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Abstract

We propose a method to capture the unusual Covid recession and recovery in our usual business cycle models. The initial outbreak is represented by a new shock process which loads on wedges underlying the usual shocks and comes with news about its propagation. We apply our method to a standard model. The loadings are estimated with 2020q2 data; the evolving news is identified using professional forecasts. On net, the Covid shock is dominated by supply effects. It accounts for most of the early dynamics, was inflationary and a persistent drag on activity, and the majority of its effects were unanticipated.

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Unusual shocks in our usual models*

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January 20, 2023

Abstract

We propose a method to capture the unusual Covid recession and recovery in our usual business cycle models. The initial outbreak is represented by a new shock process which loads on wedges underlying the usual shocks and comes with news about its propagation. We apply our method to a standard model. The loadings are estimated with 2020q2 data; the evolving news is identified using professional forecasts. On net, the Covid shock is dominated by supply effects. It accounts for most of the early dynamics, was inflationary and a persistent drag on activity, and the majority of its effects were unanticipated.

JEL Classification Numbers: C51, E10, E31, E32, E52
Keywords: Covid-19, pandemic, DSGE models, Survey of Professional Forecasters, business cycles

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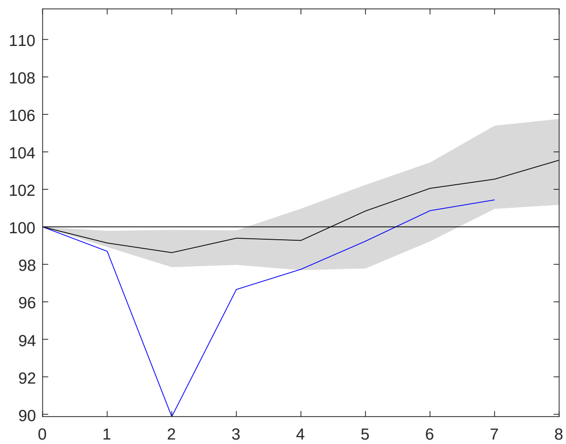
Dynamic stochastic general equilibrium (DSGE) models have proven to be a valuable empirical framework for understanding aggregate economic dynamics. Since these models are estimated using historical data, they are suitable to study recurrent dynamics, such as the business cycle. Can these models remain useful in the face of large unprecedented shocks? We propose a new methodology to incorporate unusual shocks into our usual models that does not require modeling the fundamentals of the shock. We apply the methodology to study the Covid-19 pandemic recession and recovery within the context of a medium-scale New Keynesian (NK) business cycle model.

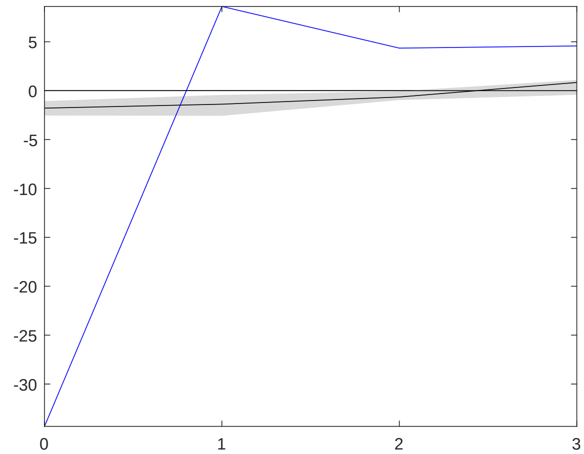
Figure 1: Recessions and recoveries in real GDP, 1947–2021



Note: The figure shows recessions and recoveries in the level of real GDP since 1947. GDP is normalized to 100 at the business cycle peak in quarter 0. The black line corresponds to medians and the shading is the 25 to 75 percentile range from before the Covid recession. The blue line corresponds to the index of the level of output during the Covid recession and recovery. Source: Haver Analytics and authors' calculations.

The Covid recession and recovery are highly unusual. Figure 1 compares the Covid recession to all previous recessions since 1947, as determined by the NBER's Business Cycle Dating Committee, with the level of real GDP normalized to 100 at the most recent peak prior to a recession. The black line corresponds to the median path of output for those pre-Covid recessions and recoveries, the gray shading indicates the 25 to 75 percentile range for output from before the Covid recession, and the blue line shows the path of output for the Covid recession and recovery. Typical recessions last a few quarters and generally involve a gradual recovery. The collapse in output in 2020q2 is an order of magnitude larger than in a

Figure 2: GDP growth forecast revisions at the onset of recessions, 1968–2021



Note: Real GDP growth forecast revisions in the quarter of the onset of a recession compared to the quarter before taken from the *Survey of Professional Forecasters*, which begins in 1968 (and is currently conducted by the Federal Reserve bank of Philadelphia). The horizontal axis indicates the horizon of the forecast. In period 0 the revision is the difference between the one-quarter-ahead growth forecast in the quarter before the recession starts and the nowcast in the quarter it starts. The black line is medians and shading is the 25 to 75 percentile range from before the Covid recession. The blue line corresponds to forecast revisions at the onset of the Covid recession. Source: Survey of Professional Forecasters and authors' calculations.

typical postwar recession. Yet output takes just a quarter longer to reach the previous peak. The Covid recession is far deeper and the subsequent recovery is much faster than those of a typical business cycle.

The pre-pandemic dynamics get reflected in private sector forecasts of that time. Figure 2 shows forecast revisions for real GDP growth in the first quarter of an NBER recession taken from the *Survey of Professional Forecasters* (SPF) starting from the beginning of the survey in 1968 (the lines and shading are constructed analogously to the previous figure).[1] The revision in period 0 is the difference between the SPF nowcast and the forecast from the period before, when the economy was at its business cycle peak. Before the Covid recession hit the US economy, forecasters would downgrade their forecasts from prior to a recession's start for a couple quarters out – they would expect a slow-to-start and gradual recovery. In contrast forecasters surveyed in May 2022 expected a fast start to a rapid recovery, with forecast revisions large and positive for three quarters out.[2]

[1] The survey is currently conducted by the Federal Reserve Bank of Philadelphia.

[2] Eichenbaum, Rebelo, and Trabandt (2021) highlight other unique aspects of the Covid recession. Unlike

We show that the usual shocks in a canonical medium-scale DSGE model estimated with data from before the pandemic struggles to capture the highly unusual dynamics shown in Figures 1 and 2. This poses a challenge to the viability of our usual models. We tackle this challenge by developing a framework to estimate a new shock called the *Covid shock*. This shock proves to play a crucial role in explaining the large contraction in output in the second quarter of 2020 and the revision to forecasts in that period.

The initial outbreak is represented as the onset of a new shock process, where the shock loads on wedges in the DSGE model in the same way as a subset of its usual shocks. We focus on wedges that propagate business cycle co-movement as well as other shocks that play a major role in explaining consumption, investment, and inflation. The chosen wedges and corresponding loadings define the *nature* of the Covid shock. Surprise Covid shocks come with news about its future path. The surprise and news structure of the Covid shock provides the flexibility not provided by the usual shocks to account for the dramatic fall in output in 2020q2 and professional forecasts at that time of a sharp rebound and rapid recovery starting in 2020q3.

We adopt an event-study approach to identify the nature of the Covid shock. The macroeconomic data in 2020q2 clearly are dominated by the public and private sector responses to the pandemic and so should be particularly informative about the nature of the new shock. We acknowledge this by estimating the parameters defining the nature of the Covid shock with data from this quarter alone.[3] We use revisions to SPF forecasts of output growth and inflation to identify this news over time. Including news and holding fixed the nature of the Covid shock enables us to separate out the macroeconomic effects of Covid from those of the usual shocks.

Observing professional forecasters' expectations is particularly helpful when studying an unusual shock. Forecasters recognize that by its very nature an unusual shock is not captured in the historical dynamics. As such their forecasts will not be restricted by the historical

a typical recession the drop in output is driven by consumption, which tracks output quite closely. Investment declines by much less, but also recovers much faster than it does typically.

[3] The economic impact of the pandemic began to take hold in March 2020 but only shows up as a small contraction in activity in 2020q1.

dynamics and will incorporate any new information they are absorbing in real time about how the shock will propagate through the economy. As forecasters update their beliefs about the propagation of the shock, this will get reflected in their forecast revisions.[4]

The nature of the Covid shock includes significant loadings on wedges that generate both demand and supply effects. On net, the supply forces dominate in 2020q2 as the Covid shock lowered output and put upward pressure on prices. The latter is notable given that prices actually fell in that quarter. The supply effects were expected to persist as the revisions to the SPF forecasts in 2020q2 attributed to the Covid shock had output remaining below pre-pandemic levels and prices higher over the next four quarters.

We use our model to disentangle the effects of the Covid shock and the usual shocks on aggregate activity and inflation over the period 2020q2 – 2021q3, taking into account possible effects from the Delta wave in 2021q3. We measure aggregate activity with de-trended per capita hours, which is a good indicator of the cyclical position of the US economy. The Covid shock explains about two-thirds of the massive decline in hours in 2020q2 and contributes considerably to the extraordinary economic rebound in the next quarter. Over the following four quarters, the shock is a significant drag on economic activity. The inflationary effects of the Covid shock are more muted mostly because of the flat Phillips curve. Nonetheless the Covid shock is inflationary throughout the sample period, significantly so in 2020q2.

An advantage of our methodology is the ability it provides to quantify the role of beliefs about the path of the pandemic. This is due to the surprise and news structure of the unusual shock and the observation of revisions to the professional forecasters' forecasts. We find that beliefs about the future path of the Covid shock reduced the magnitude of the 2020q2 contraction in GDP by 10 percentage points but were a drag on activity for the remainder of 2020. We also find that the economic effects of the pandemic were mostly hard to anticipate. In other words, the professional forecasters were continually surprised by the abrupt turns the pandemic took, both in terms of the surprise component and the news

[4]Professional forecasts are also valuable when the effects of an unusual shock are studied in real time. Given the large uncertainty and the scarcity of data that characterize the initial periods of an unusual episode, observing such data allows for the best possible real-time estimation of the effects of the unusual events.

component of the Covid shock.

The news structure of the novel shock is helpful when unusual events repeat themselves after their first occurrence. In the case of the Covid pandemic, we observed recurrent waves of infections as well as the emergence of new variants. In every repetition of the unusual event, agents become more knowledgeable about the effects of the event. We show that once the nature of the shock is estimated at the onset of the event, the news structure of the shock can be used to make different assumptions regarding how much agents have learned from previous experiences. For instance, when we estimate the effect of the Delta wave we assume that agents have perfect ex-ante knowledge about the path of the Covid shock as given by the estimates from the initial wave.

We estimate the Delta shock to be a small fraction of the original shock, and so it has little effect. A substantial caveat to this result and our analysis in general is that in 2020q2 the pandemic (including the lockdowns) came with a large fiscal intervention that because of the simplicity of the fiscal block in the canonical NK business cycle model confounds our identification of the Covid shock. The Delta wave did not involve any new fiscal interventions.

One advantage of our methodology is its generality; that is, it can be applied to any DSGE model, including models that are more suitable to study the propagation of fiscal shocks. However, there is no consensus on how to model the propagation of fiscal shocks. Moreover, most existing models focus on taxes and spending and so seem poorly suited to study the unusual nature of the pandemic fiscal interventions, in which transfers played a major role. Given the large fiscal interventions involved, there will be plenty to learn about the propagation of fiscal shocks from the Covid episode, but it will be necessary to account for the pandemic dynamics to separately identify their effects. Our framework makes it possible to do that without having to model the epidemiology of a pandemic.

In the next section of the paper we review the related literature. Next, we describe the unusual shock, how we use it to isolate the role of beliefs about the pandemic, and how we estimate it. We then discuss the effects of the estimated shock within the context of an off-the-shelf medium-scale NK DSGE model. We finish with some concluding remarks.

1. Related literature

Primiceri and Tambalotti (2020) identify a surprise Covid shock in a monthly vector autoregressive model (VAR). Their shock is defined as a linear combination of the VAR's reduced form shocks with weights estimated using data from March and April 2020, when aggregate dynamics clearly were dominated by the Covid shock. We differ in two respects: First, our shock is a linear combination of wedges in a structural model, and second, we include forward-looking information to identify news shocks that come with the surprise. Including the news shocks turns out to be crucial to our estimation of the Covid shock.

Lenza and Primiceri (2022) model Covid in a VAR by scaling the variances of the usual independently and identically distributed (i.i.d.) residuals by a common scaling parameter that decays exponentially over time. The scaling parameter and its rate of decay are estimated using data from March, April, and May 2020. They use their framework to demonstrate that one obtains similar VAR parameter estimates by dropping those observations. This will be helpful going forward to estimate VARs with data that includes the pandemic period. We provide a way to estimate structural models with these data. Note that Lenza and Primiceri (2022) do not exploit the information in private sector forecasts. Our structure allows us to measure the sensitivity of private sector decisions to expectations about the future effects of the pandemic that are revealed through news.

We synthesize our Covid shock from wedges in a structural model and so contribute to the large literature that studies structural wedges in various contexts. Recent work in this literature by Inoue, Kuo, and Rossi (2020) is particularly relevant for us. They use wedges in a medium-scale DSGE model to measure model misspecification. Our approach acknowledges misspecification through wedges but attributes it all to the Covid shock.

We find that both supply and demand components of the Covid shock play large roles in accounting for the effects of Covid. Guerrieri, Lorenzoni, Straub, and Werning (2022) show how a supply shock can cause demand shortages in a two-sector NK model. In our setting the endogenous effects of the supply shock on demand they describe would be captured by the loadings of the common factor on the model's wedges. The wedges also can be viewed as a reduced-form characterization of the interaction of demand and supply shocks in the NK model with input-output linkages studied by Baqaee and Farhi (2022).

Our analysis is complementary to the now large literature that embeds epidemiological models within otherwise standard business cycle models to study the Covid pandemic, for example Eichenbaum et al. (2021) and Acemoglu, Chernozhukov, Werning, and Whinston (2021). These "epi-mac" models yield important new insights but add considerable complexity. Our framework does not involve changing our usual models, but leverages their existing structure to synthesize a new shock that can capture the dynamics resulting from the unusual event. By basing our analysis on a standard DSGE model we can assess the empirical relevance of the new shock relative to the usual shocks that have proved to be useful in accounting for U.S. business cycles. This assessment is possible because of the relatively large size of our model that fits the data comparably to how VARs fit them. So our methodology allows us to single out the effects of the Covid shock from the effects of the usual business cycle shocks that fit the data in normal times. This suggests our framework can be used to configure a benchmark for the epi-mac literature.

Our approach addresses the absence of a major pandemic in the postwar data before Covid that might otherwise be used to identify the effects of Covid. Alternatively one could use additional time-series data to learn from history about the possible effects of the Covid shock on the economy. The only comparable major pandemic was the Spanish influenza of 1918 and 1919. Barro, Ursua, and Weng (2020), Barro (2020), and Velde (2020) use this episode to shed light on the economic effects of a pandemic. Ludvigson, Ma, and Ng (2020) project the economic impact of Covid based on estimates of the impact of deadly disasters in recent U.S. history.

2. The Covid shock

This section describes how we introduce the new Covid shock into a DSGE model. First, we define the shock and introduce enough structure to separately identify the shock and news about its evolution. We then discuss some implications of the surprise and news structure of the Covid shock. Finally, we describe how we estimate the shock. A key feature of our methodology is that it allows agents' beliefs about the evolution of the Covid shock to vary over time. We use data from the SPF on forecasts of output and inflation to identify revisions to these beliefs.

2.1. Definition

The Covid shock Ψ_t is defined as

$$\Psi_t = \sum_{j=0}^{N} \psi_{t-j}^{j}, \ N \geq 0, \tag{1}$$

where the random variables ψ_{t-j}^{j} are shocks that are anticipated at time t to hit the economy in period $t+j$ and N is the anticipation horizon of agents. This information can be divided into two components. The first component — called *surprise* — contains all the information about the current effects of Covid that were not anticipated in previous periods. The surprise in period t is ψ_t^0. The second component — called *news* — represents all the information about the future effects of Covid received by agents in the current period. The news received in period t is $\{\psi_t^1, \psi_t^2, \ldots, \psi_t^N\}$.

The shocks ψ_t^j equal date t revisions to expectations about future Covid shocks Ψ_{t+j}. Specifically, from (1) we have

$$\psi_t^j = E_t \Psi_{t+j} - E_{t-1} \Psi_{t+j} \ j \in \{0, 1, \ldots, N\}. \tag{2}$$

That is, the news shocks ψ_t^j equal time t revisions to agents' expectations about the effects of Covid in period $t+j$.

The news shocks have a factor structure given by

$$\psi_t^j \quad = \quad \lambda_j(t) f_t, \ j \in \{0, 1, \ldots, N\}, \tag{3}$$

where the common factor f_t is an independent Gaussian random variable with a mean of zero and a standard deviation of $\sigma(t)$. We assume $f_t = 0$ for $t < t^*$, where t^* is the date of the onset of the pandemic. The time-varying factor loadings $\lambda(t) = \{\lambda_j(t)\}_{j=0}^{N}$ and variance of f_t are *not* random variables from the perspective of agents in the model. Agents treat them as parameters, and we estimate them.

Notice that given the structure of shocks summarized by equations (1) and (3) we can write the Covid shock as

$$\Psi_t = \sum_{j=0}^{N} \lambda_j(t-j) f_{t-j}. \tag{4}$$

It follows that the Covid shock Ψ_t is serially correlated as it depends on current and past realizations of f_t.

