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Abstract

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JEL Classification: D40, D83, L10

Keywords:

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Searching Online and Product Returns*

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Abstract

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Keywords: product returns, consumer search, search efficiencies, product matches
JEL codes: D40, D83, L10

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1 Introduction

E-commerce has come with a sharp increase in products being returned after purchase. In the USA the National Retail Federation estimates that across different retail channels $428 billion of merchandise value is returned in 2020, which is around 10% of total retail sales. Focusing on online retail only, these numbers are approximately $102 billion and 18% of online sales. They also mention that "online returns more than doubled and are a major driver of the overall growth of returns".[1] Given this development, retail firms have begun to treat product returns strategically by considering optimal return policies. Moreover, product returns are a social concern in that they potentially point to a large social cost of e-commerce, including environmental costs that are paid by agents not involved in the transaction.[2]

An important source for higher return rates in online markets, we argue, is that online consumer search differs from that in traditional markets along two dimensions. First, search for objective information is much easier online. With only a few clicks, consumers can get access to prices and other objective product features. However, and second, it is certainly not that easy to learn subjective features of product match online: How will a certain pair of glasses fit, or, how easy is it to operate a digital camera? To overcome this latter aspect of online search, firms may choose lenient return policies providing incentives for consumers to buy the product without spending effort *before* purchase to determine whether the product fits their needs. Instead, consumers may check product fit in a more comfortable home environment and return the product if they learn the fit is not good enough.

Product returns are also important for regulatory agencies whose policies towards

[1] See, https://9d4f6e00179f3c3b57f1-4eec5353d4ae74185076baef01cb1fa1.ssl.cf5.rackcdn.com/Customer%20Returns%20in%20the%20Retail%20Industry%20-%20short%20pdf.pdf. This tendency is confirmed by other sources, see e.g., https://www.invespcro.com/blog/ecommerce-product-return-rate-statistics/ and https://www.nosto.com/ecommerce-statistics/return-rate/.

[2] These environmental costs include greenhouse gas emissions, non-recycled packaging and products filling up landfills (see, e.g. Tian and Sarkis (2022)). Some websites (see, e.g., https://www.fastcompany.com/90701492/how-businesses-can-fight-the-environmental-menace-of-free-returns?msclkid=8b9dc58fc80511ec9233fbf51095482e) estimate that only 54 percent of all packaging gets recycled and 5 billion pounds of returned goods end up in landfills each year.

returns differ across the world. In the USA, there is no general regulation in place, but some individual states mandate that retailers give refunds on products that are returned within 20 or 30 days of purchase if they themselves do not provide a clearly stated policy.[3] In the European Union, refunds on online purchases are mandatory.[4] Even though full refunds seem to be the norm (either because they are mandated or because of a firm's voluntary, strategic choice), the small print may make all the difference as refund policies differ according to (i) who should pay for the return cost and (ii) whether firms charge a so-called restocking fee (which can be up to 20% of the purchase price) and these aspects are typically not regulated.

This leads us to the following questions. In the absence of regulations, is it optimal for firms to stimulate product returns? Do consumers or firms benefit from product returns? Are product returns socially wasteful or do they provide an efficient solution to the trade-off between search cost reduction and cost increases due to returns? Are regulations redundant if firms voluntarily offer product returns? How do the welfare results change if there are externalities in the form of pollution and other environmental issues?

To answer these questions, we build on the seminal consumer search paper by Wolinsky (1986). Our methodological contribution consists of extending it in a number of fundamental ways. First, firms choose two prices: the price at which consumers purchase the product and the refund consumers get when they return the product, i.e., firms think strategically about their return policy. Second, we divide the search cost into two parts: a very small cost of learning a firm's price and refund policy and a much larger inspection cost to learn the personal match value. Third, we allow consumers to purchase a product without inspecting its match value before purchase. If they do so, they will typically inspect the product after purchase and can do so at a lower than before-purchase inspection cost or not inspect at all. Fourth, if consumers return the product, it has a salvage value for the firm that is typically smaller than the production cost. Thus, there are two important aspects

[3] See, e.g., https://www.findlaw.com/consumer/consumer-transactions/customer-returns-and-refund-laws-by-state.html

[4] "If you bought a product or a service online or outside of a shop..., you have the right to cancel and return your order within 14 days, for any reason and without a justification". See, https://europa.eu/youreurope/citizens/consumers/shopping/guarantees-returns.

to offering product returns:[5] (i) the difference between production cost and salvage value represents the (social) cost of product returns, while (ii) the difference in search costs before and after purchase represents a potential (social) benefit.

We have three main sets of results. First, and most importantly, we show that whenever the market provides returns, the outcome is almost always inefficient: there are either too few or too many returns. In a market context, efficiency requires that for every unit that is bought and then returned, the consumer pays the cost related to the return and firms make no profit, i.e., the difference between price and refund should equal the difference between production cost and salvage value. As the shipping fees are one reason why the salvage value is lower than the production cost, efficiency requires consumers to pay for shipping. The market provides too few returns if, and only if, the difference between price and refund that is offered is larger than the difference between production cost and salvage value. Too few returns arise, for example, when production costs are relatively low or when the search cost is relatively high. If there is a social cost of product returns, for example in the form of a pollution externality, that are not taken into account by the two sides of a transaction the market outcome may get closer to being efficient.

The evaluation of product returns in a market context is very different from that under monopoly. First, the notion of efficiency itself is significantly different as under monopoly efficiency requires that the refund that is offered equals the salvage value. The difference lies in the fact that if a consumer does not buy at the monopolist, they do not buy at all, whereas in a market they continue to search, potentially generating both more profits for firms and more value for consumers. Second, in a search market, but not under monopoly, firms may offer a refund that is inefficiently low. This is surprising as one would expect that competition forces firms to offer more generous refunds. However, by marginally increasing the refund, a firm increases sales at other firms and improves consumer welfare, an externality they do not take into account. When offering refunds, a monopolist optimally chooses a refund that is larger than the salvage value to deter consumers

[5] We abstract away from fraud. Especially in online transactions, fraud may come from both sides of the market: firms may ship broken or otherwise non-functional products, while consumers may buy products for a certain occasion and then return them. Our model deals with firms that care about their reputation and who keep track of -and ban- consumers engaged in fraudulent behavior.

from inspecting before purchase. Thus, a monopolist always promotes too many returns from a social welfare perspective. One clear indication that the monopoly context is very different is that when the search cost is high firms have more market power and the results should get closer to monopoly. However, as indicated above, higher search costs make it more likely the market provides too few returns.

The second set of results relate to the overall equilibrium characterization. We show that if the inspection cost is not too large, there always exists a unique equilibrium with trade. Depending on the parameter values, it is such that either (*i*) all firms offer a refund resulting in consumers inspecting after purchase, or (*ii*) all firms effectively incentivize consumers to inspect before purchase (which results in the Wolinsky equilibrium), or (*iii*) some firms offer an effective return policy, while others do not.[6] Refund equilibria exist when there are large efficiency gains of inspecting at home, and the social waste of returns (measured as the difference between production cost and salvage value) is relatively small. The Wolinsky equilibrium exists in the opposite case where the efficiency gains of inspecting at home are small, while the social waste of returns is large. In between these two parameter regions, the equilibrium has some firms offering returns, while others do not.[7] We use this characterization results to indicate in what type of markets we may expect more product returns and why product returns are more frequent in online markets.

Third, we fully characterize the properties of the equilibrium with refunds. In such an equilibrium, it has to be the case that the price and refund policy firms offer are such that the consumer is indifferent between inspecting before and after purchasing the good. A firm would not want to offer a more generous refund than is

[6]This equilibrium characterization partially explains why one observes more product returns in some industries than in others. The National Retail Federation reports that retail categories such as apparel and footwear have a relatively high product return rate, whereas the product return rate for other categories such as beauty and health care is relatively low (see, https://9d4f6e00179f3c3b57f1-4eec5353d4ae74185076baef01cb1fa1.ssl.cf5.rackcdn.com/Customer%20Returns%20in%20the%20Retail. For the first two categories, product match is important and large efficiencies can be gained by inspecting at home, while the salvage value is reasonably close to the production cost. For the second set of categories, the salvage value is almost zero and return policies are therefore unattractive.

[7]Thus, even though firms are ex ante identical, in an asymmetric equilibrium one may observe different behaviors. If we would perturb the model slightly so that firms are heterogeneous to start with, it would probably be easier to get different behaviorial patterns to co-exist in equilibrium.

strictly necessary for consumers to be willing to inspect afterwards as this reduces profit. Firms also do not want to offer a lower refund as this triggers consumers to inspect before purchase lowering a firm's profits. Firms can exploit the fact that as the consumer's threat of inspecting before purchase becomes less attractive when the search efficiency gains of inspecting at home get larger by increasing the difference between price and refund. Thus, firms may benefit and capture the gains from less costly search.

We also consider the more practical, regulatory issue of whether regulators should mandate firms to offer a minimal threshold percentage of the sales price as a refund, while leaving the pricing decision to the discretion of firms. Restricting attention to cases where the market offers refunds that are too small from a social welfare perspective, we have two sets of results. First, the threshold harms consumers and benefits firms, while leaving social welfare unaffected, if it is chosen just marginally above the market equilibrium outcome. The reason is that firms react to the regulation by increasing prices. Second, if the regulation is more bold and the threshold is substantially above the refund percentage that the market would choose, then the regulator may achieve the first best, although a full refund always creates excessive returns.[8]

The above results indicate that inspection after purchase and product returns are key elements to understand the functioning of online markets. The results explain that product returns are an integral part of online markets and that, despite the appearance of inefficiency, they can be efficient in reducing search/inspection cost and improving match values. However, the market outcome is generally inefficient as there are either too many or too few returns.

The existing literature on product returns typically focuses on a monopoly where consumers do not have an outside option, are uncertain about the product's value to them and learn the value after purchase at no cost. As discussed above, our results indicate that studying refunds in a market context where consumers search other firms if they return the product is important as it yields very different insights. The paper in the monopoly literature that is closest to ours is Matthews and Persico

[8]The only reason the EU mandate of full returns does not necessarily lead to inefficiently many returns is that it does not stipulate that restocking or shipping fees cannot be levied towards consumers.

(2007). They show that if a monopolist offers refunds, it always promotes too many returns from a social welfare perspective. Even though in a market context firms also offer a refund that is larger than the salvage value, this is only one aspect of the efficiency considerations and it does not imply the market offers too many returns. Other papers in this literature include Che (1996), Inderst and Ottaviani (2013), Inderst and Tirosh (2015) and Jerath and Ren (2022).[9] Che (1996) shows that a monopolist may offer a generous refund to induce risk-averse buyers to buy, whereas Inderst and Ottaviani (2013), Inderst and Tirosh (2015) study an alternative reason why a firm may offer refunds, namely as a way to signal product quality. Jerath and Ren (2022) consider whether a monopolist has an incentive to facilitate search before purchase in relation to the complexity of the return policies they choose and show that their model may account for different refund practices. To stress the important, different insights that obtain by allowing consumers to continue to search other firms once they return a product and to keep the analysis tractable, we abstract away from other roles product returns may have, such as signaling quality.

Our paper is also related to several recent branches of the consumer search literature. First, Armstrong (2017), Choi et al. (2018), Haan et al. (2018), among others, build on Wolinsky (1986), but allow consumers to direct their search based on the prices firms charge. This literature is inspired by the observation that in online markets price information is more easy to acquire than other types of information. These papers stick, however, to the standard consumer search set-up that consumers cannot buy without inspecting before purchase. Doval (2018) is the first paper in this literature to introduce the option of buying without inspection, but she only considers the optimal consumer search problem and studies how the optimal stopping rule differs from the classic Weitzman (1979) rule, while Chen et al. (2021) introduce this option in the Choi et al. (2018) model.[10] None of these papers allows consumers to inspect the product after purchase, however, (which is necessary for them to be able to return the product) or the question whether the market stimulates

[9]Hinnosaar and Kawai (2020) explore robust pricing with refunds, where consumers' value either match or do not match with the monopolist's product and where the firm does not know the consumer's prior over his match value.

[10]Fishman and Lubensky (2016) also introduce the option of purchasing without inspection in a Wolinsky-type model, but like Chen et al. (2021) they do not study product returns.

firms to offer lenient return policies leading to product returns. Petrikaitė (2018) introduces the option of product returns in a model where consumers learn one component of their match value after purchase, but the key issues of this paper, namely whether firms stimulate product returns and whether this enhances market efficiency, are not addressed in this paper as firms cannot choose their return policy and the salvage value is assumed to be equal to the production cost.

The rest of the paper is organized as follows. The next section introduces the model, while Section 3 characterizes the efficient allocation. Sections 4 and 5 characterize the market equilibria, where Section 4 focuses on market equilibria where firms offer a refund that incentivizes consumers to inspect the product after purchase, while Section 5 investigates more generally what type of market outcomes can be sustained as equilibria and when these different types of equilibria exist. Combining the results on efficiency and the characterization of market equilibria Section 6 analyzes whether the market provides too little or too many returns and the effect of regulation. Section 7 concludes with a discussion, while proofs withheld from the text are given in the (online) Appendix.

2 Model and Preliminary Results

The market is comprised of a unit mass of consumers with a unit demand for a product and a unit mass of firms who supply the product. Following the consumer search literature based on Wolinsky (1986), a consumer's match value with a firm is drawn from a distribution G with support $[\underline{v}, \bar{v}] \subset \overline{\mathbb{R}}$, density g, with both G and $1 - G$ logconcave. Match values are independent across firms and consumers. At the start of the game, firms simultaneously set not only their price $p \geq 0$, but also their *refund* $\tau \geq 0$, which is the amount of money given back to a consumer who purchases the product and then decides to return it. We often refer to the pair (p, τ) as the firm's "contract". Firms face a constant marginal cost of production $c \geq 0$ and a *salvage value* $\eta \in [0, c]$ for items that have been purchased and subsequently returned. The difference $c - \eta$ captures the cost of product returns.[11]

[11]Our welfare analysis in Section 3 accommodates externalities, for example, in the form of pollution produced when transporting goods between firms and consumers. Since market participants

Consumers are uncertain of their match values as well as firms' contracts, but they can learn these through costly search. A consumer starts out by incurring a small but positive *search cost* $\epsilon > 0$ to visit a firm to learn the price it charges and the return policy it offers. The consumer then has three options: He can incur an *inspection cost* $s > 0$ to learn his match value with the firm, he can buy the product without inspecting it, or he can decide to leave and visit another firm (or leave the market altogether). If the consumer buys the product without first inspecting it, he has the option to incur an inspection cost of βs, with $\beta \in [0, 1]$, to learn his match value after purchase. If the consumer learns the match value after purchase, he can decide whether to keep the product or to return it to the firm, receiving τ and then possibly continue his search at another firm. The term $(1 - \beta)s$ measures the search efficiency of inspecting after purchase and captures the reduction in inspection cost if the consumer inspects after purchase relative to inspecting before purchase. Let $\Omega \subset \mathbb{R}^5_+$ denote the set of parameter values $\omega = (c, \beta, \eta, s, \epsilon)$.

Throughout the paper we focus on Perfect Bayesian Equilibria where firms choose their strategies to maximize expected profits given their information and consumers choose an optimal sequential search strategy. In addition, if consumers observe a firm's deviation from equilibrium behavior, they continue to believe that firms that are not yet visited play their equilibrium strategies as these do not depend on the information consumers have.

2.1 Some comments on the Interpretation of the Model

We now discuss different aspects of our model. First, how our model relates to online versus brick-and-mortar shops and the associated difference in inspection costs before and after purchase can be interpreted in different ways. For example, one may interpret s as the cost of going to a brick-and-mortar store and βs as the cost of inspecting at home after having purchased the product online. Arguably, the inspection cost at home is (much) lower, but may still be positive given that a

overlook externalities when making choices, the presence or magnitude of the externalities has no effect on our comparative static and equilibrium characterization results in Sections 4 and 5.

product typically has to be returned within a certain narrow time period.[12] Another interpretation is that all possible inspection (both before and after purchase) is performed online, taking into account that consumers may learn about products from online reviews. Also in this interpretation, it often is easier to interpret online reviews when having the product at hand after it has been shipped home.

