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Abstract

Why do firms engage in costly, voluntary disclosure of information which is subsumed by a later announcement? We consider a model in which the firm's manager can choose to disclose short-term information which becomes redundant later. When disclosure costs are sufficiently low, the manager discloses even if she only cares about the long-term price of the firm. Intuitively, by disclosing, she causes early investors to trade less aggressively, reducing price informativeness, which in turn increases information acquisition by late investors. The subsequent increase in acquisition more than offsets the initial decrease in price informativeness and, consequently, improves long term prices.

JEL Classification: D21, D82, D83, D84, G32

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On the Voluntary Disclosure of Redundant Information*

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November 6, 2022

Abstract

Why do firms engage in costly, voluntary disclosure of information which is subsumed by a later announcement? We consider a model in which the firm's manager can choose to disclose short-term information which becomes redundant later. When disclosure costs are sufficiently low, the manager discloses even if she only cares about the long-term price of the firm. Intuitively, by disclosing, she causes early investors to trade less aggressively, reducing price informativeness, which in turn increases information acquisition by late investors. The subsequent increase in acquisition more than offsets the initial decrease in price informativeness and, consequently, improves long term prices.

Keywords: costly voluntary disclosure, information acquisition, redundant information
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1 Introduction

Will voluntary disclosure of information (e.g., the existence of ongoing projects), which will be completely subsumed by a later announcement (e.g., the ultimate cash flows from the projects), have an incremental impact on a firm's stock price at the time of the later announcement? A Bayesian economist might be tempted to respond that the answer is no. After all, the early information becomes redundant at the later date.

This reaction reflects a broader attitude about the impact on long-term valuations of trading on short-term information. Stiglitz (1989) considers an example of investors acquiring information today that becomes publicly available tomorrow, and argues that while there may be a private benefit to trading on such information in the short-term, there is no long-term effect on prices.[1] In this case, what are the firm's incentives to disclose such short-term information?

Importantly, the settings we have in mind are ones in which disclosure is truly redundant. Specifically, it does not lead to feedback effects that would impact firm investment decisions, nor does it convey any information about persistent unobservable factors that could impact firm value beyond the subsequent disclosure at the later date. Trueman (1986) forcefully argues that in such a setting:

> "*the disclosure would simply advance the time at which investors learn something about the firm's earnings. The market value of the firm at the end of the period, after the actual earnings had been reported, however, would be unaffected by the forecast release (since the estimated earnings becomes irrelevant for valuation at that time).*"

While the above argument is intuitive, in this paper we show that strategic early disclosure can increase firm market value at future dates even after the disclosure becomes redundant. Our insight is that disclosure affects investors' information acquisition decisions, and consequently, influences the information environment at later dates. Disclosure directly impacts early investors' information choices, which changes the public information available to later investors (via the information revealed by short-term prices), and consequently affects their information acquisition and demand for the stock.

To highlight the economic channel, we restrict attention to a stylized setting. Specifically, we study a model with two trading periods in which the firm's terminal value depends on the payoffs to a long term project (i.e., the assets-in-place of the firm) and, possibly, a short term project. If it exists, the short-term project's payoff is publicly revealed after trade in

[1] In fact, Stiglitz (1989) argues that it may be desirable to tax short-term turnover to make prices less volatile, even though this reduces or eliminates the incorporation of short-term information into prices.

the first period and before trade in the second period. The long-term project's payoff is revealed only after trade in the second period, when it is paid out as part of the terminal value.

We assume that the manager knows with certainty whether the short-term project exists, and can disclose this information truthfully before the first round of trade by paying a cost (as in Verrecchia (1983)). The manager's objective at the disclosure stage is to maximize the long-run (second-period) market value. The short-term project's payoff is publicly revealed before the second round of trading, independently of the manager's disclosure decision, and makes the manager's disclosure completely redundant. Before trading in each round, investors choose whether or not to acquire costly information about the long-term project (as in Grossman and Stiglitz (1980)).

Our main result is that, provided that the cost of disclosure is not too high, the manager will voluntarily disclose the existence of the short-term project. By disclosing, the manager affects information acquisition and trading by investors. When early (first-period) investors learn that there is a short-term project, they face more uncertainty about the second-period price, which, in general, has two effects on first period price informativeness. First, increased uncertainty reduces how aggressively informed investors trade on their information, which tends to reduce price informativeness. Second, increased uncertainty can either increase or decrease the fraction of early investors who choose to acquire information.[2] However, we show that the impact of lowering trading intensity dominates and, as a result, disclosure always reduces first-period price informativeness. Once the short-term project's payoff is revealed prior to the second round of trade, the increased uncertainty about the long-term project leads more second-period investors to become informed about it. We show that the impact of more information acquisition at the later date dominates the impact of higher uncertainty in the short term. Hence, the resulting long term prices more precisely reflect the firm's true value and hence are higher on average, due to a lower risk premium.

Our analysis is particularly relevant when the projects that a firm may have undertaken are themselves subject to high uncertainty. For instance, firms that engage in multi-stage R&D investments (e.g., clinical trials for pharmaceuticals) often choose to disclose this information in early stages even though such disclosures become redundant once the outcomes are realized. Our model implies that the impact of such disclosures on investors' information acquisition and longer term prices is likely to be stronger when there is high uncertainty about the ultimate payoffs from the project. Consistent with our mechanism, Cookson, Moon, and

[2] As we discuss below in Section 3.3, the fact that the fraction informed can increase or decrease with residual uncertainty also holds in the single period model of Grossman and Stiglitz (1980) – see their Section II.H.

Noh (2022) show that forward-looking, speculative disclosures are associated with a gradual, longer-term increase in average prices, a gradual increase in liquidity, and more informed trading. Moreover, they show that these effects are stronger when such disclosures are about R&D and for firms with higher idiosyncratic volatility.

There is an extensive literature on understanding the rationales for disclosure. Diamond (1985) shows how pre-commitment to publicly disclosing information can improve welfare by improving risk sharing and saving real resources which would otherwise be devoted to private information acquisition. In the presence of proprietary disclosure costs, sufficiently good news is disclosed and bad news is withheld (Verrecchia (1983)), and improvements in the quality of managers' information increases disclosure (Verrecchia (1990)). A similar threshold disclosure characterization exists if investors are uncertain as to whether the manager has information (Dye (1985), Jung and Kwon (1988)). Diamond and Verrecchia (1991) show disclosure changes risk for market makers, which affects their willingness to provide liquidity.[3] We contribute to this literature by introducing a distinct rationale for firms to voluntarily disclose information.

Our paper is related to the literature on earnings guidance, which focuses on the manager's incentives to influence investors' expectations about future earnings. In Trueman (1986), early voluntary disclosure that is later validated by a mandatory disclosure helps the manager signal to investors about her persistent skill in identifying optimal investment decisions.[4] In contrast, our model is designed so that at the time of the mandatory disclosure, any earlier voluntary disclosures become completely redundant. Yet, our analysis highlights a channel whereby the earlier disclosure still increases firm value at the later date.

The literature on feedback effects highlights a related complementarity between disclosure and informed trading by investors.[5] In such models, the manager chooses to strategically disclose information to encourage investors to trade more aggressively on their private information, or acquire more information. This results in more informative prices and, consequently, better informed real decisions by the manager. For instance, Goldstein and Yang (2015), Goldstein and Yang (2019) and Goldstein, Schneemeier, and Yang (2020) highlight how disclosure along one dimension of fundamentals can crowd in more informed trading along another dimension. Yang (2020) considers an oligopoly setting where firms disclose information about consumer demand to encourage investors to trade on their private information about an orthogonal component.

[3] See also surveys in Verrecchia (2001), Dye (2001) and Beyer, Cohen, Lys, and Walther (2010).

[4] In his model, the manager's ability to predict the firm's future optimal production level is the unobservable characteristic. By releasing a forecast that is subsequently validated, the manager signals to investors that he has that skill, which improves subsequent firm investment decisions, and hence increases firm value.

[5] See Bond, Edmans, and Goldstein (2012) and Goldstein and Yang (2017) for recent surveys.

In contrast to this literature, the disclosure in our setting is not about the realization of cash flows per se, but the exposure to an additional source of risk (the short-term project). As such, the most closely related papers are Smith (2020) and Lassak (2020). Smith (2020) shows that disclosure about a firm's riskiness can induce investors to acquire more information about fundamentals. Lassak (2020) studies a setting where disclosure about cash flows can increase uncertainty, and shows that the firm discloses information only if it crowds in more learning by investors.

The economic mechanism in our model is distinct from this work. First, the manager's motivation for disclosure does not rely on any feedback effects: the manager does not learn from, or make investment decisions based on, the equilibrium price. Second, in our setting, the direct effect of the firm's disclosure about the short-term project is to initially *discourage* informed trading by early investors and so make short-term prices *less* informative. However, we show that through the endogenous information acquisition choices of later investors, this leads to more informative prices in the long term. Importantly, in our model, the manager's disclosure decisions reduce price informativeness in the short-term, yet there is an amplification effect in the long term, whereby long-term prices are more informative with the disclosure than without.

Our analysis also highlights the importance of considering the consequences of risk and investor risk-aversion for voluntary disclosure. Two related papers are Dye and Hughes (2018), who study a how firms' voluntary disclosure decisions to risk-averse investors are affected by systematic risk, and Banerjee, Marinovic, and Smith (2021), who study how a firm's disclosure decision is affected by the presence of diversely-informed risk-averse investors. In these papers, importantly, there is only one trading date (i.e., there is no notion of redundancy), and the manager's disclosure and the information that investors are endowed with are both about the firm's total cash flow. In contrast, our analysis features endogenous information acquisition by investors over multiple periods. This multi-period setting is critical for the mechanism we focus on, because a reduction in price informativeness in the short run leads to improved price informativeness in the long run.

A different rationale for strategically increasing uncertainty to induce information collection is to reduce agency costs, as featured in Strobl (2014). He considers a static model with moral hazard and adverse selection, and shows that managerial investment behavior under the optimal contract tends to lead to increased uncertainty about output, and consequently more information collection by investors. Importantly, there is no notion of redundancy and no disclosure decision, which are crucial elements of our analysis. Moreover, there is no systematic increase in expected prices as a result of information acquisition, a key result of our paper, and investors collect information only once, so there is no notion of shifting

Figure 1: Timeline of events

$t = 0$	$t = 1$	$t = 2$	$t = 3$
Manager observes x, chooses D or ND	Frac. λ_1 acquire info and observe θ Inv. demand X_{i1} Asset trades at P_1	$x\eta$ publicly revealed Frac. λ_2 acquire info and observe θ Inv. demand X_{i2} Asset trades at P_2	θ, u revealed Asset pays V

the distribution of information collection over time, unlike the economic mechanism in our model.

2 Model

In this section, we describe the model and discuss some important assumptions. Figure 1 summarizes the timing of events.

Payoffs. There are four dates $t \in \{0, 1, 2, 3\}$ and two securities. The gross return on the risk-free security is normalized to one. The risky security is a claim to a public firm with terminal value V, which will be realized at $t = 3$. The value V is given by

$$V = \bar{V} + x\eta + \theta + u, \tag{1}$$

where $\bar{V}$ is a known constant, η, θ, and u are independently normally distributed and $x \in \{0, 1\}$ is independent with prior probability p on $x = 1$. The aggregate supply of the risky security is given by $Z_t = \bar{Z} + z_t$ where z_t are normally distributed, and are independent of each other and other random variables. We denote the date t price of the risky security by P_t, and note that $P_3 = V$. We denote the mean of η by $\bar{\eta}$, normalize the means of θ, u and z_t to zero without loss of generality, and denote the variance (precision) of these shocks by $\sigma^2_{(\cdot)}$ ($\tau_{(\cdot)}$, respectively).

The event where $x = 1$ corresponds to the case where the short-term project exists and $x = 0$ to the case it does not. The payoff to the short-term project $x\eta$ is publicly revealed at date $t = 2$, and the payoff $\theta + u$ is publicly revealed at date $t = 3$ when the asset pays off.

Investors. There are overlapping generations (OLG) of investors. Generation t consists of a continuum of investors, indexed by $i \in [0, 1]$ with CARA utility and risk-aversion γ. Investor i in generation t can pay a cost c to observe θ immediately before trading at date t, and submits demand schedule X_{it} to maximize her expected utility over wealth at date

$t+1$. Importantly, the price at date $t+1$ is determined by the trading demand by investors in generation $t+1$. With some abuse of notation, we denote investors who choose to acquire information about θ by $i = I$, those who choose to remain uninformed by $i = U$, and the fraction who choose to become informed at date t by λ_t. Let $\mathcal{F}_{It} = \sigma\left(\theta, \{P_k\}_{k \le t}\right)$ and $\mathcal{F}_{Ut} = \sigma\left(\{P_k\}_{k \le t}\right)$ denote the information sets at time t for informed and uninformed traders, respectively. We will use $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$, $i \in \{I, U\}$, to denote the relevant conditional expectation and conditional variance operators..

Manager. The firm's manager knows x at date $t = 0$ and chooses whether or not to verifiably disclose it at a cost $c_D > 0$.[6] Let $d = D$ and $d = ND$ correspond to the choice of disclosure and non-disclosure, respectively. The manager optimally chooses her disclosure strategy to maximize the expected date 2 price. Let

$$U_d(x) = \mathbb{E}\left[P_2 \middle| d, x\right]$$

denote the expected price conditional on the realized value of x and the disclosure decision d.[7] Formally, a type x manager's problem is

$$U(x) \equiv \max_{d \in D, ND} U_d(x) - c_D \mathbf{1}_{\{d=D\}}. \tag{2}$$

3 Analysis

Our focus in this section is to show that there exists an equilibrium in which a manager with a short-term project ($x = 1$) discloses this information at date 0, while a manager without a project ($x = 0$) does not disclose. Importantly, in this equilibrium, investors at date 1 and 2 infer $x = 0$ (with probability 1) in the event that the manager does not disclose.

We shall establish this by working backwards. First, taking a disclosure $d \in \{D, ND\}$ and investor's information acquisition choices λ_1 and λ_2 as given, we solve for the equilibrium prices P_1 and P_2 in Section 3.1. Next, given a disclosure policy d, we solve for the optimal information acquisition choices at dates 1 and 2 in Section 3.2. Finally, we establish sufficient conditions under which our conjectured disclosure policy is the unique equilibrium policy in Section 3.3.

[6] Importantly, note that the manager does not observe the realization of η until date 2.

[7] Note that because $\mathbb{E}[x\eta|x] = 0$, and because ηx is publicly-disclosed before the $t = 2$ trading round and therefore enters any price function linearly, the expected price in the event of no disclosure is identical across values of x, $U_{ND}(0) = U_{ND}(1)$.