We assume that each of the ψ_t^j map into M DSGE wedges $\Sigma_t(i),\ i \in \{1, 2, \ldots, M\}$, that enter into our DSGE model identically to shocks already present — for example, technology and discount factor shocks. We assume the wedges have i.i.d. surprise and news elements that relate directly to the news about the Covid shock. Specifically,

$$\Sigma_t(i) = \sum_{j=0}^{N} \epsilon_{t-j}^{j}(i), \tag{5}$$

where

$$\epsilon_t^j(i) \quad = \quad \phi_i \psi_t^j. \tag{6}$$

Combining (1), (5), and (6) we can see that the wedges are proportional to the Covid shock, that is,

$$\Sigma_t(i) = \phi_i \Psi_t.$$

The scalar parameters ϕ_i are the loadings of the Covid shock Ψ_t onto the wedges. We refer to the choice of wedges and the loadings as the *nature* of the Covid shock. Note that while the underlying wedge shocks are i.i.d., the surprise and news structure allows agents to forecast persistence. Also note that the weights $\phi = \{\phi_i\}_{i=1}^{M}$ do not depend on the anticipation horizon of the wedges so that the combination of the DSGE wedges, which is used to approximate expectations about the evolution of the pandemic, does not vary across anticipation horizons. We think this assumption is natural but it also allows us to economize on the number of parameters that we need to estimate. Combined with constant ϕ this assumption identifies the Covid news separately from news of the usual shocks if it is already present in the DSGE model, provided that news of each usual shock is not perfectly correlated as the Covid wedges are in our framework.

To sum up, we capture the dynamics of the Covid shock with the loadings ϕ and $\lambda(t)$ and the common factor f_t. The vector ϕ describes the nature of the Covid shock, defined as a particular combination of wedges that enter into the DSGE model in the same way as a subset of the usual shocks. The loadings $\lambda(t)$ capture evolving beliefs about the Covid shock. The variance $\sigma(t)$ summarizes the uncertainty underlying these beliefs. Individual realizations of the exogenous variable f_t account for revisions to agents' expectations of the future path of the Covid shock.[5]

2.2. Surprise, News, and Perfect Foresight

As shown in equation (2), the news shocks ψ_t^j equal time t revision to agents' expectations about the economic effects of Covid in period $t + j$. This anticipation structure allows us to construct counterfactual exercises under various assumptions regarding the flow of information about the effects of Covid received by agents. For instance, it is conceivable that agents are more aware about what to expect from a second pandemic wave than what they were at the onset of the pandemic. To illustrate how this can be implemented using our methodology, we assume that agents have perfect foresight about the effects of the second

[5] In a short note on inflation based on the NY Fed's DSGE model Del Negro, Gleich, Goyal, Johnson, and Tambalotti (2022) describe how they account for the pandemic in that model. Their approach also involves introducing shocks to some of the model's existing wedges, but it differs in several respects: they do not exploit the parsimony of a factor structure, they do not use data on professional forecasts, and they calibrate key parameters, including scaling down the volatility the usual shocks when they estimate their i.i.d wedge shocks' variances using data in 2020q2. Cardani, Croitorov, Giovannini, Pfeiffer, Ratto, and Vogel (2022) also introduce a novel Covid shock into an off-the-shelf DSGE model, in their case a model of the Euro area, but their shock is based on modifying the model's structure to include forced savings and labor hoarding rather than leveraging the model's pre-existing wedges.

wave. In this scenario, agents have fully learned what going through a pandemic wave means and will commit no errors in forecasting the effects of the second wave.

To implement this scenario, we assume that when the second wave hits in period t^{**}, the following Covid news is realized: $\psi^j_{t^{**}} = \delta \cdot \Psi_{t^*+j}$ for $j \in \{0, 1, ..., T\}$, where Ψ_{t^*+j} denote the Covid shocks j periods after the start of the first wave in t^* and T denotes the duration of the first wave. The parameter δ is a scaling factor, which determines whether the second wave is more severe ($\delta > 1$) or less severe ($\delta < 1$) than the first wave. In the subsequent periods ($t^{**} + 1, t^{**} + 2, \dots, t^{**} + T$), there will be neither surprise nor news since all the effects of the second wave were correctly anticipated from the start and hence there is no revision to agents' expectations after the first period. In symbols, $\psi^j_{t^{**}+i} = 0$ for any $i \in \{1, 2, \dots, T\}$ and $j \in \{1, 2, \dots, N\}$.[6]

2.3. Estimation

To identify the Covid shock Ψ_t we need to estimate ϕ and $\Xi(t) = [\lambda(t), \sigma(t)]$ for the number periods we determine *a priori* that agents update their beliefs about the Covid shock. We apply an event-study approach to identify ϕ. In 2020q2 there was an unusually large drop in economic activity — far beyond the bounds of a typical business cycle peak to trough — and an unusual expected rebound. We expect the dramatic variation in 2020q2 is due chiefly to the Covid shock. This suggests 2020q2 data on current and expected future activity and inflation will be particularly informative about ϕ.

Let Θ denote the usual parameters in our DSGE model, which are taken as given. Note that this includes the volatilities of the usual shocks. This is important because it means our estimation in effect lets the data speak about the relative volatility of the Covid shock. We use Bayes' theorem to obtain a distribution of $\Xi(t)$ and ϕ conditional on the usual data

[6]Note that, in this example, $\psi^0_{t^{**}} \neq 0$ implies that agents do not anticipate the start of the second wave and, in fact, are surprised by that. However, at the beginning of the second wave, they can perfectly foresee its effects. It is straightforward to relax that assumption and assume that agents can correctly anticipate the start of a new wave k periods in advance: $\psi^{j+k}_{t^{**}-k} = \delta\Psi_{t^*+j}$ and $\psi^{j+k}_{t^{**}-k+i} = 0$ in any subsequent period $i = \{1, 2, \dots, T\}$.

up to date t, denoted X^t. At date $t = t^*$ we have,

$$p\left(\Xi(t), \phi | X^t, \Theta, s_{t-1}; \mathcal{M}\right) \propto \mathcal{L}\left(X^t | \Xi(t), \phi, \Theta, s_{t-1}; \mathcal{M}\right) p\left(\Xi(t), \phi\right), \tag{7}$$

where $\mathcal{M}$ denotes our DSGE model and s_{t-1} is the model's state vector estimated one quarter earlier. The density $p(\cdot)$ is our prior on the new parameters capturing the nature of and beliefs about the Covid shock. The density $\mathcal{L}(\cdot)$ is the likelihood function associated with the data X^t. We expect the dramatic movements in the data at date $t = t^*$ to be particularly informative about the nature of the Covid shock, ϕ. After that period we have

$$\begin{aligned} & p\left(\Xi(t^*+j) | X^{t^*+j}, \phi, \Theta, s_{t^*+j-1}; \mathcal{M}\right) \\ & \qquad \propto \mathcal{L}\left(X^{t^*+j} | \Xi(t^*+j), \phi, \Theta, s_{t^*+j-1}; \mathcal{M}\right) p\left(\Xi(t^*+j)\right), \end{aligned} \tag{8}$$

for $j = 1, 2, \ldots, N-1$.

We estimate ϕ and $\Xi(t)$ sequentially by maximizing the posterior modes in (7) and (8). For $t = t^*$ the intuition is to find the combination of the wedges $\Sigma_t(i)$ that, along with the usual shocks, best explain the one-step-ahead forecast error of the usual data, that includes current activity and professional forecasts. For $t > t^*$ the $\Xi(t)$ are identified by the revisions to the professional forecasts of output and inflation.

We use the Kalman smoother to estimate f_t. With f_t and our estimates of ϕ and $\lambda(t)$ we obtain estimates of the Covid shock and its anticipated and unanticipated components from (1) and (3).

3. The DSGE Model

We study the Covid shock within Campbell, Fisher, Justiniano, and Melosi (2016)'s medium-scale NK model. Most of the model is familiar as it is a variant of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), and so here we just provide a brief overview.[7] The main differences between our model and theirs is the inclusion of a preference

[7]Our model is closest to Justiniano, Primiceri, and Tambalotti (2013).

for government bonds, anticipated deviations from the monetary policy rule, and shocks to investment-specific technological change.[8]

The representative household's preferences are non-separable with respect to consumption and hours worked and separable with respect to real government bonds.[9] Preferences are buffeted by shocks to the discount factor and to the preference for government bonds.[10] A positive discount factor shock reduces output, consumption, and hours but increases investment because it raises the preference for future consumption relative to current consumption. Justiniano, Primiceri, and Tambalotti (2010) and others show that it is an important driver of consumption fluctuations.[11] We refer to the shock to the preference for government bonds as the liquidity preference shock. This shock is a source of co-movement between output, consumption, investment, and hours. A positive liquidity preference shock increases the demand for government bonds relative to private capital and consumption and so creates a desire to consume less today compared to tomorrow. This drives down both consumption and investment and, therefore, overall activity. The two preference shocks follow AR(1) processes.

The specification of the production side of the economy is standard. It includes perfectly competitive producers that aggregate intermediate goods into the final good; monopolistic competitive intermediate goods producers with identical Cobb-Douglas production functions that require labor and capital as inputs; and labor compositors that package the differentiated labor of households into a homogeneous labor input supplied to the intermediate goods producers. The intermediate goods producers and suppliers of differentiated labor are subject

[8]The details of our model and its estimation are provided in the appendix. Those details are not crucial to understanding our analysis.

[9]Fisher (2015) and Campbell et al. (2016) discuss how including a preference for government bonds drives a steady state wedge between interest rates on private and government bonds that is otherwise absent from standard models. Doing so brings discounting into the household's linearized inter-temporal Euler equation for consumption, which somewhat mitigates the forward guidance puzzle highlighted by Del Negro, Giannoni, and Patterson (2015). Such preferences are now common in the literature, for example Michaillat and Saez (2021), Eichenbaum, Johannsen, and Rebelo (2021), and Anzoategui, Comin, Gertler, and Martinez (2019).

[10]Fisher (2015) showed the latter shock provides a simple micro-foundation for Smets and Wouters (2007)'s shock to the consumption Euler equation.

[11]It is often used to motivate why monetary policy might become constrained by the effective lower bound on nominal interest rates (see, for example, Eggertsson and Woodford (2003)), and so it is particularly relevant for our analysis which includes episodes when that this constraint is binding.

to Calvo price and wage setting frictions and charge markups that are subject to shocks. These "cost-push" shocks follow ARMA(1,1) processes.

The model also includes variable capital utilization with capital depreciation that is an increasing function of utilization; stochastic investment adjustment costs; and permanent shocks to neutral and investment-specific technologies. We refer to the shock to investment adjustment costs as the marginal efficiency of investment (MEI) shock. A positive MEI shock increases the yield of capital from an additional unit of investment. This drives investment up and consumption down. Investment rises by more than consumption falls, so output and hours also rise. Justiniano et al. (2010) and others show that this shock is an important driver of investment.

The neutral technology shock shifts the production functions of intermediate goods producers. In our model this shock is a major source of business cycle co-movement. A positive neutral technology shock raises the desired stock of capital and makes households richer. Therefore consumption, investment, and output rise. Because the substitution effect dominates the wealth effect, hours also rise.

A positive investment-specific technology shock increases the rate at which final goods can be transformed into investment goods. This shock turns out to be relatively unimportant for cyclical fluctuations.[12] The model includes stochastic government spending. These shocks also are unimportant for business cycles. The shocks to the growth rates of the two technology shifters and the government spending shock are all assumed to be AR(1) processes.

There is a central bank that sets its policy rate (the interest rate on one-period risk free government bonds) with a conventional policy rule. There are two shocks to this rule: surprise and news. Surprise is an addition or subtraction to the rule's constant that occurs in the period of the shock. News shocks add to or subtract from the constant in future periods, as in Campbell, Evans, Fisher, and Justiniano (2012) and Campbell et al. (2016)

[12]This is a common finding with empirical NK models. It contrasts with Fisher (2002), who found using structural VAR methods that these shocks are a significant driver of aggregate fluctuations.

who build on Laséen and Svensson (2011) and Gürkaynak, Sack, and Swanson (2005).[13] Without news shocks, agents' expectations of future policy rates could violate the effective lower bound on nominal interest rates (ELB). However, our estimation prevents this from happening because it matches data on expected future funds rates with the model's private expectations of future policy.[14] Note that including monetary policy news has the added benefit of allowing the model to explain strategic deviations from the rule, such as a policy of "lower for longer" in which lift-off from the ELB is delayed and slower than otherwise predicted by the monetary policy rule. That said, the news does not have to reflect explicit forward guidance by the central bank. The conventional policy rule includes gap terms that depend on publicly observable measures of output and inflation, and a near unit root random constant term to address inflation's low-frequency dynamics.

Government spending is financed by lump sum taxes and government bonds are in zero net supply. This simple fiscal block is standard in the literature. However, it is a limitation of our analysis of the Covid episode because the first wave of the virus spurred a substantial fiscal intervention that included both transfers and spending. This means our estimated Covid shock and its propagation will likely confound some of the effects of these policies. For example, the revisions to forecasters' expectations about GDP and inflation we use to identify the Covid shock and its propagation will include updated views of how fiscal policy will play out. While we recognize this limitation, it is a consequence of choosing an off-the-shelf model to demonstrate our approach.

Our empirical strategy involves the solution to the model's log-linearized equilibrium conditions and applies econometric techniques that rely on linearity to estimate the model's parameters. Using a linearized model to study the dramatic variation due to Covid is another

[13] Using insights from Chahrour and Jurado (2018), Campbell, Ferroni, Fisher, and Melosi (2019) show how including monetary policy news is equivalent to an environment in which the central bank communicates about future policy deviations via noisy signals where agents' use Bayes' rule to update their beliefs about those deviations.

[14] Since the ELB is not imposed explicitly, distributions of interest rates over states on given dates include negative values. Our model solution is certainty equivalent, so this does not influence agents' decisions. As such, our solution method does not take into account that the probability distributions of future outcomes are non-symmetric in models with occasionally binding constraints and that this asymmetry affects agents' beliefs and thereby equilibrium outcomes.

limitation of our analysis. However, our methods can be applied to non-linear settings.

4. Estimation of the DSGE model with the Covid shock

We now describe the estimation of our model's usual structural parameters with pre-pandemic data and the unusual shock and its propagation with data up to 2021q3. We estimate the model's usual structural parameters using Bayesian methods with data prior to the pandemic and then use the onset of the pandemic to estimate the nature and propagation of the Covid shock as described in Section 2.[15]

4.1. Pre-pandemic period

Our pre-pandemic estimation follows Campbell et al. (2019), and we refer the reader to that paper for most of the details. The pre-pandemic sample period is 1993q1–2016q4 and we assume a sample break in 2008q4. The sample break is motivated by the evidence of lower interest rates and trend economic growth later in the sample, the greater use of forward-looking communications by the Fed following the Great Financial Crisis, and the stabilization of inflation and inflation expectations in the mid-2000s. The sample break is characterized by unanticipated and permanent reductions in the return on government bonds and steady state growth, an increase in the horizon of anticipated deviations from the policy rule from four to ten quarters, and setting the variance of the inflation drift term to zero.[16]

The model is estimated with a rich array of data, including 26 time series in the first sample and an additional 6 quarters of interest rate futures prices in the second sample to identify anticipated deviations from the monetary policy rule over a longer horizon.[17] These data include GDP, consumption and investment growth, hours, multiple wage and

[15] A general overview of Bayesian estimation is provided in Herbst and Schorfheide (2015) and Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016)

[16] See Del Negro, Giannone, Giannoni, and Tambalotti (2017) for a more flexible way of modeling the trends in interest rates and output growth.

[17] The interest rate futures data is from the Chicago Fed. Unless otherwise noted all other data are from Haver Analytics. Our identification of the anticipated deviations with only 33 observations in the second sample relies on its factor structure and our priors. Our priors are informed by estimating a factor model over the second sample using Gürkaynak et al. (2005)'s high-frequency estimation strategy.

price inflation series, and series on expected future inflation, output, and interest rates. The expectations data are of one- to four-quarter-ahead expected core inflation as measured by both the Consumer Price Index (CPI) and the Price Index of Personal Consumption Expenditures (PCE), GDP growth, ten-year-ahead-average expected CPI and PCE inflation from the SPF, and one- to four-quarter-ahead interest rate futures. The second sample estimation is restricted to estimating the parameters of the monetary policy news, holding fixed the remaining model parameters at their values estimated using the first sample.[18] Our estimation forces data on real activity, wages, and prices to coexist with the interest rate futures data, and our model includes a preference for government bonds. These features yield plausible estimates of the effects of monetary news shocks.[19]

4.2. The first wave of the pandemic

We assume the synthetic Covid shock is composed of liquidity preference, permanent neutral technology, marginal efficiency of investment (MEI), discount factor, and inflation cost-push shocks ($M = 5$). The first two shocks are major sources of co-movement in the model, while the MEI and discount factor shocks are important determinants of consumption and investment but drive them in different directions. We include the price markup shock because we expect there is a cost-push aspect of Covid. We assume that agents try to anticipate the effects of the Covid shock up to four quarters ahead ($N = 4$). Note that the time horizon of the SPF forecasts ranges from one quarter to four quarters out, exactly matching the horizon of the anticipated Covid shocks in the model. As we shall explain, observing these expectations is key to identifying the Covid shocks.

We set $t^* = 2020\text{q}2$ and use the Kalman filter, data prior to that quarter, and our pre-pandemic parameter estimates to obtain the state vector in that quarter.[20] Since we find

[18]We also include two auxiliary inflation measures (which do not enter the DSGE model) to map the model concepts of output and inflation to the their empirical counterparts. We estimate the AR(1) processes for these variables separately for the two samples.

[19]The interest rate futures data are from the Chicago Fed. Unless otherwise noted all other data are from Haver Analytics.

[20]We also reduce the variance of the measurement error shocks in the equations bridging the model forecasts of GDP and core PCE inflation one to four quarters ahead with the SPF observed counterparts by

that $\lambda(t)$ is poorly identified when we try to estimate it in 2021q1, we assume $f_t = 0$ in 2021q1 and 2021q2.[21] We follow the strategy outlined in Section 2.3 and use data from 2020q2–2020q4 to obtain estimates of the Covid parameters, $\hat{\phi}$ and $\hat{\Xi}(t)$.

