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Abstract

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JEL Classification: E32, C32

Keywords: Business cycle, Frequency domain, Structural dynamic factor models

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An American Macroeconomic Picture

Supply and Demand Shocks in the Frequency Domain *

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29th March 2023

Abstract

We provide a few new empirical facts that any theoretical model of the US macroeconomy should feature in order to be consistent with the data. 1) There are two classes of shocks: demand and supply. Supply shocks have long-run effects on economic activity, demand shocks do not. 2) Both supply and demand shocks are important sources of business cycles fluctuations. 3) Supply shocks are the primary driver for consumption fluctuations, demand shocks for investment. 4) The demand shock is closely related to the credit spread, while the supply shock is essentially a news technology shock. The results are obtained using a novel frequency domain method to identify demand and supply shock.

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1. Introduction

Figuring out what is the correct or most reliable theory underlying the data has always been the cornerstone of macroeconomic research. The empirical business cycle literature has tried to inform and support the theory by providing various stilized facts and representations of the macroeconomy.

At the origins of the modern empirical macroeconomic debate, Blanchard and Quah (1989) (BQ henceforth) draw a sketch of the macroeconomy as driven by two shocks, a permanent shock and a transitory one, interpreted as supply and demand, respectively. Both shocks are depicted as important sources of business cycle fluctuations.

In the following 30 years, empirical research moved away from the idea of a comprehensive representation of the macroeconomy, focusing mainly on partial identification and the study of single, more specific sources of fluctuation, such as technology shocks (Galì, 1999), news shocks (Beaudry and Portier, 2006), noise shocks (Blanchard et al., 2013), uncertainty shocks (Bloom, 2009), credit shocks (Gilchrist and Zakrajšek, 2012), to name just a few of the most important.

A couple of recent papers, however, departing from the widespread partial identification approach, go back to seeking a general and parsimonious representation of the macroeconomy. Angeletos et al. (2020) (ACD henceforth) look for the shock that most explains the business cycle —the so called "main business cycle shock". The authors, using a frequency-domain identification method in the context of structural VARs, argue that the bulk of cyclical fluctuations in real economic activity can be explained by a single shock. This shock is not the technology shock of the RBC model (Kydland and Prescott, 1982), since it has no long run effects on output. However, it cannot be considered a standard demand shock either, because it has no effect on prices.

The second paper is Avarucci et al. (2021) (ACFZ henceforth). Within a large factor model framework, ACFZ finds that just two shocks are enough to describe all macroeconomic variables, thus confirming, albeit with a different method, a previous important result by Onatski (2009). Moreover, it suggests that such shocks could be a temporary demand shock and a permanent supply shock.

The present paper is close in spirit to BQ, ACD and ACFZ. What we do is to provide a general picture of the main forces driving the US macroeconomy, at both cyclical and long run frequencies, with the goal of identifying empirical regularities which theoretical models should feature in order to be consistent with the data.

Our working hypothesis is that there are two main shocks, as suggested by the above factor model literature, and that these can be identified as textbook-type demand and supply shocks. The former should move prices and quantities in the same direction and have only transitory effects on real activity variables, while the latter should move prices and quantities in the opposite directions and have permanent effects. What we have in mind is a simple AD-AS model, or a New Keynesian model where the macroeconomy is described in terms of an aggegate demand curve (AD) and a Phillips curve (NKPC) which we refer to as the "traditional view". In a nutshell, our main result is that this hypothesis is confirmed by the data.

We use a dataset of 114 quarterly US time series, covering the period 1961-I to 2019-IV and assume that the data follow a large-dimensional Structural Dynamic Factor model, as introduced by Stock and Watson (2005) and Forni et al. (2009), which is naturally designed to describe a large number of time series with a relatively small number of common shocks. Having a large dataset, we can study the impulse response functions of all relevant macroeconomic variables within a unified framework; moreover, the rich information environment enables us to avoid the well-known noninvertibility problem affecting SVAR analysis (Hansen and Sargent, 1991; Lippi and Reichlin, 1993, 1994).

From a methodological point of view, we contribute to frequency domain analysis by providing a fairly comprehensive treatment of structural identification in the frequency domain. We extend the approach used in ACD[1] (see also Sarno et al., 2007; DiCecio and Owyang, 2010; Giannone et al., 2019) in several directions. In particular, in order to implement our identification scheme, we show how to jointly target variances of different variables and target covariances on a given frequency band.

Our identification strategy unfolds in two steps. In the first step, we select the two shocks maximizing the explained variance of the main macroeconomic variables, at all frequencies of macroeconomic interest, that is, excluding fluctuations with period of less than 18 months, of little interest for macroeconomic analysis. In so doing, we do not target a single variable at a time, as in ACD, but target jointly several variables. More specifically, we include in the target the variances of the main trending real activity variables (GDP, consumption, investment, TFP and labour productivity) as well as the variances of other important real and nominal variables (the unemployment rate, hours worked, the inflation rate, the federal funds rate and the S&P500 stock price index).

We find that these two shocks are successful in explaining the bulk of the variance of the main macroeconomic aggregates at both business cycle and long run frequencies, providing a fairly complete picture of the US macroeconomy. Adding a third shock increases only marginally the explained variance of the main real and nominal variables.

In the second step, we rotate the two main shocks in order to give them an economic interpretation. We implement two different identification schemes. In the first one (Identification I) we define a demand shock and a supply shock with a completely novel criterion. The demand shock is obtained by maximizing the covariance of GDP and inflation at business-cycle frequencies. The supply shock is automatically identified, by the orthogonality condition, as the shock minimizing the above covariance. In the second scheme (Identification II) we define a permanent and a transitory shock. Precisely, we define the permanent shock as the one that explains most of the long run variance of trending real activity variables (i.e. GDP, TFP, consumption, investment and labor productivity). The transitory shock is automatically identified by the orthogonality condition as the one minimizing the explained long run variance of the above variables.

In a sense, this procedure is close in spirit to BQ. Just like BQ, we provide a general picture of the forces driving the macroeconomy. By reducing the number of shocks of interest in the first stage, and identifying all of these shocks in the second stage, our method can be regarded as a global identification exercise, as opposed to the prevailing partial identification approach.

Our main results are the following. First, the two identification schemes provide the same outcomes. The inflationary demand and the deflationary supply shocks of Identification I are almost identical to the transitory and permanent shocks of Identification II,

[1] ACD show how to identify the shock which maximizes the explained variance of a given variable on a specific frequency band. This method is the frequency domain version of Uhlig (2004), who identifies two shocks that maximize the majority of the k-step ahead prediction error variances in real GNP for horizons between 0 and 5 years.

respectively. Hence, we show empirically that demand shocks have transitory effect on real economic activity. Second, both shocks, demand and supply, explain sizable fractions of business cycle fluctuations. Third, the demand shock is the most important cyclical shock for output, investment and unemployment, while private consumption fluctuations are mostly explained by supply shocks. Finally, our demand shock is to a large extent a credit shock, since it explains almost all cyclical variance of the risk spread and is the main driver of interest rates at all frequencies; moreover, the supply shock has the features of a news technology shock. It accounts for almost all the long run and the long cycles (between 8 and 20 years) of real activity and is the main driver of the consumer confidence index.

The above findings are broadly consistent with BQ's ones, but complete BQ's scketch with a large body of new evidence about prices, interest rates, consumption, investment and other macroeconomic variables. Differently from BQ, where long run neutrality of demand shocks is assumed, here it shows up as a result. Several papers have shown that special demand side shocks, such as monetary policy shocks or financial shocks, have transitory effects on output. But no one, to our knowledge, have shown that shocks identified as standard demand shocks have no long run effects on real activity.

By focusing on just two shocks, demand and supply, we do not want to deny that there is a plurality of sources of fluctuations, nor deny the importance of specific shocks analyzed in the literature. Rather, we think that such shocks can be grouped into the broader supply and demand categories: for instance, the technology shock is of course a supply shock, whereas uncertainty and credit shocks are best seen as transitory demand shocks. Our idea is that shocks having different nature but belonging to the same group, demand or supply, do have similar effects on the main macroeconomic aggregates, so that grouping them can produce meaningful results, in terms of impulse response functions and variance decomposition.

Our results are in line with the evidence in ACFZ and partially at odds with the picture emerging from ACD. With respect to the latter paper, we agree that the demand shock is the most important cyclical shock and is disconnected with long run real activity. On the other hand, our demand shock is inflationary and our supply shock explains a sizable fraction of the cyclical variance of output. Our paper is also related to Furlanetto et al. (2021), since our identification scheme, albeit based on frequency domain techniques, is similar to theirs from a substantive economic point of view. In contrast with their findings, where the demand shock is found to have long run effects, our demand shock does not affect real per-capita GDP and labour market in the long run.

The paper is structured as follows. In Section 2 we present the factor model setup and a comprehensive treatment of frequency domain identification. In Section 3 we present the design of our empirical analysis, with special focus on our two-stage identification procedure. In Section 4 we present the results. Section 5 concludes.

2. Identification in the frequency domain

2.1. The Structural Dynamic Factor Model

Let x_t be a n-dimensional, stationary vector of observable economic variables. The vector x_t is part of an infinite dimensional panel of time series. Each variable x_{it}, $i = 1, \ldots, n$, is decomposed into the sum of two mutually orthogonal unobservable components, the common component, χ_{it}, and the idiosyncratic component, ξ_{it}:

$$x_{it} = \chi_{it} + \xi_{it}. \tag{1}$$

The idiosyncratic components are interpreted as sources of variation that are specific to one or just a small group of variables, like regional or sectoral shocks, plus measurement error. In particular, for macroeconomic variables like GDP, investment or consumption, in which all local and sectoral shocks have been averaged out, the idiosyncratic part can be interpreted essentially as only containing measurement error. The idiosyncratic components are allowed to be mildly cross-sectionally correlated, thus they have a covariance matrix which is not necessarily diagonal (see Forni et al., 2009, Assumption 5).[2] The common components, on the contrary, account for the bulk of the co-movements among macroeconomic variables. This is because they are different linear combinations of the same $r < n$ common factors, not depending on i, i.e. they span a r-dimensional vector space (see Stock and Watson, 2002a,b; Bai and Ng, 2002). Then there exist an r-dimensional weakly stationary vector process $F_t = (F_{1t} \ldots F_{rt})'$, orthogonal to $\xi_t = (\xi_{1t} \ldots \xi_{nt})'$, and loadings λ_{ij}, $j = 1, \ldots, r$, such that

$$\chi_{it} = \lambda_{i1} F_{1t} + \ldots + \lambda_{ir} F_{rt} \quad \text{or} \quad \chi_t = \Lambda F_t. \tag{2}$$

The unobservable coordinates of F_t are called the static factor and Λ, the factor loading matrix, is of size $n \times r$. We require the factors to be pervasive i.e. to have non-negligible effects on most of the variables x_{it} (see Forni et al., 2009, Assumption 4). Combining (1) and (2), we get a static equation linking the n observable variables x_{it} to the r factors and the idiosyncratic components

$$x_{it} = \lambda_{i1} F_{1t} + \ldots + \lambda_{ir} F_{rt} + \xi_{it} \quad \text{or} \quad x_t = \Lambda F_t + \xi_t. \tag{3}$$

Equation (3) is the static factor representation, where the factors have only contemporaneous effect on the common components. The dynamic nature of the model comes from the fact that the static factors F_t follow a VAR(p) driven by a q-dimensional vector of orthonormal structural white noise, or common shocks $u_t = (u_{1t}, \ldots, u_{qt})'$, with $q \leq r$.

[2] A factor structure with mildly correlated idiosyncratic components is more realistic than a structure with orthogonal ones. However, in this case common and idiosyncratic component can be disentangled only as $n \to \infty$. This is what characterizes the large approximate dynamic factor model and motivates the assumption of an infinite number of variables. In the traditional dynamic factor model (Sargent and Sims, 1977; Geweke, 1977), on the other hand, the idiosyncratic components are orthogonal to each other; $\xi_t = (\xi_{1t} \ldots \xi_{nt})'$ has no cross-sectional dependence, a more restrictive assumption but estimation is possible even if the cross-sectional dimension is finite.

Precisely:

$$x_t = \Lambda F_t + \xi_t \tag{4a}$$
$$C(L)F_t = \epsilon_t \tag{4b}$$
$$\epsilon_t = Ru_t \tag{4c}$$

where ϵ_t is the residual of the VAR on F_t, $E(\epsilon_t\epsilon_t') = \Sigma_\epsilon$, $C(L)$ is an $r \times r$, stable polynomial matrix and R is $r \times q$ and has maximum rank q. As a consequence, R has a left inverse and the vector u_t belongs to the space spanned by $F_{t-s},\ s \geq 0$, that is, u_t is fundamental for F_t. By inverting the matrix $C(L)$ we get $F_t = C(L)^{-1}\epsilon_t = C(L)^{-1}Ru_t$, so that the dynamic relationship between u_t and the common components is

$$\chi_t = \left[\Lambda C(L)^{-1}R\right] u_t = B(L)u_t. \tag{5}$$

Then, by merging (1) and (5), we have the structural dynamic representation

$$x_{it} = b_i(L)u_t + \xi_{it} \quad \text{or} \quad x_t = B(L)u_t + \xi_t, \tag{6}$$

where the macroeconomic variables are represented as driven by a few pervasive structural shocks, loaded with the impulse response functions in $B(L)$, plus measurement error. We are interested in the effect of structural shocks on the common components χ_t of some key series, i.e. on the variables obtained by removing measurement errors, so we are neglecting the idiosyncratic components. Notice that representation (6) is not unique, since the impulse response functions are not identified. Forni et al. (2009) (Proposition 2), show that identification is achieved up to orthogonal rotations, just like in structural VAR models.

Let us consider the linear mapping in (4c), $\epsilon_t = Ru_t$. We define $R = SH$, where S is the Cholesky factor of Σ_ϵ , such that $SS' = \Sigma_\epsilon$, and H is an orthonormal matrix, namely a matrix such that $H^{-1} = H'$. We can then rewrite (5) as

$$\chi_t = \left[\Lambda C(L)^{-1}S\right] Hu_t = D(L)Hu_t = B(L)u_t \tag{7}$$

where $D(L) = \Lambda C(L)^{-1}S$ encapsulates the Cholesky impulse response functions and $B(L) = D(L)H$ collects the structural IRFs. Then, the effect of the j-th structural shock on the k-th variable is given by the (k, j) element of the matrix $B(L) = D(L)H$, that is, the product of the k-th row of $D(L)$ and the j-th column of H. On the other hand, the structural shocks are related to the VAR residuals by the relation $u_t = R^{-1}\epsilon_t = H'S^{-1}\epsilon_t = H'\eta_t$, η_t being the vector of the Cholesky shocks. Hence the j-th structural shock is given by the product of the j-th row of H' and η_t. Since we are interested in identifying the shocks, we deal with the choice of H. This is usually done as in standard SVAR analysis, which mainly employs an appropriate number of exclusion or sign restrictions motivated on economic grounds. Here we discuss an alternative approach: shock identification in the frequency domain.[3]

[3]This is not the first paper using frequency domain techniques to identify structural shocks —in addition to ACD, let us mention Christiano et al. (2006), Sarno et al. (2007), DiCecio and Owyang

2.2. Frequency band targets

We discuss two approaches to identify structural shocks in the frequency domain. The former is based on the maximization/minimization of the contribution of the structural shock to the variance or the comovements of a set of variables of interest in a given frequency band, which we refer to as *targeted frequency band covariances.* This is the approach we follow in this paper. The latter is based on restrictions on the sign of the comovements within a given frequency band. In this subsection we define the objects to be restricted to reach identification. In the two following subsections we show how to implement the identification.

Let us go back to representation (7). Letting $\left[\underline{\theta}, \overline{\theta}\right]$ be a band of frequencies such that $0 \leq \underline{\theta} \leq \overline{\theta} \leq \pi$, the comovements between the components of χ_t with period between $2\pi/\overline{\theta}$ and $2\pi/\underline{\theta}$ are measured by the *frequency band covariance matrix*

$$V\left(\underline{\theta}, \overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \Re\Big(D\left(e^{-i\theta}\right) D\left(e^{i\theta}\right)'\Big)\, d\theta$$

where $\Re\left(z\right)$ denotes the real part of z.[4] The matrix $V(\underline{\theta}, \overline{\theta})$ captures the entire frequency band volatility of the variables. The variance (or covariance) contribution of any generic shock $h'\eta_t$, where h is such that $h'h = 1$, to $V\left(\underline{\theta}, \overline{\theta}\right)$ is:

$$\Psi\left(\underline{\theta}, \overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right) hh' D\left(e^{i\theta}\right)'\right)\, d\theta. \tag{8}$$

Our identification approach consists of imposing restrictions on the contribution of the shock to the elements of the frequency band covariance matrix. The l, k element of $\Psi\left(\underline{\theta}, \overline{\theta}\right)$, is simply $\Psi_{lk}\left(\underline{\theta}, \overline{\theta}\right) = \mathcal{E}_l \Psi\left(\underline{\theta}, \overline{\theta}\right) \mathcal{E}_k'$ where $\mathcal{E}_l$ is the l-th row of the n-dimensional identity matrix. Using equation (8), we have[5]

$$\Psi_{lk}\left(\underline{\theta}, \overline{\theta}\right) = h' \left[\int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right)' \mathcal{E}_l' \mathcal{E}_k D\left(e^{i\theta}\right)\right) d\theta\right] h.$$

This is the objective function to be restricted to reach identification, in the case of a single target. The specification of the objective function can be properly defined for different targets (l, k) and/or frequency band, according to the identification scheme. For instance, if the interval $[\underline{\theta}, \overline{\theta}]$ is the cyclical band, the diagonal element $\Psi_{11}\left(\underline{\theta}, \overline{\theta}\right)$ is the cyclical variance of x_{1t} attributable to the combination $h'\eta_t$. This is the objective function used in ACD to identify the business cycle shock. The off-diagonal term $\Psi_{12}\left(\underline{\theta}, \overline{\theta}\right)$ is the

(2010), Giannone et al. (2019), Dieppe et al. (2021). It is however, to our knowledge, the first paper providing a comprehensive theory of identification in frequency domain.

[4]The diagonal elements of the spectral density matrix are real while the off-diagonal elements, the cross-spectra, are typically complex, with a real part, called co-spectrum, and an imaginary part. The integral of the co-spectrum of two variables over a given frequency band is the covariance of the two variables over that band, while the integral of the cross-spectrum is the cross covariance.

[5]To see this, notice that $\mathcal{E}_l D\left(e^{-i\theta}\right) h$ is a scalar so that it is equal to $h' D\left(e^{-i\theta}\right)' \mathcal{E}_l'$. The same reasoning applies to $h' D\left(e^{i\theta}\right)' \mathcal{E}_k'$

cyclical covariance between variable x_{1t} and x_{2t} attributable to the same shock. In the empirical section below, one of our identification schemes targets the covariance between GDP growth and inflation.

