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Abstract

We analyze vertical integration between platforms providing operating systems to manufacturers of devices when there are indirect network effects between buyers of devices and developers of applications. Vertical integration creates market power over developers, and over non-integrated manufacturers but only under certain circumstances. That market power enables to coordinate pricing decisions across both sides of the market, which leads to a better internalization of network effects. Vertical integration does not systematically lead to foreclosure and can benefit all parties, even in the absence of efficiency gains. Its competitive impact depends on the strength and the structure of indirect network effects.

JEL Classification: L40, L10, D43

Keywords: Vertical integration, Platform markets, Network effects, Foreclosure

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The Competitive Effects of Vertical Integration in Platform Markets*

Jérôme Pouyet† Thomas Trégouët‡

Abstract

We analyze vertical integration between platforms providing operating systems to manufacturers of devices when there are indirect network effects between buyers of devices and developers of applications. Vertical integration creates market power over developers, and over non-integrated manufacturers but only under certain circumstances. That market power enables to coordinate pricing decisions across both sides of the market, which leads to a better internalization of network effects. Vertical integration does not systematically lead to foreclosure and can benefit all parties, even in the absence of efficiency gains. Its competitive impact depends on the strength and the structure of indirect network effects.

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1. Introduction

Motivation. Software platform industries have recently witnessed many changes in the nature of the relationship between software and hardware producers. Traditional suppliers of operating systems have ventured into the hardware market and prominent hardware manufacturers have developed their own operating systems. In the smartphone market, while Apple further intensified its hallmarked integration between hardware and software,

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Google launched in 2016 the Pixel, its first device conceptualized and engineered in-house. It also acquired a major handset manufacturer, HTC, in 2018 and recently started designing mobile processors, the Google Tensors.[1] Samsung and Huawei, subjugated to Google for the use of its Android platform while delivering Google substantial money through services installed on their phones, have started to equip some of their devices with their own operating systems.[2] In the online retail sector, Amazon sells devices powered by FireOS, an operating system built on Android's technology but stripped from Google's applications. Microsoft, once praised for its software-only model, has ventured in the electronic devices market with the Surface brand. Other industries are witnessing a similar momentum of integration along the value chain.[3] These changes are scrutinized closely by regulators and competition authorities, and whether the usual competitive assessment of integration could readily be applied to platform markets remains an on-going debate in the antitrust arena.[4]

In this article, we address the following question: what are the competitive effects of vertical integration between platforms offering operating systems and device manufacturers? We show that indirect network effects, which are prevalent in digital markets, substantially impact the competitive assessment of vertical integration. Although vertical integration still creates some market power, as the common wisdom has it, the sources of such market power are different than in one-sided markets. Perhaps more importantly, the exercise of such market power does not necessarily harm either consumers or non-integrated competitors. For instance, when indirect network effects are sufficiently strong and asymmetric (in a sense to be defined properly later on), vertical integration can benefit all parties, even in the absence of efficiency gains. A recurrent intuition of our analysis is that vertical integration enables to coordinate several pricing decisions and such coordination sometimes allows a better internalization of indirect network effects.

The Model. Several platforms compete to license their operating systems to two manufacturers of devices. Manufacturers equip their devices with an operating system, pay some fee (whose precise nature is detailed later on) to the corresponding platforms, and then compete to sell devices to buyers. Developers pay fees to platforms to publish their applications on the operating systems. Therefore, a device gives its buyers access to applications developed for the operating system it is equipped with. This interaction between buyers of devices and developers of applications is the source of indirect network effects in our analysis.

Our benchmark is the situation in which none of the platforms are integrated with a manufacturer. There, competition between equally-efficient platforms leads to a Bertrand-like outcome in which neither the developers nor the manufacturers pay anything to the

[1] Google initially maintained arm's-length relationships with several smartphone producers to build the Nexus range, even after the acquisition of Motorola in 2011. Whereas some experts argued that Google's primary objective was to strengthen its patent portfolio, many now retrospectively think that this was also a test of the feasibility of a more integrated business model.

[2] The attempt was not fully conclusive for Samsung, which recently stopped offering its in-house operating system (except for smart TVs). By contrast, Huawei has continued to promote its own operating system, Harmony, against Android, partly because of the U.S. embargo that prevents some Chinese companies from using Google technologies since 2019.

[3] For instance, whereas Google is actively promoting its self-driving technology through its division Waymo, traditional car manufacturers such as BMW have developed their own in-house technology.

[4] See, for instance, OECD (2018) and the FTC Hearing #3 regarding 'Competition and Consumer Protection in the 21st Century' (www.ftc.gov/news-events/events-calendar/2018/10/ftc-hearing-3-competition-consumer-protection-21st-century).

platforms. Competition prevents platforms from exerting any market power, either on manufacturers or on developers.

VERTICAL INTEGRATION. We then consider vertical integration between a platform and a manufacturer. Because it faces competition from equally-efficient platforms, the vertically-integrated platform cannot exert any market power on the non-integrated manufacturer; a result that is standard from the literature on strategic vertical integration (see, e.g., Salop and Scheffman, 1983, Ordover et al., 1990 or Chen, 2001).[5]

Vertical integration creates, however, market power over developers because the integrated platform has monopoly power over the access to the buyers of its device. This is a new source of market power, which comes from the two-sided nature of our model. The next step of our analysis consists in assessing how the integrated firm exercises such market power. To do so, observe that the integrated platform has two pricing instruments: the fee paid by developers to publish their applications on its operating system and the price paid by buyers for its device. The integrated platform's prices are guided by two forces: a one-sided logic, according to which increasing prices (above their pre-merger levels) allows to extract more profit from developers and from buyers; a two-sided logic, according to which setting an asymmetric pricing structure allows to better internalize network effects between buyers and developers. Which logic prevails overall depends on the strength and the structure of indirect network effects. For instance, and in the spirit of the literature on two-sided markets (Armstrong, 2006, Rochet and Tirole, 2006 and Caillaud and Jullien, 2003), when buyers value strongly applications, the integrated platform finds it optimal to decrease the developer fee and increase the price for its device.

Next, we analyze the consequences of that market power.

FORECLOSURE. Because vertical integration does not create market power on the non-integrated manufacturer, foreclosure cannot be the result of a 'raise the rival's cost' effect. Foreclosure may arise, or not, because the integrated firm has some market power over developers, which ultimately impacts the non-integrated manufacturer's profit. For instance, when buyers value strongly applications, the integrated platform subsidizes developers and increases the price of its device, which boosts the non-integrated manufacturer's demand. As a result, the vertical merger benefits the non-integrated manufacturer. A reverse conclusion obtains when developers value more the participation of buyers. Summarizing, foreclosure of the non-integrated manufacturer is neither systematic nor the result of a 'raise the rival's cost' effect. It is, rather, the mere collateral damage of the integrated firm's market power over developers that, sometimes, depending on the strength and the structure of network effects, leads to an asymmetric pricing structure.

WELFARE. Assuming a linear specification of our model, we fully characterize the impact of vertical integration on buyer and developer surpluses. In a nutshell, when indirect network effects are strong and sufficiently asymmetric, a situation that may characterize more infant platform markets, large social gains can be generated by implementing an asymmetric pricing structure that internalizes these effects. This is precisely what the integrated platform does, and vertical integration benefits buyers and developers. Otherwise, when network effects are balanced or weak, a situation that may characterize more

[5]We focus on the literature that determines under which circumstances vertical integration creates some market power and leads to harmful foreclosure of non-integrated competitors. Another strand, following Hart and Tirole (1990), shows that vertical integration may be used as a mean to restore the upstream market power that was eroded by a lack of commitment; see Rey and Tirole (2007) and Riordan (2008) for surveys.

mature platform markets, the pricing structure chosen by the integrated platform aims more to directly extract surpluses from buyers and developers through price increases. In these cases, developers and buyers tend to be harmed by the vertical merger.

Importantly, there is no obvious correlation between buyer/developer harm and foreclosure of the non-integrated manufacturer. For instance, when network effects are much stronger on the developers side than on the buyers side of the market, buyers and developers may benefit from the merger; internalization of indirect network effects by the integrated platform may require to increase the developer fee, which hurts the non-integrated manufacturer.

Efficiency Gains. Next, we consider that vertical integration creates synergies. In a traditional one-sided framework, synergies have two facets. They are pro-competitive because they are passed through partly to buyers in the form of a lower price for the integrated platform's device. They are anti-competitive because they create some market power that allows the integrated firm to command some payment from the non-integrated manufacturer, thereby softening competition on the buyers' market through a 'raise the rival's cost' effect.

The analysis becomes more complex in our two-sided framework. Although a more efficient integrated platform is able to command a higher fee from the non-integrated manufacturer, it is not always willing to do so. This holds because, again, the integrated platform uses its pricing instruments to extract surplus (from buyers, developers and the non-integrated manufacturer) but also to internalize network effects across both sides of the market. Such internalization requires, sometimes, to lower the fee paid by the non-integrated manufacturer below its pre-merger level. To illustrate, when network effects are stronger on the developers' side than on the buyers' side, subsidizing buyers can be done by setting a low price for the integrated platform's device and charging a low fee to the non-integrated manufacturer. Whether the fees charged by the integrated platform increase or decrease following the vertical merger depends, again, on the structure and the strength of indirect network effects.

We then study whether vertical integration leads to foreclosure and harms buyers or developers. Overall, and in line with the situation without efficiency gains, vertical integration tends again to be beneficial (respectively, detrimental) to welfare when indirect network effects are strong and asymmetric (respectively balanced or weak).

Coordination Motives and Porting Costs. Finally, we discuss the impact of vertical integration when platform users gain if manufacturers adopt the same operating system (perhaps because of direct network effects between users), or when developers have a cost to port their applications on different platforms. In these situations, there are motives of coordination between manufacturers. Much as in the case of efficiency gains, coordination motives create market power over the non-integrated manufacturer, because vertical integration somewhat forces the coordination of manufacturers on the integrated firm's operating system. However, such market power is not necessarily detrimental to welfare; this depends on the strength and the structure of network effects.

Related literature. To the best of our knowledge, our paper is the first to link, on the one hand, the literature on two-sided markets and, on the other hand, the literature on strategic vertical integration, in the specific context of platform-manufacturer relationships.

From the literature on two-sided markets, we borrow the general insight that indirect network effects are key to understanding platform pricing and competition (Caillaud and Jullien, 2003; Armstrong, 2006; Rochet and Tirole, 2006; Weyl, 2010). That literature has considered the effect of exclusive dealing between a platform and content providers (that is, developers in our model): Evans (2013) discusses the antitrust of such vertical relations in platform industries; Doganoglu and Wright (2010) and Hagiu and Lee (2011) provide a rationale for why platforms sign exclusive contracts with content providers; Church and Gandal (2000) describe the incentives of a manufacturer that is integrated with a developer to make its applications compatible with the hardware of a rival manufacturer; Hagiu and Spulber (2013) show that investment in first-party content (that is, vertical integration with one side of the market) depends on whether a platform faces a 'chicken-and-egg' coordination problem; in the video game industry, Lee (2013) finds that exclusivity tends to be pro-competitive, in that it benefits an entrant platform more than an incumbent platform. While we share with these papers the issue of the competitive impact of vertical restraints in two-sided markets, our work also differs substantially, for we are interested in the interactions between platforms/operating systems and manufacturers when devices are an essential link to connect buyers and developers.

Our analysis also belongs to the strategic approach of vertical integration initiated by Ordover et al. (1990). A message conveyed by that literature is that vertical integration can lead to input foreclosure and be detrimental to consumer surplus. Analyses that feature trade-offs between the pro- and the anti-competitive effects of vertical integration include the following: Ordover et al. (1990) and Reiffen (1992), in which integration generates an extra commitment power; Riordan (2008) and Loertscher and Reisinger (2014), in which the integrated firm is dominant; Chen (2001), in which manufacturers have switching costs; Choi and Yi (2000), in which upstream suppliers can choose the specification of their inputs; Chen and Riordan (2007), in which exclusive dealing can be used in combination with integration; Nocke and White (2007) and Normann (2009), in which upstream suppliers tacitly collude; Hombert et al. (2019), in which there are more manufacturers than upstream suppliers; and Hunold and Stahl (2016), in which integration can be either controlling or passive.[6] None of these papers address multi-sided markets, and our analysis provides several new insights. For instance, vertical integration does not systematically lead to input foreclosure and may benefit both consumers and non-integrated manufacturers even in the absence of efficiency gains.

ORGANIZATION OF THE PAPER. Section 2 describes the model. Section 3 provides several useful benchmarks. Sections 4 and 5 analyze the impact of vertical integration without and with efficiency gains respectively. Section 6 discusses the role of coordination motives and porting costs. Section 7 concludes. All proofs are relegated to an Appendix.

2. MODEL AND PRELIMINARY RESULTS

We consider a two-sided market where buyers of devices and developers of applications may interact. These interactions require: buyers to purchase devices from manufacturers; developers to decide how much applications to develop and for which operating systems; manufacturers to choose an operating system and a price for their devices; platforms to license their operating systems to device manufacturers and set fees to publish applications

[6]For empirical analyses, see, e.g., Lafontaine and Slade (2007) and Crawford et al. (2018) and the references therein.

on their operating systems.

2.1. Technologies, Preferences and Timing

Platforms. Platforms compete to license their operating systems to manufacturers of devices and attract developers of applications. Platforms are symmetric: the marginal cost to provide an operating system is normalized to 0 without loss of generality. There are $N+1$ (with $N \geq 2$) such platforms, denoted by I, E_1, ..., E_N. In the following, platform I will be the one contemplating a merger with a manufacturer and we sometimes refer to platforms E_1, ..., E_n as the fringe of platforms. Let $\mathcal{P}$ denote the set of platforms. There are two manufacturers, denoted by M_1 and M_2.

The contractual relationship between a manufacturer and a platform typically specifies which party owns user-generated data and, accordingly, who can monetize these data through targeted advertising for instance. Let r be the per-user benefit generated by a buyer of a device equipped with an operating system. A platform decides how this benefit is shared with manufacturers: $\beta_i^k \in [0,1]$ (respectively, $1-\beta_i^k$) is the share of r kept by a manufacturer k (respectively, platform i) if it equips its devices with platform i's operating system.[7]

Platforms also charge fees to application developers.[8] Denote by a_i the fee charged by platform i to allow a developer to make its application available on that platform's operating system. That fee can be either positive or negative.

The profit of a platform $i \in \mathcal{P}$ can be expressed as follows

$$\Pi_i = \sum_{k=1,2} \mathbb{1}_{\{M_k \text{ adopts } i\}}(1-\beta_i^k) r Q_B^k + a_i q_i,$$

where (i) $\mathbb{1}_{\{M_k \text{ adopts } i\}}$ is the indicator function equal to 1 when manufacturer M_k chooses platform i's operating system and 0 otherwise, (ii) Q_B^k is the number of buyers of device k, and (iii) q_i is the number of applications available on platform i's operating system.

Manufacturers and Buyers of Devices. Manufacturers are symmetric and produce at the same constant marginal cost normalized without loss of generality to 0. The number of buyers of manufacturer M_k's device depends on the prices charged by manufacturers to buyers, denoted by p_k and p_ℓ, with $k \neq \ell \in \{1,2\}$, and on the number of

[7]As a broader illustration, in the recent Google and Alphabet v Commission case, the European Commission found that Google has infringed Article 102 TFEU by entering into several agreements with Original Equipment Manufacturers and Mobile Network Operators (Commission Decision C(2018) 4761 final of 18 July 2018, available at https://eur-lex.europa.eu/legal-content/GA/TXT/?uri=CELEX:52019XC1128(02)). Of particular relevance were: (i) the mobile application distribution agreements, which required OEMs and MNOs wishing to pre-install Google Play on their devices to also pre-install other Google applications; (ii) the portfolio-based revenue sharing agreements, according to which Google provided payments to OEMs and MNOs in return for having the Google Search application exclusively pre-installed on a given portfolio of smart mobile devices. The 14th September 2022, the Court of Justice of the European Union largely confirmed the Commission's decision that Google imposed unlawful restrictions on manufacturers of Android mobile devices and mobile network operators in order to consolidate the dominant position of its search engine (see https://curia.europa.eu/jcms/upload/docs/application/pdf/2022-09/cp220147en.pdf).

[8]Software platforms often charge developers on participation (Google charges developers $25 for each application published on the Play Store) or on transaction each time an application is sold on the platform (both Apple and Google charge a roughly 30% royalty on each transaction on their respective applications stores).

applications running on the devices, denoted by n_S^k and n_S^ℓ. Hence, it may be written as $Q_B^k(p_k, p_\ell, n_S^k, n_S^\ell)$. Assume that these so-called 'quasi-demand functions' are symmetric: $Q_B^k(p_k, p_\ell, n_S^k, n_S^\ell) = Q_B^\ell(p_\ell, p_k, n_S^\ell, n_S^k)$. The profit of a manufacturer M_k when it chooses platform i's operating system can thus be written as follows[9]

$$\pi_k = (p_k + \beta_i^k r) Q_B^k.$$

From the buyer side, assume that given some numbers of applications running on the manufacturers' devices (n_S^1, n_S^2): devices are demand substitutes for buyers, or $\partial Q_B^k/\partial p_k < 0 < \partial Q_B^k/\partial p_\ell$; the direct price effect is stronger than the indirect one, or $\partial Q_B^k/\partial p_k + \partial Q_B^k/\partial p_\ell < 0$; buyers of device k value positively the number of applications available on their devices, or $\partial Q_B^k/\partial n_S^k > 0$, but negatively the number of applications available on the other device, or $\partial Q_B^k/\partial n_S^\ell < 0$.[10] Last, to compute the buyer surplus, we consider that there exists a representative buyer with utility function $U_B(q_1, q_2, n_S^1, n_S^2)$ such that Q_B^1 and Q_B^2 are solutions of $\max_{(q_1 \geq 0, q_2 \geq 0)} U_B(q_1, q_2, n_S^1, n_S^2) - p_1 q_1 - p_2 q_2$. Let $V_B(p_1, p_2, n_S^1, n_S^2)$ denote the corresponding indirect utility.

APPLICATION DEVELOPERS. We assume that there is a representative developer which bears a strictly increasing convex cost $C_S(q_S)$ to develop q_S applications. Once applications are developed, the developer bears the platform-specific cost $C_i(q_i) = c_i q_i$, with $c_i \geq 0$, to make q_i applications available on platform i (with $0 \leq q_i \leq q_S$). For the moment, we assume that there are no porting costs for all platforms, or $c_i = 0$ for all $i \in \mathcal{P}$. This assumption will be relaxed in Section 6 where we show that porting costs create a specific source of market power.

Assume that platforms set some developer fees $(a_i)_{i \in \mathcal{P}}$. Let n_B^i be the number of buyers using a device running platform i's operating system. When the developer creates q_S applications and publishes q_i of these on platform i, its profit is given by $\sum_{i \in \mathcal{P}} (u_S n_B^i q_i - a_i q_i) - C_S(q_S)$. In words, q_i applications published on platform i with a number of users n_B^i yields a gross benefit of $u_S n_B^i q_i$. Parameter u_S relates to the strength of indirect network effects from the developer side of the market.

Since there are no porting costs, all applications are published on platform i (that is, $q_i = q_S$) as soon as $u_S n_B^i - a_i \geq 0$. The developer's gross profit thus simplifies to $U_S(q_S, (n_B^i)_{i \in \mathcal{P}}) = q_S \sum_{i \in \mathcal{P}} (u_S n_B^i) \mathbb{1}_{\{u_S n_B^i - a_i \geq 0\}} - C_S(q_S)$ and its net profit writes as follows

$$q_S \sum_{i \in \mathcal{P}} (u_S n_B^i - a_i) \mathbb{1}_{\{u_S n_B^i - a_i \geq 0\}} - C_S(q_S). \tag{2.1}$$

Let $Q_S((n_B^i, a_i)_{i \in \mathcal{P}})$ be the number of applications q_S that maximizes (2.1) and denote by $V_S((n_B^i, a_i)_{i \in \mathcal{P}})$ the corresponding developer profit.

Figure 1 summarizes the structure of the model.

TIMING. In stage 1, platforms set the shares of the per-user benefit left to manufacturers in exchange of using their operating systems and the fees charged to developers. In stage 2, manufacturers choose the operating system for their devices. Once operating systems have been chosen, manufacturers set the prices of their devices in stage 3. Last, in stage

[9]The sharing parameter β_i^k acts thus like a negative perceived marginal cost for manufacturers.

[10]The numbers of applications available on the various devices can thus be viewed as endogenous quality attributes.

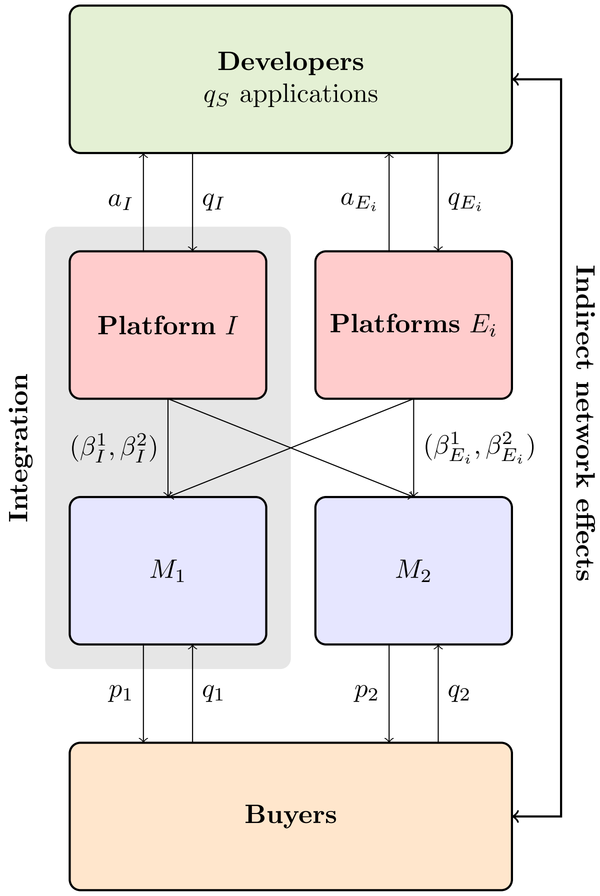


Figure 1: The model.