3.1 Financial market equilibrium

For given disclosure and information choices, in such an equilibrium, the financial market equilibrium either places probability 1 on $x = 1$ (in the event that the $x = 1$ manager discloses) or places probability 1 on $x = 0$ (in the event that the $x = 0$ manager discloses or either manager does not disclose and is inferred to be the $x = 0$ type). Hence, the derivation of the financial market equilibrium follows from the standard "conjecture and verify" approach. Fix the fraction λ_t of investors in generation t who acquire information about fundamentals θ. We conjecture that prices are of the form:

$$P_1(d) = A_1(d) + B_1(d)\, s_{p1}, \quad \text{and} \tag{3}$$

$$P_2(x,d) = A_2(d) + B_2(d)\, s_{p2} + C_2(d)\, s_{p1} + x\eta, \tag{4}$$

where the price signal $s_{pt} \equiv \theta + b_t(d)\, z_t$ for $t \in \{1,2\}$. Note that the equilibrium price coefficients $\{A_t, B_t, C_t, b_t\}$ depend on the manager's date zero disclosure decision d. However, in what follows, we will suppress this dependence for notational convenience unless necessary. The above conjecture implies that the date t price provides a noisy, linear signal s_{pt} about fundamentals θ to the uninformed investors of that generation. Moreover, the uninformed investors at date 2 can condition on the date 1 price to infer s_{p1}. This implies that the conditional beliefs of an uninformed investor at date $t = 1$ are given by:

$$\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1} s_{p1}}{\tau_\theta + \tau_{p1}}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}, \quad \text{where } \tau_{p1} \equiv \tau_z / b_1^2. \tag{5}$$

Similarly, the conditional beliefs of an uninformed investor at date $t = 2$ are given by:

$$\mathbb{E}_{U2}[\theta] = \frac{\tau_{p1} s_{p1} + \tau_{p2} s_{p2}}{\tau_\theta + \tau_{p1} + \tau_{p2}}, \quad \mathbb{V}_{U2}[\theta] = \frac{1}{\tau_\theta + \tau_{p1} + \tau_{p2}} \equiv \frac{1}{\tau_{U2}}, \quad \text{where } \tau_{p2} \equiv \tau_z / b_2^2. \tag{6}$$

Note that investor i in generation t chooses optimal demand X_{it} to maximize CARA utility over wealth at date $t+1$ i.e.,

$$X_{it} \equiv \arg\max_X \mathbb{E}_{it}[-e^{-\gamma\{W_t + X(P_{t+1} - P_t)\}}] \tag{7}$$

$$= \frac{\mathbb{E}_{it}[P_{t+1}] - P_t}{\gamma \mathbb{V}_{it}[P_{t+1}]} \tag{8}$$

where the date 3 price is $P_3 = V$. This implies that the optimal demand for date 2 informed and uninformed investors are given by

$$X_{I2} = \frac{1}{\gamma}\frac{\bar{V} + x\eta + \theta - P_2}{1/\tau_u}, \quad \text{and} \quad X_{U2} = \frac{1}{\gamma}\frac{\bar{V} + x\eta + \mathbb{E}_{U2}[\theta] - P_2}{1/\tau_u + 1/\tau_{U2}}, \tag{9}$$

respectively. The market clearing condition at date 2 is:

$$\lambda_2 X_{I2} + (1-\lambda_2) X_{U2} = \bar{Z} + z_2. \tag{10}$$

Re-arranging terms, we see that the market clearing price verifies the conjecture in (3). Similarly, the optimal demand for date 1 informed and uninformed investors are given by

$$X_{I1} = \frac{1}{\gamma}\frac{A_2 + B_2\theta + C_2 s_{p1} + x\bar{\eta} - P_1}{x^2/\tau_\eta + B_2^2/\tau_{p2}}, \quad \text{and} \quad X_{U1} = \frac{1}{\gamma}\frac{A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} + x\bar{\eta} - P_1}{x^2/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}}, \tag{11}$$

respectively, where investors understand $x = 1$ if $d = D$ and $x = 0$ if $d = ND$. Again, the date 1 market clearing condition, which is given by:

$$\lambda_1 X_{I1} + (1-\lambda_1) X_{U1} = \bar{Z} + z_1, \tag{12}$$

implies that the market clearing price verifies the conjecture in (3)-(4).

The following result characterizes the financial market equilibrium.

Lemma 1. *Fix the fraction of informed at each date i.e., $\lambda_1, \lambda_2 \in [0,1]$. There exists an equilibrium in which date 1 and 2 equilibrium prices are given by (3)-(4), where the price signals $s_{pt} \equiv \theta + b_t z_t$,*

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u}, \quad b_1 = -\frac{\gamma}{B_2 \lambda_1}\left(\frac{B_2^2\gamma^2}{\lambda_2^2\tau_u^2\tau_z} + \frac{x^2}{\tau_\eta}\right), \tag{13}$$

and the price coefficients A_1, A_2, B_1, B_2 and C_2 are characterized in the appendix.

3.2 Information acquisition choices

Given the characterization of the financial market equilibrium in the previous section, one can characterize the optimal information acquisition choices for generation t investors by comparing their expected utility with and without private information.

Let $\mathbb{E}_{t^-}[\cdot]$ refer to the expectation of generation t investors before they have acquired any information or observed the date t price. Then, the expected utility from acquiring information is given by:

$$U_{I,t} \equiv \mathbb{E}_{t^-}[\mathbb{E}_{It}[-e^{-\gamma\{W_t+X_{It}(P_{t+1}-P_t)-c\}}]], \tag{14}$$

while the expected utility from not acquiring information is given by:

$$U_{U,t} \equiv \mathbb{E}_{t^-}[\mathbb{E}_{Ut}[-e^{-\gamma\{W_t+X_{Ut}(P_{t+1}-P_t)\}}]]. \tag{15}$$

Standard calculations show that the relative expected utility can be expressed as:

$$\Gamma_t(\lambda_1, \lambda_2) \equiv \frac{U_{I,t}}{U_{U,t}} = e^{\gamma c}\sqrt{\frac{\mathbb{V}_{It}[P_{t+1}]}{\mathbb{V}_{Ut}[P_{t+1}]}} \tag{16}$$

just as in Grossman and Stiglitz (1980). Note that if $\Gamma_t(\lambda_t = 1) < 1$, then all investors in generation t choose to become informed (i.e., $\lambda_t = 1$), while if $\Gamma_t(\lambda_t = 0) > 1$, then no investors acquire information (i.e., $\lambda_t = 0$).

As is standard in the literature, in what follows we focus on equilibria featuring "interior" information choices i.e., $\lambda_1, \lambda_2 \in (0, 1)$ to keep the analysis transparent. In Appendix B.1, we characterize conditions under which such interior equilibria obtain. Consistent with intuition, the information equilibrium is interior when information costs neither "too high" (so that no investors acquire information) nor "too low" (so that all investors acquire information). Moreover, to ensure that date 1 information choices are interior when the firm has a short-term project (i.e., $x = 1$), the prior uncertainty about this projects payoff must not be too high (i.e., τ_η cannot be too low), because otherwise, no investors acquire information.

3.3 Disclosure decision

We begin by showing that the expected date 2 price is increasing in λ_2.

Lemma 2. *For a fixed x, the expected date 2 price $\mathbb{E}[P_2]$ is an increasing function of the fraction of investors who acquire information at date 2 (i.e., λ_2).*

Proof. Note that

$$\mathbb{E}[P_2] = \bar{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2)}{\mathbb{V}_{U2}[P_3]}}\bar{Z} + x\bar{\eta}. \tag{17}$$

Furthermore, when λ_2 is interior, it is pinned down by the information acquisition condition

$\Gamma_2(\lambda_2) = 1$, which implies

$$\mathbb{V}_{U2}[P_3] = e^{2\gamma c}\mathbb{V}_{I2}[P_3] \tag{18}$$

Hence, the weighted average precision can be expressed as:

$$\frac{\lambda_2}{\mathbb{V}_{I2}[P_3]} + \frac{(1-\lambda_2)}{\mathbb{V}_{U2}[P_3]} = \frac{1}{\mathbb{V}_{I2}\left[P_3\right]}\left(\lambda_2 \left(1 - e^{-2c\gamma}\right) + e^{-2c\gamma}\right). \tag{19}$$

Since $\mathbb{V}_{I2}\left[P_3\right] = 1/\tau_u$, implies that the expected price $\mathbb{E}\left[P_2\right]$ is an increasing function of λ_2. $\square$

The above result highlights a key feature of our setting: in equilibrium, the expected price increases in the fraction of investors informed at date 2. Intuitively, as more investors become informed, the equilibrium price becomes more informative about fundamentals. This implies that the weighted average precision increases, which in turn implies that the risk premium (price discount) is lower.

There are two notable features in the above analysis. First, the equilibrium posterior variance of uninformed and informed investors are proportional (i.e., equation (18) holds). This is an implication of equilibrium in information acquisition — an investor must be indifferent between paying the cost to acquire information and remaining uninformed — and arises generally in models with fixed costs of information (e.g., Grossman and Stiglitz (1980) and related models).

Second, the posterior variance of informed investors at date 2 ($\mathbb{V}_{I2}\left[P_3\right]$) does not depend on the fraction of informed investors λ_2. This is because informed investors observe a perfect signal about fundamentals (θ) if they choose to acquire information and consequently have nothing to learn from the price. However, this is not critical for the relation between expected price and the fraction λ_2 of informed investors, and similar results hold when informed investors observe a signal with noise.

Next, we show that the fraction of investors who acquire information at date 2 (i.e., λ_2) is higher with disclosure ($d = D$) than not ($d = ND$).

Lemma 3. *The date 1 price is less informative (i.e., the precision of the date-1 price signal, τ_{p1}, is lower) and date 2 information acquisition is higher (i.e., the date-2 fraction of informed investors, λ_2, is higher) when the manager discloses.*

This result is intuitive. When $d = D$, investors infer that $x = 1$, and so face higher unlearnable uncertainty at date 1. This leads informed investors to trade less aggressively on their information and makes the date 1 price less informative — this is apparent from the expression for date 1 demands in Equation (11).

The increase in uncertainty can also lead to more or less information acquisition at date 1 (i.e., λ_1 may be higher or lower). There are two forces that operate in different directions on the fraction informed when unlearnable uncertainty increases. First, because risk-averse traders anticipate trading less aggressively on their signals when uncertainty is higher, it makes acquiring information about the learnable component less valuable. This tends to reduce the fraction informed. On the other hand this reduction in trading aggressiveness tends to reduce the informativeness of the price-signal and so encourage more traders to acquire private information.Note that this is analogous to what Grossman and Stiglitz (1980) show for the effect of changes in residual uncertainty in their single period setting (see their Section II.H).[8]

However, the Lemma establishes that the impact on trading aggressiveness always dominates, and therefore the date 1 price is always less informative about fundamentals when the manager discloses the short term project.. In turn, this implies that prior to acquiring information, date 2 investors have a higher conditional variance about fundamentals θ, which leads to more information acquisition prior to the date 2 trading round.

We are now ready to establish the existence of the conjectured equilibrium.

Proposition 1. *Suppose the cost of disclosure $c_D > 0$ is not too large. Then, there exists an equilibrium in which a manager with a short-term project (i.e., $x = 1$) discloses this information (i.e., chooses $d = D$), but a manager without the project (i.e., $x = 0$) does not disclose (i.e., $d = ND$). Moreover, the expected long term price $\mathbb{E}[P_2]$ is higher with disclosure than without.*

Proof. To establish that the conjectured equilibrium is in fact an equilibrium, it suffices to show that each manager type prefers to play her conjectured strategy given the strategy of the other type. Consider first the manager without a project ($x = 0$). Taking the $x = 1$ manager's disclosure strategy as given, then if the $x = 0$ manager follows her conjectured strategy and does not disclose she is inferred to be the low type. If she deviates and discloses $x = 0$ then she is still identified as the low type and also pays the disclosure cost. Hence, playing her conjectured strategy is optimal.

Consider now the $x = 1$ manager. Taking the $x = 0$ manager's strategy of non-disclosure as given, we need to establish that the $x = 1$ manager prefers to disclose. Supposing that she instead deviates and refrains from disclosing, then in the conjectured equilibrium the

[8]Specifically, denote the terminal payoff by $\theta + u$ in their single period model. The fraction of informed investors is pinned down by $e^{\gamma c} = \sqrt{\frac{\tau_u}{\tau_U}}$, where $\tau_U = \left(\frac{1}{\tau_u} + \frac{1}{\tau_\theta + \tau_p}\right)^{-1}$ and $\tau_p = \frac{\lambda^2 \tau_u^2 \tau_z}{\gamma^2}$. The indifference condition implies that the (interior) equilibrium λ can be expressed as $\lambda = \frac{\gamma}{\sqrt{\tau_u \tau_z}} \sqrt{\frac{1}{e^{2\gamma c} - 1} - \frac{\tau_\theta}{\tau_u}}$, and so is hump-shaped in τ_u: it is increasing in τ_u when τ_u is low, but decreasing in τ_u when τ_u is high.

market will infer her to be an $x = 0$ type. The payoff from disclosure is

$$U_D(1) - c_D = \mathbb{E}[P_2(1, D)] - c_D \tag{20}$$

while the payoff from non-disclosure (given that $x\eta$ is publicly revealed at date two) is: $U_{ND}(1) = \mathbb{E}[P_2(1, ND)]$. The incremental benefit from disclosing versus not disclosing is

$$(U_D(1) - c_D) - U_{ND}(1) = \mathbb{E}[P_2(1, D)] - \mathbb{E}[P_2(1, ND)] - c_D. \tag{21}$$

Since $\lambda_2(D) > \lambda_2(ND)$ by Lemma 3 and since $\mathbb{E}[P_2]$ is increasing in λ_2 by Lemma 2, we have that

$$\mathbb{E}[P_2(1, D)] - \mathbb{E}[P_2(1, ND)] > 0, \tag{22}$$

which establishes the result about the expected price. Moreover, this implies that as long as the cost of disclosure c_D is not too high, it is optimal for the $x = 1$ manager to disclose. $\square$

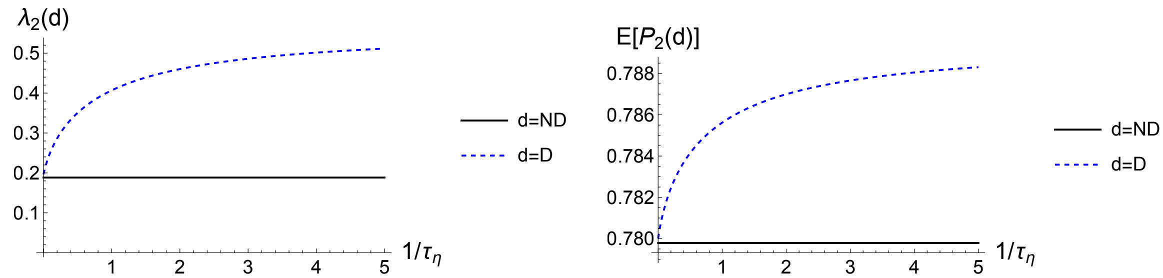
As discussed above, the key mechanism that causes disclosure to increase the expected price at date 2, $\mathbb{E}[P_2]$, is that the presence of the project increases the risk faced by investors at date 1. Consequently, they trade less aggressively, which decreases price informativeness and therefore incentivizes information collection at date 2. Figure 2 provides an illustration of this channel. Panel (a) plots the equilibrium fraction of investors who acquire information at date 2, and panel (b) plots the equilibrium expected date 2 price $\mathbb{E}[P_2]$, in the event of disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid), as a function of the variance of the project payoff, $1/\tau_\eta$. Note that the effect of disclosure on date 2 information acquisition and, consequently, on the expected date 2 price is higher when the project is risker. Intuitively, this is because the riskier the project is, the more that disclosing its presence harms date 1 price informativeness and therefore increases the value of acquiring information at date 2. It follows that the incentive to disclose is stronger for riskier projects.

