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Estimating and Testing Investment-based Asset Pricing Models*

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March 2023

Abstract

Most investment-based asset pricing models predict a close link between a firm's stock return and firm-characteristics at any point in time. Yet, previous work typically examines the weaker prediction that this link should hold on average. We show how to incorporate the time-series predictions in the estimation and testing of investment-based models using the generalized method of moments. We find that standard specifications of the investment-based model with one physical capital input fail to match the time series properties of stock returns in the data, and discuss the implications of the findings for future research.

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1 Introduction

The neoclassical investment-based asset pricing model links firm characteristics to stock returns. With homogeneous of degree one technology, the model predicts that a firm's realized investment return, which is a function of firm characteristics, should be equal to its stock return at any point in time (Cochrane 1991, and Restoy and Rockinger 1994). Under more general specifications of the model in which this equality does not hold, the close link between stock returns and firm-characteristics at any point in time is still (approximately) preserved, as we show here. Despite this strong time-series prediction, most of the structural work in investment-based asset pricing to date, tests the model by the generalized methods of moments (GMM) using the weaker prediction that a firm's stock return and firm-characteristics should be related on average. Following this procedure, Liu, Whited, and Zhang (2009) (henceforth LWZ) shows that a standard neoclassical investment-based model with one capital input and homogeneous of degree one technology matches surprisingly well the cross-sectional dispersion in average stock returns of a large range of portfolio sorts. In this paper, we show how to incorporate the time-series predictions in the estimation and testing of investment-based models using GMM. We find that both the standard investment-based model in LWZ and a more general specification without homogeneous technology fail to capture the time-series properties of stock returns in the data, and we discuss the implications of the findings for future research.

We first apply our estimation approach to the standard specification of the investment-based model with one capital input and homogeneous of degree one technology, as the one examined in LWZ, where the equality between levered investment and stock returns holds. As in a standard nonlinear least squares estimation approach, we add the squared distance between stock returns and model-implied investment returns at each point in time as a new set of target moment conditions in the GMM estimation of the model, which we label as time-series moments. In addition, as in previous work, we also consider cross-sectional moments, that is, the model prediction that investment and stock returns should be equal on average.

We follow LWZ and estimate the baseline model by the GMM at the portfolio-level, using ten book-to-market portfolios (we also consider other test assets in the appendix), to reduce the impact of estimation noise. The estimation targets twenty moments: ten cross-sectional moments as in LWZ, and our novel ten time-series moments. To understand the role of the two set of moments on the results, we investigate how the estimates and model fit change when we vary the relative weight of the two set of moments in the GMM weighting matrix.

Most of our analyses focuses on the economic evaluation of the fit of a candidate investment-based model as a benchmark for model validation. In particular, we investigate how well a given model is able to simultaneously capture the average cross-sectional and time-series variation of stock returns across portfolios with economically reasonable parameter values. To that end, we discuss the model fit using descriptive measures such as cross-sectional R^2 (henceforth $XS - R^2$) of the scatter plot of average portfolio-level stock returns against average portfolio-level estimated investment returns, the time-series R^2 (henceforth $TS - R^2$) measured as the time-series R^2 of a linear projection of the realized stock return against the estimated investment return, and the magnitude of the pricing errors, measured as the average of the residuals across portfolios (the standard alpha, or abnormal return, in empirical asset pricing), and over time for a given portfolio.

The estimation results of the baseline investment-based model can be summarized as follows. First, consistent with previous work, when we only use cross-sectional moments in the GMM estimation, the model matches these cross-sectional moments very well, with low cross-sectional pricing errors (alpha of about 1.4% per year) and a high $XS - R^2$ of 73%. The time-series fit of model is very poor, however, with an average $TS - R^2$ of -90% across portfolios. Thus, most of the time series variation of realized stock returns is captured by the estimation residuals, not by the predicted investment returns. We conclude that when estimating the baseline investment-based model using the cross-sectional moments only, the model fails to capture its time series implications.

Second, our estimation approach uncovers a novel trade-off between cross-sectional fit and time-series fit in the baseline investment-based model: when we incorporate the model-implied time series moments in the GMM estimation to improve its time series fit we find that the model cannot fit both sets of moments simultaneously. As we increase the relative weight of the time-series moments in the estimation, the fit on the cross-sectional moments deteriorates significantly: the $XS - R^2$ decreases from 73% when only cross-sectional moments are used in the estimation, to –137% when only time-series moments are used in the estimation. As expected, the time-series fit of the model improves when the weight of the time-series moments increases. But more importantly, and perhaps surprisingly, even when only the time-series moments are used in the estimation, the model's time-series fit remains poor, with a $TS - R^2$ of -3%. That is, the baseline investment-based model is not able to capture the time-series behavior of stock returns in the data even when the estimation is designed to maximize its time series fit.

We investigate two potential empirical reasons for the poor fit of the baseline investment-based model in the time-series, despite its good fit in the cross-section. First, we investigate the role of possible issues in the portfolio aggregation for the results. As noted in Belo, Gala, Salomao, and Vitorino (2022) (henceforth BGSV), and Gonçalves, Xue, and Zhang (2020) (henceforth GXZ), the portfolio-level aggregation procedure in LWZ suffers from an aggregation bias, and we investigate if this bias can be the cause the poor fit of the baseline investment-based model in the time series (we discuss the source of this aggregation bias in detail in Section 6.1). Following BGSV and GXZ, we estimate the model using portfolio-level investment returns properly aggregated from firm-level investment return data. Using simulated data from a calibrated version of the baseline investment-based model, we confirm that, in theory, the $TS - R^2$ should be perfect when the portfolio-level moments are properly aggregated, and that the biased portfolio-level aggregation does lead to very low $TS - R^2$, and hence this bias could be a potential explanation for the low $TS - R^2$ observed in the data. However, when we re-estimate the model with a proper bias-free portfolio-level aggregation in the real data, the fit of the model in the time series remains very poor, $TS - R^2 = -13\%$, even when the estimation focuses only on the time-series moments to maximize its time-series fit. Thus, portfolio-level aggregation issues do not appear to be the main cause for the inability of the baseline investment-model to capture the time series behavior of stock returns in the data.

Second, we address the possibility that quantities (e.g. investment) and asset prices are misaligned in the data, which might explain the observed low contemporaneous correlation between stock returns and investment returns. For example, stock prices (and hence stock returns) might respond instantaneously to aggregate shocks, whereas investment might take more time to adjust, in which case investment returns lag stock returns (see, for example, Lamont 2000 for a more formal analysis of this issue). To address this concern, we investigate the time-series fit of the model using time-smoothed data, that is, annualized 5-year compounded returns. If the misalignment in the data is relatively short lived, the misalignment between investment and stock returns should be less pronounced at longer-horizons. We provide support for this conjecture using again simulated data from a calibrated version of the baseline investment-based model in which we parameterize the data misalignment and show that, in theory, the $TS - R^2$ is indeed significantly higher when we use annualized compounded returns in the presence of data misalignment ($TS - R^2 = 71\%$ at longer horizons versus $TS - R^2 = 11\%$ at annual frequency with misalignment). However, in the real data, the $TS - R^2$ using annualized 5-year compounded stock and investment returns remains negative, $TS - R^2 = -10\%$. Thus, the data misalignment in asset prices and real quantity data also does not appear to be the main cause for the inability of the baseline investment-based model to capture the time series behavior of stock returns in the data.

Next, we show how to extend our approach to evaluate the performance of investment-based models in which the potentially restrictive assumption of homogeneous of degree one technologies does not hold. In these specifications of the model, in general, the stock return and investment return equality does not hold and so it cannot be used to estimate the model. However, as we show here, most investment-based asset pricing models imply a strong relationship between realized stock returns and firm-characteristics, such as, a firm's current marginal product of capital, or the current and lagged investment rates, both in the cross-section and in time-series. Such relationships can be assessed in the data for any model by running simple stock return-firm characteristics regressions, and a successful candidate investment-based model should generate regression results that are consistent with the corresponding regressions in the real data. This approach can be used simply as an external validity test of a calibrated existing model, or it can be incorporated in the estimation and test of any model as additional target moment conditions using, for example, a simulated method of moments approach.

Using this insight, we apply our approach to a more general specification of the investment-based model with decreasing returns to scale, non convex and asymmetric adjustment costs, and operating fixed costs. Using data simulated from the model, we document that this more general version of the investment-based model also implies a strong relationship between stock returns and firm characteristics both in cross-section and time-series that can be well approximated by a linear specification. When applied to the real data, however, the time-series relationship between realized stock returns and the same firm characteristics is weak, thus we can also reject this more general version of the model on economic grounds based on its poor fit in the time-series.

Our approach highlights the importance of examining the time-series, in addition to cross-sectional, predictions of investment-based models to better evaluate this class of models. As we document here, matching the time-series implications of investment-based models represents a higher hurdle for these models than matching cross-sectional moments. Thus, the use of time-series moments can help researchers identify areas of the model fit that require improvement, and guide researchers on how to improve their specification.

Our findings have implications for future research in investment-based asset pricing. Going forward, additional capital inputs such as intangible capital as in Peters and Taylor (2017) and BGSV, quasi-fixed labor inputs as in BGSV, or short-term and long-term assets as in GXZ, might be incorporated in the model to improve its fit in the time-series. In addition, accounting for firm- or industry-level heterogeneity in the technologies, which are assumed to be similar across firms in the baseline analyses, as well as more general functional forms, should be investigated.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 describes the econometric methodology. Section 5 reports the estimation and tests results of the baseline investment-based model. Section 6 investigates potential reasons for the poor time-series fit. Section 7 proposes an extension of the approach that can be used to estimate and test more general specifications of the investment-based model. Section 8 discusses the results from alternative estimation approaches and robustness checks. Finally, Section 9 concludes. A separate appendix with additional results is available online.

2 Related literature

Our work is closely related to Liu, Whited, and Zhang (2009) who is the first to estimate the baseline neoclassical investment-based model on the cross-section of stock returns. Different from LWZ, our estimation procedure requires the model to also match the realized time series of the observed stock returns as close as possible (in the same spirit, BGSV requires the model to match the time series of valuation ratios), and not just on average.[1] LWZ use their estimates to document that the implied stock and investment returns have low correlation, which they label a correlation puzzle. Similarly, the correlation puzzle is documented at the aggregate level in Kuehn (2009). We show that this puzzle persists even when the estimation is designed to maximize the time-series fit of the model. Our analysis is broader in that we show how to incorporate the time series implications of investment-based models directly in the estimation and evaluation of these models, and we discuss potential alternative empirical

[1] In robustness analysis, LWZ includes the cross-section of portfolio variance moments in the estimation, but, as we show in sub-section 8.2, matching variance moments does not help improving the time-series fit of the model.

reasons for the observed poor time-series fit.

Li, Ma, Wang, and Yu (2021) estimate an investment-based model with two capital inputs using firm-level data and Bayesian estimation methods. Different from our work, they focus only on a specification of the investment-based model in which the homogeneous of degree one assumption holds, and they estimate the model to match firm-level stock returns. In addition, and departing from baseline neoclassical investment-based models with stable technologies, they estimate industry-specific and time-varying technological parameters. They show that allowing for time-varying parameters of the firm's technology helps to significantly improve the fit of the model in explaining the returns of several anomaly portfolios.

Gonçalves, Xue, and Zhang (2020) document the aggregation bias in the original LWZ portfolio-level aggregation approach (see also BGSV and Zhang, 2017, for earlier discussions of this aggregation bias in LWZ). Our analysis shows that the aggregation bias alone cannot explain the poor fit of the investment-based model with one-capital input in the time series.

Delikouras and Dittmar (2021) estimate and test standard investment-based models using GMM with cross-sectional moments and investment Euler equations, which requires the specification of a stochastic discount factor, and also find that the baseline investment-based model is unable to match both sets of moments jointly. Our paper shares the goal of testing investment-based models across a larger set of model implications, but differs in the approach. Our analysis focuses on the properties of the firm's technology and does not require the specification of a stochastic discount factor (at least for the estimation of homogeneous of degree one models), thus avoiding the joint hypothesis testing problem. Naturally, the ultimate goal is to obtain a specification of firm's technology and of the stochastic discount factor that simultaneously matches the cross-sectional, the time-series, and the investment Euler equations as in Delikouras and Dittmar (2021). However, focusing only on the properties of the firm's technology allow us to narrow down how its specification alone affects the time-series and cross-sectional fit of the model. Our paper thus complements their approach by providing the first step (focus on firm's technology) towards that goal.

Finally, our paper contributes to the literature on developing and improving the methodology in the estimation and testing of structural models in financial economics.[2] In addition, our paper is closely related to the strand of production-based asset pricing literature that links firm characteristics to asset returns.[3] We contribute to these literatures by improving the econometric methodology for estimating and testing this class of models.

3 The neoclassical investment-based model

We briefly present the baseline neoclassical investment-based model of the firm with one (physical) capital input and homogeneous of degree one technology as in LWZ. We use their notation whenever possible. Time is discrete and the horizon infinite. Firms choose costlessly adjustable inputs each period, taking their prices as given, to maximize operating profits (revenues minus expenditures on these inputs). Taking operating profits as given, firms choose investment and debt to maximize the market equity.

Operating profits for firm i at time t are given by $\Pi\left(K_{it}, X_{it}\right)$, in which K_{it} is physical capital and X_{it} is a vector of exogenous aggregate and firm-specific shocks. The firm has a Cobb-Douglas production function with constant returns to scale. As such, $\Pi\left(K_{it}, X_{it}\right) = K_{it}\partial\Pi\left(K_{it}, X_{it}\right)/\partial K_{it}$, and the marginal product of capital, $\partial\Pi\left(K_{it}, X_{it}\right)/\partial K_{it} = \alpha Y_{it}/K_{it}$, in which α is the capital's share in output and Y_{it} is sales.

Capital depreciates at an exogenous rate of δ_{it}, which is firm-specific and time-varying:

$$K_{it+1} = I_{it} + \left(1 - \delta_{it}\right) K_{it}, \tag{1}$$

where I_{it} is investment. Firms incur adjustment costs when investing. The adjustment costs function, denoted $\Phi\left(I_{it}, K_{it}\right)$, is increasing and convex in I_{it}, is decreasing in K_{it}, and has

[2]Examples include, Cochrane (1996), Hansen and Jagannathan (1997), Bazdresch, Kahn, and Whited (2018), Chen, Dou, and Kogan (2019), Gala, Gomes, and Liu (2020), Cheng, Dou, and Liao (2022).

