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Abstract

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JEL Classification: J31, H24

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The motherhood wage and income traps*

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Abstract

We present a simple dynamic model based on on-the-job human capital accumulation affecting the dynamic of wage rates and labor earnings. We show how these dynamics are determined by the interplay between the supply and demand sides of the labor market. The model can generate and explain the different dynamics of women's earnings after childbirth documented in the empirical literature on child penalties. We show that the temporary negative shock in labor supply due to childbearing may create a wage trap and a permanent divergence of labor earnings between genders. Even when the wage trap is avoided, and working mothers are on a path toward a high-wage equilibrium, slow convergence can permanently lose earnings. We use this model to study the impact of different policies on the gender wage gap and child penalties. We show that mandatory maternal leave exacerbates the shock which pleads against long leaves. Similarly, cash transfers to mothers via the income effect on labor supply aggravate gender wage differences. By contrast, temporary subsidies to mothers' wages (possibly in the form of Income Tax Credits) are not only useful to exit the wage trap, but also to speed up recovery and reduce the child penalty when the shock in labor supply is small enough to avoid the wage trap. Other family policies, like formal childcare subsidies and in-kind provision of formal childcare, are potentially useful because they reduce the mothers' cost of labor supply, but they affect mothers' choices only indirectly.

Keywords: child penalty, mothers' earnings dynamics, multiple equilibria, wage and income traps.

JEL classification: J31, H24.

1 Introduction

An increasing amount of evidence suggests that children and parenthood may explain most of the remaining gender inequality in modern societies. Specifically, significant and persistent earnings gaps open up between men and women after childbirth. Recent studies have documented that these gaps arise from drastically reduced labor supply of mothers: after their first child, mothers choose to work fewer hours, or in lower-paid but more child-friendly jobs, or not at all, when their children are very young. In the Economics of Gender and Family literature, motherhood or child penalty is defined as the percentage loss of average earnings due to having children. The motherhood penalty exists in all the analyzed countries, but to quite different extents; see, for example, Kleven et al. (2019a) and (2019b). The debate about mechanisms explaining this gender gap in the dynamic of earnings is still ongoing and inconclusive.

A large expansion of family policies has occurred over the last 50 years. Maternity and parental leave and childcare provision and subsidization have been among the most widespread policies and also among the ones more often analyzed in the (mostly empirical) literature; see Olivetti and Petrongolo (2017). The impact of these policies on gender gaps continues to be debated. On the one hand, there is a widespread belief that family policies could be helpful, on the other hand, the concern that some policies may have backfired is increasing among researchers; see, among others, Fernández-Kranz and Rodríguez-Planas (2021), and Kleven et al. (2020).

Designing effective policies requires a clear understanding of the mechanism driving women's decisions regarding labor supply and career. Different mechanisms contribute to explaining mothers' low attachment to their pre-motherhood labor supply. In this paper, we propose a simple dynamic model based on on-the-job human capital accumulation affecting the dynamic of wage rates and labor earnings. We show how these dynamics are determined by the interplay between the supply and demand sides of the labor market.

Interestingly, our model accommodates the different dynamics of earnings documented by

Kleven et al. (2019a), (2019b). We show that a temporary negative shock in labor supply (or more generally effort) due to childbearing may create a wage trap and a permanent divergence of labor earnings between otherwise identical individuals who did not experience the negative shock. Even when the wage trap is avoided and working mothers enter the virtuous trend toward a high-wage equilibrium, a permanent loss of earnings is observed.[1]

Furthermore, our model can account for workers' socioeconomic status (SES) and education level and thus provides interesting insights into the relationship between human capital accumulated before motherhood and mothers' dynamics of wages and earnings. Specifically, the model predicts that women with high SES are less likely to enter the wage trap. Intuitively, the effort shock generated by motherhood might be less severe for a highly educated woman because it is partially absorbed by the human capital she has accumulated before her maternity leave. Pora and Wilner (2019) use French administrative data and document the heterogeneity of the consequences of childbirth along the distribution of pre-childbirth wages; their evidence is fully in line with our results. However, the fact that women with low SES are more likely to enter the wage trap does not necessarily imply that they also suffer the higher child penalty overall. Indeed, even though women with high SES do not fall into the trap, total losses in earnings during the slow and incomplete convergence to the high steady state may imply larger losses than the ones experienced by a poorly-educated women entering the wage trap.

Last but not least, our model allows identifying those family policies that are successful in reducing earning gaps induced by motherhood's shock in labor supply. Specifically, tem-

[1]The concept of traps, and particularly poverty traps has been widely studied in the macroeconomic literature. The seminal paper is Azariadis and Drazen (1990), who show how poverty traps can emerge when human capital is subject to threshold externalities. While traps are often modelled as a low-level equilibrium in a static model of coordination failures, we discuss the concept in a dynamic setting, following Matsuyama (2008). In a static setting, we would be unable to distinguish poverty traps from (possibly temporary) bad market outcomes, that are also often modelled as low-level equilibriums in a static model of coordination failures; see Matsuyama (2008) for further discussion and references. Furthermore, studying the persistence of a child penalty for years after the initial shock is inherently a dynamic problem.

porary subsidies to mothers' wages (possibly in the form of Income Tax Credits) appear to be a cost-effective policy to address working mothers' low attachment to their pre-motherhood labor supply. These subsidies are useful to exit the wage trap and also to speed up recovery and reduce the child penalty when the shock in labor supply is small enough to avoid the wage trap. Intuitively, by encouraging mothers' labor supply, those subsidies mitigate the loss in human capital accumulation and activate a positive spiral of increasing labor supply and increasing wage rates which alleviates the gender gap in earnings in the short run with positive spillover effects in the long run. Other family policies like formal childcare subsidies and/or in-kind provision of formal childcare are potentially useful because they reduce the cost of providing labor supply by mothers, but they are affecting mothers' choices only indirectly. Policies that backfire are instead (long) parental leave for mothers and cash transfers that temporarily alleviate mothers' earnings losses, but also reduce the incentive to provide effort and labor supply, and thus, contribute to the negative spiral generating a low effort and low wage rates, leading to a possible wage trap for mothers.

Our paper relates to a few theoretical studies of mechanisms explaining gendered labor market outcomes. Albanesi and Olivetti (2009) model employers' behaviors and show how gendered equilibria might emerge with no fundamental differences in productivity or preferences across genders. If firms believe that hours of market work are lower for women because of informal child care and home production, employers will offer them labor contracts with lower earnings, performance pay, and effort. Then, the opportunity cost of informal home care is lower for women, and wives will allocate more time to home production, thus confirming firms' beliefs. Our mechanism with ex-ante identical workers is complementary to Albanesi and Olivetti's (2009) because it relies on a different description of the interplay between the supply and demand sides of the labor market. Differently from Albanesi and Olivetti (2009), our model studies the long-lasting effects of motherhood on the dynamics of women's wages and earnings.[2]

[2]In Albanesi and Olivetti (2009), a gendered equilibrium realizes from the beginning of a woman's working life and irrespective of the labor supply shock due to childbearing. Hence, their equilibria do not reproduce

In a theoretical model used to guide their empirical analysis, Andresen and Nix (2022) account for different possible explanations for child penalties: women's health shock of giving birth and breast-feeding, partners' heterogeneity in preferences for spending time with children, partners' heterogeneity in comparative advantages for household work, gender norms and discriminations by employers. They estimate and compare child penalties in heterosexual nonadopting, adopting, and same-sex couples to understand what causes the child penalty in heterosexual nonadopting couples. They rule out giving birth and the father's advantage in the labor market as mechanisms, leaving preferences, gender norms, and discrimination as the main explanations of child penalties. The focus of their theoretical model is very different from ours because they consider a static framework with fixed wage rates. Our mechanism to explain child penalties is however complementary to theirs.

The paper is organized as follows. Section 2 presents a dynamic model of wage rates and labor earnings based on on-the-job human capital accumulation. Section 3 illustrates the existence and possible multiplicity of steady states. Section 4 describes mothers' income dynamics and shows the existence and importance of wage and income traps. Subsections 4.2 and 4.3 show that these wage and income traps reproduce the empirical evidence on child penalties and describe how they change with mothers' socioeconomic status. Finally, Section 5 evaluates the impact of family policies on the model's dynamic mechanism and their effectiveness. Subsection 5.1 identifies ineffective and potentially counterproductive policies, while Subsection 5.2 highlights policies that have the potential to be effective.

2 The model

Consider an economy where all individuals are identical ex-ante, except for their gender. The gender difference is of no relevance in period 0, but it may become important in later periods. Specifically, female workers may experience a shock in their labor supply due, for instance, to motherhood.

Kleven et al.'s (2019a), and (2019b) earnings dynamics.

Our model has three main ingredients. The first one, which is standard, is the optimization problem of a representative individual choosing their effort level in any period t. The second component is the determination of an individual's wage in any period t as a function of their effort in the previous period. Third and last, combining the first two elements with an added assumption of possible adaptive expectations leads to the representation of wages and labor income dynamics.

2.1 Individual problem

Earning in period t, $w_t e_t$, depends on effort e_t and wage rate w_t. We interpret effort e_t as a combination of labor supply, ℓ_t, and human capital, h_t. This allows us to distinguish between the earnings dynamic of individuals characterized by high and low socioeconomic status, as we explain in Subsection 4.3. As an intuition, in the case of highly-educated workers, the human capital component is relatively large, and a higher level of effort can be achieved.

Desired effort $\widehat{e}_t$ is obtained by maximizing

$$\Lambda[e_t] = u[w_t e_t] + v[1 - e_t], \tag{1}$$

where $u[c_t]$ and $v[z_t]$ represent utility from consumption c_t, being equivalent to per period labor income $y_t = w_t e_t$, and from leisure $z_t = 1 - e_t$, respectively. Both functions are increasing and concave.

