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HOW DO PREDICTION MARKET FEES AFFECT PRICES AND PARTICIPANTS?

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Discussion Paper DP17972
First Published 10 March 2023
This Revision 16 March 2023

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Abstract

The CFTC has recently licensed a commercial prediction market to operate in the US. Previous theoretical work has not incorporated that prediction markets charge fees. We examine the impact of fees by introducing them to a model in which the market price equals the true probability when there are no fees. Fees charged on winnings mean this property no longer generally holds but trading fees that charge equal amounts on both sides of contracts preserve this feature. Prediction markets with fees feature a form of favorite-longshot bias: Post-fee loss rates depend negatively on the probability of the event being backed.

JEL Classification: G13, G14, G23

Keywords: N/A

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How Do Prediction Market Fees Affect Prices and Participants?

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March 2023

Abstract

The CFTC has recently licensed a commercial prediction market to operate in the US. Previous theoretical work has not incorporated that prediction markets charge fees. We examine the impact of fees by introducing them to a model in which the market price equals the true probability when there are no fees. We find that fees charged on winnings generally mean contract prices for low probability outcomes are below the true probability but the impact of fees means prediction markets feature a form of favorite-longshot bias: Post-fee loss rates depend negatively on the probability of the event being backed.

Keywords: Prediction Markets, Commission Fees, Favorite-Longshot Bias

JEL Classification: G11, G13, G14, G23

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1. Introduction

Prediction markets facilitate people to place wagers that events will happen by matching them with other people who wager that the event will not happen. These markets have long been considered potentially useful for their ability to summarize the public’s information. Governments can use them to obtain information about the likelihood of events to assist with policy. Businesses can design their strategy using information about the likelihood of certain events. Investors can use them to hedge various event-related risks directly rather than indirectly via portfolios expected to move a certain way if events occur. Popular books by Surowiecki (2004) and Sunstein (2006) have promoted the idea that these markets should be much more widely used.

The development of prediction markets has been limited, however, by regulatory restrictions in the United States. A 2008 article in *Science* by 22 high profile social scientists including four economics Nobel prize winners, recommended reform of these restrictions and argued the case for “The Promise of Prediction Markets.” This promise may be about to be delivered on because the Commodity Futures Trading Commission (CFTC) decided in November 2020 to license a market of this type offered by startup firm Kalshi.¹ Kalshi has begun trading as a licensed predictions market—officially its products are considered swaps—and it seems likely that competitors will soon join the market.

An important difference between this new era of commercial prediction markets and the Iowa Electronic Markets (IEM) site that has been widely studied in the previous literature is that firms like Kalshi are commercial operations and they charge trading fees, while trading with the IEM was commission-free. Longer running commercial prediction market operators outside the US also earn fees by charging commissions on winnings. Previous theoretical work has assumed the absence of fees, so in this paper, we examine how fees will affect prices and participants in prediction markets.

We first look at whether fees impact the ability of prediction market prices to accurately summarize the public’s beliefs. In theory, the conditions for prices in prediction markets without fees to equal the public’s average belief are restrictive. Manski (2006) showed that when observing a price p in a prediction market, the average belief could be as low as p^2 or as high as $2p - p^2$. He and Treich (2017) show that if participants have CRRA utility functions, prediction market prices equal the average belief of participants if and only they have log utility, even if the distribution of beliefs is symmetric. Ottaviani and Sørensen (2015) also show that if agents combine prior beliefs with a common public signal, prediction market prices will under-react to the signal.

Despite these theoretical results, the evidence suggests that prediction markets without fees work extremely well. In particular, the extensively studied IEM has consistently produced accurate forecasts that have beaten other sources of predictions such as opinion polls.² Since prediction markets without fees appear to work well in practice, we are going to assume they work in theory. In our

¹The CFTC decision on Kalshi can be found at <https://www.cftc.gov/PressRoom/PressReleases/8302-20>

²See for example, studies such as Berg, Nelson and Rietz (2008) and Forsythe et al (1992).

model, agents have log utility and symmetric beliefs that are correct on average about probabilities so the prediction market price equals the true probability when the market has no fees. We then illustrate the implications of adding fees to the model.

We find that prediction markets prices are affected by fees and will generally not equal the corresponding average value of the public's beliefs, though the precise pattern depends on the form of the fee structure. For example, a flat trading fee across all contract types and a flat commission rate on winnings both lead to prices being lower than probabilities when the true probability of success is below a half with the opposite applying for more likely outcomes. However, we also provide an example of a fee structure that leads to prices for outcomes that have a probability below a half being higher than the true probability. Overall, however, we find that the deviations of prices from the public's average beliefs are generally modest for realistic commission rates.

We also examine how fees impact the returns on money invested by participants. Without fees, prediction markets just exchange money between two parties so on average there are no profits or losses but once the market charges fees, participants lose money on average. We show that the impact of these losses follows a pronounced pattern: Contracts backing low probability outcomes lose a greater percentage of their investment than contracts backing higher probability outcomes. This result echoes the well-known "favorite-longshot bias" from the literature on sports betting. However, while researchers have come up with many possible behavioral explanations for this bias, the explanation in this context is somewhat different.³

Consider a fixed fee F on both sides of a contract. We show that in this case the price, p , of a contract tends to be pretty close to the true probability, π . This means that pre-fee average returns will be approximately zero so the average loss on all contracts is approximately F . Average loss rates on investments backing events with $\pi < 0.5$ are greater than for taking the other side because F is larger relative to the financial investment of $\pi + F$ than it is relative to the other side of the contract which has an investment of $1 - \pi + F$. This result generalizes to more complex fee structures. While the model does not contain asymmetries so that people systematically overweight small probabilities—beliefs about probabilities are symmetric around the true value—those who buy contracts with a low probability of winning turn out to be proportionately more out of line in their assessment of their contract's rate of return than those who buy contracts likely to succeed.