It is also possible to target more than one element of $\Psi\left(\underline{\theta},\overline{\theta}\right)$. This multiple-target approach is a key point to implement the identification strategy used in the empirical section below. Letting (M_1, N_1), (M_2, N_2), ..., (M_m, N_m) be the m entries of interest, we can target a weighted sum of such entries. For instance, we can take the simple sum of the variances of different variables, or a weighted sum, with weights equal to the reciprocals of the standard deviations (which is equivalent to taking the sum of the variances of the standardized variables). The contribution of the shock $h'\eta_t$ to a weighted sum is given by

$$h'\left[\int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right)' \sum_{k=1}^{m} \omega_k \mathcal{E}'_{M_k}\mathcal{E}_{N_k} D\left(e^{-i\theta}\right)\right) d\theta\right] h$$

where ω_k are the weights, to be chosen by the researcher.

Finally, notice that $\sum_{k=1}^{m} \omega_k \mathcal{E}'_{M_k}\mathcal{E}_{N_k} = P'_M \Omega P_N$, where $P_M = \left(\mathcal{E}'_{M_1}, \mathcal{E}'_{M_2}, \ldots, \mathcal{E}'_{M_m}\right)'$ and $P_N = \left(\mathcal{E}'_{N_1}, \mathcal{E}'_{N_2}, \ldots, \mathcal{E}'_{N_m}\right)'$ are $n \times m$ matrices, and $\Omega = \operatorname{diag}\left(\omega_1, \omega_2, \ldots \omega_m\right)$ is a $m \times m$ matrix. Hence the above equation can be re-written as

$$\sum_{k=1}^{m} \omega_k \Psi_{M_k N_k}\left(\underline{\theta},\overline{\theta}\right) = h' O_{MN}\left(\underline{\theta},\overline{\theta}\right) h \tag{9}$$

where

$$O_{MN}\left(\underline{\theta},\overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{i\theta}\right)' P'_M \Omega P_N D\left(e^{-i\theta}\right)\right)\, d\theta.$$

This is the objective function of our identification problem, in the case of multiple targets. Of course, this objective function reduces to the single target objective function in the case $m = 1$.

An example of multiple-target identification is the cyclical variance of a set of real economic activity variables: one could jointly maximize the cyclical variance of GDP growth and unemployment. Assuming that GDP growth and unemployment are the first two variables in x_t, we have $m = 2$, $M_1 = N_1 = 1$ and $M_2 = N_2 = 2$,

$$P_M = P_N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}, \quad \Omega = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}.$$

In this case, a reasonable choice for the weights is to take the reciprocals of the cyclical variances of the variables, i.e. $\omega_1 = \frac{1}{V_{11}\left(\underline{\theta},\overline{\theta}\right)}$ and $\omega_2 = \frac{1}{V_{22}\left(\underline{\theta},\overline{\theta}\right)}$.

2.3. Quantitative constraints

The first identification strategy is based on quantitative restrictions and is the one pursued in this paper.

Let us assume that the shock of interest is the first one, u_{1t}, and that such shock is the one maximizing $\Psi_{lk}\left(\underline{\theta}, \overline{\theta}\right)$, in the case of a single target, or $\sum_{k=1}^{m} \omega_k \Psi_{M_k N_k}\left(\underline{\theta}, \overline{\theta}\right)$, in the case of multiple target. In this case h_1, the first column of the matrix H, is formally given by

$$h_1 = \underset{h \in \mathbb{R}^n}{\arg\max}\ h'\, O_{MN}\left(\underline{\theta}, \overline{\theta}\right) h \qquad \text{s.t.} \quad h'h = 1. \tag{10}$$

It is easily seen that h_1 is equal to the eigenvector associated to the largest eigenvalue of the matrix $O_{MN}(\underline{\theta}, \overline{\theta})$ (Uhlig, 2004), and delivers the shock $u_{1t} = h_1' \eta_t$. This is a generalization of the approach used in ACD to identify the business cycle shock. In that paper, a single target is used, with $k = l$, so that the objective function is $\Psi_{ll}\left(\underline{\theta}, \overline{\theta}\right)$. We can then retrieve the corresponding structural IRFs as

$$B(L) = D(L)h_1 = \left[\Lambda C(L)^{-1} S\right] h_1. \tag{11}$$

If the researcher is interested in identifying more than one shock, the procedure can be extended to identify multiple shocks sequentially: first, obtain the shock with the largest contribution to the frequency band covariance, then obtain the shock orthogonal to the first, solving another maximization problem, and so on. Suppose, without loss of generality, that the shocks $u_{1t}, u_{2t}, ..., u_{qt}$, have to be identified. The vector h_1 is found according to equation (10). The vectors h_j with $1 < j \leq q$ are found solving the following maximization problem:

$$h_j = \underset{h \in \mathbb{R}^n}{\operatorname{argmax}}\ h'\, O_{MN}\left(\underline{\theta}, \overline{\theta}\right) h \qquad \text{s.t.} \quad \begin{cases} h'h = 1, \\ h'h_\ell = 0, \quad \ell < j. \end{cases} \tag{12}$$

Notice that the objective function can in principle be appropriately redefined for each shock by changing the targets (M, N) and/or the frequency band $\left[\underline{\theta}, \overline{\theta}\right]$, according to the identification scheme (even if for notational simplicity we avoid to explicit the possible dependence on j of $M, N, \underline{\theta}$ and $\overline{\theta}$).

Here are some examples.

For instance, we could identify the aggregate supply shock as the one maximizing the long run variance of GDP growth and then identify the aggregate demand shock as the shock orthogonal to the supply shock, which maximizes the cyclical variance of GDP growth. In this case, we change the frequency band of interest in the two maximization problems. Another example is the identification of a real and a nominal shock. We could first maximize the variance of GDP growth and then maximize the variance of inflation. In this case, the target would change in the two maximization problems. Moreover, we might be interested in identifying the two main business cycle shocks: first, the shock with the largest contribution to the frequency band covariance, then the shock orthogonal to the first with the second largest contribution. In this case, the target and the frequency band are assumed to be the same for all shocks.

It is also possible to use the sequential procedure just explained to nest two sets of quantitative constraints, i.e. two step procedure, by maximizing the appropriate target functions on the corresponding frequency band. For instance, in the first step, two main shocks are obtained by maximizing the appropriate target function on the band $[0\ 2\pi/6]$, which excludes fluctuations of less than 18 months, of little interest for macroeconomic analysis. In the second step two structural shocks are found by combining the two shocks obtained in the first step. This is the route we follow in this paper and the specific approach will be discussed below.

Of course, in the above problems, the argmax can be replaced by the argmin. For instance if we want to identify a shock that has only transitory effects on a given variable, the long run variance of such a variable has to be minimized.

2.4. Qualitative constraints

The second identification strategy we discuss relies on qualitative restrictions imposed on the entries of the matrix $\Psi(\underline{\theta}, \overline{\theta})$. More specifically, sign restrictions can be imposed on the off-diagonal elements of the frequency covariance matrix. This is a quite natural approach to restrict the sign of cyclical or long run comovements.

The implementation of this identification strategy is very similar to the one in time domain. We draw rotation vectors, i.e. h, (or rotation matrices) and then retain the draws satisfying the desired restrictions on the elements of interest of the frequency band covariance.

3. Empirical Approach

3.1. Data and estimation procedure

Coming to the empirical application, we use the quarterly dataset for high dimensional macroeconomic analysis recently developed by (Granese, 2023).

The $N \times T$ dataset is made up of 114 US quarterly series, covering the period 1961-I to 2019-IV. Most series are from the FRED-QD database.[6] TFP data series are from John Fernald's website (Fernald, 2012) while the Confidence data are available on the Michigan survey of consumer website.[7] Following standard practice, consumption includes non-durables and services, while investment has been broadly defined to include consumer durables. Both measures are deflated. Monthly data, like the macroeconomic uncertainty measure estimated by Jurado et al. (2015), have been aggregated to get quarterly figures. Finally, it is worth noting that most series are expressed in per capita terms, dividing by population aged 16 years or more (civilian non-institutional population series) and stock market data have been deflated by the GDP deflator. We transform each series to reach

[6] The FRED-QD is a large (248 series) quarterly macroeconomic database developed by McCracken and Ng (2020).

[7] http://www.sca.isr.umich.edu/

stationarity. The complete list of variables and transformations is provided in Appendix (B).

The analysis focuses on a subset of 13 macroeconomic series of interest: (1) the log difference of the real per capita GDP; (2) the log difference of real per capita consumption, defined as the sum of non-durable consumption and services; (3) the log difference of real per capita investment, computed as the sum of fixed investment and durable consumption; (4) the unemployment rate, (5) the log of real per capita hours worked; (6) the inflation rate, defined as the log difference of the GDP deflator; (7) labour productivity; (8) the cumulated sum of the utility-adjusted total factor productivity; (9) the Federal Funds rate; (10) the risk spread between Moody's Baa Corporate Bond Yeald and the 10-Year Treasury Constant Maturity Rate; (11) Shiller's real S&P500 stock price index; (12) the measure of macroeconomic uncertainty by Jurado et al. (2015) at the three-month horizon and (13) the Michigan University confidence index component concerning expected business conditions for the next five years (BC5Y).[8]

In order to compute the spectra and the objective function for our maximization problems we proceed as follows. We estimate the first two equations (4a)-(4b) using the two step estimation technique discussed in Forni et al. (2009), which we briefly review here.

First Step. We set a value for the number r of the static factors, using the criterion by Bai and Ng (2002) with the penalty modification proposed in Alessi et al. (2010), finding a number of static factors $\hat{r} = 11$.[9] The static factors $F_t = (F_{1t} \dots F_{rt})'$ are estimated by the first $\hat{r}$ principal components of the variables in our dataset, and the factor loadings, λ_{ij}, $j = 1 \dots r$, by the associated eigenvectors. Thus, the estimated loading matrix, $\hat{\Lambda}$, is the $n \times \hat{r}$ matrix having on the columns the normalized eigenvectors corresponding to the $\hat{r}$-largest eigenvalues of the sample covariance matrix of the data, $\hat{\Sigma}_x$. The estimated common component vector is given by $\hat{\chi}_t = \hat{\Lambda}\hat{F}_t$.

Second Step. We run a VAR(p) for the estimated factors $\hat{F}_t$ to get estimates $\hat{C}(L)$ and $\hat{\epsilon}_t$ of $C(L)$ and the VAR innovations ϵ_t. The estimated Moving Average representation is $\hat{F}_t = \hat{C}(L)^{-1}\hat{\epsilon}_t$. The number of lags p is determined according to the BIC criterion ($\hat{p}_{BIC} = 1$). In the robustness section we repeat the analysis with different lags order. To orthogonalize the shocks we use the Cholesky factor $\hat{S}$ of $\hat{\Sigma}_\epsilon$. Therefore, the Cholesky IRFs of the common components are obtained according to (7) as

$$\hat{D}(L) = \hat{\Lambda}[\hat{C}(L)^{-1}\hat{S}].$$

From this matrix we estimate the spectral density of the common components at the Fourier frequencies $\theta = 2\pi s/T$, $s = 1, \dots, T$, and take the real part, so that the resulting off-diagonal terms are co-spectra rather than cross-spectra. This is useful when we take an off-diagonal term as a target, since the integral of the co-spectrum of two variables over a given frequency band is the covariance of the two variables over that band. Finally,

[8]BC5Y summarizes responses to the following forward-looking question: "Turning to economic conditions in the country as a whole, do you expect that over the next five years we will have mostly good times, or periods of widespread unemployment and depression, or what?". The anticipation properties of this variable on future movements in economic activity in general and TFP in particular are widely discussed in Barsky and Sims (2012) and Beaudry and Portier (2006).

[9]In the robustness section, we take into account the uncertainty in estimating the number of static factors, and repeat the analysis with different specifications of $\hat{r}$.

we compute $V\left(\underline{\theta},\overline{\theta}\right)$ by replacing the integral with the simple average of the real part of the spectral density matrix, across the frequencies belonging to the relevant interval. $\Psi(\underline{\theta},\overline{\theta})$ and $O_{MN}\left(\underline{\theta},\overline{\theta}\right)$ are estimated in a similar way.

We do not apply the rank reduction step (see the on-line Appendix A) as this will be part of the identification strategy discussed below.

To conclude this section, let us look at the common-idiosyncratic variance decomposition of the key variables above with $\hat{r} = 11$ static factors, shown in Table 1. The common variance of the main macroeconomic aggregates like GDP, consumption, investment and unemployment rate are 94, 82, 90 and 94 percent of total variance, respectively. These numbers seem compatible with the measurement error interpretation of the idiosyncratic components.

3.2. Identification: A two-step procedure

Aim of this work is to provide a global and parsimonious description of the main forces driving the macroeconomy overall, at both cyclical and long run frequencies. There are two main questions we want to address. First, how many shocks are needed to explain the bulk of fluctuations in the main macroeconomic aggregates? Second, what are they and what are their effects? To address these two questions we develop a two-step strategy based on the econometric theory presented in the previous section.

First Step. First of all, we find the q shocks which explain the bulk of cyclical and long run variance of the main macroeconomic aggregates, both real and nominal. To do this, we solve maximization problems (10) and (12) with a multiple target and in the frequency interval $[\underline{\theta}\ \overline{\theta}] = [0\ 2\pi/6]$ (the trend-cycle band henceforth), which corresponds to periodicities greater than 18 months, thus excluding high frequency fluctuations of less than 18 months, of little interest for macroeconomic analysis.[10] More specifically we include in the target the variances of the growth rates for trended real activity variables (i.e. GDP, consumption, investment, TFP, labour productivity) as well as the variances of other real and nominal variables (i.e. unemployment rate, hours worked, inflation rate, Federal Funds Rate and S&P500 stock price index). The weights are given by the reciprocals of the (frequency band) variances of the variables, computed as the average of the spectral densities in the relevant frequency interval. Let us set M_1 and N_1 equal to the position of GDP in the data set, M_2 and N_2 equal to the position of consuption, etc., and call g_j, for $j = 1, ..., q$, the q vectors solving the maximization problem $g_j = \arg\max g' O_{MN}\left(\underline{\theta},\overline{\theta}\right) g$ subject to $g'g = 1$ and $g'g_l = 0$ for $l < j$; we obtain a matrix $G = [g_1\ g_2 \dots g_q]$ of dimension $r \times q$. We show below that two shocks are enough to explain the bulk of cyclical and long run fluctuations in the main macroeconomic aggregates.

Second Step. The shocks $g_1'\eta_t, ..., g_q'\eta_t$ lack of any economic interpretation: they are simply the largest contributors to the frequency band variances ordered in decreasing order of importance. We therefore move on to the second step and identify two structural shocks. We use two identification schemes.

Identification I. We identify a demand shock and a supply shock using a novel ap-

[10]The band $[0\ 2\pi/6]$ includes: business-cycle frequencies, $[2\pi/32\ 2\pi/6]$, corresponding to cycles between 18 months and 8 years, long cycles, $[2\pi/80\ 2\pi/32)$, which includes waves ranging from 8 and 20 years, and the long run, $[0\ 2\pi/80)$, corresponding to cycles of 20 years or more, with quarterly data.

proach. The demand shock is obtained by maximizing the covariance of GDP growth and the inflation rate at business cycle frequencies. The supply shock is automatically identified by the orthogonality condition as the shock minimizing such covariance. This identification scheme is related to the one recently used by Furlanetto et al. (2021), in that the demand shock is defined on the basis of the comovements of output and inflation and can in principle affect output in the long run.[11]

Identification II. We identify a permanent and a transitory shock. The permanent shock is identified as the one that explains most of the long run variance[12] of trending real activity variables, i.e. GDP growth, TFP, consumption growth, investment growth and labor productivity. The transitory shock is automatically identified by the orthogonality condition as the one minimizing the explained long run variance of the above variables. The effects on cyclical variance are left unrestricted, so that the two shocks can explain whatever fraction of business cycle fluctuation in the real activity variables, as well as the cyclical volatility of inflation and interest rate.

To impose the identifying restrictions in the second step we solve a problem very similar to the one of equation (10). The only difference is that now we rotate just the $q = 2$ main shocks obtained from the first step rather than the $\hat{r}$ Cholesky shocks. Formally, let $G = [g_1\ g_2]$ and consider the $n \times q$ matrix $D^*(L) = D(L)G$. We combine the columns of $D^*(L)$ and the shocks $G'\eta_t$ by solving the following maximization problem:

$$
\begin{aligned}
h_1^* &= \underset{h^* \in \mathbb{R}^q}{\operatorname{argmax}}\ h^{*\prime}\ O_{MN}^*\left(\underline{\theta}, \overline{\theta}\right) h^* \qquad \text{s.t.} \quad h^{*\prime} h^* = 1 \\
O_{MN}^*\left(\underline{\theta}, \overline{\theta}\right) &= \int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D^*\left(e^{-i\theta}\right)' P_M' \Omega P_N D^*\left(e^{-i\theta}\right)\right)\ d\theta
\end{aligned}
\tag{13}
$$

where now h^* and h_1^* are 2-dimensional orthonormal vectors. In a context with two structural shocks, the solution to (13) is enough to identify simultaneously both $h_1 = Gh_1^*$ and, similarly, $h_2 = Gh_2^*$ since the vector h_2^* is pinned down by the orthogonality restrictions. The structural impulse-response function are the entries of $B(L) = D(L)H$, where $H = [h_1\ h_2]$ and the structural shocks are $u_t = H'\eta_t$. For the two identifications, the specification of the objective function is the following:

Identification I: the frequency interval is $[\underline{\theta}\ \overline{\theta}] = [2\pi/32\ 2\pi/6]$. M is the position of GDP and N the position of inflation in vector x_t.

Identification II: the frequency interval is $[\underline{\theta}\ \overline{\theta}] = [0\ 2\pi/80]$, and $M = N$ is the vector whose elements are the positions of the real variables in the vector x_t.

[11] Note that unlike our identification scheme, the one used by Furlanetto et al. (2021) is implemented in the time domain.

[12] The long run is defined as frequencies in the interval $[0\ 2\pi/80)$, corresponding to cycles of 20 years or more.

4. Results

4.1. Two shocks

As explained above, in the first step of our procedure we select the two shocks maximizing the explained variance of the main macroeconomic variables on the trend-cycle band, that is, on a frequency band that includes all the frequencies of main interest for macroeconomic analysis. Table 2 reports, for each variable, the percentage of variance jointly explained by the two shocks on the whole trend-cycle band, on the business-cycle frequencies and on the long run, along with the variance explained by the shock with the third largest contribution. The aim is to see how large is the explained variance when only two shocks are selected and how large is the variance we lose with respect to the specification with three shocks.

The percentage of cyclical variance jointly explained by the two shocks is about 76 for real per capita GDP growth, 70 for consumption, about 79 for investment and unemployment rate. We also see that two shocks are enough to capture about 86% of cyclical inflation fluctuations, about 76% of the federal funds rate and more than 82% of the risk spread, the JLN uncertainty measure and BC5Y. We conclude that two shocks are enough to provide an accurate description of the business cycle fluctuations in both real and nominal variables.

Turning to the long run, we see that the percentage of variance jointly explained by the two shocks is 81 for real per capita GDP growth, 82 for unemployment rate, about 76 for consumption and about 66 for TFP. Two shocks account for about 85% of inflation fluctuations, 86% of the FFR and risk spread, and about 91% of uncertainty. Thus, two shocks not only account for the bulk of business cycles fluctuations, but also explain the long run.

The variance that we lose by selecting two shock instead of three is negligible for almost all variables, so the third shock is not large or pervasive enough to be considered as a main driver of the US economy. The third shock capturs essentially the cyclical fluctuations of TFP, which are of little interest for our analysis, because we are mainly interested in the long-run fluctuations of TFP.

