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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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JEL Classification: C10

Keywords: N/A

Kim Christensen - kim@econ.au.dk
Aarhus University, CREATES, Danish Finance Institute

Allan Timmermann - atimmerm@ucsd.edu
University of California, San Diego and CEPR

Bezirgen Veliyev - bveliyev@econ.au.dk
Aarhus University

Warp Speed Price Moves: Jumps after Earnings Announcements*

Kim Christensen†‡ Allan Timmermann§ Bezirgen Veliyev†‡

March 27, 2023

Abstract

Corporate earnings announcements unpack large bundles of information that should, if markets are efficient, almost surely trigger jumps in stock prices immediately after the news release. Testing this implication is difficult in practice because most earnings announcements occur in the after-hours market where prices are contaminated by high levels of microstructure noise. We develop a new noise-robust jump test statistic and demonstrate that stock prices almost always jump immediately after earnings announcements. Finally, we develop a trading-based approach that allows us to estimate exactly how long it takes for markets to incorporate earnings news and quantify the importance of transaction costs.

JEL Classification: C10; C80.

Keywords: after-hours trading; earnings announcements; jump testing; high-frequency data; market efficiency; price discovery.

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†Department of Economics and Business Economics, Aarhus University, Fuglesangs Allé 4, 8210 Aarhus V, Denmark.

‡Research fellow at Danish Finance Institute.

§Rady School of Management and Department of Economics, UCSD, 9500 Gilman Drive, La Jolla, CA 92037, USA.

1 Introduction

Earnings announcements constitute perhaps the single most important source of firm-level information for stock market investors and play an essential role in the price discovery process. Such announcements are the key occasions on which firms release audited results and communicate their views on economic prospects to investors and the public. With several headline numbers being released simultaneously, earnings announcements can be thought of as points in time where draws from the news process have a very high variance. Following such news, investors are likely to materially revise their posterior estimates of firm values which, in turn, increases the likelihood of large changes in stock prices.

The widespread presence of high-frequency trading for the most liquid stocks means that in an efficient market where trading frictions are not prohibitively large, earnings announcements will almost surely trigger jumps in prices because they release so much information and resolve so much uncertainty that investors typically revise their pre-announcement beliefs by a significant amount. Effectively, a very high frequency of jumps in stock prices immediately after earnings announcements is a *necessary* condition for markets to efficiently incorporate the new information.[1]

Building on this idea, we propose to use jump tests based on high-frequency tick-by-tick price data as a new way to examine market efficiency. Our approach is fundamentally different from approaches used in existing studies of market efficiency. Existing work typically studies predictability in price movements in one- or five-minute intervals after news announcements, see, e.g., Ederington and Lee (1993). While relatively short in an absolute sense, these time intervals are excessively long in a world with high-frequency trading that gets executed in a matter of milliseconds after major news releases. In the presence of such high-frequency trading, only jump tests conducted at the tick-by-tick level can reveal whether markets react efficiently to major news events.

Testing for jumps in prices after earnings announcements creates special challenges. In recent years, the vast majority of publicly traded U.S. firms have made their earnings announcements in the after-hours market outside the regular trading session from 9:30am–4:00pm.[2] After-hours trading sessions on days with earnings announcements are therefore central to the price discovery process and it is important to understand price dynamics in after-hours markets in these sessions, especially around earnings announcements. We do so in this paper by studying transactions, bid-ask spreads, price discovery, and return predictability at a much higher tick-by-tick frequency than in previous studies. All told, we examine more than 19.07 billion after-hours quotes and 5.81 billion after-hours transactions for 25 stocks over a 12-year sample. For these stocks, our analysis provides

[1]The condition that prices jump after earnings announcements is, however, clearly not sufficient because the price could systematically over- or undershoot its new equilibrium level, thus introducing predictable dynamics in the post-announcement returns. We explore this issue in Section 5.

[2]Jiang, Likitapiwat, and McInish (2012), Lyle, Stephan, and Yohn (2021), and Michaely, Rubin, and Vedrashko (2013) report that more than 95% of US firms announce earnings outside the regular exchange trading session. This fraction is as high as 99.1% in the recent analysis by Grégoire and Martineau (2021) which studies companies belonging to the S&P 1500 stock index between 2011–2015.

the most complete analysis of trades and price formation in the after-hours market to date.

Unfortunately, after-hours transaction prices are severely affected by market microstructure noise, which tends to distort standard jump tests. The irregular format of data from the after-hours trading sessions poses serious challenges to empirical work and has limited jump tests to focus on data recorded for the regular trading sessions.[3] To address these challenges, we develop a new jump test that is suitable for illiquid markets, by generalizing the classical bipower variation-based jump test of Barndorff-Nielsen and Shephard (2006). We construct a noise-robust test statistic that relies on a pre-averaging technique to lessen the detrimental impact of the noise (see, e.g., Aït-Sahalia, Jacod, and Li, 2012; Jacod, Li, Mykland, Podolskij, and Vetter, 2009; Lee and Mykland, 2012; Podolskij and Vetter, 2009a,b). We extend the existing theory by generalizing the microstructure noise to allow for heteroscedasticity and dependence. We also show how to estimate the asymptotic variance of the test statistic, which is itself a non-trivial problem.[4] Through simulations and empirical results, we demonstrate that the conventional jump test is severely distorted by the type of market microstructure noise that is prevalent in after-hours markets, leading both to too many false positives and false negatives.

Using our new noise-robust test statistic, we find strong empirical evidence in support of our hypothesis that earnings announcements trigger jumps in prices. Specifically, the probability that stock prices jump in an after-hours session with an earnings announcement exceeds 95% while the corresponding jump probabilities for regular and after-hours sessions without earnings announcements are 3.5% and 4.5%, respectively. Oversimplifying our results a bit, we find that prices rarely jump during regular trading sessions but almost always jump after earnings announcements.

Having examined the *necessary* link between earnings announcements and the likelihood of jumps in the after-hours post-announcement stock price, we next study whether prices adjust *sufficiently* fast and by an appropriate amount to efficiently incorporate the new information. In doing this, we inspect all trades and quotes, using a more detailed and complete data set than that examined in previous studies. The fine granularity of our data means that we can pinpoint exactly how long it takes for new information to get incorporated into prices as well as identify the factors - including earnings surprises, market liquidity and analyst coverage - that determine price movements in the immediate aftermath of earnings announcements.

Examining a simple trading rule based on price forecasts computed off the surprise component in earnings announcements, over the 12-year sample (2008-2020) we find a highly significant mean return of 1.67% per transaction in a no-friction scenario where investors trade on the first transaction price after the announcement. Executing trades instead at the mid-point of the bid-ask spread, mean returns drop to 1.32% per trade which remains highly significant. This finding continues to hold

[3]See, e.g., Aït-Sahalia and Jacod (2009b); Aït-Sahalia, Jacod, and Li (2012); Andersen, Bollerslev, and Dobrev (2007); Caporin, Kolokolov, and Renò (2017); Christensen, Oomen, and Podolskij (2014); Corsi, Pirino, and Renò (2010); Jiang and Oomen (2008).

[4]We propose an jump- and noise-robust version of the subsampling approach studied in Christensen, Podolskij, Thamrongrat, and Veliyev (2017), which delivers a consistent estimator with desirable finite sample properties.

for trades executed at the actual spreads obtained from the bid-ask quotes after the announcement (mean of 0.81%). Delaying trades by 5 seconds reduces mean excess returns to 0.52% per trade, which remains significant, whereas further delays beyond 5 seconds lead to mean excess returns that are no longer statistically significant.

To analyze whether market efficiency has changed over time, we split our data into two sub-samples. In the early sub-sample (2008-2015), mean excess returns are a highly significant 2.02% and 1.58% per trade in the frictionless and mid-point trading scenarios, respectively. Even with a trading delay of 15 seconds, mean excess returns remain significant and economically large (0.64% per trade). Conversely, in the second sub-sample (2016-2020), the only significant values for mean excess returns are obtained in the frictionless and midquote scenarios while mean excess returns become small and insignificant in the presence of bid-ask spreads or timing delays. These findings suggest that the post-earnings announcement price discovery process in the after-hours market has become extremely fast and increasingly efficient over time for the most liquid U.S. stocks.

Our analysis is related to a number of previous studies. Like us, Grégoire and Martineau (2021) examine how earnings announcement news get transmitted to stock prices in the after-hours market. They study a much wider set of stocks than we do, but examine the dynamics of price formation for each firm in far less detail, aggregating instead price changes into five-minute intervals. Interestingly, they conclude that bid-ask spreads are so wide that investors could not have covered their round-trip trading costs even if they knew the post-announcement closing price.[5] We arrive at an entirely different conclusion for our sample of stocks, finding instead that post-announcement prices often jump well outside the range covered by the pre-announcement bid-ask spread which remains relatively narrow.

Our analysis is also related to studies on price formation in the extended-hour trading session. In a seminal contribution, Barclay and Hendershott (2003) study price discovery and trading in the after-hours market, finding some evidence of inefficient price formation. Our results show that trading activity and price discovery in the after-hours market is a tale of two markets. Even for the most liquid stocks like those examined here, after-hours trading volume is almost entirely dormant on days without earnings announcements. In contrast, on days with earnings announcements, after-hours trading activity explodes. Average after-hours trading volume on days with earnings announcements exceeds its counterpart on days without earnings announcement by almost two full orders of magnitude.[6]

Another literature has examined jumps in stock prices and their relation to news and effect on returns. Jeon, McCurdy, and Zhao (2022) examine the relation between news articles and price jumps. However, their analysis is based on general news and daily data, whereas we look at earnings announcements and high-frequency data at a much higher level of granularity. Bollerslev, Li, and

[5]Specifically, Grégoire and Martineau (2021) write that "pre-announcement bid-ask spreads are wide enough to include the post-announcement closing price, eliminating the profits of informed liquidity takers."

[6]Averaged over firms and years, this ratio equals 93.57 over our sample.

Todorov (2016) compute significantly positive risk premia for jump and overnight market betas but not for continuous market betas. They find that stocks with higher jump and overnight market betas earn higher returns even when controlling for continuous market betas. This suggests that investors command a risk premium mainly for exposure to discontinuous variations in the aggregate market, but not for exposure to continuous variations. Along similar lines Cremers, Halling, and Weinbaum (2015) find evidence of higher risk premia for exposure to changes in jump risk than for exposure to changes in volatility risk.

The outline of the paper is as follows. Section 2 establishes the link between price jumps and market efficiency before introducing our new noise-robust jump test statistic. Section 3 explains our data sources while Section 4 implements the jump test statistic on a unique set of high-frequency data that includes information about the transaction record from outside the official opening hours of the exchange. Section 5 studies price dynamics after earnings announcements and analyzes returns from a simple trading strategy while Section 6 concludes. Supplemental appendices at the back of the paper provide additional results and technical analysis.

2 Earnings announcements, market efficiency and price jumps

We begin by motivating why, in efficient markets, we should expect stock prices to jump immediately after earnings announcements. To test this implication, we next introduce a new jump test that is robust to the high levels of market microstructure noise that typically affect prices in the after-hours trading session. We finish the section by reporting results from Monte Carlo simulations comparing the performance of our new noise-robust test to that of existing jump tests.

2.1 Earnings announcements and market efficiency

Earnings announcements can be thought of as points in time where large bundles of information get disclosed simultaneously to all investors. Moreover, the timing of such information disclosures is typically known ahead of time, giving investors ample opportunity to prepare for the announcement and respond instantaneously.[7]

The asset pricing setup that most closely resembles earnings announcements is perhaps the continuous-time model with regime-switching dynamics and jumps developed by David and Veronesi (2014). In this model, discrete shifts between a set of unobserved regimes determine the drift of the fundamental variables. The real earnings process contains a Poisson jump component which feeds into the pricing kernel and so is a source of systematic risk with jumps in the earnings process translating into jumps in stock prices. This is consistent with earnings announcements, which we can think of as discrete jumps in the earnings news process, inducing jumps in stock prices.[8]

[7] Most large-cap companies inform markets in advance about their intention to release a financial report either Before Market Open (BMO), during Regular Trading Hours (RTH), or After Market Close (AMC), see, e.g., `https://www.nasdaq.com/market-activity/earnings`.

[8] David and Veronesi (2014) look at the aggregate (economy-wide) earnings process, whereas we analyze earnings

Even in the absence of jumps in the earnings process, large revisions to investors' learning process about the underlying state can induce large, discrete changes to prices. For example, in the models developed by Veronesi (1999, 2000), the dividend growth process switches discretely between high-growth and low-growth states. Investors monitor dividends and infer the underlying state from news about dividends. A switch in the underlying dividend growth regime will generally lead to a significant revision in investors' dividend growth estimates and, in turn, the stock price. Assuming that the growth processes of dividends and earnings are closely correlated, a similar effect will emerge in a model in which investors observe earnings. In this setting, earnings announcements are events with significant releases of news about the earnings process at a single point in time. With a high likelihood, this triggers significant revisions to investors' beliefs about future earnings and, in turn, leads to large price movements at the time of the earnings announcement.

In other continuous-time models such as Kyle (1985), the terminal date can be interpreted as a public earnings announcement or the release of other "major" public news such as the outcome of a merger. In such a setting, the price is driven by order flows (and sometimes an explicit, diffusive public news process) prior to the terminal date and evolves continuously during that time. Then, assuming non-trivial information is released, the price typically jumps at the terminal announcement point.

Investors in the Kyle model are risk-neutral. Accounting for risk aversion, prices could jump even when the earnings news does not surprise investors due to the uncertainty resolution effect as earnings announcements remove short-term uncertainty about firm earnings. Hence, the stock price is more likely to move up (due to the lower risk premium) than down following an earnings announcement that is close to investors' expectations.[9]

In discrete-time models with disagreement among investors (heterogeneous priors) such as Banerjee and Kremer (2010), earnings announcements can be thought of as periods in which the variance of the public information signal is an order of magnitude bigger than normal. In these models, the revelation of big bundles of information is likely to cause investors to significantly revise their posteriors. Such revisions in beliefs will, in turn, typically trigger large changes in the underlying asset price whose variance is proportional to the variance of the public signal.[10]

Discrete-time asset pricing models tend to have a stylized time dimension and so do not directly spell out whether movements in asset prices take the form of rapid diffusion or discrete jumps. However, if markets are efficient, prices should adjust very quickly and are thus more likely to

announcements at the firm level. However, as long as firms' earnings contain some information about the economy-wide, systematic earnings process, firm-level prices should jump as well. We show in the data analysis that this condition holds empirically.

[9] Preference for uncertainty resolution is not the only way through which news announcements move prices even in the absence of surprises. Ai and Bansal (2018) argue that macroeconomic news announcements can earn a substantial risk premium provided that preferences have a property dubbed generalized risk sensitivity. In turn, this requires that news contain information about future continuation utility which is equivalent to the news carrying information about the prospect of future economic growth.

[10] See, e.g., equation (10) in Banerjee and Kremer (2010).

jump, i.e. move by a large amount more or less instantaneously, in the immediate aftermath of the release of large amounts of public information.

This discussion suggests the following hypothesis.

Hypothesis 1. *Necessary conditions for market efficiency after an earnings announcement include that:*

1. *The stock price almost always jumps immediately after the earnings announcement.*
2. *Due to the presence of a common/systematic news component in earnings announcements, the price of the market index, as well as the prices of other stocks with related business activities (e.g., in the same industry), are significantly more likely to also jump immediately after earnings announcements than at times without such announcements.*
3. *Due to uncertainty resolution, the stock price is more likely to jump up than down following an earnings announcement that is in line with analyst expectations.*

The higher jump probability in stock prices after an earnings announcement is a weak implication in the sense that it is a necessary but not sufficient condition for market efficiency. This holds because stock prices could over- or undershoot. For example, prices could jump too far, giving rise to a mean-reverting component that drifts towards the pre-announcement price in the period following the jump. Conversely, if prices do not jump far enough, we should instead expect to see post-announcement prices drift systematically in the direction of the jump.

These examples suggest that a sufficient condition for market efficiency after earnings announcements is that investors cannot detect any return predictability on the entire post-announcement price path and exploit such predictability after accounting for trading costs and risk adjustments:[11]

Hypothesis 2. *A sufficient condition for market efficiency after an earnings announcement is that, at each point in time of the post-announcement price path:*

1. *post-announcement returns do not display any predictive patterns large enough to allow investors to earn abnormal returns after accounting for transaction costs and risk premia.*

Importantly, Hypothesis 2 does not mean that the price discovery process is settled instantaneously. We would expect elevated volatility of the price process lasting several minutes after the release of the news as information gets processed and trading positions are adjusted. However, this price discovery process should not leave any (local) biases that allow investors to predict and exploit future price movements.

The condition that predictability of prices should not be exploitable in an economic sense is important and goes back to Jensen (1978).[12] Trading costs and bid ask spreads may induce negative

[11] Equilibrium prices are only determined up to the marginal cost of acquiring and processing information and implementing transactions (Grossman and Stiglitz, 1980; Pedersen, 2015).

[12] He defines market efficiency in this way: "A market is efficient with respect to information set θ_t if it is impossible to make economic profits by trading on the basis of information set θ_t."

autocorrelation in returns. However, this should not generate abnormal profits after accounting for round-trip transaction costs. Similarly, time-varying risk premia might induce some predictability in the price process but are unlikely to matter much at the very high frequency that we analyze here compared to the longer horizons conventionally studied in the literature on return predictability.

Testing the sufficient conditions in Hypothesis 2 is challenging because it has to hold for all possible trading rules that condition on the released information. Short of inspecting the profitability of all possible trading and prediction rules based on this information, it is difficult to conduct an exhaustive test of this condition. In practice, analysis typically focuses on simple prediction rules based on news measures such as earnings surprises. Conclusions regarding market efficiency are therefore limited to the class of prediction and trading rules being examined.

2.2 Existing tests of market efficiency

Our approach to testing market efficiency by examining the presence of jumps immediately after the disclosure of large information bundles is fundamentally different from common practice in the finance literature on market efficiency in the aftermath of macroeconomic news announcements (e.g., Ederington and Lee, 1993; Andersen, Bollerslev, Diebold, and Vega, 2003) or corporate earnings announcements (e.g., Beaver, 1968; Chambers and Penman, 1984; Jiang, Likitapiwat, and McInish, 2012; Grégoire and Martineau, 2021; Lyle, Stephan, and Yohn, 2021).

This literature typically studies predictability in post-announcement returns over fixed time intervals such as one or five minutes. For example, Ederington and Lee (1993) conclude that *"Following an announcement, traders with immediate access to the market apparently form an estimate of the release's implication for market prices almost immediately, and the actual price adjusts to this level within one minute. The price level at the end of one minute of trading is a relatively unbiased estimate of the final equilibrium price."* Similarly, using data on exchange rates sampled at 5-minute intervals, Andersen, Bollerslev, Diebold, and Vega (2003) observe that *"The general pattern is one of very quick exchange-rate conditional mean adjustment, characterized by a jump immediately following the announcement, and little movement thereafter."* Importantly, the evidence of jumps presented by Andersen, Bollerslev, Diebold, and Vega (2003) is based on visual inspection of price movements over five-minute intervals and their sampling scheme does not allow them to formally test for instantaneous jumps in exchange rates.

In contrast, our definition in Hypothesis 2 does explicitly *not* reference time intervals such as 30 or 60 seconds. Hence, the conditions have to hold at each point in time, i.e., on the entire price path, regardless of whether we are studying fixed or varying intervals, calendar or trading time. This is an important advantage because choosing the length of the time interval used in the conventional market efficiency tests is a difficult and ultimately arbitrary task.

Even more importantly, profitable transactions can occur at much higher frequencies than the traditional intervals of 1-5 minutes. In current markets, thousands of trades are executed at intervals measured in milliseconds. Viewed in this perspective, the absence of serial correlation in, e.g., one-minute returns is no guarantee that prices move efficiently. In a world dominated by high-frequency trading where trade intervals can shrink to an almost arbitrary limit, ultimately only jump tests can reveal whether markets react efficiently to big news events.

To illustrate the extreme speed with which transactions occur in the immediate aftermath of an announcement, Panel A of Figure 1 shows tick-by-tick evidence on the price movement and transaction count from the announcement (time 0) to 60 seconds after Apple released its earnings report on 07/30/2020; Panel B zooms further in on the first 100 milliseconds. The instantaneous transaction count exceeds 800 in the first post-announcement second and begins only 15 milliseconds after the announcement. This can arguably be attributed to algorithmic traders sniping stale quotes in the limit order book. Trading activity then lies dormant for a few seconds before picking up significant steam again around the time where slower financial intermediaries are likely to have entered the market. In the first ten seconds, we register an average of around 300 trades per second while in the one-minute post-announcement window, more than 165 trades get executed per second, on average.[13]

This example highlights the speed with which prices of the most liquid stocks adjust after earnings announcements. It also motivates the need for market efficiency tests that account for such extremely fast trading, yet possess robustness to the high levels of noise typically encountered in after-hours markets. This is the topic we next turn to.

2.3 A noise-robust jump test

The vast majority of corporate earnings announcements occur in the after-hours market where trading volume is typically far lower and bid-ask spreads much wider compared to the regular trading session.[14] As we demonstrate below, in the presence of such microstructure frictions, conventional jump tests tend to spuriously identify false jumps, while often failing to detect true jumps. This shortcoming makes it important to have a testing procedure that is robust to the noised-up features of high-frequency data from the after-hours market.[15]

Given the dominance in empirical work of the jump test of Barndorff-Nielsen and Shephard (2006), we develop a noise-robust generalization which retains the intuitive nature of their test.

[13] Towards the end of our sample, overall trading activity in the after-hours market is much higher than in the early years with high-speed electronic trading often exceeding 200 trades per second right after earnings announcements.

[14] Empirical work based on high-frequency data from the regular trading session is inconclusive about the importance of jumps relative to the diffusive volatility (as often modeled by the inclusion of a Brownian motion in the price process), see, e.g., Christensen, Oomen, and Podolskij (2014). This is another reason why we explore high-frequency data from the extended trading session in search for jumps.

[15] Aït-Sahalia, Jacod, and Li (2012) construct a noise-robust jump test from power variations sampled at different frequencies. However, it is not obvious how to implement such sampling in the after-hours market that is typically quite illiquid with limited trading. Lee and Mykland (2008, 2012) construct a noise-robust point-in-time jump test, which employs high-frequency returns standardized by a localized bipower variation (see also Lee and Hannig, 2010). A limitation of the usage of this test in our setting is that the exact timing of earnings announcements is not always known with certainty.

2.3.1 The efficient price

We start by introducing a general continuous-time setup for modeling noisy high-frequency data.[16] Consider a security price observed on the time interval [0,1] which we interpret as a complete daily trading session further described in Section 3. The fundamental theorem of asset pricing implies that in an efficient market without frictions and no arbitrage the price process, $P = (P_t)_{t \geq 0}$, must be a semimartingale (e.g., Delbaen and Schachermayer, 1994). Normalizing the log-price process $p = (p_t)_{t \geq 0}$ with $p_t = \log(P_t)$ to start at zero ($p_0 = 0$), we can decompose the cumulative intraday log-return at time t (r_t) into a continuous part (r_t^c) and a jump, or discontinuous, part (r_t^d):

$$r_t = r_t^c + r_t^d. \tag{1}$$

Consistent with no-arbitrage, $r^c = (r_t^c)_{t \geq 0}$ takes the form of a drift and diffusion part:

$$r_t^c = \underbrace{\int_0^t a_s \mathrm{d}s}_{\text{drift}} + \underbrace{\int_0^t \sigma_s \mathrm{d}W_s}_{\text{volatility}}, \tag{2}$$

where the drift $a = (a_t)_{t \geq 0}$ is a predictable and locally bounded process, the volatility $\sigma = (\sigma_t)_{t \geq 0}$ is an adapted and càdlàg process, and $W = (W_t)_{t \geq 0}$ is a standard Brownian motion.

In turn, we can decompose $r^d = (r_t^d)_{t \geq 0}$ into small and big jumps:[17]

$$r_t^d = \underbrace{\int_0^t \int_{\mathbb{R}} \delta(s,x) 1_{\{|\delta(s,x)| \leq 1\}} (\mu - \nu)(\mathrm{d}s, \mathrm{d}x)}_{\text{“small” jumps}} + \underbrace{\int_0^t \int_{\mathbb{R}} \delta(s,x) 1_{\{|\delta(s,x)| > 1\}} \mu(\mathrm{d}s, \mathrm{d}x)}_{\text{“big” jumps}}, \tag{3}$$

where μ is a Poisson random measure on $\mathbb{R}_+ \times \mathbb{R}$ with compensator $\nu(\mathrm{d}s, \mathrm{d}x) = \mathrm{d}sF(\mathrm{d}x)$.[18]

This framework is nonparametric since we do not constrain the dynamics of the efficient price process. Our approach is therefore compatible with many salient features of high-frequency financial data such as time-varying expected returns, stochastic volatility, leverage effects, and jumps in the price and volatility coefficient. Price jumps can be of both infinite activity and have infinite variation.

The null hypothesis that we wish to test is that P is continuous. This can be formulated as $\mathcal{H}_0 : \omega \in \Omega_0$, where $\Omega_0 \subseteq \Omega$ is the subset:

$$\Omega_0 = \{\omega \in \Omega : t \longmapsto P_t(\omega) \text{ is continuous on } [0,1]\}. \tag{4}$$

We test this null against the alternative $\mathcal{H}_a : \omega \in \Omega_1$ with $\Omega_1 = \Omega_0^\complement$ which is equivalent to the

[16] As usual, randomness is described by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$. The construction of this space in a noisy high-frequency setting is outlined in Jacod and Protter (2012).

[17] The separation into “small” and “large” jumps is not crucial in the analysis.

[18] Here F is a σ-finite measure, δ is a predictable function, and $(\tau_k)_{k \geq 1}$ is a sequence of $\mathcal{F}_t$-stopping times increasing to ∞ such that $|\delta(\omega, s, x)| \wedge 1 \leq \psi_k(x)$ for all (ω, s, x) with $s \leq \tau_k(\omega)$ and $\int_{\mathbb{R}} \psi_k^\beta(x) F(\mathrm{d}x) < \infty$ for all $k \geq 1$ and $\beta \in [0,2]$. β relates to the activity of the price jumps and can be interpreted as a generalized version of the Blumenthal and Getoor (1961) index for a Lévy process, see Aït-Sahalia and Jacod (2009a). A higher value of β increases the frequency of the small jumps. Figure B.1 in Appendix B provides an illustration of this characteristic.

restriction $r_t^d = 0$ for all t.

To develop a test of $\mathcal{H}_0$, we examine the composition of the quadratic return variation:

$$[r]_1 = \int_0^1 \sigma_s^2 \mathrm{d}s + \sum_{0\leq s\leq 1} (\Delta r_s^d)^2, \tag{5}$$

where $\Delta r_s^d = r_s^d - r_{s-}^d$ is the jump size at time s and $r_{s-}^d = \lim_{t\uparrow s} r_t^d$. The first term in (5) is the quadratic variation of the continuous log-return, also known as the integrated variance. The second term represents the quadratic variation of the jump process, which is zero under $\mathcal{H}_0$.

Quadratic variation can equivalently be defined as follows:

$$[r]_1 = \operatorname*{plim}_{n\to\infty} \sum_{i=1}^{n} r_i^2, \tag{6}$$

where $r_i = p_{t_i} - p_{t_{i-1}}$ is the log-return over $[t_{i-1}, t_i]$ and the convergence in probability holds for every partition $0 = t_0 < t_1 < \ldots < t_n = 1$ such that $\max_{1\leq i\leq n}(t_i - t_{i-1}) \to 0$ as $n \to \infty$, motivating why tick-by-tick high-frequency data is used to estimate quadratic variation.

2.3.2 The observed price

In practice, the efficient market hypothesis is violated because of market frictions—or microstructure noise—such as price discreteness and bid–ask spreads. In addition, even though the price process evolves in continuous time, trading is discrete. For simplicity, suppose the observed log-price is recorded at equidistant time points $t_i = i\Delta_n$, for $i = 0, 1, \ldots, n$, where $\Delta_n = 1/n$ is the time gap, which we collect in $p^* = (p^*_{i\Delta_n})_{i=0}^n$. To account for microstructure noise, we follow Hasbrouck (1995) and model $p^*_{i\Delta_n}$ as

$$p^*_{i\Delta_n} = p_{i\Delta_n} + \epsilon_{i\Delta_n}, \tag{7}$$

where $\epsilon_{i\Delta_n}$ is the microstructure effect which we characterize through the following assumption.

Assumption (N): The microstructure noise is of the form

$$\epsilon_{i\Delta_n} = \omega_{i\Delta_n} \pi_i, \tag{8}$$

where

(i) $\pi = (\pi_i)_{i=0}^n$ is a sequence of i.i.d. random variables with mean zero and unit variance. Moreover, the distribution of π_0 is symmetric and has moments of arbitrary order.

(ii) $\omega = (\omega_t)_{t\geq 0}$ is of the form:

$$\omega_t = \omega_0 + \int_0^t \bar{a}_s \mathrm{d}s + \int_0^t \bar{\sigma}_s \mathrm{d}W_s + \int_0^t \bar{v}_s \mathrm{d}B_s, \tag{9}$$

where ω_0 is an $\mathcal{F}_0$-measurable random variable, $\bar{a} = (\bar{a}_t)_{t\geq 0}$, $\bar{\sigma} = (\bar{\sigma}_t)_{t\geq 0}$, $\bar{v} = (\bar{v}_t)_{t\geq 0}$ are adapted, càdlàg processes, while $B = (B_t)_{t\geq 0}$ is a standard Brownian motion and $B \perp\!\!\!\perp W$.[19]

(iii) $\pi \perp\!\!\!\perp (p, \omega)$.

(iv) $\limsup_{|t|\to\infty} |\chi(t)| < 1$, where χ is the characteristic function of π_0.

The conditions we impose on the noise process are weak and allow for complicated dynamics, for example the noise can be heteroscedastic and *nonlinearily* dependent.[20] Specifically, Assumption (N) resembles Assumption 4 in Li, Todorov, and Tauchen (2017) and Assumption (N) in Jacod, Li, and Zheng (2017). It generalizes previous work on pre-averaging based on i.i.d. and exogenous noise (Christensen, Oomen, and Podolskij, 2014; Jacod, Li, Mykland, Podolskij, and Vetter, 2009; Podolskij and Vetter, 2009a,b) to more realistic processes.

The key challenge here is to draw inference about the presence of jumps in the efficient price from a high-frequency record of noisy prices. To see this, note that the observed log-return measures the efficient log-return with error since it incorporates the increment to the microstructure component as can be seen from equation (7):

$$r_i^* = p_{i\Delta_n}^* - p_{(i-1)\Delta_n}^* = r_i + \epsilon_{i\Delta_n} - \epsilon_{(i-1)\Delta_n}. \tag{10}$$

2.3.3 The pre-averaging approach

To mitigate the effect of microstructure noise, we adopt the pre-averaging procedure of Jacod, Li, Mykland, Podolskij, and Vetter (2009); Podolskij and Vetter (2009a,b). Specifically, we replace the observed noisy log-return by a pre-averaged log-return:

$$\bar{r}_i^* = \sum_{j=1}^{k_n-1} g_j^n r_{i+j}^*, \quad \text{for } i = 0, \dots, n - 2k_n + 1, \tag{11}$$

where $g_j^n = g(j/k_n)$ is a weight function, $k_n = \theta\sqrt{n} + o\big(n^{-1/4}\big)$ is the pre-averaging horizon, and $\theta > 0$ is a tuning parameter.[21]

The averaging in (11) attenuates the microstructure noise and enhances the efficient log-return. If k_n is even and $g(x) = \min(x, 1-x)$, (11) can be rewritten as

$$\bar{r}_i^* = \frac{1}{k_n} \sum_{j=k_n/2+1}^{k_n} p_{\frac{i+j}{n}}^* - \frac{1}{k_n} \sum_{j=1}^{k_n/2} p_{\frac{i+j}{n}}^*. \tag{12}$$

Thus, $(\bar{r}_i^*)_{i=1}^{n-k_n+2}$ constitutes a sequence of log-returns constructed by simple averaging of the noisy log-price. This means that, with minor modifications, standard estimators of quadratic variation can be tweaked to exploit the pre-averaged log-return series. We therefore define the pre-averaged

[19] The symbol $A \perp\!\!\!\perp B$ means A and B are stochastically independent.

[20] Meanwhile, the i.i.d assumption on π implies the noise is *linearily* independent, i.e. uncorrelated. To further allow for serial correlation, we can follow Jacod, Li, and Zheng (2017) who assume π is a stationary process with suitable mixing conditions, but the distinction is not important here.