The rest of the paper is organized as follows. Section 2 introduces the model and its predictions in the absence of fees. Section 3 describes the impact of a flat trading fee for all contracts. Section 4 examines the fee formula currently used by Kalshi in which contract fees depend on the price and shows this is less distortionary than flat fees. Section 5 analyzes the more common approach of charging a commission rate only on winnings and Section 6 provides an example of a fee structure similar

³For surveys of the literature on favorite-longshot bias, see Ottaviani and Sørensen (2008) and Snowberg and Wolfers (2008).

to Kalshi's but applied only to winnings. Compared with a fixed commission rate on winnings, this fee structure is more generous to longshot contracts but we show these contracts still have larger loss rates in this case because the prices of longshot contracts are raised above their true probability.

2. The Model Without Fees

Here we introduce the model without fees and the subsequent sections then examine the impact of different fee structures. We model a prediction market as follows, following similar presentations in Gjerstad (2004) and Wolfers and Zitzewitz (2006). There is an event in which an outcome has a probability π of occurring. There is a continuum of potential market participants of size 1 who each have a subjective belief $\tilde{\pi}$ about the true probability π that is independently distributed across the population. As in Ali (1977), these beliefs are characterized by a continuous symmetric cumulative distribution function $F(\tilde{\pi})$ with mean π , meaning the potential participants are, on average, correct in their belief about π .

The prediction market operator arranges trading in an Arrow-Debreu security that has price p and pays out 1 if the event occurs and zero otherwise. Trading takes place via peer-to-peer matching arranged by the prediction market operator. This means that for every "back contract" on the event happening, there is a corresponding "lay contract" on it not happening. The person buying the back contract commits p in funds and the person buying the lay contract commits $1 - p$ and then one of them wins the combined funds of one dollar. Lay contracts are thus Arrow-Debreu securities with price $1 - p$ that pay out 1 when the event does not happen and zero when it does.

Participants have log utility and wealth of w_i that is uncorrelated with their beliefs about π , so we assume the same distribution of wealth $G(w_i)$ for all values of $\tilde{\pi}_i$. In making their decisions on purchasing contracts, participants take any trading fees or commissions on winnings into account. The trading process is a double auction with agents submitting the quantities of each type of contract they would buy at each price. The equilibrium price is the one that matches demand for back contracts with demand for lay contracts.

With no fees, participant i solves the problem

$$\text{Max}_{x_i} \left[\tilde{\pi}_i \log(w_i + x_i(1-p)) + (1 - \tilde{\pi}_i) \log(w_i - x_i p) \right] \quad (1)$$

where a positive x_i denotes purchases of back contracts and a negative x_i represents purchases of lay contracts. The optimal quantity of contracts is

$$x_i = \left(\frac{\tilde{\pi}_i - p}{p(1-p)} \right) w_i \quad (2)$$

Demand for contracts depends positively on the perceived expected profit (the numerator) and neg-

actively on the perceived variance (the denominator). The equilibrium price equates the volume of back and lay contracts so the average value of x_i is zero.

$$\int_0^1 \left(\frac{\tilde{\pi}_i - p}{p(1-p)} \right) w_i f(\tilde{\pi}) g(w) d\tilde{\pi} = 0 \quad (3)$$

The independence of wealth and beliefs means this can be simplified to

$$\int_0^1 \left(\frac{\tilde{\pi}_i - p}{p(1-p)} \right) f(\tilde{\pi}) d\tilde{\pi} = 0 \quad (4)$$

so the equilibrium price equals the true probability

$$p = \int_0^1 \tilde{\pi}_i f(\tilde{\pi}) d\tilde{\pi} = E(\pi) = \pi \quad (5)$$

The expected losses on these two contracts are

$$L^B = (1 - \pi)p - \pi(1 - p) = p - \pi = 0 \quad (6)$$

$$L^L = \pi(1 - p) - (1 - \pi)p = \pi - p = 0 \quad (7)$$

so both contracts break even on average.

3. A Flat Trading Fee on All Contracts

We first model the impact of a trading fee F charged on each contract. Optimal contract demand now solves

$$\text{Max}_x \left[\pi \log(w + x(1-p) - F|x|) + (1 - \pi) \log(w - xp - F|x|) \right] \quad (8)$$

The first-order condition is

$$\frac{\pi(1-p - FI_x)}{w + x(1-p) - F|x|} = \frac{(1-\pi)(p + FI_x)}{w - xp - F|x|} \quad (9)$$

where

$$I_x = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0 \end{cases} \quad (10)$$

The first-order condition can be re-expressed as a formula determining the ratio of the marginal utilities of wealth for the two outcomes

$$\frac{w - xp - F|x|}{w + x(1 - p) - F|x|} = \frac{1 - \pi}{\pi} \frac{p + FI_x}{1 - p - FI_x} \quad (11)$$

The left side equals one when $x = 0$ and depends negatively on x . To have $x > 0$, this ratio of marginal utilities has to be less than one, which requires

$$p + FI_x < \pi \Rightarrow \pi > p + F \quad (12)$$

Similarly, $x < 0$ requires

$$p + FI_x > \pi \Rightarrow \pi < p - F \quad (13)$$

This means that people with beliefs such that $p - F \leq \pi \leq p + F$ will not buy any contracts. Ironically, given the hope that prediction markets will accurately summarize the public's information, it turns out to be those with the most accurate beliefs who chose not to participate in the market.

For those who purchase contracts, the condition determining the quantity demanded is the same as before except now the effective price of contracts are $p + F$ and $1 - p + F$ rather than p and $1 - p$. Contract demands are

$$x_i^B = \left(\frac{\tilde{\pi}_i - p - F}{(p + F)(1 - p - F)} \right) w_i \quad (14)$$

$$x_i^L = \left(\frac{p - \tilde{\pi}_i - F}{(p - F)(1 - p + F)} \right) w_i \quad (15)$$

and the equilibrium price satisfies

$$\int_{p+F}^1 \left(\frac{\tilde{\pi}_i - p - F}{(p + F)(1 - p - F)} \right) f(\tilde{\pi}) d\tilde{\pi} = \int_0^{p-F} \left(\frac{p - \tilde{\pi}_i - F}{(p - F)(1 - p + F)} \right) f(\tilde{\pi}) d\tilde{\pi} \quad (16)$$

If these two denominators were the same, we could use the symmetry of the probability distribution of beliefs and the symmetry of the numerators of the demand functions to show that the equilibrium price continues to be $p = \pi$ despite the introduction of the trading fee, F . The different denominators means this is not generally true. However, it is true for $\pi = 0.5$ and for most of the range of feasible values of π the ratio of these two denominators is close to one. So for most values of π we will have $p \approx \pi$.