[3]Examples include, Zhang (2005b), Belo (2010), Belo, Lin, and Bazdresch (2014), İmrohoroğlu and Tüzel (2014), Kogan and Papanikolaou (2014), Kung and Schmid (2015), Croce (2014), and Deng (2021).

constant returns to scale in I_{it} and K_{it}. We use a standard quadratic functional form:

$$\Phi\left(I_{it}, K_{it}\right) = \frac{c}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}, \tag{2}$$

where $c > 0$ is the slope adjustment cost parameter.

Firms finance investment with one-period debt. At the beginning of period t, firm i issues an amount of debt, denoted B_{it+1}, that must be repaid at the beginning of $t+1$. Let r_{it}^B denote the gross corporate bond return on B_{it}. We can write taxable corporate profits as operating profits minus depreciation, adjustment costs, and interest expense: $\Pi\left(K_{it}, X_{it}\right) - \delta_{it}K_{it} - \Phi\left(I_{it}, K_{it}\right) - \left(r_{it}^B - 1\right)B_{it}$. Let τ_t denote the corporate tax rate. We define the payout of firm i as:

$$D_{it} \equiv \left(1-\tau_t\right)\left[\Pi\left(K_{it}, X_{it}\right) - \Phi\left(I_{it}, K_{it}\right)\right] - I_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it} K_{it} + \tau_t\left(r_{it}^B - 1\right)B_{it}, \tag{3}$$

where $\tau_t\delta_{it}K_{it}$ is the depreciation tax shield, and $\tau_t\left(r_{it}^B - 1\right)B_{it}$ is the interest tax shield.

Let M_{t+1} denote the stochastic discount factor from period t to $t+1$, which is correlated with the aggregate component of the productivity shock X_{it}. The firm chooses optimal capital investment and debt to maximize the cum-dividend market value of equity:

$$V_{it} \equiv \underset{\{I_{it+s}, K_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}}{max} E_t\left[\sum_{s=0}^{\infty} M_{t+s}D_{it+s}\right], \tag{4}$$

subject to a transversality condition given by $lim_{T\to\infty}E_t\left[M_{t+T}B_{it+T+1}\right] = 0$.

Firms' equity value maximization implies that $E_t\left[M_{t+1}r_{it+1}^I\right] = 1$, in which r_{it+1}^I is the investment return, defined as

$$r_{it+1}^I \equiv \frac{\left(1-\tau_{t+1}\right)\left[\alpha\frac{Y_{it+1}}{K_{it+1}} + \frac{c}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} + \left(1-\delta_{it+1}\right)\left[1 + \left(1-\tau_{t+1}\right)c\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]}{1 + \left(1-\tau_t\right)c\left(\frac{I_{it}}{K_{it}}\right)}. \tag{5}$$

The investment return is the ratio of the marginal benefits of investment at period $t+1$ to the marginal costs of investment at t.

The first-order condition of maximizing Equation (4) with respect to B_{it+1} implies that $E_t\left[M_{t+1}r_{it+1}^{Ba}\right] = 1$, in which $r_{it+1}^{Ba} \equiv r_{it+1}^{B} - \left(r_{it+1}^{B} - 1\right)\tau_{t+1}$ is the after-tax corporate bond return. Define $P_{it} \equiv V_{it} - D_{it}$ as the ex-dividend equity value, $r_{it+1}^{S} \equiv \left(P_{it+1} + D_{it+1}\right)/P_{it}$ as the stock return, and $w_{it} \equiv B_{it+1}/\left(P_{it} + B_{it+1}\right)$ as the market leverage. Under constant returns to scale, the investment return equals the weighted average of the stock return and the after-tax corporate bond return:

$$r_{it+1}^{I} = w_{it}r_{it+1}^{Ba} + \left(1 - w_{it}\right)r_{it+1}^{S}. \tag{6}$$

Equivalently, the stock return equals the levered investment return, denoted r_{it+1}^{Iw}:

$$r_{it+1}^{S} = r_{it+1}^{Iw} \equiv \frac{r_{it+1}^{I} - w_{it}r_{it+1}^{Ba}}{1 - w_{it}}. \tag{7}$$

4 Econometric methodology

Section 4.1 describes the portfolio approach used to estimate the baseline model. Section 4.2 describes the generalized method of moments (GMM) estimation methodology, discusses the cross-sectional moment conditions used in prior studies, and presents our new time-series moment conditions. Section 4.3 discusses the metrics used to evaluate the fit of the model.

4.1 Portfolio approach

Equation (7) establishes an exact relationship between a firm's observed stock return and its model-implied levered investment return at each point in time. However, using equation (7) and firm-level data to directly estimate the model parameters can be challenging because the firm-level data can be very noisy. Thus, we estimate the model parameters using portfolio-level moments as in LWZ. The use of portfolio-level moments, a common practice in the asset pricing literature, has several attractive features in our context. First, it allows us to reduce the noise in the firm-level data, and hence obtain more accurate parameter estimates and measures of model fit such as R^2. Second, portfolio-level moments are less sensitive, and hence more stable, to firm entry and exit, and to missing firm-level observations. Finally, the use of portfolio-level moments allows us link our approach to the large empirical asset pricing literature, and to characterize the data in a more parsimonious manner as the number of portfolios is naturally smaller than the number of firms in the data.

Following LWZ, we construct the portfolio-level investment returns as follows. We first compute the portfolio-level characteristics in the investment return formula given by equation (5) as $I_{it+1}/K_{it+1} = \sum_{j=1}^{N} I_{ijt+1}/\sum_{j=1}^{N} K_{ijt+1}$, $Y_{it}/K_{it} = \sum_{j=1}^{N} Y_{ijt}/\sum_{j=1}^{N} K_{ijt}$, etc, where a portfolio is indexed by $i = 1, ..., N$, and a firm is indexed by j. We then plug each characteristic in the investment return formula given by equation (5) to compute the portfolio-level investment return. As portfolios, we use the standard ten book-to-market (BM) portfolios (we consider other test assets in the appendix), and describe the construction of the portfolios in the Data subsection 5.1.

4.2 GMM estimation and target moments

We estimate the investment-based model using the generalized method of moments (GMM). We define the moment conditions as follows. As in previous work (e.g. LWZ), we consider a set of cross-sectional moments implied by Equation (7). In particular, this equation implies a weaker model prediction that stock returns should be equal to levered investment returns on average,

$$g_i^{XS} = E_T\left[r_{it+1}^S - r_{it+1}^{Iw}\right] = 0, \tag{8}$$

in which $E_T[\cdot]$ is the sample mean of the series in brackets. We can stack the previous moment for each portfolio into a column vector g^{XS}, which we label as cross-sectional moment conditions.

According to equation (7), the investment-based model has rich implications not only for the cross-section of stock returns, but also for the time-series of stock returns. Hence, we also require the estimation to match the equality of stock and levered investment returns at each point in time as closely as possible, that is, we add a new set of time-series moment conditions. Consistent with a standard nonlinear least squares estimation approach (henceforth NLLS), we specify the time-series moment condition for each portfolio as the time-series average of the squared differences between stock returns and levered investment returns:

$$g_i^{TS} = E_T \left(r_{it+1}^{S} - r_{it+1}^{Iw} \right)^2 = 0. \tag{9}$$

We can stack the previous moment for each portfolio into a column vector g^{TS}, which we label as time-series moment conditions.

We note that we use the NLLS objective function (sum of squared residuals) as the target time-series moment for each portfolio. Under the null that the model is correctly specified, this moment should be zero, and hence it is a valid moment condition for GMM. Naturally, if there is some measurement error or other noise in the data, the moment will deviate from zero even if the model is correctly specified, but it should not deviate too much and the deviation should be random. The fact that the deviation might be different from zero due to noise might affect the interpretation of the chi-square tests for assessing the validity of the model (described in the next section), but this is not a concern in our analysis given that our focus is mostly on the interpretation of the model fit on economic grounds, with less emphasis on the statistical tests of the model. Finally, the use of NLLS objective function as the moment condition has the benefit of being parsimonious and easy to interpret.[4]

We then estimate the model parameters $\theta \equiv (\alpha, c)$, using one-step GMM to minimize

[4] In robustness checks, we confirm that the main results reported here are similar to those obtained when we use the NLLS first order conditions as test moments. Also, there are several alternative ways of incorporating the time-series implications in the estimation and testing of the model, such as, for example, the use of conditional moments. We discuss the robustness checks and the alternative approaches in the subsection 8.1 .

three different set of moments: i) a weighted average of g_i^{XS}; ii) a weighted average of g_i^{TS}; or iii) a weighted average of g_i^{XS} and g_i^{TS}. Specifically, we stack the cross-sectional and the time-series moment conditions in the matrix $g \equiv \left[g^{XS}; g^{TS}\right]$. We then estimate the model parameters θ by minimizing a weighted combination of the sample moments, denoted by g_T:

$$\min_{\theta}\; g_T' W g_T, \tag{10}$$

in which W is the prespecified weighting matrix. As suggested in Cochrane (2009), a prespecified weighting matrix can force the estimation and evaluation to pay attention to economically interesting moments, in contrast to an optimal (or other) weighting matrix. In our context, we use several different specifications of the weighting matrix, which vary on the relative weights of the cross-sectional and time-series moments in the estimation. If the model is valid, this choice of target moments should not affect the parameter estimates nor the fit of the model: the same set of model parameters should match both the cross-sectional and the time-series moments, because both are proper moment conditions and GMM is a consistent estimator.

Specifically, $W = [I, 0]$ indicates that we put identity weights on g^{XS} and zero weights on g^{TS} (cross-sectional moments only, which we label as *Only XS* in the tables). This is a special case of our methodology and identical to the GMM estimation in existing studies as LWZ. When $W = [0, I]$, we assign identity weights on g^{TS} and no weights on g^{XS} (time-series moments only, which we label as *Only TS* in the tables). More generally, when $W = [I, Z]$, we assign identity weights on g^{XS} and positive weights Z on g^{TS} (cross-sectional and time-series moments, which we label as *Both XS and TS* in the tables). In this setup, by increasing Z we can specify relatively more weight on g^{TS} over g^{XS}, thus forcing the estimation to pay relatively more attention to the time-series features of the data.

4.3 Model evaluation

We evaluate the fit of the investment-based model on economic grounds, with less emphasis on the statistical tests of the model. As discussed in Cochrane (1991), the baseline investment-based model should be rejected at any level of significance because the model predicts that stock returns and investment returns should be equal at each point in time without any error term, which is not possible to achieve in the data. Nevertheless, if the model provides a reasonably description of the real world, the investment return generated by a successful candidate model should match well the behavior of the stock returns in the real data, and this matching can be assessed with standard goodness of fit measures.

Specifically, we investigate how well the model is able to capture the average cross-sectional variation and the time-series variation of stock returns across portfolios. Accordingly, our analyses focuses on first stage GMM estimates. (In the online appendix we show that our conclusions are similar if we use second stage GMM.) In addition, the evaluation of the model is based on the properties of easy-to-interpret goodness of fit measures such as the $XS-R^2$ of the scatter plot of average portfolio-level stock returns against average portfolio-level estimated levered investment returns, the $TS-R^2$ of the realized stock return against the realized estimated levered investment returns of the portfolios, and the magnitude of the cross-sectional and time-series pricing errors of each portfolio.

Specifically, we compute the different goodness of fit measures as follows. Denote the time series average of stock return and model-implied levered investment return for each portfolio i as $\overline{r_i^S}$ and $\overline{r_i^{Iw}}$, respectively, and define $\overline{\overline{r^S}} = \frac{1}{N}\sum_{i=1}^{N}\overline{r_i^S}$. We compute the $XS-R^2$

and the $TS - R^2$ (with pooled data of the portfolios across time) as follows:[5]

$$XS - R^2 = 1 - \frac{\sum_{i=1}^{N} \left(\overline{r_i^S} - \overline{r_i^{Iw}} \right)^2}{\sum_{i=1}^{N} \left(\overline{r_i^S} - \overline{\overline{r^S}} \right)^2}, \tag{11}$$

and

$$TS - R^2 = 1 - \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} \left(r_{it+1}^S - r_{it+1}^{Iw} \right)^2}{\sum_{i=1}^{N} \sum_{t=1}^{T} \left(r_{it+1}^S - \overline{\overline{r^S}} \right)^2}. \tag{12}$$

In addition, we compute average pricing errors, both in the cross-section and in the time-series. We define the model cross-sectional pricing error of portfolio i, which we can interpret as the standard alpha (abnormal return) in the asset pricing literature, as:

$$e_i^{XS} \equiv E_T \left[r_{it+1}^S - r_{it+1}^{Iw} \right], \tag{13}$$

in which $E_T [\cdot]$ is the sample mean of the series in brackets. The mean absolute cross-sectional error across portfolios is then defined as

$$\left| e^{XS} \right| = \frac{1}{N} \sum_{i=1}^{N} \left| e_i^{XS} \right|. \tag{14}$$

Similarly, we compute the alpha of the spread high-minus-low portfolio ($H - L$, which is typically examined in the asset pricing literature) as:

$$\left| e_{H-L}^{XS} \right| = \left| e_H^{XS} - e_L^{XS} \right|. \tag{15}$$

To obtain the pricing errors in the time-series, we first compute the time-series pricing

[5]The $TS - R^2$ is based on pooled data. There are many alternative ways of measuring the time-series fit of the model. For example, one can compute the time-series R^2 for each portfolio separately, and report an average R^2, or compute the correlation of each portfolio with its stock return and report the average correlation. Although the numbers naturally vary slightly across the different measures, the interpretation of the results is consistent across these alternative measures. Hence, we use a simple time-series R^2 based on pooled data for most of the analysis.

error for each portfolio i at each point in time as:

$$e^{TS}_{it+1} \equiv r^{S}_{it+1} - r^{Iw}_{it+1}. \tag{16}$$

We then define the mean absolute time-series error averaged over time and across portfolios as:

$$\left|e^{TS}\right| = \frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left|e^{TS}_{it+1}\right|. \tag{17}$$

Note that the units of all the pricing errors discussed here are in annual returns per year.

