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DYNAMIC INCENTIVES FOR BUY-SIDE ANALYSTS

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Abstract

We develop a dynamic adverse selection model where a career-concerned buy-side analyst advises a fund manager about investment decisions. The analyst's ability is privately known, as is any information she learns over time. The manager wants to elicit information to maximize fund performance while also identifying and retaining high-skill analysts. We characterize the optimal dynamic contract, show that it has several features supported by empirical evidence, and derive novel testable implications. The fund manager's optimal contract both maximizes the value of information and screens out low-skill analysts by incentivizing the analyst to always provide honest advice.

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ABSTRACT

We develop a dynamic adverse selection model where a career-concerned buy-side analyst advises a fund manager about investment decisions. The analyst's ability is privately known, as is any information she learns over time. The manager wants to elicit information to maximize fund performance while also identifying and retaining high-skill analysts. We characterize the optimal dynamic contract, show that it has several features supported by empirical evidence, and derive novel testable implications. The fund manager's optimal contract both maximizes the value of information and screens out low-skill analysts by incentivizing the analyst to always provide honest advice.

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Buy-side analysts gather and provide critical information to fund managers for their exclusive use within the firm (Cheng, Liu, and Qian (2006); Frey and Herbst (2014)). Analysts' advice, ideas, and recommendations play an important role in determining fund performance and in meeting investment goals. However, analysts are often motivated by reputational career concerns (Hong and Kubik (2003); Clement and Tse (2005)), and so their incentives need not align with those of their fund managers. In particular, analysts may manipulate their recommendations in order to appear competent, but in so doing, may negatively impact the fund's profits. In this paper, we develop a general framework that captures these divergent incentives and allows us to characterize the fund manager's optimal dynamic incentive contracts.

We develop a dynamic adverse selection environment in which a buy-side analyst (the agent) with privately known ability or skill (high or low) is tasked with providing advice about possible investment decisions by some target date T . This advice typically takes the form of investment recommendations, while the deadline might correspond to a target date for portfolio rebalancing, a pitch meeting with a potential investor, quarterly earnings season, or simply a regulatory filing due date. A fund manager (the principal) decides how to act in response to this advice, and also chooses whether or not to retain the services of (or, equivalently, whether or not to promote) the analyst after observing the performance of the analyst's advice. The manager has two competing objectives. First, she wants to maximize the fund's profits in the near term by obtaining and optimally using information relevant to her portfolio decisions. Second, she also wants to maximize the fund's long-term performance by optimally managing the human capital of her organization via analyst recruitment and retention—in particular, she only wants to retain (and eventually promote) the analyst if she is of high ability. Conversely, the analyst is concerned with maintaining her job; she cares about fund performance to the extent that it reflects on her ability (and thereby impacts her retention).

Formally, in our model, there is an unknown state of the world corresponding to the fund manager's time- T ex post optimal investment decision. Over time, the analyst may observe a private signal that reveals the underlying state of the world; the arrival rate of this signal is greater when the analyst is highly skilled. Since learning is private and only the analyst observes when a signal arrives, she is free to make any recommendation at any time to the fund manager regardless of what she may or may not actually know. The fund manager is free to make and revise her investment decisions at any time, taking into account the knowledge that the analyst's advice (if any) is strategic. The resulting payoff from any such actions serves as a noisy signal of the quality of the analyst's advice, and may depend on how early the recommendation arrives: we allow (but do not require) the fund manager to have a preference for early resolution of uncertainty. The fund manager's decision to retain the analyst is a function of the recommendation, the time at which it was made, and its realized ex post performance. We study the following problem: how should the fund manager use the retention decision to screen analyst ability while accounting for strategic, untruthful recommendations that negatively affect fund performance?

While our model is novel, its individual ingredients find support from the literature on analysts and fund managers. As mentioned above, career concerns matter for financial analysts and have repeatedly been shown to influence their forecasts and advice; for example, Brown, Call, Clement, and Sharp (2016) note that a majority of buy-side analysts view job security as a "very important" motivator, while Crawford, Gray, Johnson, and Price (2018) document reputation-building activities by buy-side analysts. Conversely, firms work hard to efficiently match responsibility and capital to skill (Berk, van Binsbergen, and Liu (2017)) and clearly have an incentive to retain the services of good analysts while terminating poor

ones (Hong, Kubik, and Solomon (2000)). The dynamic information structure where an analyst learns about the underlying fundamentals via private signals is a natural way to model uncertainty in this environment (see for instance, Clarke and Subramanian (2006) and Crane and Crotty (2020)).

In our setting, the fund manager wants to simultaneously elicit honest advice and screen the skilled from the unskilled analyst. At first glance, these goals appear to conflict with one another. A contract designed to weed out a low-ability analyst poses the risk of inducing strategic recommendations that attempt to mimic her high-ability counterpart. Conversely, a contract designed to elicit accurate recommendations may do a poor job of screening (a trivial example is the contract that commits to never terminate the analyst, regardless of her performance). Our first result shows that the fund manager’s dual objectives need not be in conflict with one another: the *optimal* dynamic contract incentivizes the analyst to always provide truthful advice. This is in sharp contrast to the literature on analysts or forecasters facing exogenously given (and suboptimal) incentives (see, e.g., Trueman (1994) or Ottaviani and Sørensen (2006)), where agents manipulate their advice in order to appear skilled.

This theoretical result finds empirical support while highlighting one of the key distinctions between buy-side and sell-side analysts. As many studies have shown, recommendations from the sell side are often overly positive due to conflicts of interest (see, for instance, Cowen, Groysberg, and Healy (2006); Kolasinski and Kothari (2008); Malmendier and Shanthikumar (2014)); they can also be distorted due to career concerns induced by the public nature of sell-side forecasts, as analysts compete with each other for “star” status (Ottaviani and Sørensen (2006)). Meanwhile, there is evidence from the buy side that supports our theoretical finding: if buy-side analysts’ recommendations are undistorted, fund managers should be more likely to respond to their advice than to sell-side information. Indeed, Groysberg, Healy, Serafeim, and Shanthikumar (2013) find that buy-side analysts are less likely to assign optimistic “strong buy” or “buy” ratings than their sell-side counterparts. Additionally, Rebello and Wei (2014) show that trades made by fund managers are highly responsive to buy-side recommendations, while reliance on sell-side research is concentrated in assets that are not followed by buy-side analysts. This prediction from our model can also be tested directly: one could compare actual recommendations and forecasts from those in anonymous surveys where strategic behavior plays no role as Marinovic, Ottaviani, and Sørensen (2013) do for professional forecasters by comparing the anonymous Survey of Professional Forecasters with the non-anonymous Business Economic Outlook.

The optimal contract has a simple and, in our view, realistic structure that highlights the key dimensions along which buy-side analysts should be screened. Because skilled and unskilled analysts in our environment are differentiated by the *speed* at which they learn about the underlying state of the world, the optimal contract screens using a combination of the timing of advice and its realized performance. The key qualitative features of the optimal contract are threefold: (i) the later the analyst makes her recommendation, the lower the likelihood of her retention; (ii) the analyst is sometimes retained despite an (ex post) suboptimal recommendation; and (iii) the analyst is retained with positive probability even if she abstains from making any recommendation at all. We discuss each of these features in turn.

The first of the three properties is perhaps the most intuitive: since the skilled analyst learns faster, contracts that effectively screen ability should reward earlier recommendations. As we have suggested above, however, the optimal contract must prevent both types of analyst from signaling by making uninformed early guesses in the absence of information. The optimal contract does so by providing an option value to waiting: the analyst is more likely to be terminated following a recommendation that yields poor results, and so uninformed guessing (which is more likely to result in poor portfolio performance) is less

attractive than waiting and potentially learning the underlying state.

In light of the above discussion, it is perhaps surprising that “bad” or ex post incorrect recommendations are not punished with certain termination, as leniency increases the incentive to guess. Our model allows for the possibility that, given the uncertainties inherent in the market, even sound advice based on concrete information can sometimes fail to yield strong results. Thus, poor performance is not a perfect signal that the underlying advice was flawed. Since the optimal contract induces truthful recommendations and the good type learns more quickly, earlier recommendations are more likely to be made by good analysts. Thus, it is not in the manager’s interest to administer draconian punishments for early recommendations that ultimately underperform; the likelihood of retention, while positive, is small enough to prevent the low-ability analyst from guessing.¹ The likelihood of termination after a poorly performing recommendation thus increases the closer the recommendation is made to the target date T .

The final novel feature of the optimal contract naturally captures the fact that buy-side analysts are typically responsible for far more firms than their sell-side counterparts. As a result, even the best analysts may not always be able to offer fully informed advice. In such situations, the fund manager is better off not adjusting her portfolio or, instead, relying on information from the sell side. However, if the analyst is fired in the absence of a concrete recommendation, she will always have an incentive to eventually provide a speculative guess. The only way to prevent this is to guarantee some small likelihood of retention even in the absence of a recommendation. An alternate interpretation is that, if the analyst does not observe an informative signal, she may make “safe” recommendations that do not require specialized in-depth research that depends on analyst ability (for instance, investing in low volatility assets). Stated this way, this feature of the optimal contract is supported by evidence in [Groysberg, Healy, Serafeim, and Shanthikumar \(2013\)](#), who show that buy-side analysts are more likely to make such safe recommendations.

One reason for the relative scarcity of research on buy-side analysts (compared to the sell side) is the difficulty of obtaining data. Most studies use proprietary data procured from a single large fund management company. Importantly, since our model studies incentive contracts *within* the firm, it can be tested on such data sets since we do not require firm heterogeneity. In particular, this flexibility permits tests of the main high-level implication of the model: career outcomes for analysts should depend not only on the accuracy but also on the *timing* of their recommendations. To the best of our knowledge, ours is the first paper to highlight this channel for analyst differentiation; previous studies have relied on either forecast accuracy (for example, [Gu and Wu \(2003\)](#)) or forecast volatility (see [Hilary and Hsu \(2013\)](#)) as a proxy for analyst performance and skill.

Our model is the first to study the optimal design of dynamic incentives for career-concerned analysts; more generally, we are unaware of other work (even in static environments) that addresses how contracts should be structured to optimally identify talent within the fund. While our model is written with buy-side analysts in mind, it is worth mentioning that our setting captures the incentives underlying a number of other principal-agent settings where a firm cares about both the quality of information provision as well as about screening. Examples include traders, quantitative analysts, or, more generally, any setting where a principal employs an “innovator.”

This paper is related to work in economics on dynamic mechanism design (see [Bergemann and Välimäki \(2019\)](#) for a recent survey) but the structure and key elements of the model differ from much of that

¹[Manso \(2011\)](#) studies a two-period setting with moral hazard (in contrast to our dynamic adverse selection setting) where, despite important modeling differences, also features an optimal contract that tolerates some early failures.

literature. The closest paper is [Deb, Pai, and Said \(2018\)](#), where a prospective political candidate decides whether or not to hire an advisor or pollster based on the dynamics of her forecasts in an earlier, unrelated contest. That paper differs along three key dimensions: (i) the principal there cares only about screening the agent and not about the actual information she provides; (ii) there are only two possible outcomes (the identity of the election winner); and (iii) attention is restricted to deterministic contracts. Generalizing along each of these three dimensions is crucial to accurately model buy-side analysts. Clearly, fund managers care directly about the informational content of analyst recommendations, and fund portfolios can be adjusted along many more than just two dimensions. Finally, the structure of our optimal contract and the resulting testable predictions highlight the importance of the *likelihood* that, and not just whether or not, an analyst is retained.

1. MODEL

In order to study the interaction of analyst incentives with information provision, we develop a dynamic adverse selection model where a fund manager (the principal) contracts with an analyst (the agent). The analyst conducts research on behalf of the manager to help optimize the fund portfolio and may recommend certain actions; the portfolio’s future performance is a noisy signal that partially reveals whether or not the analyst’s advice was in fact correct.

1.1. THE ENVIRONMENT

STATE: The underlying state of the world $\omega \in \Omega$ corresponds to the optimal strategy that the fund should follow. This state is initially unknown to both the manager and the analyst. We assume that there are finitely many states of the world and that each state is equally likely; hence, the commonly known prior distribution is $\Pr(\omega) = \frac{1}{n}$, where $n := |\Omega| < \infty$. (We defer a discussion of the more general case with asymmetry to [Section 6](#).)

OUTCOME: After the target date $T < \infty$, a publicly observed outcome $r \in \Omega$ is realized. This can be interpreted as, for instance, realized market prices, and so the firm learns what its optimal strategy *should* have been. This outcome is correlated with the true state ω : the outcome r matches the true state ω with probability $\gamma \in (\frac{1}{n}, 1)$; any other outcome is observed with uniform probability. This implies that

$$\Pr(r|\omega) = \begin{cases} \gamma & \text{if } r = \omega, \\ \frac{1-\gamma}{n-1} & \text{otherwise.} \end{cases}$$

We assume that $\gamma < 1$ to account for the possibility of imperfect foresight, even with maximal information. This accounts for the possibility of, for instance, unanticipated shocks or market frictions that create a wedge between ex ante expectations and ex post outcomes and optimal strategies. For completeness, however, we discuss the case of perfect ex post information (that is, when $\gamma = 1$) in [Section 6](#).

1.2. THE ANALYST

PRIVATE TYPE: There is a single agent (the analyst) whose ability or skill is her private information: the analyst has a privately known type $\theta \in \{h, l\}$ corresponding to high ability (h , or “skilled”) or low ability (l , or “unskilled”). The commonly known prior probability that the analyst is skilled is $\Pr(\theta = h) = \rho \in (0, 1)$.

PRIVATE LEARNING: The analyst conducts research that may, over time, reveal the underlying state of the world. We model the arrival of information as a type-dependent Bernoulli process: in each discrete period $t = 1, \dots, T$, the agent learns the unknown state ω with probability α_θ and otherwise remains uninformed.² Letting ϕ denote the absence of new information, we assume that the agent privately observes a signal $s_t \in \mathcal{S} := \Omega \cup \{\phi\}$ in each period t , where

$$\Pr(s_t | \omega, s_{t-1}) = \begin{cases} 1 & \text{if } s_t = \omega \text{ and } s_{t-1} = \omega, \\ \alpha_\theta & \text{if } s_t = \omega \text{ and } s_{t-1} \neq \omega, \\ 1 - \alpha_\theta & \text{if } s_t = \phi \text{ and } s_{t-1} \neq \omega, \\ 0 & \text{otherwise.} \end{cases}$$

We further assume that $0 < \alpha_l < \alpha_h < 1$, so that a high-ability (type- h) analyst is a better researcher (and hence a faster learner) than a low-ability (type- l) analyst.

In words, this corresponds to an “all-or-nothing” information structure: the analyst either learns the state of the world perfectly or learns nothing at all. The analyst’s ability determines the *rate* at which an informative signal arrives, and both types are increasingly likely to have observed an informative signal as the target date T approaches. This modeling assumption is natural and is supported by evidence from buy-side analyst compensation. For instance, [Groysberg, Healy, and Chapman \(2008\)](#) document that buy-side analysts are “rewarded for providing support to portfolio managers and *new ideas* that differ from the Street consensus” (emphasis added). Meanwhile, [Crane and Crotty \(2020\)](#) document substantial heterogeneity in analyst skill and show that the heterogeneity in this skill is “driven by differences in *information production ability*” (emphasis added). Clearly, the speed at which an analyst learns or generates new ideas is critical.

This framework corresponds to a pure adverse selection environment, and we do not consider any issues that might arise from moral hazard. Essentially, we are assuming that the analyst’s cost of research is sufficiently low that the optimal contract always induces full effort.

PREFERENCES: The agent’s preferences are type-independent: both types of analyst have reputation-motivated career concerns and therefore act to maximize the probability with which they are retained. We model this by assuming that the agent receives a payoff of 1 if she is retained by the fund and a payoff of 0 if not.