4, buyers decide whether to buy a device, and, simultaneously, developers decide how much applications to develop and on which platforms to publish. All decisions are public and we look for the subgame-perfect equilibrium of the game.

Running Example. We sometimes use the following specification of the model, in particular to compute the welfare impact of vertical integration.

- *Buyers.*

  Demand for device k is given by

$$Q_B^k(p_k, p_\ell, n_S^k, n_S^\ell) = \Big(v - p_k - \gamma\Big(p_k - \frac{p_k + p_\ell}{2}\Big)\Big) + \Big(u_B n_S^k + \frac{\gamma}{2} u_B (n_S^k - n_S^\ell)\Big), \tag{2.2}$$

  Terms in the first parenthesis in Equation (2.2) correspond to a standard product-market interaction with imperfectly substitutable products. Terms in the second parenthesis illustrate how indirect network effects between device users and application developers impact the demand for the manufacturers' products.

  The utility function of the representative buyer is given by[11]

$$U_B(q_1, q_2, n_S^1, n_S^2) = q_0 + \sum_{k=1,2} (v q_k + u_B n_S^k q_k) - \frac{1}{2}\frac{1}{2(1+\gamma)}\Big(2\sum_{k=1,2} q_k^2 + \gamma(\sum_{k=1,2} q_k)^2\Big),$$

[11]This is Shubik and Levitan (1980)'s linear demands system, to which we append indirect network effects additively.

where q_0 is the numéraire and q_k is the quantity of device k bought.

- *Developers.*

  The development cost is $C_S(q_S) = \frac{1}{2}q_S^2$. Hence,

$$Q_S((n_B^i, a_i)_{i\in\mathcal{P}}) = \sum_{i\in\mathcal{P}}(u_S n_B^i - a_i)\mathbb{1}_{\{u_S n_B^i - a_i \geq 0\}}.$$

2.2. *Participation Decisions*

Consider now the last stage of the game. At that stage, platforms have set their fees $(\beta_1^i, \beta_2^i, a_i)_{i\in\mathcal{P}}$ and manufacturers have chosen their operating system and the prices (p_1, p_2) for the devices.

DEVELOPERS' CHOICES OF OPERATING SYSTEMS. The first question we address concerns the choice made by developers to publish their applications on the various operating systems.

Before doing so, we can simplify the exposition with the following observation. Any platform i should never set a developer fee a_i that discourages the developer from publishing on its operating system, for such strategy is weakly dominated by setting a zero fee. Hence, we can consider without loss of generality that $u_S n_B^i - a_i \geq 0$ for any platform $i \in \mathcal{P}$. Therefore, the representative developer is willing to publish all its applications on all the platforms (that is, $q_i = q_S$ for all $i \in \mathcal{P}$). This implies that whatever the choices of operating systems by the manufacturers, the developer is able to interact with all the buyers of devices, or $\sum_{i\in\mathcal{P}} n_B^i = n_B^1 + n_B^2$. The developer's profit can thus be rewritten more simply as $(u_S(n_B^1 + n_B^2) - a)q_S - C_S(q_S)$ where $a \equiv \sum_{i\in\mathcal{P}} a_i$ denotes the 'total developer fee.' The number of applications n_S that maximizes this profit is given by

$$n_S = Q_S(u_S(n_B^1 + n_B^2) - a),$$

where $Q_S = (C_S')^{-1}$.

Another immediate consequence of the fact that $u_S n_B^i - a_i \geq 0$ for any platform i is that whatever their choices of operating systems, manufacturers benefit from the same number of applications running on their devices: $n_S^1 = n_S^2 \equiv n_S$. The demand for device k may now be written more simply as $n_B^k = Q_B^k(p_k, p_\ell, n_S)$.

BUYERS' AND DEVELOPERS' PARTICIPATION DECISIONS. Given the prices of devices p_1 and p_2 and a total fee a paid by developers, the number of buyers of each device and the number of applications must be consistent with each other and solve

$$\left\{\begin{array}{lcl} n_B^1 & = & Q_B^1(p_1, p_2, n_S), \\ n_B^2 & = & Q_B^2(p_2, p_1, n_S), \\ n_S & = & Q_S(u_S(n_B^1 + n_B^2) - a). \end{array}\right. \tag{2.3}$$

Assume that the solution of (2.3) is unique and interior for the relevant range of prices.[12] That solution defines, as functions of the prices of the devices and the developer fee, the buyers' demands for devices, denoted by $D_k(p_k, p_\ell, a)$ with $k \neq \ell \in \{1, 2\}$, and the number of applications developed (also called the developers' demand), denoted by $D_S(p_1, p_2, a)$.

[12] As shown in Appendix A.1, this requires that indirect network effects are not too strong.

As shown in Appendix A.1, the following usual properties hold: the developers' demand is decreasing in the prices of devices and in the developer fee ($\partial D_S/\partial p_k < 0$ and $\partial D_S/\partial a < 0$); the demand for a device is decreasing in its own price and in the developer fee ($\partial D_k/\partial p_k < 0$ and $\partial D_k/\partial a < 0$). We further impose that the demand for a device is more responsive to its own price than to the price of the other device ($\partial D_k/\partial p_k + \partial D_k/\partial p_\ell < 0$). In the running example, all these properties hold provided that $2u_B u_S < 1$.

Impact of Networks Effects on Product Market Interactions. Perhaps more surprising is the fact that indirect network effects impact the nature of the interaction between manufacturers on the product market. The demand faced by a manufacturer may, indeed, either increase or decrease with the price of the rival manufacturer, depending on the strength of indirect network effects relative to the degree of product market competition. Formally, using the system (2.3), it follows immediately that (omitting arguments)

$$\frac{\partial D_k}{\partial p_\ell} = \frac{\partial Q_B^k}{\partial p_\ell} + \frac{\partial Q_B^k}{\partial n_S}\frac{\partial D_S}{\partial p_\ell},$$

which can be positive or negative. The intuition is as follows. If p_ℓ increases, then some buyers are diverted from M_ℓ, and M_k's demand increases by $\partial Q_B^k/\partial p_\ell$. This is a standard rivalry effect created by product market competition between manufacturers. The increase in p_ℓ has, moreover, a negative impact on the total number of buyers, since the direct price effect on buyers of device ℓ is stronger than the indirect price effect on buyers of device k ($\partial Q_B^\ell/\partial p_\ell + \partial Q_B^k/\partial p_\ell < 0$). Since there are less buyers overall, there are fewer applications too, for developers find it less attractive to develop ($\partial D_S/\partial p_\ell < 0$). Because buyers value applications, this negatively affects M_k's demand by $\partial Q_B^k/\partial n_S$. We therefore expect that when indirect network effects are small (that is, when $(\partial Q_B^k/\partial n_S)(\partial D_S/\partial p_\ell) \approx 0$), the rivalry effect created by product market competition dominates and $\partial D_k/\partial p_\ell \geq 0$, that is, devices are demand substitutes. By contrast, when product market competition is weak (that is, when $\partial Q_B^k/\partial p_\ell \approx 0$), then the interaction created by indirect network effects dominates and $\partial D_k/\partial p_\ell \leq 0$, that is, devices are demand complements. In our running example, devices are demand substitutes when $\gamma - 2u_B u_S(1+\gamma) > 0$, and demand complements otherwise.

In the sequel, we shall focus on the case studied by the bulk of the literature on strategic vertical integration, namely the case where manufacturers' products are demand substitutes:

Assumption 1. *Indirect network effects are not too strong relative to product market competition so that manufacturers' products are demand substitutes: for $k \neq \ell$, for all (p_k, p_ℓ, a)*

$$\frac{\partial D_k}{\partial p_\ell}(p_k, p_\ell, a) \geq 0.$$

In the running example, this amounts to $\sigma \equiv \gamma - 2u_B u_S(1+\gamma) \geq 0$.

2.3. Competition between Manufacturers

In stage 3, manufacturers compete on the product market. Given a share β_k of the per-user benefit that M_k receives from the platform it has chosen and a total fee a paid by developers, let $\pi_k(\beta_k, p_k, p_\ell, a) = (p_k + \beta_k r)D_k(p_k, p_\ell, a)$ denote M_k's profit. We make some assumptions on manufacturers' best responses in prices that ensure the price competition subgame is 'well-behaved.'

M_k's best response, denoted by $R_k(\beta_k, p_\ell, a)$, is uniquely characterized by the first-order condition $\frac{\partial \pi_k}{\partial p_k}(\beta_k, R_k, p_\ell, a) = 0$. Moreover, $0 \leq \partial R_k/\partial p_\ell < 1$ for all $(\beta_k, p_k, p_\ell, a)$, so that prices of devices are strategic complements and best responses satisfy the usual stability assumption.[13] Last, M_k's best response decreases with a, that is, $\partial R_k/\partial a \leq 0$ for all $(\beta_k, p_k, p_\ell, a)$.[14] This assumption seems reasonable since an increase in the developer fee negatively impacts the demand for device k. Together, these assumptions ensure that there exists a unique pair of prices $(\hat{p}_1(\beta_1, \beta_2, a), \hat{p}_2(\beta_2, \beta_1, a))$ that form the Nash equilibrium of stage 3 of the game, and that the equilibrium price of a manufacturer is decreasing in its share of the per-user benefit and in the developer fee, or $\partial \hat{p}_k/\partial \beta_k < 0$ and $\partial \hat{p}_k/\partial a \leq 0$. We further impose that $|\partial \hat{p}_k/\partial \beta_k| < r$.[15,16]

Let $\hat{\pi}_k(\beta_k, \beta_\ell, a) = \pi_k(\beta_k, \hat{p}_k(\beta_k, \beta_\ell, a), \hat{p}_\ell(\beta_\ell, \beta_k, a), a)$ denote M_k's profit at the equilibrium of the subgame starting at stage 3. From the assumptions made above, we obtain the following: (i) $\frac{\partial \hat{\pi}_k}{\partial a}(\beta_k, \beta_\ell, a) \leq 0$ for all (β_k, β_ℓ, a) because an increase in the developer fee reduces the number of applications and acts thus as a negative shock on the demands faced by manufacturers; (ii) a manufacturer's profit increases with the share of the per-user benefit it receives, or $\frac{\partial \hat{\pi}_k}{\partial \beta_k}(\beta_k, \beta_\ell, a) > 0$ for all (β_k, β_ℓ, a). Roughly speaking, the assumptions on the manufacturers subgame ensure that the direct shift in their profit functions (associated to a change in the sharing parameter or in the developer fee) is stronger than the indirect shift in the marginal profit, which in turn changes the equilibrium between manufacturers.[17]

3. BENCHMARKS

We study three relevant benchmarks: the welfare-maximizing outcome; the case of a perfectly competitive platform; the situation of 'separation', in which none of the platforms are integrated with manufacturers.

3.1. Ramsey Pricing

Let $\Pi(p_1, p_2, a) = (p_1 + r)D_1(p_1, p_2, a) + (p_2 + r)D_2(p_2, p_1, a) + aD_S(p_1, p_2, a)$ denote the industry profit, that is, the sum of the platforms' and the manufacturers' profits. Welfare is then given by $W(p_1, p_2, a) = V_B(p_1, p_2, D_S(p_1, p_2, a)) + V_S(a, D_1(p_1, p_2, a) + D_2(p_2, p_1, a)) + \Pi(p_1, p_2, a)$.

WELFARE-MAXIMIZING PRICES. Maximizing welfare requires to set prices for the devices and a developer fee that are below the corresponding perceived marginal costs in order to internalize network effects across buyers and developers. Simple computations show

[13] This holds when $0 < \frac{\partial^2 \pi_k}{\partial p_k \partial p_\ell} < -\frac{\partial^2 \pi_k}{\partial p_k^2}$ for all $(\beta_k, p_k, p_\ell, a)$. See Seade (1980) and Dixit (1986).

[14] Observe that $\frac{\partial R_k}{\partial a} < 0$ amounts to $\frac{\partial^2 \pi_k}{\partial p_k \partial a} = \frac{\partial D_k}{\partial a} + D_k \frac{\partial^2 D_k}{\partial p_k \partial a}(-\frac{\partial D_k}{\partial p_k})^{-1} < 0$. Hence, R_k decreases with a if either $\frac{\partial^2 D_k}{\partial p_k \partial a} < 0$ or if $\frac{\partial^2 D_k}{\partial p_k \partial a} > 0$ but small enough.

[15] This is the equivalent of the usual assumption that cost pass-throughs are smaller than one. It can be linked to the log-curvature of demand functions as discussed in Weyl and Fabinger (2013) and Ritz (2015) for instance.

[16] All these assumptions are satisfied in our running example as we show in Appendix A.2.

[17] Although intuitive, these properties may not always hold. In a Cournot oligopoly, Seade (1985), Kimmel (1992) and Linnemer (2003), among others, find conditions under which an increase in the marginal cost of several firms increases or decreases equilibrium profits; see Février and Linnemer (2004) for a unifying framework. Cowan (2004) extends the analysis to demand shocks. See also Dixit (1986) and Leahy and Neary (1997) for the case of Bertrand oligopolies.

indeed that, in an interior optimum, the socially optimal prices are given by (omitting some notations)[18]

$$\begin{aligned} p_1 + r = p_2 + r &= -\frac{\partial U_S}{\partial n_B}(D_S, D_1 + D_2), \\ a &= -\frac{\partial U_B}{\partial n_S}(D_1, D_2, D_S). \end{aligned}$$

Although these prices maximize welfare, they provide the industry with a loss. Absent the possibility to make transfers between, on the one hand, buyers and developers, and, on the other hand, platforms and manufacturers, prices of devices and the developer fee must ensure that the industry breaks even.

RAMSEY PRICES. Imposing that the industry breaks even prevents from setting prices below marginal costs on both sides of the market. Suppose that the break-even constraint binds

(3.1) $$(p_1 + r)D_1 + (p_2 + r)D_2 + aD_S = 0.$$

This implies that either devices are sold at a negative margin ($p_1 + r < 0$ and $p_2 + r < 0$) and application developers are charged a strictly positive fee ($a > 0$); or, devices are sold at a strictly positive margin ($p_1 + r > 0$ and $p_2 + r > 0$) and developers are given a subsidy ($a < 0$). Intuitively, developers should be subsidized when they create sufficiently strong an externality on buyers (that is, when buyers have a stronger valuation for applications than developers' valuation for the number buyers), while keeping an eye on how such subsidies impact the break-even constraint (3.1). This intuition is made rigorous in Appendix A.3, in which we show that developers are subsidized if and only if (omitting some notations)

(3.2) $$\frac{1}{\eta_S}\left(\frac{\partial U_S}{\partial n_B} + \frac{\partial U_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}\right) < \frac{1}{\eta_B}\left(\frac{\partial U_B}{\partial n_S} + \frac{\partial U_S}{\partial n_B}\frac{\partial Q_B}{\partial n_S}\right),$$

where $\eta_B = -\frac{1}{n_B}(\frac{\partial Q_B}{\partial p_1}(p, p, n_S) + \frac{\partial Q_B}{\partial p_2}(p, p, n_S))$, $\eta_S = -\frac{1}{n_S}\frac{\partial Q_S}{\partial a}(a, n_B)$ and $Q_B = Q_B^1 + Q_B^2$. There are three terms in the left-hand side of Equation (3.2). The first term ($1/\eta_S$) is the inverse of the semi-elasticity of the developers' demand. The second term ($\frac{\partial U_S}{\partial n_B}$) measures the extent to which developers benefit directly from an increase in the participation of buyers. The third term ($\frac{\partial U_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}$) measures the extent to which buyers benefit from an increase in their own participation through a feedback effect: an increase in the number of buyers boosts the number of applications, which ultimately benefits buyers. Therefore, Equation (3.2) shows that developers are subsidized (and buyers are thus taxed) when they are the 'high-elasticity group of users', that is, when, relative to buyers, their demand is more price elastic, and when they benefit less from the participation of buyers and more from their own participation through the feedback effect.

The next result confirms this intuition in our running example.[19]

LEMMA 1. *In the running example, the price of devices p^R and the developer fee a^R that maximize welfare W subject to the industry break-even constraint* (3.1) *are such that $a^R > 0$ and $p^R + r < 0$ if and only if $u_B < 2u_S^3$.*

[18]See Appendix A.3.

[19]Some assumptions are needed to ensure that the Ramsey problem, and the other maximization problems analyzed below, are quasi-concave. In the running example, these conditions are fully characterized in Appendix A.3.

Proof. See Appendix A.3. □

At the optimum, the industry's break-even constraint binds even though there are no fixed costs of production. This emphasizes that indirect network effects makes the social planner willing to operate cross-subsidies between the various sides of the market.

3.2. A Perfectly Competitive Platform

The next benchmark assumes that there is only one platform that maximizes the manufacturers' profits subject to a break-even constraint. Because manufacturers are symmetric, we can assume that the platform offers the same sharing of the per-user benefits β. Let a denote the developer fee. The perfectly competitive platform's problem may now be written as follows (omitting some notations)

$$\max_{(\beta,a)} \sum_{k=1,2} \hat{\pi}_k(\beta,\beta,a)$$
$$\text{s.t.} \sum_{\substack{k=1,2\\ \ell\neq k}} (1-\beta) r D_k(\hat{p}_k,\hat{p}_\ell,a) + a D_S(\hat{p}_1,\hat{p}_2,a) \geq 0.$$

Manufacturers benefit when developers are subsidized since this boosts the number of applications running on their devices. However, setting a negative developer fee (that is, $a<0$) requires to leave some of the per-user benefits to the platform (that is, $\beta<1$) to ensure the break-even condition. Intuitively, subsidizing developers is worth for manufacturers only when this sufficiently increases their demands, which arises when buyers strongly value the number of applications running on the devices. The next result makes this intuition formal in the case of our running example.

LEMMA 2. *In the running example, a perfectly competitive platform makes no profit and sets*

- $\beta^C = 1$ *and* $a^C = 0$ *if* $u_B \leq u_S$;
- $\beta^C < 1$ *and* $a^C < 0$ *if* $u_B > u_S$.

Proof. See Appendix A.3. □

3.3. Separation

The situation of 'separation,' in which none of the manufacturers are integrated with either platforms, serves as our benchmark to assess the impact of vertical integration. Because several operating systems are available to manufacturers, there may exist several Nash equilibria in the subgame of choice of operating systems by the manufacturers (stage 2 of our game). We want to avoid situations where a platform obtains some unduly market power created by the mere lack of coordination between manufacturers. To that end, we impose the following selection on the equilibrium set: If there exists several Nash equilibria in the subgame starting at stage 2, we select the one that maximizes the manufacturers' joint profit. This is a mild yet meaningful restriction commonly found in the literature.[20]

[20] To see its purpose in our setting, suppose that platform I sets $(\beta_I = 1, a_I > 0)$ and platforms from the fringe set $(\beta_E = 1, a_E = 0)$. There are several Nash equilibria in the subgame starting at stage

Then, we obtain the following result.

Lemma 3. *In equilibrium under separation, developers pay no fee ($a^S = 0$), manufacturers obtain all the per-user benefit ($\beta^S = 1$), platforms make no profits and manufacturers are indifferent between any of the platforms' operating systems.*

Proof. See Appendix A.4. □

To provide some intuition, suppose all platforms set $\beta_1 = \beta_2 = 1$ and a nil developer fee. Platforms make no profit, developers publish their applications on all platforms and manufacturers are indifferent between all the operating systems. Consider now a deviation by, say, platform I, which sets $a_I < 0$ and $\beta_I < 1$ such that its profit remains nil. As illustrated in Lemma 2, this deviation could be profitable for manufacturers. However, each manufacturer individually has the incentives to choose a platform from the fringe. Indeed, the developer can costlessly port its applications on platforms from the fringe while still enjoying the subsidy offered by platform I; even if a manufacturer chooses a platform from the fringe, it will still benefit from the same number of applications. And platforms from the fringe offer a higher share of the per-user benefit, which attracts each manufacturer individually. Put differently, a platform can freeride on the subsidy offered to the developer by another platform.

Figure 2 summarizes the Ramsey and the separation benchmarks for the running example. Under separation, competition between symmetric platforms prevents the exercise of any market power at the platform level. Competition also limits the ability of platforms to implement some implicit redistribution between the different groups of users and prevents platforms from harnessing indirect network effects between the various groups of users through an asymmetric price structure. This will prove important to understand the consequences of vertical integration, which we study now.

4. Vertical Integration

Consider now that platform I is integrated with manufacturer M_1. To streamline the exposition, we directly consider that competition between non-integrated platforms $E_1, ..., E_N$ leads them to set a nil developer fee and a sharing parameter equal to 1, a result that follows from the logic of Lemma 3.

Our analysis proceeds as follows. First, we show that vertical integration creates market power over developers (Section 4.1). Second, we study how such market power is exercised (Section 4.2) and its consequences on welfare (Section 4.3).

4.1. Price Competition and Choice of Operating System

At stage 3 of the game, the integrated platform and the non-integrated manufacturer compete in prices to sell their devices to buyers. The outcome of that price competition subgame depends on whether the non-integrated manufacturer chooses one the non-integrated platform's operating system (in the following, we will refer to this case by saying that M_2 chooses 'platform E') or that of the integrated platform:

2, one in which both manufacturers choose I's operating system (because if a manufacturer deviates, the developer still pays a_I to reach buyers of the other manufacturer's device), another one in which each manufacturer adopts the operating system of a platform from the fringe. Clearly, the former is Pareto-dominated (for manufacturers) by any of the latter.