In addition, if the model provides a good description of reality, the estimation residuals should be small, and largely random either across portfolios or over time, that is, they should not exhibit a systematic behavior. Hence, we also study the properties of the time-series error terms (residuals) implied by the estimation. In particular, we perform a principal components analysis (PCA) of the residuals of the portfolios over time, and report the fraction of the variation of the residuals that is explained by each principal component.

For completeness, we also perform standard statistical tests of the model. We assume that stock return and investment returns are observed with an error. The general distribution theory applies to GMM with prespecified weighting matrices (Cochrane 2009). Let $D = \partial g_T/\partial\theta$. We estimate S, a consistent estimate of the variance-covariance matrix of the sample errors g_T, with a standard Bartlett kernel with a lag length of two. The estimate of θ, denoted $\hat{\theta}$, is asymptotically normal with the following variance-covariance matrix:

$$var\left(\hat{\theta}\right) = \frac{1}{T}\left(D'WD\right)^{-1} D'WSWD\left(D'WD\right)^{-1}. \tag{18}$$

To construct standard errors for individual model errors, we use:

$$var\left(g_T\right) = \frac{1}{T}\left[I - D\left(D'WD\right)^{-1}D'W\right] S \left[I - D\left(D'WD\right)^{-1}D'W\right]', \tag{19}$$

which is the variance-covariance matrix for g_T. We follow Hansen (1982) to form a χ^2 test on the null hypothesis that all of the model errors are jointly zero:

$$g_T' \left[var\left(g_T\right)\right]^+ g_T \sim \chi^2\left(\#moments - \#paras\right), \tag{20}$$

in which χ^2 denotes the chi-square distribution, and the superscript + denotes pseudo-inversion.[6]

5 Estimation and tests results

Section 5.1 describes the data. Section 5.2 reports the estimation results of the baseline investment-based model from matching the cross-sectional moments only as in previous studies. Section 5.3 reports the estimation and tests results from matching the cross-sectional and the time-series moments jointly.

5.1 Data

To facilitate the comparison with prior studies, our sample construction largely follows LWZ and GXZ. Our sample consists of all common stocks traded on NYSE, Amex, and Nasdaq from 1963 to 2020. The firm-level data are from the Center for Research in Security Prices (CRSP) monthly stock files and the annual Standard and Poor's Compustat files. We exclude firms in financial and utility sectors and firms with non positive total assets, capital, or sales. We include only firms with a fiscal year end in December to align the accounting data across firms.[7]

[6]In the online appendix, we develop an external validity specification test of the model. Specifically, we estimate the model using cross sectional moments only, and evaluate how the fitted model matches the time series moments. The conclusions from this approach are similar to those reported here.

[7]We also replicate our main results using the publicly available data from LWZ, which is from 1963 to 2005. In the online appendix we show that the results using this shorter sample are broadly similar to those reported here.

5.1.1 Testing portfolios

As noted, we use ten book-to-market (BM) portfolios as the test assets. Following Fama and French (1992), at the end of June of each year t, we sort all stocks on BM, which is defined as the book equity for the fiscal year ending in calendar year $t-1$ divided by the market equity (from CRSP) at the end of December of $t-1$, into deciles based on the NYSE breakpoints. We calculate equal-weighted annual returns from July of each year t to June of year $t+1$ for the portfolios, which are re-balanced at the end of each June. Our conclusions are similar if other testing portfolios are used, as reported in the online appendix and discussed in sub-section 8.1.

5.1.2 Variable measurement and timing alignment

Compustat records both stock and flow variables at the end of year t. In the model, however, stock variables dated t are measured at the beginning of year t, and flow variables dated t are over the course of year t. As such, for the year $t = 2015$, for example, we take time-t stock variables from the 2014 balance sheet, and time-t flow variables from the 2015 income or cash flow statement.

We measure output, Y_{it}, as sales (item SALE). Capital stock, K_{it}, is net property, plant, and equipment (item PPENT). Depreciation rate, δ_{it}, is the amount of depreciation and amortization (item DP) subtracted by the amortization of intangibles (item AM, zero if missing) and then divided by the capital stock, K_{it}. We measure investment, I_{it}, directly as $K_{it+1} - (1 - \delta_{it}) K_{it}$. Total debt, B_{it+1}, is long-term debt (item DLTT, zero if missing) plus short-term debt (item DLC, zero if missing). The market leverage, w_{it}, is the ratio of total debt to the sum of total debt and market equity (from CRSP). The tax rate, τ_t, is the statutory corporate income tax rate from the Commerce House's annual publications. We measure the pretax cost of debt, r^B_{it+1}, as the ratio of total interest and related expenses (item XINT) scaled by total debt, B_{it+1}.

To mitigate the impact of outliers, we winsorize 2% of the extreme observations every year. We winsorize the unbounded variables such as investment rate, I_{it}/K_{it}, at the $1\%-99\%$ level. For variables that are bounded below at zero, such as sales-to-capital, Y_{it+1}/K_{it+1}, and the depreciation rate, δ_{it}, we winsorize at the $0\% - 98\%$ level. Finally, we do not winsorize the market leverage, w_{it}, because it is bounded between zero and one.

To match levered investment returns with stock returns, we need to align their timing. As noted, we use the Fama-French portfolio approach in forming testing portfolios at the end of June of each year t. Portfolio stock returns are calculated from July of year t to June of year $t+1$. To construct the matching annual investment returns, we use capital at the end of fiscal year $t-1$ (K_{it}), the tax rate, investment, and capital at the end of year t (τ_t, I_{it}, and K_{it+1}), as well as other variables at the end of year $t+1$ (τ_{t+1}, Y_{it+1}, I_{it+1}, and δ_{it+1}). Because stock variables are measured at the beginning of the year and flow variables are realized over the course of the year, the investment returns go approximately from the middle of year t to the middle of year $t+1$. As such, the investment return timing matches the stock return timing as close as possible.

5.2 Matching cross-sectional moments only

We first estimate the investment-based model using only the cross-sectional moments given by Equation (8), as in LWZ. Column (1) in Table 1 reports the parameter estimates and goodness of fit measures. The parameter estimates, capital share, $\alpha = 0.25$, and the adjustment cost parameter, $c = 10.94$, are economically plausible. In addition, consistent with LWZ, the model does a very good job in explaining the cross-sectional variation in average returns of the 10 BM portfolios (value premium). The mean absolute cross sectional error, $\left|e^{XS}\right|$, given by equation (14) is 1.35% per annum. The mean absolute high-minus-low cross sectional error, $\left|e^{XS}_{H-L}\right|$, given by equation (15) is 2.32% per annum. The $XS-R^2$ is 73%, and hence the model is able to capture the value spread in the cross-section quite well. Finally, the χ^2 test examines the joint errors of 10 cross-sectional moments, and the model is not rejected statistically (p-value of 0.60).

[Table 1 here]

In contrast with the good cross-sectional fit, the model is unable to explain the time-series variation of the stock returns of the 10 BM portfolios. Panel A in Table 2 shows that, although the mean and standard deviation of stock and investment returns match reasonably well, the stock and investment returns have a very low correlation for every portfolio, with an average time-series correlation of 8%. This is in sharp contrast with the model prediction that stock returns and investment returns should be perfectly correlated. As a result, the stock returns and the residuals are strongly positively correlated in the time series (correlation is on average 66% across portfolios) which means that most of the time-series variation of stock returns is captured by the residuals, not by the predicted investment returns.

[Table 2 here]

Column (1) in Table 1 provides additional evidence for the poor time-series fit of the model when the model is estimated using cross-sectional moments only. The mean absolute time series error, $\left|e^{TS}\right|$, given by Equation (17) is about 24% per annum which is greater than the magnitude of the average returns of the portfolios. The $TS-R^2$ is negative, -90%.

The principal components analysis of the residuals also show that the residuals of the model exhibit a strong systematic component. The results in the last column of Table 2 reveal that about 72% of the time series variation of the residuals can be explained by the first principal component. This large systematic component in the residuals suggests that the poor time-series fit of the model is unlikely due to random noise in the data.

Figure 1 plot the stock returns of 10 BM portfolios at each point in time against the levered investment returns (left panel) and the error terms (right panel). This figure illustrates in a clear manner the model's overall poor time-series fit. If the model performs perfectly, all the observations in the left panel should lie on the 45-degree line. However, the left panel shows that the scattered points of stock and investment returns are largely random with a pooled correlation of 0.11. In sharp contrast with this pattern, the scattered points of stock returns and the error terms on the right panel are well aligned along the 45-degree line, with a pooled correlation of 0.64. Thus, almost all of the time series variation in stock returns of each portfolio is explained by the model residual, not by the model-implied fitted investment return.

[Figure 1 here]

5.3 Matching cross-sectional and time-series moments jointly

To improve the time-series fit of the model, we add the time-series moments to the GMM estimation and re-evaluate the model fit.

Table 1 reports the results. As we increase the relative weight of the time series moments in the estimation from column (1) (zero weight on TS moments, all weight on XS moments) to column (8) (all weight on TS moments, zero weight on XS moments), the fit on the cross-sectional moments deteriorates significantly: the mean absolute cross-sectional error (alpha) monotonically increases from 1.35% per annum when only cross-sectional moments are used in the estimation (column 1), to 3.93% when only time series moments are used in the estimation (column 8); the mean absolute high-minus-low cross-sectional error increases from 2.32% per annum (column 1) to 16% (column 8), and the $XS-R^2$ decreases from 73% (column 1) to -137% (column 8). Going in the opposite direction, as expected, there is an improvement in the fit of the model in the time-series: the mean absolute time series error (almost) monotonically decreases from 23.90% per annum (column 1) to 17.98% (column 8) and the $TS-R^2$ increases from -90% (column 1) to -3% (column 8). Thus, these results uncover a novel trade-off between cross-sectional fit and time-series fit: the baseline investment-based model cannot fit both sets of moments simultaneously. The top ($XS-R^2$ and $TS-R^2$) and bottom (mae, mean absolute error) panels in Figure 2 illustrates this trade-off.

[Figure 2 here]

More importantly, even when only the time-series moments are used in the estimation (column 8), the model fit on the time series is still poor, with a $TS - R^2 = -3\%$. Thus, the standard investment-based model with one (physical) capital input and quadratic adjustment costs is not able to capture the time series behavior of stock returns in the data, even when the estimation is designed to maximize its time series fit.

6 Potential empirical reasons for the poor time series fit

We investigate two potential empirical reasons for the poor fit of the baseline investment-based model in the time-series, despite its good fit in the cross-section. We focus on empirical reasons here because to take the model to the data, it is necessary to map the model variables to the data, and the required empirical assumptions might not be correct even if the underlying theoretical model is valid. We then discuss the broader model implications of our findings for future research in investment-based asset pricing in the Conclusion Section 9.

6.1 Aggregation bias

We start by investigating the role of portfolio-level aggregation for the results. As noted in BGSV and GXZ, the portfolio aggregation procedure in LWZ suffers from an aggregation bias because the portfolio-level investment return and stock returns, which the estimation tries to match, are not computed in a consistent manner because the investment-returns are not properly aggregated. Specifically, as discussed in section 4.1, the portfolio-level investment return in LWZ is obtained by first computing each portfolio-level characteristic separately (e.g., the portfolio-level investment rate, marginal product of capital), and then plug each of these characteristics directly in the investment return formula to obtain the portfolio-level investment return at any given point in time. In turn, the estimation matches this portfolio-level investment return to the equal-weighted average of firm-level stock returns in the portfolio. However, given the non linearity of the investment return formula, these two variables should not be equal, and hence the estimation procedure does not recover the structural parameters of the model.

To properly estimate the model and recover the structural parameters, as in GXZ, the portfolio-level investment return should be computed in the same way as the portfolio-level stock return. Specifically, given the nonlinear nature of investment returns, we should first compute the investment return for each firm (indexed by the subscript j), and then compute the portfolio-level (indexed by the subscript i) investment return as the equal-weighted average of the firm-level investment returns of the firms in the portfolio as $r^I_{it} = \frac{1}{N} \sum_{j=1}^{N} r^I_{ijt}$. In turn, this should be matched with the corresponding portfolio i equal-weighted stock return computed as $r^S_{it} = \frac{1}{N} \sum_{j=1}^{N} r^S_{ijt}$. (We focus on equal-weighted returns here to be consistent with the approach in LWZ and GXZ, but our results are similar if we use value-weighted returns.)

To investigate if the aggregation bias induced by the LWZ portfolio-level aggregation can help explain the poor time-series fit of the baseline model documented in the previous section, we perform two sets of analysis. First, we use simulated data in which the model holds by design at the firm-level, and re-estimate the model to evaluate the theoretical impact of the aggregation-bias on the model fit. Second, we do a proper portfolio-level aggregation in the real data, and re-estimate the model to re-evaluate its time-series fit.

6.1.1 The impact of aggregation bias in simulated data

We simulate data from a model economy in which the assumptions of the baseline investment-based model hold, and hence firm-level stock and investment returns are equal by construction. To generate the data in a simple manner, we use the real data on firm-level characteristics (such as, sales-to-capital, investment rate) as inputs to construct the model-implied investment and stock returns. We set the true model parameter values for capital share as $\alpha = 0.04$, and the adjustment cost parameter as $c = 0.8$, and compute the model-implied firm-level investment return using Equation (5) and the actual data on firm characteristics.

These investment returns are also the firm-level stock return according to Equation (7). As in the empirical analysis, we then create 10 BM portfolios which we use to replicate the GMM estimation of the model. The parameter values used generate a value premium in the artificial data that is similar to the one in the real data (11% per annum, not tabulated).[8]

To examine the impact of aggregation bias on the evaluation of the model, we replicate the LWZ portfolio aggregation method using the simulated data. Panel A in Table 3 confirms the aggregation bias in the parameter estimates, consistent with the results of BGSV: the estimation fails to recover the true parameter values. Another evidence of the aggregation bias is that the parameter estimates vary with the set of moments used (columns 1 to 5). The aggregation bias is also able to break the perfect correlation between stock return and investment return: Panel A in Table 3 shows that the maximum $TS - R^2$ the model can achieve is 1%. This poor fit using the simulated data gives hope that the aggregation bias alone might be an important contributor for the poor empirical time-series fit of model when estimated using the LWZ portfolio aggregation procedure.

