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JEL Classification: D72, D82, E62, H62, H63

Keywords: Fiscal transparency

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Debt ceilings with fiscal intransparency and imperfect electoral accountability

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This version: March 2023

Abstract

Motivated by the recent debate about the EU fiscal framework, we analyse public debt ceilings in a political-economy model with uncertainty about both the type of policymaker (benevolent or selfish) and the state of the economy (good or bad). Pooling, hybrid and separating equilibria may exist. The presence of elections generates disciplining and selection effects, of which the relative importance differs across these equilibria. Shifts in the debt ceiling may lead to switches in the prevailing type of equilibrium, thereby inducing jumps in expected welfare. The optimal debt ceiling trades off the implied distortion in the intertemporal allocation of resources under a benevolent policymaker against excessive debt creation under a self-interested policymaker so as to finance diversion of resources for private use. Increased transparency in terms of voters learning with a higher probability the state of the economy before casting their vote affects neither the optimal ceiling nor expected welfare. If instead increased transparency allows for imposing state-contingent debt ceilings, for instance monitored by an independent fiscal institution, welfare may be improved. We identify the circumstances under which it is optimal to make debt ceilings state-contingent. State-contingent ceilings induce more frequent pooling, resulting in less excessive debt creation. The potential benefits of state-contingency support the greater role that recent European Commission proposals assign to the national independent fiscal institutions and the differentiation in debt reduction paths of EU Member States.

Keywords: Fiscal transparency, non-contingent and contingent debt ceilings, welfare, selection and disciplining effect of elections, independent fiscal institutions.
JEL Codes: E62, H62, H63, D72, D82.

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1 Introduction

In the years following the global financial crisis (GFC), government debt in many parts of the rich world surged to levels unprecedented in peacetime.[1] As a result, existing fiscal rules were strengthened and new rules were introduced. This was in particular the case in the European Union (EU), where countries are subject to the numerical fiscal rules of the Stability and Growth Pact (SGP). In addition, to the extent that they did not already have them in place, EU Member States established new fiscal oversight bodies referred to as "independent fiscal institutions" (IFIs). Both fiscal rules and IFIs are intended to improve fiscal policymaking, usually by enforcing fiscal discipline and sometimes also by encouraging more counter-cyclical fiscal stances. Fiscal rules take the form of numerical limits on budgetary aggregates, typically the government debt or deficit, and sometimes public expenditures.[2] IFIs are generally intended to reduce informational asymmetries between the government and the public, thereby enhancing the enforceability of fiscal rules and improving electoral accountability. In particular, they may be in charge of monitoring and publicly reporting the government's adherence to existing fiscal rules. They may also be mandated to provide independent and unbiased information relevant to the public budget. Specifically, in the EU they have to either construct the macroeconomic projections underlying the government's budgetary plans or consent the projections produced by the government. Endowed with this task, IFIs help to enhance budgetary transparency, which puts pressure on governments to pursue better fiscal policies and aids voters in making informed choices at the ballot box. Unfortunately, however, the revisions to the fiscal framework following the GFC have not prevented EU governments from having debts above the 60% of GDP ceiling. In fact, the corona crisis that started in early 2020 has caused another upward jump in public debt levels, the jump being largest for the countries with the highest debt just prior to corona (European Fiscal Board, 2021). Under its severe economic downturn clause, the SGP's enforcement is temporarily halted. Prominent in the recent debate about a revision of the SGP is the question whether debt ceilings and adjustment speeds towards these ceilings should be differentiated based on starting positions and the state of the economy.

In this paper we explore optimal public debt ceilings when both the type of incumbent policymaker and the state of the economy are unobserved by voters. This information asymmetry captures that, in reality, policymakers' motives are typically not perfectly known to voters.[3] Neither is the actual state of the economy – ex-post revisions of macroeconomic figures are common (Beetsma et al. (2009) and Beetsma et al. (2022a)) and sometimes these are quite substantial. In our two-period model policymakers can either be benevolent, maximizing voter welfare, or selfish, only interested in resources diverted to private use. In the first period the incumbent politician allocates resources over public good provision and private rents. Available resources follow from tax revenues and the issuance of public debt, where the tax revenues depend on the state of the economy (either bad or good). Voters observe

[1] Reinhart and Sbrancia (2015) report aggregate figures for the central government debt-to-GDP ratio for advanced economies from 1900 onward. In 2011, this aggregate ratio reached a record high until then during peacetime and almost surpassed the post-World War II level. During the corona crisis, debt levels have surpassed this old record debt level by a wide margin.

[2] See Debrun et al. (2013) and Debrun and Kinda (2017) for an extensive description of fiscal rules and their evolution.

[3] What motivates candidates for political office is ultimately an empirical question. In reality, most candidates likely represent a mixture of the two types. However, the insights from our analysis on the welfare trade-off associated with the selection and disciplining effects of elections will be sharpest when we assume the existence of only these two extreme types.

the level of public good provision and the amount of public debt created, but do not observe the tax revenues (i.e. the state of the economy) nor the amount of private rents extracted. At the end of period 1 they either re-elect the incumbent or elect the challenger; the winner then makes the policy decisions in period 2. In period 2 a selfish policymaker extracts all available resources for private use. The relevant decision for a selfish incumbent in period 1 is thus whether to extract all the rents now, or postpone (potentially, viz. if re-elected) doing so to period 2 in which the rent extraction possibilities might be better. We assume that the prevailing fiscal rules impose a perfectly enforceable debt ceiling in period 1. Our analysis focuses on setting this ceiling optimally as to maximize voter welfare.

Different types of equilibria may arise: a pooling equilibrium in which in the first period the selfish incumbent mimics the benevolent type irrespective of the state of the world, a hybrid one in which the selfish incumbent mimics the benevolent type when the state is bad, but grabs all the resources when the state is good and available resources are plenty, and a separating equilibrium in which the selfish incumbent appropriates all resources irrespective of the state. Such first-period rent-seeking implies creating debt for the wrong reasons, i.e. it corresponds to *excessive* debt creation. Curbing excessive debt creation is in our model thus equivalent to curbing (first-period) rent-seeking. The presence of elections exerts both disciplining and selection effects, of which the relative importance varies across the different types of equilibria. The optimal non-contingent debt ceiling trades off curbing diversion of resources by the selfish type (requiring a tight ceiling) against minimizing the distortion in the intertemporal allocation of resources by the benevolent type (facilitated by a loose ceiling). In finding the optimal ceiling, we need to take account of the possibility that a change in the debt ceiling may cause a shift in the prevailing type of equilibrium, thereby producing a discrete jump in welfare. Hence, the optimal debt ceiling is found by comparing welfare levels across the various possible types of equilibria.

An important question concerns the benefit of enhancing fiscal transparency. This benefit depends on the extent to which transparency can be improved and whether debt ceilings can be made state-contingent. We show that the optimal non-contingent debt ceiling and equilibrium welfare remain unaffected by merely raising the probability with which the true state can be observed by voters before elections take place. This follows because the selfish incumbent modifies her equilibrium strategy accordingly in that she no longer hides behind the state so as to avoid being unmasked. Loosely put, enhanced electoral accountability *per se* has no impact on the effectiveness of an (optimally chosen) constant ceiling. Translated to the context of the EU, the assignment of the task to IFIs of clarifying the state of the economy through own projections or consenting those of the government would not yield any benefits if debt ceilings cannot be made contingent on this information. Indeed, so far the installation of the national IFI's in the EU has done little to compel governments to build buffers by limiting public debt accumulation.

However, if transparency about the state can be raised to an extent that state-contingent debt ceilings can be installed, equilibrium welfare may well increase. The net benefit depends on the area within the parameter space for which it is optimal to differentiate the debt ceiling between the states. In the EU context, establishing with sufficient precision the state of the economy for this purpose could be the task of a well-designed IFI. It would be a natural extension of its current assignment of constructing or consenting the macroeconomic projections. Imposing state-contingent debt ceilings in the EU would be politically difficult, however, because countries could accuse the European Commission of a horizontally unequal treatment. Even if differentiation in debt ceilings is economically fully justified, countries with tighter ceilings imposed may try to play the card of unequal treatment or refer to adverse economic circumstances.[4] The Commission would likely find it difficult to resist such pressure. Assessment and verification of the economic state by an IFI would strengthen the hand of the Commission if it sought to differentiate debt ceilings.

Within our framework installing state-contingent debt ceilings does not produce a strict welfare improvement over a constant ceiling only in the exceptional case where the optimal constant ceiling would never bind the benevolent type and would induce the selfish type to always postpone rent-seeking, i.e. a pooling equilibrium with no debt creation for rent-seeking purposes. In all other instances a strict welfare gain is made. The most typical case here is where the optimal non-contingent ceiling induces a hybrid equilibrium, as the alternative of pooling necessitates a ceiling so restrictive that it particularly hurts a benevolent incumbent when the state is bad. State-contingent ceilings eliminate this downside and the induced outcome typically shifts away from rent-seeking in the short run towards more frequent pooling. This results in a less frequent short-term loss of resources under state-contingent ceilings, but also makes it more likely that resources under a benevolent incumbent are suboptimally allocated. The benefit of the former dominates the disadvantage of the latter. We find further that state-contingent ceilings are not valuable inasmuch they allow to induce a different type of rent seeking behavior (now vs. later) under different economic circumstances, but rather to induce typically the same type of behavior in a tailor made way. In loose terms, optimal state-contingent ceilings tackle rent seeking – and thus excessive debt creation – in a uniform manner.

Our finding on the strict optimality of state-contingent debt ceilings supports an SGP that links the required improvement in the public finances to the state of the economy. This is effectively the case when a country is under an Excessive Deficit Procedure and confronted with an unexpected deterioration of the economy, in which case its plan to correct the public finances is allowed to be revised. This is also the case under the preventive arm of the SGP, for example when growth is negative. The recent proposals by the European Commission (2022) contain a path for public spending that serves as a reference for the Member State to come up with its own proposal.[5] While these proposals need to be worked out further, it is likely that the Member State is allowed to factor in the state of the economy when it makes its own proposal, as the Commission's proposals would require debt of countries with a high sustainability risk to be only on a sustainable downward path at the end of a reference period of four years (or seven years in the case of structural reforms or public investments sufficient to alleviate future budgetary pressures).

Our paper extends the two-period model in Besley and Smart (2007) by assuming that public resources are stochastic, by introducing public debt and (most importantly) by exploring optimal debt ceilings, which may be constant or state-contingent in the case of an IFI able to detect the state of the economy.[6] It also relates to three other strands of the literature. First, it speaks to the literature on

[4]In the past, the Commission has regularly sought to avoid starting a procedure based on the preventive arm of the SGP by arbitraging between different requirements (on the structural balance versus the so-called expenditure benchmark) imposed on Member States.

[5]More precisely, the requirements on the Member State are to be expressed in terms of a net primary expenditure path that corrects primary expenditure for discretionary revenue measures and cyclical unemployment expenditures.

[6]In Besley and Smart (2007) the government can maximally extract a fixed amount X from the voters in each period.

political agency and accountability, which was first formalized by Barro (1973) and Ferejohn (1986).[7] Political agency models featuring both moral hazard and adverse selection can be split into signaling and career concern type of models. As this paper highlights the role of different political motives, which should arguably be known to the politicians themselves, we resort to the signaling approach.[8]

Second, this paper connects to the literature on the political economy of public debt and the so-called debt bias in fiscal policy making. In Alesina and Tabellini (1990) the debt bias arises from competition between different interest groups for scarce public resources. Instead, in Rogoff and Sibert (1988) and Rogoff (1990), the incumbent uses high public expenditures to signal her competence. In more recent contributions, such as Halac and Yared (2022), the debt bias is often attributed to the myopic behavior of politicians with a discount rate higher than that of the electorate.

Lastly, this paper relates directly to the literature on fiscal constraints and fiscal transparency. Resistance to numerical fiscal rules is often motivated with the argument that they hamper the government's ability to smooth business cycles.[9] Moreover, Rogoff (1990) and Besley and Smart (2007) argue against fiscal constraints, as they prevent effective signaling. In contrast, Reinhart and Rogoff (2010) and Obstfeld (2013) make a case for fiscal constraints by highlighting the perils of debt overaccumulation.[10] Furthermore, Fatas and Mihov (2003) and Fatas and Mihov (2006) build an empirical case for fiscal constraints by showing that they tend to mitigate output volatility. Contributions focussing on fiscal rules in monetary unions are Beetsma and Uhlig (1999), Chari and Kehoe (2007), Chari and Kehoe (2008), Debrun et al. (2008), Krogstrup and Wyplosz (2010), Hatchondo et al. (2022a) and Hatchondo et al. (2022b). Iara and Wolff (2011) and Reuter (2015) investigate national fiscal rules in the euro area and Halac and Yared (2018) analyse the coordination of fical rules. Dovis and Kirpalani (2020) and Halac and Yared (2022) explore fiscal rules in the absence of commitment to enforce these rules. Turning to the effect of fiscal transparency, Besley and Smart (2007) argue that more information might enhance selection, but undermine voters' ability to fiscally discipline incumbents. A similar result is obtained by Beetsma et al. (2022b). Milesi-Ferretti (2004) raises the concern that more fiscal transparency might result in more creative accounting strategies to prevent the detection of poor fiscal

Hence, in view of time discounting and the possibility of being voted out of office, in their setup it is always optimal for a selfish incumbent to extract an amount X of rents in the first period of her tenure if the unit costs of public good provision are high. Only if the latter are low, the selfish type may try to stay in office by foregoing some rents in the first period of her tenure and pretend the costs of public good provision are high (thereby extracting some rents already), in order to (potentially) reap the additional maximum rent extraction opportunities in the future. In contrast, in our setup also a selfish type that encounters circumstances that in principle favor immediate rent seeking may refrain from doing so, given that future rent seeking possibilities may be even better. Whether this happens depends on the spread in resources between economic states and the amount of debt carried into the future.

[7]The theoretical nexus between electoral accountability and fiscal policy has subsequently also been explored empirically. See, for instance, Besley and Case (1995a) and Alt et al. (2011), who exploit changes in gubernational term limits in the US to show that fiscal outcomes improve when office-holding politicians can run again for office compared to when they face a term limit. Further, Bonfiglioli and Gancia (2013) provide empirical evidence underscoring that the policy choices of incumbents are sensitive to the degree of accountability.

[8]In contrast to signaling models, in career concern type of models the agent ex-ante does not know her own type. This modeling approach is often chosen when the competence of political candidates is allowed to differ, since competence is arguably not perfectly observable by the candidate herself. Rogoff (1990), Coate and Morris (1995) and Besley and Case (1995b) developed early political agency signaling models with both moral hazard and adverse selection. The career concern type of political agency model à la Holmstrom (1999) was proposed by Persson and Tabellini (2002). Beetsma et al. (2022b) explore the benefits of fiscal transparency using a career concern type of model.

[9]This argument goes back to Lucas and Stokey (1983) and has been revisited empirically by e.g. Blanchard and Perotti (2002).

[10]Reinhart and Rogoff (2010) establish that, beyond a certain level, higher public debt is associated with lower economic growth. Obstfeld (2013) argues that budgetary prudence is necessary for governments to retain space to react to unforeseen shocks causing large-scale macroeconomic downturns.

behavior. However, using a panel of the OECD member states Alt and Lassen (2006) show empirically that more fiscal transparency reduces public debt accumulation. Finally, Bordignon and Minelli (2001) and Dovis and Kirpalani (2021) explore the optimal degree of transparency of (fiscal) rules.

The remainder of the paper is structured as follows. In Section 2, we introduce the baseline model. Section 3 contains the equilibrium analysis underpinning the ensuing derivation of our results. In Section 4 we derive expected welfare and study optimal debt ceilings in the absence of any fiscal transparency. Section 5 studies the implications of fiscal transparency about the state of the economy for optimal debt ceilings and expected welfare. Finally, Section 6 concludes the main body of the paper.

2 The Model

We present a two-period model with both moral hazard and adverse selection, in which voters delegate their decision power over public spending and debt accumulation to an elected politician. Politicians come in two different types: either benevolent or selfish. A benevolent politician derives utility from public good provision only, while a selfish politician only derives utility from private rents. Voters do not directly observe a politician's type.

In the first period the incumbent politician allocates resources to public good provision or to private rents. The available first-period resources are determined by tax revenues and the issuance of public debt. Tax revenues depend on the unobserved stochastic state of the economy. Between the two periods an election takes place in which voters either re-elect the incumbent or elect the challenger. The election serves two purposes: disciplining a potentially selfish politician and (in expectation) selecting the best candidate from the voters' perspective. In the second period a benevolent politician spends on public good provision only, while a selfish government, no longer constrained by a re-election motive, spends only on private rents.

Our set-up introduces important extensions to the workhorse model in Besley and Smart (2007) that will be crucial for the outcomes of our analysis. In particular, we introduce stochastic tax revenues and public debt, which allows us to explore optimal debt ceilings. Public debt can serve as an instrument to re-allocate resources over time and to signal the incumbent's type. Because tax revenues are stochastic, there may be circumstances in which a selfish incumbent foregoes rent extraction in the first period, in order to improve her future re-election prospects and appropriate all government resources during a potential second term in office. Moreover, in contrast to Besley and Smart (2007), in our setup a selfish incumbent has more than one option to mimick the benevolent type, because she can also pretend that the state of the economy is different than it actually is.

We now turn to the specifics of the model. Each period $t = 1, 2$, the office-holding politician allocates public spending over public goods g_t and private rents s_t. Tax revenues are τy_t, where y_t is stochastic income and τ is the exogenous tax rate, which is common knowledge and which for convenience we assume to be constant over the two periods. Income can either be high (y_H) or low (y_L), with $p \equiv Pr(y_t = y_H)$ and $y_L < y_H$. It is identically and independently distributed over the two periods. Because the tax rate and stochastic income never feature in isolation in the analysis below, we save on notation by simply setting $\tau = 1$. The government budget constraints in periods 1, respectively 2, then equal:

$$\begin{aligned} g_1 + s_1 &= y_1 + d \\ g_2 + s_2 &= y_2 - d \end{aligned} \tag{1}$$

where d is the amount of public debt issued in period 1 and which must be repaid in period 2. Without loss of generality, we assume that the (net) rate of interest on the debt is zero. The repayment restriction and the fact that the government cannot accumulate more assets than the tax revenues in period 1 imply that d must satisfy $-y_1 \leq d \leq y_L$. Moreover, prevailing fiscal rules additionally impose a debt ceiling $\overline{d}$. Hence the amount of debt chosen should necessarily satisfy:

$$-y_1 \leq d \leq \min\left\{y_L, \overline{d}\right\} \tag{2}$$

The population consists of a large number of voters with mass normalized to unity. They only obtain utility from public good consumption. Hence, the preferences of a representative voter are given by

$$W(g_1, g_2) = u(g_1) + \beta u(g_2), \tag{3}$$

where $0 < \beta \leq 1$ is the discount factor and the felicity function $u(\cdot)$ is assumed to be increasing and strictly concave. We also assume that the Inada condition $\lim_{g\to 0} u'(g) = \infty$ holds, which ensures an internal solution $-y_1 < d < y_L$ to the benevolent incumbent's debt optimization problem in the absence of an effective debt ceiling $\overline{d}$ (i.e., when $\min\left\{y_L, \overline{d}\right\} = y_L$).

