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Partial Secrecy in Vertical Contracting*

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This version: March 21, 2023

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This paper introduces a notion of partial secrecy in bilateral contracting games between one upstream firm and several competing downstream firms. The supplier's offer quantities are subject to trembles, and each downstream firm observes a noisy signal about the offer received by its competitor before deciding whether to accept its offer. A downstream firm's belief about its competitor's quantity is determined endogenously as a weighted average of the competitor's expected equilibrium quantity and the signal about the actual quantity that the competitor was offered. The degree of contract secrecy is captured by the weight that this belief puts on the competitor's expected equilibrium quantity. We find that a higher degree of secrecy implies a more competitive equilibrium outcome, both in a game with simultaneous offers and in a dynamic game with alternating offers similar to the one in Do and Miklós-Thal (2022, "Opportunism in Vertical Contracting: A Dynamic Perspective," CEPR Discussion Paper No. DP16951).

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1 Introduction

Models of bilateral contracting between an upstream monopolist and multiple competing downstream firms are widely used in the industrial organization literature, and their analyses have generated important insights about the effects of vertical restraints and vertical mergers. A well-known result of this literature is that the equilibrium outcomes of games in which the upstream supplier makes simultaneous contract offers to the downstream firms depend on whether offers are public (i.e., observed by all downstream firms) or secret (i.e., observed only by the firm receiving an offer). When offers are public, the supplier exerts its full market power in equilibrium. When offers are secret, however, the supplier faces a temptation to behave opportunistically by offering contracts that raise the bilateral surplus with one downstream firm at the expense of other downstream firms. Equilibrium outcomes then depend on a downstream firm's beliefs about other downstream firms' offers after obtaining an out-of-equilibrium offer; under the commonly used passive-beliefs assumption, the supplier is unable to fully exert its market power. Vertical mergers and vertical restraints like exclusive dealing can re-establish the supplier's market power in that case, to the detriment of consumers and social welfare.[1]

The extant literature treats contract secrecy as binary. Contract offers are either public or secret. In the former case, all downstream firms observe all offers without any noise. In the latter case, a downstream firm receives no information at all about the actual offers received by its competitors. In practice, however, neither of these two extremes, public or secret contract offers, is likely to be a good description of the informational environment faced by firms. Public offers lack realism because negotiations of bilateral contracts typically involve private communication between the contracting parties and suppliers, even if they wanted to, cannot credibly commit to making all offers publicly observable. Secret offers, on the other hand, ignore that information leakages across downstream firms can occur even if contracts offers are made privately. Moreover, in dynamic settings, accepted supply contracts are unlikely to remain fully secret as time passes, because downstream firms can draw inferences about their competitors' current supply terms from observed market

[1] On the importance of contract observability and beliefs following out-of-equilibrium offers, see O'Brien and Shaffer (1992), McAfee and Schwartz (1994), Rey and Vergé (2004), and the survey article by Rey and Tirole (2007). On the various ways of "solving" the supplier's opportunism problem in vertical contracting, see also Hart and Tirole (1990), Marx and Shaffer (2004), Montez (2015), and Gaudin (2019).

behavior and market outcomes.

This paper takes a first step towards modeling *partial secrecy* in vertical contracting games and analyzing its implications for equilibrium outcomes.[2] Our analysis builds on two new modeling elements. First, the supplier's offer quantities are subject to mistakes or trembles. Such mistakes could arise due to coordination failures or communication frictions within the supplier's organization, for instance, or due to human error. Second, each downstream firm obtains a noisy signal about its competitor's quantity before deciding whether to accept its offer. Given these two elements, a downstream firm's posterior mean belief about its competitor's quantity, after observing the signal, becomes a weighted average of the competitor's expected equilibrium quantity and the signal about the actual quantity that the competitor was offered. The degree of contract secrecy is captured by the weight that this belief puts on the competitor's expected equilibrium quantity, which depends on the ratio of the variance of the signal over the variance of the trembles in the offer quantities. As the signal about the actual quantity becomes more precise (the signal variance falls), the degree of secrecy falls and the belief moves closer to the true quantity that the competitor was offered. As the signal becomes less precise, the degree of secrecy rises and the belief moves closer to the competing firm's expected equilibrium quantity.

Since all quantities are offered with positive—albeit possibly very small—probability in equilibrium, our analysis and results do not rely on any assumptions about beliefs following out-of-equilibrium offer quantities.[3] Passive beliefs, which are widely used in the literature but have been lacking a theoretical foundation, arise endogenously as the limiting case in which the ratio of the variance of the trembles over the variance of the signals goes to zero.

We analyze the implications of partial secrecy in two vertical contracting games: first, a game with simultaneous offers, extending the standard modeling approach in the literature;

[2]The only notion of partial secrecy in the literature that we are aware of is that of interim (or ex post) observability, whereby a downstream firm observes its competitors' contracts after accepting its contract but before competing downstream (McAfee and Schwartz, 1994; Rey and Vergé, 2004). Our analysis deals with partial secrecy before contract acceptance decisions are made, introducing a continuous measure of contract secrecy. Since contracts fix the downstream firms' strategic decisions (quantities) in our model, the question of interim observability does not arise.

[3]Other attemps to endogenize beliefs in vertical contracting games with secret offers can be found in White (2007), who considers a game in which the supplier has private information about its marginal cost, and In and Wright (2018), who propose the refinement that beliefs should be invariant to the order in which offers are made. In contrast, the beliefs following any observed offer quantity and signal are obtained through Bayesian updating in our game.

and second, a dynamic game with alternating offers, following our modeling approach in Do and Miklós-Thal (2022). In the dynamic game, the supplier alternates between making offers to two competing downstream firms over an infinite horizon, and the analysis focuses on Markov perfect equilibria. While Do and Miklós-Thal (2022) assume that each downstream firm is aware of its competitor's current contracted quantity when deciding whether to accept an offer, either because contract offers are public or because the firm can perfectly infer its competitor's current quantity from observed market outcomes in the time that elapses between offers, in the present paper we assume instead that contract offers are secret and that each downstream firm's inferences about its competitor's current quantity are imperfect.

In both the simultaneous-offers game and the alternating-offers games, we find that a greater degree of secrecy leads to a more competitive outcome with a higher expected total quantity sold in equilibrium. In other words, a greater degree of secrecy implies a greater degree of equilibrium opportunism. Intuitively, this is because a change in the quantity offered to one downstream firm has a lesser effect on the competing downstream firm's posterior belief when the degree of secrecy is higher, which implies that the supplier's equilibrium offers internalize less of the competitive externalities across downstream firms.

The equilibrium degree of opportunism varies continuously with the degree of secrecy in the two games analyzed in this paper. In the simultaneous-offers game, the expected equilibrium outcomes span the entire range between the integrated monopoly outcome, corresponding to the equilibrium under simultaneous public offers, and the competitive (pairwise-proof) outcome, corresponding to the equilibrium under secret offers and passive beliefs. In the alternating-offers game, the expected steady-state equilibrium outcome is more competitive than the integrated monopoly outcome even in the limit case corresponding to public offers, and it approaches the competitive (pairwise-proof) outcome for any discount factor as the degree of contract secrecy rises. Moreover, the combination of secrecy and dynamic recontracting implies stronger opportunism than secrecy alone. For any given degree of secrecy, the equilibrium outcome is more competitive at the steady state of the dynamic model with recontracting than in the static model with simultaneous offers. The findings thus complement the results in Do and Miklós-Thal (2022), where we analyze how the equilibrium degree of opportunism varies with the firms' discount factor and the speeds at which the supplier's contract with one retailer reacts to changes in the other retailer's contract.

Understanding the equilibrium degree of opportunism is useful for vertical merger policy and for public policy on vertical restraints. This is because when opportunism is greater, then the competitive damage arising from strategies like vertical mergers or vertical restraints that lead to the monopoly outcome will be worse, and the supplier's incentives to use such strategies will be stronger. Our findings suggest that a greater degree of secrecy raises both the supplier's incentive to employ strategies aimed at dampening competition and the harm to competition from allowing the supplier to do so. This is true both with and without the possibility of future recontracting.

The rest of this paper will be organized as follows. Section 2 introduces partial secrecy into an otherwise standard model of simultaneous contract offers. Section 3 analyzes the implications of partial secrecy in an infinite-horizone dynamic model with alternating offers. Section 4 concludes. All formal proofs are relegated to Appendix A.

2 Simultaneous offers

Industry setting We consider vertical contracting between one upstream supplier, U, and two competing downstream firms D_i ($i = A, B$, also called retailers). The downstream firms purchase an input from the supplier, transform it into a final good using a one-to-one technology, and sell the final good to consumers. Upstream marginal costs are constant and equal to $c \in [0, 1)$, downstream marginal costs are constant and normalized to zero.

Consumers have a linear inverse demand function $P(q_A + q_B) = 1 - (q_A + q_B)$ for the product, where q_i denotes the quantity put on the market by downstream firm D_i.[4] The variable profit of downstream firm D_i (gross of any payments to the supplier) is given by

$$\pi(q_i, q_{-i}) = q_i P(q_A + q_B).$$

We use subscripts to denote derivatives, e.g., $\pi_1(q_i, q_{-i}) = \frac{\partial \pi(q_i, q_{-i})}{\partial q_i}$ and $\pi_2(q_i, q_{-i}) =$

[4] As in Do and Miklós-Thal (2022), using $P(q) = 1 - q$ rather than $P(q) = a - bq$ is a simple normalization and without loss of generality. If $P(q) = a - bq$, the term $1 - c$ in our results would simply be replaced by $\frac{a-c}{b}$.

$\frac{\partial \pi(q_i, q_{-i})}{\partial q_{-i}}$. The sum of all three firms' profits is given by the industry profit

$$\Pi\left(q_A + q_B\right) = \left(q_A + q_B\right)\left(P\left(q_A + q_B\right) - c\right).$$

The quantity that maximizes industry profits is denoted by $Q^M = \arg\max_Q \Pi(Q)$, and $q^M = \frac{Q^M}{2}$ denotes the quantity per retailer when they split the total monopoly quantity Q^M equally. Moreover,

$$q^C = \arg\max_q \left(q\left(P\left(q + q^C\right) - c\right)\right) = \arg\max_q \left(\Pi\left(q + q^C\right) - \pi\left(q^C, q\right)\right)$$

denotes the per-firm Cournot quantity given the marginal production cost c. Given the linear demand specification, $q^M = \frac{1-c}{4}$ and $q^C = \frac{1-c}{3}$.

