiment. Even if they do deviate from the collusive outcome, however, their actions will lead either to the other firm or both firms earning lower profits. This will trigger further experimentation that may result in more intense competition for a period. In the long run, however, the market can only settle down if both firms end up earning at least the average in the economy. This can only happen in all markets if all markets consist of firms that are colluding.

This paper ignores one of the potentially most important sources of competition; entry. Entry might not destroy the result, however, but only temper it. Very simply, free entry can be seen as ensuring that profits net of entry costs are about zero. The fact that the post-entry game is collusive will lead to excess entry. The economy will then be exemplified by collusion between incumbents (price exceeds marginal cost), and zero profits (price equals average cost). This will lead to a highly inefficient welfare outcome. Entry can only counteract the tendency to collusion if the entry is *ex post*, as in the case of contestable markets or limit-pricing theory. This raises many issues that are not addressed in this paper, but will be in sequel works.

"The best monopoly profit is a quiet life" John Hicks (1935).

"This is the criterion by which the economic system selects survivors: those who realise positive profits are the survivors; those who suffer losses disappear" Armen Alchian (1950, p.213)

It has long been argued that firms must earn at least normal-profits to survive in the long-run¹. Failure to achieve this will activate some market mechanism such as bankruptcy, the possible replacement of managers by shareholders, or takeover. The level of normal-profits is taken to be the average level of profits in the economy. This paper explores the implications of this hypothesis in the context of an economy consisting of many identical oligopolistic markets: what conclusions can we reach about the level of competition that will emerge in such an economy?

We model the behaviour of firms using an aspiration based model of bounded rationality. Firms at any time adopt a pure-strategy. If they are achieving their aspiration level, then they are likely to continue with the same strategy. If however, they are below their aspiration level then they are likely to experiment and try something new. This approach has been put forward as a good model of individual decision making in the mathematical psychology literature ((Lewin (1936), Siegel (1957))), and as a model of organisational decision making (Cyert and March (1963). Kornai (1971) and Simon (1947)). Furthermore, we adopt a formulation which allows the aspiration level to be endogenous (as in Borgers and Sarin (1994), Karandikar et al (1997), Palomino and Vega-Redonodo (1997)), reflecting the past experience of the firm and the current profitability of the economy.

The key feature of this paper is to link together the aspirations of firms with the level of normal profit by requiring that in the long run the aspiration level of all firms is to have at least normal profits.

The structure of the economy envisaged is that of an economy consisting of a large number of identical duopolies. Firms have a finite strategy set, and we need assume very little about the structure of the payoff matrix of the constituent duopoly game, except that the joint payoff can be maximized by a payoff-symmetric outcome. We need to make some assumptions about aspirations and experiments: if firms are achieving

¹There are obvious exceptions here, such as non-profit organisations and owner-managed firms. We are considering the "typical" managerial public corporation.

their current aspiration level, then the probability that they experiment goes to zero over time, whilst the probability is bounded away from zero if they are below aspiration. In the case that firms decide to experiment and try out a new strategy, we need assume only that the probabilities of choosing certain strategies are bounded away from zero over time.

The main result of the paper is the Theorem, which states that *the collusive (joint-profit maximizing) outcome is the (almost) global attractor for this economic system.* What is novel about this result is that *cooperation is not only possible, but almost inevitable.* In the case of the PD the dominant strategy of defection will disappear, and all firms will end up cooperating to produce the symmetric joint profit maximum.

In section 1 of the paper we outline the basic model in terms of payoffs and strategies. In section 2, we consider how the economy/population evolves over time, and state the main results. In section 3, we look at two concrete examples to illustrate the model: the PD and Cournot duopoly. In section 4 we discuss the recent related literature, and then conclude.

1: The Model.

Time is discrete and eternal, with $t=0\dots\infty$. Although the model is more generally applicable, we will talk about "firms" rather than "agents". There are K pure-strategies, $k=\{1\dots K\}$. Π is the $K \times K$ matrix of payoffs π_{ij} , where π_{ij} is the payoff when i plays j . Payoffs are bounded, with $\pi_{ij} \in \pi$, where $\pi \subseteq \mathbb{R}$ is the smallest convex set covering the points $\{\pi_{ij}\}$. We can define the set of unordered pairs of strategies as L : where $L \equiv \{(i,j): (i,j) \in \{1,2,\dots,K\}^2 \text{ and } i \neq j\}$, so that $L = \#L = K(K-1)/2$. Elements of L ($r, q \in L$) may sometimes be referenced by the underlying pair (i,j) . A is the set of subsets A of L : $A \equiv \{A: A \subseteq L\}$. In particular, the set of payoff-symmetric pairs is $Sym \equiv \{q \in L: \pi_{i_j} = \pi_{j_i}\}$. The average payoff earned by a pair q is: $\pi(q) \equiv (\pi_{i_j} + \pi_{j_i})/2$. The only assumption we need to introduce about the payoffs Π are the following:

Definition 1:

- (a) $Maxav = \max_{q \in L} \pi(q)$
- (b) $\Pi S = \max_{q \in Sym} \pi(q)$
- $S = \operatorname{argmax}_{q \in Sym} \pi(q)$

Assumption 1: $\Pi S = \text{Maxav}$

Assumption 1 requires that the maximum joint-payoff can be attained at a payoff-symmetric pair of strategies². For example, consider the standard Prisoner's Dilemma (PD) with:

$$\Pi_{PD} = \begin{bmatrix} 2 & 0 \\ a & 1 \end{bmatrix}$$

where 2 is the cooperative payoff; 1 is the payoff when both defect; 0 is the sucker's payoff; $a > 2$ is the double crosser's payoff. A1 is satisfied iff $a \leq 4$: that is the combined defect/sucker payoff is less than the combined cooperative payoff.

The economy consists of a continuum of markets $\lambda \in [0,1]$, each consisting of a duopoly. We can define the general characteristic function over all subsets A of markets $[0,1]$: for any A in A^* , the σ -field of $[0,1]$, $J_t(A, \lambda) = 1$ iff $\lambda \in A$ at time t (0 otherwise). The probability measure P can then be defined:

$$P_t(A) = \int_0^1 J_t(A, \lambda) d\lambda$$

For most of the paper, we will want to classify duopolies in terms of the pair of strategies q which they are playing. A particular market λ is type $q = (i, j)$ at period t iff one firm plays i and one firm j in period t . The notion of duopoly "type" can be interpreted as the competitiveness of the market, since this is summarised completely by q . Define the mapping $\Lambda_t: A \rightarrow A^*$, from subsets of strategy pairs subsets of markets:

$$\Lambda_t(A) = \{ \lambda \in [0,1] : \lambda \text{ is type } q \text{ at time } t \text{ and } q \in A \}$$

The proportion of duopolies in the set $\Lambda_t(A)$ at time t is then simply:

$$P_t(A) \equiv P_t(\Lambda_t(A))$$

$P_t(A)$ gives the proportion of markets that have duopolies of type $q \in A$ in A at time t . In particular, we can write the $L \times 1$ vector $P_t = [P_t(q)]$ where $P_t \in \Delta^L$ gives the proportions of each $q \in L$.