In each of the two periods, the governing politician i can either be benevolent with unconditional probability $Pr(i = b) = \theta$ or selfish with unconditional probability $Pr(i = s) = 1 - \theta$, where b and s label the benevolent and selfish type, respectively. The type of the challenger at the election is drawn independently from the same unconditional distribution as the incumbent.

A benevolent politician shares voters' preferences. However, for tractability we assume that such an incumbent behaves "myopically" by maximizing voters' expected utility under the myopic expectation that she will be re-elected with certainty.[11] Hence, in period 1, a benevolent incumbent solves:

$$\max_{g_1, g_2, s_1, s_2, d} u(g_1) + \beta E\left[u(g_2)\right] \quad \text{s.t.} \quad (1) \text{ and } (2) \tag{4}$$

Here, and in the sequel, $E(.)$ is always the expectation taken over the state of the economy (so never over the type of politician). A selfish politician is only interested in the amount of rent extraction. Specifically, her utility increases linearly in the amount of rents extracted and, hence, she solves:

$$\max_{g_1, g_2, s_1, s_2, d} s_1 + \pi\beta E\left[s_2\right] \quad \text{s.t.} \quad (1) \text{ and } (2) \tag{5}$$

where π denotes the re-election probability, which is endogenously determined in equilibrium.

The timing and information structure are as follows. At the start of period 1, nature draws the

[11] In Besley and Smart (2007), the benevolent politician acts in a similarly myopic way, i.e., she only maximizes current-period welfare without considering the implications for future welfare. See Lockwood (2005) for a discussion on the implications of introducing a benevolent type who acts strategically in their framework.

incumbent's type as well as the state of the economy, both of which are private information to the incumbent. Then, the incumbent chooses period-1 public good provision, rent extraction and public debt. The voters only observe public good provision and public debt; they cannot directly observe whether some resources were used for private rents. Based on the observed public debt and public good provision, voters thus form expectations on the amount of rent extraction and update their beliefs about the incumbent's type. (Since the challenger cannot take any verifiable actions to reveal information about its type, the voters' best guess about the challenger j equals their prior belief, i.e., $Pr(j = b) = \theta$.) Subsequently, elections take place, in which voters re-elect the incumbent if, based on their updated beliefs, they think she is more likely to be benevolent than the challenger. Finally, the newly-elected politician observes the period-2 state of the economy, after which she allocates the period-2 resources that remain after debt repayment over public good provision g_2 and rents s_2. Throughout, voters are assumed to behave sequentially rational and in a Bayes-consistent way.

3 Equilibrium analysis

In this section we first characterize the benevolent type's optimal debt choices (Section 3.1) and the selfish type's incentives to mimic these (Section 3.2), before fully characterizing all equilibria in the final subsection.

3.1 Optimal debt choices of the benevolent type

First, consider optimization problem (4) of a benevolent incumbent in the absence of a (effective) debt ceiling, i.e., when $\min\left\{y_L, \overline{d}\right\} = y_L$. The standard Euler condition for an interior solution then reads:

$$u'\left(y_1 + d\right) = \beta E\left[u'(y_2 - d)\right] \tag{6}$$

Part (a) of Lemma 1 below characterizes the solution to this equation. As one intuitively would expect, the incumbent's most preferred debt level is strictly decreasing in y_1. However, the prevailing debt ceiling $\overline{d}$ may prevent her from choosing this most preferred level. In that case the benevolent type is forced to accumulate no more debt than the ceiling allows.

Lemma 1. (*Optimal debt choice benevolent type.*)

(a) For each $y_1 \in \{y_L, y_H\}$ equation (6) has a unique solution $d\left(y_1\right)$ satisfying $\frac{\partial d(y_1)}{\partial y_1} < 0$. Moreover, these preferred debt levels satisfy (i) $-\frac{(y_H - y_L)}{2} < d\left(y_H\right) < d\left(y_L\right) < y_L$ and (ii) $0 < d\left(y_L\right) < d\left(y_H\right) + \left(y_H - y_L\right)$;

(b) The benevolent type's optimal choice of debt equals $d^*\left(y_1\right) \equiv \min\left\{d\left(y_1\right), \overline{d}\right\}$ for $y_1 \in \{y_L, y_H\}$. Public goods spending becomes $g_1^*\left(y_1\right) \equiv y_1 + d^*\left(y_1\right)$, with $g_1^*\left(y_L\right) < g_1^*\left(y_H\right)$.

From Lemma 1 it follows that the benevolent type will choose policy combination $\left(d^*\left(y_1\right), g_1^*\left(y_1\right)\right)$ in the first period. Compared to the case where economic circumstances are good, a (weakly) higher debt is chosen and less public goods are provided when the state of the economy is bad.

In general, there are various motives for the benevolent type to create debt in our setting. First, there is a public goods smoothing motive resulting from decreasing marginal returns to public good provision (as the felicity function $u(\cdot)$ is concave). The larger y_1 is, the less debt is created per this motive per se. Second, the discounting of future utility ($\beta \leq 1$) creates an incentive to borrow against the future so as to have more public goods today rather than tomorrow. Third, future income y_2 is uncertain while current income y_1 is not. Risk aversion – also captured by $u(\cdot)$ being concave – then provides incentives to shift part of the certain period 1 income to the uncertain future (if risk aversion does not increase with wealth). This is the pre-cautionary motive.

The contribution of each of these drivers depends on the concavity of the felicity function and in general is hard to perfectly isolate. Yet to gain intuition, suppose there is no uncertainty, such that risk considerations do not play a role. Without uncertainty, period 2 income is fixed; denote this income level as $E(y_2)$ to make it easily comparable with the existing notation. In that case the benevolent type would prefer a debt level that weakly exceeds the one that would perfectly smooth the net present values of public good spending across periods, i.e. $d(y_1) \geq c_1$ with c_1 given by:[12]

$$c_1 \equiv \frac{\beta E\left(y_2\right) - y_1}{1+\beta} \quad \text{for } y_1 \in \{y_L, y_H\} \tag{7}$$

Note that $c_H < 0$ necessarily (c_L can take either sign). Without uncertainty perfect smoothing of public good spending would occur if $\beta = 1$. However, if $\beta < 1$, the benevolent type has a positive rate of time preference, causing period 1 public consumption to exceed the discounted value of period 2 public consumption when uncertainty is absent. From $d(y_1)$ decreasing in β and c_1 increasing in β, it follows that $d(y_1) - c_1$ is decreasing in β.

Only risk considerations in the presence of uncertainty may pull the preferred debt below threshold c_1. Assuming that risk aversion does not increase with income, intuitively one would then expect this downward pulling force to be weaker the larger first-period income y_1 is. Lemma 2 confirms this intuition by providing a sufficient condition on $u\left(\cdot\right)$ satisfied by both CARA and CRRA (for $x \geq 0$) preferences (as well as for logarithmic utility $u\left(x\right) = \ln x$).

Lemma 2. (*Sufficient condition for monotonic first-best incentives to create debt beyond public goods smoothing.*) $d\left(y_1\right) - c_1$ *is increasing in* y_1 *if the felicity function satisfies* $u''' > 0$ *and* $\frac{u''''}{u'''} \leq \frac{u'''}{u''}$. *An immediate implication is then that* $d\left(y_L\right) > c_L \Rightarrow d\left(y_H\right) > c_H$.

For now Lemma 2 only serves intuition, but is not actively employed. In Section 5.2 we will make fruitful use of this lemma when we compare the current case of a constant debt ceiling $\overline{d}$ with the one in which the ceiling can be made state-contingent.

[12]Starting from the Euler equation (6) defining the preferred debt $d(y_1)$ we have in the absence of uncertainty:

$$u'\left(y_1 + d\right) = \beta E\left[u'(y_2 - d)\right] = \beta\left[u'(Ey_2 - d)\right] \leq u'(Ey_2 - d) \leq u'(\beta Ey_2 - \beta d)$$

Here the second equality just notation wise reflects the absence of uncertainty. The first inequality follows from $\beta \leq 1$ and $u' > 0$, while the second inequality follows from $u'' \leq 0$ and $y_2 - d \geq 0$ (and thus $\beta y_2 - \beta d \leq y_2 - d$). Hence, by $u'' \leq 0$ we get $y_1 + d \geq \beta Ey_2 - \beta d$ and thus $d(y_1) \geq c_1$.

Policy combination	$y_1 = y_L$	$y_1 = y_H$	$\Pr[chosen]$ in eq.
$(d^*(y_L), g_1^*(y_L))$	$\pi_L \cdot \beta(E(y_2) - d^*(y_L))$	$(y_H - y_L) + \pi_L \cdot \beta(E(y_2) - d^*(y_L))$	$\lambda(y_1)$
$(d^*(y_H), g_1^*(y_H))$	$-(y_H - y_L) + \pi_H \cdot \beta(E(y_2) - d^*(y_H))$	$\pi_H \cdot \beta(E(y_2) - d^*(y_H))$	$\gamma(y_1)$
$(\overline{d}, 0)$	$y_L + \overline{d}$	$y_H + \overline{d}$	$1 - \lambda(y_1) - \gamma(y_1)$

Table 1: Expected payoffs of the selfish incumbent from the three relevant policy combinations

3.2 Mimicking incentives of the selfish type

A selfish politician only gains utility from extracting rents. Hence, the question is not whether this type will eventually extract rents, but rather *when* she will do so. Clearly, the latter choice is made in tandem with the choice of debt. To gain intuition, first consider the timing choice in isolation, assuming a given level of debt d. Comparing the net present values of the available expected resources for rent extraction of $y_1 + d$ in period 1 and of $\beta(E(y_2) - d)$ in period 2 (and in view of the linearity of its preferences), the selfish type prefers full rent extraction in the first period over full rent extraction in the second period whenever $d > c_1$. Note that threshold c_1 is decreasing in y_1 (cf. equation (7)), implying that the selfish type in principle has a stronger incentive to front-load rent extraction the higher y_1 is.

The decision when to extract rents is not independent of the choice of debt though. Postponing (some) rent extraction to the second period is possible only if the selfish incumbent is actually re-elected. Voters will only be inclined to re-elect the incumbent if they observe a policy combination that is also chosen by the benevolent type, i.e. either $(d^*(y_L), g_1^*(y_L))$ or $(d^*(y_H), g_1^*(y_H))$. A deviant policy combination (d, g_1) fully reveals that the incumbent is selfish and leads to her being voted out of office. In that case, the best a selfish incumbent can do is to extract rents in the first period to the fullest extent possible (i.e. $s_1 = y_1 + \overline{d}$) and choose policy $(\overline{d}, 0)$. Table 1 provides the expected payoffs from the three relevant policy combinations a selfish incumbent could effectively choose, conditional on the actual economic state in the first period. We use π_1 to denote the probability that voters re-elect the incumbent after observing $(d^*(y_1), g_1^*(y_1))$.

Note that, for a given policy combination, the payoffs in economic state $y_1 = y_H$ always exceed those in state $y_1 = y_L$ by the fixed amount $(y_H - y_L)$. Hence, the selfish incumbent's preference ranking is exactly the same in both economic states. This ranking depends (among other things) on the level of the debt ceiling $\overline{d}$ and the re-election probabilities π_L and π_H.

Lemma 3. (*Mimicking incentives selfish type.*)

(a) If $\overline{d} > c_1 + \max\{0, \beta(c_1 - d(y_1))\}$, a selfish incumbent always strictly prefers policy combination $(\overline{d}, 0)$ over $(d^*(y_1), g_1^*(y_1))$ and thus never mimics the benevolent type;

(b) If $\pi_L \geq \pi_H$, a selfish incumbent strictly prefers policy combination $(d^*(y_L), g_1^*(y_L))$ over policy combination $(d^*(y_H), g_1^*(y_H))$.

The most the perfect mimicking of the benevolent type – i.e. choosing $(d^*(y_1), g_1^*(y_1))$ when first-period income equals y_1 – can yield to the selfish incumbent is that she is re-elected for sure $(\pi_1 = 1)$.

In that case the selfish type would obtain $\beta\left(E\left(y_2\right)-d^*\left(y_1\right)\right)$ from full rent extraction in the second period. This payoff (weakly) decreases with debt ceiling $\overline{d}$. Fully extracting rents today by choosing $\left(\overline{d},0\right)$ brings the selfish type $y_1+\overline{d}$. Therefore, policy combination $\left(\overline{d},0\right)$ becomes best if $\overline{d}$ is sufficiently large. Part (a) of Lemma 3 provides the relevant threshold for $\overline{d}$.

Apart from perfect mimicking, the selfish type may also deceive in two dimensions simultaneously; pretend to be of the benevolent type while at the same time suggesting that the state is different than it actually is. That is, the selfish type could also 'hide behind the state' and opt for $\left(d^*\left(y_L\right),g_1^*\left(y_L\right)\right)$ when $y_1=y_H$, or for $\left(d^*\left(y_H\right),g_1^*\left(y_H\right)\right)$ when $y_1=y_L$. In the former case the selfish incumbent then already grabs rents $s_1=\left(y_H-y_L\right)$ in period one when economic conditions are good and hopes to be able to grab an additional $\beta\left(E\left(y_2\right)-d^*\left(y_L\right)\right)$ in rents in the next period. In the latter case the selfish incumbent actually invests $\left(y_H-y_L\right)$ out of her own pocket when current economic conditions are bad as to facilitate grabbing $\beta\left(E\left(y_2\right)-d^*\left(y_H\right)\right)$ in the future.[13] Intuitively, one would expect that such an investment makes sense only if by doing so the selfish type increases her probability of re-election, i.e. if $\pi_H>\pi_L$. Part (b) of Lemma 3 confirms this intuition. It shows more generally that the selfish incumbent may prefer $\left(d^*\left(y_H\right),g_1^*\left(y_H\right)\right)$ only if it leads to a larger probability of being re-elected. This can be immediately understood as follows. Compared to $\left(d^*\left(y_L\right),g_1^*\left(y_L\right)\right)$, policy combination $\left(d^*\left(y_H\right),g_1^*\left(y_H\right)\right)$ reduces resources left for rent extraction today by $\left(y_H-y_L\right)$ and increases potential future resources for doing so by $d^*\left(y_H\right)-d^*\left(y_L\right)$. By Lemma 1 the latter always fall short of the former.

Lemma 3 does not fully pin down the optimal debt choice of the selfish type, as this depends on the exact values of the re-election probabilities π_L and π_H. These are endogenously determined in equilibrium, to which we turn next.

3.3 Equilibrium characterization

In a perfect Bayesian equilibrium both the (selfish) incumbent and the voters behave rationally and best respond against each other's behavior. Let $\lambda\left(y_1\right)$ for $y_1\in\left\{y_L,y_H\right\}$ denote the probability with which the selfish type of incumbent chooses policy combination $\left(d^*\left(y_L\right),g_1^*\left(y_L\right)\right)$ and $\gamma\left(y_1\right)$ for $y_1\in\left\{y_L,y_H\right\}$ the probability with which she chooses $\left(d^*\left(y_H\right),g_1^*\left(y_H\right)\right)$. With the remaining probability $1-\lambda\left(y_1\right)-\gamma\left(y_1\right)$ she then opts for policy combination $\left(\overline{d},0\right)$. The final column of Table 1 reflects this notational convention for ease of reference.

Starting from their prior belief θ voters use Bayes' rule to update their beliefs about the incumbent's type, based on the policy combination actually chosen. If this posterior belief exceeds prior θ voters re-elect the incumbent, while if it falls short of θ they elect the challenger. Only if their posterior equals the prior voters are indifferent and willing to use a mixed strategy.

Proposition 1 below fully characterizes the three different classes of equilibria that exist. Here an equilibrium is labelled *separating* if voters can perfectly infer the incumbent's type based on the policy chosen and labelled *pooling* if they cannot infer anything (such that the posterior belief always equals

[13] We thus assume that the selfish incumbent is not cash constrained and can indeed pay $\left(y_H-y_L\right)$ out of her own pocket. This may not be particularly realistic. As we show in Section 4, however, our analysis of equilibrium welfare and optimal debt ceilings holds irrespective of whether (in equilibrium) the selfish incumbent actually ever chooses $\left(d^*\left(y_H\right),g_1^*\left(y_H\right)\right)$ when $y_1=y_L$.

the prior).[14] Equilibria that fall in between these extremes are called *hybrid.* Intuitively (cf. Lemma 3), the level of the debt ceiling $\overline{d}$ crucially determines the type of equilibrium that applies.

Proposition 1.[15] *(Equilibria under a constant debt ceiling.)*

(a) If $\overline{d} > c_L + \max\{0, \beta(c_L - d(y_L))\}$: There exists a unique *separating* equilibrium in which the selfish type always chooses full rent extraction $(\overline{d}, 0)$ in the first period. Equilibrium strategies satisfy:

(a.1) $\lambda(y_1) = \gamma(y_1) = 0$, for $y_1 \in \{y_L, y_H\}$;

(a.2) $\pi_L = \pi_H = 1$.

(b) If $c_H + \max\{0, \beta(c_H - d(y_H))\} < \overline{d} \leq c_L + \max\{0, \beta(c_L - d(y_L))\}$: There exists a class of *hybrid* equilibria in which the selfish type is (for all $y_1 \in \{y_L, y_H\}$) indifferent between choosing $(d^*(y_L), g_1^*(y_L))$ and full rent extraction $(\overline{d}, 0)$. Equilibrium strategies satisfy:

(b.1) $(1-p)\lambda(y_L) + p\lambda(y_H) = (1-p)$ and $\gamma(y_L) = \gamma(y_H) = 0$;

(b.2) $\pi_L = \frac{y_L + \overline{d}}{\beta(E(y_2) - d^*(y_L))}$ and $\pi_H = 1$.

(c) If $\overline{d} \leq c_H + \max\{0, \beta(c_H - d(y_H))\}$: There exists a class of *pooling* equilibria in which the selfish type is (for all $y_1 \in \{y_L, y_H\}$) indifferent between policy combinations $(d^*(y_L), g_1^*(y_L))$ and $(d^*(y_H), g_1^*(y_H))$; full rent extraction $(\overline{d}, 0)$ does not occur. Equilibrium strategies satisfy:

(c.1) $(1-p)\lambda(y_L) + p\lambda(y_H) = (1-p)$ and $(1-p)\gamma(y_L) + p\gamma(y_H) = p$;

(c.2) $y_L + \overline{d} \leq -(y_H - y_L) + \pi_H \cdot \beta(E(y_2) - d^*(y_H)) = \pi_L \cdot \beta(E(y_2) - d^*(y_L))$.