As has been shown in [Hong, Kubik, and Solomon \(2000\)](#) and subsequent work, the threat of termination and the promise of promotion are powerful generators of career concerns. Indeed, analysts are viewed as “portfolio managers in training” at many firms, and the best analysts are “eventually promoted to portfolio manager, which is typically a more highly paid position than that of analyst” ([Groysberg, Healy, and Chapman \(2008\)](#)). Thus, for the purposes of our model, it is equivalent to think of the analyst caring either about retention (instead of termination) or about promotion (as opposed to being passed over).

Although reputational considerations are shared by both buy- and sell-side analysts, it is important to note a key distinction between the two groups that drives our modeling assumptions: since buy-side advice is private and not disseminated outside the firm, the career concerns of buy-side analysts are much

²This process is the discrete-time analogue of a continuous-time Poisson arrival process, a technology that has been previously used to model research and innovation by, for instance, [Akcigit and Liu \(2015\)](#).

simpler than their sell-side counterparts. There are no conflicts of interest with firms that maintain investment banking relationships with the analyst’s employer, or contest incentives associated with industry-wide awards and the desire to achieve “star” status.

1.3. THE FUND MANAGER’S MECHANISM

The fund manager designs the mechanism mediating the analyst–fund interaction. We focus on the case where the principal fully *commits* to this mechanism. This design commitment can be thought of as a choice of organizational policy and institutional rules. It is worth emphasizing that this design aspect differentiates our paper from the remainder of the literature where analyst incentives are exogenously fixed.

We limit the fund manager’s contracting power, however, by ruling out the use of direct revelation mechanisms.³ In our employment contract setting, a direct revelation mechanism is *prima facie* impractical: it would require the fund manager to first solicit the analyst’s type and (in order to incentivize truthful disclosure) commit to retaining her despite a revelation that she is unskilled. (This is even more implausible if the manager makes a promotion—as opposed to retention—decision.) Even if a manager *could* commit to such a contract, the analyst would surely be concerned about how self-reporting a lack of skill will affect her future career prospects, as fund managers may share such information across firms.⁴ As a result, such direct revelation contracts are not feasible in the real world, and so we instead deliberately limit the manager to a more practical set of “indirect” contracts.⁵

COMMUNICATION: In each period $t = 1, \dots, T$, the analyst (strategically) reports her information $\tilde{s}_t \in \mathcal{S}$, possibly as the realization of a type-dependent mixed strategy σ^θ .⁶ Note that no communication occurs in period 0, as the agent’s only private information at that time is her initial type θ . This is a natural restriction on the set of game forms permitted to the manager: in practice, fund managers do not ask their analysts to *self-report* their ability or skill, nor do they give any credibility to such self-reports given the strong career concerns in the industry.

Buy-side analysts typically make recommendations that have substantial influence on fund investment decisions. Here, we are deliberately agnostic as to the specific interpretation of a reported signal: it can be thought of as a single buy, sell, or hold recommendation, a set of such recommendations for a basket of securities, or some other form of investment strategy advice. Note that we are free to take such a broad

³The revelation principle for dynamic environments is due to Myerson (1986); as per Sugaya and Wolitzky (2021), it holds without caveat in our environment. Any outcome that the manager can achieve with an arbitrary game form is also attainable using an incentive compatible direct revelation mechanism.

⁴Such privacy concerns would persist if the agent was asked to choose from a menu of contracts—the “traditional” alternative to direct revelation—instead of being asked to make an explicit type report, as her choice would indirectly reveal her type. For a broader discussion of these considerations in mechanism design, see Pai and Roth (2013).

⁵This is a similar approach to the dynamic pricing literature that characterizes optimal price paths (as opposed to more general optimal mechanisms; see Board (2008), for example), or to work in auction theory comparing the revenue generated by various standard auction formats (as opposed to deriving the optimal auction; see DeMarzo, Kremer, and Skrzypacz (2005), among others). Note, however, that we will formally prove with Lemma A.1 that this restriction does not reduce the fund’s ability to separate high-skill analysts from their low-skill counterparts.

⁶Formally, in each period t , the agent has a private history $h^t := (s^t, \tilde{s}^{t-1})$ that contains the t privately observed signals $s^t := (s_1, \dots, s_t)$ and the $t - 1$ reports $\tilde{s}^{t-1} := (\tilde{s}_1, \dots, \tilde{s}_{t-1})$ made prior to period t . Letting $\mathcal{H} = \bigcup_{t=1}^T \mathcal{S}^t \times \mathcal{S}^{t-1}$ denote the set of all histories, the type- θ agent’s strategy $\sigma^\theta : \mathcal{H} \rightarrow \Delta(\mathcal{S})$ determines the distribution of reports at each history. We will use the signal subscript $\sigma_s^\theta(h^t)$ to denote the probability that the agent reports signal $s \in \mathcal{S}$ at history h^t .

stance because we impose minimal structure on the fund manager’s payoff function (see below).

RETENTION DECISION: The fund manager chooses (at the beginning of the interaction) a retention rule $\chi(\tilde{s}^T, r) \in [0, 1]$ that specifies the probability with which the manager retains the analyst as a function of her T reported signals \tilde{s}^T and the observed outcome r . Although this retention rule χ is not a direct revelation mechanism—we reiterate that the communication protocol does *not* solicit the agent’s private type θ along with her signals—we will still refer to it as a mechanism.

As we mentioned above, it is equivalent to think of the manager’s retention decision as a promotion decision instead. To maintain consistent terminology, however, we will only refer to this decision as retention throughout the paper.

PREFERENCES: The fund manager has dual concerns: she wishes to maximize the fund’s profits in both the short term (by maximizing the value of information relevant to her portfolio decisions) as well as in the long term (by optimally managing the human capital of her organization via analyst recruitment and retention).

Based on the analyst’s reports $\tilde{s}^t := (\tilde{s}_1, \dots, \tilde{s}_t)$ through period t , the manager has a posterior belief π_t about the underlying state of the world. This belief is formed using Bayes’ rule, given the analyst’s equilibrium (type-dependent) reporting strategy $\sigma := (\sigma^h, \sigma^l)$. The manager’s ex post value of information $V(\pi_1, \dots, \pi_T) \in \mathbb{R}$ is a function of her beliefs leading up to period T . At any period $t \leq T$, with past and present beliefs (π_1, \dots, π_t) , the manager’s value of information is the expectation $\mathbb{E}[V(\pi_1, \dots, \pi_T) | \pi_1, \dots, \pi_t, \sigma]$ of V , taken with respect to the expected future beliefs (which in turn depend on the analyst’s strategy).

This is an extremely general formulation of the fund manager’s value for information that captures numerous different objective functions. Perhaps the most natural objective corresponds to an environment where, in *every* period t , the manager chooses whether, and if so how, to adjust the fund portfolio. Her decision depends on her beliefs about the underlying state (which in turn informs the optimal ex post trading strategy). The generality not only allows the manager to anticipate future information and accommodate transaction costs, it also captures the fact that information can have different values in different periods: For instance, buying a security that performs well at period T (say because of unexpectedly high reported earnings) can be more profitable if it is purchased before the market price incorporates the information via consensus forecasts (typically released closer to T) and reduces potential gains. Likewise, our formulation also captures the setting studied in [Kadan and Manela \(2019\)](#) where the principal derives both instrumental and psychic value from information used to trade Arrow-Debreu securities in a dynamic environment with learning; there, V would correspond to the fraction of wealth the principal is willing to forgo in return for the earlier resolution of uncertainty.

We assume that high-ability analysts are net positive contributors to the fund’s long-run profitability, while low-ability analysts detract from the fund’s expected long-run value. Indeed, the fund manager can rely on information from the sell side instead of employing a low-ability in-house analyst. Therefore, the principal prefers to retain high-ability analysts and to terminate low-ability ones, and we denote by W the extent to which the manager separates the two. We allow for asymmetry in the long-run costs and benefits of the two types due to differences in, for example, their expected future earnings and their future

contributions to fund profits. Thus, we normalize the value of human capital by letting

$$W := \mathbb{E} [\chi(\tilde{s}^T, r) (\mathbb{1}_{\theta=h} - c\mathbb{1}_{\theta=l})], \quad (1)$$

where $c > 0$ is the loss incurred by retaining (or promoting) low-skill analysts.

Given the manager's dual concerns, we can write her payoff as a function

$$\Pi(V, W),$$

of the value of information V and the degree of type separation W . We assume that Π is strictly increasing in both V and W , but impose no further restrictions on Π .

2. BENCHMARK: PUBLICLY OBSERVED SIGNALS

To develop some intuition and provide a contrast to the optimal contract we characterize, we first consider the “first-best” benchmark in which the analyst's learning occurs publicly: the fund manager observes any signals as they arrive, and the agent's only private information is her initial type. Of course, the lack of private information about the state implies that the fund manager is able to maximize the value of information and associated trading profits without regard to incentive provision to the agent. Likewise, the agent-retention decision can be entirely decoupled from the portfolio decision. This implies that the fund manager essentially maximizes her objective $\Pi(V, W)$ pointwise with respect to both the value of information V and long-run human capital W , yielding a payoff

$$\Pi^{FB} := \Pi(V^{FB}, W^{FB}),$$

where V^{FB} is the first-best expected value of information and W^{FB} is the first-best expected degree of type separation.

To more fully characterize the benchmark W^{FB} , recall from (1) that our measure of type separation can be written as

$$\begin{aligned} W &= \sum_{s^T \in \mathcal{S}^T} \sum_{r \in \Omega} \Pr(s^T, r) \chi(s^T, r) [\Pr(\theta = h | s^T, r) - \Pr(\theta = l | s^T, r)c] \\ &= \sum_{s^T \in \mathcal{S}^T} \sum_{r \in \Omega} [\rho \Pr(s^T, r | \theta = h) - (1 - \rho) \Pr(s^T, r | \theta = l)c] \chi(s^T, r). \end{aligned} \quad (2)$$

In words, this is the difference between the expected probability of retaining the high-ability analyst and the expected probability of retaining the low-ability analyst, accounting for the cost c of retaining the latter. Maximizing this expression pointwise with respect to $\chi(s^T, r)$ yields the fund manager's first-best rule: retain the analyst if her expected contribution to the future value of the fund is positive, and terminate the relationship otherwise. This benchmark rule may be written as

$$\chi^{FB}(s^T, r) = \begin{cases} 1 & \text{if } \rho \Pr(s^T, r | \theta = h) \geq (1 - \rho) \Pr(s^T, r | \theta = l)c, \\ 0 & \text{otherwise.} \end{cases}$$

Of course, the conditional probabilities featured in the first-best retention rule are easily expressed in

terms of the underlying primitives. Since signals are conditionally i.i.d. as long as the analyst is uninformed, only the arrival time of the first informative signal (if any) plays a role. With this in mind, note that the probability of type θ observing a signal profile s^T in which the first informative signal arrives in period k is

$$\delta_{k,\theta} := \begin{cases} \alpha_\theta(1 - \alpha_\theta)^{k-1} & \text{if } k = 1, \dots, T, \\ (1 - \alpha_\theta)^T & \text{if } k = \infty, \end{cases} \quad (3)$$

where we denote by $k = \infty$ the event in which the analyst *never* observes an informative signal. This implies that, given such an arrival time k , the expected contribution of the analyst to the fund's human capital is positive if, and only if,

$$\Delta_k := \beta\delta_{k,h} - \delta_{k,l} \geq 0, \text{ where } \beta := \frac{\rho}{(1 - \rho)c}. \quad (4)$$

We now provide a fuller characterization of this first-best retention policy:

THEOREM 1. *Suppose the analyst's signals are publicly observed. There exist thresholds $\underline{\beta} < 1 < \bar{\beta}$ such that the analyst is never retained if $\beta \leq \underline{\beta}$ and is always retained if $\beta \geq \bar{\beta}$. When $\beta \in (\underline{\beta}, \bar{\beta})$, the first-best retention policy χ^{FB} is a cutoff policy: the analyst is retained if, and only if, an informative signal arrives in some period $t \leq \bar{k} := 1 + \ln\left(\frac{\beta\alpha_h}{\alpha_l}\right) / \ln\left(\frac{1-\alpha_l}{1-\alpha_h}\right)$.*

In words, [Theorem 1](#) states that if β is sufficiently small (so that the analyst is ex ante likely to have low ability, or the cost of erring in retention is large), the agent is never retained, regardless of whether or when information arrives. If, instead, β is sufficiently large (so the agent is ex ante likely to have high ability, or the relative cost of erring and retaining the low-ability type is small), the analyst is always retained, regardless of whether or when information arrives. But when β takes an intermediate value, screening is valuable and the principal employs a cutoff \bar{k} such that the analyst is retained only if she receives an informative signal by period \bar{k} .

The absence of dynamic adverse selection has two important consequences reflected in [Theorem 1](#)'s first-best policy. First, the optimal policy is deterministic: depending on the arrival time of an informative signal, the analyst is either sure to be retained or sure to be fired, with a sharp delineation between these two outcomes at time \bar{k} . Second, since the agent's signals are public, there is no need for the optimal policy to use the additional information conveyed by the eventual realized outcome to provide incentives; therefore, we have $\chi^{FB}(s^T, r) = \chi^{FB}(s^T, r')$ for all outcomes $r, r' \in \Omega$. Since signals are perfectly informative of the underlying state for both types, there is no further type-dependent information that is contained in the final outcome.

An immediate consequence of these two properties is that whenever the first-best policy discriminates between the two types, the analyst's career concerns provide a strong incentive for strategic manipulation. In particular, if the analyst learns *privately*, then she can guarantee retention by simply "guessing" and reporting the arrival of an *arbitrary* informative signal in any period $t \leq \bar{k}$. This strategic reporting of private information by *both* types of analyst precludes the implementation of the first-best retention policy since the agent is always retained—and it also has a deleterious effect on the fund's trading profits since the fund manager no longer has access to informed recommendations.

3. OVERVIEW OF MAIN RESULTS

As established above, the “first-best” incentive contract χ^{FB} for $\beta \in (\underline{\beta}, \overline{\beta})$ fails spectacularly in the presence of private learning: it fails to deliver useful information for trading purposes, and it fails to induce any separation between high-ability and low-ability analysts. Unsurprisingly, we can improve on this contract by providing meaningful incentives to the analyst. Consider, for instance, a contract that promises to always retain the agent regardless of whether and when they report any news. Such a promise maximally insures the analyst and, hence, makes her indifferent between *all* reporting strategies; in particular, it incentivizes truthful reporting. Thus, the principal can attain the first-best value of information V^{FB} despite achieving no separation between high-skill and low-skill analysts. For precisely this reason, we will henceforth focus only on the more interesting (nontrivial) case where $\beta \in (\underline{\beta}, \overline{\beta})$, so that the manager has a meaningful incentive to separate the two types.

In this case, the fund has dual objectives, and it may therefore be willing to sacrifice on informativeness in the short-run in order to decrease the distortions in human capital management. Surprisingly, however, our first main result shows that such a sacrifice is *not* necessary. Indeed, the fund manager’s optimal mechanism attains the first-best value of information V^{FB} while minimizing (though not completely eliminating) the distortions in analyst retention.

THEOREM 2. *Suppose $\beta \in (\underline{\beta}, \overline{\beta})$. The fund manager’s expected payoff from the optimal mechanism is $\Pi^* := \Pi(V^{FB}, W^*)$, where $W^* < W^{FB}$ is the maximal separation of types possible under private analyst learning.*

It is worth reemphasizing the implication of the above result as it draws a sharp contrast between buy-side and sell-side analysts. A robust empirical finding is that sell-side analysts’ recommendations are typically overly optimistic, and furthermore that they tend to herd. By contrast, [Theorem 2](#) implies that it is possible for funds to construct appropriate contracts to incentivize truthful information revelation by buy-side analysts; indeed, we show that doing so is *optimal* and that this can be balanced with the fund manager’s screening motives. As mentioned earlier, this is because there are fewer conflicts of interest on the buy side and analysts are not competing with each other via public forecasts. Eliciting such bias-free advice is one possible reason why buy-side analysts are employed in the first place (instead of funds simply relying on sell-side advice).