A supply contract consists of a quantity and a fixed fee. If D_i accepts a contract (q_i, f_i), then it pays the fixed fee f_i to U, transforms all q_i input units into its final output, and puts q_i units on the market.[5]

Benchmark: Public offers The first benchmark game is the one in which U makes simultaneous public offers to the retailers, then the retailers simultaneously and independently decide whether to accept their offers. In this game, the supplier fully exerts its monopoly power and obtains the entire monopoly profit in (a subgame perfect) equilibrium. For instance, U can achieve this by offering the contract $\left(q^M, \pi\left(q^M, q^M\right)\right)$ to each retailer. Both retailers will accept, because the fixed fee does not exceed an individual retailer's variable profit given that both retailers were offered q^M, and together they will sell the monopoly quantity Q^M. The intuition for why the monopoly outcome arises in equilibrium is that the supplier internalizes the effects on *all* retailers' profits when making offers: Any change in the quantity offered to D_i affects the fixed fee that the supplier can obtain from retailer D_{-i} by an amount equal to the effect of the change on D_{-i}'s variable profit.

Benchmark: Secret offers and passive beliefs The second benchmark game is the one in which U makes simultaneous secret offers to the retailers, then the retailers simultaneously and independently decide whether to accept their offers. When D_i cannot observe

[5] For simplicity, supply contracts are assumed to be quantity-fixing, that is, they fix how much quantity the downstream firm transforms into the final output and puts on the market. See Do and Miklós-Thal (2022, fn. 8) for a discussion of this assumption.

the contract offered to D_{-i}, the (perfect Bayesian) equilibrium of the game is sensitive to D_i's beliefs about the contract offered to D_{-i} when D_i receives an out-of-equilibrium offer. A reasonable and widely-used assumption in Cournot settings like the one we consider is that retailers hold *passive beliefs*, whereby a retailer that receives an out-of-equilibrium offer continues to believe that its rival was offered the equilibrium contract.[6]

Let (q_A^*, q_B^*) denote the equilibrium quantities. With passive beliefs, retailer D_i is willing to accept an offer (q, f) if and only if $f \leq \pi\left(q, q_{-i}^*\right)$. The equilibrium offer to D_i must therefore maximize the bilateral surplus of the pair $U - D_i$ given q_{-i}^*, which in our setting means that D_i's equilibrium quantity must be the Cournot best response to q_{-i}^*:

$$q_i^* = \arg\max_q \left(\pi\left(q, q_{-i}^*\right) - cq\right) = \arg\max_q \left(\Pi\left(q + q_{-i}^*\right) - \pi\left(q_{-i}^*, q\right)\right).$$

In the unique equilibrium given passive beliefs, the retailers thus sell the Cournot quantities $\left(q^C, q^C\right)$ and the supplier earns $\Pi\left(2q^C\right) < \Pi\left(2q^M\right)$. Intuitively, the supplier is unable to fully exert its market power because when making an offer to D_i, it does *not* internalize the negative effect that a higher q_i has on D_{-i}'s variable profit.[7]

Simultaneous-offers game with partial secrecy The simultaneous-offers game with partial secrecy proceeds as follows:

1. U chooses target quantities $\{\widehat{q}_A, \widehat{q}_B\}$, which are unobserved by the downstream firms.

2. An offer quantity q_i is drawn from the distribution $N\left(\widehat{q}_i, \sigma_q^2\right)$ for each $i = A, B$ and observed privately by U and D_i.

3. For each $i = A, B$, a signal $s_i = q_i + \varepsilon_i$ is realized and observed by all firms, where ε_i is distributed according to $N(0, \sigma_s^2)$.[8]

[6] The passive beliefs refinement is appealing in Cournot-like settings because U has no incentive to change the offer to D_{-i} when it changes the offer to D_i. See Hart and Tirole (1990), Rey and Vergé (2004), or Rey and Tirole (2007) for more detailed discussions.

[7] Some papers in the vertical contracting literature (e.g., O'Brien and Shaffer (1992)) use the "contract equilibrium" concept pioneered by Cremer and Riordan (1987), which requires contracts to be pairwise stable (i.e., each contract must maximize bilateral surplus given the contracts of other retailers) but does not rule out multi-lateral deviations. In the model with Cournot competition and quantity-fixing contracts considered here, the quantities in a passive-beliefs perfect Bayesian equilibrium coincide with the quantities in such a contract equilibrium.

[8] The results remain unchanged if the signal s_i about D_i's offer quantity q_i is privately observed by U and D_{-i}.

4. U offers fixed fees $\{f_A, f_B\}$ to D_A and D_B. Each downstream firm D_i observes its own fixed fee offer, but not the offer made to the rival downstream firm.

5. The downstream firms simultaneously decide whether to accept ($g_i = 1$) or reject ($g_i = 0$) their offers (q_i, f_i). Downstream firms with accepted contracts sell to final consumers and payoffs are realized.

This game makes two main departures from the benchmark simultaneous-offers games described earlier. First, while U's target quantity choices in stage 1 determine the expected quantities in the contracts offered to the two retailers, the actual quantities offered are subject to stochastic mistakes. These mistakes can be thought of as resulting from communication or coordination frictions within the supplier's organization, or as human errors. Second, each retailer receives a signal about its competitor's quantity before deciding whether to accept its contract. In the static model, these signals can be thought of as information leakages between firms.

The assumption that fixed fees are offered after the quantity signals are observed (and that the fixed fees are not subject to mistakes) ensures that the supplier can extract each retailer's full expected variable profit in equilibrium.[9] Since the benchmark cases of simultaneous public or secret offers feature full surplus extraction in equilibrium, having a model in which the supplier can extract the full downstream variable profits in expectation allows us to isolate the effect of the degree of secrecy on equilibrium outcomes in the absence of additional departures from the standard models.

Equilibrium concept The solution concept is perfect Bayesian equilibrium. To be more precise, let us denote a history at the beginning of stage τ by h^τ and a private history of D_i by h_i^τ. For instance, $h^4 = (\widehat{q}_A, \widehat{q}_B, q_A, q_B, s_A, s_B)$, and $h_i^5 = (q_i, s_A, s_B, f_i)$. Then, a strategy of U consists of target quantities $\widehat{q}_i \in \mathbb{R}$ and fixed fees $F_i : h^4 \mapsto f_i \in \mathbb{R}$; and a strategy of D_i is an acceptance decision, $G_i : h_i^5 \mapsto g_i \in \{1, 0\}$. Denoting by $\beta_i(h_i^\tau)$ the belief of D_i over histories at h_i^τ, a perfect Bayesian equilibrium consists of a strategy

[9] An alternative formulation that would also lead to full surplus extraction would be that in stage 1, U sets fixed fees $(f_A(q_A, s_B), f_B(q_B, s_A))$ that condition on the signal realizations and the realized offer quantities; in stages 2 and 3, the offer quantities and signals are realized; and in stage 4, the downstream firms simultaneously decide whether to accept or reject U's offers.

profile, $((\widehat{q}_i, F_i), G_i)_{i=A,B}$, and a belief system, $(\beta_i)_{i=A,B}$, that satisfy sequential rationality and consistency.

In a perfect Bayesian equilibrium, the supplier will not condition its fixed-fee offer to D_i on $(\widehat{q}_A, \widehat{q}_B, q_{-i}, s_i, f_{-i})$, because there are no upstream externalities and the target quantities are payoff-irrelevant. Thus, there exists a unique on-path fixed fee offered to D_i following (q_i, s_{-i}), which is the highest fixed fee, denoted by $F_i(q_i, s_{-i})$, that D_i is willing to accept given its beliefs after observing (q_i, s_{-i}). Perfect Bayesian equilibrium does not impose any restrictions on D_i's beliefs following an offer $f_i' \neq F_i(q_i, s_{-i})$. We are going to assume that observing an offer $f_i' \neq F_i(q_i, s_{-i})$ does not alter D_i's beliefs. This refinement is reasonable, because whether a deviation that involves offering a different fixed fee to D_i is profitable or unprofitable for U is independent of (q_{-i}, f_{-i}).

Equilibrium analysis: Beliefs Denoting the equilibrium target quantities by $(\widehat{q}_A^*, \widehat{q}_B^*)$, the retailers' prior beliefs are that the actual quantities offered by the supplier follow $N\left(\widehat{q}_A^*, \sigma_q^2\right)$ and $N\left(\widehat{q}_B^*, \sigma_q^2\right)$. Once D_i observes its offer q_i at stage 2, its belief about its own offer becomes q_i, while the belief about the other retailer's offer remains unchanged (since any quantity is offered with positive probability on the equilibrium path).

At stage 3, after observing the signal s_{-i}, D_i revises its belief about the quantity q_{-i} that was offered to D_{-i} from the prior to the following posterior:

$$q_{-i} \sim N\left(\frac{\sigma_s^2}{\sigma_s^2+\sigma_q^2}\widehat{q}_{-i}^* + \frac{\sigma_q^2}{\sigma_s^2+\sigma_q^2}s_{-i}, \frac{\sigma_q^2\sigma_s^2}{\sigma_s^2+\sigma_q^2}\right).$$

We will denote the weight that the posterior mean puts on the prior mean by

$$\alpha \equiv \frac{1}{1+\frac{\sigma_q^2}{\sigma_s^2}},$$

and the posterior mean by

$$\mu_{-i} \equiv E\left(q_{-i} \left| s_{-i}\right.\right) = \alpha\widehat{q}_{-i}^* + (1-\alpha)\, s_{-i}.$$

As α rises, that is, as the variance of the mistake falls relative to the variance of the signal, each downstream firm's posterior mean belief puts more weight on the prior and responds less to the signal about the actual quantity that its competitor was offered. The limit case of $\alpha \to 0$ ($\frac{\sigma_q^2}{\sigma_s^2} \to \infty$) corresponds to the benchmark case of public offers, in which each downstream firm observes its competitor's quantity. The limit case of $\alpha \to 1$ ($\frac{\sigma_q^2}{\sigma_s^2} \to 0$) corresponds to the benchmark case of secret offers and passive beliefs, in which each downstream firm obtains no information about its competitor's quantity. In what follows, we will refer to α as the *degree of contract secrecy*.