The exception is when p is either very high or low. The denominator of the demand function measures the variance of profits or losses on the contract. For contracts with a low price, the addition of even a small fee raises this variance a lot more than it does for contracts with a high price. So, for example, when $p = 0.1$, the variance of the "longshot" contract is 16 percent higher than for the

corresponding favorite contract. For the same gap between beliefs and prices, demand for the back contract at $p = 0.1$ will be 16 percent lower than the corresponding demand for the matching lay contract that costs $1 - p = 0.9$. This lower demand for longshot contracts means the equilibrating market prices will be lower than π when $\pi < 0.5$ and higher than it when $\pi > 0.5$.⁴

In the absence of an analytical solution, we need to specify the probability density function of beliefs $f(\tilde{\pi})$ and solve the model numerically to illustrate its properties. We use the assumption that beliefs are uniform on $[\pi - \sigma\pi(1 - \pi), \pi + \sigma\pi(1 - \pi)]$ where $\sigma = 1$. With this specification, when $\pi = 0.5$ beliefs go from $\tilde{\pi} = 0.25$ to $\tilde{\pi} = 0.75$ and this range becomes gradually smaller as π moves towards either zero or one. This calibration of disagreement is chosen because intuitively there is less room for sensible people to disagree if an event is almost certain to happen or not happen than if it is a toss-up and it avoids assigning people probability beliefs that are either negative or greater than one. However, the same patterns shown here emerge for all reasonable specifications examined. We use a figure of one cent for the fee ($F = 0.01$) which, as we discuss below, is a realistic baseline.

The upper panel of Figure 1 shows that for most values of π , prices are close to the true probability but for contracts that are less likely to succeed, prices drop in a nonlinear fashion below the true probability. In this calibration, prices for the lowest probability contracts head towards about 15% below the true probability. This figure suggests there may be an asymmetry because back contracts with $\pi < 0.5$ are a larger percentage below their true probabilities than contract with $\pi > 0.5$. However, the pricing of back and lay contracts is still symmetric because the risk and reward features of a back contract with probability $\pi = 0.5 - \nu$ are identical to those of a lay contract at probability $1 - \pi = 0.5 + \nu$. The ratio of the price a back contract to π when $\pi = 0.3$ is equal to the ratio of the price of a lay contract to $1 - \pi$ when $\pi = 0.7$.

The average outcome for participants depends on both the price they paid and the fees. The average profit for a back contract without considering fees will be $\pi - p$, so before fees are taken into account, low probability back contracts will make profits, as illustrated by the negative loss rates in the middle panel. However, once fees are factored in, this effect is offset and low probability contracts actually experience much larger loss rates relative to the cost of the investment (either $p + F$ or $1 - p + F$). The bottom panel shows these loss rates tending towards about 25 percent in this calibration.

The intuition for this result is as follows. Suppose $p \approx \pi$. In this case average losses on contracts would be approximately the same π as in equations 6 and 7 but with the additional fee deducted. This means both contracts lose approximately F on average. Relative to the fee-inclusive cost of the

⁴This result echoes Page and Clemen's (2013) model in which participants receive payments with a delay and discount their expected payout if successful. In their model, discounting produces an inaction region of beliefs and prices do not equal probabilities but their model features risk-neutral investors and they find the opposite of the result here, with prices being higher than probabilities for $\pi < 0.5$.

investments, the loss rates on the two contracts are

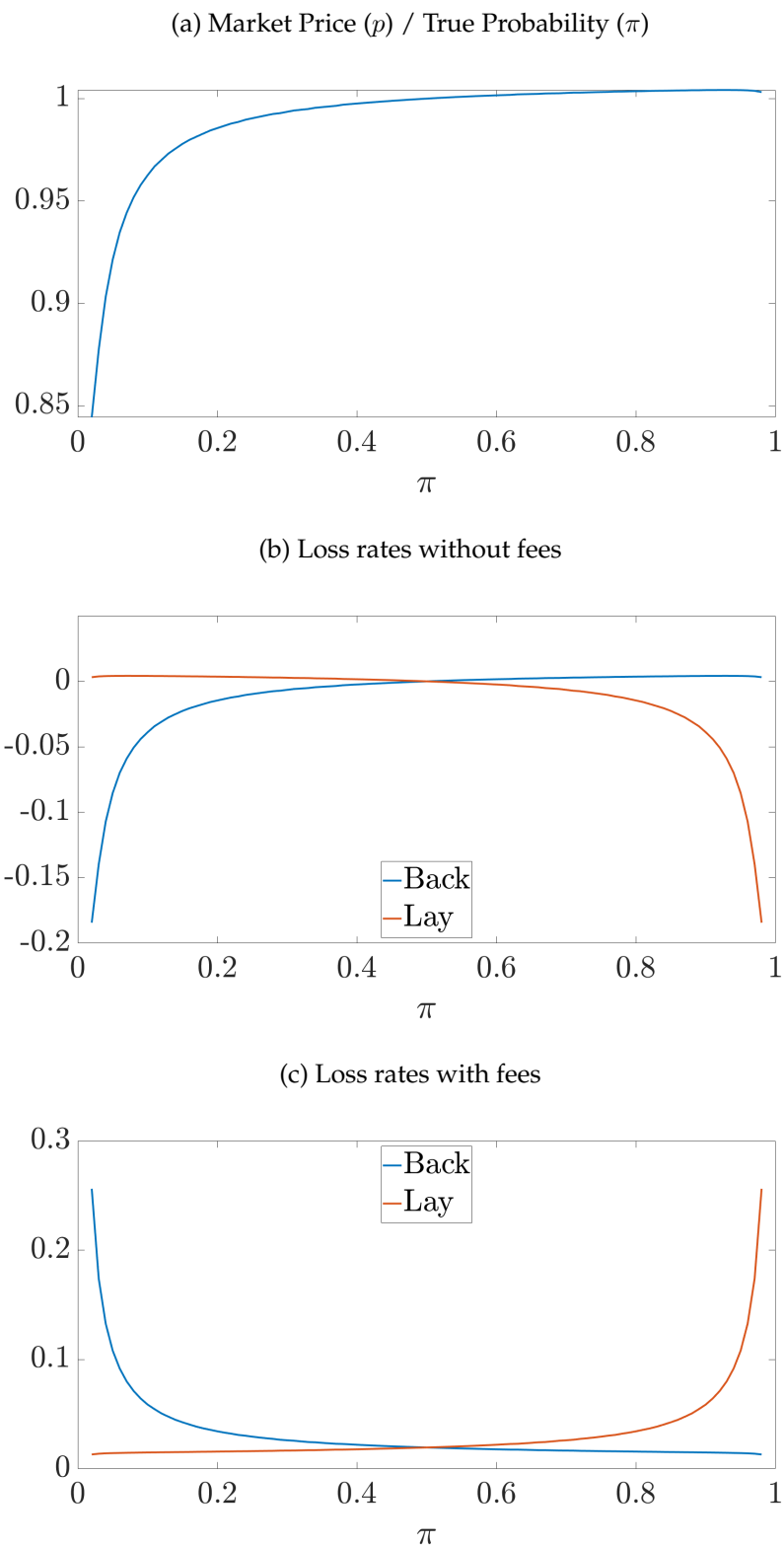
$$\frac{L^B}{p + F} \approx \frac{F}{\pi + F} \quad \frac{L^L}{1 - p + F} \approx \frac{F}{1 - \pi + F} \quad (17)$$