3.4 Equilibrium uniqueness

Proposition 1 establishes the existence of an equilibrium of our conjectured form. However, it does not speak to the existence of other equilibria in which, e.g., both types do not disclose with positive probability. We will show below that within the class of equilibria in which the financial market is linear, the equilibrium we characterize is unique. To do so, we must entertain the possibility of managers following mixed disclosure strategies, so it is helpful to make explicit the dependence of expected P_2 on the market's belief about the manager's type. Hence, let $U_{ND}(x; q) = \mathbb{E}[P_2 | d = ND, x]$ denote the expected price as a function of

Figure 2: λ_2 and $\mathbb{E}[P_2]$ as a function of the project variance, $1/\tau_\eta$

The figure plots the fraction of informed investors at date 2 (i.e., λ_2) and the expected date 2 price as a function of the variance of the project payoff $1/\tau_\eta$ for disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid). The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_z = 1$,, $\bar{\eta} = 0$.



(a) λ_2 versus $1/\tau_\eta$ (b) $\mathbb{E}[P_2]$ versus $1/\tau_\eta$

x in the event that the manager does not disclose and the market assigns probability q to $x = 1$ in the event of no disclosure. Suppose that in the case that both types disclose with probability one, and hence the conditional probability given nondisclosure is not defined by Bayes rule, the market assigns off-equilibrium belief $q_{OFF} = 0$. That is, in the event of off-equilibrium non-disclosure the market assigns the manager the lowest type. The following Lemma establishes that we can rule out any equilibria with interior λ in which an $x = 0$ manager discloses with positive probability.

Lemma 4. *There do not exist equilibria in which an $x = 0$ manager discloses with strictly positive probability.*

Intuitively, in any conjectured equilibrium in which an $x = 0$ manager discloses, she can make herself strictly better off by refraining from disclosing, saving the disclosure cost, and, at worst, still being perfectly identified at an $x = 0$ type. Owing to Lemma 4, the only remaining candidate equilibria (with interior λ's) are those in which the $x = 0$ manager never discloses and the $x = 1$ manager discloses with probability r_1 that is strictly less than one, $r_1 \in [0, 1)$.

When the $x = 1$ manager mixes between disclosure and not, then when the market observes no disclosure, traders at date 1 perceive the future asset price as following a normal mixture distribution and there does not exist a linear equilibrium in the financial market, which we record in the following Proposition.

Proposition 2. *There do not exist equilibria in which an $x = 1$ manager discloses with probability less than one and the financial market equilibrium is linear.*

Hence, there are no equilibria in which the manager follows a mixed disclosure strategy (or never discloses) and asset prices are linear functions of the underlying shocks.

While we have focused on linear equilibria, we note that due to the inherent nonlinearity of the problem, it is unclear whether or not any noisy rational expectations equilibrium outside of the linear class even exists in the case of mixed strategy disclosure, and if so, what its properties are.[9] However, under an additional economically natural continuity assumption on the (admittedly endogenous) expected price in the event of nondisclosure, the following Proposition rules out mixed strategies when disclosure costs are sufficiently low and the prior probability p is sufficiently low.

Proposition 3. *Suppose that $U_{ND}(1;q)$ is continuous in q at $q=0$. Then if the disclosure cost c_D and the prior probability p that $x=1$ are sufficiently small, there does not exist an equilibrium in which an $x=1$ manager discloses with probability $r_1<1$.*

Intuitively, when p is very low, if an $x=1$ manager does not disclose the market assigns probability close to zero that she is the $x=1$ type. We know from Proposition 1 that, as long as costs are sufficiently small, if the market assigns probability *equal to* zero that $x=1$, then an $x=1$ manager finds it optimal to disclose and thereby identify herself to the market. Hence, under continuity of non-disclosure expected utility U_{ND} at $q=0$, it remains optimal for the $x=1$ manager to disclose when this probability is positive but small.

4 Discussion and Extensions

Our model is stylized for tractability. In this Section, we provide discussion of our assumptions and how our mechanism extends to more general settings.

We focus on a manager who is concerned with the "long run", post-disclosure price P_2 so that the disclosure is unambiguously redundant by the time her "utility" is realized. This is the starkest setting in which to illustrate that, despite the redundancy, she may still find it optimal to engage in costly disclosure. When the manager also cares about the "short run" price P_1, which is directly affected by disclosure, then she may have additional incentives to do so (e.g., if η had a sufficiently positive mean).[10] One could also allow the manager's

[9]Such a setting does not meet any known conditions for characterizing equilibria outside of the linear class, such as the "exponential family" condition of Breon-Drish (2015). More generally, characterizing the existence and uniqueness of rational expectations equilibria in settings where payoffs follow more general non-exponential family distributions (e.g., normal mixture distributions, as in this case) is a difficult open problem in the literature, and is beyond the scope of this paper.

[10]As discussed earlier, the literature has also considered alternative settings in which disclosure encourages short-term information acquisition, which would partially mitigate the economic channel we focus on. However, in such settings, the disclosed information is not redundant in the sense we capture.

objective to depend on the terminal price $P_3 = \bar{V} + x\eta + \theta + u$, but because this quantity is exogenous, excluding it is without loss of generality. While it would be interesting to extend our analysis to solve for the manager's objective as part of an optimal contract, such an extension is beyond the scope of the current paper. However, we expect that our mechanism will be present, qualitatively, in any situation in which the optimal contract places positive weight on the long-run price.

It is not necessary for the payoff $x\eta$ to be perfectly publicly revealed at date 2 for the initial disclosure to become redundant and for our mechanism to operate – a noisy signal at date 2 about $x\eta$ would have qualitatively similar implications. Similarly, one could allow the disclosure of $x = 1$ at date 1 to be accompanied by a noisy, public signal about the realization of η e.g., $s_\eta = \eta + \varepsilon_\eta$ with ε_η independently normally distributed. In this case, our results would be qualitatively unchanged – we would only have to change the notation in the analysis to replace the unconditional moments of η by their conditional counterparts (i.e., replace $\bar{\eta}$ by $\mathbb{E}[\eta|s_\eta]$ and σ_η^2 by $\mathbb{V}[\eta|s_\eta]$).

All of our results go through as stated for a more general setting $x \in \{x_L, x_H\}$ for any nonnegative $x_L < x_H$, and where we interpret the manager's disclosure as being directly about the riskiness of the short-term project. This is because the manager does not choose whether to make the investment, but only to disclose whether the firm has the project. [11] We expect our results to be qualitatively similar if x follows a more general discrete distribution with more than two states and the disclosure cost is sufficiently small. In this case, we conjecture that all managers above the lowest possible realization of x will find it optimal to disclose and the lowest possible type will refrain from disclosing. Moreover, while we expect that our mechanism is qualitatively robust to even more general distributions and cost functions, such settings are generally intractable.[12]

We consider a setting with short-lived investors in order to transparently and tractably illustrate the important economic forces. In Section 4.1 below, we consider a fully dynamic version of our model, in which investors are long-lived and can acquire a signal at a time of their choosing. We show that our main result obtains for a range of parameter values: that is, disclosing higher x leads to more information acquisition at date 2, and consequently, higher expected price.

Similarly, the assumption that asset supply is i.i.d. is for simplicity. In Section 4.2 below,

[11] In this setting, we would still have that disclosing x_H leads investors to face more uncertainty at date 1, which makes date 1 prices less informative, and consequently, leads to more information acquisition at date 2. As such, firms with $x = x_H$ would disclose this information, while firms with $x = x_L$ would be indifferent between disclosing and not.

[12] In particular, as we have discussed in Section 3.4, if, in equilibrium, investors place positive probability on more than one value of x, then there is no longer a linear equilibrium in the financial market, and it is not possible to characterize the equilibrium, or even demonstrate existence.

we show that our results are robust to correlated asset supply (i.e., $Z_t = \overline{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t$ for $\phi \neq 0$). What is important is that disclosure about the existence of the project reduces price informativeness in the first trading round by making the asset riskier (and thereby inducing informed investors to trade less aggressively), which increases the value of acquiring information prior to the $t = 2$ trading round.

The details of the analysis for these extensions are presented in Appendix B.

4.1 Long lived investors

In Appendix B.2 we show our results generalize for a range of parameter values in a setting in which investors are long-lived and can acquire information at either date 1 or 2. Specifically, we assume that the asset payoff and supply dynamics are the same as in the benchmark setting. However, in contrast to the assumption of short-lived investors in the benchmark setting, we assume that there is a unit mass of long-lived investors with CARA (γ) utility over $t = 3$ wealth who participate at both trading dates. Each investor i can pay a cost to observe θ immediately before trading at the date of her choosing (i.e., at $t = 1^-$ or $t = 2^-$).[13]

In order to sustain an equilibrium with information acquisition at $t = 2^-$ we assume time-dependent information costs c_t with $c_1 > c_2$. This is because the gross value of information decreases deterministically in time. If c was constant in time, then in any conjectured interior equilibrium with nonzero acquisition at the second date, we necessarily have a profitable deviation for any investor who is currently acquiring at $t = 2^-$ to instead acquire at $t = 1^-$, pay the same cost, and yet obtain strictly higher expected utility.

Investors submit demand schedules $X_{it}, t \in \{1, 2\}$ to maximize expected utility over terminal wealth. We subscript quantities associated with an investor informed at $t = 1^-$ by $I1$, at $t = 2^-$ by $I2$, and those who choose to remain uninformed by $U1$ and $U2$. The fraction of investors who are informed at date t is denoted by λ_t.

As in the earlier analysis, we conjecture and verify that there exists a financial market equilibrium in which prices are of the form:

$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{23}$$

where the $s_{pt} \equiv \theta + b_t(Z_t - \bar{Z})$ for $t \in \{1, 2\}$. Relative to the benchmark analysis in Section 3, there are two notable changes. First, the optimal demand for investors at date $t = 1$ reflects a dynamic hedging demand. Specifically, while the optimal demand for investor i at date 2

[13] Since the information available to investors is the same in either period, investors will not choose to acquire information in both periods.

is given by

$$X_{i1} = \frac{\mathbb{E}_{i2}[V] - P_2}{\gamma \mathbb{V}_{i2}[P_3]}, \tag{24}$$

as before, we show that the optimal demand for investor i at date 1 can be expressed as:

$$X_{i1} = \frac{1}{\gamma} \frac{\mathbb{E}_{i1}[P_2 - P_1 - \beta_{i1}(V - P_2)]}{\mathbb{V}_{i1}(P_2 - P_1 - \beta_{i1}(V - P_2))} \tag{25}$$

where $\beta_{i1} = \frac{\mathbb{C}_{i1}(V - P_2, P_2 - P_1)}{\mathbb{V}_{i1}(V - P_2)}$ is the conditional regression coefficient of $P_2 - P_1$ on $V - P_2$ given investor i's information set. Using these demands, we can solve for the equilibrium price coefficients by imposing market clearing at both dates.

Second, while the date $t = 2^-$ information equilibrium condition (for an interior equilibrium) is given by

$$\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}} e^{\gamma c_2} = 1, \tag{26}$$

as in the benchmark analysis, the date $t = 1^-$ information equilibrium condition reduces to

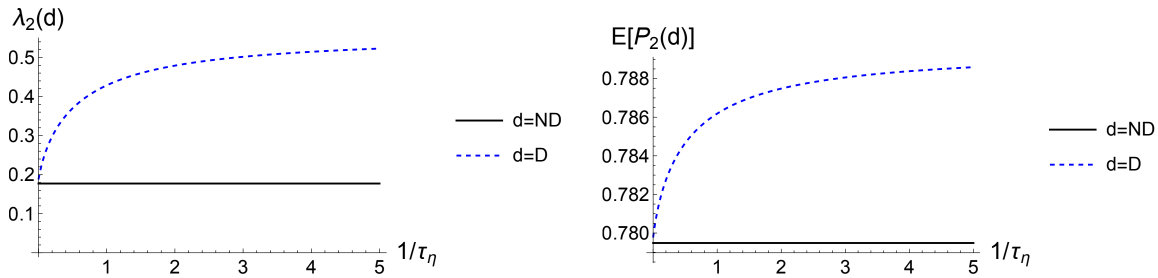
$$e^{\gamma(c_1 - c_2)} \sqrt{\frac{\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))}} = 1. \tag{27}$$

This reflects the fact that investors have the option to wait until date $t = 2^-$ to acquire information, and so will only acquire information at date 1 (i.e., pay a cost c_1 instead of c_2), if the reduction in the variance of the orthogonal part of the date 1 return (i.e., the part of return $P_2 - P_1$ that is conditionally independent of $V - P_2$) is sufficiently large relative to the incremental information cost $c_1 - c_2$. Together, equations (26) and (27) pin down the equilibrium fractions of informed traders, λ_1 and λ_2, in any interior equilibrium.

Note that Lemma 2 applies directly in this setting, since the date 2 price has the same functional form as in the benchmark model and the date $t = 2^-$ information condition is the same. Hence, the expected date 2 price increases with disclosure if and only if the date 2 fraction of informed traders λ_2 increases with disclosure. While an analytical proof of Lemma 3 is not tractable, we can numerically show that the key result obtains i.e., $\lambda_2(D) > \lambda_2(ND)$ in this setting for a range of parameter values. As a result, the analog of Proposition (1) applies even when investors are not myopic and can choose when to acquire information. Figure 3 provides an illustration of these results. As in our benchmark analysis, both the fraction of informed investors (Panel (a)) and the expected price at date 2 (Panel (b)) are higher with disclosure than without, and the effect of disclosure on both the fraction informed and the expected price is stronger when the project is riskier ($1/\tau_\eta$ is higher).

Figure 3: λ_2 and $\mathbb{E}[P_2]$ as a function of the project variance, $1/\tau_\eta$, with long-lived investors

The figure plots the fraction of informed investors at date 2 (i.e., λ_2) and the expected date 2 price as a function of the variance of the project payoff $1/\tau_\eta$ for disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid). The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c_1 = 0.7$, $c_2 = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_z = 1$, $\bar{\eta} = 0$.