Equilibrium analysis: Quantities To solve for the equilibrium target quantities (which are also the expected quantities in equilibrium), we first consider D_i's contract acceptance decision at stage 5 of the game. D_i is willing to accept (q_i, f_i) if and only if the fixed fee does not exceed D_i's expected profit:

$$f_i \leq E\left(\pi\left(q_i, q_{-i}\right)|s_{-i}\right) = q_i\left(1 - q_i - \mu_{-i}\right) = q_i\left(1 - q_i - \alpha\widehat{q}^*_{-i} - (1-\alpha)\,s_{-i}\right),$$

where the expectation is taken over q_{-i} conditional on s_{-i} and the strategies. In equilibrium, therefore, U sets $F_i\left(q_i, s_{-i}\right) = q_i\left(1 - q_i - \alpha\widehat{q}_{-i} - (1-\alpha)\,s_{-i}\right)$ for each D_i and both retailers accept their offers.[10]

Moving backward, consider U's optimal choice of the target quantities in stage 1. The equilibrium target quantities $(\widehat{q}^*_A, \widehat{q}^*_B)$ must solve the following problem:

$$\max_{\widehat{q}_A, \widehat{q}_B} \sum_{i=A,B} E\left(q_i\left(1 - q_i - \alpha\widehat{q}^*_{-i} - (1-\alpha)s_{-i}\right) - cq_i | \widehat{q}_A, \widehat{q}_B\right),$$

where the expectation is taken over q_i and s_{-i} for $i = A, B$. The choice of $\widehat{q}_i$ affects the supplier's expected payoff both through its direct effect on the distribution of D_i's actual quantity (q_i) and through its indirect effect on the distribution of D_{-i}'s beliefs about D_i's

[10]Given the normality assumptions, q_i and the expected margin $E\left(P\left(q_i + q_{-i}\right)|s_{-i}\right) - c$ with the expectation being taken over q_{-i} can take on negative values. If this is the case, U would prefer to induce retailer D_i to reject U's offer (by setting a high fixed fee at stage 4). For convenience, we ignore this possibility, both in the simultaneous-offers game and in the later alternating-offer game. As noted by Vives (1984, fn. 2) in the context of an oligopoly model in which the intercept of a linear demand function is normally distributed and the firms obtain normally distributed signals about its value, "The probability of this event [negative prices and quantities] can be made arbitrarily small by appropriately choosing the variances of the model." In our setting, by setting the variances of both the mistakes in the quantity offers and the signal sufficiently small, the probability of a negative quantity or a negative expected margin can be made arbitrarily small. The degree of contract secrecy only depends on the ratio of the two variances, which can take on any value in $(0, 1)$ while both variances are close to zero.

An alternative way to avoid this issue is to replace the normal specification by other distributions that preserve the linearity of the conditional expectations. In Appendix B, we illustrate this by using the beta-binomial conjugate: the actual quantity with a mistake is drawn from a beta distribution, and the signal is drawn from a binomial distribution. In this setup, (i) q_i and the expected margin remain nonnegative; (ii) the conditional expectation of the rival's quantity is a weighted average of the realized signal and the target quantity; and (iii) the weight is characterized by the relative volatilities of mistakes and signals.

quantity (through the distribution of the signal s_i). The degree of secrecy regulates the strength of the latter effect.

This supplier's problem is equivalent to

$$\max_{\widehat{q}_A,\widehat{q}_B} \sum_{i=A,B} \left(\pi\left(\widehat{q}_i, \alpha\widehat{q}^*_{-i} + (1-\alpha)\widehat{q}_{-i}\right) - c\widehat{q}_i - \sigma_q^2\right),$$

because

$$\begin{aligned}
&E\left(q_i\left(1 - q_i - \alpha\widehat{q}^*_{-i} - (1-\alpha)s_{-i}\right) - cq_i|\widehat{q}_A, \widehat{q}_B\right)\\
=&\widehat{q}_i - E\left(q_i^2|\widehat{q}_i\right) - \alpha\widehat{q}^*_{-i}\widehat{q}_i - (1-\alpha)E\left(q_i s_{-i}|\widehat{q}_A, \widehat{q}_B\right) - c\widehat{q}_i\\
=&\widehat{q}_i - \left(\sigma_q^2 + \widehat{q}_i^2\right) - \alpha\widehat{q}^*_{-i}\widehat{q}_i - (1-\alpha)\widehat{q}_i\widehat{q}_{-i} - c\widehat{q}_i\\
=&\widehat{q}_i\left(1 - \widehat{q}_i - \alpha\widehat{q}^*_{-i} - (1-\alpha)\widehat{q}_{-i}\right) - c\widehat{q}_i - \sigma_q^2\\
=&\pi\left(\widehat{q}_i, \alpha\widehat{q}^*_{-i} + (1-\alpha)\widehat{q}_{-i}\right) - c\widehat{q}_i - \sigma_q^2,
\end{aligned}$$

where the second equality follows from

$$E\left(q_i^2|\widehat{q}_i\right) = Var\left(q_i|\widehat{q}_i\right) + E\left(q_i|\widehat{q}_i\right)^2 = \sigma_q^2 + \widehat{q}_i^2,$$

and

$$E\left(q_i s_{-i}|\widehat{q}_A, \widehat{q}_B\right) = E\left(q_i|\widehat{q}_i\right)E\left(s_{-i}|\widehat{q}_{-i}\right) = \widehat{q}_i\widehat{q}_{-i}.$$

The first-order conditions become, for each $i = A, B$,

$$\pi_1\left(\widehat{q}_i^*, \widehat{q}^*_{-i}\right) - c + (1-\alpha)\,\pi_2\left(\widehat{q}^*_{-i}, \widehat{q}_i^*\right) = 0.$$

The first-order conditions show that when setting D_i's target quantity, the supplier internalizes a fraction $(1-\alpha)$ of the effect of a change in D_i's quantity on D_{-i}'s profit (evaluated at the expected equilibrium quantities). The degree of secrecy thus determines the extent to which the competitive externalities across retailers are internalized in equilibrium. As the degree of secrecy rises, a smaller share of the competitive externalities is internalized.

Solving the first-order conditions for the target quantities yields our main result in the simultaneous-offers game:[11]

[11] The second-order condition are satisfied, because for any given $(\widehat{q}_A^*, \widehat{q}_B^*)$, the Hessian of the objective function,

$$\begin{bmatrix} \pi_{11} + (1-\alpha)^2\pi_{22} & (1-\alpha)(\pi_{12} + \pi_{21}) \\ (1-\alpha)(\pi_{12} + \pi_{21}) & \pi_{11} + (1-\alpha)^2\pi_{22} \end{bmatrix} = \begin{bmatrix} -2 & -2(1-\alpha) \\ -2(1-\alpha) & -2 \end{bmatrix},$$

is negative definite.

Proposition 1 *The equilibrium target quantities in the simultaneous-offers game are given by*

$$\widehat{q}_A^* = \widehat{q}_B^* = \widehat{q}^* \equiv \frac{1-c}{4-\alpha} \in \left(q^M, q^C\right).$$

The equilibrium target quantities vary continuously in the degree of secrecy α, spanning the entire range between the monopoly quantities (for $\alpha \to 0$, corresponding to the benchmark of public offers) and the Cournot quantities (for $\alpha \to 1$, corresponding to the benchmark of secret offers and passive beliefs). As the degree of contract secrecy rises, the equilibrium target quantities increase and thus the expected price paid by consumers falls.

The analysis extends straightforwardly to the more general case with $n \geq 2$ downstream firms under the assumption that all firms observe the same signal realizations.[12] In this case, each downstream firm D_i ($i = 1, 2, ..., n$), after observing the common signal s_j about the quantity q_j offered to D_j, $j \neq i$, revises its belief from the prior $N\left(\widehat{q}_j^*, \sigma_q^2\right)$ to the posterior:

$$q_j \sim N\left(\frac{\sigma_s^2}{\sigma_s^2+\sigma_q^2}\widehat{q}_j^* + \frac{\sigma_q^2}{\sigma_s^2+\sigma_q^2}s_j, \frac{\sigma_q^2\sigma_s^2}{\sigma_s^2+\sigma_q^2}\right).$$

Hence, the expression for the posterior mean, $\mu_j = E\left(q_j|s_j\right) = \alpha\widehat{q}_j^* + (1-\alpha)s_j$, remains the same as in the case of $n = 2$.

The first-order conditions for the target quantities become, for each $i \in \{1, 2, ..., n\}$,

$$\frac{\partial\pi_i\left(\widehat{\mathbf{q}}^*\right)}{\partial q_i} - c + (1-\alpha)\sum_{j\neq i}\frac{\partial\pi_j\left(\widehat{\mathbf{q}}^*\right)}{\partial q_i} = 0,$$

where $\pi_i(\mathbf{q}) = q_i P\left(\sum_j q_j\right)$ is D_i's profit given the quantity vector $\mathbf{q} = (q_1, q_1, ..., q_n)$ and $\widehat{\mathbf{q}}^* = (\widehat{q}_1^*, \widehat{q}_2^*, ..., \widehat{q}_n^*)$ is the vector of equilibrium target quantities. The equilibrium target quantities become

$$\widehat{q}_i^* = \frac{1-c}{2n-(n-1)\alpha} \quad \text{for all } i = 1, 2, ..., n.$$

Again, we observe that the equilibrium quantity converges to the Cournot quantity, $\frac{1-c}{n+1}$, as $\alpha \to 1$ (corresponding to secret offers) and to the monopoly quantity, $\frac{1-c}{2n}$, as $\alpha \to 0$ (corresponding to public offers).

[12] The analysis of cases in which the downstream firms observe different signals (e.g., privately observed independent draws) is left for future research.

3 Alternating offers

Framework Our second game, building on Do and Miklós-Thal (2022), considers an infinite-horizon dynamic model with alternating offers.[13] Time is discrete ($t = 1, 2, ...$), and U makes contract offers to D_A (resp. D_B) in odd periods (resp. even periods). An accepted contract lasts for two periods, and the firms share a common discount factor $\delta \in [0, 1)$ between periods. A contract (q_i, f_i) now specifies the quantity q_i to be put on the market by D_i in each of the two periods that the contract covers and a one-time upfront fixed fee f_i paid at the time of contract acceptance. As in the simultaneous-offers game, the supplier's quantity offers are subject to mistakes. Moreover, prior to receiving an offer, a retailer obtains a signal of its competitor's current quantity.

Formally, the timing within each period t is as follows ($i = A$ in odd periods, $i = B$ in even periods):

1. U chooses a target quantity $\widehat{q}_i^t$, which is unobserved by the downstream firms.
2. An offer quantity q_i^t is drawn from $N\left(\widehat{q}_i^t, \sigma_q^2\right)$ and observed privately by U and D_i.
3. U offers a fixed fee f_i^t to D_i, observed privately by D_i.
4. D_i decides whether to accept ($g_i^t = 1$) or reject U's offer ($g_i^t = 0$). D_i's decision is unobserved by D_{-i}. Downstream firms with accepted contracts sell to final consumers.
5. A signal $s_i^t = q_i^t \cdot g_i^t + \epsilon_i^t$ is realized and observed publicly by all firms, where ϵ_i^t is distributed according to $N\left(0, \sigma_s^2\right)$.

In the dynamic setting, it is natural to think of each retailer as learning about its competitor's current quantity from observed market outcomes (e.g., the price, which could be modeled as a stochastic function of the total quantity). To simplify the analysis, we model the information that each retailer obtains as a public signal of the competitor's quantity

[13] The dynamic model in Do and Miklós-Thal (2022) and the present paper are inspired by a series of seminal papers on dynamic oligopoly games with asynchronous moves (Maskin and Tirole, 1987, 1988a,b). The key distinguishing feature of our analyses is that we consider a vertical industry with a strategic input supplier.

only, as in the simultaneous-offers game, and assume that retailers draw no additional inferences about their competitor's quantities from prices/own payoffs.[14] This ensures that a retailer's belief about its competitor's current quantity (when the firms play Markov perfect public strategies) depends on the sequence of publicly observed signals only, and not on the retailer's own quantity.