[21] The theory imposes weak regularity conditions on g, namely $g : [0,1] \to \mathbb{R}$ is continuous and piecewise continuously differentiable with piecewise Lipshitz derivative g', $g(0) = g(1) = 0$, and $\int_0^1 g(s)^2 \mathrm{d}s > 0$.

realized variance and pre-averaged bipower variation as follows:

$$
\begin{aligned}
RV_n^* &= \frac{1}{k_n \psi_2^n} \sum_{i=0}^{n-2k_n+1} |\bar{r}_i^*|^2 - \frac{\psi_1^n}{\psi_2^n \theta^2} \hat{\omega}_n^2, \\
BV_n^* &= \frac{1}{k_n \psi_2^n} \frac{\pi}{2} \sum_{i=0}^{n-2k_n+1} |\bar{r}_i^*||\bar{r}_{i+k_n}^*| - \frac{\psi_1^n}{\psi_2^n \theta^2} \hat{\omega}_n^2,
\end{aligned}
\tag{13}
$$

where

$$
\hat{\omega}_n^2 = -\frac{1}{n-1} \sum_{i=2}^{n} r_i^* r_{i-1}^*, \tag{14}
$$

and

$$
\psi_1^n = k_n \sum_{j=0}^{k_n-1} (g_{j+1}^n - g_j^n)^2 \quad \text{and} \quad \psi_2^n = \frac{1}{k_n} \sum_{j=1}^{k_n} (g_j^n)^2. \tag{15}
$$

The key here is that RV_n^* is the sum of squared pre-averaged returns, whereas BV_n^* is based on cross-products that are k_n terms apart. As we prove in Theorem 1, the extra distancing ensures that BV_n^* is asymptotically jump-robust, whereas RV_n^* is not. Both estimators are normalized for the effect of pre-averaging and bias-corrected for the remaining microstructure noise.[22]

2.3.4 Asymptotic theory

We next develop the theoretical foundation for the construction of our noise-robust jump test procedure. To do this, we need a structural condition on the diffusive volatility process:

Assumption (V): σ is of the form:

$$
\sigma_t = \sigma_0 + \int_0^t \tilde{a}_s \mathrm{d}s + \int_0^t \tilde{\sigma}_s \mathrm{d}W_s + \int_0^t \tilde{v}_s \mathrm{d}B_s, \tag{16}
$$

where σ_0 is an $\mathcal{F}_0$-measurable random variable, while $\tilde{a} = (\tilde{a}_t)_{t\geq 0}$, $\tilde{\sigma} = (\tilde{\sigma}_t)_{t\geq 0}$ and $\tilde{v} = (\tilde{v}_t)_{t\geq 0}$ are adapted, càdlàg processes.

Assumption (V) together with Assumption (N,ii) allow us to establish the order of various approximating terms appearing in the proofs, once we freeze the volatility and noise processes locally on small blocks of high-frequency data.[23]

Theorem 1. *Assume that r follows the process in* (1) *with $\beta = 2$ and that Assumptions (V) and (N) hold. Then, as $n \to \infty$,*

$$
RV_n^* \xrightarrow{p} [r]_1 \qquad \text{and} \qquad BV_n^* \xrightarrow{p} \int_0^1 \sigma_s^2 \mathrm{d}s. \tag{17}
$$

[22]The bias-correction employs the jump-robust estimator in (14) proposed by Oomen (2006) and designed to estimate the variance of the noise in i.i.d. settings. Consistent with Kalnina (2011), we show in Appendix A that in the general framework of Assumption (N), $\hat{\omega}_n^2 \xrightarrow{p} \int_0^1 \omega_s^2 \mathrm{d}s$ as $n \to \infty$.

[23]Assumption (V) can be extended to discontinuous volatility processes following Barndorff-Nielsen, Graversen, Jacod, Podolskij, and Shephard (2006).

Moreover, provided that $\mathcal{H}_0$ is true, as $n \to \infty$,

$$n^{1/4} \begin{pmatrix} RV_n^* - \int_0^1 \sigma_s^2 \mathrm{d}s \\ BV_n^* - \int_0^1 \sigma_s^2 \mathrm{d}s \end{pmatrix} \xrightarrow{d} N(0, \Sigma), \tag{18}$$

where Σ is the 2×2 asymptotic covariance matrix.[24]

Theorem 1 establishes a weak law of large numbers for the pre-averaged realized variance and pre-averaged bipower variation in the noisy jump-diffusion model under $\mathcal{H}_a$. It shows that RV_n^* is consistent for the quadratic variation, including the jump part, while BV_n^* converges to the integrated variance, excluding the jump part. Their difference therefore estimates the quadratic jump variation, $RV_n^* - BV_n^* \xrightarrow{p} \sum_{0 \leq s \leq t} (\Delta r_s^d)^2$ which is zero in the absence of jumps. The last part of Theorem 1 establishes a bivariate asymptotic normal distribution of RV_n^* and BV_n^* as estimators of integrated variance under $\mathcal{H}_0$.

Applying the delta rule to (18), it follows from Theorem 1 that, under $\mathcal{H}_0$,:

$$\mathcal{J}_n^{\text{inf.}} = \frac{n^{1/4}\big(RV_n^* - BV_n^*\big)}{\sqrt{v^\top \Sigma v}} \xrightarrow{d} N(0, 1), \tag{19}$$

where $v = [1, -1]^\top$. In contrast, $\mathcal{J}_n^{\text{inf.}} \xrightarrow{p} \infty$ under the alternative, $\mathcal{H}_a$.

2.3.5 A noise- and jump-robust covariance estimator

The test statistic $\mathcal{J}_n^{\text{inf.}}$ is not feasible since it depends on the covariance matrix Σ, which is a function of the latent volatility and microstructure noise processes. We therefore next develop a noise- and jump-robust plug-in estimator of Σ.[25]

We propose to set

$$\Sigma_n^* = \frac{1}{L} \sum_{l=1}^{L} \left(\frac{n^{1/4}}{\sqrt{L}} \begin{bmatrix} \check{BV}_l^*(2,0) - \check{BV}_n^*(2,0) \\ \check{BV}_l^*(1,1) - \check{BV}_n^*(1,1) \end{bmatrix} \right) \left(\frac{n^{1/4}}{\sqrt{L}} \begin{bmatrix} \check{BV}_l^*(2,0) - \check{BV}_n^*(2,0) \\ \check{BV}_l^*(1,1) - \check{BV}_n^*(1,1) \end{bmatrix} \right)^\top, \tag{20}$$

where

$$\begin{aligned}
\check{BV}_n^*(q,r) &= \frac{1}{k_n \psi_2^n} c(q,r) \sum_{i=0}^{n-2k_n+1} |\check{r}_i^*|^q |\check{r}_i^*|^r - \frac{\psi_1^n}{\psi_2^n \theta^2} \hat{\omega}^2, \\
\check{BV}_l^*(q,r) &= \frac{Lpk_n}{n} \sum_{i=1}^{n/Lpk_n} v_{(i-1)L+l}(q,r)^n, \\
v_i(q,r)^n &= \frac{n}{pk_n - 2k_n + 2} \frac{1}{k_n \psi_2^n} c(q,r) \sum_{j, j+2k_n-1 \in B_i(p)} |\check{r}_j^*|^q |\check{r}_{j+k_n}^*|^r - \frac{\psi_1^n}{\psi_2^n \theta^2} \hat{\omega}^2, \\
B_i(p) &= \big\{ j : (i-1)pk_n \leq j \leq ipk_n \big\},
\end{aligned} \tag{21}$$

[24] An explicit expression for Σ is presented in (41) in Appendix A.

[25] In principle, we only need to estimate Σ given that $\mathcal{H}_0$ is true. However, by using a jump-robust estimator of the asymptotic covariance matrix we avoid losing power under $\mathcal{H}_a$.

and $c(q,r)$ is a constant.

$\check{BV}^*_n(q,r)$ is the truncated pre-averaged bipower variation of Christensen, Hounyo, and Podolskij (2018). It modifies the pre-averaged bipower variation in (13) by raising the pre-averaged log-returns to the exponents q and r. It also employs jump-truncated pre-averaged returns, $\check{r}^*_i$, which are defined in (49) in Appendix A. Apart from the truncation, $\check{BV}^*_n(q,r)$ nests the pre-averaged realized variance for $(q,r) = (2,0)$, where $c(2,0) = 1$, and the pre-averaged bipower variation for $(q,r) = (1,1)$, where $c(1,1) = \pi/2$.[26]

Σ^*_n is the sample covariance matrix of subsampled pre-averaged bipower variation estimators that are calculated on smaller batches of high-frequency data covering blocks of length pk_n. These are centered around the full sample statistic and suitably normalized. L is the number of subsample estimates, and we assume n is a multiple of Lpk_n for notational convenience.

With this in place, we have the following result:

Theorem 2. *Assume that r follows the process in (1) with $\beta \leq 1$ and that Assumptions (V) and (N) hold. Moreover, assume that $L \asymp n^{(1-\delta)/2}$, where $\delta > 0$ is a small number such that*

$$\frac{3/2+\delta}{8-\beta} < \bar{\omega} < \frac{1}{4} - \delta. \tag{22}$$

Then, as $n \to \infty$, $p \to \infty$, $L/p \to \infty$, $\sqrt{n}/Lp^2 \to \infty$, and for $\delta < 1/16$,

$$\Sigma^*_n \xrightarrow{p} \Sigma. \tag{23}$$

Theorem 2 shows that Σ^*_n is consistent both under the null (diffusion only) and under the alternative (diffusion and jumps). We confine attention to jump processes of bounded variation, which is standard when inference is based on the continuous part of the process.

By Slutsky' theorem, we obtain the following feasible jump test that holds under the null $\mathcal{H}_0$:

$$\mathcal{J}_n = \frac{n^{1/4}\big(RV^*_n - BV^*_n\big)}{\sqrt{v^\top \Sigma^*_n v}} \xrightarrow{d} N(0,1). \tag{24}$$

In the presence of jumps, $\mathcal{J}_n \xrightarrow{p} \infty$. Hence large values of $\mathcal{J}_n$ are indicative of price jumps.

The finite sample properties of $\mathcal{J}_n$ under the alternative can be further improved if we also replace BV^*_n in the numerator of (24) with a truncated version:[27]

$$\mathcal{J}_n = \frac{n^{1/4}\big(RV^*_n - \check{BV}^*_n\big)}{\sqrt{v^\top \Sigma^*_n v}}, \tag{25}$$

[26] The latter is therefore also sometimes referred to as $(1,1)$-pre-averaged bipower variation

[27] On the one hand, "big" jumps induce an upward bias in BV^*_n for realistic sample sizes, causing a downward bias in the estimated jump variation and reducing the power of the jump test. However, such biases can readily be handled through truncation. On the other hand, the bipower mechanism—multiplication of adjacent log-returns—is an effective tool to get rid of "small" jumps that survive any truncation. This approach of cracking down twice on the jump component was proposed by Corsi, Pirino, and Renò (2010) for bipower variation in the noise-free setting. In Appendix A, we show that the substitution of BV^*_n with $\check{BV}^*_n$ in the numerator does not alter the asymptotic theory in Theorem 1.

where $\check{BV}_n^* \equiv \check{BV}_n^*(1,1)$. Exploiting that our test is asymptotically unbiased and consistent, we can compare the test statistic $\mathcal{J}_n$ against critical values from the standard normal distribution to test for jumps.

The final form of $\mathcal{J}_n$ in (25) serves as our noise-robust jump test statistic.[28] A one-sided test procedure is appropriate here so the decision rule is to reject $\mathcal{H}_0$ at significance level α if $\mathcal{J}_n > \Phi^{-1}(1-\alpha)$, where $\Phi(x)$ is the standard normal distribution function:

$$\mathbb{P}\Big(\mathcal{J}_n > \Phi^{-1}(1-\alpha)\Big) \to \begin{cases} \alpha, & \text{under } \mathcal{H}_0, \\ 1, & \text{under } \mathcal{H}_a. \end{cases} \tag{26}$$

2.4 Monte Carlo Simulations

In Appendix B, we conduct a thorough Monte Carlo study that examines the finite-sample properties of our noise-robust jump test, which we compare to the noise-free test of Barndorff-Nielsen and Shephard (2006). We develop a general setting that examines levels of microstructure noise that match those seen in the after-hours trading sessions in our data.

Our key findings from these simulations can be summarized as follows.

1. Even when the observed price is affected by a very high level of noise, our jump test exhibits size control and does not systematically over- or under-reject the null hypothesis of no jumps.

2. Our jump test has good power properties and correctly identifies jumps under the alternative in a variety of scenarios for the microstructure noise, pre-averaging, subsampling, and jump-truncation.

3. In sharp contrast, the performance of the conventional jump test based on 5-minute realized variance and bipower variation degrades in the presence of the levels of microstructure noise typical for after-hours trading. In the presence of such noise, the classical test spuriously identifies too many jumps, falsely rejecting the null when no jumps are present. Conversely, it frequently fails to reject the null and thus fails to identify true jumps when these are genuinely present.

In summary, our simulations demonstrate both the need for and benefits from utilizing our noise-robust jump test on the high-frequency after-hours transaction price data examined in our empirical application, which we next proceed to.

3 Data

Earnings announcements should trigger jumps in stock prices if investors efficiently and instantaneously process the new earnings information. In this section, we introduce the data sources

[28] Barndorff-Nielsen and Shephard (2006) advocate a log- and ratio-based transformation of the noise-free version of (25) via the delta method. We implemented both these alternative versions of the noise-robust jump test statistic, which does not impact any of our conclusions.

employed to study if this prediction is supported empirically.

3.1 Transaction and quotation data

Jump tests benefit crucially from having data at the highest possible resolution, so our analysis employs tick-by-tick transaction and quotation data on individual firms' stock prices to track their response to corporate earnings announcements.

U.S. stock exchanges are only open for trading on normal business days from 9:30am–4:00pm Eastern time.[29] However, orders can be submitted and trades executed at any time via private trading systems such as electronic communication networks (ECNs). On most days, official pre-market trading is available from 4:00am–9:30am, while the after-hours market runs from 4:00pm–8:00pm.[30] During this time, the Consolidated Tape collates real-time tick-by-tick data that is available for purchase via private vendors such as the New York Stock Exchange (NYSE) Trade and Quote (TAQ) database.

Transaction volumes in after-hours markets have grown substantially, more than tripling between 2008 and 2020 for the stocks we analyze in our paper. Over the same period, after-hours trading quadrupled from less than one-fifth of a percent to nearly one percent of total transaction volume (including trading from the regular trading session). However, on days with earnings announcements this fraction is substantially higher, frequently exceeding 10%. Appendix C summarizes the evolution over time in trading volume and spreads in the after-hours market.

As noted earlier, the vast majority of U.S. companies publish their financial results outside the regular trading session.[31] In Appendix D, we examine the trading activity in the pre-market (6:00am–9:30am) and after-hours market (4:00pm–6:30pm). We find that trading activity levels are much lower in the pre-market, even on announcement days.[32] This impairs the implementation of our jump test so our analysis focuses on the after-hours market.[33] Because individual transaction counts are prohibitively large in that segment and liquidity evaporates fast as we move to stocks with lower after-hours market liquidity, we focus our analysis on the 25 firms with the largest after-hours trading volume on announcement days.[34] These stocks account for a disproportionately large

[29] The major national stock exchanges close at 1:00pm the day before big public holidays, such as July 4th, Thanksgiving, and Christmas.

[30] The trading schedule of NASDAQ- and NYSE-listed securities can be gauged at `https://www.nasdaq.com/stock-market-trading-hours-for-nasdaq` and `https://www.nyse.com/markets/nyse-arca/market-info`.

[31] Companies prefer to announce at these times because the after-hours markets are mostly comprised of professional investors and informed traders, see Jiang, Likitapiwat, and McInish (2012). This speeds up the price discovery process and helps convey information to the general public.

[32] Figure D.1 in Appendix D plots the volume distribution for constituents of the S&P 500 during the pre-market (6:00am–9:30am, Panel A) and the after-hours market (4:00pm–6:30pm, Panel B).

[33] In practice, there is usually limited trading activity after 6:30pm, so we do not consider trading after this time.

[34] To identify these firms, we sort the constituents of the S&P 500 index as of 12/31/2020 by their post-close (4:00pm–6:30pm) transaction volume on quarterly earnings announcement days, averaged over the five-year period from 01/01/2015–31/12/2019. We also require that the firms report at least 75% of their earnings announcements after the close of the regular trading session during our full sample period 06/02/2008–12/31/2020. We exclude Paypal (PYPL, ranked 19th) from the original list since this firm was only exchange-listed (for the 2nd time) on

fraction of trading activity, or 41.14% of the total transaction count, in the after-hours market among all firms in the S&P 500 index during our sample.

We download high-frequency data from the NYSE TAQ database for the selected stocks over the sample period 06/02/2008 through 12/31/2020, a total of $T = 3{,}169$ business days. The data is cleaned using state-of-the-art procedures (e.g. Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2009; Christensen, Oomen, and Podolskij, 2014). The main distinction here is that we retain observations with a timestamp outside the official exchange trading hours if they otherwise fulfill the conditions required to be tagged non-erroneous.[35] We use tick-time sampling to remove transactions that merely repeat the previous price and aggregate remaining observations with identical timestamps. The latter are instead replaced with an average price, a procedure that is known to further alleviate microstructure noise. In total, we preserve over 19 billion (19,074,128,835) quotations and nearly six billion (5,817,606,415) transactions across stocks.

Table 1 reports ticker symbols for our list of stocks and summary statistics for their trading volume in the after-hours session.[36] We show the sample average, standard deviation, and the 25th, 50th, 75th and 99th percentiles of the transaction count distribution for days without earnings announcements (columns 2–8) and days with earnings announcements (columns 9–15). For many firms, the after-hours transaction count is two orders of magnitude larger on days with earnings announcements relative to days without announcements. For example, Facebook (FB) averaged 76,129 after-hours transactions on earnings announcement days, compared with only 891 transactions on days with no such announcements. It is clearly uncommon for individual firms to exhibit high transaction counts in the after-hours trading session on days without earnings announcements.

3.2 Announcement timing

While most companies announce their financial results at a fixed time, for others the timing can vary quarter-by-quarter. Even slight inaccuracies in the recording of the announcement time can impede our calculation of the post-announcement price reaction, so accurate timing is imperative and we spend considerable effort on double checking the time stamps.

We begin by gathering announcement times from a data set provided by Wall Street Horizon that offers a comprehensive suite of corporate events data, including detailed information on the timing of earnings announcements.[37] Wall Street Horizon stores the timestamp included with the press release issued by the company when it publicly announces its quarterly results.[38]

07/20/2015 and therefore has too short a sample. PayPal is replaced by Ulta Beauty (ULTA).

[35] A complete list of sale conditions that can be associated with a transaction is available in the documentation of the Daily TAQ Client Specification manual available for download at: `http://www.nyxdata.com/Data-Products/Daily-TAQ`. In particular, the label "T" identifies pre-market and after-hours trades. These are virtually always ignored in the high-frequency volatility literature, since the first rule of the widely used Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009) filtering algorithm instructs researchers to "*P1. Delete entries with a time stamp outside the 9:30am–4:00pm window when the exchange is open.*"

[36] The corresponding table for the pre-market trading activity can be found in Appendix D.

[37] See `https://www.wallstreethorizon.com/`.

[38] A publicly traded company with classes of securities registered in Section 12 or subject to Section 15(d) of the

As a double-check, we hand collect a separate set of announcement times from the Factiva news archive by searching on the announcement day for press releases marked with the company ticker code and a designated "Earnings" subject.[39]

Comparing the merged set of announcement times with the audit trail of the complete tick-by-tick transaction price history suggests that occasionally both Wall Street Horizon and Factiva get it wrong. To correct seemingly erroneous timestamps, we design a conservative screening algorithm that infers the announcement time based on the price volatility. In particular, we calculate the log-return for each minute from 4:00pm–6:30pm. If the largest absolute one-minute log-return exceeds 2.5% and it precedes the announcement time from both Wall Street Horizon and Factiva, we assume the earnings were released at the earlier time. The filter affects only a very limited number of announcements (46 in total, or 3.95%) with a high concentration in Tesla (TSLA), Intel (INTL), and Micron Technology (MU) which together account for more than half of the updated announcement times. For other companies, only a minuscule amount of announcement times are modified (18 in total, or 1.74%), and here the median change is one minute, suggesting that the algorithm primarily captures small rounding effects for some of the most important earnings reports that lead to substantial revisions of the security price.[40]

The third-to-last column in Table 1 shows the modal announcement time for each firm, which ranges from as early as 4:00pm (reports released immediately after the close of the regular exchange-trading session) to as late as 4:30pm, with many firms announcing around 4:05pm.

3.3 Earnings surprises

To compute the surprise element in earnings announcements, we employ the Institutional Brokers Estimate System (I/B/E/S) database available through subscription to Refinitiv (formerly part of Thomson Reuters).[41] I/B/E/S aggregates earnings information for over 22,000 companies. In addition to the actual reported earnings per share (EPS), adjusted for non-recurring items and stock option expenses, I/B/E/S surveys earnings forecasts from leading professional analysts covering individual companies. Prior to each announcement, we compile this information into a consensus

Securities Exchange Act of 1934 is legally required to file regular reports with the SEC, including an annual form 10-K and quarterly form 10-Q's, in addition to proxy reports and other reporting requirements. Most companies file form 10-Q either 40 or 45 days after the end of the fiscal quarter, but many large corporations choose to summarize their financial statements at an earlier date by issuing a press release and filing a form 8-K.

[39] A limitation of both data sets is that announcement times are rounded to the nearest minute. However, most of the earnings reports in our sample are released to the public through professional dissemination services, such as BusinessWire or PR Newswire which are under strict requirements to ensure immediate and equal access to company information as noted by the SEC's Regulation Fair Disclosure. Combined with the audit-trail information left from the high-frequency data after each announcement suggests that the rounding effect is typically negligible.

[40] Importantly, our noise-robust jump test statistic is unaltered by the screening algorithm since it does not depend on the announcement time. Moreover, employing Wall Street Horizon and Factive information only does not cause any discernible change in the post-announcement return regression (Section 5.1). Only the excess returns from our trading strategy (Section 5.2) deteriorate slightly with the removal of the inferred announcement time but remain significant in the baseline implementation.

[41] `https://www.refinitiv.com/`.

estimate (μ_{EPS}) and a standard deviation of that estimate calculated over the distribution of analyst expectations (σ_{EPS}). Following Berkman and Truong (2009), Michaely, Rubin, and Vedrashko (2013), among others, our measure of the standardized unexpected earnings is given by[42]

$$z_{\text{EPS}} = \frac{\text{Actual EPS} - \mu_{\text{EPS}}}{\sigma_{\text{EPS}}}. \tag{27}$$

We purge announcements with no analyst forecasts and those where σ_{EPS} is less than 0.001 (one tenth of a cent) which result in an unstable z_{EPS} measure. This removes five announcements, all from Ulta Beauty (ULTA) that was still a relatively overlooked small-cap stock with limited broker coverage at the beginning of our sample. We also exclude announcements where share trading was temporarily suspended by the exchange as it leads to a disconnected trade sequence.[43] Finally, following Grégoire and Martineau (2021) we discard announcements where z_{EPS} exceeds ten in absolute value. Such outliers are often caused by extraordinary items in the profit statement.[44] The last step removes 18 observations (about 1.5% of the sample).

The number of quarterly earnings announcements retained for each company is shown in the fourth-to-last column in Table 1. We report summary statistics of z_{EPS} in the second-to-last column. Standardized unexpected earnings are substantially higher than zero on average, consistent with the notion that managers engage in earnings smoothing to avoid negative surprises.

4 Empirical results

We next proceed with our empirical investigation.[45] Price dynamics and trading activity in the after-hours market is not as thoroughly studied and understood as that in the regular trading session and it has evolved significantly since the seminal analysis of Barclay and Hendershott (2003). We therefore begin by briefly summarizing some of the key features of the after-hours market, emphasizing the contrast between days with and without earnings announcements. We start by

[42]Some studies employ the median analyst estimate instead of the sample average (Dellavigna and Pollet, 2009; Grégoire and Martineau, 2021; Hartzmark and Shue, 2018). Others replace the standard deviation with the lagged closing price of the stock, e.g. from a week before the announcement or at the end of the previous quarter (Grégoire and Martineau, 2021; Lyle, Stephan, and Yohn, 2021). Our main findings are not altered by adopting these alternative ways of defining the standardized unexpected earnings.

[43]Trading halts are registered in the daily TAQ master files that contain static information for securities traded by participants of the Consolidated Tape Association (CTA), mainly stocks with NYSE as primary exchange, and Unlisted Trading Privileges (UTP), or NASDAQ-listed issues. The file has a letter code in the "Halt Delay Reason" if trading was paused (blank otherwise). Resumption of trading is flagged in the transaction data with a sale condition "5" defined as "Market Center Re-Opening Trade" (CTA) or "Re-opening Prints" (UTP).

[44]For example, on 08/25/2020 Salesforce (CRM) posted a quarterly EPS of \$1.44 after adjusting for non-recurrent items. With a consensus estimate of \$0.66 and a standard deviation of about four cents, this yields a twenty-sigma event. The surprise was motivated by mark-to-market accounting for the company's investments in nCino, which saw a \$617 million unrealized gain in the quarter.

[45]We employ the weight function $g(x) = \min(x, 1-x)$ and pre-averaging horizon $k_n = \lfloor \theta\sqrt{n} \rfloor$ with $\theta = 1/2$. We set $L = p = 10$ for the subsampler, and $c = 5$ and $\bar{\omega} = 0.24$ for the truncation device. In practice, fixing θ means k_n is time-varying since it changes with the actual number of high-frequency data available in each trading session, n. As a robustness check, we also fixed $k_n = 25$ and $k_n = 50$ without any noticeable change in the results.

analyzing the pattern in trading volumes and bid-ask spreads in a short interval around earnings announcements before turning to the evidence on the presence of price jumps.

4.1 Trading volume and bid-ask spread

To get a broad picture of post-announcement trades, for each five-second interval in a one-hour window centered around the announcement time, we calculate transaction counts and bid-ask spreads (in basis points) as

$$\text{Spread} = 10000 \times \frac{\text{ask} - \text{bid}}{\text{midquote}}, \tag{28}$$

where midquote = (bid + ask)/2.

As a control sample, for each announcement we select a random no announcement date and calculate transaction counts and Spread on a corresponding window.

Panel A in Figure 2 reports the transaction count per five-second interval on days with and without earnings announcements. The modal announcement time across companies is 4:05pm. Hence, the market typically closes five minutes prior to the announcement (labeled time 0), as highlighted in the figure. The transaction volume right before the regular market close is notably higher on days with announcements than on days without announcements, but both drop sharply at the close. From this point onward, the graphs evolve very differently, however. Whereas the after-hours transaction count remains close to zero on no-announcement days, it spikes to more than 20 contracts shortly after the announcement time on days with earnings announcements before gradually tailoring off over the next 30 minutes.

In Panel B of Figure 2, we compare bid-ask spreads in the after-hours market on days with and without earnings announcements. Again, we see distinct differences. On no announcement days, the median quoted spread starts at 5 bps at the end of the regular trading session before rising to 25–30 bps within the first five minutes of the post-trading session. It then gradually inclines for the duration of the after-hours session as we approach the overnight period. On announcement days, the median quoted spread jumps from 5 bps at close (4:00pm) before quickly rising to 40 bps at the time of the announcement, then drops to 20 bps within 10–20 minutes of the announcement.

For comparison, the average quoted spread during post-close trading is 36.1 bps, while the average spread in regular trading equals 6.2 bps, implying that the bid-ask spread on average is six times greater in the after-hours session compared with the regular trading session.[46]

4.2 Jump proportion

The theoretical analysis in Section 2 highlights that jumps in prices increase the realized variance without affecting the bipower variation. This insight suggests an intuitive way to check whether

[46]This is shown in Figure C.3 in Appendix C, which reports summary statistics on the after-hours trading volume and bid-ask spreads. Like the trading volume, after-hours bid-ask spreads follow a distinct intraday pattern and are generally lower on announcement days, except during the first 10–15 minutes after an announcement. After-hours spreads remain lower the following day compared to next-day spreads on no announcement days.

earnings announcements cause jumps.

First, since earnings announcements almost exclusively fall outside the hours of the exchange-traded session, in Panel A of Figure 3 we plot the pre-averaged realized variance computed for the regular trading session from 9:30am–4:00pm against its corresponding value computed for extended trading session from 9:30am–6:30pm, both converted to annualized standard deviations. Their difference measures the incremental volatility observed in the after-hours market from 4:00pm–6:30pm. On no announcement days (indicated by a red dot) the extra volatility during after-hours trading is typically minuscule and has a negligible impact on the cumulative pre-averaged realized variance measure. Except for a few outliers, the vast majority of these observations form a cloud close to the 45-degree line. Conversely, on days with a quarterly earnings announcement (indicated by a blue cross), the discrepancies are much more pronounced with the pre-averaged realized variance substantially exceeding its corresponding regular trading session value. Earnings releases clearly trigger substantial price volatility in the after-hours market, which is otherwise dormant.

Second, and more directly related to our jump test, in Panel B of Figure 3 we plot the pre-averaged bipower variation against the pre-averaged realized variance, both based on high-frequency data from the extended trading session (9:30am-6:30pm). Realized variance less bipower variation isolates the contribution of jumps to return variation, which is zero (up to sampling error) if the price path is continuous but is strictly positive if there are price jumps. Hence, the farther to the right of the 45-degree line a data point is, the more it indicates the presence of jumps. Consistent with this interpretation, the pre-averaged realized variance and bipower variation are more or less perfectly aligned on no announcement days. Conversely, on days with earnings announcements we observe large differences indicating jumps in price movements at these times. Table 2 provides further stock-level details on realized variance and bipower variation for the regular and extended trading session broken into days with and without earnings announcements. Appendix E provides an even more detailed analysis for Apple which is perhaps the most prominent stock in our sample.

An alternative approach to gauge the relative importance of jumps is to examine the jump proportion which estimates the fraction of the return variation originating from the jump component. Following Christensen, Oomen, and Podolskij (2014), this is defined as[47]

$$\text{Jump proportion} = 1 - \frac{\text{Bipower variation}}{\text{Realized variance}}. \tag{29}$$

Figure 4 plots kernel density estimates of the distribution of the jump proportion. As indicated by the peak near zero for the graphs based on our noise-robust pre-averaging estimators, on most days without announcements (Panel A) the jump component accounts for none of the variation in the stock price.[48] Conversely, on days with earnings announcements (Panel B) more than fifty percent

[47]The jump proportion is computed without pre-averaging if sampling at a 5-minute frequency or with pre-averaging if sampling at the tick-by-tick frequency.

[48]Note that small negative numbers can be observed due to estimation error.

of the total variation in price movements stems from the jump component.[49]

In contrast, the jump proportion extracted with five-minute sampling peaks near 10%, regardless of whether there is an announcement or not.[50] Although the jump proportion for the five-minute measure is slightly more right-skewed on days with announcements, in general it overestimates the importance of jumps on days with no announcements and underestimates it on days with announcements. These findings are consistent with the biases in the conventional jump measure documented in our Monte Carlo simulation results.

4.3 Jump frequency

Figure 4 shows that the jump component is important in after-hours trading sessions on days with earnings announcements but largely absent on days without announcements. To examine this point more closely, we compute the jump indicator:

$$J_{it} = \begin{cases} 1, & \mathcal{J}_{n,it} > q, \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

where $\mathcal{J}_{n,it}$ is the noise-robust jump test statistic in (25) for company i on day t. We set the critical value q in a way that reduces the likelihood of identifying false jumps by applying a Bonferroni correction to account for multiple testing and keep the family-wise error rate at 1%.[51]

Table 2 reports the jump frequency across firms in our sample during the regular trading session in Panel A versus the extended trading session in Panel B. The sample is further split into days with and without an earnings announcement. Applying the pre-averaged jump test statistic to the regular trading session, we identify 2,739 firm-days with a significant jump, corresponding to 3.46% of all stock-trading days. While jumps do occur at a higher frequency than expected by random chance (the size of the test is 1%), the difference is not very large and so it is relatively rare for jumps to occur during the regular trading session.[52] Among after-hours trading sessions without earnings announcements, we find 3,474 firm-days with jumps, corresponding to a jump frequency of 4.45% which is only slightly higher than the previous estimate. Finally, for after-hours trading sessions with earnings announcements, we identify 1,109 days with jumps, corresponding to a jump frequency of 95.27% which is far higher than the previous values.

In essence, while it is rare to identify a jump during regular trading sessions or in extended trading sessions without announcements, it is rare *not* to find a jump in extended sessions with earnings announcements. This finding provides strong empirical support for Hypothesis 1, part 1.

[49] The slight attenuation observed with $\theta = 1$ is consistent with our Monte Carlo evidence in Appendix B, where we note that excessive pre-averaging tends to deflate the power of our microstructure noise-robust jump test.

[50] The cross-sectional average jump proportion for the five-minute realized variance and bipower variation calculated on the regular trading session 9:30am–4:00pm is 7.3%. This number is consistent with Table 1 in Christensen, Oomen, and Podolskij (2014).

[51] Bajgrowicz, Scaillet, and Treccani (2016) report that up to 90% of jumps identified with the five-minute jump test are spurious due to multiple hypothesis testing.

[52] This result is consistent with Christensen, Oomen, and Podolskij (2014).

The corresponding numbers for the five-minute jump test statistic of Barndorff-Nielsen and Shephard (2006) are 4,385 (a 5.54% jump frequency) in regular trading sessions, 9,193 (11.79%) in extended trading sessions without earnings announcements, and 482 (41.41%) in extended trading sessions with earnings announcements. This again illustrates that the conventional jump test identifies too many jumps (false positives) during regular trading sessions and extended trading sessions without earnings announcements but too few jumps (false negatives) during extended trading with earnings announcements. This finding is backed up by our simulations in Appendix B, which show that the five-minute jump test overrejects in the presence of little or no microstructure noise (as in the regular trading sessions) but lacks power (underrejects) in the presence of high levels of noise contamination (as in the after-hours market).

It is often easy to explain the rare occasions where prices do not appear to jump after an earnings announcement (less than 5% of cases). Table 2 shows that failure to find jumps after earnings announcements occurs predominantly for the three least liquid companies, namely Gilead (GILD), Applied Materials (AMAT), and Ulta Beauty (ULTA). Together, these three firms account for about half of the announcements with no jumps, with a high concentration of these cases located in the early part of our sample. In these instances, there are often relatively few post-announcement observations to base the analysis on, rendering it practically impossible for our noise-robust test statistic to work as intended.