(a) λ_2 versus $1/\tau_\eta$ (b) $\mathbb{E}[P_2]$ versus $1/\tau_\eta$

4.2 Persistent aggregate supply shocks

In Appendix B.3, we consider an extension of our benchmark model that retains the assumption of short-lived investors but extends the setting to allow for persistence in the supply shocks. Specifically, suppose that the aggregate supply of the stock follows an AR(1) process i.e.,

$$Z_t = \overline{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t; \qquad Z_0 \equiv \bar{Z}, \tag{28}$$

for $\phi \in (0, 1)$ and z_t independently normally distributed with precisions τ_{zt}. This nests our benchmark setting as the special case in which $\phi = 0$ and $\tau_{z1} = \tau_{z2} = \tau_z$.

We conjecture and verify that there exists a financial market equilibrium in which prices are of the form:

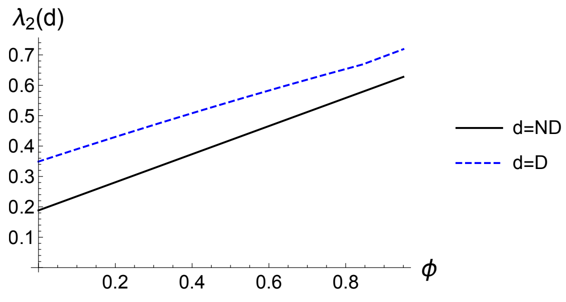
$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{29}$$

where the $s_{pt} \equiv \theta + b_t(Z_t - \bar{Z})$ for $t \in \{1, 2\}$. While much of the analysis follows from that in the benchmark model, a key difference is that the price signals s_{p1} and s_{p2} are now correlated, which affects the date 1 investors' beliefs about date 2 prices, and date 2 investors' beliefs about the terminal payoff. The explicit calculations are provided in Appendix B.

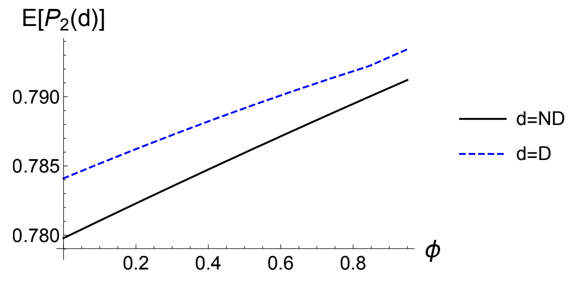
Given these differences, however, the functional forms for demand functions and the information conditions are analogous to those in Section 3, and a version of Lemma 1 obtains. Moreover, Lemma 2 applies directly in this setting. Although we expect the analog to Lemma 3 to hold, analytically establishing this result is intractable. However, we can demonstrate

Figure 4: λ_2 and $\mathbb{E}[P_2]$ as a function of persistence ϕ of supply shocks

The figure plots the fraction of informed investors at date 2 (i.e., λ_2) and the expected date 2 price as a function of ϕ, for disclosure (i.e., $d = D$, dashed) versus no-disclosure (i.e., $d = ND$, solid). The other parameter values are as follows: $\bar{V} = 1$, $\gamma = 0.2$, $c_1 = c_2 = 0.3$, $\tau_\theta = 1$, $\tau_u = 1$, $\tau_{z1} = \tau_{z2} = 1$, $\tau_\eta = 2$, $\bar{\eta} = 0$.



(a) λ_2 versus ϕ



(b) $\mathbb{E}[P_2]$ versus ϕ

numerically that the result obtains i.e.,

$$\lambda_2(D) \geq \lambda_2(ND), \tag{30}$$

for a wide range of parameters, and consequently, Proposition 1 applies in this setting.

Figure 4 provides an illustration. Panel (a) plots the fraction of investors who acquire information at date 2^-, and panel (b) plots the expected price at date 2. As in our benchmark model, disclosure leads to more information acquisition and higher expected prices. Notably, the fraction of investors who acquire information at date 2 tends to increase with ϕ. Intuitively, this is because when noise is persistent, the date 2 price is less (incrementally) informative given the public information, since the noise in the date 1 and date 2 signals are correlated. As a result, the incremental value of acquiring information is higher in this case. Similar results hold in the long-lived investor setting of Section 4.1 above, with persistent supply shocks.[14]

[14]The setting in Appendix B.2 already incorporates both long-lived investors and potentially persistent shocks.

5 Concluding Remarks

We propose a novel rationale for voluntary disclosure, by studying how voluntary, costly disclosure affects information acquisition in a dynamic model of trading. We show that a manager finds it optimal to disclose information that becomes redundant at a later date, even if she intends to maximize long-term share prices. By disclosing information about the presence of a short-term risky project, the manager increases perceived risk and reduces price informativeness in early periods. Once the payoffs of this short-term project are revealed, later investors acquire information more aggressively than they would have if the manager had not disclosed the project earlier. We show that increased information acquisition by later investors can dominate the short-term increase in uncertainty, and lead to long-term prices that are more informative and higher on average. Furthermore, the impact of disclosure on investors' information acquisition and longer term prices is likely to be larger for firms in industries where cashflows of typical projects are more uncertain and dispersed.

Our analysis suggests a number of natural extensions. For instance, it would be interesting to consider the strategic timing of voluntary disclosure (as in Guttman, Kremer, and Skrzypacz (2014)) in a setting with dynamic information acquisition. It might also be interesting to study how voluntary disclosure is affected by dynamic information acquisition in the presence of real investment and feedback effects. Finally, one could endogenize the manager's objective as part of an optimal contracting problem and study disclosure policies in the resulting equilibrium. We leave these questions for future work.

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A Proofs

A.1 Proof of Lemma 1

We can solve for the coefficients b_1 and b_2 by observing that $s_{p,t}$ is a linear transformation of the informed residual demand $\lambda_t X_{It} - z_t$. This implies:

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u}, \quad b_1 = -\frac{\gamma}{\lambda_1} \frac{x^2/\tau_\eta + B_2^2/\tau_{p2}}{B_2}. \tag{31}$$

Next, we solve for the price coefficients by imposing market clearing and matching coefficients. Specifically, note that B_2 is given by:

$$B_2 = \frac{\lambda_2 \left(\tau_{U1} + \tau_u\right) + \tau_{p2}}{\tau_{U1} + \lambda_2 \tau_u + \tau_{p2}}, \tag{32}$$

where $\tau_{U1} = \tau_\theta + \tau_{p1}$, and $\tau_{pt} = \tau_z / b_t^2$. Substituting, this implies that B_2 is a solution to $H(B_2) = 0$, where

$$H(B_2) = \frac{\lambda_2 \left(\gamma^2 \left(\frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2} + \tau_\theta + \tau_u \right) + \lambda_2 \tau_u^2 \tau_z \right)}{\gamma^2 \left(\frac{B_2^2 \lambda_1^2 \tau_\eta^2 \tau_z^3}{\left(\frac{B_2^2 \gamma^3 \tau_\eta}{\lambda_2^2 \tau_u^2} + \gamma x^2 \tau_z \right)^2} + \tau_\theta + \lambda_2 \tau_u \right) + \lambda_2^2 \tau_u^2 \tau_z} - B_2 \tag{33}$$

Note that

$$\begin{aligned} H(0) &= \frac{\lambda_2 \left(\gamma^2 \left(\tau_\theta + \tau_u\right) + \lambda_2 \tau_u^2 \tau_z\right)}{\gamma^2 \left(\tau_\theta + \lambda_2 \tau_u\right) + \lambda_2^2 \tau_u^2 \tau_z} > 0 \\ H(1) &= \frac{(\lambda_2 - 1)\left(\tau_\theta \left(\gamma^3 \tau_\eta + \gamma \lambda_2^2 x^2 \tau_u^2 \tau_z\right)^2 + \lambda_1^2 \lambda_2^4 \tau_\eta^2 \tau_u^4 \tau_z^3\right)}{\tau_\theta \left(\gamma^3 \tau_\eta + \gamma \lambda_2^2 x^2 \tau_u^2 \tau_z\right)^2 + 2\gamma^2 \lambda_2^3 x^2 \tau_\eta \tau_u^3 \tau_z (\gamma^2 + \lambda_2 \tau_u \tau_z) + \lambda_2^5 x^4 \tau_u^5 \tau_z^2 (\gamma^2 + \lambda_2 \tau_u \tau_z) + \lambda_2 \tau_\eta^2 \tau_u \left(\gamma^6 + \gamma^4 \lambda_2 \tau_u \tau_z + \lambda_1^2 \lambda_2^3 \tau_u^3 \tau_z^3\right)} \\ &\leq 0, \end{aligned}$$

since $\lambda_2 \leq 1$, which implies there exists a solution to $H(B_2) = 0$ for $B_2 \in (0, 1)$.

Given B_2, we can solve for (b_1, b_2), and then solve for the other coefficients using the following system:

$$A_2 = \bar{V} - \frac{\gamma \bar{Z} \left(\gamma^2 \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \tau_u \right) + \lambda_2^2 \tau_u^2 \tau_z \right)}{\gamma^2 \tau_u \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \lambda_2 \tau_u \right) + \lambda_2^2 \tau_u^3 \tau_z} \tag{34}$$

$$A_1 = A_2 - \frac{\gamma \bar{Z}}{\frac{1-\lambda_1}{B_2^2 \left(\frac{1}{\frac{\tau_z}{b_1^2} + \tau_\theta} + \frac{b_2^2}{\tau_z} \right) + \frac{x^2}{\tau_\eta}} + \frac{\lambda_1 \tau_\eta \tau_z}{b_2^2 B_2^2 \tau_\eta + x^2 \tau_z}} + x \bar{\eta} \tag{35}$$

$$C_2 = \frac{b_2^2 (1 - \lambda_2) \tau_z}{b_1^2 \left(b_2^2 \left(\frac{\tau_z}{b_1^2} + \tau_\theta + \lambda_2 \tau_u \right) + \tau_z \right)} \tag{36}$$

$$B_1 = \frac{b_1^2 \tau_\theta (B_2 \lambda_1 + C_2) (b_2^2 B_2^2 \tau_\eta + x^2 \tau_z) + (B_2 + C_2) \tau_z (B_2^2 (b_1^2 \lambda_1 + b_2^2) \tau_\eta + x^2 \tau_z)}{B_2^2 \tau_\eta (b_2^2 (b_1^2 \tau_\theta + \tau_z) + b_1^2 \lambda_1 \tau_z) + x^2 \tau_z (b_1^2 \tau_\theta + \tau_z)}. \tag{37}$$

□

A.2 Proof of Lemma 3

We first establish the claim about the fraction informed, λ_2, and then show that it implies the claim about the precision τ_{p1}. For an interior λ_1, λ_2, note that the information equilibrium conditions imply

$$W = \frac{\mathbb{V}_{Ut}[\mathbb{E}_{It}[P_{t+1}]]}{\mathbb{V}_{It}[P_{t+1}]} \tag{38}$$

where $W \equiv e^{2\gamma c} - 1$. Substituting in explicitly, the information equilibrium conditions for dates $t = 2$ and $t = 1$ are given by

$$W = \frac{1/\tau_{U2}}{1/\tau_u}, \quad \text{and} \quad W = \frac{B_2^2/\tau_{U1} + x^2/\tau_\eta}{B_2^2/\tau_{p2} + x^2/\tau_\eta}, \tag{39}$$

respectively. Plugging in $\tau_{U2} = \tau_{U1} + \tau_{p2}$ and rearranging terms gives:

$$\tau_e = W (\tau_{U1} + \tau_{p2}) \tag{40}$$

$$\frac{B_2^2}{(B_2^2/\tau_{p2} + x^2/\tau_\eta)} = W \tau_{U1} \tag{41}$$

Next, recall that since B_2 is given by

$$B_2 = \frac{\lambda_2 (\tau_{U1} + \tau_u) + \tau_{p2}}{\tau_{U1} + \lambda_2 \tau_u + \tau_{p2}} \tag{42}$$

we can substitute in to express it as

$$B_2 = \frac{\lambda_2 \tau_u (1+W) + W\tau_{p2}(1-\lambda_2)}{(1+\lambda_2 W)\tau_u} \tag{43}$$

$$= \frac{\lambda_2 \left(\gamma^2 (W+1) + (1-\lambda_2)\lambda_2 \tau_z W \tau_u\right)}{\gamma^2 (\lambda_2 W + 1)} \tag{44}$$

Combining (40) and (41), plugging in (44), and rearranging, characterizes the equilibrium relation between λ_2 and x in any interior equilibrium:

$$\frac{x^2}{\tau_\eta} = \frac{B_2^2}{\tau_{p2}} \frac{(W+1)\tau_{p2} - \tau_u}{\tau_u - W\tau_{p2}} \tag{45}$$

$$= \frac{\lambda_2^2 \left(\gamma^2 (1+W) + (1-\lambda_2)\lambda_2 W \tau_u \tau_z\right)^2}{\gamma^4 (1+\lambda_2 W)^2} \frac{(W+1)\tau_{p2} - \tau_u}{\tau_{p2}(\tau_u - W\tau_{p2})} \tag{46}$$

$$= \frac{\left(\gamma^2 (W+1) + (1-\lambda_2)\lambda_2 W \tau_u \tau_z\right)^2 \left(\gamma^2 - \lambda_2^2 (W+1)\tau_u \tau_z\right)}{\gamma^2 \tau_u^2 (\lambda_2 W + 1)^2 \tau_z \left(\lambda_2^2 W \tau_u \tau_z - \gamma^2\right)} \equiv G(\lambda_2) \tag{47}$$

where the last line defines the function $G(\lambda_2)$ in order to condense notation. Note that $G(0) = -\frac{\gamma^2 (W+1)^2}{\tau_u^2 \tau_z} < 0$ and $G(1) = -\frac{\gamma^2\left(\gamma^2 - (W+1)\tau_u \tau_z\right)}{\tau_u^2 \tau_z (\gamma^2 - W\tau_u \tau_z)}$. For the equilibrium $\lambda_2 \in (0,1)$ to exist, we need to have: $G(1) > \frac{x^2}{\tau_\eta} > 0 > G(0)$, which is equivalent to restricting

$$\frac{\gamma^2}{(1+W)\tau_z} < \tau_u < \frac{\gamma^2}{W\tau_z} \tag{48}$$

$$\Leftrightarrow \gamma^2 > W\tau_u \tau_z, \tag{49}$$

$$\gamma^2 < (1+W)\tau_u \tau_z \tag{50}$$

Moreover, tedious algebra establishes that

$$G(\lambda_2) = \frac{\left(\frac{W+1}{W} - \frac{\gamma^2}{\lambda_2^2 W \tau_u \tau_z}\right)\left(1 - \frac{(1-\lambda_2)\left(\frac{1}{W} - \frac{\lambda_2^2 \tau_u \tau_z}{\gamma^2}\right)}{\lambda_2 + \frac{1}{W}}\right)^2}{\tau_u \left(\frac{1}{W} - \frac{\lambda_2^2 \tau_u \tau_z}{\gamma^2}\right)} \equiv g_1(\lambda_2)\, g_2(\lambda_2)\, g_3(\lambda_2) \tag{51}$$

By the implicit function theorem, in order to show that λ_2 is increasing in x^2, we need to show that $G(\lambda_2)$ above is increasing in λ_2. Now,

$$g_3(\lambda_2) = \frac{1}{\tau_u \left(\frac{1}{W} - \frac{\lambda_2^2 \tau_u \tau_z}{\gamma^2}\right)} > 0 \tag{52}$$

from (48), and $g_3(\lambda_2)$ is increasing in λ_2. Next,

$$g_2(\lambda_2) = \left(1 - \frac{(1-\lambda_2)\left(\frac{1}{W} - \frac{\lambda_2^2 \tau_u \tau_z}{\gamma^2}\right)}{\lambda_2 + \frac{1}{W}}\right)^2 > 0 \tag{53}$$

and since $\frac{1}{W} - \frac{\lambda_2^2 \tau_u \tau_z}{\gamma^2} > 0$, $g_2(\lambda_2)$ is increasing in λ_2. Finally, $g_1(\lambda_2)$ is increasing in λ_2. Moreover, it must be positive in an interior equilibrium since $G(\lambda_2) = \frac{x^2}{\tau_\eta} \geq 0$ and we already know that g_2 and g_3 are positive. This implies

$$\frac{dG}{d\lambda_2} = g_1' g_2 g_3 + g_1 g_2' g_3 + g_1 g_2 g_3' > 0. \tag{54}$$

To summarize this shows that $G(\lambda_2)$ is increasing in λ_2. Since $G(\lambda_2) = \frac{x^2}{\tau_\eta}$, this implies that the equilibrium λ_2 is increasing in x^2 as long as λ_1 and λ_2 are interior.