The results using the simulated data also show that the time-series moments are more powerful at detecting the aggregation error, than the cross-sectional moments. When estimating and testing using only the cross-sectional moments as in column (1) in Table 3, the mean absolute cross sectional error and the mean absolute high-minus-low cross sectional error are both low, the $XS - R^2$ is high, about 51%, and the model is not rejected by the χ^2 test with a p-value on testing the joint errors of 10 cross-sectional moments of 0.52. That is, the incorrectly estimated model can easily match the cross-sectional moments. In contrast, when estimating and testing the model using the time-series moments as in column (5) in Table 3, both the cross-sectional fit and time-series fit of the model are poor, and the p-value of the χ^2 test on the joint errors of 10 time-series moments is only 0.17, indicating a poor model fit stemming from, in this case, aggregation bias.

[8]In untabulated results, we confirm that GMM with a correctly specified model (homogeneous of degree one) and with proper firm-level aggregation recovers the true model parameters. In addition, the model fit is perfect, and using cross sectional moments or time series moments in the estimation makes no difference in the estimation and evaluation of the model.

[Table 3 here]

6.1.2 The impact of aggregation bias in real data

Next, we investigate the model fit in the data using a proper portfolio-aggregation (which we label as firm-level aggregation because portfolio-level returns are properly constructed from firm-level returns). Panel A in Table 4 reports the estimation results from this analysis. Although the time-series fit of the model improves with the correct aggregation, it remains very poor. Even when the estimation only targets the time-series moments, the $TS - R^2$ is -13%.

[Table 4 here]

Taken together, the results from these analyses suggest that the portfolio-level aggregation-bias issue does not appear to be the main cause for the inability of the baseline model to capture the time-series behavior of stock returns in the real data.

6.2 Misalignment between asset price data and real quantities

As a second possible empirical reason for the poor time series fit of the baseline investment-based model, we address the possibility that real quantities (e.g. investment) and asset prices are misaligned in the data. For example, stock prices (and hence stock returns) might respond instantaneously to aggregate shocks, whereas firm investment might take more time to adjust, in which case investment returns lag stock returns (see, for example, Lamont 2000, for a more formal analysis of this issue). But if the misalignment in the data is relatively short lived, the misalignment should be less relevant for longer-horizon returns (i.e., for time-smoothed data). Thus, we conjecture that multi-year compounded investment and stock returns should be significantly less affected by data misalignment issues than one year horizon returns.

6.2.1 The impact of data misalignment in simulated data

We use simulated data to test the conjecture that compounded (here, using a five-year horizon) investment and stock returns should be less affected by the data misalignment issues than annual return data.

As in the previous sub-section, we first generate simulated data from a specification of the model in which the equality between investment and stock returns holds (using the same parameter values as in the previous analysis), and introduce misalignment in the estimation. To evaluate the impact of data misalignment, we then estimate the model using annual returns and (annualized) five-year compounded returns, and compare the results. To identify the impact of misalignment on the results, we estimate the model at the portfolio-level using the proper aggregation method. In this case, the only reason for the imperfect time series fit of the model is due to data misalignment, and not due to the aggregation bias.

We introduce misalignment in the simulated data as follows. If investment lags stock returns in the real data, we can capture this feature in the simulated data in a simple manner by estimating the model matching stock returns at $t+1$ (r^S_{it+1}) with lagged investment returns at t (r^{Iw}_{it}), and use this timing to construct the cross-sectional and the time-series moments. Panel B in Table 3 shows that the data misalignment alone breaks significantly the expected perfect time-series fit of the model. The mean absolute time series error becomes sizable, and the $TS - R^2$ is very low, 11% (in column 10, even when the model is estimated on time-series moments only), which is more consistent with the data. This suggest that data misalignment is potentially a good candidate explanation for the poor empirical time-series fit of the model using annual return data.

In the presence of data misalignment, using annualized compounded returns in the estimation should mitigate the impact of the misalignment and improve the time-series fit of the model. To verify this conjecture, we perform the estimation of the model using annualized 5-year compounded stock and investment returns in the time-series moments. In particular, annualized 5-year compounded stock and investment returns are computed as

$\left(r^S_{it+1}r^S_{it+2}...r^S_{it+5}\right)^{1/5}$ and $\left(r^{Iw}_{it+1}r^{Iw}_{it+2}...r^{Iw}_{it+5}\right)^{1/5}$, respectively. Panel C in Table 3 report the results. The mean absolute time series error drops significantly, and the $TS-R^2$ increases significantly to around 71% (in column 15). Taken together, this result show that, consistent with our conjecture, the issues introduced by data misalignment should be significantly mitigated if we estimate the model using compounded returns, that is, if we focus on the model-implied relationship between stock and investment returns at longer horizons (i.e. time-smoothed data).

6.2.2 The impact of data misalignment in real data

In light of the previous analysis using simulated data, we investigate the time series fit of the baseline investment-based model when we use annualized 5-year compounded returns in the real data. Panel B in Table 4 reports the results. When we use long horizon compounded returns in the estimation, the time-series fit of the model remains far from satisfactory. The $TS-R^2$ remains very low at -10% (see column 10, even when only the time-series moments are used in the estimation). Taken together, the results from this analysis suggest the misalignment between price and investment data (at least the type of misalignment specification examined here) also does not appear to be the main cause for the inability of the baseline model to capture the time series behavior of stock returns in the real data.

7 Specification-free tests

The analysis so far has been based on the specification of the investment-based model with homogeneous of degree one operating profit and adjustment cost functions (the so-called Hayashi (1982) conditions), in which case stock returns and levered investment returns should be equal at each point in time as in Equation (7). This result underlies the moment conditions in Section 4.2 which we use to estimate and test the baseline investment-based model. Because these Hayashi conditions are strong assumptions and should be more properly interpreted as an approximation of the reality, it is natural to question its validity, especially in light of the poor time-series fit of the baseline investment-based model reported in the previous sections.[9]

7.1 A specification-free estimation approach

Here, we propose a more general approach to evaluate the performance of investment-based asset pricing models that can be used even when the Hayashi conditions do not hold. We label this more general method as a specification-free approach to evaluate investment-based models because it does not rely so heavily on functional form restrictions. As a result, this approach is useful to test a broader set of specifications of the investment-based model.

The Hayashi conditions might not hold due to several reasons such as decreasing returns to scale, fixed adjustment costs and operating costs, and time-to-build. In these cases, in general, the stock-investment return equality does not hold and hence it cannot be used to construct the moment conditions used to estimate and test the model in the previous sections.

Most investment-based asset pricing models, however, imply a close relationship between a firm's equilibrium stock return and its characteristics X_{it+1}. We can investigate the strength of these relationships using simple linear regressions of the form:

$$r_{it+1}^{S} = \beta X_{it+1}, \tag{21}$$

in which X_i is a vector of firm (or portfolio-level) characteristics.[10] Analogous to the estimation of the model in the previous sections, this stock return-firm characteristics regression

[9]Building on Abel and Eberly (1994), Zhang (2005a) shows that the stock-investment return equality still holds as long as the operating profits and the adjustment-cost functions are homogeneous of the same degree, not necessarily one. In addition, Zhang (2005a) provides a broad discussion of the necessary conditions for the equality to hold.

[10]We approximate the model-implied link between stock returns and firm characteristics using a linear specification without much loss of generality because higher order terms can always be included as an additional firm-characteristic.

can be estimated both in the time-series (using panel data), and in the cross-section (using the time series average of stock returns and firm characteristics). In the baseline neoclassical model in which the Hayashi conditions hold, the key firm characteristics X_{it+1} in the investment return formula in Equation (5) include current profitability (sales-to-capital ratio), and current and lagged investment rate. In other investment-based models, the key relevant firm characteristics might be different, but as long as the firm characteristics can be measured both in the data and in the model, we can evaluate if a given model generates a relationship between realized stock returns and firm characteristics that is consistent with the data (in terms of slope coefficients and goodness of fit).

Similar arguments have been made in the investment-q literature. Eberly, Rebelo, and Vincent (2008) show that when the Hayashi conditions do not hold, Tobin's q is not a sufficient statistic for investment, but the optimal investment from a model featuring decreasing returns to scale and a fixed cost can still be well approximated by a log-linear function of q. Gala, Gomes, and Liu (2020) show that even under very general assumptions about the nature of markets, production and investment technologies, optimal investments are functions of and well captured by the relevant state variables such as firm size and productivity.

The stock return-firm characteristics regressions can be used to evaluate investment-based models in at least two different ways. First, it can be used as out-of-sample moments to evaluate the time-series implications for stock returns of a given calibration of the model. We follow this approach here. More generally, the stock return-firm characteristics regressions can be used to simultaneously estimate the structural parameters and test the model. The estimation can be done using alternative estimation methods such as the Simulated Method of Moments (SMM), adding the stock return-firm characteristics regression results estimated in the actual data as a set of target moments for the model estimation. The estimation using SMM is outside the scope of our paper, however. Our goal here is to show how future research can incorporate the time-series and cross-sectional implications of investment-based models to evaluate and test these models along a broader set of moments.

7.2 The specification-free approach in practice

To show how the specification-free approach can be used to evaluate investment-based models, we estimate the time-series and cross-sectional relationships between stock returns and firm characteristics defined in Equation (21) both in the data, and also in simulated data generated from two different specifications of the investment-based model.

The first specification is the baseline investment-based model in which, as in the previous sections, the Hayashi conditions hold, and thus stock returns and investment returns are equal. We label it as homogeneous of degree one model, or HD1 model. We generate data from this model in the same way as in the previous Section 6. This specification of the model is useful here because we already know that in this model stock returns and firm characteristics are closely linked by Equation (7). We can then investigate if this nonlinear relationship is preserved in the linear approximation of this relationship captured by the stock return-firm characteristics regressions.

The second model is an off-the-shelf investment-model based on Lin and Zhang (2013) in which the Hayashi conditions do not hold. We focus on this model given its good fit on matching key moments of real quantities and asset prices in simulated data, hence it serves as a natural laboratory to show how to implement our approach in practice. Specifically, a firm i's operating profit function features decreasing returns to scale and a positive fixed cost:

$$\Pi_{it} = X_t Z_{it} K_{it}^{\alpha} - f, \tag{22}$$

in which $0 < \alpha < 1$ is the curvature parameter, and $f > 0$ is a positive fixed cost. X_t and Z_{it} are aggregate and idiosyncratic profitability shocks respectively. Capital investment entails the following adjustment costs:

$$\Phi\left(I_{it},K_{it}\right)=\begin{cases}a^{+}K_{it}+\frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2}K_{it} & I_{it}>0\\ 0 & I_{it}=0\\ a^{-}K_{it}+\frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2}K_{it} & I_{it}<0\end{cases}, \tag{23}$$

where $a^{-}>a^{+}>0$ and $c^{-}>c^{+}>0$ capture non convex and asymmetric adjustment costs.

We label this model as non-homogeneous of degree one model, or non-HD1 model. To generate simulated data from this model, we calibrate the model as in Lin and Zhang (2013) to match average quantities and asset prices moments both at the aggregate-level and in the cross-section, including the value premium. In the online appendix we provide a detailed description of the calibration of this model, including the specification of the stochastic processes.

Table 5 reports the stock return-firm characteristics regression results in the real data, and using simulated data from the two investment-based models. Consistent with the previous analyses, we run the regressions at the portfolio-level (using 10 book-to-market portfolios) to reduce noise in the firm-level data, and in which each portfolio-level characteristic is computed as the equal-weighted average of the characteristic across firms in the portfolio, to avoid the aggregation bias. We consider the following characteristics: profitability ($YK_{it+1}\equiv\frac{Y_{it+1}}{K_{it+1}}$), investment rate ($IK_{it+1}\equiv\frac{I_{it+1}}{K_{it+1}}$) and its squared term, lagged investment rate ($IK_{it}\equiv\frac{I_{it}}{K_{it}}$), and size ($K_{it}\equiv logK_{it}$) at each point in time, and also in the cross-section, in which case we average all variables (stock returns and characteristics) over time for each portfolio. Since the non-HD1 model does not feature leverage, we use unlevered stock returns in the real data and in the HD1 model in all the analyses here to allow for a meaningful comparison. We unlever portfolio stock returns in the data by multiplying levered returns with one minus portfolio-level average leverage. We normalize both the dependent and the independent variables using pooled data.

[Table 5 here]

Columns (1) and (2) in Table 5 report the results using the simulated data from the baseline investment-based model in which the Hayashi conditions hold (HD1 model). As discussed before, this model predicts that stock returns and investment returns are equal at any given point in time, and hence the model implies a perfect non-linear relationship between stock returns and firm-characteristics. This tight relationship is preserved in a linear approximation of the relationship, given the high $XS - R^2$ (91%), and the high $TS - R^2$ (90%). So, the simple linear functional form relationship preserves the model implied strong (non-linear) link between stock returns and firm-characteristics.

Columns (3) and (4) in Table 5 report the results using simulated data from the Lin and Zhang (2013) non-homogeneous of degree one model. Interestingly, even though this model does not predict the equality between stock returns and investment returns, this model also implies that stock returns and firm-characteristics are highly correlated, both in the cross-sectional ($XS - R^2$ of 96%), and in the time-series ($TS - R^2$ of 87%). This result shows that the strong link between stock returns and firm characteristics (at least using data aggregated at the portfolio-level) appears to be a more general feature of investment-based models, that is, it is not restricted to specifications of the model in which the Hayashi condition holds.

The next step in the evaluation of the investment-based model is to compare the stock return-firm-characteristic regression results in the model, with those in the real data. Columns (9) and (10) in Table 5 report the regression results in the real data. Consistent with both the baseline investment-based model and the Lin and Zhang (2013) non-homogeneous of degree one model, the cross-sectional fit in the real data is quite high, about 71%. But the time-series fit of the stock return-firm characteristics regression in the real data is very poor, with a $TS - R^2$ of about 5%, and the coefficients on the firm-characteristics are mostly insignificant.

