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THE COHERENCE SIDE OF RATIONALITY: RULES OF THUMB, NARROW BRACKETING, AND MANAGERIAL INCOHERENCE IN CORPORATE FORECASTS

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JEL Classification: D84, D22, L2, M2, G32

Keywords: Coherence, Rules of thumb, Narrow bracketing, Firm expectations

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The Coherence Side of Rationality

Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts*

Pamela Giustinelli† and Stefano Rossi‡

This version: January 2023

Abstract

We develop a theory of forecast coherence in a firm production setting, which yields a normative ex ante benchmark of first-best coherent forecasts and statistical tests to detect incoherence ex post. Under the null, the forecast errors of output and inputs are "close" to one another. Using the Duke Survey of top executives of large US corporations, we reject the null of coherence for 55% of CFOs in our sample. In a positive version of our model, incoherence reflects intra-personal frictions in coordinating multiple forecasts, implying that some of the rules of thumb proposed by the managerial education literature to make contemporaneous forecasts may emerge as second-best optimal. Consistent with our model, we find that corporate performance correlates negatively with incoherence, being lowest for firms whose CFOs provide "narrow bracketing" forecasts—projecting past capital growth into the future while ignoring output and labor. We also find that the use of incoherent rules of thumb correlates negatively with corporate investment spending and positively with corporate leverage.

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I Introduction

Coherence refers to "the consistency of the elements of the person's judgment" (Hammond (2007), p. xvi). When forecasting multiple variables at the same time, coherence requires the forecaster to fully assess all the connections among the variables under consideration, with the purpose of delivering rational forecasts. Coherence constitutes one of the two standards for evaluating the rationality of forecasts, together with accuracy (Hammond, 1990, 1996). While there are numerous theoretical and empirical analyses of forecast accuracy, centering on whether forecast errors are systematically predictable from information known at the time of the forecast (Tversky and Kahneman (1974); see Benjamin (2019) for a recent survey), coherence has received much less attention.[1] Our main contribution is to provide the first theory of coherence in a firm production setting and the first evidence that top financial executives make severely incoherent forecasts of their own firm's output and input growth.

Incoherence may be very costly. Under a standard production technology, a firm that wishes to double its output will likely have to plan using a lot more of its inputs such as capital and labor, lest the desired output proves unattainable. Ignoring the technological constraint implies that the firm could end up using a suboptimal mix of capital and labor, thereby attaining a lower profit than it would have been otherwise possible with its production technology and budget constraint. More generally, corporate planning and internal forecasting underlie all resource allocation and investment decisions inside the firm and are still not well understood (Graham, 2022). The managerial education literature (e.g., Ruback (2004), Titman and Martin (2016), Welch (2017), Holthausen and Zmijewski (2020), and Koller et al. (2020)) acknowledges the challenges of making plans about multiple variables at the same time and provides a number of rules of thumb to help CFOs make rational forecasts. To the best of our knowledge, none of these rules of thumb have been assessed to date, be it theoretically or empirically.

[1] Tversky and Kahneman (1974) discuss coherence in the context of subjective expected utility and argue that to fully assess coherence it is not sufficient to elicit the individual's subjective probabilities, but one would also need to elicit "*the entire web of beliefs held by the individual*" (p. 1130). By contrast, we study coherence in the context of firms' production plans and thus in the context of objective technological relationships among inputs and output.

One such rule of thumb, the "plain growth forecast" (Welch (2017), p. 593), is to forecast the growth rate of a firm's input (e.g., capital) by projecting that input's past growth rates into the future, while disregarding information about other inputs (e.g., labor). This is reminiscent of the "narrow-bracketing" behavior of decision makers who, facing multiple choices at the same time, make each choice in isolation. Narrow bracketing yields lower utility than the first best of broad bracketing (Thaler (1985); Read et al. (1999)). One mechanism underlying narrow bracketing is mental accounting (Thaler (1985); Tversky and Kahneman (1981)), whereby decision makers hold a separate mental account for each decision, as opposed to a single budget constraint for their total expenditures. For example, when considering two consumption goods—say food and gasoline—narrow bracketing implies allocating resources suboptimally by treating food consumption as independent of gasoline consumption, and vice versa. In principle, narrow bracketing could be even more problematic in a firm setting, for two reasons. First, corporate planning features not only a budget constraint but also a production technology. Second, top executives need to make very detailed and explicit forecasts (plans) for several years. As a result, narrow bracketing could lead to incoherent and suboptimal allocation of resources to future capital expenditures while ignoring labor costs, or vice versa.

We provide the first theory of forecast coherence in a firm production setting, which yields several tests of forecast coherence, and we present the first evidence that top financial executives make severely incoherent forecasts of their own firm's output and input growth. We implement our tests in a population of senior financial executives dealing with corporate forecasts and production decisions. We use the Duke Survey of top executives of large- and mid-size US corporations, who provide forecasts of annual growth rates of multiple firm-level balance-sheet variables simultaneously (e.g., Ben-David et al. (2013) and Graham (2022)). Besides being finance professionals, most of these forecasters are top financial executives and chief financial officers (CFOs) that are actively involved in setting the corporate investment and financing policies of their firms, which allows us to jointly assess corporate forecasts and corporate policies.

Our theoretical model of firm forecasts provides a normative benchmark of ex ante coherent forecasts that are first-best optimal. In a positive version of our model, we study the second-best optimal forecasting rules of agents who observe noisy signals about output and input prices and we compare these second-best forecasting rules with the rules of thumb suggested by the managerial education literature. We then use our model to measure the extent to which financial executives are incoherent and the mechanisms underlying incoherence. We examine empirically whether executives' forecasts reflect the rules of thumb suggested by the managerial education literature, and whether the use of such rules of thumb is linked to coherence. We also examine whether incoherence is related to CFOs' personal characteristics, firm's performance, the intensity of investment spending, and debt policy—all corporate activities in which CFOs play a key role, over and above chief executive officers (CEOs); see Graham et al. (2015) and Malmendier et al. (2022). We find that the use of incoherent rules of thumb correlates negatively with corporate performance and investment spending, and positively with leverage.

Using CFOs' forecasts of input and output variables over the period 2001-2018, we document a large cross-sectional dispersion in input forecasts conditional on output forecast. More than one quarter of CFOs predict at the same time an increase in output (i.e., sales revenues) and a decrease in input (e.g., capital expenditures); about one third of CFOs predict at the same time a decrease in output and an increase in capital expenditures. In principle, such dispersion in the contemporaneous forecasts of sales and capital expenditures might rationally reflect heterogeneity across firms, for example in investment lags or inventory. If so, one would expect the forecast errors to be (close to) zero ex post. However, this is not the case in the data. Strikingly, more than 40% of CFOs make forecast errors in input growth and output growth that have opposite signs. This pattern of a large dispersion in contemporaneous *forecast errors* is inconsistent with the view that the previously shown dispersion in contemporaneous forecasts merely reflects persistent firm-level heterogeneity, because the latter is first-differenced away when computing forecast errors. Rather, these patterns point to the existence of severe incoherence in CFO forecasts. At the same time, in the absence of a theoretical model and of a well-specified testable hypothesis, these patterns are still inconclusive.

Motivated by this evidence, we develop a formal theory of forecast coherence in firm production. In our model, two inputs (capital and labor) combine to produce output according to a standard production technology. The optimal forecasts of inputs and output coherently reflect both the production technology and the budget constraint. In the first best, we establish that a forecaster who is asked to produce forecasts of growth rates of output and inputs should provide forecasts that are linked cross-sectionally by parameters reflecting the contribution of capital and labor to the firm's production technology and budget constraint.

Our framework yields natural tests of forecast coherence across balance-sheet variables. We find that most CFOs forecast a growth of output that is larger than the output growth implied by feeding into a general CES production function the same CFOs' forecasts of capital and labor input growth, thereby violating an inequality implied by our model. Under more stringent assumptions to account for uncertainty, we develop a test based on the idea that under the null hypothesis of coherence the forecast errors of output and inputs cannot be "too far" from one another. We find that for 55% of CFOs in our sample we reject the null hypothesis of coherence at the 95% confidence level.

Furthermore, our framework provides a benchmark for an ex ante optimal coherent forecast, which we use to evaluate the rules of thumb that the managerial literature has proposed to help managers make balance-sheet forecasts. We establish conditions under which these rules of thumb yield optimal coherent forecasts. We find that some (but not all) of these rules of thumb represent an optimal second-best forecast rule when CFOs observe noisy signals about the firm's production technology. In particular, narrow-bracketing forecasts projecting past capital expenditures into the future are second-best optimal in the limit in which the CFO observes infinitely noisy signals about the output and the other input (e.g., labor).

To assess our model empirically, we develop a continuous, ex ante measure of managerial incoherence given by the (absolute value of the) orthogonal distance between the actual forecast and the theoretically optimal coherent one. This distance measure is predetermined relative to corporate performance and can thus be used to assess our model's predictions, unlike our test statistic that also makes use of realizations. We validate our distance measure by showing that it strongly predicts the calculated test statistic. Consistent with our model, we find that (1) the narrow-bracketing rule of thumb is the most distant from the optimal coherent forecast, and (2) corporate performance (ROA) correlates negatively with managerial incoherence and is lowest for firms whose CFOs provide narrow-bracketing forecasts. We also show that the use of incoherent rules of thumb correlates negatively with investment spending and positively with leverage.

Finally, we examine corporate performance, investment, and debt around the date when the CFO takes office for the subset of CFOs who disclose their identity. We use hand-collected data to track these critical event dates and perform an event-study analysis. Our results show that performance decreases in the years following the start date of an incoherent CFO, and such decline is larger, the more incoherent the CFO. In the same spirit, investment spending declines in the years following the start date of a narrow-bracketing CFO. Although these results are consistent with our model, given our data, we are not able to conclude that these empirical relationships are necessarily causal. Our results indicate that incoherence is pervasive among top financial executives, and suggest that incoherence may come with the use of a suboptimal mix of capital and labor.

The idea that coherence is a pillar of rationality goes back at least to Aristotle.[2] Ethical and political philosophers and legal scholars have often seen incoherence in the context of human institutions such as moral and legal systems as concerning.[3] Rational choice theory typically maintains that individuals are endowed with coherent systems of beliefs and preferences, and defines coherence as the principal criterion of rationality (e.g., Becker (1996) and Posner (2014)). Sen (1993) formalizes the notion of "internal consistency" (i.e., coherence) in the context of individual decision making and argues that there is no way of determining whether a choice function is coherent or not without referring to something external to choice behavior such as objectives, values, or norms. Tversky and Kahneman (1974) similarly argue that to fully assess coherence one should elicit "*the entire web of beliefs held by the individual*" (p. 1130).

By studying coherence in the context of production theory and firm plans, we bypass the need to elicit beliefs, norms, or values. As long as firms' executives agree that firms' profits should be maximized, the features of the production technology, which are given in the short run, should guide firm planners in making coherent forecasts. Therefore,

[2]See, e.g., Fogelin (2003)'s illustration using the law of noncontradiction.
[3]E.g., Raz (1994), Rawls (1999), Dworkin (1986)), and Sunstein et al. (2002).

our findings of pervasive incoherence imply that firms may leave money on the table by using a suboptimal mix of inputs, consistent with recent work on behavioral firms making inefficient choices, see DellaVigna (2018), DellaVigna and Gentzkow (2019), and Strulov-Shlain (2022).

The paper proceeds as follows. Section II discusses our contribution to the literature. Section III describes the Duke Survey and presents some motivating empirical evidence. Section IV presents our model of optimal coherent firm forecasts and derives our statistical tests of coherence. Section V presents our empirical results. Section VI concludes.

II Related Literature

Coherence and Accuracy Requirements of Rationality. The psychology literature has long recognized that rationality in probabilistic judgments and forecasts involves both accuracy (sometimes called 'correspondence') and coherence (sometimes called 'consistency').[4] This literature typically maintains that accuracy and coherence are separate properties, but has struggled to provide a formal framework or direct evidence to assess such a claim.[5] Rather, the literature has focused on predicting systematic inaccuracy from specific violations of statistics and probability laws.

In a series of famous experiments, Tversky and Kahneman (1971, 1974, 1983) document systematic misconceptions of statistics and probability theory, including the law of large numbers, the conjunction rule, the law of total probability, and Bayes' theorem. Since then, a number of theories and experiments have shown how the extent of such misconceptions predicts systematic inaccuracy in specific prediction tasks, see, e.g., Rabin (2002), Benjamin et al. (2016), Wright et al. (1994), Berg et al. (2022), Zhu et al. (2020), and Zhu et al. (2022).

Tversky and Kahneman (1974) have famously labeled this research program as

[4]See, e.g., Hammond (1996), Gigerenzer et al. (1999), Mandel (2005), Gigerenzer and Gaissmaier (2011), Lee and Zhang (2012).

[5]A theoretical literature in philosophy develops axiomatic definitions of coherence using probability theory, e.g., see Schippers (2014) and references therein. A general theme in this literature is to define two propositions as coherent with each other if they are positively correlated according to some suitably defined measure of correlation. In the data, however, multiple forecasts may be correlated for reasons other than coherence. Furthermore, none of these papers disentangle coherence from accuracy.

"heuristics and biases", whereby the use of heuristics generates systematic and predictable forecast errors (see also Thaler (2018)). More recently, however, a number of authors have recognized that at least some of these results can also be cast in terms of the coherence-accuracy framework. For example, Hammond (1996), Tentori et al. (2013), Jönsson and Shogenji (2019), and others discuss how Tversky and Kahneman (1983)'s conjunction fallacy can be understood as a violation of coherence with respect to probability laws. Similar arguments can be made with respect to the disjunction fallacy and violations of Bayes' theorem. Therefore, documenting a form of incoherence such as the violation of the law of total probability or Bayes' theorem in one domain typically serves the purpose of predicting future systematic inaccuracy in another domain, without aiming at disentangling coherence from accuracy.

Relative to this literature, our key contribution is to provide a formal framework in a forecasting setting in which coherence and accuracy are defined with respect to the same forecasting task, which allows us to jointly assess forecast accuracy and coherence. We show that to do so one needs both theory and data. In terms of theory, an economic model provides a benchmark against which to judge coherence. This is similar to the use of probability theory for assessing coherence of probabilistic judgments, but crucially economic theory allows us to nest forecast coherence and accuracy in a setting with optimizing agents. In terms of data, observing both forecasts and realizations—and thus forecast errors—allows us to jointly assess accuracy and coherence, and also to disentangle them at the individual level. Accuracy is assessed by testing whether forecast errors of each variable are 'sufficiently' close to zero; coherence is assessed by testing whether forecast errors of different variables (inputs and output in our setting) are 'sufficiently' close to one another. In both cases, the extent of 'sufficiently' is pinned down by economic theory. In our data, we find that 12% of individuals are incoherent but accurate, and 13% of individuals are inaccurate but coherent.

Disentangling coherence and accuracy is crucial, because—at least under some conditions—coherence can be assessed ex ante. This is a similar intuition to that of the debiasing research program in psychology (e.g., Fischhoff (1982)), with the key difference that in our framework one can use economic theory and regression analysis to determine the ex ante coherent rule. In fact, the managerial education literature (e.g., Ruback (2004), Titman and Martin (2016), Welch (2017), Holthausen and Zmijewski (2020), and Koller et al. (2020)) recognizes the challenge of forecasting many firm variables at the same time and provides a number of rules of thumb, without however relying on theory or evidence to guide the choice among them. We show both theoretically and empirically that not all rules of thumb are equivalent to one another. To the contrary, while some rules of thumb do come close enough to the ex ante optimal coherent forecast, others provide severely incoherent forecasts. Notably, the "plain growth forecast" (Welch (2017), p. 593) that we dub the 'narrow-bracketing' rule yields severely incoherent forecasts.

Bracketing. Lab experiments and empirical research in psychology and behavioral economics show that decision makers often narrowly bracket and make interrelated decisions in isolation (e.g., Tversky and Kahneman (1981), Read et al. (1999), Rabin and Weizsäcker (2009), and Ellis and Freeman (2020)). To rationalize this evidence, Thaler (1985, 2018) and Heath and Soll (1996) argue that individuals hold a mental account of each decision without considering the interdependence among decisions implied by the budget constraint and the marginal rates of substitution in the utility function.

Economic theories of narrow bracketing and mental accounting include Barberis et al. (2006), Rabin and Weizsäcker (2009), Hastings and Shapiro (2013, 2018), and Lian (2021).[6] Our theory is closest to Lian (2021), who models a narrow thinker making consumption decisions about individual goods with imperfect knowledge of the other goods and faces difficulties at coordinating multiple decisions, thereby endogenizing narrow bracketing.

We add to this literature by developing the first model of narrow bracketing in a firm setting featuring a production technology, and by providing field evidence of narrow bracketing in a sample of senior financial executives directly involved with corporate forecasts and production decisions. Similar to Lian (2021), there are no explicit mental budgets, which avoids the need to take a stand on where such mental budgets come from.

[6]These models study the monetary gambles, stock market participation, and consumption decisions of narrow-bracketing agents. They show that narrow bracketing can lead to stochastically dominated choices, including low stock market participation, stochastically dominated gambles, and suboptimal consumption bundles.