Assuming an interior solution, effort $\widehat{e}_t \in (0, 1)$ is defined by the FOC, $\Lambda^{'}[\widehat{e}_t] = 0$, which is given by

$$\varrho[\widehat{e}_t, w_t] \equiv w_t u^{'}[w_t \widehat{e}_t] - v^{'}[1 - \widehat{e}_t] = 0. \tag{2}$$

Since $\varrho^{\prime}_{\widehat{e}} = \partial \varrho[\widehat{e}_t, w_t]/\partial e_t = w_t^2 u^{''}[w_t \widehat{e}_t] + v^{''}[1 - \widehat{e}_t] < 0$, (2) defines a function $\varphi[.]$ such that

$$\widehat{e}_t = \varphi[w_t], \tag{3}$$

where

$$\varphi^{\prime}[w_t] = \frac{u^{'}[w_t \widehat{e}_t] + w_t \widehat{e}_t u^{''}[w_t \widehat{e}_t]}{-\varrho^{\prime}_{\widehat{e}}}. \tag{4}$$

Equation (4) is textbook microeconomics and shows that income and substitution effects have opposite signs; hence the total effect is ambiguous. When, for any x, the relative risk aversion coefficient $R_u[x] = -xu''[x]/u'[x]$ is smaller than 1, then $\varphi[.]$ is a monotonic and increasing function of w_t. Note that our general analysis allows for cases in which $\varphi[.]$ is not a monotonic function (see, for example, our Illustration B).

Note also that

$$\varphi''[w_t] = \frac{\widehat{\varrho}''_{ww}(\widehat{\varrho}'_e)^2 - 2\widehat{\varrho}'_w\widehat{\varrho}'_e\widehat{\varrho}''_{we} + \widehat{\varrho}''_{ee}(\widehat{\varrho}'_w)^2}{-(\widehat{\varrho}'_e)^3}, \tag{5}$$

with $\widehat{\varrho}'_w = u'[w_t\widehat{e}_t] + w_t\widehat{e}_t u''[w_t\widehat{e}_t]$, $\widehat{\varrho}''_{ww} = 2\widehat{e}_t u''[w_t\widehat{e}_t] + w_t\widehat{e}_t{}^2 u'''[w_t\widehat{e}_t]$, $\widehat{\varrho}''_{we} = 2w_t u''[w_t\widehat{e}_t] + w_t^2\widehat{e}_t u'''[w_t\widehat{e}_t]$ and $\widehat{\varrho}''_{ee} = w_t^3 u'''[w_t\widehat{e}_t] - v'''[1-\widehat{e}_t]$.

Equation (5) shows that the sign of $\varphi''[.]$ is ambiguous even if $\varphi[.]$ is monotonic. This sign depends crucially on Kimball's coefficient of relative prudence $P_u^R[x] = -xu'''[x]/u''[x]$, the relative risk aversion coefficient $R_u[x]$ and $\widehat{\varrho}''_{ee}$ (i.e. the comparison between the third derivatives of $u[.]$ and $v[.]$).[3]

2.2 Wage determination

We assume that an individual's wage rate in any given period is determined by their effort in the previous period. Formally, the increasing function $\psi[.]$ relates today wage w_t, to last period's effort e_{t-1}

$$w_t = \psi[e_{t-1}]. \tag{6}$$

A learning-by-doing human capital product technology thus defines wages at an aggregate level:[4] current effort indirectly increases future wage rates by entering the production function of human capital investment. Examples of (6) have been used by Heckman (1976), Shaw (1989), and Imai and Keane (2004), among others. It is generally assumed that the

[3]As $\widehat{\varrho}''_{ww}/\widehat{e}_t = \widehat{\varrho}''_{we}/w_t = 2u''[w_t\widehat{e}_t] + w_t\widehat{e}_t u'''[w_t\widehat{e}_t]$, if $R_u[x] < 1$ and $P_u^R < 2$ for all x and $\widehat{\varrho}''_{ee} < 0$, then $\varphi[.]$ is an increasing and concave function.

[4]Since learning-by-doing is interpreted at the aggregate level, equation (6) does not enter the individual's maximization problem stated in (1).

initial stock of human capital depreciates, but human capital is simultaneously augmented by combining the current capital stock and effort to form new human capital.

The function $\psi[.]$ is monotonically increasing. In Illustration A, we consider a sigmoïd function $\psi_A[.]$, i.e., a function that is first convex and then concave. The specific shape of this function implies that, according to the level of accumulated effort, human capital and on-the-job training, a negative shock in labor supply might have a larger (in the convex part) or a smaller (in the concave part of the function) impact on current effort and wages. In our Illustration B we show that all our results remain valid when $\psi_B[.]$ is strictly convex.

2.3 Effective and desired effort

The third ingredient of our model is the assumption of adaptive anticipations about effective effort e_t. This reflects the fact that a "period" may be too short for the worker to adjust their labor supply to the desired level immediately.

Specifically, we assume that effective effort e_t is defined by

$$e_t = \alpha e_{t-1} + (1-\alpha)\widehat{e}_t, \tag{7}$$

where $\alpha \in [0, 1)$ and $\widehat{e}_t = \varphi[w_t]$, defined by (3), is the desired effort maximizing utility $\Lambda[e_t]$ for a given wage.

The parameter α represents the frequency at which effort decisions can be revised. When $\alpha = 0$ or is low, frequent adjustments are possible while a larger level of α reflects a larger degree of rigidity in the labor market.

3 Dynamics of effort, wage rate and labor income

3.1 Effort

Combining these elements, we can determine the dynamics of effort, wage rate, and labor income. From (6) and (7), we can determine a function $\Delta[.]$ so that the dynamics of effort

e_t is described by

$$e_t = \alpha e_{t-1} + (1-\alpha)\varphi[\psi[e_{t-1}]] \equiv \Delta[e_{t-1}]. \tag{8}$$

The function $\Delta[.]$ has the following properties

$$\Delta'[e_{t-1}] = \alpha + (1-\alpha)\psi^{'}[e_{t-1}]\varphi'[w_t], \tag{9}$$

and

$$\Delta^{''}[e_{t-1}] = (1-\alpha)\big\{\psi^{''}[e_{t-1}]\varphi^{'}[\psi[e_{t-1}]] + (\psi^{'}[e_{t-1}])^2\varphi^{''}[\psi[e_{t-1}]]\big\}. \tag{10}$$

Consequently, when $\varphi'[.]$ is always positive then $\Delta[.]$ is always increasing, implying that the dynamics of effort are always monotonic.[5] Note that this is a sufficient and not a necessary condition. When $\varphi'[.]$ is first positive and then negative, the dynamics may or may not be monotonic.

Without further assumptions, our model can imply various types of dynamics. According to (5), the sign of $\varphi^{''}[.]$ is ambiguous even when $\varphi^{'}[.]$ is positive. Thus, whatever the sign of $\psi^{''}[.]$, expression (10) indicates that the sign of $\Delta^{''}[.]$ is also ambiguous.[6]

To study child penalties, we will concentrate on setting with multiple steady states of which the (locally stable) lower one represents a "trap". In other words, once the relevant variable is below the low steady state or above it and sufficiently close, it will remain trapped in this equilibrium.

The dynamics of effort will determine the dynamics of the wage rate and labor income. In particular, the steady states of $e^\star$ will determine the ones of the wage rate $w^\star$ and of labor income $y^\star$.

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[5]Increasing or decreasing depending on the initial level; see our Illustration A and the discussion provided in Subsection 4.1 for more details.

[6]For instance, assuming that $\psi[.]$ is first convex and then concave (as in Illustration A) does not guarantee that $\Delta[.]$ is also convex and then concave. Conversely, Illustration B shows that $\Delta[.]$ may be convex and then concave even when $\psi[.]$ is strictly convex. In other words, the properties of $\psi[.]$ are important, but different shapes may yield a trap and multiple steady states.

3.2 Wage rate

Given that $w_{t+1} = \psi[e_t]$, the dynamics of wage w_t is described by a function $\Omega[.]$ that writes

$$w_{t+1} = \psi\left[\alpha\psi^{-1}[w_t] + (1-\alpha)\varphi[w_t]\right] \equiv \Omega[w_t]. \tag{11}$$

The function $\Omega[.]$ has the following property

$$\Omega'[w_t] = \left(\frac{\alpha}{\psi'[\psi^{-1}[w_t]]} + (1-\alpha)\varphi'[w_t]\right)\psi'\left[\alpha\psi^{-1}[w_t] + (1-\alpha)\varphi[w_t]\right]. \tag{12}$$

Comparing (9) and (12) shows that $\Delta'[e_{t-1}]\psi'[e_{t-1}] = \Omega'[w_t]\psi'[e_t]$. Consequently, the monotonicity of the dynamics of effort and wage rate are related. It thus follows that when $\varphi'[.]$ is always positive the dynamics of the wage rate is also monotonic.[7]

Finally recall that each steady state of effort $e^\star$ leads to the corresponding wage rate equilibrium $w^\star = \psi[e^\star]$.

3.3 Labor income

The dynamics described in the previous subsections also determine that of labor income, that is $y_t = w_t e_t$. Given that $w_t = \psi[e_{t-1}]$ and $e_t = \Delta[e_{t-1}]$, there exists a function $\Gamma[.]$ such that $y_t = w_t e_t = \psi[e_{t-1}]\Delta[e_{t-1}] \equiv \Gamma[e_{t-1}]$. Consequently, we have $e_{t-1} = \Gamma^{-1}[y_t]$ and we can write

$$y_{t+1} = \Gamma[e_t] = \Gamma[\Delta[e_{t-1}]] = \Gamma[\Delta[\Gamma^{-1}[y_t]]] \equiv \Phi[y_t], \tag{13}$$

thereby expressing y_{t+1} as a function of y_t and defining the function $\Phi[.]$ which expresses this dynamics.