Equilibrium concept We focus on perfect Bayesian equilibria in symmetric (stationary) Markov perfect public strategies, which depend on the history of the game only through a payoff-relevant state variable. More precisely, the state variable in a period when U makes an offer to D_i will be D_i's expectation (mean belief) of D_{-i}'s current quantity, which will be denoted by μ_{-i}. A Markov perfect public strategy of U is then given by a pair of mappings $(R(\cdot), F(\cdot))$, where $R(\mu_{-i}) \in \mathbb{R}$ is the target quantity chosen in state μ_{-i} and $F(q_i; \mu_{-i}) \in \mathbb{R}$ is the fixed fee offer in state μ_{-i} after an actual quantity q_i is realized. We will refer to $R(\cdot)$ as the *dynamic target-quantity reaction function.* A Markov perfect public strategy of D_i is given by a function $G(q_i, f_i; \mu_{-i}) \in \{1, 0\}$, where $G(q_i, f_i; \mu_{-i}) = 1$ (resp. $G(q_i, f_i; \mu_{-i}) = 0$) means that D_i accepts (resp. rejects) the offer (q_i, f_i) in state μ_{-i}. Note that the state variable is common knowledge because the downstream firms' beliefs are functions of the sequence of publicly observed signals, as described in more detail later. Also, we continue to assume that no off-path deviation alters D_i's beliefs (that is, an out-of-equilibrium fixed fee offer does not affect a downstream firm's beliefs).

Henceforth, a perfect Bayesian equilibrium in symmetric (stationary) Markov perfect public strategies is referred to simply as *equilibrium*, and we will say that an equilibrium is linear if the dynamic target-quantity reaction function is linear. All firms are risk neutral and seek to maximize expected presented discounted payoffs.

Equilibrium analysis: Beliefs To understand how the state variable evolves over time, suppose that in period $t-1$, U made an offer to D_{-i} and the state variable was μ_i. Then, D_i believes that U set a target quantity for D_{-i} of $R(\mu_i)$ in period $t-1$. Therefore, D_i's

[14]The assumption that retailers draw no additional inferences from prices/own payoffs, while restrictive, may be appropriate in settings in which public signals are sufficient statistics for privately observed ex-post payoffs (this would be the case, for instance, if $p_i^t = 1 - q_i^t - s_{-i}^t$, so that for retailer D_i, s_{-i}^t is a sufficient statistic for (p_i^t, s_{-i}^t)), or if the ex-post payoffs are realized with a time lag (see Mailath and Samuelson 2006, p. 226). It is reminiscent of the common assumption in repeated games with imperfect monitoring that firms receive no information about other firms' actions beyond a public signal.

posterior belief about q_{-i}^{t-1}, after observing s_{-i} at the end of period $t-1$, is given by

$$N\left(\frac{\sigma_s^2}{\sigma_s^2+\sigma_q^2}R\left(\mu_i\right)+\frac{\sigma_q^2}{\sigma_s^2+\sigma_q^2}s_{-i},\frac{\sigma_q^2\sigma_s^2}{\sigma_s^2+\sigma_q^2}\right).$$

As in the game with simultaneous offers, $\alpha=\left(1+\frac{\sigma_q^2}{\sigma_s^2}\right)^{-1}$ measures the degree of secrecy, where $\alpha=0$ (resp. $\alpha=1$) corresponds to public offers (resp. secret offers and passive beliefs). The state variable thus evolves from μ_i to

$$\mu_{-i}=\alpha R\left(\mu_i\right)+\left(1-\alpha\right)s_{-i}.$$

As in the simultaneous-offers game, the degree of secrecy α determines how much weight a retailer's posterior belief about its competitor's quantity puts on the prior belief versus the signal about the competitor's actual quantity.

Equilibrium analysis: Strategies To solve for an equilibrium, we define the following value functions. Given the Markov strategies $((R,F),G)$, let $W\left(\mu_{-i}\right)$ (resp. $W\left(q_i;\mu_{-i}\right)$) denote the expected present discounted value of U's profits when U chooses a target quantity (resp. a fixed fee after q_i is realized) in state μ_{-i} and all firms play according to their Markov strategies strategies henceforth. Similarly, let $V\left(q_i,f_i;\mu_{-i}\right)$ denote the expected present discounted value of D_i when D_i accepts the contract offer (q_i,f_i) it received at state μ_{-i} and all firms play according to their Markov strategies henceforth.

The supplier U's optimization problems are then given by

$$\begin{aligned} W\left(q_i;\mu_{-i}\right)=&\max_{f_i}E\Big(f_i-c\left(1+\delta\right)q_i+\delta W\left(\alpha R\left(\mu_{-i}\right)+\left(1-\alpha\right)s_i\right)\big|q_i\Big).\\ &\text{s.t. } V\left(q_i,f_i;\mu_{-i}\right)\geq V\left(0,0;\mu_{-i}\right), \end{aligned}$$

and

$$W\left(\mu_{-i}\right)=\max_{\widehat{q}_i}E\Big(W\left(q_i,\mu_{-i}\right)\big|\widehat{q}_i\Big),$$

where the constraint $V\left(q_i,f_i;\mu_{-i}\right)\geq V\left(0,0;\mu_{-i}\right)$ guarantees acceptance by the downstream firms, and the expectations in $W\left(q_i;\mu_{-i}\right)$ and $W\left(\mu_{-i}\right)$ are respectively taken over s_i and q_i.

We first show that D_i's acceptance decision reduces to to an upper bound on the fixed fee.

Lemma 1 *It is optimal for D_i to accept offer (q_i, f_i) in state μ_{-i}, i.e., $V\left(q_i, f_i; \mu_{-i}\right) \geq V\left(0, 0; \mu_{-i}\right)$, if and only if*

$$f_i \leq \bar{F}\left(q_i; \mu_{-i}\right) \equiv \pi\left(q_i, \mu_{-i}\right) + \delta E\left(\pi\left(q_i, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right)\right) | q_i\right),$$

where the expectation is taken over s_i.

The fixed payment $\bar{F}\left(q_i; \mu_{-i}\right)$ extracts the present discounted value of all variable profits that D_i expects to earn during the contract duration. The first term captures D_i's expected profit from selling q_i in the current period when it believes that its competitor sells a quantity of μ_{-i} in expectation. The second term captures D_i's expected profit from selling q_i in the next period. This term takes into account that D_{-i} will form its belief about D_i's quantity based on the realization of s_i and the supplier's equilibrium target quantity $R\left(\mu_{-i}\right)$, resulting in the next period state, $\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i$. Therefore, the supplier will set the target quantity to D_{-i} in the next period at $R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right)$, which determines D_i's expected profit in the next period.

Since the supplier's payoff strictly increases in f_i, $F\left(q_i; \mu_{-i}\right) = \bar{F}\left(q_i; \mu_{-i}\right)$ in equilibrium. Substituting this binding constraint into the objective function, the supplier chooses the target quantity to solve the following problem:

$$\begin{aligned} W\left(\mu_{-i}\right) = \max_{\widehat{q}_i} E\Big(&\pi\left(q_i, \mu_{-i}\right) + \delta\pi\left(q_i, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right)\right) - c(1+\delta) q_i \\ &+ \delta W\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right) | \widehat{q}_i\Big), \end{aligned} \tag{1}$$

with the expectation being taken over q_i and s_i. The strategies $((R, F), G)$ form an equilibrium if and only if there exist value functions $W(\cdot)$ such that, for every $\mu_{-i} \in \mathbb{R}$, (1) holds,

$$\begin{aligned} R\left(\mu_{-i}\right) \in \arg\max_{\widehat{q}_i} E\Big(&\pi\left(q_i, \mu_{-i}\right) + \delta\pi\left(q_i, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right)\right) - c(1+\delta) q_i \\ &+ \delta W\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right) | \widehat{q}_i\Big), \end{aligned}$$

$F\left(q_i; \mu_{-i}\right) = \bar{F}\left(q_i; \mu_{-i}\right)$, and $G\left(q_i, f_i; \mu_{-i}\right) = 1$ if and only if $f_i \leq \bar{F}\left(q_i; \mu_{-i}\right)$.

We find that, as in the dynamic game with public offers in Do and Miklós-Thal (2022), the game has a unique linear equilibrium.

Proposition 2 *The alternating-offers game has a unique linear equilibrium. In this equilibrium, $R\left(\mu_{-i}\right) = a^* - b^*\mu_{-i}$, with $a^* \geq 0$ and $0 < b^* \leq \frac{1}{2}$.*

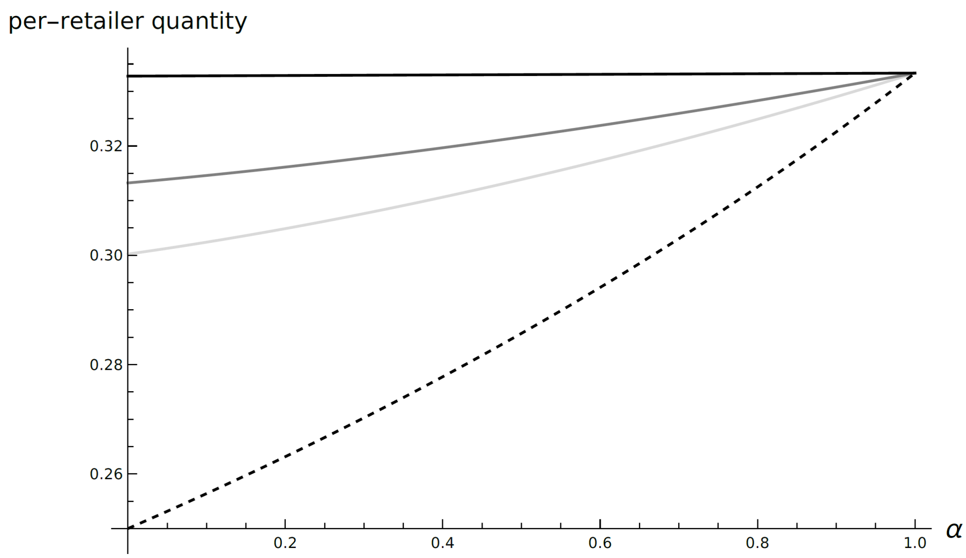

Figure 1: The top three lines show the expected steady-state quantity μ^0 as a function of the degree of contract secrecy α for three different discount factors (black: $\delta = 0.01$, dark gray: $\delta = 0.5$, light gray: $\delta = 0.99$). The dashed line shows that equilibrium target quantity $\widehat{q}^*$ in the simultaneous-offers game.

Equilibrium steady state The equilibrium target-quantity reaction function $R(\cdot)$ induces a long-run steady-state expected quantity/belief that satisfies $\mu^0 = R(\mu^0)$:

$$\mu^0 \equiv \lim_{t\to\infty} E\left(q_i^t\right) = \frac{a^*}{1+b^*} \text{ for each } i = A, B.$$

Our next proposition shows that the steady-state expected quantity rises in the degree of secrecy.