4.3. The Delta wave

With the arrival of the Delta variant in 2021q3 we assume another Covid shocks hits the model economy under the assumption that agents have learned from the first wave and so have perfect foresight of its propagation, as described in Section 2.2. The the size of the shock, δ, is estimated with 2021q3 data. A potential drawback of our approach to Delta is that the first wave of Covid came with a substantial fiscal intervention which potentially influenced our estimate of the composition and propagation of the synthetic shock. The Delta wave did not involve any new substantial fiscal interventions. We expect that a model with a richer fiscal structure would better isolate the pandemic-specific features of the initial wave.[22]

5. The estimated effects of Covid

In this section, we study the estimated Covid shock. First, we describe the estimated parameters and their identification. We then study the contributions of the unusual and usual shocks to the one-quarter-ahead forecast errors and forecast revisions of output and inflation in 2020q2 and the importance of including the Covid shock to explain the dynamics in Figure 2. Next we examine the effects of the unusual and usual shocks on aggregate activity and inflation over the pandemic period 2020q2 to 2021q3. Lastly, we study the role of beliefs in the propagation of the Covid shock.

Table 1: Parameter Estimates

	2020Q2	2020Q3	2020Q4	2021Q3
ϕ_b	0.0038	0.0038	0.0038	0.0038
ϕ_s	1	1	1	1
ϕ_i	0.1696	0.1696	0.1696	0.1696
ϕ_ν	-0.444	-0.444	-0.444	-0.444
ϕ_p	0.0103	0.0103	0.0103	0.0103
λ_0	1	1	1	1
λ_1	-0.1298	-0.3922	-0.0109	-0.5074
λ_2	-0.1044	-0.3535	-0.1051	-0.1193
λ_3	-0.1405	-0.225	-0.2956	-0.0052
λ_4	-0.1068	0.1724	0.2786	0.1021
σ_f	11.6863	11.6737	9.7799	NA

Note: The parameters ϕ_b, ϕ_s, ϕ_i, ϕ_ν, and ϕ_p, denote the loadings of the Covid shock onto its components which include the discount rate, liquidity preference, marginal efficiency of investment, permanent neutral technology, and the cost-push shocks. The entries in the rightmost column replicate the perfect anticipation of the path of the Covid shock over the horizon of the SPF forecasts. Since this is a deterministic path the variance is not applicable. The estimates are modal values of the posterior distribution based on normally distributed loose priors.

5.1. Parameter estimates and their identification

Our estimates of the Covid shock's parameters are displayed in Table 1. The first three columns shows the estimated ϕ_is and λ_js for 2020q2–2020q4. The final column shows the λ_is that replicate the perfectly anticipated propagation of the Covid shock. The parameters ϕ_s, ϕ_b, ϕ_i, ϕ_ν, and ϕ_p, denote the loadings of the Covid shock on the liquidity preference, discount rate, MEI, neutral technology, and cost-push components of Covid. These parameters are estimated using just 2020q2 data and they do not change across quarters. We normalize the loading for the liquidity preference component to 1. Covid loads more on this component than the others. Neutral technology loads negatively with near half the weight of liquidity

a factor of 10 starting in 2020q2. This is to more closely tie revisions to the SPF forecasts to our estimates of $\lambda(t)$.

[21] By poorly identified we mean the marginal likelihoods of the parameters become very flat.

[22] In principle we could re-estimate the nature and propagation of the Delta shock in 2021q3. We explored this approach but found the nature of the shock was poorly identified.

preference. The next largest loading is MEI which is more modest sized and positive. The discount factor and cost-push components have much smaller loadings that are positive.

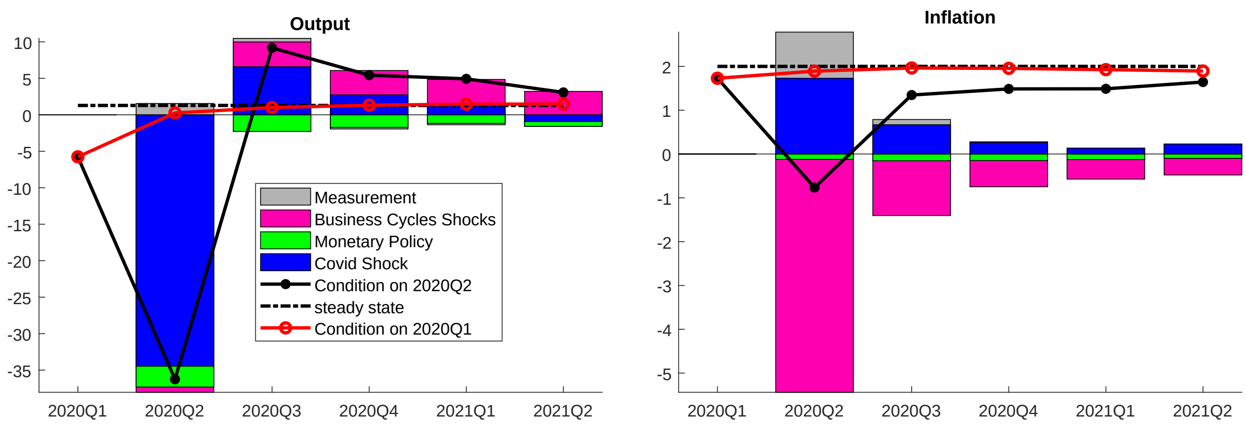
The large loadings on the liquidity preference and neutral technology wedges is broadly consistent with the widely held view that the Covid shock has both demand and supply side aspects to it. A shock to the liquidity preference wedge is a demand shock — it moves output and prices in the same direction. Recall that a positive liquidity preference shock increases the demand for government bonds over private capital and creates a desire to consume less today compared to tomorrow. This means the positive shock reduces both consumption and investment and is therefore broadly contractionary.[23] A positive liquidity preference shock has a small impact on inflation because of the flat Phillips curve. A shock to the neutral technology wedge is a supply shock as it moves output and prices in opposite directions. A negative neutral technology shock lowers consumption because of the wealth effect, investment because of capital becoming less productive, and hours because the of the substitution effect on labor supply dominating the wealth effect. Negative neutral technology shocks are significantly inflationary because they lower productivity directly.

The loadings λ of the common factor onto the Covid news shocks are re-estimated each quarter. From equation (2) we see that Covid news j-steps ahead reflects one-step-ahead forecast errors of the Covid shock. The loading of the common factor f_t onto the unanticipated component of the Covid shock, λ_0, is normalized to one. In 2020q2 the Covid shock is entirely unanticipated by construction. The negative values of the λs in that quarter indicate expectations of the Covid shock going forward were a persistent reversal from the unanticipated shock. The table shows there are similar revisions to expectations in the remaining quarters of 2020.

We can gain insight into the sign reversal by studying the one-step-ahead forecast errors and revisions to expectations for GDP growth and core PCE inflation in 2020q2. These are displayed in Figure 3. The red lines indicate the forecast is conditioned on 2020q1 data and the black line indicates conditioning on data in 2020q2. Note that the forecasts for

[23]This contrasts with a positive discount factor shock that causes investment to rise by more than consumption falls.

Figure 3: One-step-ahead forecast error decomposition in 2020q2



Note: This figure features the decomposition of one-step-ahead forecast error of output and inflation into the parts attributed to the Covid shock, usual business cycle shocks, surprise and news monetary policy shocks, and measurement. The black lines indicate 2020q2 output and inflation and SPF forecasts of these variables one to four quarters ahead. The red line is the forecast conditioned on 2020q1 data. The colored bars show the contribution to the forecast error of the indicated shocks. Source: Authors' calculations, SPF, and Haver Analytics.

GDP growth and core PCE growth in 2020q2 are essentially equal to the SPF data in that quarter.[24] The units are percentage points at an annual rate. In 2020q2 output collapsed and prices fell, but forecasters expected a quick rebound of both in 2020q3. The expected rebound explains the sign reversal in our estimates of the λs in 2020q2. Figure 3 also reveals output growth and inflation were expected to revert to the steady state and target gradually (output from above and inflation from below).

5.2. Contribution of the Covid shock to forecast errors and revisions in 2020q2

The colored bars in Figure 3 show the decomposition of the forecast errors and revisions into contributions of the Covid shock, including both surprise and news (blue), the usual business cycle shocks (pink), surprise and news shocks to monetary policy (green), and measurement (grey).[25] The left plot in Figure 3 shows that Covid explains almost all the

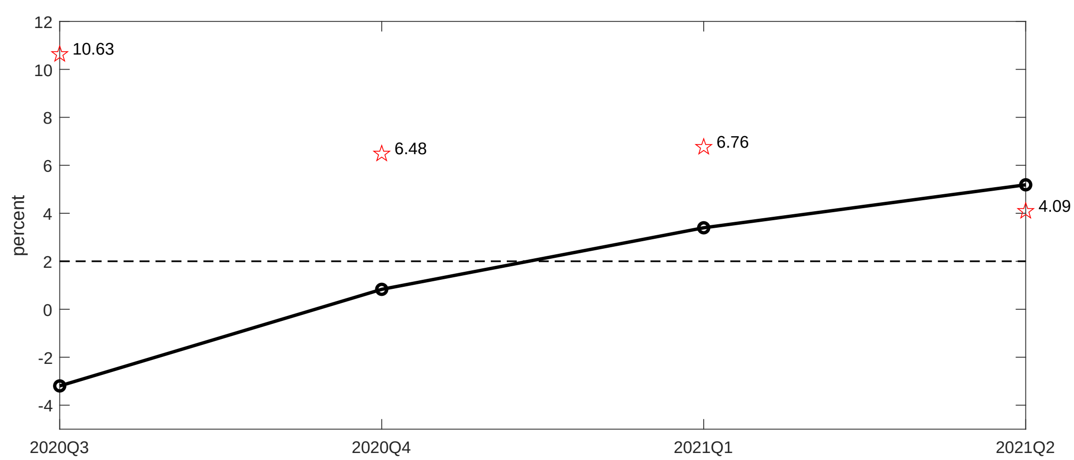
[24] When we estimate the DSGE model we allow for measurement error in the SPF forecasts. For the Covid period we reduce the variances of the measurement error by a factor of 10 so there is very little difference between the SPF forecasts and the model forecasts.

[25] Measurement includes shocks to variables from outside the model used to map model-consistent GDP and core PCE inflation into their BEA counterparts and classical measurement error on wage and price inflation and interest rate futures.

drop in output in 2020q2 and accounts for a substantial fraction of the rebound anticipated to occur 2020q3. Monetary policy is expected to be a drag on activity throughout the forecast horizon, presumably because of expectations that effective lower bound on nominal interest rates will be binding. The usual shocks reappear as substantial drivers of expected activity and inflation in 2020q3.

We do not scale down the volatility of the usual shocks in q2 (our estimation is conditional on the model's parameters that were estimated using data from before the pandemic) and so let the data speak about the relative contribution of the Covid shock. If the usual shocks were useful to explain the dynamics of output and inflation and the other observables in 2020q2, the likelihood would have attributed a junior role to the unusual shock. But this is not what Figure 3 shows us. The Covid shock explains almost all of the dramatic contraction in output in 2020q2 and more than half of the expected rebound in the next quarter (the atypical pattern we highlighted in Figure 2).

Figure 4: Forecast in 2020q2 of GDP growth from the SPF and the DSGE model that excludes the Covid shock



Note: Red stars indicate the median SFP forecast in 2020q2 of GDP growth in 2020q3–2021q2. The black line shows the DSGE model's inferred forecast without the Covid shock but with otherwise identical parameters. The difference between the black line and the red stars is due to measurement error identified by the Kalman filter. Source: Survey of Professional Forecasters and authors' calculations.

Why does the likelihood function attribute such a large role to the unusual shock? Figure 4 provides some insight into this question. The red stars denote SPF forecasts in 2020q2 for GDP growth over the next four quarters.[26] The black line shows the forecast based on inferring the usual shocks in 2020q2 from the model without the Covid shock. The difference between them is measurement error, which provides a visual characterization of the model's struggle to explain the data when the Covid shock is shut down. This error is very large to explain the SPF forecasts of GDP growth over the next three quarters; specifically, the error is three orders of magnitude larger than its standard deviation in 2020q3. This standard deviation is estimated using data from the more recent pre-pandemic recessions in Figures 1 and 2 and so the model infers shocks that inherit these dynamics, which are very different from the Covid dynamics. This implies that our medium-scale DSGE models with only their usual set of shocks cannot understand the unusual pandemic recession and its anticipated recovery shown in Figure 2.

The right plot of Figure 3 shows Covid pushed prices higher in 2020q2 and was expected to put upward pressure on prices through 2021q2. The model attributes the decline in current and expected inflation to the usual business cycle shocks. The fall in output and rise in prices attributed to Covid in 2020q2 indicate that the model interprets Covid on net as a supply shock. The accumulated effect of Covid on the levels of output and prices indicate that in 2020q2 agents expected the relatively strong supply effects of Covid to persist.

5.3. The effects of the shocks from 2020q2 through 2021q3

We now study the Covid shock's contributions to aggregate outcomes alongside the usual shocks over the period 2020q2–2021q3. We measure the shocks with the Kalman smoother (the results in the previous section use the filter). For ease of interpretation we group the model's usual shocks into five categories: demand, transitory supply, persistent supply, monetary policy, and other. The composition of each category is summarized in Table 2.

We will focus on the contributions of all the shocks to log per capita hours worked and core PCE inflation. Our empirical measure of hours is de-trended from outside the model using underlying trends in labor force participation and average hours per worker, as well as

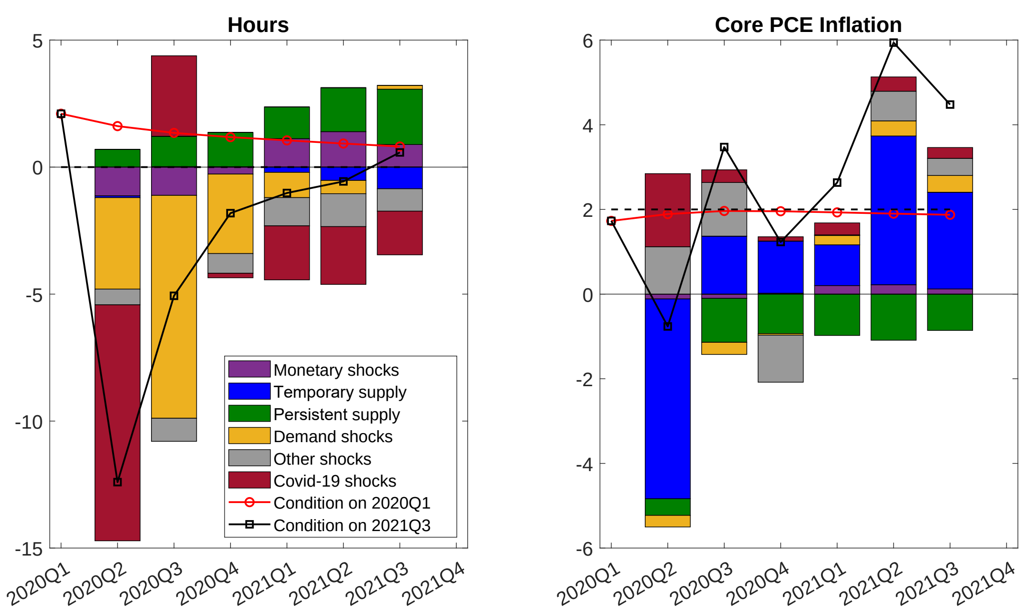
[26]We reduce the measurement error (estimated with pre-pandemic data) to ensure the model with Covid corresponds closely to the data

Table 2: Categories of usual shocks

Category	Usual shock
Demand	Liquidity preference + Discount factor
Transitory supply	Wage and price cost push
Persistent supply	Neutral technology + IS technology + MEI
Monetary policy	Unanticipated and anticipated
Other	Residual (government) spending + measurement

Note: IS denotes investment-specific. Residual spending includes net exports, inventory investment, and government spending. Measurement includes measurement error in core PCE, as well as shocks to consumer durable inflation and inflation in the consumption price of residual output. The latter two variables are used in the measurement equations to map model GDP and inflation onto the U.S. Bureau of Economic Analysis' GDP and core PCE inflation.

Figure 5: The estimated effects of the Covid shock and the usual shock, 2020q2–2021q3



Note: This figure features the Kalman smoothed decomposition of the contributions of the Covid shock and the model's usual shocks over the period 2020q2–2021q3. The black lines are data, and the red line is the forecast as of 2020q1. The units are percent. Log per capita hours is zero when at trend. Steady-state inflation equals two. Source: Haver Analytics, Board of Governors, Chicago Fed, and authors' calculations.

estimates of the natural rate of unemployment.[27] This measure of hours seems to be a good indicator of the cyclical position of the US economy.

The contributions are displayed in Figure 5. The red lines show the forecast of log hours (in percentage point deviations from its trend) and inflation conditioned on 2020q1 data. The black lines show the realizations of these variables from 2020q2 to 2021q3. The colored bars indicate contributions of the shocks to deviations from steady state. The sum of the colored bars corresponds to the difference between the red and the black lines.

The left-hand plot in Figure 5 shows the sharp contraction and fast recovery of the labor market. The Covid shock is the largest factor contributing to the sharp downturn in 2020q2. This shock is also largely responsible for the initial recovery in 2020q3. If not for demand shocks, hours would have been 10 percentage points higher. The Covid shock is a persistent drag on hours, significantly so in 2021. This seems consistent with the impact on labor supply often attributed to Covid (for example, the lower supply due to some people's Covid-related concerns about in-person public-facing jobs). The impact of Delta is very small (we estimate $\delta = .03$). While overall the contributions of Covid are substantial, the usual shocks play an important role as well. Demand shocks are a large drag on the labor market early on. Persistent supply shocks provide a notable boost to activity later on. Monetary policy is initially contractionary, but its effects turn positive at the beginning of 2021.