The basic tradeoff a selfish type faces is to either fully extract all currently available resources $y_1 + \overline{d}$ and be voted out of office as a result, or to keep the option of potentially even better rent-seeking possibilities in the next period by pretending to be benevolent in the current period. In the pooling equilibria of Proposition 1(c) the selfish type forgoes complete rent-seeking today even when economic conditions are very good for grabbing, i.e. when $y_1 = y_H$. In the hybrid equilibria of part (b) she only does so when current economic conditions limit extensive rent-seeking, i.e. when $y_1 = y_L$, while in the separating equilibrium of part (a) she never forgoes current possibilities for rent extraction.

In general, the higher the level of the debt limit $\overline{d}$ is, the higher the payoffs of rent seeking today for the selfish type become. Likewise, a higher debt limit weakly increases the debt levels $d^*(y_L)$ and $d^*(y_H)$ chosen by the benevolent type and thereby makes mimicking in principle less attractive. If the debt limit $\overline{d}$ increases, we thus move upwards from case (c) to case (b) to case (a) in Proposition 1. In

[14] Note that in our pooling equilibria the selfish type does not necessarily make the exact same choices as a benevolent type would do, i.e. she need not perfectly pool with this type in terms of actual behavior (as she may 'hide behind the state'). Yet the strategy the selfish type uses in the pooling equilibria, together with voters not knowing the actual state of economy y_1, makes her behavior from the voters' viewpoint observationally fully equivalent to that of a benevolent type.

[15] Exactly at the border between cases (a) and (b) and the one between cases (b) and (c), the adjacent types of equilibria exist side-by-side. Solely for technical convenience (in particular, as to ensure upper semi-continuity of expected equlibrium welfare, see the proof of Proposition 1), the proposition assumes that in these instances the hybrid and the pooling equilibria apply, respectively.

terms of voter welfare this has the potential downside of enlarging the possibilities for grabbing today, yet at the same time has the potential upside of improved electoral selection and thereby reduced rent seeking tomorrow. The choice of an optimal debt ceiling $\overline{d}$ should weigh these two opposing effects and is explored in the next section.

4 Welfare and the optimal debt ceiling

The possibility to oust an incumbent after the first period has two potential beneficial welfare effects. First, it may restrain a selfish incumbent in grabbing resources for private use in the first period (a *disciplining* effect) and it may lead to the replacement of a selfish incumbent by a benevolent challenger (a *selection* effect). This section first decomposes expected voter welfare in terms of the discipline and selection effects for each of the possible equilibria that may occur. This is then followed by an analysis of the debt ceiling that maximizes expected voter welfare.

4.1 Welfare effects of electoral discipline and selection

To gauge the welfare effects of electoral discipline and selection, we compare equilibrium welfare with a benchmark in which these effects are absent. Let W_T denote expected voter welfare if the incumbent is tenured for sure, i.e. serves a two-period term without elections.[16] We then have:

$$\begin{aligned} W_T \equiv (1-\theta) W_T^s + \theta W_T^b &= (1-\theta)(1+\beta) u(0) \\ &+ \theta \left\{ (1-p) \left[u\left(g_1^*(y_L)\right) + \beta E u\left(y_2 - d^*(y_L)\right) \right] + p \left[u\left(g_1^*(y_H)\right) + \beta E u\left(y_2 - d^*(y_H)\right) \right] \right\} \end{aligned}$$

where W_T^b (W_T^s) is expected welfare when the incumbent is benevolent (selfish). Because in this bechmark there are no elections, the selfish incumbent will not spend anything on public goods.

Compared to W_T, elections induce a *disciplining effect* if selfish incumbents abstain from rent seeking in the first period and spend resources on public good provision instead to promote their re-election prospects. For an amount g spent on public goods in the first period, the size of this effect is measured by:

$$D(g) \equiv u(g) - u(0)$$

that is, by voters' first-period utility difference between public good spending of level g and no spending on public goods in the first period at all. Similarly so, elections induce a (positive) *selection effect* if a selfish incumbent is replaced by a challenger of the benevolent type. The size of this effect is measured by voters' discounted utility difference between spending of the remaining resources $y_2 - d$ on public goods and no public good spending in the second period at all:

$$S(d) \equiv \beta \left(E u(y_2 - d) - u(0) \right)$$

[16] Besley and Smart (2007) use an alternative benchmark in their analysis, viz. the expected welfare that results when politicians are always removed from office after having served a single term. In that case there may still be a 'selection effect' ex post (when the newcomer and the incumbent appear to be of different types), yet in expectation this effect vanishes, because replacement of the politician is mechanic. Our benchmark is arguably more natural as there is no replacement at all (and thus not even a selection effect ex post). It also facilitates the unambiguous assignment of the discipline and selection effects to the respective periods 1 and 2.

Obviously, in the reverse case where a benevolent incumbent is succeeded by a selfish challenger, a negative selection effect equal to $-S(d)$ materializes.

Expected equilibrium welfare now equals benchmark welfare W_T plus the expected discipline and selection effects occurring.[17] In particular, we have:

$$\begin{aligned}
EW(\lambda(y_1), \gamma(y_1), \pi_1) = W_T &+ (1-\theta)\cdot[(1-p)\lambda(y_L) + p\lambda(y_H)]\cdot D(g_1^*(y_L)) \\
&+ (1-\theta)\cdot[(1-p)\gamma(y_L) + p\gamma(y_H)]\cdot D(g_1^*(y_H)) \\
&+ \theta\cdot(1-\theta)\cdot\{[((1-p)\lambda(y_L) + p\lambda(y_H)) - (1-p)]\cdot(1-\pi_L)\}\cdot S(d^*(y_L)) \\
&+ \theta\cdot(1-\theta)\cdot\{[((1-p)\gamma(y_L) + p\gamma(y_H)) - p]\cdot(1-\pi_H)\}\cdot S(d^*(y_H)) \\
&+ \theta\cdot(1-\theta)\cdot\{(1-p)(1-\lambda(y_L)-\gamma(y_L)) + p(1-\lambda(y_H)-\gamma(y_H))\}\cdot S(\overline{d})
\end{aligned} \tag{8}$$

Intuitively, the disciplining benefit $D(g_1^*(y_L))$ is obtained only when the incumbent is selfish, which happens with probability $(1-\theta)$, and chooses policy combination $(d^*(y_L), g_1^*(y_L))$ in the first period. The latter occurs with probability $[(1-p)\lambda(y_L) + p\lambda(y_H)]$. The intuition for the disciplining effect in the second row is similar and arises when the selfish incumbent chooses policy combination $(d^*(y_H), g_1^*(y_H))$. The final three rows correspond to selection effects, potentially occurring after each of the three policy combinations that may be chosen in the first period in equilibrium. These can be understood as follows. Selection effects may only occur in case the incumbent and challenger are of opposite types, which happens with probability $\theta\cdot(1-\theta)$. In that case, relative to the bechmark of keeping the tenured incumbent in office for two periods, elections then improve selection with probability $\Pr(\text{Not re-elected} \mid \text{selfish incumbent})$ and worsen selection with probability $\Pr(\text{Not re-elected} \mid \text{benevolent incumbent})$. This implies a net overall improvement in selection equal to $\{\Pr(\text{Not re-elected} \mid \text{selfish incumbent}) - \Pr(\text{Not re-elected} \mid \text{benevolent incumbent})\}$. The terms in curly brackets reflect these net probabilities for each choice of policy combination in period 1. For a given choice of debt d in the first period, the second-period benefit of electing a good type rather than a bad type politician equals $S(d)$. This gives the final terms of the third to fifth rows.

In all equilibria, the equilibrium strategies of the selfish incumbent and the voter are such that the selection effects after choosing policy combinations $(d^*(y_L), g_1^*(y_L))$ and $(d^*(y_H), g_1^*(y_H))$ disappear. The terms in the third and fourth row thus vanish. For the former term this follows from the fact that either $\pi_L = 1$ as in the separating equilibrium of Proposition 1(a), so compared to the tenure bechmark there is no selection, or $(1-p)\lambda(y_L) + p\lambda(y_H) = (1-p)$ as in the hybrid and pooling equilibria of Proposition 1(b) and 1(c). In the latter case the overall probability that the selfish incumbent chooses the benevolent type's optimal debt level in the bad state is equal to the probability of the bad state materializing. Hence, it is equal to the benevolent incumbent choosing the optimal bad-state debt level. But this implies that observing the incumbent's bad-state debt level does not provide any additional information on the incumbent's type, making the voter indifferent between incumbent and challenger in expectation (as induced by her prior). The same reasoning holds when the good economic state materializes, i.e. for the term in the fourth row.

[17] Formally, benchmark welfare W_T itself can also be expressed in terms of discipline and selection effects: $W_T = (1+\beta)\cdot u(0) + \theta\cdot\left\{(1-p)\left[D\left(g_1^*(y_L)\right) + S(d^*(y_L))\right] + p\left[D\left(g_1^*(y_H)\right) + S(d^*(y_H))\right]\right\}$. As these effects derive from the behavior of the benevolent incumbent, their interpretation is then conceptually different though.

Equilibrium	Type of effect	Size of effect
(a) Separating	Disciplining	0
	Selection	$\theta \cdot (1-\theta) \cdot S\left(\overline{d}\right)$
(b) Hybrid	Disciplining	$(1-\theta) \cdot (1-p) \cdot D\left(g_1^*(y_L)\right)$
	Selection	$\theta \cdot (1-\theta) \cdot p \cdot S\left(\overline{d}\right)$
(c) Pooling	Disciplining	$(1-\theta) \cdot \left[(1-p) \cdot D\left(g_1^*(y_L)\right) + p \cdot D\left(g_1^*(y_H)\right)\right]$
	Selection	0

Note: Equibrium welfare EW_i for $i \in \{s, h, p\}$ equals W_T plus the disciplining and selection effects listed above.

Table 2: Disciplining and selection effects under the different equilibria

Based on the expression for expected welfare in (8) and Proposition 1, the equilibrium welfare gains from elections are summarized in Table 2. For the hybrid equilibria of case (b), the effect sizes in the above table are independent of the specific values for $\lambda(y_L)$ and $\lambda(y_H)$ (and clearly the same holds true for W_T). All equilibria belonging to case (b) of Proposition 1 are thus welfare equivalent to each other. The same holds true for the expected payoffs of the selfish incumbent; given her willingness to mix and thus indifference, her expected payoffs do not depend on the exact mixing probabilities chosen. Finally, note that also for the benevolent type of politician all the mixed equilbria of case (b) are payoff equivalent, as re-election probabilities π_L and π_H are independent of $\lambda(y_1)$ (cf. Proposition 1). Similar observations regarding payoff equivalence can be made for the multiple pooling equilibria belonging to case (c). Hence, for the comparative statics of welfare and expected payoffs in general, only the *type* of equilibrium – separating, hybrid, or pooling – matters.

4.2 The optimal debt ceiling

Viewed from the decomposition discussed above, an increase in the debt ceiling $\overline{d}$ impacts a voter's equilibrium welfare in various ways. First, an increase in $\overline{d}$ weakly increases the benchmark welfare level W_T. Intuitively, without the threat of being ousted out of office, a selfish incumbent would always extract maximum rents and spend nothing on public goods provision, irrespective of $\overline{d}$. A benevolent incumbent, on the other hand, would like to serve voter interests perfectly, but might be contrained in doing so by the debt limit in place. The less restrictive this limit is, the less distorted the intertemporal allocation of resources by the benevolent type becomes. Beyond $\overline{d} = d(y_L)$ the debt ceiling is no longer binding for the benevolent type and benchmark welfare W_T is independent of $\overline{d}$.

Second, variations in $\overline{d}$ affect the disciplining effect of elections. For an equilibrium of a given type, an increase in $\overline{d}$ increases the benefits obtained from disciplining the selfish type. Disciplining entails that the selfish type is induced to behave 'as if' being of the benevolent type in the first period. With fewer restrictions on debt, the genuine benevolent type is less constrained in choosing the socially-optimal intertemporal allocation of resources. This in turn implies that mimicking of her behavior by the selfish type brings larger improvements in voter welfare. Formally, $D(g_1^*(y_1)) = D\left(y_1 + \min\left\{d(y_1), \overline{d}\right\}\right)$ is weakly increasing in $\overline{d}$ for $y_1 \in \{y_L, y_H\}$.

Third, again for an equilibrium of a given type, the selection benefits of elections decrease with debt limit $\overline{d}$. Elections bring the potential upside of replacing a selfish incumbent with a benevolent challenger. In that case the remaining resources available in period two are used for public good provision, rather than for rent extraction. These benefits increase with the amount of resources left to period two and thus decrease with $\overline{d}$. Formally, $S\left(\overline{d}\right)$ is decreasing in $\overline{d}$.

The overall joint impact of the three 'intensive margin' effects related to W_T, $D\left(g_1^*\left(y_i\right)\right)$ and $S\left(\overline{d}\right)$ follows from how equilibrium welfare EW_i for $i \in \{s, h, p\}$ varies with $\overline{d}$, where EW_s, EW_h, and EW_p denote expected welfare in a separating, hybrid, and pooling equilibrium, respectively. The following lemma gauges this joint intensive margin effect, by characterizing the shape of EW_i.

Lemma 4. (*Joint intensive margin effects.*) Consider the expressions for expected welfare in the various equilibria on the *entire* interval $\overline{d} \in [-y_1, y_L]$ (so ignoring for now that the different equilibria only exist on adjacent, mutually exclusive and jointly exhaustive, subintervals):

(a) The expression for EW_s is strictly concave in $\overline{d}$ and attains its global maximum at some $\tilde{d}_s < d\left(y_L\right)$;

(b) The expression for EW_h is strictly concave in $\overline{d}$ and attains its global maximum at $\overline{d} = d\left(y_L\right)$ if $p \leq \frac{1}{1+\theta}$ and at some $\tilde{d}_h < d\left(y_L\right)$ if $p > \frac{1}{1+\theta}$;[18]

(c) The expression for EW_p is weakly concave in $\overline{d}$, increasing up to $\overline{d} = d\left(y_L\right)$ and being constant afterwards.

Because by assumption $u(\cdot)$ is concave and expected welfare is a weighted average of terms involving $u(\cdot)$, expected welfare is also (weakly) concave. The shape of EW_p immediately follows from both W_T and $D\left(g_1^*\left(y_1\right)\right)$ being weakly increasing in $\overline{d}$ up till $\overline{d} = d\left(y_L\right)$ and being flat afterwards. In a hybrid equilibrium voters obtain – relative to benchmark welfare W_T – a disciplining benefit $D\left(g_1^*\left(y_1\right)\right)$ with probability $(1-\theta)\cdot(1-p)$ and a selection benefit $S\left(\overline{d}\right)$ with probability $\theta\cdot(1-\theta)\cdot p$ (cf. Table 2). The disciplining benefit increases with $\overline{d}$, while the selection benefit decreases with $\overline{d}$; yet for $\overline{d} \leq d\left(y_L\right)$ the increase in D outweighs the decrease in S.[19] Therefore, if the former occurs weakly more often, i.e. if $p \leq \frac{1}{1+\theta}$, EW_h increases in $\overline{d}$ up till $\overline{d} = d\left(y_L\right)$. Otherwise, net marginal benefits turn negative at some lower debt limit $\tilde{d}_h$. The latter intuition also holds for EW_s, where only the terms W_T and $S\left(\overline{d}\right)$ play a role.

Apart from its intensive margin effects, an increase in $\overline{d}$ may also have an 'extensive margin' impact by inducing a shift in the type of equilibrium, such that effectively a disciplining benefit $D\left(g_1^*\left(y_i\right)\right)$ is given up for a selection benefit $S\left(\overline{d}\right)$. Consider, for instance, moving from a pooling equilibrium to a hybrid one. The equilibrium outcome then essentially only differs in case the incumbent turns out to be selfish and the state of the economy is good, which happens with probability $(1-\theta)\cdot p$. By moving to the hybrid equilibrium voters in that case gain selection benefit $S\left(\overline{d}\right)$ with probability θ (viz. the probability that the challenger is benevolent), in exchange for losing discipline benefit $D\left(g_1^*\left(y_H\right)\right)$. The following lemma shows that these extensive margin effects are necessarily negative at the relevant borders of existence between two different types of equilibria.

[18] It holds that $\tilde{d}_s < \tilde{d}_h < d\left(y_L\right)$; hence the global maximizer of EW_h is always strictly larger than the global maximizer of EW_s (and the global maximizer of EW_p is weakly larger than the one of EW_h).

[19] $D'\left(y_L + \overline{d}\right) + S'(\overline{d}) \geq 0$ for $\overline{d} \leq d\left(y_L\right)$ follows from the Euler equation (6) defining $d\left(y_L\right)$.

Lemma 5. (*Extensive margin effects.*) The expected welfare differences between adjacent types of equilibria equal:

$$EW_s - EW_h = (1-\theta)\cdot(1-p)\cdot\left[\theta\cdot S\left(\overline{d}\right) - D\left(g_1^*\left(y_L\right)\right)\right]$$
$$EW_h - EW_p = (1-\theta)\cdot p\cdot\left[\theta\cdot S\left(\overline{d}\right) - D\left(g_1^*\left(y_H\right)\right)\right]$$

The terms $\left[\theta\cdot S\left(\overline{d}\right) - D\left(g_1^*\left(y_1\right)\right)\right]$ for $y_1 \in \{y_L, y_H\}$ are decreasing in $\overline{d}$ and necessarily negative at the border of existence $c_1 + \max\{0, \beta\left(c_1 - d\left(y_1\right)\right)\}$ between two different types of equilibria.

At the borders between two different types of equilibria the selfish type is indifferent (for some $y_1 \in \{y_L, y_H\}$) between full rent extraction today and postponing it to the next period. Lemma 5 implies that in that case voters strictly prefer the selfish type to postpone. Apart from having opposed interests about the occurence of rent seeking per se, voters thus also have diverging interests regarding its timing (given that it eventually occurs anyway).

Lemmas 4 and 5 together restrict the list of potential candidates for the overall optimal debt ceiling $\overline{d}_{opt}$. The global maximizers characterized in Lemma 4 only qualify for $\overline{d}_{opt}$ if they belong to the subinterval for which the equilibrium in question actually exists. As a result, only one of these maximizers is relevant at most (and we label it the *eligible* global maximizer). To see this, note from Proposition 1 that by steadily increasing the debt ceiling $\overline{d}$, we move from the pooling range to the hybrid range and finally to the separating range. The global maximizers are ranked reversely though: $\tilde{d}_s < \tilde{d}_h < d\left(y_L\right)$. Therefore, if one of the global maximizers falls within the proper range for existence of that equilibrium type, the other two global maximizers necessarily do not. Besides the single eligible global maximizer (if it exists), in principle also the two borders between equilibrium types qualify for $\overline{d}_{opt}$.

Completely characterizing $\overline{d}_{opt}$ – by identifying the eligible global maximizer (if it exists) and comparing its welfare level with the welfare levels generated if the ceiling is set at one of the two respective borders – turns out to be hard analytically.[20] Nevertheless, some straightforward but insightful observations about the tightness and effectiveness of the optimal fiscal rule $\overline{d}_{opt}$ can be made.