The other main reason why we sometimes fail to detect a post-announcement price jump is that although our jump test has excellent power, it is not bullet proof. This is particularly relevant for announcements that are succeeded by whipsawing in the price. Intuitively, a jump is less likely to be detected when it is surrounded by extremely high diffusive volatility which reduces the relative contribution of the jump to return variation. We elaborate further on this connection in the context of our binary choice model below.

4.4 Determinants of the jump frequency

To more formally test Hypothesis 1, we next develop a regression framework to examine the determinants of price jumps. We begin by inspecting the role of earnings surprises since this is the measure most directly related to earnings announcements.

To capture the relation between the jump probability versus the sign and magnitude of earnings surprises, we first sort earnings announcements by the value of z_{EPS}, form decile portfolios from lowest to highest values of the earnings surprise, and plot the corresponding average jump frequencies implied by our noise-robust jump test in Panel A of Figure 5. The jump probability exceeds 90% across deciles. Moreover, it does not depend systematically on the size of the earnings surprise. In fact, prices are as likely to jump on days with the most negative earnings surprises (decile 1) as they are on days with the most positive surprises (decile 10).

Our finding that prices jump even when investors are not surprised by the reported earnings per share is consistent with Hypothesis 1 and can be explained as follows. First, it is possible that information other than the headline earnings figure gets released (e.g., forward guidance on future earnings) and that this triggers a jump.[53] While this helps explain why prices *can* jump even on days where markets do not get surprised by the earnings figure, we would still expect a lower *average* jump probability on such days. This points to the second explanation, namely that prices jump because of the resolution of uncertainty associated with the earnings announcement (Banerjee and Kremer, 2010). An implication of this story is that prices are more likely to move up (due to the lower post-announcement risk premium) than down following an earnings announcement that is close to investor expectations. We further explore this prediction in Section 5.1.

To establish a more formal connection between price jumps and earnings announcements and examine whether other factors affect jump probabilities, we next estimate logit regressions. Our analysis includes up to five regressors. To estimate the effect of an earnings announcement on the jump probability and test Hypothesis 1, we consider an announcement dummy variable:

$$EA_{it} = \begin{cases} 1, & \text{on announcement days,} \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

Since z_{EPS} proxies for the magnitude of the news component in the announcement, we expect these to matter for the jump probability as well. As our second and third covariates, we therefore include the absolute value of the earnings surprise, measured separately for positive and negative surprises, i.e., we define $z^+_{\mathrm{EPS},it} = z_{\mathrm{EPS},it} EA^+_{it}$ and $z^-_{\mathrm{EPS},it} = z_{\mathrm{EPS},it} EA^-_{it}$, where $EA^+_{it} = \mathbb{1}(z_{\mathrm{EPS},it} > 0)$ and $EA^-_{it} = \mathbb{1}(z_{\mathrm{EPS},it} < 0)$ are indicator variables that capture the direction of the earnings surprise.

Fourth, to see whether high volatility in prices prior to the announcement makes it more or less likely to find jumps, we include the pre-averaged realized volatility, $\sqrt{RV^*_n}$, computed over the regular trading session from 9:30am–4:00pm on the day of the announcement. We expect to find a negative effect since high volatility makes it harder for our noise-robust jump test to detect jumps.

Fifth, as a measure of the resources available to facilitate the price discovery process, we include the number of analysts covering the stock, N_A.[54] We would expect to find a positive coefficient since the price discovery process, and thus the speed of price adjustments, should be more efficient for stocks with the greatest analyst coverage.

Next, we estimate the logit model using observations for stock i on day t:

$$P(J_{it} = 1) = F(a + b_1 EA_{it} + b_2 |z^+_{\mathrm{EPS},it}| + b_3 |z^-_{\mathrm{EPS},it}| + b_4 \sqrt{RV^*_{n,it}} + b_5 N_{A,it}), \tag{32}$$

where F is the logistic distribution function.[55]

[53] For example, on 10/24/2011 Netflix (NFLX) reported third-quarter EPS of $1.16 which beat the consensus expectation of $0.94, corresponding to a modest 1.6 standard deviation surprise. However, the company also significantly lowered its forward guidance, and the stock crashed more than 20% in after-hours trading.

[54] We also considered the dispersion in EPS forecasts across analysts, σ_{EPS}, to get a measure of subjective uncertainty surrounding a particular earnings announcement. However, this measure fails to be significant in our logit regression.

[55] The coefficients in the logit model are estimated based on the entire sample of days with and without earnings announcements. Hence, the extra explanatory variables in (32) should be understood as being interacted with the EA variable and assume a value of zero on no-announcement days. With this design, all covariates measure the

Table 3 reports estimation results for the logit regression applied to the extended trading session.[56] The estimated intercept in column (2) with only the announcement dummy yields a jump probability on days without announcements ($EA = 0$) of 4.45% compared to 95.27% for announcement days ($EA = 1$), which agrees with the results in Section 4.3. Further, and consistent with Hypothesis 1 and Figure 5, the earnings announcement dummy is highly statistically significant and positive, showing that the likelihood of a jump increases by a huge amount and is near unity in the wake of an earnings announcement.

As reported in column 3, the estimated coefficients on both positive ($|z^+_{\text{EPS}}|$) and negative ($|z^-_{\text{EPS}}|$) earnings surprises are positive but insignificant, despite the much larger coefficient estimate for negative surprises. Because earnings surprises are predominantly positive, the insignificant slope estimate on $|z^-_{\text{EPS}}|$ may merely be an artefact of the much smaller sample available to estimate this coefficient.

The effect of $\sqrt{RV^*_n}$ is significantly negative so higher pre-announcement return volatility reduces the jump probability. For example, setting the realized volatility at its 10th and 90th percentile our coefficient estimates imply a conditional jump probability of 98.42% and 96.93% (97.16% and 92.73%) for an average negative (positive) announcement. A plausible explanation is that it is harder to detect a post-announcement price jump amid high levels of volatility which weakens the testing power. Moreover, a high level of volatility can act as a substitute for price jumps, since it widens the disperson of the return distribution and enables the price to move rapidly, albeit continuously, in response to earnings announcements.

The effect of the number of analysts covering a stock, $N_{A,it}$, is significantly positive, so jumps are more likely on earnings announcement days for those stocks covered by the largest number of analysts. For example, the coefficient estimate implies a conditional jump probability of 94.75% and 99.13% (90.00% and 98.55%) for an average negative (positive) announcement when the number of analysts is at its 10th and 90th percentile, respectively (keeping pre-averaged realized volatility fixed at its average value). Broader analyst coverage is likely to speed up the price discovery process and increases the discreteness in price movements and the chance of observing jumps.

4.5 Jump Spillovers

Earnings announcements may reveal information about the state of the economy and are likely to be particularly informative about the earnings prospects of closely related firms within the same industry. In Savor and Wilson (2016), investors use earnings of announcing firms (AFs) to revise their expectations of future dividends for non-announcing firms (NAFs), which increases the covariance between firm-specific and market-wide cash-flow news. It is therefore likely that an

marginal effect (or the logarithm of the odds ratio) on the jump probability *conditional* on an announcement.

[56] The results of our logistic jump probability regression and the linear return regression in the next subsection are robust to the inclusion of stock fixed effects.

earnings announcement from the AF can trigger co-jumps in the prices of NAFs.[57] To examine this, and test Hypothesis 1 part 3, we intersect announcement days in the AF with no-announcement days in NAFs. Evidence of spillover effects should show up as a higher conditional jump probability in the NAFs, given an earnings release from the AF, relative to the conditional jump probability when no firms are announcing.

Table 4 reports the proportion of extended trading sessions with a jump in the NAF (listed in columns) when the AF (in rows) reports financial results. For example, conditional on a Facebook (FB) announcement, the price of Apple (AAPL), Amazon (AMZN), and Google (GOOG) jump on 13.3%, 25.8%, and 28.1% of their no announcement days. The conditional jump probability generally rises for companies within the tech sector and is much lower for companies from other sectors. This is consistent with the presence of a common jump factor for the tech industry. Moreover, on days where Intel (INTC) is the AF, we record a conditional jump probability of 20.0%, 44.0%, and 30.0% in the price of Nvidia (NVDA), Advanced Micro Devices (AMD), and Micron Technology (MU), pointing to a common jump component tracking the microconductor industry. These numbers are much higher than the no announcement jump probability of 4.18%, which we calculate as the cross-sectional average jump frequency across common no announcement days of our firms.[58] Our analysis therefore provides strong support Hypothesis 1.2.

Conversely, there is little evidence of spillover effects from some of the other companies in our sample without obvious industry peers. For example, conditional on an announcement from Walt Disney (DIS), the probability of a jump in the other stocks is always less than 10% and often close to zero, which suggests that the jump component is tracking idiosyncratic news.[59]

As a more formal test of whether individual firms' earnings announcements contain a systematic component, we also compute the probability of a jump in the S&P 500 index (proxied by SPY) in the extended trading session in which the firm announces its earnings. This can be viewed as the conditional probability of a jump in the market portfolio given that a particular firm has announced its earnings. If firms' earnings announcements contain a systematic component, we would expect this jump frequency to be significantly higher than the jump frequency in the market index in the absence of an earnings announcement. Moreover, we expect the effect to be strongest among firms operating in industries more central to the economy.

The right-most column in Table 4 offers strong support for the presence of a systematic, or "market", component in individual stocks' price jumps. Specifically, the frequency of jumps in the S&P 500 index is very high conditional on a jump in the price of important stocks such as Apple, Facebook, Amazon, Microsoft, Oracle, and Starbucks. For example the market index jumps 30% of

[57]Theory and empirical results on tests for co-jumps is provided in Jacod and Todorov (2009); Caporin, Kolokolov, and Renò (2017).

[58]The conditional jump probability on common no announcement days is slightly lower than the conditional no announcement day jump probability reported in Table 2 since the latter does not discriminate between days where other firms are or are not announcing.

[59]It may of course exert influence on the conditional jump probability of related companies not included in our sample such as Hasbro (HAS).

the time after earnings announcements by Apple. This is much higher than the jump probability in the market index on common no-announcement days (3.98%).

Conversely, the market jump probability is much lower when firms such as Tesla and Salesforce announce earnings, suggesting that price movements in both of these stocks were largely driven by idiosyncratic news unrelated to the broader economy.

We would also expect to find a higher jump probability for the market index, the more firms announce earnings on a given day. In our sample there are between zero and five earnings announcements per day (with only a handful of the latter). The conditional jump probability in the market index is strictly increasing in the number of announcements, rising from 3.98% on common no-announcement days to 4.77%, 5.06%, 12.96%, 15.00%, and 37.50% as the number of announcements rises from one to five. These observations are consistent with Hypothesis 1. part 2.

These results are particularly strong because it is often found that it is very difficult to detect jumps in the market index even when the price of the underlying stocks jump because the individual stocks typically take only a small weight in the index basket.[60] Our results point towards an important information effect that makes jumps in many individual stocks' prices caused by earnings announcements relevant for the broader market.

5 Price dynamics after earnings announcements

Our results so far provide insights into price dynamics (jumps) in the immediate aftermath of earnings announcements. However, they do not show by how much prices typically move or if prices over- or undershoot. In this section, we explore whether post-announcement returns are predictable and whether this can be exploited in simple trading strategies to generate abnormal returns. Both points help us better understand the efficiency of the post-announcement price formation, specifically the "sufficiency" part of conditions for market efficiency.

5.1 Determinants of the jump size

While our noise-robust test statistic speaks to the presence (or absence) of a jump, it does not reveal by how much the price jumps. We therefore next construct a proxy for the jump size based on a return measured over a small time interval following the earnings announcement. In particular, we compute the one-minute post-announcement return (r_{1m}^{EA}) as the log-price a minute after the earnings release less the latest log-price prior to the announcement.

The last column of Table 1 reports summary statistics for the one-minute post-announcement returns for each company. While the cross-sectional sample average of 0.11% is rather large, the standard deviation of 3.52% is even bigger, indicating that one-minute post-announcement returns,

[60] Note that we also do not adjust for the sequencing of the earnings announcements. As shown by Savor and Wilson (2016), firms that announce their fiscal results later in the earnings cycle tend to carry less new information about the economy and so are less likely to trigger a jump in the market.

while positive on average, are highly dispersed. One-minute returns are slightly negatively skewed (-0.29), although this is mainly driven by a few extremely negative return observations in our sample as evidenced by a large coefficient of kurtosis (5.09).

In Panel B of Figure 5, we plot the average one-minute post-announcement return against the standardized earnings surprise. We observe a near-monotonic relationship between the sign and magnitude of earnings surprises and subsequent returns. Specifically, large negative (positive) earnings surprises are associated with large negative (positive) post-announcement returns.

More than 80% of earnings announcements surprise positively in our sample, so firms are expected to beat consensus estimates. The average value of the standardized unexpected earnings is $\bar{z}_{\text{EPS}} = 1.49$ (median value of 1.12), which is located in the sixth decile in Figure 5. The average one-minute post-announcement return in that decile is 0.34% (median value of 0.07%), which is consistent with Hypothesis 1 part 3, that earnings announcements in line with expectations incur positive returns due to uncertainty resolution.

We next examine the determinants of the change in stock prices in the aftermath of earnings announcements. To understand which factors drive post-announcement returns, we estimate a regression model that includes the predictor variables from the logit model in 32 except the EA dummy variable. In addition, we add the cumulative net order imbalance, which we construct following Grégoire and Martineau (2021):

$$OI_{it} = \frac{B_{it} - S_{it}}{B_{it} + S_{it}}, \tag{33}$$

where B_{it} and S_{it} correspond to the buyer- and seller-initiated trading volume over the one-minute post-announcement window. To get signed trade volume, we assign an aggressiveness indicator to each transaction based on whether it was buyer- or seller-initiated. We employ a "level-1 algorithm" known as the tick rule (e.g. Chakrabarty, Pascual, and Shkilko, 2015).[61] Note that in contrast to the other variables, the net order imbalance is not known ahead of the announcement.

We then estimate a pooled panel regression for the one-minute post-announcement return for stock i on day t:

$$r^{\text{EA}}_{1m,it} = a + b_1 z^{+}_{\text{EPS},it} + b_2 z^{-}_{\text{EPS},it} + b_3 \sqrt{RV^{*}_{n,it}} D_{it} + b_4 N_{A,it} D_{it} + b_5 OI_{it} + \epsilon_{it}, \tag{34}$$

where $D_{it} = \text{sign}(z_{\text{EPS},it})$ is the sign function. The idea behind interacting the sign of z_{EPS} with pre-averaged realized volatility and the number of analysts is that the latter are strictly positive and merely capture the speed with which information gets impounded into stock prices whereas earnings surprises are directional. The bigger the return volatility and number of analysts, the more we expect positive news (a positive sign indicator) to move returns upwards and negative news (a

[61] Quoting from their paper: *"The tick rule classifies a trade as buyer-initiated if the trade price is above the preceding trade price (an uptick trade) and as seller-initiated if the trade price is below the preceding trade price (a downtick trade). If the trade price is identical to the previous trade price (a zero-tick trade), the rule looks for the closest prior price that differs from the current trade price. Zero-uptick trades are classified as buys, and zero-downtick trades are classified as sells."*

negative sign indicator) to move returns downwards. A priori, we therefore expect all coefficients to be positive in this regression.

Table 5 reports our estimation results. In column (2), we look at the restricted model with only z^{+}_{EPS} and $z^{-}_{EPS,it}$ but no additional controls. Both estimates of b_1 and b_2 are positive, implying that larger earnings surprises translate into larger announcement returns. The marginal effect of negative earnings surprises is almost three times greater than that of positive earnings surprises, showing that negative earnings surprises trigger a much larger drop in the stock price relative to the rise in price for a positive surprise of corresponding magnitude. In this regression, the coefficients are also significantly different from each other (P-value of 0.0024). However, adding controls in column (2) and (3), the estimates of b_1 and b_2 remain positive and $b_2 > b_1$, but the hypothesis $H_0 : b_1 = b_2$ is no longer rejected.

The slope estimate on the pre-averaged realized volatility, $\sqrt{RV^{*}_{n}}$, is positive, though statistically insignificant. Higher volatility is therefore (weakly) associated with wider dispersion in returns as post-announcement prices move further away from their initial value during periods of higher volatility.

The coefficient estimate on the number of analysts, N_A, is positive and significant. Hence, a larger number of analysts monitoring a company leads to larger one-minute post-announcement returns following positive surprises and smaller (more negative) returns for negative surprises.

Finally, we consider the order imbalance variable, OI. While the other variables in the regression are known ex-ante (at or before the earnings announcement), order imbalance is only known ex-post. Bearing this in mind, our estimates on OI are positive and highly significant for both positive and negative earnings surprises, implying that larger net order imbalances are associated with bigger movements in returns. Intuitively, buying pressure (positive OI) pushes prices up, while selling pressure (negative OI) reduces prices.[62]

We conclude from this evidence that both small and large earnings surprises almost always trigger a jump in the stock price. However, while small earnings surprises are associated with small returns, large earnings surprises tend to move prices by a bigger amount in a direction that is consistent with the sign of the earnings surprise. This effect is stronger for negative earnings surprises. Moreover, prices move by more, the more analysts cover a given stock and the magnitude of price movements is also affected by the pre-announcement price volatility.[63]

[62]As a robustness check, we reestimated the return regression in (34) based on 5-, 10-, 30-, and 60-minute post-announcement returns. Although some of the marginal effects were found to be larger simply because the longer-horizon returns tend to be larger than one-minute returns, none of the conclusions regarding the sign and significance of the coefficient estimates changed.

[63]Ai, Bansal, Im, and Ying (2021) examine empirically the effect of macroeconomic announcements on asset prices. They find that more than two-thirds of the equity risk premium accrues on the 30 days per year with pre-scheduled macroeconomic announcements. Consistent with Savor and Wilson (2016), they find a positive and significant expected return-β relation on days with announcements and a zero relation on no announcement days. Lucca and Moench (2015) document large positive average excess returns on U.S. equities in the 24-hour window leading up to pre-scheduled Federal Open Market Committee meetings, but not in other major U.S. macroeconomic announcements. Ai and Bansal (2018) develop a theoretical model to explain how asset prices can rise in anticipation

5.2 Trading strategy

We next examine if the one-minute post-announcement return can be predicted and, if so, whether the resulting forecasts can be exploited in a simple trading strategy. To maintain a parsimonious design, we employ the pooled panel regression in (34) based on the standardized earnings surprise as the only conditioning information:

$$r^{\mathrm{EA}}_{1m,it} = a + b_1 z^{+}_{EPS,it} + b_2 z^{-}_{EPS,it} + \epsilon_{it}. \tag{35}$$

In Figure 6, we plot the one-minute post-announcement return against the earnings surprise measure across firms and earnings announcements in our sample. We superimpose the fitted piecewise linear model from (35). Although a positive relationship is evident from the figure, it is surrounded by a substantial amount of noise.

We next recursively estimate the parameters in (35) using an initial warm-up sample of one month of announcement days, adding new data as it becomes available. This ensures our forecasts are available in real time and do not suffer from look-ahead bias.

Our trading strategy then opens a long (short) position if the predicted return in (35) is positive (negative). Our baseline scenario assumes the position is held until the end of the trading day (6:30pm, or EOD) and closed at the last available observation. However, the reported results are robust to leaving the position on the book until the exchange opens the next morning or closes the following afternoon. To avoid chasing too small expected profits, we set an entrance barrier of 0.5% in absolute value for the predicted return, but our results are robust to using a range of other threshold values. With a 0.5% required return, the strategy generates 581 trading signals from 1,164 earnings announcements, of which 447 are long and 134 are short.

To gauge the importance of transaction costs, we consider four different implementations of our trading strategy. Our first approach sets the entry level of the trade equal to the first post-announcement transaction price (Trade) and so ignores any trading frictions. Our second approach employs the midquote (Midquote). Whereas the Trade strategy can snipe stale quotes that were resting in the limit order book prior to the announcement, the Midquote strategy employs an updated quote, which incorporates the information from the announcement. Trading at the midquote is often possible for large institutional traders, but it may not be feasible for other entities. Therefore, the third strategy incorporates the quoted bid-ask spread (Best Bid and Offer, or BBO). It initiates a buy order at the prevailing best ask, whereas a sell order is initiated at the prevailing best bid. The latter therefore incorporates a full spread into the returns generated from the trading strategy.[64] Finally, our fourth approach makes the odds even more unfavorable by introducing a latency delay, which is the minimum number of seconds an investor must wait before entering a position (BBO+Xs). Such trading delays cause investors to miss out on the initial reaction in the

of macroeconomic news, generating a pre-announcement drift. We also examined our data set but failed to identify any significant pre-corporate earnings announcement drift.

[64] We do not control for market impact since our data is not rich enough to include such information. Hence, our results are mostly relevant for small trading sizes.

stock price but it could be beneficial to wait a few seconds before starting a trade because spreads narrow following an announcement.

Figure 7 shows the evolution over time in the cumulative return based on payoffs from individual trades. The slope of the graph reflects the rate at which returns accrue. Hence, the steeper the curve, the larger the returns. Conversely, flat spots or a declining curve indicate that the trading strategy fails to earn, or even outright loses, money.

The figure highlights that the transaction price and midquote approaches yield positive returns throughout our sample with no notable periods of inferior performance (full blue and dashed red lines). As expected, introducing execution costs or latency delays reduces the rate of accumulation. In particular, from 2016 onward we start to see flat spots (no returns, on average) in the strategy that pays the bid-ask spread (dashed-dotted line). Adding a further 5- or 10-second latency delay (dashed lines) reduces returns even more and the strategy loses money after 2016. This indicates that the speed with which earnings announcement information gets incorporated into prices has increased in the second part of our sample.

To inspect the profitability of our trading strategy, Figure 7 also reports (in parenthesis) the sample average return and the test statistic for examining if the mean return is different from zero. In a frictionless market where investors can enter the position at the first post-announcement transaction price, a highly significant return of 1.54% per trade (t-statistic of 6.69) is earned over the full sample. Trading at the midquote, the average return drops to 1.18% per trade which remains highly significant (t-statistic of 5.31). Even if trading is executed at the bid-ask spreads, the average return of 0.66% continues to be significant (t-statistic of 2.91). A 5-second latency delay reduces the return to 0.41% per trade, which is now only borderline significant (t-statistic of 1.89). Forcing a 10-second delay on trade entrance, the average return is still positive at 0.29% per trade, which is insignificant (t-statistic of 1.36).

Panel A of Table 6 reports a more comprehensive set of summary statistics. On top of the sample average returns and test statistics computed over the full sample (2008–2020), we also show results separately for the two subsamples 2008–2015 and 2016–2020. In this way, we can examine if the price discovery process has changed over time as indicated by the change in profitability observed in Figure 7.

Across the board, average returns are both larger and more significant in the first subsample (2008–2015) compared to the full sample. For example, the average return per trade in the transaction price setting is 2.07% (t-statistic of 6.73) over 2008-2015 but only 0.88% (t-statistic of 2.56) over 2016-2020. Moreover, the average return inferred from the midquote strategy drops to 0.62% (t-statistic of 1.84) in the second subsample – more than a full percentage point lower than its early subsample counterpart (1.64%). Trading at the bid-ask spread further reduces the average return and adding a latency delay even renders them slightly negative, although not significant, in the second subsample. These findings are consistent with a more efficient price discovery process in the most recent subsample, with average returns declining notably to the point where they are not significantly different from zero after accounting for bid-ask spreads or latency delay.

Panel B of Table 6 shows the effect of changing the investment horizon. Instead of closing positions at 6:30pm, the duration of the position is now measured either in minutes (physical time) or by the number of tick updates (tick time) after the announcement.[65]

In Panels B.1–B.5, we extend the duration of the position from 30 seconds to 5 minutes. In the full sample, average returns from the no-friction strategy based on transaction prices increase monotonically from 0.73% (t-statistic of 7.55) for positions closed after 30 seconds to 1.30% (t-statistic of 7.43) per trade for positions closed after five minutes. In the midquote strategy, the average return is slightly compressed but continues to be positive and statistically significant. Trades executed at the BBO reduces the average return which becomes slightly negative at -0.16% for the 30 seconds termination rule and borderline significant (t-statistic of -1.67) but rises to 0.42% at the 5-minute mark (t-statistic of 2.46). Latency delays lead to further reductions in trading performance as none of the approaches generate positive *and* significant average returns. Once again the results change meaningfully between the first and second subsamples. As the price discovery process improves, average returns become lower and less significant in the second subsample compared to the earlier subsample. Interestingly, the average return over the 2016–2020 sample for the baseline scenario in Panel A is close to the corresponding value for the 3-minute stopping rule in Panel B.4. This suggests that, in the most recent subsample, the price discovery process has more or less been completed at that time.

Next, we consider what happens to trading performance in transaction time if we close the position after between 50 and 1,000 tick updates. As shown in Panels B.6–B.10, we continue to generate positive and significant average returns for trades executed at the transaction price or midquote. However, executing at the BBO again produces insignificant mean returns in the latest subsample. Interestingly, trades stopped after 1,000 tick updates in Panel B.10 yield mean returns nearly identical to those from the positions held until 6:30pm (Panel A) in the first subsample, but not in the second. This reflects the lower trading activity in the early parts of our sample, which means that 1,000 tick updates cover a much longer time interval in the early subsample.

Figure 8 offers a visual summary of these results by plotting the cumulative return from trading strategies that close out the position after a fixed number of tick updates (Panels A–C) or after a fixed number of seconds (Panels D–F), employing either the transaction price, midquote or BBO. The figure illustrates both how fast the price adjusts in the very short period after an earnings announcement (both in trade time and in physical time) and also that the price settles at its new level even faster in the second subsample.

[65]We also estimated the predictive return regression with the post-announcement return (dependent variable) measured over a 5-, 10-, 30-, and 60-minute horizon. Average trading returns decline monotonically as the length of the measurement window is extended, suggesting that the price response to earnings surprises is very rapid and that the "signal" is strongest for the one-minute post-announcement return. However, the decline is relatively slow. For example, compared to the number in Table 6, moving from a one-minute to a 60-minute horizon causes a decline of only 13 basis points in average return for the transaction price approach.

Overall, our results demonstrate that price discovery in the post-announcement window has gotten far speedier during our twelve-year sample. Prior to 2016, investors were able to capture significant profits from post-earnings announcement trades executed at the BBO even with a latency delay of 10 seconds. In contrast, during the last subsample, after-hours prices move so fast after earnings announcements that it is no longer possible to generate significant outperformance after accounting for bid-ask spreads.

While proving that the sufficiency condition for market efficiency holds would ultimately require us to inspect a much broader class of trading strategies, the evidence presented here for the most recent subsample is not contradictory of condition 1 in Hypothesis 2 and is certainly consistent with trading in the after-hours market becoming notably more efficient over time.

5.3 Risk-adjusted return performance

The large average returns of some of the trading strategies described above may reflect a risk premium, i.e. exposure to the market portfolio or other priced risk factors. To examine the importance of this, we create a monthly return series by summing trading strategy log-returns within a given month. On days without active trading signals, we assume the risk-free rate is earned which yields the monthly log-return:[66]

$$r_m = \sum_{t=1}^{\#\mathrm{EA}_m} r_t^{\mathrm{EA}} + (N_m - \#\mathrm{EA}_m) r_{f,d}, \quad \text{for } m = 1, \ldots, M, \tag{36}$$

where $\#\mathrm{EA}_m$ is the number of announcement days with trading signals in month m, r_t^{EA} is the return generated from each trade, $r_{f,d}$ is the daily risk-free rate, N_m is the number of days in month m, and M is the number of months.

The first row in Panel A of Table 7 reports the sample mean of the (raw) monthly excess returns computed under our different trading scenarios. For trades executed at the transaction price (column 1 labeled "Trade"), the mean return is 6.14% per month. This figure is both economically large and highly statistically significant. Trades executed at the midquote earn a mean return of 4.71% per month, which is again economically large and significant. Trading at the best bid or offer reduces the mean return to 2.61% per month, which remains highly significant, however. Introducing latency of five or ten seconds brings it further down to 1.62% and 1.16% per month with only the former being significant at the 5% level. These results are in close alignment with our earlier trading strategy results from Table 6, so aggregation to the monthly horizon does not materially affect our conclusions.

We also compute the Sharpe ratio as the average monthly excess returns divided by the sample standard deviation of the monthly return series, both converted to annualized figures. The Sharpe ratio (shown in the third row) follows a pattern similar to that of mean excess returns, starting out

[66]We proxy the daily interest rate through the yield-to-maturity of a one-month T-bill converted into a continuously compounded daily interest rate.

at 1.85 for the transaction price setting and 1.51 for trades executed at the midquote. It then drops to 0.85, 0.57 and 0.41 for the BBO scenarios with a zero, five and ten second latency delay.

Panels B and C in Table 7 show sub-sample results with the top row reporting mean (raw) returns and the third row showing Sharpe ratios. The results are distinctly different across the two subsamples. The mean return in the scenario with no trading frictions (7.786% per month) is more than twice as high in the 2008-2015 period as compared to the 2016-2020 subsample (3.778%). Moreover, the mean return in the first subsample is statistically significant across all trading scenarios even after accounting for bid and ask prices and a 10 second latency delay. In contrast, in the second subsample the mean return is only 0.759% per month in the BBO scenario which is statistically insignificant. Our estimated Sharpe ratios are also much smaller, in some cases negative, in the second subsample as compared to the first period. These results are again consistent with our daily returns analysis.

To adjust for risk exposure, we next subtract the monthly risk-free rate, $r_{f,m}$, from r_m and regress the resulting excess return on an intercept, the five factors from the extended Fama and French (2015) model, and the momentum factor of Carhart (1997):

$$r_m - r_{f,m} = \alpha + \beta_{\mathrm{MKT}} r_m^{\mathrm{MKT}} + \beta_{\mathrm{HML}} r_m^{\mathrm{HML}} + \beta_{\mathrm{SMB}} r_m^{\mathrm{SMB}} + \beta_{\mathrm{RMW}} r_m^{\mathrm{RMW}} + \beta_{\mathrm{CMA}} r_m^{\mathrm{CMA}} + \beta_{\mathrm{MOM}} r_m^{\mathrm{MOM}} + \epsilon_m, \tag{37}$$

where r_m^{MKT} is the monthly excess return on the market portfolio (MKT), and the remaining covariates are excess returns on the factor-mimicking portfolios, where r_m^{HML} is High Minus Low (HML), r_m^{SMB} is Small Minus Big (SML), r_m^{RMW} is Robust Minus Weak (RMW), r_m^{CMA} is Conservative Minus Aggressive (CMA), and r_m^{MOM} is Momentum (MOM).[67]

The monthly alpha estimates from the risk-adjustment regression in (37) are bigger than the corresponding mean raw returns, with differences ranging from 0.6% to nearly 1% per month. This means that the alpha estimate is positive (at 1.77% per month) and statistically significant even for the BBO strategy with a 10-second latency delay.

All our trading strategies generate slightly negative market betas with values ranging from -0.28 to -0.04, though none of these estimates are statistically significant. The only risk factor that our trading strategy returns load significantly on is the profitability factor, RMW, which generates significantly negative estimates around -1.8 to -1.9. Since earnings announcements contain information that is relevant to profitability, it is perhaps not too unexpected that the RMW factor stands out in terms of significance and magnitude of the betas. Our trading rule goes long in stocks whose actual earnings numbers surprise analysts the most while it shorts stocks whose earnings disappoint. The negative loadings on the RMW factor is therefore consistent with firms with relatively weak profitability outperforming analyst expectations the most, whereas companies with relatively robust earnings disappoint the most.

Turning to the subsample analysis, Panels B and C show that alpha estimates are far bigger,

[67] The data are downloaded from Kenneth French's website: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`. The one-month T-bill rate is from Ibbotson Associates.

and always statistically significant in the early period (2008-2015) as compared to the later sample (2016-2020). In fact, in the second subsample alpha estimates are only statistically significant under the no-friction and midquote scenarios.

We conclude from these findings that while risk-adjusting the returns from our trading strategy slightly strengthens our performance results, the basic conclusions from the earlier analysis remain the same. In the early subsample (2008-2015), there is strong evidence that information on earnings surprises could have been exploited to generate significantly positive mean returns both on a raw and risk-adjusted basis. Conversely, market efficiency seems to have improved after 2016 to the point where our trading strategy is no longer generating significantly positive raw or risk-adjusted returns after accounting for transaction costs.

6 Conclusion

High-frequency trading has become ever more widespread in financial markets, suggesting that new public information should be incorporated in prices almost instantaneously after the news release. In an efficient market without large trading frictions, stock prices should therefore jump, almost surely, in the immediate aftermath of releases of large bundles of information such as firms' earnings announcements. This suggests using jump tests on highly granular "tick-by-tick" data as a new way to test a necessary (but not sufficient) condition for market efficiency. This testing strategy is very different from the conventional practice of considering post-announcement returns defined over a fixed time interval such as one or five minutes.

Because the vast majority of earnings announcements almost exclusively occur after market close, an analysis of the post-announcement price discovery process requires that we examine a relatively unexplored set of high-frequency data that includes quotations and transactions from the after-hours trading sessions. Irregular trading patterns and high levels of market micro structure noise in the after-hours trading session pose severe challenges to conventional jump tests which we show tend to distort inference about jumps.

To address these critical shortcomings of conventional jump tests, we develop a new jump test that is robust to the unusually noisy price data observed in after-hours markets. Our noise-robust generalization extends the classical bipower variation-based jump test of Barndorff-Nielsen and Shephard (2006) to a pre-averaged version that can be implemented on noisy high-frequency data. Moreover, we develop a subsample estimator of the asymptotic variance-covariance matrix that is jump-robust under the alternative, extending earlier work of Christensen, Podolskij, Thamrongrat, and Veliyev (2017).