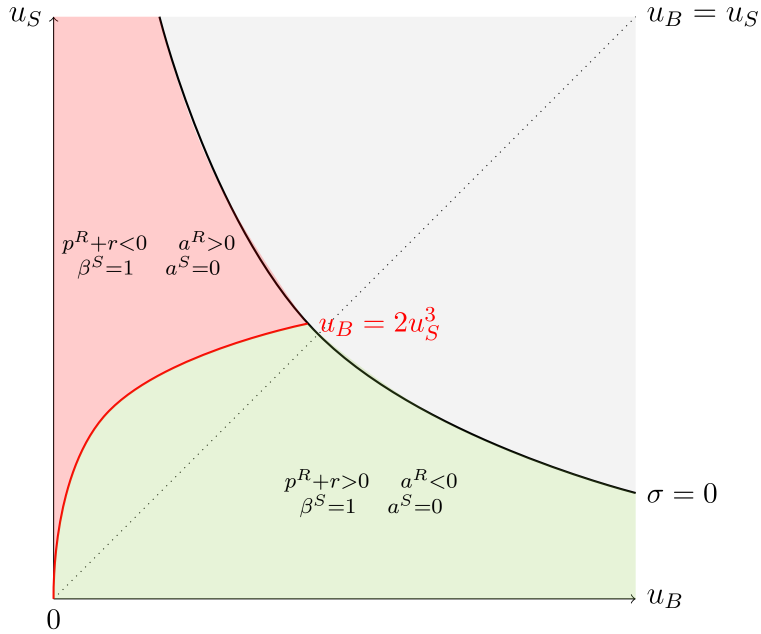


Figure 2: Ramsey and separation benchmarks in the running example. Note: the grey area corresponds to non admissible values of the parameters ($\sigma < 0$).

(E). When M_2 chooses E's operating system, its profit is $(p_2 + r)D_2$ and the integrated platform's profit is

$$(p_1 + r)D_1 + a_I D_S \tag{4.1}$$

because developers are willing to reach buyers of the integrated manufacturer's device.

(I). When M_2 chooses the integrated platform's operating system, its profit writes now as $(p_2 + \beta_I r)D_2$ and that of the integrated platform is given by

$$(p_1 + r)D_1 + a_I D_S + (1 - \beta_I)rD_2 \tag{4.2}$$

because it perceives some per-user benefit from the non-integrated manufacturer on top of the revenues earned from developers.

For both cases, we adopt implicitly the same assumptions on best responses as those made in Section 2.3 to ensure that the price competition subgame is 'well-behaved.' In case (E) (resp. case (I)), price competition on the product market then leads to equilibrium prices denoted by $p_1^E(1, 1, a_I)$ and $p_2^E(1, 1, a_I)$ (resp. $p_1^I(1, \beta_I, a_I)$ and $p_2^I(\beta_I, 1, a_I)$). Let $\pi_1^E(1, 1, a_I) = (p_1^E + r)D_1(p_1^E, p_2^E, a_I) + a_I D_S(p_1^E, p_2^E, a_I)$ and $\pi_2^E(1, 1, a_I) = (p_2^E + r)D_2(p_2^E, p_1^E, a_I)$ (resp. $\pi_1^I(1, \beta_I, a_I) = (p_1^I + r)D_1(p_1^I, p_2^I, a_I) + (1 - \beta_I)rD_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$ and $\pi_2^I(\beta_I, 1, a_I) = (p_2^I + (1 - \beta_I)r)D_2(p_2^I, p_1^I, a_I))$ denote profits corresponding to case (E) (resp. (I)).

Let us then focus on the integrated firm's pricing incentives at stage 3 of the game. These incentives depend on the revenues raised from developers, namely $a_I D_S$. Intuitively, a low price for device 1 boosts the overall number of devices, which then increases the number of applications. This effect is present both when the integrated firm supplies the non-integrated manufacturer (see Equation (4.2)) and when it does not (see Equation (4.1)).

Pricing incentives also depend on whether or not the integrated firm licenses its operating system to the non-integrated manufacturer. Indeed, when this is the case, increasing the price of device 1 increases the demand for device 2 since manufacturers' products are demand substitutes, and, therefore, increases the revenues $(1-\beta_I)rD_2$ generated by the licensing of the operating system.[21] This is the so-called 'accommodation effect' found in the literature on strategic vertical integration. This effect suggests that the non-integrated manufacturer may be willing to accept a sharing parameter smaller than 1 to make the integrated manufacturer a softer competitor. Equivalently, the integrated platform has some market power over the non-integrated manufacturer.

That market power is, however, constrained by the competitive pressure exerted by platforms from the fringe. Indeed, the non-integrated manufacturer always has the option to choose the fringe's operating system. Therefore, the non-integrated manufacturer adopts the integrated platform's operating system if

$$\pi_2^I(\beta_I, 1, a_I) \geq \pi_2^E(1, 1, a_I). \tag{4.3}$$

4.2. The Integrated Platform's Pricing Policy

At the first stage of the game, the integrated platform's profit writes as $\pi_1^I(1, \beta_I, a_I) = (p_1^I + r)D_1(p_1^I, p_2^I, a_I) + (1-\beta_I)rD_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$. Formally, β_I and a_I are solution of[22]

$$\begin{aligned} \max_{(\beta_I, a_I)} \quad & \pi_1^I(1, \beta_I, a_I) \\ \text{s.t.} \quad & \pi_2^I(\beta_I, 1, a_I) \geq \pi_2^E(1, 1, a_I), \\ & 0 \leq \beta_I \leq 1. \end{aligned} \tag{4.4}$$

To study this problem, we proceed in two steps. First, we consider a relaxed problem in which none of the constraints are taken into account. This allows to understand the logic underlying the exercise of market power by the integrated platform. Second, we solve for the constrained problem.[23]

The Relaxed Problem. Maximizing the integrated firm's profit requires to extract revenues from developers and from the non-integrated manufacturer. This calls for increasing a_I and decreasing β_I, while keeping an eye on how this impacts competition on the buyer's market. But it also requires to take advantage of network effects across both sides of the market. When, for instance, buyers value strongly the applications offered by developers, the integrated platform wants to boost the number of applications available

[21] Formally, the integrated firm's best response in price changes from $D_1 + (p_1 + r)\partial D_1/\partial p_1 + a_I \partial D_S/\partial p_1 = 0$ when M_2 buys from the fringe, to $D_1 + (p_1 + r)\partial D_1/\partial p_1 + a_I \partial D_S/\partial p_1 + (1-\beta_I)r\partial D_2/\partial p_1 = 0$ when M_2 chooses the integrated firm's operating system. Since devices are demand substitutes, $\partial D_2/\partial p_1 \geq 0$ and the integrated firm's best response in price shifts upward when it licenses its operating system.

[22] In the main text (but not in the numerical simulations), we neglect the constraint $\beta_I \geq 0$. When it binds, it simply means the integrated firm wants to extract as much as possible of the per-user benefit associated to licensing its operating system to the non-integrated manufacturer.

[23] Some conditions are required to ensure that the maximization problems are quasi-concave. In the running example, these conditions are fully characterized in Appendix A.5.

on its device with a low developer fee and extract some of the buyer surplus through a low sharing parameter asked to the non-integrated manufacturer.

Let us assume that the solution of this relaxed problem is interior and denote by (β_I^*, a_I^*) the solution of the system formed by the two first-order conditions $\partial\pi_1^I/\partial\beta_I = 0$ and $\partial\pi_1^I/\partial a_I = 0$. In Appendix A.5, we characterize this solution for the running example. Figure 3 below represents the two curves $\beta_I^* = 1$ and $a_I^* = 0$ in the (u_B, u_S)-space to allow a comparison with the separation benchmark.

It is instructive to consider first the case analyzed in the extant literature, namely the case with no network effects or $u_B = u_S = 0$. The integrated platform sets the developer fee to 0 because developers have no intrinsic value to participate and bring no value to buyers. It sets a sharing parameter strictly below 1 because, first, this increases the revenues from licensing the operating system and, second, this raises the perceived marginal cost of the non-integrated manufacturer and thus relaxes competition on the buyer market.

Considering now network effects between both sides of the market leads to distinguish three broad cases:

- 'Strong and buyer-skewed network effects.' This corresponds to the green region in Figure 3, in which buyers' valuation for number of applications is larger than developers' valuation for the number of buyers (that is, $u_B > u_S$). There, the solution of the relaxed problem calls for setting a negative developer fee (that is, $a_I^* < 0$) to boost the participation of developers and extracting the buyer surplus thereby created with a low sharing parameter (that is, $\beta_I^* < 1$).

- 'Strong and developer-skewed network effects.' This corresponds to the red region in Figure 3, in which $u_S \gg u_B$. There, the solution of the relaxed problem leads to a high sharing parameter (that is, $\beta_I^* > 1$) to boost the participation of buyers and a high developer fee to extract the developer surplus (that is, $a_I^* > 0$).

- In the blue region, network effects are rather balanced and weak across both sides of the market. There, the integrated platform sets a positive developer fee (that is, $a_I^* > 0$) and a low sharing parameter (that is, $\beta_I^* < 1$).

The comparison between the outcome under separation $(a^S = 0, \beta^S = 1)$ and the optimal pricing policy obtained in the relaxed problem illustrates a central feature of our model. The way that the integrated firm exercises its market power on developers and on the non-integrated manufacturer depends on the strength and the structure of network effects across both sides of the market. In a nutshell, when network effects are strong and asymmetric, a two-sided market logic is at work: the integrated firm implements an asymmetric pricing structure, subsidizing one side and taxing the other, to harness those network effects. When network effects are weak and balanced, a one-sided market logic is at work: the integrated platform exercises its market power by raising the developer fee and decreasing the sharing parameter.

Optimal Pricing Policy. Let us now come back to the platform's problem as defined in (4.4) and denote by (β_I^{**}, a_I^{**}) its solution. With respect to the relaxed problem, the first constraint is that the sharing parameter must be positive and smaller than 1.

The second constraint is the participation constraint of the non-integrated manufacturer, namely (4.3). Observe that the non-integrated manufacturer's profit if it buys

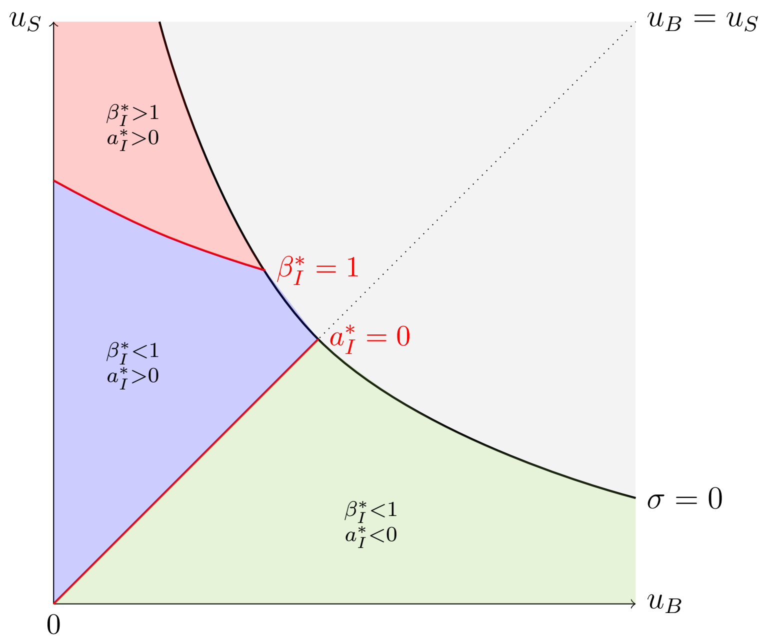


Figure 3: The solution of the relaxed problem (β_I^*, a_I^*) in the running example.

the fringe's operating system, namely $\pi_2^E(1, 1, a_I)$, coincides with its profit if it chooses the integrated platform's operating system for a sharing parameter $\beta_I = 1$: this holds because there is no accommodation effect when the integrated platform gives up all the per-user benefit. Therefore, $\pi_2^E(1, 1, a_I) = \pi_2^I(1, 1, a_I)$, which implies that[24]

$$\pi_2^I(\beta_I, 1, a_I) \geq \pi_2^E(1, 1, a_I) \Leftrightarrow \beta_I \geq 1. \tag{4.5}$$

In words, the competitive pressure exerted by the fringe prevents the integrated platform from exerting any market power on the non-integrated manufacturer, or $\beta_I^{**} = 1$. This illustrates another standard result from the literature on strategic integration: absent efficiency gains, vertical integration does not create market power over non-integrated manufacturers.

The integrated platform still has some market power over the developer who wants to access the buyers of its device. The optimal developer fee maximizes its profit, which writes as[25] $\pi_1^I(a_I) = (p_1^I(a_I) + r)D_1(p_1^I(a_I), p_2^I(a_I), a_I) + a_I D_S(p_1^I(a_I), p_2^I(a_I), a_I)$, and is given by the first-order condition $d\pi_1^I(a_I^{**})/da_I = 0$, or

$$\left(\left[D_S + a_I\left(\frac{\partial D_S}{\partial a_I} + \frac{\partial D_S}{\partial p_2}\frac{dp_2^I}{da_I}\right)\right] + \left[(p_1^I + r)\left(\frac{\partial D_1}{\partial a_I} + \frac{\partial D_1}{\partial p_2}\frac{dp_2^I}{da_I}\right)\right]\right)\Big|_{a_I = a_I^{**}} = 0. \tag{4.6}$$

Increasing the developer fee has both a direct impact on the number of applications and the demand for the integrated manufacturer's device, and an indirect impact through the strategic effect on the non-integrated manufacturer's price. Remember that, under our assumptions, $dp_k^I/da_I \leq 0$. Hence, Equation (4.6) shows that increasing the developer fee allows to capture revenues from developers (first bracketed term) but depreciates the profit earned from buyers (second bracketed term). The first effect calls for increasing

[24]Remind that under our assumption the profit of a manufacturer is increasing in its sharing parameter.
[25]Prices and profits are written as function of this variable only from now on.

the developer fee above the marginal cost, but the second one calls for decreasing it. The integrated platform may still be willing to subsidize the developer if this boosts sufficiently the demand for its own device.

PROPOSITION 1. *The integrated platform's optimal pricing policy is as follows:*

- *All the per-user benefits is left to the non-integrated manufacturer:* $\beta_I^{**} = 1$*;*
- *The developer fee is the solution* a_I^{**} *of* (4.6)*.*

In the running example, $a_I^{**} > 0$ *if and only if* $u_B < \underline{h}(u_S)$ *(with* $\underline{h}(u_S) > u_S$ *for all* $u_S > 0$*).*

Proof. See Appendix A.6. □

Figure 4 represents graphically Proposition 1. The integrated platform boosts the number of applications with a negative developer fee only when network effects are sufficiently strong and buyer-skewed. Otherwise, it sets a positive developer fee. Indeed, with respect to the relaxed problem, the competitive pressure exerted by platforms from the fringe forces the integrated platform to give up all the per-user benefits to the non-integrated manufacturer. It is therefore less profitable to subsidize developers because such a loss cannot be recouped with benefits earned from the licensing of its operating system.

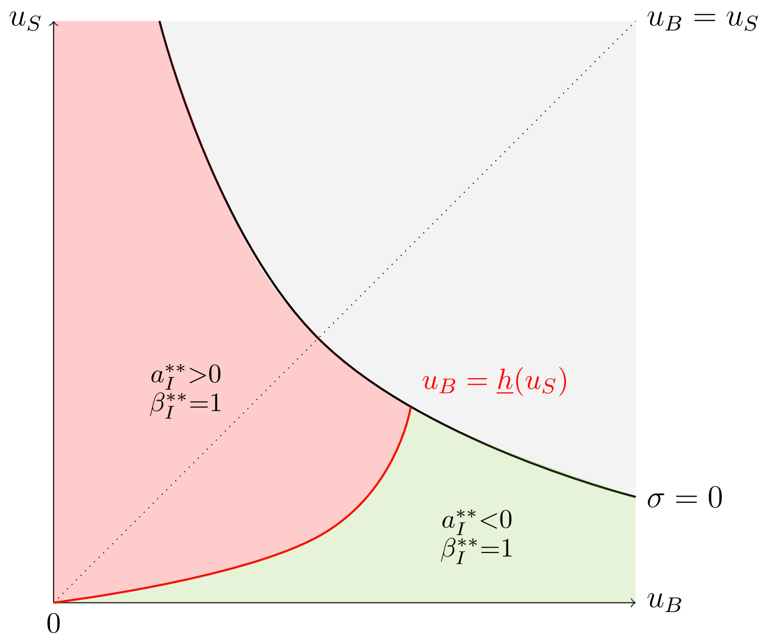


Figure 4: The integrated platform's optimal pricing policy (β_I^{**}, a_I^{**}) in the running example.

4.3. Competitive Impact of Vertical Integration

We now assess the welfare impact of vertical integration. Since the integration outcome coincides with that under separation when $a_I = 0$, we only need to study how the non-integrated manufacturer's profit $\pi_2^I(a_I) = (p_2^I(a_I) + r)D_2(p_2^I(a_I), p_1^I(a_I), a_I)$, the buyer surplus $V_B(p_1^I(a_I), p_2^I(a_I), D_S^I(p_1^I(a_I), p_2^I(a_I), a_I))$, and the developer surplus $V_S(a_I, D_1(p_1^I(a_I), p_2^I(a_I), a_I) + D_2(p_2^I(a_I), p_1^I(a_I), a_I))$ vary with the developer fee a_I.

To study whether vertical integration leads to foreclosure of the non-integrated manufacturer, we can differentiate the non-integrated manufacturer's profit with respect to the developer fee to get

$$\frac{d\pi_2^I}{da_I}(a_I) = (p_2^I + r)\left(\frac{\partial D_2}{\partial a_I} + \frac{\partial D_2}{\partial p_1}\frac{dp_1^I}{da_I}\right) < 0.$$

Since prices of devices decrease with the developer fee under our assumptions, we obtain immediately the next proposition.

PROPOSITION 2. *Vertical integration creates foreclosure if and only if the developer fee increases above the pre-merger level. Therefore, in the running example, foreclosure arises if and only if* $u_S > \underline{h}(u_B)$.

Proof. Immediate from the text. □

This foreclosure effect is different from the one found in the literature for two reasons.

First, it does not stem from a 'raise the rival's cost' effect. In our setting, foreclosure is a collateral damage of the integrated platform's market power on developers, but not the result of its desire to soften competition on the buyer's market.

Second, foreclosure is not systematic. When network effects are strong and buyer-skewed, Proposition 1 has shown that the integrated firm lowers the developer fee (with respect to the pre-merger level), which increases the non-integrated manufacturer's profit. A reverse result obtains when the integrated firm raises the developer fee above the pre-merger level.

Consider now the impact of vertical integration on buyer and developer surpluses. The analysis is slightly more involved since these surpluses are intertwined through indirect network effects. We consider a small variation of the developer fee around its value under separation (that is, 0). Simple manipulations lead to (omitting some arguments)

$$\left.\frac{dV_B}{da_I}\right|_{a_I=0} = -Q_B\left(\frac{dp_1^I}{da_I} + \frac{dp_2^I}{da_I}\right) + \frac{\partial U_B}{\partial n_S}\frac{dD_S}{da_I}, \tag{4.7}$$

$$\left.\frac{dD_S}{da_I}\right|_{a_I=0} = \frac{1}{1 - 2\frac{\partial Q_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}}\left(\frac{\partial Q_S}{\partial n_B}\left(\frac{\partial Q_B}{\partial p_k} + \frac{\partial Q_B}{\partial p_\ell}\right)\left(\frac{dp_1^I}{da_I} + \frac{dp_2^I}{da_I}\right) + \frac{\partial Q_S}{\partial a}\right). \tag{4.8}$$

Equation (4.8) describes how the number of applications varies when the developer fee charged by the integrated platform increases. First, since publishing applications becomes more costly, developers are less willing to participate; this corresponds to the term $\partial Q_S/\partial a < 0$. Second, the prices paid by buyers decrease, so that there are more buyers overall, which benefits developers through indirect network effects; this corresponds to the term $(\partial Q_B/\partial p_k + \partial Q_B/\partial p_\ell)(dp_1^I/da_I + dp_2^I/da_I) > 0$.

Equation (4.7) describes how the surplus of buyers varies when the developer fee increases. There are two effects again. First, the prices paid by buyers decrease, which boosts the demand from those buyers (this corresponds to the first term in the right-hand side). Second, fewer or more applications are developed, which impacts negatively or positively buyers through indirect network effects.

This suggests that the impact of vertical integration on buyers and on developers is a priori ambiguous. For instance, taxing developers with a positive fee may actually improve the surpluses of buyers and developers, if doing so sufficiently reduces the prices of devices and boosts the number of buyers. Next proposition provides a complete characterization of the impact of vertical integration on buyer and developer surpluses in our running example.

PROPOSITION 3. *In the running example, vertical integration*

- *increases buyer surplus if either* $u_B \geq \underline{h}(u_S)$ *or* $u_S \geq \overline{h}_B(u_B)$*;*
- *increases developer surplus if and only if either* $u_B \geq \underline{h}(u_S)$ *or* $u_S \geq \overline{h}_S(u_B)$*.*[26]

Proof. See Appendix A.7. □

Figure 5 represents graphically Proposition 2 and Proposition 3.