This approximation shows loss rates tends towards close to zero as the probability of the contract increases towards one and to infinity as π falls towards zero. In our example, the equilibrium price does fall notably below π for low probability contracts but not enough to offset the basic pattern that fees have a higher impact on cheap contracts than on expensive ones.

This pattern of higher loss rates on longshot contracts is superficially similar to the finding that bets on longshots in sports tend to have larger loss rates than bets on favorites. Various behavioral explanations have been suggested to explain this phenomenon but they do not apply to this example. For example, one explanation is that bettors like the high variance that comes with betting on longshots but our agents are risk averse. Another argument is Ali's (1977) demonstration that a symmetric distribution of beliefs among risk-neutral bettors implied favorite-longshot bias in pari-mutuel betting markets, which pool the funds placed by gamblers and pay them out (minus a fraction for the operator) proportionately to those who picked the winner. Ali explained how odds on favorites need to be better than odds on longshots to be consistent with pari-mutuel betting's property that more money is placed on the favorite than the longshot. However, prediction markets by definition place most of the money at risk on favorites and, without fees, deliver equal returns to both sides of contracts in our model.

A closer analogy is the idea that people may overweight small probabilities when assessing the potential return on bets, as in Kahneman and Tversky's (1979) prospect theory. In our case, there is no asymmetry of the kind where people systematically overweight small probabilities. However, those who do overweight these probabilities have the worst outcomes in terms of the return on investment. Purchasers of both contracts have beliefs that over-estimate the return on their chosen contract: Back contract buyers think its probability of success is at least $\frac{\pi+F}{\pi} = 1 + \frac{F}{\pi}$ times higher than it is while those who purchase lay contracts think its probability of success is at least $\frac{1-\pi+F}{1-\pi} = 1 + \frac{F}{1-\pi}$ times higher than it is. When $\pi < 0.5$, the former mis-estimation is larger than the latter and leads to bigger percentage losses.

Figure 1: Ratios of equilibrium prices to true probabilities and pre- and post-fee loss rates with flat fees of $F = 0.01$. Agents have uniform beliefs on $[\pi - \pi(1 - \pi), \pi + \pi(1 - \pi)]$



4. Kalshi's Fee Formula

The new prediction market Kalshi charges the same fee on both sides of each contract but it varies the fees with the price. Specifically, Kalshi charge a trading fee of $F = \theta p(1 - p)$ where at present $\theta = 0.07$.⁵ This pricing schedule has an inverse-U shaped cost per contract peaking at $\pi = 0.5$ and falling towards zero as the price of the contract heads towards either zero or one.

Kalshi's pricing approach implies smaller distortions in prices than the flat fee on all contracts. The source of this distortion was the relative increase in the variance of low probability contracts but Kalshi's fees being lower for cheap contracts reduces this element. Solving numerically under the same assumptions about beliefs as before but with F replaced with $\theta p(1 - p)$ where $\theta = 0.07$, the upper panel of Figure 2 shows the same pattern of prices being lower than probabilities when $\pi < 0.5$ and above them otherwise but the distortion in prices is much smaller, with this calibration showing even the lowest prices only 3 percent below the true probability.

In terms of the incidence of losses, the more accurate prices under Kalshi's formula mean lower pre-fee profits for longshot contracts (the middle panel) so that loss rates incorporating fees are still greater for contracts with $\pi < 0.5$. However, this pricing model does not show post-fee loss rates falling monotonically as π rises as it did with a flat fee rate on all contracts (the bottom panel). Instead, loss rates rise slightly as π moves above zero, from about 3.5% in this example for the lowest probabilities to 3.8% for $\pi = 0.25$ before falling again as π rises, though with nothing like as nonlinear a pattern as with the flat 1 cent fee. Compared with Figure 1, the lower loss rates for extreme longshot contracts come at the expense of higher loss rates for contracts in the middle of the probability distribution.