The average level of profits at t in markets with pair $q \in A$ is:

²Another way of putting A1 is that the "cooperative" outcome payoff ΠS does not lie strictly in the interior of the convex hull of Π in duopoly payoff space $\{ \pi_{ij}, \pi_{ji} \} \in \pi^2$.

$$\bar{\pi}_t(A) = \sum_{q \in A} P_t(q) \cdot \pi(q) / P_t(A)$$

The average level of profits in the economy at t is $\bar{\pi}_t = \bar{\pi}_t(L)$. Sometimes for convenience we will write $\bar{\pi}_t = \bar{\pi}(P_t)$, where $\bar{\pi}: \Delta^L \rightarrow \pi$ is time-invariant. $\bar{\pi}$ is fixed and the dynamics of P_t over time will completely determine the dynamics of the average profits $\bar{\pi}_t$. Analysing the model in terms of pairs of strategies used is similar to the methodology adopted by Atkinson and Suppes (1958)³.

1A: Aspirations and Learning.

Each firm follows the following simple learning rule. It has an aspiration level⁴ α_{ft} . If it is earning less than α_{ft} , then it decides to experiment with probability β_{ft} . If the firm is earning at least α_{ft} , then it is achieving its aspiration level, and it will experiment with probability ε_{ft} . The essence of the aspiration model is that $\beta_{ft} > \varepsilon_{ft}$. Define for each t $\varepsilon_t \equiv \sup_f \{\varepsilon_{ft}\}$ and $\beta_t \equiv \inf_f \{\beta_{ft}\}$, $\alpha_t \equiv \inf_f \{\alpha_{ft}\}$. Let us denote the information set of firm f at time t as h_{tf} : whilst we will not define it formally here, since the model requires little particular structure, it could include the past history of the firm and its industry, and the current and past mean-profit levels. The aspirations of each firm f can be based on its own information, $\alpha_{ft} = \alpha_f(h_{tf}, t)$, in a possibly idiosyncratic manner which varies over time.

Assumption 2: There exists t_0 such that for $t > t_0$,

- (a) $\alpha_{ft} \leq \bar{\pi}_t$.
- (b) $\alpha_t \geq \bar{\pi}_t - \eta_t$, where $\eta_t \geq 0$ and $\eta_t \rightarrow 0$ (uniform convergence).
- (c) $\exists \bar{\beta} > 0$ such that for all f and t , $\beta_{ft} \geq \bar{\beta}$.
- (d) $\exists \bar{\varepsilon} \in [0, 1)$ s.t. $\varepsilon_t \leq \bar{\varepsilon}$ for all t and $\sum_{t=0}^{\infty} \varepsilon_t$ is bounded.

³In their "finite markov model", agents choose pure-strategies but have a probability of switching between pure strategies. The key difference with the present paper is that we use an explicit aspiration based model.

⁴See Kornai (1971, p157-158) for a discussion of the origins and meaning of the term "aspiration level" in psychology and economics. The use of the aspiration based model of corporate decision making has a long history in economics - Cyert and March (1963), Simon (1947). There is also empirical support from experiments - see Lant (1992).

Assumption A2(a) requires that (eventually) firms have aspirations which are not overoptimistic⁵. Assumption A2(b) requires that in the limit (the long-run) the aspiration level is at least equal to average profits. One possibility satisfying A2(a,b) is to have $\alpha_{ft} = \alpha_t = \bar{\pi}_t$ (Dixon (1995), Palomino and Vega-Redondo (1996)). Under A2(c)-(d) there is eventually a discontinuity in the probability of experimenting at α_{ft} : at or above the aspiration level, the probability of experimenting goes rapidly to zero A2(d); below it the probability of experimentation remains bounded away from zero indefinitely (A2(c)); eventually $\epsilon_{ft} < \beta \leq \beta_{ft}$ ⁶. There is nothing that precludes ϵ_{ft} and β_{ft} from depending upon various factors (the distance of current profits from the aspiration level⁷, the payoff-history of the firm etc.) so long as A2 is satisfied.

1B: Switching Probabilities.

Given that a firm decides to experiment, we can define its *conditional switching probabilities*. $s_{ft}(i,g)$ is the period t probability that firm f switches from strategy i to strategy g , conditional upon deciding to experiment. For an individual firm, we can define these in a very general way: in particular they might depend upon the history of the individual firm, the current and past value of average profits and so on. We assume:

$$s_{ft}(i,g) = s_f(h_{ft},t)(1-\delta_{ft}) + (\delta_{ft}/K)$$

with: $\sum_{g=1}^K s_{ft}(i,g) = 1$ and $\delta_{ft} \in [0,1]$ for all t and f .

$s_f(h_{ft},t)$ represents the systematic part which can represent any model of learning or experimental behaviour (including reinforcement learning, best-response dynamics, imitation and so on), which can be firm specific. Note that we allow for t to enter into the function directly. $\delta_{ft} \in [0,1]$

⁵If $\alpha_{ft} > \bar{\pi}_t$, then the firms would be aiming at a level of profits that is unattainable in the economy as a whole.

⁶This captures the essence of Hick's notion of a "quiet life", if we loosely interpret "the monopoly profit" as being one above the aspiration level.

⁷In Karandikar et al (1997), $\epsilon_{ft} = 0$, and β_{ft} is a function of $\alpha_{ft} - \pi_{ft}$. Consistent with A2(c), they assume that "inertia" means that β_{ft} is bounded away from 1: unlike this paper, however they assume that β_{ft} tends to zero as $\alpha_{ft} - \pi_{ft}$ tends to zero from above, which violates A2(c).

represents the random or "drift" term in the decision: a proportion δ_{ft} of the systematic probability is reallocated uniformly over all K strategies. this can be firm and time specific. At present, we need not specify further beyond this very general model: we will give specific examples at the end of this section.