Using this new jump test, we show that prices of the stocks with the highest after-hours trading volume almost always jump after earnings announcements. Conversely, jumps in stock prices are rare both during the regular trading session and during after-hours trading sessions without earnings announcements. We also find strong evidence of a jump spillover effect: conditional on an important company's stock price jumping after its earnings announcement, there tends to be a significantly higher chance that prices of non-announcing firms in the same industry, as well as the market index, will jump in the same after-hours trading session.

That stock prices in the after-hours market nearly always jump after the release of earnings announcements is indicative of a very rapid price discovery process and, thus, consistent with efficient markets. However, the price jump in the immediate aftermath of an earnings release may be too big or too small, potentially introducing profitable trading opportunities. We examine if in fact prices are unbiased predictors of their future steady-state values by studying the performance of a trading rule subject to different degrees of trading frictions.

In the absence of any transaction costs, we find that investors could have earned highly significant, positive risk-adjusted mean returns in the early years of our sample (2008-2015). Conversely, in the second part of our sample (2016-2020), we find that even minor trading frictions such as bid-ask spreads reduce average risk-adjusted returns to the point where they are no longer statistically significant. We conclude from this analysis that the after-hours market incorporates information on the earnings announcements of the largest US firms extremely fast, very rapidly, particularly after 2016.

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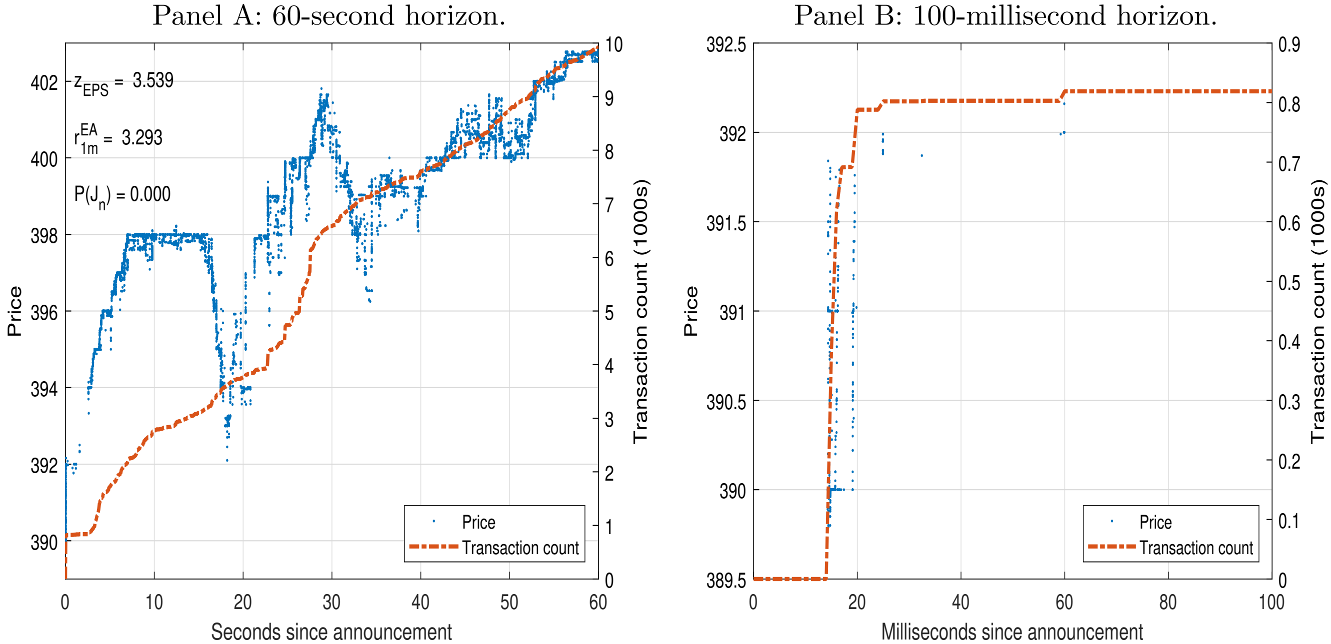
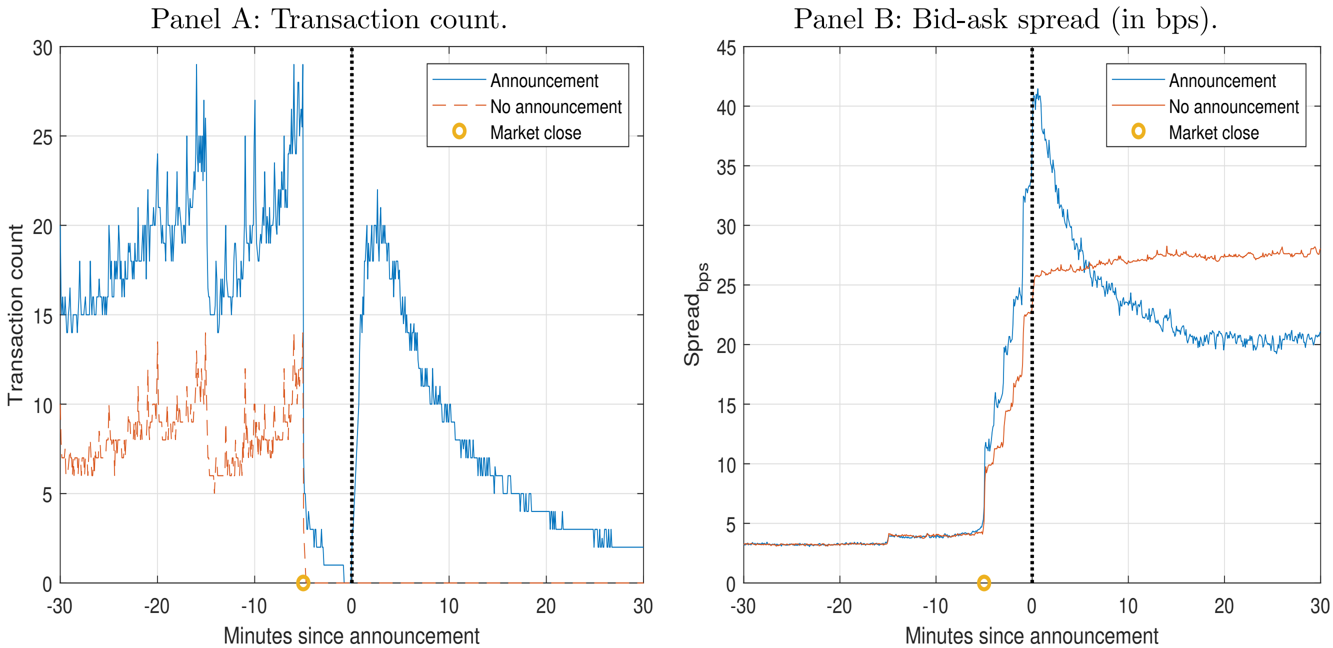
Figure 1: Apple's third-quarter 2020 earnings announcement on 07/30/2020.



Note. The figure shows the post-announcement price (left y-axis) and cumulative transaction count in 1000s (right y-axis) for Apple's third-quarter 2020 earnings announcement, which was released at 4:30pm on 07/30/2020. In Panel A, we plot the data from the first 60 seconds since the announcement, whereas Panel B zooms further in on the first 100 milliseconds since the announcement.

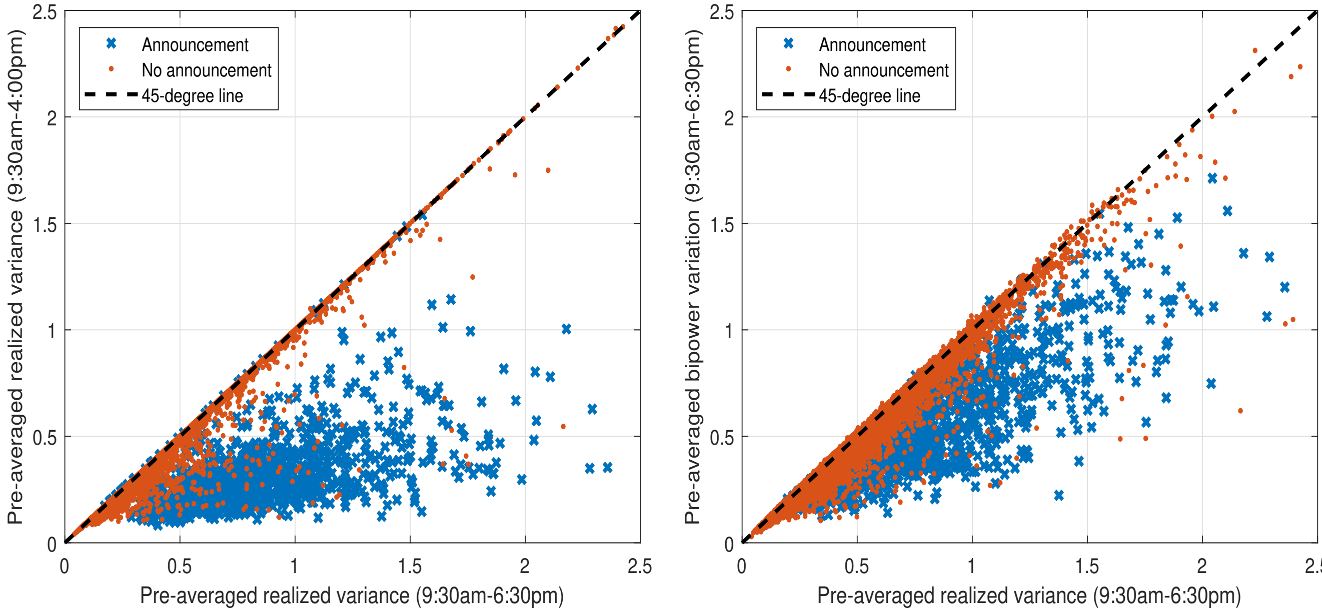
Figure 2: Trading volume and bid-ask spread in the after-hours market.



Note. In Panel A, we show the cross-sectional sample average trading volume (measured by transaction counts) for each five-second interval in a one-hour window centered around the earnings announcement, which occurs at time 0. In Panel B, we report the associated median bid-ask spread (in basis points), which is computed as $\text{Spread}_{\text{bps}} = 10000 \times (\text{ask} - \text{bid})/\text{midquote}$, where $\text{midquote} = (\text{bid} + \text{ask})/2$. As a control sample, for each announcement we select a random no announcement date without replacement and calculate trading volume and $\text{Spread}_{\text{bps}}$ on the corresponding time interval. The modal announcement time across companies is 4:05pm. Hence, the exchange typically closes five minutes prior to an announcement (highlighted by an orange circle).

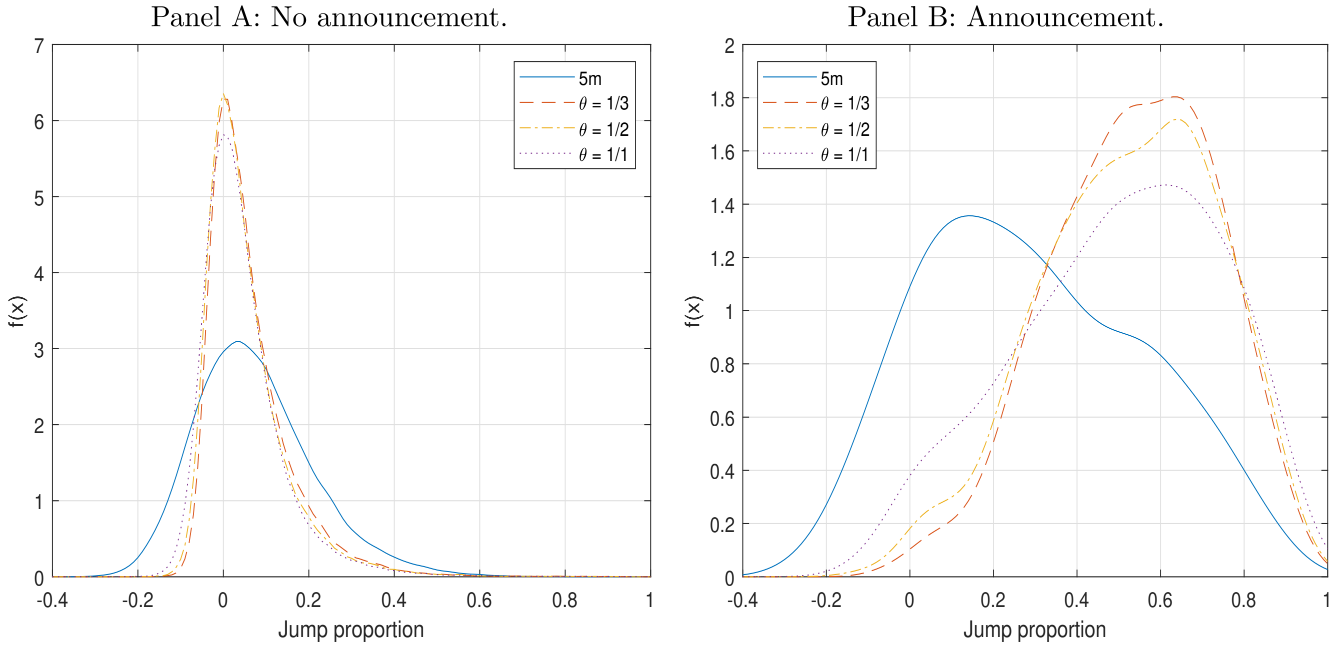
Figure 3: Pre-averaged realized variance and bipower variation.

Panel A: Incremental return variation.   Panel B: Incremental jump variation.



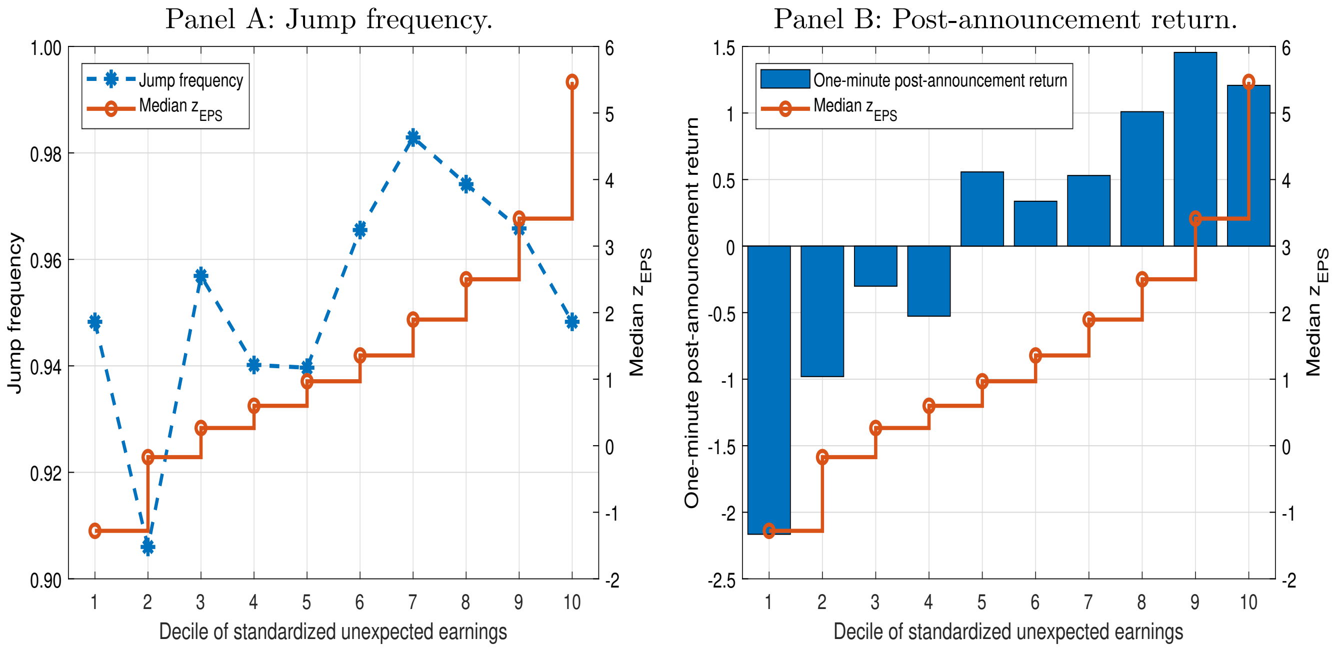
Note. We show point estimates of the pre-averaged realized variance and pre-averaged bipower variation, converted to an annualized standard deviation. The axis label shows which estimator is plotted. In parenthesis, we further indicate for which part of the day high-frequency data are employed to calculate the estimate.

Figure 4: Distribution of the estimated jump proportion.



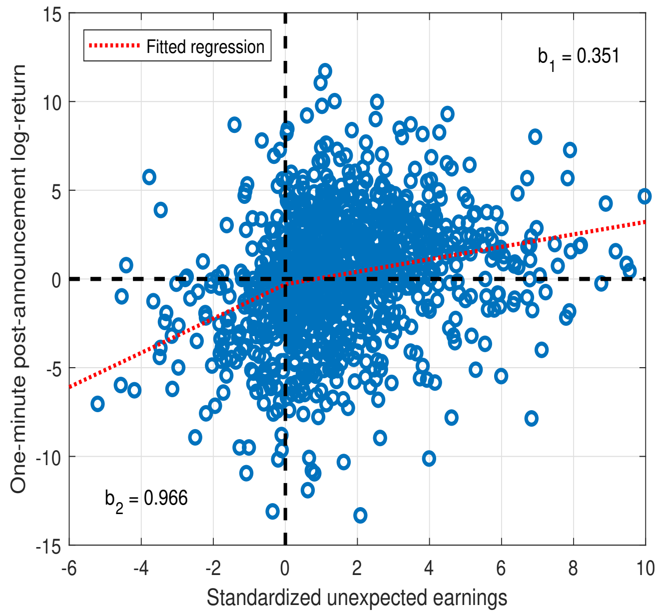
Note. This figure shows kernel smoothed densities of the proportion of quadratic return variation from the jump component, which is estimated by: Jump proportion = 1 - Bipower variation / Realized variance. The latter are computed without pre-averaging if sampling at a 5-minute frequency (5m) or with pre-averaging if sampling at the tick-by-tick frequency (rest). θ controls the size of the pre-averaging horizon $k_n = \lfloor \theta\sqrt{n} \rfloor$ for calculating pre-averaged returns. n is the number of tick-by-tick data. The sample is split into days without (in Panel A) and with earnings announcements (in Panel B).

Figure 5: Jump frequency and post-announcement return against standardized earnings surprise.



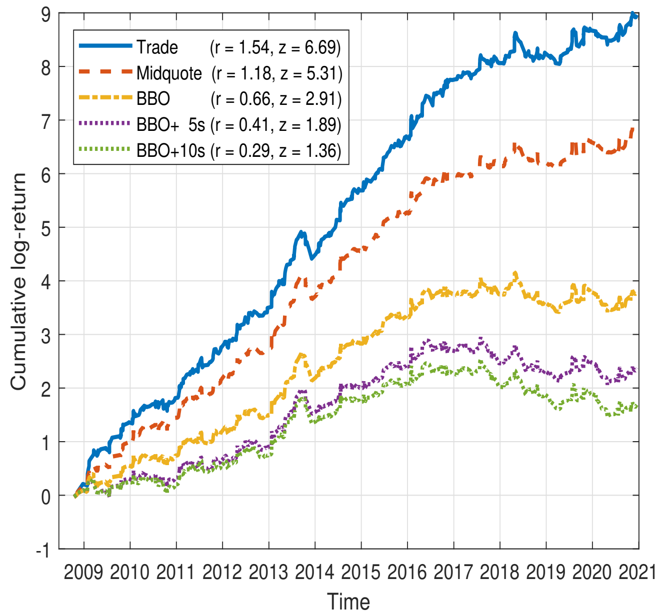
Note. We sort our earnings announcements by the value of the standardized unexpected earnings, z_{EPS}, and form decile portfolios from lowest to highest values of the earnings surprise. In Panel A, we then plot the corresponding announcement jump frequency implied by our noise-robust jump test (averaged within decile). In Panel B, we report the one-minute post-announcement return (averaged within decile). The median value of z_{EPS} within each decile is plotted against the right-hand y-axis.

Figure 6: Post-announcement return against standardized earnings surprise.



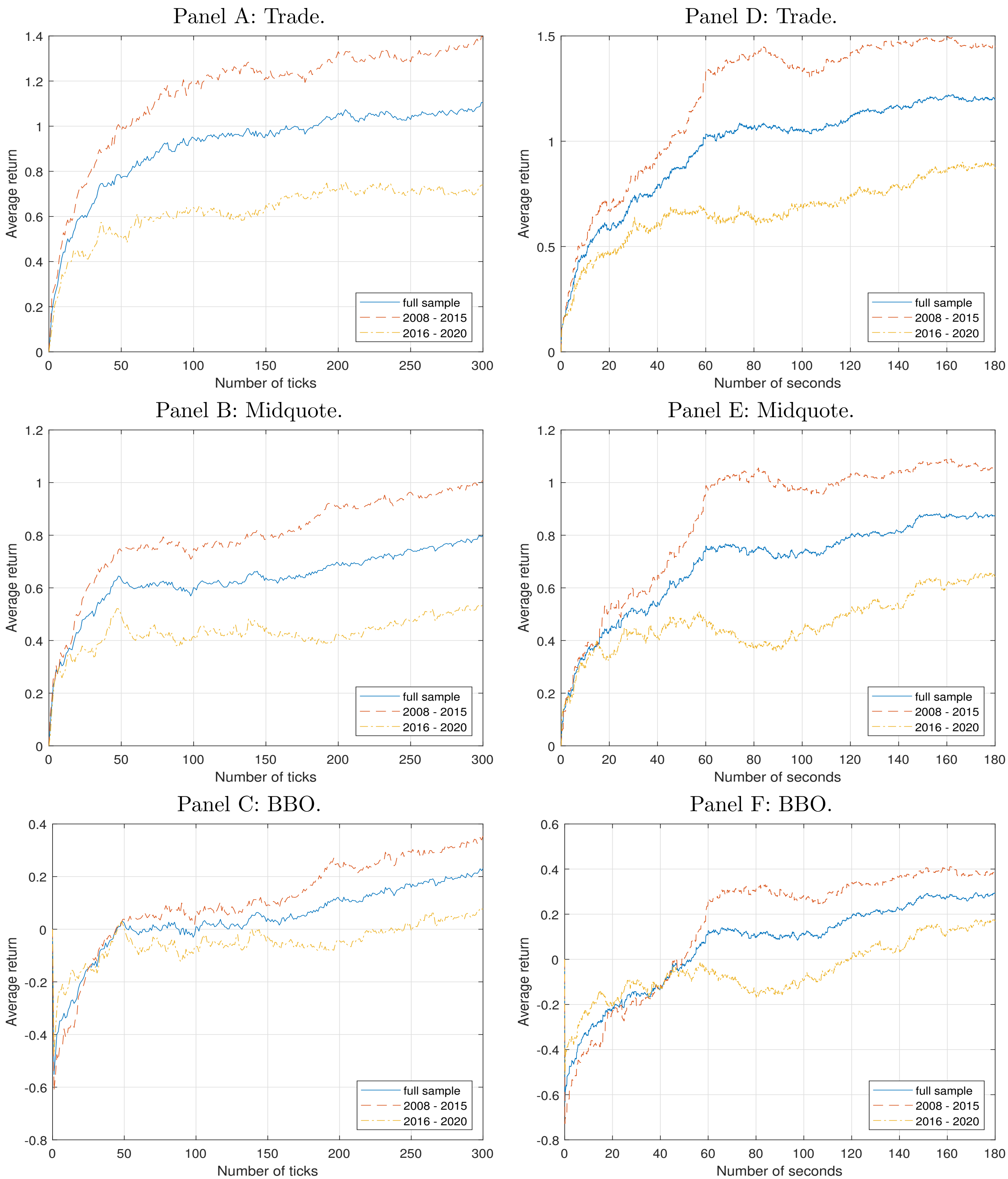
Note. We show the one-minute post-announcement return, r^{EA}_{1m}, against the standardized earnings surprise, z_{EPS}. The fit from a pooled panel regression $r^{\mathrm{EA}}_{1m,it} = a + b_1 z^{+}_{\mathrm{EPS},it} + b_2 z^{-}_{\mathrm{EPS},it} + \epsilon_{it}$ is plotted as a red dotted line for visual reference.

Figure 7: Cumulative return from trading strategy.



Note. The figure shows the cumulative return from a trading strategy that employs the standardized earnings surprise to predict the one-minute post-announcement return: $r^{\mathrm{EA}}_{1m,it} = a + b_1 z^{+}_{\mathrm{EPS},it} + b_2 z^{-}_{\mathrm{EPS},it} + \epsilon_{it}$. A long (short) position in the stock is entered if the predicted return is greater (smaller) than 0.5% (-0.5%) and held until 6:30pm. The sample average return and test statistic for testing that the mean return is zero, based on robust standard errors, is reported in parenthesis. "Trade" employs the transaction price, "Midquote" the midquote, and "BBO" the best bid and offer. "+Xs" enforces a latency delay of X seconds before entrance.

Figure 8: Cumulative returns from trading strategy.



Note. The figure shows the cumulative return in percent from a trading strategy that employs the standardized earnings surprise to predict the one-minute post-announcement return: $r^{\mathrm{EA}}_{1m,it} = a + b_1 z^{+}_{\mathrm{EPS},it} + b_2 z^{-}_{\mathrm{EPS},it} + \epsilon_{it}$. A long (short) position in the stock is entered if the predicted excess return is greater (smaller) than 0.5% (-0.5%). Panel A–C shows the cumulative return when closing out the position after a fixed number of tick updates, whereas Panel D–F shows the associated results when closing out the position after a fixed number of seconds. "Trade" employs the transaction price, "Midquote" the midquote, and "BBO" the best bid and offer.

Table 1: S&P 500 companies by after-hours (4:00pm–6:30pm) trading activity.

	Conditional on no announcement							Conditional on announcement							Announcement information			
				Quantile							Quantile							
Ticker	Mean	Fraction	Std.	0.25	0.50	0.75	0.99	Mean	Fraction	Std.	0.25	0.50	0.75	0.99	#EA	Time	z_{EPS}	r_{1m}^{EA}
FB	891	0.52	1,258	307	526	1,000	5,380	76,129	20.80	29,991	59,075	70,024	88,936	211,431	34	4:05pm	1.750 (1.252)	0.336 (4.446)
AAPL	1,552	0.61	3,950	386	649	1,235	18,311	51,346	15.94	29,927	32,891	44,509	56,832	194,220	40	4:30pm	1.669 (1.584)	0.806 (3.395)
NFLX	367	0.55	606	52	128	434	2,732	33,328	20.89	26,447	14,460	25,303	46,619	122,817	47	4:05pm	1.028 (1.621)	−0.236 (6.769)
AMZN	808	0.82	1,442	74	152	866	6,751	33,612	22.16	18,872	22,063	28,423	38,523	94,020	50	4:01pm	0.687 (1.790)	−0.369 (5.714)
NVDA	405	0.36	860	39	74	428	3,912	16,909	8.45	20,656	1,903	4,456	36,972	68,790	48	4:20pm	1.626 (1.787)	0.714 (3.988)
TSLA	2,156	0.86	10,399	25	195	824	32,536	27,210	15.41	35,016	1,287	15,304	30,270	139,936	40	4:05pm	0.355 (1.696)	1.248 (3.422)
AMD	579	0.38	1,486	40	80	612	5,722	15,417	7.37	21,607	1,711	4,849	25,714	86,880	47	4:15pm	0.746 (1.342)	−0.859 (4.201)
MU	251	0.21	480	53	102	280	1,959	13,508	6.89	13,340	1,990	11,078	21,868	59,740	50	4:01pm	0.262 (1.259)	0.196 (2.927)
MSFT	609	0.27	1,425	112	195	479	6,460	20,152	7.31	17,196	9,919	16,144	24,788	104,119	46	4:03pm	2.257 (2.698)	−0.148 (1.787)
GOOG	198	0.60	370	61	100	201	1,524	15,221	20.81	7,253	9,630	14,043	19,716	30,074	49	4:01pm	0.930 (1.849)	0.347 (2.973)
INTC	278	0.20	549	92	159	281	2,029	22,705	9.72	17,642	12,198	17,908	27,180	102,918	42	4:01pm	2.320 (2.250)	−0.034 (2.425)
DIS	328	0.38	898	26	57	276	3,666	10,043	7.87	12,298	2,116	5,545	11,760	56,136	48	4:15pm	1.263 (2.192)	−0.194 (2.200)
CRM	102	0.24	296	18	34	90	1,066	10,131	12.71	13,701	4,216	6,285	10,431	86,704	46	4:05pm	1.761 (1.728)	0.874 (3.484)
NKE	98	0.23	171	16	33	120	715	7,339	10.79	6,527	2,679	5,947	9,337	36,950	48	4:15pm	2.309 (2.632)	0.421 (2.903)
CSCO	194	0.17	239	74	124	219	1,210	20,912	10.00	16,488	11,140	17,694	24,999	91,696	50	4:05pm	2.331 (1.428)	−0.109 (2.588)
SBUX	145	0.24	314	39	61	153	1,269	6,607	7.46	4,012	3,670	5,691	9,331	16,886	50	4:05pm	1.126 (1.872)	−0.347 (2.998)
IBM	79	0.21	124	30	49	87	469	8,385	11.85	5,432	5,161	7,474	9,924	30,725	48	4:08pm	1.203 (1.337)	0.030 (2.058)
CMG	47	0.45	157	9	24	53	263	4,695	18.30	4,002	1,222	3,821	7,024	16,027	48	4:10pm	1.231 (2.324)	−0.443 (4.224)
ORCL	102	0.11	554	39	61	96	612	8,994	6.34	5,863	4,131	8,979	11,217	26,529	49	4:00pm	1.757 (2.804)	−0.220 (2.402)
ADBE	84	0.24	202	26	40	81	624	5,389	9.51	4,273	2,675	4,628	6,477	23,037	49	4:05pm	2.235 (2.084)	0.102 (2.856)
QCOM	145	0.18	581	49	74	130	947	9,695	8.21	7,473	5,000	7,594	12,302	33,231	49	4:00pm	1.355 (1.444)	0.500 (2.639)
EBAY	77	0.13	86	38	55	85	374	8,664	8.15	7,007	4,180	7,488	11,404	42,753	49	4:15pm	1.624 (1.518)	−0.489 (3.514)
GILD	208	0.23	2,014	46	69	112	1,685	4,125	4.07	5,449	566	1,830	7,064	25,630	48	4:05pm	0.671 (1.703)	0.081 (1.362)
AMAT	88	0.15	106	37	59	101	476	3,368	3.85	3,221	666	2,268	4,998	12,834	48	4:01pm	1.573 (1.878)	0.324 (2.278)
ULTA	33	0.37	52	8	17	41	201	3,368	11.32	4,549	427	2,576	4,311	22,330	41	4:03pm	3.569 (2.586)	0.773 (5.370)

Note. In the left-hand side of the table, we show descriptive statistics of the twenty-five most liquid companies from the S&P 500 index based on their after-hours market (4:00pm–6:30pm) trading volume, which is separated into no announcement days (columns 2–8) and annnouncement days (columns 9–15). Ticker is the stock listing symbol. Mean and Std. are the transaction count sample average and standard deviation. Quantile are selected quantiles from the empirical transaction count distribution. Fraction is the number of transactions executed in the after-hours market divided by the total transaction count for the extended trading session from 9:30am–6:30pm. In the right-hand side of the table, we report announcement information. #EA is the number of earnings annoucements. Time is the modal announcement time (Eastern Standard Time). z_{EPS} and r_{1m}^{EA} are the sample average standardized unexpected earnings and one-minute post-announcement return (standard deviation across announcements reported below in parentheis).

Table 2: Sample average of realized variance, bipower variation, and jump proportion.