Moreover, our agent also makes decisions and forecasts based on different, non-nested information, which differentiates both us and Lian (2021) from the rational inattention literature (e.g., Sims (2003), Mackowiak and Wiederholt (2009), Matějka and McKay (2015), and Kőszegi and Matějka (2020)), in which different decisions are made based on the same nested information.[7] Unlike Lian (2021), we study a production model where the interconnection among different decisions come from the budget constraint and the production technology, which delivers novel predictions about corporate forecasts and corporate performance, which we then test in our data.

Survey Expectations of Firms. Our paper is also related to the recent and growing empirical literature studying beliefs and forecasts of corporate top executives both about the macroeconomy and about own variables.[8] Ben-David et al. (2013) and Boutros et al. (2020) show that top executives are miscalibrated, as they provide probability distributions of stock market returns that are too narrow, consistent with managerial overconfidence (see also Campello et al. (2010), Campello et al. (2011, 2012) on financial constraints and the financial crisis). Bloom et al. (2021) show that forecasting firms' own variables is even harder than forecasting the aggregate economy. Gennaioli et al. (2016) show that corporate investment plans as well as actual investment are explained by CFOs' expectations of earnings growth. Graham (2022) documents that the revenue growth forecast is most important in terms of its consequences for the firm and its plans (see also Altig et al. (2022)). Bachmann and Bayer (2013, 2014) find that the dispersion and volatility of expectations and expectation errors are countercyclical. We confirm that firms make on average accurate sales growth forecasts, but we also show that expectations of other variables, for example capital expenditures, are much less predictive of realized growth rates, consistent with incoherence. We add to this literature

[7]Rational inattention models also use noisy signals to capture decision makers' inability to incorporate all relevant information when making each decision. Related to rational inattention but using a deterministically imperfect perception of fundamentals rather than noisy signals, Gabaix (2014, 2019) develops a sparsity model in which, similar to the rational inattention approach, the sparse agent's multiple decisions are made based on the same, imprecise perception of the fundamental. However, in all these models, different decisions are made based on the same nested information.

[8]See selected chapters on firm expectations in the recent Handbook of Economic Expectations, e.g., Born et al. (2023) and Candia et al. (2023).

by documenting heterogeneity in the extent to which corporate managers provide forecasts of multiple balance-sheet items at the same time. We are able to directly control for the measures of overconfidence and optimism used in this literature and we show that, unlike overconfidence that predicts more aggressive corporate investment spending, incoherence correlates with reduced investment spending and increased leverage, consistent with the idea from psychology that incoherence and overconfidence are different traits.

III Data and Motivating Evidence

A. Data

We use two main sources of data, one on CFO expectations and one on firm realizations. CFO expectations come from the Duke Survey, launched by John Graham and Campbell Harvey in July 1996. Each quarter, the study surveys between 2,000 and 3,000 CFOs, asking their views about the US economy and corporate policies, as well as their expectations of future firm performance and operational plans. The usual response rate is 5% to 8%; most responses arrive within the first two days of the survey invitation date. Since the end of the 1990s, the survey consistently asks respondents their expectations of the future twelve-month growth of key corporate variables, including revenues, capital expenditures, employment, and earnings.[9]

Our data comprises 72 quarterly surveys conducted between March 2001 and December 2018. We observe corporate forecasts as a single number per variable, which we interpret as the CFO's expected value, corresponding to the firm's base case scenario. For many firms, the base case is the only scenario that gives rise to fleshed out forecasts in their internal planning process.[10]

Forecasts are elicited for all variables jointly as follows:

Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, −2%, etc.) [Leave blank if not applicable] Revenues: _____; Capital

[9]Historical surveys as well as aggregated responses can be accessed at https://cfosurvey.fuqua.duke.edu/.

[10]Firms that internally consider additional scenarios typically consider a downside scenario to plan for contingencies and an upside scenario to lay out stretch goals. However, these additional scenarios are often developed in less detail than the base case and do not necessarily lead to fleshed-out forecasts. See Graham (2022), p. 1997, for more details.

spending: _____; R&D spending: _____; Technology spending: _____; Prices of your product: _____; Earnings: _____; Cash on balance sheet: _____; Number of domestic full-time employees: _____; Wage: _____; Dividends: _____. Advertising: _____. Share repurchases: _____.

Figure A1 of the Online Appendix displays an actual screenshot of the above questions. Table 1 reports summary statistics on CFO twelve-month ahead growth forecasts (Panel A) and on growth realizations in a matched Duke-Compustat sample (Panel B).[11]

Firm realizations come from Compustat, which extracts the information from the Security and Exchange Commission (SEC)-required public filing of financial statements. Compustat covers all publicly traded firms across all sectors of the US economy since 1955. We exclude firms with negative assets and we winsorize at the 1% level.

When matching Duke and Compustat data there are four sources of attrition: (1) due to privacy restrictions associated with these data, not all Duke respondents report their firm ID, so they cannot be matched to Compustat; (2) not all Duke respondents give forecasts about all variables in each survey; (3) not all variables elicited in the Duke survey have a precise counterpart in Compustat, namely, technology spending, outsourced employees, health spending, productivity, product prices, and share repurchases; and (4) not all variables for which there is a precise counterpart in Compustat have full coverage, chiefly among those, wages are missing for about 90% of Compustat firms and R&D and advertising expenditures are also missing for a large fraction of Compustat firms.[12]

Table A2 in the Appendix reports summary statistics on the full Compustat sample and on the matched Duke-Compustat sample. Firms in the Duke data are on average larger than Compustat firms in terms of sales and assets. Firms in the Duke data are also more profitable and hoard more cash than Compustat firms, but are otherwise similar in terms of market-to- book ratio, investment, and leverage. These patterns broadly concur with prior work using the Duke data (e.g., Ben-David et al. (2013)).

[11]We match the Duke and Compustat datasets by firm ID, implying that for some firm-year pairs there might be multiple CFO forecasts. Table A1 in the Appendix reports the same statistics in the full Compustat population.

[12]The matched Duke-Compustat sample mostly refers to the earlier part of the sample, until about 2011Q4. This is not a problem since we will conduct most of our regression analysis in the pre-financial crisis period. Points (1) and (2) imply a potential selection problem. If anything, however, our respondents are positively selected among those more likely to give coherent and accurate forecasts of all variables, under the assumption that missing forecasts reflect lack of knowledge about the variables. Points (3) and (4) imply that our analysis of forecast errors needs to be limited to variables for which there is full coverage in both Duke and Compustat.

Comparing Panel A and B of Table 1 shows that CFOs are on average slightly more optimistic about output (i.e., revenues), although the medians of forecasts and realizations are quite close to one another, consistent with the observation in Graham (2022) that CFOs care about getting revenues forecasts right. Conversely, CFOs are on average more conservative about input (i.e., capital expenditures), with the distribution of capital expenditures realizations shifted to the right relative to the distribution of capital expenditures forecasts. However, these simple comparisons mask substantial cross-sectional heterogeneity. In the next subsection, we examine the joint distributions of output and input forecasts and forecast errors.

B. Motivating Evidence

Figure 1 shows the scatter plot of contemporaneous forecasts of output growth (i.e., revenues growth) and input growth (i.e., capital expenditures growth). Panel A refers to the whole Duke sample, whereas Panel B refers to the matched Duke-Compustat sample. According to the managerial education literature (e.g., Ruback (2004) and others), one would expect a strong positive association between output and input forecasts. Yet, both scatter plots in Panel A and B of Figure 1 show huge amounts of dispersion in the contemporaneous forecasts of output and capital. In particular, more than one quarter of CFOs predict at the same time an increase in output (i.e., sales revenues) and a decrease in input (e.g., capital expenditures); about one third of CFOs predict at the same time a decrease in output and an increase in capital expenditures. Furthermore, while the univariate regression coefficient of sales growth forecast on capital expenditures growth forecast in Panel A is positive (0.157) and strongly statistically significant as expected, there remains substantial unexplained variation (the R^2 is 4%). Similar patterns obtain in Panel B, where the regression coefficient is 0.171 and the R^2 is 8%.

While suggestive of incoherence, these patterns could still be reflecting rational coherent forecasts. Take for example the upper-left quadrant in which CFOs forecast higher sales but lower capital expenditures. These forecasts would make sense if, for example, the firm had a lot of accumulated inventory, so that over the following 12 months that firm could increase its sales while being able to accommodate a decrease in capital expenditures over the same horizon. Similarly, production lags could imply a build up of fixed capacity while at the same time not an increase in output in the following 12 months, explaining observations in the lower-right quadrant in which CFOs forecast higher capital expenditures and lower sales.

More generally, the dispersion in CFO forecasts documented in Figure 1 could reflect firm-level heterogeneity in realized sales and capital expenditures. For some firms, high sales growth might come with low capital expenditures growth, and for others the opposite might occur. Indeed, Figure 2 shows that the realizations of sales growth and capital expenditures growth also display a significant cross-sectional dispersion. Comparing the two scatter plots of forecasts and realizations, however, is insufficient because the comparison is not made at the individual firm level. If the forecasts shown in Figure 1 rationally reflect heterogeneity across firms, then one would expect that the same firm whose CFO predicts high sales growth and low capital expenditures should later end up with high realized sales growth and low realized capital expenditures. That is, one would expect that the forecast errors of sales growth and capex growth should be (close to) zero.

This is not the case in the data. Figure 3 plots the forecast errors in sales against the forecast errors in capital expenditures. Our sample size shrinks considerably, because for this exercise we need to match CFO forecasts to realizations in Compustat data. However, the pattern is very similar to that in Figure 1. While the univariate regression coefficient is positive (0.149) and strongly significant as expected, the R^2 is only 11%. Most important, 42% of observations still lie in the upper-left and lower-right quadrants, indicating that many CFOs make forecast errors of contemporaneous output and input of opposite sign. These patterns are inconsistent with mere heterogeneity across firms in sales and capital expenditures growth. Rather, these patterns suggest pervasive incoherence across CFOs.[13]

At the same time, in the absence of a theoretical model and a well-specified testable hypothesis, these patterns are still inconclusive. We need a theoretical framework to formalize the notion of coherence and to derive statistical tests to detect incoherence in the data. In the next section, we provide such a framework.

[13]In the body of the paper, we focus on the joint distributions of output and input forecasts and forecast errors to provide a close mapping with theory. In unreported tests, we find similar empirical patterns in the joint distribution of forecasts and forecast errors of output and profit, and of input and profit.

IV Theoretical Framework

When preparing corporate plans, CFOs typically start from output by making a sales revenue forecast (aka top line forecast) for a number years, and then proceed to make forecasts of all other balance sheet variables, including capital and labor expenditures (e.g., see Welch (2017) and Graham (2022)). Therefore, CFOs face a challenging multidimensional forecasting problem, which requires making forecasts of multiple balance sheet items that are individually accurate and collectively coherent with one another.

The managerial education literature proposes a number of rules of thumb to make corporate forecasts. Welch (2017), p. 593-594, provides the following taxonomy:

(R1) A **plain growth** forecast, projecting the past growth rates of each individual item. Welch (2017) implements this rule by computing the average of the two most recent past annual growth rates and taking this average as the predictor of future growth.

(R2) A pure **proportion of sales** forecast, forecasting each item, e.g., capital expenditures, as a fixed proportion of sales. Welch (2017) implements this rule by assigning each item the same growth rate as sales.

(R3) An **economies-of-scale** forecast, positing for each item a fixed component and a variable component, the latter itself a proportion sales. Welch (2017) implements this rule by estimating a univariate mean linear regression of each balance sheet item's growth on contemporaneous sales growth using Compustat data. The estimated regression intercept is the fixed component and the estimated slope multiplied by the sales forecast is the variable component.

(R4) An **industry-based** forecast, drawing on information from other firms in the same industry. Welch (2017) implements this rule exactly as (R3), but using only data from firms in the same industry as the firm under consideration.

(R5) A **disaggregated** forecast, recognizing that each item may comove not only with sales but also with the other items. Welch (2017) implements this rule by expanding the specification of the (R3) regression to include additional contemporaneous items and using all Compustat data.

The literature has not reached a consensus on which of the above rules, if any, constitutes best practice. For example, Ruback (2004) advocates using methods (R2) and (R3). Harvard Business School case studies typically suggest a combination of methods (R1), (R2), and (R4), e.g., see Luehrman and Heilprin (2009) and Stafford and Heilprin (2011). Koller et al. (2020) advocate (R2), writing that "*net Property, Plant and Equipment should be forecast as a percentage of revenues*" (p. 286). Titman and Martin (2016), Chapter 2, describe a method akin to (R3). Holthausen and Zmijewski (2020) describe an elaborate process to generate a forecast of capital expenditures that is akin to (R5).

This literature lacks a formal framework designed to offer guidance as to whether the above methods are equivalent to one another or differ along key dimensions and, if so, which method is best and under what conditions.

In this section, we address this gap. We have two objectives. First, in Subsection *A.* we develop a normative theoretical framework to derive rational coherent corporate forecasts ex ante and statistical tests for detecting incoherence ex post. Second, in Subsection *B.* we develop a positive framework describing how CFOs make second-best optimal forecasts in the presence of noisy signals about the firm's technology. Our framework nests the above rules of thumb and shows conditions under which some of them emerge as second-best optimal.

A. A Benchmark Model of Optimal Corporate Forecasts

Consider a general class of constant elasticity of substitution (CES) production functions and a budget constraint,

$$\begin{aligned} y = f\left(x_1, x_2\right) &= \left(\frac{a}{a+b} x_1^{\xi} + \frac{b}{a+b} x_2^{\xi}\right)^{\frac{a+b}{\xi}} \\ p_1 x_1 + p_2 x_2 &\leq Z, \end{aligned}$$

where y is the output, x_1 and x_2 are input quantities (capital and labor), p_1 and p_2 are the input prices, the output price p_y is normalized to 1, Z is a real-valued budget constraints, $\nu \equiv a+b > 0$ are parameters governing the returns to scale (constant for $\nu = 1$, increasing for $\nu > 1$, and decreasing for $\nu < 1$), and the elasticity of substitution between x_1 and x_2 is $\chi = \frac{1}{1-\xi}$. We assume that factor-augmenting productivities are constant over time and we normalize them to one.[14] We also assume that the technological relationship is stable over time and not subject to aggregate shocks.[15] This formulation is very general (Moysan and Senouci, 2016) and it nests a number of widely used specifications as special cases.[16] Finally, denote $\log p_i = \pi_i$, where $i = 1, 2$, and assume for now input prices are i.i.d., $\{\pi_{i,t}\}_{t\geq 1} \sim \mathcal{N}\left(0, \sigma_i^2\right)$, with $corr\left(\pi_1, \pi_2\right) = \rho_{1,2}$.[17]

Consider a forecaster who at time t issues a forecast F_t of the future realization of a generic variable, x_{t+1}, to minimize a quadratic loss function,

$$\min_{F_t} \mathbb{E}\left[\left(x_{t+1} - F_t\right)^2 |\Omega_t\right],$$

where Ω_t denotes the information set at t and at solution $F_t^* = \mathbb{E}\left[x_{t+1}|\Omega_t\right] \equiv \mathbb{E}_t\left[x_{t+1}\right]$.

A.1 Optimal Forecasts and Tests of Coherence

Proposition 1 (Inequality). *Forecast coherence requires that the forecasts of output and inputs, $\mathbb{E}_t\left[y_{t+1}\right]$, $\mathbb{E}_t\left[x_{1,t+1}\right]$, and $\mathbb{E}_t\left[x_{2,t+1}\right]$, satisfy an inequality, whose direction depends on whether the CES production function is concave or convex. For $\xi \leq 1$ and $a + b \leq 1$, the CES function is concave and forecast coherence requires*

$$\mathbb{E}_t\left[y_{t+1}\right] \leq f\left(\mathbb{E}_t\left[x_{1,t+1}\right], \mathbb{E}_t\left[x_{2,t+1}\right]\right) = \left(\frac{a}{a+b}\mathbb{E}_t\left[x_{1,t+1}\right]^{\xi} + \frac{b}{a+b}\mathbb{E}_t\left[x_{2,t+1}\right]^{\xi}\right)^{\frac{a+b}{\xi}}. \quad (1)$$

[14] This is without loss of generality because in our setting a TFP shock would be observationally equivalent to input price shocks in the same direction.

[15] This is plausible because we focus on cross-sectional differences in coherence across forecasters, and we implement most of our tests over 2001-2007 at the peak of the 'great moderation', a time when aggregate volatility was not a concern.

[16] For $\chi \to +\infty$ the inputs are perfect substitutes and the production function is linear; for $\chi \to 0$ there is no substitution and the production function is Leontieff; and for $\chi = 1$ we have a Cobb-Douglas. The empirical literature suggests as a plausible $\chi \in (0.5, 1]$ (e.g., see Berndt (1976) and Oberfield and Raval (2021)), implying $\xi \in (-1, 0]$.

[17] While our theory can be readily extended to the general case of n inputs, we shall focus on the case of a production function $F(K, L)$ with two inputs, capital (K) and labor (L), as it allows a tight mapping with our data. In principle, we could consider a production function $F(K, L, M)$ with three inputs, capital (K), labor (L), and materials (M). However, CFOs are asked to forecast future sales and future expenditures on capital and wages, but not materials. Similarly, Compustat contains data on realized sales, capital expenditures, and wages, but not materials.