All in all, our findings depict a picture of the US macroeconomy where two shocks provide a complete and parsimonious characterization at both cyclical and long run frequencies. This is in line with existing factor model evidence. As pointed out in the introduction, Onatski (2009), using his test for the number of shocks in a large dynamic factor model, cannot reject the null that there are 2 shocks against the alternative that there are from 3 to 7. ACFZ propose a new consistent estimator for the number of shocks, the "Dynamic eigenvalue Difference Ratio estimator" (DDR), that can be applied to single frequencies as well as to frequency bands, and finds that the US macroeconomy is well described by two major shocks. These results are in line with the evidence provided in papers such as Sargent and Sims (1977) and Giannone et al. (2005). To further corroborate our results, we apply the DDR estimator to our dataset on the whole interval $[0\ \pi]$ and on the trend-cycle band. The criterion selects two shocks on both bands.[13]

[13]To compute the DDR estimator, we set the bandwidth parameter $M_T = \lfloor a\sqrt{T} \rfloor$ with $a = 0.5$.

4.2. Identification I: explained cyclical and long run variances

Table 3 presents the results for Identification I, where we identify a supply and a demand shock based on the cyclical covariance between the inflation rate and per capita GDP growth. The table reports the cyclical and long run variances explained by the identified shocks. Notice that under this identification scheme both the long run and cyclical variance contributions are left unrestricted. Thus, we can verify whether the supply shock is permanent or not and whether the demand shock is transitory or not.

A first key result is that the demand shock explains a negligible fraction of the long run variance of trending real activity variables. It account for about 3% of GDP growth, less than 9% of consumption and hours worked, about 5% of investment, 11% of unemployment and less than 1% of TFP. Hence, unlike Furlanetto et al. (2021), we do not find evidence of hysteresis effects on output and labor market. On the other hand, our demand shock explains most of the long run variance in the inflation rate (about 65%) and the federal funds rate (about 84%).

The supply shock explains the bulk of the long run variance of real activity variables. It explains 78% of output growth, about 70% of consumption, investment and unemployment, and 55% of hours worked. Note that the percentage of TFP long run variance explained by the supply shock is about 65%, in line with the view that supply shocks include an important technological component.

Turning to the explained variances at business cycle frequencies, we see that the demand shock is the main source of cyclical fluctuations in output growth. It accounts for about 49% of GDP fluctuations. Still, the supply shock explains a sizable fraction of GDP cyclical variance, about 27%. As for inflation fluctuations, both demand and supply shocks explain an important part of cyclical variance. The former captures about 44% while the latter explains 42%.

An interesting result emerges when comparing the importance of the two shocks for GDP, consumption, investment, unemployment and hours worked. The supply shock is dominant for consumption. It accounts for about 41% of business cycle fluctuations, whereas the demand shock explains less than 30%. This result can easily be explained in the light of permanent income theory: consumption is mainly driven by permanent income, and permanent shocks have much larger effects on permanent income than transitory shocks (Quah, 1990).

The demand shock is also dominant for unemployment and investment. The cyclical variance of unemployment explained by our demand shock is about 50%, whereas the variance due to the supply shock is 29%. This result is in line with the evidence in Blanchard and Quah (1989), where the aggregate demand shock, the transitory one, plays a major role for unemployment fluctuations. As for investment, the demand shock accounts for about 55% of the cyclical variance, whereas the permanent shock accounts for only 24%. A possible explanation is that private investment is closely related to credit market conditions, which in turn are largely driven by demand. Indeed the demand shock explains almost all cyclical variance of the risk spread – about 77%, as against a scanty 11% explained by the supply shock. These numbers suggest that our demand shock is to a large extent a credit shock.

A few additional observations are in order. First, the forward-looking measure of consumer confidence (BC5Y) is mostly explained by the supply shock, both at business cycle frequencies and in the long run. This finding seems consistent with Barsky and Sims

(2012) and with the "news" interpretation of confidence indicators: consumer confidence is likely to reflect information about future productivity rather than animal spirits.

Second, the federal funds rate is explained almost exclusively by the demand shock, both at cyclical frequencies and in the long run. This is consistent with the idea that monetary policy follows a systematic rule according to which the nominal rate reacts positively to current inflation and real activity changes, in order to stabilize cyclical fluctuations. Supply shocks induce negative comovements of inflation and GDP growth, so that monetary policy react weakly to them.

Finally, both demand and supply have a sizable role in explaining JLN uncertainty at cyclical frequencies. Demand shocks explain 46% while supply shocks explain about 37%. If we interpret exogenous uncertainty shocks as demand shocks, we are left with a lower bound of approximately 40% of endogenous uncertainty fluctuations, induced by non-uncertainty shocks (that is, supply shocks and other demand-side shocks, such as credit or monetary policy shocks). Therefore, JLN macroeconomic uncertainty can be considered endogenous to a considerable extent. This finding is broadly consistent with Ludvigson et al. (2021).

Figure 1 summarizes the above findings by reporting the variance decomposition for the variables of interest. The figure reports the percentage of explained variance of each shock, frequency by frequency. The pink area is the long run frequency band, the lilac area is the business cycle frequency band. The blue line refers to the permanent shock and the red line to the transitory shock. The yellow line is the sum of the two.

The figure also provides additional information about the "long cycles" frequency band, i.e. fluctuations of periodicity between eight and twenty years that fall in the white area between the long run and the business cycle frequency bands. The upper-left panel refers to GDP growth: long cycles are explained almost exclusively by the supply shock. The same result applies to all real activity variables but unemployment. It follows that if the business cycle were defined by including longer cycles, e.g. cycles with periodicity between 6 and 50 quarters as suggested by Beaudry et al. (2020), the importance of the supply shock in explaining real activity fluctuations would increase.[14]

4.3. Identification I: impulse response functions

Figure 2 reports the impulse response functions to the supply shock, Identification I. The black solid lines are the point estimates, while the dark and light gray areas are the 68% and 90% confidence band, respectively. The shock has a large positive permanent effect on GDP and its components and generates a temporary hump-shaped response of unemployment and hours worked. GDP increases immediately by around 0.2%, peaks around the 10th quarter and converges to 1.2% in the long run. The effect on consumption appears to be slightly larger and persistent, reaching a maximum of about 2%. Unemployment behaves counter-cyclically and reaches a minimum of about -0.2% around the 8th quarter. The supply shock generates a negative comovement between inflation rate and output growth. The former immediately falls by around -0.2% and the effect is relatively short lived. The response of stock prices is positive and persistent, peaking at

[14] Beaudry et al. (2020) show that many macroeconomic aggregates appear to have a peak in their spectral densities at periodicities between 32 and 50 quarters and that the implied movements coincide with NBER cycle dating. For this reason, they argue that the definition of the business cycle should be modified accordingly.

0.9 percent, while the risk premium, after a nearly zero impact effect, decreases with a temporary hump-shape, reaching a minimum of about -0.14%.

A few additional observations are in order. First, we see that systematic monetary policy, as proxied by the federal funds rate, reacts negatively to the supply shock on impact, with an insignificant response after about one year. This suggests that systematic policy reacts more to inflation than real activity. However, the effect of the unit variance supply shock is really small, the maximum being about 10 basis points, as against the 21 basis points of the demand shock (Figure 3). Second, the response of TFP to the supply shock has an S shape which resembles the one typically found for the news technology shock, with a relatively small impact effect (about 0.4) and a much larger long run effect (about 1.2). This suggests that the supply shock includes an important news shock component as in Beaudry and Portier (2006). The significant positive impact effect of the supply shock on the consumer confidence component BC5Y, documented above, is in line with this interpretation, given the anticipation properties of this variable about future technology. Finally, JLN uncertainty decreases immediately in response to positive supply shocks, with a maximum effect at horizon one of about -0.25%. These movements in macro uncertainty persist for about two years after the shock.

Figure 3 reports the impulse response functions to the demand shock, Identification I. The responses of real economic activity variables are temporary and hump-shaped, peaking at horizon 3 or 4 (one year after the shock). The effects are no longer statistically significant after about 2-3 years. GDP has a positive impact effect of 0.4% and a peak of about 0.8%. Unemployment falls at a minimum of around -0.2%, then shows a significant and short lived rebound effect between the 12th and the 20th quarter, with a peak of about 0.1%. Investment shows a similar, albeit less pronounced and not significant rebound effect.

The response of inflation and the interest rate are very similar, in terms of both shape and magnitude. The former increases on impact by about 0.15%, peaks at 0.2% and converges to zero afterward. The effect appears to be more persistent than that of the permanent shock. The interest rate increases in a hump-shaped pattern, reaching a maximum of about 0.23%. As noted above, this suggests a very active behavior of monetary policy, consistent with standard Taylor rules, implying a systematic policy reaction to inflation and output. As expected, TFP essentially does not react to the unit variance demand shock, the effect being not significant at all horizon. For stock prices the effect is positive but very short lived, being significant only on impact (about 0.5%). Thus, the stock market reacts more to supply shocks than demand shocks. The effects on the risk premium are much larger and short lived for demand shocks than for supply shocks. The shape of the impulse response function of the risk premium, with a maximum effect on impact and at lag 1 (about -0.35%), closely resembles the one of the excess bond premium obtained in Gilchrist and Zakrajšek (2012). This again suggests that shocks related to credit and financial conditions are an important component of the demand shock.

4.4. Identification II

Let us now turn to Identification II, where we identify a permanent and a transitory shock on real variables. Here the co-spectrum of inflation and GDP growth is left unrestricted, so that, looking at the impulse-response functions, we can verify whether the permanent shock is a supply shock and the transitory shock is a demand shock.

More importantly, the two identification schemes provide very similar outcomes. The matching is really striking: the correlation of the demand (supply) shock of identification I and the transitory (permanent) shock of Identification II is higher than 0.99.

Table 4 presents results for the variance decomposition. Notice first that Identification II is successful in isolating a transitory shock. Indeed, the percentage of GDP growth, consumption and TFP long run fluctuations accounted for by the transitory shock is negligible (1.7, 5.9 and 1.6% respectively). The variance decompositions in the table are very similar to the ones of Identification I. Once again, both shocks are important sources of business cycle fluctuations in real economic activity. The permanent shock is more important for consumption, while the transitory shock is more important for output growth, unemployment and investment. Concerning inflation, both transitory and permanent shocks explain a large percentage of cyclical fluctuations. In particular, the transitory shock is not disconnected from inflation, in that it accounts for about 49% of cyclical variance, contrary to what found in ACD. This result is not at all implied by our identification.

Turning to the impulse response functions, Figure 4 and Figure 5 compare results of Identification II with those of Identification I. Figure 4 overlaps the responses to the supply shock of Identification I and the permanent shock of Identification II, whereas Figure 5 overlaps the responses to the demand shock of Identification I and the transitory shock of Identification II. The solid black lines are the point estimates for Identification I, the cyan dashed lines are the point estimates for Identification II and the dark and light gray areas are the 68% and 90% confidence band, respectively, relative to Identification I.[15] The correspondence between the two identification schemes is striking. The key message is that our expansionary transitory shock raises inflation, whereas our expansionary permanent shock reduces inflation, in line with New Keynesian textbook models and thus supporting the traditional view.

4.5. Discussion

The general picture emerging from our empirical analysis is the following. US data are consistent with a view of the macroeconomy as driven by two main shocks: a deflationary supply shock having long-lasting effects on real economic activity and an inflationary demand shock having only transitory effects. Both shocks explain a sizable part of business-cycle fluctuations.

This picture is clearly incompatible with the standard RBC model and largely in line with BQ, where transitory shocks are found important in explaining the business-cycle fluctuations of economic activity. Our findings are also incompatible with the view put forward by Beaudry and Portier (2006) that news shocks capture the bulk of cyclical fluctuations in real activity. Rather, they are consistent with Barsky and Sims (2011) and Forni et al. (2014), where the news technology shock explains a minority, albeit sizable, part of business cycle fluctuations.

Our evidence, far from being at odds with the partial identification literature, provides evidence in favor of some of the studies cited in the introduction. In particular, the response of TFP to the supply shock has an S shape which resembles the one typically

[15] The IRFs of Identification II with their confidence bands are reported in Appendix C.

found for the news technology shock (Beaudry and Portier, 2006), suggesting that news shocks are the dominant component of supply shocks. Moreover, the explained variance and the shape of the impulse response function of the risk premium to the demand shock are very much similar to the ones found in the credit shock literature (Gilchrist and Zakrajšek, 2012) consistently with the idea that credit shocks are the dominant component of demand shocks (even if they could include an exogenous uncertainty component).

As already observed, our results are partially at odds with the picture emerging from ACD. The finding that the bulk of cyclical fluctuations are not driven by a permanent shock is in line with ACD: the demand shock is the most important business cycle shock for output growth and is largely disconnected from the long run of real economic activity. On the other hand, the ACD's hypothesis that most of the business cycle fluctuations of real activity can be explained by just one shock, a non-inflationary demand shock affecting all real activity variables with the same dynamics, is rejected here: our supply shock explains a sizable part of cyclical fluctuations and is the main business-cycle driver for consumption, suggesting that at least two shocks are needed to explain the bulk of cyclical fluctuations in real economic activity variables. This important point is studied in detail in Granese (2023). Moreover, the demand shock is not disconnected from inflation at both cyclical and long run frequencies. These last two results are broadly in line with ACFZ.

4.6. Robustness

In this subsection we conduct a few robustness exercises for Identification I. Robustness results for Identification II are similar and are reported in Appendix C.

First, we test robustness to the inclusion of additional lags with respect to the one lag baseline specification. We estimate the model with two, three (as suggested by the AIC) and four lags, respectively. Table 5 reports the cyclical (top panel) and long run (bottom panel) variances accounted for by the identified supply and demand shocks. The first two columns correspond to our baseline specification, $p = 1$, while the remaining are for the alternative specifications, $p = 2, 3, 4$. In addition, Panel (a) of Table 7 summarizes the above findings by reporting, for each variable and shock, the maximum and minimum shares of explained variance, as the lag order changes.

As for the business cycle, baseline results appear to be quite robust with respect to changes in specification. The GDP growth variance explained by the supply shock ranges from a minimum of 27% (baseline) to a maximum of 30% (4 lags specification), while for the demand ranges from 47% (3 lags) to 51% (4 lags). The investment variance explained by the supply shock ranges from a minimum of 24% (baseline) to a maximum of 34% (4 lags specification), while for the demand ranges from 49% (4 lags) to 55% (baseline). The finding that consumption fluctuations are mostly explained by supply shocks is a fully robust result. In the 3 lags specification, it explains 51% of the consumption cyclical variance, while only 20% is explained by the demand shock, a difference of 31 percentage points. All in all, the demand shock is still the most important cyclical shock for real activity, but the increase in the number of lags seems to enhance the cyclical footprint of the supply shock, reinforcing our view that the business cycle is driven by two main shocks.

The only sensitivity analysis worth noting is the following. As lags increase, the demand shock appears less tightly connected, in terms of variance contributions, to inflation fluctuations. The cyclical variance explained by the demand shock ranges between a minimum of 17% (4 lags specification) to a maximum of 44% (baseline) while for the supply shock it ranges from 42% (baseline) to 63% (4 lags). The demand shock is partially disconnected from inflation only in the 4 lags specification in which, however, it accounts for 17% of inflation, as against the 7% found in ACD. For the transitory shock of Identification II, the percentage of explained variance of inflation is somewhat more robust across lag specifications, ranging between 29 and 49% (see Appendix C).

Turning to the long run, the variance decomposition displays figures fairly close to the baseline for most of the variables. For example, the output growth long run variance explained by the supply shock varies from 67% (4 lags) to 78% (baseline), while for the demand shock ranges from about 3% (3 lags and baseline cases) to 11% (4 lags). The main conclusions about the long run contribution of the two shocks are confirmed, except one: the finding that demand shock explains most of the long run fluctuations in inflation (64% vs. 20% of the supply shock) is not robust: for the 2, 3 and 4 lags specifications, demand explains 36, 21 and 13% percent, respectively, while supply explains 34, 26 and 36%.

Figures 6 and 7 display the impulse response functions to the supply and the demand shocks, respectively, for different lag specifications. The solid black lines (point estimates) and confidence bands are those obtained in the baseline exercise. All in all, the dynamic responses to supply shocks are similar to those obtained in the baseline exercise, most of them lying within the baseline confidence bands. As for the demand shock, the magnitude of responses is slightly smaller only for inflation and interest rate, with similar shapes.

Finally, we check the robustness of the results as the number of static factors increases. In particular, we compare the results of our baseline specification ($r = 11$) with four alternatives: $r = 13, 15, 17, 20$. Table 6 reports the cyclical (top panel) and long run (bottom panel) variances accounted for by the identified supply and demand shocks. As the number of static factors changes, the contribution of the identified shocks to the cyclical and long run variances of the main macroeconomic variables does not change much. As in the previous exercise, panel (b) of Table 7 summarizes the above findings by reporting, for each variable and shock, the maximum and minimum shares of explained variance obtained as the factor specification changes. For example, the percentage of cyclical variance explained by the demand shock varies between 49 and 52 for GDP, depending on the specification of r, 25 and 29 for consumption, 53 and 55 for investment, and so on. The results become slightly sensitive only when the number of static factors becomes very large with respect to the benchmark. For example, the consumption cyclical variance explained by the supply shock ranges between a minimum of about 26% ($r = 17$ and $r = 20$) to a maximum of 41% (baseline case): when $r = 17$ and $r = 20$, supply is no longer dominant for consumption, although demand alone still cannot explain most of the cyclical fluctuations.

The same robustness is found when considering the IRFs. Figures 8 and 9 display the impulse response functions to the supply and the demand shocks, respectively, obtained in this exercise. The responses are very much similar to the baseline. All in all the results are fairly robust to different specifications.

5. Summary and conclusions

In this paper we provide a comprehensive and stylized description of the U.S. macroeonomy and investigate whether the traditional view has support in the data. The evidence shows that this is the case.

The result is obtained assuming that data follow a Structural Dynamic Factor Model and using a novel identification technique in the frequency domain. Our identification strategy unfolds in two steps. In the first step, we select the two shocks with the largest contribution to the cyclical and long run variance of the main real and nominal macroeconomic variables. We show that adding a third shock would only marginally increase the explained variance. In the second step, we rotate the two main shocks in order to give them an economic interpretation. We implement two different identification schemes: in the first one we define a demand and a supply with a completely novel criterion based on the covariance between inflation and output, while in the second scheme we define a permanent shock on real activity and a transitory one in a way that is very close to BQ.

The two identification schemes provide strikingly similar outcomes in terms of both variance decomposition and impulse response functions. The US macroeconomy is driven by two main forces: a supply shock, which is permanent and generates a negative comovement between prices and quantities, and a demand shock, which is transitory and generates a positive comovement between prices and quantities. We show empirically that demand shocks have only transitory effect on real economic activity. Both demand and supply are important sources of business cycle fluctuations. The demand shock is closely related to credit market conditions and is the main business-cycle shock for output, investment and unemployment, while the supply shock is to a large extent a news technology shock and is the main business cycle shock for private consumption. Finally, supply shocks not only account for almost all the long run fluctuations of real activity, but also for long cycles (between 8 and 20 years).