Whilst we have interpreted switching behaviour as the same firm in two periods changing behaviour, the formal model would be exactly the same if we think of a different firm in each period. For example, a firm in a particular market might exit (due to bankruptcy, or death). In this case the switching probability would pertain to the "place" of the firm: the probability that next period the firm taking the place of the existing firm would play a particular strategy.

Let us define the set of potentially joint-profit maximizing strategies σ : $\sigma \equiv \{i \in K: \exists j \text{ such that } \pi_{ij} = \Pi S\}$. Furthermore, let us denote firm f's competitor as firm f'.

Assumption 3: Switching probabilities. There exists $\gamma > 0$ such that for all f and t, $s_{ft}(i, g) > \gamma$ if either

(a) $g \in \sigma$

or (b) f' is playing g at time t.

This is a very general assumption which applies to any learning or experimentation process if the noise term is bounded away from zero:

Observation: If there exists $\delta > 0$ such that for all $f \in F$ and $t = 0, \dots, \infty$ $\delta_{ft} > \delta$, then A3 is satisfied for any $s_f(h_t, t)$.

To see why, under the antecedent condition (1) becomes:

$$s_t(i, g) \geq \frac{1}{K} \delta_{ft} \geq \frac{\delta}{K} > 0$$

which satisfies A3. In the no-noise case ($\delta_{ft} = 0$ for all f and t), then some learning models might satisfy A3, whilst others will not. We will be using A3 as a *sufficient* condition.

Let us briefly consider some examples of noiseless switching rules

with $\delta=0$. First, we can have an *imitation rule*⁸. Most simply the experimenting firm randomly observes another firm in the economy and copies it. In this case, the conditional switching probabilities are the population proportions for that strategy \hat{P}_t , so that:

$$s_{ft}(i,g) = \hat{P}_t(g)$$

Imitation may fail to satisfy A3 if $\hat{P}_t(g)$ tends to 0 and $g \in \sigma$. Secondly, consider best-response dynamics. Let us define the best response mapping:

$$BR(j) = \operatorname{argmax}_{i=1..K} \pi_{ij}$$

The best response dynamic results in the following switching probabilities:

$$s_{ft}(i,g) = \begin{cases} \frac{1}{\#BR(j)} & \text{if } g \in BR(j) \\ 0 & \text{otherwise.} \end{cases}$$

where we have assumed that if there is more than one best response they are equally likely to be chosen. Thirdly, we can have *random switching*:

$$s_{ft}(i,g) = 1/K$$

These three rules are all independent of history: alternatives are fictitious play, reinforcement learning which are history dependent.

2: The Evolution of the Population.

At any time t , we can divide the set of markets into two groups: (i) the "above aspiration" markets, where both firms are above aspiration and who experiment with a probability below ϵ_t ; (ii) the "below aspiration" markets in which one or both firms have profits strictly below aspiration. Define the subset of above-aspiration markets:

$$AA_t = \{\lambda \in [0,1]: \pi_{1j} \geq \alpha_{ft} \text{ and } \pi_{j1} \geq \alpha_{f't}\}$$

where firm f at λ plays i and firm f' plays j . The below-aspiration markets are the complement of AA_t : $BA_t = [0,1] - AA_t$. BA_t can be partitioned into two, depending whether one or both firms are earning below aspiration:

⁸This is an old idea - see Alchian (1950). For recent applications, see Weibull (1995, pp.186-190), Schlag (1996a,b).

$$BA2_t = \{\lambda \in [0,1]: \pi_{1j} < \alpha_{ft} \text{ and } \pi_{j1} < \alpha_{r,t}\}$$

$$BA1_t = BA_t - BA2_t$$

Since the payoffs of each firm are determined by the pair of strategies played at its market, we can classify markets according to the pair of strategies played at that market. In the proofs, it is useful to divide up the pairs \mathbb{L} into two subsets using the least-aspiration α_t :

$$AA_t = \{(i,j) \in \mathbb{L}: \pi_{ij} \geq \alpha_t \text{ and } \pi_{ji} \geq \alpha_t\}$$

$$BA_t = \mathbb{L} - AA_t$$

Since α_t may vary with time, the set of pairs $AA_t \subseteq \mathbb{L}$ will in general vary over time. The two subsets S and Sym are time invariant. Clearly, under A1, $S \subseteq AA_t$ for all t . Furthermore, define $BSym_t = BA_t \cap Sym$. Since α_t is the lower bound on aspirations at t , the following inequalities hold:

$$\begin{cases} P_t(BA_t) \geq P_t(BA_t) & (\Lambda_t(BA_t) \subseteq BA_t) \\ P_t(BA2_t) \geq P_t(BSym_t) & (\Lambda_t(BSym_t) \subseteq BA2_t) \\ P_t(AA_t) \geq P_t(AA_t) \geq P_t(S) & (AA_t \subseteq \Lambda_t(AA_t) \subseteq \Lambda_t(S)) \end{cases} \quad (1)$$

In the special case where $\alpha_{rt} = \alpha_t = \bar{\pi}_t$, we have $P_t(BA_t) = P_t(BA_t)$ and $P_t(AA_t) = P_t(AA_t)$. We are now in a position to derive our results: we start with two Lemmas, and then use these to derive the Theorem. All proofs are in the Appendix.

Lemma 1. A1-2. (a) There exists $P^* \in [0,1]$ such that $P_t(S) \rightarrow P^*$.

(b) If $P_t(S) > 0$ for some t , then $P_\tau(S) > 0$ for $\tau \geq t$ and $P^* > 0$.

Lemma 1 establishes that there exists a limit for the proportion of markets that are maximizing joint profits. This is trivial in the case where $\varepsilon_t = 0$ for $t = 0, \infty$: once joint-profit maximization is established in a market, neither firm will ever experiment. Hence S is an *absorbing state*, and $P_t(S)$ is monotonic and bounded so that it possesses a limit. The result still holds so long as ε_t converges to zero fast enough (A2(d)). Furthermore, from Lemma (b), once (strictly) positive $P_t(S)$ remains so.

Lemma 2: A1-3. $\sum_{t=0}^{\infty} P_t(BA_t)$ is bounded.

Lemma 2 implies that the proportion of markets where one or both firms are below aspiration tends to zero. The reasoning here is in two stages. First, the proportion of payoff-symmetric markets with both firms below least aspiration has to go to zero. The proportion of these markets which become collusive is bounded away from zero (A2(c) and A3(a)): since $P_t(S)$ is bounded, it follows that this flow must go to zero, and hence the proportion of payoff-symmetric pairs must go to zero. The second step is to show that for the proportion of payoff-symmetric pairs to go to zero, so must $P_t(BA_t)$. In fact in both cases, convergence to zero must be quick enough to ensure that the infinite sum is bounded.