For $\xi \geq 1$ and $a+b \geq 1$, the CES function is convex and coherence requires

$$\mathbb{E}_t\left[y_{t+1}\right] \geq f\left(\mathbb{E}_t\left[x_{1,t+1}\right], \mathbb{E}_t\left[x_{2,t+1}\right]\right) = \left(\frac{a}{a+b}\mathbb{E}_t\left[x_{1,t+1}\right]^{\xi} + \frac{b}{a+b}\mathbb{E}_t\left[x_{2,t+1}\right]^{\xi}\right)^{\frac{a+b}{\xi}}. \tag{2}$$

All Proofs are in the Appendix.

Proposition 1 provides a first restriction on contemporaneous forecasts that a coherent forecaster must satisfy. Empirical implementation of this first test of forecast coherence requires data on $\mathbb{E}_t\left[y_{t+1}\right]$, $\mathbb{E}_t\left[x_{1,t+1}\right]$, and $\mathbb{E}_t\left[x_{2,t+1}\right]$ for each CFO available in Duke Survey and knowledge of parameters a and b. Since typically $a+b \leq 1$, we focus on inequality (1). We rely on the literature to obtain a range of plausible values for the elasticity of substitution, χ, for which it is often the case that $\chi \in (0.5, 1]$ (e.g., Berndt (1976) and Oberfield and Raval (2021)). With this information, in Subsection V.A. we verify how many CFOs report coherent forecasts of output and inputs in the sense of inequality (1).

In general, the CES is a non-linear function of the inputs. Because the rules of thumb (R1)-(R5) are instead linear, and one of our objectives is to rationalize these rules of thumb at least in a second-best sense, we shall now focus on the limit case of $\xi \to 0$, corresponding to a Cobb-Douglas production function,

$$\lim_{\xi \to 0}\left(\frac{a}{a+b}x_1^{\xi} + \frac{b}{a+b}x_2^{\xi}\right)^{\frac{a+b}{\xi}} = x_1^a \cdot x_2^b.$$

Corollary 1 (Cobb-Douglas). *In the limit case in which $\xi \to 0$,*

$$\mathbb{E}_t \log\left[y_{t+1}\right] = a \cdot \mathbb{E}_t \log\left[x_{1,t+1}\right] + b \cdot \mathbb{E}_t \log\left[x_{2,t+1}\right].$$

Similarly,

$$\mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] = a \cdot \mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right].$$

Because the Cobb-Douglas production function is linear in logs, the coherence requirement of Proposition 1 holds with equality, both for forecasts expressed in levels and in growth rates.

The Cobb-Douglas specification is also useful because it allows us to construct two additional tests of coherence. We now assume that prices follow an AR(1) process, $\pi_{i,t+1} = \gamma_i \pi_{i,t} + \epsilon_{i,t+1}$, with $0 < \gamma_i < 1$ ($\gamma_i = 0$ denotes the i.i.d. case), where the error terms are i.i.d., normally distributed, and uncorrelated, namely, $\{\epsilon_{1,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_1^2)$, $\{\epsilon_{2,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_2^2)$, and $\{\epsilon_{1,t}\}_{t\geq 1} \perp \{\epsilon_{2,t}\}_{t\geq 1}$.

Proposition 2 (Test Statistics). *If $\xi \to 0$, $\rho_{1,2} = 0$, and $p_1 x_1 + p_2 x_2 = Z$, under the null hypothesis of coherent forecasts it holds that*

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a\mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1) \tag{3}$$

and

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1), \tag{4}$$

where $FE_t \log y_{t+1} = \log y_{t+1} - \mathbb{E}_t \log y_{t+1}$ and $FE_t \log x_{1,t+1} = \log x_{1,t+1} - \mathbb{E}_t \log x_{1,t+1}$.

Proposition 2 derives two test statistics at the individual CFO level. These statistics have an intuitive interpretation. Under the null of coherence, the forecasts of the output and of one input cannot be "too far" from each other (3). Similarly, the forecast errors of the output and of one input cannot be "too far" from each other (4).

Similar to Proposition 1, empirical implementation of Proposition 2 requires knowledge of technology parameters a and b. Additionally, Proposition 2 further requires knowledge of γ_2 or σ_2, as well as the more stringent assumptions that $\xi \to 0$ and that input prices follow the assumed processes.[18]

Unlike Proposition 1, Proposition 2 does not require observing $\mathbb{E}_t[x_{2,t+1}]$ or realization $x_{2,t+1}$, so it can be implemented when $\mathbb{E}_t[x_{2,t+1}]$, $x_{2,t+1}$, or both are not observable. This observation is important for our empirical analysis, as one limitation of Compustat is its limited coverage of wages: about 90% of all firm-year observations on wages are missing.

Comparing the C1 and C2 statistics of Proposition 2 is also instructive. On the one hand, computation of the C1-stat in (3) does not require observing realizations, so it can be potentially implemented with expectations data only. On the other hand, the C1-stat requires information about the budget, Z, which is not only the cash and liquid securities

[18] When some of these parameters need to be estimated, the test statistics will no longer be normally distributed. We further discuss implementation details in Subsection V.A.

from the firm's balance sheet but also the external resources that firms can access from financial markets at short notice. While known to CFOs, this information is not easily observable by the econometrician for most firms. Furthermore, the C1-stat does not allow to distinguish between forecast coherence and forecast accuracy.

Conversely, the C2-stat in (4) requires observing both forecasts and realizations, but it does not require observing the budget, Z. Therefore, in our analysis we focus on the C2-stat. Moreover, the C2-stat enables us to distinguish between coherence and accuracy, because in our model $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1} \sim \mathcal{N}(0,1)$ and $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y} \sim \mathcal{N}(0,1)$.

Figure 4 depicts the theoretical connection between forecast accuracy and forecast coherence as implied by the C2-stat and shows that there are four conceptual cases, corresponding to the four areas of the figure. In the first area, the forecaster is both accurate and coherent. This occurs when both $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero and also close to each other. In the second area, $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$, $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$, or both are statistically different from zero but quite close to each other, so the forecaster is inaccurate but coherent. In the third area, both $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero but sufficiently apart from each other, so the forecaster is accurate but incoherent. In the fourth area, $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$, $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$, or both are statistically different from zero and also far apart from each other, so the forecaster is both inaccurate and incoherent.

Much research in psychology and elsewhere has been cast in terms of whether the accuracy or the coherence paradigm is the correct one, e.g., see Hammond (2007) and references therein. These interpretations are incomplete or even misleading. By clarifying the theoretical connection between coherence and accuracy, Figure 4 reveals that coherence and accuracy are two distinct but related concepts, both of which are necessary to understand rationality of individual forecasts. Figure 4 further shows that one needs an analytical framework nesting both accuracy and coherence to disentangle them and to understand their relationship.

A.2 Optimal Forecasts and Rules of Thumb

We now consider the forecasting problem in the case a and b are unknown to the forecaster.

Proposition 3. *If parameters a and b are unknown, a forecaster can estimate them using* *a linear projection operator, with the forecasted variables in logs.*

Corollary 2. *In a multivariate linear projection,* $\mathbb{E}_t \log[x_{1,t+1}] = \alpha + \beta_1 \cdot \mathbb{E}_t \log[y_{t+1}] + \beta_2 \cdot \mathbb{E}_t \log[x_{2,t+1}]$*, the parameters are*

$$\alpha = \mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0, \qquad \beta_1 = \frac{1}{a}, \qquad \beta_2 = -\frac{b}{a},$$

where $\mathbb{E}\log[y_{t+1}]=\mu_y$ *and* $\mathbb{E}\log[x_{i,t+1}]=\mu_i$*, for* $i = 1, 2$*, are the unconditional means. The same result obtains with the variables in growth rates.*

Corollary 2 rationalizes how a rule of thumb akin to (R5) described above delivers the first-best optimal forecast. Specifically, Corollary 2 prescribes implementing (R5) using data on output growth and labor growth forecasts to provide forecasts of capital growth and using parameters derived from a linear projection of the firm's input on the output and the other input. Note also that (R5), as well as (R1)-(R4), are defined in the managerial education literature as linear functions of growth rates (not in logs). Our analysis implies that such linear rules will be correct up to a first-order Taylor approximation. Importantly, Corollary 2 holds both in levels and in growth rates.

Corollary 3. *In a univariate linear projection,* $\mathbb{E}_t\log[x_{1,t+1}] = \alpha + \beta \cdot \mathbb{E}_t \log[y_{t+1}]$*, the parameters are*

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}\mu_y, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}.$$

In a univariate linear projection with the variables in growth rates, $\mathbb{E}_t\log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] = \alpha + \beta \cdot \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right]$*, the parameters are*

$$\alpha = 0, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}.$$

Corollary 3 shows that rule of thumb (R3), which makes use of information on the output and one input but neglects the other input, in general yields different forecasts from those of (R5). Therefore (R3) yields in general incoherent forecasts. We now examine a special case in which (R3) yields coherent forecasts.

Corollary 4. *If* $\rho_{1,2} = 1$ *and* $\sigma_1^2 = \sigma_2^2 = \sigma_{1,2} = \sigma^2$*, then for a linear regression in growth rates,* $\mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\right] = \alpha + \beta \cdot \mathbb{E}_t\left[\frac{x_{i,t+1}}{x_{i,t}}\right] + e_{i,t+1}$*, with* $i = 1, 2$*, we have* $\alpha > 0 \iff 0 < \beta <$

$1 \iff \nu < 1$. *The same is true for i.i.d. shocks, setting* $\gamma_i = 0\ \forall i$.

Corollary 4 shows that (R3) can be optimal under rather special circumstances, that is, when input prices are perfectly correlated and thus there is no added benefit from a multivariate rule like (R5) relative to the univariate rule (R3).

Furthermore, Corollary 4 indicates that rule (R2), which amounts to setting $\mathbb{E}_t\left[\frac{x_{i,t+1}}{x_{i,t}}\right] = \mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\right]$, is optimal when $\alpha = 0$ and $\beta = 1$, that is, under constant returns to scale $\nu = 1$. In this case, (R2) is exactly equivalent to (R3). If returns to scale are not constant, rule (R2) is suboptimal. More generally, whenever $\rho_{1,2} \in (-1, 1)$ and $\sigma_1^2 \neq \sigma_2^2$, both (R3) and (R2) yield incoherent forecasts and the forecaster would do better by relying on information provided by all inputs and the output.

Finally, rule (R1) amounts to extrapolating past information of the input being forecasted, while disregarding information about the output and the other input. Because (R1) treats each item in isolation, we interpret (R1) as an example of narrow bracketing. In general, (R1) amounts to setting the forecast of $x_{i,t+1}$ equal to the average of k past growth rates, $\log F_{i,t}^{R1} = \frac{1}{k}\sum_{j=1}^{k} \log \frac{x_{i,t+1-j}}{x_{i,t-j}}$. Welch (2017) advocates $k = 2$. We establish:

Corollary 5 (Losses Under Narrow Bracketing Forecasts). *Under (R1) and* $k \to +\infty$, $\mathbb{E}_t\left[L_{t+1}^{R1}\right] = \mathbb{E}_t\left[L_{t+1}^{o}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2 > \mathbb{E}_t\left[L_{t+1}^{o}\right]$ *for* $\gamma_i < 1$, *where* $\mathbb{E}_t\left[L_{t+1}^{R1}\right]$ *and* $\mathbb{E}_t\left[L_{t+1}^{o}\right]$ *denote the expected losses under (R1) and the optimal forecast, respectively.*

Corollary 5 implies that (R1) is optimal if and only if one uses a lot of data, $k \to +\infty$, and if the input price follows a random walk, $\gamma_i = 1$, otherwise it is strictly inferior.

Under optimal forecasts individuals should be broad bracketers, as they should compute forecasts taking into account the structure of the firm's problem and all available data on inputs, $x_{i,t}$, and output, y_t. Under narrow bracketing, individuals ignore the structure of the firm's problem and, when forecasting growth of item x_i, they examine data about x_i in isolation and ignore data on items $x_{\neg i}$ and y.

In reality, narrow bracketing could be a second-best optimal response to imperfect information. Moreover, individuals may be producing forecasts between the two extremes of broad and narrow bracketing. They may be better informed about the price of input 1 and have difficulty accessing information about the price of input 2.

Following Lian (2021), in the next subsection we capture these possibilities by introducing noisy signals, and we recast the forecasting problem under narrow bracketing as multiple selves playing an incomplete information, common interest game.[19] With two inputs, capital and labor, the CFO "capital self" makes forecasts of capital expenditures growth by observing imprecise signals of output and labor growth. Conversely, the CFO "labor self" makes forecasts of labor expenditures growth by observing imprecise signals of output and capital growth. In equilibrium, each self does not perfectly know other selves' signals (states of mind) and, thus, makes forecasts with imperfect knowledge of other selves' forecasts. In this sense, narrow thinking introduces intra-personal frictions in coordinating multiple forecasts.

B. A Model of Narrow Bracketing in Corporate Forecasts

We consider a forecaster self that makes a forecast for input 1, $F \log x_1$, to minimize a quadratic loss function,[20]

$$\min_{F \log x_1} \mathbb{E} \left(\log x_1 - F \log x_1\right)^2,$$

where for simplicity we drop the time subscript, t, because the problem is stationary. The firm's production technology is $y = x_1^a x_2^b$ and the budget constraint is $p_1 x_1 + p_2 x_2 = Z$. We assume that the forecaster observes two noisy signals, $\eta_y = \log y + \epsilon_y$ and $\eta_2 = \log x_2 + \epsilon_2$, where $\epsilon_y \sim \mathcal{N}\left(\mu_y, s_y^2\right)$ and $\epsilon_2 \sim \mathcal{N}\left(\mu_2, s_2^2\right)$.

Proposition 4. *The optimal forecast of input x_1 given signals η_y and η_2 is*

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right) + \beta_2\left(\eta_2 - \mu_2\right),$$

where

$$\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2 + s_2^2}}, \quad \beta_2 = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.$$

[19] In the multiple selves literature, multiple selves have conflicting interests (e.g., Marschak and Radner (1972), Piccione and Rubinstein (1997), and Benabou and Tirole (2002)); whereas in our paper and in Lian (2021) they have common interests. Despite common interests, since different selves do not share their information, they have difficulty in coordinating their decisions in response to shocks to the fundamentals.

[20] This formulation is without loss of generality as it can be cast in terms of generic inputs i and $\neg i$.

Proposition 4 shows that, upon observing signals η_y and η_2, the optimal forecast of input x_1 is given by a linear projection of (the deviation of the signals from the prior means of) output y and input x_2. In such a linear projection, the constant term is the prior mean of x_1 and the slope coefficients are functions of the fundamental uncertainty and of the precision of the signals. Proposition 4 rationalizes rule of thumb (R5) as the optimal coherent forecast also in a second-best sense. Here, Proposition 4 clarifies that in a second-best world the accuracy of this linear projection will depend on the precision of the signals. Note that in our model the forecaster makes forecasts based on different, non-nested information, that is, in the sense of Blackwell, neither input i's signal is more informative than input $\neg i$'s signal nor input $\neg i$'s signal is more informative than input i's signal (see also Lian (2021)). Next, we examine a number of special cases.

Corollary 6 (Narrow Bracketing). *When* s_y^2,$s_2^2 \rightarrow +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1$.

When both signals are infinitely noisy, the optimal forecast of input x_1 ignores the signals and instead projects the prior mean μ_1 into the future. Corollary 6 rationalizes rule of thumb (R1) as an optimal forecast when the CFO observes infinitely noisy signals about the output and the other input.

Corollary 7 (Univariate Projections). *When* $s_2^2 \rightarrow +\infty$ *and* $0 < s_y^2 < +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right)$, *where* $\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}$.

When the signal of the other input is infinitely noisy and the signal of the output is noisy but informative, the optimal forecast of input x_1 is a univariate linear projection of (the deviation from the prior means of) the output, whereby the constant term is still the prior mean of x_1 and the slope coefficient is a function of the fundamental uncertainty and of the precision of the signal. This corollary rationalizes rule of thumb (R3) as an optimal forecast when the CFO observes infinitely noisy signals about the other input. Note that in general β_y, $\beta_2 \neq 1$; hence, (R2) is in general not optimal, even in a second-best world.

Finally, rule of thumb (R4) can be thought of as a version of (R3) in which the linear projection is estimated for a subsample of firms in the same industry as the firm under consideration. On the one hand, using a smaller sample might hurt the performance of the linear projection, in which case (R3) may be superior to (R4). On the other hand, firms in an industry might differ from firms in other industries, for example, because a_j or b_j are specific to industry j, in which case (R4) may be superior to (R3). We will evaluate these possibilities empirically in the next section.

For completeness, we provide the following corollary, stating that when both signals are infinitely precise, we fall back to the case of Corollary 2.

Corollary 8 (Precise Signals). *When* $s_y^2, s_2^2 \to 0$, *the optimal forecast is* $\mathbb{E}[\log x_1 | \eta_y, \eta_2] = \frac{1}{a}(\eta_y - b\eta_2)$.