All in all, the evidence strongly support the very standard view of the macroeconomy where fluctuations in real economic activity and prices arise from shifts in the aggregate demand and aggregate supply curves. From our perspective, theory should look at the U.S. macroeconomy through the lens of a two-shock, New Keynesian textbook framework, in order to be consistent with the data.

Tables

Variables	χ	ξ
GDP	94.33	5.67
Consumption	81.62	18.38
Investment	89.54	10.46
Unemployment Rate	94.17	5.83
Hours Worked	83.53	16.47
Inflation	90.47	9.53
Labor Productivity	89.31	10.69
TFP	80.91	19.09
FFR	97.92	2.08
Baa-GS10 Spread	78.05	21.95
S&P500	94.47	5.53
JLN Uncertainty 3M	83.81	16.19
BC5Y	75.87	24.13

Table 1: Percentage of the variance explained by the estimated common and idiosyncratic components of selected variables. Baseline specification: $r = 11$ static factors. We run the test proposed by Alessi et al. (2010).

Variables	Trend-Cycle band		Cyclical band		Long Run band	
	First two	Third	First two	Third	First two	Third
GDP	77.9	1.9	76.2	2.0	81.0	0.7
Consumption	70.8	1.0	69.7	0.6	75.6	1.6
Investment	79.9	0.5	78.9	0.6	72.3	0.2
Unemployment Rate	83.7	3.9	78.5	1.6	82.0	7.3
Hours Worked	65.3	14.6	58.1	12.6	63.5	16.6
Inflation	85.5	6.3	86.1	7.2	85.4	5.8
Labor Productivity	47.3	30.8	46.9	31.0	63.4	10.8
TFP	31.6	54.0	27.4	58.0	66.1	20.0
FFR	83.8	1.1	75.5	3.6	85.9	0.3
Baa-GS10 spread	85.0	0.8	87.8	0.3	86.1	1.0
S&P 500 real	55.0	2.0	57.1	1.3	30.9	6.0
JLM uncertainty 3M	85.4	1.2	82.9	1.3	91.8	2.0
BC5Y	85.5	6.8	89.1	2.4	83.4	9.2

Table 2: Percentage of variance explained by the first two main shocks and by the third for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

Variables	Cyclical variance			Long Run variance		
	Supply	Demand	Sum	Supply	Demand	Sum
GDP	27.1	49.1	76.2	77.7	3.3	81.0
Consumption	40.6	29.2	69.7	66.9	8.7	75.6
Investment	23.6	55.3	78.9	67.8	4.5	72.3
Unemployment Rate	29.0	49.5	78.5	70.9	11.0	82.0
Hours Worked	26.3	31.9	58.1	54.7	8.8	63.5
Inflation	41.8	44.3	86.1	20.0	65.4	85.4
Labor Productivity	22.5	24.4	46.9	60.1	3.3	63.4
TFP	21.0	6.4	27.4	65.2	0.9	66.1
FFR	13.3	62.2	75.5	2.3	83.6	85.9
Baa-GS10	10.8	77.0	87.8	44.0	42.1	86.1
S&P500	33.3	23.8	57.1	30.4	0.5	30.9
JLN Uncertainty 3M	37.4	45.5	82.9	54.5	37.3	91.8
BC5Y	69.1	20.1	89.1	74.8	8.6	83.4

Table 3: Identification I. Percentage of variance explained by the supply (deflationary) shock and the demand (inflationary) shock for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

Variables	Cyclical variance			Long Run variance		
	Perm	Trans	Sum	Perm	Trans	Sum
GDP	29.6	46.6	76.2	79.3	1.7	81.0
Consumption	43.9	25.9	69.7	69.7	5.9	75.6
Investment	25.5	53.4	78.9	67.2	5.1	72.3
Unemployment Rate	30.1	48.4	78.5	68.7	13.2	82.0
Hours Worked	29.2	29.0	58.1	57.3	6.3	63.5
Inflation	37.2	48.8	86.1	15.5	69.9	85.4
Labor Productivity	23.0	23.9	46.9	58.3	5.1	63.4
TFP	20.7	6.7	27.4	64.5	1.6	66.1
FFR	10.9	64.5	75.5	0.9	85.0	85.9
Baa-GS10	12.9	74.8	87.8	49.2	36.9	86.1
S&P500	36.2	20.9	57.1	30.2	0.7	30.9
JLN Uncertainty 3M	39.8	43.1	82.9	49.2	42.5	91.8
BC5Y	71.2	17.9	89.1	71.5	11.9	83.4

Table 4: Identification II. Percentage of variance explained by the permanent shock and the transitory shock for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

Variables	P=1		P=2		P=3		P=4	
	Percentage of Explained Cyclical Variance							
	Supply	Dem	Supply	Dem	Supply	Dem	Supply	Dem
GDP	27.1	49.1	26.5	49.7	29.0	47.4	30.4	51.2
Consumption	40.6	29.2	45.6	21.2	50.7	20.1	46.5	25.9
Investment	23.6	55.3	25.2	53.3	30.4	49.9	34.2	49.2
Unemployment	29.0	49.5	31.8	51.4	37.3	44.2	41.8	40.1
Hours Worked	26.3	31.9	23.2	40.1	28.0	32.5	27.3	34.1
Inflation	41.8	44.3	54.2	33.2	57.9	23.1	62.8	16.5
Labor Productivity	22.5	24.4	25.1	30.6	21.9	38.5	15.9	40.7
TFP	21.0	6.4	20.5	8.7	16.6	14.0	12.7	10.9
FFR	13.3	62.2	21.6	55.6	27.0	41.0	32.2	36.8
Baa-GS10	10.8	77.0	14.0	72.9	22.1	60.1	23.4	55.9
S&P500	33.3	23.8	32.3	21.1	26.0	33.6	25.1	35.2
JLN Uncertainty 3M	37.4	45.5	41.5	42.4	44.0	41.3	47.6	38.3
BC5Y	69.1	20.1	68.4	20.9	68.8	19.9	69.4	21.1
	Percentage of Explained Long Run Variance							
	Supply	Dem	Supply	Dem	Supply	Dem	Supply	Dem
GDP	77.7	3.3	69.6	5.2	71.4	2.3	66.5	11.3
Consumption	66.9	8.7	52.8	10.9	57.9	2.8	52.0	9.8
Investment	67.8	4.5	74.5	1.1	77.3	1.1	76.7	4.2
Unemployment	70.9	11.0	81.2	6.0	84.6	4.6	85.7	4.9
Hours Worked	54.7	8.8	50.5	21.2	63.3	13.1	53.9	24.1
Inflation	20.0	65.4	33.6	36.3	26.3	20.8	36.3	13.2
Labor Productivity	60.1	3.3	65.1	0.5	76.4	0.2	74.0	5.0
TFP	65.2	0.9	60.3	0.7	67.1	1.3	63.8	5.5
FFR	2.3	83.6	12.4	66.5	9.2	42.6	18.8	39.0
Baa-GS10	44.0	42.1	23.6	35.3	27.9	14.7	21.2	18.0
S&P500	30.4	0.5	37.5	0.1	43.1	0.8	46.2	1.5
JLN Uncertainty 3M	54.5	37.3	70.8	21.5	68.8	17.6	80.5	9.5
BC5Y	74.8	8.6	85.5	1.0	88.2	1.3	91.8	0.4

Table 5: Identification I: Percentage of variance explained by the supply shock and the demand shock for a few selected variables, by frequency band, according to different lags order: $p = [1\ 2\ 3\ 4]$. Baseline specification: $p = 1$. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

Variables	R=11		R=13		R=15		R=17		R=20	
	Percentage of Explained Cyclical Variance									
	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem
GDP	27.1	49.1	22.2	51.8	23.8	49.0	18.2	50.9	18.0	50.8
Consumption	40.6	29.2	30.4	28.4	31.0	25.4	26.1	28.6	26.6	27.5
Investment	23.6	55.3	24.3	54.1	25.6	52.8	22.1	55.2	21.6	53.7
Unemployment	29.0	49.5	30.6	46.7	30.9	44.6	27.4	51.8	28.2	49.9
Hours Worked	26.3	31.9	19.8	32.3	23.5	28.4	18.2	29.7	19.2	31.5
Inflation	41.8	44.3	45.0	30.6	40.8	29.1	43.5	30.7	44.5	28.9
Labor Productivity	22.5	24.4	18.1	29.0	20.3	27.2	16.6	32.5	17.8	33.8
TFP	21.0	6.4	14.5	5.7	16.4	5.8	20.4	3.9	17.9	3.8
FFR	13.3	62.2	24.9	52.3	23.8	52.9	15.7	47.6	17.0	45.7
Baa-GS10	10.8	77.0	13.0	72.1	13.1	67.4	12.7	49.1	12.6	49.3
S&P500	33.3	23.8	25.6	32.3	26.9	31.7	19.7	38.0	16.7	36.9
JLN Uncertainty 3M	37.4	45.5	43.8	36.7	43.5	36.9	43.2	33.7	42.9	33.7
BC5Y	69.1	20.1	54.1	20.2	47.8	15.5	45.3	14.4	43.7	13.3
	Percentage of Explained Long Run Variance									
	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem
GDP	77.7	3.3	74.7	6.1	75.4	5.6	67.9	4.8	69.1	6.5
Consumption	66.9	8.7	60.9	9.5	61.0	8.3	56.7	8.8	57.0	10.2
Investment	67.8	4.5	68.4	2.5	68.1	2.9	64.9	1.4	64.4	1.7
Unemployment	70.9	11.0	78.0	7.8	73.0	8.6	74.0	10.0	74.8	9.3
Hours Worked	54.7	8.8	52.8	12.6	50.9	11.0	55.8	10.3	53.8	10.9
Inflation	20.0	65.4	22.6	47.7	20.4	48.6	19.1	47.4	20.7	46.0
Labor Productivity	60.1	3.3	62.0	1.4	62.2	1.8	69.8	0.6	70.3	0.2
TFP	65.2	0.9	65.5	0.1	65.3	0.1	70.4	0.3	68.7	0.7
FFR	2.3	83.6	6.0	70.5	5.0	71.6	3.8	69.8	4.9	67.9
Baa-GS10	44.0	42.1	38.5	33.4	37.2	34.1	28.8	23.6	27.6	25.6
S&P500	30.4	0.5	29.7	1.9	29.0	1.8	22.6	1.2	22.4	1.0
JLN Uncertainty 3M	54.5	37.3	67.0	23.7	61.5	25.3	54.3	29.3	57.7	26.5
BC5Y	74.8	8.6	82.7	4.2	79.9	4.8	80.8	4.5	79.8	3.5

Table 6: Identification I: Percentage of variance explained by the Demand shock and the Supply shock for a few selected variables, by frequency band, according to the number of static factors: $r = [11\ 13\ 15\ 17\ 20]$. Baseline specification: $r = 11$ static factors. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

(a) Robustness Identification I: Maximum and Minimum percentage value of explained variance according to different lags order: $p = [\ 1\ 2\ 3\ 4\ ]$. Baseline specification: $p = 1$ lag.

Variables	Cyclical Variance				Long Run Variance			
	Supply		Demand		Supply		Demand	
	Min	Max	Min	Max	Min	Max	Min	Max
GDP	26.5	30.4	47.4	51.2	66.5	77.7	2.3	11.3
Consumption	40.6	50.7	20.1	29.2	52.0	66.9	2.8	10.9
Investment	23.6	34.2	49.2	55.3	67.8	77.3	1.1	4.5
Unemployment	29.0	41.8	40.1	51.4	70.9	85.7	4.6	11.0
Hours Worked	23.2	28.0	31.9	40.1	50.5	63.3	8.8	24.1
Inflation	41.8	62.8	16.5	44.3	20.0	36.3	13.2	65.4
Labor Productivity	15.9	25.1	24.4	40.7	60.1	76.4	0.2	5.0
TFP	12.7	21.0	6.4	14.0	60.3	67.1	0.7	5.5
FFR	13.3	32.2	36.8	62.2	2.3	18.8	39.0	83.6
Baa-GS10	10.8	23.4	55.9	77.0	21.2	44.0	14.7	42.1
S&P500	25.1	33.3	21.1	35.2	30.4	46.2	0.1	1.5
JLN Uncertainty 3M	37.4	47.6	38.3	45.5	54.5	80.5	9.5	37.3
BC5Y	68.4	69.4	19.9	21.1	74.8	91.8	0.4	8.6

(b) Robustness Identification I: Maximum and minimum value of explained variance according to the number of static factors: $r = [\ 11\ 13\ 15\ 17\ 20\ ]$. Baseline specification: $r = 11$ static factors.

Variables	Cyclical Variance				Long Run Variance			
	Supply		Demand		Supply		Demand	
	Min	Max	Min	Max	Min	Max	Min	Max
GDP	18.0	27.1	49.0	51.8	67.9	77.7	3.3	6.5
Consumption	26.1	40.6	25.4	29.2	56.7	66.9	8.3	10.2
Investment	21.6	25.6	52.8	55.3	64.4	68.4	1.4	4.5
Unemployment	27.4	30.9	44.6	51.8	70.9	78.0	7.8	11.0
Hours Worked	18.2	26.3	28.4	32.3	50.9	55.8	8.8	12.6
Inflation	40.8	45.0	28.9	44.3	19.1	22.6	46.0	65.4
Labor Productivity	16.6	22.5	24.4	33.8	60.1	70.3	0.2	3.3
TFP	14.5	21.0	3.8	6.4	65.2	70.4	0.1	0.9
FFR	13.3	24.9	45.7	62.2	2.3	6.0	67.9	83.6
Baa-GS10	10.8	13.1	49.1	77.0	27.6	44.0	23.6	42.1
S&P500	16.7	33.3	23.8	38.0	22.4	30.4	0.5	1.9
JLN Uncertainty 3M	37.4	43.8	33.7	45.5	54.3	67.0	23.7	37.3
BC5Y	43.7	69.1	13.3	20.2	74.8	82.7	3.5	8.6

Table 7: Percentage of variance explained by the supply shock and the demand shock (Identification I) for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

Figures

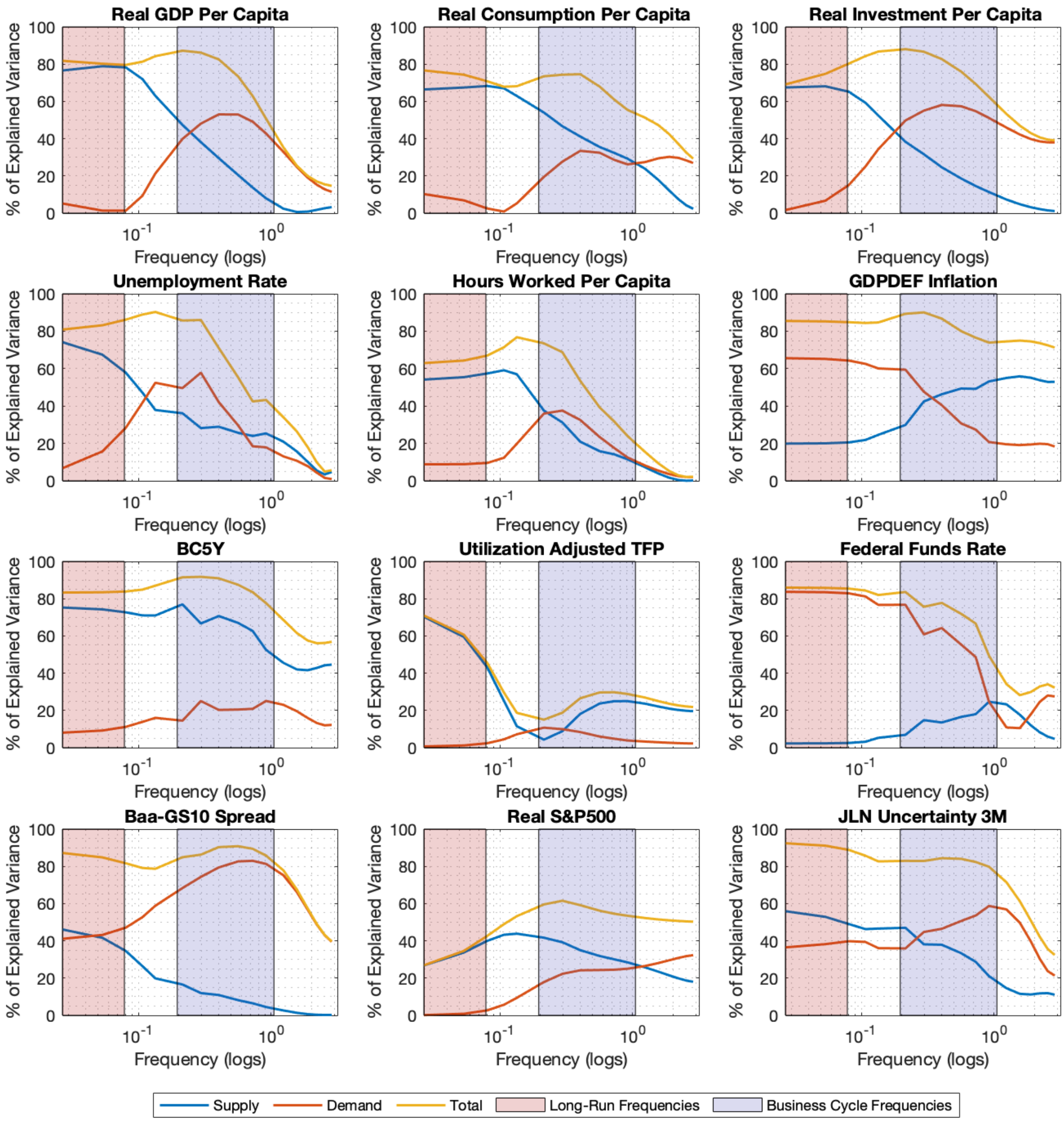

Figure 1: Identification I: Spectral Decomposition for a few selected variables, frequency by frequency. The figure reports the percentage of explained variance. Blue line: Contribution of the supply shock; Red line: Contribution of the demand shock; Yellow line: sum. Pink shadowed area: Long run frequencies (>80 quarters); Lilac shadowed area: Business Cycle frequencies (6-32 quarters).