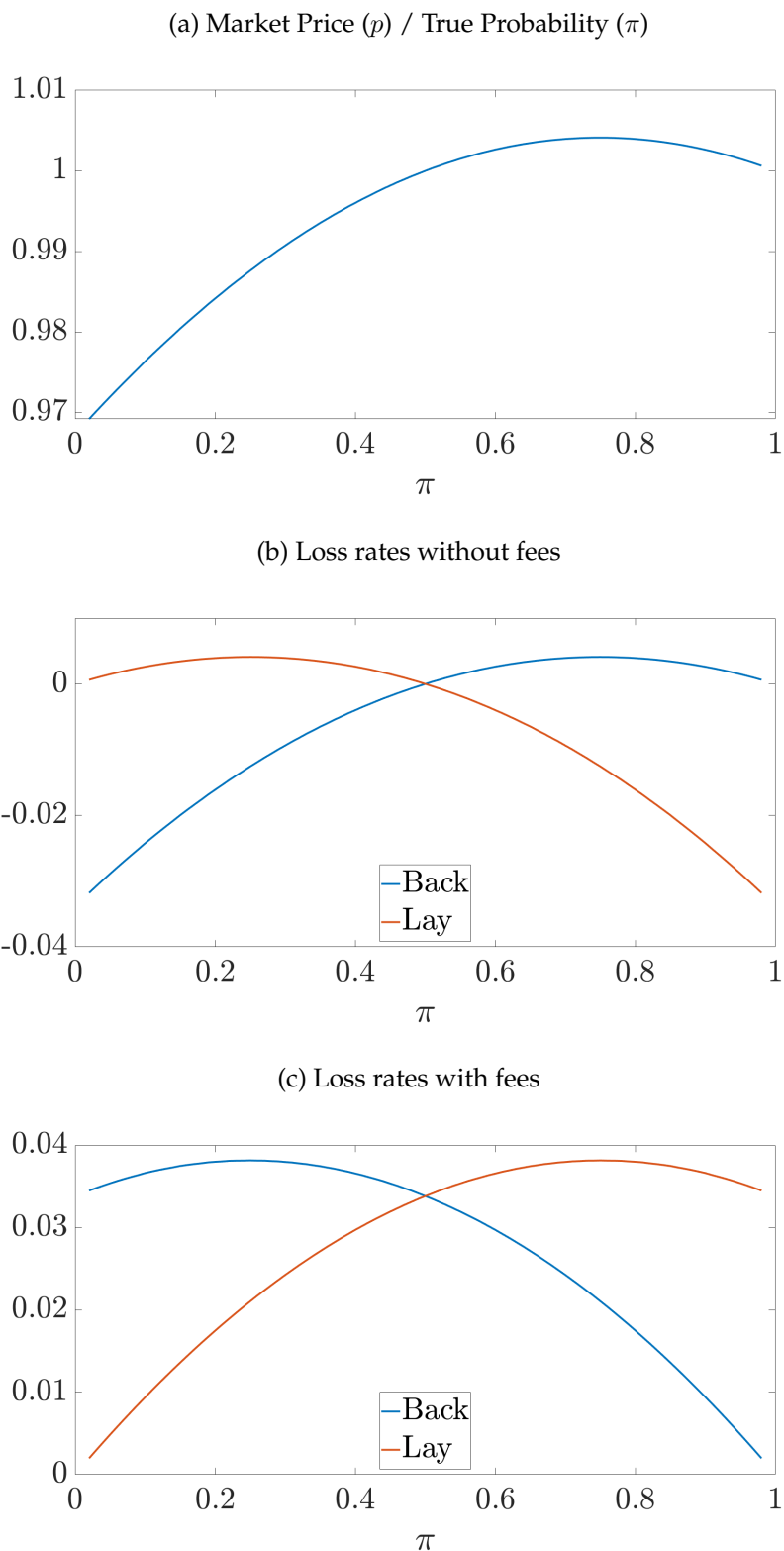
To justify the comparison between the numbers in Figures 1 and 2, note that if the events being speculated on were equally likely to have probabilities of success anywhere between zero and one, then total fees across all contracts from Kalshi's approach could be calculated as

$$2\theta \int_0^1 \pi(1 - \pi) d\pi = 2\theta \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\theta}{3}. \quad (18)$$

Hence, a flat fee with a charge of $\frac{\theta}{6}$ would raise the same fee income across all contracts as Kalshi's formula. At their current pricing of $\theta = 0.07$, this shows Kalshi's approach would raise approximately the same revenue as a flat 1 cent fee on contracts but does so without causing the same pattern of significant price distortion or extremely high loss rates for longshot contracts that Figure 1 shows this 1 cent fee would induce.

⁵Technically, Kalshi charge a fee that is a round up to the next cent of $x\theta p(1 - p)$ where x is the number of contracts purchased. See <https://kalshi.com/docs/kalshi-fee-schedule.pdf> for details. This pricing structure means fees for very small purchases will tend to higher than the formula but even for investments as small as \$50, fees will correspond either exactly or almost exactly to the formula.

Figure 2: Ratios of equilibrium prices to true probabilities and pre- and post-fee loss rates with Kalshi fees of $\theta p(1-p)$ and $\theta = 0.07$. Agents have uniform beliefs on $[\pi - \pi(1-\pi), \pi + \pi(1-\pi)]$



5. A Flat Commission Rate on Winnings

We now consider how prediction markets work when they only charge a fee on winnings, as operated by commercial prediction markets outside the US, such as UK-based firms Smarkets and Betfair. There are some behavioural arguments that suggest a charge on winnings as being a more likely model to emerge in future commercial prediction markets. A charge on all contracts, whether they are going to win or lose, is highly salient to participants. Losing money on a contract is bad enough, having to pay a trading fee as well may be further discouraging, particularly if agents are motivated by loss aversion. Conversely, a charge on your winnings may not feel so bad: You still came out ahead and the market operator can hardly be expected to do their job for free. The fact that Betfair—owned by the large international betting firm Flutter which also owns Paddy Power and FanDuel—implement this method indicates that experienced and sophisticated participants in this market believe it is the best approach.

In this version of the model, the market operator charges a commission rate $1 - \mu$ on winnings. People with beliefs $\tilde{\pi}_i$ perceive expected post-commission winnings from purchasing the back contract to be $\tilde{\pi}_i \mu (1 - p)$ and expected losses to be $(1 - \tilde{\pi}_i) p$. They will be willing to purchase this contract if

$$\tilde{\pi}_i \mu (1 - p) \geq (1 - \tilde{\pi}_i) p \implies \tilde{\pi}_i \geq \frac{p}{p + (1 - p) \mu} = \tilde{\pi}^B \quad (19)$$

Similarly, participants will perceive expected post-commission winnings from purchasing the lay contract to be $(1 - \tilde{\pi}_i) p \mu$ and expected losses as $\tilde{\pi}_i (1 - p)$. They will be willing to purchase this contract if

$$\tilde{\pi}_i (1 - p) \leq (1 - \tilde{\pi}_i) p \mu \implies \tilde{\pi}_i \leq \frac{p \mu}{1 - p + p \mu} = \tilde{\pi}^L \quad (20)$$