Proposition 3 *The expected quantity μ^0 in the long-run steady state satisfies the following properties:*

(1) μ^0 is strictly increasing in the degree of secrecy α, with $\lim_{\alpha\to 1}\mu^0 = q^C$ for all δ,

(2) μ^0 is strictly decreasing in the discount factor δ, with $\lim_{\delta\to 0}\mu^0 = q^C$ for all α.

Figure 1 illustrates how the equilibrium steady-state expected quantity varies with the degree of secrecy for three different discount factors. For each discount factor, the steady-state expected quantity is increasing in the degree of secrecy. For $\alpha \to 0$, the model coincides with the symmetric retailer case in Do and Miklós-Thal (2022).[15] As the degree of secrecy α rises, the rival retailer's belief in the next period depends less on the offer made in the current period, because the signal becomes less informative and a change in today's offer will have less impact on the distribution of states in the next period. Intuitively, when deciding on its target offer quantity for D_i, U therefore becomes more focused on maximizing the bilateral surplus with D_i given D_i's current belief and less concerned with favorably influencing D_{-i}'s belief in the next period by setting a lower target quantity. In the limit, for $\alpha \to 1$, the rival retailer's belief becomes independent of the current-period offer, and the first-order condition at the steady state becomes

$$0 = (1+\delta)\left(\pi_1\left(\mu^0, \mu^0\right) - c\right),$$

which holds when μ^0 is equal to the Cournot quantity q^C.

Figure 1 also illustrates the finding that, for a given degree of contract secrecy, the equilibrium steady-state expected quantity is lower the higher the discount factor. This is consistent with our earlier finding in Do and Miklós-Thal (2022), in a dynamic model of asynchronous recontracting with public offers, that patience reduces the degree of opportunism. Intuitively, when deciding on its target offer quantity for D_i, U is more concerned with maximizing the bilateral surplus with D_i given D_i's current belief and less concerned with inducing high industry profits in future periods (that the supplier can fully extract through future fixed fees) when the discount factor is lower.

Do and Miklós-Thal's (2022) finding that asynchronous dynamic recontracting causes opportunism also extends from public to partially secret offers. Comparing the equilibrium steady-state of the dynamic alternating-offers to the equilibrium outcome of the simultaneous-offers game, we obtain the following results:

Proposition 4 *For any $\alpha \in (0,1)$ and $\delta \in (0,1)$,*

$$\mu^0 > \widehat{q}^*.$$

Moreover, $\lim_{(\alpha,\delta)\to(0,1)} \mu^0 = \frac{3(1-c)}{10} > \lim_{\alpha\to 0} \widehat{q}^* = q^M$.

[15] In Do and Miklós-Thal (2022), we analyze a continuous-time game in which recontracting events arise stochastically and the arrival rates of the recontracting events can differ across retailers. The limit case corresponding to public offers ($\alpha \to 0$) in this paper is equivalent to the model with symmetric arrival rates in Do and Miklós-Thal (2022).

The equilibrium outcome of the static game, illustrated by the dashed line in Figure 1, is less competitive than the equilibrium steady-state of the dynamic alternating-offers game for any any given degree of partial secrecy and any discount factor. Moreover, in the alternating-offers game, unlike in the simultaneous-offers game, the steady-state quantity is bounded below by a quantity strictly greater than the monopoly quantity even as secrecy vanishes.

4 Conclusion

In this paper, we have taken a first step towards modeling *partially secret* supply contracts. We have introduced a continuous measure of the degree to which a downstream firm's supply terms can be observed by competing downstream firms. Using this measure, we have analyzed how the degree of secrecy affects equilibrium outcomes in two vertical contracting games. First, a game in which the supplier makes simultaneous offers to the downstream firms, as commonly considered in the literature. Second, an infinite-horizon dynamic game in which the supplier makes alternating offers to two downstream firms, building on our recent work in Do and Miklós-Thal (2022). Our analysis does not rely on any assumptions about beliefs following out-of-equilibrium offer quantities, and passive beliefs arise endogenously as a limiting case.

We find that, in both games, a greater degree of contract secrecy is associated with a more competitive equilibrium outcome in a continuous fashion. The results thus suggest that as downstream firms get better at predicting their competitors' contract terms—be it through ties between downstream firms due to a partnership (e.g., a joint venture), personal connections, competitive intelligence, or by drawing inferences from observed market outcomes—suppliers will be able to exert their market power more fully and induce higher final prices. Horizontal information leakages in the downstream market lead to less competition because of how they affect equilibrium supply contracts.

There are several limitations to this research. First, we have focused on quantity competition in the downstream market and quantity-fixing supply contracts. Future research could explore partial contract secrecy in settings with price competition where downstream firms obtain signals about their competitors' wholesale prices. Second, we have focused on settings with symmetric downstream firms. Future research could potentially analyze the role of asymmetries among downstream firms, for instance, in the precision of the signals that competitors obtain about their contracts.

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Appendix A: Proofs

Proof of Lemma 1. Let $V\left(\mu_{-i}\right)$ be the equilibrium value of the downstream firm who contracts with U, and $\widetilde{V}\left(\mu_{-i}\right)$ the equilibrium value of another downstream firm. If D_i accepts (q_i, f_i), it obtains

$$V\left(q_i, f_i; \mu_{-i}\right) = \pi\left(q_i, \mu_{-i}\right) - f_i + \delta E\left(\pi\left(q_i, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)\right)|q_i\right) \\ +\delta E\left(\widetilde{V}\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)|q_i\right), \qquad (2)$$

where the expectations are taken over s_i. Instead, if D_i rejects, it obtains

$$V\left(0, 0; \mu_{-i}\right) = \delta E\left(\widetilde{V}\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)|0\right).$$

Thus, it is optimal for D_i to accept (q_i, f_i) if and only if

$$f_i \leq F\left(q_i; \mu_{-i}\right) = \pi\left(q_i, \mu_{-i}\right) + \delta E\left(\pi\left(q_i, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)\right)|q_i\right) \\ +\delta\left(E\left(\widetilde{V}\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)|q_i\right) - E\left(\widetilde{V}\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)|0\right)\right).$$

By substituting $F\left(q_i, \mu_{-i}\right)$ in (2), we have

$$V\left(q_i, F\left(q_i, \mu_{-i}\right); \mu_{-i}\right) = \delta E\left(\widetilde{V}\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)|0\right),$$

and so, for all μ_{-i},

$$V\left(\mu_{-i}\right) = \delta E\left(\widetilde{V}\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)|0\right).$$

In addition, since $\widetilde{V}\left(\mu_i\right) \leq \delta\bar{V}$ for all μ_i where $\bar{V} \equiv \sup_{\mu_{-i}} V\left(\mu_{-i}\right) < \infty$, this implies

$$V\left(\mu_{-i}\right) \leq \delta\bar{V}$$

for all μ_{-i}. Therefore, $V\left(\mu_{-i}\right) = \widetilde{V}\left(\mu_i\right) = 0$ for all μ_{-i} and μ_i, and we conclude

$$F\left(q_i; \mu_{-i}\right) = \pi\left(q_i, \mu_{-i}\right) + \delta E\left(\pi\left(q_i, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)s_i\right)\right)|q_i\right).$$

Proof of Proposition 2. We consider a linear dynamic target-quantity reaction function $R\left(\mu_{-i}\right) = a - b\mu_{-i}$. Our first intermediate result shows that, under this linear $R(\cdot)$, the supplier's problem (1) can be simplified as follows:

Lemma A1 *Suppose that $R(\cdot)$ is linear. Then, the supplier's problem can be rewritten as*

$$\begin{aligned} W\left(\mu_{-i}\right) = \max_{\widehat{q}} \Big(& \pi\left(\widehat{q}, \mu_{-i}\right) + \delta\pi\left(\widehat{q}, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)\widehat{q}\right)\right) - c(1+\delta)\widehat{q} \\ & + \delta W\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)\widehat{q}\right) + constant \Big). \end{aligned} \tag{3}$$

Proof. Note that

$$\begin{aligned} E\left(\pi\left(q, \mu_{-i}\right) - c(1+\delta) q \,|\widehat{q}\right) =& E\left(\left(1 - q - \mu_{-i}\right) q - c(1+\delta) q \,|\widehat{q}\right) \\ =& \widehat{q}\left(1 - \widehat{q} - \mu_{-i}\right) - c(1+\delta)\widehat{q} - \sigma_q^2 \\ =& \pi\left(\widehat{q}, \mu_{-i}\right) - c(1+\delta)\widehat{q} - \sigma_q^2, \end{aligned}$$

where the expectation being taken over q. Similarly, one can check that

$$\begin{aligned} & E\left(\pi\left(q, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right)\right) |\widehat{q}\right) \\ =& \pi\left(\widehat{q}, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)\widehat{q}\right)\right) - (1 - b(1-\alpha))\sigma_q^2, \end{aligned}$$

where the expectation is taken over q and s_i. To see this, observe that

$$\begin{aligned} E\left(\pi\left(q, R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right)\right) |\widehat{q}\right) =& E\left(q - q^2 - R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right) q \,|\widehat{q}\right) \\ =& \widehat{q} - \widehat{q}^2 - \sigma_q^2 - E\left(R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right) q \,|\widehat{q}\right), \end{aligned} \tag{4}$$

and

$$\begin{aligned} E\left(R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha) s_i\right) q \,|\widehat{q}\right) =& E\left(\left(a - b\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)\left(q + \epsilon_i\right)\right)\right) q \,|\widehat{q}\right) \\ =& E\left(aq - b\alpha R\left(\mu_{-i}\right) q - b(1-\alpha) q^2 - b(1-\alpha)\epsilon_i q \,|\widehat{q}\right) \\ =& E\left(aq - b\alpha R\left(\mu_{-i}\right) q - b(1-\alpha) q^2 \,|\widehat{q}\right) \\ =& a\widehat{q} - b\alpha R\left(\mu_{-i}\right)\widehat{q} - b(1-\alpha)\left(\widehat{q}^2 + \sigma_q^2\right) \\ =& \widehat{q}\left(a - b\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)\widehat{q}\right)\right) - b(1-\alpha)\sigma_q^2 \\ =& \widehat{q} R\left(\alpha R\left(\mu_{-i}\right) + (1-\alpha)\widehat{q}\right) - b(1-\alpha)\sigma_q^2. \end{aligned} \tag{5}$$

By substituting (5) in (4), we have

$$\begin{aligned}
&E\left(\pi\left(q, R\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)s_i\right)\right)|\widehat{q}\right)\\
=&\widehat{q}-\widehat{q}^2-\sigma_q^2-\widehat{q}R\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)+b(1-\alpha)\sigma_q^2\\
=&\widehat{q}\left(1-\widehat{q}-R\left(\alpha\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)\right)-(1-b(1-\alpha))\sigma_q^2\\
=&\pi\left(\widehat{q}, R\left(\alpha\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)\right)-(1-b(1-\alpha))\sigma_q^2.
\end{aligned}$$