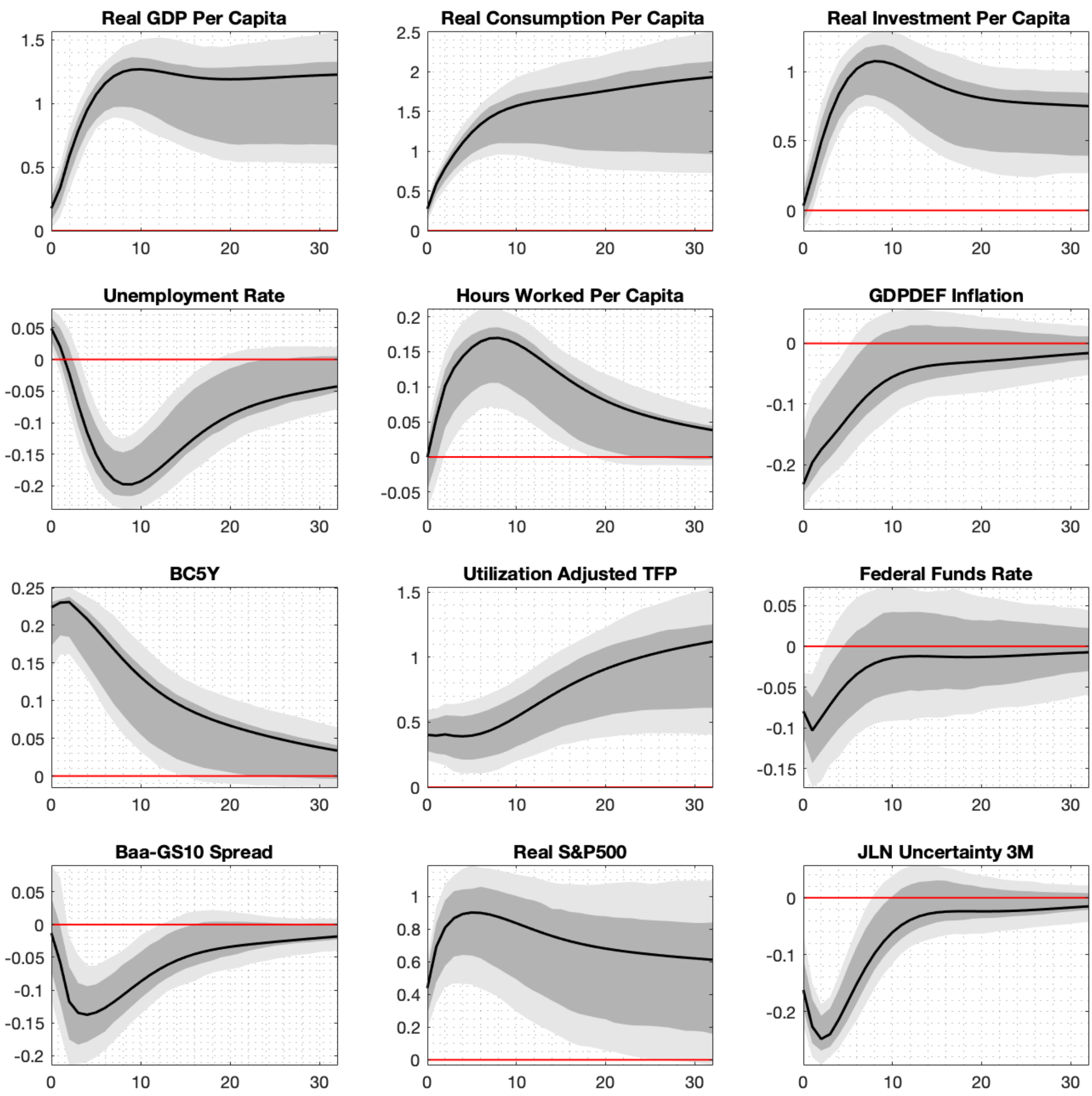

Figure 2: Identification I: Point estimates of the Impulse Response Functions of the Supply Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

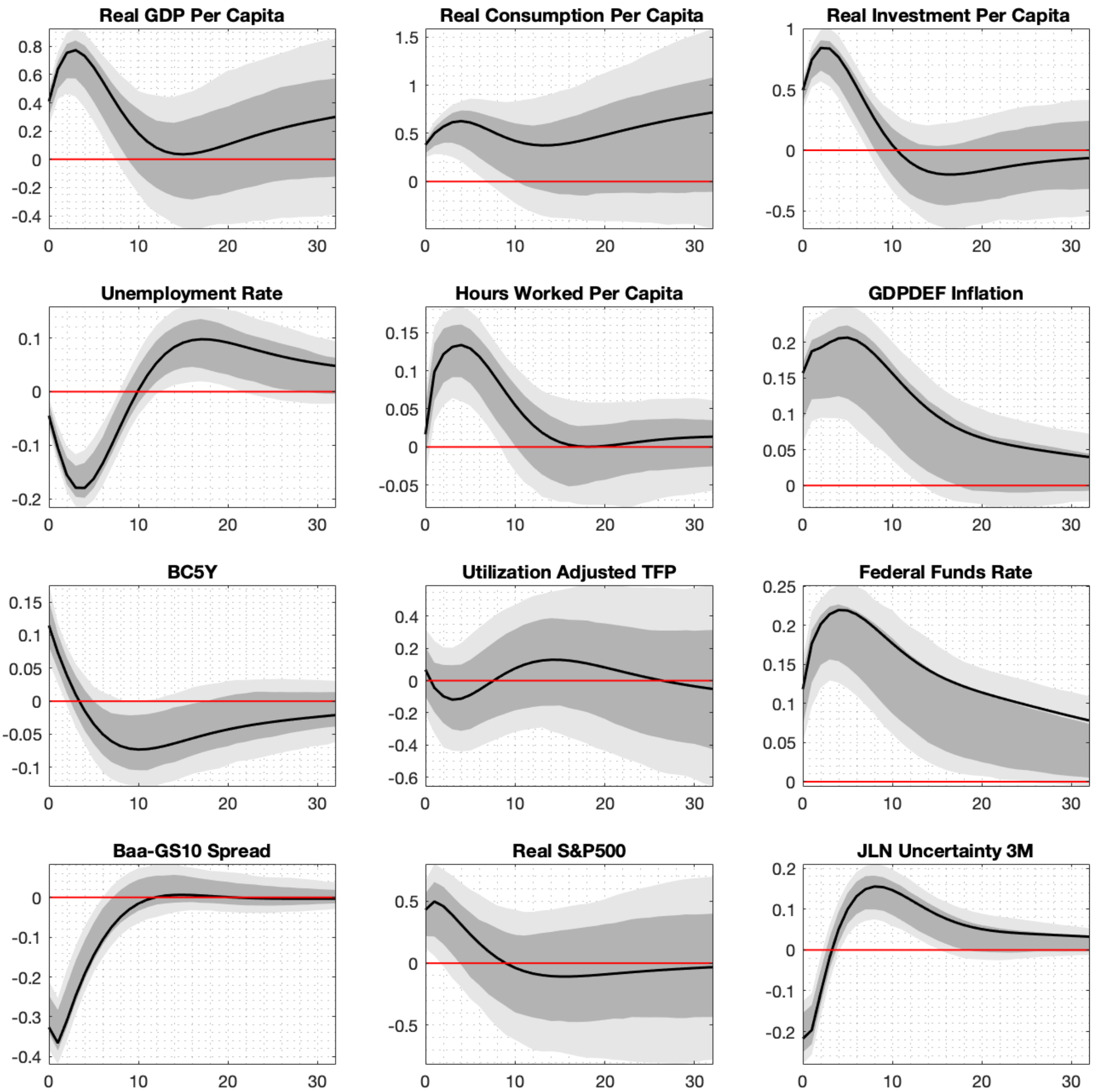

Figure 3: Identification I: Point estimates of the Impulse Response Functions of the Demand Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

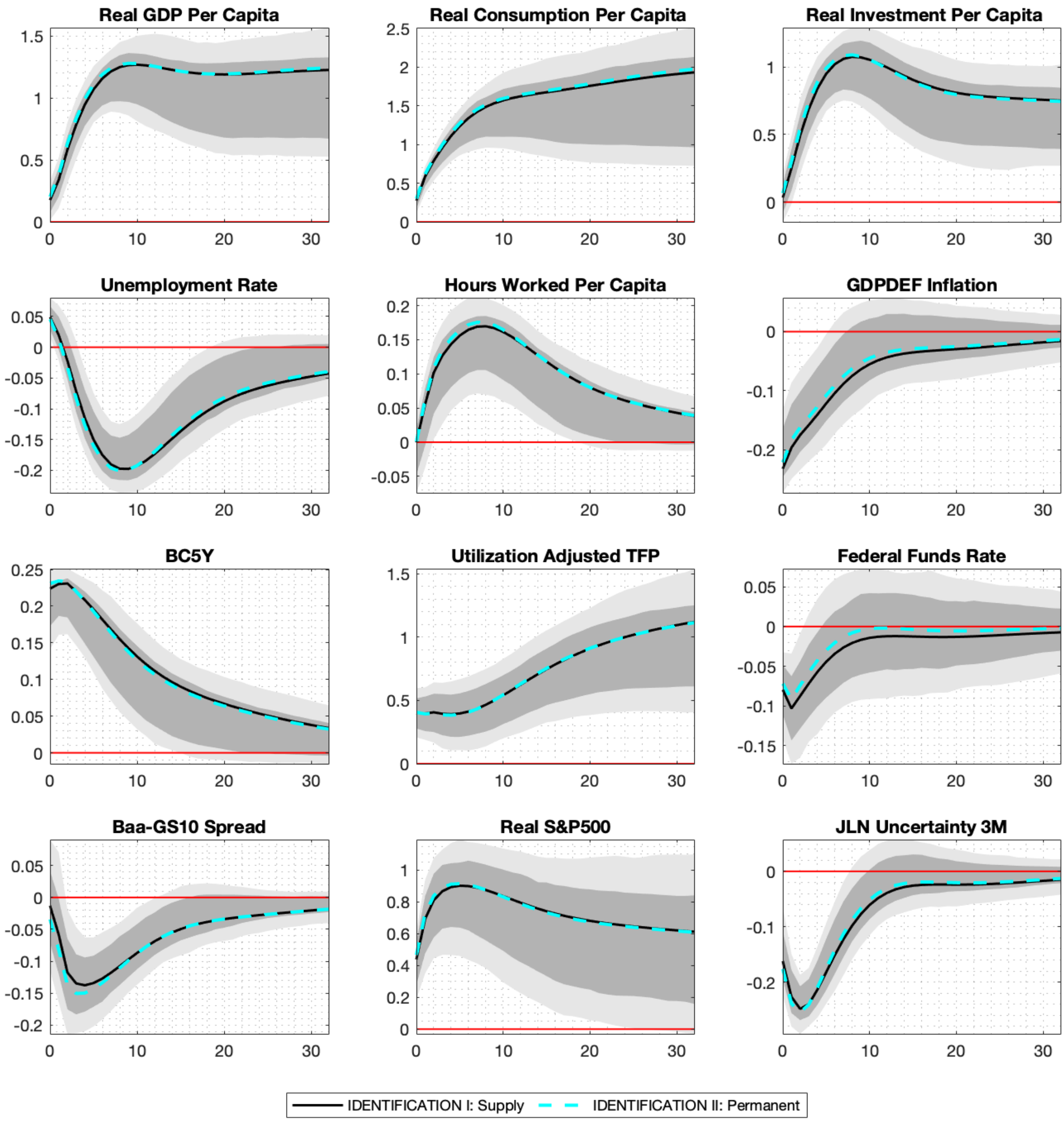

Figure 4: Impulse response functions of the Supply shock (Identification I, black line) and the Permanent shock (Identification II, cyan dashed line). The dark gray and light gray areas are the 68% and 90% confidence bands, respectively, for Identification I.

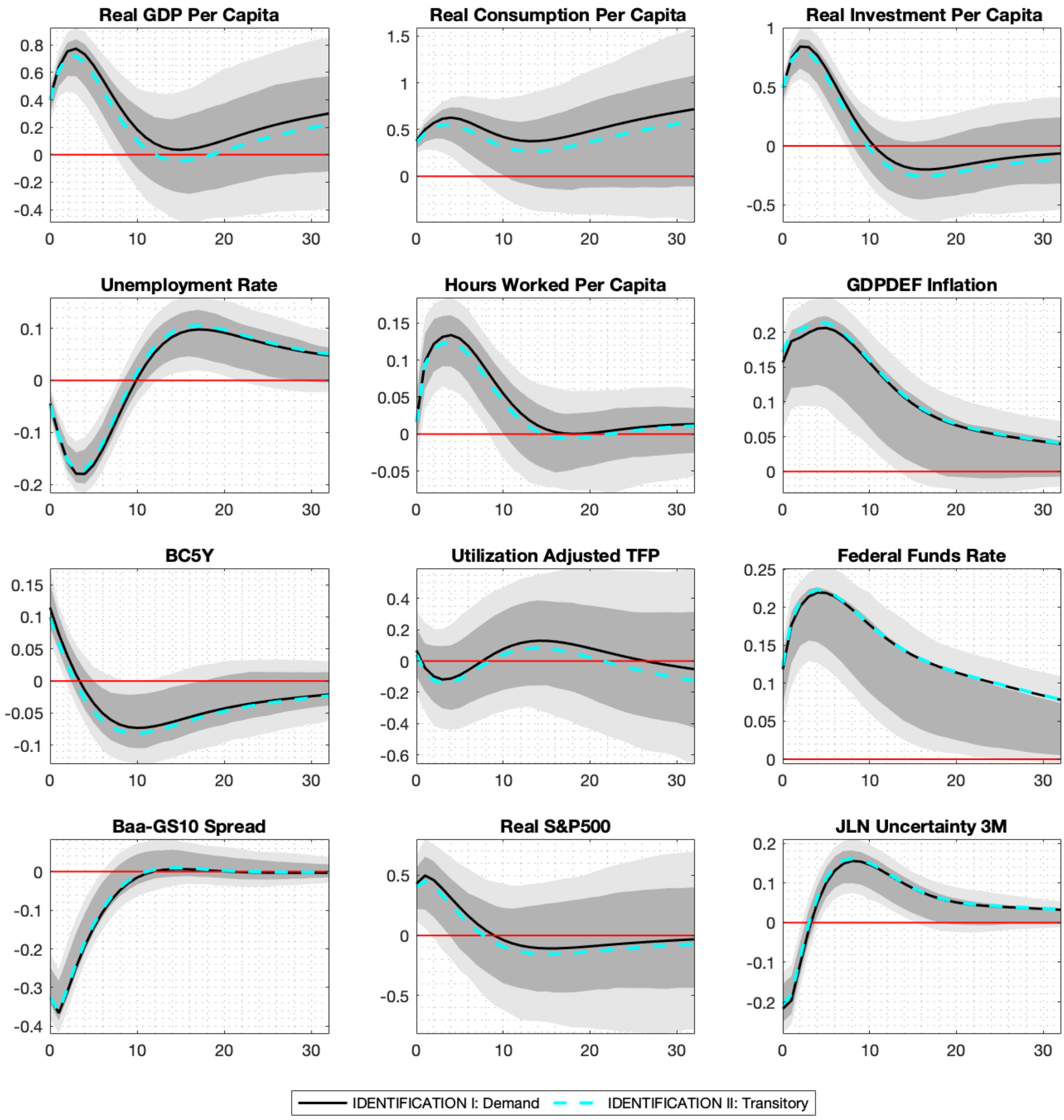

Figure 5: Impulse response functions of the Demand shock (Identification I, black line) and the Transitory shock (Identification II, cyan dashed line). The dark gray and light gray areas are the 68% and 90% confidence bands, respectively, for Identification I.

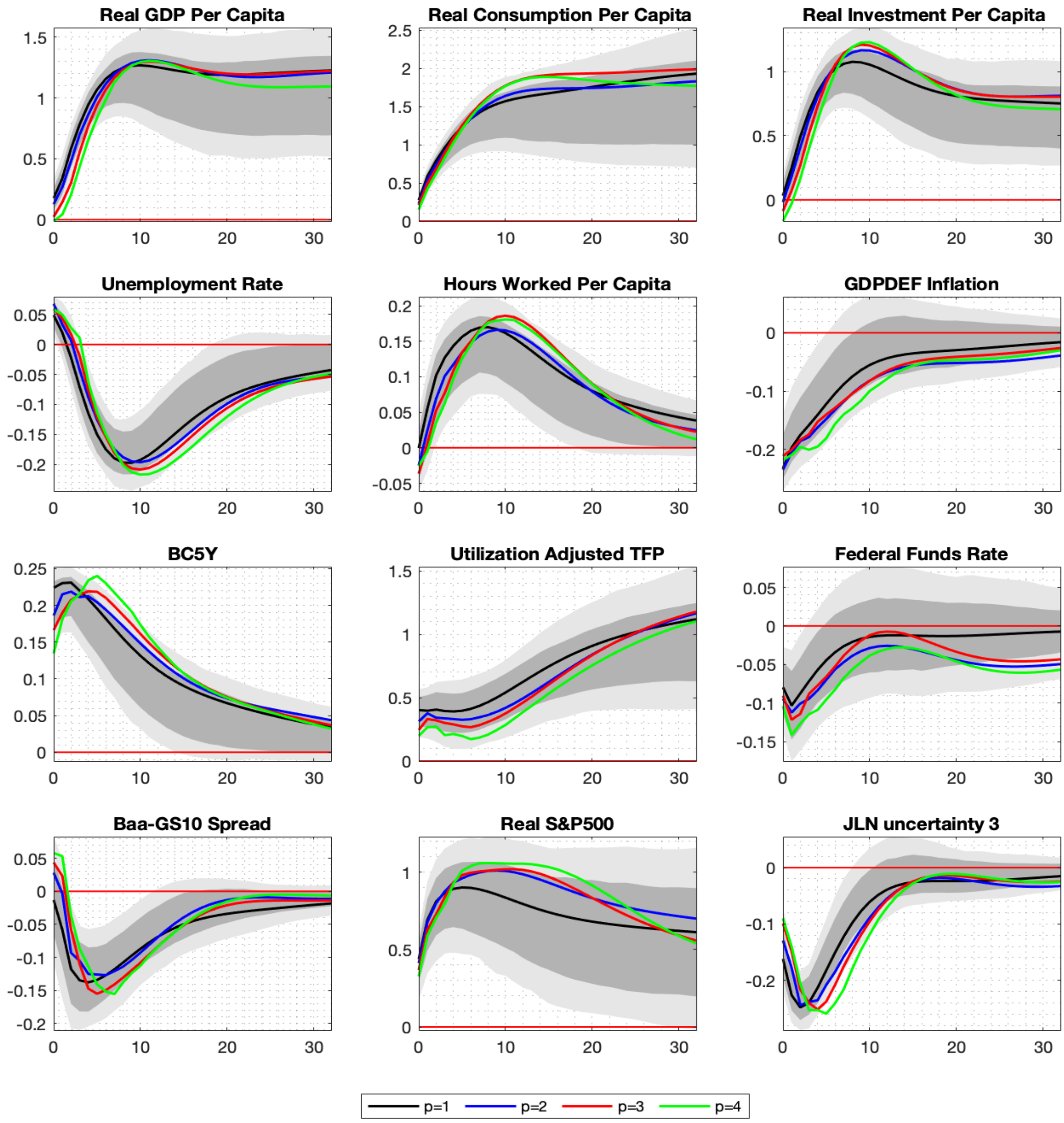

Figure 6: Identification I: Impulse response functions of the Supply shock, according to different lags order: $p = [1\ 2\ 3\ 4]$. Baseline specification: $p = 1$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