In deciding the quantity of back contracts x_i^B that participant i with belief $\tilde{\pi}_i \geq \tilde{\pi}^B$ and wealth w_i wants to buy, she solves the problem

$$\text{Max}_{x_i} \tilde{\pi}_i \log(w_i + \mu(1 - p)x_i^B) + (1 - \tilde{\pi}_i) \log(w_i - px_i^B) \quad (21)$$

Again, demand for contracts is determined by the ratio of the perceived expected profit to the perceived variance

$$x_i^B = \left[\frac{\tilde{\pi}_i \mu (1 - p) - (1 - \tilde{\pi}_i) p}{\mu p (1 - p)} \right] w_i = D^B(\tilde{\pi}_i, p, \mu) w_i \quad (22)$$

while the demand for lay contracts for participant i with belief $\tilde{\pi}_i \leq \tilde{\pi}^L$ is

$$x_i^L = \left[\frac{(1 - \tilde{\pi}_i) p \mu - \tilde{\pi}_i (1 - p)}{\mu p (1 - p)} \right] w_i = D^L(\tilde{\pi}_i, p, \mu) w_i \quad (23)$$

The equilibrium price satisfies

$$\int_{\tilde{\pi}^B}^1 \left[\frac{\tilde{\pi}_i \mu (1-p) - (1-\tilde{\pi}_i) p}{\mu p (1-p)} \right] f(\tilde{\pi}) d\tilde{\pi} = \int_0^{\tilde{\pi}^L} \left[\frac{(1-\tilde{\pi}_i) p \mu - \tilde{\pi}_i (1-p)}{\mu p (1-p)} \right] f(\tilde{\pi}) d\tilde{\pi} \quad (24)$$

Again, to illustrate the model fully, we need a numerical solution. However, we can describe the general properties of the equilibrium price that will hold for all symmetric belief distributions. The equilibrium price is characterized by

$$\int_{\tilde{\pi}^B}^1 [\tilde{\pi}_i \mu (1-p) - (1-\tilde{\pi}_i) p] f(\tilde{\pi}) d\tilde{\pi} = \int_0^{\tilde{\pi}^L} [(1-\tilde{\pi}_i) p \mu - \tilde{\pi}_i (1-p)] f(\tilde{\pi}) d\tilde{\pi} \quad (25)$$

The left side depends negatively on p and the right side depends positively on it, so there is a unique equilibrium price. And again the pricing of back and lay contracts will be symmetric.

Two other features from the previous results also hold here. First, we can again show that when $\pi = 0.5$, the equilibrium price is $p = 0.5$. In this case, the belief thresholds for purchasing contracts are $\tilde{\pi}^L = \frac{\mu}{1+\mu}$ and $\tilde{\pi}^B = \frac{1}{1+\mu}$. These thresholds sum to one, which means that for this value of π (and only this value) the thresholds are equidistant from the true probability. Also, for $\pi = 0.5$, the price $p = 0.5$ makes the volumes $D^B(\tilde{\pi}^B + \epsilon, p, \mu)$ and $D^L(\tilde{\pi}^L - \epsilon, p, \mu)$ at equal distances above and below the thresholds equal to each other. These two demands being equal means

$$\mu(1-p)(\tilde{\pi}^B + \epsilon) - (1-\tilde{\pi}_i^B - \epsilon)p = (1-\tilde{\pi}_i^L + \epsilon)\mu p - (\tilde{\pi}_i^L - \epsilon)(1-p) \quad (26)$$

Inserting $p = 0.5$, $\tilde{\pi}^L = \frac{\mu}{1+\mu}$ and $\tilde{\pi}^B = \frac{1}{1+\mu}$ we can see the identity holds. The symmetry of belief thresholds and demands together mean a price of $p = 0.5$ equates total demand for the two types of contracts.

Finally, we again can show that for $\pi < 0.5$, the equilibrium price will be lower than the true probability and vice versa for $\pi > 0.5$. There are two reasons for this. The introduction of fees increases the threshold for taking the back contract and reduces the threshold for taking the lay contract, increasing the range of beliefs in which people purchase neither contract. However, one can easily show from calculating the partial derivatives $\frac{\partial \tilde{\pi}^B}{\partial \mu}$ and $\frac{\partial \tilde{\pi}^L}{\partial \mu}$ that the size of these effects is only equal for $\pi = 0.5$ and for $\pi < 0.5$, the fee crowds out more back contract buyers than lay contract buyers with the opposite applying when $\pi > 0.5$.

In addition, one can also show from calculating $\frac{\partial D^B}{\partial \tilde{\pi}}$ and $\frac{\partial D^L}{\partial \tilde{\pi}}$ that apart from at $p = 0.5$, once there are fees on winnings, contract demands no longer change in a symmetric manner as beliefs move above and below the belief thresholds. As beliefs move further above or further below the threshold levels, participants will be willing to buy more contracts. However, when $p < 0.5$, the fraction of wealth allocated to back contracts is less sensitive to changes in $\tilde{\pi}$ than the fraction allocated to