Now consider the $t = 1$ price-signal precision τ_{p1}. Once again using the $t = 2$ information equilibrium condition, we can write λ_2 explicitly in any interior equilibrium:

$$W = \frac{1/\tau_{U2}}{1/\tau_u} \tag{55}$$

$$\Rightarrow e^{2\gamma c} - 1 = \frac{\tau_u}{\tau_\theta + \tau_{p1} + \tau_z \left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} \tag{56}$$

$$\Rightarrow \lambda_2 = \gamma \frac{1}{\sqrt{\tau_u \tau_z}} \left(\frac{1}{e^{2\gamma c} - 1} - \frac{\tau_\theta + \tau_{p1}}{\tau_u}\right)^{1/2} \tag{57}$$

the only term on the right-hand side of eq. (57) that depends on the manager's disclosure x is the $t = 1$ price-signal precision τ_{p1} and we have already shown that λ_2 increases in x^2. Hence, it must be the case that τ_{p1} decreases in x^2.

□

A.3 Proof of Lemma 4

Suppose to the contrary that there exists an interior equilibrium in which an $x = 0$ manager discloses with probability $r_0 \in (0, 1]$, an $x = 1$ manager discloses with probability $r_1 \in [0, 1]$, and we do not have $r_0 = r_1 = 1$ (we consider this case separately below). In such an equilibrium, the market assigns probability

$$q(r_0, r_1) = \frac{p(1 - r_1)}{(1 - p)(1 - r_0) + p(1 - r_1)}$$

that $x = 1$ in the event of no disclosure. Consider first the case $r_1 = 1$. In this case the market assigns probability $q = 0$ and therefore the expected price for an $x = 0$ manager is identical whether she discloses or not, $U_{ND}(0; q) = U_D(0)$. Hence,

$$U_D(0) - c_D < U_{ND}(0; q),$$

which implies that the $x = 0$ manager strictly prefers not disclosing. Consider next the case in which $r_1 \in [0, 1)$. In this case, because the $x = 1$ manager does not disclose with positive probability, then we know that she is either indifferent (in the case $r_1 \in (0, 1)$) or strictly prefers not disclosing (in the case $r_1 = 0$), which implies:

$$\begin{aligned} &U_D(1) - c_D - U_{ND}(1; q) \leq 0 \\ \Rightarrow\, &U_D(0) - c_D - U_{ND}(0; q) < 0 \end{aligned}$$

where the second line follows from Lemma 2, which establishes that $U_D(1) > U_D(0)$, and the observation that $U_{ND}(1; q) = U_{ND}(0; q)$, since the expected non-disclosure price only depends on the market's beliefs q and not the realized value of x. This implies that the $x = 0$ manager strictly prefers not disclosing. Finally, consider the case in which $r_0 = r_1 = 1$. In this case, Bayes rule does not pin down the probability that the market assigns to $x = 1$ in event of nondisclosure. Given off-equilibrium belief $q_{OFF} = 0$, we again have $U_{ND}(0; q_{OFF}) = U_D(0)$ and hence

$$U_D(0) - c_D < U_{ND}(0; q)$$

so that the $x = 0$ manager strictly prefers not disclosing. □

A.4 Proof of Proposition 2

Suppose that there exists an equilibrium in which the manager follows a disclosure strategy $r_0 = 0$, $r_1 \in [0, 1)$ and asset prices in the event of nondisclosure $d = ND$ are linear functions of fundamentals

$$\begin{aligned} P_1(ND) &= A_1(ND) + B_1(ND)\, s_{p1}, \quad \text{and} \\ P_2(x, ND) &= A_2(ND) + B_2(ND)\, s_{p2} + C_2(ND)\, s_{p1} + x\eta \end{aligned}$$

where the price signals $s_{pt} \equiv \theta + b_t\,(ND)\,z_t$ for $t \in \{1, 2\}$, and we define the precisions $\tau_{pt} \equiv \tau_z / b_t^2$. In the analysis that follows we will suppress the explicit dependence of the coefficients on the event of nondisclosure in order to reduce clutter. Let $\mathcal{F}_{It} = \sigma\left(d, \theta, \{P_k\}_{k \le t}\right) = \sigma\left(d, \theta, \{s_{pk}\}_{k \le t}\right)$ and $\mathcal{F}_{Ut} = \sigma\left(d, \{P_k\}_{k \le t}\right) = \sigma\left(d, \{s_{pk}\}_{k \le t}\right)$ denote the information sets at time t for informed and uninformed investors, respectively, with conditional expectation and variance operators $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$, $i \in \{I, U\}$.

In an equilibrium of the posited form, all investors at the date 1 trading round assign probability

$$q(r_0 = 0, r_1) = \frac{p(1 - r_1)}{(1 - p) + p(1 - r_1)} \in (p, 1)$$

that the firm has a project, $x = 1$. Consider the problem of an arbitrary informed investor at date 1

$$\max_X \mathbb{E}_{I1}\left[-e^{-\gamma X (P_2 - P_1)}\right]$$

Computing the expected utility in the objective function yields

$$\begin{aligned}
&\mathbb{E}_{I1}\left[-e^{-\gamma X (P_2(x) - P_1)}\right] \\
&= q\mathbb{E}_{I1}\left[-e^{-\gamma X (P_2(x) - P_1)} \middle| x = 1\right] + (1 - q)\,\mathbb{E}_{I1}\left[-e^{-\gamma X (P_2(x) - P_1)} \middle| x = 0\right] \\
&= -qe^{-\gamma X (\mathbb{E}_{11}[P_2(1)] - P_1) + \frac{1}{2}\gamma^2 X^2 \mathbb{V}_{I1}(P_2(1))} - (1 - q)\,e^{-\gamma X (\mathbb{E}_{11}[P_2(0)] - P_1) + \frac{1}{2}\gamma^2 X^2 \mathbb{V}_{I1}(P_2(0))} \\
&= -qe^{-\gamma X (A_2 + B_2\theta + C_2 s_{p1} + \bar{\eta} - P_1) + \frac{1}{2}\gamma^2 X^2 \left(1/\tau_\eta + B_2^2/\tau_{p2}\right)} - (1 - q)\,e^{-\gamma X (A_2 + B_2\theta + C_2 s_{p1} - P_1) + \frac{1}{2}\gamma^2 X^2 B_2^2/\tau_{p2}}
\end{aligned}$$

where the first equality uses the law of iterated expectations to condition down on x and the second equality uses the fact that, given x, the second period price is conditionally Normally distributed under the informed investor information set, and the final equality plugs in for the conditional means and variances. The investor's maximization problem is strictly concave and defined for demands X on the entire real line. Hence, there is a unique optimal demand

X_{I1}, for which the FOC is necessary and sufficient:

$$\begin{aligned}0 &= q\left(A_2 + B_2\theta + C_2 s_{p1} + \bar{\eta} - P_1 - \gamma X_{I1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}}\right)\right) e^{-\gamma X_{I1}(A_2+B_2\theta+C_2 s_{p1}+\bar{\eta}-P_1)+\frac{1}{2}\gamma^2 X_{I1}^2\left(1/\tau_\eta + B_2^2/\tau_{p2}\right)} \\ &\quad + (1-q)\left(A_2 + B_2\theta + C_2 s_{p1} - P_1 - \gamma X_{I1}\tfrac{B_2^2}{\tau_{p2}}\right) e^{-\gamma X_{I1}(A_2+B_2\theta+C_2 s_{p1}-P_1)+\frac{1}{2}\gamma^2 X_{I1}^2 B_2^2/\tau_{p2}} \\ &= q\left(A_2 + B_2\theta + C_2\frac{P_1 - A_1}{B_1} + \bar{\eta} - P_1 - \gamma X_{I1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}}\right)\right) e^{-\gamma X_{I1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} \\ &\quad + (1-q)\left(A_2 + B_2\theta + C_2\frac{P_1 - A_1}{B_1} - P_1 - \gamma X_{I1}\tfrac{B_2^2}{\tau_{p2}}\right)\end{aligned} \tag{58}$$

where the second equality divides out terms that are common across the two exponentials and plugs in explicitly for s_{p1} in terms of the price P_1 using the initial functional form conjecture. Equation (58) does not have a closed form solution, but it uniquely characterizes the informed demand function $X_{I1}(\theta, P_1)$.

Similarly, consider the problem of an arbitrary uninformed investor at date 1

$$\max_X \mathbb{E}_{U1}\left[-e^{-\gamma X(P_2 - P_1)}\right].$$

Under the conjectured price functions and resulting uninformed information set, for the date 1 uninformed investors, the fundamental θ is conditionally normally distributed with conditional mean and variance

$$\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1} s_{p1}}{\tau_\theta + \tau_{p1}} = \frac{\tau_{p1}}{\tau_\theta + \tau_{p1}}\frac{P_1 - A_1}{B_1}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}. \tag{59}$$

Hence, computing the expected utility in the objective function yields

$$\begin{aligned}&\mathbb{E}_{U1}\left[-e^{-\gamma X(P_2(x) - P_1)}\right] \\ &= q\mathbb{E}_{U1}\left[-e^{-\gamma X(P_2(x)-P_1)}\middle| x = 1\right] + (1-q)\,\mathbb{E}_{U1}\left[-e^{-\gamma X(P_2(x)-P_1)}\middle| x = 0\right] \\ &= -qe^{-\gamma X(\mathbb{E}_{U1}[P_2(1)]-P_1)+\frac{1}{2}\gamma^2 X^2 \mathbb{V}_{U1}(P_2(1))} - (1-q)\,e^{-\gamma X(\mathbb{E}_{U1}[P_2(0)]-P_1)+\frac{1}{2}\gamma^2 X^2 \mathbb{V}_{U1}(P_2(0))} \\ &= -qe^{-\gamma X(A_2+B_2\mathbb{E}_{U1}[\theta]+C_2 s_{p1}+\bar{\eta}-P_1)+\frac{1}{2}\gamma^2 X^2\left(1/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)} \\ &\qquad - (1-q)\,e^{-\gamma X(A_2+B_2\mathbb{E}_{U1}[\theta]+C_2 s_{p1}-P_1)+\frac{1}{2}\gamma^2 X^2\left(B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)}\end{aligned}$$

where the first equality uses the law of iterated expectations to condition down on x and the second equality uses the fact that, given x, the second period price is conditionally Normally distributed under the uninformed investor information set, and the final equality plugs in for the conditional means and variances. As for an informed investor, an uninformed investor's maximization problem is strictly concave and defined for demands X on the entire real line. Hence, there is a unique optimal demand X_{U1}, for which the FOC is necessary and sufficient:

$$\begin{aligned}
0 &= q\left(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} + \bar{\eta} - P_1 - \gamma X_{U1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}} + \tfrac{B_2^2}{\tau_{U1}}\right)\right) \\
&\qquad \times e^{-\gamma X_{U1}(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} + \bar{\eta} - P_1) + \frac{1}{2}\gamma^2 X_{U1}^2\left(1/\tau_\eta + B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)} \\
&\quad + (1-q)\left(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} - P_1 - \gamma X_{U1}\left(\tfrac{B_2^2}{\tau_{p2}} + \tfrac{B_2^2}{\tau_{U1}}\right)\right) \\
&\qquad e^{-\gamma X_{U1}(A_2 + B_2\mathbb{E}_{U1}[\theta] + C_2 s_{p1} - P_1) + \frac{1}{2}\gamma^2 X_{U1}^2\left(B_2^2/\tau_{p2} + B_2^2/\tau_{U1}\right)} \\
&= q\left(A_2 + \left(B_2\frac{\tau_{p1}}{\tau_\theta + \tau_{p1}} + C_2\right)\frac{P_1 - A_1}{B_1} + \bar{\eta} - P_1 - \gamma X_{U1}\left(\tfrac{1}{\tau_\eta} + \tfrac{B_2^2}{\tau_{p2}} + \tfrac{B_2^2}{\tau_{U1}}\right)\right) e^{-\gamma X_{U1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{U1}^2 \frac{1}{\tau_\eta}} \\
&\quad + (1-q)\left(A_2 + \left(B_2\frac{\tau_{p1}}{\tau_\theta + \tau_{p1}} + C_2\right)\frac{P_1 - A_1}{B_1} - P_1 - \gamma X_{U1}\left(\tfrac{B_2^2}{\tau_{p2}} + \tfrac{B_2^2}{\tau_{U1}}\right)\right)
\end{aligned} \tag{60}$$

where the second equality divides out terms that are common across the two exponentials and plugs in explicitly for $\mathbb{E}_{U1}[\theta]$ and s_{p1} in terms of the price P_1 using eq. (59) and the initial functional form conjecture. Equation (60) does not have a closed form solution, but it uniquely characterizes the uninformed demand function $X_{U1}(P_1)$.