Theorem: A1 - A3. If $P_t(S) > 0$ for some t , then as $t \rightarrow \infty$:

- (a) $P(S)_t$ tends uniformly to 1,
- (b) $\bar{\pi}_t$ tends uniformly to ΠS .

The intuition behind the Theorem is fairly simple. Consider the simplest case where $\epsilon_t = 0$ and $\alpha_{ft} = \alpha_t = \bar{\pi}_t$: the pair(s) S then constitute an *absorbing state* in the Markov process. From Lemma 2, the proportion of firms with one or more firms below aspiration will tend to zero, so that all firms will be at or above average profits. The only way that this is possible is to have all firms earning ΠS . We require $P_t(S) > 0$ for some t in order to avoid the process getting stuck at a position where all markets earn exactly the average at a level below ΠS ⁹. In the next section, we examine the Prisoner's Dilemma and Cournot Duopoly to illustrate each of these points in a concrete way.

Whilst the intuition is fairly clear, the exact evolution of P_t and $\bar{\pi}_t$ is open to a wide variety of possibilities under A1-3. In particular, the path of both can be highly non-monotonic, and the Theorem does little to tie down the nature of the path towards the long-run stationary state.

⁹If we assume that $\epsilon_t > 0$ for all t , then it follows that $P_t(S) > 0$ for all $t > 1$. However, if $\epsilon_t = 0$ for all t , then we need to rule out $P_t(S) = 0$ for all t .

This would require the examination of specific models for the evolution of aspirations and the switching probabilities. However, the Theorem does establish the long run properties of a very wide class of learning processes.

One interpretation of the result is a model of *group selection*. However, it should be noted that individual firms cannot choose whom they play against: they can only choose their own behaviour. Groups are selected, but only indirectly by the market mechanism: in duopolies that are too competitive, profits of one or both firms are eventually below aspiration, becoming unsustainable. Thus the process outlined in this paper can be interpreted as one where nature (the economy) selects the optimum degree of competitiveness (the cooperative solution). Note that Alchian's original argument (Alchian (1950)) was conducted at the level of the *individual* firm: either the atomistic competitor or a monopoly. However, in duopoly the individual firm's profits depends upon the *joint-strategy* of the firms: hence it is the joint-strategy that is chosen. Whilst the motivation of our arguments is similar in spirit, the conclusions reached in a strategic environment are very different.

3: The PD and Cournot Duopoly.

Let us consider the examples of the PD and Cournot Duopoly with the "simple learning model": where $\epsilon_t=0$, $\beta_t=1$ and $\alpha_{r_t}=\alpha_t=\bar{\pi}_t$. In the simple learning model, S is an absorbing state, $\Lambda_t(BA_t)=BA_t$ and $\Lambda_t(AA_t)=AA_t$.

3A: Prisoner's Dilemma. We consider two cases of the PD:

$$\Pi_{PD} = \begin{bmatrix} 2 & 0 \\ a & 1 \end{bmatrix}; \quad \Pi_{PD}^* = \begin{bmatrix} 2 & a \\ -a & 1 \end{bmatrix}$$

where $a>2$, $k=\{c,d\}$, $\mathbb{L}=\{cc, cd, dd\}$, $S=\{cc\}$, $\Pi S=2$, $K=2$, $L=3$. With Π_{PD} , A1 is satisfied if $a\leq 4$: with Π_{PD}^* any $a>2$.

Figure 1 here

The evolution of the population in Π_{PD} can be represented on the unit simplex in Fig 1, where each point represents a 3-vector of proportions of markets playing each strategy pair. We represent the iso-average payoff loci on the simplex: these are linear and parallel since the average

payoff is a linear combination of the payoffs for each strategy pair, with slope $(a-2)/2$. $\bar{\pi}=a/2$ is the dotted straight line passing through corner all-cd; $\bar{\pi}=2$ passes through all-cc, $\bar{\pi}=1$ passes through all-dd.

The dynamics of this system are straightforward with the simple learning model. Average profits must lie in the interval $\pi=[1,2]$. Except in the case where $P(dd)=1$ and $\bar{\pi}_t=1$, we thus have: $AA_t=\{cc\}$; $BSym_t=\{dd\}$; $BA_t-BSym_t=\{cd\}$. From any point except where $P(dd)=1$, all trajectories will lead to the apex where $P(cc)=1$.

With $\bar{\pi}_{PD}^*$ the sucker-payoff is $-a$, so that $\pi(cd)=0$. This extends the possible range of average profits to $\pi=[0,2]$. The iso-profit loci are downward sloping as in Fig 2: the minimum is represented by the dotted line through the cd vertex, and the maximum by the dotted line through cc. The line passing through dd is the $\bar{\pi}=1$ line.

Fig 2 here

The dynamics here are different depending whether the economy is in region A or B. In region A $\bar{\pi}>1$, so that: $AA_t=\{cc\}$; $BSym_t=\{dd\}$; $BA_t-BSym_t=\{cd\}$. In region B $\bar{\pi}\leq 1$, however we have: $AA_t=\{cc,dd\}$; $BA_t=\{cd\}$. With the simple learning model, only firms playing c at cd markets will experiment, so that the trajectories must be horizontal lines in B. Hence we can see why we need to assume that $P(cc)>0$ in the Theorem: all-dd has a basin of attraction along the Southern edge of the simplex¹⁰.

The Theorem gives a sufficient condition for all-cc to be the attractor. We will now illustrate how dropping some of the key assumptions will open the possibility that all-cc ceases to be the attractor. We have already seen why $P(cc)>0$ is necessary to rule out all-dd in $\bar{\pi}_{PD}^*$. Next consider a violation of A3: the noiseless best-response dynamic. In both $\bar{\pi}_{PD}^*$ and $\bar{\pi}_{PD}$, the best response is d (the strictly dominant strategy), which violates A3(a). In this case the pairs divide into two disconnected sets: there can be no flow into or out of cc, and dd is an absorbing state. The only flows in this system are from cd to dd. The resultant dynamics are represented in figure 3, where the attractor is the Northeastern edge of the simplex, where there are no cd markets. The paths to this are simply the horizontal lines: the economy starts off with an initial proportion of cc markets, and eventually all the rest will become dd. However, whilst the example of noiseless best response

¹⁰This is not robust: if we have $\varepsilon_t > 0$ for all t, then this basin disappears.

dynamics is an interesting illustration of what can happen when A3 is violated, it is not at all robust. Any level of switching noise $\delta > 0$, no matter how small, will lead to the Theorem becoming valid again, and cc absorbing all markets so long as $P(S) > 0$ for some t.