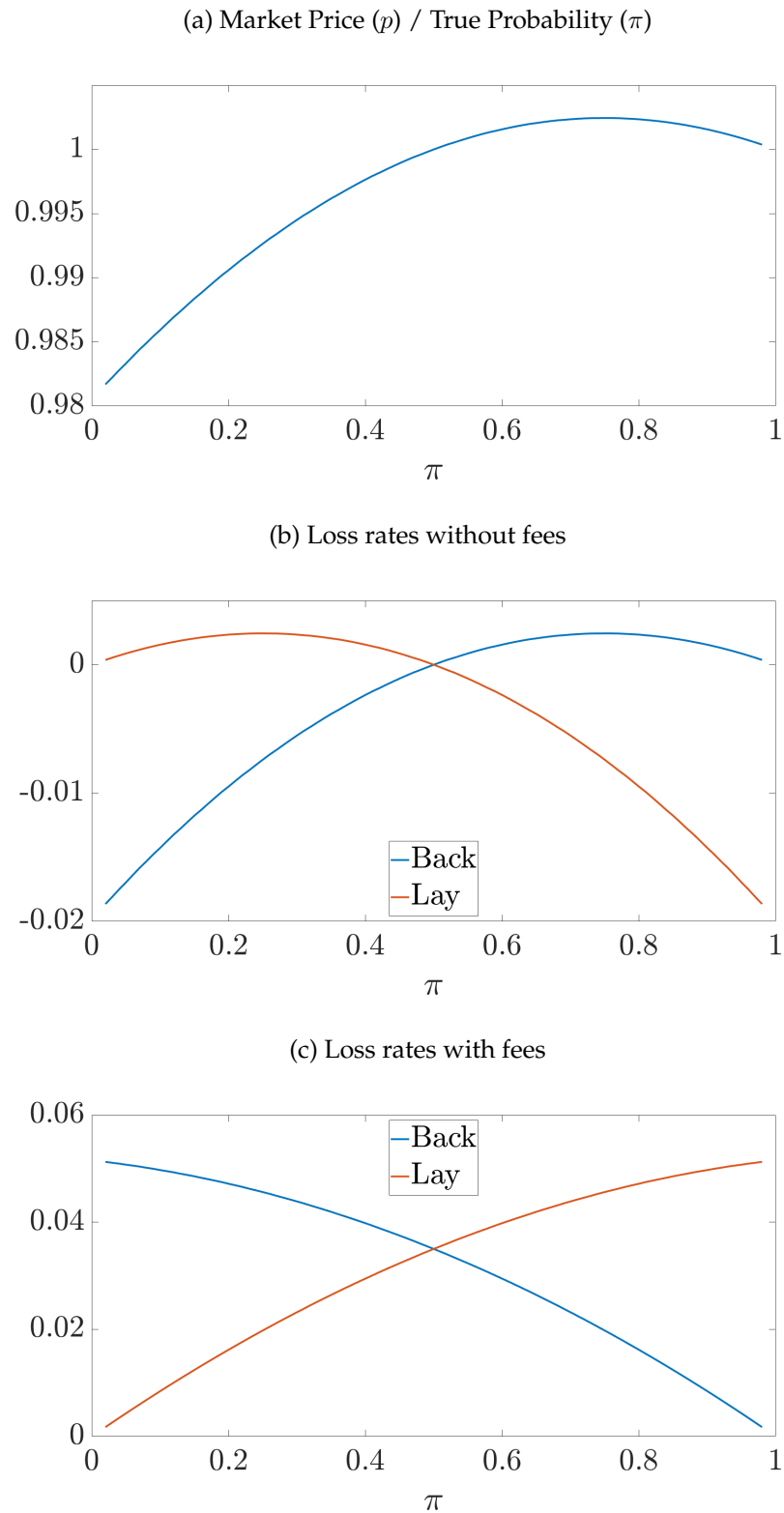
the corresponding lay contracts. Because the equilibrium price needs to equate volumes on both contracts, it needs to be lower than the no-fee case when $\pi < 0.5$ to encourage more longshot backers and discourage some favorite backers.

Solving the model with the same formulation of beliefs as before and with $1 - \mu = 0.07$, Figure 3 shows a somewhat smaller distortion of prices to the previous two cases. The implications for returns on contracts are relatively similar to the case of Kalshi's pricing, though the loss rates for longshot contracts are a bit higher and they fall monotonically as π rises. Since the approximation $p \approx \pi$ works well, we can provide a simple comparison of how Kalshi's pricing compares with commission fees charged on winnings. Total commission fees on winnings are

$$C = (1 - \mu) (\pi (1 - p) + (1 - \pi) p) \approx 2 (1 - \mu) \pi (1 - \pi) \quad (27)$$

In other words, a commission on winnings at rate $1 - \mu$ earns approximate the same amount of fees as Kalshi's trading fees with $\theta = 1 - \mu$. So, Kalshi's complex-looking formula with $\theta = 0.07$ can be understood as raising the same amount of fees as a 7 percent fee on winnings, which is why we use 7 percent here. Compared with the 2 percent fee on winnings charged by BetFair and Smarkets, Kalshi's current pricing looks expensive.

Figure 3: Ratios of equilibrium prices to true probabilities and pre- and post-fee loss rates with a commission rate on winnings of $1 - \mu = 0.07$. Agents have uniform beliefs on $[\pi - \pi(1 - \pi), \pi + \pi(1 - \pi)]$