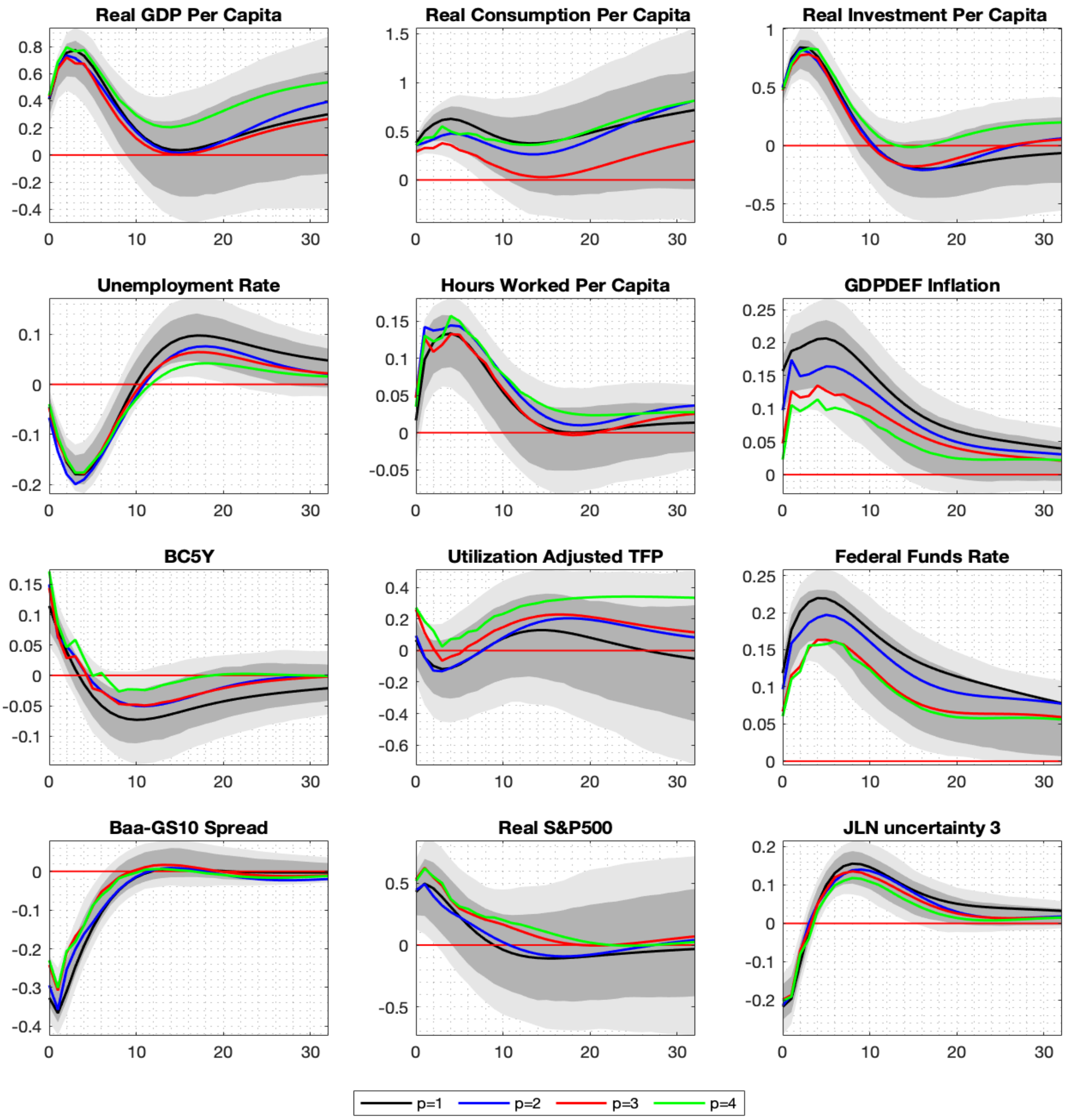

Figure 7: Identification I: Impulse response functions of the Demand shock, according to different lags order: $p = [1\ 2\ 3\ 4]$. Baseline specification: $p = 1$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

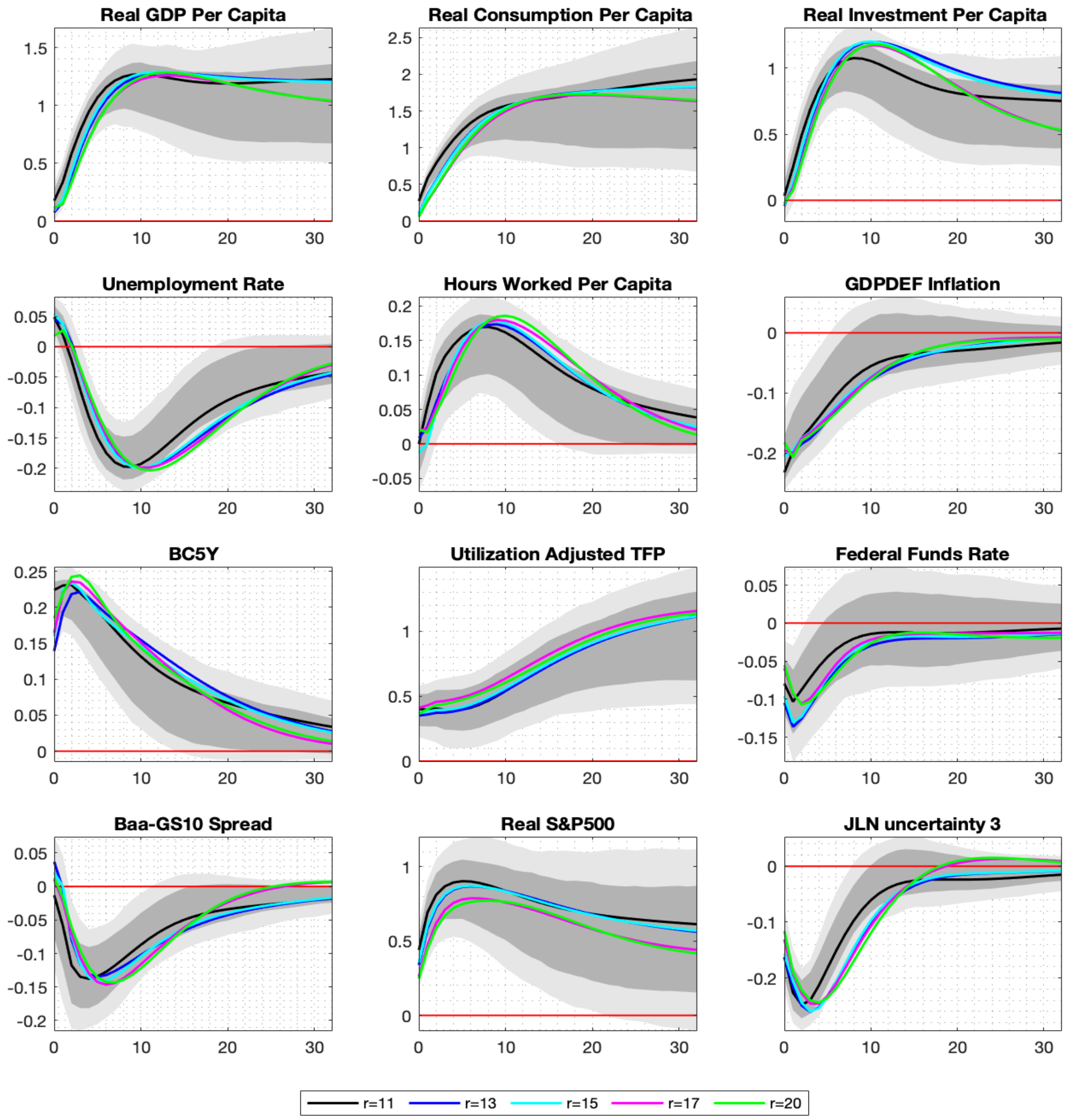

Figure 8: Identification I: Impulse response functions of the Supply shock, according to different number of static factors: $r = [11\ 6\ 9\ 13\ 15]$. Baseline specification: $r = 11$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

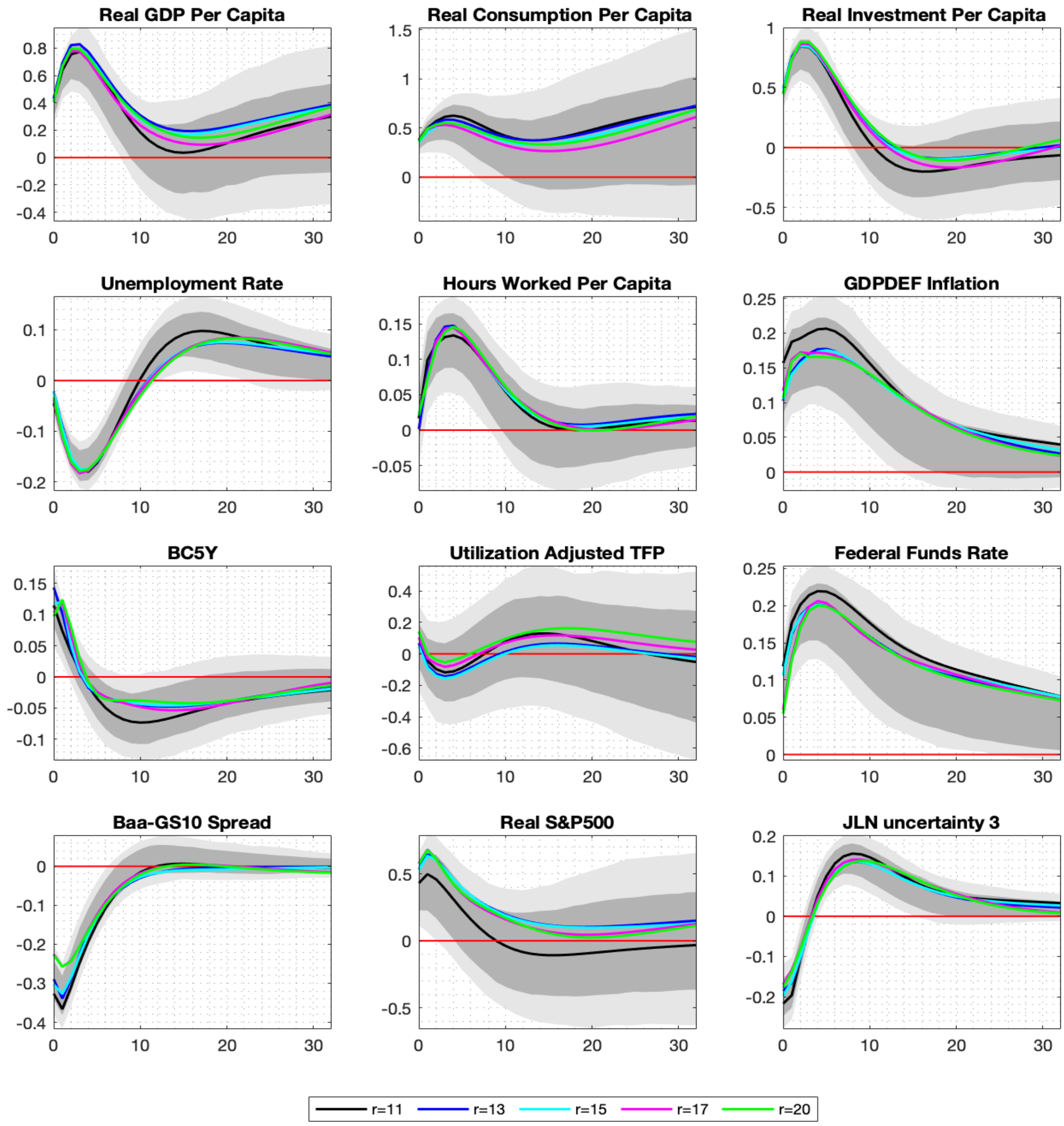

Figure 9: Identification I: Impulse response functions of the Demand shock, according to different number of static factors: $r = [11\ 6\ 9\ 13\ 15]$. Baseline specification: $r = 11$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

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Appendix

A. Rank Reduction Step

In the standard estimation procedure the identification techniques are applied to the residuals of the VAR estimated for F_t after estimating q, the number of common shocks, and the rank reduction. The estimated factors $\hat{F}_t$ are not exactly singular, as they contain a residual of the idiosyncratic components that disappears completely only asymptotically. As a consequence, the vector $\hat{\epsilon}_t$ has rank $r > q$, although the last $r - q$ eigenvalues of $\hat{\Sigma}_\epsilon$ are close to zero (Forni et al., 2020). In the standard procedure, singularity is forced on $\hat{\epsilon}_t$ by means of rank-reduction techniques. In Forni et al. (2009), the rank reduction is obtained by using the spectral decomposition of $\hat{\Sigma}_\epsilon$, so that the vector $\hat{\epsilon}_t$ is replaced by the $\hat{q}$-dimensional vector $V^{-1}\hat{\epsilon}_t$, where V^{-1} is the matrix whose rows are the normalised eigenvectors corresponding to the q-largest eigenvalues of the variance-covariance matrix of $\hat{\epsilon}_t$. This is equivalent to assume that the static rank of the common components is r, which is the rank of its covariance matrix, while the dynamic rank is q, which is the rank of its spectral density. In empirical situation, the number q of dynamic factors or common shocks is unknown and has to be determined by existing information criteria. For instance, the criterion proposed by Hallin and Liška (2007) is based on the properties of dynamic eigenvalues of the data and looks for the value q that minimizes the contribution of the idiosyncratic component. Alternative methods are proposed by Onatski (2009), Amengual and Watson (2007) and Bai and Ng (2007). Recently, Avarucci et al. (2021) introduce a novel consistent criterion to estimate the number of common shocks that can be applied to single frequencies as well as to frequency bands. Such criteria, albeit consistent, often give different results each other.

Forni et al. (2020) shown that the rank reduction step can be ignored with no consequences on the (IRFs) estimation accuracy. Since different information criteria often give different results, the estimation of q and the rank reduction can be a potential source of error, in particular whether $\hat{q}$ underestimates the true q, leading to large estimation errors implied by a possible mis-specification of q. Therefore, we apply the identification techniques to the not exactly singular Cholesky-transformed residuals of the estimated VAR without reducing the rank.

Moreover, by reducing the number of shocks of interest in the first stage of our identification strategy, where we select the two shocks maximizing the explained variance of targeted variables on the band $[0\ 2\pi/6]$, rather than across all frequencies, we do not need to implement the rank reduction step in our estimation procedure.

B. Data Description and Data Treatment

For the description of each variable see McCracken and Ng (2020). For variables not in the FRED-QD dataset, refer to the Mnemonic and note. Treatment codes: 1 = no treatment; 2 = first difference, Δx_t; 4 = $\log(x_t)$; 5 = log of the first difference, $\Delta \log(x_t)$.

ID	FRED-QD ID	Mnemonic	Treatment code	Note
1	1	GDPC1/CNP16OV	5	
2	2	PCECC96/CNP16OV	5	
3	3	PCDGx/CNP16OV	5	
4	4	PCESVx/CNP16OV	5	
5	5	PCNDx/CNP16OV	5	
6	6	GPDIC1/CNP16OV	5	
7	7	FPIx/CNP16OV	5	
8	8	Y033RC1Q027SBEAx/CNP16OV	5	
9	9	PNFIx/CNP16OV	5	
10	10	PRFIx/CNP16OV	5	
11	11	A014RE1Q156NBEA	1	
12	12	GCEC1/CNP16OV	5	
13	13	A823RL1Q225SBEA	1	
14	14	FGRECPTx/CNP16OV	5	
15	15	SLCEx/CNP16OV	5	
16	16	EXPGSC1/CNP16OV	5	
17	17	IMPGSC1/CNP16OV	5	
18	18	DPIC96/CNP16OV	5	
19	19	OUTNFB/CNP16OV	5	
20	20	OUTBS/CNP16OV	5	
21		(PCESVx+PCNDx)/CNP16OV	5	
22		(PCDGx+FPIx)/CNP16OV	5	
23	22	INDPRO/CNP16OV	5	
24	23	IPFINAL/CNP16OV	5	
25	24	IPCONGD/CNP16OV	5	
26	25	IPMAT/CNP16OV	5	
27	28	IPDCONGD/CNP16OV	5	
28	30	IPNCONGD/CNP16OV	5	
29	31	IPBUSEQ/CNP16OV	5	
30	35	PAYEMS/CNP16OV	2	
31	36	USPRIV/CNP16OV	2	
32	38	SRVPRD/CNP16OV	2	
33	39	USGOOD/CNP16OV	2	
34	51	USGOVT/CNP16OV	2	
35	57	CE16OV/CNP16OV (EMRATIO)	2	
36	58	CIVPART	2	
37	59	UNRATE	1	
38	60	UNRATESTx	1	
39	61	UNRATELTx	1	
40	62	LNS14000012	1	
41	63	LNS14000025	1	
42	64	LNS14000026	1	
43	74	HOABS/CNP16OV	4	
44	76	HOANBS/CNP16OV	4	
45	77	AWHMAN	1	
46	79	AWOTMAN	1	
47	81	HOUST/CNP160V	5	
48	95	PCECTPI	5	
49	96	PCEPILFE	5	

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ID	FRED-QD ID	Mnemonic	Treatment code	Note
50		GDPDEF	5	GDP: Implicit Price Deflator
51	97	GDPCTPI	5	
52	98	GPDICTPI	5	
53	120	CPIAUCSL	5	
54	121	CPILFESL	5	
55	122	WPSFD49207	5	
56	123	PPIACO	5	
57	124	WPSFD49502	5	
58	126	PPIIDC	5	
59	129	WPU0561	5	
60	130	OILPRICEx	5	
61	135	COMPRNFB	5	
62	138	OPHNFB	5	
63	139	OPHPBS	5	
64	140	ULCBS	5	
65	142	ULCNFB	5	
66	143	UNLPNBS	5	
67		dtfp	1	Fernald's TFP growth
68		dtfp util	1	Fernald's TFP growth CU adjusted
69		dtfp I	1	Fernald's TFP growth - Inv
70		dtfp C	1	Fernald's TFP growth - Con
71		dtfp I util	1	Fernald's TFP growth CU - Inv
72		dtfp C util	1	Fernald's TFP growth CU - Con
73	144	FEDFUNDS	1	
74	145	TB3MS	1	
75	146	TB6MS	1	
76	147	GS1	1	
77	148	GS10	1	
78	150	AAA	1	
79	151	BAA	1	
80	152	BAA10YM	1	
81	156	GS10TB3Mx	1	
82		BAA-AAA	1	
83		GS10-FEDFUNDS	1	
84		GS1-FEDFUNDS	1	
85		BAA-FEDFUNDS	1	
86	158	BOGMBASEREALx/CNP16OV	5	
87	160	M1REAL/CNP16OV	5	
88	161	M2REAL/CNP16OV	5	
89	163	BUSLOANSx/CNP16OV	5	
90	164	CONSUMERx/CNP16OV	5	
91	166	REALLNx/CNP16OV	5	
92	168	TOTALSLx/CNP16OV	5	
93	188	UMCSENTx	1	
94		Business Condition 12 Months	1	Michigan Consumer Survey
95		Business Condition 5 Years	1	Michigan Consumer Survey
96		Current Index	1	Michigan Consumer Survey
97		Expected Index	1	Michigan Consumer Survey
98		News Index: Relative	1	Michigan Consumer Survey
99	197	UEMPMEAN	1	
100	201	GS5	1	
101	210	CUSR0000SAC	5	
102	211	CUSR0000SAD	5	
103	212	CUSR0000SAS	5	
104	213	CPIULFSL	5	
105	245	S&P 500	5	

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ID	FRED-QD ID	Mnemonic	Treatment code	Note
106	246	S&P: indust	5	
107		S&P 500/GDPDEF	5	
108		S&P: indust/GDPDEF	5	
109		JLN Macro Unc 1-month	1	JLN Uncertainty
110		JLN Macro Unc 3-month	1	JLN Uncertainty
111		JLN Macro Unc 12-month	1	JLN Uncertainty
112		DPCCRC1Q027SBEAx/CNP16OV	5	Real PCE Excluding food and energy
113		DFXARC1M027SBEAx/CNP16OV	5	Real PCE: Food
114		DNRGRC1Q027SBEAx/CNP16OV	5	Real PCE: Energy goods

C. Additional Results and Robustness

Tables

Frequencies	DDR	DGR	DER
$0 \leq \omega \leq 2\pi/6$	2	2	1
$0 \leq \omega \leq 2\pi/8$	2	2	1
$0 \leq \omega \leq \pi$	2	1	1

Table C.1: Number of estimated dynamic factors by DDR, DGR and DER evaluated at selected frequencies or frequency bands. The size of the spectral window - bandwidth parameter - is $M_T = a\sqrt{T}$ with $a = 0.5$. DDR: Dynamic Difference Ratio Estimator; DGR: Dynamic Growth Ratio Estimator; DER: Dynamic Eigenvalue Ratio Estimator.