	Panel A: Regular trading session (9:30am–4:00pm)								Panel B: Extended trading session (9:30am–6:30pm)							
	no EA				EA				no EA				EA			
	RV	BV	JP	JF	RV	BV	JP	JF	RV	BV	JP	JF	RV	BV	JP	JF
FB	25.0	24.9	0.6	1.0	26.8	26.6	1.6	0.0	25.4	25.1	1.6	3.2	97.0	71.0	42.6	97.1
AAPL	21.4	21.1	1.6	1.7	20.4	20.0	4.1	5.0	21.7	21.3	2.2	3.5	63.4	42.0	51.7	100.0
NFLX	35.6	35.3	0.8	1.4	38.8	38.6	0.9	2.1	35.9	35.4	1.2	2.5	143.0	89.2	57.3	97.9
AMZN	25.5	25.4	0.0	0.6	28.9	28.7	1.2	0.0	25.7	25.5	0.7	2.5	110.1	72.6	50.4	100.0
NVDA	33.1	32.5	3.8	2.1	37.6	37.1	3.4	2.1	33.5	32.6	4.5	3.8	101.5	65.5	54.2	97.9
TSLA	40.1	39.9	0.2	0.6	42.4	42.1	0.2	5.0	40.8	40.4	0.9	2.0	111.4	86.2	37.7	90.0
AMD	43.0	39.1	14.8	11.3	44.7	39.5	16.7	19.1	43.6	39.4	15.3	13.0	106.0	71.1	49.0	93.6
MU	40.2	38.4	7.6	4.7	44.8	42.4	7.8	2.0	40.6	38.5	8.4	6.6	93.1	70.7	40.7	96.0
MSFT	20.0	19.5	6.4	5.8	22.0	21.4	6.1	4.3	20.2	19.6	6.9	7.3	57.9	40.0	49.8	100.0
GOOG	19.8	19.7	0.6	0.9	22.0	22.0	0.4	0.0	20.0	19.8	1.1	2.2	75.7	50.4	51.1	93.9
INTC	22.2	21.5	7.8	6.8	25.8	25.1	7.1	0.0	22.5	21.6	8.4	8.7	70.8	49.0	50.6	100.0
DIS	19.5	19.1	4.3	2.8	23.8	23.2	5.9	4.2	19.6	19.1	4.5	3.1	56.5	34.0	61.6	97.9
CRM	28.3	27.8	3.7	1.6	36.2	35.3	5.0	2.2	28.5	27.8	4.0	2.1	92.3	57.3	59.0	95.7
NKE	20.5	20.0	4.5	2.2	23.8	23.2	5.3	8.3	20.5	20.1	4.5	2.3	68.1	39.1	63.5	100.0
CSCO	20.5	19.6	9.9	9.2	23.5	22.4	9.9	14.0	20.7	19.7	10.3	10.4	71.8	51.1	47.5	100.0
SBUX	22.3	21.7	4.6	2.7	26.3	25.5	6.5	2.0	22.4	21.8	4.9	3.5	69.5	45.8	52.7	100.0
IBM	16.8	16.5	3.2	1.7	18.8	18.5	2.9	2.1	16.8	16.5	3.3	1.9	52.0	30.6	62.5	100.0
CMG	26.7	26.2	2.7	1.8	29.2	28.7	3.1	0.0	26.8	26.2	2.8	2.0	96.0	54.7	60.5	89.6
ORCL	19.9	19.2	7.7	5.6	25.5	24.1	9.5	10.2	19.9	19.2	7.7	5.6	66.8	39.8	60.2	93.9
ADBE	23.5	23.0	3.8	2.0	27.2	26.6	4.0	2.0	23.7	23.1	4.0	2.4	78.2	49.7	54.2	98.0
QCOM	22.8	22.2	5.2	3.7	25.0	24.3	5.7	4.1	23.0	22.3	5.6	4.7	76.3	47.4	58.2	95.9
EBAY	24.6	23.8	7.0	4.1	30.5	29.4	7.2	4.1	24.7	23.8	7.0	4.3	91.9	56.6	57.5	95.9
GILD	23.6	22.9	5.1	3.8	27.4	26.7	6.0	4.2	23.7	23.0	5.4	4.6	51.2	35.1	46.7	81.2
AMAT	27.2	26.2	7.8	4.5	29.7	28.6	8.7	4.2	27.3	26.2	8.1	5.1	64.7	45.0	46.9	83.3
ULTA	32.5	31.3	5.7	2.5	37.4	35.6	5.4	7.3	32.5	31.3	5.7	2.7	100.1	63.8	50.4	80.5
Average	26.2	25.5	4.8	3.4	29.5	28.6	5.4	4.3	26.4	25.6	5.2	4.4	82.6	54.3	52.7	95.1

Note. This table reports the pre-averaged realized variance (RV) and pre-averaged bipower variation (BV). We also construct the jump proportion (JP), which is defined as Jump Proportion = 1 - Bipower variation/Realized Variance. We further compute the jump frequency (JF), which is derived from the jump indicator in (30). The realized variance and bipower variation are converted to annualized standard deviation in percent for convenience. The full sample is split into days without (no EA) and with (EA) earnings announcements. The measures are then averaged across subsamples by company. In Panel A, we report the analysis for the regular trading session (9:30am–4:00pm), whereas Panel B displays the associated results for the extended trading session (9:30am–6:30am). The bottom row reports the grand mean ("Average") over all stock-days.

Table 3: Logit estimates for the stock price jump probability.

Variable	(1)	(2)	(3)
Intercept	-2.789	-3.066	-3.066
	(0.015)	(0.017)	(0.017)
EA		6.070	4.286
		(0.139)	(0.577)
$\lvert z^{+}_{\mathrm{EPS}}\rvert$			0.121
			(0.092)
$\lvert z^{-}_{\mathrm{EPS}}\rvert$			0.353
			(0.303)
$\sqrt{RV^{*}_{n}}$			-0.334
			(0.095)
N_A			0.088
			(0.018)
Pseudo R^2		0.1759	0.1772

Note. We estimate the logit regression: $P(J_{it} = 1) = F(a + b_1 EA_{it} + b_2|z^{+}_{\mathrm{EPS},it}| + b_3|z^{-}_{\mathrm{EPS},it}| + b_4\sqrt{RV^{*}_{n,it}} + b_5 N_{A,it})$, where F is the logistic distribution function. J_{it} is equal to one if there is a jump in the price of company i's stock on day t, zero otherwise. EA_{it} is equal to one if there is an earnings announcement for company i on day t, zero otherwise. $z^{+}_{\mathrm{EPS},it}$ ($z^{-}_{\mathrm{EPS},it}$) is the standardized unexpected earnings for positive (negative) announcements, $\sqrt{RV^{*}_{n,it}}$ is the pre-averaged realized volatility calculated over the regular trading session on the announcement day, and $N_{A,it}$ is the number of analyst earnings forecasts for the announcement. The other covariates are interacted with EA and take a value of zero on no announcement days. The table reports parameter estimates of the full model in column (3) and of restricted versions in column (1)–(2). Standard errors are shown in parenthesis below the parameter estimate. The number of observations is 79,160 of which 77,996 are no announcement days and 1,164 are announcements days. Pseudo R^2 is the likelihood ratio index, $R^2 = 1 - L_1/L_0$, where L_0 is the log-likelihood of the constant model in (1) and L_1 is the log-likelihood of the models in (2)–(3).

Table 4: Co-jump analysis.

	Non-announcing firm (NAF)																									
Announcing firm (AF)	Fb	Aapl	Nflx	Amzn	Nvda	Tsla	Amd	Mu	Msft	Goog	Intc	Dis	Crm	Nke	Csco	Sbux	Ibm	Cmg	Orcl	Adbe	Qcom	Ebay	Gild	Amat	Ulta	MKT
Fb		16.1	3.0	25.8	17.6	0.0	12.5	8.8	10.3	28.1	5.9	0.0	0.0	0.0	17.6	0.0	2.9	0.0	5.9	2.9	16.7	0.0	10.7	8.8	2.9	14.7
Aapl	24.3		2.6	19.4	15.0	2.5	17.6	20.0	18.4	5.4	12.8	0.0	2.5	5.0	20.0	5.9	5.0	2.9	2.5	5.0	23.1	8.6	16.1	7.5	5.0	30.0
Nflx	13.0	6.7		33.3	6.4	2.1	15.6	8.5	8.9	19.1	15.2	0.0	2.1	4.3	19.1	6.8	2.6	2.3	4.3	2.1	7.0	2.4	10.9	6.4	0.0	6.4
Amzn	17.0	6.5	12.5		12.0	0.0	20.0	18.0	11.1	35.1	11.9	6.0	2.0	0.0	16.0	5.7	6.0	2.4	2.0	8.0	16.3	4.0	2.5	8.0	2.0	12.0
Nvda	0.0	6.2	2.1	0.0		2.2	35.4	10.4	0.0	0.0	10.4	2.6	0.0	4.2	8.9	4.3	0.0	0.0	2.1	0.0	4.3	4.2	0.0	2.8	0.0	2.1
Tsla	6.1	5.0	0.0	12.5	10.5		15.4	5.0	6.1	7.5	15.0	5.0	0.0	2.5	16.2	0.0	0.0	2.7	2.5	0.0	2.9	0.0	2.5	10.3	0.0	5.0
Amd	13.3	9.8	2.2	12.8	19.1	2.2		19.1	15.8	6.7	19.1	2.1	0.0	0.0	10.6	4.9	0.0	0.0	6.4	2.1	6.7	4.7	4.8	0.0	6.4	8.5
Mu	0.0	2.0	2.0	0.0	8.0	2.0	20.0		8.0	0.0	12.0	2.0	4.0	2.5	10.0	2.0	2.0	2.0	10.2	4.0	0.0	4.0	6.0	16.0	0.0	6.0
Msft	17.1	6.8	4.5	31.2	6.5	0.0	13.5	15.2		20.0	17.1	6.5	2.2	0.0	13.0	2.6	2.3	5.6	2.2	2.2	7.0	2.3	0.0	13.0	2.2	10.9
Goog	31.9	17.4	2.0	27.8	8.2	2.0	15.6	10.2	10.5		13.6	8.2	0.0	0.0	8.2	4.8	4.8	0.0	6.1	8.2	8.2	2.0	2.3	2.0	4.1	8.2
Intc	9.5	7.3	4.9	5.9	11.9	2.4	38.1	26.2	24.3	10.8		2.4	4.8	2.4	9.5	5.9	2.7	0.0	2.4	2.4	10.0	2.6	5.0	14.3	0.0	7.1
Dis	4.2	0.0	2.1	0.0	0.0	0.0	8.3	4.2	4.2	0.0	6.2		2.1	2.1	8.5	2.1	0.0	0.0	2.1	0.0	2.1	2.1	6.8	10.4	0.0	2.1
Crm	2.2	2.2	0.0	0.0	4.3	4.3	10.9	8.7	2.2	0.0	8.7	6.5		0.0	13.3	2.2	2.2	0.0	4.3	0.0	2.2	6.5	2.2	0.0	0.0	2.2
Nke	2.1	6.2	4.2	0.0	4.2	2.1	14.6	7.9	8.3	4.2	8.3	0.0	4.2		16.7	8.3	6.2	2.1	13.3	2.1	6.2	6.2	10.4	4.2	4.2	6.2
Csco	0.0	4.0	0.0	2.0	0.0	4.3	18.0	2.0	8.0	0.0	4.0	4.1	0.0	4.0		4.0	4.0	0.0	6.0	2.0	2.0	2.0	4.0	2.4	0.0	4.0
Sbux	13.0	4.5	4.3	8.6	10.2	0.0	22.7	22.0	9.5	11.6	9.5	2.0	4.0	2.0	8.0		0.0	0.0	0.0	4.0	15.2	2.3	4.3	6.0	2.0	10.0
Ibm	4.2	8.3	2.6	14.6	6.2	2.1	7.3	18.8	15.2	7.3	16.3	0.0	4.2	2.1	18.8	0.0		0.0	2.1	0.0	8.5	2.3	2.1	4.2	4.2	4.2
Cmg	13.0	14.0	11.1	25.6	8.3	0.0	21.1	18.8	18.4	9.5	19.1	4.5	0.0	0.0	16.7	4.4	4.3		4.2	4.2	4.8	4.5	0.0	10.4	2.1	6.2
Orcl	2.0	6.1	2.0	2.0	0.0	2.0	8.2	10.4	4.1	2.0	10.2	10.2	4.1	4.3	8.2	6.1	8.2	2.0		7.3	6.1	6.1	10.2	6.1	0.0	12.8
Adbe	4.1	10.2	4.1	0.0	0.0	2.0	6.1	8.2	6.1	0.0	10.2	4.1	0.0	4.1	8.2	2.0	2.0	4.1	4.9		2.0	12.2	2.0	4.1	2.2	4.3
Qcom	0.0	8.3	4.4	12.5	6.2	2.3	10.6	12.2	10.9	12.2	12.8	2.1	4.1	2.0	4.2	0.0	2.1	2.3	2.0	2.0		2.9	4.3	10.2	2.0	10.2
Ebay	9.3	4.5	2.3	30.6	4.1	6.4	17.8	14.3	17.4	6.1	10.9	2.0	4.1	8.2	14.3	0.0	4.5	6.7	4.1	6.1	5.7		4.2	8.2	2.0	8.2
Gild	19.0	12.8	4.3	10.5	12.5	2.1	11.6	14.6	8.7	11.9	8.7	4.5	4.2	2.1	18.8	4.5	6.2	2.4	6.2	6.2	10.9	2.1		4.2	2.1	8.3
Amat	0.0	4.2	0.0	0.0	2.8	2.1	14.6	10.4	6.2	0.0	12.5	0.0	4.5	4.2	12.5	6.2	2.1	0.0	8.3	2.1	4.2	4.2	4.2		4.2	2.1
Ulta	2.4	0.0	0.0	2.4	2.4	2.4	9.8	4.9	7.3	2.4	4.9	9.8	0.0	4.9	19.5	9.8	0.0	0.0	5.1	0.0	9.8	2.4	4.9	7.3		4.9

Note. We report the proportion of extended trading sessions with a jump in the non-announcing firm (NAF, in columns) when an announcing firm (AF, in rows) report financial results. The column MKT reports the jump frequency of the equity market (as proxied by SPY), conditional on the various firms' announcement days.

Table 5: Post-announcement return regression.

Variable	(1)	(2)	(3)	(4)
Intercept	0.112	−0.297	−0.704	−0.729
	(0.103)	(0.148)	(0.182)	(0.182)
z^{+}_{EPS}		0.351	0.278	0.269
		(0.055)	(0.057)	(0.055)
z^{-}_{EPS}		0.966	0.432	0.424
		(0.188)	(0.225)	(0.224)
$\sqrt{RV^{*}_{n}}$			0.050	0.058
			(0.095)	(0.093)
N_A			0.020	0.020
			(0.008)	(0.008)
OI				1.869
				(0.329)
$\bar{R}^2$		0.0735	0.0855	0.1044
P-value		0.0023	0.5021	0.4934

Note. We estimate the linear regression: $r^{\text{EA}}_{1m,it} = a + b_1 z^{+}_{\text{EPS},it} + b_2 z^{-}_{\text{EPS},it} + b_3\sqrt{RV^{*}_{n,it}}D_{it} + b_4 N_{A,it}D_{it} + b_5 OI_{it} + \epsilon_{it}$. $r^{\text{EA}}_{1m,it}$ is the one-minute post-announcement return, $z^{+}_{\text{EPS},it}$ ($z^{-}_{\text{EPS},it}$) is the standardized unexpected earnings for positive (negative) announcements, $\sqrt{RV^{*}_{n,it}}$ is the pre-averaged realized volatility calculated over the regular trading session on the announcement day, $N_{A,it}$ is the number of analyst earnings forecasts for the announcement, OI_{it} is the one-minute post-announcement cumulative net order imbalance, and $D_{it} = \text{sign}(z_{\text{EPS},it})$ is the sign function. The table reports parameter estimates of the full model in column (4) and of restricted versions in column (1)–(3). Standard errors are shown in parenthesis below the parameter estimate. The number of observations is 1,164. $\bar{R}^2$ is the adjusted coefficient of multiple determination. The P-value is for testing the hypothesis $H_0 : b_1 = b_2$ against $H_1 : b_1 \neq b_2$.

Table 6: Average return from trading strategy.

Panel A: Baseline termination rule

	Terminate at 6:30pm (EOD).		
	Full sample	2008–2015	2016–2020
Trade	1.54 (6.69)	2.07 (6.73)	0.88 (2.56)
Midquote	1.18 (5.31)	1.64 (5.52)	0.62 (1.84)
BBO	0.66 (2.91)	1.04 (3.47)	0.18 (0.52)
BBO+5s	0.41 (1.89)	0.75 (2.59)	-0.02 (-0.07)
BBO+10s	0.29 (1.36)	0.64 (2.23)	-0.15 (-0.47)

Panel B: Alternative termination rules

	Terminate after fixed number of minutes.			Terminate after fixed number of ticks.		
	Full sample	2008–2015	2016–2020	Full sample	2008–2015	2016–2020
	Panel B.1: Stop after 30 seconds			*Panel B.6: Stop after 50 ticks*		
Trade	0.73 (7.55)	0.83 (6.41)	0.62 (4.19)	0.77 (8.88)	0.98 (7.99)	0.51 (4.29)
Midquote	0.51 (5.47)	0.57 (4.60)	0.44 (3.08)	0.63 (5.92)	0.74 (5.21)	0.50 (3.08)
BBO	-0.16 (-1.67)	-0.21 (-1.60)	-0.11 (-0.72)	0.02 (0.21)	0.04 (0.26)	0.00 (0.02)
BBO+5s	-0.41 (-4.96)	-0.49 (-4.22)	-0.31 (-2.66)	-0.22 (-2.27)	-0.25 (-1.79)	-0.20 (-1.40)
BBO+10s	-0.53 (-7.12)	-0.60 (-5.57)	-0.43 (-4.46)	-0.34 (-3.68)	-0.36 (-2.73)	-0.32 (-2.49)
	Panel B.2: Stop after 1 minute			*Panel B.7: Stop after 100 ticks*		
Trade	1.03 (7.91)	1.33 (7.86)	0.66 (3.28)	0.95 (9.20)	1.19 (8.04)	0.64 (4.70)
Midquote	0.76 (6.23)	0.99 (6.17)	0.47 (2.54)	0.60 (4.87)	0.75 (4.44)	0.42 (2.31)
BBO	0.13 (1.01)	0.27 (1.62)	-0.05 (-0.27)	0.01 (0.08)	0.07 (0.41)	-0.06 (-0.35)
BBO+5s	-0.12 (-1.05)	-0.01 (-0.09)	-0.25 (-1.52)	-0.24 (-2.07)	-0.21 (-1.31)	-0.26 (-1.68)
BBO+10s	-0.24 (-2.19)	-0.12 (-0.82)	-0.38 (-2.47)	-0.35 (-3.25)	-0.32 (-2.07)	-0.39 (-2.66)
	Panel B.3: Stop after 2 minutes			*Panel B.8: Stop after 250 ticks*		
Trade	1.12 (7.57)	1.42 (7.48)	0.75 (3.22)	1.03 (8.02)	1.28 (6.92)	0.71 (4.18)
Midquote	0.80 (5.71)	1.03 (5.73)	0.51 (2.32)	0.75 (5.28)	0.96 (5.12)	0.47 (2.22)
BBO	0.19 (1.33)	0.34 (1.81)	0.01 (0.03)	0.16 (1.11)	0.29 (1.54)	-0.01 (-0.03)
BBO+5s	-0.05 (-0.41)	0.06 (0.32)	-0.19 (-0.96)	-0.09 (-0.66)	0.01 (0.04)	-0.21 (-1.06)
BBO+10s	-0.17 (-1.33)	-0.05 (-0.31)	-0.32 (-1.67)	-0.20 (-1.60)	-0.10 (-0.58)	-0.33 (-1.78)
	Panel B.4: Stop after 3 minutes			*Panel B.9: Stop after 500 ticks*		
Trade	1.20 (7.21)	1.45 (6.86)	0.88 (3.35)	1.12 (7.81)	1.45 (7.04)	0.71 (3.71)
Midquote	0.87 (5.52)	1.06 (5.27)	0.64 (2.55)	0.86 (5.52)	1.07 (5.31)	0.60 (2.46)
BBO	0.29 (1.81)	0.39 (1.90)	0.17 (0.66)	0.31 (1.96)	0.43 (2.11)	0.16 (0.64)
BBO+5s	0.05 (0.31)	0.11 (0.56)	-0.03 (-0.13)	0.06 (0.43)	0.15 (0.75)	-0.04 (-0.17)
BBO+10s	-0.07 (-0.47)	0.00 (0.00)	-0.16 (-0.68)	-0.05 (-0.37)	0.04 (0.19)	-0.17 (-0.75)
	Panel B.5: Stop after 5 minutes			*Panel B.10: Stop after 1,000 ticks*		
Trade	1.30 (7.43)	1.63 (7.25)	0.89 (3.25)	1.10 (6.91)	1.54 (7.03)	0.54 (2.41)
Midquote	0.97 (5.70)	1.23 (5.69)	0.63 (2.36)	0.94 (5.40)	1.19 (5.25)	0.62 (2.32)
BBO	0.42 (2.46)	0.62 (2.79)	0.18 (0.66)	0.41 (2.32)	0.57 (2.48)	0.20 (0.75)
BBO+5s	0.18 (1.10)	0.34 (1.59)	-0.02 (-0.09)	0.16 (0.98)	0.29 (1.31)	0.00 (0.02)
BBO+10s	0.06 (0.38)	0.23 (1.08)	-0.15 (-0.61)	0.05 (0.28)	0.18 (0.82)	-0.12 (-0.49)

Note. The table shows the average return in percent from a trading strategy that employs the standardized earnings surprise to predict the one-minute post-announcement return: $r^{\mathrm{EA}}_{1m,it} = a + b_1 z^{+}_{\mathrm{EPS},it} + b_2 z^{-}_{\mathrm{EPS},it} + \epsilon_{it}$. A long (short) position in the stock is opened if the predicted excess return is greater (smaller) than 0.5% (-0.5%). In the baseline termination rule in Panel A, the position is held until 6:30pm (EOD). In Panel B, alternative termination rules are inspected. In Panels B.1–B.5, the position is closed after a fixed number of minutes, whereas in Panels B.6–B.10 the position is closed after a fixed number of tick updates. "Trade" employs the transaction price, "Midquote" the midquote, and "BBO" the best bid and offer. "+Xs" enforces a latency delay of X seconds before entrance. The test statistic for testing that the average return is zero, based on robust standard errors, is reported in parenthesis.

Table 7: Properties of monthly trading returns.

	Trade	Midquote	BBO	BBO+5s	BBO+10s
	Sample average return, Jensen's alpha, and Sharpe ratio				
Panel A: Full sample					
$\bar{r}_m$	6.139***	4.705***	2.606***	1.625**	1.161
	(0.951)	(0.891)	(0.873)	(0.820)	(0.815)
α	7.101***	5.398***	3.270***	2.263***	1.769**
	(0.965)	(0.912)	(0.890)	(0.832)	(0.830)
Sharpe ratio	1.845	1.509	0.853	0.566	0.407
Panel B: 2008 – 2015					
$\bar{r}_m$	7.786***	6.143***	3.895***	2.829***	2.418***
	(1.099)	(1.010)	(0.987)	(0.927)	(0.920)
α	8.244***	6.329***	4.085***	3.102***	2.706***
	(1.160)	(1.084)	(1.054)	(0.979)	(0.975)
Sharpe ratio	2.631	2.259	1.466	1.134	0.976
Panel C: 2016 – 2020					
$\bar{r}_m$	3.778**	2.644*	0.759	−0.100	−0.641
	(1.647)	(1.576)	(1.555)	(1.460)	(1.450)
α	5.429***	3.985**	2.011	0.999	0.374
	(1.594)	(1.602)	(1.580)	(1.493)	(1.461)
Sharpe ratio	1.017	0.744	0.217	-0.030	-0.196
	Full sample Fama-French regression				
β_{MKT}	−0.279	−0.136	−0.105	−0.048	−0.037
	(0.227)	(0.205)	(0.200)	(0.179)	(0.181)
β_{HML}	0.757*	0.535	0.551	0.569	0.525
	(0.426)	(0.398)	(0.422)	(0.398)	(0.405)
β_{SMB}	−0.511	−0.470	−0.408	−0.590*	−0.543
	(0.390)	(0.373)	(0.370)	(0.355)	(0.351)
β_{RMW}	−1.886***	−1.704***	−1.680***	−1.628***	−1.638***
	(0.579)	(0.575)	(0.595)	(0.543)	(0.535)
β_{CMA}	−0.569	−0.219	−0.243	−0.141	0.029
	(0.630)	(0.613)	(0.644)	(0.634)	(0.641)
β_{MOM}	0.070	0.122	0.190	0.271	0.279
	(0.178)	(0.173)	(0.176)	(0.182)	(0.182)
R^2	0.090	0.073	0.076	0.091	0.092
P-value	0.040	0.096	0.084	0.037	0.035

Note. We construct a monthly return series (in percent), $r_m = \sum_{t=1}^{\#\text{EA}_m} r_t^{\text{EA}} + (N_m - \#\text{EA}_m) r_{f,d}$, for $m = 1, \ldots, M$, where $\#\text{EA}_m$ is the number of announcement days with trading signals in month m, r_t^{EA} is the return generated from each trade, $r_{f,d}$ is the daily risk-free rate, N_m is the number of days in month m, and M is the number of months. In Panel A, $\bar{r}_m$ is the sample average monthly return. We subtract the monthly risk-free rate, $r_{f,m}$, from r_m to create an excess return. The Sharpe ratio is the average excess return divided by the sample standard deviation of the return series, which we convert to an annualized figure by multiplying with $\sqrt{12}$. We regress the excess return on an intercept, the five factors from the extended Fama and French (2015) asset pricing model, and the momentum factor of Carhart (1997): $r_m - r_{f,m} = \alpha + \beta_{\text{MKT}} r_m^{\text{MKT}} + \beta_{\text{HML}} r_m^{\text{HML}} + \beta_{\text{SMB}} r_m^{\text{SMB}} + \beta_{\text{RMW}} r_m^{\text{RMW}} + \beta_{\text{CMA}} r_m^{\text{CMA}} + \beta_{\text{MOM}} r_m^{\text{MOM}} + \epsilon_m$, where r_m^{MKT} is the monthly excess return on the market (MKT), r_m^{HML} is High Minus Low (HML), r_m^{SMB} is Small Minus Big (SML), r_m^{RMW} is Robust Minus Weak (RMW), r_m^{CMA} is Conservative Minus Aggressive (CMA), and r_m^{MOM} is Momentum (MOM). The table reports parameter estimates from the regression in Panel B. Standard errors are shown in parenthesis below the parameter estimate. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level. The number of observations is 146. R^2 is the coefficient of determination. The P-value is for testing the hypothesis $H_0 : \beta_{\text{MKT}} = \cdots = \beta_{\text{MOM}} = 0$.

A Proof of theoretical results

In this appendix, we prove our mathematical results. We assume that a, σ, $\tilde{a}$, $\tilde{\sigma}$, $\tilde{v}$, ω, $\bar{a}$, $\bar{\sigma}$, $\bar{v}$ and the jump components of p_t are bounded. This follows from a localization procedure (Jacod and Protter, 2012, Section 4.4.1) and is without loss of generality. Moreover, we employ a generic constant C whose value may change from line to line.

A.1 An extended central limit theorem

We start by presenting an extended version of Theorem 1 for a generalized pre-averaged bipower variation estimator. We abuse the notation from the main text slightly and here define the estimator with an alternative normalization. We further omit the bias-correction part. This delivers an estimator that is much more convenient to work with in the derivations:

$$BV_n(q,r) = \frac{1}{n} \sum_{i=0}^{n-2k_n+1} |n^{1/4} \bar{r}_i^*|^q |n^{1/4} \bar{r}_{i+k_n}^*|^r, \tag{38}$$

where $q, r \in \mathcal{S} \equiv \{0\} \cup [1, \infty)$.

Following Podolskij and Vetter (2009a), we expect that under $\mathcal{H}_0$

$$BV_n(q,r) \xrightarrow{p} V(q,r) = \mu_q \mu_r \int_0^1 \left(\theta \psi_2 \sigma_s^2 + \frac{1}{\theta} \psi_1 \omega_s^2 \right)^{\frac{q+r}{2}} \mathrm{d}s,$$

where $\mu_s = \mathbb{E}\big[|N(0,1)|^s\big]$ and

$$\psi_1 = \int_0^1 [g'(s)]^2 \mathrm{d}s \quad \text{and} \quad \psi_2 = \int_0^1 g(s)^2 \mathrm{d}s.$$

Theorem 3. *Assume that r follows the process in (1) with $r_t^d \equiv 0$ (for all t), and that Assumptions (V) and (N) hold. As $n \to 0$, for $q_1, q_2, r_1, r_2 \in \mathcal{S}$,*

$$n^{1/4} \begin{pmatrix} BV_n(q_1, r_1) - V(q_1, r_1) \\ BV_n(q_2, r_2) - V(q_2, r_2) \end{pmatrix} \xrightarrow{\mathcal{D}_s} N(0, \tilde{\Sigma}),$$

where $\tilde{\Sigma}$ is defined in (40) and $\xrightarrow{\mathcal{D}_s}$ denotes stable convergence in law.

To present an expression for the asymptotic covariance matrix in the above central limit theorem, we need to introduce some additional notation. In particular, for $i, j \in \{1, 2\}$, we set

$$h_{ij}(x, y, z) = \mathrm{cov}(|H_1|^{q_i} |H_2|^{r_i}, |H_3|^{q_j} |H_4|^{r_j}), \tag{39}$$

where x, y are two-dimensional vectors, whereas z is a four-dimensional vector, and (H_1, H_2, H_3, H_4) are centered multivariate normal distributed random variables with covariance structure:

(i) $\mathbb{E}\big[|H_i|^2\big] = y_1 x_1^2 + y_2 x_2^2, \quad i \in \{1, 2, 3, 4\}$,

(ii) $H_1 \perp H_2, \quad H_1 \perp H_4, \quad H_3 \perp H_4$,

(iii) $\text{cov}(H_1, H_3) = \text{cov}(H_2, H_4) = z_1 x_1^2 + z_2 x_2^2$,

(iv) $\text{cov}(H_2, H_3) = z_3 x_1^2 + z_4 x_2^2$.

We also introduce the following functions:

$$f_1(s) = \frac{1}{\theta}\phi_1(s), \quad f_2(s) = \theta\phi_2(s), \quad f_3(s) = \frac{1}{\theta}\phi_3(s), \quad f_4(s) = \theta\phi_4(s),$$

for $s \in [0, 2]$, where

$$\phi_1(s) = \int_0^{1-s} g'(u)g'(u+s)\mathrm{d}s, \quad \phi_2(s) = \int_0^{1-s} g(u)g(u+s)\mathrm{d}s,$$
$$\phi_3(2-s) = \int_0^{1-s} g'(u)g'(u+s-1)\mathrm{d}s, \quad \phi_4(s) = \int_0^{2-s} g(u)g(u+s-1)\mathrm{d}s.$$

We compactly write these in vectorized form as $f(s) = (f_1(s), \dots, f_4(s))$. The (i,j)-th component of $\tilde{\Sigma}$ is then given by

$$\tilde{\Sigma}_{ij} = 2\theta \int_0^1 \int_0^2 h_{ij}\big((\omega_u, \sigma_u), (\psi_1/\theta, \theta\psi_2), f(s)\big)\mathrm{d}s\mathrm{d}u. \tag{40}$$

Compared to the estimators introduced in this appendix, the pre-averaged realized variance and bipower variation in (13) are merely rescaled and bias-corrected versions of (38) with $(q,r) = (2,0)$ and $(q,r) = (1,1)$, since (see also Lemma 11)

$$RV_n^* = c_1 BV_n(2,0) + o_p\big(n^{-1/4}\big) \quad \text{and} \quad BV_n^* = c_2 BV_n(1,1) + o_p\big(n^{-1/4}\big),$$

where

$$c_1 = \frac{1}{\theta\psi_2} \quad \text{and} \quad c_2 = \frac{\pi}{2\theta\psi_2}.$$

Hence, the relation between $\tilde{\Sigma}$ and Σ is as follows:

$$\Sigma_{ij} = c_i c_j \tilde{\Sigma}_{ij}. \tag{41}$$

Proof of Theorem 3

For any $m \leq i$, we define

$$\bar{r}_{i,m}^* = \sum_{j=1}^{k_n} g(j/k_n)\big(\sigma_{m\Delta_n}(W_{(i+j)\Delta_n} - W_{(i+j-1)\Delta_n}) + \omega_{m\Delta_n}(\pi_{i+j} - \pi_{i+j-1})\big). \tag{42}$$

We note that $\bar{r}_{i,m}^*$ serves to approximate $\bar{r}_i^*$ by freezing the processes σ and ω locally at the time point $m\Delta_n$.

We also denote

$$\eta(q,r)_i^n = |n^{1/4}\bar{r}_i^*|^q |n^{1/4}\bar{r}_{i+k_n}^*|^r,$$
$$\eta(q,r)_{i,m}^n = |n^{1/4}\bar{r}_{i,m}^*|^q |n^{1/4}\bar{r}_{i+k_n,m}^*|^r.$$

Lemma 4. *Under the maintained assumptions of Theorem 3, for any $q, r \in \mathcal{S}$, and real number $v \geq 1$, it holds that*

$$\max\Big(\mathbb{E}\big[|n^{1/4}\bar{r}_i^*|^v \mid \mathcal{F}_{i\Delta_n}\big], \mathbb{E}\big[|n^{1/4}\bar{r}_{i,m}^*|^v \mid \mathcal{F}_{m\Delta_n}\big]\Big) \leq C,$$

and

$$\max\Big(\mathbb{E}\big[|\eta(q,r)_i^n|^v \mid \mathcal{F}_{i\Delta_n}\big], \mathbb{E}\big[|\eta(q,r)_{i,m}^n|^v \mid \mathcal{F}_{m\Delta_n}\big]\Big) \leq C,$$

uniformly in i and m.

Proof. The first bound is a standard result for the pre-averaged returns, which follows from the boundedness of g, σ and ω. Then, the second bound is due to the Cauchy-Schwarz inequality. ■

Lemma 5. *Under the maintained assumptions of Theorem 3, for any $q, r \in \mathcal{S}$, it holds that*

$$\mathbb{E}\big[\eta(q,r)_{i,m}^n \mid \mathcal{F}_{m\Delta_n}\big] = \mu_q\mu_r\Big(\theta\psi_2\sigma_{m\Delta_n}^2 + \frac{1}{\theta}\psi_1\omega_{m\Delta_n}^2\Big)^{\frac{q+r}{2}} + o_p(n^{-1/4}),$$

uniformly in i and m.

Proof. This follows from the proof of Lemma 4 in Podolskij and Vetter (2009a). ■

The next preliminary result concerns the error of $\alpha_{i,m}^n \equiv n^{1/4}(\bar{r}_i^* - \bar{r}_{i,m}^*)$.