To sum up, our theory yields a normative benchmark for an ex ante optimal coherent forecast, and a number of restrictions and tests to detect incoherence ex post. Our results imply that ex ante forecasts that differ from the normative benchmark are incoherent. Therefore, the main empirical implication of our model is that expected corporate profits, $\mathbb{E}[\Psi] = \mathbb{E}[p_y y - p_1 x_1 - p_2 x_2]$, should decrease with the extent of incoherence. Furthermore, our positive framework formalizes a mechanism through which incoherence may arise as a result of narrow thinking, namely, intra-personal frictions in coordinating multiple forecasts. Our results deliver a pecking order of the rules of thumb proposed in the managerial education literature and a key additional empirical implication: the channel through which incoherence arises is by the use of certain rules of thumb, most notably, (R1) and (R2).[21] In the next section, we evaluate empirically these predictions.

V Empirical Analysis

In this section, we present our findings. In Subsection *A.*, we implement our tests of coherence. In Subsection *B.*, we introduce a continuous measure of incoherence, we establish which rules of thumb are reflected in CFOs forecasts, and how the use of such rules of thumb correlates with incoherence. In Subsection *C.*, we examine how incoherence and the rules of thumb correlate with firm performance. In Subsection *D.*, we examine

[21]Specifically, Proposition 3 says that the optimal ex ante coherent forecast is a specific version of (R5), because it implies the use of an optimal mix of inputs. Corollary 6 implies that (R1) is the most extreme—among those considered—deviation from (R5) when all signals are infinitely noisy. Corollary 7 implies that (R2) uses information about the output, but in a way that is not optimal. Corollary 7 further implies that (R3) and (R4) use optimally the information about the output but ignore the other input. As a result, (R5) should be the optimal coherent forecast rule; (R1) and (R2) should be the worst; (R3) and (R4) should be better than either (R1) or (R2), but not necessarily approximating (R5).

how the use of rules thumb correlate with firm investment and debt policy. In Subsection *E.*, we investigate how firm behavior changes around the years in which CFOs take office.

A. Test Implementation and Results

We implement the inequality restriction in Proposition 1, developed under a general CES function. We view this inequality as imposing on the data as little restriction as possible. We compute a and b using the universe of industries from the Bureau of Economic Analysis and find that $a + b \leq 1$. Furthermore, the elasticity of substitution between capital and labor in the US economy, denoted with χ, is typically documented in the literature to be between 0.5 and 1 (e.g., see Berndt (1976) and Oberfield and Raval (2021)), where $\chi = 1$ defines the Cobb-Douglas production function. Therefore, the CES function is weakly concave and thus inequality (1) is the relevant one. We account for heterogeneity by allowing a and b to vary by industry and by presenting our results for three different values of the elasticity of substitution between capital and labor, $\chi = 0.5, 0.7, 0.9$. We implement our inequality restriction both in levels and in growth rates.[22]

Table 2 reports our results. Panel A shows that most CFOs' forecasts violate the inequality restriction of Proposition 1. In levels, almost all CFOs give joint forecasts of capital, labor, and output that jointly violate the inequality. However, as just discussed, results in levels should be seen with caution, as they refer to a much smaller sample given the limitations of Compustat data on wages. In growth rates, about 73% of CFOs' forecasts violate the inequality. These results are quite stable across different values of the elasticity of substitution between capital and labor. If anything, moving toward $\chi = 1$ (Cobb-Douglas) appears to give a slightly better shot at CFOs to give coherent forecasts.

Panel B reports summary statistics of the difference between the left-hand side and the right-hand side of the inequality of Proposition 1. Most CFOs forecast a growth of output that is larger than the output growth implied by feeding into a CES production function the CFOs' forecasts of capital and labor input growth. Interestingly, the extent

[22] We observe CFO forecasts of growth rates, not of levels. Moreover, while we observe the CFO forecast of the growth rate of labor expenditures, $\mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$, for a large sample, in Compustat we observe few realizations of $x_{2,t+1}$. Therefore, when we compute $\mathbb{E}_t [x_{2,t+1}] = x_{2,t} \cdot \mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$ to implement the inequality restriction in levels, we end with much fewer observations in levels than in growth rates.

of these violations varies widely across CFOs. One possibility is that this variation is due to CFOs facing different conditions and amounts of uncertainty, which cannot be directly assessed based on the point forecasts in our data. To account for uncertainty, one needs to put more structure on the problem.

We do this in Proposition 2, where we derive two test statistics, the C1-stat and the C2-stat, based on the assumptions that the production function is Cobb-Douglas and the input prices are normally distributed. The C2-stat, which is based on forecast errors, is the best statistic to implement, as it does not require observing realizations of wages, which have limited coverage in Compustat, and it is robust to firm-level unobserved heterogeneity, e.g., the total resources Z. In fact note how, when moving from the C1-stat in equation (3) to the C2-stat in equation (4), Z drops out due to differencing away the realizations of the variables at the numerator.

We implement our C2-stat using the Duke data. To do so, we need to make a few remarks. First, the quantities $\mathrm{FE}_t \log x_{1,t+1}$ and $\mathrm{FE}_t \log y_{t+1}$ are directly observable in the data if and only if the forecasts are already elicited in logs. Then, using the logs of realizations $x_{1,t+1}$ and y_{t+1} one can directly compute the forecast errors $\mathrm{FE}_t \log x_{1,t+1}$ and $\mathrm{FE}_t \log y_{t+1}$. However, if, as it is the case in the Duke Survey, the forecasts are not elicited in logs and we only observe $\mathbb{E}_t x_{1,t+1}$ and $\mathbb{E}_t y_{t+1}$, one must use the following transformation, here written for a generic variable, x:

$$\mathbb{E}_t \log x_{t+1} = \log \mathbb{E}_t x_{t+1} - \frac{1}{2} V_t \log x_{t+1}, \tag{5}$$

where $V_t \log x_{t+1}$ is the conditional variance of $\log x$. Therefore, $V_t \log x_{i,t+1} = \sigma_i^2 = (1 - \gamma_i^2)\, V \log x_{i,t+1}$ for $i = 1, 2$, where $V \log x_{i,t+1}$ is the unconditional variance and γ_i is the coefficient of an AR(1) regression of $\log x_{i,t}$. The conditional variance of the output is then $V_t \log y_{t+1} = a^2\sigma_1^2 + b^2\sigma_2^2$.

Therefore, for each CFO we observe four items, two forecasts and two realizations, and we estimate three parameters, a, b, and σ_2, from aggregate industry data from the Bureau of Economic Analysis and Bureau of Labor Statistics. As a result, our C2-stat is distributed according to a Student t distribution with $N - K = 4 - 3 = 1$ degree of freedom. Table 3 reports our results. Panel A of Table 3 shows that for 55.7% of CFOs in our sample we reject the null hypothesis of coherence at the 95% confidence level.[23] This striking result corroborates the view that our previous evidence in Figures 1 and 3 and Table 2 is inconsistent with coherence and indicates that a majority of CFOs in our sample provide incoherent forecasts of their firm output and capital input.

Panel A of Table 3 further shows that, by contrast, CFOs are fairly accurate with respect to their output forecasts. In fact, we reject the null of accuracy for output forecasts at the 95% confidence level for 27.2% of CFOs. CFOs are substantially less accurate with respect to their capital expenditures forecasts, as we reject the null of accuracy in capital expenditures forecasts for 47.9% of CFOs in our sample. These results are consistent with the observations in Graham (2022) that top executives care the most about their output forecasts. When considering output and capital input forecasts together, we reject the null of accuracy for 57.0% of CFOs in our sample.

Panel B of Table 3 assesses coherence and accuracy together. It shows that 31.1% of CFOs in our sample are both coherent and accurate; 13.2% are coherent but inaccurate; 12.0% are accurate but incoherent; and the remaining 43.7% are both incoherent and inaccurate (all at the 95% confidence level).[24] The summary statistics of the cross-sectional distribution of our calculated C2-stat and of the forecast errors for output and capital input are shown in Panel C.

One concern with these results is the extent to which the computed C2-stat, as well as the accuracy statistics, are sensitive to the uncertainty coming from estimating the parameters a, b, and σ_2.[25] To address this concern, we perform a non-parametric bootstrap procedure, as follows. For each CFO, we generate 1,000 bootstrap repetitions

[23] At the 99% confidence level, we reject the null of coherence for 7.7% of CFOs. The difference between the rejection regions at 95% and 99% confidence level reflects the distribution of the Student t with one degree of freedom.

[24] The figures at the 99% confidence level are 89.4%, 2.9%, 3.4%, and 4.3%, respectively.

[25] To be precise, the C2-stat in (4) depends on a, b, and σ_2 *directly*, where σ_2 is the square root of the conditional variance of the log of x_2 at $t+1$ given the information set at t. Furthermore, the C2-stat depends *indirectly* on σ_1 (defined analogously to σ_2) through the forecast error of the log of x_1, which depends on (5). Finally, the C2-stat further depends indirectly on σ_y through the forecast error of the log of y, which in turn depends on a, b, σ_1^2, and σ_2^2.

of the C2-stat.[26] Using these 1,000 replications, we compute the fraction of cases out of 1,000 for which we reject the null of coherence at the 95% and 99% confidence levels. Hence, for each CFO and confidence level, the computed statistic is a number between 0 and 1, where 0 means that the null of coherence was never rejected across all 1,000 repetitions and 1 means that the null of coherence was rejected for all 1,000 repetitions.

In Figure 5, we plot the value of this statistic (on the vertical axis) against its empirical cumulative distribution function across CFOs (on the horizontal axis). The top plot refers to the calculation of the statistic at the 95% confidence level, while the bottom plot refers to its calculation at the 99% confidence level. The graph on the top shows that for about 40% of CFOs the null of coherence is rejected in all bootstrap repetitions; for about 15% of CFOs the null of coherence is never rejected; and for the remaining 45% of CFOs the fraction of rejections across bootstrap repetitions is strictly between 0 and 1. Consistent with the results in Table 3, the proportion of CFOs for whom the null of coherence is rejected more than half of the times is approximately 55%.[27] We conclude that our results in Table 3 are robust to estimation uncertainty.

To further understand our approach, consider the following cross-sectional regression test suggested by equation (4) in Proposition 2:

$$\mathrm{FE}_t \log y_{t+1} = \alpha + \beta \cdot \mathrm{FE}_t \log x_{1,t+1} + \varepsilon,$$

whereby under forecast coherence one would expect α=0 and β=a, namely, the slope coefficient should equal the capital share of GDP. When estimating this regression in our

[26]Specifically, for each of the 15 BEA industries and pair of consecutive years between 1987 and 2018, we resample observations with replacement 1,000 times (aka bootstrap replications). At each replication, we obtain an estimate of σ_1 based on the residual sum of squares (RSS) of the regression of total capital compensation on its lag and an estimate of σ_2 based on the RSS of the regression of total labor compensation on its lag, using cluster bootstrap with 6 clusters corresponding to the following year windows: 1988-1992, 1993-1997, 1998-2002, 2003-2007, 2008-2012, and 2013-2018. We additionally generate bootstrap estimates of a and b as the capital and labor shares of total factor compensation. Endowed with bootstrap estimates of a, b, σ_1^2, and σ_2^2, we derive corresponding estimates for σ_y^2, for the forecast errors of the log of output (y) and of the log of capital input (x_1), and thus for the C2-stat.

[27]The graph on the bottom of Figure 5 shows that for slightly less than 10% of CFOs the null of coherence is rejected in all bootstrap repetitions; for about 60% of CFOs the null of coherence is never rejected across all bootstrap repetitions; and for the remaining 30% of CFOs the fraction of rejections across bootstrap repetitions is strictly between 0 and 1. The proportion of CFOs for whom the null of coherence is rejected more than half of the times is slightly over 10%, that is, slightly higher but in the same ballpark as the proportion of rejections calculated in Table 3.

data, we find $\widehat{\alpha} = -0.007$, not significantly different from zero, and $\widehat{\beta}$ =0.196, precisely estimated and significantly smaller than the capital share of 0.4. Thus, we reject the null of coherence at the 99% confidence level.[28] However, these results only establish that on average CFO forecasts violate coherence, without revealing which CFOs violate it and which ones do not, and without disentangling coherence and accuracy. This is why we emphasize our individual-level tests, whose results are shown in Table 3.

B. Rules of Thumb Indicators and Measure of Incoherence

We now characterize in our data the rules of thumb used by CFOs to produce corporate forecasts. We focus on forecasts about sales revenues (i.e., output) and capital expenditures because they have a clear mapping with theory and we observe their realizations in Compustat, which allows computing forecast errors. In the Online Appendix we report data on other items.

We consider the five rules of thumb discussed in Section IV. Whenever a rule of thumb can be implemented in multiple ways, we follow Welch (2017). Therefore, (R1) uses the average growth of the past two years as the forecast. (R2) uses the same growth rate for sales revenues and capital expenditures, equivalent to using a univariate regression model, Sales Growth $= \alpha + \beta \cdot$ Capital Expenditures Growth $+ \varepsilon$, where $\alpha = 0$ and $\beta = 1$. (R3) uses the same regression, but estimates it in the population of Compustat firms. Using 2000-2019 data, $\widehat{\alpha}$ =0.106 and $\widehat{\beta}$ =0.077, precisely estimated (see column 1 of Online Appendix Table A3). (R4) estimates the same regression by industry. We do so at the 1-digit SIC code level to make sure that each industry has enough observations.[29]

Finally, (R5) recognizes that a more sophisticated approach would use coefficients from a multivariate regression model, but does not fully specify which variables to include, possibly because no benchmark theory is available in the literature. Our model of Section

[28]In fact, we reject the null for any estimate of the capital share of 0.30 or above. When we allow the capital share to vary by industry, we reject the null of coherence for 7 out of 13 industries, representing 86% of the total observations.

[29]SIC 1-digit codes roughly correspond to the following sectors: Agriculture, forestry, and fishing; Mining; Construction; Manufacturing; Transportation, communications, and public utilities; Wholesale trade; Retail trade; Finance, insurance, real estate; Services; and Public administration. SIC codes also allow a close mapping to the analogous classification of the Bureau of Economic Analysis.

IV indicates that the full multivariate model delivering the optimal rational coherent forecast should be as follows:

$$\text{Sales Growth} = \alpha + \beta \cdot \text{Capital Expenditures Growth} + \theta \cdot \text{Labor Costs Growth} + \varepsilon.$$

However, as previously mentioned, Compustat provides only scant information about wages. Furthermore, to measure incoherence one needs to observe both forecasts and realizations of both output and all input variables, implying that using capital expenditures and wages as our input variables we end up with about 50 observations, too few to allow for a meaningful empirical analysis. On the other hand, there are two variables containing information about the operating costs of the goods sold for which we do observe both forecasts and realizations, namely Earnings, corresponding in our model to $F(K, L) - p_L L$, and Advertising, a specific operating cost.[30]

Therefore, in the main text we replace Labor Costs Growth with Earnings Growth, because we observe both realizations and forecasts of earnings for a large number of firms. Column 8 of Table A3 shows that in this case $\widehat{\alpha}$ =0.106, $\widehat{\beta}$ =0.074, and $\widehat{\theta}$ =0.030, precisely estimated. We take these estimates as representing both Rule (R5) and as characterizing the rational coherent benchmark. In the Online Appendix we replace Labor Costs Growth with Advertising Growth. Column 9 of Table A3 shows that in this case $\widehat{\alpha}$ =0.081, $\widehat{\beta}$ =0.057, and $\widehat{\theta}$ =0.121, precisely estimated.

We can now assign a unique type to each CFO. We do so in two steps. First, for each CFO we compute the orthogonal distance between the actual forecast and that implied by each of the five rules of thumb (R1)-(R5). Second, for each CFO we compute the minimum distance among those five distances and we assign a type, $\tau = 1, .., 5$, corresponding to the rule that is closest to the actual forecast.

Table 4 shows that, among the 396 CFOs for whom we observe the identity and the joint forecasts and realizations of all variables, a plurality of about 40% makes a forecast that is closest to (R2), and 27% use exactly (R2). This is perhaps not surprising, since

[30] Alternatively, Compustat has extensive coverage of the cost of goods sold (COGS). The COGS item bundles together all expenses directly attributable to the production of the goods sold by the firm and includes materials and intermediate inputs, labor cost, energy, and so on. However, the Duke survey does not ask CFOs to forecast COGS, or other non-labor operating expenses, with the exception of advertising.

(R2) is a simple rule to implement, as it entails assigning the same forecast to the two items under consideration. About 15% of CFOs are closest to the rational coherent rule, (R5), and 7.6% of CFOs are closest to the narrow-bracketing one, (R1). Finally, about 11% are closest to (R3) and 27% to (R4). These results underscore the large heterogeneity in forecasting rules used by the CFOs, reflecting the fact that providing coherent forecasts is a challenging task and the managerial education literature has not achieved a consensus in recommending either rule of thumb.[31]

Next, we compute our ex ante measure of incoherence as the orthogonal distance between the three-dimensional point corresponding to the forecasts of sales growth, capital expenditure growth, and the growth of our proxy for labor costs, and the hyperplane corresponding to (R5). We implement (R5) via a multivariate regression of sales growth on capital expenditures growth and the growth of the same proxy for labor costs,

$$y_{i,t} = \beta_0 + \beta_1 x_{1i,t} + \beta_2 x_{2i,t} + \varepsilon_{i,t}. \tag{6}$$

Specifically, we define

$$\text{Incoherence}_{i,t} = \frac{\left| F_{i,t}\left[y_{i,t+1}\right] - \widehat{\beta}_1 F_{i,t}\left[x_{1i,t+1}\right] - \widehat{\beta}_2 F_{i,t}\left[x_{2i,t+1}\right] - \widehat{\beta}_0 \right|}{\sqrt{1^2 + \widehat{\beta}_1^{\,2} + \widehat{\beta}_2^{\,2}}}, \tag{7}$$

where $\widehat{\beta}_0$, $\widehat{\beta}_1$, $\widehat{\beta}_2$ are the estimated coefficients of (6).