The dynamics of wages and effort jointly determine the ones of income. As long as $\varphi[.]$ is increasing, $\Phi[.]$ is the composite of three increasing and positive functions[8] and is thus

[7] Increasing or decreasing, depending on e_{-1} or equivalently w_0.

[8] Formally this is established by a simple argument. We have shown above that when $\varphi[.]$ is an increasing function $\Delta[.]$ is increasing (and positive). Since $\psi[.]$ is also an increasing and positive function, it follows that $\Gamma[.] = \psi[.]\Delta[.]$, which is the product of these two functions, is also increasing and positive and the same applies to the inverse function, $\Gamma^{-1}[.]$, given that $\Gamma'[.] \neq 0$. Consequently, $\Phi[.] = \Gamma[\Delta[\Gamma^{-1}[.]]]$ is the composite of three increasing and positive.

increasing. To sum up, when $\varphi[.]$ is a monotonically increasing function of the wage rate, the dynamics of income y_t is monotonically (increasing or decreasing depending on its initial level y_0).

Concerning the steady states, each equilibrium effort $e^\star$ results obviously in a corresponding steady state of labor income $y^\star = \Gamma[e^\star]$.

4 Income dynamics, child penalties and policies: an illustration

To study the existence and possible multiplicity of steady states $e^\star$, we could keep a rather general specification and introduce assumptions on $\Delta[0]$, $\Delta[1]$, $\Delta'[e]$ and/or $\Delta''[e]$.[9] However, rather than focusing on the properties $\Delta[.]$, we think that a better understanding of mothers' income dynamics can be reached by presenting the fundamentals of our model (i.e., the specification of $u[.]$, $v[.]$ and $\psi[.]$). They illustrate the existence and importance of wage and income traps and can be used to study potential policies to mitigate or eradicate child penalties. In the main text, we present Illustration A. To discuss its robustness, an alternative specification, Illustration B, is presented in Appendix A.1.

4.1 Dynamics, steady-states and convergence

For the sake of illustration, we use the utility function specified by

$$u_A[c_t] = a\sqrt{c_t} \quad \text{and} \quad v_A[z_t] = \sqrt{b + z_t}, \tag{14}$$

where $a > 0$ and $b \geq 0$. Desired effort $\widehat{e}_t$ is then given by

$$\widehat{e}_t = \varphi[w_t] \equiv \frac{(1+b)a^2 w_t}{1 + a^2 w_t}, \tag{15}$$

which is an increasing function of w_t and has an interior solution when $w_t < 1/(a^2 b)$.

[9] When $\Delta[0] > 0$ and $\Delta[1] < 1$, it is, for instance, straightforward to show that there exists at least one steady state $e^\star \in (0, 1)$.

Assuming $\alpha = 0$ so that $e_t = \widehat{e}_t$, and proceeding like in Section 3 shows that the dynamics of effort e_t and wage rates w_{t+1} for $e_{t-1} > 0$ and $w_t > 0$ are represented by

$$e_t = \Delta[e_{t-1}] \equiv \frac{(1+b)a^2\psi[e_{t-1}]}{1+a^2\psi[e_{t-1}]}, \tag{16}$$

and

$$w_{t+1} = \Omega[w_t] \equiv \psi\left[\frac{(1+b)a^2 w_t}{1+a^2 w_t}\right]. \tag{17}$$

Note that these conditions apply as long as the solution is interior that is if and only if $\psi[1] < 1/(a^2 b)$.

Turning to y_t, we show in Appendix A.2 that when $y_t > 0$, its dynamics is expressed by[10]

$$y_{t+1} = \Phi[y_t] \equiv \frac{(1+b)a^2\psi^2\left[\dfrac{2a(1+b)}{a+\sqrt{a^2+4(1+b)/y_t}}\right]}{1+a^2\psi\left[\dfrac{2a(1+b)}{a+\sqrt{a^2+4(1+b)/y_t}}\right]}. \tag{18}$$

Expressions (16)–(18) are valid for any function $\psi[.]$. To obtain a fully-fledged specification; we now assume that $\psi[.]$ is given by the following sigmoïd (convex and then concave) function

$$\psi_A[e_{t-1}] = \varepsilon + \frac{\xi e_{t-1}^n}{\theta + e_{t-1}^n}, \tag{19}$$

where $\varepsilon \geq 0$, $n > 1$, $\xi > 0$ and $0 < \theta < (1+n)/(n-1)$.[11]

To provide a graphical representation[12] of our three dynamics described by (16)–(19) we consider the following parameters: $\alpha = 0$, $a = b = 0.65$, $\varepsilon = 1.7$, $\xi = 1.5$, $\theta = 0.01$ and $n = 25$. The dynamics of effort, $\Delta[.]$, of wage, $\Omega[.]$, and of labor income, $\Phi[.]$, are represented in Figure 1.

Panel (a) in Figure 1 represents the dynamics of effort. As expected this dynamics is monotonic, and there are three steady states $e_a^* \simeq 0.69342$, $e_b^* \simeq 0.82375$ and $e_c^* \simeq 0.93985$.

[10] With some abuse $\psi^2[.]$ is used to denote $(\psi[.])^2$.

[11] Since $\psi_A''[\varpi] = 0$ for $\varpi^n = (n-1)\theta/(1+n)$, a necessary and sufficient condition to have $\psi_A[.]$ first convex and then concave in $(0,1)$ is that $1 > (n-1)\theta/(1+n)$; meaning that $\theta < (1+n)/(n-1)$. Obviously, $\theta \in (0,1)$ represents a sufficient condition.

[12] If the three dynamics described by (16)–(19) are defined for $e_{t-1} \neq 0$, $w_t \neq 0$ and $y_t \neq 0$, it is easy to show that $\Delta[0] = (1+b)a^2\varepsilon/(1+a^2\varepsilon)$, $\Omega[0] = \varepsilon$ and $\Phi[0] = (1+b)a^2\varepsilon^2/(1+a^2\varepsilon)$.

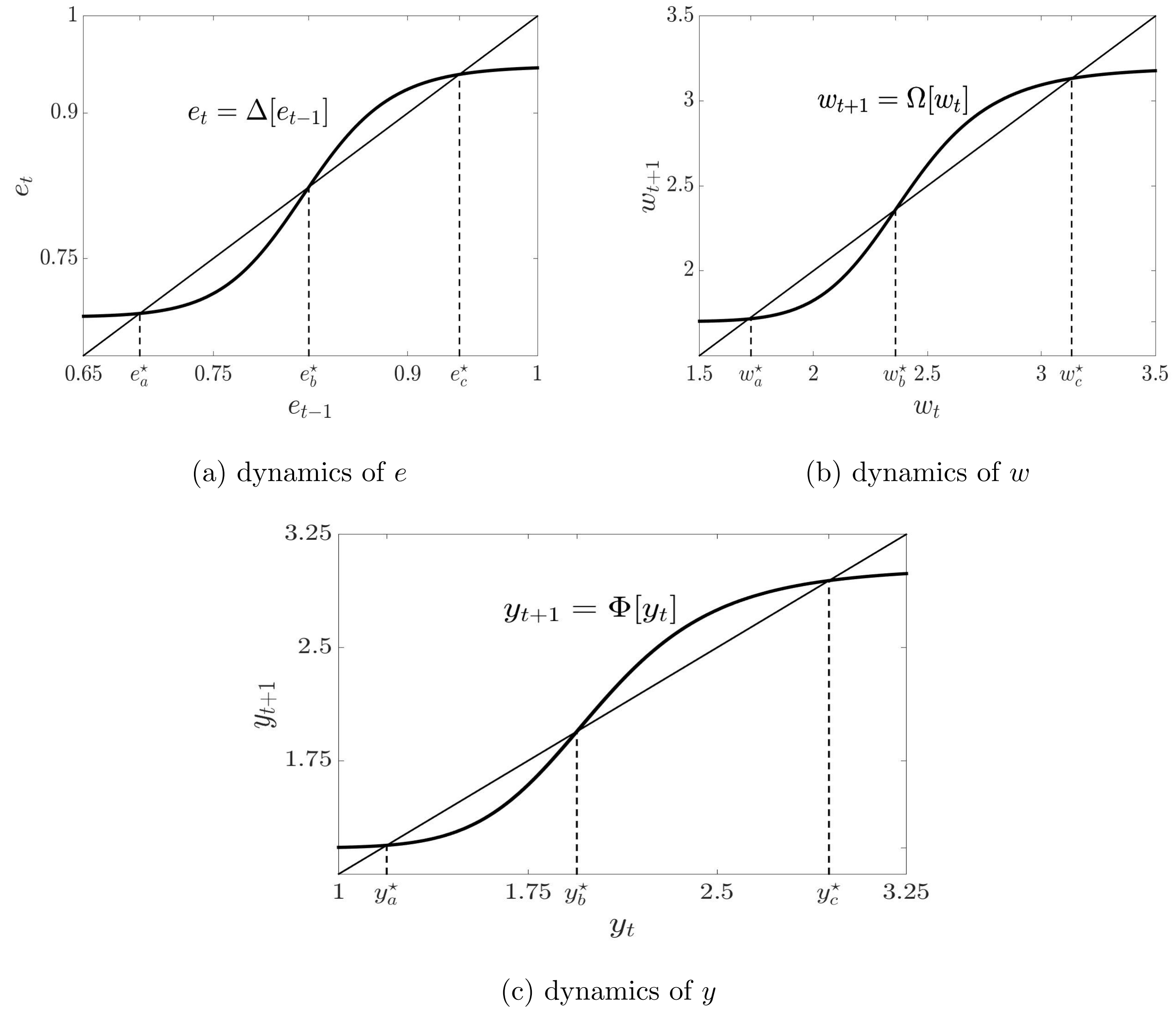


Figure 1: The different dynamics of our Illustration A. Representation of $\Delta[.]$, $\Omega[.]$ and $\Phi[.]$ as specified by (16)–(19) with parameters $\alpha = 0$, $a = b = 0.65$, $\varepsilon = 1.7$, $\xi = 1.5$, $\theta = 0.01$ and $n = 25$.