Lemma 6. (*Optimal constant debt ceiling.*) For the welfare maximizing constant debt ceiling $\overline{d}_{opt}$ it holds that:

(a) If $d\left(y_L\right) < c_H + \beta\left(c_H - d\left(y_H\right)\right)$, any non-binding debt ceiling falling within the pooling range is welfare maximizing, i.e. $\overline{d}_{opt} \in \left[d\left(y_L\right), c_H + \beta\left(c_H - d\left(y_H\right)\right)\right]$;

(b) If $c_H + \beta\left(c_H - d\left(y_H\right)\right) < d\left(y_L\right)$, it holds that $c_H + \max\{0, \beta\left(c_H - d\left(y_H\right)\right)\} \leq \overline{d}_{opt} \leq d\left(y_L\right)$ and $\overline{d}_{opt} < d\left(y_L\right)$ at least when either $c_L < d\left(y_L\right)$ or $p > \frac{1}{1+\theta}$;

(c) $\overline{d}_{opt} < d\left(y_H\right) \Longrightarrow \overline{d}_{opt} < d\left(y_L\right)$, but not necessarily vice versa. Put differently, the optimal ceiling is less likely to strictly bind when $y = y_H$ than when $y = y_L$;

[20] Note that, while $EW_s - EW_h$ jumps down when the debt ceiling crosses the border c_L between the hybrid and the separating equilibrium (cf. Lemma 5) and $EW_s - EW_h$ keeps on decreasing as we raise $\overline{d}$, it could still be case that EW_s and EW_h are both individually increasing in $\overline{d}$ at this border and that EW_s evaluated at its global maximizer $\tilde{d}_s > c_L$ exceeds the value of the function EW_h at the border point c_L. A similar observation holds with respect to $EW_h - EW_p$ and the comparison of the pooling and the hybrid range.

(d) At (any given $\overline{d}$ and thus also at) $\overline{d} = \overline{d}_{opt}$, a selfish type is more likely to postpone rent-seeking to period 2 when $y = y_L$ than when $y = y_H$.

In case (a) a debt ceiling equal to $\overline{d} = d(y_L)$ still falls within the pooling interval. Setting a lower ceiling then strictly worsens expected welfare per Lemma 4(c). Intuitively this follows because such a lower ceiling would constrain the benevolent type in her policy choice, while it does not reduce rent-seeking by the selfish type (as she always grabs all the resources left in period 2). Choosing a higher ceiling also does not bring any positive intensive margin effects. It would only induce negative extensive margin effects if the higher ceiling would lead to a shift in the type of equilibrium, but it is immaterial otherwise. Therefore, the optimal debt ceiling can be chosen arbitrarily lax such that it never restricts the benevolent type, as long as it stays within the pooling range. Case (a) may only occur if $d(y_H) < \frac{1+\beta}{\beta} \cdot c_H = E(y_2) - \frac{y_H}{\beta}$ and thus is very negative, i.e. if a benevolent type would save a lot of resources for the next period in case the current economic state is good. This requires that the benevolent type finds the future sufficiently important and that there is uncertainty about period 2 income (cf. Section 3.1).[21]

If a debt ceiling equal to $d(y_L)$ does not fall within the pooling interval (case (b) in Lemma 6), $d(y_L)$ still can be the welfare maximizing choice. Necessary requirements for this are that $\overline{d} = d(y_L)$ corresponds to the global maximizer of EW_h (which requires $p \leq \frac{1}{1+\theta}$ by Lemma 4) and falls in the hybrid equilibrium range (for which $d(y_L) < c_L$ is necessary). As eligible global maximizer $\overline{d} = d(y_L)$ should then be compared in terms of welfare to the border $\overline{d} = c_H + \max\{0, \beta(c_H - d(y_H))\}$ between the pooling and the hybrid range (the other border c_L is irrelevant given the eligible global maximizer being located in the hybrid range). If these necessary requirements are not met, the optimal ceiling always strictly binds the benevolent incumbent at least when economic circumstances are bad.

The final two parts of Lemma 6 report two observations that trivially hold for a constant debt ceiling. These are just included to facilitate the comparison with the case of a state-contingent debt ceiling considered in the next section for which the statements do not trivially hold from the outset.

5 Increasing fiscal transparency

In this section we allow for the possibility that voters receive information about the state of the economy. We refer to this development as an increase in "fiscal transparency", because it makes it possible to more accurately assess the government's budgetary situation and how its budget is allocated. In the European Union providing information on the state of the economy would typically be the work of an IFI. They either construct the macroeconomic projections that form the basis for the government's budget or they consent the macroeconomic projections produced by their government. Below, we first consider the case in which the actual state of the economy is observed by voters with some probability. In that case 'hiding behind the state' becomes less attractive for a selfish type and, as it appears, will no longer occur. Nevertheless, welfare is unaffected when voters are better informed

[21] From Lemma 1(a.i) we have that $\frac{-(y_H - y_L)}{2} < d(y_H)$. The requirement $d(y_H) < E(y_2) - \frac{y_H}{\beta}$ thus can only be satisfied if $\frac{y_H}{\beta} < y_L + \left(p + \frac{1}{2}\right)(y_H - y_L)$. Therefore, necessary conditions for case (a) of Lemma 6 to apply are $p > \frac{1}{2}$ and $\beta > \frac{2}{2p+1}$ (and thus at least $\beta > \frac{2}{3}$).

Policy combination	$y_1 = y_L$	$y_1 = y_H$
$\left(d^*(y_L), g_1^*(y_L)\right)$	$\pi_L \cdot \beta\left(E(y_2) - d^*(y_L)\right)$	$(y_H - y_L) + (1-\xi) \cdot \pi_L \cdot \beta\left(E(y_2) - d^*(y_L)\right)$
$\left(d^*(y_H), g_1^*(y_H)\right)$	$-(y_H - y_L) + (1-\xi) \cdot \pi_H \cdot \beta\left(E(y_2) - d^*(y_H)\right)$	$\pi_H \cdot \beta\left(E(y_2) - d^*(y_H)\right)$
$\left(\overline{d}, 0\right)$	$y_L + \overline{d}$	$y_H + \overline{d}$

Table 3: Expected payoffs of selfish incumbent under fiscal transparency

in this way, irrespective of the probability with which the state is observed. By immediate implication this carries over to the optimal debt ceiling. We then turn to the case where, based on the economic analysis of an IFI, a state-contingent debt ceiling $\overline{d}(y_1)$ can be imposed. This may improve welfare, as the trade-off between curbing diversion of public resources for private use by the selfish type and giving the benevolent type discretion in the intertemporal allocation of public resources can then be approached in a tailor-made, state-contingent fashion.

5.1 Fiscal transparency: enhancing electoral accountability

Suppose that with probability $\xi > 0$ the state of the economy becomes observable to voters before the election occurs. If this happens, voters can perfectly infer the actual spending on private rents. The relevant payoff matrix is now given by Table 3. Note that it no longer holds that the expected payoffs of a given policy combination in state $y_1 = y_H$ exceed those in state $y_1 = y_L$ by a fixed amount. In particular, for $\xi > 0$ the payoffs obtained from simultaneously deceiving in two dimensions – i.e. pretending to be of the benevolent type *and* suggesting that the state is different than it actually is – become lower. That is, choosing $(d^*(y_L), g_1^*(y_L))$ when $y_1 = y_H$ now yields the selfish type less, and so does choosing $(d^*(y_H), g_1^*(y_H))$ when $y_1 = y_L$. This effectively makes that in equilibrium the selfish type will no longer do so and, in turn, will always use a pure strategy. Apart from that, the structure of the equilibria remains essentially unaffected.

In Proposition A.1 in Appendix A we provide a comprehensive equilibrium characterization. Proposition 2 below summarizes the main conclusions that follow in terms of the politician's policy choices and expected equilibrium welfare.

Proposition 2 (*Impact of enhanced electoral accountability.*) Consider the setting in which with probability $\xi > 0$ the state of the economy y_1 becomes observable to voters before they cast their vote. It holds that:

(a) The selfish type never hides behind the state: $\lambda(y_H) = \gamma(y_L) = 0$;

(b) For all $\overline{d}$ the equilibria essentially correspond to those in Proposition 1 with the additional requirement that $\lambda(y_H) = \gamma(y_L) = 0$;

(c) Expected equilibrium welfare and, hence, the optimal debt ceiling are unaffected.

In a hybrid equilibrium the selfish incumbent now always chooses $(d^*(y_L), g_1^*(y_L))$ when $y = y_L$ (as does the benevolent incumbent) and always $\left(\overline{d}, 0\right)$ in case $y = y_H$. Hence, there are no instances in which fiscal transparency (the state becoming publicly known) actually raises the chances to unmask a selfish incumbent. Similarly, in a pooling equilibrium the selfish incumbent now always chooses $(d^*(y_L), g_1^*(y_L))$ when $y = y_L$ and always $(d^*(y_H), g_1^*(y_H))$ when $y = y_H$. Again, there are no instances in which fiscal transparency helps in exposing a selfish incumbent. This also intuitively explains why expected equilibrium welfare and thus the optimal debt ceiling are unaffected. In the original setting without fiscal transparency, a class of hybrid equilibria exists as well as a class of pooling equilibria (cf. Proposition 1). As observed in Section 4.1, all the equilibria within a given class are welfare equivalent. Fiscal transparency essentially restricts the class of equilibria (of a given type) to a single equilibrium.[22] But with welfare equivalence, this restriction is inconsequential and has no implications for the optimal debt ceiling.

In sum, although fiscal transparency *per se* may induce changes in the policy choices made by the selfish type, it does not affect the amount of rent extraction and thereby welfare. To enhance welfare transparency should be complemented with additional policy measures, to which we turn next.

5.2 Fiscal transparency: enabling state-contingent debt ceilings

Suppose now that through careful analysis an IFI has the ability to perfectly observe the state of the economy and report on it in a verifiable way. This allows for the possibility to impose a state-contingent debt ceiling $\overline{d}(y_1)$. Using the announced state-contingent debt ceiling voters can back out the state of the world y_1 before they cast their vote. The analysis is thus 'as if' y_1 is commonly known from the outset and, as in the previous subsection, hiding behind the state no longer plays a role.

In the presence of a state-contingent debt ceiling $\overline{d}(y_1)$, the debt level chosen by the benevolent type now equals $d^*(y_1) \equiv \min\left\{d(y_1), \overline{d}(y_1)\right\}$. The resulting equilibrium behavior of the selfish incumbent for $y_1 \in \{y_L, y_H\}$ is characterized by:

Proposition 3. (*Equilibria under state-contingent debt ceilings.)* For the subgame in which the state of the world equals $y_1 \in \{y_L, y_H\}$, the equilibrium outcome is as follows:

(a) If $\overline{d}(y_1) > c_1 + \max\{0, \beta(c_1 - d(y_1))\}$: There exists a unique *separating* outcome in which the selfish type always chooses full rent extraction $\left(\overline{d}(y_1), 0\right)$ in the first period. After policy combination $(d^*(y_1), g_1^*(y_1))$ the incumbent is re-elected for sure: $\pi_1 = 1$;

(b) If $\overline{d}(y_1) \leq c_1 + \max\{0, \beta(c_1 - d(y_1))\}$: There exists a class of *pooling* outcomes in which the selfish type chooses $(d^*(y_1), g_1^*(y_1))$ for sure. After this policy combination the incumbent is re-elected with probability $\pi_1 \geq \frac{y_1 + \overline{d}(y_1)}{\beta(E(y_2) - d^*(y_1))}$.

The equilibria of the entire game follow from combining the equilibrium outcomes in the two respective subgames for $y_1 = y_L$ and $y_1 = y_H$, respectively (again leading to pooling, hybrid and separating type of equilibria in the overall game).

With the state of the economy being common knowledge, the analysis can be done conditional on y_1. Proposition 3 then reveals that for a contingent debt ceiling $\overline{d}(y_1)$ that is sufficiently low, the

[22] Here we write 'essentially', because formally speaking there still exist multiple equilibria within the hybrid and pooling classes; voters' equilibrium mixing probabilities π_L and π_H may then take multiple values. (Clearly, given that voters are indifferent in these instances, these mixing probabilities are also immaterial for equilibrium welfare and the optimal debt ceiling.) Our focus in the main text is on uniqueness in terms of the selfish politician's behavior.

selfish type pools with the benevolent type, while for $\overline{d}(y_1)$ sufficiently high the selfish type separates. From the border $c_1 + \max\{0, \beta(c_1 - d(y_1))\}$ being decreasing in y_1 (cf. Proposition 1) it follows that the range of debt ceilings for which the separating outcome applies when $y = y_L$ is a strict subset of the one for $y = y_H$. This reflects the earlier observation that the selfish type has a stronger incentive to front-load rent seeking the higher y_1 is.

The analysis of the welfare effects and the optimal contingent debt ceilings closely resembles the one of the constant ceiling case. In particular, as long as the pooling range of Proposition 3(b) applies, increasing the ceiling $\overline{d}(y_1)$ strictly benefits expected welfare if the debt ceiling is below $d(y_1)$ and thus curtails the benevolent type in choosing the optimal intertemporal resource allocation, while it brings no harm once it exceeds $d(y_1)$ but remains within the pooling range. Lemma 4(c) thus continues to apply mutatis mutandis. At the border there is again a discrete fall in welfare due to the jump from the pooling to the separating interval, as measured by the extensive margin effect $\left[\theta S\left(\overline{d}(y_1)\right) - D\left(g_1^*(y_1)\right)\right]$, just as in Lemma 5. In case the selfish type separates for given y_1, an increase in $\overline{d}(y_1)$ improves the intertemporal resource allocation by the benevolent type as long as the limit is below $d(y_1)$, while it worsens first-period extraction by the selfish type. The unique debt level $\widetilde{d}_s(y_1)$ for which the marginal benefit of increasing the contingent debt ceiling equals the marginal cost is characterized by the solution to:

$$\theta\left[D'\left(y_1 + \overline{d}(y_1)\right) + S'\left(\overline{d}(y_1)\right)\right] = -(1-\theta)\,\theta S'\left(\overline{d}(y_1)\right) \tag{9}$$

By the Euler equation (6) for optimal resource allocation, the term within square brackets on the left-hand side equals 0 for $\overline{d}(y_1) = d(y_1)$, while the term on the right-hand side is strictly positive. Intuitively, at $\overline{d}(y_1) = d(y_1)$ a marginal reduction of the debt limit has only a second-order distortive effect on the debt policy of the benevolent type, but a first-order benefit in terms of reducing first-period rent extraction. The debt level $\widetilde{d}_s(y_1)$ that equates the marginal benefit with the marginal cost is thus strictly below $d(y_1)$.[23] Similar to the constant ceiling case, $\widetilde{d}_s(y_1)$ is the single eligible global maximizer if it falls within the separating range. The optimal state-contingent debt ceiling $\overline{d}_{opt}(y_1)$ then follows from comparing the welfare levels belonging to $\overline{d}(y_1) = \widetilde{d}_s(y_1)$ and border $\overline{d}(y_1) = c_1 + \max\{0, \beta(c_1 - d(y_1))\}$. Otherwise, only the latter border value remains as candidate. Lemma 7 is the analogon of Lemma 6 in the presence of state-contingent ceilings.

Lemma 7. (*Optimal contingent debt ceiling.*) For the welfare maximizing contingent debt ceiling $\overline{d}_{opt}(y_1)$ it holds that:

- **(a)** If $d(y_1) < c_1$, any non-binding debt ceiling falling within the pooling range is welfare maximizing, i.e. $\overline{d}_{opt}(y_1) \in [d(y_1), c_1 + \beta(c_1 - d(y_1))]$;
- **(b)** If $c_1 < d(y_1)$, it holds that $\overline{d}_{opt}(y_1) \in \left\{c_1, \widetilde{d}_s(y_1)\right\}$ and $\overline{d}_{opt}(y_1)$ always strictly binds the benevolent type: $c_1 \leq \overline{d}_{opt}(y_1) < d(y_1)$;
- **(c)** Under the conditions of Lemma 2 it holds that $\overline{d}_{opt}(y_L) < d(y_L) \Longrightarrow \overline{d}_{opt}(y_H) < d(y_H)$, but not neccesarily vice versa. Put differently, the optimal ceiling is more likely to strictly bind when

[23] It also necessarily holds that $\widetilde{d}_s(y_H) < \tilde{d}_s < \widetilde{d}_s(y_L)$, with $\tilde{d}_s$ the global maximizer of EW_s in the constant ceiling case (cf. Lemma 4(a)).

$y = y_H$ then when $y = y_L$;

(d) At $\overline{d}_{opt}(y_1)$, a selfish type need not necessarily be more likely to postpone rent-seeking to period 2 when $y = y_L$ than when $y = y_H$.

Comparing Lemma 7(c) with Lemma 6(c), we observe that moving to a contingent debt ceiling reverses the economic circumstances under which the optimal ceiling is more likely to bind, from $y_1 = y_L$ under a constant ceiling to $y_1 = y_H$ under a contingent one. For a constant debt ceiling this follows directly from $d(y_H) < d(y_L)$. With a contingent ceiling the intuition is as follows. For debt ceilings sufficiently high to be potentially optimal, voters prefer the selfish type to postpone rent seeking to period 2. If $d(y_1) < c_1$ this can be done costlessly by setting a ceiling equal to $d(y_1)$. If not, the optimal ceiling necessarily strictly binds the benevolent type (Lemma 7(b)), either to curb rent seeking today somewhat or to induce postponement to period 2. Now, under the conditions of Lemma 2, $d(y_1) < c_1$ is more likely to hold when y_1 is low. This explains the reversal under contingent ceilings.

Parts (d) of Lemmas 6 and 7 reveal that such a reversal might potentially also happen regarding when the selfish type is more likely to extract all rents immediately in period 1. With a constant ceiling she is more likely to do so when $y_1 = y_H$. This also explains the nature of the hybrid equilibrium (cf. Proposition 1): mimicking behavior for $y_1 = y_L$, immediate rent-extraction when $y = y_H$. With contingent debt ceilings such behavior can in principle be reversed by setting a sufficiently lower limit for $y_1 = y_H$, such that the selfish type extracts rents in the first period when $y_1 = y_L$, but postpones it to the second period when $y_1 = y_H$. This would happen if $\overline{d}_{opt}(y_L) = \widetilde{d}_s(y_L)$ and $\overline{d}_{opt}(y_H) = c_H$. A priori this case cannot be excluded given the assumptions made, although intuitively one would expect it to be very unlikely.[24]

Our final proposition summarizes both the impact on the induced behavior of the two types of politicians and the welfare implications of moving to contingent debt ceilings.