When we proxy $x_{2i,t}$ with the growth of earnings, i.e., net income, as in column 8 of Table A3, we end up with 396 CFOs in our final sample. In the Online Appendix, where we proxy $x_{2i,t}$ with advertising growth using the estimated coefficients reported in column 9 of Table A3, we end up with 130 CFOs. In the latter case, we obtain similar results. Proxying $x_{2i,t}$ with wages growth results in a sample of about 50 CFOs, too few to allow for a meaningful analysis.

Rather than the specific variable used to proxy for labor and other operating costs, what turns out to be important is moving from a univariate regression of sales growth

[31] In the Online Appendix, we perform a number of robustness tests. Table A4 finds very similar results when distance is measured relative to the earnings forecast. Table A5 finds very similar results when using advertising instead of earnings to measure (R5).

on capital expenditures growth to a multivariate regression including some proxy of labor costs. That is, it is important that a forecaster making forecasts about individual items takes into account the contemporaneous relationship between multiple items.

We validate our ex ante measure of incoherence by showing that it predicts our C2-stat, which is instead an ex post measure containing information about realizations through the forecast errors. Specifically, we estimate a univariate regression of the absolute value of the calculated C2-stat on the ex ante measure of incoherence, and find

$$|\widehat{\text{C2-stat}}| = \underset{(0.022)}{0.229} + \underset{(0.197)}{0.629} \cdot \text{Incoherence},$$

where standard errors are reported in parentheses under the point estimates. Clearly, our ex ante measure of incoherence strongly predicts the absolute value of the C2-stat.

Our model predicts a pecking order of rules of thumb, according to which (R5) is the first best optimal one (see Corollary 2), the narrow-bracketing rule (R1) should be the farthest from the optimal one (see Corollary 6), and the univariate rule of thumb (R3) should be somewhere in the middle (see Corollary 7).

We evaluate these predictions by regressing our ex ante measure of incoherence on dummy variables for the CFO type. Columns 1 through 4 of Table 5 present estimates of univariate regressions including one dummy at the time, while column 5 presents the full specification where (R5) is used as the reference group. Consistent with our theory, the estimates in column 5 show that the narrow-bracketing rule of thumb (R1) is the farthest away from (R5), followed by (R2). Both (R1) and (R2) deliver significantly different forecasts from (R5) and are the most distant from the optimal forecast (R5), implying the highest incoherence. By contrast, (R3) and (R4) deliver forecasts that are on average statistically indistinguishable from the rational coherent one.[32]

Next, we explore how our ex ante measure of incoherence varies with CFOs personal characteristics. We also include the Optimism and Miscalibration measures of Ben-David et al. (2013) to examine how incoherence is related to those. Panel A of Table 6 shows

[32]Similarly, in Table A6 of the Online Appendix in which we use advertising instead of earnings to measure (R5), we find that (R1) is the most distant forecast from (R5), followed by (R2), and both (R1) and (R2) are significantly different from (R5).

descriptive statistics: 45% of the CFOs in our sample has an MBA, 9% are females, and on average they are 50.4 years old and have been on the job for 4.3 years. These figures are in line with those reported in prior work (e.g., Ben-David et al. (2013)).

Panel B of Table 6 reports our regression results. Perhaps the most interesting finding is that having an MBA does not correlate with incoherence, likely reflecting the twin facts that some rules of thumb are quite simple, thus CFOs may come up with them on their own, and that there is no consensus in MBA textbooks on which of the different possible rules of thumb should be used. In fact, as we have just seen in Table 5, the rules of thumb perform very differently in terms of forecast coherence.[33]

C. *Incoherence, Rules of Thumb, and Firm Performance*

We now examine the main prediction of our theoretical model: corporate performance should decrease with managerial incoherence, because incoherence implies the use of a suboptimal mix of inputs. We estimate the regression model,

$$\mathrm{ROA}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \mathrm{Incoherence}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where i indexes the CFO-firm pair, j indexes the industry, t indexes time, the dependent variable $\mathrm{ROA}_{i,j,t}$ is the percent return on the firm's assets, λ_j are industry fixed effects, δ_t are survey fixed effects, and $X_{i,j}$ includes firm-level controls—firm size, market-to-book, and dividends—and CFO-level variables such as miscalibration and optimism, measured at both short- and long-term horizons. Based on our model, we hypothesize $\beta < 0$. We also assume that the technological relationship is stable over time and not subject to aggregate shocks. Therefore, for this part of our empirical analysis we limit ourselves to the 2001-2007 period, to abstract from the impact of the financial crisis, which is arguably an aggregate shock and it has been documented to have an impact on managerial beliefs (e.g., Boutros et al. (2020)). Furthermore, the 2001-2007 period was the peak of the 'great moderation', a time when aggregate volatility was not a concern. We compute bootstrap standard errors following Cameron et al. (2008) and we cluster them at the

[33] CFO tenure does not correlate with incoherence. This, combined with the fact that longer tenured CFOs delegate less (Graham et al., 2015), suggests that incoherence is unrelated to delegation of authority.

firm level. Given the above and the fact that we have no source of exogenous variation in incoherence, the empirical results should simply be interpreted as correlations.

Table 7 reports our results. Column 1 indicates that a one standard deviation increase in incoherence (0.079 from Table 5) is associated with a 3-percent lower ROA. This correlation is significant at the 5% level. Columns 2 and 3 show that the results are robust and quantitatively stable when we condition on measures of managerial miscalibration and optimism. Column 4 shows that the results are also stable when we include industry and survey fixed effects. Columns 5-8 report the same specifications when we add firm-level regressors. There is some attrition so sample size shrinks reflecting the availability of regressors, but the main result remains statistically significant and quite stable.

Next, we investigate the extent to which the previous results reflect the use of different rules of thumb. We estimate a specification similar to the previous one, but instead of incoherence we include dummies for CFO types, corresponding to the use of rules of thumb (R1)-(R4), so our results should be interpreted relative to the corporate performance of firms whose CFOs use rule of thumb (R5). Our model predicts that performance should be lowest for CFOs using the narrow-bracketing rule of thumb (R1). Table 8 reports our results. Consistent with our model, Table 8 shows that in all specifications (R1) is associated with the lowest corporate performance, with an estimated coefficient that implies a 5%-to-6% lower ROA for firms whose CFO uses (R1) relative to firms whose CFO uses (R5). These are very large differences in economic terms. With respect to the other rules, Table 8 shows both (R2) and (R3) are associated with a 2%-3% lower ROA relative to (R5), whereas performance of firms whose CFOs use (R4) is indistinguishable from the performance of those firms whose CFOs use (R5). We conclude that, consistent with our model, corporate performance correlates negatively with incoherence and is on average lowest for firms whose CFOs use the narrow-bracketing rule of thumb.

D. Incoherence, Rules of Thumb, and Corporate Policies

We now examine the channels underlying the observed negative correlation between incoherence and performance, and between the narrow-bracketing rule of thumb and performance. According to our theoretical model, incoherence reflects the use of a suboptimal mix of inputs, which leads to lower earnings than it would otherwise be possible given the firm's technology and budget constraint. Given our results in Tables 2 and 3 and the patterns shown in Figures 1 and 3, we conjecture that one way through which the suboptimal mix of inputs may come up is by having a lower level of investment spending relative to the one needed to achieve the hoped-for growth in output and sales revenues. We investigate this conjecture by estimating the following regression model:

$$\mathrm{Y}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \text{Rules of Thumb}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where Rules of $\text{Thumb}_{i,j,t}$ is a vector of binary indicators for the four rules (R1)-(R4), and the dependent variable $\mathrm{Y}_{i,j,t}$ in columns (1)-(3) of Table 9 is the ratio of capital expenditures divided by assets, and then in columns (4)-(6) of Table 9 the ratio of corporate long-term book debt divided by assets. Table 9 reports the results. Columns 1 and 2 of Table 9 show that both (R1) and (R2) are associated with 1.3%-1.6% lower levels of capital expenditures relative to (R5). The difference is large in economic terms, and for (R2) also statistically significant. Columns 4 and 6 of Table 9 show also that (R1) and (R2) are associated with 4% and 9% higher leverage relative to (R5). Again, for (R2) the difference is statistically significant. Interestingly, our results are robust to conditioning for miscalibration and optimism and, consistent with Ben-David et al. (2013), we find that miscalibration and optimism are correlated with higher investment spending, underscoring that incoherence and miscalibration are different phenomena. These results show that the most incoherent rules of thumb—the narrow-bracketing (R1) and (R2)—are associated with lower investment and higher leverage and suggest that, in line with our theory, managerial incoherence comes with suboptimal investment and financing policies.

E. Change in Firm Behavior when CFOs Take Office

We now search for hints about the direction of causality. On the one hand, high incoherence might lead to lower investment and lower performance. Alternatively, lower investment levels might induce CFOs to be incoherent and forecast too high a growth in sales revenue. Relatedly, incoherent CFOs might be selected, or might self select to work in firms with low investment spending and poor performance.

To shed light on the direction of causality, we exploit within-firm variation across time. We examine how corporate performance, investment, and leverage evolve in the years surrounding a CFO's hiring. We extract the dates when CFOs join firms from Execucomp and Boardex data and supplement this information by hand-collecting data from 10-K filings. A CFO is considered to take office in a firm when he or she first signs the financial reports. We match corporate performance, investment, leverage, and characteristics from Compustat for the year of taking office. The dependent variables in our regressions are the difference in average ROA, investment, and leverage between the two years following the CFO taking office and the two years prior to the event.

Table 10 presents our results. Column 1 shows that corporate performance declines following the appointment of an incoherent CFO, in particular, a CFO who uses (R1). The use of the narrow-bracketing rule of thumb is associated with a 2.2% lower investment intensity in the two years after that CFO takes office relative to an average investment intensity of 4.5 percentage points. On the other hand, we find no change in corporate leverage around the years an incoherent CFO takes office.

Although we cannot rule out reverse causality, our findings are consistent with CFO incoherence and the use of a narrow-bracketing rule of thumb leading to a decrease in corporate investment and a decrease in corporate performance.

VI Conclusion

We develop a theory of forecast coherence in firm production, which yields a statistical test whereby under the null of coherence the forecast errors of output and inputs are not "too far" from one another. Using the Duke Survey of top US executives, we document that for 55% of CFOs in our sample we reject the null hypothesis of coherence.

Our baseline model provides a normative benchmark of an ex ante coherent forecast. In a positive version of our model in which agents observe noisy signals about output and inputs, some of the rules of thumb suggested by the managerial education literature may emerge as second-best optimal. In particular, the narrow-bracketing rule—making forecasts about one input projecting past growth rates of that input—is second-best optimal if the agent observes infinitely noisy signals about output and the other input.

Consistent with our model, (1) the narrow-bracketing rule of thumb is the most distant from the ex ante coherent forecast, and (2) corporate performance correlates negatively with managerial incoherence and is on average lowest for firms whose CFOs make narrow-bracketing forecasts. We also provide evidence that the use of rules of thumb correlates negatively with investment spending and positively with leverage.

Much research in psychology and elsewhere has been cast in terms of whether the use of heuristics or rules of thumb to help probabilistic judgment and forecasting is always necessarily good or bad, e.g., see Hammond (2007). We highlight that these interpretations are incomplete or even misleading, because heuristics per se can help individuals in challenging forecasting tasks, but not all heuristics are necessarily equally good or equivalent to one another. We highlight that one needs both theory and data to evaluate alternative heuristics with respect to their proposed goals.

Our results show that some rules of thumb do help. At the same time, not all rules of thumb are equally good; in fact, some are actually quite bad. We provide conditions under which some rules of thumb approximate the optimal forecast, and conditions under which they do not. Our results indicate that some rules of thumb lead to narrow-bracketing forecasts and thus to severe incoherence, which in turn correlates with low performance. Therefore, such rules of thumb should not belong in the toolkit of future corporate executives. Our results do inform the managerial education literature by providing a pecking order of the rules of thumb to be taught going forward.

Under our maintained assumptions, coherence can be achieved ex ante, unlike accuracy, and some rules of thumb can help approach coherence. One such assumption is that the firm's production technology is stable over time. This is likely appropriate in our sample period, but of course it might not hold in general, and it will not hold when disruptive technological innovation or unexpected aggregate shocks occur. At the same time, disruptive innovation and aggregate shocks will also threaten accuracy. In future work, it will be important to relax these assumptions and to establish more generally the promise of coherence as an overarching principle to help making better forecasts.

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Figure 1: **Contemporaneous Forecasts of Output and Capital**