Lemma 6. *Under the maintained assumptions of Theorem 3, for any real number $v \geq 2$, it holds that*

$$\mathbb{E}\big[|\alpha_{i,m}^n|^v \mid \mathcal{F}_{m\Delta_n}\big] \leq C\Big(n^{-v/4} + \big(\mathbb{E}\big[\Gamma(\sigma,m,i)^n + \Gamma(\omega,m,i)^n \mid \mathcal{F}_{m\Delta_n}\big]\big)^v\Big),$$

$$\mathbb{E}[|\eta(q,r)_i^n - \eta(q,r)_{i,m}^n|^v|\mathcal{F}_{m\Delta_n}] \leq C\Big(n^{-v/4} + \big(\mathbb{E}\big[\Gamma(\sigma,m,i)^n + \Gamma(\omega,m,i)^n \mid \mathcal{F}_{m\Delta_n}\big]\big)^v\Big),$$

where for any process γ we set

$$\Gamma(\gamma,m,i)^n = \sup_{m\Delta_n \leq s \leq (i+2k_n)\Delta_n} |\gamma_s - \gamma_{m\Delta_n}|.$$

Proof. By construction

$$\alpha_{i,m}^n = n^{1/4}\int_{i\Delta_n}^{(i+k_n)\Delta_n} g_n(s - i\Delta_n)\big[a_s\mathrm{d}s + (\sigma_s - \sigma_{m\Delta_n})\mathrm{d}W_s\big]$$
$$+ n^{1/4}\sum_{j=0}^{k_n-1}\big[g(j/k_n) - g(j+1/k_n)\big](\omega_{(i+j)\Delta_n} - \omega_{m\Delta_n})\pi_{i+j},$$

where $g_n(s) = \sum_{j=1}^{k_n} g(j/k_n)1_{\{(j-1)\Delta_n, j\Delta_n\}}(s)$. Note that g, a and σ are bounded and $|g(j/k_n) - g(j+1/k_n)| \leq C/k_n$. Furthermore, ω is bounded and independent of π.

The first result then follows from the Burkholder-Davis-Gundy inequality and Γ being bounded. The second applies Hölder's inequality and the inequalities $||y|^q - |x|^q| \leq C|x-y|\max(|x|^{q-1}, |y|^{q-1})$ and $|y_1y_2 - x_1x_2| \leq |y_1 - x_1||y_2| + |x_1||y_2 - x_2|$. ■

We proceed by applying the "big blocks–small blocks" technique of Jacod, Li, Mykland, Podolskij, and Vetter (2009). To this end, we fix an integer $p \geq 2$, which in the later stages of the proof tends to infinity, and introduce the notation

$$\begin{aligned} a_j(p) &= 2j(p+1)k_n, \quad b_j(p) = 2j(p+1)k_n + 2pk_n, \\ A_j(p) &= \mathbb{Z} \cap [a_j(p), b_j(p)), \quad B_j(p) = \mathbb{Z} \cap [b_j(p), a_{j+1}(p)). \end{aligned}$$

$A_j(p)$ is the big block, which has size $2pk_n$, while the small block $B_j(p)$ only has a size of $2k_n$. The small blocks are going to separate the big blocks. This means that some important terms (defined later) are conditionally independent, since the summands in the pre-averaged bipower estimator employ $2k_n$ underlying high-frequency returns. We let $j_n(p) \equiv \lfloor n/2(p+1)k_n \rfloor$ be the number of such big block–small block pairs. We utilize these pairwise blocks until observation $i_n(p) \equiv j_n(p)2(p+1)k_n$, while leaving some residual unused data at the end.

Now, we define

$$\zeta(p,q,r,1)_j^n = \sum_{u=a_j(p)}^{b_j(p)-1} \tilde{Y}_u^n(q,r) \qquad \text{and} \qquad \zeta(p,q,r,2)_j^n = \sum_{u=b_j(p)}^{a_{j+1}(p)-1} \tilde{Y}_u^n(q,r).$$

with

$$\tilde{Y}_u^n(q,r) = \begin{cases} n^{-1/2}\left(\eta(q,r)_{u,a_j(p)}^n - \mathbb{E}\big[\eta(q,r)_{u,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\big]\right), & u \in A_j(p), \\ n^{-1/2}\left(\eta(q,r)_{u,b_j(p)}^n - \mathbb{E}\big[\eta(q,r)_{u,b_j(p)}^n \mid \mathcal{F}_{b_j(p)\Delta_n}\big]\right), & u \in B_j(p), \\ n^{-1/2}\left(\eta(q,r)_{u,i_n(p)}^n - \mathbb{E}\big[\eta(q,r)_{u,i_n(p)}^n \mid \mathcal{F}_{i_n(p)\Delta_n}\big]\right), & u \in i_n(p), \end{cases}$$

and denote

$$\begin{aligned} M(p,q,r)^n &= n^{-1/2} \sum_{j=0}^{j_n(p)-1} \zeta(p,q,r,1)_j^n, \\ N(p,q,r)^n &= n^{-1/2} \sum_{j=0}^{j_n(p)-1} \zeta(p,q,r,2)_j^n, \\ C(p,q,r)^n &= n^{-1/2} \sum_{u=i_n(p)}^{n} \tilde{Y}_u^n(q,r). \end{aligned}$$

Next, we introduce the decomposition

$$n^{1/4}(BV_n(q,r) - V(q,r)) = n^{1/4}\left[M(p,q,r)^n + N(p,q,r)^n + C(p,q,r)^n\right] + F(p,q,r)^n, \tag{43}$$

which implicitly defines $F(p,q,r)^n$ as a remainder term. In a series of lemmas, we will show that $M(p,q,r)^n$ is the leading term, while the remaining parts are asymptotically negligible.

Lemma 7. *Let $p \geq 2$ be fixed. Under the maintained assumptions of Theorem 3, it holds that*

$$\mathbb{E}\left[\left(n^{1/4}N(p,q,r)^n\right)^2\right] \leq C/p.$$

Proof. Write

$$L_k^n = n^{-1/2} \sum_{j=0}^{k} \zeta(p,q,r,2)_j^n.$$

We observe that the process $(L_k^n)_{k=0}^{j_n(p)-1}$ is a discrete-time martingale under the filtration $(\mathcal{F}_{b_j(p)\Delta_n})_{j=0}^{j_n(p)-1}$. Then, Doob's inequality yields that

$$\mathbb{E}\left[\left(n^{1/4}N(p,q,r)^n\right)^2\right] \leq Cn^{-1/2} \sum_{j=0}^{j_n(p)-1} \mathbb{E}\left[\left(\zeta(p,q,r,2)_j^n\right)^2\right]. \tag{44}$$

Lemma 4 implies that $\mathbb{E}\left[\left(\tilde{Y}_u^n(q,r)\right)^2\right] \leq C/n$ uniformly in j, p, q, and r. Consequently, we deduce that $\mathbb{E}\left[\left(\zeta(p,q,r,2)_j^n\right)^2\right] \leq C$, and then using $j_n(p) \leq Cn^{1/2}/p$ ends the proof. ■

Lemma 8. *Let $p \geq 2$ be fixed. Under the maintained assumptions of Theorem 3, it holds that*

$$\mathbb{E}\left[\left|n^{1/4}C(p,q,r)^n\right|\right] \leq Cpn^{-1/4}.$$

Proof. Note that $n - i_n(p) \leq Cpn^{1/2}$ and $\mathbb{E}\left[\left|\tilde{Y}_u^n(q,r)\right|\right] \leq Cn^{-1/2}$. This immediately leads to the conclusion that $\mathbb{E}\left[\left|n^{1/4}C(p,q,r)^n\right|\right] \leq Cpn^{-1/4}$. ■

Lemma 9. *Under the maintained assumptions of Theorem 3, for any $\delta > 0$, it holds that*

$$\lim_{p\to\infty} \limsup_{n\to\infty} \mathbb{P}\left[\left|F(p,q,r)^n\right| > \delta\right] = 0.$$

Proof. Recalling the decomposition in (43), we write that

$$F(p,q,r)^n = A(p,q,r)^n + B(p,q,r)^n + D(p,q,r)^n - n^{1/4}C(p,q,r)^n, \tag{45}$$

where

$$A(p,q,r)^n = n^{1/4}BV_n(q,r) - n^{1/4}\frac{1}{n} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \eta(q,r)^n_{a_j(p)+u,a_j(p)},$$

$$B(p,q,r)^n = n^{1/4}\frac{1}{n} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[\eta(q,r)^n_{a_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n}\right] - n^{1/4}V(q,r),$$

$$\begin{aligned} D(p,q,r)^n = {} & n^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2k_n-1} \eta(q,r)^n_{b_j(p)+u,a_j(p)} - \mathbb{E}\left[\eta(q,r)^n_{b_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n}\right] \\ & - n^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2k_n-1} \eta(q,r)^n_{b_j(p)+u,b_j(p)} - \mathbb{E}\left[\eta(q,r)^n_{b_j(p)+u,b_j(p)} \mid \mathcal{F}_{b_jp)\Delta_n}\right] \end{aligned}$$

The $C(p,q,r)^n$ term was covered in Lemma 8, while the $D(p,q,r)^n$ term is dealt with following Lemma 7. So we proceed to $A(p,q,r)^n$. Ignoring asymptotically negligible remainder terms, and employing the shorthand notation $\tilde{\eta}(q,r)^n_{i,m} \equiv \eta(q,r)^n_i - \eta(q,r)^n_{i,m}$, it is enough to study

$$A_1^n = n^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[\tilde{\eta}(q,r)^n_{a_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n}\right],$$

$$A_2^n = n^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \tilde{\eta}(q,r)^n_{a_j(p)+u,a_j(p)} - \mathbb{E}\left[\tilde{\eta}(q,r)^n_{a_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n}\right].$$

Proceeding as in the proof of Lemma 7, applying Doob's inequality and Lemma 6 yields

$$\begin{aligned}
\mathbb{E}\left[\left(A_2^n\right)^2\right] &\le Cn^{-3/2}pk_n \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[\left(\tilde{\eta}(q,r)^n_{a_j(p)+u,a_j(p)}\right)^2\right] \\
&\le Cn^{-3/2}pk_n \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \left(n^{-1/2} + \mathbb{E}\left[\left(\Gamma(\sigma, a_j(p), a_j(p)+u)^n + \Gamma(\omega, a_j(p), a_j(p)+u)^n\right)^2\right]\right) \\
&\le Cpn^{-1/2} + Cn^{-1/2}p^2 \sum_{j=0}^{j_n(p)-1} \mathbb{E}\left[\left(\Gamma(\sigma, a_j(p), a_{j+1}(p))^n + \Gamma(\omega, a_j(p), a_{j+1}(p))^n\right)^2\right],
\end{aligned}$$

which is negligible by Lemma 5.4 in Jacod, Li, Mykland, Podolskij, and Vetter (2009).

Moving to the A_1^n term, we observe that from the inequality $|y_1y_2 - x_1x_2| \le |y_1 - x_1||y_2| + |x_1||y_2 - x_2|$, and Lemmas 4 and 6, it suffices to set $r = 0$ and look at the term

$$n^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[\eta(q)^n_{a_j(p)+u} - \eta(q)^n_{a_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n}\right].$$

Now, take $f(x) = |x|^q$ and recall that $\alpha^n_{i,m} = n^{1/4}\left(\bar{r}^*_i - \bar{r}^*_{i,m}\right)$. By Taylor's theorem and Lemmas 4 and 6, we can reduce the problem to studying the term

$$n^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[f'(\eta(q)_{a_j(p)+u,a_j(p)})\alpha^n_{a_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n}\right]. \tag{46}$$

At this stage, we need an extra decomposition:

$$\alpha^n_{i,m} = \alpha^n_{i,m}(1) + \alpha^n_{i,m}(2),$$

where

$$\begin{aligned}
\alpha^n_{i,m}(1) &= n^{1/4} \sum_{j=1}^{k_n-1} g(j/k_n) \left[\Delta_n a_{m\Delta_n} + \int_{(i+j-1)\Delta_n}^{(i+j)\Delta_n} \left[\tilde{\sigma}_{m\Delta_n}(W_s - W_{m\Delta_n}) + \tilde{v}_{m\Delta_n}(B_s - B_{m\Delta_n})\right] \mathrm{d}W_s\right] \\
&\quad + n^{1/4} \sum_{j=0}^{k_n-1} \left[g(j/k_n) - g(j+1/k_n)\right] \left[\bar{\sigma}_{m\Delta_n}(W_{(i+j)\Delta_n} - W_{m\Delta_n}) + \bar{v}_{m\Delta_n}(B_{(i+j)\Delta_n} - B_{m\Delta_n})\right] \pi_{i+j}, \\
\alpha^n_{i,m}(2) &= n^{1/4} \sum_{j=1}^{k_n-1} g(j/k_n) \left[\int_{(i+j-1)\Delta_n}^{(i+j)\Delta_n} (a_s - a_{m\Delta_n}) \mathrm{d}s + \int_{(i+j-1)\Delta_n}^{(i+j)\Delta_n} \int_{m\Delta_n}^{s} \tilde{a}_u \mathrm{d}u \mathrm{d}W_s\right]
\end{aligned}$$

$$+ n^{1/4} \sum_{j=1}^{k_n-1} g(j/k_n) \left[\int_{(i+j-1)\Delta_n}^{(i+j)\Delta_n} \left[\int_{m\Delta_n}^{s} (\tilde{\sigma}_u - \tilde{\sigma}_{m\Delta_n}) \mathrm{d}W_u + \int_{m\Delta_n}^{s} (\tilde{v}_u - \tilde{v}_{m\Delta_n}) \mathrm{d}B_u \right] \mathrm{d}W_s \right]$$

$$+ n^{1/4} \sum_{j=0}^{k_n-1} \left[g(j/k_n) - g(j+1/k_n) \right] \left[\int_{m\Delta_n}^{(i+j)\Delta_n} \bar{a}_s \mathrm{d}s + (\bar{\sigma}_s - \bar{\sigma}_{m\Delta_n}) \mathrm{d}W_s + (\bar{v}_s - \bar{v}_{m\Delta_n}) \mathrm{d}B_s \right] \pi_{i+j}.$$

Returning to (46), we first note that

$$\mathbb{E}\left[f'(\eta(q)_{a_j(p)+u,a_j(p)}) \alpha^n_{a_j(p)+u,a_j(p)}(1) \mid \mathcal{F}_{a_j(p)\Delta_n} \right] = 0,$$

because $(W, B, \pi) \overset{d}{=} -(W, B, \pi)$, and since $f'(\eta(q)_{a_j(p)+u,a_j(p)})$ is odd and $\alpha^n_{a_j(p)+u,a_j(p)}(1)$ is even in (W, B, π). Furthermore,

$$\mathbb{E}\left[\left(\alpha^n_{a_j(p)+u,a_j(p)}(2) \right)^2 \right] \leq C(pk_n\Delta_n)^2 + Cpn^{-1/2} \mathbb{E}\left[(\gamma^n_j(p))^2 \right],$$

where

$$\gamma^n_j(p) = \sup_{a_j(p)\Delta_n \leq s \leq a_{j+1}(p)\Delta_n} \left[|a_s - a_{a_{j(p)}}| + |\tilde{\sigma}_s - \tilde{\sigma}_{a_{j(p)}}| + |\tilde{v}_s - \tilde{v}_{a_{j(p)}}| + |\bar{\sigma}_s - \bar{\sigma}_{a_{j(p)}}| + |\bar{v}_s - \bar{v}_{a_{j(p)}}| \right].$$

Then, for any fixed $p \geq 2$, it follows from the Cauchy-Schwarz inequality that

$$\begin{aligned} \mathbb{E}\big[|A^n_1|\big] &\leq Cn^{-3/4} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[\left(\alpha^n_{a_j(p)+u,a_j(p)}(2) \right)^2 \right]^{1/2} \\ &\leq Cn^{-3/4} n(pk_n\Delta_n) + Cpn^{-1/2} \sum_{j=0}^{j_n(p)-1} \mathbb{E}\big[(\gamma^n_j(p))^2\big]^{1/2} \to 0, \end{aligned}$$

as $n \to \infty$, which follows from Lemma 5.4 in Jacod, Li, Mykland, Podolskij, and Vetter (2009).

Now, we proceed to the last term, $B(p, q, r)^n$. This can be further divided into

$$B(p, q, r)^n = B(p, q, r)^n_1 + B(p, q, r)^n_2,$$

with

$$B(p, q, r)^n_1 = n^{1/4} \frac{1}{n} \sum_{j=0}^{j_n(p)-1} \sum_{u=0}^{2(p+1)k_n-1} \mathbb{E}\left[\eta(q, r)_{a_j(p)+u,a_j(p)} \mid \mathcal{F}_{a_j(p)\Delta_n} \right] - n^{1/4} \mu_q \mu_r \sum_{j=0}^{j_n(p)-1} \int_{a_j(p)\Delta_n}^{a_{j+1}(p)\Delta_n} \upsilon_{a_j(p)} \mathrm{d}t,$$

$$B(p, q, r)^n_2 = n^{1/4} \mu_q \mu_r \sum_{j=0}^{j_n(p)-1} \int_{a_j(p)\Delta_n}^{a_{j+1}(p)\Delta_n} \big(\upsilon_{a_j(p)} - \upsilon_t \big) \mathrm{d}t - n^{1/4} \mu_q \mu_r \int_{i_n(p)\Delta_n}^{1} \upsilon_t \mathrm{d}t,$$

and where $\upsilon_t = \theta \psi_2 \sigma^2_t + \frac{1}{\theta} \psi_1 \omega^2_t$.

In view of Lemma 5,

$$\lim_{p \to \infty} \limsup_{n \to \infty} \mathbb{P}\big[|B(p, q, r)^n_1| > \delta| \big] = 0.$$

Concerning the term $B(p, q, r)^n_2$, we remark that υ_t is a continuous Itô process due to Assumptions (V) and (N). Hence, a known result on the error of Riemann integration (see, e.g., the proof of

Lemma A.1 (iv) in Christensen, Podolskij, Thamrongrat, and Veliyev (2017)) states that

$$\mathbb{E}\left[|B(p,q,r)_2^n|\right] \leq Cn^{1/4}(pk_n\Delta_n + pn^{-1/2}) \leq Cpn^{-1/4},$$

which completes the proof. ■

Lemma 10. *Let $p \geq 2$ be fixed. Under the maintained assumptions of Theorem 3, it holds that*

$$n^{1/4}(M(p,q_1,r_1)^n, M(p,q_2,r_2)^n) \xrightarrow{\mathcal{D}_s} MN(0,\Sigma(p)),$$

where the (i,j)-th component of the 2x2 covariance matrix $\Sigma(p)$ is defined as

$$\Sigma(p)_{ij} = \theta\frac{p}{p+1}\int_0^1\int_0^2\left(2-\frac{s}{p}\right)h_{ij}\big((\omega_u,\sigma_u),(\psi_1/\theta,\theta\psi_2),f(s)\big)\mathrm{d}s\mathrm{d}u.$$

Proof. We set $\zeta(p)_j^n = \big(\zeta(p,q_1,r_1,1)_j^n, \zeta(p,q_2,r_2,1)_j^n\big)^\top$ for notational convenience. In view of Theorem IX.7.28 in Jacod and Shiryaev (2003), it suffices to verify the following four conditions:

$$n^{-1/2}\sum_{j=0}^{j_n(p)-1}\mathbb{E}\left[\zeta(p)_j^n(\zeta(p)_j^n)^\top \mid \mathcal{F}_{a_j(p)}\right] \xrightarrow{p} \Sigma(p),$$

$$n^{-1/2}\sum_{j=0}^{j_n(p)-1}\mathbb{E}\left[\|\zeta(p)_j^n\|^4 \mid \mathcal{F}_{a_j(p)}\right] \xrightarrow{p} 0,$$

$$n^{-1/2}\sum_{j=0}^{j_n(p)-1}\mathbb{E}\left[\zeta(p)_j^n(W_{b_j(p)\Delta_n} - W_{a_j(p)\Delta_n}) \mid \mathcal{F}_{a_j(p)}\right] \xrightarrow{p} 0,$$

$$n^{-1/2}\sum_{j=0}^{j_n(p)-1}\mathbb{E}\left[\zeta(p)_j^n(N_{b_j(p)\Delta_n} - N_{a_j(p)\Delta_n}) \mid \mathcal{F}_{a_j(p)}\right] \xrightarrow{p} 0,$$

for any bounded martingale N that is orthogonal to W.

The last three convergences are omitted, since they can be proved exactly as in Podolskij and Vetter (2009a) and Jacod, Li, Mykland, Podolskij, and Vetter (2009). So we only prove the convergence in the first condition, which is done separately for each component of $\tilde{\Sigma}$. We only spell out the details for the first term.

To this end, for any $u, v \in A_j(p)$ and $u \leq v$, we note that

$$\mathbb{E}\left[\tilde{Y}_u^n\tilde{Y}_v^n \mid \mathcal{F}_{a_j(p)\Delta_n}\right] = \frac{1}{n}\Big(\mathbb{E}\big[\eta_{u,a_j(p)}^n\eta_{v,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\big] - \mathbb{E}\big[\eta_{u,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\big]\mathbb{E}\big[\eta_{v,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\big]\Big).$$

We suppress the dependence on q and r for brevity. Let $\tilde{\eta}_{i,m}^n$ be a version of the variable $\eta_{i,m}^n$, where the noise variable π is replaced by a Gaussian distributed one. In view of Lemma 5,

$$\mathbb{E}\left[\eta_{u,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\right] = \mathbb{E}\left[\tilde{\eta}_{u,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\right] + o_p(n^{-1/4}).$$

This, together with Lemma 4 and the Cauchy-Schwarz inequality, implies that

$$\left|\mathbb{E}\left[\eta_{u,a_j(p)}^n\eta_{v,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\right] - \mathbb{E}\left[\tilde{\eta}_{u,a_j(p)}^n\tilde{\eta}_{v,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\right]\right| \leq \left|\mathbb{E}\left[(\eta_{u,a_j(p)}^n - \tilde{\eta}_{u,a_j(p)}^n)\eta_{v,a_j(p)}^n \mid \mathcal{F}_{a_j(p)\Delta_n}\right]\right|$$

$$+\left|\mathbb{E}\left[(\eta^n_{v,a_j(p)}-\tilde{\eta}^n_{v,a_j(p)})\tilde{\eta}^n_{u,a_j(p)}\mid \mathcal{F}_{a_j(p)\Delta_n}\right]\right|$$
$$= o_p(n^{-1/8}).$$

Further, recalling the notation in (39), we have that

$$\mathbb{E}\left[\tilde{\eta}^n_{u,a_j(p)}\tilde{\eta}^n_{v,a_j(p)}\mid \mathcal{F}_{a_j(p)\Delta_n}\right]-\mathbb{E}\left[\tilde{\eta}^n_{u,a_j(p)}\mid \mathcal{F}_{a_j(p)\Delta_n}\right]\mathbb{E}\left[\tilde{\eta}^n_{v,a_j(p)}\mid \mathcal{F}_{a_j(p)\Delta_n}\right]$$
$$= h_{11}\big((\omega_{a_j(p)\Delta_n},\sigma_{a_j(p)\Delta_n}),\psi^n,f^n((v-u)/k_n)\big),$$

where $\psi^n=(n^{1/2}\psi^n_1/k_n,k_n\psi^n_2/n^{1/2})$ and $f^n(s)=\big(f^n_1(s),\ldots,f^n_4(s)\big)$ with

$$f^n_1(s)=n^{1/2}\sum_{j=0}^{k_n(1-s)}\left(g\left(\frac{j}{k_n}\right)-g\left(\frac{j+1}{k_n}\right)\right)\left(g\left(\frac{j+sk_n}{k_n}\right)-g\left(\frac{j+1+sk_n}{k_n}\right)\right),$$
$$f^n_2(s)=n^{-1/2}\sum_{j=0}^{k_n(1-s)}g\left(\frac{j}{k_n}\right)g\left(\frac{j+sk_n}{k_n}\right),$$
$$f^n_3(s)=n^{1/2}\sum_{j=0}^{k_n(2-s)}\left(g\left(\frac{j}{k_n}\right)-g\left(\frac{j+1}{k_n}\right)\right)\left(g\left(\frac{j+sk_n-k_n}{k_n}\right)-g\left(\frac{j+1+sk_n-k_n}{k_n}\right)\right),$$
$$f^n_4(s)=n^{-1/2}\sum_{j=0}^{k_n(2-s)}g\left(\frac{j}{k_n}\right)g\left(\frac{j+sk_n-k_n}{k_n}\right).$$

As a consequence, for $u,v\in A_j(p)$ and $0\le v-u<2k_n$, we obtain

$$\mathbb{E}\left[\tilde{Y}^n_u\tilde{Y}^n_v\mid \mathcal{F}_{a_j(p)\Delta_n}\right]=\frac{1}{n}h_{11}\big((\omega_{a_j(p)\Delta_n},\sigma_{a_j(p)\Delta_n}),\psi^n,f^n((v-u)/k_n)\big)+o_p(n^{-1}),$$

while this term vanishes for $v-u\ge 2k_n$. In turn, this implies that

$$\mathbb{E}\left[(\zeta(p,1)^n_j)^2\mid \mathcal{F}_{a_j(p)\Delta_n}\right]=R^n_1+R^n_2,$$

where

$$R^n_1=2\sum_{u=a_j(p)}^{b_j(p)-2k_n-1}\sum_{v=u}^{u+2k_n-1}\mathbb{E}\left[\tilde{Y}^n_u\tilde{Y}^n_v\mid \mathcal{F}_{a_j(p)\Delta_n}\right]-\sum_{u=a_j(p)}^{b_j(p)-2k_n-1}\mathbb{E}\left[(\tilde{Y}^n_u)^2\mid \mathcal{F}_{a_j(p)\Delta_n}\right],$$
$$R^n_2=2\sum_{u=b_j(p)-2k_n}^{b_j(p)-1}\sum_{v=u}^{b_j(p)-1}\mathbb{E}\left[\tilde{Y}^n_u\tilde{Y}^n_v\mid \mathcal{F}_{a_j(p)\Delta_n}\right]-\sum_{u=b_j(p)-2k_n}^{b_j(p)-1}\mathbb{E}\left[(\tilde{Y}^n_u)^2\mid \mathcal{F}_{a_j(p)\Delta_n}\right].$$

Concerning the first term,

$$n^{-1/2}\sum_{j=0}^{j_n(p)-1}R^n_1=n^{-3/2}2(2pk_n-2k_n)\sum_{j=0}^{j_n(p)-1}\sum_{v=0}^{2k_n-1}h_{11}\big((\omega_{a_j(p)\Delta_n},\sigma_{a_j(p)\Delta_n}),\psi^n,f^n(v/k_n)\big)+o_p(1)$$
$$\xrightarrow{p}2\theta\frac{p-1}{p+1}\int_0^1\int_0^2 h_{ij}\big((\omega_u,\sigma_u),(\psi_1/\theta,\theta\psi_2),f(s)\big)\mathrm{d}s\mathrm{d}u,$$

where the argument is as in Podolskij and Vetter (2009a) via continuity of the function h_{11} and

Lebesgue's theorem.

Furthermore, we deduce that

$$n^{-1/2}\sum_{j=0}^{j_n(p)-1} n^{-1/2}R_2^n = n^{-3/2}2\sum_{j=0}^{j_n(p)-1}\sum_{v=0}^{2k_n-1}(2k_n-v)h_{11}\big((\omega_{a_j(p)\Delta_n},\sigma_{a_j(p)\Delta_n}),\psi^n,f^n(v/k_n)\big)+o_p(1)$$

$$\xrightarrow{p}\frac{\theta}{p+1}\int_0^1\int_0^2(2-s)h_{ij}\big((\omega_u,\sigma_u),(\psi_1/\theta,\theta\psi_2),f(s)\big)\mathrm{d}s\mathrm{d}u.$$

The statement then follows by summing these two terms. ■

The proof of Theorem 3 now follows directly from (43) and Lemmas 7 – 10.

Proof of Theorem 1

The proof is mostly based on Theorem 3 above, after we take into account that the error in the noise variance estimator introduced in (14).

Lemma 11. *Assume that r follows the process in* (1) *and that Assumptions (V) and (N) hold. Then, as $n\to\infty$,*

$$\sqrt{n}\left(\hat{\omega}_n^2-\int_0^1\omega_s^2\mathrm{d}s\right)=O_p(1). \tag{47}$$

Proof. We can write

$$\hat{\omega}_n^2-\int_0^1\omega_s^2\mathrm{d}s=N_1^n+N_2^n+N_3^n, \tag{48}$$

where

$$N_1^n=-\int_0^1\omega_s^2\mathrm{d}s-\frac{1}{n-1}\sum_{i=2}^n(\epsilon_{i\Delta_n}-\epsilon_{(i-1)\Delta_n})(\epsilon_{(i-1)\Delta_n}-\epsilon_{(i-2)\Delta_n}),$$

$$N_2^n=-\frac{1}{n-1}\sum_{i=2}^n\left[(p_{i\Delta_n}-p_{(i-1)\Delta_n})(\epsilon_{(i-1)\Delta_n}-\epsilon_{(i-2)\Delta_n})+(\epsilon_{i\Delta_n}-\epsilon_{(i-1)\Delta_n})(p_{(i-1)\Delta_n}-p_{(i-2)\Delta_n})\right],$$

$$N_3^n=-\frac{1}{n-1}\sum_{i=2}^n(p_{i\Delta_n}-p_{(i-1)\Delta_n})(p_{(i-1)\Delta_n}-p_{(i-2)\Delta_n}).$$

In view of (2.1.44) in Jacod and Protter (2012), we find that $\sup_{1\le i\le n}\mathbb{E}\left[(p_{i\Delta_n}-p_{(i-1)\Delta_n})^2\right]\le C/n$. Combining this with the identity $\sup_{1\le i\le n}\mathbb{E}\left[(\epsilon_{i\Delta_n}-\epsilon_{(i-1)\Delta_n})^2\right]\le C$ leads to

$$\mathbb{E}\left[|N_n^2|+|N_n^3|\right]\le Cn^{-1/2}.$$

Concerning the main term, we apply a further decomposition $N_1^n=N_{11}^n+N_{12}^n+N_{13}^n$, where

$$N_{11}^n=-\int_0^1\omega_s^2\mathrm{d}s+\frac{1}{n-1}\sum_{i=2}^n\omega_{(i-1)\Delta_n}^2\pi_{i-1}^2,\qquad N_{12}^n=\frac{1}{n-1}\sum_{i=2}^n\omega_{(i-1)\Delta_n}^2(\pi_{i-1}^2-1),$$

$$N_{13}^n=-\frac{1}{n-1}\sum_{i=2}^n\left[\omega_{i\Delta_n}\pi_i\omega_{(i-1)\Delta_n}\pi_{i-1}-\omega_{i\Delta_n}\pi_i\omega_{(i-2)\Delta_n}\pi_{i-2}+\omega_{(i-1)\Delta_n}\pi_{i-1}\omega_{(i-2)\Delta_n}\pi_{i-2}\right].$$

Alluding to Assumption (N)(iii), a standard Riemann integration error argument yields $N_{11}^n = O_p(n^{-1/2})$. In the second term above, we note that

$$\mathbb{E}\left[\omega_{(i-1)\Delta_n}^2(\pi_{i-1}^2 - 1)\omega_{(j-1)\Delta_n}^2(\pi_{j-1}^2 - 1)\right] = 0,$$

for $i \neq j$ by Assumption (N)(i) and (iv). It follows that $\mathbb{E}\left[(N_{12}^n)^2\right] \leq C/n$. In the third term, the summands are again uncorrelated, which leads to $\mathbb{E}\left[(N_{13}^n)^2\right] \leq C/n$. ■

We now turn to the proof of Theorem 1. In relation to the convergence in probability part, it follows from the above preliminary results that

$$BV_n(2,0) \xrightarrow{p} \int_0^1 \Big(\theta\psi_2\sigma_s^2 + \frac{1}{\theta}\psi_1\omega_s^2\Big)\mathrm{d}s + \theta\psi_2 \sum_{0\leq s\leq 1}(\Delta r_s)^2,$$

$$BV_n(1,1) \xrightarrow{p} \int_0^1 \Big(\theta\psi_2\sigma_s^2 + \frac{1}{\theta}\psi_1\omega_s^2\Big)\mathrm{d}s.$$

This can be shown by proceeding as in the proof of Theorem 2 in Podolskij and Vetter (2009a). Now, the consistency in Theorem 1 follows from Lemma 11. The central limit theorem is derived under the further assumption that p is continuous. Hence, it follows from Theorem 3 [choosing the exponents $(q_1, r_1) = (2, 0)$ and $(q_2, r_2) = (1, 1)$] and Lemma 11. ■

A.2 A jump- and noise-robust estimator of $\tilde{\Sigma}$

Here, we present an estimator of $\tilde{\Sigma}$, which appears in the extended central limit theorem in Theorem 3. Its connection with Σ is explained in (41).

We build on previous work of Christensen, Podolskij, Thamrongrat, and Veliyev (2017), who propose a subsampling estimator of $\tilde{\Sigma}$ (see also Politis, Romano, and Wolf, 1999; Kalnina and Linton, 2007; Kalnina, 2011; Mykland and Zhang, 2017). An appealing feature of their estimator is that it is positive semi-definite and has good small sample properties.[68] In that paper, the subsampling estimator is shown to be consistent if there is either price jumps or microstructure noise, but not in the presence of both. Microstructure noise is further restricted to be either heteroscedastic or dependent, but not both. Here, we extend their framework to account for all of these features at once.