In a more abstract environment, [Theorem 2](#) would not be surprising. A “standard” dynamic mechanism design approach to this class of problems makes use of the full power and generality of the revelation principle: an incentive compatible direct revelation mechanism where the agent is induced to *always* reveal *all* private information truthfully, regardless of her type or the nature of her private information. The fund would therefore always be able to attain the first-best value of information.

Our result is therefore surprising precisely because, despite *not* employing a direct revelation mechanism, the principal is able to elicit sufficient information to attain her first-best trading outcomes. In fact, we show that the payoff that the fund manager obtains from our optimal contract coincides with that of the optimal direct mechanism, and so we effectively show that restricting attention to these real-world contracts is without loss.

To better understand this striking result, it is helpful to develop some of the underlying intuition. The first key insight is that direct revelation mechanisms are *not* necessary to solve the principal’s screening

problem (in isolation, setting aside the value of information).⁷ To see why, consider a direct mechanism $\bar{\chi}(\theta, s^T, r) \in [0, 1]$ that specifies the probability of retaining the analyst given her reported ability θ , her profile of reported signals s^T , and the final outcome r . Suppose that this direct mechanism is incentive compatible, and denote by \bar{W} the manager’s payoff from screening induced by $\bar{\chi}$. Since $\bar{\chi}$ is incentive compatible, the analyst always finds it optimal to report her ability truthfully; therefore, mechanically *forcing* her to misreport her type yields the analyst a lower payoff, regardless of how she might compensate with additional future misreports. Since (holding all else fixed) the manager’s payoff is decreasing in the probability the low-ability type- l analyst is retained, reducing the latter’s payoff is beneficial to the manager. This can be achieved with the indirect mechanism $\chi(\cdot, r) := \bar{\chi}(h, \cdot, r)$. Of course, since the original mechanism $\bar{\chi}$ was incentive compatible, it remains optimal for the high-ability type- h analyst to report information truthfully in χ , and her retention probability remains unchanged; thus, this indirect mechanism induces at least \bar{W} type separation, if not strictly more.

Of course, solving the fund’s screening problem and maximizing W alone is generally insufficient for solving the fund’s joint profits-and-screening problem: attempting to maximize type separation alone might provide the analyst with an incentive to strategically manipulate her recommendations, thereby negatively impacting the fund manager’s ability to maximize the value of this advice. There are three types of such manipulations that must be prevented in order to maximize the fund’s value of information and associated trading profits:

- (NM) *No Misreporting*. The analyst must have an incentive to *truthfully report* the state when she learns it instead of misreporting it as some other state.
- (ND) *No Delay*. The analyst must have an incentive to *immediately report* when she has learned the state instead of delaying and reporting in a future period (or never reporting at all).
- (NG) *No Guessing*. The analyst who remains uninformed must be willing to *wait for information* to arrive instead of blindly guessing the state.

Recalling that the difference between a skilled analyst and an unskilled analyst in our environment is the speed at which they learn and not the quality of their information, it is immediately clear that the first two conditions (NM) and (ND) above apply identically to both types of agents—once the analyst has learned the underlying state, her true ability no longer matters for her incentives. On the other hand, the third condition above *does* depend on the analyst’s type, as the option value of waiting for information depends on the agent’s perceived likelihood of that information actually arriving. But since the high-ability analyst is more likely to learn the state by waiting than her low-ability counterpart, satisfying condition (NG) for the type- h analyst does *not* guarantee that it is satisfied for the type- l analyst—the unskilled agent may prefer at some point to make an uninformed recommendation instead of waiting for information.

The second key insight necessary to show [Theorem 2](#) is that the fund manager finds it optimal to ensure that condition (NG) is indeed satisfied for the type- l low-ability analyst at all histories. To see why this must be the case, suppose for the moment that there is some period \bar{t} where an uninformed type- h analyst is content to wait for news but an uninformed type- l analyst *strictly* prefers to guess the state instead of waiting for more news. Of course, this would imply that *only* type- h analysts are ever willing to wait for news beyond time \bar{t} , and therefore that rewarding delayed learning with increased retention probabilities

⁷We formally state and prove this result as [Lemma A.1](#); this result also appears in the context of evaluating strategic forecasters in [Deb, Pai, and Said \(2018\)](#).

can only increase the likelihood of retaining the high-skill type- h analyst. Critically, this can be done in a way that does not affect the behavior of the unskilled type- l analyst (or her resulting retention probability) since her preference for preemption was strict. Since this increases the principal’s type separation payoff W while leaving the value of information V unchanged, the fund manager’s overall payoff Π strictly increases.⁸

Enumerating the manipulability conditions above also highlights some of the tensions inherent in incentivizing the analyst. For instance, inducing immediate reporting requires the principal to reward earlier reports of news, whereas the need to provide incentives for waiting require the principal to moderate any punishment of later reports. These two countervailing forces must be carefully balanced in any optimal contract.

The conditions above also help illuminate some key qualitative features of the optimal contract. First, note that since $\gamma < 1$, even the high-ability analyst’s best advice is imperfect, and the fund manager may find it optimal to retain the agent despite an erroneous recommendation. However, the manager’s lenience for bad advice decreases over time, as declining retention probabilities generate better incentives for immediate reporting. Likewise, the manager cannot always terminate the analyst if she fails to offer any advice at all by the target date T , as doing so guarantees that even a high-ability analyst will occasionally find it optimal to gamble and provide an uninformed recommendation in order to maintain her position. It is therefore critical to sometimes retain the analyst in the absence of a recommendation.⁹

4. THE OPTIMAL MECHANISM

We now turn to a formal characterization of the fund manager’s optimal mechanism in this environment, proceeding as follows. We will derive the mechanism that only maximizes the fund manager’s payoff W from screening. We will then show that this mechanism features the property that both analyst types report their signals truthfully. This in turn implies that the mechanism that maximizes W also maximizes the principal’s joint payoff $\Pi(V, W)$.

Three preliminary observations are particularly useful:

1. As explained above (and formally demonstrated in [Lemma A.1](#)), we can restrict attention to mechanisms that induce the high-ability type- h analyst to be fully transparent while leaving the low-skill type- l analyst unconstrained.
2. Since learning is “all-or-none” in this environment, only the analyst’s *first* informative signal matters: after observing such a signal, the agent perfectly learns the underlying state of the world. Since we want full transparency from the type- h analyst, we need only condition on the first reported non-null signal $s_t \neq \phi$.
3. Finally, since the underlying states and signals are symmetric, it is without loss to symmetrize the principal’s mechanism.

These three observations imply that we need only examine indirect mechanisms $\chi(s^T, r)$ that are incen-

⁸We formally state and prove this result in the appendix as [Lemma B.1](#).

⁹In a related contribution, [Backus and Little \(2020\)](#) discuss the difficulties in incentivizing an expert to admit to uncertainty. Unlike our commitment to retaining the agent, they rely on the ex post verification of “problem difficulty” to induce such confessions.

tive compatible for the high-skill type- h analyst, and that can be described by numbers

$$(p_t, q_t)_{t=1}^T \text{ and } p_\infty$$

such that:

- the agent is retained with probability p_t if they first report having observed a non-null signal $s_t = \omega$ in period t and the eventual outcome is $r = \omega$;
- the agent is retained with probability q_t if they first report having observed a non-null signal $s_t = \omega$ in period t and the eventual outcome is $r \neq \omega$; and
- the agent is retained with probability p_∞ , regardless of outcome, if they never report having observed a non-null signal.

Given such a mechanism, if the analyst truthfully reports her first non-null signal in period t , she is retained with probability $\gamma p_t + (1 - \gamma)q_t$; if she “guesses” a state in period t while uninformed (i.e., reports observing signal $\tilde{s}_t = \omega$ in period t despite having actually only observed null signals ϕ), she is retained with probability $\frac{1}{n}p_t + \frac{n-1}{n}q_t$.

It is now straightforward to formally specify the constraints that arise from requiring incentive compatibility for the high-ability type- h analyst. First, an informed agent who has learned the true state of the world ω must prefer to report that state truthfully instead of misreporting it as some $\omega' \neq \omega$. This implies that we must have

$$\gamma p_t + (1 - \gamma)q_t \geq \gamma q_t + (1 - \gamma) \left(\frac{1}{n-1}p_t + \frac{n-2}{n-1}q_t \right) \text{ for all } t = 1, \dots, T.$$

Because $\gamma > \frac{1}{n}$, this “no-misreporting” condition can be rewritten as

$$p_t \geq q_t \text{ for all } t = 1, \dots, T. \quad (\text{NM})$$

In addition, the analyst must have an incentive to immediately report observing an informative signal instead of delaying and reporting it in a future period (or never reporting it at all); this implies that we must satisfy the “no-delay” condition

$$\gamma p_t + (1 - \gamma)q_t \geq \gamma p_{t+1} + (1 - \gamma)q_{t+1} \text{ for all } t = 1, \dots, T. \quad (\text{ND})$$

(We mildly abuse notation here and let $p_{T+1} := p_\infty$ and $q_{T+1} := p_\infty$ for convenience.) Notice that the incentive constraints (ND) and (NM) apply only to informed agents, and are therefore type-independent; therefore, when (ND) and (NM) are satisfied for a high-ability type- h analyst, they are necessarily also satisfied for a low-ability type- l analyst.

An *uninformed* analyst’s expectations about the future depend on her type, however, as differentially able analysts learn about the underlying state of the world at different rates. We therefore define U_t^θ as the expected payoff of an uninformed type- θ agent in period t who reports their null signal $s_t = \phi$ truthfully and waits for an additional period. In that next period, the agent receives (and truthfully reports) an informative signal with probability α_θ (yielding a payoff $\gamma p_{t+1} + (1 - \gamma)q_{t+1}$), and remains uninformed

(and then proceeds optimally) with probability $(1 - \alpha_\theta)$; thus, we inductively define

$$U_t^\theta := \alpha_\theta (\gamma p_{t+1} + (1 - \gamma)q_{t+1}) + (1 - \alpha_\theta) \max \left\{ U_{t+1}^\theta, \frac{1}{n}p_{t+1} + \frac{n-1}{n}q_{t+1} \right\}, \text{ with } U_T^\theta := p_\infty \quad (5)$$

as there are no additional private signals to observe after period T . For an uninformed agent to prefer to wait for an additional signal instead of pretending to be informed and misreporting an arbitrarily chosen state, the mechanism must satisfy a “no-guessing” constraint

$$U_t^h \geq \frac{1}{n}p_t + \frac{n-1}{n}q_t \text{ for all } t = 1, \dots, T, \quad (\text{NG}^h)$$

$$U_t^l \geq \frac{1}{n}p_t + \frac{n-1}{n}q_t \text{ for all } t = 1, \dots, T. \quad (\text{NG}^l)$$

Lemma A.1 implies that (NG^h) must be satisfied in any solution to the fund manager’s problem, but that she is free to choose a mechanism where (NG^l) is violated—an uninformed low-ability type- l analyst may find it in her best interest to *act* as though informed and simply manufacture news even though her uninformed high-skill type- h counterpart finds it optimal to remain silent and wait for future learning opportunities.

Finally, note that U_0^θ , as defined in (5), corresponds to the type- θ analyst’s ex ante expected probability of being retained. Therefore, we can write the fund manager’s screening problem as

$$\begin{aligned} W^* &= \max_{(p_t, q_t)_{t=1}^T, p_\infty} \{ \rho U_0^h - (1 - \rho)U_0^l c \} \\ \text{s.t., for all } t &= 1, \dots, T, \\ &(\text{ND}), (\text{NM}), (\text{NG}^h), \\ &0 \leq p_t, q_t \leq 1, \text{ and } 0 \leq p_\infty \leq 1. \end{aligned} \quad (\mathcal{P})$$

We are now in a position to describe the manager’s optimal retention rule.¹⁰

THEOREM 3. *Suppose $\beta \in (\underline{\beta}, \bar{\beta})$. The mechanism that maximizes the fund manager’s payoff from screening W is also the optimal mechanism that jointly maximizes the payoff $\Pi(V, W)$ from information and screening. It is characterized by a cutoff period t^* such that:*

- *If the analyst correctly reports ω in period t , she is retained with probability*

$$p_t = \begin{cases} 1 & \text{if } t \leq t^*, \\ (1 + \alpha_l(\gamma n - 1))^{t^* - t} & \text{if } t > t^*. \end{cases}$$

- *If the analyst incorrectly reports ω in period t , she is retained with probability*

$$q_t = \begin{cases} 1 - \left(1 - \frac{\alpha_l(\gamma n - 1)}{n-1}\right)^{t^* - t} & \text{if } t \leq t^*, \\ 0 & \text{if } t > t^*. \end{cases}$$

¹⁰We defer the proof of [Theorem 3](#) to [Appendix B](#).

- If the analyst never reports a state ω (i.e., only reports null signals), she is retained with probability

$$p_\infty = \frac{1}{n} (1 + \alpha_l(\gamma n - 1))^{t^* - T}.$$

This result shows that, in the optimal mechanism, the principal rewards the arrival of earlier information. Qualitatively, the nature of this reward takes a different form before and after the cutoff period t^* . If the agent makes a recommendation before t^* , she is retained (with certainty) if her advice is ex post optimal (i.e., her reported state correctly matches the outcome) and with positive probability that decreases over time if the recommendation underperforms. If the agent instead makes a recommendation after t^* , she is only retained (with declining probabilities over time) when that recommendation correctly matches the eventual outcome.

This property also provides an interesting contrast to the first-best optimal retention rule from [Theorem 1](#). Since the analyst *cannot* lie in the benchmark environment, the only screening-relevant information is when information arrives, not how it performs. Thus, the principal is maximally forgiving and does not hold the analyst responsible for unhedgeable risks that lead the final outcome to differ from the underlying state. In the full problem, however, forgiveness is easily manipulated and provides incentives for guessing. The principal therefore *must* penalize these unhedgeable outcomes—despite the agent’s honest provision of information—to provide incentives for patience in information acquisition. Of course, the extent of such penalties is tempered by the fact that even high-skilled analysts’ best recommendations are also subject to this risk.

Finally, it is critical to point out that the retention probabilities described in [Theorem 3](#) satisfy the unskilled type- l analyst’s no-guessing constraint (NG^l). But because a skilled analyst is more likely to observe an informative signal after waiting than is an unskilled one, the type- h agent is *strictly* willing to wait whenever the type- l agent is willing to do so—if constraints (NG^l) are satisfied, then so are constraints (NG^h). This implies that the mechanism described above is fully incentive compatible for *both* types, and the principal receives *exactly* the same information as in the first-best problem where all learning is public. This observation immediately yields [Theorem 2](#): the fund attains its first-best value of information V^{FB} .

5. DISCUSSION AND IMPLICATIONS

This paper is the first that attempts to characterize optimal contracts for financial analysts. One possible reason for this is that the bulk of the research on analysts concentrates (largely due to the availability of public data) on the sell side. Sell-side analysts have a variety of conflicts of interest related to the public nature of their forecasts, which significantly complicates the modeling of their competing incentives. By contrast, one of our important insights is that the behavior and incentives of buy-side analysts can be more cleanly modeled, thereby opening the door for new theoretically driven empirical analyses of their optimal compensation structure. To the best of our knowledge, such empirical analyses have not been conducted on the buy side; however, our model offers several testable implications that can readily be taken to within-firm data from fund management companies.

We first observe that the optimal contract is not deterministic; the likelihood that the agent is retained decreases over time and can be interior. This contrast to the first-best highlights the role that adverse selection plays in our model. When the agent cannot strategically distort her information, the principal’s payoff from type separation (2) is linear in her retention decision, and so randomization has no value.

When the agent can engage in strategic misreporting, the optimal contract still induces truthful reporting, but this has to be *incentivized*. The only way the principal can provide the appropriate incentives is by conditioning the retention decision on the final realized outcome r and by distorting the retention probability. This finding is testable using data on the performance of analyst recommendations and promotion or retention outcomes; likewise, interpreting the scope of an analyst’s coverage responsibilities as a proxy for the probability of retention allows an additional test of this implication.