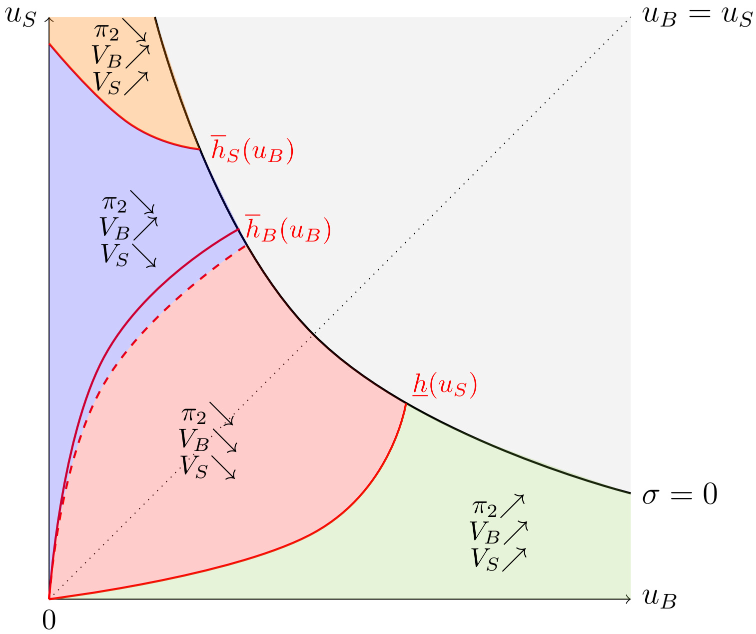


Figure 5: Impact of vertical integration on the non-integrated manufacturer's profit (π_2), buyer surplus (V_B), and developer surplus (V_S) in the running example.

Several comments are worth making. First, and quite remarkably, vertical integration can improve buyer and developer surpluses simultaneously even in the absence of efficiency gains. The intuition is that, when indirect network effects are sufficiently strong and asymmetric, the integrated firm's market power over developers leads to an asymmetric pricing structure that better internalizes network effects. The price structure is thus closer to the one that would be socially optimal, that is, prices under vertical integration are closer to their Ramsey counterparts than under separation.

[26]In Appendix A.7, we show that $\overline{h}_B(u_B) = ((\gamma+2)u_B)/(2(\gamma+1)u_B^2+1)$ and $\overline{h}_S(u_B) = (1/2)(\sqrt{2(\gamma+4)+(\gamma+3)^2u_B^2} - (\gamma+3)u_B)$.

More precisely, with buyer-skewed network effects ($u_B > \underline{h}(u_S)$), the integrated platform subsidizes developers (see Proposition 1). Although the prices of devices increase, buyers benefit from an increase in the number of applications, which explains that their surplus increases following the merger. By contrast, with developer-skewed network effects ($u_S > \overline{h}_B(u_S)$), the integrated platform taxes developers, which tends to reduce the number of applications. However, the prices of devices decrease and buyer surplus increases following the merger. In these two cases of strongly-skewed indirect network effects, which correspond to the green and orange regions in Figure 5, the surpluses of buyers and developers increase because the integrated firm implements an asymmetric pricing structure that is more in line with the Ramsey optimum (see Figure 2).

Second, the impact of vertical integration on foreclosure is now disconnected from its impact on buyers or on developers. We already know from Proposition 2 that foreclosure is related to the developer fee chosen by the integrated platform. Proposition 3 shows that what matters for buyer and developer surpluses is the asymmetry between network effects. Figure 5 illustrates that buyers and developers gain from the vertical merger when network effects are either strongly buyer-skewed ($u_B > \underline{h}(u_S)$), a region where the developer fee decreases, or strongly developer-skewed ($u_S > \overline{h}_B(u_S)$, a region where the developer fee increases.

Third, when network effects become smaller or more symmetric, there is less value to create through an asymmetric pricing structure and the integrated firm's market power is more likely to be detrimental. To illustrate, suppose that $u_B = u_S$. The Ramsey optimum requires then to subsidize developers and tax buyers (see Figure 2). By contrast, the integrated firm exercises its market power over developers by increasing the developer fee, which leads to a lower price for its device (see Figure 4). In Appendix A.7, we show that, when $u_B < \underline{h}(u_S)$ and $u_S < \overline{h}_B(u_S)$, the buyer surplus may either increase or decrease.[27]

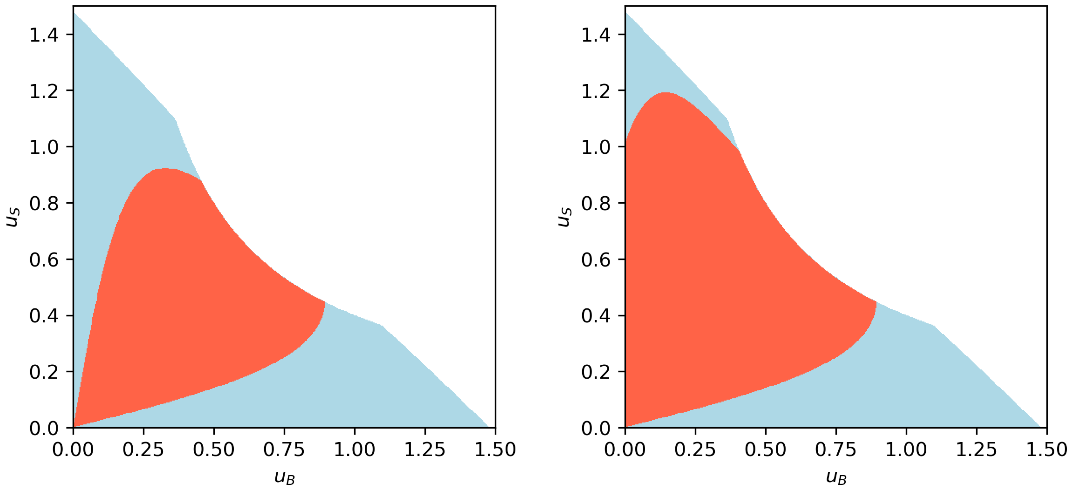
4.4. Discussion

Our analysis provides therefore a new efficiency defense for vertical integration in platform markets. Vertical integration creates market power on the developer side of the market. That market power is used by the integrated firm to extract more profit from developers and from the non-integrated manufacturer; a potentially harmful effect for buyers. In a two-sided market, however, that market power is also used to internalize indirect network effects between the two sides of the market; a potentially beneficial effect for buyers.

Although consumer surplus seems to be the standard pursued by antitrust authorities, it is also interesting to briefly look at total welfare. Figure 6 computes total welfare for a particular specification of our running example.[28] It shows that welfare also improves following vertical integration when network effects are sufficiently strong and asymmetric.

[27] In Proposition 3, the conditions for the buyer surplus to increase are sufficient only. In Appendix A.7, we show that the intermediate region $\{(u_B, u_S) : u_B \leq \underline{h}(u_S) \text{ and } u_S \leq \overline{h}_B(u_B)\}$ can be divided into two subsets, one in which the buyer surplus increases and the other in which it decreases. The frontier between these two subsets is represented by the red dashed curve in Figure 5.

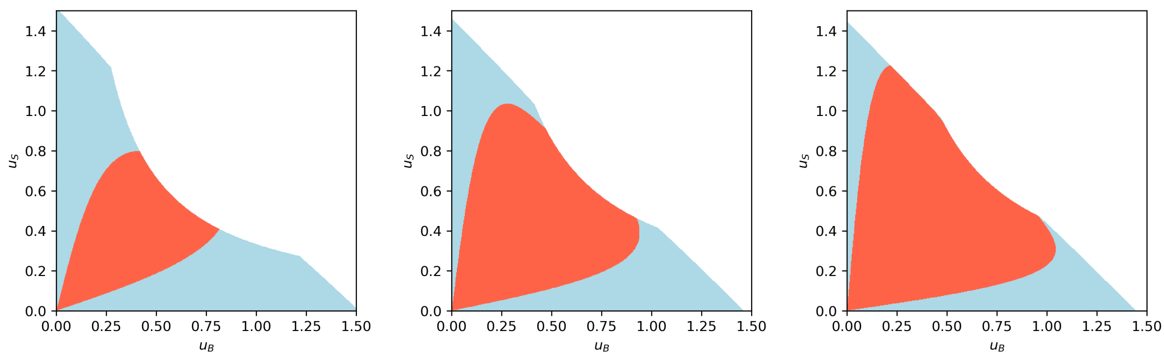
[28] The Python code of the simulations is available on the authors' webpages, as well as other simulations. If otherwise not specified, we use the following set of parameters values: $\gamma = 4$, $v = 2$, $r = 0.5$.



(a) ΔV_B (b) ΔW

Figure 6: Impact of vertical integration on the buyers surplus (V_B, left panel) and on total welfare (W, right panel): V_B and W decrease (resp. increase) following integration in the red area (resp. the blue area).

These simulations can be used to study the role of parameter γ that describes the degree of substitutability between manufacturers' products on the downstream market (Figure 7). Intuitively, as γ increases, products become more substitutes and price competition between manufacturers intensifies. As a result, prices become more rigid and cost-based. For the integrated platform, this implies that internalizing network effects through an asymmetric price structure becomes less interesting: there is no point in subsidizing developers if the price charged to buyers cannot be raised. Therefore, as the simulations reported in Figure 7 suggest, vertical integration becomes more harmful to buyers when manufacturers' products are more demand substitutes. Both network effects and the intensity of competition between manufacturers matter to determine whether vertical integration benefits or hurts buyers.



(a) $\gamma = 2$ (b) $\gamma = 6$ (c) $\gamma = 10$

Figure 7: Impact of vertical integration on buyers surplus (V_B) for different degrees of substitutability between manufacturers (γ): V_B decreases (resp. increases) following integration in the red area (resp. the blue area).

5. Efficiency Gains

We now study vertical mergers that bring efficiency gains. Efficiency gains are modeled as follows: following the merger, the per-buyer benefit associated to the integrated platform's operating system becomes $r_0 > r$. Put differently, the integrated platform I is better able to create value from user-generated data obtained from the manufacturers that use its operating system.[29] The separation benchmark is unchanged (see Section 3.3).

From now on, we consider that platform I is integrated with manufacturer M_1. Under vertical integration, we can still apply the logic of Lemma 3 to show that, in equilibrium, platforms from the fringe set $\beta_E = 1$ and $a_E = 0$.

As in Section 4, pricing incentives at stage 3 of the game depend on the choice of operating system by the non-integrated firm. To track changes in the per-user benefit, we adopt now the following notations. When the non-integrated manufacturer adopts the fringe's operating system, M_2 obtains a net per-user benefit of $1 \cdot r$. Let $(p_1^E(r_0, r, a_I), p_2^E(r, r_0, a_I))$ and $(\pi_1^E(r_0, r, a_I), \pi_2^E(r, r_0, a_I))$ be the prices of devices and the profits in that case. When the non-integrated manufacturer adopts the integrated firm's operating system, M_2 obtains a net per-user benefit of $\beta_I \cdot r_0$. Let $(p_1^I(r_0, \beta_I r_0, a_I), p_2^I(\beta_I r_0, r_0, a_I))$ and $(\pi_1^I(r_0, \beta_I r_0, a_I), \pi_2^I(\beta_I r_0, r_0, a_I))$ be the prices of devices and the profits in that case. For future reference, we have in particular $\pi_1^I(r_0, \beta_I r_0, a_I) = (p_1^I + r_0)D_1(p_1^I, p_2^I, a_I) + (1 - \beta_I)r_0 D_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$, $\pi_2^I(\beta_I r_0, r_0, a_I) = (p_2^I + \beta_I r_0)D_2(p_2^I, p_1^I, a_I)$ and $\pi_2^E(r, r_0, a_I) = (p_2^E + r)D_2(p_2^E, p_1^E, a_I)$.

The roadmap of our analysis is as follows. First, we show that efficiency gains provide the integrated firm with some market power over the non-integrated manufacturer (Section 5.1). Second, we study the way such market market power is exercised (Section 5.2) and show that it is not necessarily detrimental either to buyers and developers or to the non-integrated manufacturer (Sections 5.3 and 5.4).

5.1. *Efficiency Gains Create Market Power*

Efficiency gains create market power vis-à-vis the non-integrated manufacturer. To understand why, observe that, at stage 3 of the game, the integrated platform's profit when it supplies the non-integrated manufacturer writes now as

$$(p_1 + r_0)D_1 + (1 - \beta_I)r_0 D_2 + a_I D_S. \tag{5.1}$$

Comparing the integrated platform's profit with efficiency gain (Equation (5.1)) and without (Equation (4.2)) shows that, with efficiency gains, even when the integrated platform provides the same value as the fringe in terms of per-user benefits, that is, when $\beta_I r_0 = r$, it earns some strictly positive profit from licensing its operating system, namely $(1 - \beta_I)r_0 D_2 = (r_0 - r)D_2 > 0$. Hence, even when both operating systems are licensed on the same terms, the non-integrated manufacturer now strictly prefers adopting the integrated firm's operating system because this makes that firm more accommodating on the buyers market: $\pi_2^I(\beta_I r_0 = r, r_0, a_I) > \pi_2^E(r, r_0, a_I)$.

This implies that the integrated platform is now able to license its operating system

[29] We could have assumed that synergies between platform I and manufacturer M_1 decreases the (marginal) cost of I's operating system. Those two formulations are equivalent.

against a sharing parameter strictly smaller than the fringe's. Put differently, efficiency gains create market power over the non-integrated manufacturer. The integrated firm's market power remains constrained by the fringe's behavior, though, and the following participation constraint must be satisfied

$$\pi_2^I(\beta_I r_0, r_0, a_I) \geq \pi_2^E(r, r_0, a_I). \tag{5.2}$$

To relate with the case of no efficiency gains, the participation constraint (5.2) rewrites as follows in the running example

$$\beta_I \geq \overline{\beta}_I(r_0, r) \equiv 1 - \frac{r_0 - r}{r}\frac{8 + \sigma(8+\sigma)}{8(1+\sigma)}. \tag{5.3}$$

Comparing with (4.5) shows that as soon as the merger creates efficiency gains ($r_0 > r$), it also empowers the integrated platform with some market power over the non-integrated manufacturer (that is, $\overline{\beta}_I(r_0, r) < 1$). The next step consists in analyzing how the integrated platform uses its market power over the developer and the non-integrated manufacturer.

5.2. The Integrated Firm's Pricing Policy

The integrated platform's problem can be written as follows

$$\begin{aligned} \max_{(\beta_I, a_I)} \quad & \pi_1^I(r_0, \beta_I r_0, a_I) \\ \text{s.t.} \quad & \pi_2^I(\beta_I r_0, r_0, a_I) \geq \pi_2^E(r, r_0, a_I), \\ & 0 \leq \beta_I \leq 1. \end{aligned} \tag{5.4}$$

It is again useful to introduce the solution of the relaxed problem (that is, when none of the constraints in the above problem are taken into account), which we denote by $(\beta_I^*(r_0), a_I^*(r_0))$. Up to the fact that the integrated firm's sharing parameter is now r_0, the outcome of the relaxed problem can be represented in a similar way as in Figure 3.

Consider now the constrained problem (5.4) in our running example. In Appendix A.8, we show that the main features of the solution to that problem, denoted by $(\beta_I^{**}(r_0), a_I^{**}(r_0))$, can be summarized with Figure 8. There are three regions of interest depending on which constraints are binding.

Suppose that network effects are buyer-skewed (green region below the 45°-line in Figure 8). In that case, absent any constraints the integrated platform would like to set a low sharing parameter and a negative developer fee ($\beta_I^*(r_0) < 1$ and $a_I^*(r_0) < 0$). With respect to Section 4, setting a sharing parameter below the pre-merger level is now feasible thanks to the efficiency gains that create upstream market power. Hence, with buyer-skewed network effects, we expect that the non-integrated manufacturer's participation constraint becomes binding and that the integrated platform charges a sharing parameter below the pre-merger level (that is, $\beta_I^{**}(r_0) = \overline{\beta}_I(r_0, r) < 1$). The integrated platform may also subsidize developers if network effects are sufficiently asymmetric.

Suppose now that indirect network effects are sufficiently developer-skewed (red region in Figure 8). The integrated firm then wants to subsidize buyers with a high sharing parameter and tax developers with a positive fee. Hence, we expect that the integrated firm gives up all the per-user benefit to the non-integrated manufacturer and taxes developers

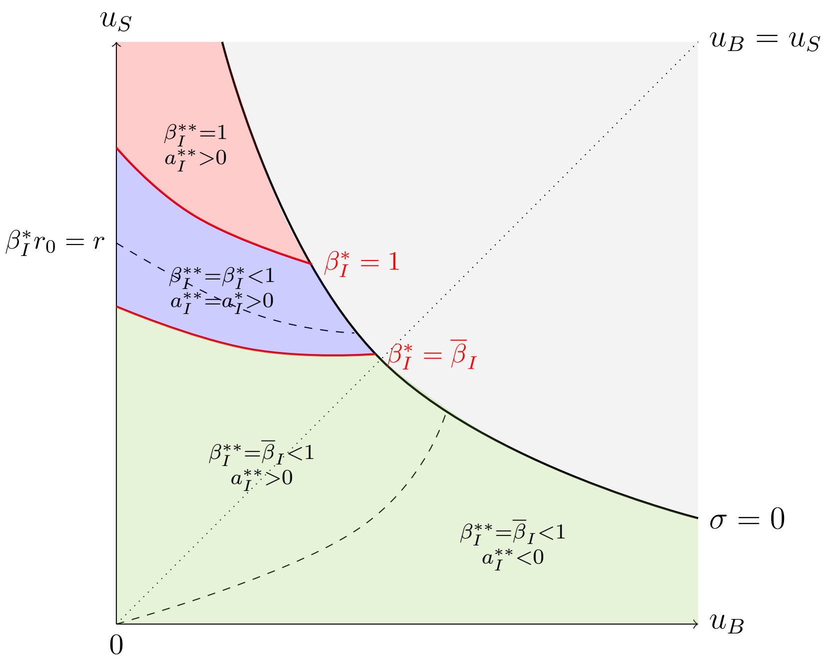


Figure 8: The integrated platform's optimal pricing policy $(\beta^{**}_I(r_0), a^{**}_I(r_0))$ in the running example with efficiency gains.

(that is, $\beta^{**}_I(r_0) > 1$ and $a^{**}_I(r_0) > 0$).

Last, it is also possible that none of the constraints are binding (blue region in Figure 8), in which case the solution of the integrated platform's problem is actually the solution of the relaxed problem. This arises with moderately developer-skewed network effects. In that case, with respect to the pre-merger outcome, the integrated firm moderately decreases the sharing parameter but taxes developers.

5.3. Competitive Impact of Vertical Integration with Efficiency Gains: Polar Cases

With efficiency gains, the impact of vertical integration on buyers, developers and the non-integrated manufacturer is less straightforward to assess because both the developer fee and the sharing parameter change with respect to the separation benchmark. For instance, when the developer fee increases beyond its pre-merger level, this does not necessarily imply foreclosure, for the sharing parameter could be lowered leading to a net gain for the non-integrated manufacturer. We explore this intuition by analyzing two polar cases.

Consider first that developers do not value the participation of buyers (that is, $u_S = 0$ in our running example). Without efficiency gains, buyers and developers, as well as the non-integrated manufacturer, benefit from integration. With efficiency gains, the integrated platform decreases the sharing parameter below the pre-merger level ($\beta^{**}_I(r_0) < 1$) and sets a negative fee for developers ($a^{**}_I(r_0) < 0$). Since developers do not value the participation of buyers, they are better off following integration because their participation is subsidized. Things are more complicated for the buyers and the non-integrated manufacturer. On the one hand, both benefit from the fact that the participation of developers is subsidized. On the other hand, the integrated platform exploits its competitive advantage and extract more from the non-integrated manufacturer. Intuitively, this latter effect prevails when network effects are weak overall, that is, when buyers also value weakly the participation of developers. Next proposition formalizes this intuition.

PROPOSITION 4. *Consider the running example and assume $u_S = 0$. Following integration,*

- *there is foreclosure if and only if u_B is small enough;*
- *buyers are better off if and only if u_B is large enough;*
- *developers are always better off.*

Proof. See the Online Appendix. □

Let us now consider another polar case in which buyers do not value the participation of developers (that is, $u_B = 0$ in our running example). The integrated platform sets a developer fee above the pre-merger level ($a_I^{**}(r_0) > 0$), as in the case without efficiency gains. The difference is that efficiency gains provide incentives to increase the sharing parameter. When u_S is small (resp. large), the integrated platform sets a sharing parameter below (resp. beyond) the pre-merger level. Therefore, intuitively, not only buyers and developers but also the non-integrated manufacturer may benefit from integration when developers value strongly the participation of buyers. Next proposition formalizes this intuition.