The equilibria e_a^* and e_c^* are locally stable while e_b^* is unstable. Consequently: if $e_{-1} < e_a^*$ then the path of e is increasing and converges to e_a^*; if $e_a^* < e_{-1} < e_b^*$ the dynamics is decreasing and converge to e_a^*; if $e_b^* < e_{-1} < e_c^*$ the dynamics is increasing and converges to e_c^*; finally if $e_{-1} > e_c^*$ the dynamics is decreasing and converges to e_c^*.

Turning to Panel (b), it shows that the function expressing the wage dynamics, $\Omega[.]$, is also increasing. The dynamics of the wage mirrors that of effort. It is monotonic and there are three steady states $w_a^* \simeq 1.71572$, $w_b^* \simeq 2.35973$ and $w_c^* \simeq 3.13246$ with $w_a^* = \psi_A[e_a^*]$,

$w_b^* = \psi_A[e_b^*]$ and $w_c^* = \psi_A[e_c^*]$. Equilibria w_a^* and w_c^* are globally stable while w_b^* is unstable. As for effort, the time path of wage rates depends on its initial level w_0, and the convergence process is similar to that of e_t.

Finally, the dynamics of labor income $\Phi[.]$ is represented in Panel (c). Since effort is increasing, the general result stated in Section 3.3 applies, and the dynamic is monotonic. Furthermore, since y is jointly determined by w and e, it is not surprising that we obtain again three steady states $y_a^* \simeq 1.18971$, $y_b^* \simeq 1.94384$ and $y_c^* \simeq 2.94405$ with $y_a^* = w_a^* e_a^*$, $y_b^* = w_b^* e_b^*$ and $y_c^* = w_c^* e_c^*$. Equilibria y_a^* et y_c^* are locally stable while y_b^* is unstable. Convergence depends on y_0, along paths similar to those described for e_t and w_t.

Remember that these dynamics, specifically the existence of three steady states and an associated trap at the lower one, are obtained from our illustration. Without further assumptions, one can obtain different patterns in our general model. In particular, only the low steady state may exist. This being said, one could suspect that the pattern we obtain (with three steady states and a trap) is due to the sigmoïd (convex and then concave) specification of $\psi[.]$ and that the monotonic dynamics require an increasing specification of $\varphi[.]$. However, while these assumptions are convenient for simplifying calculations, neither is necessary. To illustrate this we use a utility function that yields an effort level that is not monotonic in w and a functional form of $\psi[.]$ which is strictly convex; see Appendix A.1.

We now can study how these specifications can explain the emergence and persistence of a child penalty. Furthermore, we can draw lessons for designing policies to mitigate gender differences in income and the associated child penalty.

4.2 Motherhood and child penalties

So far we have dealt with a representative individual assuming that ex-ante there is no difference across genders about wage determination. In other words, we ignore gendered equilibria emerging in the labor market as a consequence of employers' self-fulfilling prophecies (see Albanesi and Olivetti, 2009), as well as plain discrimination persisting in empirical evidence (see Blau and Kahn, 2017, and Bertrand, 2020, among others). Introducing the demand side of the labor market explicitly or adding discrimination could only reinforce our arguments.[13]

We now consider a male and a female worker who initially are identical and close to the high steady-state ($e_0 = 0.94 \simeq e_c^* \simeq 0.93985$ in our Illustration A) so that their earnings are more or less constant over time. Assuming assortative matching in the couple, one can think of the two workers as spouses.[14] In each of the panels of Figure 2, we represent the path of earning during 15 periods using the same parameter values as in the previous figures. In period 5, the female labor supply is affected by an exogenous shock (due to birth of a child), which can be more or less severe. In the subsequent periods, we return to the dynamics described by our model.

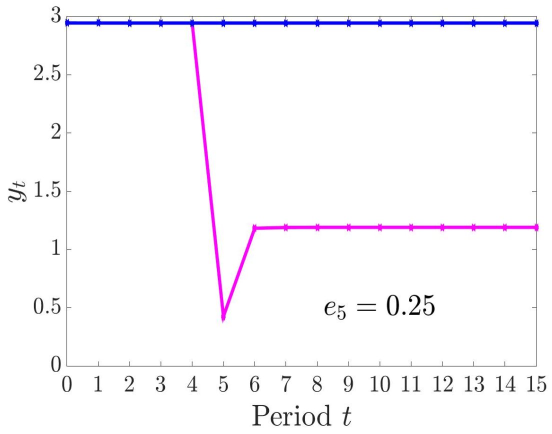
We define child penalties as the impact of children on the gross labor earnings of women relative to men. This is in line with the definition used in the empirical literature; see Kleven et al. (2019a) and (2019b), and Andresen and Nix (2022). We now show that, depending on the extent of the shock, our model yields the different earning paths and the corresponding child penalty that match those described in the empirical literature.

Panel (a) of Figure 2 represents the case where the shock is rather drastic and implies that woman's effort falls below the lower steady state. Specifically, we assume that the shock brings the female worker to $e_5 = 0.25 < e_a^*$. In the following periods, effort and earning increase but the mother remains trapped in the low steady state. This dynamic is very similar to the one depicted in Figure 3 by Kleven et al. (2019a), which reproduces the relatively high child penalty documented in German-speaking countries (Germany and Austria).

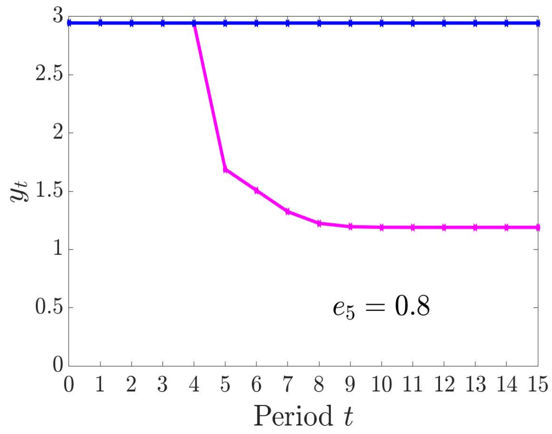
Panel (b) represents the case where the initial shock is less drastic, and the effort falls to $e_5 = 0.8$. While the shock is smaller than in the previous case, it brings the female

[13]A very simple way to introduce discrimination could be to make the function $\psi[.]$, defined by (19), gender-specific.

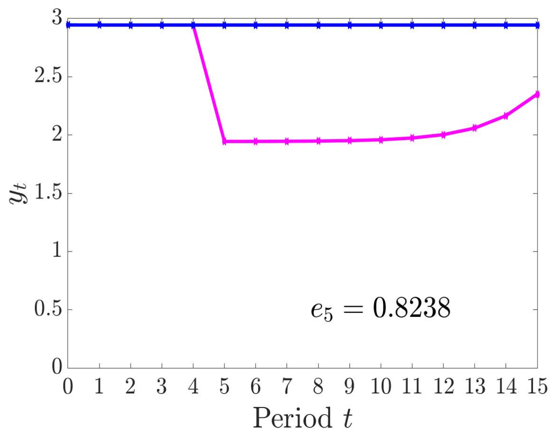
[14]Empirical evidence shows that most couples match assortatively in terms of education and income. See Chiappori et al. (2020a) for evidence from the U.K., Chiappori et al. (2020b) for the U.S., and Chiappori et al. (2022) for the case of Dutch couples.



(a) drastic shock



(b) less drastic shock



(c) small shock

Figure 2: Shock on female effort and child penalty.

worker below e_b^* so that eventually, the female worker continues to be trapped in the low steady state. Consequently, her income will continue to decline in the following periods and eventually reach the low steady state. This dynamic is similar to the one shown in Figure 2 by Kleven et al. (2019a) representing the child penalty for English-speaking countries (United States and United Kingdom).

A third possible pattern is represented in Panel (c). Now the shock is sufficiently small to ensure that $e_5 = 0.8238 > e_b^*$, so the dynamic converges to the high steady state. Consequently, the female worker is not trapped in the low steady state. Nevertheless, the one-period shock has a persistent impact. While income would eventually converge to the high steady state (and catch up with men), this is not achieved within the ten remaining periods. The female income keeps increasing but remains below the male worker's level, for instance, until retirement. This figure is similar to Figure 1 in Kleven et al. (2019a), especially in the case of Denmark. Here, the authors are reporting the relatively low child penalty in two countries where gender inequality is relatively small (Denmark and Sweden).

Our illustration shows that a negative shock in the effort, like the one generated by motherhood via labor supply, might be sufficient to activate a negative spiral of low wage rates leading to a persisting wage trap. In the case of assortatively matched couples, the motherhood trap is possibly very relevant despite women being ex-ante similar to their spouses regarding future earnings prospects. In addition, the gap in earnings might be substantial in size (like in the German-speaking countries) and, more importantly, it may be persistent, even when the trap is avoided.[15]

Many factors contribute to defining whether the income trap realizes and how dramatic it is. First, as Kleven et al. (2019a) pointed out, social norms prevailing in the relevant social group might amplify the effects of motherhood. For example, a mother may decide to extend her maternity leave because, in her social group, mothers are expected not to work full-time when their children are below school age. A second important ingredient we discuss in Section 5 below is the institutional environment and the specific design of family policies (e.g., the generosity and length of parental leave and/or the size of childcare subsidies), which make it more or less costly to provide labor supply after the motherhood shock. Casting these effects in terms of our examples, the two factors contribute to determining the extent of the shock. Finally, a third source of heterogeneity is mothers' socioeconomic status, which we discuss in the next section.

[15] But, at least in our example, the speed of convergence crucially depends on the extent of the shock. As Figure 2 shows, $e_5 = 0.8238$ yields very slow convergence.

4.3 Socioeconomic status

We drop for the time being the assumption that everyone is identical ex-ante and distinguish between highly educated women and less educated women. As we mentioned in Subsection 2.1, in our interpretation e_t is a increasing function of labor supply, ℓ_t, and human capital, h_t.