The optimal contract also shows that the career consequences of a given forecast depend not just on its content, but also on its timing. The optimal probability of retention following a forecast (whether proven ex post to be correct or not) is decreasing in the time t it is made. While this is intuitive, the underlying economics driving the result are subtle. As shown in the solution to the public signal benchmark, later recommendations are more likely to arise from the unskilled analyst, and so the manager would like to punish late recommendations regardless of outcome. However, a substantial increase in the likelihood of termination from period t to period $t + 1$ will lead an as-yet uninformed analyst to guess in period t . Therefore, the only way to penalize late recommendations without destroying the quality of the analyst’s reported information is to lower it *gradually* over time (as can be seen from the explicit closed-forms in [Theorem 3](#)). Note that our model is able to capture this feature precisely because we permit the manager to use stochastic contracts; this generality is often disallowed in dynamic mechanism design problems as it can significantly complicate the analysis.¹¹

Additionally, recall from [Theorem 2](#) that the optimal contract incentivizes both the high- and low-ability analyst to reveal signals truthfully. Both the high- and low-ability analyst’s reports are equally likely to match (or mismatch) the public outcome, so that does not reveal any information about the analyst’s true underlying ability. Nevertheless, the analyst is punished if the reported signal and the eventual outcome do not match; this is because, unlike the first best, the contract must provide incentives for the agent to report truthfully, and this targeted punishment is the most effective way to prevent guessing. This difference is perhaps the most stark when the analyst reports a signal after t^* ; in this case, the analyst is terminated for sure if the final outcome does not match but is retained with positive probability when no signal arrives at all!¹² The conclusion that both forecast accuracy and forecast timing should affect termination decisions also manifests in the evaluation of sell-side analysts. [Clarke and Subramanian \(2006\)](#) show that there is a significantly negative relationship between the probability of future termination of security analysts and the error with which they forecast end-of-quarter earnings. Additionally, they show that recent performance has a more significant impact on the probability of future termination.

We end this section by suggesting one way in which our results can be tested with data on buy-side analysts. Analyst recommendations can be separated into two sets: one consisting of “safe” recommendations that mirror consensus sell-side recommendations, and another containing contrarian recommendations that differ from the consensus or involving assets for which no consensus exists. In terms of our model, the latter corresponds to signal reports whereas the former can be identified as not reporting a signal. The timing of the recommendation can be computed with respect to relevant events like earnings announcement for the asset or disclosure deadlines for the fund. The model predicts consistent variation in the likelihood of termination (or, for some firms, promotion) related to the timing and forecast error

¹¹For instance, see [Courty and Li \(2000\)](#), [Krähmer and Strausz \(2011\)](#), or [Boleslavsky and Said \(2013\)](#), who rely on sufficient conditions to ensure the optimality of deterministic contracts.

¹²This would remain the case even if the high-ability analyst was less likely to mismatch than her low-skill counterpart, as the fund manager would still need to disincentivize guessing.

for recommendations in the second group and the proportion of recommendations that fall in the first group.

6. EXTENSIONS

In the previous sections, we made certain assumptions to simplify the exposition. However, the results and intuitions discussed above extend directly to more general environments. In this section, we present two extensions of our results: first, to the case of fully revealing outcomes; and second, to the asymmetric case where states and signals are not uniformly distributed.

6.1. FULLY REVEALING OUTCOMES

We assumed throughout our analysis in Sections 3 and 4 that $\gamma \in (\frac{1}{n}, 1)$, so that the public outcome was informative about the true underlying state, but not perfectly so. While we view this to be the more realistic assumption, there are some contexts where the volume of information that is available ex post may in fact permit perfect identification of the underlying state, and therefore the assumption that $\gamma = 1$ is more appropriate.

Our first main result (Theorem 2) continues to hold when $\gamma = 1$: the fund manager faces no loss in the quality of information she can obtain despite facing a dynamic adverse selection problem. The key contrast is with respect to the characterization of the manager's optimal retention mechanism in Theorem 3. In particular, note that when $\gamma = 1$, incentivizing the analyst to truthfully report the state when she learns it (instead of misreporting it as some other state) becomes trivial: the eventual outcome always perfectly reveals the true state, and so bad advice *always* corresponds to a misreported state or a guess. In such cases, the principal is free to maximally punish the analyst. This yields the following optimal contract:

THEOREM 4. *Suppose $\beta \in (\underline{\beta}, \bar{\beta})$. The optimal mechanism is such that:*

- *The analyst is never retained if her report and the eventual outcome do not match, so that $q_t = 0$ for all $t = 1, \dots, T$.*
- *There is a cutoff period $t^* < \bar{k}$ such that, if the analyst correctly matches the outcome in period t , she is retained with probability*

$$p_t = \begin{cases} 1 & \text{if } t \leq t^*, \\ (1 + \alpha_l(n-1))^{t^*-t} & \text{if } t > t^*. \end{cases}$$

- *The analyst is retained with probability $p_\infty = \frac{1}{n}p_T > 0$ if she never reports any information.*

This contract yields the fund manager her first-best value of information V^{FB} .

6.2. ASYMMETRIC STATES AND SIGNALS

A helpful assumption in the analysis above was that all states were equally likely. In practice, however, that is typically not the case. We might expect, for example, that a particular asset is more likely to appreciate in value over a given time horizon than to depreciate, or that a new firm's product is likely to experience costly production disruptions that negatively impact its profitability and viability. Our model can easily accommodate such asymmetry, and indeed delivers some interesting additional characteristics of the

optimal contract. We restrict attention to two states in order to illustrate the impact of asymmetry in a relatively parsimonious fashion.

We assume that there are two states of the world $\omega \in \{a, b\}$. The commonly known prior belief the state is ω given by $\pi_\omega \in (0, 1)$, where $\pi_a + \pi_b = 1$. In addition, we assume that the probability the eventual outcome matches the true state is $\gamma_\omega \in (\pi_\omega, 1)$ for each $\omega \in \{a, b\}$. All other elements of the model and environment remain as described in [Section 1](#).

Unlike in a fully symmetric environment, the fund manager may offer a mechanism that conditions the retention decision not only on whether the analyst's report matches the eventual outcome, but also the specific content of that report. In particular, the manager chooses probabilities

$$(p_t^\omega, q_t^\omega)_{t=1}^T \text{ and } p_\infty^\omega \text{ for each } \omega \in \{a, b\}$$

such that an analyst first reporting $s_t = \omega$ in period t is retained with probability p_t^ω if the eventual outcome is $r = \omega$ and with probability q_t^ω otherwise; and that the analyst is retained with probability p_∞^ω if the eventual outcome is $r = \omega$ and the analyst never reports observing a non-null signal $s_t \neq \phi$.

Importantly, observe that the first-best contract in this asymmetric environment remains identical to that described in [Theorem 1](#). This is because what has changed is the distribution of signals and not their rate of arrival. Since the principal can observe these signals in the first-best benchmark, the only information she uses to screen continues to be the time at which the analyst's first signal arrives.

Based on the above observation, one might be tempted to think that the optimal mechanism might be symmetric despite the underlying asymmetry in the environment. Recall that the distortions in the optimal mechanism relative to the first-best only arise in order to satisfy incentive constraints, and note that the optimal mechanism in [Theorem 3](#) continues to satisfy both the no-misreporting and no-delay constraints. However, the asymmetry in the environment implies that the uninformed analyst will prefer to guess the outcome (and not the state) that is unconditionally more likely.

For an uninformed analyst who has not yet received a signal, the likelihood that outcome $r = \omega$ occurs is given by

$$\eta_\omega := \pi_\omega \gamma_\omega + \pi_{\omega'} (1 - \gamma_{\omega'}), \text{ where } \omega' \neq \omega.$$

The no-misreporting condition implies that the analyst must be punished for outcomes that do not match her reported signal. Therefore, if the analyst were to guess, she would report the state corresponding to the outcome that she thinks is more likely to eventually arise; that is, she would report $s_t = \omega$ if $\eta_\omega > \eta_{\omega'}$. Now suppose the optimal mechanism were symmetric. This would imply that the value to guessing one of the two states would (generically) be strictly greater than the other. This in turn allows the principal to improve screening by distorting the mechanism corresponding to the less likely state back towards the first-best. We show that this can be done without inducing the agent to guess the other state, while also satisfying the remaining constraints and (strictly) raising the manager's payoff.

THEOREM 5. *Suppose $\beta \in (\underline{\beta}, \bar{\beta})$. The optimal mechanism is such that:*

- *There are cutoff periods t_ω^* for each $\omega = a, b$ such that $p_t^\omega = 1$ and $q_t^\omega > 0$ for $t < t_\omega^*$; $p_t^\omega = 1$ and $q_t^\omega \geq 0$ for $t = t_\omega^*$; and $p_t^\omega < 1$ and $q_t^\omega = 0$ for $t > t_\omega^*$.*
- *If $\eta_\omega > \eta_{\omega'}$, then $t_\omega^* \leq t_{\omega'}^*$. Moreover, $p_t^\omega \leq p_t^{\omega'}$ and $q_t^\omega \leq q_t^{\omega'}$ for all t .*

- *The analyst is retained with probability $p_\infty^\omega = \eta_\omega p_T^\omega + (1 - \eta_\omega) q_T^\omega > 0$ if she never reports any information and the eventual outcome is $r = \omega$.*

This contract yields the fund manager her first-best value of information V^{FB} .

The key novel economic implication of [Theorem 5](#) is that the analyst is rewarded for riskier recommendations. This reward is both in the form of a greater likelihood of retention when her advice pans out *and* a lower likelihood of termination when it does not. The optimal contract ensures that, despite these asymmetric rewards, the analyst only offers the risky advice when she is certain that it is the best course of action. This result finds support from [Clarke and Subramanian \(2006\)](#), who show that analysts make bolder forecasts when they face lower probabilities of future termination.

The complete analysis leading to the above result (which we defer to [Appendix D](#)) proceeds in a fashion similar to that of [Section 4](#). One key difference, however, is that the set of constraints an optimal mechanism must satisfy is doubled: an analyst’s willingness to truthfully and immediately report the state now depends directly on which state is observed. We can still argue, however, that these constraints must *always* hold for both the high- and low-skill analyst, and so the fund manager extracts the same information from the analyst as in the first-best world where signals are public; that is, our first main result ([Theorem 2](#)) continues to hold.

7. CONCLUDING REMARKS

Our dynamic adverse selection model captures key features of the relationship between buy-side analysts and their fund managers. Reputationally motivated analysts have career concerns and wish to appear competent, while profit-oriented managers want to maximize the quality of information, both in the short run (by eliciting accurate advice) and in the long run (by employing and retaining only high-skill analysts). We show that the manager’s optimal incentive contract maximizes the value of information by incentivizing all analysts, regardless of their ability, to report new information immediately and accurately. The optimal contract rewards both speed and accuracy, but does not punish their absence too harshly. In particular, late-arriving information and admissions of uncertainty are rewarded, though less generously than early-arriving advice.

We conclude by suggesting some avenues for further inquiry and investigation. One natural candidate for generalization is the all-or-nothing information structure we employ, as it rules out the possibility of forecast revisions; indeed, [Boulland, Ornthanalai, and Womack \(2022\)](#) provide evidence that lower-skilled sell-side analysts revise their forecasts more frequently than higher-skilled analysts. Accounting for this property requires informative signals that are not perfectly revealing. For instance, analysts might differ on two dimensions: the rate at which information arrives and the quality or precision of their information once it does. Another possibility is to examine environments with costly information acquisition and endogenously determined quality of analyst information. We hope to explore these and other related issues in future research.

A. OMITTED RESULTS AND PROOFS

PROOF OF THEOREM 1. Suppose first that $\Delta_\infty := \beta(1 - \alpha_h)^T - (1 - \alpha_l)^T \geq 0$, or equivalently, that

$$\beta \geq \left(\frac{1 - \alpha_l}{1 - \alpha_h} \right)^T =: \bar{\beta}.$$

Then for any $k = 1, \dots, T$, we have

$$\begin{aligned} \Delta_k &:= \beta\alpha_h(1 - \alpha_h)^{k-1} - \alpha_l(1 - \alpha_l)^{k-1} \geq \left(\frac{1 - \alpha_l}{1 - \alpha_h} \right)^T \alpha_h(1 - \alpha_h)^{k-1} - \alpha_l(1 - \alpha_l)^{k-1} \\ &= (1 - \alpha_l)^{k-1} \left(\alpha_h \left(\frac{1 - \alpha_l}{1 - \alpha_h} \right)^{T-k+1} - \alpha_l \right) > 0, \end{aligned}$$

where the final inequality holds because $0 < \alpha_l < \alpha_h < 1$. Thus, if $\Delta_\infty \geq 0$, $\Delta_k > 0$ for all $k = 1, \dots, T$ and the principal retains the agent, regardless of her signals.

On the other hand, suppose that $\beta < \bar{\beta}$, so that $\Delta_\infty < 0$. The first-best does not retain the agent if an informative signal *never* arrives, but *will* retain her if the arrival time k is such that $\Delta_k \geq 0$, or equivalently, whenever $\beta\alpha_h(1 - \alpha_h)^{k-1} \geq \alpha_l(1 - \alpha_l)^{k-1}$. Taking logs of both sides and rearranging yields the condition

$$\ln \left(\frac{\beta\alpha_h}{\alpha_l} \right) \geq (k - 1) \ln \left(\frac{1 - \alpha_l}{1 - \alpha_h} \right).$$

Since $0 < \alpha_l < \alpha_h < 1$, the logarithm on the right-hand-side is positive, and so the inequality above can be rewritten as $k \leq \bar{k}$, where \bar{k} is as defined in the statement of [Theorem 1](#).

Note, however, that if

$$\beta \leq \frac{\alpha_l}{\alpha_h} := \underline{\beta},$$

the inequality above implies that $\bar{k} \leq 1$, and so the principal finds it optimal to never retain the agent, regardless of whether or when an informative signal arrives. \blacksquare

LEMMA A.1. *There is a mechanism that maximizes type separation W that does not depend on the reported type. Specifically, for any incentive compatible direct mechanism $\bar{\chi}$, there is an indirect mechanism χ with the following properties:*

1. *the separation induced by χ is (weakly) higher than the separation induced by $\bar{\chi}$;*
2. *the type- h agent has an incentive to report her signals truthfully; and*
3. *the type- l agent is free to misreport her signals optimally.*

PROOF. Fix any incentive compatible direct revelation mechanism $\bar{\chi}(\theta, s^T, r)$ that induces type separation \bar{W} , and define the alternative mechanism $\chi'(s^T, r)$ by

$$\chi'(s^T, r) := \bar{\chi}(h, s^T, r) \text{ for all } s^T \in \mathcal{S}^T \text{ and all } r \in \Omega.$$

Denote by $\mu(\tilde{s}^T | s^T, \sigma)$ the probability that an agent who observes signals s^T and follows strategy $\sigma \in \Sigma$ reports the sequence \tilde{s}^T , where Σ is the set of all dynamic reporting strategies adapted to the signal process

(as defined in Section 1.3). The separation induced by χ' is then

$$W' = \rho \sup_{\sigma^h \in \Sigma} \left\{ \sum_{(s^T, r)} \Pr(s^T, r|h) \sum_{\tilde{s}^T} \mu(\tilde{s}^T | s^T, \sigma^h) \bar{\chi}(h, \tilde{s}^T, r) \right\} \\ - (1 - \rho)c \sup_{\sigma^l \in \Sigma} \left\{ \sum_{(s^T, r)} \Pr(s^T, r|l) \sum_{\tilde{s}^T} \mu(\tilde{s}^T | s^T, \sigma^l) \bar{\chi}(h, \tilde{s}^T, r) \right\},$$

as both the type- h and type- l agents are free to optimize their signal reporting. Note, however, that incentive compatibility of the original mechanism implies that the type- h agent finds truthful reporting of signals to be optimal, implying that

$$W' = \rho \sum_{(s^T, r)} \Pr(s^T, r|h) \bar{\chi}(h, s^T, r) - (1 - \rho)c \sup_{\sigma^l \in \Sigma} \left\{ \sum_{(s^T, r)} \Pr(s^T, r|l) \sum_{\tilde{s}^T} \mu(\tilde{s}^T | s^T, \sigma^l) \bar{\chi}(h, \tilde{s}^T, r) \right\}.$$