Variables	P=1		P=2		P=3		P=4	
	Percentage of Explained Cyclical Variance							
	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans
GDP	29.6	46.6	29.6	46.6	28.8	47.7	27.1	54.5
Consumption	43.9	25.9	50.4	16.5	52.7	18.1	51.3	21.0
Investment	25.5	53.4	27.1	51.4	30.0	50.3	30.2	53.2
Unemployment	30.1	48.4	31.8	51.4	36.4	45.1	36.4	45.5
Hours Worked	29.2	29.0	27.7	35.6	29.6	31.0	31.7	29.6
Inflation	37.2	48.8	44.0	43.4	52.1	28.9	43.6	35.7
Labor Productivity	23.0	23.9	26.1	29.5	22.0	38.5	17.4	39.2
TFP	20.7	6.7	19.1	10.0	17.3	13.3	16.3	7.3
FFR	10.9	64.5	15.0	62.2	23.1	44.9	17.9	51.2
Baa-GS10	12.9	74.8	16.2	70.7	21.6	60.6	20.0	59.3
S&P500	36.2	20.9	37.2	16.2	29.5	30.1	35.4	24.9
JLN Uncertainty 3M	39.8	43.1	43.7	40.2	44.0	41.2	45.0	40.9
BC5Y	71.2	17.9	72.5	16.8	70.6	18.1	71.0	19.5
	Percentage of Explained Long Run Variance							
	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans
GDP	79.3	1.7	72.9	2.0	73.0	0.7	76.5	1.4
Consumption	69.7	5.9	57.6	6.1	59.2	1.5	59.4	2.4
Investment	67.2	5.1	73.3	2.2	76.9	1.5	79.6	1.3
Unemployment	68.7	13.2	80.1	7.1	83.9	5.3	83.9	6.7
Hours Worked	57.3	6.3	58.8	12.9	68.0	8.4	71.3	6.7
Inflation	15.5	69.9	24.2	45.8	22.0	25.0	22.3	27.2
Labor Productivity	58.3	5.1	64.1	1.5	76.4	0.2	78.9	0.1
TFP	64.5	1.6	60.2	0.7	67.4	0.9	68.3	1.1
FFR	0.9	85.0	5.6	73.3	5.9	45.8	5.6	52.2
Baa-GS10	49.2	36.9	31.6	27.3	31.2	11.3	31.5	7.7
S&P500	30.2	0.7	36.9	0.8	43.6	0.2	46.2	1.6
JLN Uncertainty 3M	49.2	42.5	59.8	32.4	62.5	23.8	61.0	29.0
BC5Y	71.5	11.9	81.6	4.9	85.7	3.8	86.8	5.4

Table C.2: Identification II: Percentage of variance explained by the permanent shock and the transitory shock for a few selected variables, by frequency band, according to different lags order: $p = [1\ 2\ 3\ 4]$. Baseline specification: $p = 1$. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

Variables	R=11		R=13		R=15		R=17		R=20	
	Percentage of Explained Cyclical Variance									
	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans
GDP	29.6	46.6	24.0	50.1	25.7	47.1	18.7	50.4	19.0	49.9
Consumption	43.9	25.9	33.3	25.4	33.3	23.1	27.1	27.6	28.3	25.8
Investment	25.5	53.4	25.5	53.0	26.8	51.6	22.4	54.9	21.9	53.4
Unemployment	30.1	48.4	30.7	46.5	31.3	44.1	26.9	52.3	27.3	50.8
Hours Worked	29.2	29.0	23.3	28.8	26.2	25.6	20.5	27.5	21.9	28.8
Inflation	37.2	48.8	36.9	38.6	34.4	35.5	36.7	37.5	34.5	38.8
Labor Productivity	23.0	23.9	18.3	28.8	20.4	27.1	17.3	31.9	18.8	32.8
TFP	20.7	6.7	13.6	6.6	15.4	6.8	19.9	4.5	17.3	4.4
FFR	10.9	64.5	18.3	58.9	18.1	58.6	11.6	51.8	10.7	52.0
Baa-GS10	12.9	74.8	14.4	70.7	14.2	66.3	13.7	48.2	13.7	48.2
S&P500	36.2	20.9	31.1	26.8	31.9	26.7	23.3	34.3	21.8	31.9
JLN Uncertainty 3M	39.8	43.1	46.7	33.8	45.8	34.6	44.3	32.5	44.2	32.4
BC5Y	71.2	17.9	56.1	18.1	50.1	13.1	47.7	12.1	46.8	10.2
	Percentage of Explained Long Run Variance									
	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans
GDP	79.3	1.7	78.7	2.1	78.8	2.1	70.3	2.5	73.2	2.4
Consumption	69.7	5.9	65.6	4.8	64.9	4.4	59.9	5.6	62.0	5.2
Investment	67.2	5.1	68.3	2.5	68.0	3.0	64.6	1.7	64.8	1.4
Unemployment	68.7	13.2	74.8	11.0	70.2	11.4	71.4	12.6	70.9	13.2
Hours Worked	57.3	6.3	58.4	7.0	55.5	6.4	60.1	6.0	59.9	4.8
Inflation	15.5	69.9	15.1	55.3	14.0	54.9	13.4	53.1	12.3	54.4
Labor Productivity	58.3	5.1	59.4	4.0	59.8	4.2	68.1	2.3	68.0	2.5
TFP	64.5	1.6	64.4	1.2	64.3	1.2	70.1	0.6	68.7	0.8
FFR	0.9	85.0	2.1	74.5	1.8	74.8	1.3	72.3	1.0	71.8
Baa-GS10	49.2	36.9	46.5	25.4	44.4	26.9	33.9	18.5	35.3	17.9
S&P500	30.2	0.7	31.0	0.6	30.2	0.7	23.4	0.4	23.1	0.3
JLN Uncertainty 3M	49.2	42.5	57.9	32.8	53.8	33.1	47.1	36.5	47.1	37.2
BC5Y	71.5	11.9	77.5	9.4	75.4	9.2	76.8	8.5	74.2	9.1

Table C.3: Identification II: Percentage of variance explained by the Transitory shock and the Permannent shock for a few selected variables, by frequency band, according to the number of static factors: $r = [11\ 13\ 15\ 17\ 20]$. Baseline specification: $r = 11$ static factors. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

(a) Identification II: Maximum and Minimum percentage value of explained variance according to different lags order: $p = [\ 1\ 2\ 3\ 4\ ]$. Baseline specification: $p = 1$ lag.

Variables	Cyclical Variance				Long Run Variance			
	Perm		Trans		Perm		Trans	
	Min	Max	Min	Max	Min	Max	Min	Max
GDP	27.1	29.6	46.6	54.4	72.9	79.3	0.7	2.0
Consumption	43.9	52.7	16.5	25.9	57.6	69.7	1.5	6.1
Investment	25.5	30.2	50.3	53.4	67.2	79.6	1.3	5.1
Unemployment	30.1	36.4	45.1	51.4	68.7	83.9	5.3	13.2
Hours Worked	27.7	31.7	29.0	35.6	57.3	71.3	6.3	12.9
Inflation	37.2	52.1	28.9	48.8	15.5	24.2	25.0	69.9
Labor Productivity	17.4	26.1	23.9	39.2	58.3	78.9	0.1	5.1
TFP	16.3	20.7	6.7	13.3	60.2	68.3	0.7	1.6
FFR	10.9	23.1	44.9	64.5	0.9	5.9	45.8	85.0
Baa-GS10	12.9	21.6	59.3	74.8	31.2	49.2	7.7	36.9
S&P500	29.5	37.2	16.2	30.1	30.2	46.2	0.2	1.6
JLN Uncertainty 3M	39.8	45.0	40.2	43.1	49.2	62.5	23.8	42.5
BC5Y	70.6	72.5	16.8	19.5	71.5	86.8	3.8	11.9

(b) Identification II: Maximum and minimum value of explained variance according to the number of static factors: $r = [\ 11\ 13\ 15\ 17\ 20\ ]$. Baseline specification: $r = 11$ static factors.

Variables	Cyclical Variance				Long Run Variance			
	Perm		Trans		Perm		Trans	
	Min	Max	Min	Max	Min	Max	Min	Max
GDP	18.7	29.6	46.6	50.4	70.3	79.3	1.7	2.5
Consumption	27.1	43.9	23.1	27.6	59.9	69.7	4.4	5.9
Investment	21.9	26.8	51.6	54.9	64.6	68.3	1.4	5.1
Unemployment	26.9	31.3	44.1	52.3	68.7	74.8	11.0	13.2
Hours Worked	20.5	29.2	25.6	29.0	55.5	60.1	4.8	7.0
Inflation	34.4	36.9	35.5	48.8	12.3	15.5	53.1	69.9
Labor Productivity	17.3	23.0	23.9	32.8	58.3	68.1	2.3	5.1
TFP	13.6	20.7	4.4	6.8	64.3	70.1	0.6	1.6
FFR	10.7	18.3	51.8	64.5	0.9	2.1	71.8	85.0
Baa-GS10	12.9	14.4	48.2	74.8	33.9	49.2	17.9	36.9
S&P500	21.8	36.2	20.9	34.3	23.1	31.0	0.3	0.7
JLN Uncertainty 3M	39.8	46.7	32.4	43.1	47.1	57.9	42.5	32.8
BC5Y	46.8	71.2	10.2	18.1	71.5	77.5	8.5	11.9

Table C.4: Percentage of variance explained by the permanet shock and the transitory shock (Identification II) for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \leq \omega \leq 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

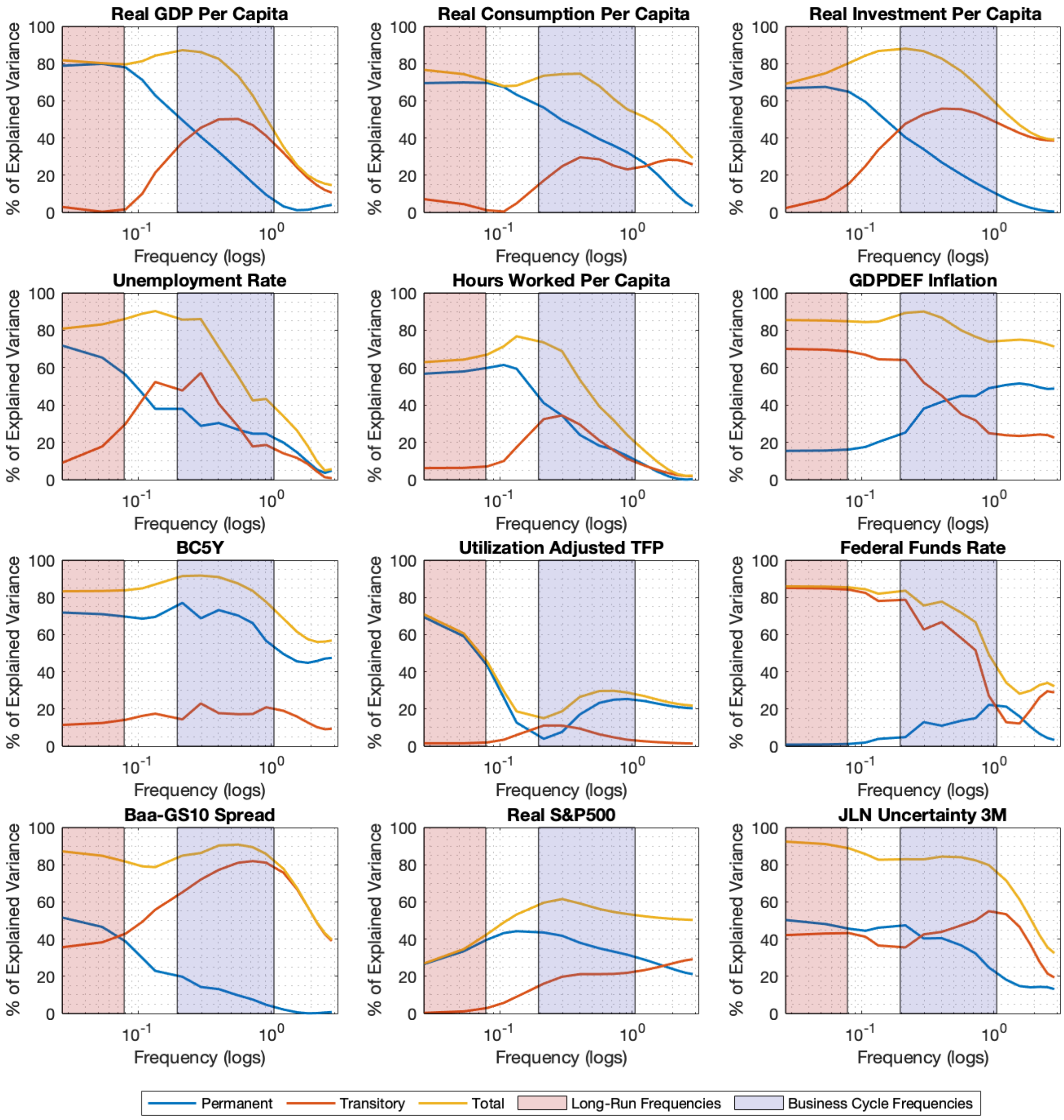

Figure C.1: IDENTIFICATION II: SPECTRAL DECOMPOSITION FOR A FEW SELECTED VARIABLES, FREQUENCY BY FREQUENCY. THE FIGURE REPORTS THE PERCENTAGE OF EXPLAINED VARIANCE. BLUE LINE: CONTRIBUTION OF THE PERMANENT SHOCK; RED LINE: CONTRIBUTION OF THE TRANSITORY SHOCK; YELLOW LINE: SUM. PINK SHADOWED AREA: LONG RUN FREQUENCIES (>80 QUARTERS); LILAC SHADOWED AREA: BUSINESS CYCLE FREQUENCIES (6-32 QUARTERS).

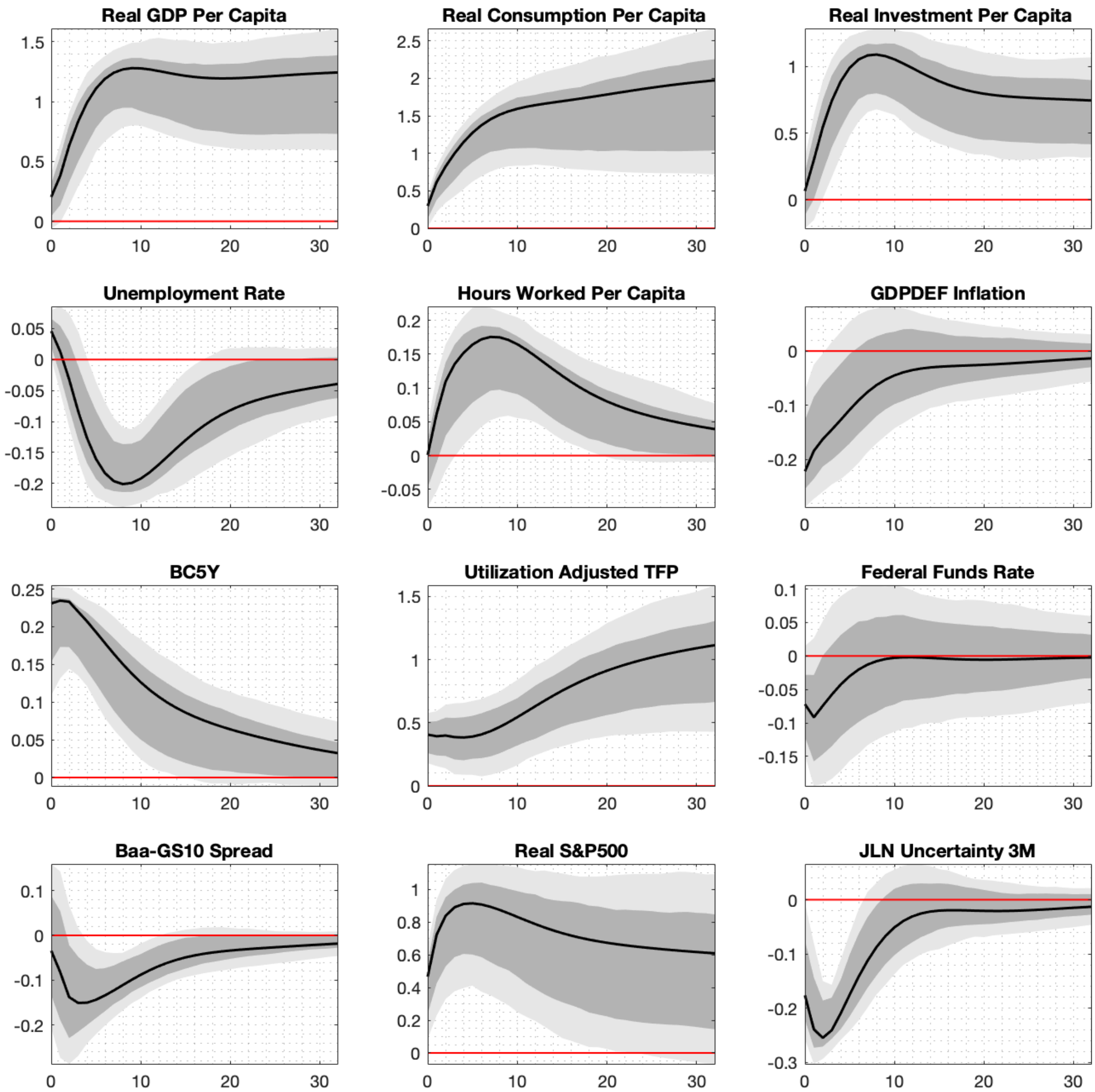

Figure C.2: Identification II: Point estimates of the Impulse Response Functions of the Permanent Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