Second, and in line with the above second interpretation, our model can capture consumers acquiring information about their match values upon incurring the initial search cost. For example, adopting an approach used by Anderson and Renault (2021) and Nocke and Rey (2023), suppose that when incurring the search cost ϵ, a consumer also learns whether or not the product is a “match" for his needs. A product that fails to be a match delivers a value of $v = 0$, while other products offer a match value drawn from the distribution G specified above. Somewhat more elaborately, we could also allow for a consumer's match value to be decomposed into two components, i.e., $v = v_1 + v_2$, whereby v_1 is the value of objective features of a product that can take on a finite[13] set of magnitudes and be learned together with price (such as for example the size of a notebook). With this modification, the only change to our model is that the search cost must factor in the expected cost of visiting firms that fail to provide the (best) match.[14]

Third, we may also consider that there are some product features that are prohibitively costly or even impossible to learn online. For example, we could interpret the decomposition of a consumer's match value differently and say that v_1 is the value offered by features that can be learned by inspecting the product, while v_2 is the value of features that are only discovered once the product is at hand. In this case, there remains uncertainty about the product's value even if one has inspected the v_1 features before purchase, and the analysis becomes more complicated, but our main insights remain.

Fourth, an implication of the assumption that consumers incur an arbitrarily small

[12]We do not explicitly address issues related to the length of the period in which consumers can return the product, but see Lyu (2022) for some of the relevant considerations.

[13]If v_1 can be measured continuously, the technical analysis will become more complicated as depending on their v_1 value different consumers may continue to search differently, but if ϵ is small enough the qualitative conclusions of our analysis should hold true.

[14]Letting $\rho > 0$ denote the probability of having a match with a firm, the expected search costs incurred before reaching a firm that provides a match is $\epsilon' = \frac{1}{1-\rho}\epsilon$.

but positive search cost to learn a firm's price and return policy is that they cannot direct their search to firms with lower prices and/or a more favorable return policy. We know from Diamond (1971) that small search costs may have very different implications from search costs being equal to zero and we think this assumption is also appropriate in many online markets where consumers do not often buy from the same shop and they do not know a return policy in advance.

Fifth, the model does not explicitly define nor address who pays the transportation costs, which are clearly important for online purchases. These aspects are easily interpreted in our model as follows. First, the cost of shipping itself is one reason why the salvage value is smaller than the production cost. Second, the difference between the price and refund is a measure for how these costs are divided between firms and consumers. For example, if the difference $c - \eta$ is entirely due to the cost of shipping, then a full refund $\tau = p$ implies that firms pay the cost while a contract satisfying $p - \tau = c - \eta$ implies that consumers pay for it.

Sixth, the model is flexible enough to cover environments where consumers incur an additional "hassle cost" to return a product. If we denote this cost by γ, then our model is isomorphic to this new model, by replacing the refund with $\tilde{\tau} \equiv \tau - \gamma$ and replacing the salvage value with $\tilde{\eta} \equiv \eta - \gamma$.

Seventh, we model search as a sequential decision problem. Alternatively, one may consider that the possibility to return a product after purchase introduces a delay for the next search and that this makes it optimal for consumers to engage in simultaneous search (cf., Morgan and Manning (1985)), In the supplementary material we show that if the delay is not substantial, sequential search continues to be optimal. As product deliveries usually only take a few days, delay does not seem to be an argument to consider simultaneous search. In the last section of the paper we discuss a more promising alternative for modeling simultaneous search in online markets.

2.2 Consumers

We now present some preliminary findings, starting with the consumer side. Consider the consumer's problem at the moment he has incurred the search cost ϵ to

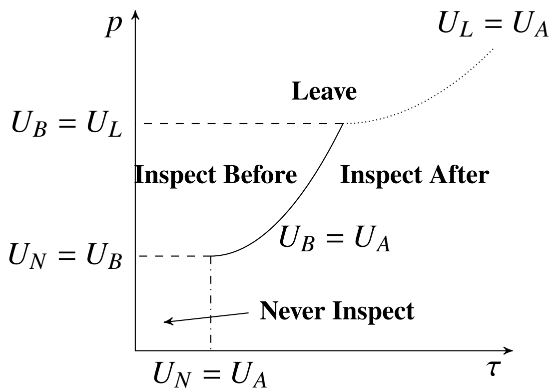


Figure 1: Inspection choices for a given price p and return policy τ.

visit a firm and learned its price and refund (p, τ). Denote the consumer's outside option by a. The outside option is endogeneous to the model and depends on the equilibrium contracts offered by other firms. The consumer has four possible paths of play. (i) If he decides to leave the current firm, he gets a. (ii) He may decide to buy the product and neither inspect it before nor after purchasing it, yielding the payoff $v - p$. (iii) If he decides to inspect the product before purchase, his payoff is $v - p - s$ if the net value exceeds the outside option $v - p \geq a$ and otherwise the payoff is $a - s$. (iv) If he buys the product and inspects afterwards, his payoff is $v - p - \beta s$ if his match value exceeds the payoff from returning the product $v \geq a + \tau$, and otherwise his payoff is $a + \tau - p - \beta s$. Thus, the expected utility from each of these options is given by the following expressions.[15]

$$\begin{cases} \text{never inspect} & U_N = E(v) - p \\ \text{inspect after} & U_A = E(\max\{v, \tau + a\}) - p - \beta s \\ \text{inspect before} & U_B = E(\max\{v - p, a\}) - s \\ \text{leave} & U_L = a \end{cases} \tag{1}$$

When presented with a price and return policy, the consumer selects the inspec-

[15]The expressions for utility in (1) are functions of the price, refund, outside option, and model primitives: $U_i(p, \tau, a, \omega)$ for $i \in \{N, A, B, L\}$. Throughout the paper, we suppress these arguments when convenient.

tion option yielding the highest expected payoff. Figure 1 illustrates how the optimal choice depends on the price and return policy of the firm that is visited. When the return policy is unfavorable and the price is moderate, consumers adopt the usual search strategy of inspecting before purchase. When the return policy is unfavorable and the observed price is too high, they leave the firm without inspecting the product, while if the product is sufficiently cheap they buy without ever intending to inspect the good. When the return policy is more favorable, the consumer will opt to first buy the product and then inspect it, returning it if it turns out that his match value is low. As we shall discuss, the sizes of the different regions depend on the underlying search parameters s and β as well as on the outside option a.

Importantly, the figure shows that that there exists a region "Inspect Before" where firms offer a positive refund that is never used by consumers as it is too low. Thus, offering such a contract results in the same market outcome as not offering a refund. In the rest of the paper, we refer to policies where firms offer a refund as those that induce consumers to inspect after purchase and there is a positive probability that the consumer returns the product. For the region "Inspect Before" we say that no refund is given.

To characterize consumer behavior, it is useful to let $S \equiv E(v - \underline{v})$ and introduce the *reservation price* $r : [0, S) \to [\underline{v}, \bar{v}]$, implicitly defined by

$$E(\max\{v - r(x), 0\}) = x. \tag{2}$$

Intuitively, $r(x)$ is the price at which consumers are indifferent between incurring the inspection cost x and taking the outside option of zero. Letting $S' \equiv E(\bar{v} - v)$, we follow Doval (2018) and introduce the *backup price* $b : [0, S') \to [\underline{v}, \bar{v}]$, implicitly defined by

$$E(\max\{b(x) - v, 0\}) = x. \tag{3}$$

Similarly, $b(x)$ corresponds to the price making consumers indifferent between incurring the cost x to inspect a firm's product and buying it without inspection when the outside option is zero. Throughout the paper, we maintain that the sum of the search and inspection costs is less than the unique value s^* equating the reservation and backup prices $r(s^*) = b(s^*)$, *i.e.*, $0 < s + \epsilon < s^*$.[16]

2.3 Firms

Given the inspection strategy adopted by consumers, we now turn to the firms' problem. Denote the probability a consumer continues to search after visiting another, randomly drawn, firm by q. Then, if the consumer's outside option is a in each round of search, a firm's expected profit from offering a particular price and refund is determined by the consumers' inspection decisions as follows.

$$\begin{cases} \text{never inspect} & \pi_N = \frac{p-c}{1-q} \\ \text{inspect after} & \pi_A = \frac{p-c+(\eta-\tau)G(a+\tau)}{1-q} \\ \text{inspect before} & \pi_B = \frac{(p-c)(1-G(a+p))}{1-q} \\ \text{leave} & \pi_L = 0 \end{cases} \tag{4}$$

If the consumer never inspects, the firm's profit is simply equal to $p - c$ over all consumers that visit it. If consumers inspect afterwards, the firm has to give the refund τ back to all consumers who return the product in exchange for the salvage value η. From the consumer's problem it is clear that a fraction $G(a + \tau)$ returns the product. Finally, if consumers inspect before purchase, the firm's profit is simply the Wolinsky profit.

The terms $1 - q$ in the different profit expressions may appear to be relatively unimportant scaling factors. That appearance is false, however, as we shall show in the next sections. The probability q endogenously depends on contracts that firms offer and thereby on the parameters in Ω. In particular, some of the comparative statics effects on profits and on welfare arise, because the market generates more product returns. For example, firms may make more profits overall despite the fact that on each direct sale they generate less profits, simply because it is possible to make money over products that are returned.

[16]Appendix S.1 verifies the existence of a unique $0 < s^* < \min\{S, S'\}$ equating $r(s^*) = b(s^*)$. Thus, our analysis is restricted to the set $\Omega = \{(c, \beta, \eta, s, \epsilon) \in \mathbb{R}_+^5 : 0 \leq \beta \leq 1, 0 \leq \eta \leq c, 0 < s + \epsilon < s^*\}$

As an equilibrium requires a firm's best response to be well-defined, we focus on strategy profiles in which consumers break indifference between inspection options in favor of the firm. To find possible best replies, we should examine all points of discontinuity whereby $U_j(p, \tau) = U_k(p, \tau)$ for $j, k \in \{N, A, B, L\}$ and $k \neq j$ and also consider interior optima, where the consumer strictly prefers an inspection option j and $\nabla \pi_j = 0$. In principle, this gives six possible classes of best responses at boundary regions where consumers are indifferent between at least two options and four possible classes of interior solutions.

3 Efficiency

The key to understanding how return policies affect competitive search markets is to examine their impact on social welfare, defined to be the sum of industry profit and consumer surplus. To this end, we begin by establishing the conditions for efficiency supposing that all firms offer a single contract (p, τ). As is depicted in Figure 1, depending on the contract, distinct search behaviors by consumers are induced.

Consider a contract with a sufficiently generous refund so that consumers opt to inspect products after purchase. In this case, consumer surplus satisfies $a = U_A - \epsilon$. Using the expression for utility (1) and the definition of the reservation price (2), consumer surplus can be solved explicitly:

$$a + \tau = r(p - \tau + \beta s + \epsilon). \tag{5}$$

When returns are made social welfare is $\mathbf{S}_R = \pi_A(p, \tau, a) + a$, where a satisfies (5). By examining (4) and (5), and realizing that in this case $q = G(a + \tau)$, we can see that $\mathbf{S}_R$ only depends on the contract (p, τ) through the difference $p - \tau$. Thus, to determine which contracts are optimal, fix the price and differentiate welfare in the refund to obtain

$$\frac{d\mathbf{S}_R}{d\tau} \propto -(p - \tau - (c - \eta)). \tag{6}$$

Welfare is therefore strictly increasing in the refund when $p - \tau > c - \eta$, strictly decreasing in the refund when $p - \tau < c - \eta$, and maximized by a contract that sets $p - \tau = c - \eta$.

Note that efficiency considerations in a monopoly context are very different as when returns are made, efficiency only requires that consumers that return the product have a value that is smaller than the salvage value of the firm. To understand the intuition behind (6), consider the effect of starting from an arbitrary contract and gradually increasing the refund. For items that are bought and kept despite the more generous refund, there is no effect on welfare. For items that would have already been returned, increasing the refund simply transfers funds from firms to consumers, with a net zero effect on welfare. The way increasing the refund does affect welfare is on the margin of changing search behavior: consumers become more likely to make a return. While the envelope theorem provides that the change in search behavior does not itself affect consumer surplus, it does affect profit. On one hand, increasing the rate of product returns is costly because it requires firms to spend more on refunds. At the same time, it is important to realize that firms not only make profit over products that consumers keep, but so long as $p - \tau - c + \eta > 0$ they also make profits over products that are returned. Thus, efficiency requires that firms do not make profits over returned items, i.e. $p - \tau - c + \eta = 0$. At such a contract, welfare is

$$\mathbf{S}_R = r(c - \eta + \beta s + \epsilon) - \eta. \tag{7}$$

Turning to the two other inspection options, consider first contracts with a less generous refund and a moderate price so that consumers choose to inspect before purchasing, i.e., $a = U_B - \epsilon$. Solving explicitly for consumer surplus yields

$$a + p = r(s + \epsilon). \tag{8}$$

So, if consumers inspect before purchase, welfare is constant and takes the standard form found in the Wolinsky framework $\mathbf{S}_W = r(s + \epsilon) - c$. Finally, if the contract has both a low price and refund, then consumers will buy the product at the first firm they visit and never inspect it, in which case welfare is $\mathbf{S}_N = E(v) - c$. Given the assumptions on the search costs, $\mathbf{S}_W > \mathbf{S}_N$.

Note that if the contract satisfies $p - \tau = c - \eta$, then by examination of (4), profit equals $\tau - \eta$ regardless of whether consumers inspect before or after purchase. Thus, what is socially optimal corresponds to what is best for consumers. Letting $\psi(\omega) = \mathbf{S}_R - \mathbf{S}_W = r(c - \eta + \beta s + \epsilon) - r(s + \epsilon) + c - \eta$, we summarize our discussion in the following proposition.

Proposition 1. *The social optimum is achieved by having all firms offer a contract* $(\hat{p}, \hat{\tau})$ *with* $\hat{p} - \hat{\tau} = c - \eta$ *and* $c \leq \hat{p} \leq \max\{\psi(\omega), 0\} + r(s + \epsilon)$. *At the social optimum, (i) if* $\psi(\omega) > 0$, *consumers inspect a good after purchasing it, (ii) if* $\psi(\omega) < 0$, *consumers inspect a good before purchasing it, and (iii) if* $\psi(\omega) = 0$, *consumers either inspect a good before or after purchasing it.*

In S.2 of the supplementary material, we show that the above result continues to hold allowing any distribution of contracts that firms offer. The requirement that the price satisfies $c \leq \hat{p} \leq \max\{\psi(\omega), 0\} + r(s + \epsilon)$ serves to ensure that profit and consumer surplus are both nonnegative. The proposition shows that interpreting $c - \eta$ as the transportation cost related to product returns, it is socially optimal to let consumers pay for this cost it is socially optimal to have product returns.

Up until now, the welfare analysis only took the various costs borne by the individual market participants into account. Repeatedly purchasing and returning products might, however, create additional social costs, for example in the form of pollution due to excessive transportation or the waste from discarding returned items. We now show that these aspects can easily be incorporated by including a social (environmental) cost from each sale $e_s \geq 0$ and each return $e_r \geq 0$.

To investigate the effect of environmental externalities on welfare, consider that firms offer a contract (p, τ) that leads consumers to inspect after purchase. As the probability a consumer returns an item after purchasing it is $G(a + \tau)$, the expected number of returns made by a consumer is $\frac{G(a+\tau)}{1-G(a+\tau)}$ and the expected number of purchases by a consumer is $\frac{1}{1-G(a+\tau)}$. Summing over consumer surplus, industry profit, and externalities, the expression for welfare becomes

$$\begin{aligned}
\mathbf{S}_R^e &= \frac{p - c - (\tau - \eta)G(a+\tau)}{1 - G(a+\tau)} + a - (e_r + e_s)\,\frac{G(a+\tau)}{1 - G(a+\tau)} - e_s \\
&= \frac{p - c - (\tau - \eta + e_r + e_s)G(a+\tau)}{1 - G(a+\tau)} + a - e_s
\end{aligned} \tag{9}$$

If the social costs of market activity are too large, then efficiency naturally requires the market to be inactive. For less extreme externalities, the efficient contract is easily computed by noticing that the maximization of (9) is the same as before, except that the salvage value is now replaced with a *social salvage value* $\eta^e = \eta - e_r - e_s$. Thus, the proposition 1 remains valid with η being replaced by η^e.

4 Refund Equilibria

We now characterize properties of equilibria where firms offer a refund such that consumers inspect the product after purchase and claim the refund if their match value is relatively small. We use these results in the next section to show when refund equilibria exist and in Section 6 to characterize the efficiency of market outcomes.

Allowing purchased items to be returned leads to a simple trade-off. Consumers benefit from waiting to inspect a product until after buying it, when it is easier to do so, and returning it when dissatisfied (i.e. when $\beta < 1$). Firms absorb a loss when returned products lose value (i.e. when the salvage value is less than the production cost $\eta < c$). In this section, we detail how this trade-off determines the return policies that are offered in the market place.