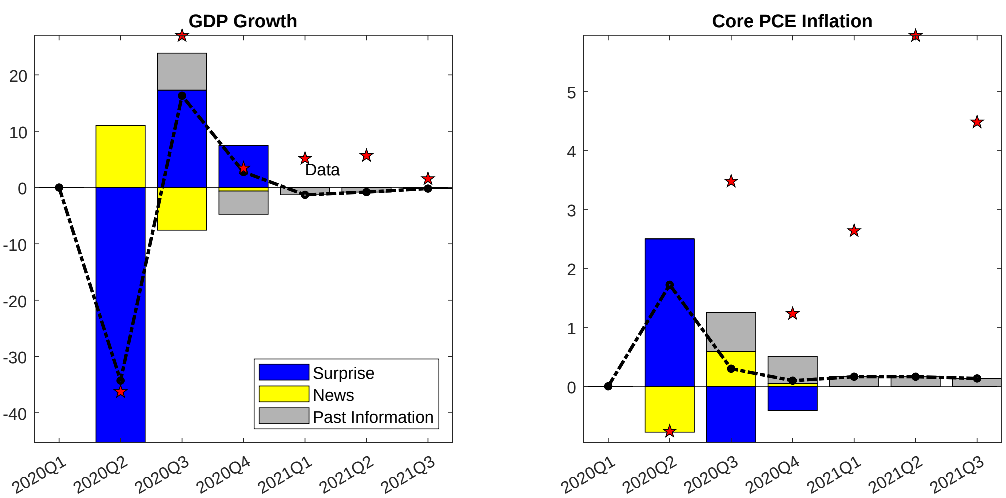
The right-hand plot in Figure 5 shows the volatility of core PCE inflation and its sharp rise in the middle of 2021. Covid is inflationary throughout, consistent with our earlier interpretation of it as being, on net, a supply shock. While it pushes up inflation, it does so by relatively little after 2020q2. The model attributes most of the gyrations in inflation to transitory supply shocks — in this case, price cost-push shocks. This latter finding is consistent with Del Negro et al. (2022) who estimate a differently specified Covid shock in a DSGE model (see footnote 5). Our estimates show Covid having a large positive impact on inflation in 2020q2 and remaining inflationary throughout the period.

[27] We use the trends that enter into the Federal Reserve Board of Governors' large scale macro-econometric model FRB/US, which are available on the Board's public web site. The natural rate is based on calculations at the Chicago Fed. Our measurement follows Campbell et al. (2016) and is described in the appendix.

5.4. The role of beliefs in the propagation of the Covid shock

The macroeconomic effects of the Covid shock in any given period can be fully decomposed into three parts: those due the current surprise (ψ_t^0), news about the future path of the Covid shock (ψ_t^1, ...,ψ_t^N), and the propagation through the economy of the past surprises and news. The latter is simply the sum of the impulse response functions of past surprise and news shocks. Figure 6 shows the contribution of the smoothed Covid shock to output growth and inflation decomposed into surprise (blue), news (yellow), and propagation (grey). The dashed line is the overall effect of Covid (the sum of the bars) and the red stars indicate data.

Figure 6: The role of beliefs about the Covid shock, 2020q2–2021q3



Note: This figure features the Kalman smoothed decomposition of the contributions of the surprise, news, and past surprise and news shocks as described in the main text. The dashed lines shows the total effects of the Covid shock. The bars decompose the total effects into three components. The blue bars show the contribution of the surprise shock in the period it is realized. The yellow bars show the contribution of news in the period it is received. The grey bars shows the contribution of past surprises and news through their propagation. The red stars indicate data. Source: Haver Analytics and authors' calculations.

The left plot shows that news about the future path of the Covid shock reduced the magnitude of the 2020q2 contraction in GDP by roughly 10 percentage points. In 2020q3 and 2020q4 the news about the pandemic dragged down activity, by 8 and .7 percentage points, respectively. This finding is consistent with concerns about the future path of the pandemic summarized in releases of Wolters Kluwer's *Blue Chip Economic Indicators* at the time. According to our model, beliefs of a recrudescence of the pandemic later on lowers current activity.

The left plot in Figure 6 also reveals that the macroeconomic consequences of the first wave of Covid were hard to anticipate. This is captured visually by the predominance of blue and yellow (compared to the gray). Still, the size of the grey bars suggests that at least some of the effects of Covid in 2020q3 and 2020q4 could be anticipated.[28] The contribution of news and surprise on inflation, as shown in the right plot, is the mirror image of that on GDP growth. This suggests that on net both surprise and news about the path of Covid were dominated by supply effects.

6. Conclusion

We proposed a new methodology for addressing unusual shocks in our usual business cycle models and used it to study the macroeconomic effects of the Covid-19 pandemic shock in a canonical medium-scale DSGE model. Our framework can be applied to estimate *any* DSGE model with data including the pandemic without necessarily having to model the detailed epidemiology. It also is easy to extend it to include more survey data that might help inform the propagation of the Covid shock.

Only when we introduce the Covid shock is the model able to account for the highly unusual dynamics exhibited in Figures 1 and 2. The Covid shock has both supply and demand side effects, but on net, the supply forces dominated. It accounts for a significant fraction of the early business cycle dynamics, was a persistent drag on aggregate activity, and was inflationary throughout. We also find that a majority of the effects of the Covid shock were unanticipated.

Our findings are based on the canonical model that has a rudimentary fiscal block. In order to better isolate the Covid shock one would need to consider a DSGE model with a sophisticated fiscal block to address the unusual fiscal policy that was implemented to cushion the blow of the pandemic. This is an important area for future research. Another important

[28] In the case of Delta, when we assume perfect foresight the yellow and blue would only appear in the period of the shock and the remainder of the path would be grey bars only.

area for future research is to consider the Covid shock in non-linear models. The size of the Covid shock suggests that non-linear effects could be important for its propagation.

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Appendix to "Unusual shocks in our usual models" by Filippo Ferroni, Jonas Fisher, and Leonardo Melosi[29]

This appendix describes the DSGE model and its estimation in detail. The first section presents the model economy's primitives. Section B gives the formulas used to remove nominal and technological trends from model variables and thereby induce model stationarity, and Sections C and D discuss the stationary economy's steady state and the log linearization of its equilibrium necessary conditions around it. Section E discusses measurement issues which arise when comparing model-generated data with data measured by the BEA and BLS. Section F describes our mixed Calibration-Bayesian Estimation empirical strategy and presents the resulting parameter values.

A. The Model's Primitives

Eight kinds of agents populate the model economy: Households, investment producers, competitive final goods producers, monopolistically-competitive differentiated goods producers, labor packers, monopolistically-competitive guilds, a fiscal authority, and a monetary authority. These agents interact with each other in markets for: final goods used for consumption and investment, investment goods used to augment the stock of productive capital, differentiated intermediate goods, capital services, raw labor, differentiated labor, composite labor, government bonds, privately-issued bonds, and state-contingent claims.

A.1. Households

Our model's households are the ultimate owners of all assets in positive net supply (the capital stock, differentiated goods producers, and guilds). They provide labor and divide their current after-tax income (from wages and assets) between current consumption, investment in productive capital, and purchases of financial assets, both those issued by the government and those issued by other households. The individual household divides its current resources between consumption and the available vehicles for intertemporal substitution (capital and financial assets) to maximize a discounted sum of current and expected future felicity.

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \varepsilon^b_{t+\tau} \left(U_{t+\tau} + \varepsilon^s_{t+\tau} L \left(\frac{B_{t+\tau}}{P_{t+\tau} R_{t+\tau}} \right) \right) \right]$$

with

$$U_t = \frac{1}{1-\gamma_c} \left((C_t - \varrho \bar{C}_{t-1})(1 - H_t^{1+\gamma_h}) \right)^{(1-\gamma_c)} \tag{9}$$

The function $L(\cdot)$ is strictly increasing, concave, and differentiable everywhere on $[0, \infty)$. In particular, $L'(0)$ exists and is finite. Without loss of generality, we set $L'(0)$ to one. The argument of $L(\cdot)$ equals the real value of government bonds in the household's portfolio:

[29] This appendix is co-authored with Jeffrey Campbell, University of Notre Dame and Tilberg University, jcampbel24@nd.edu

their period $t+1$ redemption value B_t divided by their nominal yield R_t expressed in units of the consumption good with the nominal price index P_t. The time-varying coefficient multiplying this felicity from bond holdings, ε_t^s, is the liquidity preference shock introduced by Fisher (2015). A separate shock influences the household's discounting of future utility to the present, ε_t^b. Specifically, the household discounts a certain utility in $t+\tau$ back to t with $\beta^\tau \mathbb{E}_t\left[\varepsilon_{t+\tau}^b/\varepsilon_t^b\right]$. In logarithms, these two preference shocks follow independent autoregressive processes.

$$\ln \varepsilon_t^b = (1-\rho_b)\ln \varepsilon_*^b + \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim \mathbb{N}(0,\sigma_b^2) \tag{10}$$

$$\ln \varepsilon_t^s = (1-\rho_s)\ln \varepsilon_*^s + \rho_s \ln \varepsilon_{t-1}^s + \eta_t^s, \eta_t^s \sim \mathbb{N}(0,\sigma_s^2). \tag{11}$$

A household's wealth at the beginning of period t consists of its nominal government bond holdings, B_t, its net holdings of privately-issued financial assets, and its capital stock K_{t-1}. The household chooses a rate of capital utilization u_t, and the capital services resulting from this choice equal $u_t K_{t-1}$. The cost of increasing utilization is higher depreciation. An increasing, convex and differentiable function $\delta(U)$ gives the capital depreciation rate. We specify this as

$$\delta(u) = \delta_0 + \delta_1(u-u_\star) + \frac{\delta_2}{2}(u-u_\star)^2.$$

A household can augment its capital stock with investment, I_t. Investment requires paying adjustment costs of the "i-dot" form introduced by Christiano, Eichenbaum, and Evans (2005). Also, an *investment demand shock* alters the efficiency of investment in augmenting the capital stock. Altogether, if the household's investment in the previous period was I_{t-1}, and it purchases I_t units of the investment good today, then the stock of capital available in the *next* period is

$$K_t = (1-\delta(u_t))\,K_{t-1} + \varepsilon_t^i\left(1 - S\left(\frac{A_{t-1}^K I_t}{A_t^K I_{t-1}}\right)\right) I_t. \tag{12}$$

In (12), A_t^K equals the productivity level of capital goods production, described in more detail below, and ε_t^i is the investment demand shock. In logarithms, this follows a first-order autoregression with a normally-distributed innovation.

$$\ln \varepsilon_t^i = (1-\rho_i)\ln \varepsilon_*^i + \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim \mathbb{N}(0,\sigma_i^2) \tag{13}$$

A.2. Production

The producers of investment goods use a linear technology to transform the final good into investment goods. The technological rate of exchange from the final good to the investment good in period t is A_t^I. We denote $\Delta \ln A_t^I$ with ω_t, which we call the *investment-specific technology shock* and which follows first-order autogregression with normally distributed innovations.

$$\omega_t = (1-\rho_\omega)\omega_\star + \rho_\omega \omega_{t-1} + \eta_t^\omega, \eta_t^\omega \sim \mathbb{N}(0,\sigma_\omega^2) \tag{14}$$

Investment goods producers are perfectly competitive.

Final good producers also operate a constant-returns-to-scale technology; which takes as inputs the products of the differentiated goods producers. To specify this, let Y_{it} denote the quantity of good i purchased by the representative final good producer in period t, for $i \in [0,1]$. The representative final good producer's output then equals

$$Y_t \equiv \left(\int_0^1 Y_{it}^{\frac{1}{1+\lambda_t^p}} di \right)^{1+\lambda_t^p}.$$

With this technology, the elasticity of substitution between any two differentiated products equals $1 + 1/\lambda_t^p$ in period t. Although this is constant across products within a time period, it varies stochastically over time according to an ARMA$(1,1)$ in logarithms.

$$\ln \lambda_t^p = (1-\rho_p) \ln \lambda_\star^p + \rho_p \ln \lambda_{t-1}^p - \theta_p \eta_{t-1}^p + \eta_t^p, \eta_t^p \sim \mathbb{N}(0, \sigma_p^2) \tag{15}$$

Given nominal prices for the intermediate goods P_{it}, it is a standard exercise to show that the final goods producers' marginal cost equals

$$P_t = \left(\int_0^1 P_{it}^{-\frac{1}{\lambda_t^p}} di \right)^{-\lambda_t^p} \tag{16}$$

Just like investment goods firms, the final goods' producers are perfectly competitive. Therefore, profit maximization and positive final goods output together require the competitive output price to equal P_t. Therefore, we can define inflation of the nominal final good price as $\pi_t \equiv \ln(P_t/P_{t-1})$.

The intermediate goods producers each use the technology

$$Y_{it} = (K_{it}^e)^\alpha \left(A_t^Y H_{it}^d \right)^{1-\alpha} - A_t \Phi \tag{17}$$

Here, K_{it}^e and H_{it}^d are the capital services and labor services used by firm i, and A_t^Y is the level of neutral technology. Its growth rate, $\nu_t \equiv \ln(A_t^Y/A_{t-1}^Y)$, follows a first-order autogregression.

$$\nu_t = (1-\rho_\nu)\,\nu_* + \rho_\upsilon \nu_{t-1} + \eta_t^\nu, \eta_t^\nu \sim \mathbb{N}(0, \sigma_\nu^2), \tag{18}$$

The final term in (17) represents the fixed costs of production. These grow with

$$A_t \equiv A_t^Y \left(A_t^I \right)^{\frac{\alpha}{1-\alpha}}. \tag{19}$$

We demonstrate below that A_t is the stochastic trend in equilibrium output and consumption, measured in units of the final good. We denote its growth rate with

$$z_t = \nu_t + \frac{\alpha}{1-\alpha} \omega_t \tag{20}$$

Similarly, define

$$A_t^K \equiv A_t A_t^I \tag{21}$$

In the specification of the capital accumulation technology, we labelled A_t^K the "productivity level of capital goods production." We demonstrate below that this is indeed the case with the definition in (21).

Each intermediate goods producer chooses prices subject to a Calvo (1983) pricing scheme. With probability $\zeta_p \in [0, 1]$, producer i has the opportunity to set P_{it} without constraints. With the complementary probability, P_{it} is set with the indexing rule

$$P_{it} = P_{it-1} \pi_{t-1}^{\iota_p} \pi_\star^{1-\iota_p}. \tag{22}$$

In (22), $\pi_\star$ is the gross rate of price growth along the steady-state growth path, and $\iota_p \in [0, 1]$.[30]

A.3. Labor Markets

Households' hours worked pass through two intermediaries, guilds and labor packers, in their transformation into labor services used by the intermediate goods producers. The guilds take the households' homogeneous hours as their only input and produce differentiated labor services. These are then sold to the labor packers, who assemble the guilds' services into composite labor services.

The labor packers operate a constant-returns-to-scale technology with a constant elasticity of substitution between the guilds' differentiated labor services. For its specification, let H_{it} denote the hours of differenziated labor purchased from guild i at time t by the representative labor packer. Then that packer's production of composite labor services, H_t^s are given by

$$H_t^s = \left(\int_0^1 (H_{it})^{\frac{1}{1+\lambda_t^w}} di \right)^{1+\lambda_t^w}.$$

As with the final good producer's technology, an ARMA$(1,1)$ in logarithms governs the constant elasticity of substitution between any two guilds' labor services.

$$\ln \lambda_t^w = (1 - \rho_w) \ln \lambda_\star^w + \rho_w \ln \lambda_{t-1}^w - \theta_w \eta_{t-1}^w + \eta_t^w, \eta_t^w \sim \mathbb{N}(0, \sigma_w^2) \tag{23}$$

Just as with the final goods producers, we can easily show that the labor packers' marginal cost equals

$$W_t = \left(\int_0^1 (W_{it})^{-\frac{1}{\lambda_t^w}} di \right)^{-\lambda_t^w}. \tag{24}$$

Here, W_{it} is the nominal price charged by guild i per hour of differentiated labor. Since labor packers are perfectly competitive, their profit maximization and positive output together

[30] To model firms' price-setting opportunities as functions of s_t, define a random variable u_t^p which is independent over time and uniformly distributed on $[0, 1]$. Then, firm i gets a price-setting opportunity if either $u_t^p \geq \zeta_p$ and $i \in [u_t^p - \zeta_p, u_t^p]$ or if $u_t^p < \zeta_p$ and $i \in [0, u_t^p] \cup [1 + u_t^p - \zeta_p, 1]$.

require that the price of composite labor services equals their marginal cost.

Each guild produces it's differentiated labor service using a linear technology with the household's hours worked as its only input. A Calvo (1983) pricing scheme similar to that of the differentiated goods producers constrains their nominal prices. Guild i has an unconstrained opportunity to choose its nominal price with probability $\zeta_w \in [0, 1]$. With the complementary probability, W_{it} is set with an indexing rule based on π_{t-1} and last period's trend growth rate, z_{t-1}.

$$W_{it} = W_{it-1} \left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota_w} \left(\pi_\star e^{z_\star}\right)^{1-\iota_w}. \tag{25}$$

In (25), $z_\star \equiv \nu_\star + \frac{\alpha}{1-\alpha}\omega_\star$ is the unconditional mean of z_t and $\iota_w \in [0, 1]$.