Figure 3 and Figure 4 here

Lastly, what happens when A1 is violated? For simplicity, let us consider the Π_{PD} payoff matrix, with $a=6$ so that $\text{Max}av=3>\Pi S=2$. The iso-payoff loci in Figure 4 are vertical, passing through all-cd ($\bar{\pi}=3$), all-cc ($\bar{\pi}=2$) and all-dd ($\bar{\pi}=1$). There are two different regions: in M_1 $\bar{\pi} < 2$, so that $AA_t=\{cc\}$, $BA_t=\{dd,cd\}$; whilst in M_2 $AA_t=\emptyset$, whilst $BA_t=\{cc,dd,cd\}$

What exactly happens depends upon the exact switching technology. With random switching, the equilibrium is a point in M_2 where $P^*(cc)=0.125$, $P^*(dd)=0.5$, $P^*(cd)=0.375$ with $\bar{\pi}=2.125$. There is a perpetual flow of markets between the three pairs of strategies.

3B: Cournot Duopoly with Random Switching.

Perhaps the simplest economic application of our model is to Cournot duopoly without costs, so that the two firms in any duopoly produce output x and y, and the price is $P=\max[0,1-x-y]$, and the profits of the firms are x.P and y.P respectively. In this case we have the set S has a unique element: it is the joint profit maximizing (JPM) pair where each firm produces 0.25 (half of the monopoly output 0.5). Furthermore, $\Pi S=\text{max}av=0.125$, so that A1 is satisfied. With the random switching rule $s_f(i,j)=1/K$ for all i,j, which satisfies A1.

We¹¹ allowed for 21 firm types, choosing a grid of granularity 0.025 over the range¹² 0.1 to 0.6, perturbing it slightly by moving 0.325 to 0.333 (1/3), so that the Cournot-Nash output was included. Hence $K=21$ and $L=231$. The simulations were initiated from the initial position with a uniform distribution over all pairs. The results of the simulation are depicted in Figures 5a and 5b. In Figure 5b, we see the path of average profits over time: in Figure 5a the evolution of population proportions of the JPM market (0.125,0.125) and the symmetric Cournot market are depicted (note that the proportions are measured on a logarithmic scale).

¹¹I would like to thank Paolo Lupi for implementing these simulations for me in Gauss.

¹²We did not allow for a wider grid range (e.g. [0,1]), because the additional strategies are often ones with very low or zero profits: they slow down the simulation without adding any extra insight.

Figures 5a,b here

From Fig 5b, we see that the average profits converge to the symmetric joint profit maximum of 0.125. However, the time path of profits is non-monotonic: at particular times there appear large drops in profit. The reason for this is quite intuitive. As the average profit level increases and surpasses that of one or both firms, which start to experiment. The profits of firms at those markets will then on average fall below the population average as the firms disperse over some or all output pairs. The effect of this can be quite dramatic: the discontinuity is particularly large when a symmetric market goes critical, since both firms at each such market begin to experiment. However, whilst the time-series of profits is non-monotonic and "discontinuous", there is a clear upward trend and convergence to 0.125.

From Figure 5a, the proportion $P(S)$ is monotonic, but not smooth. Corresponding to the discontinuous falls in population average profit, there are jumps in the proportion of firms at the JPM market, corresponding to the jumps in average profit. The proportion of firms at the Cournot pair $(1/3, 1/3)$ is a highly non-monotonic time series. The first thing to note is that in the initial stages of the simulation, the proportion of Cournot markets exceeds the proportion of JPM markets. This can occur because during this period the Cournot pair is also in the set AA_t : until average profits reach $1/9$, the Cournot pair will "absorb" markets from BA_t . The fact that the Cournot pair attracts more than JPM is due to the fact that early on more markets in BA_t can reach the Cournot pair than JPM. However, after 50 iterations, the Cournot pair has a smaller proportion than the JPM pair, and is in BA_t most of the time. The time-series of the Cournot market type is not atypical: most pairs except JPM have a similar time-series profile. The convergence of the proportion of markets towards type JPM is steady but slow: this is because the probability of hitting JPM from other locations is small throughout the simulation: from each market in which both firms experiment there is a probability of $1/442$ of moving to JPM. Convergence is in general quicker with fewer strategies and non-random switching rules. We explore more general learning rules in the Cournot model using simulations in Dixon and Lupi (1997).

4: Related Literature.

There are several recent papers related to ours. The closest is Palomino and Vega-Redondo (1996). This paper considers a population of players who are randomly matched, and play the prisoner's dilemma. The mean payoff is known in each period, and this determines the aspiration level¹³. If a player is earning below aspiration with its current strategy, then it switches with a positive probability to the other strategy. They find that for certain parameters, all paths converge to a situation with a strictly positive proportion of cooperators. Our paper differs in that we do not have random matching, and that we consider a very general class of (of which the PD is one example).

Bendor et al (1994) and Kandrikar et al (1997) both consider a two player game, where individual behaviour is driven by a similar aspiration based model. In Bendor et al aspiration levels are constant over time, and they impose the condition that the individual aspiration levels are equal to the long-run individual average payoff¹⁴ (*consistent aspirations*); in Kandrikar et al aspirations can evolve, but are determined by individual payoff histories. In Both papers, there are multiple long-run equilibria, which in general include cooperative outcomes. The key difference between our own paper and these papers is the social dimension: here it is the *population* average which ultimately determines the aspiration level¹⁵.

The local interaction literature (Ellison (1993), Fudenberg and Ellison (1995), Oliphant (1994)) is similar in that here firms only interact with their market competitors. However, the key interactions in our paper are not only local, but also social via the population average payoff. Our results hold even if the individual firms ignore the existence of their competitors, and consider themselves to be solving a non-strategic problem. This feature also differentiates our paper from other learning models (e.g. Blume and Easley (1992), Marimon and Grattan (1995)). More similar to our approach are papers where there is global

¹³In fact, they assume a partial adjustment model, so that the aspiration level changes in accordance with the difference between the current level and average profits.

¹⁴"A minimal requirement for a model of endogenous aspirations is that in the long-run, aspirations should not be out of line with the average payoffs accumulated from experience", Bendor et al (1994, p.9).