In addition, incentive compatibility of the original mechanism implies that forcing the type- l agent to misreport her initial type and then re-optimize reduces her expected utility; this implies that

$$W' \geq \rho \sum_{(s^T, r)} \Pr(s^T, r|h) \bar{\chi}(h, s^T, r) - (1 - \rho)c \sum_{(s^T, r)} \Pr(s^T, r|l) \bar{\chi}(l, s^T, r) =: \bar{W}.$$

Thus, since type separation W is *decreasing* in the utility of the type- l agent, the new mechanism $\chi'(\cdot, r)$ improves this dimension of the principal's objective. As $\bar{\chi}(\theta, \cdot, r)$ was an arbitrary incentive compatible mechanism, there exists a mechanism that maximizes type separation W that solicits only the agent's signals and incentivizes the type- h analyst to report them truthfully. ■

PROOF OF THEOREM 2. The optimal contract described in Theorem 3 satisfies the incentive compatibility constraints (ND), (NM), (NG^h), and (NG^l), so both skilled and unskilled analysts reveal all new information immediately and truthfully, and neither type fabricates information. Therefore, the fund manager observes exactly the same information as in the first-best, and so the value of information generated by the analyst is V^{FB} . ■

B. PROOF OF THEOREM 3

Recall that the fund manager's objective function in (\mathcal{P}) is written in terms of the analyst's expected continuation values U_t^θ . Notice, however, that these objects' definition in (5) implicitly assumes that an uninformed agent who chooses to wait another period—in hopes of obtaining an informative signal and learning the state—reports any such signal *immediately* and *honestly*. Of course, this is precisely the behavior that is guaranteed by the satisfaction of the no-delay and no-misreporting constraints (ND) and (NM). Since these conditions are already incorporated into the objective function, we can work with the simpler relaxed problem where we drop these constraints and verify their satisfaction ex post. Thus,

we will examine—after factoring out the $(1 - \rho)c$ term—the relaxed problem

$$\begin{aligned} & \max_{(p_t, q_t)_{t=1}^T, p_\infty} \{ \beta U_0^h - U_0^l \} \\ & \text{s.t., for all } t = 1, \dots, T, \\ & \text{(NG}^h), 0 \leq p_t, q_t \leq 1 \text{ and } 0 \leq p_\infty \leq 1. \end{aligned} \tag{P'}$$

But recall that U_t^θ , as defined in (5), embeds a maximization term that reflects the analyst's ability to strategically preempt their learning process and guess the state of the world. Constraint (NG^h) guarantees that the high-skill type- h analyst does not wish to do so, but the possibility remains that the slower-learning type- b analyst may in fact find guessing worthwhile. In particular, there may exist some period \bar{t} such that type l prefers to guess in period \bar{t} if she remains uninformed at that time. We show, however, that this is suboptimal: it is without loss to impose the type- l no-guessing constraint (NG^l) on the relaxed problem (P').

LEMMA B.1. *There exists a solution to (P') where the type- l no-guessing constraint (NG^l) is satisfied.*

PROOF. Suppose that $\{(p_t, q_t)_{t=1}^T, p_\infty\}$ solves problem (P'), so that the type- h no-guessing constraints (NG^h) are satisfied, and there is some period $\bar{t} \leq T$ such that an uninformed type- l agent *strictly* prefers to guess at period \bar{t} instead of waiting. The uninformed type- l agent will always guess at period \bar{t} , and so in subsequent periods the principal can only be facing a type- h agent. (Recall from the definition of U_t^θ that the informed agent—of either type—never delays or misreports.)

Suppose first that $p_\infty < 1$, and consider $\{(p'_t, q'_t)_{t=1}^T, p'_\infty\}$ with $p'_\infty = p_\infty + \varepsilon$, $p'_t = p_t$, and $q'_t = q_t$ for all $t \leq T$. Since the type- l agent strictly prefers guessing at \bar{t} in the original mechanism, she continues to do so in the new mechanism for $\varepsilon > 0$ small enough. On the other hand, $p'_\infty > p_\infty$ implies waiting is more attractive for type h , so the no-guessing constraints (NG^h) remain satisfied.

Suppose instead that $p_\infty = 1$, and note that this implies that there exists $t > \bar{t}$ such that $\gamma p_t + (1 - \gamma)q_t < 1$. (If not, $U_{\bar{t}}^l = 1$ and so (NG^l) cannot be strictly violated.) Let \hat{t} denote the *latest* such period (so $\gamma p_{\hat{t}+1} + (1 - \gamma)q_{\hat{t}+1} = 1$ and $U_{\hat{t}}^h = 1$), and consider a perturbation that increases $\gamma p_{\hat{t}} + (1 - \gamma)q_{\hat{t}}$ by $\varepsilon > 0$. For ε small enough, the type- l agent still strictly prefers to guess at \bar{t} , while (since $U_{\hat{t}}^h = 1$) the type- h agent still prefers to truthfully report being uninformed.

Since only type- h agents report being uninformed at period \bar{t} and continue in the mechanism, increasing the probabilities of being retained after period \bar{t} strictly increases the principal's payoff, violating the conjectured optimality of the original solution. \diamond

Since the type- h agent has a higher probability of observing an informative signal when waiting than does a type- l agent, the type- h agent is *strictly* willing to wait whenever the type- l agent is willing to do so; that is, constraints (NG^h) are implied by (NG^l). Moreover, when these constraints are satisfied, we can write the payoff of an uninformed type- θ analyst in period t as

$$\begin{aligned} U_t^\theta &= \alpha_\theta (\gamma p_{t+1} + (1 - \gamma)q_{t+1}) + (1 - \alpha_\theta)U_{t+1}^\theta \\ &= \sum_{\tau=t+1}^T \alpha_\theta (1 - \alpha_\theta)^{\tau-t-1} (\gamma p_\tau + (1 - \gamma)q_\tau) + (1 - \alpha_\theta)^{T-t} p_\infty. \end{aligned}$$

Recalling that, for all $t = 1, \dots, T$, we had

$$\Delta_t := \beta\alpha_h(1 - \alpha_h)^{t-1} - \alpha_l(1 - \alpha_l)^{t-1} \text{ and } \Delta_\infty := \beta(1 - \alpha_h)^T - (1 - \alpha_l)^T,$$

we can rewrite problem (\mathcal{P}') as

$$\begin{aligned} & \max_{(p_t, q_t)_{t=1}^T, p_\infty} \left\{ \sum_{t=1}^T \Delta_t(\gamma p_t + (1 - \gamma)q_t) + \Delta_\infty p_\infty \right\} \\ & \text{s.t., for all } t = 1, \dots, T, \\ & \quad (\text{NG}^l), 0 \leq p_t, q_t \leq 1 \text{ and } 0 \leq p_\infty \leq 1. \end{aligned}$$

We relax this problem further by dropping the feasibility constraints that lower-bound p_t and p_∞ , as well as those that upper-bound p_∞ and q_t . Multiplying both sides of the period- t no-guess constraint (NG^l) by $(1 - \alpha_l)^t$, we arrive at a relaxed primal problem in standard form:

$$\begin{aligned} & \max_{(p_t, q_t)_{t=1}^T, p_\infty} \left\{ \sum_{t=1}^T \Delta_t(\gamma p_t + (1 - \gamma)q_t) + \Delta_\infty p_\infty \right\} \\ & \text{s.t., for all } t = 1, \dots, T, \\ & (1 - \alpha_l)^t \left(\frac{1}{n}p_t + \frac{n-1}{n}q_t \right) - \sum_{\tau=t+1}^T \alpha_l(1 - \alpha_l)^{\tau-1}(\gamma p_\tau + (1 - \gamma)q_\tau) - (1 - \alpha_l)^T p_\infty \leq 0, \\ & p_t \leq 1, \text{ and } q_t \geq 0. \end{aligned} \tag{\mathcal{R}}$$

This corresponds to the following dual problem, where λ_t is the dual variable for the feasibility constraint on p_t and μ_t is the dual variable for the type- l agent's period- t no-guessing constraint:

$$\begin{aligned} & \min_{(\lambda_t, \mu_t)_{t=1}^T} \left\{ \sum_{t=1}^T \lambda_t \right\} \\ & \text{s.t., for all } t = 1, \dots, T, \\ & \lambda_t + \frac{1}{n}(1 - \alpha_l)^t \mu_t - \alpha_l(1 - \alpha_l)^{t-1} \gamma \sum_{\tau=1}^{t-1} \mu_\tau = \gamma \Delta_t, \\ & \frac{n-1}{n}(1 - \alpha_l)^t \mu_t - \alpha_l(1 - \alpha_l)^{t-1}(1 - \gamma) \sum_{\tau=1}^{t-1} \mu_\tau \geq (1 - \gamma)\Delta_t, \\ & - (1 - \alpha_l)^T \sum_{\tau=1}^T \mu_\tau = \Delta_\infty, \\ & \lambda_t \geq 0, \text{ and } \mu_t \geq 0. \end{aligned} \tag{\mathcal{D}}$$

Notice that, because we have dropped the nonnegativity constraints on p_t and p_∞ in the relaxed primal problem (\mathcal{R}) , their complementary constraints in the dual (\mathcal{D}) are equalities.

LEMMA B.2. *In any solution, the type- l agent's no-guessing constraints (NG ^{l}) must bind in every period.*

PROOF. Let $\{(p_t, q_t)_{t=1}^T, p_\infty\}$ be a candidate solution to the relaxed primal problem (\mathcal{R}), and suppose that the type- l agent's no-guessing constraint (NG ^{l}) is slack for some period \bar{t} .

Let $\{(\lambda_t, \mu_t)_{t=1}^T\}$ be the nonnegative solution to the dual problem (\mathcal{D}) that complements the candidate primal solution. Complementary slackness implies that we must have $\mu_{\bar{t}} = 0$, and we can write the period- \bar{t} dual constraint corresponding to $q_{\bar{t}}$ as

$$\alpha_l(1 - \alpha_l)^{\bar{t}-1} \sum_{\tau=1}^{\bar{t}-1} \mu_\tau \leq -\Delta_{\bar{t}}.$$

Suppose first that $\bar{t} < T$. The dual constraint complemented by $p_{\bar{t}+1}$ can be written as

$$\begin{aligned} \lambda_{\bar{t}+1} + \frac{1}{n}(1 - \alpha_l)^{\bar{t}+1} \mu_{\bar{t}+1} &= \gamma \left(\Delta_{\bar{t}+1} + \alpha_l(1 - \alpha_l)^{\bar{t}} \sum_{\tau=1}^{\bar{t}} \mu_\tau \right) = \gamma \left(\Delta_{\bar{t}+1} + \alpha_l(1 - \alpha_l)^{\bar{t}} \sum_{\tau=1}^{\bar{t}-1} \mu_\tau \right) \\ &\leq \gamma (\Delta_{\bar{t}+1} - (1 - \alpha_l)\Delta_{\bar{t}}) = \gamma\beta\alpha_h(1 - \alpha_h)^{\bar{t}-1}(\alpha_l - \alpha_h) < 0, \end{aligned}$$

where the second equality follows from $\mu_{\bar{t}} = 0$ and the final inequality from the fact that $\alpha_h > \alpha_l$. Of course, this yields a contradiction, since $\lambda_{\bar{t}+1}$ and $\mu_{\bar{t}+1}$ are both nonnegative.

If, instead, $\bar{t} = T$, note that the dual constraint complemented by p_∞ can be written as

$$\begin{aligned} 0 &= \Delta_\infty + (1 - \alpha_l)^T \sum_{\tau=1}^{\bar{t}} \mu_\tau = \Delta_\infty + (1 - \alpha_l)^T \sum_{\tau=1}^{\bar{t}-1} \mu_\tau \\ &\leq \Delta_\infty - \frac{1 - \alpha_l}{\alpha_l} \Delta_{\bar{t}} = \beta(1 - \alpha_h)^{T-1} \left((1 - \alpha_h) - \frac{\alpha_h}{\alpha_l}(1 - \alpha_l) \right) = \beta(1 - \alpha_h)^{T-1} \left(1 - \frac{\alpha_h}{\alpha_l} \right) < 0, \end{aligned}$$

where the second equality follows from $\mu_{\bar{t}} = 0$ and the final inequality from the fact that $\alpha_h > \alpha_l$. This is, of course, another contradiction. \diamond

LEMMA B.3. *In any solution $\{(p_t, q_t)_{t=1}^T, p_\infty\}$ to the relaxed primal (\mathcal{R}), we have $q_t = 0$ whenever $p_t < 1$.*

PROOF. Suppose not; that is, suppose that there exists a solution $\{(p_t, q_t)_{t=1}^T, p_\infty\}$ with $1 > p_{\bar{t}}$ and $q_{\bar{t}} > 0$ for some \bar{t} . Consider a perturbation to this solution that increases $p_{\bar{t}}$ by $\varepsilon > 0$ and decreases $q_{\bar{t}}$ by $\frac{\gamma}{1-\gamma}\varepsilon$. For sufficiently small ε , this perturbation is feasible. Moreover, it does not affect the objective value of the relaxed primal problem (\mathcal{R}) or any of the period- t no-guessing constraints (NG ^{l}) for any $t \neq \bar{t}$. But since $\gamma > \frac{1}{n}$, this perturbation relaxes the period- \bar{t} no-guessing constraint. This of course contradicts Lemma B.2, which showed that the no-guessing constraints (NG ^{l}), for *all* $t = 1, \dots, T$, are binding in any solution. \diamond

Recalling that the type- l continuation value when uninformed in period- t is

$$U_t^l = \sum_{\tau=t+1}^T \alpha_\theta (1 - \alpha_\theta)^{\tau-t-1} (\gamma p_\tau + (1 - \gamma) q_\tau) + (1 - \alpha_\theta)^{T-t} p_\infty,$$

the binding type- l no-guessing constraint implies that

$$U_t^l = \frac{1}{n} p_t + \frac{n-1}{n} q_t$$

for all $t = 1, \dots, T$. Moreover, [Lemma B.3](#) above implies that this can be rewritten as

$$p_t = \min \{1, nU_t^l\} \text{ and } q_t = \frac{nU_t^l - p_t}{n-1}.$$

It is easy to verify that U_t^l is strictly decreasing in t . This implies that there is a cutoff period t^* such that the optimal contract $\{(p_t^*, q_t^*)_{t=1}^T, p_\infty^*\}$ is such that $p_t^* = 1$ and $q_t^* > 0$ for $t < t^*$; $p_{t^*}^* = 1$ and $q_{t^*}^* \geq 0$; $p_t^* < 1$ and $q_t^* = 0$ for $t > t^*$; and $p_\infty^* = \frac{1}{n} p_T^* + \frac{n-1}{n} q_T^* \in (0, 1)$.

LEMMA B.4. *There is an optimal contract $\{(p_t^*, q_t^*)_{t=1}^T, p_\infty^*\}$ with $q_{t^*}^* = 0$.*

PROOF. The optimal contract properties described above are sufficient to pin down the dual variables that complement an optimal solution. Note that $q_t^* > 0$ for all $t < t^*$ implies that the dual program (\mathcal{D}) constraint complemented by q_t binds. Dividing through by $\frac{n-1}{n}(1 - \alpha_l)^t$, this can be written as

$$-\frac{\alpha_l(1 - \gamma)n}{(1 - \alpha_l)(n - 1)} \sum_{\tau=1}^{t-1} \mu_\tau + \mu_t = \frac{(1 - \gamma)n\Delta_t}{(1 - \alpha_l)^t(n - 1)}. \quad (\text{B.1})$$

In addition, $p_t^* < 1$ for all $t > t^*$ implies that $\lambda_t = 0$ and therefore that the dual program (\mathcal{D}) constraint complemented by p_t can be rewritten as

$$-\frac{\alpha_l \gamma n}{1 - \alpha_l} \sum_{\tau=1}^{t-1} \mu_\tau + \mu_t = \frac{\gamma n \Delta_t}{(1 - \alpha_l)^t}, \quad (\text{B.2})$$

where we have divided through by $\frac{1}{n}(1 - \alpha_l)^t$. Finally, we can rewrite the binding dual program (\mathcal{D}) constraint complemented by p_∞ as

$$\sum_{\tau=1}^T \mu_\tau = -\frac{\Delta_\infty}{(1 - \alpha_l)^T}. \quad (\text{B.3})$$

Jointly, equations (B.1), (B.2), and (B.3) form a system of T equations in the T unknowns $\{\mu_t\}_{t=1}^T$.