With the optimal demand functions pinned down, the market clearing condition requires

$$\lambda_1 X_{I1}(\theta, P_1) + (1 - \lambda_1)\, X_{U1}(P_1) = \bar{Z} + z_1. \tag{61}$$

The demand functions characterized by eqs. (58) and (60), and the market clearing condition (61) fully characterize the $t = 1$ equilibrium price. In order for our conjecture that P_1 is linear to be consistent, it must be the case that the P_1 that satisfies this set of equilibrium conditions is a linear function of the form $P_1 = A_1 + B_1 s_{p1} = A_1 + B_1(\theta + b_1 z_1)$. We will proceed by enforcing the initial conjecture that the price is a linear function of this form and showing that this leads to a contradiction.

Specifically, we will show that a linear P_1 so defined has non-constant derivative, which contradicts linearity. By the implicit function theorem, the demand functions characterized in eqs. (58) and (60) are continuously differentiable in their arguments, and it therefore follows from another application of the implicit function theorem that the equilibrium price defined by eq. (61) is a continuously differentiable function of the underlying random variables θ and z_1. Differentiating the market clearing condition totally yields

$$\frac{\partial}{\partial z_1} P_1 = \frac{1}{\lambda_1 \frac{\partial}{\partial P_1} X_{I1}(\theta, P_1) + (1-\lambda_1) \frac{\partial}{\partial P_1} X_{U1}(P_1)}. \tag{62}$$

Furthermore, computing the partial derivative of the informed demand function with respect to P_1 using the implicit function theorem on eqs. (58) yields

$$\frac{\partial}{\partial P_1} X_{I1}(\theta, P_1) = -K_{I1}^{-1} \tag{63}$$

where

$$\begin{aligned}
K_{I1} = & \frac{q\gamma\left(\frac{1}{\tau_\eta} + \frac{B_2^2}{\tau_{p2}}\right) e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)\gamma \frac{B_2^2}{\tau_{p2}}}{\left(1 - \frac{C_2}{B_1}\right)\left(q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)\right)} \\
& + \frac{q\left(A_2 + B_2\theta + C_2 \frac{P_1 - A_1}{B_1} + \bar{\eta} - P_1 - \gamma X_{I1}\left(\frac{1}{\tau_\eta} + \frac{B_2^2}{\tau_{p2}}\right)\right)\left(\gamma\bar{\eta} - \gamma^2 X_{I1}\frac{1}{\tau_\eta}\right) e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}}}{\left(1 - \frac{C_2}{B_1}\right)\left(q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)\right)} \\
= & \frac{q\gamma\left(\frac{1}{\tau_\eta} + \frac{B_2^2}{\tau_{p2}}\right) e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)\gamma \frac{B_2^2}{\tau_{p2}}}{\left(1 - \frac{C_2}{B_1}\right)\left(q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)\right)} \\
& + \gamma \frac{\left(\bar{\eta} - \gamma X_{I1}\frac{1}{\tau_\eta}\right)^2}{1 - \frac{C_2}{B_1}} \frac{q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}}}{q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)} \left(1 - \frac{q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}}}{q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)}\right)
\end{aligned}$$

where the final equality uses the FOC from eq. (58) to substitute

$$\left(A_2 + B_2\theta + C_2 \frac{P_1 - A_1}{B_1} - P_1 - \gamma X_{I1} \frac{B_2^2}{\tau_{p2}}\right) = -\left(\bar{\eta} - \gamma X_{I1}\frac{1}{\tau_\eta}\right) \frac{q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}}}{q e^{-\gamma X_{I1}\bar{\eta} + \frac{1}{2}\gamma^2 X_{I1}^2 \frac{1}{\tau_\eta}} + (1-q)}$$

in the second term and simplifies the resulting expression. Similarly, using the implicit function theorem to compute the partial derivative of uninformed demand using eq. (60) gives

$$\frac{\partial}{\partial P_1} X_{U1}(P_1) = -K_{U1}^{-1} \tag{64}$$

where

$$K_{U1} = \frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\gamma\left(\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\right)} \tag{65}$$

$$+\frac{q\left(A_2+\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{P_1-A_1}{B_1}+\bar{\eta}-P_1-\gamma X_{U1}\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)\right)\left(\gamma\bar{\eta}-\gamma^2 X_{U1}\frac{1}{\tau_\eta}\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\right)}$$

$$= \frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)e^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\gamma\left(\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)\right)} \tag{66}$$

$$+\gamma\frac{\left(\bar{\eta}-\gamma X_{U1}\frac{1}{\tau_\eta}\right)^2}{1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}}\frac{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)}\left(1-\frac{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}}{qe^{-\gamma X_{U1}\bar{\eta}+\frac{1}{2}\gamma^2 X_{U1}^2\frac{1}{\tau_\eta}}+(1-q)}\right) \tag{67}$$

Now, if the equilibrium price is linear $P_1 = A_1 + B_1(\theta + b_1 z_1)$, then there is a continuum of (θ, z_1) values at which the informed investors perceive the asset as having zero risk premium and, as a consequence of their FOC (eq. (58)) have an equilibrium demand of zero shares. Define this set of fundamentals

$$M = \left\{(\theta, z_1) : A_2 + B_2\theta + C_2\frac{P_1 - A_1}{B_1} + q\bar{\eta} - P_1 = 0\right\}$$
$$= \{(\theta, z_1) : A_2 + B_2\theta + C_2(\theta + b_1 z_1) + q\bar{\eta} - (A_1 + B_1(\theta + b_1 z_1)) = 0\},$$

pick any point $(t, \zeta) \in M$, and let $\hat{p} = P_1(t, \zeta)$ denote the associated price. At such a realization of fundamentals, we have that $X_{I1} = 0$ and consequently eq. (63) yields that

$$\left.\frac{\partial}{\partial P_1}\right|_{P_1=\hat{p}} X_{I1}(\theta, P_1) = -\left(\frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}\right)+(1-q)\gamma\frac{B_2^2}{\tau_{p2}}}{\left(1-\frac{C_2}{B_1}\right)} + \frac{\gamma q(1-q)\bar{\eta}^2}{\left(1-\frac{C_2}{B_1}\right)}\right)^{-1}.$$

Note that $\left.\frac{\partial}{\partial P_1}\right|_{P_1=\hat{p}} X_{I1}(\theta, P_1) \equiv G_{I1}$ is constant with respect to values of$(t, \zeta) \in M$.

Similarly, by the market clearing condition, since informed demand satisfies $X_{I1} = 0$, the equilibrium uninformed demand must be $X_{U1} = \frac{\bar{Z}+\zeta}{1-\lambda_1}$, and consequently eq. (65) yields that the derivative of the uninformed demand function, evaluated at $\hat{p}$ is pinned down by

$$K_{U1}\bigg|_{P_1=\hat{p}} = \frac{q\gamma\left(\frac{1}{\tau_\eta}+\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)e^{-\gamma\frac{\bar{Z}+z_1}{1-\lambda_1}\bar{\eta}+\frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}+(1-q)\,\gamma\left(\frac{B_2^2}{\tau_{p2}}+\frac{B_2^2}{\tau_{U1}}\right)}{\left(1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}\right)\left(qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta}+\frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}+(1-q)\right)} \tag{68}$$

$$+\gamma\frac{\left(\bar{\eta}-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\frac{1}{\tau_\eta}\right)^2}{1-\left(B_2\frac{\tau_{p1}}{\tau_\theta+\tau_{p1}}+C_2\right)\frac{1}{B_1}}\frac{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta}+\frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}}{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta}+\frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}+(1-q)}\left(1-\frac{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta}+\frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}}{qe^{-\gamma\frac{\bar{Z}+\zeta}{1-\lambda_1}\bar{\eta}+\frac{1}{2}\gamma^2\left(\frac{\bar{Z}+\zeta}{1-\lambda_1}\right)^2\frac{1}{\tau_\eta}}+(1-q)}\right).$$

Note that, since $q \in (0,1)$, K_{U1} is a nontrivial function of the particular value of ζ, so that by eq. (64), we have $\frac{\partial}{\partial P_1}\Big|_{P_1=\hat{p}} X_{I1}(\theta, P_1) = G_{U1}(\zeta)$ for non-constant function $G_{U1}(\zeta) \equiv -\left(K_{U1}\Big|_{P_1=\hat{p}}\right)^{-1}$. Finally, returning to eq. (62), we have that the partial derivative of P_1 with respect to z_1, evaluated at the point $\theta = t, z_1 = \zeta$ is

$$\frac{\partial}{\partial z_1}\bigg|_{\theta=t,z_1=\zeta} P_1 = \frac{1}{\lambda_1 G_{I1} + (1-\lambda_1)\,G_{U1}(\zeta)}.$$

Because the partial derivative depends on the particular realization $z_1 = \zeta$, it is not constant and therefore the function P_1 cannot be linear. This is a contradiction and completes the proof. □

A.5 Proof of Proposition 3

We know from Proposition 1 that for sufficiently small $c_D > 0$ a high-type manager strictly prefers disclosing to not disclosing and being assigned probability 0 of being the $x = 1$ type:

$$U_{ND}(1;0) < U_D(1) - c_D \Rightarrow U_D(1) - c_D - U_{ND}(1;0) > 0 \tag{69}$$

Fix such a sufficiently small c_D and suppose that there also exists an equilibrium in which the $x = 1$ manager discloses with probability $r_1 < 1$ and the $x = 0$ manager never discloses, $r_0 = 0$. In such an equilibrium, the market assigns probability

$$q(0, r_1) = \frac{p(1-r_1)}{(1-p)+p(1-r_1)} < p$$

that $x = 1$ in the event of no disclosure. Under the assumed continuity of U_{ND}, for every $\varepsilon > 0$ there exists $q_\varepsilon > 0$ such that for $q \in [0, q_\varepsilon)$ we have

$$-\varepsilon < U_{ND}(1, q) - U_{ND}(1, 0) < \varepsilon.$$

Now, pick any ε such that $0 < \varepsilon < U_D(1) - c_D - U_{ND}(1; 0)$, which is guaranteed to exist owing to eq. (69). For any $q \in [0, q_\varepsilon)$ we have

$$\begin{aligned} U_{ND}(1; q) &< U_{ND}(1; q) + \varepsilon \\ &< U_D(1) - c_D \end{aligned}$$

where the first line follows from the continuity of U_{ND} and the second line follows from the choice of ε. Because $q(0, r_1) < p$ for any value of r_1, this implies that as long as $p < q_\varepsilon$ we have

$$U_{ND}(1; q(0, r_1)) < U_D(1) - c_D$$

which implies that the $x = 1$ manager strictly prefers to disclose, which is a contradiction. $\square$

B Additional Analysis

B.1 Conditions for interior equilibria

We begin with a characterization of conditions under which interior information equilibria obtain.

Lemma 5. *Fix $x \in \{0, 1\}$. If there exist $\lambda_1 \in (0, 1)$ and $\lambda_2 \in (0, 1)$ that solve the following system of two equations, where the coefficients $B_2(\lambda_1, \lambda_2)$ and $b_1(\lambda_1, \lambda_2)$ are as defined in Lemma 1, then there exists an interior equilibrium in the information market.*

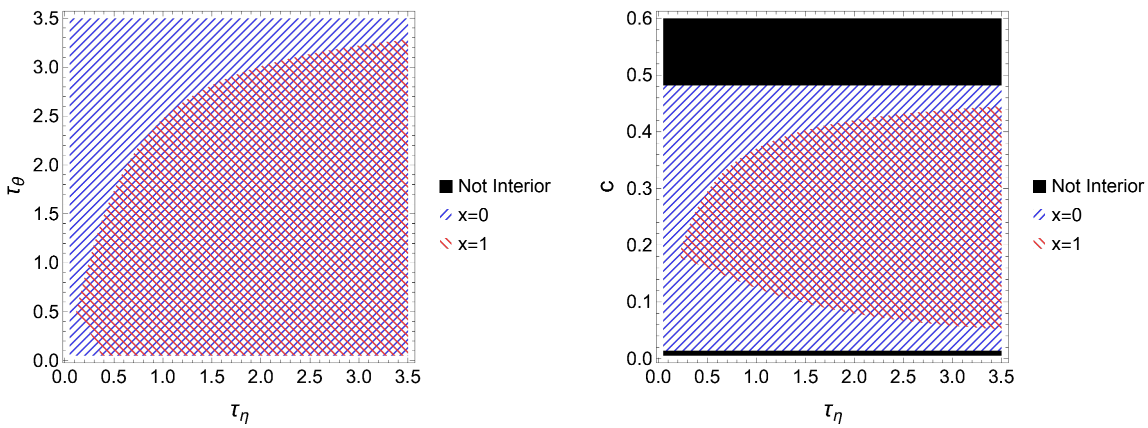
$$\frac{\frac{B_2^2(\lambda_1,\lambda_2)}{\tau_\theta + \tau_z / b_1^2(\lambda_1,\lambda_2)}}{\frac{B_2^2(\lambda_1,\lambda_2)}{\tau_z \left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} - 1 \tag{70}$$

$$\frac{\tau_u}{\tau_\theta + \frac{\tau_z}{b_1^2(\lambda_1,\lambda)} + \tau_z \left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} = e^{2\gamma c} - 1 \tag{71}$$

Proof. In an interior equilibrium, λ_1 and λ_2 are characterized by the conditions $\Gamma_t(\lambda_1, \lambda_2) = 1$ for $t \in \{1, 2\}$, where Γ_t is defined in eq. (39). Plugging in to the $t = 1$ condition and

Figure 5: Parameter regions in which $\lambda_t \in (0,1)$

The figures plot the region of the parameter space in which $\lambda_1, \lambda_2 \in (0,1)$ for $x=0$ and $x=1$. Unless specified, the other parameters are set to $\gamma=0.5$, $c=0.2$, $\tau_\theta=1$, $\tau_u=1$, $\tau_\eta=1$ and $\tau_z=1$.



(a) τ_θ versus τ_η

(b) τ_η versus c

rearranging yields

$$\frac{\mathbb{V}_{U1}[P_2]}{\mathbb{V}_{I1}[P_2]} = e^{2\gamma c} \Leftrightarrow \frac{B_2^2 b_2^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta} + B_2^2 \mathbb{V}_{U1}(\theta)}{B_2^2 b_2^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} \Leftrightarrow \frac{\frac{B_2^2}{\tau_\theta + \tau_z / b_1^2}}{B_2^2 \left(\frac{\gamma}{\lambda_2 \tau_u}\right)^2 \frac{1}{\tau_z} + \frac{x^2}{\tau_\eta}} = e^{2\gamma c} - 1$$

where the first equivalence follows from substituting the price function from eq. (3), and the second equivalence follows from rearranging and substituting in for the equilibrium values of $\mathbb{V}_{U1}(\theta)$ and b_2. Similarly, plugging in to the $t=2$ condition and rearranging yields

$$\frac{\mathbb{V}_{U2}[V]}{\mathbb{V}_{I2}[V]} = e^{2\gamma c} \Leftrightarrow \frac{\frac{1}{\tau_u} + \frac{1}{\tau_{U2}}}{\frac{1}{\tau_u}} = e^{2\gamma c} \Leftrightarrow \frac{\tau_u}{\tau_\theta + \frac{\tau_z}{b_1^2} + \tau_z \left(\frac{\lambda_2 \tau_u}{\gamma}\right)^2} = e^{2\gamma c} - 1$$

where the first equivalence follows from substituting in for the variances in terms of precision, and the second equivalence follows from rearranging and substituting in the equilibrium value of τ_{U2}. □

Due to the highly nonlinear nature of the information market equilibrium conditions, the equilibrium λ_t's are not generally available in closed-form and it is difficult to pin down analytical conditions on primitives that ensure that the equilibrium is interior. However, it is straightforward to numerically solve for equilibrium and check whether the conditions in Lemma 5 are satisfied.