Proposition 4 (*Constant vs. contingent case.*) Comparing the case of optimal contingent debt ceilings with the one of optimal constant debt ceilings, it holds that:

(a) If $d(y_L) \leq c_H + \beta(c_H - d(y_H))$, a contingent ceiling does not improve welfare and any ceiling $\overline{d}_{opt} \in [d(y_L), c_H + \beta(c_H - d(y_H))]$ is optimal in both cases. The benevolent type is never restricted by the optimal ceiling and the selfish type always postpones rent-seeking;

(b) If $c_H + \beta(c_H - d(y_H)) < d(y_L)$, a contingent debt ceiling strictly improves welfare. Moreover,

(b.1) if $d(y_H) < c_L$, then $\overline{d}_{opt}(y_H) \leq \overline{d}_{opt} \leq \overline{d}_{opt}(y_L)$ necessarily, with at least one of the inequalities strict. Relative to the constant ceiling case, the contingent ceilings thus (weakly) limit the benevolent type less when $y_1 = y_L$ and lead to (weakly) less first-period rent-seeking when $y_1 = y_H$;

[24] Note that under the conditions of Lemma 2 inducing postponement is both more difficult ($d(y_L) - c_L < d(y_H) - c_H$) and relatively more costly ($\widetilde{d}_s(y_L) - c_L < \widetilde{d}_s(y_H) - c_H$) when y_1 is high. Therefore, if it pays to do so when $y_1 = y_H$, it seems rather likely that it also does so in case $y_1 = y_L$.

(b.2) if $d(y_H) > c_L$, besides $\overline{d}_{opt}(y_H) \leq \overline{d}_{opt} \leq \overline{d}_{opt}(y_L)$, it may occur for θ sufficiently close to 1 that either (i) $\overline{d}_{opt}(y_H) < \overline{d}_{opt}(y_L) < \overline{d}_{opt}$, (ii) $\overline{d}_{opt} < \overline{d}_{opt}(y_H) < \overline{d}_{opt}(y_L)$, or (iii) $\overline{d}_{opt}(y_L) < \overline{d}_{opt}(y_H)$. Relative to the constant ceiling case, the contingent ceilings thus can move both the restrictions on the benevolent type when $y_1 = y_L$ and the amount of first-period rent-seeking when $y_1 = y_H$ in either direction.

The basic tradeoff in choosing the optimal debt ceiling is between a more lax one as to not severely restrict the benevolent type and a more strict one as to reduce rent-seeking by the selfish type. If a constant ceiling exists that never restricts the benevolent type even in bad economic circumstances where she prefers a high debt (i.e. $\overline{d} \geq d(y_L)$), but at the same time always reduces rent-seeking to the fullest extent possible even when conditions for grabbing are good (i.e. $\overline{d} \leq c_H + \beta(c_H - d(y_H))$), this tradeoff is effectively always empty. Contingent ceilings then bring no welfare benefits at all. Note that the condition in Proposition 4(a) for this to happen is independent of θ and thus independent of the expected extent of the rent-seeking problem. It also necessarily requires that the future is sufficiently important ($\beta > \frac{2}{3}$) and that there is some uncertainty (in particular, that $\frac{1}{2} < p < 1$).

In case there necessarily is a tradeoff with a constant ceiling (cf. Proposition 4(b)), contingent ceilings can address this tradeoff to a better extent and realize a strict welfare gain. Intuitively one would expect that they do so by moving the ceiling upwards when $y_1 = y_L$ and downwards when $y_1 = y_H$, i.e. that $\overline{d}_{opt}(y_H) \leq \overline{d}_{opt} \leq \overline{d}_{opt}(y_L)$ necessarily. Somewhat surprisingly, this need not always be the case. This can be understood as follows. If $d(y_H) < c_L$ as in Proposition 4(b.1), the smallest ceiling that could potentially be optimal when $y_1 = y_L$ is higher than the highest ceiling that could potentially be optimal when $y_1 = y_H$. The basic tradeoff is then always "well ordered" across states – with higher ceilings being weakly better when economic circumstances are bad rather than good – and the optimal constant ceiling then necessarily falls in between the optimal contingent ones.[25] The case $d(y_H) < c_L$ necessarily arises when β is sufficiently close to one.[26]

If instead $d(y_H) > c_L$, however, it holds for sufficiently high θ that $\widetilde{d}_s(y_H) > c_L$. The smallest ceiling that could potentially be optimal when $y_1 = y_L$ (viz. c_L) is then lower than the highest ceiling that could potentially be optimal when $y_1 = y_H$ (viz. $\widetilde{d}_s(y_H)$). In that case debt ceiling levels $\overline{d}$ and $\overline{d}'$, with $\overline{d}' > \overline{d}$, do exist for which from a welfare perspective the lower limit $\overline{d}$ is better when $y_1 = y_L$, while the higher limit $\overline{d}'$ is better in case $y_1 = y_H$.[27] Put differently, the tradeoff flips direction across economic circumstances for some range of ceilings. This may result in the globally optimal ceilings no

[25] Consider two debt ceilings with $\overline{d}' > \overline{d}$. Well-ordered across states is then defined as follows: if from a welfare perspective $\overline{d}'$ is better than $\overline{d}$ when $y_1 = y_H$, then this also holds when $y_1 = y_L$. (By contraposition this then also implies that in case $\overline{d}'$ is worse than $\overline{d}$ when $y_1 = y_L$, this also holds when $y_1 = y_H$.) Hence, if some increase in the constant debt ceiling is welfare improving when $y_1 = y_H$, it necessarily also is when $y_1 = y_L$ (but not vice versa). For the case $d(y_H) < c_L$ we specifically have that for ceilings below $\overline{d}_{opt}(y_H)$, increasing the ceiling to $\overline{d}_{opt}(y_H)$ is beneficial in both economic states, for ceilings in between $\overline{d}_{opt}(y_H)$ and $\overline{d}_{opt}(y_L)$ a higher ceiling improves welfare when $y_1 = y_L$ but harms it when $y_1 = y_H$, and increasing the ceiling always harms welfare for ceilings above $\overline{d}_{opt}(y_L)$.

[26] This follows from $d(y_1)$ decreasing in β, c_1 increasing in β, and $d(y_1) \leq c_1$ for $\beta = 1$ under the conditions of Lemma 2. To see the latter, from the Euler equation we get for $\beta = 1$:

$$u'(y_1 + d) = \left[Eu'(y_2 - d)\right] \geq u'(Ey_2 - d)$$

The final inequality follows from Jensen's inequality and u' convex by the assumptions of Lemma 2. Hence, by $u'' \leq 0$ we get $y_1 + d \leq Ey_2 - d$ and thus $d(y_1) \leq c_1$.

[27] This for instance applies for ceilings $\overline{d} \leq c_L < \overline{d}'$ sufficiently close to c_L.

longer satisfying $\overline{d}_{opt}(y_H) \leq \overline{d}_{opt} \leq \overline{d}_{opt}(y_L)$.

A first type of counterintuitive outcome is when, even though $\overline{d}_{opt}(y_H) < \overline{d}_{opt}(y_L)$, the optimal constant ceiling $\overline{d}_{opt}$ does not fall in between these two. One possibility here is that $\overline{d}_{opt}$ exceeds $\overline{d}_{opt}(y_L)$; this occurs when the optimal contingent ceilings induce the selfish type to always pool with the benevolent type in the first period ($\overline{d}_{opt}(y_1) = c_1$), while the optimal constant ceiling induces her to always extract full rents in the first period ($\overline{d}_{opt} = \tilde{d}_s$). The other possibility is that $\overline{d}_{opt}$ falls short of $\overline{d}_{opt}(y_H)$. In that case the optimal contingent ceilings always induce the selfish type to extract all rents in the first period ($\overline{d}_{opt}(y_1) = \tilde{d}_s(y_1)$), but the optimal constant ceiling does not (at least) when $y_1 = y_L$ (i.e. $\overline{d}_{opt} \leq c_L$). A second type of counterintuitive outcome arises when the optimal state-contingent debt ceiling is *more tight* when the state of the economy is bad than when it is good. This can potentially happen only when $\overline{d}_{opt}(y_L) = c_L < \tilde{d}_s(y_H) = \overline{d}_{opt}(y_H)$. With these ceilings behavior corresponds to that in a hybrid equilibrium of the constant ceilings case. The numerical analysis that we discuss in the next subsection reveal that, although each of the three counterintuive cases may indeed occur, overall they are quite rare.

5.2.1 Numerical analysis

Our formal analysis provides a partial analytical characterization of the optimal debt ceilings, by restricting the set of candidate optima to just a few. A full characterization is complicated by the fact that expected equilibrium welfare is not continuous, with discrete drops when moving from one equilibrium type to the other (cf. Lemma 5). A direct comparison of the welfare levels obtained under each of the remaining few candidates is therefore needed. In general this is hard to solve analytically. In this subsection we therefore provide a numerical evalution by assuming a power felicity function $u(x) = x^{\varphi}$, where φ captures the constant degree of relative risk aversion (CRRA). With this felicity function, the Inada conditions are fulfilled.

Parameter	Range
$u(x) = x^{\varphi}$	$\varphi \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$
$Pr(i = benvolent) = \theta$	$\theta \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$
$p \equiv Pr(y_1 = y_H)$	$p \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$
discount factor β	$\beta \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$
y_L	$y_L = 1$
y_H	$y_H \in \{1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2\}$ $\cup \{2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 4.0, 5.0, 10.0\}$

Table 4: Combinations of parameters explored in the numerical analysis

In our simulations we numerically solve for the optimal debt ceilings for a large number of parameter combinations. We vary parameter combinations over a broad range within the relevant parameter space: the curvature of utility φ (9 different values), the likelihood θ of a benevolent policymaker (9 values), the probability of a good economic state p (9 values), the discount factor β (10 values) and the variation in output captured by the spread $y_H - y_L$ (23 values). Because what matters most is the spread between y_L and y_H, we normalize $y_L = 1$. This yields $9^3 \cdot 10 \cdot 23 = 167,670$ cases, given by the

Carthesian product over the sets of parameter values listed in Table 4.

A detailed percentage wise overview of the nature of the optimal ceilings in the constant versus contingent case is provided in Table 8 in Appendix B. Here we provide a more aggregate overview. In particular, we characterize the optimal debt ceilings in terms of *induced behavior* of both the selfish and the benevolent type. The optimal ceiling induces the selfish type to postpone rent seeking to period 2, only when it is (weakly) below threshold c_1. The benevolent type's choice is affected by the optimal ceiling only if this ceiling binds and is strictly below her preferred debt level $d(y_1)$. Panels A and B in Tables 5 and 6 characterize the optimal constant, respectively contingent, ceilings along these lines.

		$\overline{d}_{opt} \leq c_L$	$\overline{d}_{opt} > c_L$	$\overline{d}_{opt} < d(y_L)$	$\overline{d}_{opt} \geq d(y_L)$
A. Rent seeking by selfish type		y_L*: period 2*	y_L*: period 1*		
$\overline{d}_{opt} \leq c_H$	y_H*: period 2*	35.05	*n.p.*		
$\overline{d}_{opt} > c_H$	y_H*: period 1*	40.87	24.09		
B. Benevolent type's debt choice				y_L*: binds*	y_L*: lax*
$\overline{d}_{opt} < d(y_H)$	y_H*: binds*			75.34	*n.p.*
$\overline{d}_{opt} \geq d(y_H)$	y_H*: lax*			19.87	4.80

Notes: Panel A characterizes the optimal debt ceiling in terms of induced behavior of the selfish type, for each $y_1 \in \{y_L, y_H\}$ either postponing rent-seeking to period 2 ($\overline{d}_{opt} \leq c_1$) or inducing rent seeking in the first period ($\overline{d}_{opt} > c_1$). Panel B classifies the optimal constant ceiling based on whether for each $y_1 \in \{y_L, y_H\}$ it binds the benevolent type ($\overline{d}_{opt} < d(y_1)$) or not ($\overline{d}_{opt} \geq d(y_1)$). n.p. means 'not possible'.

Table 5: Percentage wise overview of the nature of optimal constant debt ceilings

Table 5 reveals that in about 35% of the cases the optimal constant ceiling induces the selfish type to always postpone rent seeking to period 2. Put differently, in these instances the optimal ceiling is in the pooling equilibrium range of Proposition 1(c). In 24% of the cases the optimal ceiling induces full separation. The most frequent case (almost 41%), however, is where the optimal ceiling falls in the hybrid equilibrium range of Proposition 1(b). The selfish type is then induced to postpone rent seeking when current economic circumstances are bad, but not so when these are good. Recall that with a constant ceiling the reverse can never happen. Panel B reveals that the optimal constant ceiling always binds the benevolent type in a large majority (75%) of the cases. Nevertheless, quite often (almost 20%) it only binds when $y_1 = y_L$. In 5% of the cases the optimal ceiling equals $\overline{d}_{opt} = d(y_L)$ and thus never binds. Finally, considering the implications for both the selfish and the benevolent type jointly, in less than 48% of all cases the qualitative behavioral implications are the same for $y_1 = y_L$ and $y_1 = y_H$.[28] More often than not, the optimal constant ceiling thus induces a different type of behavior under different economic circumstances.

Turning to the contingent ceiling case in Table 6, a first notable feature from panel A is that the far majority of observations – more than 98% – are on the diagonal. A hybrid-like outcome in which the selfish type postpones rent seeking in some, but not all economic circumstances occurs in less than

[28]This percentage cannot be read directly off Table 5, but can be inferred from the detailed overview in Table 8 in Appendix B. In particular, the overall 35.05% of cases appearing in panel A with a pooling equilibrium, can be split into 7.28% of cases where the optimal constant ceiling only binds when $y_1 = y_L$ and 27.77% of cases where it binds for both $y_1 \in \{y_L, y_H\}$. In a similar vein, the 24.09% of cases with a separating equilibrium can be split into 4.26% and 19.83%, respectively. Hence in $27.77 + 19.83 = 47.60\%$ of the cases the qualitative behavioral implications of the optimal ceiling are the same when $y_1 = y_L$ and $y_1 = y_H$.

		$\overline{d}_{opt}(y_L) \leq c_L$	$\overline{d}_{opt}(y_L) > c_L$	$\overline{d}_{opt}(y_L) < d(y_L)$	$\overline{d}_{opt}(y_L) \geq d(y_L)$
A. Rent seeking by selfish type		y_L*: period 2*	y_L*: period 1*		
$\overline{d}_{opt}(y_H) \leq c_H$	y_H*: period 2*	77.81	0		
$\overline{d}_{opt}(y_H) > c_H$	y_H*: period 1*	1.84	20.36		
B. Restrictions on benevolent type				y_L*: binds*	y_L*: lax*
$\overline{d}_{opt}(y_H) < d(y_H)$	y_H*: binds*			85.42	4.44
$\overline{d}_{opt}(y_H) \geq d(y_H)$	y_H*: lax*			0	10.15

Notes: Panel A characterizes the optimal contingent ceilings in terms of induced behavior of the selfish type, for each $y_1 \in \{y_L, y_H\}$ either postponing rent-seeking to period 2 ($\overline{d}_{opt}(y_1) \leq c_1$) or inducing rent seeking in the first period ($\overline{d}_{opt}(y_1) > c_1$). Panel B classifies the optimal contingent ceilings based on whether for each $y_1 \in \{y_L, y_H\}$ it binds the benevolent type ($\overline{d}_{opt}(y_1) < d(y_1)$) or not ($\overline{d}_{opt}(y_1) \geq d(y_1)$).

Table 6: Percentage wise overview of the nature of optimal contingent debt ceilings

2% of the cases. A second observation is that there are no observations above the diagonal. Although Lemma 7(d) does not formally exclude this possibility, the numerical analysis confirms our earlier intuition that such a reversal in rent seeking behavior (postpone rent seeking when $y_1 = y_H$ but not when $y_1 = y_L$) is very unlikely.[29] Similar observations pertain to panel B. Again most observations are on the diagonal; in over 85% of the cases the optimal contingent ceiling is always binding, while in about 10% of the cases it is never so. And now there are no observations below the diagonal, resulting from the implication derived in Lemma 7(c). Combining the above observations from panels A and B of Table 6, in 94% of all cases the qualitative behavioral implications are the same for $y_1 = y_L$ and $y_1 = y_H$ (cf. Table 8 in the Appendix B). The overall picture that emerges is thus that the optimal contingent ceiling induces the same type of behavior under different economic circumstances in a tailor made way. In particular, the most common case (over 63%) is a binding contingent ceiling equal to c_L respectively c_H that induces the selfish type to always postpone rent-seeking to period 2.

The above findings suggest that contingent ceilings typically allow for a welfare improving shift from a hybrid equilibrium under a constant ceiling towards a pooling outcome. This is illustrated in Table 7, which displays the induced equilibria under optimal constant ceilings (columns) against the nature of the induced equilibria under optimal contingent ceilings (rows). Contingent debt ceilings induce a clear movement towards more pooling. Intuitively this can be easily understood. Inducing a pooling outcome with a constant debt ceiling requires a restrictive uniform debt limit (cf. Proposition 1(c)) that disproportionately hurts a benevolent incumbent when $y_1 = y_L$. With a contingent ceiling this downside vanishes as the debt limit could be set higher (only) when $y_1 = y_L$ and still induce pooling. More generally, the extensive margin effect of making ceilings contingent is that it may induce a shift in equilibrium, typically lowering the *incidence* of rent seeking (when moving from separating to hybrid or pooling, or from hybrid to pooling).[30] The most common shift is from hybrid to pooling (39.83%), but

[29] For our CRRA specification we did not find any instances even when expanding the parameter configurations beyond those in Table 4; yet for CARA we did encounter such cases (and for parameter values that are not extreme). The latter also indicates that the condition in Lemma 2 is not sufficient for excluding this case.

[30] The incidence of rent seeking in a pooling outcome equals $(1-\theta)$; there is no rent seeking in the first period, while in the second period it occurs with probability $(1-\theta)$. (If the incumbent is re-elected, $(1-\theta)$ reflects the prior probability that the incumbent is selfish; in case the incumbent is not re-elected, $(1-\theta)$ gives the probability of encountering a selfish challenger.) Similarly so, the incidence of rent-seeking in a separating outcome equals $(1-\theta) + (1-\theta)^2 = (1-\theta)(2-\theta)$. For the hybrid outcome the incidence then equals the (state-probability) weighted combination of the two: $(1-\theta)[(1-p) + p(2-\theta)]$.

also shifts from a separating outcome towards a pooling one are now and then observed (in 2.93% of cases). The latter corresponds to the couterintuive outcome in case (i) of Proposition 4(b.2), where the optimal constant ceiling is strictly higher than both contingent ones. Only in 0.1% of the cases a shift in the opposite direction of increasing the incidence of rent-seeking is observed, from a hybrid outcome under a constant ceiling to a separating one under the contingent ceilings. This corresponds to case (ii) in Proposition 4(b.2) in which the optimal constant ceiling always falls short of the optimal contingent ones. Here the overall welfare gain essentially follows from a compensating intensive margin effect that makes separation induced via contingent ceilings sufficiently more beneficial than separation induced via a constant one. Finally, all 0.9% of instances where a shift from separation under a constant ceiling towards a hybrid outcome under contingent ones is observed, belong to case (iii) in Proposition 4(b.2) where the contingent ceiling is more strict when economic circumstances are bad than when these are good.[31]

Contigent ceiling case	*Constant ceiling case*			
	Pooling	*Hybrid*	*Separating*	*Total*
Pooling outcome	35.05	39.83	2.93	77.81
Hybrid outcome	0	0.94	0.90	1.84
Separating outcome	0	0.10	20.26	20.36
Total	35.05	40, 87	24.09	

Table 7: Percentage wise overview of outcomes under constant vs. contingent optimal ceilings

Even in all instances where the type of outcome stays the same (cf. the diagonal in Table 7), a strict welfare gain is made by being able to use contingent ceilings. That is, in all these instances the optimal contingent ceilings differ between states ($\overline{d}_{opt}(y_L) \neq \overline{d}_{opt}(y_H)$). For the parameter configurations considered, we thus do not obtain any cases that belong to Proposition 4(a) (see the column labelled 'lax' in Table 8 in Appendix B). Only when we would increase y_H to an arguably unreasonable extent (well above 10), a few cases are found where Proposition 4(a) applies. This situation thus seems to be the very rare exception rather than the rule.