PROPOSITION 5. *Consider the running example and assume $u_B = 0$. Following integration,*

- *there is foreclosure if and only if u_S is either large enough or small enough;*
- *buyers are better off if and only if u_S is large enough;*
- *developers are better off if and only if u_S is large enough.*

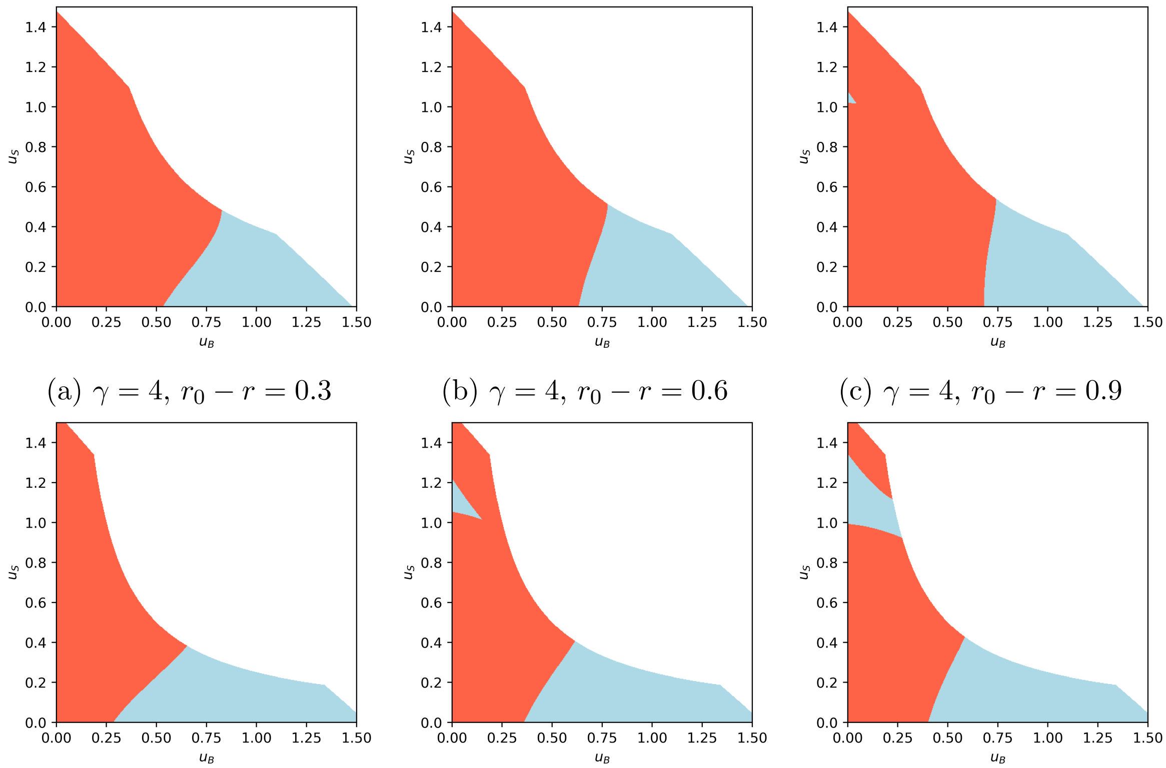
Proof. See the Online Appendix. □

5.4. *Competitive Impact of Vertical Integration with Efficiency Gains: Numerical Simulations*

In this Section, we extend Propositions 4 and 5 using numerical simulations.[30] The results are depicted in Figures 9, 10 and 11, which represent the range of parameters u_B and u_S for which integration harms or benefits buyers, developers and the non-integrated manufacturer respectively.

Overall, the results of the simulations are in line with those of Section 4. First, there is no foreclosure when network effects are skewed toward buyers (see Figure 9). Second, buyers and developers benefit from integration when network effects are sufficiently asymmetric (see Figures 10 and 11 respectively). Third, when network effects are

[30]The Python code of the simulations is available on the authors' webpages, as well as other simulations. As in Section 4.4, unless specified in the figure caption, we use the following set of parameters values: $\gamma = 4$, $v = 2$, $r = 0.5$.



(a) $\gamma = 4$, $r_0 - r = 0.3$ (b) $\gamma = 4$, $r_0 - r = 0.6$ (c) $\gamma = 4$, $r_0 - r = 0.9$

(d) $\gamma = 1$, $r_0 - r = 0.3$ (e) $\gamma = 1$, $r_0 - r = 0.6$ (f) $\gamma = 1$, $r_0 - r = 0.9$

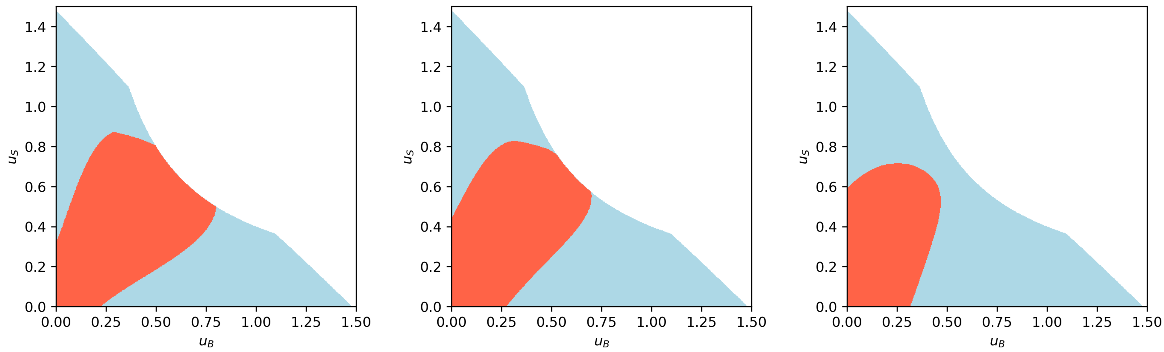
Figure 9: Impact of vertical integration on the non-integrated manufacturer's profit (π_2) for different levels of the efficiency gains ($r_0 - r$): π_2 decreases (resp. increases) following integration in the red area (resp. the blue area).

rather balanced, buyers, developers and the non-integrated manufacturer all lose from the vertical merger (see Figures 9, 10 and 11 respectively).

Consider first that network effects are developer-skewed. As efficiency gains increase, Figure 9 suggests that foreclosure becomes less of an issue. This arises because the integrated platform sets a sharing parameter close or equal to 1. Put differently, efficiency gains are passed through almost entirely to the non-integrated manufacturer. This effect can be strong enough to compensate for the increase in the developer fee. This happens when efficiency gains are sufficiently large and when manufacturers' products are weak substitutes, that is when γ is small (Figures 9d, 9e and 9f). In this case, competition on the downstream market is weak and the non-integrated manufacturer benefits fully from the high sharing parameter.

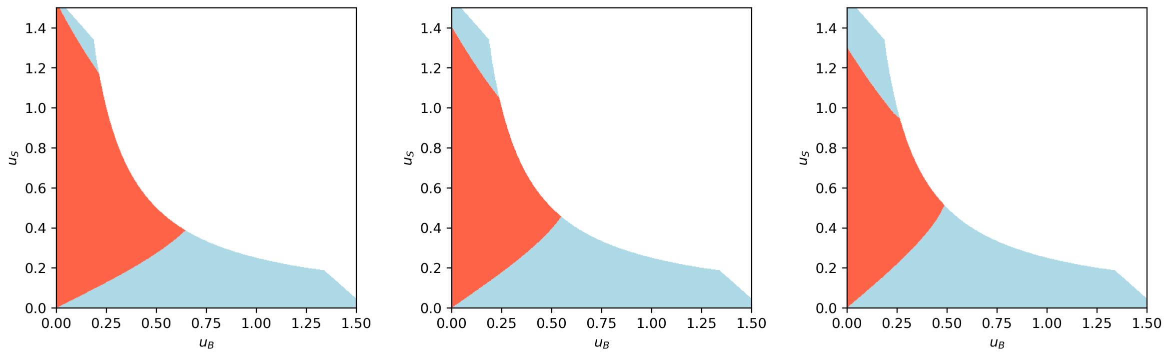
Second, the non-integrated manufacturer is foreclosed when network effects are balanced and weak, that is in the neighborhood of $(u_B, u_S) = (0, 0)$ (see Figure 9). This is reminiscent of the literature on the strategic effects of vertical integration: when vertical integration creates efficiency gains, the non-integrated rival is (partially) foreclosed (see, e.g., Chen, 2001).

Third, buyers and developers are better off following integration when network effects are strong and asymmetric, that is when either u_S is large and u_B is small or the opposite (see Figures 10 and 11). These are situations where a platform would like to internalize network effects through an asymmetric price structure. When it creates efficiency



(a) $r_0 - r = 0.3$ (b) $r_0 - r = 0.6$ (c) $r_0 - r = 0.9$

Figure 10: Impact of vertical integration on buyer surplus (V_B) for different levels of efficiency gains ($r_0 - r$): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).



(a) $r_0 - r = 0.3$ (b) $r_0 - r = 0.6$ (c) $r_0 - r = 0.9$

Figure 11: Impact of vertical integration on developer surplus (V_S) for different levels of efficiency gains ($r_0 - r$): V_S decreases (resp. increases) following integration in the red area (resp. the blue area).

gains, vertical integration allows the platform to implement an even more asymmetric price structure because it now has some control over its sharing parameter. Buyers and developers benefit as well from such a more asymmetric price structure.

The results from the simulations reinforce the main message of our analysis. A vertically integrated platform is empowered with some market power on users on both sides of the market and on the non-integrated manufacturer. However, because vertical integration allows for a better internalization of network effects, the exercise of this market power is not necessarily at the expense of the non-integrated firm. Buyers and developers may also benefit from integration.

6. COORDINATION MOTIVES AND PORTING COSTS

COORDINATION MOTIVES. Although a quintessential feature of platform markets is the presence of indirect network effects linking various groups of agents, these markets also frequently exhibit direct network effects of paramount importance. For instance, operating systems often feature applications or software programs that aim to take advantage of direct network effects between buyers.[31] Hence, when more manufacturers adopt the same operating system, this creates extra benefits for buyers of devices using the same operating system.[32] Ultimately, part of these benefits may end up being pocketed by manufacturers, which creates coordination motives between manufacturers. Our goal is to analyze how such coordination motives impact on our assessment of vertical integration.

To do so, we consider our model without efficiency gains (in which vertical integration does not create any market power over the non-integrated manufacturer) and modify it as follows. If manufacturers choose different operating systems, then quasi-demands are given as before by (2.2) and the analysis is the same as in Section 4. If manufacturers choose the same operating system, then quasi-demands are now given by[33]

$$\left\{\begin{array}{lcl}\tilde{Q}_B^1 & = & Q_B^1 + \alpha_B,\\ \tilde{Q}_B^2 & = & Q_B^2 + \alpha_B,\\ \tilde{Q}_S & = & Q_S.\end{array}\right. \tag{6.1}$$

Parameter α_B is positive and used as a shortcut to capture the magnitude of the extra gains for buyers when manufacturers adopt the same operating systems. Demands $(\tilde{D}_1, \tilde{D}_2, \tilde{D}_S)$ that solve (6.1) are clearly increasing in α_B. Let us further assume that the profits of manufacturers associated with these demands, denoted by $(\tilde{\pi}_1^i, \tilde{\pi}_2^i)$ with $i \in \mathcal{P}$, are also increasing in α_B.[34]

The analysis under separation is immediate. In any equilibrium, manufacturers coordinate on one platform $i \in \mathcal{P}$, obtain the whole per-user benefit, and the developer is charged a nil fee by platforms. Essentially, the addition of the extra benefit α_B simply leads manufacturers to choose the same platform because there is now a coordination motive.

Consider now that platform I is vertically-integrated with manufacturer M_1. If M_2 chooses platform E from the fringe, manufacturers equip their devices with different operating systems and there are no extra benefits. M_2's profit is then given by $\pi_2^E(1, 1, a_I)$. If, instead, M_2 chooses I's operating system, there are extra benefits and M_2's profit is denoted by $\tilde{\pi}_2^I(\beta_I, 1, a_I)$.[35] We have $\tilde{\pi}_2^I(1, 1, a_I) > \pi_2^E(1, 1, a_I)$ because, if platform I sets the same sharing parameter as the fringe, M_2 now strictly prefers I's operating system to take advantage of the extra benefits. Therefore, the integrated firm can now offer $\beta_I < 1$ while still ensuring that M_2 adopts I's operating system. To illustrate, in the running example the participation constraint (4.5) becomes with coordination motives

$$\beta_I \geq \tilde{\beta}_I(\alpha_B) \equiv 1 - \frac{\alpha_B}{r}\frac{4 + 3\sigma}{4(1 + \sigma)}. \tag{6.2}$$

Much as in the case of efficiency gains (see in particular Equation (5.3)), coordination motives create market power over the non-integrated manufacturer. That market power arises even though platforms are symmetric (i.e., there are no efficiency gains) because

[31] iMessage, FaceTime and Airdrop are prime examples for iOS.

[32] One could argue similarly that developers care about the community of programmers using a particular operating system because this may help lowering development costs.

[33] This formulation where only buyers received a fixed benefit when manufacturers choose the same operating system is chosen for its tractability.

[34] Although intuitive, this property does not always hold; see the discussion in footnote 17.

[35] This profit is defined as before (see Section 4.1) except that demands (D_1, D_2, D_S) are replaced by $(\tilde{D}_1, \tilde{D}_2, \tilde{D}_S)$.

vertical integration somewhat forces the coordination of the manufacturers on the integrated firm's operating system.

The integrated platform's problem can be written as follows:

$$\begin{aligned} \max_{(\beta_I, a_I)} \quad & \tilde{\pi}_1^I(1, \beta_I, a_I) \\ \text{s.t.} \quad & \tilde{\pi}_2^I(\beta_I, 1, a_I) \geq \pi_2^E(1, 1, a_I), \\ & 0 \leq \beta_I \leq 1. \end{aligned} \tag{6.3}$$

This problem is quite similar to the one analyzed in Section 5 and its solution will feature the same main properties. For the running example, the complete resolution is provided in Appendix A.9. One noticeable difference is the following: since vertical integration does not create efficiency gains, the non-integrated manufacturer M_2 and, indirectly, the developers, do not benefit from a more advantageous sharing parameter when the integrated platform decides to subsidize buyers. This happens when network effects are developer-skewed and the integrated platform prefers to tax developers, which harms both M_2 and the developers. We therefore expect, contrary to Section 5, that when network effects are developer skewed, there is always foreclosure and developers are worse off.

We can confirm these two intuitions by studying two polar cases (Propositions 6 and 7) and by performing numerical simulations (Figures 12, 13 and 14).[36]

PROPOSITION 6. *Consider the running example with coordination motives and assume* $u_S = 0$. *Following integration,*

- *there is foreclosure if and only if* u_B *is small enough;*
- *buyers are better off if and only if* u_B *is large enough;*
- *developers are always better off.*

Proof. See the Online Appendix. □

PROPOSITION 7. *Consider the running example with coordination motives and assume* $u_B = 0$. *Following integration,*

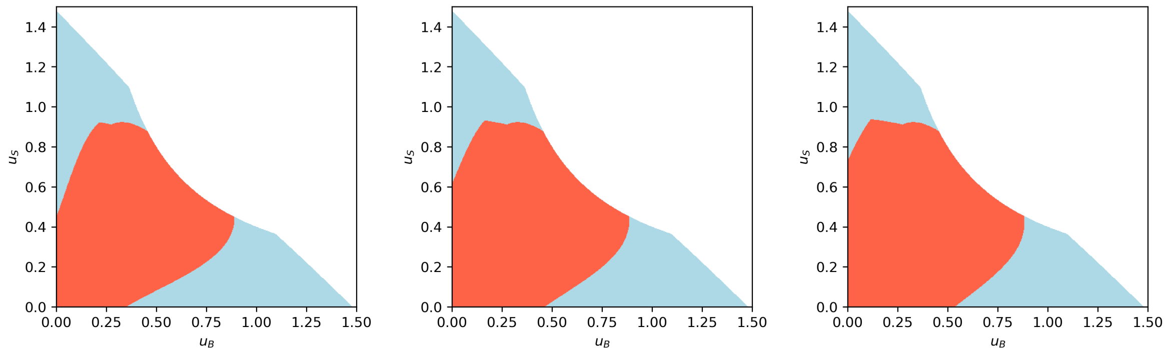
- *there is always foreclosure;*
- *buyers are better off if and only if* u_S *is large enough;*
- *developers are always worse off.*

Proof. See the Online Appendix. □

PORTING COSTS. Throughout our analysis, we have maintained the assumption that there are no platform-specific cost for the developer. In practice, the programming languages used in Android and iOS strongly differ[37] and it appears that Android application development is usually longer and more costly because of the fragmentation issue that impacts this operating system. We now discuss informally how platform-specific costs to

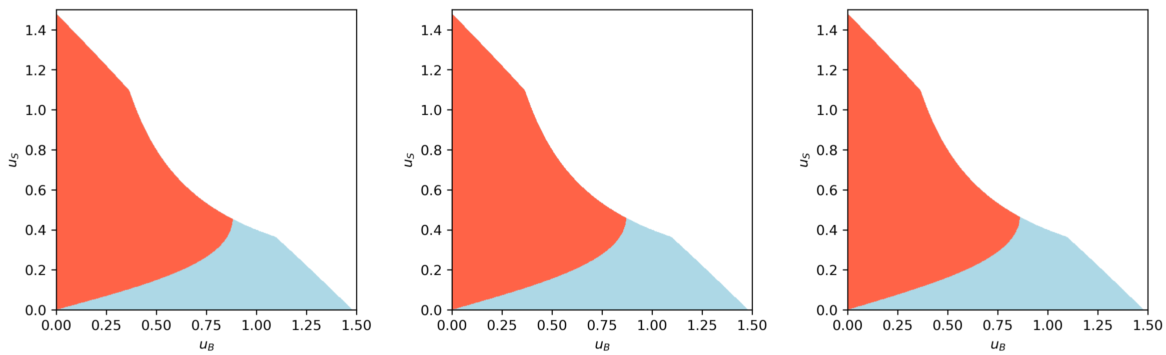
[36]The Python code of the simulations is available on the authors' webpages, as well as other simulations. As in Sections 4.4, and 5.4 we use the following set of parameters values: $\gamma = 4$, $v = 2$, $r = 0.5$.

[37]Mainly Java, C, C++ and Kotlin for the former and Objective C or Swift for the latter.



(a) $\alpha_B = 0.1$ (b) $\alpha_B = 0.2$ (c) $\alpha_B = 0.3$

Figure 12: Impact of vertical integration on buyer surplus (V_B) for different levels of the gain for buyers when manufacturers adopt the same operating system (α_B): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).



(a) $\alpha_B = 0.1$ (b) $\alpha_B = 0.2$ (c) $\alpha_B = 0.3$

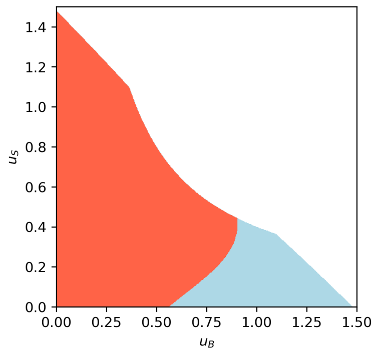
Figure 13: Impact of vertical integration on developer surplus (V_S) for different levels of the gain for buyers when manufacturers adopt the same operating system (α_B): V_S decreases (resp. increases) following integration in the red area (resp. the blue area).

port applications on operating systems impact on our analysis. Roughly speaking, we show that porting costs create a coordination motive for manufacturers.

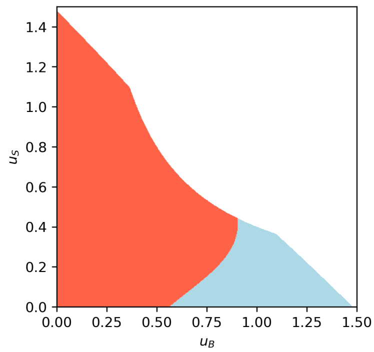
Let us assume now that the developer bears a unit cost $c_i > 0$ to port each application on platform i's operating system. To streamline the analysis, let us further assume that $c_i = c$ for all $i \in \mathcal{P}$. At the last stage of the game, the developer decides to publish its applications on platform i if $u_S n_B^i \geq a_i + c$. Therefore, the number of applications developed is now given by $Q_S(\sum_{i\in\mathcal{P}}(u_S n_B^i - (a_i + c))\mathbb{1}_{\{u_S n_B^i - (a_i+c) \geq 0\}})$. Last, assume that c is sufficiently small so that $u_S n_B^i > c$ when $n_B^i > 0$; this ensures that all applications are published on any platform i that has attracted at least one manufacturer and that sets a nil developer fee.[38]

Consider that I and M_1 are integrated. If M_2 chooses E's operating system, the

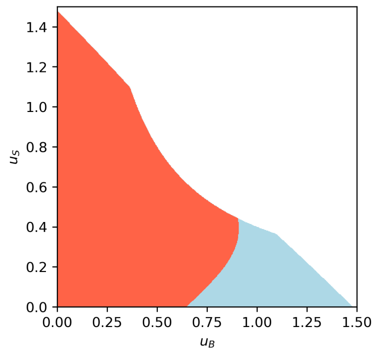
[38] When c becomes sufficiently large, and when manufacturers have chosen different operating systems, the developer might find it profitable to publish on only one operating system, i.e., to single-home rather than to multi-home.



(a) $\alpha_B = 0.1$



(b) $\alpha_B = 0.2$



(c) $\alpha_B = 0.3$

Figure 14: Impact of vertical integration on the non-integrated manufacturer's profit (π_2) for different levels of the gain for buyers when manufacturers adopt the same operating system (α_B): π_2 decreases (resp. increases) following integration in the red area (resp. the blue area).

number of applications is given by

(6.4) $$Q_S^E\Big(u_S(n_B^1+n_B^2) - \sum_{i=I,E}(a_i+c)\mathbb{1}_{\{u_S n_B^i-(a_i+c)\geq 0\}}\Big).$$

If M_2 chooses instead I's operating system, the number of applications is then given by

(6.5) $$Q_S^I\Big(u_S(n_B^1+n_B^2) - (a_I+c)\mathbb{1}_{\{u_S n_B^I-(a_I+c)\geq 0\}}\Big).$$

Comparing (6.4) and (6.5) immediately leads to $Q_S^I > Q_S^E$: With porting costs, choosing the integrated platform's operating system leads to more applications because it saves on the developer's cost. This therefore leads to higher demands for both devices. Hence, porting costs create a coordination motive for the non-integrated manufacturer. Put differently, and using notations that should be familiar by now, we have now $\pi_2^I(1,1,a_I) > \pi_2^E(1,1,a_I)$ and the integrated platform is thus empowered with some market power over the non-integrated manufacturer. To illustrate further, considering the running example and assuming that platforms from the fringe offer ($\beta_E = 1, a_E = 0$), the participation constraint (4.5) becomes with porting costs

$$\beta_I \geq 1 - \frac{c\,u_B}{r}\frac{4+3\sigma}{4(1+\sigma)}.$$

Given the analysis above (see in particular equations (5.3) and (6.2)), we expect that such market power may not always be detrimental to welfare.[39]

[39]Porting costs also introduce a novelty, which is best seen by considering the case of separation. If non-integrated manufacturers choose the same platform, say I, setting a nil developer fee (i.e., $a_E = 0$) no longer ensures that the developer publishes on E's operating system because of the porting costs. This means that platform I can now set any developer fee such $a_I + c \geq 0$ without being threaten that a rival platform attracts the developer with a lower developer fee. Although platforms still compete fiercely and should make no profits in equilibrium, some partial cross-subsidization between both sides of the market becomes possible, even under separation. A complete characterization of the equilibrium outcome under separation and integration with porting costs is left for future research.