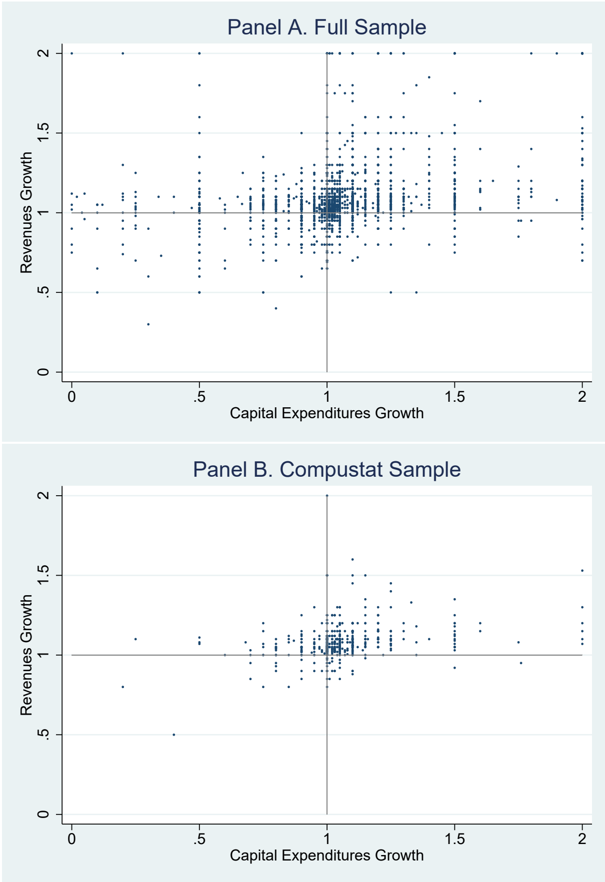

Figure 2: **Contemporaneous Realizations of Output and Capital**

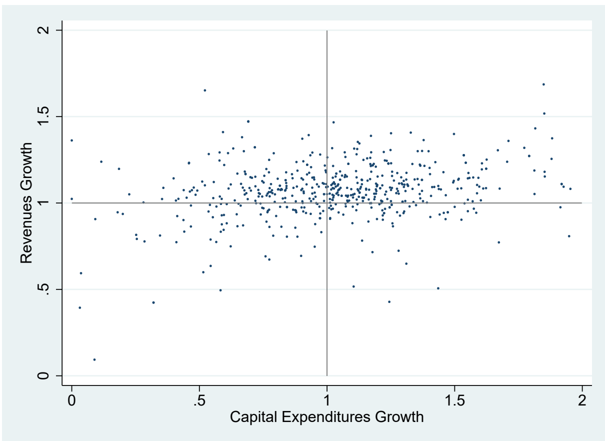

Figure 3: **Contemporaneous Forecasts Errors of Output and Capital**

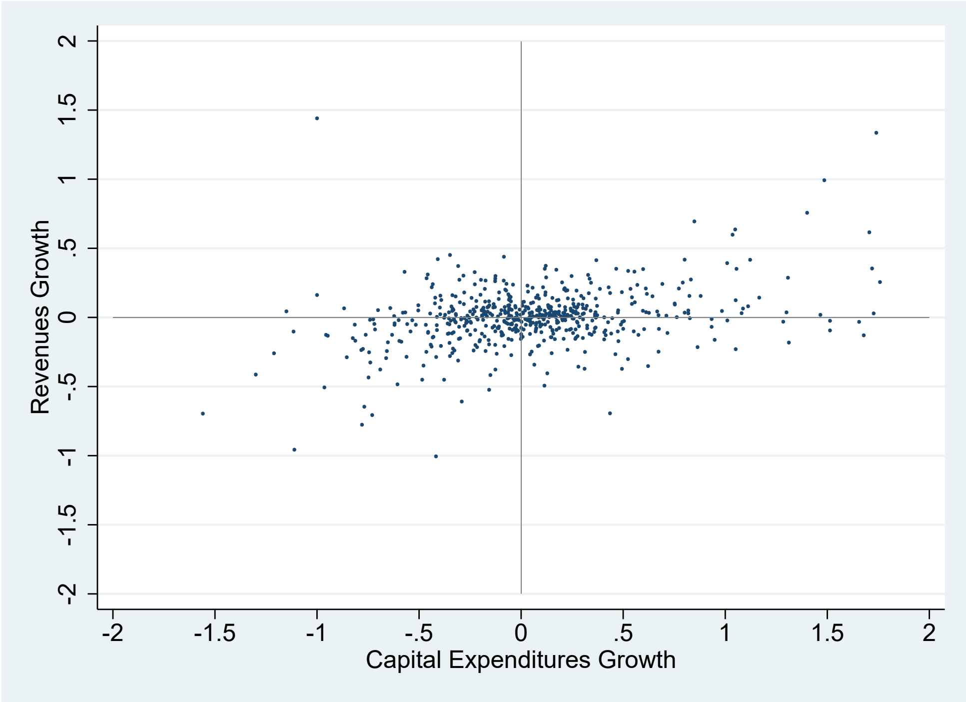

Figure 4: **(In)Coherence and (In)Accuracy Areas**

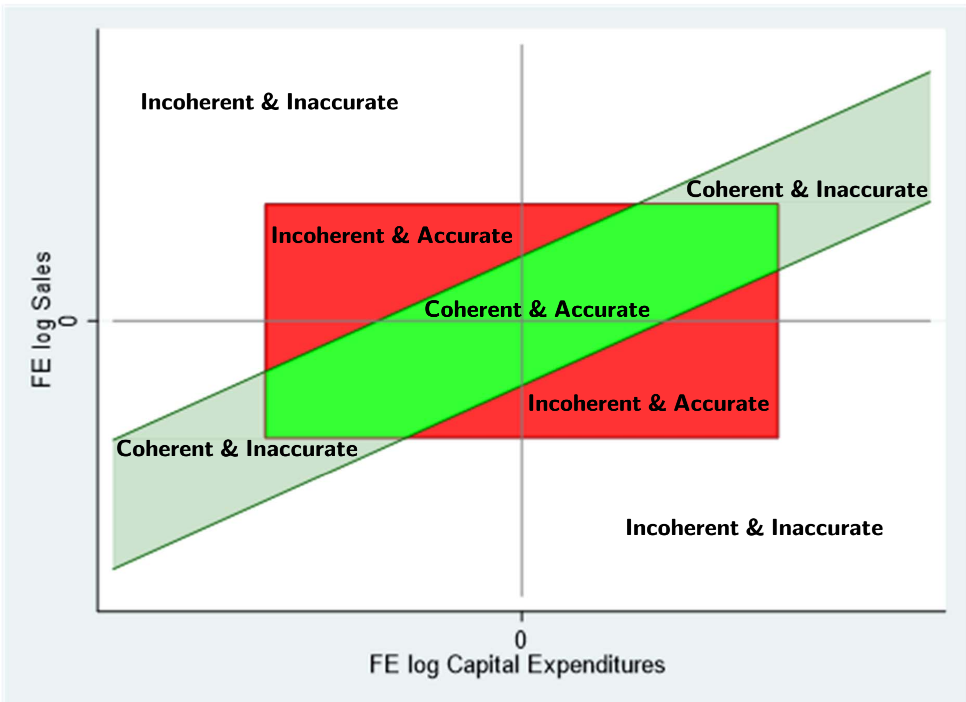

Figure 5: **Bootstrap of Coherence Test Statistic C2-stat**

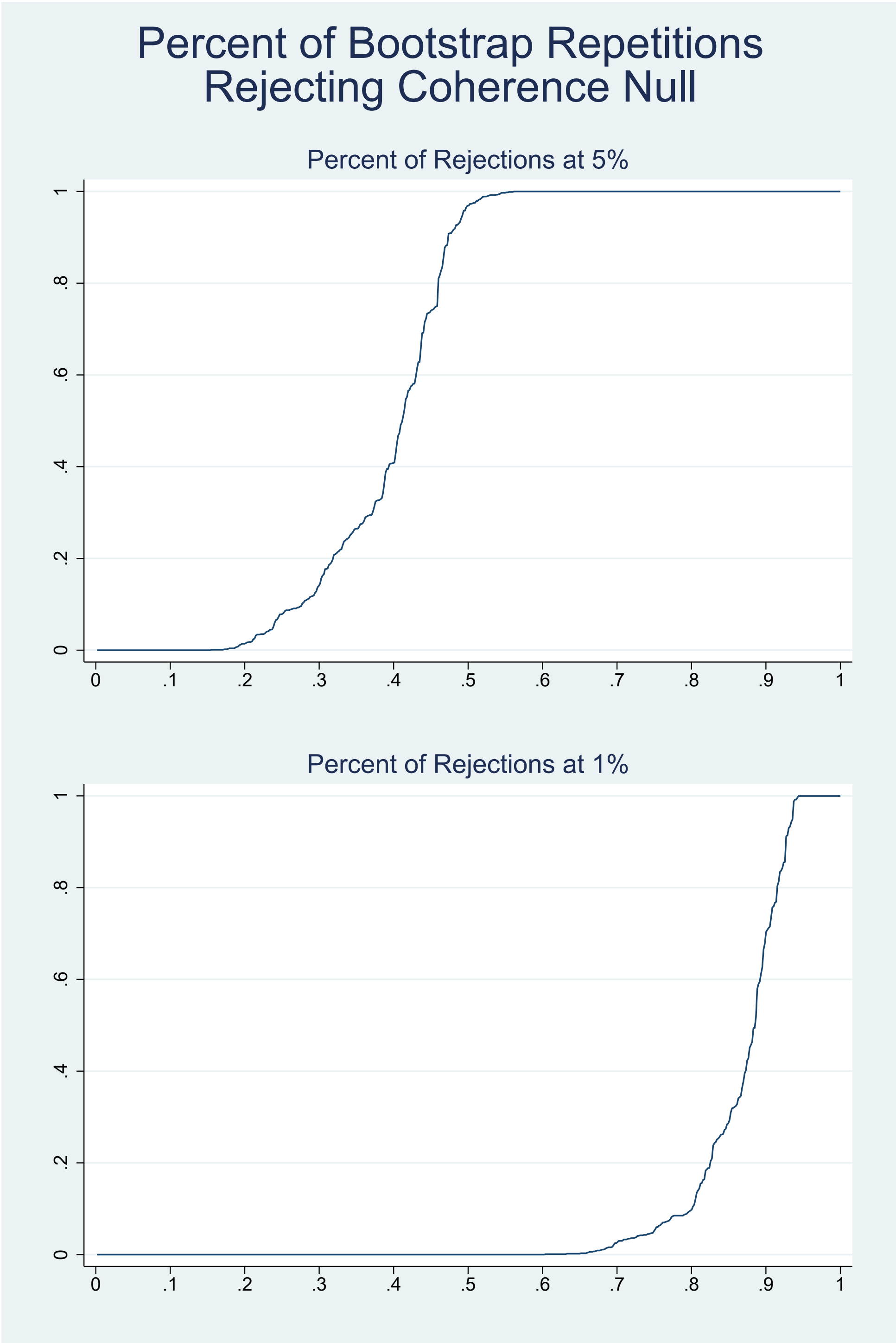

Table 1: **CFO Growth Forecasts and Realizations of Selected Balance Items**

Panel A – CFO Growth Forecasts (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Expected Growth in Revenues and in Earnings						
Revenues	9.30	27.13	-5.00	5.00	20.00	14,490
Earnings	11.00	42.37	-10.00	5.00	30.00	25,472
Expected Growth in Capital-Related Expenditures						
Capital Expenditures	8.11	43.90	-15.00	3.00	25.00	25,305
R & D	4.51	21.65	0.00	0.00	15.00	8,325
Technology Spending	6.68	28.02	-5.00	3.00	20.00	22,404
Expected Growth in Labor-Related Costs						
Wages	3.90	12.41	0.00	3.00	7.00	27,472
Employees	3.95	30.16	-5.00	1.00	10.00	25,471
Outsourced Employees	3.74	21.19	0.00	0.00	10.00	10,990
Health Spending	8.59	11.65	1.00	8.00	15.00	25,064
Expected Growth in Productivity, Product Prices, and Advertising						
Productivity	3.91	9.38	0.00	3.00	10.00	18,197
Product Prices	2.08	8.22	-3.00	2.00	7.00	24,499
Advertising	4.75	21.83	-5.00	2.00	15.00	20,989
Expected Growth in Cash Holdings and Corporate Payout						
Cash	5.02	38.56	-20.00	0.00	20.00	16,876
Dividends	4.54	30.52	0.00	0.00	15.00	5,227
Share Repurchases	1.55	24.40	0.00	0.00	5.00	5,487

Panel B – Realizations, Matched Compustat-Duke Sample (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Actual Growth in Revenues and in Earnings						
Revenues	6.80	21.32	-13.56	5.23	27.25	14,549
Earnings	-10.36	307.02	-161.71	2.36	124.59	14,580
Actual Growth in Capital-Related Expenditures						
Capital Expenditures	15.87	67.07	-42.59	3.96	75.70	13,770
R & D	7.09	29.57	-19.85	4.27	33.33	6,456
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Labor-Related Costs						
Wages	7.02	14.65	-7.23	5.35	22.10	2,836
Employees	2.98	16.95	-11.88	1.19	17.71	14,359
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	8.03	42.14	-26.76	2.79	38.46	5,735
Actual Growth in Cash Holdings and Corporate Payout						
Cash	35.42	132.66	-46.26	5.76	113.78	14,520
Dividends	12.68	52.88	-12.22	6.15	38.44	8,762
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table 2: **Violations of Coherence Inequality Restrictions**

Panel A – Inequality Test of Coherence

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
% Incoherent	100.00	100.00	99.07
% Coherent	0.00	0.00	0.93
% Total	100.00	100.00	100.00
N. Obs.	107	107	107
Inequality in Growth Rates			
% Incoherent	73.31	73.14	72.96
% Coherent	26.69	26.86	27.04
% Total	100.00	100.00	100.00
N. Obs.	577	577	577

Panel B – Summary stats of difference, LHS − RHS

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
Mean	15,419.59	15,201.16	15,033.78
Std. Dev.	28,574.83	28,233.56	27,990.89
Q10	213.3352	207.60	194.3495
Median	3,252.30	3228.92	3,205.96
Q90	39,737.28	39,505.75	39,360.82
N. Observations	107	107	107
Inequality in Growth Rates			
Mean	0.047	0.044	0.042
Std. Dev.	0.122	0.125	0.128
Q10	-0.061	-0.067	-0.067
Median	0.034	0.033	0.033
Q90	0.164	0.163	0.162
N. Observations	577	577	577

Note: χ denotes the elasticity of substitution between capital and labor.

Table 3: **The Coherence and Accuracy Sides of Rationality**

Panel A – Separate Assessment of Coherence and Accuracy (Percent of Rejections of Null)

Significance level α	Coherence Sales-CapEx	Accuracy Sales	Accuracy CapEx	Accuracy Both
	(1)	(2)	(3)	(4)
5%	55.7%	27.2%	47.9%	57.0%
1%	7.7%	1.8%	6.4%	7.1%

Panel B – Joint Assessment of Coherence and Accuracy

Significance level α	Coherent + Accurate	Coherent + Inaccurate	Incoherent + Accurate	Incoherent + Inaccurate
	(1)	(2)	(3)	(4)
5%	31.1%	13.2%	12.0%	43.7%
1%	89.4%	2.9%	3.4%	4.3%

Panel C – Test Statistics: Summary Statistics

	Mean	Std.Dev.	P05	Median	P95	N Obs.
C-statistic	-0.193	4.846	-8.335	-0.135	7.871	560
FE Sales	-0.538	19.07	-23.53	0.554	22.24	563
FE CapEx	-0.988	31.28	-54.18	1.186	41.20	560

Notes: In Panel B, Accuracy means both accurate; and inaccuracy means at least one inaccurate. Critical values are those of the t-student with one degree of freedom, +/-12.706 at the 5% and +/-63.657 at the 1%. Sales are the output. Capital Expenditures (CapEx) are input 1. Labor Expenditures are input 2 (unobserved).

Table 4: **Minimum Distance of CapEx Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.033	0.058	0.030	0.019	0.031	0.043
Std. Dev.	0.059	0.100	0.064	0.017	0.035	0.069
Frac. Zeros	0.106	0.000	0.268	0.000	0.000	0.000
P10	0.000	0.008	0.000	0.005	0.002	0.003
P25	0.007	0.015	0.000	0.006	0.007	0.008
P50	0.019	0.028	0.014	0.010	0.023	0.023
P75	0.036	0.064	0.035	0.028	0.048	0.043
P90	0.071	0.114	0.071	0.048	0.072	0.089
N of Observations	396	30	157	43	107	59
Fraction	1.000	0.076	0.396	0.109	0.270	0.149

Notes: Cross-sectional analysis with 396 CFOs.

Table 5: **Incoherence and Rules of Thumb: Distance from Optimal Forecast**

	(1)	(2)	(3)	(4)	(5)
Rule 1 (CapEx)	0.081***				0.104***
	(0.014)				(0.016)
Rule 2 (CapEx)		0.039***			0.053***
		(0.008)			(0.011)
Rule 3 (CapEx)			-0.055***		-0.020
			(0.012)		(0.014)
Rule 4 (CapEx)				-0.027***	0.010
				(0.009)	(0.012)
Constant	0.066***	0.057***	0.079***	0.080***	0.043***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.009)
R^2	0.071	0.057	0.045	0.023	0.175
N observations	396	396	396	396	396

Summary Statistics of the dependent variable	
Mean	0.073
Std. Dev.	0.079
P10	0.012
Median	0.059
P90	0.139

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table 6: **Incoherence and Personal CFO Characteristics**

Panel A – Summary statistics

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
CFO has MBA	0.452	0.498	0.000	0.000	1.000	396
Age	50.43	6.937	42.00	50.00	60.00	396
Age 40-	0.063	0.244	0.000	0.000	0.000	396
Age 41-50	0.460	0.499	0.000	0.000	1.000	396
Age 51-60	0.402	0.491	0.000	0.000	1.000	396
Age 61+	0.076	0.265	0.000	0.000	0.000	396
Tenure	4.273	4.099	0.000	3.000	9.000	396
Tenure > Median	0.462	0.499	0.000	0.000	1.000	396
Gender	0.088	0.284	0.000	0.000	0.000	396
Miscalibration ST	0.035	0.920	-1.166	0.329	0.985	360
Optimism ST	0.052	0.981	-0.918	-0.077	1.285	373
Miscalibration LT	0.039	0.979	-1.095	0.262	0.917	362
Optimism LT	0.033	1.088	-1.008	-0.078	1.077	374

Panel B – Incoherence and CFO characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
CFO has MBA	0.004			0.005		
	(0.009)			(0.009)		
Age 40-		-0.007		-0.008		
		(0.023)		(0.023)		
Age 41-50		-0.024		-0.025		
		(0.016)		(0.016)		
Age 51-60		-0.022		-0.023		
		(0.016)		(0.016)		
Tenure > Median		0.007		0.007		
		(0.008)		(0.009)		
Gender			-0.000	0.003		
			(0.011)	(0.011)		
Miscalibration ST					-0.012	
					(0.008)	
Optimism ST					-0.012	
					(0.007)	
Miscalibration LT						-0.005
						(0.004)
Optimism LT						0.001
						(0.004)
Constant	0.050**	0.070***	0.053***	0.067***	0.052*	0.046**
	(0.021)	(0.023)	(0.019)	(0.025)	(0.027)	(0.021)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.109	0.117	0.108	0.118	0.154	0.126
N of Observations	396	396	396	396	360	362

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Robust standard errors are reported in parenthesis.