¹⁵This also differentiates our paper from Borgers and Sarin (1994).

interaction through the population average action (Banerjee (1992), Canning (1992)). Lastly, there is a theoretical and experimental literature on learning in oligopoly settings (Kirman (1995)), which again focus on isolated markets, and do not have the social dimension.

5: Conclusion.

In this paper we have formulated a simple model of social learning which is based on an information structure and matching technology suggested by the economic application of oligopoly, and with a learning model in which aspirations are linked in the long run to the population average payoff. The results of the paper are very simple and very powerful: the model predicts perfect collusion (cooperation), even in the case where collusion implies the use of a dominated strategy (as in the prisoner's dilemma). The model does not require strong assumptions on the learning process or payoff matrix, and the Theorem will certainly hold in symmetric versions of most economic models.

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Appendix: Proofs

Lemma 1. A1-2. (a) There exists $P^* \in [0,1]$ such that $P_t(S) \rightarrow P^*$. (b) If $P_t(S) > 0$ for some t , then $P^* > 0$.

Proof:
$$P_t(S) = P_0(S) + \sum_{\tau=1}^t [P_\tau(S) - P_{\tau-1}(S)] \quad (a1)$$

Let us define the two sequences:

$$\begin{aligned} \omega^+_\tau &= \max \left[0, P_\tau(S) - P_{\tau-1}(S) \right] \\ \omega^-_\tau &= \min \left[0, P_\tau(S) - P_{\tau-1}(S) \right] \end{aligned}$$

Hence we can rewrite (A1) as:

$$P_t(S) = P_0(S) + \sum_{\tau=1}^t \omega^+_\tau + \sum_{\tau=1}^t \omega^-_\tau \quad (a2)$$

Under A2(d), $0 \geq \sum_{\tau=1}^t \omega^-_\tau \geq \bar{\epsilon} \sum_{\tau=1}^t \epsilon_\tau P_\tau(S) \geq \bar{\epsilon} \sum_{\tau=1}^t \epsilon_\tau$

Hence there exists $\Omega < \infty$ such that $\sum_{\tau=1}^{\infty} \omega^-_\tau = -\Omega$.

Define
$$\hat{P}_t(S) = P_0(S) + \sum_{\tau=1}^t \omega^+_\tau - \Omega \quad (a3)$$

Comparing (a2) and (a3), $1 \geq P_t(S) \geq \hat{P}_t(S)$ and as $t \rightarrow \infty$ $\hat{P}_t(S) \rightarrow P_t(S)$. Since $\hat{P}_t(S)$ is bounded and strictly monotonic, it possesses a limit $P^*(S)$. Hence $P_t(S)$ posses the same limit. From the definition of $\hat{P}_t(S)$, $P^* \in [0,1]$.

(b) follows from the fact that $P^* \geq \prod_{\tau=t}^{\infty} (1 - \epsilon_\tau) P_t(S)$. Hence:

$$\begin{aligned} \log P^* &\geq \log P_t(S) + \sum_{\tau=t}^{\infty} \log(1 - \epsilon_\tau) \\ &\geq \log P_t(S) - \sum_{\tau=t}^{\infty} \epsilon_\tau + Z \end{aligned}$$

$$\text{where } Z \cong \sum_{n=1}^{\infty} \sum_{r=t}^{\infty} \left[\frac{c_r^{2n}}{2^n} - \frac{c_r^{2n+1}}{2^{n+1}} \right] \geq 0 \text{ since } 0 \leq c_r < 1.$$

$$\text{Hence } P^* \cong P_t(S) \cdot \exp\{-\Omega\} > 0$$

where Ω is a finite upper bound on $\sum c_t$ from A2(d). □

$$\text{Lemma 2: A1-3. } \sum_{t=0}^{\infty} P_t(BA_t) \text{ is bounded.}$$

Proof: The proof uses the inequalities (1), and in particular the first two inequalities provide lower bounds for $P(BA_t)$ and $P_t(BA2_t)$.

First, we establish that $P_t(BSym_t) \rightarrow 0$. Consider the change in the proportion of firms in doupolies with strategy pairs in S , $P_t(S) - P_{t-1}(S)$. This change is the result of inflows less outflows. A lower bound on inflows is (from A2(c) and A3(a) and (1)) $\Gamma \cdot P_{t-1}(BSym_{t-1})$, where $\Gamma = (\bar{\beta} \cdot \bar{\gamma})^2$. Industries in $BSym_{t-1}$ will have both firms below aspiration level (since both firms have the same profit, which is below least-aspiration α_t): $\Gamma > 0$ is the lower bound on the probability of both firms experimenting and choosing new strategies that result in a pair $q \in S$. An upper bound on the outflows from S is c_{t-1} (since $P_{t-1}(S) \leq 1$). Hence:

$$P_t(S) - P_{t-1}(S) \geq \Gamma \cdot P_{t-1}(BSym_{t-1}) - c_{t-1}$$

$$P_T(S) = P_0(S) + \Gamma \cdot \sum_{t=1}^T P_{t-1}(BSym_{t-1}) - \sum_{t=1}^T c_t$$

From Lemma 1, the Limit of $P_T(S)$ exists and is less than 1:

$$1 \geq P_0(S) + \Gamma \cdot \sum_{t=1}^{\infty} P_{t-1}(BSym_{t-1}) - \sum_{t=1}^{\infty} c_t \geq 0$$

From A2(d), $\sum c_t$ is bounded. Hence $\sum P_t(BSym_t)$ is bounded and $P_t(BSym_t) \rightarrow 0$.

An analogous argument shows that if $P_t(\text{BSym}_t)$ tends to zero, so must $P_t(\text{BA}_t)$. Again, finding a lower bound for inflows, and an upper bound for outflows into Sym:

$$\begin{aligned} P_t(\text{Sym}) - P_{t-1}(\text{Sym}) &\geq \Gamma \cdot P_{t-1}(\text{BA}_{t-1}) - \varepsilon_{t-1} \cdot P_{t-1}(\text{AA}_{t-1}) - P_{t-1}(\text{BSym}_{t-1}) \\ &\geq \Gamma \cdot P_{t-1}(\text{BA}_{t-1}) - \varepsilon_{t-1} - P_{t-1}(\text{BSym}_{t-1}) \end{aligned}$$