In short, the numerical analysis for the CRRA specification suggests that contingent ceilings tailored to the state of the economy bring a strict welfare gain that typically derives from inducing the same type of behavior by the selfish and benevolent types in a tailor made way.

6 Concluding discussion

The debate on the EU fiscal framework has been revived by the fact that, following the changes induced by the global financial crisis and the eurozone debt crisis, it has become even more complicated and intransparent, with extensive use of flexibility. These changes did not induce the highest-debt countries to meaningfully reduce their public debt levels during the relatively good times preceding the corona crisis (European Fiscal Board, 2019). The corona crisis triggered the so-called "general escape clause" of the SGP allowing for the maximum conceivable degree of freedom, and put the potential reform of

[31] Moreover, also 0.4% in the overall 0.94% cases where a hybrid outcome pertains under both types of ceilings belong to that counterintuive case.

the framework temporarily on hold. The European Fiscal Board (2022) has argued that the temporary parking of the rules should be exploited to reform the SGP or that, if the time available does not permit a reform to be completed, the competent authorities at least come up with a proposal to that extent. With debt ratios having jumped to even higher levels during the corona crisis, and most in the countries with the highest pre-corona debt, the question on the appropriate debt reduction requirements features prominently in the debate.

The European Fiscal Board has in several reports (e.g. European Fiscal Board, 2018, 2019) argued for a reform of the SGP with a debt ceiling as the single, long-run fiscal anchor. The exact way in which this ceiling is set then becomes paramount. This paper has analysed public debt ceilings in the presence of uncertainty about both the type of incumbent in power, who can either be benevolent or only interested in extracting public resources for her own benefit, and the state of the economy. In effect, debt ceilings curb excessive debt accumulation, i.e. debt generated for diversion to private use. Generally, there are three possible equilibria: a separating one in which the selfish incumbent grabs a maximum amount of resources in the first period and is for sure not re-elected, a hybrid equilibrium in which the selfish incumbent mimics the benevolent one when the current level of resources is low, but grabs all the available resources when current resources are high, and a pooling equilibrium in which the selfish incumbent mimics the benevolent one in all the states of the world. The equilibria differ in the relative importance of the disciplining and selection effects associated with the possibility of voting the incumbent out of office.

In setting the optimal non-contingent debt ceiling a trade-off is made between reducing the debt ceiling in order to lower the amount of resources that can be grabbed by a selfish incumbent, versus raising the debt ceiling in order to provide a benevolent policymaker with discretion in choosing the socially-optimal intertemporal resource allocation. A shift in the debt ceiling may have an intensive margin effect, i.e. the equilibrium type is unchanged, and an extensive margin effect, in which the type of equilibrium itself changes. This complicates the search for the optimal debt ceiling, as it is necessary to find the optimal ceiling for each possible type of equilibrium that may exist and then compare welfare for the different local optima.

The benefit of increasing transparency about the state of the economy depends on the degree to which such transparency can be enhanced. Increasing the probability with which the true state can be observed before elections take place leaves the optimal (constant) debt ceiling and equilibrium welfare unaffected. The selfish incumbent modifies her equilibrium strategy in that she no longer hides behind the state of the economy so as to avoid being unmasked. Improved electoral accountability per se therefore does not make given, state-independent fiscal rules more effective. This may perhaps explain why the establishment of independent fiscal institutions arguably did little to combat excessive debt accumulation in the eurozone. However, if transparency about the state can be increased to such an extent that it is possible to install state-contingent debt ceilings, equilibrium welfare may increase.

Then, a key question is when – and how likely it is that – it is optimal to differentiate the debt ceiling between the states, rather than to impose a constant debt ceiling. This is important, because there is a cost to installing an independent fiscal institution with sufficient resources and a mandate to objectively and precisely establish or verify the state of the economy. Allowing for state-contingent debt ceilings is beneficial in the most typical case where the optimal constant ceiling induces a hybrid equilibrium, because the alternative of pooling would require a rather restrictive uniform debt ceiling that particularly hurts a benevolent incumbent when the state is bad. With state-contingent ceilings this downside vanishes and the induced outcome typically shifts to always mimicking in the first period. As a result, less (excessive) debt is created for rent-seeking purposes. Numerical analysis shows that the optimal state-contingent debt ceiling frequently constrains the benevolent incumbent's debt choice, which is the price paid for limiting the larger evil of the selfish incumbent grabbing all the resources for private benefit.

Our findings provide a rationale for revising the SGP such that it allows debt reduction requirements to be tighter in a good than in a bad economic state. For the context of the European Union our analysis suggests that differentiating debt reduction requirements across macroeconomic circumstances may be beneficial by inducing governments to reduce debt more in good times, thereby alleviating the financial burden in bad times. Such a modification helps to alleviate the problem of procyclical fiscal policy (European Fiscal Board, 2019). At a more general level, the issue is also addressed in the recent Commission proposals (European Commission, 2022), in which the Commission proposes a country-specific reference path for spending and the country is invited to come up with its own proposal that ensures debt of high-debt countries to be sustainably falling at the end of a reference period. The macroeconomic circumstances could then be implicitly taken into account in a Member State's proposed spending path that fulfills this requirement.

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Appendix A: Proofs

Proof of Lemma 1. Part (a). From u' strictly decreasing, the left-hand side (l.h.s.) of (6) is strictly decreasing in d and the right-hand side (r.h.s.) strictly increasing. Given $\lim_{g\to 0} u'(g) = \infty$ there is a unique debt level $d(y_1)$ that solves (6), with $-y_1 < d(y_1) < y_L$, for otherwise either the l.h.s. or the r.h.s. of (6) would go to infinity and equality would not be met. Moreover, from u' strictly decreasing it also immediately follows that $d(y_1)$ is strictly decreasing with y_1. Together these give $-y_H < d(y_H) < d(y_L) < y_L$ and $-y_L < d(y_L)$.

To obtain the tighter lower bounds in (i) and (ii), note that:

$$u'(y_1 + d) = \beta E\left[u'(y_2 - d)\right] \leq E\left[u'(y_2 - d)\right] < u'(y_L - d)$$

where the first inequality follows from $\beta \leq 1$ and the second one from $u' < 0$, $p > 0$ and $y_H > y_L$. Again from $u' < 0$ we then immediately have $y_1 + d(y_1) > y_L - d(y_1)$, hence $d(y_1) > \frac{y_L - y_1}{2}$. For the tighter upper bound in (a.ii), differentiate (6) towards y_1 to obtain $\left(1 + \frac{\partial d(y_1)}{\partial y_1}\right) \cdot u''(y_1 + d) = -\frac{\partial d(y_1)}{\partial y_1} \cdot \beta E\left[u''(y_2 - d)\right]$. This gives $\frac{\partial d(y_1)}{\partial y_1} = \frac{-u''}{u'' + \beta E u''}$. With $u'' < 0$ and $Eu'' < 0$ this implies $-1 < \frac{\partial d(y_1)}{\partial y_1} < 0$, in turn yielding $d(y_L) < d(y_H) + (y_H - y_L)$.

Part (b). Expected utility $u(y_1 + d) + \beta E\left[u(y_2 - d)\right]$ is strictly increasing up till $d(y_1)$ and strictly decreasing for all $d > d(y_1)$. The optimal debt choice thus equals $d^*(y_1) \equiv \min\left\{\overline{d}, d(y_1)\right\}$. From part (a) we get $y_L + d^*(y_L) = \min\left\{y_L + \overline{d}, y_L + d(y_L)\right\} < \min\left\{y_H + \overline{d}, y_H + d(y_H)\right\} = y_H + d^*(y_H)$. ■

Proof of Lemma 2. We proceed in three steps.

Step 1. Note that $\frac{\partial(d(y_1) - c_1)}{\partial y_1} = \frac{\partial(d(y_1))}{\partial y_1} + \frac{1}{1+\beta} \geq 0$ if $\frac{\partial(d(y_1))}{\partial y_1} \geq -\frac{1}{1+\beta}$. Differentiating (6) defining $d(y_1)$ towards y_1 and rearranging we obtain

$$\frac{\partial d(y_1)}{\partial y_1} = \frac{-u''(y_1 + d(y_1))}{u''(y_1 + d(y_1)) + \beta E u''(y_2 - d(y_1))} = \frac{-1}{1 + \beta\left[\frac{Eu''(y_2 - d(y_1))}{u''(y_1 + d(y_1))}\right]}$$

Hence $\frac{\partial(d(y_1))}{\partial y_1} \geq -\frac{1}{1+\beta}$ if $\frac{Eu''(y_2 - d(y_1))}{u''(y_1 + d(y_1))} \geq 1$. With $u'' < 0$ (and thus $Eu'' < 0$) this corresponds to:

$$u''(y_1 + d(y_1)) \geq E\left[u''(y_2 - d(y_1))\right] \tag{10}$$

Step 2. We next show that it is sufficient to prove (10) for the case where $\beta = 1$.

The optimal debt level $d(y_1)$ depends on β; to make this explicit, in the remainder of the proof we write $d(y_1;\beta)$. From the Euler equation $u'(y_1 + d(y_1;\beta)) = \beta E[u'(y_2 - d(y_1;\beta))]$ defining $d(y_1;\beta)$, we have $\frac{\partial d(y_1;\beta)}{\partial \beta} = \frac{E[u'(y_2 - d(y_1;\beta))]}{u''(y_1 + d(y_1;\beta)) + \beta E[u''(y_2 - d(y_1;\beta))]} < 0$ as (only) the denominator is negative. Differentiating the l.h.s of (10) towards β we obtain:

$$\frac{\partial u''(y_1 + d(y_1;\beta))}{\partial \beta} = u'''(y_1 + d(y_1;\beta)) \cdot \frac{\partial d(y_1;\beta)}{\partial \beta} < 0$$

when $u''' > 0$. Similarly, differentiating the r.h.s. of (10) towards β we get:

$$\frac{\partial E[u''(y_2 - d(y_1;\beta))]}{\partial \beta} = -E[u'''(y_2 - d(y_1;\beta))] \cdot \frac{\partial d(y_1;\beta)}{\partial \beta} > 0$$

when $u''' > 0$. Therefore, inequality (10) holds for all $\beta \leq 1$ if it holds for $\beta = 1$.

Step 3. We finally show that (10) holds for $\beta = 1$. The Euler equation when $\beta = 1$ reads:

$$u'(y_1 + d(y_1;1)) = E[u'(y_2 - d(y_1;1))] \tag{11}$$

Define $h(x) \equiv u'' \circ u'^{-1}(x) = u''\left(u'^{-1}(x)\right)$, i.e. the composite function of u'' and the inverse of u'. It follows that $h'(x) = \frac{u'''(u'^{-1}(x))}{u''(u'^{-1}(x))} < 0$ and that (see Jehle and Reny, 2011; pp. 114-115):

$$h''(x) = \frac{u'''\left(u'^{-1}(x)\right) \cdot \left\{ \frac{u''''(u'^{-1}(x))}{u'''(u'^{-1}(x))} - \frac{u'''(u'^{-1}(x))}{u''(u'^{-1}(x))} \right\}}{\left[u''(u'^{-1}(x))\right]^2}$$

which is non-positive if the term within {} is. Thus, if $\frac{u''''}{u'''} \leq \frac{u'''}{u''}$, h is concave.

Now apply h to both sides of (11):

$$\begin{aligned} h(u'(y_1 + d(y_1;1))) &= h(E[u'(y_2 - d(y_1;1))]) \\ &\Longrightarrow \\ u''(y_1 + d(y_1;1)) &= h(E[u'(y_2 - d(y_1;1))]) \\ &\geq E[h(u'(y_2 - d(y_1;1)))] \\ &= E[u''(y_2 - d(y_1;1))] \end{aligned}$$

where the inequality follows from Jensen's inequality and h concave. ■

Proof of Lemma 3.

Part (a). Comparing the relevant payoffs in Table 1, a selfish incumbent strictly prefers policy combination $\left(\overline{d}, 0\right)$ over $(d^*(y_1), g_1^*(y_1))$ for any $\pi_1 \in [0,1]$ if $\overline{d} > \beta(E(y_2) - d^*(y_1)) - y_1$, i.e. if

$$\overline{d} > \beta\left(E(y_2) - \min\left\{d(y_1), \overline{d}\right\}\right) - y_1$$

The l.h.s. of this inequality (grows without bounds and) is strictly increasing in $\overline{d}$, while the r.h.s. is weakly decreasing in $\overline{d}$. Hence the inequality can be rewritten as $\overline{d} > K$ for some unique cutoff K. First, suppose that the debt limit strictly binds the benevolent type at this cutoff, such that $\min\left\{d(y_1), \overline{d}\right\} = \overline{d}$ at $\overline{d} = K$. The inequality then reduces to $\overline{d} > c_1$. The supposition that the debt limit strictly binds at this cutoff requires $c_1 < d(y_1)$. Next suppose the debt limit does not strictly bind at cutoff K, so $\min\left\{d(y_1), \overline{d}\right\} = d(y_1)$ at $\overline{d} = K$. The inequality then reduces to $\overline{d} > \beta(E(y_2) - d(y_1)) - y_1 = c_1 + \beta(c_1 - d(y_1))$. The supposition that the debt limit does not strictly bind the benevolent type at this cutoff requires $c_1 + \beta(c_1 - d(y_1)) \geq d(y_1)$, i.e. $d(y_1) \leq c_1$. Combining the two cases we obtain $K = c_1 + \max\{0, \beta(c_1 - d(y_1))\}$.

Part (b). Comparing the relevant payoffs in Table 1, a selfish incumbent strictly prefers policy combination $(d^*(y_L), g_1^*(y_L))$ over $(d^*(y_H), g_1^*(y_H))$ if

$$-(y_H - y_L) + \pi_H \cdot \beta(E(y_2) - d^*(y_H)) < \pi_L \cdot \beta(E(y_2) - d^*(y_L))$$

Rearranging this can be rewritten as:

$$\pi_H \cdot \beta[d^*(y_L) - d^*(y_H)] - (\pi_L - \pi_H) \cdot \beta(E(y_2) - d^*(y_L)) < (y_H - y_L)$$

From $d(y_H) < d(y_L) < d(y_H) + (y_H - y_L)$ per Lemma 1 it follows that $[d^*(y_L) - d^*(y_H)] < (y_H - y_L)$. Hence, since $\pi_H \cdot \beta \leq 1$ and $d^*(y_L) < E(y_2)$, the above inequality necessarily holds if $\pi_L \geq \pi_H$. ■

Proof of Proposition 1. We first show that necessarily $0 < \pi_L \leq \pi_H \leq 1$ in equilibrium. Suppose $\pi_L = 0$. Then choosing $\left(\overline{d}, 0\right)$ yields the selfish incumbent strictly more than choosing $(d^*(y_L), g_1^*(y_L))$. As the latter combination is then only chosen by the benevolent type, by Bayes' rule we obtain for the posterior belief $\hat{\theta}((d^*(y_L), g_1^*(y_L))) = 1$ and thus $\pi_L = 1$, a contradiction. Hence $\pi_L > 0$ necessarily. Next suppose $\pi_L > \pi_H$. Then by Lemma 3(b) policy combination $(d^*(y_H), g_1^*(y_H))$ is never chosen by the selfish type. Bayes' rule then implies $\hat{\theta}((d^*(y_H), g_1^*(y_H))) = 1$ and thus $\pi_H = 1$, contradicting $\pi_L > \pi_H$. Hence $0 < \pi_L \leq \pi_H \leq 1$ necessarily.

We next show that $c_H + \max\{0, \beta(c_H - d(y_H))\} < c_L + \max\{0, \beta(c_L - d(y_L))\}$, yielding the three intervals for $\overline{d}$ to be considered.[32] Rewrite the inequality as

$$\max\{0, \beta(c_H - d(y_H))\} - \max\{0, \beta(c_L - d(y_L))\} < c_L - c_H = \frac{(y_H - y_L)}{1 + \beta}$$

From Lemma 1 we have that $d(y_L) - (y_H - y_L) < d(y_H)$. Hence, the inequality certainly holds if $\max\{0, \beta([c_H + (y_H - y_L)] - d(y_L))\} - \max\{0, \beta(c_L - d(y_L))\} \leq \frac{(y_H - y_L)}{1+\beta}$. This latter inequality is implied by $\beta[c_H + (y_H - y_L)] - \beta c_L = \frac{(y_H - y_L)}{1+\beta}$.

Now first suppose $\overline{d} > c_L + \max\{0, \beta(c_L - d(y_L))\}$. From Lemma 3(a) it immediately follows that policy $\left(\overline{d}, 0\right)$ is a dominant choice for the selfish incumbent in this case. By Bayes' rule then $\pi_L = \pi_H = 1$, yielding the (unique) separarting equilibrium of part (a).

Next suppose $c_H + \max\{0, \beta(c_H - d(y_H))\} < \overline{d} < c_L + \max\{0, \beta(c_L - d(y_L))\}$. By Lemma 3(a) policy $\left(\overline{d}, 0\right)$ strictly dominates $(d^*(y_H), g_1^*(y_H))$, leading to $\pi_H = 1$ by the implication of Bayes' rule. The selfish type thus effectively only chooses between $\left(\overline{d}, 0\right)$ and $(d^*(y_L), g_1^*(y_L))$. Suppose

[32] With c_1 decreasing in y_1 (cf. equation (7)) the inequality immediately follows under the conditions of Lemma 2. The argument here in the main text does not depend on the assumptions of this lemma.

she would always choose $\left(\overline{d},0\right)$. Then necessarily $\pi_L = 1$ as well. But for $\pi_L = 1$ we have $\pi_L \cdot \beta\left(E\left(y_2\right) - d^*\left(y_L\right)\right) > y_L + \overline{d}$ given $\overline{d} < c_L + \max\left\{0, \beta\left(c_L - d\left(y_L\right)\right)\right\}$ and the selfish incumbent would prefer $\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)$ over $\left(\overline{d},0\right)$, a contradiction. Similarly so, if the selfish type would always choose $\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)$ (and in particular, thus also when $y_1 = y_H$), then the posterior belief upon observing this combination would be strictly below the prior (i.e. $\hat{\theta}\left(\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)\right) < \theta$), and voters would elect the challenger for sure; $\pi_L = 0$. But in that case the selfish type would rather prefer policy $\left(\overline{d},0\right)$, again a contradiction. Hence the selfish type must be willing to use a strictly mixed strategy in equilibrium and be indifferent between $\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)$ and $\left(\overline{d},0\right)$. This requires $\pi_L = \frac{y_L + \overline{d}}{\beta\left(E\left(y_2\right) - d^*\left(y_L\right)\right)}$, and thus that the voter is indifferent between electing the incumbent and the challenger. The latter in turn requires that the posterior belief equals the prior, i.e. $\hat{\theta}\left(\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)\right) = \theta$, which corresponds to $(1-p) = (1-p)\lambda\left(y_L\right) + p\lambda\left(y_H\right)$. This gives the class of hybrid equilibria of part (b).