We also re-investigate the impact of data misalignment between price and investment data on the time-series relationship between stock returns and firm characteristics at short-and long-horizons. Analogous to the analysis in Section 6, Columns (5) and (6) show the annual regression results when we introduce misalignment in simulated data from the baseline investment-based model in which the Hayashi conditions hold. The $XS-R^2$ remains high at about 93%, and the $TS-R^2$ drops significantly, from 90% without misalignment as reported in column (2) to 13% as reported in column (6), thus getting the model closer to the real data, which has a $TS-R^2$ of 5% as reported in column (10). As before, we then replicate the regressions using long-horizon (time smoothed) variables to mitigate the impact of data misalignment. Columns (7) and (8) show that when we use annualized 5-year compounded returns and 5-year average characteristics in the simulated data with misalignment, the $XS-R^2$ remains high, and the $TS-R^2$ improves significantly to 66%, as reported in column (8). However, when we replicate the same long-horizon regressions in the real data, column (12) shows that the $TS-R^2$ using long horizon data is still very low, about 10%.

Taken together, the specification-free approach that we propose here allows us to investigate if a candidate investment-based model match the relationship between realized stock returns and firm-characteristics observed in the data. Our findings confirm that the baseline investment-based model with one (physical) capital input and homogeneous of degree one technology, and a more general version of the one capital input model with decreasing returns to scale, non-convex and asymmetric adjustment costs, and operating fixed costs, fail to capture the time-series properties of stock returns in the data.

8 Alternative approaches and robustness checks

We present alternative approaches to estimate and test investment-based models, such as the use of alternative time-series moments, higher order moments, and also the estimation of investment-based models in levels (valuation ratios), and discuss why we adopted our approach. In addition, we discuss the results from several robustness checks including a sub-sample analysis, and the use of alternative test assets. To save space, we report most of the results from these analyses in the online appendix, and we briefly summarize the main conclusions here.

8.1 Alternative time-series moments

There are several alternative ways of incorporating time-series implications of investment-based models in the estimation. We discuss two alternative approaches here.

A first alternative approach is to use the first order conditions of the NLLS optimization problem as the moment conditions in GMM, instead of the NLLS objective function. Specifically, NLLS minimizes the sum of the squared differences between stock returns and levered investment returns across N portfolios:

$$\theta = argmin \frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(r^{S}_{it+1} - r^{Iw}_{it+1}\right)^{2}, \tag{24}$$

in which $\theta \equiv (\alpha, c)$. The first order conditions of this minimization problem are:

$$g^{TS-FOC} = \frac{1}{N}\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[\left(r^{S}_{it+1} - r^{Iw}_{it+1}\right)\frac{\partial r^{Iw}_{it+1}}{\partial \theta}\right] = 0. \tag{25}$$

These first order conditions, by definition, should be zero at the minimum, and hence can directly be used as moment conditions in GMM. Indeed, that is the standard approach used to map NLLS into GMM (see, for example, Cochrane (2009), Chapter 11).

Following this procedure, when the estimation is just identified (using 2 first order conditions as moments to estimate 2 parameters), the weighting matrix does not matter for the results because all the moment conditions can be zero. Hence, the GMM estimates using the NLLS first order conditions yield exactly identical estimates to the NLLS estimation (applied to all portfolios in a pooled sample), and the results are easy to interpret given that the parameters are simply the NLLS estimates. But when the model is over-identified, as is the case in our applications here because we also use cross-sectional moments in the estimation and apply NLLS separately to each portfolio, it is impossible to force all the NLLS first order conditions to hold at the same time, and hence the choice of the weighting matrix affects the results. As a result, the model fit is difficult to interpret because the deviations from the first order condition (residuals) do not have a natural interpretation. For completeness, however, we also re-estimate the model using the alternative NLLS first order moment conditions as the time-series moments, and obtain results that are very similar to the ones obtained here.

A second potential alternative way of incorporating time-series restrictions in the estimation of investment-based models is to augment the set of moment conditions by adding instruments (variables that should be orthogonal to the error terms) in a manner that is analogous to the estimation of conditional asset pricing moments. This approach is well suited for model-implied moment conditions in which the errors at each point in time are expectation errors such as in, for example, the moment conditions implied by investment Euler equations. This is because when the error term is an expectation error, this error term should be orthogonal to any variable (called instrument) in the agent's information set. In turn, this orthogonality condition gives rise to a set of unconditional moment conditions that incorporate the time-series implications of the model, and can be estimated using standard GMM. In our approach, however, the error term inside the moment conditions are not expectation errors because the model predicts that stock and levered investment returns should be equal state-by-state without any error. The error terms in our approach arise due to, for example, measurement or misspecification errors, and hence the theory does do not imply orthogonality conditions in the same way that expectation errors do. Therefore, we do not follow this approach here.

8.2 Higher-order moments

The investment-based model also has implications for higher order moments of stock and levered investment returns which can also be tested in the data. For example, LWZ investigates if the baseline investment-based model can match the cross-section of stock return variances, which should be equal to the cross-section of levered investment return variances.

The variance moment conditions can be defined as:

$$g_i^{Var} = E\left[\left(r_{it+1}^S - E\left[r_{it+1}^S\right]\right)^2 - \left(r_{it+1}^{Iw} - E\left[r_{it+1}^{Iw}\right]\right)^2\right] = 0. \quad (26)$$

The use of higher order moments might suggest that this moment can help improve the fit of the model in the time series as well. But a simple inspection of the previous moment condition reveals that minimizing the variance errors is not equivalent to minimizing the sum of squared residuals because the variance moment ignores the time-series correlation between stock returns and investment returns. As an extreme example, take two time series with the same variance but that move in opposite direction to each other. The model fit on the variance moments will be perfect, but the time series fit will be poor because the correlation between the two variables is -1.

To show that the variance moments do not significantly improve the model fit in the time-series in the data, we re-estimate the model combining the cross-sectional moments and the variance moments in the estimation of the baseline investment-based model using GMM. Panel C in Table 4 reports the results. As we increase the weights of the variance moments in the estimation from column (11) (zero weights on variance moments, label *Only XS*) to column (15) (all weights on variance moments, label *Only Variance*), the improvements on the mean absolute time series error and the $TS - R^2$ are quite limited compared with the improvements observed when we use our time-series moments, as reported in Table 1.

8.3 Estimation in levels

Following BGSV, we can also estimate and test the investment-based model by examining its ability to match the behavior of valuation ratios, which can be interpreted as testing the model in levels, as opposed to examining the ability of the model to explain stock returns, which can be interpreted as testing the model in first-differences. Arguably, levels are more precisely measured than first-differences, and hence focusing on these moments can potentially provide a more powerful set of tests of investment-based models. Thus, we investigate the model's fit both in the cross-section and in the time-series of valuation ratios at the portfolio level.

We proceed as follows. The first-order condition from maximizing the objective function in Equation (4) with respect to I_{it} implies the standard result that the market value of the firm is given by:

$$P_{it} + B_{it+1} = \left[1 + (1 - \tau_t)\, c\left(\frac{I_{it}}{K_{it}}\right)\right] K_{it+1}, \tag{27}$$

where $P_{it} \equiv V_{it} - D_{it}$ is the ex-dividend equity value. Based on this result, we define the observed valuation ratio (aka Tobin's q) as $q_{it} \equiv (P_{it} + B_{it+1})/A_{it}$, in which A_{it} is total assets, which we compare with the model-implied valuation ratio, denoted as $\hat{q}_{it} = \left(1 + (1 - \tau_t)\, c\left(\frac{I_{it}}{K_{it}}\right)\right) \frac{K_{it+1}}{A_{it}}$. We use q_{it} and $\hat{q}_{it}$ instead of stock returns and levered investment returns to construct the moment conditions as in the baseline estimation approach, and investigate if the model can match the cross-section and the time-series of the valuation ratios across 10 book-to-market portfolios. We find that the time-series fit of the baseline investment-based model on valuation ratios is poor, and the model is rejected by the χ^2 test with a p-value of 0.04. In the main analyses, we focus our tests on the implications of the model for stock returns because that has been the focus of the investment-based asset pricing literature, allowing us to also connect our findings to the large empirical asset pricing literature.

8.4 Sub-period analysis

Andrei, Mann, and Moyen (2019) show that the relation between aggregate investment and Tobin's q has become remarkably tight in recent years by running investment regressions. Thus, we investigate whether we observe similar improvement in the stock and investment return relation. We split the sample into 1963-1994 and 1995-2020 sub-periods, and find that the time-series fit of the baseline model also improves in the more recent sample period, from $TS - R^2 = -3.6\%$ to $TS - R^2 = 10.71\%$, when the model is estimated on time-series moments only. This $TS - R^2$ in the more recent period is still too low, so the conclusion that the investment-based model is unable to capture the time-series properties of stock returns is consistent across the two sub-periods.

8.5 Other test assets

Finally, we also estimate and test the investment-based model using alternative testing portfolios. Following LWZ and GXZ, we use ten portfolios sorted on standardized unexpected earnings (SUE), ten portfolios sorted on corporate investment (CI), and ten portfolios sorted on asset growth (AG). The results are largely consistent with our main findings. The time-series fit of the baseline investment-based model is poor across these three set of portfolios, with negative $TS - R^2$'s.

9 Conclusion

Investment-based models imply that realized firm-level stock returns and firm characteristics are strongly linked at any point in time, not just on average, as examined in previous work. We incorporate this time-series prediction in the estimation and testing of investment-based model using GMM. When applied to a standard specification of the investment-based model with one-capital input and quadratic adjustment costs, our estimation results confirm that, as in LWZ, this simple model is very successful at capturing the cross-sectional variation in average stock returns across several portfolio sorts. However, we show that this model fails to capture the time-series properties of stock returns in the data, generating a $TS - R^2$ that is negative, even when the estimation tries to maximize the time-series fit of the model. Using stock return-characteristics regressions, our estimation approach can be extended to general specifications of investment-based models, and we show that the poor time-series fit is also present in a specification of the investment-based model with decreasing returns to scale, non-convex and asymmetric adjustment costs, and operating fixed costs. We show that aggregation bias in portfolio-data and possible misalignment between price and investment data issues are not main causes for the poor time-series fit of these models in the data.

Our findings have implications for future research. Because we only test two specifications of the investment-based model, our findings do not mean that the investment-based paradigm cannot match the time-series stock return data well. Our findings suggest, however, that a different specification of the model, or a change in the econometric procedures, is probably needed to capture the time-series dimension of stock returns in the real data, which we argue is an important dimension to match. Going forward, additional capital inputs such as intangible capital as in Peters and Taylor (2017) and BGSV, quasi-fixed labor inputs as in BGSV, or short-term and long-term assets as in GXZ, might be incorporated in the model to improve its fit in the time-series. In addition, accounting for firm- or industry-level heterogeneity in the technologies, which are assumed to be similar across firms in the baseline analyses, as well as more general functional forms, should be investigated. Finally, it could be useful to examine the impact of financial frictions as in Gomes, Yaron, and Zhang (2006), or time-to-build features as in Kuehn (2009), on the correlation between realized stock returns and firm characteristics. Taken together, by incorporating the time-series implications into the structural estimation of investment-based models, our methodology can be useful to better detect model misspecifications and hence help guide the improvements in the specification of this class of models in future research.

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Table 1. GMM estimation and tests of the investment-based model

This table reports the one-step GMM results from estimating jointly the cross-sectional moments and the time-series moments given by Equation (8) and (9) respectively, using book-to-market deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross-sectional moments and the second component refers to the weights on the time-series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross-sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS - R^2$ is the cross-sectional R^2. $TS - R^2$ is the time-series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the portfolio level as in LWZ. The sample is from 1963 to 2020.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ 0.3]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 2]$	$[I\ 10]$	$[0\ I]$
				Parameter estimates				
α	0.25	0.19	0.17	0.15	0.14	0.14	0.13	0.13
$[t]$	3.65	6.38	7.83	7.96	7.53	6.95	6.56	4.68
c	10.94	5.63	3.22	2.33	1.47	0.96	0.52	0.41
$[t]$	1.92	2.52	2.07	1.62	1.04	0.66	0.37	0.37
				Goodness of fit				
$\left\|e^{XS}\right\|$	1.35	1.76	2.32	2.71	3.20	3.53	3.85	3.93
$\left\|e^{XS}_{H-L}\right\|$	2.32	7.05	10.25	11.73	13.41	14.56	15.68	16.00
$\left\|e^{TS}\right\|$	23.90	21.11	19.37	18.72	18.17	18.00	17.96	17.98
$XS - R^2$	73.17	50.08	6.87	-21.07	-59.16	-89.12	-122.92	-137.02
$TS - R^2$	-90.28	-44.91	-21.69	-13.69	-7.13	-4.37	-3.10	-2.98
χ^2	6.40	16.44	17.29	17.00	15.89	15.48	15.49	13.00
d.f.	8.00	18.00	18.00	18.00	18.00	18.00	18.00	8.00
p	0.60	0.56	0.50	0.52	0.60	0.63	0.63	0.11

Table 2. Description of stock returns, investment returns, and errors

This table reports the mean, standard deviation (Std), time-series correlation for stock returns (r_{it}^S), investment returns (r_{it}^{Iw}), and errors (ϵ_{it}), in percentages. Errors are computed as $\epsilon_{it} = r_{it}^S - r_{it}^{Iw}$. Investment returns are based on estimation results from one-step GMM on the cross-sectional moments given by Equation (8), using book-to-market deciles as the testing portfolios. Aggregation is at the portfolio level as in LWZ. Weighting matrix is an identity matrix. The last column shows the principal components analysis (PCA) of the residuals, and reports the percent variability explained by each principal component. The sample is from 1963 to 2020.