Proposition 2. *When a symmetric refund equilibrium exists, it is unique and has the following properties: (i) the price and refund are set so that consumers are indifferent between inspecting products before and after purchasing them,* $U_A(p, \tau, a) = U_B(p, a)$*, and (ii) the refund exceeds the salvage value,* $\tau > \eta$.

The first property is easily understood. In symmetric refund equilibria, a more generous refund makes consumers not only return the product more often, it also transfers more money to consumers who make a return, reducing profit. Thus, a firm would not want to offer a more generous refund than is strictly necessary for consumers to be willing to inspect afterwards. As Figure 1 illustrates, this means that equilibrium contracts must lie along one of the boundary regions equating $U_A = U_L$, $U_A = U_B$, or $U_A = U_N$. Reasoning along the lines of Diamond (1971), as consumers do not yet know their match value before buying, there cannot be an active market with consumers being indifferent with leaving. Likewise, an argument akin to Diamond's rules out an active market with consumers being indifferent between inspecting after purchase and not inspecting at all (see Lemma B.5). Thus, in symmetric refund equilibria consumers must be indifferent between inspecting before and after making a purchase and strictly prefer these to the other options.

The second property shows that firms incentivize consumers to return products by offering a refund that is higher than their salvage value of returned products. This can be understood by examining the firm's optimal choice of refund along the price curve in Figure 1 for which $U_A = U_B$ binds. Moving up the curve involves both an increase in the refund and the price and thus must also involve a reduction in utility since U_B is decreasing in price. By the envelope theorem, the reduction in utility arises as the expected expenditures for consumers increases, holding the choice of when to return fixed. For the firm, this means that increasing the refund along $U_A = U_B$ yields a larger profit on items where the return decision is unchanged, but also intentivizes more consumers to place a return with a net effect of $\eta - \tau$ for these items. At an interior optimum, it must be that these two effects sum to zero, which implies that increasing the rate of returns must decrease profit, i.e., $\tau > \eta$.[17]

The proof of the proposition also establishes that a symmetric refund equilibrium is unique if it exists. We next present comparative statics results.

Proposition 3. *In a symmetric refund equilibrium:*

1. *Increasing the production cost c has no effect on the price and refund, reduces profit, has no effect on consumer surplus, and reduces total welfare.*

[17]Formally, let $p(\tau)$ be the price binding the consumer participation constraint $U_A = U_B$ for a given refund τ. The firm's first order condition for an optimal refund is $\tau - \eta = \frac{1-G(a+p(\tau))}{G(a+p(\tau))}\frac{G(a+\tau)}{g(a+\tau)} > 0$.

2. *Increasing the salvage value η leads to higher profit, less consumer surplus, and larger total welfare.*

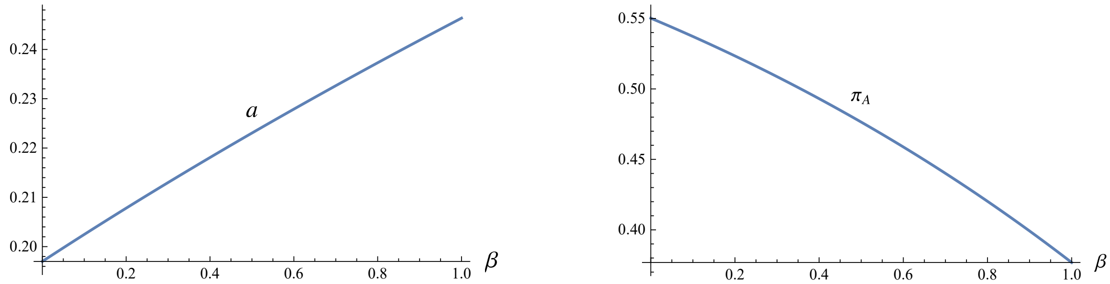
3. *Reducing β leads to higher total welfare, larger difference between price and refund while the refund is smaller, and higher consumer surplus if and only if the price is lower.*

4. *Increasing either the search cost ϵ or inspection cost s leads to a larger difference between the price and refund.*

That a firm fully absorbs increases in the production cost is a consequence of the fact that the production cost enters additively into the firm's profit function, while the consumers' indifference condition is unaffected. This is not the case for the salvage value η, however, as this value is only of relevance for returned products and its impact therefore depends also on the firms' choice of τ. A larger salvage value leads to an increase in both the price and refund, while leaving their difference unchanged. Such a shift is harmful for consumers and beneficial to firms for the simple reason that the price is paid for each item sold, while the refund is only given back to the fraction who do not like the product. Combining the two effects, an increase in the salvage value improves welfare.

It will not come as a surprise that overall welfare improves when post-purchase inspection is relatively less costly. After all, in a refund equilibrium, it is this inspection cost that determines consumer behavior. What is surprising is how this increase in total welfare is distributed between the two sides of the market. In equilibrium, a firm is kept from raising its price or lowering its refund by the threat of a consumer inspecting its product before buying it. When β is smaller, the threat to inspect before purchase is less attractive so that firms can introduce a larger wedge between the price and refund and also reduce the refund itself. If this implies that the price increases, consumers are worse off.

Whether or not consumer harm arises depends on the match value distribution. By displaying how consumer surplus and profits are affected by a change in the relative inspection cost Figure 2 illustrates that consumer surplus decreases with a reduction in β when values are uniformly distributed between zero and one. One



Consumer Surplus Profit

Figure 2: Consumer surplus a and profit π_A as functions of the relative inspection cost β in a refund equilibrium. Values are uniformly distributed with $s = 0.07$ and $\epsilon = 0.001$.

can see that changes in β have quantitatively significant effects and that total surplus is decreasing in β.

Another way to consider the implications of the difference in the cost of inspecting before and after purchase is to explore what the equilibrium contracts look like if the relative ease of inspecting after purchase is large, i.e., if β is close to 0 and s is relatively large, but not too large so that consumers are still active in the market. This case is interesting as it nicely illustrates the importance of the consumers' "outside option" of threatening to continue searching before purchase in shaping equilibrium outcomes. When s is relatively large, this threat is almost non-existent as it is too costly for consumers to inspect before. Thus, firms could abuse this situation by offering almost no refund, while keeping a high price and consumers will still only inspect after purchase.

The next result formalizes this insight by showing that in a candidate refund equilibrium, firms can expropriate almost all ex-ante surplus (that is without knowing consumers' match values) by setting prices close to $E(v)$, while offering almost no refund.

Proposition 4. *Assume that inspection after purchase requires zero cost $\beta = 0$ and that $E(v) - \underline{v} > c - \eta$. Then for ϵ small enough, welfare in a refund equilibrium is decreasing in the pre-purchase inspection cost s in a neighborhood of s^*. Moreover,*

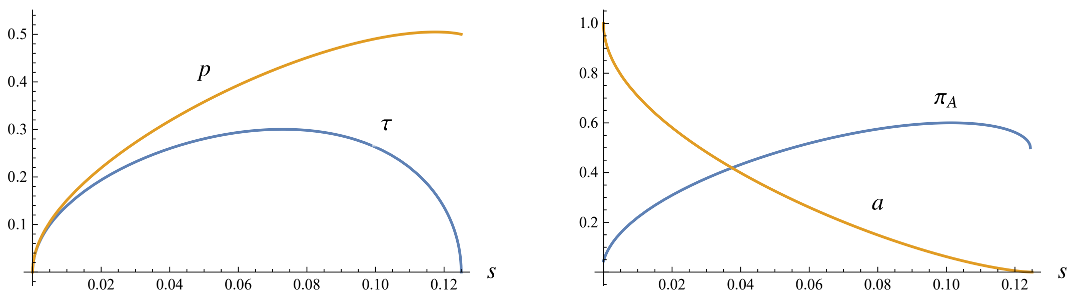


Figure 3: Price p and refund τ, consumer surplus a and profit π_A as functions of s in a refund equilibrium. Values are uniformly distributed and $\beta, c, \eta, \epsilon \to 0$.

as $s \to s^*$ and $\epsilon \to 0$, firms capture the ex ante surplus, i.e., $p \to E(v) - \underline{v} + \eta$.

What is interesting about this result is that even though the inspection cost is never paid in a refund equilibrium, it plays an important role in shaping the equilibrium. The reason welfare is decreasing in s lies in the fact that as it becomes increasingly unattractive for consumers to inspect before purchase, firms lower their refunds, implying consumers generally keep products with lower match values. Whether firms, consumers or both are worse off depends on how firms change their prices as s increases towards s^*, which in turn depends on the shape of the match value distribution. Figure 3 confirms that for the uniform distribution both firms and consumers are worse off when the inspection cost is larger. Even though the equilibrium generates almost full *ex-ante* surplus if s is close to s^* and firms extract this surplus, it is not the case that the first best optimum is achieved. If β is small, it is socially efficient for consumers to inspect products and return them even if their match value is not very low. When s is smaller than s^*, firms are forced to offer a much more generous refund policy and as they make a profit of $p - \tau - (c - \eta)$ on returned items, they benefit overall if more consumers return their products. Consumers also benefit even if prices are first increasing when s decreases starting from s^* as the match values of the products they eventually buy are much higher. Thus, this result shows the importance of taking a market perspective on product returns: both firms and consumers may benefit from products being returned.

Note that the proposition does not state that an equilibrium exists (as existence will be explored more generally in the next section). However, certainly for the uniform distribution a unique refund equilibrium exists in a left neighborhood of s^* and if c is small enough as can be concluded from the expressions below.

Example (Uniformly Distributed Values)**.** To conclude the section, we explicitly solve for the equilibrium expressions of the refund equilibrium when the match value follows the uniform distribution on $[0, 1]$. From Proposition 2 it follows that $\int_{a+\tau}^{a+p} v dv = (1-\beta)s$ and $\tau - \eta = \frac{(1-(a+p))(a+\tau)}{a+p}$. Thus, a refund equilibrium is described by these two equations along with the condition that $a = r(p-\tau+\beta s+\epsilon)-\tau$. Letting ϵ tend to zero, these equations can be easily solved for a, p and τ as[18]

$$a = \frac{2\left(1-\sqrt{8s}\right)}{2-\sqrt{8s}}\sqrt{1-\sqrt{8s}+2\beta s} - \eta$$

$$p = \frac{2-\sqrt{8s}}{2} - \frac{2\left(1-\sqrt{8s}\right)}{2-\sqrt{8s}}\sqrt{1-\sqrt{8s}+2\beta s} + \eta$$

and

$$\tau = \frac{\sqrt{8s}}{2-\sqrt{8s}}\sqrt{1-\sqrt{8s}+2\beta s} + \eta.$$

These equations are also used to compute Figures 2 and 3. It is straightforward to confirm that the refund equilibrium indeed exists for the constellation that is considered in Proposition 3 (see supplementary material S.4).

5 The Structure of Equilibrium Contracts

In this section, we characterize what type of equilibria exist and for which parameter values they exist. To this end, let us define for each value a, $\mathcal{R}(a) \equiv \{(p, \tau) \in \mathbb{R}^2_+ : U_A(p, \tau, a) = U_B(p, a) \geq U_N(p), U_L(a)\}$ to be the set of contracts making consumers indifferent between inspecting before and after and preferring these to the other inspection options. Similarly, define $\mathcal{W}(a) = \{(p, \tau) \in \mathbb{R}^2_+ : U_B(p, a) \geq U_A(p, \tau, a), U_N(p), U_L(a)\}$ as the set of contracts where consumers inspect before purchase and that they prefer this to other inspection options. For each τ, let $p(\tau)$

[18] The expressions for $\epsilon > 0$ are somewhat more involved and are available upon request.

denote the price in the pair $(p(\tau), \tau) \in \mathcal{R}(a)$. The lowest $\underline{\tau}$ and highest $\bar{\tau}$ values the refund takes in the set satisfy $p(\underline{\tau}) = (b(s) - a)_+$ and $p(\bar{\tau}) = (r(s) - a)_+$.[19] Throughout this section we maintain the following assumption.

Assumption. $\pi_A(p(\tau), \tau, a)$ is quasiconcave in τ when $\eta < \tau < \bar{\tau}$ and $0 \leq a$.[20]

Lemma B.1 shows that under this assumption *if* for a given a firms find it optimal to offer a refund inducing consumers to inspect after purchasing the product, there is always a unique way to do so. This goes beyond Proposition 2 as the latter only shows that if a *symmetric* refund equilibrium exists it is unique, but it does not guarantee that individual firms would not find it profitable to offer a different refund contract. We denote the *refund contract* by $(p_R(a), \tau_R(a))$ and the profit it yields by $\Pi_R(a)$. Define a *candidate refund equilibrium* to be the unique triple $(p_R(a_R), \tau_R(a_R), a_R)$ for which consumer surplus satisfies $a_R = U_A(p_R(a_R), \tau_R(a_R), a_R) - \epsilon$.

Within the set of contracts inducing inspection before purchase $\mathcal{W}(a)$, the logconcavity of $1 - G$ guarantees that firms have a unique optimal price. As indicated in Section 2, consumers will inspect before purchase if a firm selects a contract within $\mathcal{W}(a)$ and therefore will not use the refund option in this case. As the refund policy is redundant, we assume that within this set the firm chooses the refund to be equal to zero. Refer to the contract charging this optimal price as the *Wolinsky price*, and we denote it by $p_W(a)$. Similarly, denote the profit from charging the Wolinsky price by $\Pi_W(a)$. By standard results, there is a unique consumer surplus solving $a_W = U_B(p_W(a_W), a_W) - \epsilon$. Refer to the pair $(p_W(a_W), a_W)$ as the *candidate Wolinsky equilibrium*.

We focus on the subset of parameter values $\Omega^* \subset \Omega$ where markets are active and consumers get positive utility from inspecting before purchase when all firms charge the Wolinsky price and from inspecting after purchase when all firms play their part in a candidate symmetric refund equilibrium. Lemmas B.3 and B.4 in the appendix formally show active markets with positive consumer utility exist if s, ϵ,

[19]The set $\mathcal{R}(a)$ is nonempty since $r(s) > b(s)$ for all $s < s^*$. Consequently, the set $\mathcal{W}(a)$ is also nonempty as it includes all contracts with a refund of zero and a price between $(b(s) - a)_+$ and $(r(s) - a)_+$.

[20]Because $p(\tau)$ is only implicitly defined, it is very difficult to analytically show that $\pi_A(p(\tau), \tau, a)$ is quasiconcave in τ for commonly employed search cost distributions. Numerical analysis for the uniform distribution shows that it is.

c and η are small enough. Lemma B.2 shows that for these parameters, if all firms charge the Wolinsky price and consumers inspect before purchase, no firm has an incentive to deviate to cut the price to such an extent that it induces consumers to buy without inspection.

To keep the following discussion clear, we proceed by varying the production cost c and post-purchase inspection parameter β while holding the other parameters constant. These two parameters essentially measure the trade-off between inspecting before and after purchase. Relative to η, c is a measure of the inefficiency dimension related to inspection after purchase due to the possibility of product returns, while β is a measure of the efficiency dimension due to the lower inspection costs. We proceed with a series of claims leading up to the main characterization result, while a more formal treatment is given in the appendix.

We start off by considering the special case where $\beta = 1$ and $c = \eta$. In this case, there is no inspection efficiency of product returns, but also no cost inefficiencies. We show that a refund equilibrium and a Wolinsky equilibrium co-exist and that they are equivalent in the sense that they yield the same profit and deliver the same consumer surplus. When $\beta = 1$, for a contract to induce returns (i.e. $(p, \tau) \in \mathcal{R}(a)$), it must offer a full refund $\tau = p$. When also $c = \eta$, then granting a full refund yields profit

$$\begin{aligned}(1-q)\pi_A(p,\tau,a) &= p - c - (\tau - \eta)G(a+\tau) = p - c - (p-c)G(a+p) \\ &= (p-c)(1 - G(a+p)) = (1-q)\pi_B(p,a).\end{aligned}$$

Thus, $p_R(a) = p_W(a)$ and $\Pi_R(a) = \Pi_W(a)$ for all a and also $a_R = a_W$. Writing ρ_R for the fraction of firms offering a return policy and $\rho_W = 1 - \rho_R$, it is clear that any $\rho_R \in [0, 1]$ constitutes an equilibrium yielding surplus $a^* = a_R = a_W$.

Next, consider the case where $\beta = 1$ and the production cost c is strictly greater than the salvage value η but also less than the highest production cost admitting an active market in a Wolinsky equilibrium c^*. As above, any contract in $\mathcal{R}(a)$ must offer a full refund as $\beta = 1$. For any price and a full refund, firms would strictly prefer, however, that consumers inspect before purchase as

$$
\begin{aligned}
(1-q)\pi_A(p,\tau,a) = p - c - (\tau-\eta)G(a+\tau) &= p - c - (p-\eta)G(a+p) \\
&< (p-c)(1-G(a+p)) = (1-q)\pi_B(p,a)
\end{aligned}
$$

so that firms always strictly prefer the Wolinsky price to the refund contract $\Pi_W(a) > \Pi_R(a)$. Hence, only a Wolinsky equilibrium exists and $a^* = a_W$.