Subtracting the period- T equation (B.2) from equation (B.3) yields

$$\left(1 + \frac{\alpha_l \gamma n}{1 - \alpha_l}\right) \sum_{\tau=1}^{T-1} \mu_\tau = -\frac{\Delta_\infty + \gamma n \Delta_T}{(1 - \alpha_l)^T}.$$

Subtracting $(1 + \frac{\alpha_l \gamma n}{1 - \alpha_l})$ times the period- $(T - 1)$ equation (B.2) from this result then yields

$$\left(1 + \frac{\alpha_l \gamma n}{1 - \alpha_l}\right)^2 \sum_{\tau=1}^{T-2} \mu_\tau = -\frac{\Delta_\infty + \gamma n \Delta_T + (1 + \alpha_l(\gamma n - 1))\gamma n \Delta_{T-1}}{(1 - \alpha_l)^T},$$

where we make use of the fact that $1 + \frac{\alpha_l \gamma n}{1 - \alpha_l} = \frac{1 + \alpha_l(\gamma n - 1)}{1 - \alpha_l}$. Similarly, subtracting $(1 + \frac{\alpha_l \gamma n}{1 - \alpha_l})^2$ times the period- $(T - 2)$ equation (B.2) from the result above yields

$$\left(1 + \frac{\alpha_l \gamma n}{1 - \alpha_l}\right)^3 \sum_{\tau=1}^{T-3} \mu_\tau = -\frac{\Delta_\infty + \gamma n \Delta_T + (1 + \alpha_l(\gamma n - 1))\gamma n \Delta_{T-1} + (1 + \alpha_l(\gamma n - 1))^2 \gamma n \Delta_{T-2}}{(1 - \alpha_l)^T}.$$

Proceeding inductively in this fashion, we arrive at

$$\left(\frac{1 + \alpha_l(\gamma n - 1)}{1 - \alpha_l}\right)^{T-t^*} \sum_{\tau=1}^{t^*} \mu_\tau = -\frac{\Delta_\infty + \sum_{\tau=t^*+1}^T (1 + \alpha_l(\gamma n - 1))^{T-\tau} \gamma n \Delta_\tau}{(1 - \alpha_l)^T}. \quad (\text{B.4})$$

Thus, we can replace the original system of equations (B.1), (B.2), and (B.3) by (B.1), (B.2), and (B.4). It is easy to see that (after reordering the equations so that equation (B.4) precedes equations (B.2) for $t > t^*$) the system corresponds to a lower triangular matrix with non-zero entries on its diagonal. Therefore, there exists a unique solution $\{\mu_t^*\}_{t=1}^T$ to the system that can be found through forward substitution. Of course, such a solution also uniquely pins down the remaining dual variables λ_t^* for all $t \leq t^*$ (recalling that $\lambda_t^* = 0$ for all $t > t^*$) via the dual constraints complemented by p_t^* .

Now consider the dual constraint complemented by $q_{t^*}^*$, evaluated at the solution to the system of equations (B.1), (B.2), and (B.4):

$$\frac{n-1}{n}(1 - \alpha_l)^{t^*} \mu_{t^*}^* - \alpha_l(1 - \alpha_l)^{t^*-1}(1 - \gamma) \sum_{\tau=1}^{t^*-1} \mu_\tau^* \geq (1 - \gamma)\Delta_{t^*}.$$

This constraint is satisfied by assumption, as $\{(p_t^*, q_t^*)_{t=1}^T, p_\infty^*\}$ is optimal. If it holds strictly, then complementary slackness implies we *must* have $q_{t^*}^* = 0$ in the optimal contract. If, instead, the constraint binds, we can define another contract $\{(\hat{p}_t, \hat{q}_t)_{t=1}^T, \hat{p}_\infty\}$ constructed as follows:

- Let $\hat{U}_{t^*}^l := \frac{1}{n}$, and inductively define \hat{U}_t^l by

$$\hat{U}_t^l := \begin{cases} \hat{U}_{t+1}^l + \frac{\alpha_l(\gamma n - 1)}{n-1}(1 - \hat{U}_{t+1}^l) & \text{for all } t < t^*, \text{ working backwards from } t^* - 1, \text{ and} \\ \frac{1}{1 + \alpha_l(\gamma n - 1)} \hat{U}_{t-1}^l & \text{for all } t > t^*, \text{ working forwards from } t^* + 1. \end{cases}$$

Note that \hat{U}_t^l is strictly decreasing in t .

- For each $t = 1, \dots, T$, let $\hat{p}_t := \min\{1, n\hat{U}_t^l\}$ and $\hat{q}_t := \frac{n\hat{U}_t^l - \hat{p}_t}{n-1}$, and let $\hat{p}_\infty := \frac{1}{n}\hat{p}_T$.

Note that this contract satisfies the type- l no-guessing constraints (NG^{*l*}) by construction, and so is feasible in the relaxed primal problem (\mathcal{R}). Furthermore, $\hat{q}_t > 0$ for all $t < t^*$, $\hat{p}_t < 1$ for all $t > t^*$, and $\hat{p}_\infty \in (0, 1)$, implying that this contract complements the same dual solution $(\lambda_t^*, \mu_t^*)_{t=1}^T$ as the original

optimal contract; this implies that this construction (in which $\hat{q}_{t^*} = 0$) solves the relaxed primal problem (\mathcal{R}) . \diamond

We can further pin down the retention probabilities in the solution to (\mathcal{R}) by combining the above result with the binding type- l no-guessing constraints (NG^l) . In particular, for any $t = 1, \dots, T$, the definition of U_t^l and the binding no-guessing constraint imply, respectively, that

$$U_t^l = \alpha_l(\gamma p_{t+1}^* + (1 - \gamma)q_{t+1}^*) + (1 - \alpha_l)U_{t+1}^l \text{ and } U_t^l = \frac{1}{n}p_t^* + \frac{n-1}{n}q_t^*.$$

Moreover, since we can write $p_t^* = \min\{1, nU_t^l\}$ and $q_t^* = \frac{nU_t^l - p_t^*}{n-1}$, the two expressions above can be combined into the recurrence relation

$$U_t^l := \begin{cases} U_{t+1}^l + \frac{\alpha_l(\gamma n - 1)}{n-1}(1 - U_{t+1}^l) & \text{for all } t < t^*, \text{ and} \\ \frac{1}{1 + \alpha_l(\gamma n - 1)}U_{t-1}^l & \text{for all } t > t^*. \end{cases}$$

where we have made use of the fact that $p_t^* = 1$ for all $t < t^*$ and $q_t^* = 0$ for all $t > t^*$. Since $p_{t^*}^* = 1$ and $q_{t^*}^* = 0$, we have $U_{t^*}^l = \frac{1}{n}$, and it is therefore easy to see that the unique solution to this recurrence relation is

$$U_t^l := \begin{cases} 1 - \frac{n-1}{n} \left(1 - \frac{\alpha_l(\gamma n - 1)}{n-1}\right)^{t^* - t} & \text{for all } t \leq t^*, \text{ and} \\ \frac{1}{n} (1 + \alpha_l(\gamma n - 1))^{t^* - t} & \text{for all } t \geq t^*. \end{cases}$$

Thus, we have

$$p_t^* = \begin{cases} 1 & \text{for } t \leq t^*, \\ (1 + \alpha_l(\gamma n - 1))^{t^* - t} & \text{for } t > t^*; \end{cases} \quad q_t^* = \begin{cases} 1 - \left(1 - \frac{\alpha_l(\gamma n - 1)}{n-1}\right)^{t^* - t} & \text{for } t \leq t^*, \\ 0 & \text{for } t > t^*; \end{cases}$$

$$\text{and } p_\infty^* = \frac{1}{n} (1 + \alpha_l(\gamma n - 1))^{t^* - T}.$$

Notice that this contract is such that $1 \geq p_t^* > q_t^* \geq 0$ for all $t = 1, \dots, T$, and furthermore that both p_t^* and q_t^* are (weakly) decreasing in t . In addition, $\gamma p_T^* + (1 - \gamma)q_T^* > p_\infty^* \in (0, 1)$. Thus, the contract satisfies the no-delay constraints (ND) , the no-misreporting constraints (NM) , and the feasibility constraints from the (unrelaxed) problem (\mathcal{P}) ; that is, this contract solves the principal's original problem, and is therefore optimal. \blacksquare

C. PROOF OF THEOREM 4

Suppose that $\gamma = 1$. The relaxed primal and dual problems (\mathcal{R}) and (\mathcal{D}) can be rewritten as

$$\begin{aligned} & \max_{(p_t, q_t)_{t=1}^T, p_\infty} \left\{ \sum_{t=1}^T \Delta_t p_t + \Delta_\infty p_\infty \right\} \\ & \text{s.t., for all } t = 1, \dots, T, \\ & (1 - \alpha_l)^t \left(\frac{1}{n} p_t + \frac{n-1}{n} q_t \right) - \sum_{\tau=t+1}^T \alpha_l (1 - \alpha_l)^{\tau-1} p_\tau - (1 - \alpha_l)^T p_\infty \leq 0, \\ & p_t \leq 1, \text{ and } q_t \geq 0. \end{aligned} \tag{\mathcal{R}'}$$

and

$$\begin{aligned} & \min_{(\lambda_t, \mu_t)_{t=1}^T} \left\{ \sum_{t=1}^T \lambda_t \right\} \\ & \text{s.t., for all } t = 1, \dots, T, \\ & \lambda_t + \frac{1}{n} (1 - \alpha_l)^t \mu_t - \alpha_l (1 - \alpha_l)^{t-1} \sum_{\tau=1}^{t-1} \mu_\tau = \Delta_t, \\ & \frac{n-1}{n} (1 - \alpha_l)^t \mu_t \geq 0, \\ & - (1 - \alpha_l)^T \sum_{\tau=1}^T \mu_\tau = \Delta_\infty, \\ & \lambda_t \geq 0, \text{ and } \mu_t \geq 0. \end{aligned} \tag{\mathcal{D}'}$$

LEMMA C.1. $q_t = 0$ for all $t = 1, \dots, T$.

PROOF. Fix any t and notice that q_t does not appear in the objective of the relaxed primal (\mathcal{R}'), and only with a positive coefficient in the period- t no-guessing constraint. Thus, lowering q_t relaxes that constraint. In addition, the dual constraint that complements q_t is now redundant—the dual (\mathcal{D}') already constrains μ_t to be nonnegative. Thus, we can take $q_t = 0$ in any solution to (\mathcal{R}'). \diamond

LEMMA C.2. *There exists a period t^* such that type- l agent's no-guessing constraint is slack for all $t < t^*$ and binds for all $t \geq t^*$.*

PROOF. Let $\{(p_t, q_t)_{t=1}^T, p_\infty\}$ be a candidate solution to the relaxed primal problem (\mathcal{R}'), and suppose that the type- l agent's no-guessing constraint binds in some period t but is slack in period $t + 1$. (Note that this implies that $t + 1 \leq T$.) The binding constraint in period t can be written as

$$\frac{1}{n} (1 - \alpha_l)^t p_t - \alpha_l (1 - \alpha_l)^t p_{t+1} = \sum_{\tau=t+2}^T \alpha_l (1 - \alpha_l)^{\tau-1} p_\tau + (1 - \alpha_l)^T p_\infty,$$

while the slack constraint in period $t + 1$ can be written as

$$\frac{1}{n}(1 - \alpha_l)^{t+1}p_{t+1} < \sum_{\tau=t+2}^T \alpha_l(1 - \alpha_l)^{\tau-1}p_{\tau} + (1 - \alpha_l)^T p_{\infty}.$$

Combining these two expressions yields

$$\frac{1}{n}(1 - \alpha_l)^t p_t - \alpha_l(1 - \alpha_l)^t p_{t+1} > \frac{1}{n}(1 - \alpha_l)^{t+1} p_{t+1}, \text{ or, equivalently, } p_t > (1 + \alpha_l(n - 1))p_{t+1}.$$

Since we must have $p_t \leq 1$ in any solution, this implies that $p_{t+1} < 1$.

Let $\{(\lambda_t, \mu_t)_{t=1}^T\}$ be the nonnegative solution to the dual (\mathcal{D}') that complements the candidate relaxed primal solution. Complementary slackness implies that we must have $\lambda_{t+1} = \mu_{t+1} = 0$, and so we can write the period- $(t + 1)$ dual constraint corresponding to p_{t+1} as

$$\alpha_l(1 - \alpha_l)^t \sum_{\tau=1}^t \mu_{\tau} = -\Delta_{t+1}.$$

Suppose first that $t + 1 < T$. The dual constraint complemented by p_{t+2} can be written as

$$\begin{aligned} \lambda_{t+2} + \frac{1}{n}(1 - \alpha_l)^{t+2}\mu_{t+2} &= \Delta_{t+2} + \alpha_l(1 - \alpha_l)^{t+1} \sum_{\tau=1}^{t+1} \mu_{\tau} = \Delta_{t+2} + \alpha_l(1 - \alpha_l)^{t+1} \sum_{\tau=1}^t \mu_{\tau} \\ &\leq \Delta_{t+2} - (1 - \alpha_l)\Delta_{t+1} = \beta\alpha_h(1 - \alpha_h)^t(\alpha_l - \alpha_h) < 0, \end{aligned}$$

where the second equality follows from $\mu_{t+1} = 0$ and the final inequality from the fact that $\alpha_h > \alpha_l$. Of course, this yields a contradiction, since λ_{t+2} and μ_{t+2} are both nonnegative.

If, instead, $t + 1 = T$, note that the dual constraint complemented by p_{∞} can be written as

$$\begin{aligned} 0 &= \Delta_{\infty} + (1 - \alpha_l)^T \sum_{\tau=1}^{t+1} \mu_{\tau} = \Delta_{\infty} + (1 - \alpha_l)^T \sum_{\tau=1}^t \mu_{\tau} \\ &\leq \Delta_{\infty} - \frac{1 - \alpha_l}{\alpha_l} \Delta_{t+1} = \beta(1 - \alpha_h)^{T-1} \left((1 - \alpha_h) - \frac{\alpha_h}{\alpha_l}(1 - \alpha_l) \right) \\ &= \beta(1 - \alpha_h)^{T-1} \left(1 - \frac{\alpha_h}{\alpha_l} \right) < 0, \end{aligned}$$

where the second equality follows from $\mu_{t+1} = 0$ and the final inequality from the fact that $\alpha_h > \alpha_l$. This is, of course, another contradiction.

Thus, if the no-guessing constraint ever binds in any period t^* , it must bind in all future periods $t' > t^*$. To see that such a period t^* must exist, assume the contrary: that is, suppose that the no-guessing constraint is slack in all periods. This implies that we must have $\mu_t = 0$ for all $t = 1, \dots, T$. Since $\Delta_{\infty} < 0$, this violates the dual constraint complemented by p_{∞} . \diamond

Since the no-guessing constraint is slack for all $t < t^*$, complementary slackness implies that $\mu_t = 0$ for all $t < t^*$. Therefore, the dual constraint complemented by p_t , for any $t < t^*$, becomes

$$\lambda_t = \Delta_t.$$

Meanwhile, the dual constraint complemented by p_{t^*} becomes

$$\lambda_{t^*} + \frac{1}{n}(1 - \alpha_l)^{t^*} \mu_{t^*} = \Delta_{t^*}.$$

Of course, since $\lambda_{t^*} \geq 0$ and $\mu_{t^*} \geq 0$, this implies that we must have $\Delta_{t^*} \geq 0$. In addition, note that

$$\begin{aligned} \Delta_{\tau+1} - \Delta_\tau &= \beta\alpha_h[(1 - \alpha_h)^\tau - (1 - \alpha_h)^{\tau-1}] - \alpha_l[(1 - \alpha_l)^\tau - (1 - \alpha_l)^{\tau-1}] \\ &= \left(\frac{\alpha_l}{1 - \alpha_l}\right) \alpha_l(1 - \alpha_l)^\tau - \beta \left(\frac{\alpha_h}{1 - \alpha_h}\right) \alpha_h(1 - \alpha_h)^\tau \\ &< \left(\frac{\alpha_h}{1 - \alpha_h}\right) \alpha_l(1 - \alpha_l)^\tau - \beta \left(\frac{\alpha_h}{1 - \alpha_h}\right) \alpha_h(1 - \alpha_h)^\tau = - \left(\frac{\alpha_h}{1 - \alpha_h}\right) \Delta_{\tau+1}, \end{aligned}$$

where the strict inequality follows from the fact that $\alpha_h > \alpha_l$. Since $\Delta_{t^*} \geq 0$, this implies that $\lambda_t = \Delta_t > 0$ for all $t < t^*$. By complementary slackness, we therefore have $p_t = 1$ for all $t < t^*$.