Finally, suppose $\overline{d} < c_H + \max\left\{0, \beta\left(c_H - d\left(y_H\right)\right)\right\}$. If $\pi_L = 1$, based on Lemma 3 the selfish type would necessarily always choose $\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)$, leading to $\pi_L = 0$ by the implication of Bayes' rule, a contradiction. Hence $0 < \pi_L < 1$. For the voter to be willing to mix after$\left(d^*\left(y_L\right), g_1^*\left(y_L\right)\right)$, it is required that $(1-p) = (1-p)\lambda\left(y_L\right) + p\lambda\left(y_H\right)$. Suppose that the selfish type chooses $\left(\overline{d},0\right)$ with positive probability, i.e. $\lambda\left(y_1\right) + \gamma\left(y_1\right) < 1$ for some $y_1 \in \left\{y_L, y_H\right\}$. Together with $(1-p) = (1-p)\lambda\left(y_L\right) + p\lambda\left(y_H\right)$ this necessarily implies that $p > (1-p)\gamma\left(y_L\right) + p\gamma\left(y_H\right)$ and thus $\hat{\theta}\left(d^*\left(y_H\right), g_1^*\left(y_H\right)\right) > \theta$ and thereby $\pi_H = 1$. But in that case choosing $\left(d^*\left(y_H\right), g_1^*\left(y_H\right)\right)$ strictly dominates $\left(\overline{d},0\right)$, a contradiction. So, the selfish type never chooses $\left(\overline{d},0\right)$, and thus $p = (1-p)\gamma\left(y_L\right) + p\gamma\left(y_H\right)$ and $\gamma\left(y_1\right) = 1 - \lambda\left(y_1\right)$. Condition (c.i) follows from the required indifference between the two policy combinations chosen by the benevolent type, condition (c.ii) from the requirement that full rent extraction should yield (weakly) less. This gives the pooling equilibria of part (c).

In the border case $\overline{d} = c_L + \max\left\{0, \beta\left(c_L - d\left(y_L\right)\right)\right\}$, both equilibria (a) and (b) exist at the same time and, in fact, all combinations of $\lambda\left(y_L\right)$ and $\lambda\left(y_H\right)$ satisfying $(1-p) \geq (1-p)\lambda\left(y_L\right) + p\lambda\left(y_H\right)$ correspond to equilibrium behavior. The proposition assumes that then the hybrid equilibria of part (b) result. Given that at the border there is a discrete fall in welfare moving from the hybrid to the separating equilibrium (cf. Lemma 5), this assumption ensures that expected equilibrium welfare is an upper semi-continuous function of $\overline{d}$ and thus always attains a well-defined maximum. This facilitates the analysis of the optimal debt ceiling without having any substantive implication. (It simply avoids that expected equilibrium welfare would be highest for $\overline{d}$ strictly below but arbitrary close to the border; in terms of the size of the optimal debt ceiling these cases are practically equivalent to having $\overline{d}$ at the border.) A similar remark holds for the other border case $\overline{d} = c_H + \max\left\{0, \beta\left(c_H - d\left(y_H\right)\right)\right\}$ at which equilibria (b) and (c) co-exist and, in fact, all combinations $\gamma\left(y_L\right)$ and $\gamma\left(y_H\right)$ satisfying $p \geq (1-p)\gamma\left(y_L\right) + p\gamma\left(y_H\right)$ correspond to equilibrium behavior. The proposition assumes that the pooling equilibria of part (c) result to ensure upper semi-continuity of expected equilibrium welfare. ■

Proof of Lemma 4. First note that benchmark welfare W_T can written as:

$$W_T \equiv (1-\theta)\, W_T^s + \theta W_T^b = W_T^s + \theta\left(W_T^b - W_T^s\right) = (1+\beta)\, u\,(0)$$
$$+\, \theta\left\{(1-p)\left[D\left(g_1^*\left(y_L\right)\right) + S\left(d^*\left(y_L\right)\right)\right] + p\left[D\left(g_1^*\left(y_H\right)\right) + S\left(d^*\left(y_H\right)\right)\right]\right\}$$

With both $D()$ and $S()$ strictly concave and $D'\left(y_1+\overline{d}\right)+S'(\overline{d})>[<]0$ for $\overline{d}<[>]d(y_1)$ from the Euler equation (6) defining $d(y_1)$, we have that W_T is increasing and strictly concave in $\overline{d}$ for $\overline{d}<d(y_L)$. For $\overline{d}\geq d(y_L)$, W_T is flat, i.e. $\frac{\partial W_T}{\partial \overline{d}}=0$. For $\overline{d}\leq d(y_L)$ and thus $d^*(y_L)=\overline{d}$ the derivative equals:

$$\frac{\partial W_T}{\partial \overline{d}}=\theta\cdot\left\{(1-p)\cdot\left[D'\left(y_L+\overline{d}\right)+S'(\overline{d})\right]+p\cdot\max\left\{\left[D'\left(y_H+\overline{d}\right)+S'(\overline{d})\right],0\right\}\right\}$$

Here the max-term follows from the fact that the derivative of $D\left(g_1^*\left(y_H\right)\right)+S\left(d^*\left(y_H\right)\right)$ equals $D'\left(y_H+\overline{d}\right)+S'(\overline{d})$ for $\overline{d}\leq d(y_H)$ and 0 for $\overline{d}>d(y_H)$, together with $D'\left(y_H+\overline{d}\right)+S'(\overline{d})$ decreasing and equal to 0 at $\overline{d}=d(y_H)$.

Part (a). From Table 2 we have $EW_s=W_T+\theta(1-\theta)S\left(\overline{d}\right)$. From the concave shape of W_T and $S\left(\overline{d}\right)$ strictly decreasing and strictly concave in $\overline{d}$, it follows that EW_s is strictly concave. It thus attains a global maximum on $[-y_L,y_L]$, which is necessarily weakly below $d(y_L)$ since EW_s is strictly decreasing in $\overline{d}$ for $\overline{d}>d(y_L)$. The first order condition characterizing the unique global maximizer $\tilde{d}_s$ equals:

$$\frac{\partial W_T}{\partial \overline{d}}=-\theta(1-\theta)S'\left(\overline{d}\right)$$

Part (b). From Table 2 we have $EW_h=W_T+(1-\theta)\left[(1-p)D\left(g_1^*\left(y_L\right)\right)+p\theta S\left(\overline{d}\right)\right]$. Term $(1-p)D\left(g_1^*\left(y_L\right)\right)+p\theta S\left(\overline{d}\right)$ has a kink at $\overline{d}=d(y_L)$ but is differentiable elsewhere. The left derivative of this term at the kink can be insightfully written as:

$$(1-p)\left[D'\left(y_L+\overline{d}\right)+S'(\overline{d})\right]+(p\theta-(1-p))S'(\overline{d})$$

At $\overline{d}=d(y_L)$ the term within square brackets equals zero and thus the value of the left derivative reduces to $(p\theta-(1-p))S'(\overline{d})$. Likewise, the value of the right derivative at $\overline{d}=d(y_L)$ is $p\theta S'(\overline{d})<0$. If the left derivative is weakly positive, term $(1-p)D\left(g_1^*\left(y_L\right)\right)+p\theta S\left(\overline{d}\right)$ reaches its maximum at the kink $\overline{d}=d(y_L)$. This is the case iff $p\theta-(1-p)\leq 0$, i.e. iff $p\leq\frac{1}{1+\theta}$. In case $p>\frac{1}{1+\theta}$ the left derivative is strictly negative at the kink and the maximizer of this term is at some debt level strictly below $d(y_L)$. From the shape of W_T these conclusions carry over to the global maximizer of EW_h. In particular, the global maximizer of EW_h corresponds to $d(y_L)$ iff $p\leq\frac{1}{1+\theta}$, and for $p>\frac{1}{1+\theta}$ to the unique solution $\tilde{d}_h$ to the first order condition:

$$\frac{\partial W_T}{\partial \overline{d}}=-(1-\theta)\left[(1-p)D'\left(y_L+\overline{d}\right)+p\theta S'\left(\overline{d}\right)\right]$$

Note that from $D'\left(y_L+\overline{d}\right)>\theta S'\left(\overline{d}\right)$ it follows that the r.h.s. falls short of $-\theta(1-\theta)S'\left(\overline{d}\right)$; this implies $\tilde{d}_h>\tilde{d}_s$.

Part (c). From Table 2 we have $EW_p=W_T+(1-\theta)\left[(1-p)D\left(g_1^*\left(y_L\right)\right)+pD\left(g_1^*\left(y_H\right)\right)\right]$. Like W_T, term $\left[(1-p)D\left(g_1^*\left(y_L\right)\right)+pD\left(g_1^*\left(y_H\right)\right)\right]$ is strictly increasing and concave in $\overline{d}$ up till $\overline{d}=d(y_L)$ and constant afterwards. This immediately yields the statement. ■

Proof of Lemma 5. The expressions for EW_s-EW_h and EW_h-EW_p immediately follow from those in Table 2. From $S'(\overline{d})<0$ and $D'\left(g_1^*\left(y_1\right)\right)\geq 0$ it also immediately follows that $\theta S(\overline{d})-D\left(g_1^*\left(y_1\right)\right)$ is decreasing in $\overline{d}$. To prove the final claim, observe that $d^*(y_1)$ maximizes $D(y_1+d)+S(d)$

subject to $d \leq \overline{d}$. Hence, by definition we necessarily have that: $D(y_1 + d^*(y_1)) + S(d^*(y_1)) \geq D(y_1 + \overline{d}) + S(\overline{d})$. Rewriting gives:

$$S(\overline{d}) - D(y_1 + d^*(y_1)) \leq S(d^*(y_1)) - D(y_1 + \overline{d})$$

Evaluating the r.h.s. at the border where $y_1 + \overline{d} = \beta(E(y_2) - d^*(y_1))$ and applying Jensen's inequality we obtain:

$$\begin{aligned} S(d^*(y_1)) - D(y_1 + \overline{d}) &= S(d^*(y_1)) - D(\beta(E(y_2) - d^*(y_1))) \\ &< \beta \cdot D(E(y_2) - d^*(y_1)) - D(\beta(E(y_2) - d^*(y_1))) \end{aligned}$$

where the strict inequality follows from $u()$ being strictly concave and applying Jensen's inequality (implying for any debt level d that $S(d) < \beta \cdot D(Ey_2 - d)$). Now, from $u' > 0$ and $u'' < 0$ it follows that $\beta \cdot u'(x) \leq \beta \cdot u'(\beta x) = \frac{\partial u(\beta x)}{\partial x}$. This, in turn, implies $\beta \cdot [u(x) - u(0)] \leq u(\beta \cdot x) - u(0)$ for all $x \geq 0$ (as for $x = 0$ the inequality is met (with an equality)) and thus $\beta \cdot D(g) \leq D(\beta \cdot g)$. Hence, overall

$$S(\overline{d}) - D(g_1^*(y_1)) < \beta \cdot D(E(y_2) - d^*(y_1)) - D(\beta(E(y_2) - d^*(y_1))) \leq 0$$

This proves the lemma. ■

Proof of Lemma 6.

Part (a). From Lemmas 4 and 5 it follows that equilibrium welfare is necessarily weakly decreasing for $\overline{d} > d(y_L)$. If at $\overline{d} = d(y_L)$ the pooling equilibrium applies, one cannot do better and the same welfare level is obtained as long as the limit stays within the pooling interval, i.e. $\overline{d} \leq c_H + \max\{0, \beta(c_H - d(y_H))\}$ (cf. Proposition 1). Note that $d(y_L) > 0$ necessarily by Lemma 1. With $c_H < 0$ we thus necessarily must have that $c_H + \max\{0, \beta(c_H - d(y_H))\} = c_H + \beta(c_H - d(y_H))$ for $d(y_L)$ falling in the pooling interval to be possible.

Part (b). In this case, at $\overline{d} = d(y_L)$ either the hybrid or separating equilibrium applies and it strictly harms to set a higher debt ceiling; hence $\overline{d}_{opt} \leq d(y_L)$. Per Lemma 4(c) the optimal ceiling cannot be in the interior of the pooling range, i.e. $\overline{d}_{opt} \geq c_H + \max\{0, \beta(c_H - d(y_H))\}$. Now, in case $c_L < d(y_L)$, a debt ceiling equal to $\overline{d} = d(y_L)$ falls within the separating interval. From Lemma 4 we then immediately have that all eligible maximizers that (potentially) qualify for $\overline{d}_{opt}$ strictly fall short of $d(y_L)$. The same holds true for the two borders $c_H + \max\{0, \beta(c_H - d(y_H))\}$ and c_L. Hence $\overline{d}_{opt} < d(y_L)$ necessarily. When $d(y_L) \leq c_L$, the hybrid equilibrium pertains for $\overline{d} = d(y_L)$ (cf. Proposition 1). For $p > \frac{1}{1+\theta}$ expected welfare is decreasing for all debt ceiling beyond $\tilde{d}_h$. Hence $\overline{d}_{opt} \leq \tilde{d}_h < d(y_L)$.

Finally, part (c) trivially follows from $d(y_H) < d(y_L)$, part (d) from $c_H + \max\{0, \beta(c_H - d(y_H))\} < c_L + \max\{0, \beta(c_L - d(y_L))\}$. ■

Proof of Propostion 2. This proposition follows immediately from Proposition A.1 below. ■

Proposition A.1 Consider the setting in which with probability $\xi > 0$ the state of the economy y_1 becomes observable to the voters before they cast their vote.

(a) If $\overline{d} > c_L + \max\{0, \beta(c_L - d(y_L))\}$: There exists a unique *separating* equilibrium in which the selfish type always chooses full rent extraction $\left(\overline{d}, 0\right)$ in the first period. Equilibrium strategies satisfy:

(a.1) $\lambda(y_1) = \gamma(y_1) = 0$, for $y_1 \in \{y_L, y_H\}$;

(a.2) $\pi_L = \pi_H = 1$.

(b) If $c_H + \max\{0, \beta(c_H - d(y_H))\} < \overline{d} \leq c_L + \max\{0, \beta(c_L - d(y_L))\}$: There exists a class of *hybrid* equilibria in which the selfish type always chooses $(d^*(y_L), g_1^*(y_L))$ if $y_1 = y_L$ and full rent extraction $\left(\overline{d}, 0\right)$ in case $y_1 = y_H$. Equilibrium strategies satisfy:

(b.1) $\lambda(y_L) = 1$, $\lambda(y_H) = 0$ and $\gamma(y_L) = \gamma(y_H) = 0$;

(b.2) $\frac{y_L + \overline{d}}{\beta(E(y_2) - d^*(y_L))} \leq \pi_L \leq \frac{1}{1-\xi} \cdot \frac{y_L + \overline{d}}{\beta(E(y_2) - d^*(y_L))}$ and $\pi_H = 1$.

(c) If $\overline{d} \leq c_H + \max\{0, \beta(c_H - d(y_H))\}$: There exists a class of *pooling* equilibria in which the selfish type always chooses $(d^*(y_L), g_1^*(y_L))$ if $y_1 = y_L$ and $(d^*(y_H), g_1^*(y_H))$ in case $y_1 = y_H$; full rent extraction $\left(\overline{d}, 0\right)$ does not occur. Equilibrium strategies satisfy:

(c.1) $\lambda(y_L) = 1$, $\gamma(y_L) = 0$, $\lambda(y_H) = 0$ and $\gamma(y_H) = 1$;

(c.2) (i) $\max\left\{-(y_H - y_L) + (1-\xi)\pi_H \beta(E(y_2) - d^*(y_H)), y_L + \overline{d}\right\} \leq \pi_L \cdot \beta(E(y_2) - d^*(y_L))$ and (ii) $\max\left\{(y_H - y_L) + (1-\xi)\pi_L \beta(E(y_2) - d^*(y_L)), y_H + \overline{d}\right\} \leq \pi_H \cdot \beta(E(y_2) - d^*(y_H))$.

Proof of Proposition A.1. We first show that for $\xi > 0$, the selfish type never deceives in two dimensions (type and state), i.e. never 'hides behind the state'. Suppose that $\lambda(y_H) > 0$. This requires that the selfish type weakly prefers combination $(d^*(y_L), g_1^*(y_L))$ over the other two strategies if $y_1 = y_H$. From the payoffs in Table 3 it follows that she then strictly prefers this combination in case $y_1 = y_L$. Hence, $\lambda(y_H) > 0 \Rightarrow \lambda(y_L) = 1$. But this in turn implies that the posterior belief after observing $(d^*(y_L), g_1^*(y_L))$ falls short of prior belief θ, and thus that the incumbent is not re-elected for sure (i.e., $\pi_L = 0$). Yet, with $\pi_L = 0$ policy combination $(d^*(y_L), g_1^*(y_L))$ is strictly dominated by combination $\left(\overline{d}, 0\right)$ and thus $\lambda(y_H) = 0$, a contradiction. Therefore, in equilibrium necessarily $\lambda(y_H) = 0$. By a similar argument for policy combination $(d^*(y_H), g_1^*(y_H))$ it follows that $\gamma(y_L) = 0$.

First suppose $\overline{d} > c_L + \max\{0, \beta(c_L - d(y_L))\}$. By Lemma 3(a) the selfish type strictly prefers policy $\left(\overline{d}, 0\right)$ over mimicking the benevolent type by choosing $(d^*(y_1), g_1^*(y_1)))$. This gives the separating equilibrium of part (a).

Next suppose $c_H + \max\{0, \beta(c_H - d(y_H))\} < \overline{d} < c_L + \max\{0, \beta(c_L - d(y_L))\}$. The first quality implies $\gamma(y_H) = 0$ by Lemma 3(a) and, together with $\gamma(y_L) = 0$ (no hiding behind the state), $\pi_H = 1$ by Bayes' rule. We show that the second inequality necessarily implies $\lambda(y_L) = 1$. Suppose to the contrary that $\lambda(y_L) < 1$. Together with $\lambda(y_H) = 0$ necessarily (no hiding behind the state), this would imply $\pi_L = 1$ by Bayes' rule and the selfish type would rather prefer $\lambda(y_L) = 1$ instead. Overall this yields $\lambda(y_L) = 1$ and $\lambda(y_H) = \gamma(y_L) = \gamma(y_H) = 0$. After observing $(d^*(y_L), g_1^*(y_L))$ the voter is indifferent and willing to mix and choose any $\pi_L \in [0, 1]$. The stated condition on π_L follows from the requirement that $\lambda(y_L) = 1$ and $\lambda(y_H) = 0$ should be a best response for the the selfish type. This gives the hybrid equilibria of part (b).