Third, for the case where $\beta < 1$ and $c = \eta$, consumers are willing to take less than a full refund to inspect after purchase: $(p,\tau) \in \mathcal{R}(a)$ implies $\tau < p$. Thus, when $c = \eta$

$$
\max_{(p',\tau')\in\mathcal{R}(a)} \pi_A(p',\tau',a) > \max_{p'\in[(b(s)-a)_+, r(s)-a]} \pi_A(p',p',a) = \max_{p'\in[(b(s)-a)_+, r(s)-a]} \pi_B(p',a)
$$

so that firms yield strictly higher profit from the refund contract than the Wolinsky price. Thus, in this case there only exists a refund equilibrium and $a^* = a_R$.

An important part of the argument leading up to the above two claims is that for the equilibrium analysis we only need to focus on firms offering a Wolinsky price or a refund contract. That is, not only do equilibria not exist where some firms offer contracts that differ from one of these two contract types, deviations to other contract types are also never optimal. Thus, only three types of equilibria may exist: (i) all firms offer the Wolinsky price or (ii) all firms offer a refund contract where consumers inspect after purchase, or (iii) an asymmetric equilibrium where some firms offer the Wolinsky price and consumers inspect before purchase, while other firms offer a refund inducing consumers to inspect after.

The formal analysis of this argument involves several steps and is presented in Lemmas B.5-B.9. It is clear that for any given $\beta < 1$ if one increases c, starting from $c = \eta$, then at some point a refund equilibrium stops existing as it is simply too costly for a firm to induce consumers to buy the product without inspection before purchase. The main question is what happens around this transition? Lemma B.7 implies that if at the boundary of the region where a refund equilibrium exists firms are indifferent between offering a refund contract or a Wolinsky price, consumers prefer the refund contract to a Wolinsky price. So, if at the boundary all firms would offer a Wolinsky price, an individual firm would be better off offering a somewhat more profitable refund contract as consumers would be happy to accept such a contract. The lemma uses the logconcavity of G and $1 - G$ and the properties, listed in Proposition 2, a candidate refund equilibrium has to satisfy. Thus, if c is so large that a refund equilibrium no longer exists, it must be that the market transits to an equilibrium where some firms offer returns and others offer a Wolinsky price. This is the content of Claim B.2.

Thus, the main result of this section can be stated as follows.

Proposition 5. *Equilibria can be characterized by two continuous functions* $\underline{c}(\beta)$ *and* $\bar{c}(\beta)$ *and a constant* $c^* > \eta$ *whereby* $\eta < \underline{c} < \bar{c}$ *for all* $0 \leq \beta < 1$. *For each point with* $\beta < 1$ *and* $c \leq \max\{\bar{c}, c^*\}$ *there is a unique equilibrium with trade:*

1. *For* $\eta \leq c \leq \underline{c}$, *all firms offer a refund contract.*

2. *For* $\underline{c} < c < \bar{c}$, *a fraction of firms offer a refund contract.*

3. *For* $\bar{c} \leq c \leq c^*$, *no firm offers a refund contract.*

The proposition is depicted in Figure 4. At $\beta = 1$ and $c = \eta$ one clearly sees that all three types of equilibria come together as the optimal refund contract has full refunds and mimics market behavior in a Wolinsky equilibrium. Moving horizontally or vertically only a refund equilibrium, respectively a Wolinsky equilibrium exists. Finally, for any $\beta < 1$ an asymmetric equilibrium with some, but not all, firms offering a refund contract mitigates between the parameter regions where a refund equilibrium or a Wolinsky equilibrium exist.

The equilibrium characterization shows when the market generates product returns in the absence of regulation and fraud. Despite that regulation and fraud exist in real world brick-and-mortar and online markets, we believe that our equilibrium characterization reveals some important underlying forces that explain why product returns are more prevalent online and in some industries than in others. In particular, while product match is important for apparel and footwear where large efficiencies can be gained by inspecting at home, and returned products can be resold, our model predicts that these markets feature product returns. On the other hand, for categories

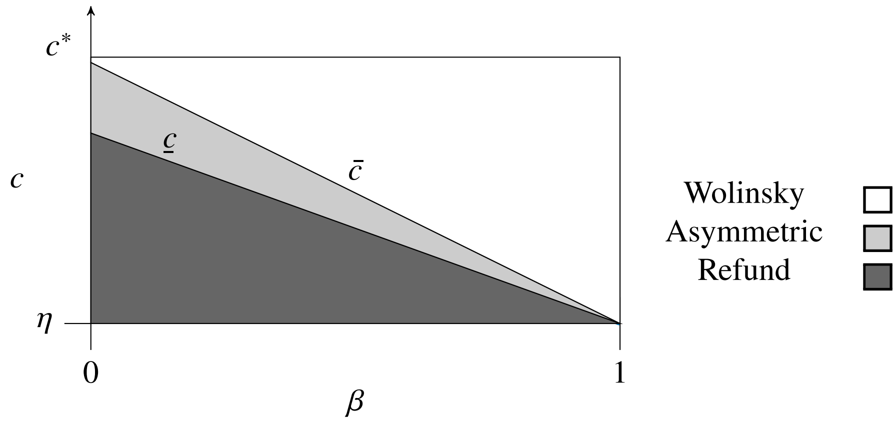


Figure 4: Equilibrium characterization.

such as beauty and health care, our model predicts that offering return policies is unattractive as the salvage value is too small relative to the production cost.

Similarly, as the next proposition summarizes and Figure 5 depicts, our model predicts that in markets where it is more difficult to inspect products online than in brick-and-mortar stores one should see more returns. This may well explain that with the rise of online markets, we also see a sharp increase in the number of returned products: in brick-and-mortar shops the inspection cost is lower and firms did not offer contracts that made it attractive to inspect only after purchase. With the higher inspection cost online, this is clearly different. Interestingly, when the inspection cost is such that consumers inspect after purchase, marginally decreasing this inspection cost further boosts returns. Recent developments in online markets where firms invest in technology so that consumers can virtually try their products (such as eye glasses) may further boost the number of product returns.

Proposition 6. *Assume $\underline{v} = \eta = \beta = 0 < c$. For any small enough ϵ there exists an inspection cost $\hat{s} \in (0, s^* - \epsilon)$ such that:*

1. *If $0 < s \leq \hat{s}$, the number of returns is zero .*
2. *If $\hat{s} \leq s \leq s^* - \epsilon$, the number of returns is initially increasing and eventually decreasing in the inspection cost.*

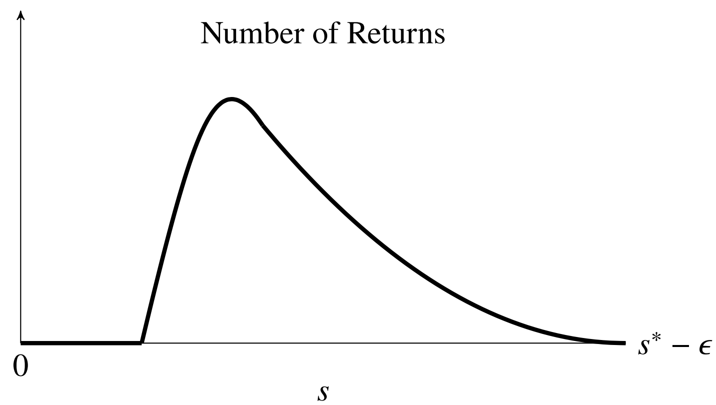


Figure 5: Equilibrium number of returns as a function of the inspection cost.

The proof of this result follows from combining arguments for Propositions 4 and 5 and is thus relegated to S.3 in the Online Appendix.

6 Market Efficiency

This section examines the efficiency of the return policies emerging from market competition and whether practical regulation can yield improvements. We begin by establishing our first important result for this section.

Proposition 7. *Refund equilibria are generically inefficient, generating either too many or too few returns.*

To understand how returns generate inefficiency, it is useful to consider the consumer's decision of whether or not to place a return. A consumer who has purchased an item and inspects afterwards will decide to place a return if their match value lies below the sum of the refund plus continuation value, $a + \tau - v > 0$, and keep the good otherwise. Returning the good offers a salvage value to the firm who sold the good, but also requires it to pay a refund, providing the firm a net change in profit of $\eta - \tau$. In a monopoly, where a could be zero, these two effects are the only two welfare considerations and thus efficiency is achieved if the consumer opts to place a return ($a + \tau - v > 0$) if and only if the effect on welfare is positive

($\eta - \tau + a + \tau - v > 0$). Thus in a monopoly, returns can only be efficient if the refund equals the salvage value $\tau = \eta$. As in a monopoly, just as in a competitive market (Proposition 2), the refund offered is larger than the salvage value, there are too many returns from an efficiency vantage point.

In a competitive market, beyond the parties involved in the transaction, a return also impacts the firms not involved in the transaction as the consumer will continue searching firms not previously visited. Specifically, when placing a return, the expected profit accrued by the remaining firms is the profit to the firm whose good the consumer decides to keep, $p - c$, plus the expected profit to those firms whose goods the consumer buys and returns, $\frac{p-c+\tau-\eta}{1-G(a+\tau)} \cdot G(a + \tau)$. Therefore, the total change in welfare resulting from placing a return is

$$\underbrace{\eta - \tau}_{\text{Transacting Firm}} \quad + \overbrace{p - c + \frac{p - c + \tau - \eta}{1 - G(a + \tau)} \cdot G(a + \tau)}^{\text{Nontransacting Firms}} + \underbrace{a + \tau - v}_{\text{Consumer}}. \tag{10}$$

Thus, returns yield two competing effects: (*i*) a loss for the firm involved in the transaction and (*ii*) a positive expected profit for the remainder of the industry. Simplifying this expression, a consumer places returns efficiently if and only if the equilibrium contracts satisfy $p - c + \tau - \eta = 0$, which generically fails to hold.

To identify when the market generates too many or too few returns, it is useful to render the efficient benchmark of Proposition 1 in graphical terms and relate it to the societal costs and benefits of returns as well as to the equilibrium characterization given in Section 5. To do this, let $f(\beta)$ be the strictly decreasing function corresponding to the cost c solving $\psi(f(\beta), \beta, \cdot) = 0$. Proposition 1 has the following corollary.

Corollary 1. *It is socially optimal to have all firms stimulate returns if (β, c) lies below the graph of f, while if (β, c) lies above the graph of f, no firm should stimulate returns.*

Figure 6 compares the market outcome with what is socially efficient for the uniform distribution. Recall from Proposition 5 that the equilibrium of the market is

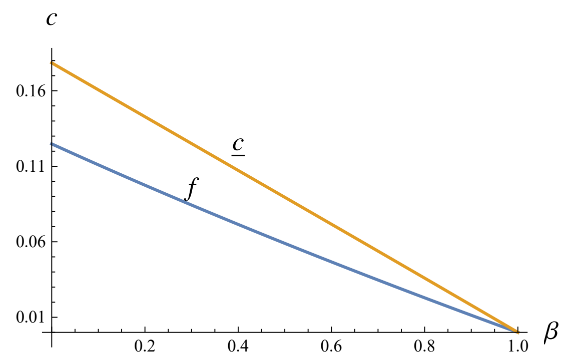


Figure 6: The curves $f(\beta)$ and $\underline{c}(\beta)$ when match values are uniformly distributed, with $\eta = 0$, $s = 0.07$, and $\epsilon = 0.001$.

a refund equilibrium if and only if the cost lies below $\underline{c}$. Combining the proposition with the above corollary, one immediately sees from 6 that the market gets it "roughly right": there is a large overlap between the regions where it is optimal (not) to have returns and where the markets offers (no) returns. In particular, when the market is active and production costs exceed $\max\{f(\beta), \bar{c}(\beta)\}$, then the market equilibrium achieves efficiency by not stimulating returns. Also, if production costs are smaller than $\min\{f(\beta), \bar{c}(\beta)\}$, then the market offers returns and it is efficient to have them.

Below this surface, there are however important inefficiencies that arise. First, Figure 6 depicts a set of production costs $f(\beta) < c < \underline{c}(\beta)$ for which a social planner would opt for consumers to inspect before purchase, but the market stimulates returns, with firms collectively incurring a loss on each item that is bought and then returned. Second, asymmetric equilibria are evidently inefficient as it is efficient to have either all or no firms offering refunds for all production costs, aside from the nongeneric case where $c = f(\beta)$. Third, the market equilibrium with returns is generically inefficient as even when it is efficient to have returns, the market provides too few returns. To see why, consider $\beta < 1$. If the market is in a refund equilibrium, $c \leq \underline{c}$, and the production cost is precisely equal to $c = f$, then the market achieves efficiency with $p - \tau = c - \eta$. As it follows from Proposition 3 that the refund contract is not affected by the production cost, if we lower cost, then it must be that $p - \tau > c - \eta$ and thus the market stimulates too few returns.

More generally, using Proposition 3 we can show when we can expect the market to provide too many or too few returns. Proposition 3 argues that the wedge between price and refund is larger, the smaller is β, or the larger is s. This immediately implies that smaller β or larger s make it more likely that the market provides too few returns. Thus, we have the following.

Proposition 8. *If $c < min\{f(\beta), \underline{c}\}$ increasing β or lowering s yields more returns, thereby producing more efficient equilibrium contracts. If, on the other hand, $f(\beta) < c < \underline{c}$, then increasing β or lowering s continues to yield more returns, but results in less efficient equilibrium contracts.*

Especially the result on s highlights the fact that the market considerations are different from the monopoly case. Typically, the higher s, the more market power firms have and the closer one should get to monopoly. Under monopoly, if it provides returns, the firm always provides too high a refund. Here, higher s makes it more likely the market provides too few returns as consumers are less likely to continue to search even if it is socially desirable to do so.

Proposition 8 holds true for the case where there are no externalities. Once we incorporate externalities into the analysis, we see that the market outcome with returns might be closer to being efficient when the market has too few returns from a social welfare perspective as it is efficient to have $\hat{p} - \hat{\tau} = c - \eta^e$ and $\eta^e < \eta$. However, if the externalities are sufficiently small, it remains the case that the market provides too few returns.

6.1 Regulating Refunds

We conclude the analysis of efficiency by addressing the practical regulatory question of whether market outcomes can be improved by requiring firms to offer a more generous return policy, but letting them set their own prices. Regulators may mandate firms to offer consumers the possibility to return their purchased item and get a refund, but they (typically) do not have a mandate to intervene with firms' pricing decisions. Our first result suggests regulators should proceed with caution in mandating product returns. Requiring firms to offer a refund above but close to the market level can be welfare neutral while resulting in a reduction in consumer surplus. Our second and more optimistic result shows that, when the societal benefits from returns strongly outweigh their costs, regulators can achieve the first best with a sufficiently bold policy.

Reconsider the social planner's problem, assuming now that he chooses the minimum (threshold) fraction of the price that firms must offer back as a refund. Specifically, when the planner selects the refund threshold θ, firms are only permitted to offer contracts in the set $X(\theta) = \{(p, \tau) \in \mathbb{R}^2_+ : \tau \geq \theta \cdot p\}$. Consider the interesting case where the unique equilibrium in the market is a refund equilibrium and the social planner can improve welfare by stimulating more returns.[21]

For a given θ, we analyze the *constrained equilibrium* in which firms and consumers both play best replies subject to the constraint that contracts belong to $X(\theta)$. Given a and $\theta < 1$, the region of permitted contracts can be represented in Figure 1 by the region to the right of the function $p = \frac{1}{\theta}\tau$ beginning at the origin and cutting through the curve $\mathcal{R}(a)$ at most once. There are two plausible symmetric constrained equilibria: one in which contracts continue to lie on the boundary $\mathcal{R}(a)$ and the other where contracts lie within the interior of the region in which consumers prefer to inspect after purchase.

If contracts lie on the boundary in a constrained equilibrium, then they must satisfy $(1 - \theta)p = p_R - \tau_R$ as there is a unique difference δ between the price and refund that makes consumers indifferent between inspecting before and after purchase (Proposition 2). Thus, given the threshold θ, the price in a boundary equilibrium can be expressed as the function $p(\theta) = \frac{\delta}{1-\theta}$. From the indifference condition, consumer surplus can likewise be expressed as $a(\theta) = r(s + \epsilon) - p(\theta)$, which is decreasing in θ. Therefore, if contracts remain on the boundary in a constrained equilibrium, lifting the mandatory refund rate is harmful for consumers. As argued in the next proposition, whenever θ is above but close enough to the equilibrium ratio τ_R/p_R, such an equilibrium exists.