A.4. Fiscal and Monetary Policy

The model economy hosts two policy authorities, each of which follows exogenously-specified rules that receive stochastic disturbances. The fiscal authority issues bonds, B_t, collects lump-sum taxes T_t, and buys "wasteful" public goods G_t. Its period-by-period budget constraint is

$$G_t + B_{t-1} = T_t + \frac{B_t}{R_t}. \tag{26}$$

The left-hand side gives the government's uses of funds, public goods spending and the retirement of existing debt. The left-hand side gives the sources of funds, taxes and the proceeds of new debt issuance at the interest rate R_t. We assume that the fiscal authority keeps its budget balanced period-by-period, so $B_t = 0$. Furthermore, the fiscal authority sets public goods expenditure equal to a stochastic share of output, expressed in consumption units.

$$G_t = (1 - 1/g_t)Y_t, \tag{27}$$

with

$$\ln g_t = (1 - \rho_g) \ln s_\star^g + \rho_g \ln g_{t-1} + \eta_t^g, \eta_t^g \sim \mathbb{N}(0, \sigma_g^2). \tag{28}$$

The monetary authority sets the nominal interest rate on government bonds, R_t. For this, it employs a Taylor rule with interest-rate smoothing and forward guidance shocks.

$$\ln R_t = \rho_R \ln R_{t-1} + (1 - \rho_R) \ln R_t^n + \sum_{j=0}^{M} \xi_{t-j}^j. \tag{29}$$

The monetary policy disturbances in (29) are $\xi_t^0, \xi_{t-1}^1, \ldots, \xi_{t-M}^M$. The public learns the value of ξ_{t-j}^j in period $t - j$. The conventional unforecastable shock to current monetary policy is ξ_t^0, while for $j \geq 1$, these disturbances are *forward guidance shocks.* We gather all monetary shocks revealed at time t into the vector ε_t^R. This is normally distributed and *i.i.d.* across time. However, *its elements may be correlated with each other.* That is,

$$\varepsilon_t^R \equiv \left(\xi_t^0, \xi_t^1, \ldots, \xi_t^M\right) \sim \mathbb{N}(0, \Sigma_\varepsilon). \tag{30}$$

The off-diagonal elements of Σ^1 are not necessarily zero, so forward-guidance shocks need not randomly impact expected future monetary policy at two adjacent dates independently. Current economic circumstances influence R_t through the notional interest rate, R_t^n.

$$\ln R_t^n = \ln r_\star + \ln \pi_t^\star + \frac{\phi_1}{4}\mathbb{E}_t \sum_{j=-2}^{1} (\ln \pi_{t+j} - \ln \pi_t^\star) + \frac{\phi_2}{4}\mathbb{E}_t \sum_{j=-2}^{1} (\ln Y_{t+j} - \ln y^\star - \ln A_{t+j})\,. \quad (31)$$

The constant $r_\star$ equals the real interest rate along a steady-state growth path, and $\pi_t^\star$ is the central bank's intermediate target for inflation. We call this the *inflation-drift shock.* it follows a first-order autoregression with a normally-distributed innovation. Its unconditional mean equals $\pi_\star$, the inflation rate on a steady-state growth path.

$$\ln \pi_t^\star = (1-\rho_\pi)\pi_\star + \rho_\pi \ln \pi_{t-1}^\star + \eta_t^\pi, \eta_t^\pi \sim \mathbb{N}(0, \sigma_\pi^2) \quad (32)$$

Allowing $\pi_t^\star$ to change over time enables our model to capture the persistent decline in inflation from the early 1990s through the early 2000s engineered by the Greenspan FOMC.

A.5. Other Financial Markets and Equilibrium Definition

All households participate in the market for nominal risk-free government debt. Additionally, they can buy and sell two classes of privately issued assets without restriction. The first is one-period nominal risk-free *private* debt. We denote the value of household's net holdings of such debt at the beginning of period t with B_{t-1}^P and the interest rate on such debt issued in period t maturing in $t+1$ with R_{t+1}^P. The second asset class consists of a complete set of *real* state-contingent claims. As of the end of period t, the household's ownership of securities that pay off one unit of the aggregate consumption good in period τ if history s^τ occurs is $Q_t(s^\tau)$, and the nominal price of such a security in the same period is $J_t(s^\tau)$.

We define an equilibrium for our economy in the usual way: Households maximize their utility given all prices, taxes, and dividends from both producers and guilds; final goods producers and labor packers maximize profits taking their input and output prices as given; differentiated goods producers and guilds maximize the market value of their dividend streams taking as given all input and financial-market prices; differentiated goods producers and guilds produce to satisfy demand at their posted prices; and otherwise all product, labor, and financial markets clear.

B. Detrending

To remove nominal and real trends, we deflate nominal variables by their matching price deflators, and we detrend any resulting real variables influenced permanently by technological change. All scaled versions of variables are the lower-case counterparts.

$$c_t = \frac{C_t}{A_t} \qquad\qquad i_t = \frac{I_t}{A_t A_t^I}$$

$$k_t = \frac{K_t}{A_t A_t^I} \qquad k_t^e = \frac{K_t^e}{A_t A_t^I}$$

$$w_t = \frac{W_t}{A_t P_t} \qquad \tilde{w}_t = \frac{\tilde{W}_t}{A_t P_t}$$

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} \qquad \pi_t = \frac{P_t}{P_{t-1}}$$

$$y_t = \frac{Y_t}{A_t} \qquad mc_t = \frac{MC_t}{P_t}$$

$$r_t^k = \frac{R_t^k A_t^I}{P_t} \qquad w_t^h = \frac{W_t^h}{A_t P_t}$$

$$\lambda_t^1 = \Lambda_t^1 A_t^{\gamma_C} \qquad \lambda_t^2 = \Lambda_t^2 A_t^{\gamma_C} A_t^I$$

$$\varepsilon_t^s = A_t^{\gamma_C} \varepsilon_t^s$$

B.1. Detrended Equations

The detrended equations describing our model are listed in the following sections.

Households' FOC

$$\begin{aligned}
\lambda_t^1 &= \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \right]^{-\gamma_c} \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \\
\lambda_t^1 w_t^h &= (1+\gamma_h) \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{(1+\sigma_h)} \right) \right]^{-\gamma_c} \left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \varepsilon_t^h h_t^{\gamma_h} \\
\frac{\lambda_t^1}{R_t^P} &= \beta E_t \left[\frac{\lambda_{t+1}^1 e^{-\gamma_C z_{t+1}}}{\pi_{t+1}} \right] \\
\frac{\lambda_t^1}{R_t} - L'(0) \frac{\varepsilon_t^b \varepsilon_t^s}{R_t} &= \beta E_t \frac{\lambda_{t+1}^1}{\pi_{t+1}} e^{-z_{t+1}\gamma_C} \\
\lambda_t^1 &= \varepsilon_t^i \lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) + \beta E_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right] \\
\lambda_t^2 &= \beta E_t \left[e^{-\gamma_C z_{t+1} - \omega_{t+1}} \left(\lambda_{t+1}^1 r_{t+1}^k u_{t+1} + \lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right) \right] \\
\lambda_t^1 r_t^k &= \lambda_t^2 \delta'(u_t) \\
k_t &= (1 - \delta(u_t)) \, k_{t-1} e^{-z_t - \omega_t} + \varepsilon_t^i \, (1 - S(\cdot)) \, i_t \\
k_t^e &= u_t k_{t-1} e^{-z_t - \omega_t}
\end{aligned}$$

Final Goods Price Index

$$1 = \left[(1-\zeta_p) \tilde{p}_t^{\frac{1}{1-\lambda_{p,t}}} + \zeta_p (\pi_{t-1}^{\iota_p} \pi^{*(1-\iota_p)} \pi_t^{-1})^{\frac{1}{1-\lambda_{p,t}}} \right]^{1-\lambda_{p,t}}$$

Intermediate Goods Firms: Capital-Labor Ratio

$$\frac{k_t^e}{h_t^d} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

Intermediate Goods Firms: Real Marginal Costs

$$mc_t = \frac{w_t^{1-\alpha}\left(r_t^k\right)^{\alpha}}{\varepsilon_t^a \alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

Intermediate Goods Firms: Price-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \lambda_{t+s}^1 \frac{\tilde{y}_{t,t+s}}{\lambda_{p,t+s}-1}\left(\frac{A_{t+s}}{A_t}\right)^{1-\gamma_C}\left[\lambda_{p,t+s} mc_{t+s} - \tilde{X}_{t,s}^p \tilde{p}_t\right]$$

where

$$\tilde{X}_{t,s}^p = \left\{ \begin{array}{ll} 1 & : s=0 \\ \frac{\prod_{j=1}^s \pi_{t+j-1}^{1-\iota_p}\pi_*^{\iota_p}}{\prod_{j=1}^s \pi_{t+j}} & : s=1,\ldots,\infty \end{array} \right\}$$

$\tilde{y}_{t,t+s}$ denotes the time $t+j$ output sold by the producers that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing producers all choose the same price, then we do not have to carry the i-subscript.

Labor Packers: Aggregate Wage Index

$$w_t = \left[(1-\zeta_w)\tilde{w}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w\left(e^{\iota_w z_{t-1} - z_t} e^{(1-\iota_w)z_*}\pi_{t-1}^{\iota}\pi_t^{-1}\pi_*^{1-\iota_w} w_{t-1}\right)^{-\frac{1}{\lambda_{w,t}}}\right]^{-\lambda_{w,t}}$$

Guilds: Wage-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \beta \lambda_{t+s}^1 \left(\frac{A_{t+s}}{A_t}\right)^{1-\gamma_C} \frac{\tilde{h}_{t,t+s}}{\lambda_{w,t+s}}\left((1+\lambda_{w,t+s})w_{t+s}^h - \tilde{X}_{t,s}^l \tilde{w}_t\right)$$

where

$$\tilde{X}^l_{t,s} = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^{s}\left(\pi_{t+j-1}e^{z_{t+j-1}}\right)^{1-\iota_w}(\pi\gamma)^{\iota_w}}{\prod_{j=1}^{s}\pi_{t+j}e^{z_{t+j}}} & : s = 1,\ldots,\infty \end{array} \right\}$$

$\tilde{h}_{t,t+s}$ denotes the time $t+j$ labor supplied by the guild that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing guilds all choose the same wage, then we do not have to carry the i-subscript.

Monetary Authority

$$R_t = R_{t-1}^{\rho_R}\left[r_*\pi_t^*\left(\prod_{j=-2}^{1}\frac{\pi_{t+j}}{\pi_t^*}\right)^{\frac{\psi_1}{4}}\left(\prod_{j=-2}^{1}\frac{y_{t+j}}{y^*}\right)^{\frac{\psi_2}{4}}\right]^{1-\rho_R}\prod_{j=0}^{M}\xi_{t-j,j}$$

The Aggregate Resource Constraint

$$\frac{y_t}{g_t} = c_t + i_t$$

Production Function

$$y_t = \varepsilon_t^a\,(k_t^e)^\alpha\,(h_t^d)^{1-\alpha} - \Phi$$

Labor Market Clearing Condition

$$h_t = h_t^d$$

C. Steady State

We normalize most shocks and the utilization rate:

$$u_\star = 1 \qquad \varepsilon^i = 1$$
$$\varepsilon^a = 1 \qquad \varepsilon^b = 1$$

Next, we set the following restriction on adjustment costs:

$$S(\cdot_*) \equiv 0$$
$$S'(\cdot_*) \equiv 0$$

C.1. Prices and Interest Rates

Given β, z_*, γ_C, and π_*, we can solve for the steady-state nominal interest rate on private bonds R_*^P by using the FOC on private bonds:

$$R_*^P = \frac{\pi_*}{(\beta e^{-\gamma_C z_*})} \tag{33}$$

From the definition of $\delta(u)$, we have

$$\begin{aligned} \delta(1) =& \delta_0 \\ \delta'(1) =& \delta_1. \end{aligned}$$

Next, given ω_*, δ_0, and the above, we can solve for the real return on capital r_*^k using the FOC on capital:

$$r_*^k = \frac{e^{\gamma_C z_* + \omega_*}}{\beta} - (1 - \delta_0) \tag{34}$$

C.2. Ratios

Moving to the production side, we can use the aggregate price equation to solve for $\tilde{p}_*$:

$$\tilde{p}_* = 1$$

Using this result and given $\lambda_{p,*}$, we can use the price Phillips curve to solve for mc_*:

$$mc_* = \frac{1}{1 + \lambda_{p,*}} \tag{35}$$

Given values for α and ε_*^a, we can use the marginal cost equation to solve for w_*:

$$w_* = \left(mc_* \alpha^\alpha (1 - \alpha)^{1-\alpha} (r_*^k)^{-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{36}$$

The definition of effective capital gives us a value for k_*^e in terms of k_*:

$$k_*^e = k_* e^{-z_* - \omega_*}$$

Calculating y_* using the labor share of output $1 - \alpha$:

$$y_* = \frac{w_* h_*}{1 - \alpha}$$

Using capital shares based off our value of α, we can calculate the output to capital ratio as follows:

$$\frac{y_*}{k_*^e} = \frac{r_*^k}{\alpha}$$

$$\frac{y_*}{k_*} = e^{-z_*-\omega_*}\frac{r_*^k}{\alpha}$$

Using the capital accumulation equation, we can get a value for $\frac{i_*}{k_*}$

$$\frac{i_*}{k_*} = 1 - (1-\delta_0)e^{-z_*-\omega_*}$$

Using the resource constraint, we can get $\frac{c_*}{k_*}$:

$$\frac{c_*}{k_*} = \frac{y_*}{k_* s_\star^g} - \frac{i_*}{k_*}$$

These ratios will give us the remaining steady-state levels and ratios:

$$k_* = y_*\left(\frac{y_*}{k_*}\right)^{-1} \qquad i_* = \frac{i_*}{k_*}k_*$$
$$c_* = \frac{c_*}{k_*}k_* \qquad g_* = g_y y_*$$

C.3. Liquidity Premium

Using the aggregate wage equation, we can get the following for $\tilde{w}_*$:

$$\tilde{w}_* = w_*$$

Combining this result with the wage Phillips curve, we get the following:

$$w_*^h = \frac{w_*}{1+\lambda_{w,*}}$$

We can use the FOC for consumption and the labor supply to pin down ε^h and λ_*^1

$$\varepsilon^b\left[c_*\left(1-\frac{\varrho}{e^z}\right)\right]^{-\gamma_c}\left(1-\varepsilon^h h_*^{(1+\gamma_h)}\right) - \lambda_*^1 = 0$$
$$-(1+\gamma_h)\varepsilon^b c_*^{(1-\gamma_c)}\left(1-\frac{\varrho}{\varepsilon^z}\right)^{(1-\gamma_c)}\left(1-\varepsilon^h h_*^{(1+\gamma_h)}\right)^{-\gamma_c}\varepsilon^h h_*^{\gamma_h} + \lambda_*^1 w_*^h = 0$$

Finally, the government bond rate is calculated from

$$\lambda_*^1 - \varepsilon_*^b\varepsilon_*^s = \beta R_*\frac{\lambda_*^1}{\pi_*}e^{-\gamma_C z}$$
$$\underbrace{\frac{\pi_*}{\beta e^{-\gamma_C z}}}_{R_*^P} - \varepsilon_*^b\varepsilon_*^s\frac{\pi_*}{\beta e^{-\gamma_C z}\lambda_*^1} = R_*$$

Noting that $R^P_* = \frac{\pi_*}{\beta e^{-\gamma_C z}}$ we can write

$$\frac{R^P_* - R_*}{R^P_*} = \frac{\varepsilon^b_* \varepsilon^s_*}{\lambda^1_*}.$$

This is the liquidity premium in steady state.

D. Log Linearization

Hatted variables refer to log deviations from steady-state ($\hat{x} = \ln\left(\frac{x_t}{x_*}\right)$). In the cases of z_t, ω_t, and ν_t, we have that $\hat{x} = x_t - x_*$ as these variables are already in logs.

Households' First Order Conditions

$$\hat{\varepsilon}^b_t - \hat{\lambda}^1_t - \gamma_c \frac{1}{1 - \frac{\varrho}{e^z}} \hat{c}_t + \gamma_c \frac{\frac{\varrho}{e^z}}{1 - \frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{37}$$

$$\hat{\lambda}^1_t + \hat{w}^h_t - \hat{\varepsilon}^b_t - \hat{\varepsilon}^h_t - \frac{1 - \gamma_c}{1 - \frac{\varrho}{e^z}} \hat{c}_t + (1 - \gamma_c) \frac{\frac{\varrho}{e^z}}{1 - \frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{38}$$

$$- \left(\gamma_h + \gamma_c (1 + \gamma_h) \frac{\varepsilon^h h_*^{1+\gamma_h}}{(1 - \varepsilon^h h_*^{1+\gamma_h})^2} \right) \hat{h}_t = 0$$

$$\hat{\lambda}^1_t = \frac{R^P_* - R_*}{R^P_*} (\hat{\varepsilon}^s_t + \hat{\varepsilon}^b_t) + \frac{R_*}{R^P_*} (\hat{R}_t + E_t[(\hat{\lambda}^1_{t+1} - \hat{\pi}_{t+1} - \gamma_C \hat{z}_{t+1}]) \tag{39}$$

$$\hat{\lambda}^1_t = E_t \left[\hat{\lambda}^1_{t+1} - \gamma_C \hat{z}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right] \tag{40}$$

$$\hat{\lambda}^1_t = \left(\ln \varepsilon^i_t + \hat{\lambda}^2_t \right) - S'' (\hat{\imath}_t - \hat{\imath}_{t-1}) + \beta e^{(1-\gamma_C)\gamma} S'' E_t (\hat{\imath}_{t+1} - \hat{\imath}_t) \tag{41}$$

$$\lambda^2_* \hat{\lambda}^2_t = \beta e^{-\gamma_C z_* - \omega_*} \left[\lambda^1_* u_* r^k_* E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}^1_{t+1} + \hat{r}^k_{t+1} + \hat{u}_{t+1} \right) \right] + \tag{42}$$

$$+ \beta e^{-\gamma_C z_* - \omega_*} \left[(1 - \delta_0) \lambda^2_* E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}^2_{t+1} \right) - \lambda^2_* \delta_1 u_* E_t \hat{u}_{t+1} \right]$$

$$\hat{\lambda}^1_t = \hat{\lambda}^2_t + \frac{\delta_2}{\delta_1} u_* \hat{u}_t - \hat{r}^k_t \tag{43}$$

$$\hat{k}_t = \left(1 - \frac{\varepsilon^i_* i_*}{k_*} \right) \left(\hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \right) + \frac{\varepsilon^i_* i_*}{k_*} \left(\hat{\varepsilon}^i_t + \hat{\imath}_t \right) - \delta_1 u_* e^{-z_* - \omega_*} \hat{u}_t \tag{44}$$

$$\hat{k}^e_t = \hat{u}_t + \hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \tag{45}$$

Capital-Labor Ratio

$$\hat{k}^e_t = \hat{w}_t - \hat{r}^k_t + \hat{h}^d_t \tag{46}$$

Real Marginal Costs

$$\widehat{mc}_t = (1-\alpha)\,\hat{w}_t + \alpha \hat{r}_t^k - \hat{\varepsilon}_t^a \tag{47}$$

The New Keynesian Phillips Curve for Inflation

$$\begin{aligned}\hat{\pi}_t = \frac{(1-\beta\zeta_p e^{(1-\gamma_C)z_*})(1-\zeta_p)}{(1+\beta\iota_p e^{(1-\gamma_C)z_*})\zeta_p}\left[\frac{\lambda_{p,*}}{1+\lambda_{p,*}}\hat{\lambda}_{p,t} + \widehat{mc}_t\right] + \\ + \frac{\iota_p}{1+\beta\iota_p e^{(1-\gamma_C)z_*}}\hat{\pi}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta\iota_p e^{(1-\gamma_C)z_*}} E_t\hat{\pi}_{t+1}\end{aligned} \tag{48}$$