7. Conclusion

We develop a model of a platform market, in which platforms interact with manufacturers of devices and there are indirect network effects between buyers of devices and developers of applications. We study the consequences of vertical integration between one of the platforms and one of the manufacturers.

The sources of upstream market power, and their consequences on foreclosure or on consumer surplus, are different from those unveiled in the extant literature. Even absent any efficiency gains, vertical integration creates market power over developers who want to access the buyers of the integrated manufacturer's device. With efficiency gains, coordination motives or porting costs, vertical integration creates additionally some market power over the non-integrated manufacturer. However, what is key is how the integrated firm exploits these sources of market power. Our analysis unveils that this depends both on the strength and on the structure of indirect network effects. When network effects are strong in level but also sufficiently asymmetric in structure, the integrated firm implements an asymmetric pricing structure, which may well improve buyer and developer surpluses as well as the non-integrated manufacturer's profit. Our analysis therefore warns policy-makers against a blind application of the standard view on foreclosure when dealing with platform markets.

As in standard markets, antitrust authorities may want to limit the anti-competitive effects of vertical integration by constraining the pricing instruments between the integrated platform and non-integrated manufacturers. In the context of platform markets, such remedy raises some issues. For instance, constraining the integrated firm's sharing parameter is likely to impact the pricing on the developer side of the market, thereby dampening the internalization of network effects.

Our setting could be extended in various directions. First, throughout the analysis, we have maintained the assumption that manufacturers' products remain demand substitutes. With (imperfect) demand complements, the integrated platform should have less incentives to foreclose the non-integrated manufacturer. Second, and relatedly, analyzing the incentives of both manufacturers and platforms to differentiate their products, and the consequences on the assessment of vertical integration, would be worth investigating. Third, there is always multi-homing on the developer side and single-homing on the buyer side of the market in our analysis. Different patterns may be more relevant depending on the platform markets under consideration and this is likely to impact on the assessment of vertical integration. These extensions are left for future research.

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A. APPENDIX

A.1. Demand Functions with Indirect Network Effects

In the last stage of the game, given prices (p_1, p_2, a), the number of buyers and the number of developers solve

$$\text{(A.1)} \qquad \begin{cases} n_B^1 &= Q_B^1(p_1, p_2, n_S), \\ n_B^2 &= Q_B^2(p_2, p_1, n_S), \\ n_S &= Q_S(u_S(n_B^1 + n_B^2) - a). \end{cases}$$

In the following, with a slight abuse of notations, let $\partial Q_S/\partial a(a, n_B) = dQ_S/da(u_S n_B - a) = -Q_S'(u_S n_B - a)$ and $\partial Q_S/\partial n_B(a, n_B) = dQ_S/dn_B(u_S n_B - a) = u_S Q_S'(u_S n_B - a)$. To avoid 'cornered-market' solutions, in which all buyers or all developers participate in equilibrium, we assume that indirect network effects are not too strong so that, in the relevant range, each manufacturer faces a demand that is locally elastic with respect to prices.

ASSUMPTION A.1 (Indirect Network Effects Are Not Too Strong). *In the relevant range of prices (p_1, p_2, a), the total number of buyers $n_B^1 + n_B^2$ and the number of developers n_S satisfy the following condition*

$$\frac{\partial Q_S}{\partial n_B}(a, n_B^1 + n_B^2)\left(\frac{\partial Q_B^1}{\partial n_S}(p_1, p_2, n_S) + \frac{\partial Q_B^2}{\partial n_S}(p_2, p_1, n_S)\right) < 1.$$

We can then show the following result.

LEMMA A.1. *In the relevant range of prices (p_1, p_2, a), system (A.1) has a unique interior solution.*

Proof. Let $D(p_1, p_2, a) = D_1(p_1, p_2, a) + D_2(p_2, p_1, a)$. From system (A.1), we have

$$\text{(A.2)} \quad D(p_1, p_2, a) = Q_B^1(p_1, p_2, Q_S(u_S D(p_1, p_2, a) - a)) + Q_B^2(p_2, p_1, Q_S(u_S D(p_1, p_2, a) - a)).$$

For a given (p_1, p_2, a), $D(p_1, p_2, a)$ is thus a fixed point of $\psi(x) = Q_B^1(p_1, p_2, Q_S(u_S x - a)) + Q_B^2(p_2, p_1, Q_S(u_S x - a))$. Notice then that

$$\psi'(x) = \frac{\partial Q_S}{\partial n_B}(a, x)\left(\frac{\partial Q_B^1}{\partial n_S}(p_1, p_2, Q_S(u_S x - a)) + \frac{\partial Q_B^2}{\partial n_S}(p_2, p_1, Q_S(u_S x - a))\right).$$

Assumption A.1 implies that $|\psi'(\cdot)| < 1$, so that $\psi(\cdot)$ is a contraction mapping and Equation (A.2) has a unique solution. It follows that $D_S(p_1, p_2, a) = Q_S(u_S D(p_1, p_2, a) - a)$ is uniquely defined, as well as $D_k(p_k, p_\ell, a) = Q_B^k(p_k, p_\ell, D_S(p_1, p_2, a))$. □

We can then show the following result.

LEMMA A.2. *The following properties hold: $\frac{\partial D_k}{\partial p_k}(p_k, p_\ell, a) < 0$, $\frac{\partial D_S}{\partial p_k}(p_k, p_\ell, a) < 0$, $|\frac{\partial D_k}{\partial p_k}(p, p, a)| > |\frac{\partial D_k}{\partial p_\ell}(p, p, a)|$, $\frac{\partial D_k}{\partial a}(p_k, p_\ell, a) < 0$ and $\frac{\partial D_S}{\partial a}(p_k, p_\ell, a) < 0$.*

Proof. By the implicit function theorem, $D(p_1, p_2, a) = D_1(p_1, p_2, a) + D_2(p_2, p_1, a)$ is continuously differentiable. Differentiating Equation (A.2) with respect to p_1 and rearranging terms, we find (omitting some arguments)

$$\frac{\partial D}{\partial p_1}\left[1 - \frac{\partial Q_S}{\partial n_B}(a, D)\left(\frac{\partial Q_B^1}{\partial n_S}(p_1, p_2, D) + \frac{\partial Q_B^2}{\partial n_S}(p_2, p_1, D)\right)\right] = \frac{\partial Q_B^1}{\partial p_1}(p_1, p_2, D) + \frac{\partial Q_B^2}{\partial p_1}(p_2, p_1, D).$$

By Assumption A.1, the term in squared brackets is positive. Therefore, $\partial D/\partial p_1$ is negative. Similarly, $\partial D/\partial p_2 < 0$. Since $\partial Q_S/\partial n_B(a, n_B) > 0$, $D_S(p_1, p_2, a) = Q_S(u_S D(p_1, p_2, a) - a)$ is decreasing in both p_1 and p_2. Then $\partial D_1/\partial p_1 = \partial Q_B^1/\partial p_1 + (\partial D/\partial p_1)(\partial Q_B^1/\partial n_S)\partial Q_S/\partial n_B$, which shows that $\partial D_1/\partial p_1 < 0$. Similarly, $\partial D_1/\partial p_2 = \partial Q_B^1/\partial p_2 + (\partial D/\partial p_2)(\partial Q_B^1/\partial n_S)\partial Q_S/\partial n_B$. For symmetric prices, $\partial D/\partial p_1 = \partial D/\partial p_2$, and therefore, $|\partial D_1/\partial p_1| - |\partial D_1/\partial p_2| = |\partial Q_B^1/\partial p_1| - |\partial Q_B^1/\partial p_2| < 0$, which is negative under our assumptions.

From Equation (A.1), the developer demand solves

$$D_S(a, p_1, p_2) = Q_S(u_S(Q_B^1(p_1, p_2, D_S(a, p_1, p_2)) + Q_B^2(p_2, p_1, D_S(a, p_1, p_2))) - a). \tag{A.3}$$

By the implicit function theorem, $D_S(a, p_1, p_2)$ is continuously differentiable. Differentiating Equation (A.3) with respect to a and rearranging terms, we find (omitting some arguments)

$$\frac{\partial D_S}{\partial a} = \frac{\frac{\partial Q_S}{\partial a}}{1 - \frac{\partial Q_S}{\partial n_B}\left(\frac{\partial Q_B^1}{\partial n_S} + \frac{\partial Q_B^2}{\partial n_S}\right)}.$$

By Assumption A.1, the denominator is positive, and therefore, $\partial D_S/\partial a$ has the sign of $\partial Q_S/\partial a$, which is negative. Since $D_k(p_k, p_\ell, a) = Q_B^k(p_k, p_\ell, D_S(p_k, p_\ell, a))$ and $\partial Q_B^k/\partial n_S > 0$, $\partial D_k/\partial a$ is also negative. □

A.2. Assumptions in the Running Example

The running example satisfies the assumptions made in Section 2. Assumption A.1, which ensures that there exists a unique and interior solution to system (2.3), writes as $2u_B u_S < 1$. Solving for n_{Bk}, $k = 1, 2$, and n_S in (2.3), the buyers' and developers' demands are given by

$$D_k(p_k, p_\ell, a) = \frac{2v - (2+\sigma)p_k + \sigma p_\ell - 2u_B a}{2(1 - 2u_B u_S)}, \tag{A.4}$$

$$D_S(p_1, p_2, a) = \frac{u_S(2v - p_1 - p_2) - a}{1 - 2u_B u_S}, \tag{A.5}$$

where $\sigma = \gamma - 2u_B u_S(1+\gamma)$, which is positive by assumption. From (A.4), and since $\sigma \geq 0$ and $0 \leq 2u_B u_S < 1$, D_k is strictly decreasing in p_k and a. Manufacturers' products are demand substitutes since $\partial D_k/\partial p_\ell = \sigma/(2(1 - 2u_B u_S)) \geq 0$. From (A.5), we have that D_S is decreasing in p_1, p_2 and a. Finally, the direct price effect is stronger than the indirect one since $-\partial D_k/\partial p_k - \partial D_k/\partial p_\ell = 1/(1 - 2u_B u_S) > 0$.

We now check that the price competition subgame is 'well-behaved' in the running example (see Section 2.3). Consider the separation benchmark. Developers pay a total fee a, and M_k earns a profit equal to $\pi_k = (p_k + \beta_k r)D_k$. $\partial^2\pi_k/\partial p_k^2 = -(2+\sigma)/(1 - 2u_B u_S) < 0$, so that π_k is strictly concave in p_k and M_k's best response is uniquely characterized by the first-order condition $\partial \pi_k/\partial p_k = 0$. The best response is given by $R_k(p_\ell, \beta_k, a) = (2(v - \beta_k r) + \sigma(p_\ell - \beta_k r) - 2u_B a)/(2(2+\sigma))$. We then have $0 < \partial R_k/\partial p_\ell < 1$ and $\partial R_k/\partial a \leq 0$. Equilibrium prices are given by $\hat{p}_k = (-2a u_B(4+3\sigma) + v(8+6\sigma) + (2+\sigma)(-\beta_\ell r\sigma - 2\beta_k r(2+\sigma)))/((4+\sigma)(4+3\sigma))$. One obtains then that: $\partial \hat{p}_k/\partial \beta_k = -r(2(2+\sigma)^2)/((4+\sigma)(4+3\sigma))$, which belongs to $(-2r/3, r/2]$ for $\sigma \geq 0$; $\partial \hat{p}_k/\partial \beta_\ell = -r(\sigma(2+\sigma))/((4+\sigma)(4+3\sigma))$, which belongs to $(-1/3, 0]$ for $\sigma \geq 0$; and $\partial \hat{p}_k/\partial a = -\frac{2u_B}{(4+\sigma)} \leq 0$.

Last, we check that these assumptions are satisfied when I and M_1 are integrated and the integrated platform sets a developer fee a_I and a sharing parameter β_I for M_2. I's profit writes as $(p_1 + r)D_1 + (1 - \beta_I)rD_2 + a_I D_S$ and its best response is $R_1(p_2, \beta_I, a_I) = (2(v - r - (u_B + u_S)a_I) + \sigma(p_2 - \beta_I r))/(2(2+\sigma))$, which is increasing with a slope smaller than 1 in p_2 and decreasing in a_I. M_2's profit writes as $(p_2 + \beta_I r)D_2$ and its best response is given by $R_2(p_1, \beta_I, a_I) = (2(v - \beta_I r) + \sigma(p_1 - \beta_I r) - 2u_B a_I)/(2(2+\sigma))$. Those best responses satisfy our assumptions. Additionally, equilibrium prices are given by $p_1^I = -\beta_I r + (-r - a_I(2u_B + u_S) + 2v + 3\beta_I r)/(4+\sigma) + (-r - a_I u_S + \beta_I r)/(4+3\sigma)$ and $p_2^I = -\beta_I r + (-r - a_I(2u_B + u_S) + 2v + 3\beta_I r)/(4+\sigma) + (r + a_I u_S - \beta_I r)/(4+3\sigma)$. These prices are decreasing in β_I at a rate smaller than r and decreasing in a_I.

A.3. Benchmarks

RAMSEY PRICES IN THE GENERAL CASE. Note that we can assume that $p_1 = p_2 \equiv p$ without loss of generality. To ease the exposition, consider the following notations: $D_B(p,a) = D_1(p,p,a) + D_2(p,p,a)$, $D_S(p,a) = D_S(p,p,a)$, $\Pi(p,a) = (p+r)D_B(p,a) + aD_S(p,a)$, $W(p,a) = V_B(p,p,D_S(p,a)) + V_S(a, D_B(p,a)) + \Pi(p,a)$. Ramsey prices solve the following problem

$$\begin{aligned} \max_{(p,a)} \quad & W(p,a) \\ \text{s.t.} \quad & \Pi(p,a) \geq 0. \end{aligned} \tag{A.6}$$

Let $\lambda \geq 0$ be the Lagrange multiplier associated to the constraint. Assume that this problem is well-behaved so that its solution can be characterized through first-order conditions.

Consider the following change of variables: $\varphi : (p,a) \mapsto (n_B, n_S) = (D_B(p,a), D_S(p,a))$. It is a $\mathcal{C}^1$-diffeomorphism since, under Assumption A.1, the system of equations $n_B = Q_B^1(p,p,n_S) + Q_B^2(p,p,n_S)$ and $n_S = Q_S(u_S n_B - a)$ has a unique solution, namely $(n_B, n_S) = (D_B(p,a), D_S(p,a))$, in the relevant range of parameters. Let $(P(n_B,n_S), A(n_B,n_S)) = \varphi^{-1}(n_B,n_S)$. Problem (A.6) then rewrites as follows

$$\begin{aligned} \max_{(n_B,n_S)} \quad W = \quad & U_B(1/2\,n_B, 1/2\,n_B, n_S) - P(n_B,n_S)n_B + U_S(n_S,n_B) - A(n_B,n_S)n_S \\ & \qquad + (P(n_B,n_S)+r)n_B + A(n_B,n_S)n_S \\ \text{s.t.} \quad & (P(n_B,n_S)+r)n_B + A(n_B,n_S)n_S = 0. \end{aligned}$$

The first-order conditions on n_B and n_S can be written as follows (omitting notations)

$$\begin{aligned} \tfrac{1}{2}\left(\tfrac{\partial U_B}{\partial q_1} + \tfrac{\partial U_B}{\partial q_2}\right) + r + \tfrac{\partial U_S}{\partial n_B} + \lambda\left(\tfrac{\partial P}{\partial n_B}n_B + P + r + \tfrac{\partial A}{\partial n_B}n_S\right) &= 0, \\ \tfrac{\partial U_B}{\partial n_S} + \tfrac{\partial U_S}{\partial q_S} + \lambda\left(\tfrac{\partial P}{\partial n_S}n_B + \tfrac{\partial A}{\partial n_S}n_S + A\right) &= 0. \end{aligned} \tag{A.7}$$

Then, noticing that the maximization problems of the representative buyer and developer give $\frac{\partial U_B}{\partial q_1} = P$, $\frac{\partial U_B}{\partial q_2} = P$, and $\frac{\partial U_S}{\partial q_S} = A$, Equations (A.7) rewrite as follows

$$\begin{aligned} P + r + \tfrac{\partial U_S}{\partial n_B} + \lambda\left(\tfrac{\partial P}{\partial n_B}n_B + P + r + \tfrac{\partial A}{\partial n_B}n_S\right) &= 0, \\ \tfrac{\partial U_B}{\partial n_S} + A + \lambda\left(\tfrac{\partial P}{\partial n_S}n_B + \tfrac{\partial A}{\partial n_S}n_S + A\right) &= 0. \end{aligned} \tag{A.8}$$

With no break-even constraint, the welfare-maximizing prices are obtained by setting $\lambda = 0$ in (A.8): $P + r = -\frac{\partial U_S}{\partial n_B}$ and $A = -\frac{\partial U_B}{\partial n_S}$ as stated in the text. This condition obviously violates the break-even constraint.

Suppose now that $\lambda > 0$. Since the constraint is binding, we now have

$$(P+r)n_B + An_S = 0, \tag{A.9}$$

which shows in particular that either $P + r \leq 0$ and $A \geq 0$, or $P + r > 0$ and $A < 0$ at the optimum. Then, combining Equations (A.8) and (A.9), we obtain the following equation on λ

$$\lambda = -\frac{n_S\frac{\partial U_B}{\partial n_S} + n_B\frac{\partial U_S}{\partial n_B}}{n_B^2\frac{\partial P}{\partial n_B} + n_S^2\frac{\partial A}{\partial n_S} + n_B n_S\left(\frac{\partial P}{\partial n_S} + \frac{\partial A}{\partial n_B}\right)}. \tag{A.10}$$

Let Den denote the denominator in (A.10). Since $\lambda > 0$ and the numerator in (A.10) is positive, we have $Den < 0$. Then, by combining Equations (A.8) and (A.9), we obtain the following expression for the margin $P + r$

$$\text{(A.11)} \qquad P + r = \frac{n_S\left(\frac{\partial U_B}{\partial n_S}(n_B\frac{\partial P}{\partial n_B} + n_S\frac{\partial A}{\partial n_B}) - \frac{\partial U_S}{\partial n_B}(n_B\frac{\partial P}{\partial n_S} + n_S\frac{\partial A}{\partial n_S})\right)}{Den - n_S\frac{\partial U_B}{\partial n_S} - n_B\frac{\partial U_S}{\partial n_B}}.$$

Since the denominator is negative, Equation (A.11) shows that $P+r$ has the sign of $f(n_B, n_S) = \frac{\partial U_S}{\partial n_B}(n_B\frac{\partial P}{\partial n_S} + n_S\frac{\partial A}{\partial n_S}) - \frac{\partial U_B}{\partial n_S}(n_B\frac{\partial P}{\partial n_B} + n_S\frac{\partial A}{\partial n_B})$. Then, noticing that $\frac{\partial P}{\partial n_B} = \frac{1}{\partial Q_B/\partial p}$ and $\frac{\partial P}{\partial n_S} = -\frac{\partial Q_B/\partial n_S}{\partial Q_B/\partial p}$ (where, with a slight abuse of notations, $\partial Q_B/\partial p = \partial Q_B/\partial p_1 + \partial Q_B/\partial p_2$), $\frac{\partial A}{\partial n_B} = -\frac{\partial Q_S/\partial n_B}{\partial Q_S/\partial a}$ and $\frac{\partial A}{\partial n_S} = \frac{1}{\partial Q_S/\partial a}$, we have after rearranging terms

$$f(n_B, n_S) = \frac{1}{\eta_B}\left(\frac{\partial U_B}{\partial n_S} + \frac{\partial U_S}{\partial n_B}\frac{\partial Q_B}{\partial n_S}\right) - \frac{1}{\eta_S}\left(\frac{\partial U_S}{\partial n_B} + \frac{\partial U_B}{\partial n_S}\frac{\partial Q_S}{\partial n_B}\right),$$

where $\eta_B = -\frac{1}{n_B}\frac{\partial Q_B}{\partial p}(p, n_S)$ and $\eta_S = -\frac{1}{n_S}\frac{\partial Q_S}{\partial a}(a, n_B)$.

Next, we study the same problem but in the context of our running example. This allows, first, to determine the conditions under which the constrained-maximization problem is concave, and, second, to obtain a neat characterization of which side is taxed/which side is subsidized as function of network effects.

Lemma 1 - Ramsey Prices in the Running Example. Consider the unconstrained problem $\max_{(p,a)} W$. Assume $2(u_B + u_S)^2 < 1$, which ensures that the Hessian is negative definite so that W is strictly concave. We have indeeed $\partial^2 W/\partial p^2 = -(2 - 4u_S(2u_B + u_S))/(1 - 2u_B u_S)^2 < 0$, $\partial^2 W/\partial a^2 = -(1 - 2u_B(u_B + 2u_S))/(1 - 2u_B u_S)^2 < 0$ and $(\partial^2 W/\partial p^2)(\partial^2 W/\partial a^2) - (\partial^2 W/\partial a \partial p)^2 = (2 - 4(u_B + u_S)^2)/(1 - 2u_B u_S)^2 > 0$.

Solving for the first-order conditions, we obtain $p = -(2u_S(u_B + u_S)v + r(1 - 2u_B(u_B + u_S)))/(1 - 2(u_B + u_S)^2)$ and $a = -(2u_B(v + r))/(1 - 2(u_B + u_S)^2)$, which yields $p + r \le 0$ and $a \le 0$.