Proposition 9. *Consider a refund equilibrium and a regulation imposing a refund threshold that is marginally higher than the equilibrium refund. Then there is*

[21] For a given $\omega \in \Omega^*$, the parameter lies in $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon), \beta < 1, c < \underline{c}(\beta), 0 < \psi(\omega')\}$.

a constrained equilibrium generating higher profit and lower consumer surplus, while keeping social welfare unchanged.

Thus, for a refund threshold to offer any improvement on market outcomes, consumers must be made to *strictly* prefer inspection after purchase. We find that as long as the difference $c - \eta$ is not too large, the planner can set a refund threshold so that the constrained equilibrium price achieves the social optimum; $(1-\theta)p = c-\eta$. Note that if $c - \eta > 0$ it should be that $\theta < 1$ as full refunds are inefficient.

Proposition 10. *If the difference between the production cost and salvage value is not too large, then there is a refund threshold $\hat{\theta}$ such that there is a constrained equilibrium that achieves maximal social welfare.*

Thus, as long as society's benefit from returns sufficiently outweigh the costs, a planner can achieve efficiency by requiring firms to offer a generous refund that is typically smaller than a full refund.

7 Conclusion

In this paper, we have addressed the trade-off that arises when firms offer consumers the possibility to receive a refund when they return a product after they have purchased it. The return option allows consumers to more easily evaluate whether their purchase satisfies their preferences. However, product returns also come at a cost as the salvage value is typically lower than the production cost. To study this trade-off, we have made a methodological contribution to the consumer search literature by augmenting the seminal search model by Wolinsky (1986) in several dimensions. We have characterized the equilibrium outcomes and have shown that the equilibrium is always unique. We have shown that whenever returns are efficient, the market generates too few returns as firms do not internalize the welfare benefit of consumers returning low match value products and continuing their search. In these cases a regulator can improve upon the market outcome.

We think our paper opens several directions for future research. First, throughout the paper we have maintained the assumption that consumers do not learn any product features unless they incur a more substantial inspection cost. Alternatively, one could make a distinction between objective product features that, certainly in online markets, are (like price) quite readily available to consumers, and other more subjective features that require a consumer to inspect more thoroughly. Consumers may then learn part of their match value without incurring the inspection cost and this will affect their decision whether to continue to inspect the product beforehand or to buy and inspect afterwards. Second, multi-product firms may stimulate consumers to engage in simultaneous search by offering to return all items they do not like. Even if consumers only want to buy one unit, this option allows them to keep the product they like best. Firms may prefer such a strategy as their price can take into account that consumers buy the best product, while the additional cost of returning multiple items is small.

These alternative ways of modeling the consumer search process and the way multi-product firms affect search may change the specific trade-offs that are studied in this paper, but the underlying theme will remain the same: there are costs and benefits of offering refund policies and for a proper understanding of especially online markets, it is important to know whether the market provides proper incentives for an efficient resolution of this trade-off.

A Appendix: Equilibria with Returns

Proof of Proposition 2. As the text argues, in a symmetric refund equilibrium, consumers are indifferent between inspecting goods before and after purchasing them and strictly prefer these to the other inspection options, i.e. $U_A = U_B > U_L, U_N$. Claims A.1 and A.2 below prove that a unique price, refund, and consumer surplus satisfies the first order conditions to be an optimal contract in this region, and thus a symmetric equilibrium with returns is unique. Examining the firm's first order conditions, Claim A.3 verifies $\underline{v} < a + \tau$ and $a + p < \bar{v}$, thus an equilibrium requires the refund to exceed the salvage value $\tau > \eta$.

Claim A.1. *There exists a unique triple $(p^*, \tau^*, a^*) \in \mathbb{R}^3$ satisfying the first order* *conditions for an interior maximum to*

$$\pi_A(p^*, \tau^*, a^*) = \max_{\{(p,\tau):U_A(p,\tau,a^*)=U_B(p,a^*)\}} \pi_A(p, \tau, a)$$

with consumer's expected utility being $a^* = U_A(p^*, \tau^*, a^*) - \epsilon$*. Moreover, the solution* (p^*, τ^*, a^*) *is continuously differentiable in all parameters.*

As a refund equilibrium requires consumers to be indifferent between inspecting before and after purchase, both (5) and (8) hold. Taken together, these conditions implicitly define the equilibrium difference between the price and refund by $\delta^* = p^* - \tau^*$ by

$$r(\delta^* + \beta s + \epsilon) + \delta^* = r(s + \epsilon). \tag{11}$$

We now show that there exists a unique δ^* solving (11). Differentiating, the left side is found to be strictly decreasing in δ, is equal to $r(\beta s + \epsilon) \geq r(s + \epsilon)$ when $\delta = 0$, and by Lemma S.2 converges to $E(v) - \beta s - \epsilon < r(s + \epsilon)$ for any increasing sequence $(\delta_n)_{n\in\mathbb{N}}$ converging to $S - \beta s - \epsilon$. Thus a unique δ^* solves (11).

As before, let $p(\tau)$ denote the price keeping consumers indifferent between inspecting before and after purchase for a given refund, i.e. $U_A(p(\tau), \tau, a) = U_B(p(\tau), a)$. From the firm's first order conditions obtained by differentiating $\pi_A(p(\tau), \tau, a)$ in τ, a unique refund τ^* solves

$$\tau^* - \eta = \frac{1 - G(r(s + \epsilon))}{G(r(s + \epsilon))} \frac{G(r(\delta^* + \beta s + \epsilon))}{g(r(\delta^* + \beta s + \epsilon))}.$$

Given (δ^*, τ^*), there is a unique price satisfying $p^* = \delta^* + \tau^*$ and unique $a^* = r(\delta^* + \beta s + \epsilon) - \tau^*$. Continuous differentiability follows from the Implicit Function Theorem.

Claim A.2. *Given the solution* (p^*, τ^*, a^*)*, we have* $U_A > U_L$ *and* $U_A > U_N$*.*

The first inequality holds trivially whenever $\epsilon > 0$. By writing the utility functions explicitly, the second inequality is seen to hold if and only if $b(s) < a^* + p = r(s + \epsilon)$, which must hold since $s + \epsilon < s^*$ implies $b(s) < E(v) < r(s + \epsilon)$.

Claim A.3. *The solution* (p^*, τ^*, a^*) *admits a positive probability of returns* $G(a^* + \tau^*) > 0$.

Having shown in the first claim that $\delta^* + \beta s + \epsilon < S$ in equilibrium, we have $a^* + \tau^* = r(\delta + \beta s + \epsilon) > \underline{v}$, and thus $G(a^* + \tau^*) > 0$. $\square$

Proof of Proposition 3. For the first part, as the solution (p^*, τ^*, a^*) in Proposition 2 is not determined by the production cost and thus firms absorb the entire increase in the cost. For the second part, notice that the value for δ^* solving (11) does not depend on the salvage value, but for a fixed δ^*, the solution for τ^* increases ($d\tau^*/d\eta = 1$); hence, the solution for p^* likewise increases. By (8), consumer surplus must decline. From the expression for profit, $\frac{p^*-c-(\tau^*-\eta)G(a^*+\tau^*)}{1-G(a^*+\tau^*)}$, as $a^* + \tau^*$ and $\tau^* - \eta$ are constant and the price is increasing in η, profit itself is increasing.

Turning to β, that the change in consumer surplus takes the opposite sign of the change in price follows immediately from Condition (8), noting that the rightmost term is independent of β and thus $a^* + p^*$ is constant in β. Implicitly differentiating (11) in β obtains $\frac{d\delta^*}{d\beta} = -\frac{s}{G(a^*+\tau^*)} < 0$. Note from (5), that $a^* + \tau^* = r(\delta^* + \beta s + \epsilon)$ is increasing in β with derivative $\frac{d(a^*+\tau^*)}{d\beta} = \frac{s}{G(a+\tau)}$. Thus, from the firm's first order conditions and the logconcavity of G

$$\frac{d\tau}{d\beta} = \frac{1 - G(a^* + p^*)}{G(a^* + p^*)} \frac{\partial}{\partial \tau} \cdot \left(\frac{G(a^* + \tau^*)}{g(a^* + \tau^*)} \right) \cdot \frac{s}{G(a^* + \tau^*)} > 0.$$

Finally, as welfare is equal to $a^* + \frac{p^*-c-(\tau-\eta)G(a^*+\tau^*)}{1-G(a^*+\tau^*)}$, differentiating in β obtains $-\frac{s}{(1-G(a^*+\tau^*))G(a^*+p^*)} < 0$.

Implicitly differentiating (11) obtains the effect of ϵ and s on δ^*. $\square$

Proof of Proposition 4. An increase in the inspection cost leads to a larger difference between the price and refund $\frac{d\delta^*}{ds} > 0$ (see Lemma S.4). The change in the welfare with respect to s is

$$\frac{d\mathbf{S}_R}{ds} = -(\delta^* - c + \eta) \frac{g(a^* + \tau^*)}{(1 - G(a^* + \tau^*))^3} \frac{d\delta}{ds} \tag{12}$$

which is negative if and only if the profit earned from returned items is positive. Given that $\lim_{(s,\epsilon)\to(s^*,0)} r(s + \epsilon) = E(v)$, (11) and Lemma S.2 provide that $\delta^* \to S$, thereby completing the proof. Finally, to verify that firms extract the ex ante surplus, note that (5) gives $a^* + \tau^* \to \underline{v}$, the firms' optimal refund condition yields $\tau^* \to \eta$, and thus (8) provides that $p^* \to E(v) - \underline{v} + \eta$. □

B Appendix: Structure of Equilibria

B.1 Technical Lemmas for Proposition 5

Lemma B.1. *There is a unique best reply in the set of points inducing inspection after purchase and it lies in $\mathcal{R}(a)$.*

Proof. Firstly, the fact that $\mathcal{R}(a)$ can admit at most one best reply follows from the objective π_A being strictly increasing in the refund when $\tau \leq \eta$ and quasiconcave thereafter.

A best reply must lie on either the boundary $\mathcal{R}(a)$ or the set making consumers indifferent with inspecting after purchase and leaving $[U_A = U_L] = \{(p, \tau) \in \mathbb{R}^2_+ : U_A(p, \tau, a) = U_L(a)\}$, otherwise a firm can increase its profit by raising its price by a small amount. In the region $[U_A = U_L]$, there are only two potential critical values for firms: one interior with $\tau = \eta$ and the other at the boundary point equating $U_A = U_B = U_L$. The second critical value belongs to $\mathcal{R}(a)$. As depicted in Figure 1, since the first critical value offers a lower refund than would be optimal in $\mathcal{R}(a)$, it must enter the region where consumers prefer to inspect before purchase. For completeness, we analytically verify what the figure depicts to complete the proof.

The regions $[U_A = U_B \geq U_L] = \{(p, \tau) : U_A(p, \tau, a) = U_B(p, a) \geq U_L(a)\}$ and $[U_A = U_L \geq U_B] = \{(p, \tau) : U_A(p, \tau, a) = U_L(a) \geq U_B(p, a)\}$ contain ordered pairs of contracts with the property that if (p, τ) and (p', τ') either both lie in the first or both lie in the second set and $\tau < \tau'$, then $p < p'$. Moreover, in the region $[U_A = U_B \geq U_L]$, pairs with higher prices deliver less utility as U_B is decreasing. As a result, refunds in $[U_A = U_L \geq U_B]$ must exceed those in $[U_A = U_B \geq U_L]$. It follows that the critical value $\tau = \eta$ lies in the region $[U_A = U_L < U_B]$ and so consumers will opt to inspect the product before purchase if a firm makes this deviation. □

*B.*1.1 *Active Market*

To ensure equilibria can support an active market, we must limit the magnitude of the search and production costs. The following three lemmas give the bounds that are needed. First, we characterize the points for which a firm would prefer to play its part in a Wolinsky equilibrium (or equivalently a refund equilibrium when $c = \eta$ and $\beta = 1$) rather than cut its price to incentivize consumers to buy without inspection. In terms of notation, denote the elements in two generic points by $\omega = (c, \beta, \eta, s, \epsilon)$ and $\omega' = (c', \beta', \eta', s', \epsilon')$.

Lemma B.2. *Let $\Omega_N \subset \Omega$ be the points for which the profit when all firms charge the Wolinsky price and consumers inspect before purchase exceeds the profit from cutting the price to induce no inspection. Then, (a) Ω_N is nonempty, and (b) if $\omega \in \Omega_N$ and $\omega' \in \Omega$ is another point with $s' \leq s$ and $\epsilon' \leq \epsilon$, then $\omega' \in \Omega_N$.*

Proof. Offering the Wolinsky price yields profit $\pi_B = p - c$ while cutting to the optimal price inducing no inspection yields profit $\pi_N = \frac{b(s)-a-c}{1-G(a+p)}$. Making use of the Envelope Theorem, we differentiate $(1 - G(a + p))(\pi_B - \pi_N)$ in s to obtain

$$\begin{aligned}\frac{d}{ds}(1 - G(a + p))\left(\pi_B - \pi_N\right) &= -(p - c)g(a + p)\frac{da}{ds} - \left(\frac{db}{ds} - \frac{da}{ds}\right)\\ &= G(a + p)\frac{da}{ds} - \frac{db}{ds}\end{aligned}$$

which is necessarily negative because $\frac{da}{ds} < 0 < \frac{db}{ds}$. Similarly, differentiating in ϵ

$$\frac{d}{d\epsilon}(1 - G(a + p))(\pi_B - \pi_N) = G(a + p)\frac{da}{d\epsilon}$$

is negative because $\frac{da}{d\epsilon} < 0$. Substituting $a + p = r(s + \epsilon)$ and $p = \frac{1-G(a+p)}{g(a+p)} + c$ we see that both expressions for profit are constant in all parameters except for s and ϵ. Finally, to prove that Ω_N is nonempty, from

$$\begin{aligned}(1 - G(a + p))(\pi_B - \pi_N) &= \frac{(1 - G(r(s + \epsilon)))^2}{g(r(s + \epsilon))} - b(s) + r(s + \epsilon) - \frac{1 - G(r(s + \epsilon))}{g(r(s + \epsilon))}\\ &= -b(s) + r(s + \epsilon) - \frac{(1 - G(r(s + \epsilon)))\, G(r(s + \epsilon))}{g(r(s + \epsilon))}\end{aligned}$$

the difference goes to $\bar{v} - \underline{v} > 0$ as $\epsilon + s \to 0$ and if the difference is positive at $(\epsilon, s) = (0, s_0)$ then it must also be positive at $(\epsilon, s) = (s_0, 0)$. $\square$

Next, consumers must receive nonnegative utility to be willing to participate in a Wolinsky equilibrium.

Lemma B.3. *Let $\Omega_W \subset \Omega$ be the points for which consumers yield positive utility from inspecting before purchase when all firms charge the Wolinsky price. Then, (a) Ω_W is nonempty, and (b) if $\omega \in \Omega_W$ and $\omega' \in \Omega$ is another point with $c' \leq c$ and $s' + \epsilon' \leq s + \epsilon$, then $\omega' \in \Omega_W$.*

Proof. Consumer surplus is $r(s+\epsilon) - \frac{1-G(r(s+\epsilon))}{g(r(s+\epsilon))} - c$ when all firms charge the Wolinsky price. Surplus is positive when $(c, s, \epsilon) \to (0, 0, 0)$ and by the logconcavity of $1 - G$, it is strictly decreasing in these three arguments. $\square$

While the parameters (β, η) continue to have no effect when consumers inspect before purchase, consumers fully absorb the production cost c. Therefore, for consumer surplus to be positive in a candidate Wolinsky equilibrium, the search costs along with the production cost cannot be too large.

Finally, consumers must receive nonnegative utility to be willing to participate in a refund equilibrium. Recall that a candidate refund equilibrium corresponds to the unique triple (p, τ, a) in which all firms choose the same contract in $\mathcal{R}(a)$ satisfying the first order conditions for a maximum and a is the utility from inspecting products after purchase when these are the prices $a = U_A(p, \tau, a) - \epsilon$.