Wage Mark-Up

$$\hat{\mu}_t^w = \hat{w}_t - \hat{w}_t^h \tag{49}$$

The New Keynesian Phillips Curve for Wages

$$\begin{aligned}\hat{w}_t = {} & \frac{1}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t+1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}(E_t\hat{\pi}_{t+1} + E_t\hat{z}_{t+1}) + \\ & \frac{\iota_w}{1+\beta e^{(1-\gamma_C)z_*}}(\hat{\pi}_{t-1} + \hat{z}_{t-1}) - \frac{1+\iota_w\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}(\hat{\pi}_t + \hat{z}_t) + \\ & \frac{1-\beta\zeta_w e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\frac{1-\zeta_w}{\zeta_w}\left[\frac{\lambda_{w,*}}{1+\lambda_{w,*}}\hat{\lambda}_{w,t} - \hat{\mu}_t^w\right]\end{aligned} \tag{50}$$

The Aggregate Resource Constraint

$$\frac{y_*}{g_*}(\hat{y}_t - \hat{g}_t) = \frac{c_*}{c_* + i_*}\hat{c}_t + \frac{i_*}{c_* + i_*}\hat{\imath}_t \tag{51}$$

The Production Function

$$\hat{y}_t = \frac{1}{mc_*}\left(\ln\varepsilon_t^a + \alpha\hat{k}_t^e + (1-\alpha)\,\hat{h}_t^d\right) \tag{52}$$

Labor Market Clearing Condition

$$\hat{h}_t = \hat{h}_t^d \tag{53}$$

Monetary Authority's Reaction Function

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[(1-\psi_1)\,\hat{\pi}_t^* + \frac{\psi_1}{4}\left(\sum_{j=-2}^{1}\hat{\pi}_{t+j}\right) + \frac{\psi_2}{4}\left(\sum_{j=-2}^{1}\hat{y}_{t+j}\right)\right] + \sum_{j=0}^{M}\hat{\xi}_{t-j,j} \tag{54}$$

E. Measurement

E.1. National Income Accounts

The model economy's basic structure, with the representative household consuming a single good and accumulating capital using a different good, differs in some important ways from the accounting conventions of the Bureau of Economic Analysis (BEA) underlying the National Income and Product Accounts (NIPA). In particular

1. The BEA treats household purchases of long-lived goods inconsistently. It classifies purchases of residential structures as investment and treats the service flow from their stock as part of Personal Consumption Expenditures (PCE) on services. The BEA classifies households purchases of all other durable goods as consumption expenditures. No service flow from the stock of household durables enters measures of current consumption. In the model, all long-lived investments add to the productive capital stock.
2. In our model all government purchases are consumption. In fact government spending includes investment goods purchased on behalf of the populace. In the model, these should be treated as additions to the single stock of productive capital.
3. The BEA sums PCE and private expenditures on productive capital (Business Fixed Investment and Residential Investment), with government spending, inventory investment, and net exports to create Gross Domestic Product. The model features only the first three of these.

To bridge these differences, we create four *model consistent* NIPA measures from the BEA NIPA data.

1. Model-consistent GDP. Since the model's capital stock includes both the stock of household durable goods and the stock of government-purchased capital, a model-consistent GDP series should include the value of both stocks' service flows. To construct these, we followed a five-step procedure.
   (a) We begin by estimating a constant (by assumption) service-flow rate by dividing the nominal value of housing services from NIPA Table 2.4.5 by the beginning-of-year value of the residential housing stock from the BEA's Fixed Asset Table 1.1. We use annual data and average from 1947 through 2014. The resulting estimate is 0.096. That is, the annual value of housing services equals approximately 10 percent of the housing stock's value each year.

(b) In the second step, we estimate estimate constant (by assumption) depreciation rates for residential structures, durable goods, and government capital. We constructed these by first dividing observations of value lost to depreciation over a calendar year by the end-of-year stocks. Both variables were taken from the BEA's Fixed Asset Tables. (Table 1.1 for the stocks and Table 1.3 for the deprecation values.) We then averaged these ratios from 1947 through 2014. The resulting estimates are 0.021, 0.194, and 0.044 for the three durable stocks.

(c) In the third step, we calculated the average rates of real price depreciation for the three stocks. For this, we began with the nominal values and implicit deflators for PCE Nondurable Goods and PCE Services from NIPA Table 1.2. We used these series and the Fisher-ideal formula to produce a chain-weighted implicit deflator for PCE Nondurable Goods and Services. Then, we calculated the price for each of the three durable good's stocks in consumption units as the ratio of the implicit deflator taken from Fixed Asset Table 1.2 to this deflator. Finally, we calculated average growth rates for these series from 1947 through 2014. The resulting estimates equal 0.0029, -0.0223, and 0.0146 for residential housing, household durable goods, and government-purchased capital.

(d) The fourth combines the previous steps' calculations to estimate constant (by assumption) service-flow rates for household durable goods and government-purchased capital. To implement this, we assumed that all *three* stocks yield the same financial return along a steady-state growth path. These returns sum the per-unit service flow with the appropriately depreciated value of the initial investment. This delivers two equations in two unknowns, the two unknown service-flow rates. The resulting estimates are 0.29 and 0.12 for household durable goods and government-purchased capital.

(e) The fifth and final step uses the annual service-flow rates to calculate real and nominal service flows from the real and nominal stocks of durable goods and government-purchased capital reported in Fixed Asset Table 1.1. This delivers an annual series. Since the stocks are measured as of the end of the calendar year, we interpret these as the service flow values in the *next* year's first quarter. We create quarterly data by linearly interpolating between these values.

With these real and nominal service flow series in hand, we create nominal model-consistent GDP by summing the BEA's definition of nominal GDP with the nominal values of the two service flows. We create the analogous series for model-consistent real GDP by applying the Fisher ideal formula to the nominal values and price indices for these three components.

2. Model-consistent Investment. The nominal version of this series sums nominal Business Fixed Investment, Residential Investment, PCE Durable Goods, and government investment expenditures. The first three of these come from NIPA Table 1.1.5, while government investment expenditures sums Federal Defense, Federal Nondefense, and

State and Local expenditures from NIPA Table 1.5.5. We construct the analogous series for real Model-consistent Investment by combining these series with their real chain-weighted counterparts found in NIPA Tables 1.1.3 and 1.5.3 using the Fisher ideal formula. By construction, this produces an implicit deflator for Model-consistent investment as well.

3. Model-consistent Consumption. The nominal version of this series sums nominal PCE Nondurable Goods, PCE Services, and the series for nominal services from the durable goods stock. The first two of these come from NIPA Table 1.1.5. We construct the analogous series for real Model-consistent consumption by combining these series with their real chain-weighted counterparts using the Fisher ideal formula. The two real PCE series come from NIPA Table 1.1.3. Again, this produces an implicit deflator for Model-consistent consumption as a by-product.

4. Model-consistent Government Purchases. Conceptually, the model's measure of Government Purchases includes all expenditures not otherwise classified as Investment or Consumption: Inventory Investment, Net Exports, and actual Government Purchases. We construct the nominal version of this series simply by subtracting nominal Model-consistent Investment and Consumption from nominal Model-consistent GDP. We calculate the analogous real series using "chain subtraction." This applies the Fisher ideal formula to Model-consistent GDP and the *negatives* of Model-consistent Consumption and Investment.

Our empirical analysis requires us to compare model-consistent series measured from the NIPA data with their counterparts from the model's solution. To do this, we begin by solving the log-linearized system above, and then we feed the model specific paths for all exogenous shocks starting from a particular initial condition. for a given such simulation, the growth rates of Model-consistent Consumption and Investment equal

$$\begin{aligned} \Delta \ln C_t^{obs} &= z_* + \Delta \hat{c}_t + z_t \text{ and} \\ \Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta \hat{i}_t + z_t + \omega_t \end{aligned}$$

The measurement of GDP growth in the model is substantially more complicated, because the variables Y_t and y_t denote model output *in consumption units.* In contrast, we mimic the BEA by using a chain-weighted Fisher ideal index to measure model-consistent GDP. Therefore, we construct an analogus chain-weighted GDP index from model data. Since such an ideal index is invariant to the units with which nominal prices are measured, we can normalize the price of consumption to equal one and employ the prices of investment goods and government purchases relative to current consumption. Our model identifies the first of these relative prices as with investment-specific technology. However, the model characterizes only government purchases *in consumption units*, because private agents do not care about their division into "real" purchases and their relative price. For this reason, we use a simple autoregression to characterize the evolution of the price of government services in consumption units. Denote this price in quarter t with P_t^g. We construct this for the US economy by dividing the Fisher-ideal price index for model-consistent government purchases by that for model-consistent consumption. Then, our model for its evolution is

$$\pi_t^{g,obs} = \ln(P_t^g/P_{t-1}^t) = (1-\beta_{2,1}-\beta_{2,2})\pi_g^* + \beta_{2,1}\ln(P_{t-1}^g/P_{t-2}^g) + \beta_{2,2}\ln(P_{t-2}^g/P_{t-3}^g) + u_t^g. \quad (55)$$

Here, $u_t^g \sim \mathbb{N}(0,\sigma_g^2)$. Given an arbitrary normalization of P_t^g to one for some time period, simulations from (55) can be used to construct simulated values of P_t^g for all other time periods. With these and a simulation from the model of all other variables in hand, we can calculate the simulation's values for Fisher ideal GDP growth using

$$\frac{Q_t}{Q_{t-1}} \equiv \sqrt{\dot{Q}_t^P \dot{Q}_t^L}, \quad (56)$$

where the Paasche and Laspeyres indices of quantity growth are

$$\dot{Q}_t^P \equiv \frac{C_t + P_t^I I_t + P_t^G(G_t/P_t^G)}{C_{t-1} + P_t^I I_{t-1} + P_t^G(G_{t-1}/P_{t-1}^g)} \text{ and} \quad (57)$$

$$\dot{Q}_t^L \equiv \frac{C_t + P_{t-1}^I I_t + P_{t-1}^G(G_t/P_t^G)}{C_{t-1} + P_{t-1}^I I_{t-1} + P_{t-1}^G(G_{t-1}/P_{t-1}^G)}. \quad (58)$$

In both (57) and (58), P_t^I is the relative price of investment to consumption. In equilibrium, this always equals A_t^I.

The above gives a complete recipe for *simulating* the growth of model-consistent real GDP growth. However, we also embody its insights into our estimation with a log-linear approximation. For this, we start by removing stochastic trends from all variables in (57) and (58), and we proceed by taking a log-linear approximation of the resulting expression. Details are available from the authors upon request.

E.1.1. Output Growth Expectations

We also discipline our model's inferences about the state of the economy by comparing expectations of one- to four-quarter ahead real GDP growth from the Survey of Professional Forecasters with the analogous expectations from our model. The Survey of Professional Forecasters did not report these expectations prior to 2007, so we use them only in the second sample. As discussed in previous section, the quarterly per-capita model-consistent real GDP growth ($\Delta \ln Q_t$) does not map one-to-one with the SPF forecast of the BEA annual real GDP growth ($\Delta \ln Y_t^{BEA}$). So we transform the former into the latter by adding back population growth to the per-capita model-consistent real GDP growth and by fitting a linear regression model of BEA real GDP growth on model-consistent real GDP growth over the sample 1993:Q1-2016Q4. In particular, we estimate the following model

$$\Delta \ln Y_t^{BEA} = \underbrace{a}_{-0.14} + \underbrace{b}_{1.06}[4 \times (\Delta \ln Q_t^{obs} + pop_t)] \quad R^2 = 0.996$$

When we bridge model and SPF forecasts, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The observation equations are

$$\Delta \ln Y_t^{l,obs} \;\; = \;\; a + 4b(\Delta \ln Q_t^{l,obs} + pop_t^l), \quad l = 1, 2, 3, 4;$$

and we assume that population forecast is at 1 percent at annual rate throughout. The two measurement errors follow mutually-independent first-order autoregressive processes.

E.2. Hours Worked Measurement

Empirical work using DSGE models like our own typically measure labor input with hours worked per capita, constructed directly from BLS measures of hours worked and the civilian non-institutional population over age 16. However, this measure corresponds poorly with business cycle models because it contains underlying low frequency variation. This fact led us to construct a new measure of hours for the model using labor market trends produced for the FRB/US model and for the Chicago Fed's in-house labor market analysis.

We begin with a multiplicative decomposition of hours worked per capita into hours per worker, the employment rate of those in the labor force, and the labor-force participation rate. The BLS provides CPS-based measures of the last two rates for the US as a whole. However, its measure of hours per worker comes from the Establishment Survey and covers only the private business sector. If we use hours per worker in the business sector to approximate hours per worker in the economy as a whole, then we can measure hours per capita as

$$\frac{H_t}{P_t} = \frac{H_t^E}{E_t^E} \frac{E_t^C}{L_t^C} \frac{L_t^C}{P_t^C}.$$

Here, H_t and P_t equal total hours worked and the total population, H_t^E / E_t^E equals hours per worker measured with the Establishment survey, E_t^C / L_t^C equals one minus the CPS based unemployment rate, and L_t^C / P_t^C equals the CPS based labor-force participation rate. Our measure of *model-relevant* hours worked deflates each component on the right-hand side by an exogenously measured trend. The trend for the unemployment rate comes from the Chicago Fed's Microeconomics team, while those for hours per worker and labor-force participation come from the FRB/US model files.

E.3. Inflation

Our empirical analysis compares model predictions of price inflation, wage inflation, inflation in the price of investment goods relative to consumption goods, and inflation expectations with their observed values from the U.S. economy. We describe our implementations of these comparisons sequentially below.

E.3.1. Price Inflation

Our model directly characterizes the inflation rate for Model-consistent Consumption. In principle, this is close to the FOMC's preferred inflation rate, that for the implicit deflator of PCE. However, in practice the match between the two inflation rates is poor. In the data, short-run movements in food and energy prices substantially influences the short-run evolution of PCE inflation. Our model lacks such a volatile sector, so if we ask it to match observed short-run inflation dynamics, it will attribute those to transitory shocks to intermediate goods' producers' desired markups driven by λ_t^p.

To avoid this outcome, we adopt a different strategy for matching model and data inflation rates, which follows that of Justiniano, Primiceri, and Tambalotti (2013). This relates three observable inflation rates – core CPI inflation, core PCE inflation, and market-based PCE inflation – to Model-consistent consumption inflation using auxiliary observation equations. For core PCE inflation, this equation is

$$\pi_t^{1,obs} = \pi_* + \pi_*^1 + \beta^{\pi,1}\hat{\pi}_t + \gamma^{\pi,j}\pi_t^{d,obs} + u_t^{\pi,1}, \tag{59}$$

In (59) as elsewhere, π_* equals the long-run inflation rate. The constant π_*^1 is an adjustment to this long-run inflation rate which accounts for possible long-run differences between realized inflation and the FOMC's goal of $\pi_\star$ (for PCE inflation π_*^1 is set to zero). The right-hand side's inflation rates, $\hat{\pi}_t$ and $\pi_t^{d,obs}$ equal Model-consistent consumption inflation and PCE Durables inflation. We refer to the coefficients multiplying them, $\beta^{\pi,1}$ and $\gamma^{\pi,1}$, as the *inflation loadings*. We include PCE Durables inflation on the right-hand side of (59) because the principle adjustment required to transform Model-consistent inflation into core PCE inflation is the replacement of the price index for durable goods services with that for durable goods purchases. The disturbance term $u_t^{\pi,1}$ follows a zero-mean first-order autoregressive process.

The other two observed inflation measures, market-based PCE inflation and core CPI inflation, have identically specified observation equations. We use 2 and 3 in superscripts to denote these equations parameters and error terms, and we use the same expressions as subscripts to denote the parameters governing the evolution of their error terms. We assume that the error terms $u_t^{\pi,1}$, $u_t^{\pi,2}$, and $u_t^{\pi,3}$ are independent of each other at all leads and lags.

To produce forecasts of inflation with these these three observation equations, we must forecast their right-hand side variables. The model itself gives forecasts of $\hat{\pi}_t$. The forecasts of durable goods inflation come from a second-order autoregression.

$$\pi_t^{d,obs} = (1 - \beta_{1,1} - \beta_{1,2})\pi_*^d + \beta_{1,1}\pi_{t-1}^{d,obs} + \beta_{1,2}\pi_{t-2}^{d,obs} + u_t^d \tag{60}$$

Its innovation is normally distributed and serially uncorrelated.

E.3.2. Wage Inflation

Although observed wage inflation does not feature the same short-run variability as does price inflation, it does include the influences of persistent demographic labor-market trends which we removed ex ante from our measure of hours worked. Therefore, we follow the same general strategy of relating observed measures of wage inflation to the model's predicted wage inflation with a error-augmented observation equation. For this, we employ two measures of compensation per hour, Earnings per Hour and Total Compensation per Hour. In parallel with our notation for inflation measures, we use 1 and 2 to denote these two wage measures of wage inflation. The observation equation for Earnings per Hour is

$$\Delta \ln w_t^{1,obs} = z_* + w_*^j + \beta^{w,1} \left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{w,1}, \tag{61}$$

where "Δ" is the first difference operator. Just as with the price inflation measurement errors, $u_t^{w,1}$ follows a zero-mean first-order autoregressive process. The observation equation for Total Compensation per Hour is analogous to (61).

E.3.3. Relative Price Inflation

To empirically ground investment-specific technological change in the model, we use an error-augmented observation equation to relate the relative price of investment to consumption, both model-consistent measures constructed from NIPA and Fixed Asset tables as described above, with the model's growth rate of the rate of technological transformation between these two goods, ω_t.

$$\pi_t^{i,obs} = \omega_* + \hat{\omega}_t + u_t^{c/i};$$

Here, $\pi_t^{i,obs}$ denotes the price of *c*onsumption relative to *i*nvestment. The measurement error $u_t^{c/i}$ follows a i.i.d. zero-mean normally-distributed innovation.