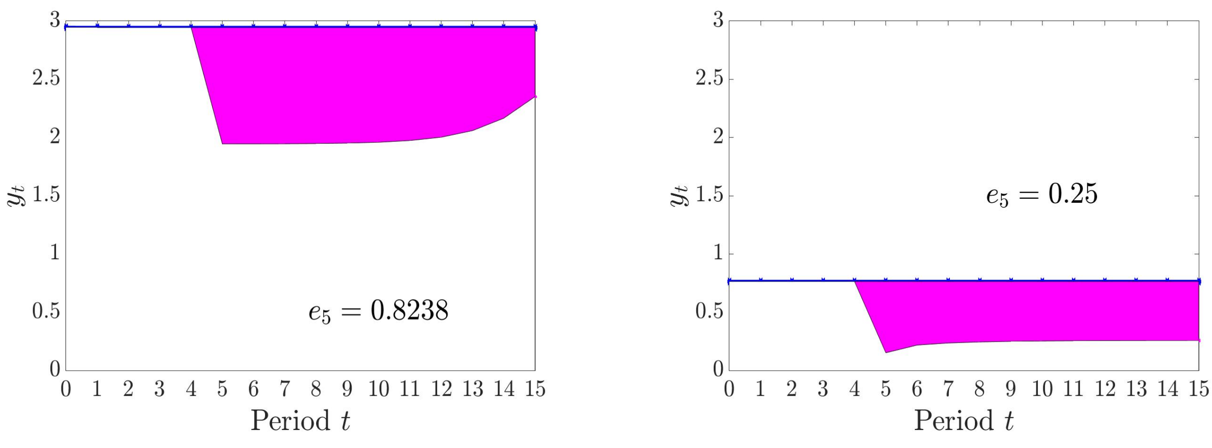
Highly educated workers are characterized by high h_t and, ceteris paribus, have access to higher wage rates than poorly educated workers. While labor supply ℓ_t falls to zero during compulsory maternity leave, human capital depreciates slowly. It is reasonable to assume that human capital remains constant if the shock in labor supply is sufficiently short (e.g. shorter than a couple of years). Hence, the effort shock generated by motherhood might be less severe for a highly educated woman because it is partially absorbed by the human capital she has accumulated before her maternity leave. Conversely, a poorly educated woman is more at risk of a poverty trap because her low human capital cannot mitigate the labor supply shock. In addition, the institutional environment might be affected by socioeconomic status as well: for example, a highly educated worker is more likely to be hired in a permanent contract, implying more maternity benefits than a short-time contract.

Let us now reinterpret Figure 2 in terms of the worker's socioeconomic status (SES). First note that a working mother with high SES reaches a higher effort and a higher wage rate before motherhood than a mother with low SES. This implies that when the shock in labor supply occurs (period 5), earnings are larger for a higher than for a poorly educated mother.

Panel (a) in Figure 2 might describe the situation of a female worker with a low SES. Human capital is low, and effort is almost pure labor supply. Consequently, the fall in labor supply due to motherhood brings the effort close to zero so that the impact of the shock is large, and the worker enters the poverty trap. Conversely, Panel (c) can describe the instance of a working mother with high SES. Human capital mitigates the labor supply shock, and the poverty trap is avoided.

Following this interpretation, the model predicts that women with low SES are more likely to enter the wage trap and suffer important earning losses. The evidence provided by Pora and Wilner (2019) is in line with this observation. They use French administrative data from 2005 to 2015 and document the heterogeneity of the consequences of childbirth, along with the distribution of pre-childbirth wages. Specifically, they show that high-wage women experience much smaller labor earnings losses due to childbirth than their lower-paid counterparts. In addition, high-wage women are much less likely to interrupt their careers or reduce their paid hours. Finally, the magnitude of such effects is completely monotone along the distribution.

However, the fact that women with low SES are more likely to enter the wage trap does not necessarily imply that they also suffer the higher child penalty. Indeed, our model can also accommodate the situation in which earning losses from motherhood are higher for mothers with high SES. This occurs when the drops in effort and wage rate after motherhood are sufficiently important for highly-educated mothers that the slow and incomplete convergence to the "high" steady state implies larger losses than the ones experienced by a poorly-educated woman entering the wage trap.



(a) high SES

(b) low SES

Figure 3: Child penalties for low and high SES women.

Child penalties for high and low SES are represented in Figure 3, which is constructed as follows. For high SES we keep the same specification as above. Thus, the representation in Panel (a) of Figure 3 is identical to Panel (c) in Figure 2. For the low SES, we keep the same utilities ($a = b = 0.65$) as in Figure 2, but we change the parameters of $\psi_A[.]$ to $\varepsilon = 0.6$, $\xi = 1$, $\theta = 0.01$ and $n = 7$. According to Appendix A.3, we continue to have three equilibria: $e_a^\star \simeq 0.37293$, $e_b^\star \simeq 0.48204$ and $e_c^\star \simeq 0.58596$ for effort and $y_a^\star \simeq 0.25776$, $y_b^\star \simeq 0.47087$ and $y_c^\star \simeq 0.76377$ for labor income. As for the high SES we consider two partners who are initially at the high equilibrium $e_c^\star$. Women are then subject to a shock entailing $e_5 = 0.25$, which will leave them trapped at the low steady state. Though at a different scale, Panel (b) in Figure 3 is the counterpart to Panel (a) in Figure 2.

In Figure 3, in each of the panels, the child penalty, which corresponds to the total earning loss (the sum of yearly losses) due to motherhood, is represented by the pink area obtained by integrating the difference between the earnings trajectories of the two partners. This shows that while the low SES women are trapped in the low steady-state, their total child penalty is smaller than for the highly educated, even though these stay on a trajectory that would eventually converge to the high steady state (which they don't reach because of the slow speed of convergence).[16]

5 Policy implications

We now consider family policies that are commonly used to increase mothers' attachment to the labor market and to reduce gender gaps in employment and earnings. Our objective is to study how these policies affect the mechanism described by our model and what this implies for their potential effectiveness.

[16] Accordingly, Andrew et al. (2021, page 1) write "In a big break from the past, the hourly wage gap between men and women is now bigger for those with degrees or A-level-equivalent qualifications than for those with lower education. It used to be that gender differences in hourly wages were especially large among less-well-educated workers."

5.1 Ineffective and potentially backfiring policies

The empirical literature has shown that some family policies, like long compulsory maternity leave and cash transfers to mothers may be ineffective or even aggravate labor market inequalities between men and women; see Fernández-Kranz and Rodríguez-Planas (2021) for a detailed list of the empirical papers finding that family-friendly policies can backfire. We now show that our model yields predictions that are consistent with these findings.

5.1.1 Maternity leave

It is a universal policy in all developed countries except the U.S., where only part of the working women is eligible, depending on their employer. We focus here on the consequences of mandatory maternity leave for working mothers. In our model, mandatory maternity leave imposes $\ell_t = 0$ for the length of the leave while providing women with a certain fraction of their pre-maternity leave salary. This translates into a reduction in effort for the considered period(s), the extent of which depends on the duration of the break relative to the total length of a period. Consequently, maternity leave is likely to amplify the negative shock generated by motherhood. This increases the gender gap (and the child penalty) and increases the risk of motherhood's wage trap.

Fernández-Kranz and Rodríguez-Planas (2021), for instance find that maternity leave has a detrimental effect on women's wages relative to men's both in Europe and the US. Similar results are reported by Olivetti and Petrongolo (2017), Schönberg and Ludsteck (2014) and Kleven et al. (2020).

Note that a potentially positive effect of maternity leave, namely that it can increase female labor force participation by helping working mothers to maintain their job after childbearing, is not accounted for in our model. For example, Del Rey et al. (2021) found an inverted U-shaped relationship between maternity leave duration and female participation because its positive effect via the time cost of work may be offset by the negative effect on wages. Their simulations show that for leaves for up to three months, the positive effect decreases.[17] Olivetti and Petrongolo (2017) report similar findings and mention studies according to which the benchmark below which women's labor market participation increases may be around 50 weeks.[18] While this effect qualifies our result, these results corroborate our finding that long leaves may be counterproductive.

To sum up, our model thus offers a possible explanation for the evidence suggesting that (long) maternity leaves exacerbate gender wage inequalities. This does, of course, not imply that maternity leave is a bad policy. Our analysis merely shows that this negative effect has to be traded off against its positive effects on mothers' and children's welfare (not accounted for in our model), particularly when determining the appropriate length of the leave.

5.1.2 Cash transfers

Cash transfers to mothers with small children are very common in developing countries, where they are typically targeted to the poorest families in which children face the most significant challenges. They are also provided in developed countries, where they are typically associated with other childcare policies, like in-kind transfer for children's early education. In OECD countries, cash transfers are frequently means-tested, and in this case, their main objective is redistribution. But cash transfers can also be unconditioned and provided to parents, irrespective of their income, for a given period of time (months or even years). In this latter case, their rationale may also include an incentive for fertility.[19]

[17] Del Rey et al. (2017) also find an ambiguous impact but for different reasons. They study investigates the effects of maternity leave on unemployment and wages in a search and matching model of the labor market. Crucial factors in their model are women's' wage bargaining power and the value of the leave for worker relative to the employer.

[18] See also Kleven et al. (2020). Girsberger et al. (2021) find a positive but very modest effect for Switzerland.

[19] Child-related cash transfers to families with children include child allowances and public income support payments which may depend on the child's age and/or family size and sometimes are income-tested. Income support payments cover periods of parental leave and aid for solo parents' families. In OECD statistics, cash, and in-kind transfers are lumped together and, on average, represent 1.163 of GDP; see OECD (2022).