Finally, it remains to show that

$$E\left(W\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)s_i\right)|\widehat{q}\right)=W\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)+constant,$$

where the expectation is taken over s_i. To see this, let us represent the supplier's value function as, for some $w_0, w_1, w_2 \in \mathbb{R}$:

$$W\left(\mu_{-i}\right)=w_0+w_1\mu_{-i}+w_2\mu_{-i}^2,$$

which follows from the linearity of $R(\cdot)$. Then,

$$\begin{aligned}
&E\left(W\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)s_i\right)|\widehat{q}\right)\\
=&E\left(w_0+w_1\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)s_i\right)+w_2\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)s_i\right)^2|\widehat{q}\right)\\
=&w_0+w_1\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)+w_2E\left(\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)(q_i+\epsilon_i)\right)^2|\widehat{q}\right),
\end{aligned}$$

with expectations being taken over s_i, q_i and ϵ_i, and

$$\begin{aligned}
&E\left(\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)(q_i+\epsilon_i)\right)^2|\widehat{q}\right)\\
=&\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)^2+(1-\alpha)^2\sigma_q^2+(1-\alpha)^2\sigma_s^2.
\end{aligned}$$

Thus,

$$E\left(W\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)s_i\right)|\widehat{q}\right)=W\left(\alpha R\left(\mu_{-i}\right)+(1-\alpha)\widehat{q}\right)+w_2(1-\alpha)^2\left(\sigma_q^2+\sigma_s^2\right).$$

This completes the proof of Lemma A1. ■

Now, we prove that there exists a unique linear equilibrium. From (3), the first-order condition is given by: at $\widehat{q}=R(\mu)$,

$$\begin{aligned}
0=&-c(1+\delta)+\pi_1(\widehat{q},\mu)+\delta\pi_1\left(\widehat{q}, R\left(\alpha R(\mu)+(1-\alpha)\widehat{q}\right)\right)\\
&+\delta\pi_2\left(\widehat{q}, R\left(\alpha R(\mu)+(1-\alpha)\widehat{q}\right)\right)R'\left(\alpha R(\mu)+(1-\alpha)\widehat{q}\right)(1-\alpha)\\
&+\delta(1-\alpha)W'\left(\alpha R(\mu)+(1-\alpha)\widehat{q}\right).
\end{aligned}\tag{6}$$

The envelope theorem implies that

$$W'(\mu) = \pi_2(R(\mu), \mu) + \delta\pi_2(R(\mu), R(R(\mu)))\, R'(R(\mu))\, R'(\mu)\, \alpha. \tag{7}$$

By substituting (7) in (6), we can rewrite the first-order condition as follows: at $\widehat{q} = R(\mu)$,

$$\begin{aligned} 0 = & -c(1+\delta) + \pi_1(\widehat{q}, \mu) \\ & + \delta\left(\pi_1(\widehat{q}, R(\widehat{q})) + \pi_2(\widehat{q}, R(\widehat{q}))(1-\alpha) R'(\widehat{q})\right) \\ & + \delta(1-\alpha)\left(\pi_2(R(\widehat{q}), \widehat{q}) + \delta\alpha\pi_2(R(\widehat{q}), R(R(\widehat{q})))\, R'(R(\widehat{q}))\, R'(\widehat{q})\right). \end{aligned} \tag{8}$$

We first show that if $b \leq 1$, the first-order condition is both necessary and sufficient. To see this, differentiate the right-hand side of (6) with respect to $\widehat{q}$ to obtain

$$K \equiv -2(1+\delta-(1-\alpha)b\delta) + \delta(1-\alpha)^2 W'',$$

where W'' is the second-order derivative of $W(\cdot)$, which is constant because $W(\cdot)$ is quadratic. Thus, if $K < 0$, the second-order condition is satisfied.

Next, note that, for all μ,

$$\begin{aligned} 0 = & \pi_1(R(\mu), \mu) + \delta\pi_1(R(\mu), R(R(\mu))) \\ & + \delta\pi_2(R(\mu), R(R(\mu)))\, R'(R(\mu))(1-\alpha) - c(1+\delta) \\ & + \delta(1-\alpha) W'(R(\mu)). \end{aligned} \tag{9}$$

Then, by taking derivative with respect to μ, we have

$$0 = -1 - (2-\alpha)\delta b^2 + 2(1+\delta)b - \delta(1-\alpha)W''b. \tag{10}$$

This is equivalent to

$$\delta(1-\alpha) W'' = \frac{-1-(2-\alpha)\delta b^2 + 2(1+\delta)b}{b}.$$

By substituting this into K, we have

$$K = \frac{-1+\alpha-\alpha b(2+2\delta-b\delta) - \alpha^2 b^2 \delta}{b}.$$

Since $-1+\alpha-\alpha^2 b^2 \delta < 0$ and $b > 0$, it is sufficient to show $2+2\delta-b\delta \geq 0$ for $K < 0$. However, this is immediate as long as $b \leq 1$.

It remains to show that there exist unique $a \geq 0$ and $0 < b \leq \frac{1}{2}$ satisfying (8). By solving (8) for $\widehat{q}$ and rearranging, we have

$$\begin{aligned}\widehat{q} = & \left(\frac{(1-c)(1+\delta)-(2-\alpha)\delta a-(1-\alpha)\alpha b^2\delta^2 a}{2+(2-(3-2\alpha)b)\delta-(1-\alpha)\alpha b^3\delta^2}\right) \\ & -\left(\frac{1}{2+(2-(3-2\alpha)b)\delta-(1-\alpha)\alpha b^3\delta^2}\right)\mu_{-i}.\end{aligned}$$

Since $\widehat{q} = R\left(\mu_{-i}\right)$, the following equations should be satisfied:

$$b = \frac{1}{2+\left(2-\left(3-2\alpha\right)b\right)\delta-\left(1-\alpha\right)\alpha b^3\delta^2} \tag{11}$$

$$a = \frac{(1-c)(1+\delta)-(2-\alpha)\delta a-(1-\alpha)\alpha b^2\delta^2 a}{2+(2-(3-2\alpha)b)\delta-(1-\alpha)\alpha b^3\delta^2}. \tag{12}$$

Note that (11) is equivalent to $\mathcal{K}\left(b\right) = 0$, where

$$\mathcal{K}\left(b\right) \equiv b\left(2+\left(2-\left(3-2\alpha\right)b\right)\delta-\left(1-\alpha\right)\alpha b^3\delta^2\right)-1.$$

Since $\mathcal{K}(0) = -1$ and $\mathcal{K}\left(\frac{1}{2}\right) = \frac{\delta\left(4+\alpha(8-\delta)+\alpha^2\delta\right)}{16} \geq 0$, the intermediate value theorem implies that there exists $b^* \in \left(0, \frac{1}{2}\right]$ such that $\mathcal{K}\left(b^*\right) = 0$. In addition, such b^* is unique since $\mathcal{K}''(b) = -2\delta\left(3-2\alpha+6\left(1-\alpha\right)\alpha b^2\delta\right) < 0$, implying strictly concave $\mathcal{K}\left(\cdot\right)$.[16]

By substituting b^* into the equation (12) and rearranging, we have

$$\begin{aligned}a^* &= \frac{\left(1-c\right)\left(1+\delta\right)}{2+(2-(3-2\alpha)b^*)\delta-(1-\alpha)\alpha b^{*3}\delta^2+\left(2-\alpha\right)\delta+\left(1-\alpha\right)\alpha b^{*2}\delta^2} \\ &\geq 0.\end{aligned}$$

This completes the construction of the unique linear equilibrium, which has the following value function:

$$W\left(\mu_{-i}\right) = constant - a^*\left(1+\delta b^{*2}\alpha\right)\mu_{-i} + \left(\frac{-1-\left(2-\alpha\right)\delta b^{*2}+2\left(1+\delta\right)b^*}{2b^*\delta\left(1-\alpha\right)}\right)\mu_{-i}^2.$$

[16]Indeed, $\mathcal{K}(1) = \left(1+\alpha\delta\right)\left(1-\left(1-\alpha\right)\delta\right) > 0$, so another real root, if it exists, must lead to a dynamically unstable equilibrium, which is not relevant for our purpose.

Proof of Proposition 3. The limit results, $\lim_{\alpha\to 1}\mu^0 = q^C = \lim_{\delta\to 0}\mu^0$, can be directly checked by evaluating μ^0 at $\alpha = 1$ and $\delta = 0$, respectively. Thus, we only prove the comparative statics results in the proposition.

First, we show that μ^0 is increasing in α. From the proof of Proposition 2, we have

$$\frac{a^*}{1+b^*} = \frac{(1-c)(1+\delta)}{\mathcal{L}(\alpha)},$$

where

$$\mathcal{L}(\alpha) \equiv (1+b^*)\left(2+(2-(3-2\alpha)b^*)\delta-(1-\alpha)\alpha b^{*3}\delta^2+(2-\alpha)\delta+(1-\alpha)\alpha b^{*2}\delta^2\right).$$

A tedious computation shows that

$$\begin{aligned}\mathcal{L}'(\alpha) = {} & (1+b^*)\left(2b^*\delta-(1-2\alpha)b^{*3}\delta^2-\delta+(1-2\alpha)b^{*2}\delta^2\right)\\ & +\frac{\partial b^*}{\partial\alpha}\times\Big(2+(2-(3-2\alpha)b^*)\delta-(1-\alpha)\alpha b^{*3}\delta^2+(2-\alpha)\delta+(1-\alpha)\alpha b^{*2}\delta^2\\ & \qquad -(1+b^*)\left((3-2\alpha)\delta+3(1-\alpha)\alpha b^{*2}\delta^2-2(1-\alpha)\alpha b^*\delta^2\right)\Big).\end{aligned}$$

For notational ease, let us write

$$\begin{aligned} L_1 & \equiv 2b^*\delta-(1-2\alpha)b^{*3}\delta^2-\delta+(1-2\alpha)b^{*2}\delta^2\\ L_2 & \equiv 2+(2-(3-2\alpha)b^*)\delta-(1-\alpha)\alpha b^{*3}\delta^2+(2-\alpha)\delta+(1-\alpha)\alpha b^{*2}\delta^2\\ & \quad -(1+b^*)\left((3-2\alpha)\delta+3(1-\alpha)\alpha b^{*2}\delta^2-2(1-\alpha)\alpha b^*\delta^2\right).\end{aligned}$$

We will prove the result by the following three claims.