We also discipline our model's inferences about the state of the economy by comparing expectations of one- to four-quarter ahead and 10-year inflation from the Survey of Professional Forecasters with the analogous expectations from our model. Just as with all of the other inflation measures, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The observation equations are

$$\begin{aligned}
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \beta^{l,j} E_t \hat{\pi}_{t+l} + u_t^{l,j,\pi}, \;\; j = 1, 2, \;\; l = 1, ...4; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l} \sum_{i=1}^{l} E_t \hat{\pi}_{t+i} + u_t^{l,j,\pi}, \;\; j = 1, 2, \;\; l = 40;
\end{aligned}$$

The measurement errors follow mutually-independent first-order autoregressive processes.

E.4. Interest Rates and Monetary Policy Shocks

Since our model features forward guidance shocks, it has non-trivial implications for the current policy rate as well as for expected future policy rates. To discipline the parameters governing their realizations, the elements of Σ_ε, using data, we compare the model's monetary policy shocks to high-frequency interest-rate innovations informed by event studies, such as that of Gürkaynak, Sack, and Swanson (2005). Those authors applied a factor structure to innovations in implied expected interest rates from futures prices around FOMC policy announcement dates. Specifically, they show that the vector of M implied interest rate changes following an FOMC policy announcement, Δr_t, can be written as

$$\Delta r_t = \Lambda f_t + \eta_t$$

Where f is a 2×1 vector of factors, Λ is a $H \times 2$ matrix of factor loadings, and η is an $H \times 1$ vector of mutually independent shocks. Denoting the 2×2 diagonal variance covariance matrix of f with Σ_f and the $H \times H$ diagonal variance-covariance matrix of η with Ψ, we can express the observed variance-covariance matrix of Δr as $\Lambda \Sigma_f \Lambda' + \Psi$.

Our model has implications for this same variance covariance matrix. For this, use the model's solution to express the changes in current and future expected interest rates following monetary policy shocks as $\Delta r = \Gamma_1 \varepsilon^R$. Here, ε_t^R is the vector which collects the current monetary policy shock with $M-1$ forward guidance shocks, and Γ_1 is an $H \times H$ matrix. In general, Γ_1 does *not* simply equal the identity matrix, because current and future inflation and output gaps respond to the monetary policy shocks and thereby influence future monetary policy "indirectly" through the interest rate rule.

We assume that a factor structure determines the cross-correlations among monetary policy shocks. Specifically, we assume

$$\varepsilon_{R,t}^j = \alpha_j f_t^\alpha + \beta_j f_t^\beta + \eta_t^j,$$

where the factors f_t^α and f_t^β and factor loadings α_i and β_i are scalars, η_t^j is a measurement error. The factors and shocks have zero means and are independent and normally distributed. In matrix notation, we have

$$\varepsilon_t^R = \boldsymbol{\alpha} f_t^\alpha + \boldsymbol{\beta} f_t^\beta + \eta_t,$$

where $\boldsymbol{\alpha} = [\alpha_0, \dots, \alpha_H]'$, $\boldsymbol{\beta} = [\beta_0, \dots, \beta_H]'$. Let $\Sigma_\eta = E(\eta_t \eta_t')$ denote the variance-covariance matrix of the idiosyncratic shocks, and σ_α^2 (σ_β^2) denote the variance of f_t^α (f_t^β). Therefore we have that

$$\Lambda \Sigma_f \Lambda' + \Psi = \Gamma_1 (\boldsymbol{\alpha}\boldsymbol{\alpha}' \sigma_\alpha^2 + \boldsymbol{\beta}\boldsymbol{\beta}' \sigma_\beta^2) \Gamma_1' + \Gamma_1 \Sigma_\eta \Gamma_1'$$

E.5. Measurement Equations Synthesis

To summarize the measurement equations are as follows:

$$\begin{aligned}
\Delta \ln Q_t^{obs} &= f\left(\hat{c}_t, \hat{c}_{t-1}, \hat{i}_t, \hat{i}_{t-1}, \hat{g}_t, \hat{\omega}_t, \hat{\pi}_t^{g,obs}\right) \equiv \Delta \ln Q_t^j; \\
\Delta \ln Y_t^{l,obs} &= a + 4b(\Delta \ln Q_t^l + pop_t^l), \quad l = 1,2,3,4; \\
\Delta \ln C_t^{obs} &= z_* + \Delta \hat{c}_t + \hat{z}_t; \\
\Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta \hat{\imath}_t + \hat{z}_t + \hat{\omega}_t; \\
\log H_t^{obs} &= \hat{H}_t; \\
\pi_t^{i,obs} &= \omega_* + \hat{\omega}_t + u_t^i; \\
R_t^{obs} &= R_* + \hat{R}_t;
\end{aligned}$$

$$
\begin{aligned}
R_t^{j,obs} &= R_* + E_t\hat{R}_{t+j},\ \ j = 1, 2, \ldots, H; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \beta^{l,j} E_t\hat{\pi}_{t+l} + u_t^{l,j,\pi},\ \ j = 1, 2,\ \ l = 1, ...4; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l}\sum_{i=1}^{l} E_t\hat{\pi}_{t+i} + u_t^{l,j,\pi},\ \ j = 1, 2,\ \ l = 40; \\
\pi_t^{j,obs} &= \pi_* + \pi_*^{j} + \beta^{\pi,j}\hat{\pi}_t + \gamma^{\pi,j}\pi_t^{d,obs} + u_t^{j,p},\ \ \text{with } \beta^{\pi,1} = 1, j = 1, 2, 3; \\
\Delta \ln w_t^{j,obs} &= z_* + w_*^{j} + \beta^{w,j}\left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{j,w},\ \ \text{with } \beta^{w,1} = 1, j = 1, 2; \\
\pi_t^{d,obs} &= (1 - \beta_{1,1} - \beta_{1,2})\pi_*^{d} + \beta_{1,1}\pi_{t-1}^{d,obs} + \beta_{1,2}\pi_{t-2}^{d,obs} + u_t^{d}; \\
\pi_t^{g,obs} &= (1 - \beta_{2,1} - \beta_{2,2})\pi_*^{g} + \beta_{2,1}\pi_{t-1}^{g,obs} + \beta_{2,2}\pi_{t-2}^{g,obs} + u_t^{g}.
\end{aligned}
$$

The left hand side variables represent data (Q denotes chain-weighted GDP). The function f in the first equation represents the linear approximation to the chain-weighted GDP formula. As previously discussed, two variables are included to complete the mapping from model to data but are not endogenous to the model. Specifically, the consumption price of government consumption plus net exports, $\pi_t^{g,obs}$, helps map model GDP to our model-consistent measure of chain-weighted GDP, and inflation in the consumption price of consumer durable goods, $\pi_t^{d,obs}$, is used to complete the mapping from model inflation to measured inflation.

The measurement equations indicate we use 21 time series to estimate the model in the first sample. In addition to the real quantities and federal funds rate that are standard in the literature our estimation includes multiple measures of wage and consumer price inflation, two measures each of average inflation expected over the next ten years and over one quarter, and $H = 4$ quarters of interest rate futures. Our second sample estimation is restricted to estimating the parameters of the stochastic process for forward guidance news with $H = 10$ plus the processes driving $\pi_t^{g,obs}$ and $\pi_t^{d,obs}$ (only the constant and the standard deviation). This estimation uses the measurement equations involving the current federal funds rate and 10 quarters of expected future policy rates plus the last two equations. We take into account the change in steady state but keep the remaining structural parameters at their first sample values. Because our estimation forces data on real activity, wages and prices to coexist with the interest rate futures data, we expect the estimation to mitigate the forward guidance puzzle. Finally, it is worth stressing that our estimation respects the ELB in the second sample. This is because we measure expected future rates in the model, the $E_t\hat{R}_{t+j}$, using the corresponding empirical futures rates, $R_t^{j,obs}$, and we use futures rates extending out 10 quarters. Finally, in the second sample we extend the use the Survey of Professional Forecasts about near term inflation expectations using the 1Q-4Q ahead CPI and PCE inflation expectations, and introduce the SPF expectations about near term real GDP growth expectations, i.e. 1Q ahead until 4Q ahead.

E.6. Data Synopsis

[31]

[31] Unless otherwise indicated all data are from Haver Analytics.

Model-Consistent Output

- The DSGE model output is the chained sum of conventional GDP with government capital services and durable goods services. This series is de-trended by population growth.

Model-Consistent Consumption

- DSGE consumption is defined as the chained sum of conventional PCE nondurable goods with PCE services and durable goods services. This series is de-trended by population growth.

Model-Consistent Investment

- Model-consistent Investment is the chained sum of durable goods purchases, fixed investment, and government investment. This series is de-trended by population growth.

Model-Consistent Residual Output Inflation

- The residual output is the chained difference of model consumption and investment from model GDP. Residual output reflects government spending and net exports.

Relative Price of Consumption to Investment

- The relative price is constructed by dividing the consumption price series and investment price series.

Deflators for Consumer Durables

- We take the log difference[32] of the PCE Durable Goods Chain Price Index for the deflators for consumer durables.

Inflation Expectations

- Our inflation expectations series are quarterly inflation expectations data from the Survey of Professional Forecasters at the Philadelphia Fed. They report inflation expectations at various horizons for both PCE and CPI measures. We use their aggregate measures of 1Q to 4Q ahead core CPI and core PCE inflation expectations, 40Q ahead average inflation expectations for CPI and PCE. The SPF did not report expectations for PCE prior to 2007, so we do not have many observations for the first sample of our data. However, we continue to include these few observations in order to initialize the kalman filter for second sample estimation. We have the full data for CPI expectations.

Real GDP Growth Expectations

[32] All log differenced series are multipled by 100.

- Our real GDP growth expectations series are annualized expectations data from the Survey of Professional Forecasters at the Philadelphia Fed. We use their BEA real GDP growth expectations from 1Q to 4Q ahead. The SPF reports these expectations throughout our sample period. We use them only in the second sample because the inflation data are only available for the second sample.

Real Wages

- We have two different measures of wages in the model - average hourly earnings and employment compensation. We take the average hourly earnings and divide by the chain price index of core PCE, then take the log difference.

- We repeat the same steps to calculate employment compensation but use the employment cost index for the compensation of civilian workers.

Price Inflation

- We use three different measures of price inflation: Core PCE, Market-Based Core PCE, and Core CPI.

Hours

- We construct our hours series with the methodology as described in *Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis* (Campbell et al., 2016).

Effective Federal Funds Rate

- For the first sample (1993q1-2008q3), we use the federal funds target rate observed as the average over the last month of the quarter.

- For the second sample (2008q4-2018q4), we use the federal funds target rate observed at the end of the quarter.

- We divide the series by 4 to convert to quarterly rates.

Expected Federal Funds Rate (FFR)

- From 1993Q1 to 2005Q4, our 4-quarter ahead path comes from Eurodollar futures. Eurodollar futures have expiration dates that lie about two weeks before the end of each quarter. Eurodollar rate is closely tied to expectations for the Federal Funds rates over the same period, so the Eurodollar futures rate corresponds with the Fed Funds rate at the middle of the last month of each quarter.

- Beginning with 2006Q1, our 4-quarter ahead, and later, 10-quarter ahead path comes from the Overnight Index Swaps (OIS). The OIS data are converted into a point estimate of the Fed Funds for a particular date using a Svensson term structure model. The dates of the OIS data reflect the middle of the quarter values, and we interpolate to obtain the end of quarter values.

- From 2014Q1, we began to use the expected Fed Funds from the Survey of Market Participant (SMP). The SMP correspond to the survey participants' expected Fed Funds at the end of the quarter.
- All expected FFR series are in quarterly rates.

F. Calibration and Bayesian Estimation

As we discussed, we follow a two-stage approach to the estimation of our model's parameters. In a calibration stage, we set the values of selected parameters so that the model has empirically-sensible implications for long-run averages from the U.S. economy. In this stage, we also enforce several normalizations and a judgemental restriction on one of the measurement error variances. In the second stage, we estimate the model's remaining parameters using standard Bayesian methods.

F.1. Calibration

Our calibration strategy is the same as in Campbell, Fisher, Justiniano, and Melosi (2016) except that we address the well-known evidence of secular declines in economic growth and rates of return on nominally risk free assets. We address these developments by imposing a change in steady state in 2008q4 (the choice of this date is motivated in the next subsection). Steady state GDP growth is governed by the mean growth rates of the neutral and investment-specific technologies, ν_* and ω_*. We adjust ω_* down to account for the slower decline in the relative price of investment since 2008q4. Given this change we then lower ν_* so that steady state GDP growth is reduced to 2%. To match a lower real risk-free rate of 1% we increase the steady state marginal utility of government bonds using ε^s_*.[33] These adjustments leave the other calibrated parameters unchanged but do change the steady state values of the endogenous variables and therefore the point at which the economy is log-linearized.[34]

We observe the long-run average of the following aggregates: nominal federal funds rate, labor share, government spending share, investment spending share, the capital-output ratio, real per-capita GDP growth (g_y), inflation in price of government, net exports and inventory investment relative to non-durables and services consumption, and the growth rate of the consumption-investment relative price.

- The labor share can be used to calibrate the parameter α.
- The government spending share determines s^g_*.
- The government price growth rate pins down π^g_*.

[33] The targets for steady state GDP growth and risk-free rate reflect a variety of evidence including the Fed's Summary of Economic Projections.

[34] Our re-calibration changes the return on private assets by a little. This small change is consistent with Yi and Zhang (2017) who show that rates of return on private capital have stayed roughly constant in the face of declines in risk free rates.

- The growth rate of the consumption-investment relative price pins down ω_*.
- The investment share pins down i_*/y_*.
- The capital output ratio pins down k_*/y_*.
- Calculate the consumption-output share

$$\frac{c_*}{y_*} = \left(1 - \frac{i_*}{y_*} - \frac{g_*}{y_*}\right). \tag{62}$$

- The growth rate of real chain-weighted GDP is used to pin down the growth rate of the common trend z_*. First

$$g_y = e^{z_*}\sqrt{\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi_*^g\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi_*^g\frac{g_*}{y_*}}}$$

All the variables in this equation are known except for z_*. So we can solve for z_*:

$$z_* \quad = \quad g_y - \frac{1}{2}\ln\left(\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi_*^g\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi_*^g\frac{g_*}{y_*}}\right) \tag{63}$$

- The growth rate of the labor-augmenting technology ν_* can be easily obtained by exploiting the following equation:

$$z_* = v_* + \frac{\alpha}{1-\alpha}\omega_*. \tag{64}$$

- We are now in a position to identify the depreciation rate δ_0 using the steady-state equation pinning down the investment capital ratio:

$$\begin{aligned}\frac{i_*}{k_*} \quad &= \quad 1 - (1-\delta_0)e^{-z_*-\omega_*} \\ &\Rightarrow \quad \delta_0 = 1 + \left(\frac{i_*}{k_*} - 1\right)e^{z_*+\omega_*}\end{aligned}$$

where the investment capital ratio is obtained combining the investment share and the capital output ratio:

$$\frac{i_*}{k_*} = \frac{i_*/y_*}{k_*/y_*}. \tag{65}$$

- From the steady-state equilibrium we have that

$$\frac{y_*}{k_*} = e^{-z_*-\omega_*}\frac{\delta_1}{\alpha}. \tag{66}$$

Therefore

$$\delta_1 \quad = \quad \alpha \left(\frac{k_*}{y_*} \right)^{-1} e^{z_* + \omega_*} \tag{67}$$

where the capital output ratio is given above.

- In steady state, the real rate of return on private bonds is derived from the first order condition for private bonds:
$$r_*^p \equiv \frac{R_*^P}{\pi_*} = \frac{e^{\gamma_c z_*}}{\beta}. \tag{68}$$
In steady state the real rental rate of capital is derived from the first order condition for capital:
$$r_*^k = \left[\frac{e^{\gamma_c z_*}}{\beta} \right] e^{\omega_*} - (1 - \delta_0) \tag{69}$$
Combining these last two equations yields
$$r_*^k = r_*^p e^{\omega_*} - (1 - \delta_0)$$
and hence
$$r_*^p = \left[r_*^k + 1 - \delta_0 \right] e^{-\omega_*}.$$
Note that $r_*^k = \delta_1$ from the first order condition for capacity utilization. It follows that
$$r_*^p \quad = \quad (1 - \delta_0 + \delta_1)\, e^{-\omega_*}$$

- The liquidity premium in steady state (i.e., $\frac{R_*/\pi_*}{r_*^p}$) can be computed now by assuming a *nominal* average federal funds rate, R_*, and an annualized average inflation rate.

- Using equation (69) and the fact that $r_*^k = \delta_1$, we can calibrate the discount factor β :
$$\beta = (1 - \delta_0 + \delta_1)^{-1} e^{\omega_*} e^{\gamma_c z_*}$$
where γ_c is a parameter of the utility function to be estimated.

F.2. Bayesian Estimation

Our Bayesian estimation uses the same split-sample strategy as in Campbell, Fisher, Justiniano, and Melosi (2016) except that we incorporate the change in steady state described above and one other change noted below. As in Campbell, Fisher, Justiniano, and Melosi (2016) our sample begins in 1993q1. This date is based on the availability and reliability of the overnight interest rate futures data. The sample period ends in 2016q4 but we impose a sample break in 2008q4. Our choice of this latter date is motivated by three main considerations. First, there is the evidence that points to lower interest rates and economic growth later in the sample. Second, it seems clear that the horizon over which forward guidance was communicated by the Fed lengthened substantially during the ELB period. Finally, the downward trends in inflation and inflation expectations from the early 1990s appear to come to an end in the mid-2000s. Splitting the sample in 2008q4 and assuming some parameters change at that date is our way of striking a balance between parsimony and addressing the multiple structural changes that seem to occur around the same time.

We estimate the full suite of non-calibrated structural parameters in the first sample under the assumption that forward guidance extends for $H = 4$ quarters. Starting in 2008q4 we assume the model environment changes in three ways. First we assume the change in the steady state described above. Second, forward guidance lengthens to $H = 10$ quarters Third, the time-varying inflation target from the first sample becomes a constant equal to the steady state rate of inflation, 2% at an annual rate. All three changes are assumed to be unanticipated and permanent.