To study the impact of this policy in our model we can introduce non-labor income, M_t, which enters the worker's utility and increases the worker's earnings as follows

$$\Lambda_M[e_t] = u[M_t + w_t e_t] + v[1 - e_t].$$

Cash transfer M_t can be provided to mothers during the maternity leave, after the maternity leave, i.e. when back to work, or in both circumstances. In Appendix A.4, we derive the dynamics of effort, wages, and earnings when cash transfers are provided. We show that they amplify the consequences of the motherhood shock because of the negative sign of the derivatives $\partial\Delta_M/\partial M_t$ and $\partial\Omega_M/\partial M_t$ (see A.8 and A.10, respectively). Hence, even a temporary cash transfer exacerbates the risk of motherhood's trap because it negatively affects the effort and the wage rate's dynamic. Intuitively, this is due to the negative income effect on e, which not surprisingly, exacerbates the shock.

In line with our reasoning, Lalive et al. (2014) estimate that longer cash benefits significantly delay the return to work of mothers when leave is job-protected, but less so once job protection has expired. Similarly, Schönberg and Ludsteck (2014) find that extensions of cash benefits beyond the job protection period produce significant long-run employment and earnings losses for affected mothers.

Note that, as observed before for maternity leave, our argument does not imply that cash transfers are overall a bad policy. Our analysis merely shows that the negative impact on wages and labor incomes has to be traded off against its positive effects on mothers' and children's welfare (not accounted for in our model).

5.2 Potentially effective policies

5.2.1 Subsidies to mothers' wages

We now consider subsidies to mothers' wages granted once maternity leave is over, and women return to work. These subsidies are a form of EITC (Earned Income Tax Credits) for young mothers. In our model, this represents the most direct way to affect women's effort and thereby reduce child penalties.

The empirical evidence on EITC is quite conclusive. For instance Olivetti and Petrongolo (2017) write that "Overall, the available micro-level studies find beneficial effects of in-work benefits on female employment, [...]."[20]

Intuitively, by encouraging mothers' effort, subsidies to mothers' wages mitigate the loss in human capital accumulation and activate a positive spiral of increasing effort and increasing wage rates which alleviates the gender gap in earnings in the short run with positive spillover effects in the long run.

Formally, assume that women's wages are subsidized at rate s during κ periods following the shock. During the κ periods, desired effort $e_t^\star$ is then obtained by maximizing

$$\Lambda_s[e_t] = u[(1+s)w_t e_t] + v[1-e_t].$$

We show in Appendix A.5 how this subsidy affects the dynamics of effort, wages, and incomes for our Illustration A. Note that these dynamics only apply to the κ periods following the shock in which female wages are subsidized. In the subsequent period the dynamics described in Subsection 4.1 and represented by expressions (16)–(19) resumes.

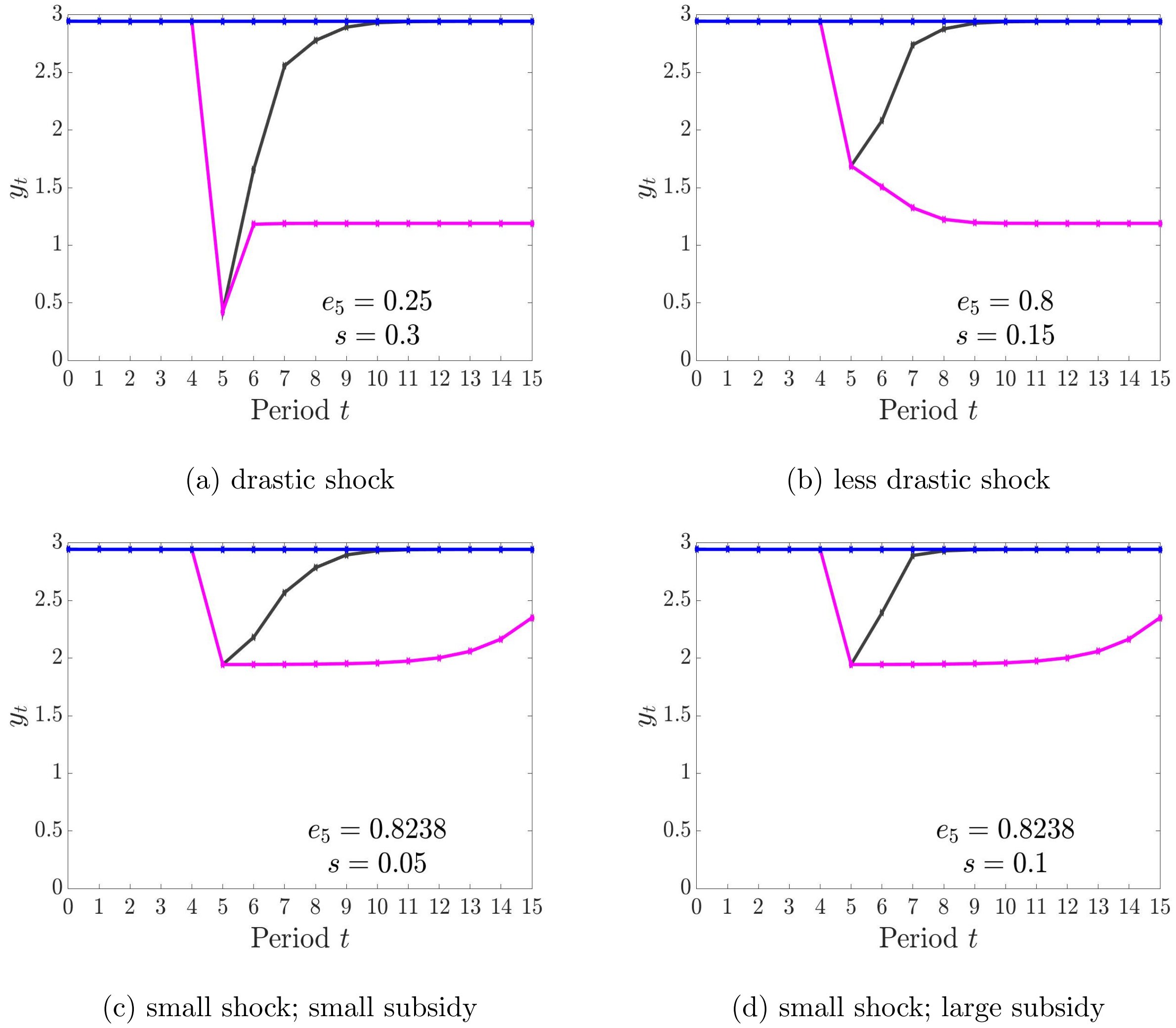
We will now reconsider the shocks represented in Figure 2 and examine how a subsidy on women's wages applied for $\kappa = 2$ periods affects the time path of women's earnings. We show that while the subsidy is transitory it has a lasting effect on female wages and it may be effective in neutralizing the effect of the initial shock. This is quite intuitive. We have shown that a one-period shock can have lasting effects; consequently one can expect a temporary subsidy to have similar long-run effects.

As before we assume that the shock occurs during period 5, at different levels, while the subsidy applies in periods 5 and 6.[21] Panel (a) in Figure 4 (like Panel (a) in Figure

[20]A large literature has examined the labor supply effects of EITC in the U.S.; see Nichols and Rothstein (2016) for a survey. In the U.K., the main in-work benefit is the Working Family Tax Credit, introduced in 1999. A general consensus exists that this policy raised the employment rate of lone mothers by 4–5 percentage points; see, among others, Francesconi and van der Klaauw (2007).

[21]Applying the subsidy in periods 6 and 7 yields similar results but with some extra delay for the recovery of women's income.

2) considers the most drastic shock leading to $e_5 = 0.25 < e_a^*$ and a subsidy of $s = 0.3$. Panel (b) in Figure 4 (the counterpart to Panel (b) in Figure 2) represents the case where the initial shock is less drastic and generates $e_5 = 0.8$, while the subsidy is also smaller and corresponds to $s = 0.15$.



(a) drastic shock

(b) less drastic shock

(c) small shock; small subsidy

(d) small shock; large subsidy

Figure 4: Impact of a wage subsidy to mothers on the dynamics of income.

In both cases, the initial shock brings the female worker below e_b^* so that she will be trapped in the low steady state. The two-period subsidy puts women on track for the high steady state again.[22]

[22]Alternatively one could consider a one-period subsidy with $\kappa = 1$. In case of the most drastic shock, a larger subsidy (for instance $s = 0.4$) is required to overcome the trap. With the less drastic shock $e_5 = 0.8$,

Finally, Panels (c) and (d) in Figure 4 reconsider the case represented in Panel (c) of Figure 2, where the shock bringing the effort to $e_5 = 0.8238$ is not sufficiently large for the women to get trapped in the low steady state. Here, recovery may be slow so that the gender gap in earnings persists until the end of active life. Now the objective of the subsidy is not to exit the trap but to speed up recovery and reduce the child penalty. We illustrate this by considering subsidies of 0.05 (Panel c) and 0.1 (Panel d). In both cases, full recovery is achieved and not surprisingly the speed of convergence is larger with the larger subsidy.

Our simulations show that a temporary subsidy to mothers' wage rate, granted immediately after maternity leave, is very effective not only in reducing the risk of a wage trap but also in mitigating the child penalty for women whose earning path converges towards the high steady state. Notably, being transitory, wage subsidies represent a relatively cheap policy, which could became even cheaper if targeted uniquely to women with low SES and at risk of the wage trap (see the Panel b of Figure 3).

We now discuss two other policies which have a more indirect effect on mother's labor supply but may still be effective.

5.2.2 Mandatory paternity leave

This policy has been progressively introduced in many countries.[23] Tamm (2019) shows that even short periods of paternity leave may have long-lasting effects on fathers' involvement in childcare and housework and that effects on maternal labor supply are also significantly positive.

In our model, compulsory paternity leave might mitigate the negative shock on mothers' labor supply or reduce its length. More indirectly, it may decrease mothers' costs of providing

a subsidy of $s = 0.15$ continues to be sufficient, but recovery will be slower.