Finally, suppose $\overline{d} < c_H + \max\{0, \beta(c_H - d(y_H))\}$. By the same (proof by contradiction) argument as in the previous paragraph, we obtain $\lambda(y_L) = 1$ and $\gamma(y_H) = 1$. The selfish incumbent then necessarily chooses $(d^*(y_L), g_1^*(y_L))$ if $y_1 = y_L$ and $(d^*(y_H), g_1^*(y_H))$ in case $y_1 = y_H$. By Bayes' rule the posterior belief after both these policy combinations then equals the prior belief θ and the voter is indifferent between the challenger and the incumbent. These probabilities should then only be such that the selfish type indeed prefers $(d^*(y_L), g_1^*(y_L))$ when $y_1 = y_L$ and $(d^*(y_H), g_1^*(y_H))$ in case $y_1 = y_H$. This gives conditions (c.1) and (c.2) in the pooling equilibria of part (c).

At the border $\overline{d} = c_L + \max\{0, \beta(c_L - d(y_L))\}$ both the separating equilibrium of part (a) and the hybrid one of part (b) exist at the same time and, in fact, all $\lambda(y_L) \in [0,1]$ correspond to equilibrium behavior. As in Proposition 1, we assume that at the border the hybrid equilibrium of part (b) results to ensure upper semi-continuity of expected equilibrium welfare. A similar remark applies to the border between cases (b) and (c), at which all $\lambda(y_L) \in [0,1]$ and $\gamma(y_H) \in [0,1]$ correspond to equilibrium behavior. We assume that there the pooling equilibrium of part (c) results. ■

Proof Proposition 3.

Part (a). By Lemma 3(a), policy combination $\left(\overline{d}(y_1), 0\right)$ is a strictly dominant choice for the selfish type. In that case $\pi_1 = 1$ by Bayes' rule.

Part (b). Suppose the selfish type chooses $\left(\overline{d}(y_1), 0\right)$ with strict positive probability. Then, the posterior belief after $(d^*(y_1), g_1^*(y_1))$ would strictly exceed θ, implying $\pi_1 = 1$. But in that case the selfish type would not want to choose $\left(\overline{d}(y_1), 0\right)$ as $(d^*(y_1), g_1^*(y_1))$ would yield him more, a contradiction. The selfish type thus chooses $(d^*(y_1), g_1^*(y_1))$ for sure. The posterior belief after this policy combination then equals the prior and any $\pi_1 \in [0,1]$ is a voter's best response. To secure that $(d^*(y_1), g_1^*(y_1))$ yields him (weakly) more than extracting maximum rents, $\pi_1 \cdot \beta(E(y_2) - d^*(y_1)) \geq y_1 + \overline{d}(y_1)$ is needed. This gives the stated condition on π_1.

At the border between cases (a) and (b), the selfish incumbent is actually indifferent between mimicking the benevolent type and extracting maximum rents. Here, we assume that then the selfish type mimics for sure, as to ensure upper semi-continuity of expected equilibrium welfare (cf. the proof of Proposition 1). ■

Proof of Lemma 7. Consider the subgame in which the state of the world equals $y_1 \in \{y_L, y_H\}$. If the selfish type separates like in Proposition 3(a) (ignoring for the moment the condition on $\overline{d}(y_1)$ under which she indeed would do so), expected welfare equals:

$$EW_s(y_1) = \theta[D(g_1^*(y_1)) + S(d^*(y_1))] + (1-\theta)\theta S\left(\overline{d}(y_1)\right)$$

From an equivalent reasoning as in Lemma 4(a), $EW_s(y_1)$ is strictly concave in $\overline{d}(y_1)$ and attains its global maximum at some $\widetilde{d}_s(y_1) < d(y_1)$.

In case the selfish type pools as in Proposition 3(b), expected welfare equals:

$$EW_p(y_1) = \theta[D(g_1^*(y_1)) + S(d^*(y_1))] + (1-\theta) D(g_1^*(y_1))$$

Equivalent to Lemma 4(c), $EW_p(y_1)$ is weakly concave in $\overline{d}(y_1)$, strictly increasing up to $\overline{d}(y_1) = d(y_1)$ and constant afterwards. From Lemma 5 it follows that $EW_s(y_1) - EW_p(y_1) = (1-\theta) \cdot$

$\left[\theta S\left(\overline{d}\left(y_1\right)\right) - D\left(g_1^*\left(y_1\right)\right)\right]$ decreases with $\overline{d}\left(y_1\right)$ and is negative at the border of existence between the separating and the pooling outcome.

Part (a). In this case, for a debt limit equal to $d\left(y_1\right)$ the pooling outcome of Proposition 3(b) still applies. Given the shape of $EW_p\left(y_1\right)$, any ceiling $\overline{d}\left(y_1\right) \geq d\left(y_1\right)$ falling within the pooling range is optimal.

Part (b). With $c_1 < d\left(y_1\right)$ the pooling range corresponds to $\overline{d}\left(y_1\right) \leq c_1$. Given the shape of $EW_p\left(y_1\right)$, the optimal ceiling within this range is at the border c_1. The optimal debt ceiling within the separating range either equals the global maximizer of $EW_s\left(y_1\right)$ denoted $\widetilde{d}_s\left(y_1\right)$ (with $\widetilde{d}_s\left(y_1\right) < d\left(y_1\right)$) if this debt limit indeed falls within the separating range, or equals the infimum of the debt limit falling in this range (i.e. equals border c_1).

Part (c). Under the conditions of Lemma 2 we have $d\left(y_L\right) > c_L \Longrightarrow d\left(y_H\right) > c_H$. The result then directly follows from part (b).

Part (d). If $\overline{d}_{opt}\left(y_H\right) = c_H$ and $\overline{d}_{opt}\left(y_L\right) = \widetilde{d}_s\left(y_L\right)$, the selfish type is *less* likely to postpone rent seeking to period 2 when $y_1 = y_L$ than when $y_1 = y_H$. Theoretically, we cannot exclude this possibility and indeed, in our simulations using a CARA specification (as alternative to CRRA), we encountered such instances (cf. Subsection 5.2.1). ■

Proof of Proposition 4.

Part (a). Suppose $d\left(y_L\right) \leq c_H + \beta\left(c_H - d\left(y_H\right)\right)$. From $c_H < 0$ and $d\left(y_L\right) > 0$ this may only happen when $d\left(y_H\right) < c_H$. Moreover, $d\left(y_L\right) \leq c_H + \beta\left(c_H - d\left(y_H\right)\right) < c_L + \max\left\{0, \beta\left(c_L - d\left(y_L\right)\right)\right\}$ implies $d\left(y_L\right) < c_L$. Hence, for both $y_1 \in \left\{y_L, y_H\right\}$ case (a) of Lemma 7 applies in the contingent case. Moreover, the respective intervals for the optimal ceilings overlap and share $\left[d\left(y_L\right), c_H + \beta\left(c_H - d\left(y_H\right)\right)\right] \neq \emptyset$. So welfare is also maximized if some common $\overline{d}_{opt}\left(y_L\right) = \overline{d}_{opt}\left(y_H\right) = \overline{d}_{opt}$ is chosen within this interval. Together with Lemma 6(a) for the constant case the result follows.

Part (b). We first show that $\overline{d}_{opt}\left(y_L\right) \neq \overline{d}_{opt}\left(y_H\right)$ necessarily, and thus that welfare is strictly harmed if these debt limits would be forced to be the same. First suppose that, besides $c_H + \beta\left(c_H - d\left(y_H\right)\right) < d\left(y_L\right)$, we have $d\left(y_H\right) < c_H$. Then Lemma 7(a) applies for $y_1 = y_H$. If $d\left(y_L\right) < c_L$ Lemma 7(a) applies for $y_1 = y_L$ as well; with $c_H + \beta\left(c_H - d\left(y_H\right)\right) < d\left(y_L\right)$ the two intervals with optimal state-contingent ceilings do not overlap and $\overline{d}_{opt}\left(y_H\right) < \overline{d}_{opt}\left(y_L\right)$ necessarily. In case $d\left(y_L\right) > c_L$ we have $\overline{d}_{opt}\left(y_L\right) \geq c_L$ necessarily from Lemma 7(b). From $c_H + \beta\left(c_H - d\left(y_H\right)\right) < c_L + \max\left\{0, \beta\left(c_L - d\left(y_L\right)\right)\right\} = c_L$ again $\overline{d}_{opt}\left(y_H\right) < \overline{d}_{opt}\left(y_L\right)$ follows. Next, let $d\left(y_H\right) > c_H$. Then $c_H \leq \overline{d}_{opt}\left(y_H\right) \in \left\{c_H, \widetilde{d}_s\left(y_H\right)\right\}$ from Lemma 7(b). If case (a) of Lemma 7 applies when $y = y_L$, again $\overline{d}_{opt}\left(y_H\right) < \overline{d}_{opt}\left(y_L\right)$ from $c_H < d\left(y_L\right)$ and $\widetilde{d}_s\left(y_H\right) < d\left(y_L\right)$. If Lemma 7(b) applies for $y = y_L$, we have $c_L \leq \overline{d}_{opt}\left(y_L\right) \in \left\{c_L, \widetilde{d}_s\left(y_L\right)\right\}$ and thus (given $c_H < c_L$ and $\widetilde{d}_s\left(y_H\right) < \widetilde{d}_s\left(y_L\right)$) generically that $\overline{d}_{opt}\left(y_L\right) \neq \overline{d}_{opt}\left(y_H\right)$.[33]

Part (b.1). With $d\left(y_H\right) < c_L$ it necessarily follows that $\widetilde{d}_s\left(y_H\right) < c_L$ and thus all candidate optima for $\overline{d}_{opt}\left(y_H\right)$ fall short of those for $\overline{d}_{opt}\left(y_L\right)$ (see parts (a) and (b) above). We thus have $\overline{d}_{opt}\left(y_H\right) < \overline{d}_{opt}\left(y_L\right)$ necessarily. We next show that for the constant case the optimal ceiling $\overline{d}_{opt}$ falls in between. From $c_H + \beta\left(c_H - d\left(y_H\right)\right) < d\left(y_L\right)$ Lemma 6(b) applies and thus $\overline{d}_{opt} \geq c_H + \max\left\{0, \beta\left(c_H - d\left(y_H\right)\right)\right\}$. Conditional on $y_1 = y_H$, i.e. in the contingent case, it would be best to set either $\overline{d}_{opt}\left(y_H\right) \in$

[33] Note that $\tilde{d}_s\left(y_H\right) = c_L$ may only hold in non-generic knife edge cases.

$\left[d\left(y_H\right), c_H+\beta\left(c_H-d\left(y_H\right)\right)\right]$ if $d\left(y_H\right) \leq c_H$ and Lemma 7(a) applies, or $\overline{d}_{opt}\left(y_H\right) \in \left\{c_H, \widetilde{d}_s\left(y_H\right)\right\}$ in case $d\left(y_H\right) > c_H$ and Lemma 7(b) applies. For $d\left(y_H\right) < c_L$ and thus $\widetilde{d}_s\left(y_H\right) < c_L$, all these debt ceilings fall in the y_L-pooling range when $y_1 = y_L$. Within this range, conditional on $y_1 = y_L$, expected welfare is increasing in $\overline{d}$. Hence the potential occurrence of $y_1 = y_L$ only makes it more attractive to set a higher debt limit in the constant case: $\overline{d}_{opt} \geq \overline{d}_{opt}\left(y_H\right)$ necessarily. Similarly, conditional on $y_1 = y_L$ it would be best to set either $\overline{d}_{opt}\left(y_L\right) \in \left[d\left(y_L\right), c_L+\beta\left(c_L-d\left(y_L\right)\right)\right]$ if $d\left(y_L\right) \leq c_L$ and Lemma 7(a) applies, or $\overline{d}_{opt}\left(y_L\right) \in \left\{c_L, \widetilde{d}_s\left(y_L\right)\right\}$ in case $d\left(y_L\right) > c_L$ and Lemma 7(b) applies. For $d\left(y_H\right) < c_L$ (and thus $\widetilde{d}_s\left(y_H\right) < c_L$) all these debt ceilings necessarily fall in the y_H-separating range. Expected welfare conditional on y_H is strictly decreasing in $\overline{d}$ beyond $\widetilde{d}_s\left(y_H\right)$, and thus $\overline{d}_{opt} \leq \overline{d}_{opt}\left(y_L\right)$. Together this gives $\overline{d}_{opt}\left(y_H\right) \leq \overline{d}_{opt} \leq \overline{d}_{opt}\left(y_L\right)$, with at least one inequality being strict.

Part (b.2). Note that $d\left(y_H\right) > c_L$ implies both $d\left(y_H\right) > c_H$ and $d\left(y_L\right) > c_L$. By Lemma 7(b) then $c_L \leq \overline{d}_{opt}\left(y_L\right) \in \left\{c_L, \widetilde{d}_s\left(y_L\right)\right\}$ and $c_H \leq \overline{d}_{opt}\left(y_H\right) \in \left\{c_H, \widetilde{d}_s\left(y_H\right)\right\}$. If $\widetilde{d}_s\left(y_H\right) < c_L$ and thus both candidate optima for $\overline{d}_{opt}\left(y_H\right)$ fall short of those for $\overline{d}_{opt}\left(y_L\right)$, the same reasoning as in part (b.1) applies and thus $\overline{d}_{opt}\left(y_H\right) \leq \overline{d}_{opt} \leq \overline{d}_{opt}\left(y_L\right)$. Consider the remaining case $\widetilde{d}_s\left(y_H\right) > c_L$ (which, given $d\left(y_H\right) > c_L$, occurs for θ sufficiently close to one). From $\widetilde{d}_s > \widetilde{d}_s\left(y_H\right)$ it then follows that $\widetilde{d}_s > c_L$ and thus $\widetilde{d}_s$ is the single eligible maximizer in the constant ceiling case. This implies $\overline{d}_{opt} \in \left\{c_H, c_L, \widetilde{d}_s\right\}$. Case (i) occurs when $\overline{d}_{opt}\left(y_H\right) = c_H$, $\overline{d}_{opt}\left(y_L\right) = c_L$ and $\overline{d}_{opt} = \widetilde{d}_s$. Case (ii) results when $\overline{d}_{opt}\left(y_H\right) = \widetilde{d}_s\left(y_H\right)$, $\overline{d}_{opt}\left(y_L\right) = \widetilde{d}_s\left(y_L\right)$ and $\overline{d}_{opt} \neq \widetilde{d}_s$, while case (iii) materializes if $\overline{d}_{opt}\left(y_H\right) = \widetilde{d}_s\left(y_H\right)$ and $\overline{d}_{opt}\left(y_L\right) = c_L$. For all other combinations of optima $\overline{d}_{opt}\left(y_H\right) \leq \overline{d}_{opt} \leq \overline{d}_{opt}\left(y_L\right)$. ■

Appendix B: More detailed overview numerical analysis

In this appendix we provide − for the numerical analysis reported in Subsection 5.2.1 − a detailed percentage wise overview of the nature of the optimal ceilings in the constant versus the contingent ceiling case. The rows in Table 8 are organized along the possible combinations of optimal contingent ceilings, while the columns are organized by both the equilibrium type and the level of the optimal constant ceiling. Consider the rows first. By Lemma 7 the optimal contingent ceiling for a given level of y_1 can either be 'lax' (cf. Lemma 7(a)), or be binding and either at border c_1 between the pooling and separating range or equal to $\widetilde{d}_s\left(y_1\right)$ in the interior of the separating range. This yields $3 \cdot 3 = 9$ potential combinations for the optimal contingent ceilings $\left\{\overline{d}_{opt}\left(y_H\right), \overline{d}_{opt}\left(y_L\right)\right\}$. Lemma 7(c) theoretically excludes combinations $\{\text{lax}, c_L\}$ and $\left\{\text{lax}, \widetilde{d}_s\left(y_L\right)\right\}$, while we do not find any observations corresponding to the combination $\left\{c_H, \widetilde{d}_s\left(y_L\right)\right\}$ in our numerical analysis. The six remaining combinations are grouped in terms of the induced behavior of the selfish type: always postpone rent-seeking to period 2, postpone only when $y_1 = y_L$, or never postpone.

The columns refers to the constant ceiling case. The optimal ceiling can either fall within the pooling, hybrid, or separating equilibrium range (cf. Proposition 1). Within the pooling range the level of the optimal ceiling can either be 'lax' (cf. Lemma 6(a)) or equal to c_H, in the hybrid range it can either equal $d\left(y_L\right)$, $\widetilde{d}_h$ or c_L, while the optimum equals $\widetilde{d}_s$ if it falls in the separating equilibrium range. For all possible levels except 'lax' and $d\left(y_L\right)$, the optimum can either bind the benevolent type when $y_1 = y_H$ or not do so. (For $y_1 = y_L$ it follows immediately from the characterization of the optimal ceiling that it strictly binds in these latter instances.) For these levels the table therefore splits the outcomes over these two possibilities.

	pooling			hybrid					separating		
	lax	c_H		$d(y_L)$	$\tilde{d}_h$		c_L		$\tilde{d}_s$		Total
		$d(y_H) < c_H$	$d(y_H) > c_H$		$d(y_H) < \tilde{d}_h$	$d(y_H) > \tilde{d}_h$	$d(y_H) < c_L$	$d(y_H) > c_L$	$d(y_H) < \tilde{d}_s$	$d(y_H) > \tilde{d}_s$	
$\{\text{lax}, \text{lax}\}$	0.00	7.27	n.p.	2.77	0.10	n.p.	n.p.	n.p.	n.p.	n.p.	10.14
$\{c_H, \text{lax}\}$	n.p.	n.p.	1.29	1.99	0.78	0.17	n.p.	n.p.	n.p.	n.p.	4.23
$\{c_H, c_L\}$	n.p.	n.p.	26.47	n.p.	0.11	0.05	7.06	26.79	0.10	2.82	63.43
$\left\{\tilde{d}_s(y_H), \text{lax}\right\}$	n.p.	n.p.	n.p.	0.04	0.11	0.06	n.p.	n.p.	n.p.	n.p.	0.21
$\left\{\tilde{d}_s(y_H), c_L\right\}$	n.p.	n.p.	0.00	n.p.	0.00	0.01	0.16	0.56	0.16	0.74	1.63
$\left\{\tilde{d}_s(y_H), \tilde{d}_s(y_L)\right\}$	n.p.	n.p.	0.00	n.p.	0.00	0.00	0.01	0.09	4.00	16.26	20.36
Total	0.00	7.28	27.77	4.80	1.10	0.29	7.23	27.45	4.26	19.83	100

Notes. The top row in the header indicates the type of equilibrium under a constant debt ceiling, the next row the location of the optimal constant debt ceiling ceiling and the third row in the header whether the optimal constant ceiling is binding or not for the benevolent incumbent when $y_1 = y_H$. The most-left column indicates the location of the optimal contingent ceilings under y_H respectively y_L. Finally, "n.p." indicates that the case is theoretically not possible.

Table 8: Optimal contingent versus optimal constant debt ceilings