We employ standard prior distributions, but those governing monetary policy shocks deserve further elaboration. Our estimation requires the variance-covariance matrix of monetary policy shocks to be consistent with the factor-structure of interest rate innovations used by Gürkaynak, Sack, and Swanson (2005), as described above. Therefore, we parameterize Σ_ε in terms of factors STD (σ_α and σ_β), factor loadings (α and β) and STD of the idiosyncratic errors ($\sigma_{\eta,j}$). We then center our priors for these parameters at their estimates from event-studies. However, we do not require our estimates to equal their prior values. Our Bayesian estimation procedure employs quarterly data on expected future interest rates, the posterior likelihood function includes them as free parameters. It is well known that factors STD and loadings are not separately identified, so we impose two scale normalizations and one rotation normalization on α and β. The rotation normalization requires that the first factor, which we label "Factor A", is the only factor influence the current policy rate. That is, the second factor, "Factor B" influences only future policy rates. Gürkaynak, Sack, and Swanson (2005) call Factors A and B the "target" and "path" factors.

F.3. Posterior Estimates

We report the results of our two-stage two-sample estimation in a series of tables. Table 3 reports our most notable calibration targets. The long-run policy rate equals 1.1 percent on a quarterly basis. We target a two percent growth rate of per capita GDP. Given an average population growth rate of one percent per year, this implies that our potential GDP growth rate equals three percent. The other empirical moments we target are a nominal investment to output ratio of 26 percent and nominal government purchases to output ratio of 15 percent. Finally, we target a capital to output ratio of approximately 10 on a quarterly basis.

Table 4 lists the parameters which we calibrate along with their given values. The table includes many more parameters than there are targets in Table 3. This is because Table 3 omitted calibration targets which map one-to-one with particular parameter values. For example, we calibrate the steady-state capital depreciation rate (δ_0) using standard methods applied to data from the Fixed Asset tables. It is also because Table 4 lists several parameters which are normalized prior to estimation. Most notable among these are the three factor loadings listed at the table's bottom. Tables 5 and 8 report prior distributions and posterior modes for the model's remaining parameters, for the first and second samples respectively. gnificantly.

Table 3: First Sample Calibration Targets

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.011
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.005
Investment to Output Ratio	I_t/Y_t	0.2597
Capital to Output Ratio	K_t/Y_t	10.7629
Fraction of Final Good Output Spent on Public Goods	G_t/Y_t	0.1532
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.371

Table 4: First Sample Calibrated Parameters

Parameter	Symbol	Value
Discount Factor	β	0.9857
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.371
Elasticity of Output w.r.t Capital Services	α	0.401
Steady-State Wage Markup	λ^w_*	1.500
Steady-State Price Markup	λ^p_*	1.500
Steady-State Scale of the Economy	H_*	1.000
Steady-State Inflation Rate (quarterly)	π_*	0.500
Steady-State Depreciation Rate	δ_0	0.0162
Steady-State Marginal Depreciation Cost	δ_1	0.0385
Core PCE, 1Q Ahead and 10Y Ahead Expected PCE		
Constant	$\pi^1_*, \pi^{l,1}_*$	0.000
Loading 1	$\beta^{\pi,1}, \beta^{l,1}$	1.000
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi^2_*, \pi^{l,2}_*$	0.122
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u^{40,j,\pi}_t$		0.010
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.418
2nd Lag Coefficient	$\beta_{1,2}$	0.379
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.311
2nd Lag Coefficient	$\beta_{2,2}$	0.0057
Compensation		
Constant	w^1_*	-0.202
Loading	$\beta^{w,1}$	1.000
Earnings Constant	w^2_*	-0.237
Loading 0 Factor A	α_0	0.981
Loading 0 Factor B	β_0	0.000
Loading 4 Factor B	β_4	0.951

Table 5: First Sample Estimated Parameters

Parameter	Symbol	Density	Prior Mean	Std.Dev	Posterior Mode
Depreciation Curve	$\frac{\delta_2}{\delta_1}$	G	1.0000	0.150	0.474
Active Price Indexation Rate	ι_p	B	0.5000	0.150	0.409
Active Wage Indexation Rate	ι_w	B	0.5000	0.150	0.077
External Habit Weight	λ	B	0.7500	0.025	0.780
Labor Supply Elasticity	γ_H	N	0.6000	0.050	0.589
Price Stickiness Probability	ζ_p	B	0.8000	0.050	0.831
Wage Stickiness Probability	ζ_w	B	0.7500	0.050	0.914
Adjustment Cost of Investment	φ	G	3.0000	0.750	5.354
Elasticity of Intertemporal Substitution	γ_c	N	1.5000	0.375	1.319
Interest Rate Response to Inflation	ψ_1	G	1.7000	0.150	1.791
Interest Rate Response to Output	ψ_2	G	0.2500	0.100	0.398
Interest Rate Smoothing Coefficient	ρ_R	B	0.8000	0.100	0.801
Autoregressive Coefficients of Shocks					
Discount Factor	ρ_b	B	0.5000	0.250	0.813
Inflation Drift	ρ_π	B	0.9900	0.010	0.998
Exogenous Spending	ρ_g	B	0.6000	0.100	0.887
Investment-Demand	ρ_i	B	0.5000	0.100	0.791
Liquidity Preference	ρ_s	B	0.6000	0.200	0.887
Price Markup	ρ_{λ_p}	B	0.6000	0.200	0.136
Wage Markup	ρ_{λ_w}	B	0.5000	0.150	0.469
Neutral Technology	ρ_ν	B	0.3000	0.150	0.492
Investment Specific Technology	ρ_ω	B	0.3500	0.100	0.303
Moving Average Coefficients of Shocks					
Price Markup	θ_{λ_p}	B	0.4000	0.200	0.307
Wage Markup	θ_{λ_w}	B	0.4000	0.200	0.391
Standard Deviations of Innovations					
Discount Factor	σ_b	U	0.5000	2.000	1.768
Inflation Drift	σ_π	I	0.0150	0.0075	0.077
Exogenous Spending	σ_g	U	1.0000	2.000	4.139
Investment-Demand	σ_i	I	0.2000	0.200	0.549
Liquidity Preference	σ_s	U	0.5000	2.000	0.341
Price Markup	σ_{λ_p}	I	0.1000	1.000	0.101
Wage Markup	σ_{λ_w}	I	0.1000	1.000	0.035
Neutral Technology	σ_ν	U	0.5000	0.250	0.530
Investment Specific Technology	σ_ω	I	0.2000	0.100	0.259
Relative Price of Cons to Inv Monetary Policy	$\sigma_{\frac{c}{i}}$	I	0.0500	2.000	0.675

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
Unanticipated	σ_{η_0}	N	0.0050	0.0025	0.012
1Q Ahead	σ_{η_1}	N	0.0050	0.0025	0.012
2Q Ahead	σ_{η_2}	N	0.0050	0.0025	0.008
3Q Ahead	σ_{η_3}	N	0.0050	0.0025	0.009
4Q Ahead	σ_{η_4}	N	0.0050	0.0025	0.012
Compensation					
Standard Deviation of $u_t^{1,w}$		I	0.0500	0.100	0.194
AR(1) Coefficient of $u_t^{1,w}$		B	0.4000	0.100	0.458
Earnings					
Loading 1	$\beta^{w,2}$	N	0.8000	0.100	0.904
Standard Deviation of $u_t^{2,w}$		I	0.0500	0.100	0.143
AR(1) Coefficient of $u_t^{2,w}$		B	0.4000	0.100	0.674
Core PCE					
Loading 2	$\gamma^{\pi,1}$	N	0.0000	1.000	0.045
Standard Deviation of $u_t^{1,p}$		I	0.0500	0.100	0.046
AR(1) Coefficient of $u_t^{1,p}$		B	0.2000	0.100	0.108
Core CPI					
Loading 1	$\beta^{\pi,2}$	N	1.0000	0.100	0.808
Loading 2	$\gamma^{\pi,2}$	N	0.0000	1.000	0.087
Standard Deviation of $u_t^{2,p}$		I	0.1000	0.100	0.077
AR(1) Coefficient of $u_t^{2,p}$		B	0.4000	0.200	0.586
Market-Based Core PCE					
Constant	π_*^3	N	-0.1000	0.100	-0.037
Loading 1	$\beta^{\pi,3}$	N	1.0000	0.100	1.121
Loading 2	$\gamma^{\pi,3}$	N	0.0000	1.000	0.015
Standard Deviation of $u_t^{3,p}$		I	0.0500	0.100	0.035
AR(1) Coefficient of $u_t^{3,p}$		B	0.2000	0.100	0.144
1Q Ahead Expected PCE					
Standard Deviation of $u_t^{1,1,\pi}$		I	0.0500	0.100	0.026
AR(1) Coefficient of $u_t^{1,1,\pi}$		B	0.2000	0.100	0.196
1Q Ahead Expected CPI					
Loading	$\beta^{1,2}$	N	1.0000	0.100	0.980
Standard Deviation of $u_t^{1,2,\pi}$		I	0.0500	0.100	0.062
AR(1) Coefficient of $u_t^{1,2,\pi}$		B	0.2000	0.100	0.198
10Y Ahead Expected PCE					
AR(1) Coefficient of $u_t^{40,1,\pi}$		B	0.2000	0.100	0.271
10Y Ahead Expected CPI					
Loading	$\beta^{40,2}$	N	1.0000	0.100	1.021

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
AR(1) Coefficient of $u_t^{40,2,\pi}$		B	0.2000	0.100	0.213
PCE Durable Goods Inflation					
Constant	π_*^d	N	-0.3500	0.100	-0.360
Standard Deviation of u_t^d		I	0.2000	2.000	0.286

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
Inflation in Relative Price of Government,					
Inventories and Net Exports to Consumption					
Constant	π_*^g	N	0.1980	1.000	-0.666
Standard Deviation of u_t^g		I	0.5000	2.000	1.861
Factor A					
Loading 1	α_1	N	0.6839	0.200	1.305
Loading 2	α_2	N	0.5224	0.200	0.877
Loading 3	α_3	N	0.4314	0.200	0.306
Loading 4	α_4	N	0.3243	0.200	-0.012
Standard Deviation	σ_α	N	0.1000	0.0750	0.040
Factor B					
Loading 1	β_1	N	0.3310	0.200	0.656
Loading 2	β_2	N	0.6525	0.200	1.104
Loading 3	β_3	N	0.8059	0.200	1.162
Standard Deviation	σ_β	N	0.1000	0.0750	0.078

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

Table 6: Second Sample Calibration Targets (Different from First Sample)

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.007
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.003
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.171

Table 7: Second Sample Calibrated Parameters (Different from First Sample)

Parameter	Symbol	Value
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.171
Steady-State Marginal Depreciation Cost	δ_1	0.038
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.122
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.020
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.000
2nd Lag Coefficient	$\beta_{1,2}$	0.000
Inflation in Relative Price of Government,		
Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.320
2nd Lag Coefficient	$\beta_{2,2}$	-0.240
Compensation Loading	$\beta^{w,1}$	1.000
Loading 5 Factor A	α_5	0.932
Loading 8 Factor B	β_8	0.210
Loading 10 Factor B	β_{10}	0.000

Table 8: Second Sample Estimated Parameters

Parameter	Symbol	Prior Mean	Prior Std.Dev	Posterior Mode
Compensation				
Constant	w_*^1	-0.2023	0.000	-0.2023
Standard Deviation of $u_t^{1,w}$		0.1941	0.100	0.284
AR(1) Coefficient of $u_t^{1,w}$		0.4579	0.000	0.4579
Earnings				
Constant	w_*^2	-0.2370	0.000	-0.237
Loading 1	$\beta^{w,2}$	0.9039	0.000	0.9039
Standard Deviation of $u_t^{2,w}$		0.1434	0.100	0.304
AR(1) Coefficient of $u_t^{2,w}$		0.6741	0.000	0.6741
Core PCE				
Loading 2	$\gamma^{\pi,1}$	0.0449	0.000	0.0449
Standard Deviation of $u_t^{1,p}$		0.0457	0.100	0.274
AR(1) Coefficient of $u_t^{1,p}$		0.1081	0.000	0.1801
Core CPI				
Loading 1	$\beta^{\pi,2}$	0.8083	0.00	0.8083
Loading 2	$\gamma^{\pi,2}$	0.0868	0.000	0.0868
Standard Deviation of $u_t^{2,p}$		0.0770	0.100	0.2517
AR(1) Coefficient of $u_t^{2,p}$		0.5856	0.000	0.5856
Market PCE				
Constant	π_*^3	-0.0367	0.000	-0.0367
Loading 1	$\beta^{\pi,3}$	1.1213	0.000	1.1213
Loading 2	$\gamma^{\pi,3}$	0.0153	0.000	0.0153
Standard Deviation of $u_t^{3,p}$		0.0349	0.100	0.2553
AR(1) Coefficient of $u_t^{3,p}$		0.1436	0.000	0.1436
1Q Ahead Expected PCE				
Standard Deviation of $u_t^{1,1,\pi}$		0.0259	0.020	0.0412
AR(1) Coefficient of $u_t^{1,1,\pi}$		0.1960	0.050	0.1832
2Q Ahead Expected PCE				
Standard Deviation of $u_t^{2,1,\pi}$		0.0259	0.020	0.0175
AR(1) Coefficient of $u_t^{2,1,\pi}$		0.1960	0.050	0.2140
3Q Ahead Expected PCE				
Standard Deviation of $u_t^{3,1,\pi}$		0.0259	0.020	0.0193
AR(1) Coefficient of $u_t^{3,1,\pi}$		0.1960	0.050	0.2202
4Q Ahead Expected PCE				
Standard Deviation of $u_t^{4,1,\pi}$		0.0259	0.020	0.0156
AR(1) Coefficient of $u_t^{4,1,\pi}$		0.1960	0.050	0.2075

Second Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
1Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.0022
Standard Deviation of $u_t^{1,2,\pi}$		0.0622	0.020	0.095
AR(1) Coefficient of $u_t^{1,2,\pi}$		0.1982	0.050	0.206
2Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.2433
Standard Deviation of $u_t^{2,2,\pi}$		0.0622	0.020	0.0411
AR(1) Coefficient of $u_t^{2,2,\pi}$		0.1982	0.050	0.2532
3Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.2662
Standard Deviation of $u_t^{3,2,\pi}$		0.0622	0.020	0.0399
AR(1) Coefficient of $u_t^{3,2,\pi}$		0.1982	0.050	0.2607
4Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.2354
Standard Deviation of $u_t^{4,2,\pi}$		0.0622	0.020	0.0406
AR(1) Coefficient of $u_t^{4,2,\pi}$		0.1982	0.050	0.2782
10Y Ahead Expected PCE				
AR(1) Coefficient of $u_t^{40,1,\pi}$		0.2711	0.000	0.2711
10Y Ahead Expected CPI				
Loading	$\beta^{40,2}$	1.0207	0.000	1.0207
AR(1) Coefficient of $u_t^{40,2,\pi}$		0.2133	0.000	0.2133
1Q Ahead Expected GDP				
Standard Deviation of $u_t^{1,1,Y}$		0.10	0.100	0.9827
AR(1) Coefficient of $u_t^{1,1,Y}$		0.20	0.100	0.1300
2Q Ahead Expected GDP				
Standard Deviation of $u_t^{2,1,Y}$		0.10	0.100	0.6263
AR(1) Coefficient of $u_t^{2,1,Y}$		0.20	0.100	0.1825
3Q Ahead Expected GDP				
Standard Deviation of $u_t^{3,1,Y}$		0.10	0.100	0.9779
AR(1) Coefficient of $u_t^{3,1,Y}$		0.20	0.100	0.1767
4Q Ahead Expected GDP				
Standard Deviation of $u_t^{4,1,Y}$		0.10	0.100	0.3664
AR(1) Coefficient of $u_t^{4,1,Y}$		0.20	0.100	0.2747
PCE Durable Goods Inflation				
Constant	π_*^d	-0.4500	0.200	-0.4858
Standard Deviation of u_t^d		0.5000	0.150	0.325
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption				

Second Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
Constant	π_*^g	0.8900	0.400	-0.1177
Standard Deviation of u_t^g		0.8143	0.150	1.267
Factor A				
Loading 0	α_0	0.0180	0.250	0.099
Loading 1	α_1	0.0574	0.250	0.202
Loading 2	α_2	0.1941	0.250	0.397
Loading 3	α_3	0.3996	0.250	0.591
Loading 4	α_4	0.6520	0.250	0.792
Loading 6	α_6	1.2266	0.250	1.116
Loading 7	α_7	1.5237	0.250	1.281
Loading 8	α_8	1.8139	0.250	1.406
Loading 9	α_9	2.0914	0.250	1.517
Loading 10	α_{10}	2.3523	0.250	2.851
Standard Deviation	σ_α	0.0442	0.100	0.056
Factor B				
Loading 0	β_0	-0.0181	0.300	0.051
Loading 1	β_1	0.2211	0.300	0.083
Loading 2	β_2	0.3679	0.300	0.125
Loading 3	β_3	0.4424	0.300	0.152
Loading 4	β_4	0.4612	0.300	0.167
Loading 5	β_5	0.4370	0.300	0.181
Loading 6	β_6	0.3817	0.300	0.192
Loading 7	β_7	0.3032	0.300	0.203
Loading 9	β_9	0.1074	0.300	0.210
Standard Deviation	σ_β	0.0334	0.100	0.449
Standard Deviations of Monetary Policy Innovations				
Unanticipated	σ_{η_0}	0.0061	0.005	0.011
1Q Ahead	σ_{η_1}	0.0021	0.005	0.010
2Q Ahead	σ_{η_2}	0.0004	0.005	0.010
3Q Ahead	σ_{η_3}	0.0019	0.005	0.010
4Q Ahead	σ_{η_4}	0.0001	0.005	0.010
5Q Ahead	σ_{η_5}	0.0025	0.005	0.009
6Q Ahead	σ_{η_6}	0.0019	0.005	0.010
7Q Ahead	σ_{η_7}	0.0011	0.005	0.009
8Q Ahead	σ_{η_8}	0.0001	0.005	0.009
9Q Ahead	σ_{η_9}	0.0014	0.005	0.010
10Q Ahead	$\sigma_{\eta_{10}}$	0.0028	0.005	0.0001