Table 7: **Incoherence and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Incoherence	-0.377**	-0.378**	-0.360**	-0.396**	-0.399**	-0.386**	-0.317*	-0.307*
	(0.157)	(0.179)	(0.162)	(0.162)	(0.186)	(0.169)	(0.192)	(0.181)
Miscalibration ST		0.003			0.001		-0.001	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.000			0.000		0.001	
		(0.006)			(0.006)		(0.005)	
Miscalibration LT			0.004			0.002		0.001
			(0.005)			(0.005)		(0.005)
Optimism LT			0.008			0.007		0.009
			(0.006)			(0.006)		(0.006)
Firm size							0.009***	0.009**
							(0.003)	(0.003)
Market-to-Book							0.028**	0.027*
							(0.014)	(0.015)
Dividends							0.022*	0.023*
							(0.012)	(0.013)
Constant	0.069***	0.069***	0.068***	0.054***	0.056***	0.057***	-0.131***	-0.123***
	(0.011)	(0.011)	(0.011)	(0.014)	(0.020)	(0.019)	(0.047)	(0.0471)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
R^2	0.046	0.042	0.047	0.071	0.064	0.068	0.177	0.185
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 8: **Rules of Thumb and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rule 1 (CapEx)	-0.057**	-0.061**	-0.059**	-0.051**	-0.059**	-0.055**	-0.053**	-0.051**
	(0.022)	(0.025)	(0.024)	(0.023)	(0.025)	(0.025)	(0.026)	(0.025)
Rule 2 (CapEx)	-0.026*	-0.027*	-0.023	-0.023	-0.028*	-0.024	-0.034	-0.031
	(0.0138)	(0.015)	(0.015)	(0.015)	(0.017)	(0.016)	(0.021)	(0.019)
Rule 3 (CapEx)	-0.031*	-0.036*	-0.034*	-0.027	-0.037*	-0.034	-0.047**	-0.045**
	(0.017)	(0.019)	(0.019)	(0.019)	(0.020)	(0.021)	(0.023)	(0.022)
Rule 4 (CapEx)	-0.012	-0.010	-0.010	-0.008	-0.008	-0.007	-0.012	-0.011
	(0.012)	(0.014)	(0.014)	(0.013)	(0.014)	(0.015)	(0.015)	(0.015)
Miscalibration ST		0.001			-0.001		-0.002	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.001			0.000		0.001	
		(0.006)			(0.005)		(0.005)	
Miscalibration LT			0.003			0.002		0.001
			(0.006)			(0.005)		(0.004)
Optimism LT			0.007			0.006		0.008
			(0.006)			(0.006)		(0.005)
Firm size							0.010***	0.009***
							(0.004)	(0.004)
Market-to-Book							0.028**	0.028*
							(0.014)	(0.015)
Dividends							0.029**	0.030**
							(0.013)	(0.014)
Constant	0.065***	0.066***	0.064***	0.040***	0.045**	0.046*	-0.147***	-0.137***
	(0.011)	(0.012)	(0.013)	(0.015)	(0.019)	(0.028)	(0.046)	(0.050)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
R^2	0.014	0.014	0.019	0.033	0.031	0.034	0.165	0.170
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 9: **Rules of Thumb and Corporate Policies**

	Investment			Leverage		
	(1)	(2)	(3)	(4)	(5)	(6)
Rule 1 (CapEx)	-0.016	-0.014	-0.015	0.055	0.041	0.047
	(0.011)	(0.011)	(0.012)	(0.092)	(0.101)	(0.092)
Rule 2 (CapEx)	-0.013**	-0.015**	-0.012	0.093*	0.098	0.092*
	(0.006)	(0.007)	(0.008)	(0.053)	(0.060)	(0.053)
Rule 3 (CapEx)	-0.007	-0.011	-0.010	-0.023	-0.015	-0.027
	(0.008)	(0.010)	(0.010)	(0.073)	(0.091)	(0.084)
Rule 4 (CapEx)	-0.003	-0.003	-0.003	-0.004	0.005	0.001
	(0.007)	(0.008)	(0.008)	(0.045)	(0.050)	(0.046)
Miscalibration ST		0.001			0.012	
		(0.003)			(0.024)	
Optimism ST		0.002			-0.006	
		(0.003)			(0.019)	
Miscalibration LT			0.002			0.013
			(0.002)			(0.018)
Optimism LT			0.004*			-0.010
			(0.002)			(0.017)
Firm size	-0.002	-0.002	-0.002	0.012	0.011	0.012
	(0.002)	(0.002)	(0.002)	(0.017)	(0.019)	(0.018)
Market-to-Book	0.006**	0.006*	0.006*	-0.082***	-0.081***	-0.081***
	(0.003)	(0.003)	(0.003)	(0.020)	(0.021)	(0.022)
Dividends	0.000	0.004	0.004	0.037	0.044	0.043
	(0.009)	(0.008)	(0.008)	(0.072)	(0.076)	(0.080)
Constant	0.044*	0.043*	0.050**	0.568**	0.666***	0.620**
	(0.025)	(0.023)	(0.023)	(0.249)	(0.228)	(0.249)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.210	0.223	0.230	0.069	0.062	0.066
N of Observations	437	397	402	437	397	402

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 10: **Change in Performance and Corporate Policies when New CFOs Take Office**

	Change in ROA		**Change in Investment**		**Change in Leverage**	
	(1)	(2)	(3)	(4)	(5)	(6)
Incoherence	-1.633*		-0.049		-0.047	
	(0.989)		(0.045)		(1.115)	
Rule 1 (CapEx)		-0.274		-0.022*		-0.011
		(0.213)		(0.012)		(0.231)
Rule 2 (CapEx)		-0.000		-0.003		-0.201
		(0.036)		(0.008)		(0.199)
Rule 3 (CapEx)		-0.057		-0.008		-0.110
		(0.051)		(0.012)		(0.153)
Rule 4 (CapEx)		0.019		0.001		-0.070
		(0.048)		(0.009)		(0.118)
Firm characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.391	0.192	0.024	0.017	0.053	0.042
N of Observations	142	142	140	140	146	146

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Online Appendix with Supplementary Material for The Coherence Side of Rationality: Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts

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A Proofs

Proof of Proposition 1. Recall that for a concave function, f, it holds that $\mathbb{E}[f(x)] \leq f(\mathbb{E}[x])$. Assume $\xi \leq 1$ and start by assuming that $a+b=1$. The CES function f is homogeneous of degree one because, for a scalar λ, we have that

$$f(\lambda \mathbf{x}) = \left[a(\lambda x_1)^{\xi} + b(\lambda x_2)^{\xi}\right]^{\frac{1}{\xi}} = \lambda\left(ax_1^{\xi} + bx_2^{\xi}\right)^{\frac{1}{\xi}}.$$

Furthermore, note that f is also quasiconcave because it is a monotone transformation of a concave function. In fact,

$$f = g^{\frac{1}{\xi}},$$

and to see that g is concave, compute its Hessian, H_g,

$$H_g = \begin{bmatrix} \frac{\partial^2 g}{\partial x_1^2} & \frac{\partial^2 g}{\partial x_2 \partial x_1} \\ \frac{\partial^2 g}{\partial x_1 \partial x_2} & \frac{\partial^2 g}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} a\xi(1-\xi)x_1^{\xi-2} & 0 \\ 0 & b\xi(1-\xi)x_2^{\xi-2} \end{bmatrix}.$$

Since H_g is negative semi-definite, we can conclude that g is concave.

Now, let $a+b \leq 1$. We have that $f = \left(g^{\frac{1}{\xi}}\right)^{a+b}$, where g is concave, as shown above. Then, f is a concave increasing function of a concave function, from which we can conclude that f is concave, which proves the first part of the proposition. The second part of the proposition on convexity follows very similar arguments.
QED

Proof of Corollary 1. Here we prove the statement in growth rates (the one in levels follows similar steps). In a Cobb-Douglas for $\xi \to 0$, assuming without loss of generality that the constraint is binding, the solution for optimal input quantities are

$$x_1^* = \frac{Z}{p_1}\frac{a}{a+b}, \qquad x_2^* = \frac{Z}{p_2}\frac{b}{a+b}.$$

Therefore, for $i = 1,2$ we have

$$\begin{aligned} \log\left[\frac{x_{i,t+1}}{x_{i,t}}\right] &= \log\left[\frac{p_{i,t}}{p_{i,t+1}}\right] \\ \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{p_{1,t}}{p_{1,t+1}}\right] + b \cdot \log\left[\frac{p_{2,t}}{p_{2,t+1}}\right]. \end{aligned}$$

Putting these together, we obtain

$$\begin{aligned} \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right] \\ \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right]. \end{aligned}$$

QED

Proof of Proposition 2. To derive the test statistic, note that we have $\mathbb{E}_t \log y_{t+1} = a\mathbb{E}_t \log x_{1,t+1} + b\mathbb{E}_t \log x_{2,t+1}$ and $\log x_{2,t+1} = \log \frac{b}{a+b}Z - \pi_{2,t+1}$, implying that the CFO forecast for input 2 is

$$\mathbb{E}_t \log x_{2,t+1} = \log \frac{b}{a+b} Z - \gamma_2 \pi_{2,t}. \tag{8}$$

Note that (8) depends on the technological parameters, a and b, and budget, Z, which we assume are known to the CFO at the time of the forecast and are stable over time. From (8) we obtain that

$$\frac{\mathbb{E}_t \log x_{2,t+1} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1).$$

We can then derive our test statistic C1-stat based on the joint forecasts of the first input and output by recalling that $\log y = a \log x_1 + b \log x_2$ as follows:

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a \mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1),$$

where the distribution here is obtained under the null hypothesis of coherent forecasts.

To derive our C2-stat, we start by defining the forecast error of a generic variable x forecasted at t and realized at $t+1$ as the difference between the realization and the forecast, $\text{FE}_t x_{t+1} = x_{t+1} - \mathbb{E}_t x_{t+1}$. We then have that

$$\begin{aligned} \text{FE}_t \log x_{2,t+1} &= \log x_{2,t+1} - \mathbb{E}_t \log x_{2,t+1} \\ &= \log \frac{b}{a+b} Z - \pi_{2,t+1} - \mathbb{E}_t \left[\log \frac{b}{a+b} Z - \pi_{2,t+1} \right] \\ &= -\text{FE}_t \pi_{2,t+1} = -\epsilon_{2,t+1}. \end{aligned}$$

As a result, the forecast error of the log of the second input is the negative of the innovation of the second log-price process. It follows that

$$\frac{\text{FE}_t \log x_{2,t+1}}{\sigma_2} \sim \mathcal{N}(0,1).$$

Noting that $\text{FE}_t \log y_{t+1} = a\text{FE}_t \log x_{1,t+1} + b\text{FE}_t \log x_{2,t+1}$, we obtain our C2-stat,

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1).$$

QED

Coherence Test Statistic: Multiple Inputs Case. Here we generalize our C2-stat to a multivariate case with N inputs. For this subsection, the production setting is

$$y = \prod_{i=1}^{N} x_i^{a_i}$$

$$\boldsymbol{p}'\boldsymbol{x} = Z,$$

where $\boldsymbol{p}$ and $\boldsymbol{x}$ are the column vectors of factor prices and quantities, respectively. As in the $N = 2$ case we have a linear relationship between the logs of inputs and output,

$$\log y = \sum_{i=1}^{N} a_i \log x_i,$$

where the same equation holds for the forecast errors. Analogously to the bivariate case, we have that $\text{FE}_t \log x_{1,t+1} = -\epsilon_{1,t+1}$ so that

$$\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1} \sim \mathcal{N}(0,1).$$

Then, using the linear relationship between the logs of inputs and output we obtain our generalized C2-stat,

$$\frac{\text{FE}_t \log y_{t+1} - \sum_{i=2}^{N} a_i \text{FE}_t \log x_{i,t+1}}{\sigma_1 a_1} \sim \mathcal{N}(0,1).$$

Proof of Proposition 3. The Proof follows directly from the observation that in our setting the conditional expectation function is

$$\mathbb{E}\left[y|x_1,x_2\right] = \mathbb{E}\left[y\right] + \beta_1\left(x_1 - \mathbb{E}\left[x_1\right]\right) + \beta_2\left(x_2 - \mathbb{E}\left[x_2\right]\right),$$

where the parameters can be derived by the Frisch-Waugh-Lovell theorem, as

$$\beta_1 = \frac{cov\left(y,x_1\right) - \left(\frac{cov(x_1,x_2)cov(y,x_2)}{var(x_2)}\right)}{var\left(x_1\right) - \frac{cov(x_1,x_2)^2}{var(x_2)}}, \ \ \beta_2 = \frac{cov\left(y,x_2\right) - \left(\frac{cov(x_1,x_2)cov(y,x_1)}{var(x_1)}\right)}{var\left(x_2\right) - \frac{cov(x_1,x_2)^2}{var(x_1)}},$$

and where $var\left(x_1\right)$, $var\left(x_2\right)$, $cov\left(y,x_1\right)$, and $cov\left(y,x_2\right)$ are functions of parameters a and b. QED

Proof of Corollary 2. In levels,

$$\mathbb{E}\left[\log x_1 | \log y, \log x_2\right] = \mu_1 + \beta_y\left(\log y - \mu_y\right) + \beta_2\left(\log x_2 - \mu_2\right),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\log y, \log x_1\right) - \left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log x_2)}{var(\log x_2)}\right)}{var\left(\log y\right) - \frac{cov(\log y,\log x_2)^2}{var(\log x_2)}},$$

$$\beta_2 = \frac{cov\left(\log x_2, \log x_1\right) - \left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log y)}{var(\log y)}\right)}{var\left(\log x_2\right) - \frac{cov(\log y,\log x_2)^2}{var(\log y)}}.$$

In detail, we have:
$var\left(\log x_2\right) = \sigma_2^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$,
$cov\left(\log x_2, \log x_1\right) = 0$,
$cov\left(\log y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2, \log x_1\right) = a\sigma_1^2$,
$cov\left(\log y, \log x_2\right) = cov\left(a\log x_1 + b\log x_2, \log x_2\right) = b\sigma_2^2$.

Substituting yields

$$\begin{aligned}
\mathbb{E}\left[\log x_1 | \log y, \log x_2\right] &= \mu_1 + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 - \frac{b^2\sigma_2^4}{\sigma_2^2}} \left(\log y - \mu_y\right) + \frac{-\left(\frac{b\sigma_2^2 a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2}\right)}{\sigma_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2}} \left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b\sigma_2^2 a\sigma_1^2}{\sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right) - b^2\sigma_2^4}\left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b}{a}\left(\log x_2 - \mu_2\right).
\end{aligned}$$

where $\mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0$ follows by Corollary 1. Proving the statement in growth rates follows similar steps.
QED

Proof of Corollary 3. In levels,

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 + \beta_y \left(\log y - \mu_y\right),$$

where coefficients equal

$$\alpha = \mu_1 - \beta_y \mu_y, \qquad \beta_y = \frac{cov\left(\log y, \log x_1\right)}{var\left(\log y\right)}.$$

We have:
$cov\left(\log x_2, \log x_1\right) = 0$,
$cov\left(\log y, \log x_1\right) = cov\left(a \log x_1 + b \log x_2, \log x_1\right) = a\sigma_1^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$.
Substituting yields

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y, \qquad \beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2},$$

and thus

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\left(\log y - \mu_y\right).$$

Proving the statement in growth rates follows similar steps and intercept α is differenced away.
QED

Proof of Corollary 4. Consider the regression

$$\frac{y_{t+1}}{y_t} = \alpha + \beta \frac{x_{i,t+1}}{x_{i,t}} + e_{t+1}.$$

Denoting variable at optimum with superscripts *, we have

$$\begin{aligned}
\beta &= \frac{cov\left(\frac{y_{t+1}}{y_t}, \frac{x^*_{i,t+1}}{x^*_{i,t}}\right)}{var\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)} = \frac{\mathbb{E}\left[\frac{y_{t+1}}{y_t} \cdot \frac{x^*_{i,t+1}}{x^*_{i,t}}\right] - \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right] \cdot \mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]}{\mathbb{E}\left[\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)^2\right] - \left(\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]\right)^2}, \\
\alpha &= \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right] - \beta \mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right].
\end{aligned}$$

Recall that

$$\left\{\begin{pmatrix}\pi_{1,t}\\ \pi_{2,t}\end{pmatrix}\right\} \overset{iid}{\sim} \mathcal{N}_2\left(\mathbf{0}, \begin{bmatrix}\sigma_1^2 & \sigma_{1,2}\\ \sigma_{1,2} & \sigma_2^2\end{bmatrix}\right).$$

Under the assumption that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, for every t we have that the Pearson correlation coefficient $\rho_{1,2} = 1$, from which it follows that $\pi_{1,t} = \pi_{2,t} + c$ almost surely, where c is a constant.[34] As a result, the setting can be recast as one in which prices are constant and the budget Z is stochastic because

$$\begin{aligned}\frac{x_{1,t+1}^*}{x_{1,t}^*} &= \frac{p_{1,t}}{p_{1,t+1}} \overset{\text{a.s.}}{=} \frac{p_{2,t}}{p_{2,t+1}} = \frac{x_{2,t+1}^*}{x_{2,t}^*}\\ \frac{y_{t+1}}{y_t} &= a\frac{p_{1,t}}{p_{1,t+1}} + b\frac{p_{2,t}}{p_{2,t+1}} \overset{\text{a.s.}}{=} (a+b)\frac{p_{1,t}}{p_{1,t+1}}\end{aligned}$$

and

$$\begin{aligned}&\left\{\frac{x_{i,t+1}^*}{x_{i,t}^*}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1, 2\\ &\left\{\frac{y_{t+1}}{y_t}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right).\end{aligned}$$

In fact, denote $z_t = \log(Z) \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \sigma^2\right)$, and assume p_1 and p_2 constant for all t. We have

$$\begin{aligned}\frac{x_{1,t+1}^*}{x_{1,t}^*} &= \frac{x_{2,t+1}^*}{x_{2,t}^*} = e^{z_{t+1}-z_t}\\ \frac{y_{t+1}}{y_t} &= e^{(a+b)(z_{t+1}-z_t)}\end{aligned}$$

and

$$\begin{aligned}&\left\{\frac{x_{i,t+1}^*}{x_{i,t}^*}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1, 2\\ &\left\{\frac{y_{t+1}}{y_t}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right),\end{aligned}$$

as it was with stochastic prices. Therefore, for clarity from now on we drop the subscript i. Now, recalling that $\nu \equiv a + b$, that $cov\left(z_{t+1}, z_t\right) = \gamma\frac{\sigma^2}{1-\gamma^2}$, and that for any scalar, c, we have

$$\mathbb{E}\left[e^{c(z_{t+1}-z_t)}\right] = e^{\frac{1}{2}c^2 var(z_{t+1}-z_t)} = e^{\frac{1}{2}c^2\left(2\frac{\sigma^2}{1-\gamma^2}-2\gamma\frac{\sigma^2}{1-\gamma^2}\right)} = e^{c^2\left(\frac{\sigma^2}{1+\gamma}\right)},$$