	Mean			Std			TS Correlation			PCA
Portfolio	r_{it}^S	r_{it}^{Iw}	ϵ_{it}	r_{it}^S	r_{it}^{Iw}	ϵ_{it}	(r_{it}^S, r_{it}^{Iw})	$(r_{it}^S, \epsilon_{it})$	$(r_{it}^{Iw}, \epsilon_{it})$	ϵ_{it}
1	11.95	14.62	-2.67	26.50	18.86	32.02	3.28	80.83	-56.19	72.36
2	12.50	15.21	-2.71	22.65	21.67	31.23	0.75	72.01	-68.85	8.47
3	14.36	13.55	0.81	23.26	17.62	30.82	-11.97	82.33	-66.21	4.38
4	16.11	14.55	1.57	23.82	23.28	33.56	-1.52	72.03	-70.46	3.53
5	14.71	12.83	1.88	22.94	22.68	29.66	15.44	65.52	-64.52	3.19
6	16.06	17.09	-1.03	22.13	24.22	29.65	18.41	59.61	-67.94	2.51
7	16.94	15.21	1.73	23.44	20.77	33.32	-13.31	78.64	-71.69	1.90
8	16.92	16.30	0.62	22.15	24.95	28.01	29.70	52.60	-65.59	1.85
9	19.53	19.44	0.09	24.14	30.05	35.05	17.76	53.66	-73.51	1.07
10	23.02	23.37	-0.35	30.01	44.19	48.55	18.73	44.76	-79.45	0.75
Average	16.21	16.22	-0.01	24.10	24.83	33.19	7.73	66.20	-68.44	-

Table 3. Simulation: Potential reasons for the poor time series fit

This table reports the one-step GMM results using simulated data from estimating jointly the cross-sectional moments and the time-series moments given by Equation (8) and (9) respectively, using book-to-market deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross-sectional moments and the second component refers to the weights on the time-series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross-sectional R^2. $TS-R^2$ is the-time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. In Panel A, aggregation is at the portfolio-level as in LWZ. In Panel B and C, portfolio-aggregation is based on firm-level investment returns, and stock returns are matched with lagged investment returns. In Panel C, stock and investment returns are annualized compounded 5-year returns.

	Panel A: Aggregation bias					Panel B: Misalignment					Panel C: Compounded returns				
	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates														
α	0.15	0.14	0.13	0.13	0.13	0.04	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04
$[t]$	5.98	7.23	5.03	2.51	0.85	15.95	13.99	8.81	3.95	2.91	70.72	71.02	71.14	56.21	8.29
c	2.53	1.48	0.79	0.67	0.66	0.81	-0.12	-0.11	-0.11	-0.11	0.84	0.83	0.79	0.63	0.45
$[t]$	1.91	1.94	0.67	0.18	0.18	2.76	-2.27	-0.54	-0.12	-0.08	9.37	9.41	9.52	7.02	0.44
	Goodness of fit														
$\left\|e^{XS}\right\|$	1.71	2.11	2.67	2.77	2.83	0.06	1.83	1.84	1.90	1.95	0.12	0.12	0.11	0.24	0.49
$\left\|e^{XS}_{H-L}\right\|$	0.54	4.00	6.80	7.44	7.57	0.00	0.39	0.04	0.13	0.22	0.54	0.55	0.60	0.76	0.92
$\left\|e^{TS}\right\|$	17.78	16.15	15.23	14.87	14.74	18.93	13.20	13.19	13.26	13.31	3.20	3.20	3.16	3.09	3.18
$XS-R^2$	50.55	23.14	-22.65	-47.92	-62.99	99.92	43.61	42.99	41.20	39.38	99.43	99.43	99.33	97.09	90.72
$TS-R^2$	-14.31	-1.81	0.61	0.73	0.77	-104.83	10.84	10.95	10.92	10.89	66.03	66.15	67.03	69.75	70.63
χ^2	7.16	15.70	16.54	16.26	11.67	0.19	15.31	15.19	15.37	10.70	0.77	15.12	14.97	14.53	11.63
d.f.	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00
p	0.52	0.61	0.55	0.57	0.17	1.00	0.64	0.65	0.64	0.22	1.00	0.65	0.66	0.69	0.17

Table 4. GMM estimation and tests of the investment-based model across alternative specifications

This table reports the one-step GMM results across alternative specifications from estimating jointly the cross-sectional moments and the time-series moments given by Equation (8) and (9) respectively in Panel A and B, and estimating jointly the cross-sectional moments and the variance moments given by Equation (8) and (26) respectively in Panel C, using book-to-market deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross-sectional R^2. $TS-R^2$ is the time-series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. In Panel A, portfolio aggregation is based on firm-level investment returns, and in Panels B and C, aggregation is at the portfolio-level as in LWZ. In Panel B, stock and investment returns are annualized compounded 5-year returns. The sample is from 1963 to 2020.

	Panel A: Firm-level aggregation					Panel B: Compounded 5-year returns					Panel C: Variance moments				
	Only XS	Both XS and TS			Only TS	Only XS	Both XS and TS			Only TS	Only XS	Both XS and Variance			Only Variance
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[I\ 40]$	$[0\ I]$	$[I\ 0]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates														
α	0.05	0.05	0.04	0.04	0.04	0.30	0.26	0.24	0.17	0.13	0.25	0.23	0.21	0.21	0.21
$[t]$	11.88	15.23	13.70	13.78	6.60	2.89	3.60	4.12	7.05	6.70	3.65	5.05	5.47	5.30	1.03
c	0.08	-0.03	-0.03	-0.03	-0.03	15.35	12.53	10.93	4.91	1.68	10.94	9.05	7.36	6.94	6.86
$[t]$	0.25	-0.08	-0.17	-0.13	-0.12	1.86	2.18	2.38	3.01	1.42	1.92	2.94	3.13	2.84	1.72
	Goodness of fit														
$\left\|e^{XS}\right\|$	1.25	1.35	2.13	2.60	2.90	1.40	1.51	1.60	2.20	3.26	1.35	1.37	1.50	1.56	1.52
$\left\|e^{XS}_{H-L}\right\|$	5.35	5.98	6.88	7.33	7.57	4.18	5.01	5.61	9.43	13.89	2.32	3.76	5.25	5.66	5.64
$\left\|e^{TS}\right\|$	19.37	19.15	19.10	19.11	19.12	9.56	9.28	9.09	8.25	7.99	23.90	23.04	22.15	21.91	21.89
$XS-R^2$	69.89	66.52	34.61	2.92	-18.29	68.97	68.16	66.60	38.71	-31.14	73.17	71.08	64.42	61.79	60.89
$TS-R^2$	-19.89	-14.66	-13.70	-13.49	-13.46	-78.72	-66.38	-58.50	-23.79	-9.87	-90.28	-75.36	-60.88	-57.11	-56.63
χ^2	9.19	17.58	17.74	17.88	14.12	8.03	16.83	16.88	17.10	15.57	6.40	14.94	15.51	15.65	8.50
d.f.	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00	8.00	18.00	18.00	18.00	8.00
p	0.33	0.48	0.47	0.46	0.08	0.43	0.53	0.53	0.52	0.05	0.60	0.67	0.63	0.62	0.39

Table 5. Specification-free test

This table reports the specification-free test results by estimating the relationship between stock returns and characteristics given by Equation (21) in both the cross-section and time-series, using book-to-market deciles as the testing portfolios. Column (1) and (2) use simulated data from a standard homogeneous of degree one (HD1) investment model. Column (3) and (4) use simulated data from a non-homogeneous of degree one model (non-HD1). Column (5) and (6) introduce misalignment in simulated data from the HD1 model. Column (9) and (10) use real data. Column (7) and (8), (11) and (12) use annualized compounded 5-year returns and 5-year average characteristics in simulated data from the HD1 model and real data respectively. R^2 is expressed as a percentage. Portfolio-aggregation is based on firm-level characteristics. The sample is from 1963 to 2020.

	Model								Data			
	HD1		non-HD1		HD1		HD1					
	Annual		Annual		Misalignment		Long-horizon		Annual		Long-horizon	
	XS	TS	XS	TS	XS	TS	XS	TS	XS	TS	XS	TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
YK_{it+1}	0.19	0.09	3.29	1.13	0.22	0.37	0.54	0.70	0.14	0.02	-0.04	-0.07
$[t]$	1.89	2.30	4.32	1.64	3.26	3.90	3.51	12.36	0.33	0.28	-0.10	-0.81
IK_{it+1}	-0.44	0.19	0.92	2.61	-0.21	-0.10	-0.44	0.67	1.41	0.47	0.42	1.42
$[t]$	-0.92	1.10	0.72	5.82	-0.66	-0.89	-1.49	4.41	0.42	2.21	0.26	4.11
IK_{it}	0.01	-0.34	-3.00	-2.41	-0.18	0.02	-1.07	-1.17	-1.29	-0.26	-0.11	-0.89
$[t]$	0.03	-5.26	-1.79	-24.60	-0.83	0.47	-5.08	-11.39	-0.45	-2.72	-0.06	-3.75
K_{it}	0.00	0.13	-0.77	-0.27	0.00	0.06	0.05	-0.02	-0.05	-0.03	-0.14	-0.01
$[t]$	0.28	2.74	-3.79	-1.22	0.13	1.01	0.96	-0.36	-0.69	-0.48	-1.12	-0.11
IK^2_{it+1}	0.69	0.79	-0.66	-1.15	0.71	-0.11	1.55	0.36	-0.08	-0.39	-0.70	-0.59
$[t]$	5.55	6.19	-1.31	-3.42	4.79	-1.25	6.82	2.76	-0.26	-2.67	-1.47	-2.28
R^2	91.05	90.04	96.16	87.24	92.72	13.38	98.58	66.15	70.76	5.03	89.16	9.63

Figure 1. Description of stock returns, investment returns, and errors

This figure scatter plots stock returns against investment returns, and stock returns against error terms, based on the estimation results from one-step GMM on the cross-sectional moments given by Equation (8), using book-to-market deciles as the testing portfolios. Aggregation is at the portfolio-level as in LWZ. The sample is from 1963 to 2020.

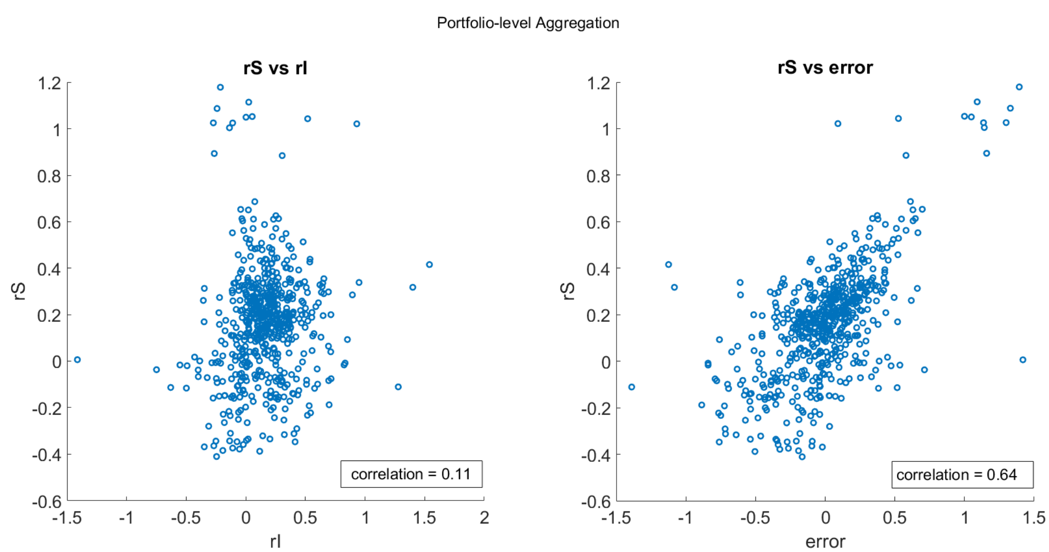
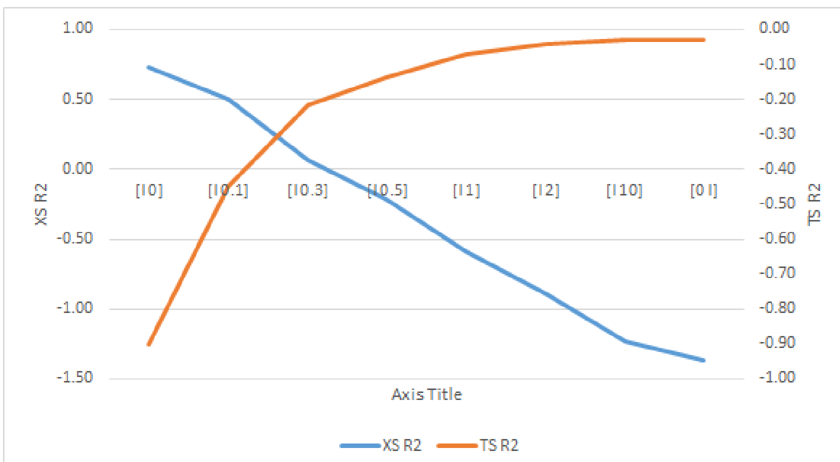
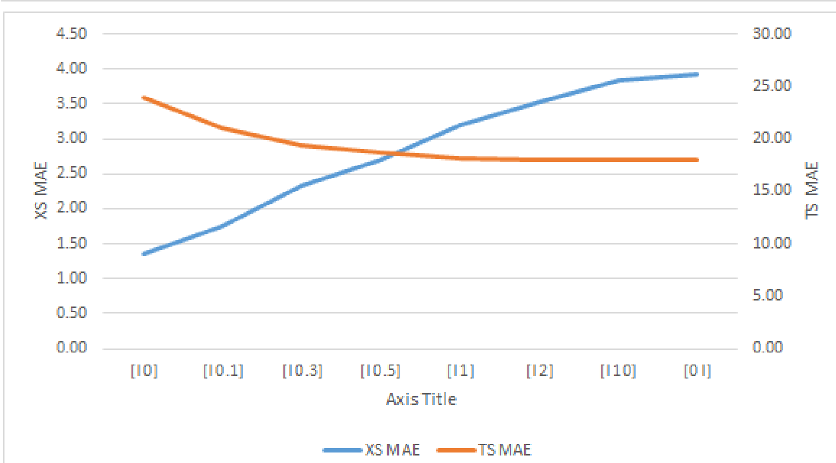


Figure 2. Cross-sectional fit versus time-series fit

The top panel plots the cross-sectional R^2 ($XS - R^2$) and the time-series R^2 ($XS - R^2$). The bottom panel plots the mean absolute cross sectional errors (in percentage per annum) given by Equation (14) and the mean absolute time series errors (in percentage per annum) given by Equation (17). Horizontal axis shows the prespecified weighting matrix, in which the first component refers to the weights on the cross-sectional moments and the second component refers to the weights on the time-series moments, so that that the axis goes from zero weight on time-series moments (only cross-sectional moments used) to all weight on time-series moments (no cross-sectional moments used). Aggregation is at the portfolio level as in LWZ. The sample is from 1963 to 2020.