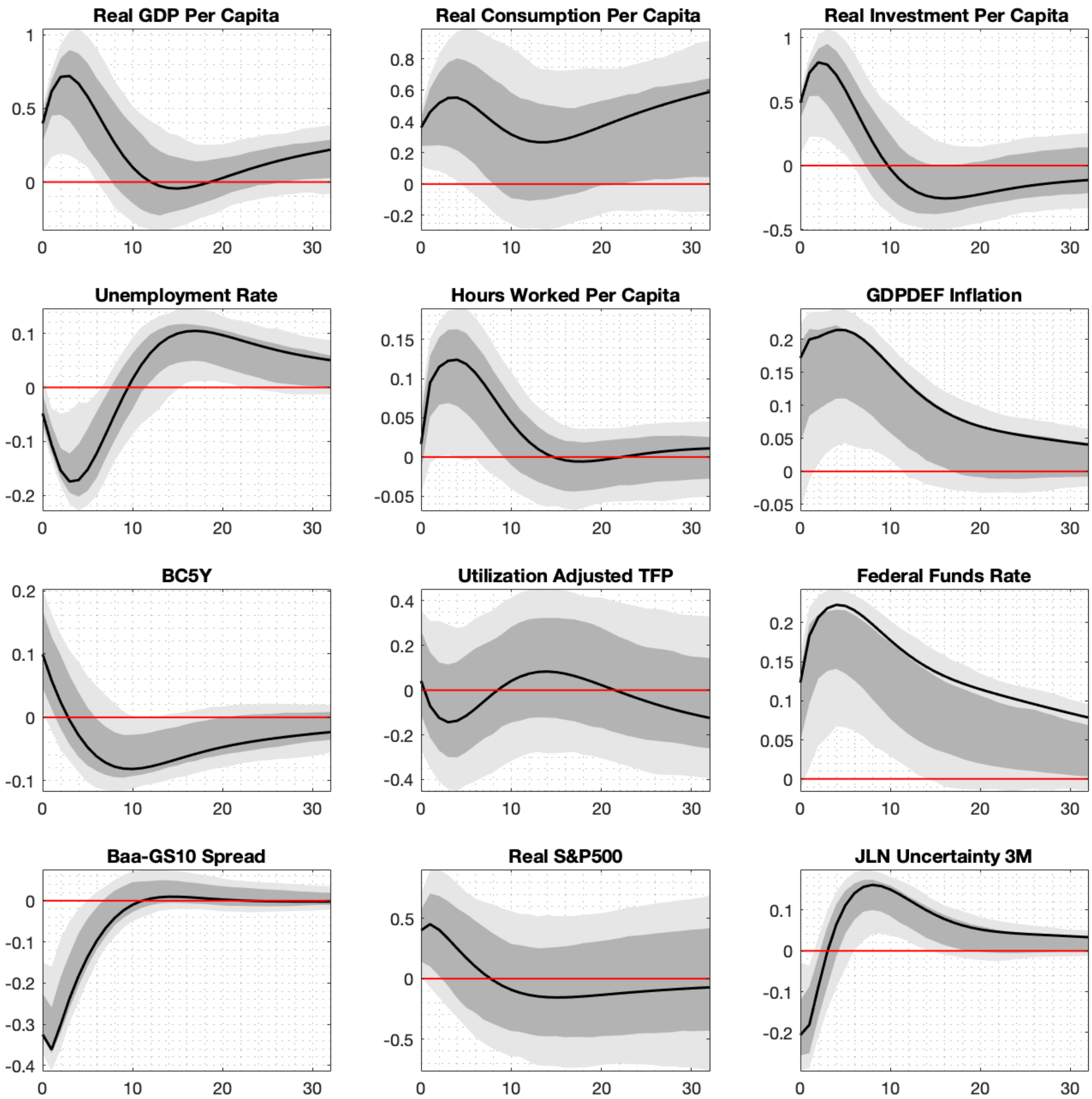

Figure C.3: Identification II: Point estimates of the Impulse Response Functions of the Transitory Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

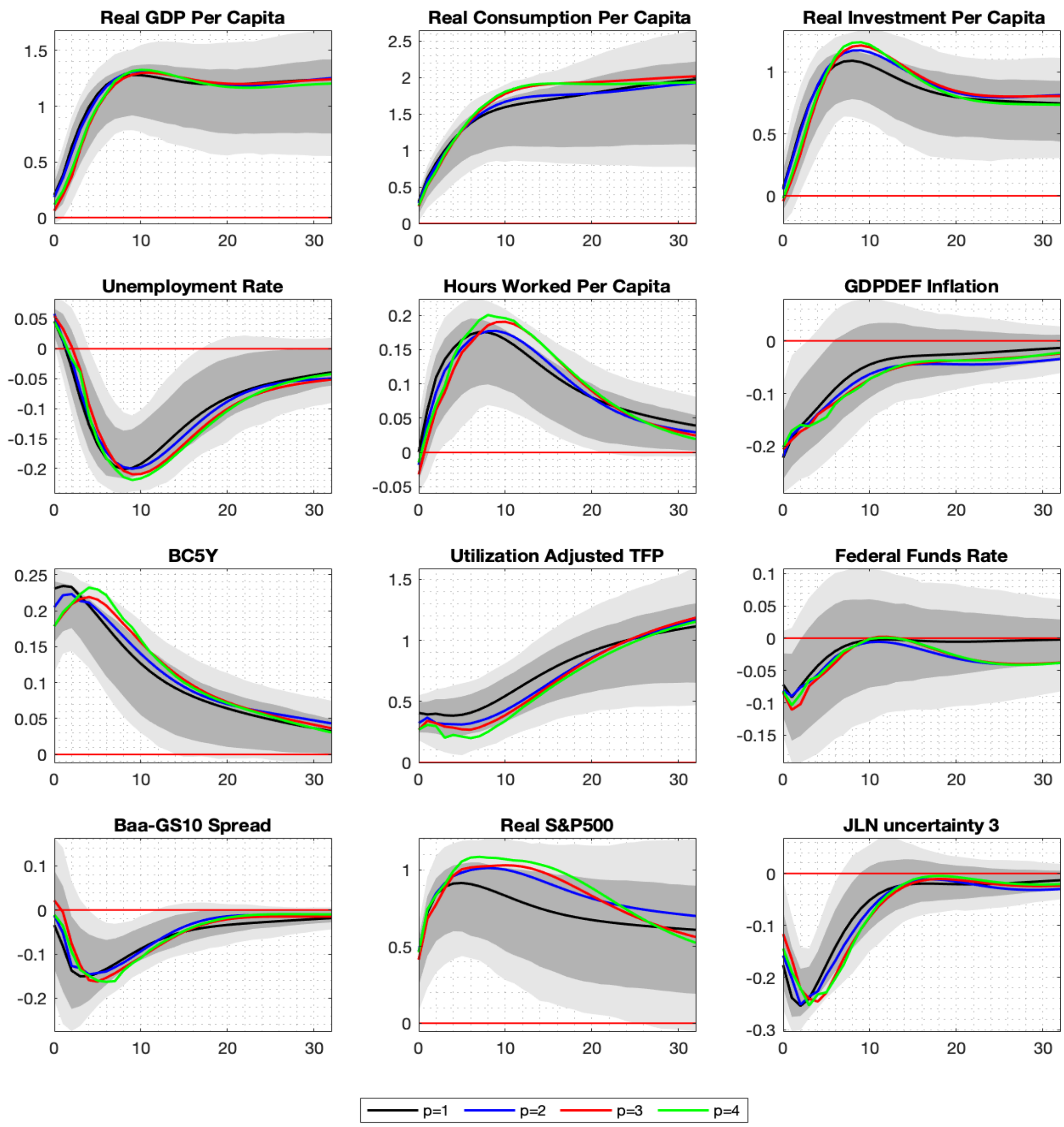

Figure C.4: Identification II: Impulse response functions of the Permanent shock, according to different lags order: $p = [1\ 2\ 3\ 4]$. Baseline specification: $p = 1$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

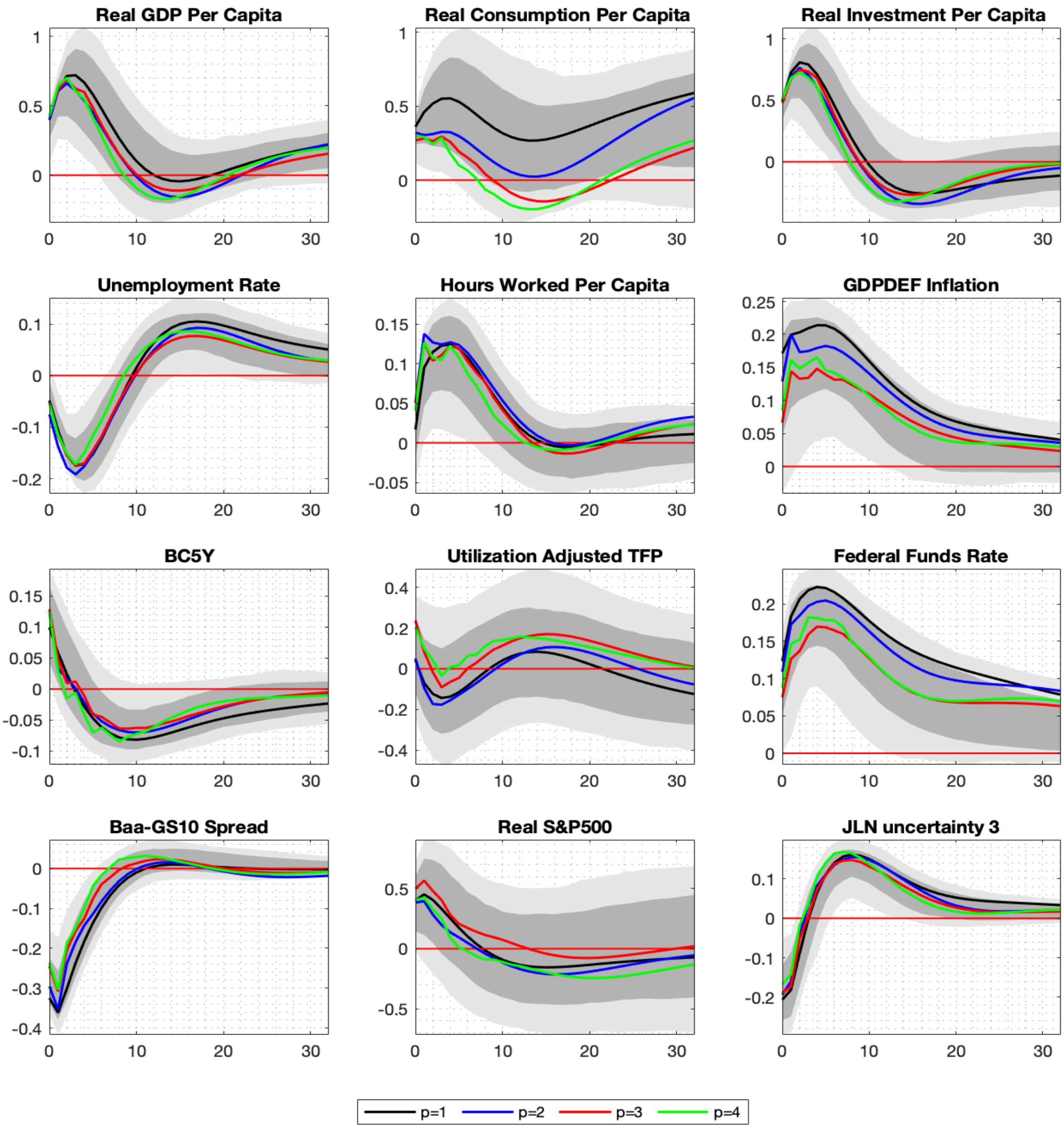

Figure C.5: Identification II: Impulse response functions of the Transitory shock, according to different lags order: $p = [1\ 2\ 3\ 4]$. Baseline specification: $p = 1$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

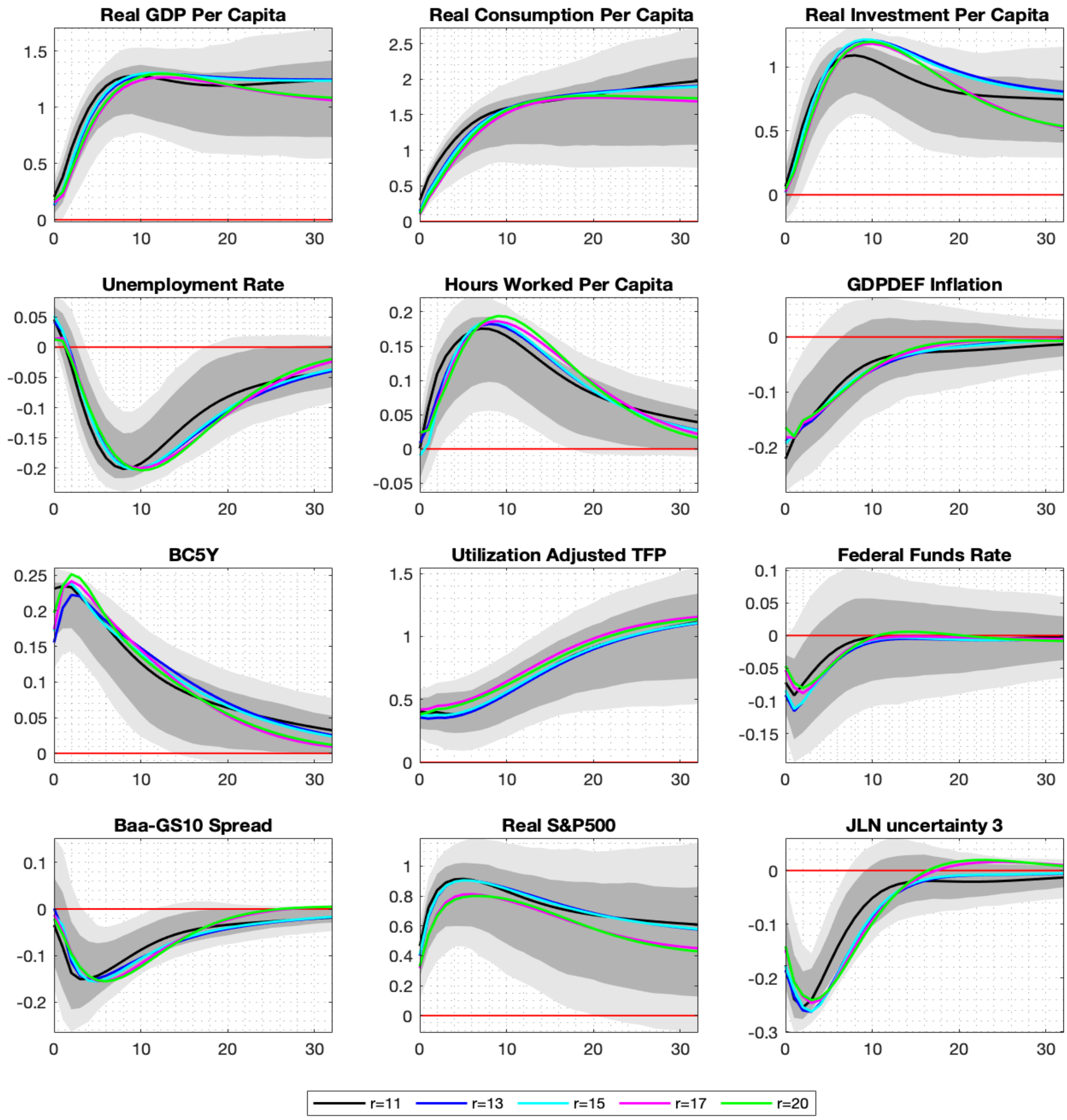

Figure C.6: Identification II: Impulse response functions of the Permanent shock, according to different number of static factors: $r = [11\ 6\ 9\ 13\ 15]$. Baseline specification: $r = 11$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

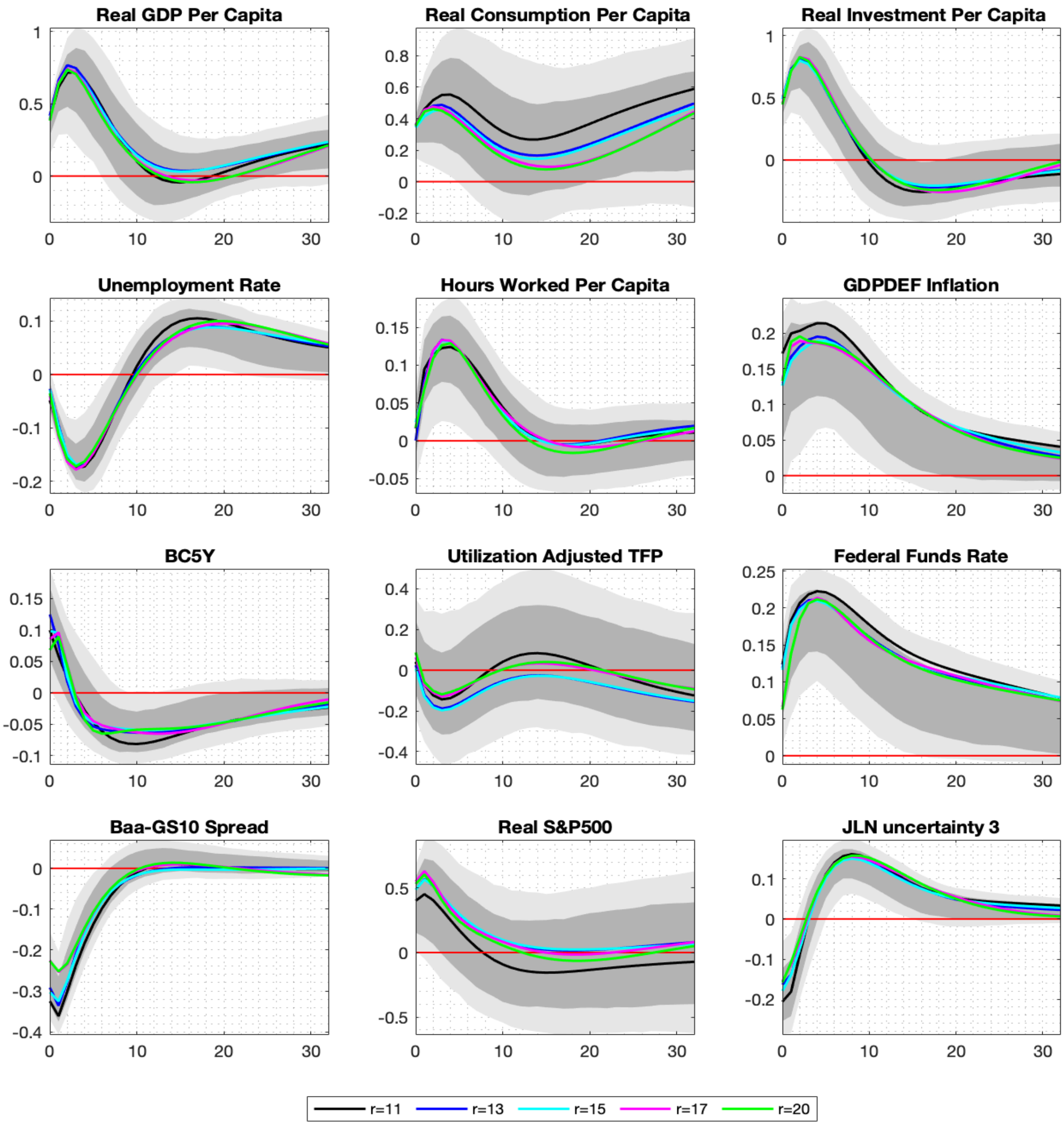

Figure C.7: Identification II: Impulse response functions of the Transitory shock, according to different number of static factors: $r = [11\ 6\ 9\ 13\ 15]$. Baseline specification: $r = 11$. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.