Lemma B.4. *Let $\Omega_R \subset \Omega$ be the points for which consumers yield positive utility from inspecting after purchase when all firms play their part in a candidate refund equilibrium. Then, (a) Ω_R is nonempty, and (b) there is a nonempty subset $\Omega'_R \subset \Omega_R$ such that, if $\omega \in \Omega'_R$ and $\omega' \in \Omega$ is another point with $\eta' \leq \eta$, $s' \leq s$ and $\epsilon' \leq \epsilon$, then $\omega' \in \Omega'_R$.*

Proof. With $\hat{r} = r(\delta + \beta s + \epsilon)$, a symmetric refund equilibrium offers consumer surplus

$$a = \hat{r} - \tau = \hat{r} - \frac{1 - G(\hat{r} + \delta)}{G(\hat{r} + \delta)} \frac{G(\hat{r})}{g(\hat{r})} - \eta \geq \hat{r} - \frac{1 - G(\hat{r})}{g(\hat{r})} - \eta. \tag{13}$$

Define Ω'_R to be the set of points for which, if we replace the value for β with 0, the right side of (13) is positive. That the set Ω'_R is nonempty follows from noting that $\hat{r} \to \bar{v}$ as $s + \epsilon \to 0$. Observe that the right side of (13) is increasing in β and decreasing in η, ϵ, and s as a result of the logconcavity of $1 - G$ and Proposition 3 (the explicit derivatives of δ and $\hat{r}$ with respect to s and ϵ are contained in S.4 of the supplementary material). From this, if the right side of (13) is positive at any point with $\beta = 0$, then it is also positive at any other point with a (weakly) smaller salvage value, search cost, and inspection cost. $\square$

Our equilibrium characterization will begin at a point in the subset satisfying the conditions of the three preceding lemmas $\Omega^* = \Omega_N \cap \Omega_W \cap \Omega_R$. That Ω^* is nonempty follows simply from taking any $\omega \in \Omega_W$, possibly lowering (η, s, ϵ) to obtain a point in $\Omega'_R \cap \Omega_W$, and then possibly lowering (s, ϵ) again to obtain a point in $\Omega_N \cap \Omega'_R \cap \Omega_W$.

B.1.2 *Equilibria*

For the remainder of this section, we fix a point $\omega \in \Omega^*$ and characterize equilibria as we vary the production cost c and the post-purchase inspection parameter β. That is, given ω, we characterize equilibria for the set of points $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon)\}$. We start by demonstrating the nonexistence of a symmetric equilibrium in which consumers make a purchase without ever inspecting the good.

Lemma B.5. *There does not exist an equilibrium with an active market in which, at each firm, consumers prefer to buy without inspection.*

Proof. Consider an active market where consumers prefer to buy without inspection. It is clear that there cannot be such an equilibrium with $U_N > U_B, U_L$ as each firm would have an incentive to raise its price. Thus, in such an equilibrium, it must be that $U_N = U_j$ for a $j \in \{B, L\}$. However, from inspecting the consumer's pay-off of the different options it follows that upon visiting a firm the consumer prefers to buy without inspection if $a \leq b(s) - p$, to inspect before purchase if $b(s) - p < a \leq r(s) - p$, and to leave if $r(s) - p < a$.[22] Thus, as the consumer

[22]Proposition 0 in Doval (2018) identifies this to be the optimal decision rule.

cannot be indifferent between not inspecting at all and leaving, an equilibrium without inspection must equate $U_N = U_B$ so that the price is $p = b(s) - a$.

Assuming all other firms charge an average price p' and all induce consumers to not inspect the product, a consumer's outside option is simply to incur the search cost to visit another firm and buy without inspection, hence, $a = E(v) - p' - \epsilon$. But this implies that a firm must charge a smaller price than the competition $p = b(s) - E(v) + \epsilon + p' < p'$[23] to ensure that consumers do not inspect the product. As each firm's best reply is to charge strictly less than the average market price, such an equilibrium cannot exist. □

Lemma B.6. *$U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ and $U_B(p_W(a), a) - \epsilon - a$ have unique roots at a_R and a_W, are positive when $a < a_i$ and negative when $a > a_i$ for $i = R, W$ respectively.*

Proof. Given that there is a unique candidate Wolinsky equilibrium and candidate refund equilibrium, the functions $U_B(p_W(a), a) - \epsilon - a$ and $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ have unique roots at a_W and a_R. The Berge Maximum Theorem[24] guarantees the Wolinsky price and refund contract to be continuous in a and thus $U_B(p_W(a), a) - \epsilon - a$ and $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ are also continuous in a. Finally to show that the functions cross zero from above, for $a \geq r(s)$, $p_W(a) = p_R(a) = 0$ and thus $U_A(p_R(a), \tau_R(a), a) - \epsilon - a = U_B(p_R(a), a) - \epsilon - a = U_B(p_W(a), a) - \epsilon - a = -\epsilon < 0$. □

Lemma B.7. *Consider an equilibrium with $\beta < 1$, $\rho_R + \rho_W = 1$ and consumer surplus a^*. If $\Pi_R(a^*) \leq \Pi_W(a^*)$, then (a) $p_R(a^*) < p_W(a^*)$, (b) $a^* \in [a_W, a_R]$ with $a_W < a_R$, and (c) If $\rho_R \in (0, 1)$, then $a_W < a^* < a_R$.*

Proof. An equilibrium must feature $a^* < r(s)$ by virtue of the fact that firms will not charge negative prices. We first argue that $p_R(a^*) < p_W(a^*)$. To the contrary, suppose $p_R \geq p_W$. If the Wolinsky price lies on the boundary $p_W = r(s) - a^*$, then by assumption $p_R = r(s) - a^*$, implying a contradiction as consumer surplus can then be computed to be $a^* - \epsilon$. Now suppose that the Wolinsky price lies in the

[23] Lemma S.3 verifies $b(s) - E(v) + \epsilon < 0$ when $s + \epsilon < s^*$.
[24] Theorem 17.31 in Aliprantis and Border (2006).

interior. Then, because τ_R is optimally chosen, we have $\tau_R - \eta$ is weakly less than

$$\frac{1-G(a^*+p_R)}{g(a^*+p_R)} \cdot \frac{g(a^*+p_R)}{G(a^*+p_R)} \cdot \frac{G(a^*+\tau_R)}{g(a^*+\tau_R)} < \frac{1-G(a^*+p_R)}{g(a^*+p_R)} \leq \frac{1-G(a^*+p_W)}{g(a^*+p_W)}$$

which is equal to $p_W - c$. The first inequality follows from the logconcavity of G and $\tau_R < p_R$ and the second inequality follows from the logconcavity of $1-G$ and the assumption $p_R \geq p_W$. Thus we have $\tau_R - \eta < p_W - c$, further implying $\tau_R < p_W$. Equilibrium profit therefore satisfies

$$\Pi_R(a^*) = \frac{p_R - c - (\tau_R - \eta)G(a^* + \tau_R)}{1-q} > \frac{(p_W - c)(1 - G(a^* + p_W))}{1-q} = \Pi_W(a^*)$$

contradicting $\Pi_R(a^*) \leq \Pi_W(a^*)$. Hence, we must have $p_W(a^*) > p_R(a^*)$, proving part (a). For (b), the implication of this is that visiting the firms charging the Wolinsky price yields less utility than visiting firms offering the refund contract.

$$U_B(p_W(a^*), a^*) < U_B(p_R(a^*), a^*) = U_A(p_R(a^*), \tau_R(a^*), a^*).$$

The conclusion for (c) follows from the equilibrium condition $a^* = \rho_R\left(U_B(p_R(a^*), a^*) - \epsilon\right) + \rho_W\left(U_B(p_W(a^*), a^*) - \epsilon\right)$ and Lemma B.6. □

Lemma B.8. *Differentiating profit in the consumer surplus yields* $\frac{d}{da}(1-q) \cdot \Pi_R(a) = -1 + G(a+\tau_R)$, $\frac{d}{da}(1-q) \cdot \Pi_W(a) = -1 + G(a+p_W)$, *and* $\frac{d}{da}(1-q) \cdot \Pi_N(a) = -1$.

The computations supporting this lemma are standard and can be found in Section S.4 of the supplementary material. Next we simplify the problem so that we need only examine a restricted version of the game. Define the *restricted game* to be the same as the original game, except that firms can only offer either the refund contract or charge the Wolinsky price. In other words, the restricted game rules out equilibria and deviations to no inspection.

Lemma B.9. *A strategy profile is an equilibrium if and only if it is an equilibrium of the restricted game.*

Proof. We begin by showing that any equilibrium must involve $\rho_N = 0$ and thus constitutes an equilibrium of the restricted game. Consider an equilibrium in which a fraction ρ_R offer the refund contract, ρ_W charge the Wolinsky price, and ρ_N induce no returns. Supposing a nonzero fraction of firms offer each contract, $\rho_i > 0$ for $i \in \{W, R, N\}$, and equilibrium utility is a^* we have $p_N(a^*) \le p_R(a^*) \le p_W(a^*)$. The relationship $p_R(a^*) \le p_W(a^*)$ follows from Lemma B.7 and the inequality $p_N(a^*) \le p_R(a^*)$ results from consumers preferring to inspect before purchase over not inspecting at all when the price is $p_R(a^*)$ and $U_N(p, a^*) - U_B(p, a^*)$ is strictly decreasing in the price. Moreover, the inequality $p_N(a^*) < p_W(a^*)$ must be strict or else firms charging $p_N(a^*)$ yield strictly higher profit than those charging $p_W(a^*)$.

Due to the order of prices, the utility from visiting a firm charging the Wolinsky price must lie strictly below a^*. Thus, from Lemma B.6, we know $a_W < a^*$. From Lemma S.5 it follows that the difference $(1-q)\,(\Pi_W(a) - \Pi_N(a))$ is increasing in a. Given that the parameter is at a point in Ω_N, we have $\Pi_W(a_W) \ge \Pi_N(a_W)$, implying $\Pi_W(a^*) > \Pi_N(a^*)$, contradicting $\Pi_W(a^*) = \Pi_N(a^*)$. In other words, firms charging the Wolinsky price must yield strictly higher profit than those inducing no inspection. By an analogous argument, there can be no equilibrium with $\rho_N > 0$ where $\rho_R = 0$ and $\rho_W > 0$ or $\rho_R > 0$ and $\rho_W = 0$. Finally, Lemma B.5 proves that there can be no such equilibrium when $\rho_R = \rho_W = 0$, i.e., no equilibrium in which all firms induce no inspection.

Next, we show that any equilibrium of the restricted game constitutes an equilibrium of the original game. To do so, it is sufficient to show that there is no incentive for firms to deviate from such an equilibrium by cutting the price to induce no inspection.

By an analogous argument to that above, if the equilibrium involves $\rho_W > 0$, then profit from the Wolinsky price is demonstrably higher than deviating and cutting the price to induce no inspection. Consider $\rho_W = 0$, so that firms are playing a symmetric refund equilibrium. Proposition 2 provides that the contract lies in the interior of $\mathcal{R}(a_R)$. By Lemma B.2, if $\beta = 1$ and $c = \eta$, there is no incentive to deviate given that the parameter belongs to Ω_N. The difference $(1-q)\cdot(\Pi_R(a_R) - \Pi_N(a_R))$ is demonstrably decreasing in β and constant in c; hence, there is is no incentive to deviate for any other value $\beta < 1$ or $c > \eta$. □

This lemma allows us to draw two conclusions. First, firms only offer either the refund contract or the Wolinsky price in equilibrium. Second, when considering whether a strategy profile in which one or both of these contracts are offered is an equilibrium, the only possible deviations are indeed to one of these two contracts.

B.2 Equilibrium Structure of Contracts

Proof of Proposition 5. Fixing $\omega \in \Omega^*$, the proof proceeds by characterizing the equilibria that arise as we vary β and c. Throughout, we use Lemma B.9 and characterize equilibria of the restricted game. Finally, we also consider the variable a to satisfy $0 \leq a < r(s)$ as this must hold in any equilibrium. Let $c^* = c^*(s, \epsilon)$ denote the production cost at which $a_W = 0$.

The difference in profit between the two strategies is expressed by

$$\Pi_R(a) - \Pi_W(a) = \frac{p_R - c - (\tau_R - \eta)G(a + \tau_R)}{1 - q} - \frac{(p_W - c)(1 - G(a + p_W)}{1 - q}.$$

As the denominator does not influence which contract is more profitable, we need only compare the numerators to identify the best reply. Differentiating their difference, Lemma B.8 yields $\frac{d}{da}(1 - q) \cdot (\Pi_R(a) - \Pi_W(a)) = G(a + \tau_R) - G(a + p_W)$.

When $\beta < 1$, Lemma B.7 guarantees $p_W > \tau_R$ when $\Pi_R(a) - \Pi_W(a)$ is non-positive and thus strictly decreasing in this region. Hence, there exists at most one root $\Pi_R(a^*) - \Pi_W(a^*) = 0$ with the difference positive for $a < a^*$ and negative for $a^* < a$. We refer to this fact as *single-crossing*. Using single-crossing and drawing repeatedly from Lemma B.7, we now prove that if there exists an equilibrium at a point with $\beta < 1$, then it is unique.

- If there is a Wolinsky equilibrium, then $\Pi_R(a_W) - \Pi_W(a_W) \leq 0$ and $a_W < a_R$. There cannot be a refund equilibrium as $a_W < a_R$ implies $\Pi_R(a_R) - \Pi_W(a_R) < 0$. Similarly, an asymmetric equilibrium requires surplus to satisfy $a^* > a_W$, implying $\Pi_R(a^*) - \Pi_W(a^*) < 0$.

- Suppose there is an asymmetric equilibrium with surplus a^*. Because $\Pi_R(a) - \Pi_W(a)$ has a unique root, there can only be one asymmetric equilibrium. Also, because $a_W < a^* < a_R$, $\Pi_R(a_W) - \Pi_W(a_W) > 0 > \Pi_R(a_R) - \Pi_W(a_R)$ and so neither a Wolinsky nor a refund equilibrium exist.

- If there is a refund equilibrium, then from the last two points it is the only equilibrium.

In the text, we prove the claims of the proposition for the regions where either $\beta = 1$ or $c = \eta$. To complete the proof, we treat the remaining regions of the parameter space.

Claim B.1. *When $\beta < 1$, there is a production cost $\underline{c}(\beta) > \eta$ for which only a refund equilibrium exists when $c \in [\eta, \underline{c}(\beta)]$.*

Recall that $a_R > 0$ throughout since varying c has no effect on consumer surplus in a refund equilibrium. At $c = \eta$, we know $\Pi_R(a_R; c) > \Pi_W(a_R; c)$. Differentiating the difference yields $\frac{d}{dc}\left(\Pi_R(a_R; c) - \Pi_W(a_R; c)\right) = -\frac{G(a_R + p_W)}{1-q}$. As $\lim_{c\to\infty} \Pi_R(a_R; c) = -\infty$, there is some cost $\underline{c}(\beta) > \eta$ for which firms prefer to stick to the refund equilibrium when $c \in [\eta, \underline{c}(\beta)]$ and deviate to the Wolinsky price when $c > \underline{c}(\beta)$. The single-crossing property provides that the refund equilibrium is the unique equilibrium in this region.

Claim B.2. *When $\beta < 1$, there exists $\bar{c}(\beta) > \underline{c}(\beta)$ for which only a unique asymmetric equilibrium exists when $c \in (\underline{c}(\beta), \bar{c}(\beta))$.*

First, we show that the difference $\Pi_R(a_W; c) - \Pi_W(a_W; c)$ is decreasing in c. Because profit in the candidate of Wolinsky equilibrium does not depend on the production cost, we have $\frac{d}{dc}\Pi_W(a_W; c) = 0$. Also $G(a_W + p_W) = G(r(s + \epsilon))$ is independent of c, thus the probability that a consumer continues to search after visiting a firm charging the Wolinsky price is independent of the production cost. Using $\frac{da_W}{dc} = -1$ and differentiating the deviation profit

$$\frac{d}{dc}\Pi_R(a_W; c) = \frac{\partial \Pi_R}{\partial a_W}\frac{da_W}{dc} + \frac{\partial \Pi_R}{\partial c} = -\frac{1 - G(a_W + \tau_R)}{1-q}(-1) - \frac{1}{1-q} < 0.$$

Let $\hat{c}$ satisfy $\Pi_W(a_W, \hat{c}) = \Pi_R(a_W, \hat{c})$. As the previous claim guaranteed no Wolinsky equilibrium at $\underline{c}$, we have $\Pi_W(a_W, \underline{c}) < \Pi_R(a_W, \underline{c})$ and thus $\underline{c} < \hat{c}$. From this and the previous claim, no Wolinsky equilibrium nor refund equilibrium exists in this region.