[23]Sweden requires firms to offer 12 months of parental leave per child, split evenly between the two parents, with three months reserved for the father. If he does not take it, the benefit is lost. This is perhaps why do Swedish men experience a small decrease in their earnings after becoming fathers—they take more paternity leave than American or German men do. However, after three months they tend to go back to work, while their wives remain home for a much longer period.

effort (for example, by affecting the function $v[1-e_t]$) which in turn would boost labor supply. These two effects concur and decrease mothers' risk of entering the wage trap. Our simple model is entirely agnostic about family arrangements, and the sharing of family duties between partners; however, fathers may alleviate their partners' labor supply shock in many ways: for example, by increasing their engagement in informal childcare and helping with household chores.[24]

Note that paternity leave will also introduce a negative show on male labor supply which will further reduce the child penalty but reduces men's utilities an effect which has to be accounted for to assess the overall welfare impact of the policy.

5.2.3 Public provision of childcare facilities and subsidies to formal childcare

These two policies potentially generate results similar to mandatory paternity leave. In-kind childcare provision and/or subsidies might reduce the extent and duration of the shock and possibly decrease mothers' costs of providing effort, again, by affecting the function $v[1-e_t]$. This may stimulate the labor supply and decreases mothers' risk of entering the wage trap.

Empirical evidence appears to support this effect. In their survey Olivetti and Petrongolo (2017) state that "both the macro and micro literature tends to find overall positive effects of subsidized childcare on female employment." For example, Lefebvre and Merrigan (2008) find that childcare subsidies for four-year-old children, introduced in Québec in 1997 and combined with wider availability and high-quality service increased mothers' labor force participation by 8 percentage points and generated 231 extra annual hours. In follow-up work, they find that these beneficial effects for mothers' outcomes persist in the long run.[25]

[24] A possible efficiency enhancing role of paternity leave is also described by Bastani et al. (2019). In their setting, parental leave rules act as a means to enhance efficiency in the labor market and alleviate the gender wage gap.

[25] Considering together parental leave and childcare subsidies and studying the impact of Austrian reforms since the 1950s on the full dynamic of male and female earnings, Kleven et al. (2020, page 1) conclude that "The enormous expansions of parental [for mothers and fathers] leave and childcare subsidies have had virtually no impact on gender convergence." Our analysis suggests a possible explanation of this result: the

On the theoretical side, Barigozzi et al. (2018) have shown that subsidizing formal childcare alleviates the norm cost (mothers' guild) perceived by mothers who choose a high career path. Consequently, more mothers will choose higher paid jobs and reduce the level of informal childcare they provide. Bastani et al. (2020) examine the desirability of subsidizing child care expenditures via tax deductions or refundable tax credits in a Mirrleesian optimal tax framework where child care services not only enable parents to work, but also contribute to children's formation of human capital. In a calibration of their model they show that both policies appear to be welfare improving.

6 Conclusion

There appears to be a widespread consensus in the literature that child penalties (*i*) only affect mothers, (*ii*) are economically significant, and (*iii*) are persistent. In addition, the share of gender inequality caused by child penalties has been increasing for decades (Kleven et al. 2019a, 2019b).

We propose a simple dynamic model to show that the temporary shock in mothers' labor supply generated by childbearing may be sufficient to create wage and income traps. Our mechanism is based on the dynamics of the interaction between effort and wages. These dynamics may imply multiple steady-states, so that women can get trapped in low effort and low labor earnings. This outcome emerges when the initial shock is sufficiently significant. This may well describe the choice of, for example, entering part-time work or, in extreme cases, exiting the labor market altogether after childbearing. As a result of the wage trap, a drastic and persistent gender gap in labor earnings realizes.

Another possible scenario is that following the shock, women remain on a dynamic path that would eventually bring them back to the high steady state. However, convergence may be slow so that they won't reach the steady state during their active life. In other words,

beneficial effect of child care and paternal leave and the negative effect of a relatively long maternity leave might have canceled out.

one observes a child penalty that decreases but is not fully eradicated during the remaining career.

By analyzing a possible mechanism that explains the different empirically observed dynamics of child penalties, our model provides a useful tool for policy analysis. In particular, it shows which policies might or might not effectively mitigate gender earning gaps generated by motherhood.

For some family policies, our model confirms the concern that they can aggravate labor market inequalities between men and women. This is particularly true for cash transfers and long mandatory maternity leave. These policies exacerbate the initial shock and thus increase the likelihood of being trapped at a low level of earnings or, at best, slow down a return to the higher steady state.

By contrast, policies that make it easier to be a working mother, like the provision of childcare facilities, subsidies to formal care, tax credits, and compulsory paternity leave, go in the right direction and may help avoid the trap or speed up convergence. Amongst these, the most direct effect on the female labor supply is achieved by an EITC subsidy on the mothers who return to work after maternity leave. This subsidy can be temporary (two periods in our illustration) and may completely eradicate the child penalty. Intuitively, its effect is a mirror image of the dynamics induced by the initial shock. The interaction between supply and demand in the labor market is the same, except that all effects are reversed. Just as a temporary shock can induce a lingering negative effect, a temporary subsidy can permanently affect the induced dynamics.

To sum up, the main lessons of our analysis are the following. First, our model shows that lingering child penalties can be avoided by appropriately designed policies. This is in line with empirical evidence which points for instance to the positive effects of EITC and other tax credits for working mothers.

Second, it sheds doubt on the commonly used argument that the most effective method for reducing the gender gap in pay is to increase workplace flexibility. By contrast, our model stresses the importance of parenthood's gendered effects on labor supply. Specifically, if human capital growth is an accumulative process that takes place at the workplace and flexibility contributes to the fall in women's labor supply after childbearing, gendered wage trajectories over the life cycle will remain even in a flexible labor market as the one in Scandinavian countries; see Angelov et al. (2016).

Third and last, the policy recommendations are clear: on the one hand, policymakers should limit cash transfers and long, fully-paid maternity leave; on the other hand, they should increase compulsory paternity leave, formal child care, and especially tax credits provided once mothers return to work.

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A Appendix

A.1 Strictly convex specification of $\psi[.]$ and non-monotonic desired effort level $\varphi[.]$

We now show through an example that neither the sigmoïd (convex and then concave) specification of $\psi[.]$ nor that of preferences implying an increasing desired effort level $\varphi[.]$ are necessary to obtain the pattern of dynamics described in Subsection 4.1. Specifically, we consider a strictly convex expression for $\psi[.]$ and a non-monotonic level of desired effort $\varphi[.]$ and continue to obtain three steady-states with a trap and monotonic dynamics.

Assume that preference are now given by

$$u_B[c_t] = -\frac{\exp[-ac_t]}{a} \quad \text{and} \quad v_B[z_t] = -\frac{\exp[-bz_t]}{b}, \tag{A.1}$$

rather than by (14), where $a > 0$ and $b > 0$. This yields a level of desired effort given by

$$\widehat{e}_t = \varphi[w_t] \equiv \frac{\ln[w_t] + b}{aw_t + b}, \tag{A.2}$$

which unlike (15) is first increasing and then decreasing in w.

Turning to the wage determination we now assume that $\psi[.]$ is given by

$$\psi_B[e_{t+1}] = \varepsilon + \xi e_{t-1}^n,$$

where $\varepsilon \geq 1$, $\xi > 0$ and $n > 1$ so that we have a strictly convex function, rather than the sigmoïd expression (19).

Assuming $\alpha = 0$ so that $e_t = \widehat{e}_t$, and proceeding like in Section 3 shows that the dynamics of effort e_t and wage rates w_{t+1} for $e_{t-1} > 0$ and $w_t > 0$ are represented by

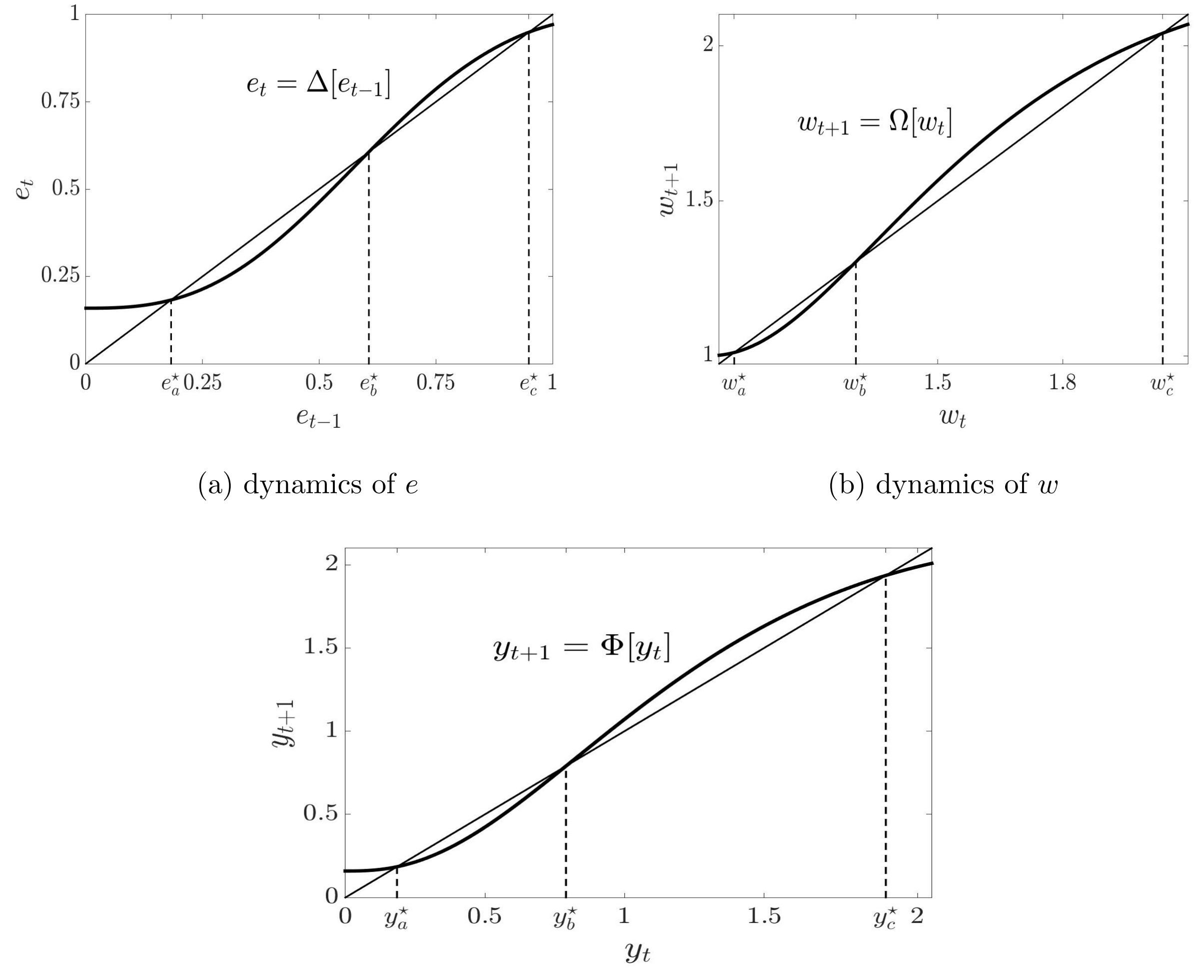
$$e_t = \Delta[e_{t-1}] \equiv \frac{\ln\left[\varepsilon + \xi e_{t-1}^n\right] + b}{a\left(\varepsilon + \xi e_{t-1}^n\right) + b}, \tag{A.3}$$