6. A Kalshi-Style Commission On Winnings

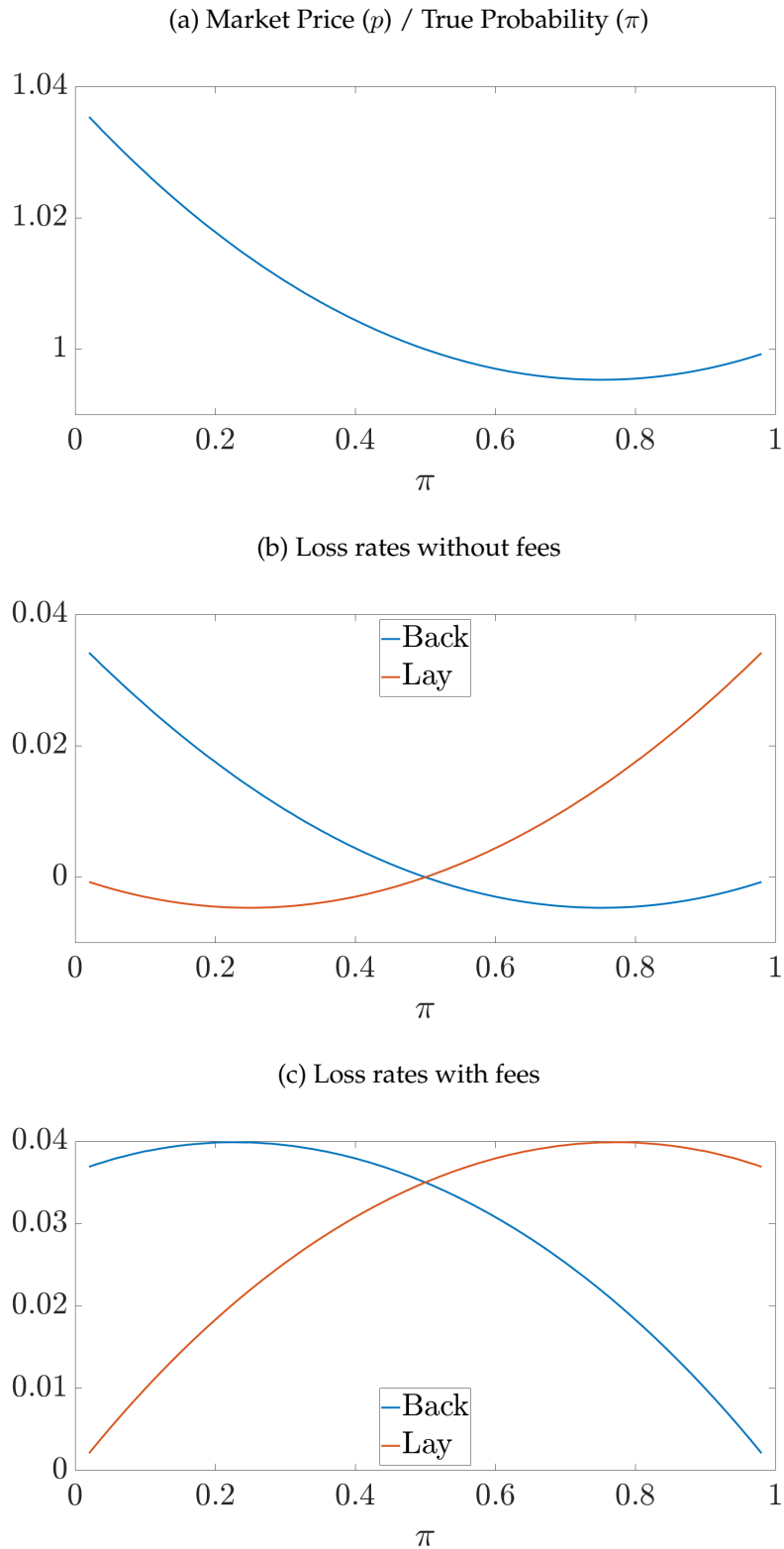
As a final example to illustrate the kinds of market patterns that may emerge as prediction markets evolve, we consider a hybrid commission fee approach, which is to charge Kalshi's fee but only on those who win. In other words, the prediction market operator does not charge a trading fee but when the back contract wins $1 - p$, its owner pays a fee of $\theta p (1 - p)$ and when the lay contract wins p , they also pay $\theta p (1 - p)$. Total commission fees would be the same as for Kalshi's trading fee model provided the value of θ is doubled.

This pricing structure would appear attractive to those interested in buying longshot contracts because the fees as a fraction of profits—which would be the appropriate way to quote them in this context—depend positively on p . With a fee of this type on winnings, the model's new belief thresholds and demands are derived by replacing μ in the formulas in the previous section with the new contract-specific post-commission payout rates on winnings, $\mu^b = 1 - \theta p$ for back contracts and $\mu^l = 1 - \theta (1 - p)$ for lay contracts.

Figure 4 shows the equilibrium prices in this case, with the same assumptions as before and $\theta = 0.14$, which would replicate the total fee amounts from the commission rate on winnings with $\theta = 0.07$ as shown in Figure 3. This approach to fees generates a different pattern for prices. Because the commission rate depends negatively on p for back contracts and positively on it for lay contracts, the results in the previous section on the impact of commission rates flip sign and the equilibrium price is now higher than the true probability when $\pi < 0.5$ and lower than it once $\pi > 0.5$ (the top panel). Equilibrium prices now over-state the lowest probabilities by similar amounts to how the fee structures in the previous two sections under-stated them but the bias is still fairly low.

The middle panel shows that the higher prices for back contracts imply pre-commission losses for these contracts when $\pi < 0.5$ with corresponding winnings for lay contracts. And despite the apparently more longshot-friendly pricing, loss rates after fees as a percentage of contract prices (the bottom panel) follow the same pattern as Kalshi's equal trading fee for each side of contracts which, at face value, was less generous to longshots. This shows that attempts to provide better value pricing for longshot contracts end up being undone by the endogenous dynamics of the market and thus that this is an innate feature of these markets.

Figure 4: Ratios of equilibrium prices to true probabilities and pre- and post-fee loss rates commission fee of $\theta p(1-p)$. Agents have uniform beliefs on $[\pi - \pi(1-\pi), \pi + \pi(1-\pi)]$



7. Conclusions

Prediction markets have the potential to play an important role in assisting governments, businesses and investors make decisions but their usefulness will depend on the accuracy of their prices and the willingness of people with a sufficiently wide range of opinions to participate in them. Our calculations suggest both good and bad news for the future of commercial prediction markets. The good news is that even in the presence of relatively high fees on winnings, prediction markets can still generate prices that are pretty close to the true probability of an event once the usual conditions hold for these markets to have this property in the absence of fees. Even better news is that the trading fee pricing formula adopted by Kalshi, the first officially licensed US prediction market, distorts prices much less than a flat fee such as 1 cent per contract.

Less good is the fact that the introduction of fees means those who have the most accurate view of the underlying probability will chose not to participate. The assumption of symmetry of beliefs around the true probability means the market in our model still delivers an accurate price because the two sets of more extreme opinions offset each other. In reality, however, cutting out the “sensible middleground” of opinion and hoping that those with more extreme opinions on either side will average out to the correct opinion may or may not work well depending on the event being predicted.

The other bad aspect is the systematic pattern of loss rates. Unlike stock markets where the average investor can be a winner, prediction markets are innately zero sum in nature and, once an operator has fees, the average outcome for participants is a negative sum. We show that this results in larger loss rates for money invested in contracts backing events that are unlikely to happen and that this finding still holds even if a commission rate formula is adjusted to be more favorable to these contracts. If people participating in prediction markets begin to realize they are experiencing significant losses on the cheaper contracts, then these markets may fail to attract people with the wide range of opinions that they need for their prices to summarize the public’s beliefs.

Ultimately, this market will work best as a way of summarizing the public’s information when there is competitive pressure to lower fees. Kalshi’s fees, which are equivalent to a 7% charge on winnings, are high relative to UK firms operating similar business models. Admittedly, Kalshi are particularly ambitious in the wide range of markets they are running but it seems likely that competitive pressures coming from future CFTC approvals will bring down fee rates and contribute to the market working more efficiently.

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