Now we show that there is an asymmetric equilibrium. In the region $c \in (\underline{c}, \hat{c})$, the absence of a Wolinsky equilibrium or refund equilibrium implies $\Pi_R(a_R) - \Pi_W(a_R) < 0 < \Pi_R(a_W) - \Pi_W(a_W)$ and so the unique root must belong to $a^* \in (a_W, a_R)$. As Lemma B.7 guarantees $p_W(a^*) > p_R(a^*)$, we have

$$U_B(p_W(a^*), a^*) - \epsilon < a^* < U_A(p_R(a^*), \tau_R(a^*), a^*) - \epsilon$$

and thus there exists a unique $\rho_R \in (0, 1)$ for which $a^* = \rho_R(U_B(p_W(a^*), a^*) - \epsilon) + (1 - \rho_R)(U_A(p_R(a^*), \tau_R(a^*), a^*) - \epsilon)$.

Finally, we need to ensure that consumers are willing to participate in the market. For a given a, increasing the production cost has the effect

$$\frac{d}{dc}(1 - q) \cdot (\Pi_R(a; c) - \Pi_W(a; c)) = -G(a + p_W) < 0.$$

As a consequence of single-crossing, if $a^*(c) \in (a_W(c), a_R)$ is the unique root of $\Pi_R(a; c) - \Pi_W(a; c)$, then $a^*(c)$ is continuous and strictly decreasing in a neighborhood of c. Let $\hat{c}'$ be the production cost at which $a^*(\hat{c}') = 0$. As $a^*(\underline{c}) > 0$, it must be that $\underline{c} < \hat{c}'$. Define $\bar{c}$ to be the cost at which either the asymmetric equilibrium becomes Wolinsky or consumer surplus in the asymmetric equilibrium drops to zero, i.e. $\bar{c} = \min\{\hat{c}, \hat{c}'\}$. As previously argued, $\bar{c} > \underline{c}$.

Claim B.3. *When $\bar{c}(\beta) < c \leq c^*$, only a Wolinsky equilibrium exists.*

As the previous claim demonstrated, $\Pi_R(a_W; c) - \Pi_W(a_W; c)$ is strictly decreasing in c and thus $\Pi_R(a_W) - \Pi_W(a_W) < 0$ in this region; hence, a Wolinsky equilibrium exists as long as it delivers nonnegative consumer surplus. □

C Appendix: Market Efficiency

Proof of Proposition 7. As argued in Proposition 1, welfare in a refund equilibrium only depends on the equilibrium contract via the difference between the price and refund $p - \tau$, which is set to make consumers indifferent between inspecting before and after purchase. As Proposition 1 proves, only in the non-generic case $\psi(\omega) = 0$ can such a contract achieve efficiency. $\square$

Proof of Proposition 9. First, let us show that for θ close enough to τ_R/p_R, there is a constrained equilibrium in which all firms charge the boundary price. Consider a firm's problem when all other firms charge the boundary price $p(\theta) = \frac{\delta}{1-\theta}$, consumer surplus is set according to these contracts $a(\theta) = r(s+\epsilon) - p(\theta)$, and it must offer a contract in the set $X(\theta)$.

Suppose the planner requires refund rate $\theta_R = \tau_R/p_R$. Given that a refund equilibrium exists at this point, charging p_R is the firm's unique best reply. Moreover, computing the change in profit in the price along the ray $\{(p,\tau) \in \mathbb{R}^2_+ : \tau = \theta_R \cdot p\}$ at this point and recalling that $\tau_R - \eta = \frac{1-G(a_R+p_R)}{G(a_R+p_R)} \frac{G(a_R+\tau_R)}{g(a_R+\tau_R)}$

$$
\begin{aligned}
\frac{\partial}{\partial p}\pi_A(p, \theta_R \cdot p, a_R)|_{p=p_R} &\propto \frac{\partial}{\partial p}\left(p - c - (\theta_R p - \eta) G(a_R + \theta_R p)\right)|_{p=p_R} \\
&= 1 - \theta_R G(a_R + \theta_R p_R) - (\theta_R p_R - \eta)\theta_R g(a_R + \theta_R p_R) \\
&= 1 - \theta_R \frac{G(a_R + \tau_R)}{G(a_R + p_R)} > 0.
\end{aligned}
$$

Let P be a compact neighborhood of p_R for which profit is increasing in the price $\frac{\partial}{\partial p}\pi_A(p, \theta_R \cdot p, a_R)|_{p=\tilde{p}} > 0$ for all $\tilde{p} \in P$. Let Θ be a neighborhood of θ_R for which $\min_{\tilde{p}\in P} \frac{\partial}{\partial p}\pi_A(p, \theta \cdot p, a(\theta))|_{p=\tilde{p}} > 0$ for all $\theta \in \Theta$. The Berge Maximum Theorem provides that there is a neighborhood $\Theta' \subset \Theta$ of θ_R for which $\theta \in \Theta'$ implies that the firm's best replies are contained in P. Thus, when $\theta \in \Theta'$, the firm's unique best reply is on the boundary.

Finally, it is immediate that consumers are made worse off since, on the boundary, the sum $a(\theta) + p(\theta) = r(s+\epsilon)$ is constant and $p(\theta)$ is increasing in θ. In this region, because $(1-\theta)p(\theta) = \delta$ is constant, social welfare

$$
\begin{aligned}
\mathbf{S}(\theta) &= p(\theta) - c + \frac{\delta - c + \eta}{1 - G(r(\delta + \beta s + \epsilon))} G(r(\delta + \beta s + \epsilon)) + a(\theta) \\
&= r(s+\epsilon) - c + \frac{\delta - c + \eta}{1 - G(r(\delta + \beta s + \epsilon))} G(r(\delta + \beta s + \epsilon))
\end{aligned}
$$

is likewise constant, implying that profit is increasing in θ. $\square$

Proof of Proposition 10. The proof proceeds by constructing a symmetric constrained equilibrium in which contracts lie interior to the region for which consumers prefer to inspect after purchase. For a symmetric constrained equilibrium to achieve the social optimum, it is necessary that the price p, threshold θ, and consumer surplus a satisfy the firm's first order conditions for an interior optimum $\theta p - \eta = \frac{1-\theta G(a+\theta p)}{\theta g(a+\theta p)}$, the equilibrium condition $a = r((1-\theta)p + \beta s + \epsilon) - \theta p$, and the optimality condition $(1-\theta)p = c - \eta$. Letting $r^* = (c - \eta + \beta s + \epsilon)$, substitute consumer surplus $a = r^* - \theta p$ and the price $p = \frac{c-\eta}{1-\theta}$ into the firm's first order condition

$$\frac{\theta}{1-\theta}(c-\eta) - \eta = \frac{1-\theta G(r^*)}{\theta g(r^*)}.$$

When $c > \eta$, the left side is strictly increasing in θ and explodes to infinity, while the right side is strictly decreasing and arbitrarily large when θ is small, implying that there exists a unique $\hat{\theta}$ solving the equation. Then $\hat{p} = \frac{1}{1-\hat{\theta}}(c-\eta)$ is the price and $a^* = r^* - \frac{1-\hat{\theta}G(r^*)}{\hat{\theta}g(r^*)}$ is the surplus. The price must be below the curve $\mathcal{R}(a^*)$ as consumers strictly prefer to inspect after purchase. To verify that consumers are not tempted to forgo inspection altogether, the price and consumer surplus satisfy $a^* + \hat{p} > r(s+\epsilon) > b(s)$; hence, we have $U_N(\hat{p}) < U_B(\hat{p}, a^*) < U_A(\hat{p}, \hat{\theta}\hat{p}, a^*)$.

We now show that firms have no incentive to deviate from this contract as long as the production cost is not too large. As $c \to \eta$, the optimal threshold $\hat{\theta}$ converges continuously to one from below. At $c = \eta$ and $\hat{\theta} = 1$, profit is precisely equal to the Wolinsky profit. For one, this means that profit is logconcave in the price, implying that $\hat{p}$ is the unique price satisfying the firm's first order conditions for an optimum within the region inducing inspection after purchase. Given the parameter belongs to a point in $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon)\}$ for some $\omega \in \Omega^*$, profit strictly exceeds that from any contract inducing no inspection (see Lemma B.2). Thus for c close to η, continuity in the profit functions and the variables $(a^*, \hat{\theta}, \hat{p})$ provides that there is no temptation to deviate to induce no inspection and that $\hat{p}$ is the unique maximizer of $\pi_A(p, \hat{\theta}p, a^*)$.

Because $\beta < 1$, when $\hat{\theta} = 1$, consumers never prefer to inspect before purchase for any price. This is to say that $X(1)$ never intersects $R(a^*)$. As the variables

$(a^*, \hat{\theta}, \hat{p})$ are continuous in the production cost, for c in a neighborhood of η, $X(\hat{\theta})$ remains bounded away from $\mathcal{R}(a^*)$; hence, there are no contracts the firm could offer to induce inspection before purchase. $\square$

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Searching Online and Product Returns

Supplementary Material

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(FOR ONLINE PUBLICATION)

S.1 The Reservation and Backup Price

The following provides a few basic properties of the reservation and backup price defined by (2) and (3). Integrating by parts, these functions are equivalently implicitly defined by

$$x = \int_{r(x)}^{\bar{v}} (1 - G(v))\,\mathrm{d}v, \ \text{ for } x \in [0, S) \tag{S.1}$$

$$x = \int_{\underline{v}}^{b(x)} G(v)\mathrm{d}v, \ \text{ for } x \in [0, S') \tag{S.2}$$

where $S = E(v - \underline{v})$ and $S' = E(\bar{v} - v)$, as defined in the text.

Lemma S.1. *Let $r(x)$ and $b(x)$ be as defined in (2) and (3).*

(a) $r'(x) < 0$ *and* $b'(x) > 0$.

(b) There exists a unique x^* such that $r(x^*) = b(x^*)$.

(c) $r(x^*) = b(x^*) = E(v)$.

(d) $r(x) > b(x)$ *for* $x < x^*$ *and* $r(x) < b(x)$ *for* $x > x^*$.

Proof. (a) follows from differentiating (S.1) and (S.2) $\frac{dr}{dx} = -(1 - G(r))^{-1}$ and $\frac{db}{dx} = G(b)^{-1}$. To prove (b), the existence of a unique x^* equating $r(x^*) = b(x^*)$ follows from (a), the continuity of r and b, observing the limiting values of $r - b$ for large and small inspection costs, and the intermediate value theorem. For (c),

$$E(v) - b = \int_{\underline{v}}^{\bar{v}} (v - b)\mathrm{d}G(v) = \int_{b}^{\bar{v}} (v - b)\mathrm{d}G(v) + \int_{\underline{v}}^{b} (v - b)\mathrm{d}G(v) \tag{S.3}$$

$$= \int_{b}^{\bar{v}} (1 - G(v))\mathrm{d}v - x. \tag{S.4}$$

From this, $E(v) = b$ if and only if

$$\int_{b}^{\bar{v}} (1 - G(v))\mathrm{d}v = x \tag{S.5}$$

which holds if and only if $b = r$. Finally, (a) and (b) imply (d). □

Lemma S.2. *For a sequence $(x_n)_{n\in\mathbb{N}}$ in $[0, S)$, the following are equivalent.*

(a) $x_n \to S$.

(b) $r(x_n) \to \underline{v}$.

(c) $x_n + r(x_n) \to E(v)$.

Proof. (a) $\implies$ (b) is immediate. For (b) $\implies$ (c), let $(x_n)_{n\in\mathbb{N}}$ be a sequence of values in $[0, S)$ converging to S so that $r(x_n) \to \underline{v}$. Using the fact that $v - r(x_n) - \max\{v - r(x_n), 0\}$ almost surely converges to a unit mass at zero and the definition of the reservation price $E(\max\{v - r(x_n), 0\}) = x_n$, we obtain

$$0 = \lim_{n\to\infty} E\left(v - r(x_n) - \max\{v - r(x_n), 0\}\right) \tag{S.6}$$

$$= \lim_{n\to\infty} E\left(v - r(x_n) - x_n\right) = E(v) - \lim_{n\to\infty} \left(r(x_n) + x_n\right) \tag{S.7}$$

from which it follows that $x_n + r(x_n) \to E(v)$. (c) $\implies$ (a) follows from the fact that $x + r(x)$ is strictly decreasing for all $x \in [0, S)$. □

Lemma S.3. *For $(s, \epsilon) \in \mathbb{R}_+^2$ satisfying $0 < s + \epsilon < s^*$, we have $\epsilon < E(v) - b(s)$.*

Proof. The conclusion follows from noting that $E(v) - b(s) - s^* + s$ is decreasing in s and is equal to zero at $s = s^*$, implying $s^* - s < E(v) - b(s)$ for $0 < s < s^*$. $\square$

S.2 Proof of Proposition 1

In this appendix, we verify that Proposition 1 continues to hold when the social planner selects a distribution $\mu \in \Delta(\mathbb{R}^2_+)$ over the contracts offered by firms. We start the welfare analysis by considering a relaxed optimization problem where the social planner not only selects the firms' contracts, but also selects the consumers' inspection option. We then verify that consumers intrinsically prefer to follow the planner's ascribed inspection option at the optimum.

Let π and a be the profit and consumer surplus induced by a contract distribution and inspection strategy. When a consumer visits a firm, the planner's continuation value, or the *continuation welfare*, is given by[25]

$$\begin{aligned}\max\Big\{ &\max_{(p,\tau)}(p-c)(1-G(a+p)) + G(a+p)\pi + \int_{a+p}^{\bar{v}}(1-G(v))\mathrm{d}v + a - s,\\ &\max_{(p,\tau)} p - c - (\tau-\eta)G(a+\tau) + G(a+\tau)\pi + \int_{a+\tau}^{\bar{v}}(1-G(v))\mathrm{d}v + \tau - p + a - \beta s,\\ &\max_{(p,\tau)} p - c + E(v) - p\Big\}.\end{aligned}$$

The outer maximization reflects the planner's choice over inspection options and the inner maximization the choice over contracts. Computing the optimal contracts, the optimal price is $\pi + c$ when goods are inspected before purchase and the optimal refund is $\pi + \eta$ when goods are inspected after purchase. The price falls out of the second and third expressions as it only serves as a transfer from consumers to firms and does not affect consumers' propensity to continue their search. Likewise, the refund is ineffective in the first and third expressions as no returns are made. Inputting the optimal contracts and simplifying terms, the continuation welfare

[25]The planner's problem can be solved directly. We take the more instructive approach and solve it as a dynamic program using the Principle of Optimality.

becomes

$$\max\left\{\pi + \int_{a+\pi+c}^{\bar{v}} (1 - G(v))\mathrm{d}v + a - s, \pi + \int_{a+\pi+\eta}^{\bar{v}} (1 - G(v))\mathrm{d}v + a - \beta s - c + \eta, E(v) - c\right\}. \tag{S.8}$$

Given the bound on how large the search and inspection costs can be, it is never optimal for consumers to forgo inspection altogether. This is verified by first noting that the maximal welfare is at least the amount gained by asking all firms to charge the Wolinsky price, $a + \pi \geq r(s + \epsilon) - c$. With this bound, the continuation welfare from inspecting before purchase (the first entry in (S.8)) is at least $r(s + \epsilon) - c + \epsilon$ and therefore strictly exceeds the continuation welfare from not inspecting at all $E(v) - c$.