Figure 5 provides illustrations of regions of the parameter space in which λ_t 's are interior.

Panel (a) illustrates how the region varies with the prior precisions of the long-term project, τ_θ, and the short-term project, τ_η. For the displayed parameter region, the equilibrium is always interior for $x = 0$. Naturally, when the short-term project does not exist ($x = 0$), the region does not vary with τ_η. When the short-term project exists, the region of interior equilibria is smaller because, when τ_η is sufficiently small (i.e., the short-term project is sufficiently risky) then no investors acquire information at $t = 1$ ($\lambda_1 = 0$). On the other hand, when τ_η grows without bound and the short-term project becomes risk-less, the $x = 1$ equilibrium is isomorphic to the $x = 0$ equilibrium (in which the project does not exist) and therefore the interior regions must coincide.

Panel (b) illustrates how the region of interior equilibria varies with the cost of information c and the prior precision of the short-term project, τ_η. Again, we see that when $x = 0$, the value of τ_η naturally has no effect on the equilibrium. For both $x = 0$ and $x = 1$, an interior equilibrium (if one exists) holds for an intermediate region of costs. If the cost is "too high" then investors do not acquire information in either period ($\lambda_1 = \lambda_2 = 0$), while if the cost is "too low", then all investors acquire information in at least one of the periods (either $\lambda_0 = 1$ or $\lambda_1 = 1$). On the other hand, for any fixed cost c, in the $x = 1$ case, we again have $\lambda_1 \to 0$ as τ_η shrinks, while the equilibria coincide when τ_η becomes sufficiently large.

B.2 Dynamic model with persistent supply shocks

The setup follows the benchmark described in Section 2 with two differences.

- Each investor is long lived and can acquire information at the date of her choosing (i.e., at $t = 1^-$ or $t = 2^-$). As discussed in Section 4.1, to sustain an equilibrium with information acquisition at $t = 2^-$ we must have time-dependent information costs c_t, with $c_1 > c_2$.

- The aggregate supply of the risky security is Z_t, $t \in \{1, 2\}$, which follows

$$Z_t = \bar{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t,$$

  where $z_t \sim N(0, \tau_{zt})$ are normally distributed, independent of each other and other random variables, and we normalize $Z_0 \equiv \overline{Z}$. The special case $\phi = 0$, $\tau_{z1} = \tau_{z2} = \tau_z$ corresponds to the supply dynamics in the benchmark model and the particular dynamic extension discussed in Section 4.1.

As in the baseline model, we search for an equilibrium in which the $x = 1$ manager always discloses and the $x = 0$ manager never discloses. We solve the model by working backwards. Specifically, Section B.2.1 characterizes the equilibrium prices at dates 1 and 2, given investors' information acquisition choices. Section B.2.2 characterizes the equilibrium information acquisition choices at each date, and Section B.2.3 characterizes the conditions necessary for our main result. Figure **??** (in the text) provides an illustration of this case. Specifically, we numerically solve a system of three equations (i.e., (114), (120), and (130)) to solve for the price signal coefficient b_1, and the fraction of informed investors at each date λ_1 and λ_2 for a given set of parameter values, with and without disclosure (i.e., for $d \in \{D, ND\}$), and then plot the date 2 fraction informed (i.e., λ_2) and the date 2 expected price (i.e., $\mathbb{E}[P_2]$ from equation (133) for different values of supply shock persistence ϕ.

B.2.1 Financial market equilibrium

For given disclosure and information choices, the derivation of the financial market equilibrium is standard. Fix the fraction λ_t of investors in generation t who acquire information about θ. We conjecture that prices are of the form:

$$P_1 = A_1 + B_1 s_{p1}, \quad \text{and} \quad P_2 = A_2 + B_2 s_{p2} + C_2 s_{p1} + x\eta, \tag{72}$$

where the $s_{pt} \equiv \theta + b_t(Z_t - \bar{Z})$ for $t \in \{1, 2\}$. In particular, the date t price provides a noisy, linear signal s_{pt} about θ to the uninformed investors of that generation. Moreover, the uninformed investors at date 2 can condition on the date 1 price to infer s_{p1}. This implies that the conditional beliefs of an uninformed investor at date $t = 1$ are given by:

$$\mathbb{E}_{U1}[\theta] = \frac{\tau_{p1} s_{p1}}{\tau_\theta + \tau_{p1}}, \quad \mathbb{V}_{U1}[\theta] = \frac{1}{\tau_\theta + \tau_{p1}} \equiv \frac{1}{\tau_{U1}}, \quad \text{where } \tau_{p1} \equiv \tau_{z1}/b_1^2. \tag{73}$$

The conditional beliefs of an uninformed investor at date $t = 2$ are more complex since the date 2 price signal is not conditionally independent of the date 1 signal. We have

$$\mathbb{E}_{U2}[\theta] = \frac{\frac{\tau_{z1}}{b_1^2} s_{p1} + \left(1 - \frac{b_2}{b_1}\phi\right) \frac{\tau_{z2}}{b_2^2} \left(s_{p2} - \frac{b_2}{b_1}\phi s_{p1}\right)}{\tau_\theta + \frac{\tau_{z1}}{b_1^2} + \left(1 - \frac{b_2}{b_1}\phi\right)^2 \frac{\tau_{z2}}{b_2^2}} \tag{74}$$

$$\mathbb{V}_{U2}^{-1}[\theta] = \tau_\theta + \frac{\tau_{z1}}{b_1^2} + \left(1 - \frac{b_2}{b_1}\phi\right)^2 \frac{\tau_{z2}}{b_2^2} \tag{75}$$

Note that if we further define the $t = 2$ 'incremental precision' τ_{p2} as

$$\tau_{p2} \equiv \left(1 - \frac{b_2}{b_1}\phi\right)^2 \frac{\tau_{z2}}{b_2^2} \tag{76}$$

then we can write these expressions concisely as

$$\mathbb{E}_{U2}[\theta] = \frac{\tau_{p1}s_{p1} + \tau_{p2}\frac{s_{p2}-\frac{b_2}{b_1}\phi s_{p1}}{\left(1-\frac{b_2}{b_1}\phi\right)}}{\tau_\theta + \tau_{p1} + \tau_{p2}} \tag{77}$$

$$\mathbb{V}_{U2}^{-1}[\theta] = \tau_\theta + \tau_{p1} + \tau_{p2} \tag{78}$$

We now proceed to construct the financial market equilibrium by backward induction.

Date $t = 2$ trading round At $t = 2$ there are no future trading rounds left, so all investors optimally behave myopically. Investor i chooses optimal demand X_{it} to maximize CARA utility over next period wealth

$$X_{i2} \equiv \arg\max_x \mathbb{E}_{i2}[-e^{-\gamma\{W_2+x(P_3-P_2)\}}] \tag{79}$$

$$= \frac{\mathbb{E}_{i2}[V] - P_2}{\gamma\mathbb{V}_{i2}[P_3]} \tag{80}$$

This yields optimized expected utility

$$\mathbb{E}_{i2}\left[-e^{-\gamma W_2+X_{i2}(V-P_2)}\right] = -e^{-\gamma W_2-\frac{1}{2}\frac{\mathbb{E}_{i2}^2[V-P_2]}{\mathbb{V}_{i2}[V]}}. \tag{81}$$

By enforcing the market clearing condition and solving for P_2 we can easily pin down conditions that define the time 2 price coefficients. Specifically, note that

$$\lambda_2\frac{\mathbb{E}_{I2}[V-P_2]}{\gamma\mathbb{V}_{I2}(V)} + (1-\lambda_2)\frac{\mathbb{E}_{I2}[V-P_2]}{\gamma\mathbb{V}_{I2}(V)} = Z_2 \tag{82}$$

$$\Leftrightarrow P_2 = \overline{V} + x\eta + \frac{\frac{\lambda_2}{\mathbb{V}_{I2}(V)}\mathbb{E}_{I2}[\theta] + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}\mathbb{E}_{U2}[\theta]}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}}Z_2 \tag{83}$$

Equating coefficients with the initial conjecture yields

$$b_2 = -\frac{\gamma}{\lambda_2 \tau_u} \tag{84}$$

$$A_2 = \overline{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \overline{Z} \tag{85}$$

$$B_2 = \frac{\left(\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{V}_{U2}[\theta] \frac{\tau_{p2}}{1-\frac{b_2}{b_1}\phi} \right)}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \tag{86}$$

$$C_2 = \frac{\frac{1-\lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{V}_{U2}[\theta] \left(\tau_{p1} - \tau_{p2} \frac{\frac{b_2}{b_1}\phi}{1-\frac{b_2}{b_1}\phi} \right)}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \tag{87}$$

Date $t = 1$ trading round Now step back to $t = 1$.

An informed investor chooses demand to solve

$$\max_x \mathbb{E}_{I1} \left[-e^{-\gamma W_1 - \gamma x (P_2 - P_1) - \frac{1}{2} \frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}} \right] \tag{88}$$

Using standard methods, it is tedious but straightforward to compute the expectation and show that her optimal demand is

$$X_{I1} = \frac{1}{\gamma} \frac{\mathbb{E}_{I1}[P_2 - P_1 - \beta_{I1}(V - P_2)]}{\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2))} \tag{89}$$

where $\beta_{I1} = \frac{\mathbb{C}_{I1}(V-P_2, P_2-P_1)}{\mathbb{V}_{I1}(V-P_2)}$ is the conditional regression coefficient of $P_2 - P_1$ on $V - P_2$. Plugging the optimal demand back into the objective function and arranging terms yields optimized expected utility

$$\mathbb{E}_{I1} \left[-e^{-\gamma W_1 - \gamma X_{I1} (P_2 - P_1) - \frac{1}{2} \frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}} \right] \tag{90}$$

$$= -\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{I1}(V - P_2)}} e^{-\gamma W_1 - \frac{1}{2} \frac{\mathbb{E}^2_{I1}[V-P_2]}{\mathbb{V}_{I1}(V-P_2)} - \frac{1}{2} \frac{\mathbb{E}^2_{I1}[P_2 - P_1 - \beta_{I1}(V-P_2)]}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}} \tag{91}$$

An uninformed investor who anticipates remaining uninformed at the second trading date chooses x to solve

$$\max_x \mathbb{E}_{U1} \left[-e^{-\gamma W_1 - \gamma x (P_2 - P_1) - \frac{1}{2} \frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}[V]}} \right] \tag{92}$$

Similarly to the informed investor, we can also show that her optimal demand is

$$X_{U1} = \frac{1}{\gamma} \frac{\mathbb{E}_{U1}[P_2 - P_1 - \beta_{U1}(V - P_2)]}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} \tag{93}$$

where $\beta_{U1} = \frac{\mathbb{C}_{U1}(V-P_2, P_2-P_1)}{\mathbb{V}_{U1}(V-P_2)}$ is the conditional regression coefficient of $P_2 - P_1$ on $V - P_2$. This demand leads to optimized expected utility

$$\mathbb{E}_{U1}\left[-e^{-\gamma W_1 - \gamma X_{U1}(P_2-P_1) - \frac{1}{2}\frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}[V]}}\right] \tag{94}$$

$$= -\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V-P_2)}} e^{-\gamma W_1 - \frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)} - \frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{95}$$

Finally, consider an uninformed investor who plans to acquire information before $t = 2$. Her problem is to choose x to maximize

$$\mathbb{E}_{U1}\left[-e^{-\gamma(W_1-c_2) - \gamma x(P_2-P_1) - \frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right] \tag{96}$$

$$= \mathbb{E}_{U1}\left[\mathbb{E}_{U2}\left[-e^{-\gamma(W_1-c_2) - \gamma x(P_2-P_1) - \frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right]\right] \tag{97}$$

$$= \mathbb{E}_{U1}\left[-e^{-\gamma(W_1-c_2) - \gamma x(P_2-P_1)}\mathbb{E}_{U2}\left[e^{-\frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right]\right] \tag{98}$$

$$= \sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}} e^{\gamma c_2}\, \mathbb{E}_{U1}\left[-e^{-\gamma W_1 - \gamma x(P_2-P_1) - \frac{1}{2}\frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{U2}[V]}}\right] \tag{99}$$

where the second line uses the law of iterated expectations, the second line pulls $\mathcal{F}_{U2}$ measurable things out of the inner expectation, and the final line computes the inner expectation, using the fact that $\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}}$ is a constant and can be pulled out of the expectation.