We propose the following jump- and noise-robust covariance matrix estimator:

$$\tilde{\Sigma}_n = \frac{1}{L}\sum_{l=1}^{L}\left(\frac{n^{1/4}}{\sqrt{L}}\begin{bmatrix} \check{BV}_l(q_1,r_1) - \check{BV}_n(q_1,r_1) \\ \check{BV}_l(q_2,r_2) - \check{BV}_n(q_2,r_2)\end{bmatrix}\right)\left(\frac{n^{1/4}}{\sqrt{L}}\begin{bmatrix} \check{BV}_l(q_1,r_1) - \check{BV}_n(q_1,r_1) \\ \check{BV}_l(q_2,r_2) - \check{BV}_n(q_2,r_2)\end{bmatrix}\right)^{\top},$$

[68] Podolskij and Vetter (2009a) develop an element-by-element estimator of $\tilde{\Sigma}$ in the i.i.d noise setting. However, this estimator is not positive semi-definite and is often ill-conditioned in practice. Moreover, it is not consistent for $\tilde{\Sigma}$ under the jump alternative.

with

$$\check{BV}_n(q,r) = \frac{1}{n}\sum_{i=0}^{n-2k_n+1} |n^{1/4}\check{r}_i^*|^q |n^{1/4}\check{r}_{i+k_n}^*|^r, \qquad \check{BV}_l(q,r) = \frac{Lpk_n}{n}\sum_{i=1}^{n/Lpk_n} v_{(i-1)L+l}(q,r)^n,$$

$$v_i(q,r)^n = \frac{1}{pk_n - 2k_n + 2}\sum_{j,j+2k_n-1\in B_i(p)} |n^{1/4}\check{r}_j^*|^q |n^{1/4}\check{r}_{j+k_n}^*|^r - \frac{\psi_1^n}{\psi_2^n\theta^2}\hat{\omega}^2,$$

and

$$\check{r}_i^* = \bar{r}_i^* \mathbb{1}(|\bar{r}_i^*| \leq u_n), \tag{49}$$

where $\mathbb{1}(A)$ is an indicator function that equals one if A is true, zero otherwise, and $u_n = \alpha n^{-\bar{\omega}}$ with $\alpha > 0$ and $\bar{\omega} \in (0, 1/4)$.

To deal with microstructure noise, $\tilde{\Sigma}_n$ is based on the pre-averaged bipower variation. To further robustify it to jumps, we exploit the truncation device of Mancini (2009) in (49). It sets large negative or positive pre-averaged log-returns to zero, since they are most likely dominated by the jump component. The threshold, u_n, is adapted to an estimate of the square-root integrated variance by setting $\alpha = c\sqrt{BV_n(1,1)}$. With this configuration, we can interpret c as the number of local diffusive standard deviations a pre-averaged log-return must exceed to be labelled a jump. The rate parameter, $\bar{\omega}$, ensures that, asymptotically, continuous pre-averaged log-returns are unaffected by the truncation. In this way, $\tilde{\Sigma}_n$ is also made jump-robust.

Theorem 12. *Assume that r follows (1) with $\beta \leq 1$ and that Assumptions (V) and (N) hold. Moreover, for each $s \in \{q_1, q_2, r_1, r_2\} \cap [1, \infty)$, we require that*

$$\frac{q + \delta - 1/2}{4q - \beta} < \bar{\omega} < \frac{1}{4} - \delta$$

where $L \asymp n^{(1-\delta)/2}$. Then, as $n \to \infty$, $\to \infty$, $L/p \to \infty$, $\sqrt{n}/Lp^2 \to \infty$ and $\delta < 1/16$, it holds that

$$\tilde{\Sigma}_n \xrightarrow{p} \tilde{\Sigma}.$$

Proof of Theorem 12

By the polarization identity, it suffices to show the result $\tilde{\Sigma}_n - \tilde{\Sigma} \xrightarrow{p} 0$ in the univariate setting. To this end, we denote by $\bar{r}_i'$ the pre-averaged return based on the continuous part of p_t and ϵ_t. The corresponding subsampling estimator is denoted as

$$\Sigma_n' = \frac{1}{L}\sum_{l=1}^{L}\left(\frac{n^{1/4}}{\sqrt{L}}\Big(BV_l'(q,r) - BV_n'(q,r)\Big)\right)^2,$$

where

$$BV_n'(q,r) = \frac{1}{n}\sum_{i=0}^{n-2k_n+1} |n^{1/4}\bar{r}_i'|^q |n^{1/4}\bar{r}_{i+k_n}'|^r, \qquad BV_l'(q,r) = \frac{Lpk_n}{n}\sum_{i=1}^{n/Lpk_n} v_{(i-1)L+l}'(q,r)^n,$$

$$v_i'(q,r)^n = \frac{1}{pk_n - 2k_n + 2} \sum_{j,j+2k_n-1 \in B_i(p)} |n^{1/4}\bar{r}_j'|^q |n^{1/4}\bar{r}_{j+k_n}'|^r.$$

Proceeding exactly as in the proof of Theorem 3.8 in Christensen, Podolskij, Thamrongrat, and Veliyev (2017), it immediately follows that $\Sigma_n' - \tilde{\Sigma} \xrightarrow{p} 0$. Hence, it suffices to show that

$$\tilde{\Sigma}_n - \Sigma_n' \xrightarrow{p} 0.$$

To do this, we note that:

$$\begin{aligned}\tilde{\Sigma}_n - \Sigma_n' = \frac{1}{L}\sum_{l=1}^{L} &\left(\frac{n^{1/4}}{\sqrt{L}}\Big(BV_l(q,r) - BV_l'(q,r) + BV_n'(q,r) - BV_n(q,r)\Big)\right) \\ &\times \left(\frac{n^{1/4}}{\sqrt{L}}\Big(BV_l(q,r) - BV_n(q,r) + BV_l'(q,r) - BV_n'(q,r)\Big)\right),\end{aligned}$$

which is an average of L terms. The main idea is to deduce that each of these L terms converges in probability to zero. In particular, we show uniform convergence in mean square:

$$\sup_{1\le l\le L} \mathbb{E}\left[\Big|\frac{n^{1/4}}{\sqrt{L}}\big(BV_l(q,r) - BV_l'(q,r)\big)\Big|^2\right] \to 0 \quad \text{and} \quad \sup_{1\le l\le L} \mathbb{E}\left[\Big|\frac{n^{1/4}}{\sqrt{L}}\big(BV_n(q,r) - BV_n'(q,r)\big)\Big|^2\right] \to 0. \tag{50}$$

Following the arguments in the proof of Lemma A.6 in Christensen, Podolskij, Thamrongrat, and Veliyev (2017) implies that

$$\sup_{1\le l\le L} \mathbb{E}\left[\Big|\frac{n^{1/4}}{\sqrt{L}}\big(BV_l'(q,r) - BV_n'(q,r)\big)\Big|^2\right] \le C. \tag{51}$$

Then, equation (50) – (51) lead to:

$$\sup_{1\le l\le L} \mathbb{E}\left[\Big|\frac{n^{1/4}}{\sqrt{L}}\big(BV_l(q,r) - BV_n(q,r)\big)\Big|^2\right] \le C.$$

The combination of the last three results and the Cauchy-Schwarz inequality implies:

$$\mathbb{E}\big[|\tilde{\Sigma}_n - \Sigma_n'|\big] \to 0,$$

so that $\tilde{\Sigma}_n - \Sigma_n' \xrightarrow{p} 0$. Thus, it is enough to show (50).

For each j, we define

$$\lambda_j^n(q,r) = |n^{1/4}\bar{r}_j^*|^q |n^{1/4}\bar{r}_{j+k_n}^*|^r \mathbb{1}_{\{|\bar{r}_j^*|\le u_n \cap |\bar{r}_{j+k_n}^*|\le u_n\}} - |n^{1/4}\bar{r}_j'|^q |n^{1/4}\bar{r}_{j+k_n}'|^r.$$

Note that

$$BV_n(q,r) - BV_n'(q,r) = \frac{1}{n}\sum_{i=0}^{n-2k_n+1} \lambda_i^n(q,r),$$

and

$$BV_l(q,r) - BV_l'(q,r) = \frac{Lpk_n}{n}\frac{1}{pk_n - 2k_n + 2}\sum_{i=1}^{n/Lpk_n}\sum_{j,j+k_n-1\in B_{(i-1)L+l}(p)} \lambda_j^n(q,r).$$

As a result of Cauchy-Schwarz, showing (50) is reduced to proving:

$$\sup_{1\leq j\leq n-2k_n+2} \mathbb{E}\left[\left|\frac{n^{1/4}}{\sqrt{L}}\lambda_j^n(q,r)\right|^2\right] \to 0.$$

Furthermore, the identity

$$\lambda_j^n(q,r) = \lambda_j^n(q,0)|n^{1/4}\bar{r}'_{j+k_n}|^r + \lambda_j^n(0,r)|n^{1/4}\bar{r}'_j|^q + \lambda_j^n(q,0)\lambda_j^n(0,r),$$

and Lemma 4 combined with Cauchy-Schwarz inequality yields:

$$\mathbb{E}\big[|\lambda_j^n(q,r)|^2\big] \leq C\Big(\mathbb{E}\big[|\lambda_j^n(q,0)|^4\big]^{1/2} + \mathbb{E}\big[|\lambda_j^n(0,r)|^4\big]^{1/2} + \mathbb{E}\big[|\lambda_j^n(q,0)|^4\big]^{1/2}\mathbb{E}\big[|\lambda_j^n(0,r)|^4\big]^{1/2}\Big).$$

Then, in view of the rate condition $n^{1/4}/\sqrt{L} \to \infty$, it is enough that

$$\sup_{1\leq j\leq n-2k_n+2} \mathbb{E}\left[\left|\frac{n^{1/4}}{\sqrt{L}}\lambda_j^n(q,0)\right|^4\right] \to 0 \quad \text{and} \quad \sup_{1\leq j\leq n-2k_n+2} \mathbb{E}\left[\left|\frac{n^{1/4}}{\sqrt{L}}\lambda_j^n(0,r)\right|^4\right] \to 0.$$

We only look at the term $\lambda_j^n(q,0)$ with $q > 0$, for which

$$\lambda_j^n(q,0) = n^{q/4}|\bar{r}'_j + \bar{r}^d_j|^q \mathbb{1}_{\{|\bar{r}^*_j|\leq u_n\}} - n^{q/4}|\bar{r}'_j|^q.$$

Now, for any $u > 0$, $a \geq 0$ and $b \geq 0$, it holds that

$$\big||x+y|^q\mathbb{1}_{\{|x+y|\leq u\}} - |x|^q\big| \leq C\left(\frac{|x|^{q+a}}{u^a} + |x|^q\frac{|y|^b}{u^b} + (|y|\wedge u)^q + \mathbb{1}_{\{q>1\}}|x|^{q-1}(|y|\wedge u)\right),$$

which can be verified by looking at the cases:

$$(1)\ |x| \geq u/2, \qquad (2)\ |x| < u/2 \text{ and } |x+y| > u, \qquad (3)\ |x| < u/2 \text{ and } |x+y| \leq u.$$

If we exploit this result with $x = \bar{r}'_j$, $y = \bar{r}^d_j$ and $u = u_n$ (a and b are selected below) in combination with a second inequality $|c-d|^4 \leq |c^4 - d^4|$, we find that:

$$|\lambda_j^n(q,0)|^4 \leq Cn^q\left(\frac{|\bar{r}'_j|^{4q+a}}{u_n^a} + |\bar{r}'_j|^{4q}\frac{|\bar{r}^d_j|^b}{u_n^b} + \big(|\bar{r}^d_j|\wedge u_n\big)^{4q} + |\bar{r}'_j|^{4q-1}\big(|\bar{r}^d_j|\wedge u_n\big)\right).$$

Note that we discard the indicator function, because it holds for any $q > 1/4$. Now, arguing as in Jacod and Protter (2012, p. 529), we deduce that

$$\mathbb{E}\left[(|\bar{r}^d_j|\wedge u_n)^2\right] \leq C\frac{\rho_n}{n^{1/2+(2-\beta)\bar{\omega}}}.$$

In view of the fact that $(|x|\wedge u)^q \leq u^{q-2}(|x|\wedge u)^2$ for $q \geq 2$ and recalling that $u_n = \alpha n^{-\bar{\omega}}$:

$$\mathbb{E}\left[(|\bar{r}^d_j|\wedge u_n)^q\right] \leq C \times \begin{cases} \dfrac{\rho_n^{q/2}}{n^{q/4+(2-\beta)\bar{\omega}q/2}} & \text{if } q \leq 2, \\ \dfrac{u_n^{q-2}\rho_n}{n^{1/2+(2-\beta)\bar{\omega}}} \leq \dfrac{\rho_n}{n^{1/2+(q-\beta)\bar{\omega}}} & \text{if } q > 2. \end{cases}$$

Moreover, for any $q > 0$:

$$\mathbb{E}\big[|\bar{r}'_j|^q\big] \leq C\frac{1}{n^{q/4}} \quad \text{and} \quad \mathbb{E}\big[|\bar{r}^d_j|^2\big] \leq C\frac{1}{n^{1/2}}.$$

Combining these inequalities with $a = 4$, $b = 1$ and and recalling $L \asymp n^{(1-\delta)/2}$ leads to

$$\mathbb{E}\left[\left|\frac{n^{1/4}}{\sqrt{L}}\lambda_j^n(q,0)\right|^4\right] \leq C\left(\frac{1}{n^{1-\delta-4\bar{\omega}}} + \frac{1}{n^{1/4-\delta-\bar{\omega}}} + \frac{\rho_n}{n^{1/2-\delta-q+(4q-\beta)\bar{\omega}}} + \frac{\rho_n^{1/2}}{n^{-\delta+(1-\beta/2)\bar{\omega}}}\right)$$

The first and second error terms converge to 0 due to the condition $\bar{\omega} < 1/4 - \delta$. The third error term converges to 0 due to the assumption $\bar{\omega} > (q + \delta - 1/2)/(4q - \beta)$. This also implies $\bar{\omega} > 1/8$, which combined with $\beta \leq 1$ and $\delta < 1/16$ deals with the fourth term. ■

Proof of Theorem 2

The result follows directly from the proof of Theorem 12 by taking $(q_1, r_1) = (2, 0)$, $(q_2, r_2) = (1, 1)$ and adjusting by the constants c_1 and c_2. It is worth highlighting that the constraint $\bar{\omega} > (q + \delta - 1/2)/(4q - \beta)$ is more binding for $q = 2$ compared to $q = 1$. ■

A.3 Irrelevance of truncation

To show that the truncation does not influence the limiting distribution in Theorem 3 and, hence, therefore also does not affect Theorem 1, we note that

$$n^{1/4}\begin{pmatrix} BV_n(q_1,r_1) - V(q_1,r_1) \\ BV_n(q_2,r_2) - V(q_2,r_2) \end{pmatrix} = n^{1/4}\begin{pmatrix} BV_n(q_1,r_1) - V(q_1,r_1) \\ \check{BV}_n(q_2,r_2) - V(q_2,r_2) \end{pmatrix} + n^{1/4}\begin{pmatrix} 0 \\ BV_n(q_2,r_2) - \check{BV}_n(q_2,r_2) \end{pmatrix}.$$

It is thus enough to show that $n^{1/4}\big(BV_n(q_2,r_2) - \check{BV}_n(q_2,r_2)\big) = o_p(1)$. An application of Boole's and Markov's inequalities and Lemma 4 in the proof of Theorem 3 yields that

$$\mathbb{P}\big[BV_n(q_2,r_2) \neq \check{BV}_n(q_2,r_2)\big] \leq \sum_{i=1}^{n-k_n+2} \mathbb{P}\left[|\bar{r}_i^*| > u_n\right] \leq \frac{C}{n^{M(1/4-\bar{\omega})-1}},$$

where $M > 0$. As M can be chosen arbitrarily large, the result follows. ■

B Monte Carlo Simulations

The jump-testing theory in Section 2 is derived using an infill asymptotic setup, which assumes that $\Delta_n \to 0$. This is unattainable in practice. In this appendix, we therefore conduct a series of Monte Carlo simulations to elaborate on the finite sample properties of the test statistic in a more realistic setting.

We simulate from the following model:

$$r_t^* = r_t + \epsilon_t, \tag{52}$$

where $r_t = r_t^c + r_t^d$, and

$$\mathrm{d}r_t^c = \sigma_t \mathrm{d}W_t, \tag{53}$$

with $r_0 \equiv 0$.

The drift term is omitted (i.e., $a_t \equiv 0$) so the continuous part of r_t evolves as a local martingale. The contribution of the drift is small over short horizons and the terms involving it—including cross products—are asymptotically negligible. It therefore tends to play a minor role in the high-frequency setting.[69] Similarly, adding a realistic expected rate of return does not affect the conclusions in any material way.

The instantaneous variance is assumed to be driven by a square-root process (e.g., Cox, Ingersoll, and Ross, 1985; Heston, 1993):

$$\mathrm{d}\sigma_t^2 = \kappa(\sigma^2 - \sigma_t^2)\mathrm{d}t + \xi\sigma_t \mathrm{d}B_t, \tag{54}$$

with $\sigma_0^2 \sim \text{Gamma}(2\kappa\sigma^2\xi^{-2}, 2\kappa\xi^{-2})$. Our choice of parameters is based on previous work such as Aït-Sahalia and Kimmel (2007). Specifically, we set $\kappa = 5$, $\sigma = 0.4$, $\xi = 0.5$, and $\rho = -\sqrt{0.5}$, where ρ is the leverage correlation between the standard Brownian motions $\mathbb{E}[\mathrm{d}W_t \mathrm{d}B_t] = \rho \mathrm{d}t$. This yields a stationary and strictly positive variance in continuous-time (i.e., the Feller condition $\xi^2 < 2\kappa\sigma^2$ holds) with an annualized mean volatility of 40%.

r_t^d is an infinite-activity tempered stable process with Lévy measure

$$\nu(\mathrm{d}x) = \tau \frac{e^{-\lambda x}}{x^{1+\beta}} \mathrm{d}x, \tag{55}$$

where $\tau > 0$ and $\lambda > 0$.

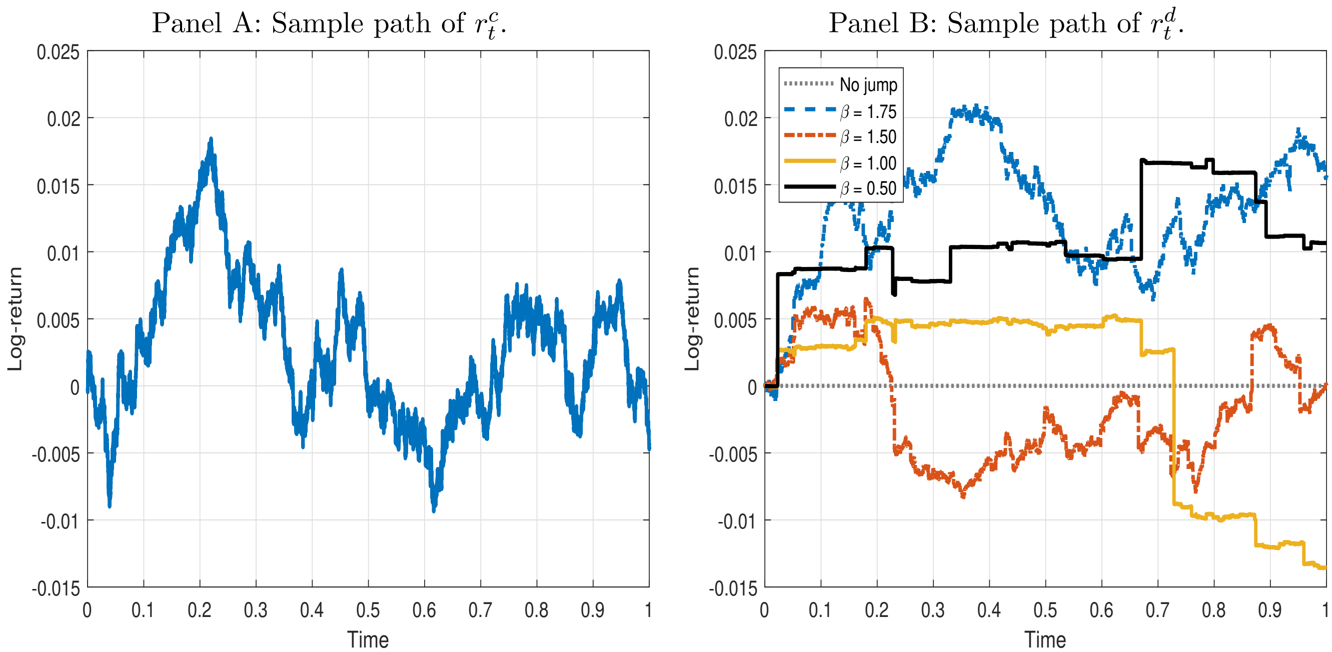
The activity of the jump process is controlled by β. As β increases, the density of the small jumps is enlarged, and the realizations of r_t^d become more vibrant and start to resemble those of a Brownian motion (see Figure B.1 for an illustration). As noted by Bollerslev and Todorov (2011), this renders the decomposition of diffusive and jump risk meaningless in practice and also makes it harder to separate the null from the jump alternative (Aït-Sahalia, Jacod, and Li (2012)). A priori, we therefore expect larger values of β to be detrimental to the rejection rate of the jump test statistic under the alternative. To gauge the impact of changing β, we follow Aït-Sahalia, Jacod,

[69] A notable exception is Christensen, Oomen, and Renò (2022).

and Li (2012) and examine $\beta = 0.50$, 1.00, 1.50 and 1.75. Moreover, we set $\lambda = 3$ and calibrate τ so that r_t^d induces 20% of the quadratic return variation, on average.

We simulate for $t \in [0, 1]$ with a time step of $\Delta_n = 1/23{,}400$. This can be interpreted as adding a new observation every second during a 6.5 hour trading session. A standard Euler discretization is employed for the continuous part.[70] As described in Todorov, Tauchen, and Grynkiv (2014), the jump component is computed as the difference between two spectrally positive tempered stable processes that are generated using the acceptance-rejection algorithm of Baeumer and Meerschaert (2010). The total number of Monte Carlo replications is $T = 10{,}000$. Figure B.1 illustrates a single sample path of r_t^c and r_t^d.

Figure B.1: A single simulation of the efficient log-return.



Note. We show a realization of the components of the efficient log-return r_t, which is a superposition of a continuous sample path Heston (1993)-type stochastic volatility model (r_t^c in Panel A) and a pure jump process of tempered stable-type (r_t^d in Panel B). The latter is plotted as a function of the activity index β using a common random seed.

We next explain how p_t is disrupted with microstructure noise, ϵ_t. In particular, we study four types of additive measurement error:

$$\epsilon_t = \begin{cases} \gamma\sqrt{\mathrm{IV}\Delta_n}\epsilon_t^N, & \text{“Gaussian”}, \\ \gamma\sqrt{\mathrm{IV}\Delta_n}\epsilon_t^T\sqrt{\dfrac{\eta-2}{\eta}}, & \text{“T-distributed”}, \\ \gamma\sqrt{\mathrm{IV}\Delta_n}\dfrac{\epsilon_t^N + \phi\epsilon_{t-1}^N}{1+\phi^2}, & \text{“Autocorrelated”}, \\ \gamma\sigma_t\sqrt{\Delta_n}\epsilon_t^N, & \text{“Heteroscedastic”}. \end{cases} \tag{56}$$

[70] There is a small probability that the variance in (54) goes negative in finite samples. We therefore enforce a full truncation floor at zero to avoid this (e.g., Andersen, 2008).

where ϵ_t^N and ϵ_t^T are i.i.d. sequences of Gaussian and t-distributed (with η degrees of freedom) random variables.

While the Gaussian i.i.d. draws are standard, t-distributed noise is less common. The latter, however, can generate notable outliers if η is small. This leads to infrequent—but large—bouncebacks in the noisy log-price series which is a common trait of raw high-frequency data. We set $\eta = 2.5$ to be consistent with this observation.

We also examine cases with autocorrelated and heteroscedastic noise. The former scenario assumes ϵ_t is MA(1) with degree of memory determined by the parameter ϕ which is fixed at $\phi = -0.77$. Throughout, the variance of the noise (on a per increment basis) changes with the level of volatility, which is a well-documented feature in practice (e.g., Bandi and Russell, 2008; Kalnina and Linton, 2008). In scenario $1-3$, the noise scales with the square root of the integrated variance $\text{IV} = \int_0^1 \sigma_s^2 \mathrm{d}s$, while in the last scenario it is a function of σ_t and thus time-varying within each simulation.

The γ parameter is the noise-to-volatility ratio of Oomen (2006) which determines the relative strength of the microstructure component. We assume $\gamma = 5$, which corresponds to heavy noise pertubation (e.g., Aït-Sahalia, Jacod, and Li, 2012; Christensen, Oomen, and Podolskij, 2014).

We follow the standard in the pre-averaging literature by setting $g(x) = \min(x, 1-x)$ and $k_n = [\theta\sqrt{n}]$. We fix the tuning parameter at $\theta = [1,2,3]/3$, which is in line with prior work (e.g., Aït-Sahalia, Jacod, and Li, 2012; Christensen, Kinnebrock, and Podolskij, 2010; Christensen, Oomen, and Podolskij, 2014). The threshold for jump-truncation is implemented with $\bar{\omega} = 0.24$ and $c = 3, 4, 5$ as a robustness check.

We compute Σ_n^* with $L = 10, 15, 20$ and $p = 10, 15, 20$ following the guidelines laid out in Christensen, Podolskij, Thamrongrat, and Veliyev (2017). However, the rejection rates of the jump test statistic are not sensitive to the concrete choice of tuning parameters. We therefore restrict attention to $L = 10$ and $p = 10$. The corresponding tables for other combinations of tuning parameters are available on request.

In Table B.1, we report the properties of the jump test statistic at the 5% significance level. We start by commenting on the results simulated under the null $\mathcal{H}_0$ of no jumps. For this case, the rejection rates are conservative relative to the nominal level of significance, except if c is small. In the latter situation, the threshold is rather narrow and the truncation device starts to eliminate continuous log-returns (drawn from states of high stochastic volatility). This instills a downward bias in the pre-averaged bipower variation, as opposed to the non-truncated pre-averaged realized variance, and leads to a widening of their difference, which measures the quadratic variation of the jump component. As expected, this also has a minuscule impact on the rejection rates with heteroscedastic noise but does not otherwise affect our results.

Next, consider the results conducted under the alternative, $\mathcal{H}_a$. A near-perfect power close to 100% is recorded for the lowest jump activity indexes. As β increases, however, the rejection rate goes down. This reduction is consistent with Aït-Sahalia, Jacod, and Li (2012) and happens because the larger is β, the more the sample path of r_t^d resembles a Brownian motion (with smaller and more erratic increments, cf. Figure B.1). This makes it tough for the test to discriminate between $\mathcal{H}_0$ and $\mathcal{H}_a$ at least over discrete intervals of fixed length Δ_n. Moreover, we see that power is a decreasing function of c. The intuition is that as c increases, $\check{BV}_n^*$ and Σ_n^* are less jump resistant. This lack of robustness tends to reduce power, as also emphasized in the no-noise version of the test from Barndorff-Nielsen and Shephard (2006). To stay conservative, we set $c = 5$ in the empirical analysis.

Finally, we observe that the rejection rates are negatively related to θ. The latter controls the pre-averaging horizon and too much smoothing reduces our ability to detect jumps on the trajectory of p. On the other hand, a larger pre-averaging window makes the estimator robust against more complicated noise structures than what we assume here. In the empirical application, we settle on $\theta = 1/2$ as a compromise.

The right columns of Table B.1 contrast our jump test to the noise-free version from Barndorff-Nielsen and Shephard (2006) implemented with a 5-minute realized variance and bipower variation. The latter are not robust to noise. As a consequence of this, the size and power of the standard jump test are distorted. In particular, the test is oversized under the null (rejects too often), while it lacks power under the alternative (rejects too little). This is true regardless of whether we allow the test statistic access to the latent efficient (no noise) log-price, p, or is computed from the observable noisy log-price, p^*. This finding is consistent with the empirical results in Table 2.

Table B.1: Rejection rate of jump test statistic ($L = 10$ and $p = 10$).

		pre-averaging									5-minute sampling	
		$\theta = 1/3$			$\theta = 1/2$			$\theta = 1/1$				
		$c = 3$	$c = 4$	$c = 5$	$c = 3$	$c = 4$	$c = 5$	$c = 3$	$c = 4$	$c = 5$	p	p^*
Panel A: Gaussian												
No jump		0.098	0.026	0.025	0.083	0.026	0.026	0.083	0.029	0.028	0.093	0.096
$\beta =$	1.75	0.591	0.364	0.282	0.357	0.193	0.135	0.258	0.126	0.086	0.111	0.104
	1.50	0.925	0.777	0.669	0.709	0.504	0.387	0.543	0.351	0.250	0.150	0.134
	1.00	0.999	0.992	0.978	0.971	0.903	0.807	0.893	0.756	0.605	0.252	0.210
	0.50	0.998	0.998	0.997	0.994	0.986	0.963	0.976	0.936	0.860	0.345	0.286
Panel B: T-distributed												
No jump		0.190	0.082	0.066	0.095	0.032	0.029	0.087	0.033	0.031	0.093	0.089
$\beta =$	1.75	0.645	0.415	0.327	0.365	0.199	0.141	0.261	0.128	0.087	0.111	0.111
	1.50	0.940	0.806	0.697	0.719	0.510	0.389	0.548	0.352	0.251	0.150	0.137
	1.00	0.999	0.995	0.982	0.971	0.907	0.812	0.893	0.756	0.607	0.252	0.223
	0.50	0.999	0.998	0.998	0.993	0.986	0.965	0.976	0.937	0.861	0.345	0.297
Panel C: Autocorrelated												
No jump		0.103	0.030	0.028	0.084	0.027	0.026	0.086	0.028	0.026	0.093	0.083
$\beta =$	1.75	0.382	0.183	0.136	0.308	0.153	0.105	0.247	0.118	0.079	0.111	0.095
	1.50	0.715	0.482	0.374	0.627	0.416	0.300	0.508	0.312	0.216	0.150	0.122
	1.00	0.974	0.900	0.799	0.942	0.833	0.702	0.870	0.704	0.545	0.252	0.192
	0.50	0.995	0.985	0.964	0.990	0.967	0.924	0.966	0.911	0.814	0.345	0.257
Panel D: Heteroscedastic												
No jump		0.112	0.021	0.020	0.092	0.023	0.022	0.082	0.028	0.027	0.093	0.006
$\beta =$	1.75	0.203	0.055	0.040	0.228	0.086	0.057	0.212	0.088	0.060	0.111	0.008
	1.50	0.356	0.132	0.087	0.470	0.245	0.156	0.428	0.231	0.152	0.150	0.008
	1.00	0.651	0.357	0.237	0.821	0.601	0.430	0.785	0.577	0.413	0.252	0.009
	0.50	0.816	0.579	0.428	0.946	0.845	0.705	0.927	0.823	0.671	0.345	0.011

Note. In the left-hand side of the table, we report the rejection rates of the feasible noise-robust pre-averaging jump test statistic in (24), i.e. the fraction of times (out of 10,000) $\mathcal{J}_n$ exceeds $\Phi^{-1}(1-\alpha)$, where $\Phi(\cdot)$ is the standard normal distribution function and the nominal level of significance is $\alpha = 0.05$. θ controls the length of the pre-averaging window $k_n = [\theta\sqrt{n}]$. c is the number of local diffusive standard deviations a pre-averaged log-return must exceed to be classified as a jump and truncated to zero. L is the number of subsamples in Σ_n^*, whereas p is the block length (in multiples of k_n). "No jump" is the null hypothesis, while β governs the activity level of the jump process under the alternative (larger values of β imply jumps are smaller and resemble Brownian motion closer). Each separate panel describes the structure of the microstructure noise. In the right-hand side, we report the rejection rates of the Barndorff-Nielsen and Shephard (2006) jump test computed at a 5-minute sampling frequency (or $n = 78$). The latter is not resistant to noise, so as a comparison we implement it both on the inaccessible noise-free log-price (p) and the observable contaminated log-price (p^*). Further details are available in Appendix B.

C Evolution of the after-hours market

In this appendix, we highlight key features of how the after-hours market has evolved over time. Our main finding is that trading volumes have increased significantly over our sample from 2008 to 2020.

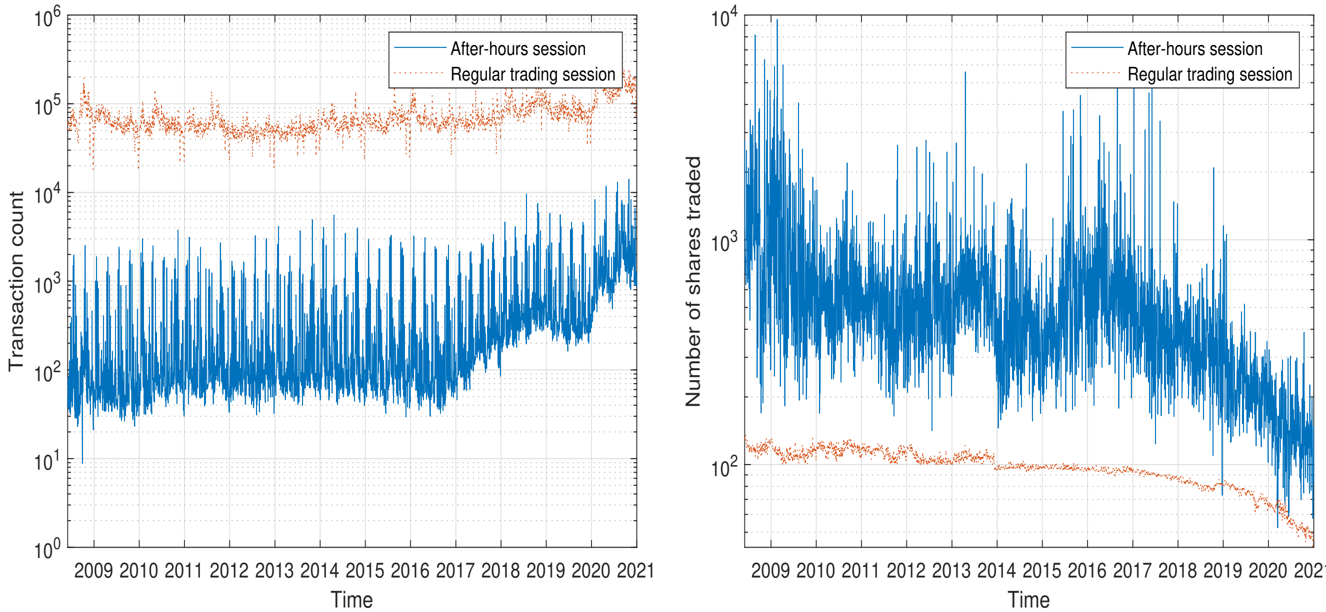
C.1 Trading Volume

The left panel in Figure C.1 shows how the daily transaction counts evolved during the regular trading session and the after-hours market over our sample which runs from 06/02/2008 to 12/31/2020, using a log-scale to improve readability. The right panel shows the median number of shares traded. In both cases, we report a single daily value computed as a cross-sectional average over the 25 companies included in our empirical analysis.