Claim 1: $L_1 \le 0$**.**

Note that

$$\begin{aligned} L_1 = {} & 2b^*\delta-(1-2\alpha)b^{*3}\delta^2-\delta+(1-2\alpha)b^{*2}\delta^2\\ = {} & \delta\left(2b^*-(1-2\alpha)b^{*3}\delta-1+(1-2\alpha)b^{*2}\delta\right)\\ = {} & \delta\left(2b^*+(1-2\alpha)b^{*2}\delta(1-b^*)-1\right).\end{aligned}$$

Also, recall that, by construction,

$$\mathcal{K}(b^*) = b^*\left(2+(2-(3-2\alpha)b^*)\delta-(1-\alpha)\alpha b^{*3}\delta^2\right)-1 = 0, \tag{13}$$

and so,

$$\begin{aligned}
&2b^* + (1-2\alpha)\, b^{*2}\delta\,(1-b^*) - 1 \\
=&2b^* + (1-2\alpha)\, b^{*2}\delta\,(1-b^*) - b^*\left(2 + (2-(3-2\alpha)\, b^*)\,\delta - (1-\alpha)\,\alpha b^{*3}\delta^2\right) \\
=&b^*\left((1-2\alpha)\, b^*\delta\,(1-b^*) - (2-(3-2\alpha)\, b^*)\,\delta + (1-\alpha)\,\alpha b^{*3}\delta^2\right) \\
=&b^*\delta\left(b^*\left((1-2\alpha)(1-b^*) + 3 - 2\alpha + (1-\alpha)\,\alpha b^{*2}\delta\right) - 2\right).
\end{aligned}$$

Note that

$$(1-2\alpha)(1-b^*) + 3 - 2\alpha + (1-\alpha)\,\alpha b^{*2}\delta < 4,$$

since

$$\begin{aligned}
&4 - \left((1-2\alpha)(1-b^*) + 3 - 2\alpha + (1-\alpha)\,\alpha b^{*2}\delta\right) \\
=&1 + 2\alpha - (1-2\alpha)(1-b^*) - (1-\alpha)\,\alpha b^{*2}\delta \\
=&1 + 2\alpha + \alpha\,(1-b^*) - (1-\alpha)\left(1 - b^* + \alpha b^{*2}\delta\right) \\
\geq&1 + 2\alpha + \alpha\,(1-b^*) - (1-\alpha)(1 - b^* + b^*) \\
>&0.
\end{aligned}$$

This implies that

$$b^*\left((1-2\alpha)(1-b^*) + 3 - 2\alpha + (1-\alpha)\,\alpha b^{*2}\delta\right) - 2 < 4b^* - 2 \leq 0,$$

which shows that Claim 1 is true.

Claim 2: $L_2 \geq 0$.

Note that $L_2 \geq 0$ is equivalent to $b^* L_2 \geq 0$:

$$\begin{aligned}
b^* L_2 =&b^*\Big(2 + (2-(3-2\alpha)\, b^*)\,\delta - (1-\alpha)\,\alpha b^{*3}\delta^2 + (2-\alpha)\,\delta + (1-\alpha)\,\alpha b^{*2}\delta^2 \\
&- (1+b^*)\left((3-2\alpha)\,\delta + 3\,(1-\alpha)\,\alpha b^{*2}\delta^2 - 2\,(1-\alpha)\,\alpha b^*\delta^2\right)\Big) \\
&\geq 0.
\end{aligned}$$

From (13), this can be rewritten as

$$\begin{aligned}
&1 + (2-\alpha)\,\delta b^* + (1-\alpha)\,\alpha b^{*3}\delta^2 \\
&- b^*(1+b^*)\left((3-2\alpha)\,\delta + 3\,(1-\alpha)\,\alpha b^{*2}\delta^2 - 2\,(1-\alpha)\,\alpha b^*\delta^2\right) \geq 0 \\
\Longleftrightarrow\ &1 + (2-\alpha)\,\delta b^* + (1-\alpha)\,\alpha b^{*3}\delta^2 \\
&- b^*(1+b^*)\,\delta\,(3 - 2\alpha + (1-\alpha)\,\alpha\delta b^*(3b^*-2)) \geq 0.
\end{aligned}$$

Note that

$$
\begin{aligned}
&1+(2-\alpha)\,\delta b^{*}+(1-\alpha)\,\alpha b^{*3}\delta^{2}\\
&\quad-b^{*}\,(1+b^{*})\,\delta\,(3-2\alpha+(1-\alpha)\,\alpha\delta b^{*}\,(3b^{*}-2))\\
\geq&1+\delta b^{*}\,(2-\alpha-(1+b^{*})\,(3-2\alpha))\\
=&1-\delta b^{*}\,(1-\alpha+(3-2\alpha)\,b^{*})\,.
\end{aligned}
$$

Again, using (13), the right-hand side becomes

$$
\begin{aligned}
&b^{*}\left(2+(2-(3-2\alpha)\,b^{*})\,\delta-(1-\alpha)\,\alpha b^{*3}\delta^{2}-\delta\,(1-\alpha+(3-2\alpha)\,b^{*})\right)\\
=&b^{*}\left(2+(1+\alpha-(6-4\alpha)\,b^{*})\,\delta-(1-\alpha)\,\alpha b^{*3}\delta^{2}\right)\\
\geq&b^{*}\left(2-(2-3\alpha)\,\delta-(1-\alpha)\,\alpha b^{*3}\delta^{2}\right).
\end{aligned}
$$

Note that this is nonnegative if $\alpha\geq\frac{2}{3}$. Otherwise,

$$
\begin{aligned}
b^{*}\left(2-(2-3\alpha)\,\delta-(1-\alpha)\,\alpha b^{*3}\delta^{2}\right)&\geq b^{*}\left(2-(2-3\alpha)-(1-\alpha)\,\alpha b^{*3}\right)\\
&=b^{*}\alpha\left(3-(1-\alpha)\,b^{*3}\right)\geq 0.
\end{aligned}
$$

Thus, Claim 2 is true.

Claim 3: $\frac{\partial b^{*}}{\partial\alpha}\leq 0$.

Recall that $\mathcal{K}\,(b)=b\left(2+(2-(3-2\alpha)\,b)\,\delta-(1-\alpha)\,\alpha b^{3}\delta^{2}\right)-1$. Thus,

$$
\begin{aligned}
\frac{\partial\mathcal{K}\,(b)}{\partial\alpha}=b\left(2b\delta-(1-2\alpha)\,b^{3}\delta^{2}\right)&=b^{2}\delta\left(2-(1-2\alpha)\,b^{2}\delta\right)\\
&\geq b^{2}\delta\left(2-b^{2}\delta\right)\\
&\geq 0.
\end{aligned}
$$

This completes the proof of Claim 3 because (i) b^{*} is given by the unique solution to $\mathcal{K}\,(b)=0$ for $b\in(0,1/2]$; (ii) $\mathcal{K}(0)<0$; and (iii) $\mathcal{K}\,(\cdot)$ is increasing in α.

Next, we show that μ^{0} is decreasing in δ using another expression for μ^{0}. By substituting $\widehat{q}=R\,(\mu^{0})=\mu^{0}$ in (8), we have

$$
\mu^{0}=\frac{(1-c)\,(1+\delta)}{3\,(1+\delta)+(1-\alpha)\,(1-b^{*})\,\delta+(1-\alpha)\,\alpha b^{*2}\delta^{2}}.
$$

Thus, μ^0 is decreasing in δ if and only if

$$3 + (1-\alpha)\frac{\delta}{1+\delta}\left(1 - b^* + \alpha b^{*2}\delta\right)$$

is increasing in δ. Since $\frac{\delta}{1+\delta}$ is increasing in δ and $1 - b^* + \alpha b^{*2}\delta \geq 0$, it suffices to show that $1 - b^* + \alpha b^{*2}\delta$ is increasing in δ.

Taking the derivative with respect to δ in $1 - b^* + \alpha b^{*2}\delta$ yields

$$-\frac{\partial b^*}{\partial \delta}\left(1 - 2\alpha b^*\delta\right) + \alpha b^{*2} \geq -\frac{\partial b^*}{\partial \delta}\left(1 - 2\alpha b^*\delta\right).$$

Thus, if $\frac{\partial b^*}{\partial \delta} < 0$, the proof is complete since $1 - 2\alpha b^*\delta > 0$.

Now, applying the implicit function theorem to (13), we have

$$\frac{\partial b^*}{\partial \delta} = -\left(\frac{b^*\left(2 - (3-2\alpha)\, b^* - 2\,(1-\alpha)\,\alpha b^{*3}\delta\right)}{2 + \left(2 - (3-2\alpha)\, b^*\right)\delta - (1-\alpha)\,\alpha b^{*3}\delta + b\left(2\alpha\delta - 3\,(1-\alpha)\,\alpha b^{*2}\delta^{*2}\right)}\right).$$

Note that both numerator and deonominator are positive. To see this, observe

$$2 - (3-2\alpha)\, b^* - 2\,(1-\alpha)\,\alpha b^{*3}\delta \geq 2 - 3b^* - \frac{1}{16} > 0,$$

and

$$2 + \left(2 - (3-2\alpha)\, b^*\right)\delta - (1-\alpha)\,\alpha b^{*3}\delta + b\left(2\alpha\delta - 3\,(1-\alpha)\,\alpha b^{*2}\delta^{*2}\right) \geq 2 - (1-\alpha)\,\alpha b^{*3}\delta^2,$$

where the inequality follows from $2 - (3-2\alpha)\, b^* > 0$ and

$$2\alpha\delta - 3\,(1-\alpha)\,\alpha b^{*2}\delta^2 = \alpha\delta\left(2 - 3\,(1-\alpha)\, b^{*2}\delta\right) \geq \alpha\delta\left(2 - 3b^{*2}\delta\right) \geq \frac{5\alpha\delta}{4}.$$

This completes the proof of the proposition.

Proof of Proposition 4. The limit results, $\lim_{(\alpha,\delta)\to(0,1)} \mu^0 = \frac{3(1-c)}{10}$ and $\lim_{\alpha\to 0} \widehat{q}^* = q^M$, can be directly checked by evaluating μ^0 and $\widehat{q}^*$ at the corresponding values of α and δ. Thus, it is sufficient to show that $\mu^0 > \widehat{q}^*$ for all $\delta \in (0,1)$ and $\alpha \in (0,1)$.

To this end, we will compare the first-order conditions in the simultaneous-offers game and the alternating-offers game at $\delta = 1$. On the one hand, the first-order condition in the simultaneous-offers game is given as:

$$\pi_1\left(\widehat{q}^*, \widehat{q}^*\right) + (1-\alpha)\,\pi_2\left(\widehat{q}^*, \widehat{q}^*\right) = c.$$

On the other hand, by substituting $\widehat{q} = R(\widehat{q}) = \mu^0$ in (8) and rearranging, we have

$$2\pi_1\left(\mu^0, \mu^0\right) + (1-\alpha)\,\pi_2\left(\mu^0, \mu^0\right)\left(1 - b^*\left(1 - b^*\alpha\right)\right) = 2c,$$

which implies that

$$\begin{aligned}\pi_1\left(\widehat{q}^*, \widehat{q}^*\right) + (1-\alpha)\,\pi_2\left(\widehat{q}^*, \widehat{q}^*\right) &= \pi_1\left(\mu^0, \mu^0\right) + (1-\alpha)\,\pi_2\left(\mu^0, \mu^0\right)\left(\frac{1 - b^*\left(1 - b^*\alpha\right)}{2}\right) \\ &> \pi_1\left(\mu^0, \mu^0\right) + (1-\alpha)\,\pi_2\left(\mu^0, \mu^0\right),\end{aligned}$$

which is equivalent to

$$-(4-\alpha)\,\widehat{q}^* > -(4-\alpha)\,\mu^0 \iff \widehat{q}^* < \mu^0.$$

This completes the proof.