Internet Appendix for

Estimating and Testing Investment-based Asset Pricing Models

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A: Additional analyses and robustness checks

This section reports additional analyses and robustness checks.

A.1 Additional results with portfolio-level aggregation

Table IA.1: Matching the cross sectional and the time series moments jointly with the original LWZ data.

Table IA.2: Matching the cross sectional and the time series moments jointly, second stage.

Table IA.3: Matching the cross sectional and the time series moments jointly using alternative testing portfolios.

Table IA.4: Matching the cross sectional and the time series moments jointly using alternative time series moments (NLLS first order conditions).

Table IA.5: Sub-sample analysis: Matching the investment return and stock return moments.

Table IA.6: Matching the valuation ratio moments.

[Table IA.1 - IA.6 here]

A.2 Additional results with firm-level aggregation

Table IA.7: Matching the cross sectional moments and the variance moments jointly, matching the cross sectional and the time series moments jointly with compounded returns, and using alternative time series moments (NLLS first order conditions).

Table IA.8: Sub-sample analysis: Matching the investment return and stock return moments.

Table IA.9: Matching the valuation ratio moments.

[Table IA.7 - IA.9 here]

B: External validity specification test

When evaluating the investment-based asset pricing model, existing studies conduct the χ^2 test using the same set of moment conditions as in the estimation. We argue that this procedure has low power to reject the model when presented with model misspecification. Thus, we develop a Wald test for model errors that are not used for estimation. It holds the model to a higher standard than a simple test of over-identifying restriction and thus accomplishes a purpose similar to that of an out-of-sample test.

Specifically, we ask the estimation to match the cross sectional moments as closely as possible and evaluate how the fitted model matches the time series moments. Following the procedure described in Cochrane (2009), we start to estimate the parameters by only using g^{XS} and obtain the distribution of all moments $var\left(g_T\right)$. Denote $var\left(g_T\right)^{TS}$ as the block of time series moments in $var\left(g_T\right)$, and we use it to compute the joint error for g^{TS} to incorporate sampling uncertainty about the parameters from their estimation stage and correlation between the estimation moments and the evaluation moments. We want to test the null hypothesis that $g^{TS} = 0$. This hypothesis constitutes a test of the external validity of the model, as it assesses the model's ability to explain patterns in the data that are not used to estimate its parameters. Under the null hypothesis that the model is correctly specified, these moments should equal zero. Formally, the χ^2 test is:

$$g_T^{TS'} \left[var\left(g_T\right)^{TS}\right]^{+} g_T^{TS} \sim \chi^2\left(\#moments - \#paras\right). \tag{28}$$

We compare standard over-identifying tests with our proposed external validity specification tests. Although the standard test has some difficulties in rejecting the model, the external validity specification test increases the power of the tests and hence can be useful in practice to detect possible model misspecifications.

Table IA.10 reports the results. Columns (1) and (2) report the standard over-identifying tests results in which the same set of moment conditions, the cross sectional moments, are used in the estimation and tests. Columns (3) and (4) report the specification tests results in which cross sectional moments are used in the estimation and time series moments are used in the tests. Columns (1) and (3) report the results based on an identity weighting matrix in the estimation, whereas columns (2) and (4) report the results based on the optimal weighting matrix. Panel A, column (1) shows that the p-value on testing the joint errors of 10 cross sectional moments is 0.60 (0.56 with an optimal weighting matrix), far from rejecting the model, despite the fact that the time series fit is very poor. In comparison, the p-value on evaluating the joint errors of the 10 time series moments is 0.24 in column (3) (0.19 with an optimal weighting matrix), getting the model much closer to rejection based on its time series fit.

The poor time series fit is more prominent with the correct portfolio aggregation as reported in Panel B in Table IA.10. Column (1) shows that the p-value on testing the joint errors of 10 cross sectional moments is 0.33 (0.32 with an optimal weighting matrix). In comparison, the p-value on evaluating the joint errors of the 10 time series moments is 0.07 in column (3) and (4), much more likely leading to a rejection of the model based on its time series fit.

[Table IA.10 here]

C: Investment model with frictions

The model is closely related to Lin and Zhang (2013). Production only takes one input, capital K, with decreasing return to scale. Firm i's operating profit function is given by

$$\Pi_{it} = X_t Z_{it} K_{it}^{\alpha} - f, \tag{29}$$

in which $0 < \alpha < 1$ is the curvature parameter, and $f > 0$ is a positive fixed cost, capturing the existence of fixed outside opportunity costs each period. Production is subject to both aggregate and idiosyncratic productivity shocks. The aggregate productivity X_t, has a stationary Markov transition function. Let $x_t = logX_t$, the transition function follows

$$x_{t+1} = \rho_x x_t + \sigma_x \mu_{t+1}, \tag{30}$$

in which μ_{t+1} is an i.i.d. standard normal shock. Firm i's productivity Z_{it} has a transition function follows

$$z_{it+1} = \bar{z}\left(1-\rho_z\right) + \rho_z z_{it} + \sigma_z \nu_{it+1}, \tag{31}$$

in which $z_{it} = logZ_{it}$, and ν_{it+1} is an i.i.d. standard normal shock. Two shocks are uncorrelated.

Firm i's capital accumulates as

$$K_{it+1} = I_{it} + \left(1-\delta\right) K_{it}, \tag{32}$$

in which δ is the rate of depreciation. Capital investment entails adjustment costs

$$\Phi\left(I_{it}, K_{it}\right) = \begin{cases} a^{+}K_{it} + \frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & I_{it} > 0 \\ 0 & I_{it} = 0 \\ a^{-}K_{it} + \frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & I_{it} < 0 \end{cases}, \tag{33}$$

where $a^{-} > a^{+} > 0$ and $c^{-} > c^{+} > 0$ capture non convex and asymmetric adjustment costs. Nonconvex part captures the cost independent of the size of investment. Convex part captures higher cost for more rapid changes. Asymmetric part captures costly reversibility. Firms face higher costs in contracting than in expanding.

The stochastic discount factor is exogenously given, denoted by M_{t+1}

$$M_{t+1} = \beta \frac{e^{\gamma(x_t - x_{t+1})}}{E_t\left[e^{\gamma(x_t - x_{t+1})}\right]}, \tag{34}$$

in which $0 < \beta < 1$, $\gamma > 0$ are constants. The risk-free rate is set to be constant.

Upon observing shocks, firms optimally choose investment to maximize the market value of equity, given by

$$V_{it} \equiv V\left(K_{it}, X_t, Z_{it}\right) = \max_{I_{it}} \left[\Pi_{it} - I_{it} - \Phi\left(I_{it}, K_{it}\right) + E_t\left[M_{t+1} V\left(K_{it+1}, X_{t+1}, Z_{it+1}\right)\right]\right]. \tag{35}$$

At the optimum, $V_{it} = D_{it} + E_t\left[M_{t+1} V_{it+1}\right]$, with $D_{it} \equiv \Pi_{it} - I_{it} - \Phi\left(I_{it}, K_{it}\right)$. Equivalently, $E_t\left[M_{t+1} r^S_{it+1}\right] = 1$ in which $r^S_{it+1} = V_{it+1} / \left(V_{it} - D_{it}\right)$ is the stock return. Similarly, $E_t\left[M_{t+1} r^I_{it+1}\right] = 1$, in which r^I_{it+1} is the investment return. However, in this investment model with frictions, Hayashi (1982) conditions do not hold, thus investment returns do not equal to stock returns.

The model is calibrated at annual frequency. The time discount factor, $\beta = 0.9718$, is set to match the real risk-free rate of 2.9% per annum. The price of risk parameter, $\gamma = 6$, is set to match the average Sharpe ratio. The persistence of aggregate corporate profits ρ_x is set to be 0.90 and conditional volatility $\sigma_x = 0.06$. For the adjustment cost parameters: $a^+ = 0.01$, $a^- = 0.1$, $c^+ = 10$, and $c^- = 200$; for the remaining parameters, $\rho_z = 0.90$, $\sigma_z = 0.10$, $\bar{z} = -0.98$, $\alpha = 0.65$, $\delta = 0.10$, and $f = 0.115$.

The model is solved with value function iterations on discrete state space. In total 1000 artificial samples are simulated from the model, each with 3000 firms and 500 years. The first 450 years are dropped to neutralize the impact of the initial condition. The remaining 50 years of simulated data are treated as from the model's stationary distribution. Empirical tests are performed on each artificial sample and cross-simulation median results are reported as model moments to compare with those in the real data. With the calibrated parameters, the model produces a value premium of 4% per annum.

Table IA.1. Matching the cross sectional and the time series moments jointly with the original LWZ data

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (8) and (9) respectively, using BM deciles as the testing portfolios, with the original LWZ data. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the portfolio level. The sample is from 1963 to 2005.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ 0.3]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 2]$	$[I\ 10]$	$[0\ I]$
				Parameter estimates				
α	0.23	0.19	0.16	0.15	0.14	0.13	0.12	0.11
$[t]$	2.74	4.57	6.04	6.59	6.93	6.90	6.55	4.13
c	8.43	4.49	2.35	1.50	0.60	0.01	-0.55	-0.71
$[t]$	1.16	1.28	1.02	0.77	0.36	0.01	-0.42	-0.42
				Goodness of fit				
$\left\|e^{XS}\right\|$	2.47	2.49	2.94	3.31	3.91	4.43	5.14	5.45
$\left\|e^{XS}_{H-L}\right\|$	1.13	4.72	7.92	9.66	11.98	13.93	16.41	17.36
$\left\|e^{TS}\right\|$	24.14	22.34	21.14	20.71	20.32	20.12	20.02	20.04
$XS-R^2$	65.20	58.12	40.40	26.49	3.54	-19.55	-56.73	-78.15
$TS-R^2$	-86.30	-61.07	-45.02	-38.61	-32.41	-29.10	-27.00	-26.76
χ^2	8.15	13.53	13.33	13.22	13.24	13.41	13.70	8.86
d.f.	8.00	18.00	18.00	18.00	18.00	18.00	18.00	8.00
p	0.42	0.76	0.77	0.78	0.78	0.77	0.75	0.35

Table IA.2. Matching the cross sectional and the time series moments jointly, second stage

This table reports the second stage GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (8) and (9) respectively, using BM deciles as the testing portfolios. Each column differs in the first stage prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the portfolio level. The sample is from 1963 to 2020.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ 0.3]$	$[I\ 0.5]$	$[I\ I]$	$[I\ 2]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates							
α	0.20	0.18	0.16	0.15	0.15	0.14	0.13	0.13
$[t]$	8.52	14.38	19.28	19.24	17.81	16.77	16.37	16.18
c	7.51	4.77	3.00	2.37	1.85	1.43	0.84	0.68
$[t]$	3.69	4.76	5.06	4.53	3.35	2.67	2.19	2.01
	Goodness of fit							
$\left\|e^{XS}\right\|$	1.88	1.99	2.45	2.72	3.00	3.22	3.61	3.72
$\left\|e^{XS}_{H-L}\right\|$	5.53	8.19	10.67	11.74	12.75	13.58	14.88	15.29
$\left\|e^{TS}\right\|$	22.18	20.49	19.18	18.72	18.35	18.12	17.95	17.93
$XS-R^2$	55.88	37.27	-0.62	-21.65	-45.03	-65.57	-99.74	-113.86
$TS-R^2$	-61.36	-36.45	-19.47	-13.82	-9.55	-6.69	-3.84	-3.34
χ^2	6.41	16.55	17.33	17.00	15.99	15.47	15.44	13.14
d.f.	8.00	18.00	18.00	18.00	18.00	18.00	18.00	8.00
p	0.60	0.55	0.50	0.52	0.59	0.63	0.63	0.11

Table IA.3. Matching the cross sectional and the time series moments jointly using alternative testing portfolios

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (8) and (9) respectively, using standardized unexpected earnings (SUE) deciles (Panel A), corporate investment (CI) deciles (Panel B), and asset growth (AG) deciles. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the portfolio level. The sample in Panel A and B uses the original LWZ data and is from 1963 to 2005, and in Panel C is from 1963 to 2020.

	Panel A: SUE				Panel B: CI				Panel C: AG			
	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates											
α	0.18	0.14	0.12	0.11	0.12	0.12	0.11	0.11	0.15	0.14	0.13	0.13
$[t]$	10.11	10.25	8.12	6.95	11.69	12.15	12.85	6.86	10.22	10.98	11.55	3.79
c	3.67	0.89	-0.91	-1.28	0.41	0.22	-0.10	-0.22	1.50	0.80	0.34	0.26
$[t]$	3.09	0.79	-0.88	-1.93	1.85	0.97	-0.34	-0.24	2.38	1.56	0.85	0.22
	Goodness of fit											
$\left\|e^{XS}\right\|$	0.69	1.70	3.45	3.98	1.59	1.77	2.21	2.48	1.19	1.34	2.01	2.30
$\left\|e^{XS}_{H-L}\right\|$	0.34	4.90	10.90	12.64	1.45	3.35	6.80	8.16	1.57	4.97	7.62	8.23
$\left\|e^{TS}\right\|$	18.00	16.99	16.69	16.74	18.80	18.74	18.64	18.61	17.13	16.82	16.88	16.91
$XS-R^2$	95.59	75.14	4.00	-25.87	-42.21	-52.68	-136.17	-207.03	80.94	69.31	41.84	27.77
$TS-R^2$	-33.87	-18.04	-9.77	-9.27	-23.19	-22.21	-20.92	-20.63	-7.47	-2.59	-1.18	-1.13
χ^2	4.61	11.13	11.69	6.86	10.41	12.60	12.49	6.16	8.06	16.61	16.33	14.27
d.f.	8.00	18.00	18.00	8.00	8.00	18.00	18.00	8.00	8.00	18.00	18.00	8.00
p	0.80	0.89	0.86	0.55	0.24	0.82	0.82	0.63	0.43	0.55	0.57	0.07

Table IA.4. Matching the cross sectional and the time series moments jointly using alternative time series moments

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the alternative time series moments given by Equation (8) and (25) respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS - R^2$ is the cross sectional R^2. $TS - R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the portfolio level. The sample is from 1963 to 2020.