Because this objective function is a constant multiple of that for an uninformed investor who plans to remain uninformed, it leads to the same optimal demand

$$X_{U1} = \frac{1}{\gamma} \frac{\mathbb{E}_{U1}[P_2 - P_1 - \beta_{U1}(P_3 - P_2)]}{\mathbb{V}_{U1}(P_2 - P_1 - \beta_{U1}(V - P_2))} \tag{100}$$

and to optimized expected utility

$$-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}}e^{\gamma c_2}\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{101}$$

$$=-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{\gamma c_2}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{102}$$

With the $t=1$ optimal demands pinned down, we can now enforce the market clearing condition to pin down the coefficients on P_1. Specifically, note that

$$\frac{\lambda_1}{\gamma}\frac{\mathbb{E}_{I1}[P_2-P_1-\beta_{I1}(V-P_2)]}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\gamma}\frac{\mathbb{E}_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}=Z_1 \tag{103}$$

$$\Rightarrow P_1=\frac{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}\mathbb{E}_{I1}[P_2-\beta_{I1}(V-P_2)]+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}\mathbb{E}_{U1}[P_2-\beta_{U1}(V-P_2)]}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}}$$

$$-\frac{\gamma}{\frac{\lambda_1}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}+\frac{1-\lambda_1}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}}Z_1 \tag{104}$$

Note that

$$\mathbb{C}_{I1}(V-P_2,P_2)=\mathbb{C}_{I1}(-B_2s_{p2},B_2s_{p2})=-B_2^2b_2^2\frac{1}{\tau_{z2}} \tag{105}$$

$$\mathbb{V}_{I1}(P_2)=\mathbb{V}_{I1}(B_2s_{p2}+x\eta)=B_2^2b_2^2\frac{1}{\tau_{z2}}+\frac{x^2}{\tau_\eta} \tag{106}$$

$$\mathbb{V}_{I1}(V-P_2)=\mathbb{V}_{I1}(u+\theta-B_2s_{p2})=\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}} \tag{107}$$

Hence

$$\beta_{I1}=\frac{\mathbb{C}_{I1}(V-P_2,P_2)}{\mathbb{V}_{I1}(V-P_2)}=\frac{-B_2^2b_2^2\frac{1}{\tau_{z2}}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}. \tag{108}$$

We can now compute

$$\mathbb{E}_{I1}[P_2-\beta_{I1}(V-P_2)] \tag{109}$$

$$=\mathbb{E}_{I1}\left[A_2+B_2s_{p2}+C_2s_{p1}+x\eta-\frac{-B_2^2b_2^2\frac{1}{\tau_{z2}}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}\left(\overline{V}+x\eta+\theta+u-(A_2+B_2s_{p2}+C_2s_{p1}+x\eta)\right)\right] \tag{110}$$

$$=\frac{\frac{1}{\tau_u}B_2\left(1-\frac{b_2}{b_1}\phi\right)+B_2^2b_2^2\frac{1}{\tau_z}}{\frac{1}{\tau_u}+B_2^2b_2^2\frac{1}{\tau_{z2}}}\theta+\text{other terms that do not depend explicitly on }\theta, \tag{111}$$

where we use $\mathbb{E}_{I1}[s_{p2}] = \mathbb{E}_{I1}[\theta + b_2\phi z_1] = \left(1 - \frac{b_2}{b_1}\phi\right)\theta + \frac{b_2}{b_1}\phi s_{p1}$, and

$$\mathbb{V}_{I1}(P_2 - P_1 - \beta_{I1}(V - P_2)) = B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta} - \frac{\left(B_2^2 b_2^2 \frac{1}{\tau_{z2}}\right)^2}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}} = \frac{\frac{1}{\tau_u} B_2^2 b_2^2 \frac{1}{\tau_{z2}}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}} + \frac{x^2}{\tau_\eta} \tag{112}$$

Substituting these into the $t = 1$ market clearing condition and grouping terms involving θ and z_1, we can pin down the linear statistic that P_1 must reveal

$$\theta - \frac{\gamma}{\lambda_1} \frac{\frac{\frac{1}{\tau_u} B_2^2 b_2^2 \frac{1}{\tau_{z2}}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}} + \frac{x^2}{\tau_\eta}}{\frac{\frac{1}{\tau_u} B_2 \left(1 - \frac{b_2}{b_1}\phi\right) + B_2^2 b_2^2 \frac{1}{\tau_z}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}}} (Z_1 - \overline{Z}) \tag{113}$$

which gives the condition

$$b_1 = -\frac{\gamma}{\lambda_1} \frac{\frac{\frac{1}{\tau_u} B_2^2 b_2^2 \frac{1}{\tau_{z2}}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}} + \frac{x^2}{\tau_\eta}}{\frac{\frac{1}{\tau_u} B_2 \left(1 - \frac{b_2}{b_1}\phi\right) + B_2^2 b_2^2 \frac{1}{\tau_{z2}}}{\frac{1}{\tau_u} + B_2^2 b_2^2 \frac{1}{\tau_{z2}}}}. \tag{114}$$

Combined with the earlier condition for B_2, this gives us enough to pin down the financial market equilibrium, given fractions of informed traders λ_t. Returning to B_2, we have

$$B_2 = \frac{\left(\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)} \mathbb{V}_{U2}[\theta] \frac{\tau_{p2}}{1 - \frac{b_2}{b_1}\phi}\right)}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{U2}(V)}} \tag{115}$$

which, after substituting in for all the variances and using the explicit expression for b_2 from earlier, is a complicated function of b_1 (and the λ's).

B.2.2 Information acquisition choices

Given the characterization of the financial market equilibrium in the previous section, one can characterize the optimal information acquisition choices each period.

Date $t = 2^-$ information acquisition Immediately after the $t = 1$ trading round, but strictly before the $t = 2$ trading round, investors who have remained uninformed must decide whether to purchase information. The forward-looking expected utilities from acquiring or not acquiring information at this stage (i.e., conditional on the information $\mathcal{F}_{U1} = \sigma(d, P_1)$

she observed in the first round) are

$$U_{I2^-} = \mathbb{E}_{U1}\left[e^{-\gamma(W_1-c_2)-\gamma X_{U1}(P_2-P_1)-\frac{1}{2}\frac{\mathbb{E}^2_{I2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right] \tag{116}$$

$$= -\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{\gamma c_2}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{117}$$

$$U_{U2^-} = \mathbb{E}_{U1}\left[e^{-\gamma W_1-\gamma X_{U1}(P_2-P_1)-\frac{1}{2}\frac{\mathbb{E}^2_{U2}[V-P_2]}{\mathbb{V}_{I2}[V]}}\right] \tag{118}$$

$$= -\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \tag{119}$$

The indifference condition for an interior equilibrium therefore requires[15]

$$\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U2}(V)}}e^{\gamma c_2} = 1. \tag{120}$$

Date $t = 1^-$ ($t = 0$) information acquisition To establish the initial information equilibrium, we need to compute the ex-ante expected utilities of all types. Let $\mu_R = \begin{pmatrix} \mathbb{E}_0[V-p_2] \\ \mathbb{E}_0[p_2-p_1] \end{pmatrix}$ be the vector of ex-ante expected returns and $\mathbb{V}_R = \mathbb{V}_0\begin{pmatrix} V-p_2 \\ p_2-p_1 \end{pmatrix}$ the ex-ante covariance matrix of returns.

The expected utility of a investor who remains uninformed at both stages is

$$U_{U0} = \mathbb{E}_0\left[-\sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{U1}(V-P_2)}}e^{-\gamma W_1-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)}-\frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}}\right] \tag{121}$$

$$= -\left(\frac{\left|\mathbb{V}_0\begin{pmatrix} V-P_2 \\ P_2-P_1 \end{pmatrix}\right|}{\mathbb{V}_{U2}(V)\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}\right)^{-1/2} e^{-\gamma W_0-\frac{1}{2}\mu_R' V_R^{-1}\mu_R'} \tag{122}$$

That of a investor who is uninformed at the first period and informed at the second period

[15] Note that here the 'interior' region in which the indifference condition characterizes the equilibrium, is the situation in which some positive mass of investors who were previously uninformed choose to acquire information at $t = 2$, but not the entire mass $1 - \lambda_1$ of such investors. The mass λ_1 from the first round do not 'forget' their information and so we necessarily have $\lambda_2 \geq \lambda_1$ in any equilibrium.

is

$$U_{UI0} = \mathbb{E}_0 \left[-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{U1}(V-P_2)}} e^{\gamma c_2} e^{-\gamma W_1 - \frac{1}{2}\frac{\mathbb{E}^2_{U1}[V-P_2]}{\mathbb{V}_{U1}(V-P_2)} - \frac{1}{2}\frac{\mathbb{E}^2_{U1}[P_2-P_1-\beta_{U1}(V-P_2)]}{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}} \right] \tag{123}$$

$$= -e^{\gamma c_2} \left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V-P_2 \\ P_2-P_1 \end{pmatrix} \right|}{\mathbb{V}_{I2}(V)\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))} \right)^{-1/2} e^{-\gamma W_0 - \frac{1}{2}\mu_R' V_R^{-1} \mu_R'} \tag{124}$$

And that of a investor who is informed at both periods is

$$U_{I0} = \mathbb{E}_0 \left[-\sqrt{\frac{\mathbb{V}_{I2}(V)}{\mathbb{V}_{I1}(V-P_2)}} e^{-\gamma W_1 - \frac{1}{2}\frac{\mathbb{E}^2_{I1}[V-P_2]}{\mathbb{V}_{I1}(V-P_2)} - \frac{1}{2}\frac{\mathbb{E}^2_{I1}[P_2-P_1-\beta_{I1}(V-P_2)]}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}} \right] \tag{125}$$

$$= - \left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V-P_2 \\ P_2-P_1 \end{pmatrix} \right|}{\mathbb{V}_{I2}(V)\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))} \right)^{-1/2} e^{-\gamma(W_0-c_1) - \frac{1}{2}\mu_R' V_R^{-1} \mu_R'} \tag{126}$$

If the $t = 2^-$ equilibrium is interior, we immediately have $U_{U0} = U_{UI0}$, which implies that a $t = 0$ interior equilibrium requires that investors be indifferent between being informed at both periods and being uninformed at both periods, $U_{U0} = U_{I0}$:

$$- \left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V-P_2 \\ P_2-P_1 \end{pmatrix} \right|}{\mathbb{V}_{U2}(V)\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))} \right)^{-1/2} e^{-\gamma W_0 - \frac{1}{2}\mu_R' V_R^{-1} \mu_R'} = - \left(\frac{\left| \mathbb{V}_0 \begin{pmatrix} V-P_2 \\ P_2-P_1 \end{pmatrix} \right|}{\mathbb{V}_{I2}(V)\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))} \right)^{-1/2} e^{-\gamma(W_0-c_1) - \frac{1}{2}\mu_R' V_R^{-1} \mu_R'} \tag{127}$$

$$\Leftrightarrow \sqrt{\mathbb{V}_{U2}(V)\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))} = \sqrt{\mathbb{V}_{I2}(V)\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))} e^{\gamma c_1} \tag{128}$$

$$\Leftrightarrow e^{\gamma c_1} = \sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{I2}(V)}} \sqrt{\frac{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}} \tag{129}$$

Again using the $t = 2$ information condition, this can be simplified to

$$e^{\gamma(c_1-c_2)} = \sqrt{\frac{\mathbb{V}_{U1}(P_2-P_1-\beta_{U1}(V-P_2))}{\mathbb{V}_{I1}(P_2-P_1-\beta_{I1}(V-P_2))}}. \tag{130}$$

B.2.3 Relation between x and $\mathbb{E}[P_2]$

The expected value of P_2 for the manager at the disclosure stage is

$$\mathbb{E}[P_2] = \overline{V} - \frac{\gamma}{\frac{\lambda_2}{\mathbb{V}_{I2}(V)} + \frac{1-\lambda_2}{\mathbb{V}_{I2}(V)}} + x\bar{\eta}. \tag{131}$$

And in an interior equilibrium we have

$$\mathbb{V}_{U2}[V] = e^{2\gamma c_2}\mathbb{V}_{I2}[V] = \frac{e^{2\gamma c_2}}{\tau_u} \tag{132}$$

so that

$$\mathbb{E}[P_2] = \overline{V} - \frac{\gamma}{\tau_u(\lambda_2(1-e^{-2\gamma c_2}) + e^{-2\gamma c_2})} \tag{133}$$

Hence, as in our benchmark analysis, it is sufficient to show that $\lambda_2(D) = \lambda_2(1) \geq \lambda_2(ND) = \lambda_2(0)$.

While analytically establishing this result is not tractable, we show numerically that the result obtains for a large region of the parameter space. Specifically, we have a system of three equations i.e., (114), (120), and (130), and three unknowns, i.e., b_1, λ_1 and λ_2, which we can solve numerically for a given set of parameter values and x.

B.3 Short-lived investors with persistent supply

In this section, we consider an extension to our benchmark analysis in which the asset supply shocks are persistent. Specifically, we assume that the aggregate supply of the risky security is Z_t, $t \in \{1,2\}$ which follows $Z_t = \bar{Z} + \phi(Z_{t-1} - \overline{Z}) + z_t$ where $z_t \sim N(0, \tau_{zt})$ are normally distributed, independent of each other and other random variables, and we normalize $Z_0 \equiv \overline{Z}$. This implies that the investors' beliefs about fundamentals θ, and the resulting intermediate steps are as in Appendix B.2. However, we need to modify the equation defining b_1 since the $t=1$ demand functions are myopic in this case. Similarly, the $t=1$ information condition simplifies due to myopic behavior.

Because the $t=2$ demand functions are identical (in functional form), the conditions defining B_2 and b_2 have the same functional forms

$$B_2 = \frac{\lambda_2\tau_u + (1-\lambda_2)\frac{e^{2\gamma c_2}-1}{e^{2\gamma c_2}}\left(1 - \frac{b_2}{b_1}\phi\right)\frac{\tau_{z2}}{b_2^2}}{\lambda_2\tau_u + (1-\lambda_2)\frac{\tau_u}{e^{2\gamma c_2}}} \tag{134}$$

$$b_2 = -\frac{\gamma}{\lambda_2\tau_u} \tag{135}$$

To pin down b_1, note that the $t = 1$ market clearing condition is

$$\frac{\lambda_1}{\gamma} \frac{\mathbb{E}_{I1}[P_2 - P_1]}{\mathbb{V}_{I1}(P_2 - P_1)} + \frac{1-\lambda_1}{\gamma} \frac{\mathbb{E}_{U1}[P_2 - P_1]}{\mathbb{V}_{U1}(P_2 - P_1)} = Z_1. \tag{136}$$

Hence, the price reveals

$$\theta - \frac{\gamma}{\lambda_1} \frac{B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}}{B_2 \left(1 - \frac{b_2}{b_1}\phi\right)} (Z_1 - \overline{Z}) \tag{137}$$

and the equation defining b_1 is

$$b_1 = -\frac{\gamma}{\lambda_1} \frac{B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}}{B_2 \left(1 - \frac{b_2}{b_1}\phi\right)}. \tag{138}$$

The $t = 2$ information condition is still

$$e^{\gamma c_2} = \sqrt{\frac{\mathbb{V}_{U2}(V)}{\mathbb{V}_{I2}(V)}} \tag{139}$$

$$\Rightarrow \tau_\theta + \frac{\tau_{z1}}{b_1^2} + \left(\frac{1}{b_2} - \frac{\phi}{b_1}\right)^2 \tau_{z2} = \frac{\tau_u}{e^{2\gamma c_2} - 1}. \tag{140}$$

The $t = 1$ information condition is

$$e^{\gamma c_1} = \sqrt{\frac{\mathbb{V}_{U1}(P_2 - P_1)}{\mathbb{V}_{I1}(P_2 - P_1)}} \tag{141}$$

$$\Rightarrow e^{2\gamma c_1} = \frac{B_2^2 \left(1 - \frac{b_2}{b_1}\phi\right)^2 \mathbb{V}_{U1}(\theta) + B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}}{B_2^2 b_2^2 \frac{1}{\tau_{z2}} + \frac{x^2}{\tau_\eta}} \tag{142}$$

As in the benchmark case, it is sufficient for us to show that

$$\lambda_2 (D) = \lambda_2 (1) \geq \lambda_2 (ND) = \lambda_2 (0). \tag{143}$$

It is intractable to establish such a result analytically. However, we can demonstrate numerically the result obtains for a wide range of parameters. Specifically, we have a system of three equations i.e., (138), (140), and (142), and three unknowns, i.e., b_1, λ_1 and λ_2, which we can solve numerically for a given set of parameter values and x.