Appendix B: Beta-Binomial Specification

Here, we propose an alternative setup in which (i) the variable profit $\left(P\left(q_i + q_{-i}\right) - c\right) q_i$ is always nonnegative; (ii) the conditional expectation of the rival retailer's quantity is a weighted average of the realized signal and the equilibrium target quantity; and (iii) the weight is simply characterized by relative volatility of mistakes and signals. Thus, the issue of negative profits under the normal specification is not crucial to our analysis and avoided without affecting our main results.

To be more precise, suppose that target and actual quantities lie on the bounded interval, $\left[0, \frac{1-c}{2}\right]$, so that an ex-post margin $\left(P\left(q_A + q_B\right) - c\right) q_i$ is always nonnegative. Given the target quantity $\widehat{q} \in \left(0, \frac{1-c}{2}\right)$, we assume that the actual quantity with mistakes is drawn from the beta distribution on the support $\left[0, \frac{1-c}{2}\right]$ with the left and right parameters

$$\left(\theta^*\left(\widehat{q}\right), \rho^*\left(\widehat{q}\right)\right) = \left(\frac{2\,(k-1)\,\widehat{q}}{1-c}, \frac{(k-1)\left(1-c-2\widehat{q}\right)}{1-c}\right),$$

for some $k > 1$. This distribution yields, for each $\widehat{q} \in \left(0, \frac{1-c}{2}\right)$,

$$E\left(q\,|\widehat{q}\right) = \widehat{q} \;\text{ and }\; Var\left(q\,|\widehat{q}\right) = \frac{1}{k}\widehat{q}\left(\frac{1-c}{2} - \widehat{q}\right).$$

Note that k governs the volatility of mistakes, and for a given k, there is a larger chance of mistakes when the target quantity is at more intermediate levels.[17]

[17]For the end points $\widehat{q} = 0$ and $\widehat{q} = \frac{1-c}{2}$, the actual quantities are assumed to be drawn from the degenerate distributions at $q = 0$ and $q = \frac{1-c}{2}$, respectively.

For the signal distribution, we assume that, given the actual quantity q, the signal is given by $s_{-i} = \frac{x(1-c)}{2n}$, where x follows the binomial distribution $\left(n, \frac{2q}{1-c}\right)$. Recall that x takes a value in $\{0, 1, 2, ..., n\}$, so s_{-i} takes a value in $\{0, \frac{1-c}{2n}, \frac{1-c}{n}, ..., \frac{1-c}{2}\}$. Note that the number of experiments n governs the precision of the signal.

Under this formulation, we obtain the following result.

Proposition 5 *Let $\widehat{q} \in \left[0, \frac{1-c}{2}\right]$ be the target quantity chosen by U. Then,*

$$E\left(q\,|\widehat{q}, s\right) = \left(\frac{1}{1+\frac{k-1}{n}}\right) s + \left(1 - \frac{1}{1+\frac{k-1}{n}}\right) \widehat{q}.$$

Proof. For simplicity, we write $\theta^* = \theta^*\left(\widehat{q}\right)$ and $\rho^* = \rho^*\left(\widehat{q}\right)$. The probability of $s_{-i} = s$ conditional on the actual quantity q is denoted by $f\left(s|q\right)$, and the density of the actual quantity q is denoted by $f\left(q|\widehat{q}\right)$. Finally, their joint probability will be written as $f\left(q, s|\widehat{q}\right)$.

First, observe that

$$\begin{aligned}
f\left(s_{-i} = s|\widehat{q}\right) &= f\left(x = \frac{2ns}{1-c}\,|\widehat{q}\right) \\
&= \int_{q=0}^{\frac{1-c}{2}} f\left(x = \frac{2ns}{1-c}\,|q\right) f\left(q\,|\widehat{q}\right) dq \\
&= \int_{q=0}^{\frac{1-c}{2}} \left(\binom{n}{\frac{2ns}{1-c}} \left(\frac{2q}{1-c}\right)^{\frac{2ns}{1-c}} \left(1 - \frac{2q}{1-c}\right)^{n-\frac{2ns}{1-c}}\right) f\left(q\,|\widehat{q}\right) dq \\
&= \binom{n}{\frac{2ns}{1-c}} \int_{q=0}^{\frac{1-c}{2}} \left(\frac{2q}{1-c}\right)^{\frac{2ns}{1-c}} \left(1 - \frac{2q}{1-c}\right)^{n-\frac{2ns}{1-c}} \left(\frac{q^{\alpha^*-1}\left(\frac{1-c}{2} - q\right)^{\beta^*-1}}{B\left(\theta^*, \rho^*\right)\left(\frac{1-c}{2}\right)^{\alpha^*+\beta^*-1}}\right) dq \\
&= \frac{1}{B\left(\theta^*, \rho^*\right)} \binom{n}{\frac{2ns}{1-c}} \int_{q=0}^{\frac{1-c}{2}} \frac{\left(\frac{2q}{1-c}\right)^{\frac{2ns}{1-c}} \left(1 - \frac{2q}{1-c}\right)^{n-\frac{2ns}{1-c}} q^{\theta^*-1}\left(\frac{1-c}{2} - q\right)^{\rho^*-1}}{\left(\frac{1-c}{2}\right)^{\theta^*+\rho^*-1}} dq,
\end{aligned}$$

where $B(\theta, \rho)$ is the beta function,

$$B(\theta, \rho) = \int_{x=0}^{1} x^{\theta-1}(1-x)^{\rho-1} dx.$$

In addition,

$$\begin{aligned}
f\left(q, s\,|\widehat{q}\right) &= f\left(s\,|q\right) f\left(q\,|\widehat{q}\right) \\
&= \binom{n}{\frac{2ns}{1-c}} \left(\frac{2q}{1-c}\right)^{\frac{2ns}{1-c}} \left(1 - \frac{2q}{1-c}\right)^{n-\frac{2ns}{1-c}} \frac{q^{\theta^*-1}\left(\frac{1-c}{2} - q\right)^{\rho^*-1}}{B\left(\theta^*, \rho^*\right)\left(\frac{1-c}{2}\right)^{\theta^*+\rho^*-1}}.
\end{aligned}$$

Therefore,

$$
\begin{aligned}
f\left(q\left|\widehat{q},s\right.\right) = \frac{f\left(q,s\left|\widehat{q}\right.\right)}{f\left(s\left|\widehat{q}\right.\right)} &= \frac{\left(\frac{2q}{1-c}\right)^{\frac{2ns}{1-c}}\left(1-\frac{2q}{1-c}\right)^{n-\frac{2ns}{1-c}} q^{\theta^*-1}\left(\frac{1-c}{2}-q\right)^{\rho^*-1}}{\int_{q=0}^{\frac{1-c}{2}}\left(\frac{2q}{1-c}\right)^{\frac{2ns}{1-c}}\left(1-\frac{2q}{1-c}\right)^{n-\frac{2ns}{1-c}} q^{\theta^*-1}\left(\frac{1-c}{2}-q\right)^{\rho^*-1} dq} \\
&= \frac{q^{\frac{2ns}{1-c}+\theta^*-1}\left(\frac{1-c}{2}-q\right)^{n-\frac{2ns}{1-c}+\rho^*-1}}{\int_{q=0}^{\frac{1-c}{2}} q^{\frac{2ns}{1-c}+\theta^*-1}\left(\frac{1-c}{2}-q\right)^{n-\frac{2ns}{1-c}+\rho^*-1} dq}.
\end{aligned}
$$

Note that, letting $y \equiv \frac{2q}{1-c}$, the denominator can be written as

$$
\begin{aligned}
&\int_{y=0}^{1}\left(\frac{1-c}{2}y\right)^{\frac{2ns}{1-c}+\theta^*-1}\left(\frac{1-c}{2}-\frac{1-c}{2}y\right)^{n-\frac{2ns}{1-c}+\rho^*-1}\left(\frac{1-c}{2}\right)dy \\
&= \left(\frac{1-c}{2}\right)^{n+\theta^*+\rho^*-1}\int_{y=0}^{1} y^{\frac{2ns}{1-c}+\theta^*-1}\left(1-y\right)^{n-\frac{2ns}{1-c}+\rho^*-1} dy.
\end{aligned}
$$

Therefore, we have

$$
f\left(q\left|\widehat{q},s\right.\right) = \frac{q^{\frac{2ns}{1-c}+\theta^*-1}\left(\frac{1-c}{2}-q\right)^{n\frac{2ns}{1-c}+\rho^*-1}}{\left(\frac{1-c}{2}\right)^{n+\theta^*+\rho^*-1}\int_{y=0}^{1} y^{\frac{2ns}{1-c}+\theta^*-1}\left(1-y\right)^{n-\frac{2ns}{1-c}+\beta^*-1} dy}.
$$

Indeed, this is the beta distribution on the support $\left[0,\frac{1-c}{2}\right]$ with the left and right parameters

$$
\left(\theta^{**},\rho^{**}\right) = \left(\frac{2ns}{1-c}+\theta^*, n-\frac{2ns}{1-c}+\rho^*\right).
$$

Thus, we have

$$
\begin{aligned}
E\left(q\left|\widehat{q},s\right.\right) &= \left(\frac{1-c}{2}\right)\left(\frac{\theta^{**}}{\theta^{**}+\rho^{**}}\right) \\
&= \left(\frac{1-c}{2}\right)\left(\frac{\frac{2ns}{1-c}+\theta^*}{n+\theta^*+\rho^*}\right) \\
&= \left(\frac{n}{n+\theta^*+\rho^*}\right)s + \left(\frac{\theta^*+\rho^*}{n+\theta^*+\rho^*}\right)\left(\frac{1-c}{2}\right)\left(\frac{\theta^*}{\theta^*+\rho^*}\right).
\end{aligned}
$$

Now, recall that

$$
\left(\theta^*,\rho^*\right) = \left(\frac{2\left(k-1\right)\widehat{q}}{1-c}, \frac{\left(k-1\right)\left(1-c-2\widehat{q}\right)}{1-c}\right),
$$

so we have $\theta^*+\rho^* = k-1$ for any $\widehat{q}$. Note also that

$$
\left(\frac{1-c}{2}\right)\left(\frac{\theta^*}{\theta^*+\rho^*}\right) = E\left(q\left|\widehat{q}\right.\right) = \widehat{q}.
$$

Therefore, we conclude that

$$
E\left(q\left|\widehat{q},s\right.\right) = \left(\frac{n}{n+k-1}\right)s + \left(1-\frac{n}{n+k-1}\right)\widehat{q}.
$$

■