[34] To see this, suppose that X, Y are two random variables such that $\rho(X, Y) = 1$. Let $V = X - \mathbb{E}[X]$ and $W = Y - \mathbb{E}[Y]$. We have $\mathbb{E}\left[(V-W)^2\right] = var(X) + var(Y) - 2cov(X, Y) = 0$, so that $V \overset{\text{a.s.}}{=} W$, from which the result $\pi_{1,t} = \pi_{2,t} + c$ follows.

we obtain the expressions

$$\begin{aligned}
\beta &= \frac{e^{\left[(\upsilon+1)^2\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\left(\upsilon^2+1\right)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(4\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}}=\frac{e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(3\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(\frac{\sigma^2}{1-\gamma}\right)}} \\
\alpha &= \frac{e^{\left[\left(\upsilon^2+2\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}-1}.
\end{aligned}$$

We can then directly verify the coimplications of the Corollary, that is,

$$\begin{aligned}
\beta &< 1 \Longleftrightarrow e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}<e^{\left(3\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(\frac{\sigma^2}{1-\gamma}\right)} \Longleftrightarrow \upsilon<1 \\
\alpha &> 0 \Longleftrightarrow e^{\left[\left(\upsilon^2+2\right)\frac{\sigma^2}{1-\gamma}\right]}>e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]} \Longleftrightarrow \upsilon<1,
\end{aligned}$$

which also holds for i.i.d. shocks, that is, for $\gamma = 0$.
QED

Proof of Corollary 5. We have

$$\log\frac{x_{i,t+1}}{x_{i,t}}=\log\frac{1/p_{i,t+1}}{1/p_{i,t}}=\pi_{i,t}-\pi_{i,t+1}.$$

Denote $\log F_t^0$ the optimal forecast of log input $x_{i,t}$ growth. We have

$$\log F_t^o=\mathbb{E}_t\left[\log\frac{x_{i,t+1}}{x_{i,t}}\right]=(1-\gamma_i)\,\pi_{i,t}.$$

Under the optimal forecast, the forecast error will be minus the innovation of the log price shock,

$$\log\frac{x_{i,t+1}}{x_{i,t}}-\mathbb{E}_t\left[\log\frac{x_{i,t+1}}{x_{i,t}}\right]=-\epsilon_{i,t+1}|\Omega_t\ \sim\mathcal{N}\left(0,1\right),$$

so that the loss and the expected loss under the optimal forecast, L_{t+1}^o and $\mathbb{E}_t\left[L_{i,t+1}^o\right]$, are

$$\begin{aligned}
L_{t+1}^o &= \epsilon_{i,t+1}^2=\sigma_i^2\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2 \\
\mathbb{E}_t\left[L_{i,t+1}^o\right] &= \sigma_i^2\mathbb{E}_t\left[\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2\right]=\sigma_i^2,
\end{aligned}$$

where the last equality follows from $\epsilon_{i,t+1}^2=\sigma_i^2=\sigma_i^2\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2$, and $\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2|\Omega_t\ \sim\chi^2$ with mean 1. Under the narrow-bracketing rule (R1), $\log F_{i,t}^{R1}=\frac{1}{k}\sum_{j=1}^{k}\log\frac{x_{i,t+1-j}}{x_{i,t-j}}$, the forecast error in logs is

$$\log\frac{x_{i,t+1}}{x_{i,t}}-\log F_{i,t}^{R1}.$$

There are several possibilities. If $k=1$, $\log F_{i,t}^{R1}=\log\frac{x_{i,t}}{x_{i,t-1}}$, then the forecast error is $\log\frac{x_{i,t+1}}{x_{i,t}}-$

$\log \frac{x_{i,t}}{x_{i,t-1}} = -\left[\epsilon_{i,t+1} - (1-\gamma_i)\,\pi_{i,t} - (\pi_{i,t} - \pi_{i,t-1})\right]$, and

$$\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \sigma_i^2\left(1 + \frac{\left[(1-\gamma_i)\,\pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1})\right]^2}{\sigma_i^2}\right) = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1})\right]^2.$$

For a general k, one obtains

$$\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t} + \frac{1}{k}\sum_{j=1}^{k}\left(\pi_{i,t+1-j} - \pi_{i,t-j}\right)\right]^2.$$

For $k \to \infty$,

$$\lim_{k\to\infty}\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2.$$

QED

Proof of Proposition 4. The Proposition is stated in the text for the case of $\rho_{1,2} = 0$. Here we prove the Proposition for the general case with correlated prices, i.e., for a generic value of $\rho_{1,2} \in [0.1]$. We have that

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right) + \beta_2\left(\eta_2 - \mu_2\right),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\eta_y, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_2)}{var(\eta_2)}\right)}{var\left(\eta_y\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_2)}},\ \ \beta_2 = \frac{cov\left(\eta_2, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_y)}{var(\eta_y)}\right)}{var\left(\eta_2\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_y)}}.$$

We have:
$cov\left(\eta_y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2 + \epsilon_y, \log x_1\right) = a\sigma_1^2 + b\rho_{1,2}$,
$cov\left(\eta_2, \log x_1\right) = \rho_{1,2}$,
$cov\left(\eta_y, \eta_2\right) = cov\left(a\log x_1 + b\log x_2 + \epsilon_y, \log x_2 + \epsilon_2\right) = b\sigma_2^2 + a\rho_{1,2}$,
$var\left(\eta_y\right) = a^2\sigma_1^2 + b^2\sigma_2^2 + \sigma_y^2 + 2ab\rho_{1,2}$,
$var\left(\eta_2\right) = \sigma_2^2 + s_2^2$.
Substituting yields

$$\beta_y = \frac{a\sigma_1^2 + b\rho_{1,2} - \frac{\rho_{1,2}\left(a\rho_{1,2} + b\sigma_2^2\right)}{\sigma_2^2 + s_2^2}}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2} - \frac{\left(a\rho_{1,2} + b\sigma_2^2\right)^2}{\sigma_2^2 + s_2^2}}$$

$$\beta_2 = \frac{\rho_{1,2} - \frac{\left(a\rho_{1,2} + b\sigma_2^2\right)\left(a\sigma_1^2 + b\rho_{1,2}\right)}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2}}}{\sigma_2^2 + s_2^2 - \frac{\left(a\rho_{1,2} + b\sigma_2^2\right)^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2}}}$$

For $\rho_{1,2}=0$, we obtain

$$\begin{aligned}
\beta_y &= \frac{a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2-\frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}, \\
\beta_2 &= \frac{-\frac{\left(a\rho_{1,2}+b\sigma_2^2\right)\left(a\sigma_1^2\right)}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}}{\sigma_2^2+s_2^2-\frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}} = -\frac{ab\sigma_1^2\sigma_2^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}\times\frac{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}{\left(\sigma_2^2+s_2^2\right)\left(a^2\sigma_1^2+b^2\sigma_2^2+s_y^2\right)-b^2\sigma_2^4} \\
&= \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4-\left(\sigma_2^2+s_2^2\right)\left(a^2\sigma_1^2+b^2\sigma_2^2+s_y^2\right)}.
\end{aligned}$$

QED

Proof of Corollary 6.

$$\begin{aligned}
\lim_{s_y,s_2\to+\infty}\beta_y &= \lim_{s_y,s_2\to+\infty}\frac{a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2-\frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}=0, \\
\lim_{s_y,s_2\to+\infty}\beta_2 &= \lim_{s_y,s_2\to+\infty}\frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4-\left(\sigma_2^2+s_2^2\right)\left(a^2\sigma_1^2+b^2\sigma_2^2+s_y^2\right)}=0.
\end{aligned}$$

QED

Proof of Corollary 7.

$$\begin{aligned}
\lim_{s_2\to+\infty}\beta_y &= \lim_{s_2\to+\infty}\frac{a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2-\frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}=\frac{a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}, \\
\lim_{s_2\to+\infty}\beta_2 &= \lim_{s_2\to+\infty}\frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4-\left(\sigma_2^2+s_2^2\right)\left(a^2\sigma_1^2+b^2\sigma_2^2+s_y^2\right)}=0.
\end{aligned}$$

QED

Proof of Corollary 8.

$$\begin{aligned}
\lim_{s_y,s_2\to0}\beta_y &= \lim_{s_y,s_2\to0}\frac{a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2-\frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}=\frac{a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2+-\frac{b^2\sigma_2^4}{\sigma_2^2}}=\frac{1}{a}, \\
\lim_{s_y,s_2\to0}\beta_2 &= \lim_{s_y,s_2\to0}\frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4-\left(\sigma_2^2+s_2^2\right)\left(a^2\sigma_1^2+b^2\sigma_2^2+s_y^2\right)}=\frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4-\sigma_2^2\left(a^2\sigma_1^2+b^2\sigma_2^2\right)}=-\frac{b}{a}.
\end{aligned}$$

QED

B Online Appendix - Further Figures and Tables

Figure A1: **Survey Questions of Firm Forecasts**

4. Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, -2%, etc.) [Leave blank if not applicable]

% Prices of your products	% Technology spending
% Overtime	% Earnings
% Advertising/Marketing spending	% Revenues
% Number of employees	% Inventory
% Productivity (output per hour worked)	% M&A activity
% Wages/Salaries	% Capital spending
% Health care costs	% Dividends

Table A1: **Growth Realizations of Selected Balance Items**

Realizations in Compustat (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Growth in Revenues and in Earnings						
Revenues	13.38	35.96	-16.64	6.89	46.24	105,866
Earnings	-16.21	432.06	-207.66	-3.92	174.42	105,841
Growth in Capital-Related Expenditures						
Capital Expenditures	35.71	132.99	-56.41	5.26	129.28	100,633
R & D	15.91	53.09	-25.24	6.75	57.64	40,715
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Growth in Labor-Related Costs						
Wages	10.57	24.22	-9.57	6.94	31.96	29,491
Employees	6.18	25.30	-14.29	2.07	29.17	107,435
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	19.12	80.07	-37.35	4.39	71.33	34,251
Growth in Cash Holdings and Corporate Payout						
Cash	76.55	308.36	-57.50	5.23	184.62	103,833
Dividends	18.74	97.74	-56.43	5.42	60.56	54,841
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table A2: **Summary Statistics**

Panel A – Matched Duke-Compustat sample

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.845	1.629	0.875	1.402	4.157	15,929
ROA	0.025	0.235	-0.154	0.037	0.167	16,591
Sales	10,617.34	28,482.18	53.66	2,043.96	49,545.00	17,799
Log(sales)	7.591	2.093	4.022	7.629	10.813	17,757
Assets	37,698.73	187,568.6	74.12	2,894.43	113,960.0	17,799
Log(assets)	7.993	2.221	4.306	7.971	11.644	17,799
Book Leverage	0.411	1.755	0.000	0.371	0.910	17,733
Capital Expenditure	0.045	0.058	0.001	0.031	0.134	16,200
R & D	0.060	0.202	0.000	0.025	0.211	9,043
Cash Flow	0.302	13.075	-1.127	0.413	3.005	17,010
Cash	4.895	62.328	0.014	0.563	15.188	17,269
Advertising	0.025	0.043	0.000	0.009	0.098	6,729
Dividends	0.117	1.420	0.000	0.060	0.349	17,391
Dividends (0/1)	0.607	0.488	0.000	1.000	1.000	17,391

Panel B – Compustat data

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.843	2.526	0.708	1.297	4.489	105,769
ROA	0.006	0.279	-0.273	0.021	0.187	123,155
Sales	3,521.58	15,220.67	17.992	315.011	14,687.00	127,307
Log(sales)	5.911	2.062	2.890	5.753	9.595	127,307
Assets	12,626.69	98,056.55	24.147	597.555	30,241.99	140,894
Log(assets)	6.529	2.170	3.184	6.393	10.317	140,894
Book Leverage	0.408	22.120	0.000	0.362	0.996	139,264
Capital Expenditure	0.063	0.159	0.000	0.032	0.212	108,909
R & D	0.075	0.135	0.000	0.027	0.290	52,955
Cash Flow	0.050	0.261	-0.222	0.061	0.256	118,905
Cash	0.192	0.396	0.002	0.081	0.685	111,475
Advertising	0.034	0.103	0.000	0.009	0.131	39,813
Dividends	0.144	7.286	0.000	0.000	0.557	122,340
Dividends (0/1)	0.431	0.495	0.000	0.000	1.000	122,340

Table A3: **Cross Sectional Regressions in Compustat Data for Rules of Thumb**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Investment growth	0.077***							0.074***	0.057***	0.052**
	(0.002)							(0.002)	(0.001)	(0.019)
Wages growth		0.657***								0.454***
		(0.017)								(0.045)
Advertising growth			0.139***						0.121***	0.061***
			(0.005)						(0.002)	(0.009)
Earnings growth				0.035***				0.030***		0.011
				(0.001)				(0.001)		(0.012)
R & D growth					0.253***					0.016
					(0.007)					(0.024)
Cash growth						0.013***				0.007
						(0.001)				(0.009)
Dividend growth							0.035***			0.011
							(0.002)			(0.021)
Constant	0.106***	0.033***	0.096***	0.133***	0.102***	0.123***	0.128***	0.106***	0.081***	0.014
	(0.001)	(0.008)	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.009)
R^2	0.081	0.350	0.128	0.027	0.132	0.012	0.004	0.102	0.177	0.376
N observations	100,441	17,523	34,182	103,738	40,711	103,363	100,247	100,040	33,202	871

Notes: The data is made of repeated cross sections. Standard errors corrected for heteroskedasticity and clustered by firm.

Table A4: **Minimum Distance of Earnings Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.026	0.045	0.021	0.028	0.032	0.031
Std. Dev.	0.033	0.054	0.028	0.046	0.035	0.030
Frac. Zeros	0.197	0.000	0.356	0.000	0.000	0.000
P10	0.000	0.002	0.000	0.002	0.005	0.004
P25	0.004	0.003	0.000	0.008	0.007	0.009
P50	0.014	0.027	0.014	0.010	0.017	0.015
P75	0.035	0.064	0.035	0.017	0.050	0.052
P90	0.068	0.101	0.057	0.099	0.071	0.073
P95	0.101	0.177	0.085	0.182	0.111	0.090
N of Observations	396	24	219	35	48	70
Fraction	1.000	0.061	0.553	0.088	0.121	0.177

Notes: Cross-sectional analysis with 396 CFOs.

Table A5: **Minimum Distance of CapEx Forecasts from Rules of Thumb: Robustness to Alternative Definition of Rule 5**

	All	R1	R2	R3	R4	R5
Mean	0.029	0.040	0.025	0.030	0.041	0.023
Std. Dev.	0.039	0.031	0.044	0.031	0.056	0.016
Frac. Zeros	0.146	0.000	0.365	0.000	0.000	0.000
P10	0.000	0.005	0.000	0.006	0.002	0.004
P25	0.005	0.017	0.000	0.009	0.003	0.009
P50	0.016	0.031	0.007	0.022	0.023	0.019
P75	0.035	0.064	0.032	0.040	0.065	0.031
P90	0.071	0.094	0.071	0.074	0.122	0.048
P95	0.106	0.094	0.106	0.094	0.222	0.049
N of Observations	130	9	52	30	18	21
Fraction	1.000	0.069	0.400	0.231	0.138	0.162

Notes: Cross-sectional analysis with 130 CFOs.

Table A6: **Incoherence and Rules of Thumb: Robustness to Alternative Definition of Rule 5**

	(1)	(2)	(3)	(4)	(5)
Rule 1	0.099***				0.134***
	(0.022)				(0.025)
Rule 2		0.018			0.053***
		(0.012)			(0.016)
Rule 3			-0.024*		0.023
			(0.014)		(0.018)
Rule 4				0.003	0.044**
				(0.018)	(0.020)
Constant	0.058***	0.058***	0.071***	0.065***	0.023*
	(0.006)	(0.008)	(0.007)	(0.007)	(0.014)
R^2	0.126	0.009	0.014	-0.008	0.175
N observations	130	130	130	130	396
Summary Statistics of the dependent variable					
Mean	0.065				
Std. Dev.	0.069				
P10	0.012				
Median	0.045				
P90	0.153				

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table A7: **Incoherence, Rules of Thumb, and Corporate Performance: Robustness Using Alternative Definition of Rule 5**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Incoherence	-0.279*	-0.335**	-0.320**	-0.317**			
	(0.156)	(0.151)	(0.159)	(0.152)			
Rule 1					-0.062**	-0.071*	-0.071*
					(0.031)	(0.042)	(0.042)
Rule 2					0.009	0.012	0.014
					(0.032)	(0.035)	(0.035)
Rule 3					0.018	0.023	0.020
					(0.037)	(0.037)	(0.038)
Rule 4					0.020	0.020	0.026
					(0.033)	(0.039)	(0.036)
Miscalibration ST		0.009				0.004	
		(0.005)				(0.007)	
Optimism ST		0.011				0.009	
		(0.011)				(0.011)	
Miscalibration LT			0.014				0.012
			(0.010)				(0.011)
Optimism LT			0.008				0.007
			(0.011)				(0.011)
Constant	0.051***	0.051***	0.048***	0.014	0.024	0.015	0.015
	(0.011)	(0.011)	(0.012)	(0.013)	(0.031)	(0.034)	(0.034)
Industry FE	No	No	No	Yes	No	No	No
Survey FE	No	No	No	Yes	No	No	No
R^2	0.055	0.075	0.073	0.140	0.017	0.020	0.026
N of Observations	136	122	121	136	136	122	121

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.