On the other hand, since the no-guessing constraint binds for all $t \geq t^*$, we can write the period- T no-guess constraint as

$$\frac{1}{n}(1 - \alpha_l)^T p_T = (1 - \alpha_l)^T p_\infty, \text{ or, equivalently, } p_\infty = \frac{1}{n} p_T.$$

Likewise, for any $t \geq t^*$, we can subtract the binding period- $(t+1)$ no-guess constraint from the binding period- t no-guess constraint, which yields

$$\frac{1}{n}(1 - \alpha_l)^t p_t - \alpha_l(1 - \alpha_l)^t p_{t+1} = \frac{1}{n}(1 - \alpha_l)^{t+1} p_{t+1}, \text{ or, equivalently, } p_{t+1} = \frac{1}{1 + \alpha_l(n - 1)} p_t.$$

Working inductively, this implies that, for all $t > t^*$,

$$p_t = (1 + \alpha_l(n - 1))^{t^* - t} p_{t^*} \text{ and } p_\infty = \frac{(1 + \alpha_l(n - 1))^{t^* - T}}{n} p_{t^*}.$$

Since $p_{t^*} \leq 1$, this implies that $p_t < 1$ for all $t > t^*$, and so (by complementary slackness) we must also have $\lambda_t = 0$ for all $t > t^*$.

The only remaining variable to determine is p_{t^*} . In particular, we must determine whether the constraint $p_{t^*} \leq 1$ binds or is slack.

LEMMA C.3. *We must have $p_{t^*} = 1$ in an optimal contract.*

PROOF. The properties of the optimal contract we describe above are sufficient to pin down the dual variables that complement an optimal solution. Recall that $\lambda_t = \Delta_t > 0$ and $\mu_t = 0$ for all $t < t^*$,

while $\lambda_t = 0$ for all $t > t^*$. This implies that we can write

$$\frac{1}{n}(1 - \alpha_l)^t \mu_t - \alpha_l(1 - \alpha_l)^{t-1} \gamma \sum_{\tau=t^*}^{t-1} \mu_\tau = \gamma \Delta_t \text{ for all } t > t^*, \text{ and } -(1 - \alpha_l)^T \sum_{\tau=t^*}^T \mu_\tau = \Delta_\infty.$$

These equations can be rewritten as

$$\mu_t - \frac{\alpha_l n}{1 - \alpha_l} \sum_{\tau=t^*}^{t-1} \mu_\tau = \frac{n \Delta_t}{(1 - \alpha_l)^t} \text{ for all } t > t^*, \text{ and} \quad (\text{C.1})$$

$$\sum_{\tau=t^*}^T \mu_\tau = -\frac{\Delta_\infty}{(1 - \alpha_l)^T}. \quad (\text{C.2})$$

Subtracting the period- T instance of equation (C.1) above from equation (C.2) yields

$$\frac{1 + \alpha_l(n - 1)}{1 - \alpha_l} \sum_{\tau=t^*}^{T-1} \mu_\tau = -\frac{\Delta_\infty}{(1 - \alpha_l)^T} - \frac{n \Delta_T}{(1 - \alpha_l)^T},$$

which can be rearranged as

$$\sum_{\tau=t^*}^{T-1} \mu_\tau = -\frac{\Delta_\infty + n \Delta_T}{(1 + \alpha_l(n - 1))(1 - \alpha_l)^{T-1}}.$$

Similarly, subtracting the period- $(T - 1)$ instance of equation (C.1) from this expression then yields

$$\frac{1 + \alpha_l(n - 1)}{1 - \alpha_l} \sum_{\tau=t^*}^{T-2} \mu_\tau = -\frac{\Delta_\infty + n \Delta_T}{(1 + \alpha_l(n - 1))(1 - \alpha_l)^{T-1}} - \frac{n \Delta_{T-1}}{(1 - \alpha_l)^{T-1}},$$

which can be rewritten as

$$\sum_{\tau=t^*}^{T-2} \mu_\tau = -\frac{\Delta_\infty + n \Delta_T + (1 - \alpha_l(n - 1))n \Delta_{T-1}}{(1 + \alpha_l(n - 1))^2(1 - \alpha_l)^{T-2}}.$$

Proceeding inductively in this manner, we arrive at

$$\mu_{t^*} = -\frac{\Delta_\varphi}{(1 + \alpha_l(n - 1))^{T-t^*}(1 - \alpha_l)^{t^*}} - \frac{n}{(1 - \alpha_l)^{t^*}} \sum_{\tau=t^*+1}^T \frac{\Delta_\tau}{(1 + \alpha_l(n - 1))^{\tau-t^*}}.$$

With this expression in hand, recall that the final remaining dual constraint (which is complemented by p_{t^*}) can be rewritten as

$$\lambda_{t^*} = \Delta_{t^*} - \frac{(1 - \alpha_l)^{t^*}}{n} \mu_{t^*} = \sum_{\tau=t^*}^T (1 + \alpha_l(n - 1))^{t^*-\tau} \Delta_\tau + \frac{(1 + \alpha_l(n - 1))^{t^*-T}}{n} \Delta_\infty. \quad (\text{C.3})$$

Thus, the value of the dual program (\mathcal{D}') is

$$\sum_{\tau=1}^T \lambda_{\tau} = \sum_{\tau=1}^{t^*-1} \Delta_{\tau} + \sum_{\tau=t^*}^T (1 + \alpha_l(n-1))^{t^*-\tau} \Delta_{\tau} + \frac{(1 + \alpha_l(n-1))^{t^*-T}}{n} \Delta_{\infty}.$$

Meanwhile, the objective of the relaxed primal program (\mathcal{R}') can now be written as

$$\sum_{\tau=1}^T \Delta_{\tau} p_{\tau} + \Delta_{\infty} p_{\infty} = \sum_{\tau=1}^{t^*-1} \Delta_{\tau} + p_{t^*} \left(\sum_{\tau=t^*}^T (1 + \alpha_l(n-1))^{t^*-\tau} \Delta_{\tau} + \frac{(1 + \alpha_l(n-1))^{t^*-T}}{n} \Delta_{\infty} \right).$$

Since the value of the primal must equal that of the dual at a solution to this linear program, we must therefore have $p_{t^*} = 1$. \diamond

Notice that the contract that solves the relaxed problem (\mathcal{R}') is such that $1 \geq p_t > q_t = 0$ for all $t = 1, \dots, T$, and furthermore that p_t is (weakly) decreasing in t . In addition, $p_T > p_{\infty} \in (0, 1)$. Thus, the contract satisfies the no-delay constraints (**ND**), the no-misreporting constraints (**NM**), and the feasibility constraints from the (unrelaxed) problem (\mathcal{P}); that is, this contract solves the principal's original problem, and is therefore optimal.

Finally, recall from the [proof of Theorem 1](#) that $\Delta_{\infty} < 0$ whenever $\beta < \bar{\beta}$ (as is the case here), and moreover that $\Delta_t > 0$ for all $t < \bar{k}$ and $\Delta_t < 0$ for all $t > \bar{k}$ (where \bar{k} is the first-best cutoff defined in [Theorem 1](#)). In addition, note from [\(C.3\)](#) that we can write

$$\lambda_{t^*} = \Delta_{t^*} + \sum_{\tau=t^*+1}^T (1 + \alpha_l(n-1))^{t^*-\tau} \Delta_{\tau} + \frac{(1 + \alpha_l(n-1))^{t^*-T}}{n} \Delta_{\infty}.$$

Since $\Delta_{\infty} < 0$, the the dual constraint $\lambda_{t^*} \geq 0$ can only be satisfied if $t^* < \bar{k}$. \blacksquare

D. PROOF OF THEOREM 5

A mechanism for asymmetric states consists of probabilities $\{(p_t^{\omega}, q_t^{\omega})_{t=1}^T, p_{\infty}^{\omega}\}_{\omega=a,b}$ such that an analyst reporting ω in period t is retained with probability p_t^{ω} if $r = \omega$ and with probability q_t^{ω} otherwise; and that the analyst is retained with probability p_{∞}^{ω} if they eventual outcome is $r = \omega$ and the analyst never reports a non-null signal.

Given such a mechanism, if the analyst truthfully reports $\omega \in \{a, b\}$ in period t , she is retained with probability $\gamma_{\omega} p_t^{\omega} + (1 - \gamma_{\omega}) q_t^{\omega}$; if she misreports $\omega \in \{a, b\}$ as $\omega' \neq \omega$ in period t , she is retained with probability $\gamma_{\omega} q_t^{\omega'} + (1 - \gamma_{\omega}) p_t^{\omega'}$; and if she “guesses” state ω in period t while uninformed, she is retained with probability $\eta_{\omega} p_t^{\omega} + (1 - \eta_{\omega}) q_t^{\omega}$, where $\eta_{\omega} := \pi_{\omega} \gamma_{\omega} + (1 - \pi_{\omega})(1 - \gamma_{\omega})$ is the (unconditional) probability of observing outcome $r = \omega$.

We now specify the constraints arising from requiring incentive compatibility for the type- h analyst. First, an informed agent who has learned the true state of the world ω must prefer to report that state truthfully instead of misreporting it as $\omega' \neq \omega$. This implies that we must have

$$\gamma_{\omega} p_t^{\omega} + (1 - \gamma_{\omega}) q_t^{\omega} \geq \gamma_{\omega} q_t^{\omega'} + (1 - \gamma_{\omega}) p_t^{\omega'} \text{ for } \omega = a, b \text{ and } \omega' \neq \omega, \text{ and all } t = 1, \dots, T. \text{ (NM}_{\omega}\text{)}$$

The analyst must also have an incentive to immediately report observing an informative signal instead of delaying and reporting it in a future period (or never reporting it at all); this implies that we must satisfy the “no-delay” condition

$$\gamma_\omega p_t^\omega + (1 - \gamma_\omega) q_t^\omega \geq \gamma_\omega p_{t+1}^\omega + (1 - \gamma_\omega) q_{t+1}^\omega \text{ for } \omega = a, b \text{ and all } t = 1, \dots, T. \quad (\text{ND}_\omega)$$

(We mildly abuse notation and let $p_{T+1}^\omega := p_\infty^\omega$ and $q_{T+1}^\omega := p_\infty^{\omega'}$ for convenience.) The constraints (ND_ω) and (NM_ω) apply only to informed agents, and are therefore type-independent.

We define U_t^θ as the expected payoff of an uninformed type- θ agent in period t who reports their null signal $s_t = \phi$ truthfully and waits for an additional period. In that next period, the agent receives (and truthfully reports) an informative signal with probability α_θ and remains uninformed (and then proceeds optimally) with probability $(1 - \alpha_\theta)$; thus, we inductively define

$$U_t^\theta := \alpha_\theta \sum_{\omega=a,b} \pi_\omega (\gamma_\omega p_{t+1}^\omega + (1 - \gamma_\omega) q_{t+1}^\omega) + (1 - \alpha_\theta) \max \left\{ U_{t+1}^\theta, \max_{\omega} \{ \eta_\omega p_{t+1}^\omega + (1 - \eta_\omega) q_{t+1}^\omega \} \right\},$$

where we let $U_T^\theta := \sum_{\omega=a,b} \eta_\omega p_\infty^\omega$, as there are no additional private signals to observe after T . For an uninformed agent to prefer to wait instead of pretending to be informed and misreporting some state ω , the mechanism must satisfy “no-guessing” constraints

$$U_t^h \geq \eta_\omega p_t^\omega + (1 - \eta_\omega) q_t^\omega \text{ for } \omega = a, b \text{ and all } t = 1, \dots, T, \quad (\text{NG}_\omega\text{-}h)$$

$$U_t^l \geq \eta_\omega p_t^\omega + (1 - \eta_\omega) q_t^\omega \text{ for } \omega = a, b \text{ and all } t = 1, \dots, T. \quad (\text{NG}_\omega\text{-}l)$$

Lemma A.1 implies that $(\text{NG}_\omega\text{-}h)$ must be satisfied in any solution to the fund manager’s problem, but that she is free to choose a mechanism where $(\text{NG}_\omega\text{-}l)$ is violated.

Finally, since U_0^θ corresponds to the type- θ analyst’s ex ante expected probability of being retained, we can write the fund manager’s screening problem as

$$\begin{aligned} W^* &= \max_{\{(p_t^\omega, q_t^\omega)_{t=1}^T, p_\infty^\omega\}_{\omega=a,b}} \{ \rho U_0^h - (1 - \rho) U_0^l c \} \\ \text{s.t.} & \text{ for all } \omega = a, b \text{ and all } t = 1, \dots, T, \\ & (\text{ND}_\omega), (\text{NM}_\omega), (\text{NG}_\omega\text{-}h), \\ & 0 \leq p_t^\omega, q_t^\omega \leq 1 \text{ and } 0 \leq p_\infty^\omega \leq 1. \end{aligned} \quad (\mathcal{P}_\omega)$$

As in [Appendix B](#), the U_t^θ terms in the manager’s objective function (\mathcal{P}_ω) already implicitly incorporate the no-delay and no-misreporting constraints (ND_ω) and (NM_ω) . Therefore, we can work with the simpler relaxed problem where we drop these constraints and verify their satisfaction ex post. After factoring out the $(1 - \rho)c$ term, we arrive at the relaxed problem

$$\begin{aligned} & \max_{\{(p_t^\omega, q_t^\omega)_{t=1}^T, p_\infty^\omega\}_{\omega=a,b}} \{ \beta U_0^h - U_0^l \} \\ \text{s.t.} & \text{ for all } \omega = a, b \text{ and all } t = 1, \dots, T, \\ & (\text{NG}_\omega\text{-}h), 0 \leq p_t^\omega, q_t^\omega \leq 1 \text{ and } 0 \leq p_\infty^\omega \leq 1. \end{aligned} \quad (\mathcal{P}'_\omega)$$

LEMMA D.1. *There exists a solution to (\mathcal{P}'_ω) where $(\mathbf{NG}_\omega-l)$ is satisfied for $\omega = a, b$.*

PROOF. Suppose that $\{(p_t^\omega, q_t^\omega)_{t=1}^T, p_\infty^\omega\}_{\omega=a,b}$ solves problem (\mathcal{P}'_ω) , so the type- h no-guessing constraints $(\mathbf{NG}_\omega-h)$ are satisfied. Suppose further that there is some period $\bar{t} \leq T$ such that an uninformed type- l agent strictly prefers to guess at period \bar{t} instead of waiting an additional period. This implies that the uninformed type- l agent will always guess at period \bar{t} , and so in subsequent periods the principal can only be facing a type- h agent. (Recall from the definition of U_t^θ that the informed agent—of either type—never delays or misreports their information.)

Suppose first that $U_T^h < 1$, implying $\min_\omega \{p_\infty^\omega\} < 1$. Then there exists $\varepsilon > 0$ sufficiently small that it is possible to increase $\sum_{\omega=a,b} \eta_\omega p_\infty^\omega$ without violating the feasibility constraint. This only makes waiting more attractive for the uninformed type- h agent in period T , so the no-guessing constraints $(\mathbf{NG}_\omega-h)$ remain satisfied. However, for ε small enough, the type- l agent's strict preference to guess remains unchanged.