and

$$w_{t+1} = \Omega[w_t] \equiv \varepsilon + \xi\left(\frac{\ln[w_t] + b}{aw_t + b}\right)^n. \tag{A.4}$$

Note that these conditions apply as long as the solution is interior.

Setting $\alpha = 0$, $a = 0.37$, $b = 0.07$, $\varepsilon = 1$, $\xi = 1.2$ and $n = 2.75$ one obtains the following dynamics



(a) dynamics of e

(b) dynamics of w

(c) dynamics of y

Figure 5: The different dynamics of our Illustration B. Representation of $\Delta[.]$, $\Gamma[.]$ and $\Phi[.]$ with parameters $\alpha = 0$, $a = 0.37$, $b = 0.07$, $\varepsilon = 1$, $\xi = 1.2$ and $n = 2.75$.

The equilibria are given by $e_a^\star \simeq 0.18265$, $e_b^\star \simeq 0.60643$ and $e_c^\star \simeq 0.94923$ for effort, $w_a^\star \simeq 1.01119$, $w_b^\star \simeq 1.30327$ and $w_c^\star \simeq 2.03979$ for wages, and $y_a^\star \simeq 0.18470$, $y_b^\star \simeq 0.79035$ and $y_c^\star \simeq 1.93622$ for labor income.

A.2 Derivation of equation (18)

Because $e_t = \widehat{e}_t$, equation (15) implies $y_t = w_t e_t = w_t \varphi[w_t] = (1+b)a^2 w_t^2/(1+a^2 w_t)$, i.e. $\mathcal{P}[w_t] \equiv (1+b)a^2 w_t^2 - a^2 y_t w_t - y_t = 0$. When $y_t > 0$, the fact that $\mathcal{P}[0] < 0$ and $\lim_{w_t \to \infty} \mathcal{P}[w_t] = +\infty$, implies

$$w_t = \phi[y_t] = \frac{a y_t + \sqrt{a^2 y_t^2 + 4(1+b) y_t}}{2a(1+b)}.$$

Using

$$y_{t+1} = w_{t+1} e_{t+1} = \psi[e_t] \varphi[\psi[e_t]] = \frac{(1+b)a^2 \psi^2[e_t]}{1 + a^2 \psi[e_t]},$$

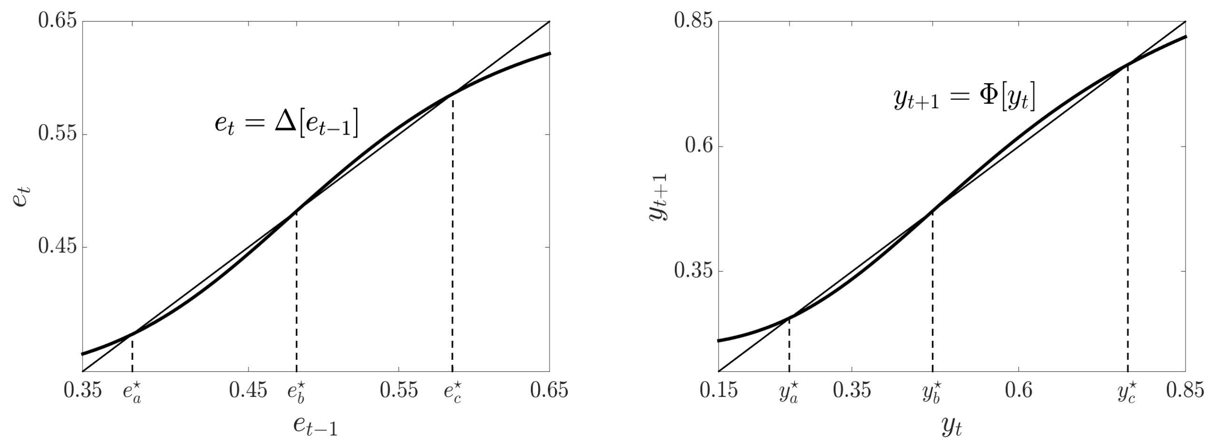
and (for $y_t > 0$)

$$e_t = \frac{y_t}{w_t} = \frac{y_t}{\phi[y_t]} = \frac{2a(1+b)}{a + \sqrt{a^2 + 4(1+b)/y_t}},$$

yields equation (18).

A.3 Dynamics of the low SES case

Considering $\alpha = 0$, $a = b = 0.65$, $\varepsilon = 0.6$, $\xi = 1$, $\theta = 0.01$ and $n = 7$, the dynamics of effort, $\Delta[.]$ and of labor income, $\Phi[.]$, described by (16)–(19) are represented in Figure 6.



(a) dynamics of e (b) dynamics of y

Figure 6: Representation of $\Delta[.]$ and $\Phi[.]$ as specified by (16)–(19) with parameters $\alpha = 0$, $a = b = 0.65$, $\varepsilon = 0.6$, $\xi = 1$, $\theta = 0.01$ and $n = 7$.

The equilibria are given by $e_a^\star \simeq 0.37293$, $e_b^\star \simeq 0.48204$ and $e_c^\star \simeq 0.58596$ for effort and $y_a^\star \simeq 0.25776$, $y_b^\star \simeq 0.47087$ and $y_c^\star \simeq 0.76377$ for labor income.

A.4 Dynamics with cash transfers

With cash transfers, M_t, the desired effort $\widehat{e}_t$ is obtained by maximizing $u[M_t+w_te_t]+v[1-e_t]$ with respect to e_t. Then, the FOC (2) becomes

$$\varrho[\widehat{e}_t, w_t, M_t] \equiv w_t u^{'}[M_t + w_t\widehat{e}_t] - v^{'}[1-\widehat{e}_t] = 0, \tag{A.5}$$

which defines the function $\varphi_M[.]$ such that

$$\widehat{e}_t = \varphi_M[w_t, M_t], \tag{A.6}$$

where $\partial\varphi_M/\partial M_t = -w_t u^{''}[M_t + w_t\widehat{e}_t]/(w_t^2 u^{''}[M_t + w_t\widehat{e}_t] + v^{''}[1-\widehat{e}_t]) < 0$.

The dynamics of effort e_t is now described by a function $\Delta_M[.]$ defined by

$$e_t = \alpha e_{t-1} + (1-\alpha)\varphi_M[\psi[e_{t-1}], M_t] \equiv \Delta_M[e_{t-1}, M_t]. \tag{A.7}$$

Consequently

$$\frac{\partial \Delta_M}{\partial M_t} = (1-\alpha)\frac{\partial \varphi_M}{\partial M_t} < 0. \tag{A.8}$$

The dynamics of wage w_t is now described by a function $\Omega_M[.]$ defined by

$$w_{t+1} = \psi\Big[\alpha\psi^{-1}[w_t] + (1-\alpha)\varphi_M[w_t, M_t]\Big] \equiv \Omega_M[w_t, M_t]. \tag{A.9}$$

Consequently

$$\frac{\partial \Omega_M}{\partial M_t} = (1-\alpha)\psi^{'}\Big[\alpha\psi^{-1}[w_t] + (1-\alpha)\varphi_M[w_t, M_t]\Big]\frac{\partial \varphi_M}{\partial M_t} < 0. \tag{A.10}$$

A.5 Dynamics with a subsidy to mother's wage rate

Proceeding like in Subsection 4.1 one easily shows that expressions (16), (17) and (18) reported there are now given by[26]

[26]Roughly speaking it is sufficient to replace w by $w(1+s)$.

$$e_t = \Delta_s[e_{t-1}] \equiv \frac{(1+b)a^2(1+s)\psi[e_{t-1}]}{1+a^2(1+s)\psi[e_{t-1}]},$$

$$w_{t+1} = \Omega_s[w_t] \equiv \psi\left[\frac{(1+b)a^2(1+s)w_t}{1+a^2(1+s)w_t}\right],$$

and

$$y_{t+1} = \Phi_s[y_t] \equiv \frac{(1+b)a^2(1+s)^2\psi^2\left[\frac{2a(1+b)}{a+\sqrt{a^2+4(1+b)/y_t}}\right]}{1+a^2(1+s)\psi\left[\frac{2a(1+b)}{a+\sqrt{a^2+4(1+b)/y_t}}\right]}.$$