Consider now problem (A.6). Π is strictly concave under the assumption $2(u_B + u_S)^2 < 1$. We have indeed $\partial^2\Pi/\partial p^2 = -4/(1 - 2u_B u_S) < 0$, $\partial^2\Pi/\partial a^2 = -2/(1 - 2u_B u_S) < 0$ and $(\partial^2\Pi/\partial p^2)(\partial^2\Pi/\partial a^2) - (\partial^2\Pi/\partial a\partial p)^2 = (8 - 4(u_B + u_S)^2)/(1 - 2u_B u_S)^2 > 0$.

The Lagrangian $\mathcal{L} = W + \lambda\Pi$, with $\lambda \ge 0$, is thus strictly concave as the sum of two strictly concave functions. The optimum is then characterized by the first-order conditions $\partial\mathcal{L}/\partial p = 0$ and $\partial\mathcal{L}/\partial a = 0$, and the complementary slackness condition $\lambda\partial\mathcal{L}/\partial\lambda = 0$. The constraint must bind at the optimum since the unconstrained outcome violates the break even constraint (except in the degenerate case $u_B = u_S = 0$). Therefore, $\lambda > 0$ at the optimum.

Using the first-order conditions $\partial\mathcal{L}/\partial p = 0$ and $\partial\mathcal{L}/\partial a = 0$, we can express the optimal price p^R and developer fee a^R as functions of the multiplier λ

$$\text{(A.12)} \qquad a^R = \frac{2(\lambda+1)(v+r)(\lambda u_B + u_B - \lambda u_S)}{-4\lambda + 2\left(\lambda^2\left((u_B+u_S)^2 - 2\right) + 2\lambda(u_B+u_S)^2 + (u_B+u_S)^2\right) - 1},$$

$$\text{(A.13)} \qquad p^R + r = \frac{(v+r)\left(-\lambda(2\lambda+1) + 2(\lambda+1)^2 u_B u_S + 2(\lambda+1)^2 u_S^2\right)}{-4\lambda + 2\left(\lambda^2\left((u_B+u_S)^2 - 2\right) + 2\lambda(u_B+u_S)^2 + (u_B+u_S)^2\right) - 1}.$$

Replacing in the constraint $\Pi = 0$, the multiplier λ satisfies

$$\text{(A.14)} \qquad (u_B + u_S)^2 = \frac{\lambda(2\lambda+1)^2}{2(\lambda+1)^3}.$$

The right-hand side in (A.14) is strictly increasing and takes values in $[0, 2)$ for $\lambda \in [0, +\infty)$. Therefore, (A.14) has a unique strictly positive solution in λ when $(u_B + u_S)^2 < 2$, which is ensured by our assumption $2(u_B+u_S)^2<1$. Denote that solution by λ^R. We can use (A.14) to replace $(u_B+u_S)^2$ as a function of λ^R in the denominator of (A.12) (which is the same as the denominator of (A.13)) to show that this denominator is equal to $-4\lambda^R-1/(\lambda^R+1)<0$. As a consequence, and using again (A.14), we obtain

$$\mathrm{Sign}(a^R)=-\mathrm{Sign}(p^R+r)=-\mathrm{Sign}(\lambda^R u_B+u_B-\lambda^R u_S).$$

Observe now that $a^R=0$ (or equivalently $p^R+r=0$) amounts to $\lambda^R=u_B/(u_S-u_B)$ with λ^R the unique positive solution of (A.14). Plugging this expression in (A.14), we obtain that the following condition between u_B and u_S must hold to have $a^R=0$ (or equivalently $p^R+r=0$): $u_B=2u_S^3$.

Last, we consider the case of a perfectly competitive platform, focusing on the running example.

A PERFECTLY COMPETITIVE PLATFORM. The manufacturers' profit is given by $\pi_M(\beta_1,\beta_2,a)=(\hat{p}_1+\beta_1 r)D_1(\hat{p}_1,\hat{p}_2,a)+(\hat{p}_2+\beta_2 r)D_2(\hat{p}_2,\hat{p}_1,a)$ where prices $(\hat{p}_1(\beta_1,\beta_2,a),\hat{p}_2(\beta_2,\beta_1,a))$ are the solution of the first-order conditions $(\hat{p}_k+\beta_k r)\frac{\partial D_k}{\partial p_k}+D_k=0$, $k=1,2$. The platform's profit is given by $\Pi(\beta_1,\beta_2,a)=(1-\beta_1)rD_1(\hat{p}_1,\hat{p}_2,a)+(1-\beta_2)rD_2(\hat{p}_2,\hat{p}_1,a)+aD_S(\hat{p}_1,\hat{p}_2,a)$.

Consider the unconstrained optimum: $\max_{(\beta_1,\beta_2,a)}\pi_M(\beta_1,\beta_2,a)$. We have

$$\frac{\partial\pi_M}{\partial a}=\sum_{\substack{k=1,2\\ \ell\neq k}}(\hat{p}_k+\beta_k r)\left(\frac{\partial D_k}{\partial p_\ell}\frac{\partial\hat{p}_\ell}{\partial a}+\frac{\partial D_k}{\partial a}\right)<0.$$

Therefore, the platform's break even constraint must bind at the optimum. When that constraint binds, we must have $a\le 0$ and $1-\beta_k\ge 0$. Without loss of generality, let us directly consider that $\beta_1=\beta_2\equiv\beta$. Let $\mathcal{L}=\pi_M(\beta,a)+\lambda\Pi(\beta,a)$ be the Lagrangian associated to the constrained maximization problem and λ be the multiplier associated to the platform's break-even constraint.

$\Pi(\beta,a)$ is strictly concave in (β,a) provided that $2-(u_B+u_S)^2\ge 0$, an assumption that we maintain from now on. $\pi_M(\beta,a)$ is not concave in (β,a). We will have to check later on that the second-order conditions are satisfied to ensure that we have characterized a local maximum.

For the moment, we neglect the constraint $0\le\beta\le 1$. Solving for the first-order conditions $\partial\mathcal{L}/\partial\beta=0$, $\partial\mathcal{L}/\partial a=0$ and $\partial\mathcal{L}/\partial\lambda=0$, we obtain

$$1-\beta=\frac{(\sigma+2)(v+r)(u_B-u_S)(u_B+u_S)}{4r\left(1-2u_Bu_S\right)+(2+\sigma)r\left(2-(u_B+u_S)^2\right)},\tag{A.15}$$

$$a=\frac{2(\sigma+2)(v+r)(u_S-u_B)}{4\left(1-2u_Bu_S\right)+(2+\sigma)\left(2-(u_B+u_S)^2\right)},\tag{A.16}$$

$$\lambda=\frac{8(1-2u_Bu_S)}{4\left(1-2u_Bu_S\right)+(2+\sigma)\left(2-(u_B+u_S)^2\right)}>0.\tag{A.17}$$

Therefore, we have $a\ge 0\Leftrightarrow 1-\beta\le 0\Leftrightarrow u_S\ge u_B$. When $u_S\ge u_B$, we have $\beta=1$ and the break-even constraint immediately gives $a=0$. When $u_S<u_B$, we have an interior solution characterized by (A.15) and (A.16).

It is also possible that β defined by (A.15) is negative. In that case, we have $\beta=0$ and $a(<0)$ is defined by $\Pi(0,a)=0$. We do not go further in the characterization of that case since it is qualitatively similar to the case $0<\beta<1$ and $a<0$.

Last, we can compute the determinant of the first and of the second principal minors of the Bordered Hessian. The first determinant is $-4r^2(\sigma+2)^2(r+v)^2/[(\sigma+4)^2(1-2u_Bu_S)^2]<0$; the second determinant is $64r^2(\sigma+2)^2(r+v)^2/[(\sigma+4)^2(1-2u_Bu_S)(4(1-2u_Bu_S)+(2+\sigma)(2-(u_B+u_S)^2))]>0$. Hence, the matrix of second derivatives of the Lagrangian is negative definite, which is a sufficient condition for a local maximum.

A.4. Proof of Lemma 3 (Separation Outcome)

Existence. Suppose all platforms offer $(\beta = 1, a = 0)$ and thus make no profit. The developer publishes all its applications on all the operating systems and manufacturers are indifferent between any of the operating systems. There are a priori three possible configurations to consider: (i) both manufacturers choose I; (ii) M_1 chooses I and M_2 chooses E; (iii) both manufacturers choose E. However, in all these configurations, all platforms make a nil profit and manufacturers' profits are given by $\hat{\pi}_1(1,1,0) = \hat{\pi}_2(1,1,0)$. These configurations are thus essentially equivalent from the platforms' perspective.

Consider now the possible deviations by platform I.

(i). First, consider a deviation $(\beta_I < 1, a_I < 0)$. Because $a_I < 0$, the developer publishes its applications on I even if no manufacturers choose I's operating system.

If both manufacturers choose I's operating system, M_1's profit is given by $\hat{\pi}_1(\beta_I, \beta_I, a_I)$. If M_1 chooses E and M_2 chooses I, M_1's profit is given by $\hat{\pi}_1(1, \beta_I, a_I)$. Since $\hat{\pi}_1(1, \beta_I, a_I) > \hat{\pi}_1(\beta_I, \beta_I, a_I)$, M_1 chooses the fringe's operating system rather than I's. Hence, following I's deviation, it is not possible that both manufacturers choose I. A configuration in which M_1 chooses I and M_2 chooses E is not an equilibrium either because $\hat{\pi}_1(\beta_I, 1, a_I) < \hat{\pi}_1(1, 1, a_I)$. Both manufacturers choosing E is the only continuation following I's deviation because $\hat{\pi}_1(1, 1, a_I) > \hat{\pi}_1(\beta_I, 1, a_I)$.

Hence, the deviation leads to both manufacturers choosing E and to a strictly negative profit for I (no manufacturers and $a_I < 0$). It is thus not profitable for I.

(ii). Second, consider a deviation $(\beta_I < 1, a_I \geq 0)$. Consider that the developer publishes on I when at least one manufacturer chooses I's operating system, or equivalently that a_I is not too large (this is the most favorable scenario for I's deviation). If both manufacturers choose I's operating system, M_1's profit is given by $\hat{\pi}_1(\beta_I, \beta_I, a_I)$. If M_1 chooses E and M_2 chooses I, profits are given by $\hat{\pi}_1(1, \beta_I, a_I)$ and $\hat{\pi}_2(\beta_I, 1, a_I)$. Therefore, both manufacturers choosing I is not part of the continuation following I's deviation. Again, it is immediate to show that, following I's deviation, both manufacturers choose E and the developer does not publish on I. The deviation is thus not profitable.

(iii). Third, consider a deviation $(\beta_I = 1, a_I \geq 0)$. Consider that the developer publishes on I when at least one manufacturer chooses I's operating system, or equivalently that a_I is not too large (again, this is the most favorable scenario for I's deviation). If both manufacturers choose I, M_1's gain is $\hat{\pi}_1(1, 1, a_I)$. If M_1 chooses E and M_2 chooses I, M_1's gain is again $\hat{\pi}_1(1, 1, a_I)$. Hence, given that the other manufacturer chooses I, each manufacturer is indifferent between I and E. Therefore, both manufacturers choosing I is a part of the continuation equilibrium. It is immediate to show that both manufacturers choosing E is also an equilibrium since $\hat{\pi}_1(1, 1, 0) > \hat{\pi}_1(1, 1, a_I)$.

Observe, though, that manufacturers collectively gain if they both choose E rather than I, for their total profit would be $\hat{\pi}_1(1, 1, 0) + \hat{\pi}_2(1, 1, 0) > \hat{\pi}_1(1, 1, a_I) + \hat{\pi}_2(1, 1, a_I)$ when $a_I > 0$. Our criterion selects both manufacturers choosing E as the continuation following the deviation by I. Hence, the deviation is not profitable for I.

Uniqueness. It remains to show that $(\beta = 1, a = 0)$ for all platforms is the unique equilibrium.

(i). Consider a situation where I sets $(\beta_I < 1, a_I)$ and attracts both manufacturers. Assume that a_I is such that the developer is not discouraged from publishing on I. Then, E can set $\beta_E = \beta_I + \varepsilon$, $\varepsilon > 0$ but small, and $a_E = 0$. Both manufacturers choose E since $\hat{\pi}_1(\beta_I, \beta_I, a_I) < \hat{\pi}_1(\beta_E, \beta_I, a_I + 0)$ (both manufacturers choosing I is not a Nash equilibrium) and $\hat{\pi}_1(\beta_E, \beta_E, 0 + \mathbb{1}_{\{a_I \leq 0\}}) \geq \hat{\pi}_1(\beta_I, \beta_E, a_I + 0)$ (both manufacturers choosing E is a Nash equilibrium).

(ii). Consider now a situation where I sets $(\beta_I < 1, a_I)$ and attracts M_1 only. Consider that M_2 chooses E that offers $(\beta_E < 1, a_E)$. Assume that a_I and a_E are such that that the developer is not discouraged from publishing on I and on E respectively. Then, E' can set $\beta_{E'} = \max(\beta_I, \beta_E) + \varepsilon$, $\varepsilon > 0$ but small, and $a_{E'} = 0$. Both manufacturers choose E' since

$\hat{\pi}_1(\beta_I, \beta_E, a_I + a_E) < \hat{\pi}_1(\beta_{E'}, \beta_E, \mathbb{1}_{\{a_I \leq 0\}} + a_E + 0)$ (both manufacturers choosing I is not a Nash equilibrium) and $\hat{\pi}_1(\beta_{E'}, \beta_{E'}, \mathbb{1}_{\{a_I \leq 0\}} + \mathbb{1}_{\{a_E \leq 0\}} + 0) \geq \hat{\pi}_1(\beta_I, \beta_E, a_I + \mathbb{1}_{\{a_E \leq 0\}} + 0)$ (both manufacturers choosing E' is a Nash equilibrium).

A consequence of (i) and (ii) is that a strategy $(\beta_I < 1, a_I)$ leads at best to a nil profit in equilibrium. That strategy is thus strictly dominated by $(\beta_I = 1, a_I = 0)$.

(iii). A strategy $(\beta_I = 1, a_I < 0)$ is never used at equilibrium because it leads at best to a nil profit. It is also strictly dominated by $(\beta_I = 1, a_I = 0)$.

(iv). It remains to study the strategy $(\beta_I = 1, a_I > 0)$. If both manufacturers choose I, then E can offer $(\beta_E = 1, a_E = a_I - \varepsilon)$, with $\varepsilon > 0$ but small. Since $\hat{\pi}_1(1, 1, a_I) > \hat{\pi}_1(1, 1, a_I + a_E)$ and $\hat{\pi}_1(1, 1, a_E) > \hat{\pi}_1(1, 1, a_I + a_E)$, there are two Nash equilibria in the subgame starting at stage 2: both manufacturers choose I; and both manufacturers choose E. According to our selection criterion, both manufacturers choose E, which leads to a nil profit for I. Similarly, if M_1 only chooses I, and M_2 chooses E that offers (β_E, a_E), E' can offer $(\beta_{E'} = 1, a_{E'} = a_I - \varepsilon)$, $\varepsilon > 0$ but small, and ensures that M_1 choose E'. The strategy $(\beta_I = 1, a_I > 0)$ is dominated by $(\beta_I = 1, a_I = 0)$.

A consequence of (i)-(ii)-(iii)-(iv) is that there is no other equilibrium than the one in which all platforms offer $(\beta = 1, a = 0)$.

A.5. Unconstrained Outcome with No Efficiency Gains in the Running Example

We study the unconstrained outcome and derive formally the curves drawn in Figure 3. Let $\pi_1^I(\beta_I, a_I) = (p_1^I + r)D_1(p_1^I, p_2^I, a_I) + (1 - \beta_I)rD_2(p_2^I, p_1^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$, where prices p_1^I and p_2^I are given in Appendix A.2.

CONDITIONS FOR CONCAVITY. We find conditions that ensure the concavity of the maximization problem $\max_{(\beta_I, a_I)} \pi_1^I(\beta_I, a_I)$. One can show that a sufficient condition is $2(u_B + u_S)^2 < 1$. This is, however, an overly restrictive condition that prevents from studying situations with quite asymmetric network effects. In the sequel, we establish a set of necessary and sufficient conditions that ensure the concavity of the previous problem. Computations show that:

(i) $\partial^2 \pi_1^I / \partial \beta_I^2 = -[4(\sigma + 1)(\sigma + 2)(\sigma(9\sigma + 32) + 32)]/[(\sigma + 4)^2(3\sigma + 4)^2(1 - 2u_B u_S)]$, which is strictly negative.

(ii) $\partial^2 \pi_1^I / \partial a_I^2 = -[2(\sigma+4)^2(3\sigma+4)^2 - 4(\sigma+2)(3\sigma+4)^2 u_B^2 - 4(3\sigma+4)(\sigma(7\sigma+32)+32)u_B u_S - 4(\sigma(\sigma(7\sigma + 40) + 64) + 32)u_S^2]/[(\sigma + 4)^2(3\sigma + 4)^2(1 - 2u_B u_S)]$.

(iii) $(\partial^2 \pi_1^I / \partial \beta_I^2)(\partial^2 \pi_1^I / \partial a_I^2) - (\partial^2 \pi_1^I / \partial a_I \partial \beta_I)^2 = -4r^2(\sigma + 1)[5\sigma^2(9u_B^2 + 22u_B u_S + 9u_S^2 - 20) + 16\sigma(5u_B^2 + 14u_B u_S + 5u_S^2 - 12) + 9\sigma^3((u_B + u_S)^2 - 2) + 16((3u_B + u_S)(u_B + 3u_S) - 8)]/((\sigma + 4)^2(3\sigma + 4)^2(1 - 2u_B u_S)^2)$.

Simplifying these expressions further, the Hessian is negative definite if and only if $-2(\sigma + 4)^2(3\sigma+4)^2 + 4(\sigma+2)(3\sigma+4)^2 u_B^2 + 4(3\sigma+4)(\sigma(7\sigma+32)+32)u_B u_S + 4(\sigma(\sigma(7\sigma+40)+64)+32)u_S^2 < 0$ and $5\sigma^2(9u_B^2 + 22u_B u_S + 9u_S^2 - 20) + 16\sigma(5u_B^2 + 14u_B u_S + 5u_S^2 - 12) + 9\sigma^3((u_B + u_S)^2 - 2) + 16((3u_B + u_S)(u_B + 3u_S) - 8) < 0$.

Instead of working with (u_B, u_S), it turns out to be easier to work with (σ, u_S) with $\sigma = \gamma - 2(1 + \gamma)u_B u_S$ (which is possible since such transformation is a $\mathcal{C}^1$-diffeomorphism). Remind that $\sigma \geq 0$ by assumption and $\sigma \leq \gamma$ by definition. Equipped with this change of variables, we have that $\partial^2 \pi_1^I / \partial a_I^2 < 0$ and $(\partial^2 \pi_1^I / \partial \beta_I^2)(\partial^2 \pi_1^I / \partial a_I^2) - (\partial^2 \pi_1^I / \partial a_I \partial \beta_I)^2 > 0$ are equivalent to

$$f(x) = \frac{1}{x(1+\gamma)(1+\sigma)(4+\sigma)^2(4+3\sigma)^2}\big(4x^2(1+\gamma)^2(32+\sigma(64+\sigma(40+7\sigma))) \\ -2x(1+\gamma)(4+3\sigma)(2(2+\sigma)(16+5\sigma(4+\sigma)) + \gamma(32+3\sigma(16+\sigma(7+\sigma)))) \\ +(\gamma-\sigma)^2(2+\sigma)(4+3\sigma)^2\big) < 0 \tag{A.18}$$

and

$$g(x) = \frac{1}{x(1+\sigma)(4+\sigma)^2(4+3\sigma)^2}\big(4x^2(1+\gamma)^2(4+3\sigma)(12+\sigma(11+3\sigma)) \\ -4x(1+\gamma)(128+\gamma(4+3\sigma)(12+\sigma(11+3\sigma))+\sigma(272+\sigma(212+\sigma(73+9\sigma)))) \\ +(\gamma-\sigma)^2(4+3\sigma)(12+\sigma(11+3\sigma))\big) < 0, \tag{A.19}$$

where $x = u_S^2$. Notice that the denominators in (A.18) and (A.19) are identical and positive. Let N_1 and N_2 denote the numerators in (A.18) and (A.19) respectively. We have

$$N_2 - N_1 = (\gamma-\sigma)^2(4+\sigma)(4+3\sigma) + 4x^2(1+\gamma)^2(16+\sigma(16+\sigma(5+2\sigma))) \\ + 2x(1+\gamma)(\gamma(4+3\sigma)(8+\sigma(26+3\sigma(5+\sigma))) + 2\sigma(48+\sigma(76+\sigma(37+6\sigma)))) > 0.$$

Put differently, Condition (A.19) is more demanding than Condition (A.18), that is $N_2 < 0$ is a necessary and sufficient condition for the concavity of the maximization problem. Then, simple computations show that condition $N_2 < 0$ amounts to

$$\frac{(u_B+u_S)^2-2}{(u_S-u_B)^2} < \frac{16+\sigma(16+5\sigma)}{(2+\sigma)(32+\sigma(32+9\sigma))}. \tag{A.20}$$

Since the right-hand side in (A.20) is positive, a sufficient condition for the concavity of the maximization problem is $(u_B+u_S)^2 < 2$. When $(u_B+u_S)^2 > 2$, condition (A.20) is more likely to be satisfied if $(u_S-u_B)^2$ large, that is, roughly speaking when network effects are sufficiently asymmetric.