Now suppose instead that $U_T^h = 1$, so $p_\infty^\omega = 1$ for all ω . But $(\mathbf{NG}_\omega-l)$ strictly violated implies that there exists $t > \bar{t}$ such that $\eta_{\hat{\omega}} p_t^{\hat{\omega}} + (1 - \eta_{\hat{\omega}}) q_t^{\hat{\omega}} < 1$ for some $\hat{\omega}$. (Otherwise, we have $U_t^\theta = 1$ for all $t > \bar{t}$ and no violation of $(\mathbf{NG}_\omega-l)$ is possible.) Let \hat{t} denote the *latest* such period (so $\eta_\omega p_{\hat{t}+1}^\omega + (1 - \eta_\omega) q_{\hat{t}+1}^\omega = 1$ for all ω and $U_{\hat{t}}^h = 1$), and consider a perturbation that increases $\eta_{\hat{\omega}} p_{\hat{t}}^{\hat{\omega}} + (1 - \eta_{\hat{\omega}}) q_{\hat{t}}^{\hat{\omega}}$ by $\varepsilon > 0$. For ε small enough, the type- l agent still strictly prefers to guess at \bar{t} , while (since $U_{\hat{t}}^h = 1$) the type- h agent still prefers to truthfully report being uninformed.

Since only type- h agents report being uninformed at period \bar{t} and continue in the mechanism, increasing the probability of retaining the type- h analyst without affecting the retention probability of the type- l analyst. This strictly increases the principal's payoff, violating the conjectured optimality of the original solution. \diamond

Since the type- h agent has a higher probability of observing an informative signal in any period than does a type- l agent, the type- h agent is *strictly* willing to wait whenever the type- l agent is willing to do so; that is, $(\mathbf{NG}_\omega-h)$ is implied by $(\mathbf{NG}_\omega-l)$. Moreover, when these constraints are satisfied, we can write the payoff of an uninformed type- θ analyst in period t as

$$\begin{aligned} U_t^\theta &= \alpha_\theta \left(\sum_{\omega=a,b} \pi_\omega (\gamma_\omega p_{t+1}^\omega + (1 - \gamma_\omega) q_{t+1}^\omega) \right) + (1 - \alpha_\theta) U_{t+1}^\theta \\ &= \sum_{\tau=t+1}^T \alpha_\theta (1 - \alpha_\theta)^{\tau-t-1} \left(\sum_{\omega=a,b} \pi_\omega (\gamma_\omega p_\tau^\omega + (1 - \gamma_\omega) q_\tau^\omega) \right) + (1 - \alpha_\theta)^{T-t} \sum_{\omega=a,b} \eta_\omega p_\infty^\omega. \end{aligned}$$

Recalling that, for all $t = 1, \dots, T$, we had

$$\Delta_t := \beta \alpha_h (1 - \alpha_h)^{t-1} - \alpha_l (1 - \alpha_l)^{t-1} \text{ and } \Delta_\infty := \beta (1 - \alpha_h)^T - (1 - \alpha_l)^T,$$

we can rewrite problem (\mathcal{P}'_ω) as

$$\begin{aligned} & \max_{\{(p_t^\omega, q_t^\omega)_{t=1}^T, p_\infty^\omega\}_{\omega=a,b}} \left\{ \sum_{t=1}^T \Delta_t \sum_{\omega=a,b} \pi_\omega (\gamma_\omega p_t^\omega + (1 - \gamma_\omega) q_t^\omega) + \Delta_\infty \sum_{\omega=a,b} \eta_\omega p_\infty^\omega \right\} \\ & \text{s.t., for all } \omega = a, b \text{ and all } t = 1, \dots, T, \\ & \text{(NG}_{\omega-l}\text{)}, 0 \leq p_t^\omega, q_t^\omega \leq 1 \text{ and } 0 \leq p_\infty^\omega \leq 1. \end{aligned}$$

We relax this problem further by dropping the feasibility constraints that lower-bound p_t^ω and p_∞^ω , as well as those that upper-bound p_∞^ω and q_t^ω . Multiplying both sides of the period- t no-guess constraint $(\text{NG}_{\omega-l})$ by $(1 - \alpha_l)^t$, we arrive at a relaxed primal problem in standard form:

$$\begin{aligned} & \max_{\{(p_t^\omega, q_t^\omega)_{t=1}^T, p_\infty^\omega\}_{\omega=a,b}} \left\{ \sum_{t=1}^T \Delta_t \sum_{\omega=a,b} \pi_\omega (\gamma_\omega p_t^\omega + (1 - \gamma_\omega) q_t^\omega) + \Delta_\infty \sum_{\omega=a,b} \eta_\omega p_\infty^\omega \right\} \\ & \text{s.t., for all } \omega = a, b \text{ and all } t = 1, \dots, T, \\ & (1 - \alpha_l)^t (\eta_\omega p_t^\omega + (1 - \eta_\omega) q_t^\omega) \tag{\mathcal{R}_\omega} \\ & - \sum_{\tau=t+1}^T \alpha_l (1 - \alpha_l)^{\tau-1} \sum_{\omega'=a,b} \pi_{\omega'} (\gamma_{\omega'} p_\tau^{\omega'} + (1 - \gamma_{\omega'}) q_\tau^{\omega'}) - (1 - \alpha_l)^T \sum_{\omega'=a,b} \eta_{\omega'} p_\infty^{\omega'} \leq 0, \\ & p_t^\omega \leq 1, \text{ and } q_t^\omega \geq 0. \end{aligned}$$

This corresponds to the following dual problem, where λ_t^ω is the dual variable for the p_t^ω -feasibility constraint and μ_t^ω is the dual variable for the type- l agent's period- t no-guessing- ω constraint:

$$\begin{aligned} & \min_{\{(\lambda_t^\omega, \mu_t^\omega)_{t=1}^T\}_{\omega=a,b}} \left\{ \sum_{t=1}^T \sum_{\omega=a,b} \lambda_t^\omega \right\} \\ & \text{s.t., for all } \omega = a, b \text{ and all } t = 1, \dots, T, \\ & \lambda_t^\omega + (1 - \alpha_l)^t \eta_\omega \mu_t^\omega - \alpha_l (1 - \alpha_l)^{t-1} \pi_\omega \gamma_\omega \sum_{\tau=1}^{t-1} \sum_{\omega'=a,b} \mu_\tau^{\omega'} = \pi_\omega \gamma_\omega \Delta_t, \tag{\mathcal{D}_\omega} \\ & (1 - \alpha_l)^t (1 - \eta_\omega) \mu_t^\omega - \alpha_l (1 - \alpha_l)^{t-1} \pi_\omega (1 - \gamma_\omega) \sum_{\tau=1}^{t-1} \sum_{\omega'=a,b} \mu_\tau^{\omega'} \geq \pi_\omega (1 - \gamma_\omega) \Delta_t, \\ & - (1 - \alpha_l)^T \sum_{\tau=1}^T \sum_{\omega'=a,b} \mu_\tau^{\omega'} = \Delta_\infty, \\ & \lambda_t^\omega \geq 0, \text{ and } \mu_t^\omega \geq 0. \end{aligned}$$

Because we have dropped the nonnegativity constraints on p_t^ω and p_∞^ω in the relaxed primal problem (\mathcal{R}_ω) , their complementary constraints in the dual (\mathcal{D}_ω) are equalities.

LEMMA D.2. *In any solution, the type- l agent's no-guessing constraints $(\text{NG}_{\omega-l})$ must bind for every possible state $\omega = a, b$ and every period $t = 1, \dots, T$.*

PROOF. Suppose that there exists some period \bar{t} such that $\mu_{\bar{t}}^{\omega} = 0$ for all ω . This implies we can write the dual constraint complementing $q_{\bar{t}}^{\omega}$ (for either ω) as

$$\alpha_l(1 - \alpha_l)^{\bar{t}-1} \sum_{\tau=1}^{\bar{t}-1} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} \leq -\Delta_{\bar{t}}.$$

If $\bar{t} < T$, then the dual constraint complementing $p_{\bar{t}+1}^{\omega}$ is

$$\begin{aligned} \lambda_{\bar{t}+1}^{\omega} + (1 - \alpha_l)^{\bar{t}+1} \eta_{\omega} \mu_{\bar{t}+1}^{\omega} &= \pi_{\omega} \gamma_{\omega} \left(\Delta_{\bar{t}+1} + \alpha_l(1 - \alpha_l)^{\bar{t}} \sum_{\tau=1}^{\bar{t}} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} \right) \\ &\leq \pi_{\omega} \gamma_{\omega} (\Delta_{\bar{t}+1} - (1 - \alpha_l) \Delta_{\bar{t}}) \\ &= \pi_{\omega} \gamma_{\omega} \beta \alpha_h (1 - \alpha_h)^{\bar{t}-1} (\alpha_l - \alpha_h) < 0. \end{aligned}$$

This of course contradicts the nonnegativity of $\lambda_{\bar{t}+1}^{\omega}$ and $\mu_{\bar{t}+1}^{\omega}$. If, instead, $\bar{t} = T$, then the dual constraint complementing p_{∞}^{ω} is

$$\begin{aligned} 0 &= \Delta_{\infty} + (1 - \alpha_l)^T \sum_{\tau=1}^T \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} \\ &= \Delta_{\infty} + \frac{(1 - \alpha_l)}{\alpha_l} \alpha_l (1 - \alpha_l)^{\bar{t}-1} \sum_{\tau=1}^{\bar{t}-1} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} \\ &\leq \Delta_{\infty} - \frac{(1 - \alpha_l)}{\alpha_l} \Delta_{\bar{t}} = \beta (1 - \alpha_h)^{\bar{t}-1} \left(1 - \frac{\alpha_h}{\alpha_l}\right) < 0. \end{aligned}$$

This is, of course, a contradiction. Thus, we must have $\sum_{\omega=a,b} \mu_t^{\omega} > 0$ for all $t = 1, \dots, T$.

Now suppose that for some t , we have $\mu_t^{\hat{\omega}} > 0 = \mu_t^{\tilde{\omega}}$. If $\mu_t^{\tilde{\omega}} = 0$, we can write the dual constraint complemented by $q_t^{\tilde{\omega}}$ as

$$\alpha_l(1 - \alpha_l)^{t-1} \sum_{\tau=1}^{t-1} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} + \Delta_t \leq 0.$$

This implies that the dual constraint complemented by $p_t^{\tilde{\omega}}$ can be written as

$$\lambda_t^{\tilde{\omega}} = \pi_{\tilde{\omega}} \gamma_{\tilde{\omega}} \left[\alpha_l(1 - \alpha_l)^{t-1} \sum_{\tau=1}^{t-1} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} + \Delta_t \right] \leq 0.$$

But recall that we must also have $\lambda_t^{\tilde{\omega}} \geq 0$. Therefore, $\mu_t^{\tilde{\omega}} = 0$ implies that $\lambda_t^{\tilde{\omega}} = 0$ and

$$\alpha_l(1 - \alpha_l)^{t-1} \sum_{\tau=1}^{t-1} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} + \Delta_t = 0.$$

Now consider the constraint complemented by $p_t^{\hat{\omega}}$, which can be written as

$$\lambda_t^{\hat{\omega}} + (1 - \alpha_l)^t \eta_{\hat{\omega}} \mu_t^{\hat{\omega}} = \pi_{\hat{\omega}} \gamma_{\hat{\omega}} \left[\alpha_l (1 - \alpha_l)^{t-1} \sum_{\tau=1}^{t-1} \sum_{\omega'=a,b} \mu_{\tau}^{\omega'} + \Delta_t \right] = 0.$$

This is a contradiction, as $\mu_t^{\hat{\omega}} > 0$ by hypothesis and $\lambda_t^{\hat{\omega}} \geq 0$ by constraint. Since $\hat{\omega} \neq \tilde{\omega}$ were arbitrarily chosen, this implies that we must have $\mu_t^{\omega} > 0$ for all $\omega = a, b$ and all $t = 1, \dots, T$; by complementary slackness, this implies that in any solution, the type- l agent's no-guessing constraints ($\text{NG}_{\omega-l}$) always bind. \diamond

LEMMA D.3. *In any solution to the relaxed primal (\mathcal{R}_{ω}), we have $q_t^{\omega} = 0$ whenever $p_t^{\omega} < 1$.*

PROOF. Suppose not; that is, suppose there exists a solution $\{(p_t^{\omega}, q_t^{\omega})_{t=1}^T, p_{\infty}^{\omega}\}_{\omega=a,b}$ with $p_{\bar{t}}^{\tilde{\omega}} < 1$ and $q_{\bar{t}}^{\tilde{\omega}} > 0$ for some \bar{t} and $\tilde{\omega}$. Consider a perturbation to this solution that increases $p_{\bar{t}}^{\tilde{\omega}}$ by $\varepsilon > 0$ and decreases $q_{\bar{t}}^{\tilde{\omega}}$ by $\frac{\gamma_{\tilde{\omega}}}{1-\gamma_{\tilde{\omega}}}\varepsilon$. For sufficiently small ε , this perturbation is feasible. Moreover, it does not affect the objective value of the relaxed primal problem (\mathcal{R}_{ω}) or any of the period- t no-guessing constraints ($\text{NG}_{\omega-l}$) for any $t \neq \bar{t}$. But since $\gamma_{\omega} > \pi_{\omega}$, this perturbation relaxes the period- \bar{t} no-guessing $-\omega$ constraint. This of course contradicts Lemma D.2, which showed that the constraints ($\text{NG}_{\omega-l}$) bind for all $\omega = a, b$ and $t = 1, \dots, T$ in any solution. \diamond

By Lemma D.2, we can write the type- l continuation value when uninformed in period- t as

$$U_t^l = \eta_{\omega} p_t^{\omega} + (1 - \eta_{\omega}) q_t^{\omega}$$

for all $\omega = a, b$ and all $t = 1, \dots, T$. Lemma D.3 above implies that this can be rewritten as

$$p_t^{\omega} = \min \left\{ 1, \frac{U_t^l}{\eta_{\omega}} \right\} \text{ and } q_t^{\omega} = \frac{U_t^l - \eta_{\omega} p_t^{\omega}}{1 - \eta_{\omega}}.$$

It is easy to verify that U_t^l is strictly decreasing in t . This implies that there are cutoff periods t_{ω}^* so that, in any solution to (\mathcal{R}_{ω}), $p_t^{\omega} = 1$ and $q_t^{\omega} > 0$ for $t < t_{\omega}^*$; $p_t^{\omega} = 1$ and $q_t^{\omega} \geq 0$ for $t = t_{\omega}^*$; and $p_t^{\omega} < 1$ and $q_t^{\omega} = 0$ for $t > t_{\omega}^*$. In addition, we are free to set $p_{\infty}^{\omega} = \eta_{\omega} p_T^{\omega} + (1 - \eta_{\omega}) q_T^{\omega} \in (0, 1)$.

Moreover, note that if $\eta_{\omega} > \eta_{\omega'}$, then we must have $t_{\omega}^* \leq t_{\omega'}^*$ and $p_t^{\omega} \leq p_t^{\omega'}$. In addition, since $\eta_{\omega'} = 1 - \eta_{\omega}$, we can write

$$\eta_{\omega} (p_t^{\omega} - q_t^{\omega'}) = (1 - \eta_{\omega}) (p_t^{\omega'} - q_t^{\omega})$$

for all $t = 1, \dots, T$. Therefore, $q_t^{\omega} \leq q_t^{\omega'}$.

In addition, note that U_t^l decreasing in t immediately implies that $(p_t^{\omega}, q_t^{\omega})$ are decreasing, and so the no-delay constraints (ND_{ω}) are satisfied. Finally, observe that $\gamma_{\omega} > \pi_{\omega}$ for both $\omega = a, b$ implies that $\gamma_{\omega} > \eta_{\omega}$; this implies that the no-misreporting constraints (NM_{ω}) are satisfied. \blacksquare

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