The lower bound for inflows comes from the fact that if at least one firm is below α_t , it may experiment and choose the same strategy as its competitor (A3(b)): if both experiment they may choose a payoff symmetric pair with a probability of at least Γ . The upper bound on outflows is based on the assumption that above-aspiration industries experiment with probability ε_t and choose a non-symmetric pair; also, all industries in the subset BSym_{t-1} leave Sym. Hence:

$$\begin{aligned} 1 &\geq P_T(\text{Sym}) \geq P_0(\text{Sym}) + \sum_{t=1}^T [P_t(\text{Sym}) - P_{t-1}(\text{Sym})] \\ 1 - P_0(\text{Sym}) &\geq \Gamma \cdot \sum_{t=1}^{\infty} P_t(\text{BA}_t) - \sum_{t=1}^{\infty} \varepsilon_t - \sum_{t=1}^{\infty} P_t(\text{BSym}_t) \end{aligned}$$

Since both $\sum \varepsilon_t$ and $\sum P_t(\text{BSym}_t)$ are bounded, and $P_t(\text{BA}_t) \geq 0$, it follows that

$$\left[1 - P_0(S) + \sum_{t=1}^{\infty} \varepsilon_t + \sum_{t=1}^{\infty} P_t(\text{BSym}_t) \right] \Gamma^{-1} \geq \sum_{t=1}^{\infty} P_t(\text{BA}_t) \geq 0$$

and hence $P_t(\text{BA}_t) \rightarrow 0$.

□

Theorem: A1-A3. If for some t $P_t(S) > 0$, then as $t \rightarrow \infty$: (a) $P(S)_t$ tends uniformly to 1, (b) $\bar{\pi}_t$ tends uniformly to $\bar{\pi}$.

Proof: From the definition for average profits, for all t :

$$\bar{\pi}_t = \bar{\pi}_t(L) = P_t(S) \cdot \bar{\pi}_t(S) + P_t(AA_t-S) \cdot \bar{\pi}_t(AA_t-S) + P_t(BA_t) \cdot \bar{\pi}_t(BA_t) \quad (a4)$$

By definition, $\bar{\pi}_t(AA_t-S) \geq \alpha_t$, and from A2(b) $\alpha_t \geq \bar{\pi}_t - \eta_t$, so that

when $P_t(S) > 0$ (a4) becomes:

$$\bar{\pi}_t \geq \frac{P_t(S) \cdot \bar{\pi} + P_t(AA_t-S) \eta_t + P_t(BA_t) \bar{\pi}_t(BA_t)}{1 - P_t(AA_t-S)} \quad (a5)$$

Since $P_t(S) + P_t(AA_t-S) + P_t(BA_t) = 1$, and the limit of $P_t(S)$ is $P^* > 0$ (Lemma 1(b)), and of $P_t(BA_t)$ is 0 (Lemma 2), the limit of $P_t(AA_t-S) = 1 - P^*$.

Hence (a5) implies:

$$\liminf_{t \rightarrow \infty} \bar{\pi}_t \geq \bar{\pi}$$

Since $\limsup \bar{\pi}_t \leq \bar{\pi}$

it follows that $\lim \bar{\pi}_t$ exists and equals $\bar{\pi}$, with $P^* = 1$. □

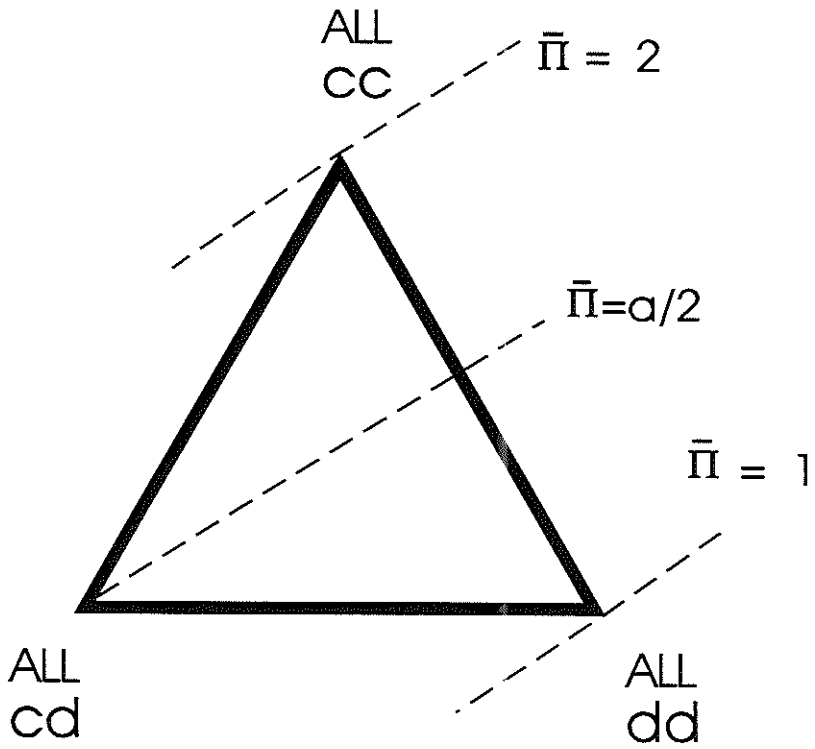


FIG 1: THE PD WITH $\bar{\Pi}_{PD}$ and $2 < \alpha < 4$.