	Only XS	Both XS and TS Moments						Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 5]$	$[I\ 10]$	$[I\ 20]$	$[I\ 40]$	$[I\ 100]$	$[0\ I]$
				Parameter estimates				
α	0.25	0.25	0.23	0.21	0.19	0.18	0.18	0.12
$[t]$	3.65	3.62	3.81	4.40	5.18	5.68	5.99	10.72
c	10.94	11.10	10.35	8.72	7.23	6.47	6.11	0.40
$[t]$	1.92	1.92	1.94	2.06	2.24	2.36	2.45	0.73
				Goodness of fit				
$\left\|e_i^{XS}\right\|$	1.35	1.41	1.83	2.29	2.65	2.84	2.96	3.93
$\left\|e_{H-L}^{XS}\right\|$	2.32	2.37	3.35	4.86	6.31	7.10	7.51	16.05
$\left\|e_i^{TS}\right\|$	23.90	23.95	23.60	22.82	22.01	21.56	21.33	17.98
$XS - R^2$	73.17	72.14	58.80	38.68	15.90	0.89	-9.01	-141.87
$TS - R^2$	-90.28	-91.13	-84.53	-71.16	-58.33	-51.52	-48.17	-2.98
χ^2	6.40	13.97	14.49	14.68	14.63	14.54	14.49	-
d.f.	8.00	10.00	10.00	10.00	10.00	10.00	10.00	-
p	0.60	0.17	0.15	0.14	0.15	0.15	0.15	-

Table IA.5. Sub-sample analysis: Matching the investment return and stock return moments

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (8) and (9) respectively, using BM deciles as the testing portfolios, in each subsample. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the portfolio level.

	Panel A: 1963 - 1994				Panel B: 1995 - 2020			
	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates							
α	0.20	0.14	0.12	0.12	0.28	0.15	0.14	0.14
$[t]$	4.25	6.94	7.24	6.92	2.59	6.38	5.39	4.39
c	6.09	0.44	-0.47	-0.59	13.95	2.69	1.54	1.40
$[t]$	1.68	0.36	-0.63	-1.41	1.74	2.15	1.05	1.20
	Goodness of fit							
$\left\|e^{XS}\right\|$	1.43	3.38	4.37	4.51	2.63	3.49	3.95	4.09
$\left\|e^{XS}_{H-L}\right\|$	2.10	13.34	17.22	17.82	5.60	15.60	17.07	17.24
$\left\|e^{TS}\right\|$	23.73	19.55	19.41	19.47	22.49	16.40	15.77	15.75
$XS-R^2$	68.73	-52.10	-156.36	-187.49	12.31	-98.08	-141.02	-150.11
$TS-R^2$	-61.41	-8.52	-3.79	-3.66	-110.47	5.47	10.72	10.71
χ^2	3.82	10.46	10.66	8.70	6.04	9.12	9.01	7.95
d.f.	8.00	18.00	18.00	8.00	8.00	18.00	18.00	8.00
p	0.87	0.92	0.91	0.37	0.64	0.96	0.96	0.44

Table IA.6. Matching the valuation ratio moments

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments for valuation ratios respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{VR-XS}\right|$ is the mean absolute cross sectional errors. $\left|e^{VR-TS}\right|$ is the mean absolute time series errors. $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. Aggregation is at the portfolio level.

	Panel A: 1963-2020				Panel B: 1963 - 1994				Panel C: 1995 - 2020			
	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates											
c	23.16	23.16	23.11	21.28	15.90	15.91	15.94	17.01	36.57	36.57	36.53	32.66
$[t]$	8.19	8.19	8.20	8.61	9.34	9.34	9.34	9.06	13.11	13.11	13.12	13.47
	Goodness of fit											
$\left\|e^{VR-XS}\right\|$	0.42	0.42	0.42	0.39	0.38	0.38	0.38	0.41	0.53	0.53	0.53	0.51
$\left\|e^{VR-TS}\right\|$	0.64	0.64	0.64	0.62	0.46	0.46	0.46	0.49	0.62	0.62	0.62	0.59
$XS-R^2$	0.22	0.22	0.22	0.20	0.26	0.26	0.26	0.25	-0.06	-0.06	-0.06	-0.10
$TS-R^2$	-0.31	-0.31	-0.30	-0.26	0.08	0.08	0.07	0.05	-0.20	-0.20	-0.20	-0.18
χ^2	17.65	18.92	18.92	16.62	10.31	10.94	10.94	10.17	8.93	9.26	9.26	9.11
d.f.	9.00	19.00	19.00	9.00	9.00	19.00	19.00	9.00	9.00	19.00	19.00	9.00
p	0.04	0.46	0.46	0.06	0.33	0.93	0.93	0.34	0.44	0.97	0.97	0.43

Table IA.7. GMM estimation and tests of the investment-based model across alternative specifications, firm-level aggregation

This table reports the one-step GMM results across alternative specifications from estimating jointly the cross sectional moments and the variance moments given by Equation (8) and (26) respectively in Panel A, estimating jointly the cross sectional moments and the time series moments given by Equation (8) and (9) respectively in Panel B, and estimating jointly the cross sectional moments and the alternative time series moments given by Equation (8) and (25) respectively in Panel C, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the firm level. In Panel B, stock and investment returns are annualized compounded 5-year returns. The sample is from 1963 to 2020.

	Panel A: Variance moments				
	Only XS	Both XS and Variance			Only Variance
Column:	(1)	(2)	(3)	(4)	(5)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 40]$	$[I\ 100]$	$[0\ I]$
	Parameter estimates				
α	0.05	0.04	0.04	0.04	-0.01
$[t]$	11.88	12.34	6.90	3.07	-0.18
c	0.08	0.43	0.45	0.46	0.55
$[t]$	0.25	2.07	1.92	1.78	1.86
	Goodness of fit				
$\left\|e^{XS}\right\|$	1.25	1.41	2.26	3.43	39.10
$\left\|e^{XS}_{H-L}\right\|$	5.35	5.45	6.62	7.86	37.28
$\left\|e^{TS}\right\|$	19.37	20.71	20.91	21.14	44.22
$XS-R^2$	69.89	58.87	33.38	-44.15	-16898.92
$TS-R^2$	-19.89	-76.22	-79.80	-81.34	-360.04
χ^2	9.19	14.72	14.83	14.93	14.52
d.f.	8.00	18.00	18.00	18.00	8.00
p	0.33	0.68	0.67	0.67	0.07

	Panel B: Compounded 5-year returns				
	Only XS	Both XS and TS			Only TS
Column:	(6)	(7)	(8)	(9)	(10)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[I\ 40]$	$[0\ I]$
	Parameter estimates				
α	0.04	0.04	0.04	0.04	0.04
$[t]$	15.33	17.28	19.04	16.12	9.33
c	0.18	0.12	-0.03	-0.06	-0.07
$[t]$	0.48	0.33	-0.08	-0.29	-0.36
	Goodness of fit				
$\left\|e^{XS}\right\|$	1.66	1.68	1.79	1.87	2.03
$\left\|e^{XS}_{H-L}\right\|$	7.60	7.76	8.61	9.17	9.60
$\left\|e^{TS}\right\|$	9.60	9.60	9.55	9.53	9.53
$XS-R^2$	56.78	56.51	51.79	45.77	35.97
$TS-R^2$	-65.65	-63.38	-58.04	-56.54	-55.84
χ^2	14.41	17.95	17.98	17.95	16.42
d.f.	8.00	18.00	18.00	18.00	8.00
p	0.07	0.46	0.46	0.46	0.04

	Panel C: Alternative time series moments				
	Only XS	Both XS and TS			Only TS
Column:	(11)	(12)	(13)	(14)	(15)
Weights:	$[I\ 0]$	$[I\ 0.1]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates				
α	0.05	0.04	0.04	0.04	0.04
$[t]$	11.88	15.30	15.00	14.76	14.72
c	0.08	-0.05	-0.05	-0.03	-0.03
$[t]$	0.25	-0.38	-1.84	-0.94	-0.69
	Goodness of fit				
$\left\|e^{XS}\right\|$	1.25	1.90	2.66	2.88	2.92
$\left\|e^{XS}_{H-L}\right\|$	5.35	6.77	7.53	7.58	7.58
$\left\|e^{TS}\right\|$	19.37	19.09	19.10	19.11	19.12
$XS-R^2$	69.89	46.72	-2.22	-17.36	-19.69
$TS-R^2$	-19.89	-14.16	-13.84	-13.47	-13.46
χ^2	9.19	11.91	12.04	12.18	-
d.f.	8.00	10.00	10.00	10.00	-
p	0.33	0.29	0.28	0.27	-

Table IA.8. Sub-sample analysis: Matching the investment return and stock return moments, firm-level aggregation

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments given by Equation (8) and (9) respectively, using BM deciles as the testing portfolios, in each subsample. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. α is the capital share and c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{XS}\right|$ is the mean absolute cross sectional errors given by Equation (14). $\left|e^{XS}_{H-L}\right|$ is the mean absolute high-minus-low cross sectional errors given by Equation (15). $\left|e^{TS}\right|$ is the mean absolute time series errors given by Equation (17). $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. $\left|e^{XS}\right|$, $\left|e^{XS}_{H-L}\right|$, $\left|e^{TS}\right|$, and R^2 are expressed as a percentage. Aggregation is at the firm level.

	Panel A: 1963 - 1994				Panel B: 1995 - 2020			
	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates							
α	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04
$[t]$	11.99	12.59	12.76	8.25	8.96	11.39	11.27	5.13
c	0.04	0.01	0.01	0.01	0.07	-0.03	-0.03	-0.03
$[t]$	0.09	0.02	0.04	0.08	0.25	-0.11	-0.14	-0.12
	Goodness of fit							
$\left\|e^{XS}\right\|$	1.30	1.34	2.32	3.16	2.18	2.15	1.98	2.03
$\left\|e^{XS}_{H-L}\right\|$	2.34	3.16	4.25	5.02	7.83	7.02	7.25	7.50
$\left\|e^{TS}\right\|$	19.93	19.89	19.75	19.71	17.71	17.08	17.06	17.07
$XS-R^2$	64.19	61.98	31.11	-16.19	29.54	27.01	23.47	15.79
$TS-R^2$	-9.61	-9.13	-8.43	-8.32	-17.02	-7.31	-7.05	-6.90
χ^2	6.89	10.72	10.74	9.42	7.59	9.04	9.05	7.96
d.f.	8.00	18.00	18.00	8.00	8.00	18.00	18.00	8.00
p	0.55	0.91	0.91	0.31	0.47	0.96	0.96	0.44

Table IA.9. Matching the valuation ratio moments, firm-level aggregation

This table reports the one-step GMM results from estimating jointly the cross sectional moments and the time series moments for valuation ratios respectively, using BM deciles as the testing portfolios. Each column differs in the prespecified weighting matrix, in which the first component refers to the weights on the cross sectional moments and the second component refers to the weights on the time series moments. c is the adjustment cost parameter. The t-statistics, denoted $[t]$, test that a given parameter equals zero. $\left|e^{VR-XS}\right|$ is the mean absolute cross sectional errors. $\left|e^{VR-TS}\right|$ is the mean absolute time series errors. $XS-R^2$ is the cross sectional R^2. $TS-R^2$ is the time series R^2. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. Aggregation is at the firm level.

	Panel A: 1963-2020				Panel B: 1963 - 1994				Panel C: 1995 - 2020			
	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS	Only XS	Both XS and TS		Only TS
Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Weights:	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$	$[I\ 0]$	$[I\ I]$	$[I\ 10]$	$[0\ I]$
	Parameter estimates											
c	21.15	21.15	21.13	20.81	15.55	15.55	15.62	16.85	28.36	28.35	28.25	25.99
$[t]$	10.27	10.28	10.38	12.18	11.17	11.18	11.22	11.26	9.36	9.36	9.40	10.71
	Goodness of fit											
$\left\|e^{VR-XS}\right\|$	0.40	0.40	0.40	0.39	0.31	0.31	0.31	0.35	0.55	0.55	0.55	0.54
$\left\|e^{VR-TS}\right\|$	0.66	0.66	0.66	0.65	0.41	0.41	0.41	0.44	0.81	0.81	0.80	0.77
$XS-R^2$	0.47	0.47	0.47	0.47	0.54	0.54	0.54	0.52	0.31	0.31	0.31	0.28
$TS-R^2$	-0.10	-0.10	-0.10	-0.09	0.23	0.23	0.22	0.18	-0.33	-0.33	-0.33	-0.26
χ^2	16.92	18.15	18.15	14.90	10.54	11.04	11.04	10.24	8.51	9.22	9.22	8.70
d.f.	9.00	19.00	19.00	9.00	9.00	19.00	19.00	9.00	9.00	19.00	19.00	9.00
p	0.05	0.51	0.51	0.09	0.31	0.92	0.92	0.33	0.48	0.97	0.97	0.47

Table IA.10. External validity specification test

This table reports the external validity specification test results based on different sets of moments used in estimation and tests. The cross sectional moments, denoted g^{XS}, are given by Equation (8). The time series moments, denoted g^{TS}, are given by Equation (9). Weighting matrix is either an identity matrix or an optimal weighting matrix. In Panels A, aggregation is at the portfolio level, and in Panel B, aggregation is at the firm level. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. The sample is from 1963 to 2020.

Estimation	g^{XS}		g^{XS}	
Tests	g^{XS}		g^{TS}	
Column:	(1)	(2)	(3)	(4)
Weights:	$[I\ 0]$	$[S^{-1}\ 0]$	$[I\ 0]$	$[S^{-1}\ 0]$
	Panel A: Portfolio-level aggregation			
χ^2	6.40	6.75	10.31	11.16
d.f.	8.00	8.00	8.00	8.00
p	0.60	0.56	0.24	0.19
	Panel B: Firm-level aggregation			
χ^2	9.19	9.24	14.63	14.66
d.f.	8.00	8.00	8.00	8.00
p	0.33	0.32	0.07	0.07