While after-hours transaction volume remains smaller than the regular trading session counterpart, it increases notably over time, particularly after 2016. The opposite shift is evident in the number of shares traded, which has trended down systematically from 2016 onward.

Figure C.1: Transaction count and number of shares traded.

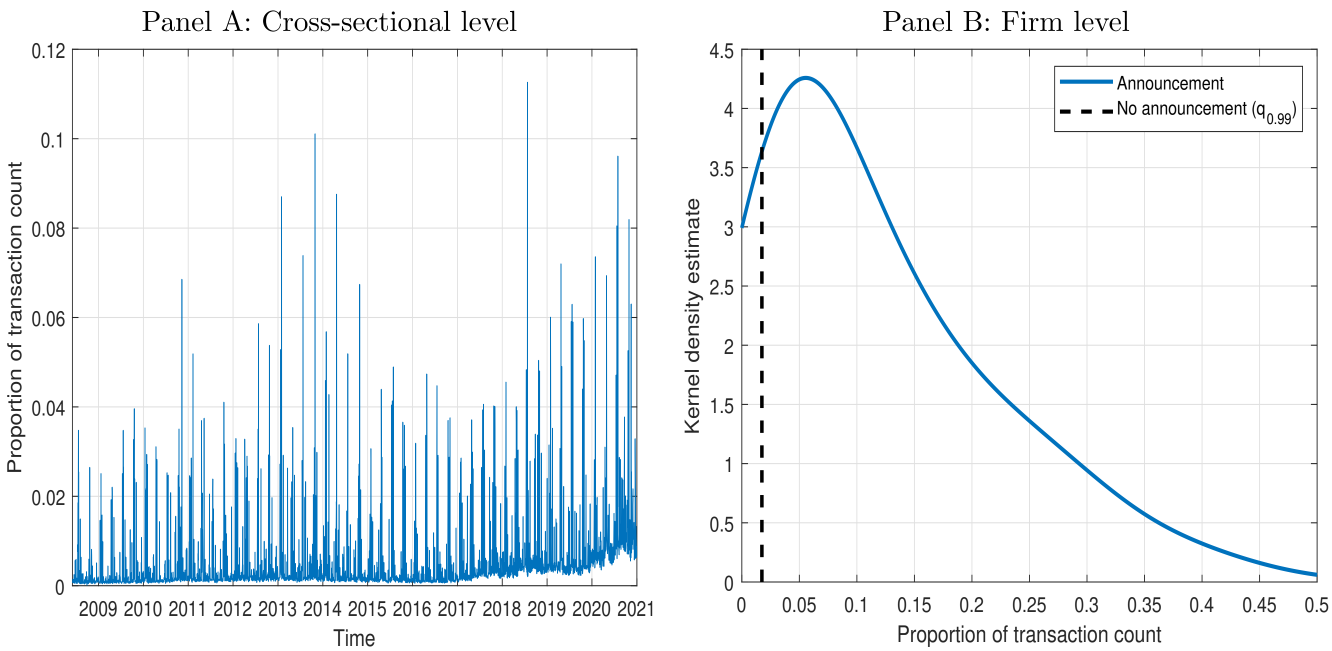
Panel A: Transaction count. Panel B: Number of shares traded.



Note. The figure plots transaction counts in Panel A and the number of shares traded in Panel B during the regular trading session and after-hours session. The statistics are computed daily for each firm in our sample and then averaged over the cross-section.

The left panel in Figure C.2 shows the daily cross-sectional average transaction count in the after-hours market relative to the regular trading session. The relative trading volume increases from 0.2% in 2008Q3–2009Q2 to 0.8% in 2020. There are notable spikes in the series, which frequently exceeds 4% and goes as high as 11%. These surges occur mostly on days where large corporations publish their financial results. To corroborate this statement, the right panel in Figure C.2 plots a kernel density estimate of the relative trading volume at the firm level on announcement days. To highlight the pronounced differences in trading activity, the figure includes the 99th percentile of the relative trading volume on no-announcement days ($q_{0.99}$), which falls far in the left tail of the distribution of relative trading volume on announcement days.

Figure C.2: Cross-sectional and firm level relative transaction count.



Note. In Panel A, we plot the daily transaction count in the after-hours session relative to the transaction count in the regular trading session, both aggregated over the cross-section of firms in our sample. In Panel B, we show the relative transaction count distribution at the firm level on announcement days. The dashed line marks the 99th percentile relative transaction count on no announcement days.

C.2 Bid-Ask Spread

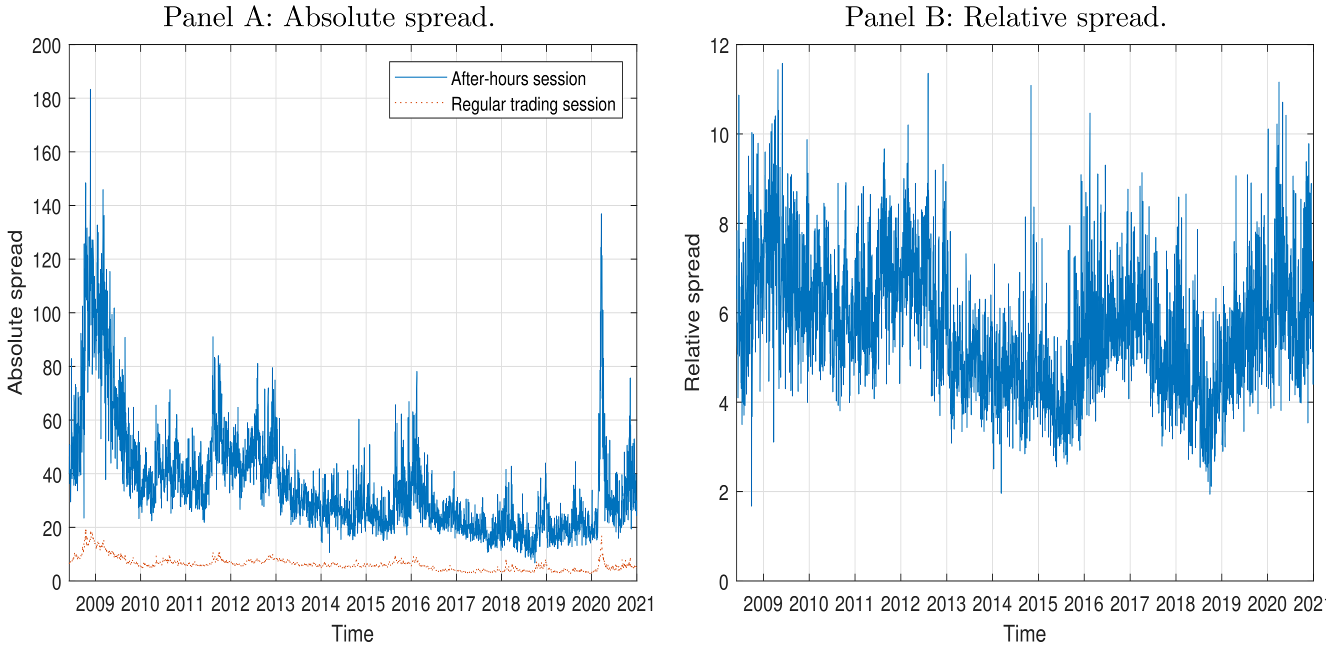
In Panel A of Figure C.3, we plot the daily cross-sectional average median quoted spread in basis points for the regular trading session and after-hours session, whereas Panel B reports the relative spread. The absolute spread (in basis points) is defined as

$$\text{Spread} = 10000 \times \frac{\text{ask} - \text{bid}}{\text{midquote}},$$

where midquote $= (\text{bid}+\text{ask})/2$. The relative spread is $\text{Spread}^{\text{after-hours session}}/\text{Spread}^{\text{regular trading session}}$.

The behavior of the quoted spread is consistent with a positive correlation between price volatility and trading costs. Spreads peak during the financial crisis in 2008–09 before starting a secular downward trend lasting until the outbreak of the Covid-19 pandemic in 2020 which saw spreads increase significantly. Relative to the spreads in the regular trading session, median spreads in after-hours markets evolve stably, mostly falling in a range of four to eight times the regular session spreads.

Figure C.3: Bid-ask spread.



Note. The figure plots the daily cross-sectional average median quoted bid-ask spread (in basis points) in the regular trading session and after-hours session in Panel A, while the relative spread is shown in Panel B.

C.3 Price Discovery

To show how lengthy the price adjustment process is after an earnings announcement, we employ a standard measure of price discovery, namely the weighted price contribution (WPC) of Barclay and Warner (1993). For company i this is defined over small intraday intervals t:

$$WPC_{it} = \sum_{a=1}^{\#EA_i} w_a \times \frac{r_{a,t}}{r_a}, \quad \text{for } t = 1, \ldots, T_A, \tag{57}$$

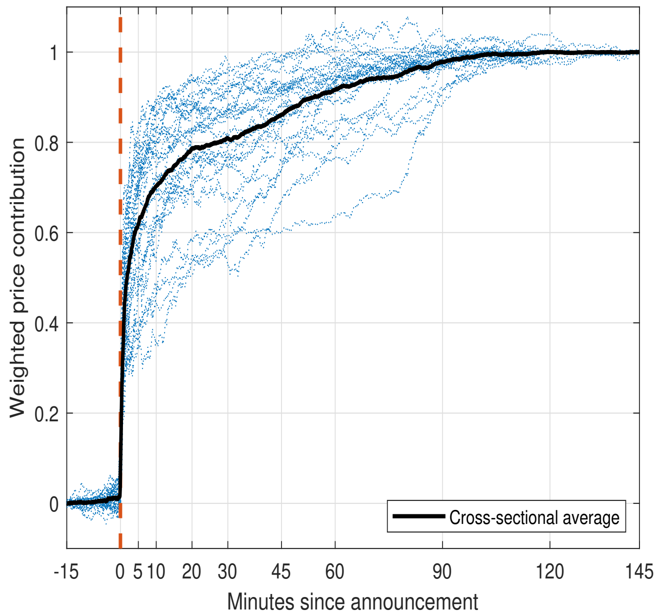
where $r_{a,t}$ is the log-return for announcement a over time interval t, T_A is the number of time intervals, $r_a = \sum_{t=1}^{T_A} r_{a,t}$ is the total announcement log-return, $w_a = |r_a| \times \left(\sum_{a=1}^{\#EA_i} |r_a|\right)^{-1}$ is announcement a's weight, and $\#EA_i$ is the number of earnings announcements for company i, as reported in Table 1. The WPC measure starts at zero and ends at one, so the price discovery process is assumed to be completed at T_A.[71]

In Figure C.4, we plot the cumulative WPC measure for each of the 25 stocks in our analysis along with their cross-sectional average. The return covers a period from 15 minutes before to 145 minutes after each announcement with the event window broken into 5-second subintervals.[72]

[71] We also calculated Hasbrouck (1995)'s information share (IS). Consistent with our results based on the WPC, the IS measure suggests that anywhere between 60%–80% of the price discovery is accounted for in the first five minutes after the announcement.

[72] We also constructed the WPC measure over a longer window that begins 15 minutes prior to each announcement and extends through the overnight period and into next day's pre-market trading up to the opening of the stock exchange at 9:30am. The extended WPC curve is essentially flat after the end of the announcement day, suggesting

Figure C.4: Weighted price contribution.



Note. We plot the weighted price contribution (WPC) of Barclay and Warner (1993), which for company i is defined over small intraday intervals t: $WPC_{it} = \sum_{a=1}^{\#EA_i} w_a \times \frac{r_{a,t}}{r_a}$, for $t = 1, \ldots, T_A$, where $r_{a,t}$ is the log-return for announcement a over time interval t, T_A is the total number of time intervals, $r_a = \sum_{t=1}^{T_A} r_{a,t}$ is the cumulative announcement log-return, $w_a = |r_a| \times \left(\sum_{a=1}^{\#EA_i} |r_a|\right)^{-1}$ is the weight for announcement a, and $\#EA_i$ is the number of earnings announcements for company i, as reported in Table 1. The returns cover a time interval from 15 minutes before to 145 minutes after each announcement with the event window broken into 5-second subintervals. The WPC is shown for each company as a blue dotted line, while the full black line is the cross-sectional average.

Implicitly, the price at $t = -15$ is viewed as a proxy for the pre-announcement fundamental value, while the price at $t = 145$ is representative of the post-announcement fundamental value. Since the typical company announces earnings at 4:05pm, the pre-announcement fundamental value is typically determined at 3:50pm during the very active regular trading session, while the post-announcement fundamental value is measured around 6:30pm. The WPC curves in Figure C.4 suggest that price discovery is very fast with close to 70% of the post-announcement equilibrium price being discovered within five minutes of an announcement.

Next, to examine how the price discovery process has shifted over time, Table C.1 reports the daily weighted price contribution (WPC) divided into four time intervals that are motivated by our empirical application and closely follow those used in Barclay and Hendershott (2003) and Jiang, Likitapiwat, and McInish (2012), namely pre-market (6:00am–9:30am), regular trading (9:30am–4:00pm), after-hours (4:00pm–6:30pm), and the overnight period (6:30pm–6:00am). In Panel A, the WPC measure calculated over the full sample and across all days indicates that 7.51% of the price discovery process occurs in the pre-market, 72.33% in regular trading, 5.42% in the after-hours market, and 14.74% overnight. On earnings announcement days, however, these fractions shift markedly as the 4:00pm–6:30pm segment accounts for 83.09% of the WPC, with only 9.14%, 5.27%, and 2.51% stemming from the remaining segments.

that very little additional price discovery takes place here.

In Panels B and C of Table C.1, we split these results into subsamples, covering 2008–2015 and 2016–2020. Over time, the after-hours segment has gained in importance. On earnings announcement days, it accounts for 88.94% of the WPC in the 2016–2020 sample. This increase has come at the expense of the price discovery during the regular trading session which on days with an earnings announcement has declined from 7.08% in the first sub-sample to 2.38% in the second sub-sample.

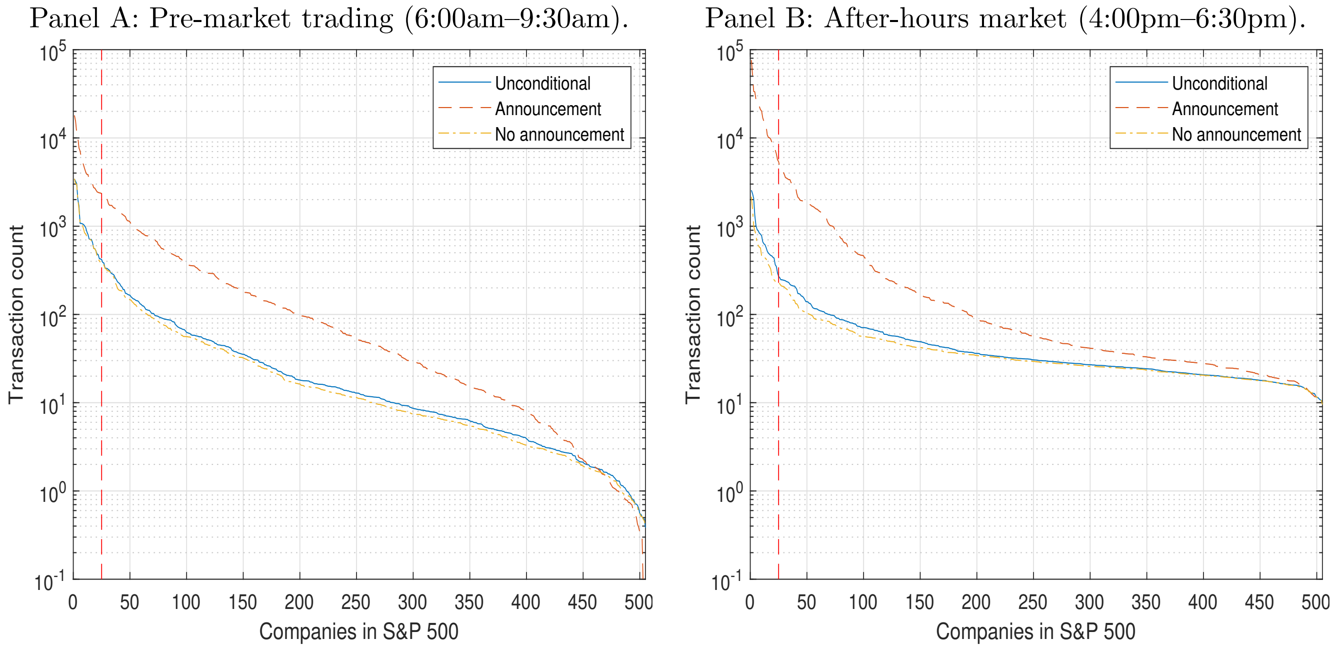
Table C.1: Weighted price contribution.

	Pre-market 9:30am–4:00pm	Regular trading 4:00pm–6:30pm	After-hours 6:30pm–6:00am	Overnight 6:00am–9:30am
Panel A: Full sample				
All	0.7233	0.0542	0.1474	0.0751
No announcement	0.7579	0.0141	0.1537	0.0742
Announcement	0.0527***	0.8309***	0.0251***	0.0914
Panel B: 2008 – 2015				
All	0.7504	0.0451	0.1304	0.0741
No announcement	0.7854	0.0065	0.1354	0.0727
Announcement	0.0708***	0.7942***	0.0332***	0.1018*
Panel C: 2016 – 2020				
All	0.6804	0.0691	0.1738	0.0767
No announcement	0.7146	0.0264	0.1822	0.0768
Announcement	0.0238***	0.8894***	0.0120***	0.0748

Note. We calculate the weighted price contribution (WPC) of of Barclay and Warner (1993) over the four time intervals pre-market (9:30am–4:00pm), regular trading (4:00pm–6:30pm), after-hours (6:30pm–6:00am), and overnight (6:00am–9:30am). The WPC measure is described in Section C.3. The table shows the unconditional WPC (All), and the conditional WPC for no announcement and announcement days. Panel A reports the full sample estimates, whereas Panels B–C report estimates separately for the subsamples 2008–2015 and 2016–2020. *, **, and *** denote that the no announcement WPC is statistically different from the announcement WPC at the 10%, 5%, and 1% level of significance, as inferred from a nonparametric permutation test with 10,000 shuffles.

D Trading activity in the pre- and after-hours market.

Figure D.1: S&P 500 extended trading session transaction count.



Note. We download NYSE Trade and Quote (TAQ) high-frequency data for the constituent members of the S&P 500 index as of 12/31/2020. We sort the companies by their average transaction counts for the sample period 06/02/2008–12/31/2020. Panel A shows the associated distribution for the pre-market session (6:00am–9:30am), whereas Panel B is for the after-hours session (4:00pm–6:30pm). Both are reported on a log-scale. We further split the unconditional transaction count distribution into days with and without earnings announcements. The vertical red dashed line indicates the transaction count for the twenty-fifth most liquid stock.

Table D.1: S&P 500 companies by pre-market (6:00am–9:30am) trading activity.

	conditional on no announcement							conditional on announcement							announcement information			
				quantile							quantile							
ticker	mean	fraction	std.	0.25	0.50	0.75	0.99	mean	fraction	std.	0.25	0.50	0.75	0.99	n_{EA}	time	r^{EA}_{30m}	z_{EPS}
GE	840	0.49	2,502	150	334	705	8,395	11,516	3.66	9,239	4,195	7,776	17,713	37,538	49	6:30am	0.752 (1.882)	1.209 (1.648)
BAC	2,990	1.25	6,085	656	1,412	3,157	25,089	18,007	4.86	14,185	6,815	11,798	23,330	52,800	50	7:00am	−0.530 (2.258)	0.934 (1.772)
WMT	147	0.18	544	12	32	87	1,908	4,954	2.99	7,725	1,237	2,377	4,912	46,543	50	7:00am	0.327 (2.220)	1.226 (1.648)
TGT	65	0.11	363	2	9	35	792	3,843	2.59	4,889	809	2,406	4,853	23,711	50	6:30am	−0.119 (1.836)	1.494 (2.354)
BBY	56	0.10	649	3	9	25	560	5,447	4.14	3,525	3,101	5,409	7,280	20,275	34	7:00am	−0.872 (3.572)	3.157 (2.473)
CAT	113	0.26	179	27	61	133	751	3,862	3.39	2,981	1,921	3,228	4,909	16,765	50	7:30am	−0.335 (2.381)	1.643 (2.768)
KR	33	0.05	359	0	3	14	348	2,050	1.41	2,593	248	796	3,392	9,053	51	8:45am	−0.742 (4.313)	1.213 (1.869)
GM	216	0.26	775	22	53	150	2,645	2,994	2.22	2,760	1,190	2,321	3,745	13,994	40	7:30am	0.155 (1.837)	1.541 (1.850)
JPM	387	0.26	920	82	170	373	3,074	5,993	2.75	4,624	3,069	4,374	7,394	22,071	50	6:59am	−0.092 (0.963)	1.596 (1.608)
HD	75	0.14	163	10	26	63	759	2,377	2.34	2,016	1,043	1,657	3,029	8,390	50	6:00am	0.046 (0.617)	2.012 (1.872)
BA	766	0.46	3,341	12	32	156	14,894	3,179	2.27	5,957	452	908	2,825	35,952	49	7:30am	0.204 (1.387)	1.271 (1.931)
BIIB	58	0.18	527	2	9	27	748	1,593	2.06	4,495	66	414	1,399	30,415	50	7:15am	0.551 (3.417)	1.769 (1.993)
C	1,566	0.74	4,860	132	367	1,218	18,126	10,189	3.31	14,867	2,714	5,757	11,118	87,389	50	7:59am	0.096 (1.605)	1.103 (1.858)
LOW	37	0.07	138	2	7	22	462	2,303	1.81	2,167	767	1,746	2,886	11,477	50	6:00am	−0.074 (0.955)	0.675 (2.109)
MCD	81	0.19	199	12	28	72	829	2,019	2.60	1,417	1,005	1,588	2,734	6,915	50	7:58am	0.184 (1.942)	1.190 (2.213)
CVS	64	0.09	374	1	6	30	637	2,007	1.18	4,258	127	390	1,917	21,491	49	7:00am	−0.017 (0.900)	1.856 (2.139)
VZ	150	0.15	741	20	45	105	1,477	1,649	1.34	1,363	777	1,256	2,244	6,216	49	7:30am	−0.132 (0.680)	0.598 (0.965)
WFC	363	0.18	1,907	36	86	221	3,786	4,484	1.74	4,485	1,738	3,165	4,726	21,858	50	8:00am	−0.645 (2.463)	0.316 (1.737)
PG	64	0.11	157	11	29	66	524	1,472	1.43	1,550	336	1,122	1,782	8,064	48	7:00am	0.110 (1.410)	1.642 (1.842)
F	756	0.62	1,566	97	263	745	6,792	7,702	3.16	10,035	747	2,526	10,591	45,122	49	7:00am	−0.035 (4.179)	1.098 (1.971)
GS	187	0.34	630	28	72	177	1,639	4,095	3.98	4,096	1,477	2,646	5,155	15,568	45	7:35am	−0.447 (1.443)	2.107 (2.145)
MS	185	0.15	1,034	15	45	132	1,448	3,340	2.11	2,721	1,446	2,449	4,361	13,841	47	7:15am	0.236 (2.410)	1.753 (2.258)
CCL	686	0.69	3,379	21	68	209	11,512	2,057	2.48	2,096	1,024	1,488	2,386	13,470	50	9:15am	0.162 (3.970)	2.626 (1.993)
DLTR	16	0.06	185	1	3	10	177	1,150	1.51	1,574	191	441	1,737	6,894	50	7:30am	0.143 (1.632)	1.364 (2.314)

Note. In the left-hand side of the table, we show descriptive statistics of the twenty-five most liquid companies from the S&P 500 index based on their pre-market (6:00am–9:30am) trading volume, which is separated into no announcement days (columns 2–8) and annnouncement days (columns 9–15). Ticker is the stock listing symbol. Mean and Std. are the transaction count sample average and standard deviation. Quantile are selected quantiles from the empirical transaction count distribution. Fraction is the number of transactions executed in the pre-market market divided by the total transaction count for the extended trading session from 6:00am–4:00pm. In the right-hand side of the table, we report announcement information. #EA is the number of earnings annoucements. Time is the modal announcement time (Eastern Standard Time). z_{EPS} and r^{EA}_{1m} are the sample average standardized unexpected earnings and one-minute post-announcement return (standard deviation across announcements reported below in parentheis).

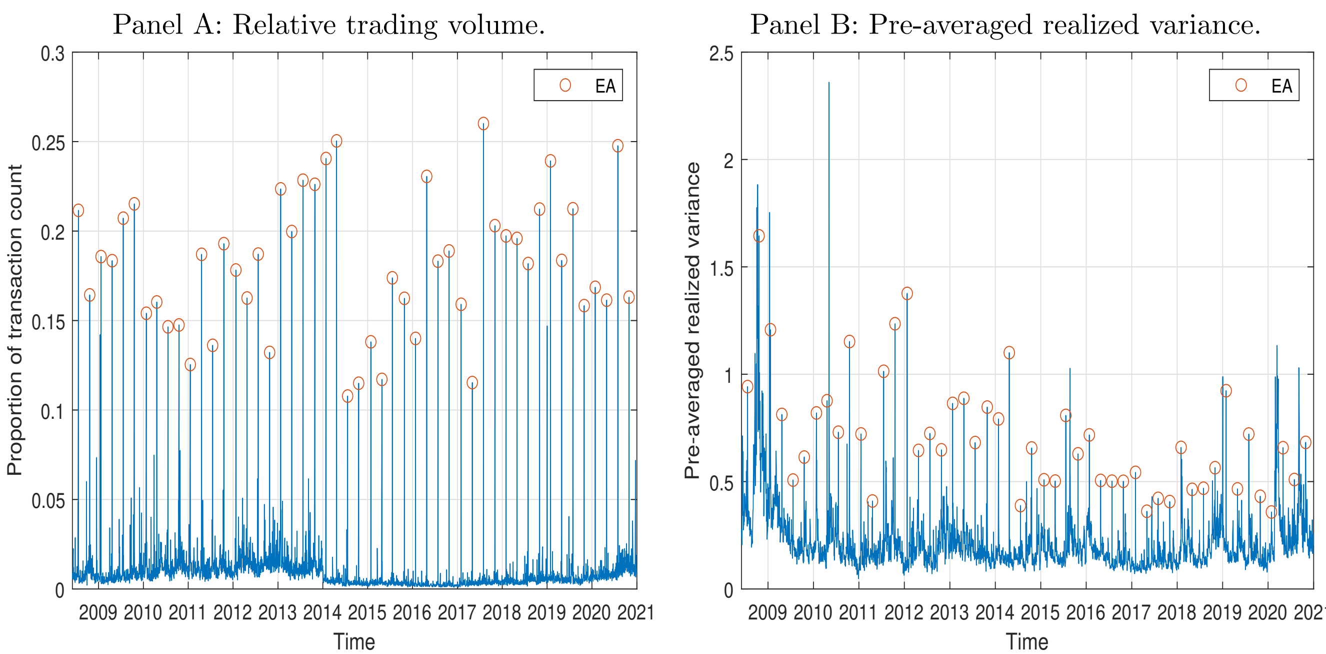
E Case study of Apple

This appendix illustrates our broader results using Apple as a case study. Apple is among the most important stocks throughout our entire sample and is followed by numerous analysts.

In Panel A of Figure E.1, we plot the proportion of the total volume (measured in transaction counts) traded during the after-hours session (4:00pm–6:30pm) relative to the extended trading session (9:30am–6:30pm). Liquidity in the after-hours market is typically small with an unconditional sample average value of 1.1% of the total volume. However, there are clear spikes on earnings announcements days where the after-hours market is highly active and the relative trading volume exceeds 18% on average on such days.

In Panel B of Figure E.1, we show the pre-averaged realized variance. The time series of the daily volatility estimates is in agreement with Panel A. It displays regularly occurring spikes that, consistent with the peaky trading volume, almost always occur on days with earnings announcements.

Figure E.1: Relative trading volume and pre-averaged realized variance of Apple.



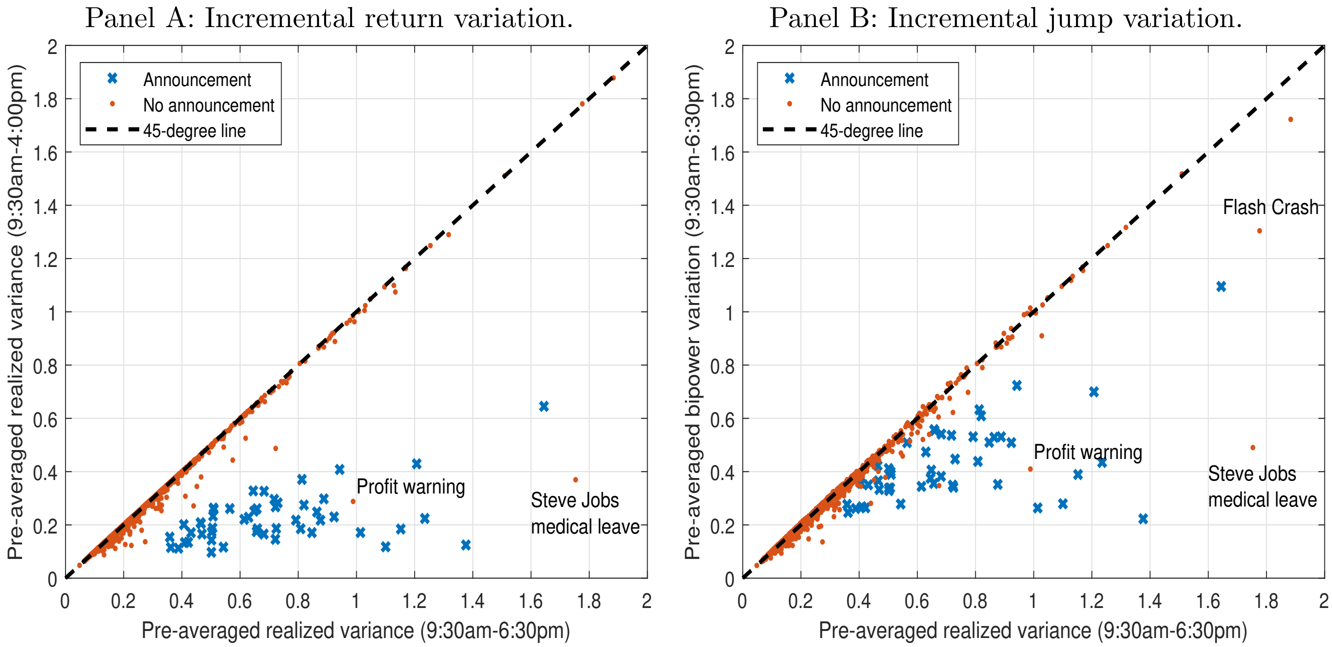
Note. In Panel A, we show the daily after-hours market (4:00pm–6:30pm) transaction count scaled by the total transaction count in the extended trading session (9:30am–6:30pm). In Panel B, we plot the daily pre-averaged realized variance, converted to an annualized standard deviation. In both panels, a red circle indicates a day on which Apple made an earnings announcement.

The left panel in Figure E.2 plots the pre-averaged realized variance computed over the regular trading session (9:30am–4:00pm) against its corresponding value computed over the extended trading session (9:30am–6:30pm). The figure also marks days on which Apple announced earnings. On days with no announcement, the estimates fall close to the 45-degree line. In contrast, on days with an announcement, the estimates differ by a lot. The only notable exception in our sample is 01/17/2009, where Apple sent out an e-mail informing about Steve Jobs' medical leave, and

01/02/2019, where the company issued a profit warning. Both these messages were released during the after-hours session.

To examine whether the incremental variance in after-hours markets took the form of jumps, Panel B in Figure E.2 plots the pre-averaged bipower variation against the pre-averaged realized variance, both computed over the extended trading session. Because bipower variation does not increase in the presence of jumps whereas the realized variance does, points on the 45-degree line are indicative of days without jumps, whereas days with such jumps should appear to the right of the line. Almost all days with price jumps in the extended trading session coincide with earnings announcement days. The primary exceptions are the day of the Steve Jobs medical leave e-mail, the profit warning, and the S&P 500 Flash Crash on 05/06/2010.

Figure E.2: Pre-averaged realized variance and bipower variation of Apple.



Note. We show point estimates of the pre-averaged realized variance and pre-averaged bipower variation of Apple, converted to an annualized standard deviation. The axis label shows which estimator is plotted. In parenthesis, we further indicate for which part of the day high-frequency data are employed to calculate the estimate.