Thus, goods are always inspected at the social optimum. An optimal contract distribution requires firms whose goods are inspected before purchase to all charge the same price, say $\hat{p}$, the remaining firms to offer the same refund $\hat{\tau}$, and for these values to satisfy $\hat{p} - c = \hat{\tau} - \eta = \pi$. Consequently, these two equalities imply that the expected price charged by the firms stimulating returns is equal to the price charged by the others. To see this, let $R \subset \mathbb{R}^2_+$ be the event in which a contract stimulates returns and $\rho = \mu(R)$ the fraction of firms who do so. Industry profit can be expressed as

$$\begin{aligned}\pi &= \rho \frac{E(p|R) - c - (\hat{\tau} - \eta)G(a + \hat{\tau})}{1 - q} + (1 - \rho)\, \frac{(\hat{p} - c)(1 - G(a + \hat{p}))}{1 - q} \\ &= \frac{\rho\,(E(p|R) - c) + \pi \cdot (1 - q - \rho)}{1 - q} = \rho \frac{E(p|R) - c - \pi}{1 - q} + \pi\end{aligned}$$

implying $E(p|R) = \pi + c$. A further implication is that industry profit does not depend on the fraction of firms the planner allocates to stimulate returns. To verify this, suppose μ and μ' are distributions over the same optimal contracts, only differing in the rate with which firms offer refund contracts: $\rho' = \mu'(R)$.[26] Then

[26]In essence, $\mu'(\cdot|R) = \mu(\cdot|R)$ and $\mu'(\cdot|R^C) = \mu(\cdot|R^C)$.

industry profit is the same under both distributions because

$$\pi' = \rho' \frac{E(p|R) - c - (\hat{\tau} - \eta)G(a' + \hat{\tau})}{1 - q'} + (1 - \rho') \frac{(\hat{p} - c)(1 - G(a' + \hat{p}))}{1 - q'}$$
$$= \pi \cdot \frac{\rho'(1 - G(a' + \hat{\tau})) + (1 - \rho')(1 - G(a' + \hat{p})}{1 - q'} = \pi.$$

As expected profit is not affected by the inspection choices, it is natural that the option delivering the highest continuation welfare coincides with the one that is best for consumers. In other words, $E(U_A(p, \tau, a)|R) = U_A(\hat{p}, \hat{\tau}, a) \geq U_B(\hat{p}, a)$, if and only if, inspecting after purchase maximizes (S.8), i.e.

$$\int_{a+\pi+\eta}^{a+\pi+c} G(v)\mathrm{d}v \leq (1 - \beta)s. \tag{S.9}$$

If either (S.9) or its reverse hold strictly at a social optimum, then it is socially efficient for all or no firms to offer refunds, respectively. If instead (S.9) holds with equality, then both consumers and firms are indifferent between the two inspection options, hence, allowing any fraction of firms to offer a refund contract achieves maximal welfare. To determine what is optimal, we only need to compare the two extreme cases. Consumer surplus equals $r(c - \eta + \beta s + \epsilon) - \hat{\tau}$ if all firms stimulate returns and it equals $r(s + \epsilon) - \hat{p}$ if no firm offers a refund.[27] Thus, defining $\psi(\omega) = r(c - \eta + \beta s + \epsilon) + c - \eta - r(s + \epsilon)$, stimulating returns is socially optimal if and only if $\psi(\omega) \geq 0$.

For the final step, we need to ensure that both sides are willing to participate in the market. Of the contract distributions solving the relaxed problem, let the planner choose a simple one in which all firms offer the same contract $(\hat{p}, \hat{\tau})$ equating $\hat{p} - \hat{\tau} = c - \eta$. Profit must be nonnegative for firms to remain in the market, requiring $\hat{p} \geq c$. To produce nonnegative consumer surplus, the price cannot be too large, giving a bound that can be expressed compactly as $\hat{p} \leq \max\{\psi(\omega), 0\} + r(s + \epsilon)$. Finally, to verify that consumers prefer the socially optimal inspection option to forgoing inspection altogether, the latter option can only be preferred when the price

[27] These are the solutions to $\hat{a} = U_A(p, \hat{\tau}, \hat{a}) - \epsilon$ when $E(p) - c = \hat{\tau} - \eta$ and $\hat{a} = U_B(p, \hat{a}) - \epsilon$ respectively.

is less than $b(s) - a$ and the price we have specified is greater than or equal to $r(s+\epsilon) - a > b(s) - a$. The following proposition summarizes our conclusions.

S.3 Proof of Proposition 6

Proof of Proposition 6. The proof proceeds by verifying that for a small enough value of ϵ, when s lies in a neighborhood of zero there exists a Wolinsky equilibrium and when s lies in a neighborhood of s^* there exists a refund equilibrium; which, from Proposition 5, must be the unique equilibria in the respective regions. We then verify that there must then be a value $\hat{s}$ such that the equilibrium is Wolinsky when $s \leq \hat{s}$ and either asymmetric or refund when $s > \hat{s}$. The conclusion that the number of returns is initially zero follows from there being no returns placed in a Wolinsky equilibrium and then that the number of returns increases and must eventually subsequently decline follows from the fact that there is a positive number of returns in asymmetric and refund equilibria and that number vanishes in the limit as $(\epsilon, s) \to (0, s^*)$ as shown in the proof of Proposition 4.

Claim S.1. *The number of returns is continuous in the inspection cost.*

For a detailed argument, we refer the reader to the proof of Proposition 5 where the same reasons that the number of returns is continuous in the production cost c provide that it is continuous in the inspection cost s. To recall the proof sketch: Firstly, the conclusion is immediate in the interior of the regions where Wolinsky and refund equilibria exist. Moreover, the fraction of firms offering a refund contract must be continuous in s. If not, say if the fraction of firms offering a refund contract featured a discontinuous increase at some inspection cost, this would lead to a discontinuous increase in consumer surplus, and thus also a discontinuous increase in the relative profit from offering the Wolinsky contract relative to the refund contract. Similarly, there cannot be a discontinuous decrease in the fraction of firms offering the refund contract.

Claim S.2. *There is a neighborhood of zero such that, if s lies in this neighborhood, then the unique equilibrium is a Wolinsky equilibrium.*

As s vanishes, the Wolinsky equilibrium consumer surplus converges to $a_W^* \equiv r(\epsilon) - \frac{1-G(r(\epsilon))}{g(r(\epsilon))}$. Because the refund contract satisfies $\int_{a+\tau_R}^{a+p_R} G(v)dv = s$, as s vanishes, so does $p_R - \tau_R$. Hence, the profit from deviating to the refund contract converges to $p_R - c - \tau_R(1 - G(a_W^* + \tau_R)) = p_R - c - p_R(1 - G(a_W^* + p_R)) < (p_R - c)(1 - G(a_W^* + p_R)) \leq (p_W - c)(1 - G(a_W + p_W))$; and thus, firms have no incentives to deviate to a refund contract.

Claim S.3. *There is a neighborhood of* $(0, s^*)$ *such that, if* (ϵ, s) *lies in this neighborhood, then the unique equilibrium is a refund equilibrium.*

From the limiting arguments in the proof for Proposition 4, we see that the profit from offering the refund contract converges to $E(v) - c$. Considering a deviation to the Wolinsky contract, in the limit, the objective is to maximize $(p - c)(1 - G(p))$ subject to the constraint $p \leq r(s^*) = E(v)$. Letting p_m denote the unconstrained maximizer of $(p-c)(1-G(p))$, notice that profit from deviating the Wolinsky contract converges to $(\min\{p_m, E(v)\} - c)\,(1 - G(\min\{p_m, E(v)\})) < \min\{p_m, E(v)\} - c \leq E(v) - c$. Thus for $\epsilon > 0$ small enough and s close enough to s^*, the refund equilibrium is the unique equilibrium.

Claim S.4. *If firms are indifferent between playing a Wolinsky equilibrium and deviating to a refund contract at an inspection cost* $\hat{s}$*, then there does not exist a Wolinsky equilibrium for all* $s > \hat{s}$*.*

Lemmas B.7 and B.8 demonstrate that $(1 - q) \cdot (\Pi_R(a) - \Pi_W(a))$ is decreasing in a in a neighborhood any value a^* satisfying $\Pi_R(a^*) = \Pi_W(a^*)$. If $\hat{s}$ is any inspection cost at which firms are indifferent between playing the Wolinsky equilibrium and deviating to the refund contract, then $\Pi_R(a_W(\hat{s})) = \Pi_W(a_W(\hat{s}))$. Because Wolinsky consumer surplus is decreasing in the inspection cost, for all $s > \hat{s}$ in a neighborhood of $\hat{s}$, $\Pi_R(a_W(s)) - \Pi_W(a_W(s)) > 0$ and so there does not exist a Wolinsky equilibrium. □

S.4 Additional Calculations

Let us first verify that a refund equilibrium exists for the parameter value in example with uniformly distributed match value in Section 4 of the text. Building on the example, we can compute that $(p-c)+(\eta-\tau)G(a+\tau)$ is given by

$$\frac{2-\sqrt{8s}}{2}-\frac{2\left(1-\sqrt{8s}\right)}{2-\sqrt{8s}}\sqrt{1-\sqrt{8s}+2\beta s}+\eta-c-\frac{\sqrt{8s}}{2-\sqrt{8s}}\left(1-\sqrt{8s}+2\beta s\right)$$

and it follows that firms' profits are proportional to this. The equilibrium is defined for $0 \le s \le 1/8$ and as indicated in Proposition 3 at $s^* = 1/8$ and $\beta = \eta = 0$ we have that $a = 0$, while $\tau = 0$ and $p = E(v) = \frac{1}{2}$.

To confirm that the refund equilibrium indeed exists for the constellation that is considered in Proposition 3, we have to consider whether a firm is better off deviating to offering a contract p' where the consumer buys without inspection. At price p' a consumer buys without inspection if buying without inspection gives consumers a higher pay-off than inspecting before purchase, *i.e.,* if $Ev - p' \ge \max\{a, \int_{a+p'}^{1}(1-v)dv+a-s\}$.[28] Thus, the best possible deviation is where $p' = \min\left\{\frac{1}{2}-a, \sqrt{2s}-a\right\}$ and the best possible deviation pay-off is $p' = \sqrt{2s} - \frac{2\left(1-\sqrt{8s}\right)}{2-\sqrt{8s}}\sqrt{1-\sqrt{8s}+2\beta s} - c$. One can easily verify that this deviation is never profitable for $\beta = 0$.

Lemma S.4 (Comparative statics in s and ϵ)**.** *Let (p,τ,a) be the solution found in Claim A.1 of Proposition 2 and denote $\delta = p-\tau$ and $\hat{r} = r(\delta+\beta s+\epsilon)$.*

(a) $\frac{d\delta}{ds} = \frac{1-G(a+\tau)-\beta(1-G(a+p))}{G(a+\tau)(1-G(a+p))} > 0$

(b) $\frac{d\hat{r}}{ds} = -\frac{1-\beta(1-G(a+p))}{G(a+\tau)(1-G(a+p))} < 0$

(c) $\frac{d\delta}{d\epsilon} = \frac{G(a+p)-G(a+\tau)}{G(a+\tau)(1-G(a+p))} > 0$

(d) $\frac{d\hat{r}}{d\epsilon} = -\frac{G(a+p)}{G(a+\tau)(1-G(a+p))} < 0$

Proof. Differentiating (11) in s and ϵ reveals (a) and (c); differentiating $\hat{r}$ in the same arguments gives (b) and (d). □

[28] It is clear that firms do not want to offer a price such that consumers inspect before purchase as in that case consumers will continue to search if their match value is smaller than the price, which for s close to s^* is true for approximately half of the consumers.

Lemma S.5. *Differentiating profit in the consumer surplus yields the following:*

$$\frac{d}{da}(1-q)\cdot\Pi_R(a) = -1 + G(a+\tau_R) \tag{S.10}$$
$$\frac{d}{da}(1-q)\cdot\Pi_W(a) = -1 + G(a+p_W) \tag{S.11}$$
$$\frac{d}{da}(1-q)\cdot\Pi_N(a) = -1 \tag{S.12}$$

Proof. Implicitly defining $\tau(p,a)$ by $\int_{a+\tau(p,a)}^{a+p} G(v)\mathrm{d}v = (1-\beta)s$, the refund contract is found by maximizing

$$\max_{p\in\mathbb{R}_+} p - c - (\tau(p,a)-\eta)G(a+\tau(p,a))$$
$$\text{s.t. } (b(s)-a)_+ \leq p \leq (r(s)-a)_+.$$

When $p_R(a)$ is interior, one can verify $\frac{d}{da}(p_R(a)+a)$ to be strictly increasing. Thus aside from at most two values of a, those equating $p_R(a)+a = b(s)$ and $p_R(a)+a = r(s)$, the function $(1-q)\cdot\Pi_R(a)$ is evidently differentiable. We shall now show that the function continues to be differentiable at these two points. Suppose p_R is interior. Noting that $\frac{\partial}{\partial a}\tau(p,a) = \frac{G(a+p)}{G(a+\tau)} - 1$ and employing the Envelope Theorem

$$\frac{d}{da}(1-q)\cdot\Pi_R(a) = \frac{\partial}{\partial a}(1-q)\cdot\pi_A(p_R,\tau(p_R,a),a)$$
$$= \frac{\partial}{\partial a}\left(p_R - c - (\tau(p_R,a)-\eta)G(a+\tau(p_R,a))\right)$$
$$= -G(a+p_R) + G(a+\tau(p_R,a)) - (\tau(p_R,a)-\eta)g(a+\tau(p_R,a))\frac{G(a+p_R)}{G(a+\tau(p_R,a))}$$
$$= -1 + G(a+\tau_R).$$

If the price is at the upper bound $p_R = r(s)-a$, then the refund is set to equate $\int_{a+\tau_R}^{r(s)} G(v)\mathrm{d}v = (1-\beta)s$ and thus $\frac{d\tau_R}{da} = -1$. Differentiating $(1-q)\cdot\Pi_R(a)$ with respect to a

$$\frac{d}{da}\left(r(s) - a - c - (\tau_R-\eta)G(a+\tau_R)\right) = -1 + G(a+\tau_R).$$

The same outcome occurs if the price at at the lower bound $p_R = b(s) - a$. Thus whether interior or at the boundary, $\frac{d}{da}(1-q)\cdot\Pi_R(a) = -1 + G(a+\tau_R)$. Since $\tau_R(a)$ is continuous, at all points $\frac{d}{da}(1-q)\cdot\Pi_R(a+) = \frac{d}{da}(1-q)\cdot\Pi_R(a-)$ and thus it is differentiable everywhere, with derivative $-1+G(a+\tau_R)$.

Turning to Wolinsky, when p_W is interior $p_W(a)+a$ is strictly increasing. It follows immediately that aside from the points equating $p_W + a = b(s)$ and $p_W + a = r(s)$, that $(1-q)\cdot\Pi_W(a)$ is differentiable. Supposing p_W to be interior, we can apply the Envelope Theorem

$$\begin{aligned}\frac{d}{da}(1-q)\cdot\Pi_W(a) &= \frac{\partial}{\partial a}(1-q)\cdot\pi_B(p_W,a) = \frac{\partial}{\partial a}(p_W-c)(1-G(a+p_W))\\ &= -(p_W-c)g(a+p_W) = -1+G(a+p_W).\end{aligned}$$

At the upper bound $p_W = r(s)-a$, $\frac{d}{da}(1-q)\cdot\Pi_W(a) = \frac{d}{da}(r(s)-a-c)(1-G(r(s))) = -1+G(a+p_W)$. The same outcome occurs if the price is at the lower bound $p_W = b(s)-a$. Thus, the derivative is always $\frac{d}{da}(1-q)\cdot\Pi_W(a) = -1+G(a+p_W)$. The desired conclusion follows.

Finally, the optimal price inducing no inspection is $p_N(a) = b(s) - a$ which profit is $(1-q)\cdot\Pi_N(a) = p_N(a) - c = b(s) - a - c$ and thus differentiating in a yields the desired conclusion. $\square$

S.5 Extension: Time Discounting

Consider a consumer who is browsing products online. The search cost to look at a product is ϵ. After looking at the product, the consumer sees its price and refund and then decides whether to (i) add the product to the cart, (ii) incur an inspection cost s to ascertain her match value, or (iii) move on to the next product.

Suppose firms all offer (p,τ) and the consumer adopts the strategy of first buying k items, having them sent to her home, and then deciding whether to keep any of the items. Letting γ be the discount factor, v^* the highest match value from the purchased items, and a the continuation value, the expected utility from following this strategy is[29]

$$\begin{aligned} U_A^k &= -k\epsilon - kp + \gamma\left[(k-1)\tau - k\beta s + E(\max\{v^*, \tau + a\}|k)\right] \\ &= -k\epsilon - kp + \gamma\left[(k-1)\tau - k\beta s + \int_{a+\tau}^{\bar{v}} (1 - G^k(v))dv + a + \tau\right]. \end{aligned}$$

For a given contract, we have

$$U_A^1 = -\epsilon - p + \gamma\left[-\beta s + \int_{a+\tau}^{\bar{v}} (1 - G(v))dv + a + \tau\right]$$

Suppose the consumer were to adopt the strategy of purchasing a new item online each day and only after k days deciding to keep the good with the highest value or to return them all and continue the search. The expected utility under this strange strategy is

$$\tilde{U}_A^k = -(\epsilon + p)\sum_{i=1}^{k} \gamma^{i-1} - \gamma^k\left[-k\beta s + k\tau + \int_{a+\tau}^{\bar{v}} (1 - G^k(v))dv + a\right]$$

As $U_A^1(\gamma) > \tilde{U}_A^k(\gamma)$ for all $\gamma \in (0, 1]$ and $\lim_{\gamma \to 1^-} U_A^k(\gamma) = \lim_{\gamma \to 1^-} \tilde{U}_A^k(\gamma)$ it follows that for γ in a neighborhood of one, $U_A^1(\gamma) > U_A^k(\gamma)$.

[29]In terms of notation, here we include the search cost explicitly in the utility.