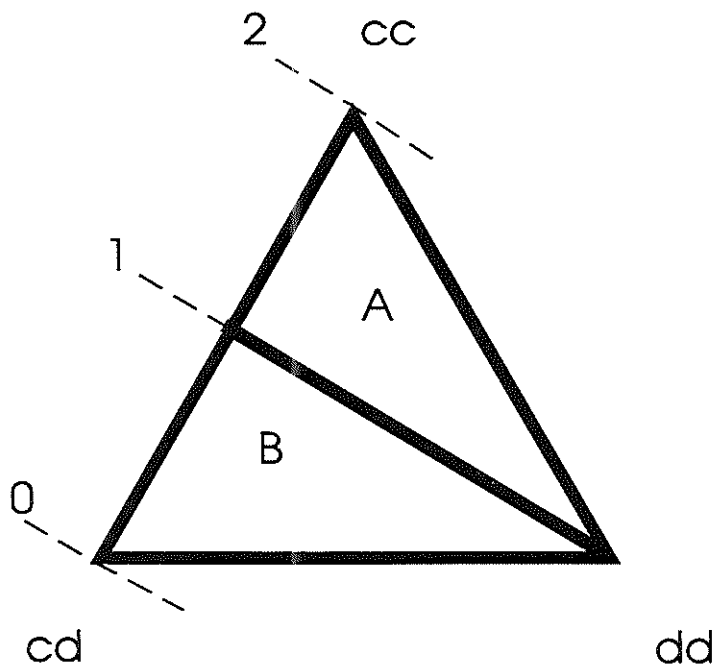


FIG 2: THE PD WITH Π_{PD}^*

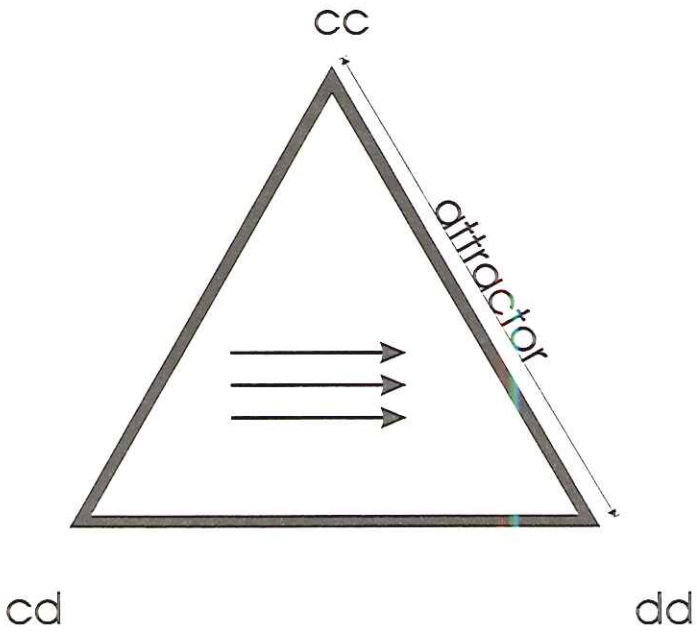


FIG 3: BEST RESPONSE DYNAMICS WITH NO NOISE

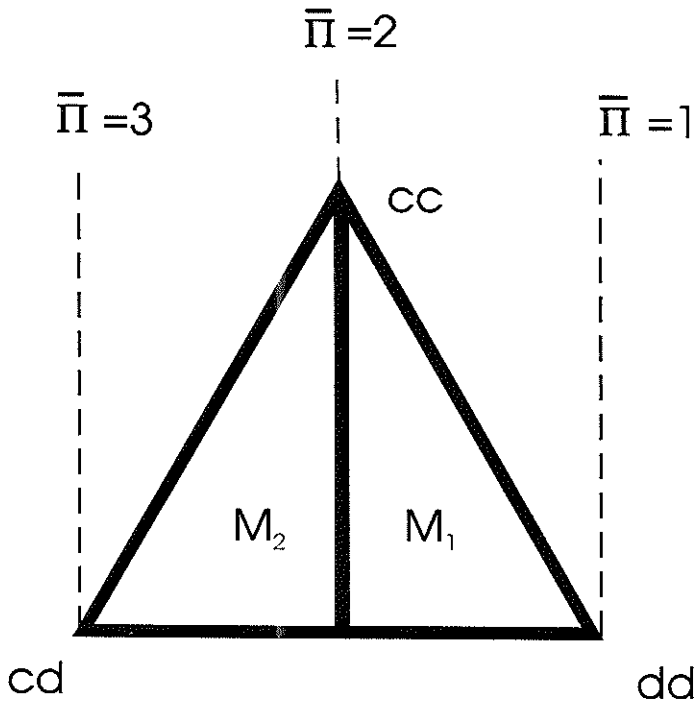
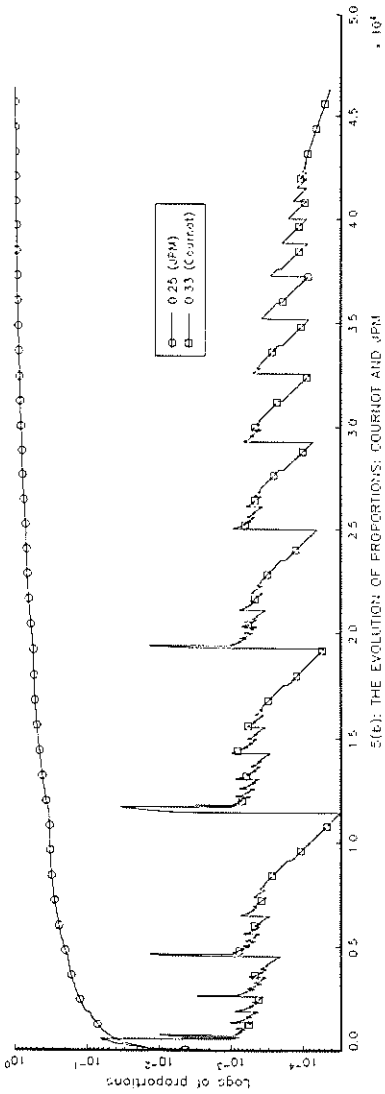
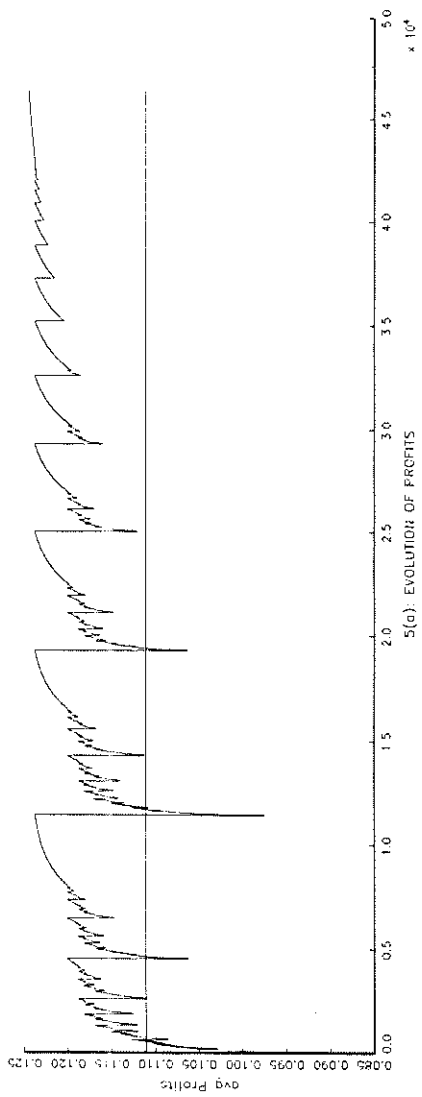


FIG 4: VIOLATION OF A1:
 PD with Π_{pd} and $a=6$

FIG. 5: THE EVOLUTION OF COMPETITION



5(b): THE EVOLUTION OF PROPORTIONS: COURNOT AND JPM



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