Let us now describe the set of (u_B, u_S) such that (A.20) is satisfied, which as we have seen amounts to $g(x) < 0$. The numerator in $g(x)$ is a polynomial of degree 2 in x, whose discriminant is equal to $64(1+\gamma)^2(1+\sigma)(2+\sigma)(32+\sigma(32+9\sigma))(\gamma(4+3\sigma)(12+\sigma(11+3\sigma))+2(32+\sigma(56+\sigma(33+7\sigma)))) > 0$. Therefore, it has two distinct real roots $\underline{x}(\sigma)$ and $\overline{x}(\sigma)$. Since the numerator in $g(x)$ is positive when $x = 0$, the smallest root is positive: $\underline{x}(\sigma) > 0$. It follows that, for a given $\gamma \geq 0$, $g(x) < 0$ amounts to $\underline{x}(\sigma) < u_S^2 < \underline{x}(\sigma)$. Define $\underline{u}_S(\sigma) = \sqrt{\underline{x}(\sigma)}$, $\underline{u}_B(\sigma) = \frac{\gamma-\sigma}{2(1+\gamma)\underline{u}_S(\sigma)}$, $\overline{u}_S(\sigma) = \sqrt{\overline{x}(\sigma)}$ and $\overline{u}_B(\sigma) = \frac{\gamma-\sigma}{2(1+\gamma)\overline{u}_S(\sigma)}$. By construction, the set of (u_B, u_S) such that (A.20) is satisfied is the set of (u_B, u_S) whose frontiers are given by the two parametric curves $\underline{\mathcal{C}} = (\underline{u}_B(\sigma), \underline{u}_S(\sigma))$ and $\overline{\mathcal{C}} = (\overline{u}_B(\sigma), \overline{u}_S(\sigma))$ for all $\sigma \in [0, \gamma]$. Curves $\underline{\mathcal{C}}$ and $\overline{\mathcal{C}}$ are represented in Figure 15.

Simple computations show that the slope of the parametric curves (given by $\dot{u}_S(\sigma)/\dot{u}_B(\sigma)$) is equal to

$$-\frac{2(1+\gamma)x(\sigma)}{(\gamma-\sigma)+2\frac{x(\sigma)}{x'(\sigma)}}, \tag{A.21}$$

where $x(\sigma) = \underline{x}(\sigma)$ for the curve $\underline{\mathcal{C}}$ and $x(\sigma) = \overline{x}(\sigma)$ for the curve $\overline{\mathcal{C}}$. Computations show that $\underline{x}'(\sigma) < 0 < \overline{x}'(\sigma)$. Plugging this in Equation (A.21) shows that the curve $\overline{\mathcal{C}}$ is downward sloping in the plane (u_B, u_S). Computations then show that $(\gamma-\sigma)+2\frac{\underline{x}(\sigma)}{\underline{x}'(\sigma)} > 0$, which proves that the curve $\underline{\mathcal{C}}$ is downward sloping in the plane (u_B, u_S).

Figure 15 represents the sufficient condition (area below the red curve) and the necessary and sufficient condition (area below the blue curves) for the concavity of the maximization problem (given that we focus on $\sigma \geq 0$).

UNCONSTRAINED OPTIMUM. Denote now by (β_I^*, a_I^*) the unique solution of the system formed

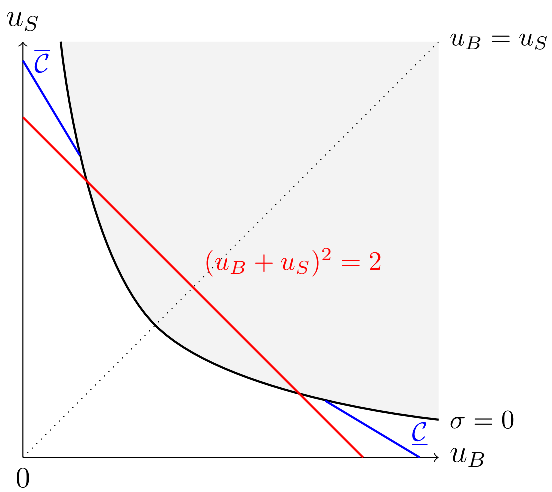


Figure 15: Necessary and sufficient conditions for the concavity of the maximization problem in the running example.

by the two first-order conditions $\partial\pi_1^I/\partial\beta_I = 0$ and $\partial\pi_1^I/\partial a_I = 0$. Simple computations lead to

(A.22)
$$r(1-\beta_I^*) = \frac{(v+r)(3\sigma+4)\left(-3\sigma(\sigma+4)+(\sigma(3\sigma+13)+20)u_Bu_S+(\sigma(3\sigma+11)+12)u_S^2-16\right)}{Den},$$

(A.23)
$$a_I^* = \frac{(v+r)(3\sigma+4)(\sigma(3\sigma+11)+12)(u_B-u_S)}{Den},$$

with $Den < 0$ since it is proportional to $-(\partial^2\pi_1^I/\partial\beta_I^2)(\partial^2\pi_1^I/\partial a_I^2) + (\partial^2\pi_1^I/\partial a_I\partial\beta_I)^2 < 0$.

First, $a_I^* = 0$ is equivalent to $u_B = u_S$.

Second, $\beta_I^* = 1$ amounts to $-3\sigma(\sigma+4)+(\sigma(3\sigma+13)+20)u_Bu_S+(\sigma(3\sigma+11)+12)u_S^2-16 = 0$ with $\sigma = \gamma - 2(1+\gamma)u_Bu_S$. Expressing u_B as a function of σ, $\beta_I^* = 1$ amounts to $(\sigma(3\sigma+13)+20)(\gamma-\sigma)/(2(\gamma+1)) - 3\sigma(\sigma+4)+(\sigma(3\sigma+11)+12)u_S^2-16 = 0$, a second degree polynomial in u_S with a positive root equal to

$$\hat{u}_S^{\beta_I^*=1}(\sigma) = \sqrt{\frac{\gamma + \frac{8\sigma}{9\sigma^2+33\sigma+36} + \sigma + \frac{8}{3}}{2(\gamma+1)}}.$$

Define $\hat{u}_B^{\beta_I^*=1}(\sigma) = (\gamma-\sigma)/(2(\gamma+1)\hat{u}_S(\sigma))$. Then, the set of (u_B, u_S) such that $\beta_I^* = 1$ is described by the curve associated to the parametric equations $(u_B = \hat{u}_B^{\beta_I^*=1}(\sigma), u_S = \hat{u}_S^{\beta_I^*=1}(\sigma))$ for all $\sigma \in [0,\gamma]$.

Straightforward computations show that $(\hat{u}_S^{\beta_I^*=1}(\sigma))^2 > (\hat{u}_B^{\beta_I^*=1}(\sigma))^2$ for all σ, so that the set of (u_B, u_S) such that $\beta_I^* = 1$ lies strictly above the 45°-degree line. We have $(\hat{u}_B^{\beta_I^*=1}(\gamma), \hat{u}_S^{\beta_I^*=1}(\gamma)) = (0, \sqrt{(3\gamma(\gamma+4)+16)/(\gamma(3\gamma+11)+12)})$. Simple computations then show that $(\hat{u}_S^{\beta_I^*=1})^2(\gamma) \in [\underline{x}(\gamma), \overline{x}(\gamma)]$ and $(\hat{u}_S^{\beta_I^*=1})^2(0) \in [\underline{x}(0), \overline{x}(0)]$.

Last, we also have

(A.24)
$$\frac{\frac{d}{d\sigma}\hat{u}_S^{\beta_I^*=1}(\sigma)}{\frac{d}{d\sigma}\hat{u}_B^{\beta_I^*=1}(\sigma)} = -\frac{C}{D}$$

with $C = ((\sigma(\sigma(9\sigma^2+66\sigma+185)+264)+176)(\gamma(\sigma(3\sigma+11)+12)+(3\sigma+4)(\sigma(\sigma+5)+8))) > 0$ and $D = (\sigma(3\sigma+11)+12)(\gamma(\sigma(\sigma(9\sigma(3\sigma+22)+571)+792)+464)+\sigma(\sigma(\sigma(3\sigma(3\sigma+38)+569)+1352)+1584)+768) > 0$; this derivative is thus strictly negative for all σ. Hence, the parametric curve $(\hat{u}_B^{\beta_I^*=1}(\sigma), \hat{u}_S^{\beta_I^*=1}(\sigma))$ for $\sigma \in [0,\gamma]$ is strictly downward-sloping in the (u_B, u_S)-plane.

Simple computations also show that: (i) $(\hat{u}_S^{\beta_I^*=1}(0))^2 = (8+3\gamma)/(6(1+\gamma))$, which belongs to $[\underline{x}(0) = (8+3\gamma-4\sqrt{4+3\gamma})/(6(1+\gamma)), \overline{x}(0) = (8+3\gamma+4\sqrt{4+3\gamma})/(6(1+\gamma))]$; (ii) $(\hat{u}_S^{\beta_I^*=1}(\gamma))^2 = (\gamma+4)/(\gamma(3\gamma+11)+12)+1 < \overline{x}(\gamma) = \frac{2}{3}(\gamma/(\gamma(3\gamma+11)+12)+4/(3\gamma+4)+3)$.

To summarize, $\beta_I^* = 1$ describes a curve in the (u_B, u_S)-space that is always above the 45°-degree line and is strictly downward-sloping, starts and ends within the sets of admissible values, as depicted in Figure 3.

A.6. Proof of Proposition 1

We consider here the constrained outcome with no efficiency gains in the running example. Let $\pi_1^I(a_I) = (p_1^I + r)D_1(p_1^I, p_2^I, a_I) + a_I D_S(p_1^I, p_2^I, a_I)$, where the prices p_1^I and p_2^I are given in Appendix A.2. From the analysis of Appendix A.5, this is a strictly concave function of a_I provided that (A.20) holds. Therefore, as shown in Appendix A.5, the objective is concave in a_I iff $u_S^2 \in [\underline{x}(\sigma), \overline{x}(\sigma)]$ for $\sigma \in [0,\gamma]$.

The first-order condition leads to

$$a_I^{**} = -\frac{(3\sigma+4)(v+r)(2(\sigma+2)(3\sigma+4)u_B - (3\sigma(\sigma(\sigma+7)+16)+32)u_S)}{H}$$

with $H = (\sigma+4)^2(3\sigma+4)^2 - 2(\sigma+2)(3\sigma+4)^2u_B^2 - 2(3\sigma+4)(\sigma(7\sigma+32)+32)u_Bu_S - 2(\sigma(\sigma(7\sigma+40)+64)+32)u_S^2 > 0$ when $u_S^2 \in [\underline{x}(\sigma), \overline{x}(\sigma)]$.

The curve $a_I^{**} = 0$ is given by $-(2(\sigma+2)(3\sigma+4)u_B - (3\sigma(\sigma(\sigma+7)+16)+32)u_S) = 0$ and can be expressed as a function of (u_S, σ) as $(3\sigma(\sigma(\sigma+7)+16)+32)u_S - (\sigma+2)(3\sigma+4)(\gamma-\sigma)/((\gamma+1)u_S) = 0$, which has a unique positive root

$$\hat{u}_S^{a_I^{**}=0}(\sigma) = \frac{\sqrt{(\sigma+2)(3\sigma+4)(\gamma-\sigma)}}{\sqrt{(\gamma+1)(3\sigma(\sigma(\sigma+7)+16)+32)}}.$$

Define $\hat{u}_B^{a_I^{**}=0}(\sigma) = (\gamma-\sigma)/(2(\gamma+1)\hat{u}_S^{a_I^{**}=0}(\sigma))$. Then, the set of (u_B, u_S) such that $a_I^{**} = 0$ is characterized by the parametric equations $(u_B = \hat{u}_B^{a_I^{**}=0}(\sigma), u_S = \hat{u}_S^{a_I^{**}=0}(\sigma))$ for all σ in the relevant range. We can check that $\hat{u}_S^{a_I^{**}=0}(\sigma) \in [\underline{x}(\sigma), \overline{x}(\sigma)]$ (so that the curve associated to the parametric equations always lies within the set of admissible values) using brute force computations that are similar to those performed in Appendix A.5 and are not reported here.

We have: $\lim_{\sigma\to\gamma}(\hat{u}_B^{a_I^{**}=0}(\gamma), \hat{u}_S^{a_I^{**}=0}(\gamma)) = (0,0)$, $(\hat{u}_B^{a_I^{**}=0}(0), \hat{u}_S^{a_I^{**}=0}(0)) = (\sqrt{\gamma/(\gamma+1)}, (1/2)\sqrt{\gamma/(\gamma+1)})$. Simple computations also show that $(\hat{u}_S^{a_I^{**}=0}(\sigma))^2 \leq (\hat{u}_B^{a_I^{**}=0}(\sigma))^2$ for all $\sigma \in [0,\gamma]$, which shows that the curve $a_I^{**} = 0$ lies below the 45°-degree line in the (u_B, u_S)-space. Its derivative is given by

$$\frac{\frac{d}{d\sigma}\hat{u}_S^{a_I^{**}=0}(\sigma)}{\frac{d}{d\sigma}\hat{u}_B^{a_I^{**}=0}(\sigma)} = \frac{I}{J},$$

with $I = 2(\sigma+2)(3\sigma+4)(\gamma(3\sigma(\sigma(\sigma(3\sigma+20)+46)+48)+64)+\sigma(3\sigma(\sigma(11\sigma+80)+200)+640)+256) > 0$ and $J = (3\sigma(\sigma(\sigma+7)+16)+32)((3\sigma(\sigma+4)+8)(3\sigma(\sigma(2\sigma+9)+16)+32) - \gamma(3\sigma(\sigma(\sigma(3\sigma+20)+46)+48)+64))$. The denominator J is equal to zero when $\gamma = [(3\sigma(\sigma+4)+8)(3\sigma(\sigma(2\sigma+9)+16)+32)]/[3\sigma(\sigma(\sigma(3\sigma+20)+46)+48)+64]$. The right-hand side is strictly increasing in σ and is equal to 4 for $\sigma = 0$. Hence, if $\gamma < 4$, the slope is always positive. If $\gamma \geq 4$, there is a vertical asymptote and the slope is first positive and then negative.

Notice also that

$$\left.\frac{\frac{d}{d\sigma}\hat{u}_S^{a_I^{**}=0}(\sigma)}{\frac{d}{d\sigma}\hat{u}_B^{a_I^{**}=0}(\sigma)}\right|_{\sigma=0} = \frac{\gamma+4}{2(4-\gamma)} \text{ and } \left.\frac{\frac{d}{d\sigma}\hat{u}_S^{a_I^{**}=0}(\sigma)}{\frac{d}{d\sigma}\hat{u}_B^{a_I^{**}=0}(\sigma)}\right|_{\sigma=\gamma} = \frac{2(\gamma+2)(3\gamma+4)}{3\gamma(\gamma(\gamma+7)+16)+32}.$$

To summarize, $a_I^{**}=0$ describes a curve in the (u_B, u_S)-space such that: it goes through $(0,0)$, it is always below the 45°-degree line, and it is strictly increasing if $\gamma < 4$ or increasing then decreasing if $\gamma \geq 4$. Therefore, there exists a uniquely defined function $\underline{h} : u_S \mapsto \underline{h}(u_S)$ such that, for all (u_B, u_S), $a_I^{**}(u_B, u_S) > 0$ if and only if $u_B < \underline{h}(u_S)$.

A.7. Proof of Proposition 3

IMPACT ON BUYERS. Let $V_B^I(a_I)$ be the buyer surplus under vertical integration when $\beta_I = 1$ and the developer fee is set at some value a_I. Let V_B^S be the buyer surplus under separation. Let $\Delta V_B(a_I) = V_B^I(a_I) - V_B^S$. Computations show that

$$\Delta V_B(a_I) = a_I \left(a_I - \frac{2(\sigma+2)(3\sigma+4)^2(v+r)((\sigma+2)u_B - u_S)}{K}\right) \frac{K}{(\sigma+4)^2(3\sigma+4)^2(1-2u_B u_S)^2}$$

where $K = (\sigma+2)^2(3\sigma+4)^2u_B^2 + \left[(\sigma+2)(\sigma(\sigma+16)+16) - 2u_B u_S(\sigma+1)(\sigma+4)^2\right] u_S^2 - 2u_B u_S(\sigma+2)(3\sigma+4)^2$. We show first that $K > 0$. Since $2u_B u_S < 1$, the term in brackets in K is strictly greater than $(\sigma+2)(\sigma(\sigma+16)+16) - (\sigma+1)(\sigma+4)^2 = (3\sigma+4)^2$. Plugging this in K then gives $K > (3\sigma+4)^2(u_S - u_B(2+\sigma))^2 \geq 0$. This implies that the sign of $\Delta V_B(a_I)$ is given by the sign of $a_I(a_I - 2(\sigma+2)(3\sigma+4)^2(v+r)((\sigma+2)u_B - u_S)/K)$.

FIRST SUFFICIENT CONDITION. Consider that $a_I^{**} < 0$, which amounts to $u_B > \underline{h}(u_S)$, with $\underline{h}$ defined in Appendix A.6 and such that $\underline{h}(u_S) > u_S$. Consequently, $a_I^{**} < 0$ implies $(\sigma+2)u_B - u_S > 0$, so that $\Delta V_B(a_I^{**}) > 0$. Therefore, we have established a first sufficient condition

$$a_I^{**} < 0 \Rightarrow \Delta V_B(a_I^{**}) > 0.$$

SECOND SUFFICIENT CONDITION. Consider that $a_I^{**} \geq 0$. If $(\sigma+2)u_B - u_S \leq 0$, then $\Delta V_B(a_I^{**}) \geq 0$. Notice that $(\sigma+2)u_B - u_S \leq 0$ is equivalent to

$$u_S \geq \overline{h}_B(u_B) \equiv \frac{(\gamma+2)u_B}{2(\gamma+1)u_B^2+1}.$$

$\overline{h}_B$ is first increasing then decreasing, reaching a maximum at $u_B = 1/\sqrt{2(\gamma+1)}$ and leading to $u_S = (\gamma+2)/(2\sqrt{2(\gamma+1)})$. These values of (u_B, u_S) satisfy $\sigma \geq 0$ iff $\gamma \geq 2$.

$\overline{h}_B$ can also be represented with the parametric equations $(u_B = \sqrt{(\gamma-\sigma)/(2(\gamma+1)(\sigma+2))}$, $u_S = \hat{\overline{u}}_S(\sigma) \equiv \sqrt{(\sigma+2)(\gamma-\sigma)/(2(\gamma+1))})$ for $\sigma \in [0, \gamma]$. This rewriting allows to get immediately that $\overline{h}_B$ is strictly above the 45°-degree line, and goes through $(0,0)$ (for $\sigma = \gamma$) and $(\sqrt{\gamma/(4(\gamma+1))}, \sqrt{\gamma/(\gamma+1)})$ (for $\sigma = 0$). This also shows that the conditions $a_I^{**} \geq 0$ (i.e., $u_B \leq \underline{h}(u_S)$) and $(\sigma+2)u_B - u_S \leq 0$ (i.e., $u_S \geq \overline{h}_B(u_B)$) define a non-empty set. Last, straightforward manipulations show that $\hat{\overline{u}}_S^2(\sigma)$ belongs to $[\underline{x}_1(\sigma), \overline{x}_1(\sigma)]$ for all σ, so that it always belongs to the admissible set.

INTERMEDIATE REGION. It remains to study the sign of $\Delta V_B(a_I^{**})$ when $a_I^{**} \geq 0$ and $(\sigma+2)u_B - u_S \geq 0$. Observe that $(\sigma+2)u_B - u_S \geq 0$ amounts to $u_S^2 \leq \overline{x}_a(\sigma) \equiv (\sigma+2)(\gamma-\sigma)/(2(\gamma+1))$. Similarly, $a_I^{**} \geq 0$ amounts to $u_S^2 \geq \underline{x}_a(\sigma) \equiv (\sigma+2)(3\sigma+4)(\gamma-\sigma)/((\gamma+1)(3\sigma(\sigma(\sigma+7)+16)+32))$ (see Appendix A.6). Simple computations show that $\overline{x}_a(\sigma) > \underline{x}_a(\sigma)$ for all σ in $[0, \gamma)$, so the interval in non empty.

On this interval, the sign of $\Delta V_B(a_I^{**})$ is given by the sign of

$$a_I^{**} - \frac{2(\sigma+2)(3\sigma+4)^2(v+r)((\sigma+2)u_B - u_S)}{K}.$$

Replacing u_B by $(\gamma-\sigma)/(2(\gamma+1)u_S)$ and up to some positive multiplicative terms (namely, $v+r$, H and K), the previous expression has the same sign as $h_1(x) = (\sigma+2)^3(3\sigma+4)^3(\gamma-\sigma)^3 + 4(\gamma+1)(\sigma+2)(3\sigma+4)x^2[\gamma^2(\sigma(\sigma(\sigma(23\sigma+183)+507)+584)+240)+\gamma(\sigma(\sigma(\sigma(\sigma(49-5\sigma)+562)+1618)+1840)+736)+\sigma(\sigma(\sigma(\sigma(37-4\sigma)+422)+1184)+1312)+512] - 4(\gamma+1)^2x^3(\gamma(3\sigma+4)(\sigma(\sigma+4)(\sigma(19\sigma+65)+88)+128) - \sigma(\sigma(\sigma+3)(\sigma+8)+16)(3\sigma(\sigma(\sigma+6)+10)+16)) - (\gamma+1)(\sigma+2)^2(3\sigma+4)^2x(\gamma-\sigma)(3\gamma(3\sigma(\sigma(\sigma+7)+16)+32)+\sigma(\sigma(\sigma(3\sigma+61)+288)+480)+256)$ with $x = u_S^2$. One can then show that $h_1(\underline{x}_a) < 0 < h_1(\overline{x}_a)$ and also $h_1'(\underline{x}_a) < 0 < h_1'(\overline{x}_a)$. Since $h_1'(x)$ is a polynomial of degree 2, it has at most two real roots and only one of these roots belongs to $[\underline{x}_a, \overline{x}_a]$. Since $h_1'(\underline{x}_a) < 0 < h_1'(\overline{x}_a)$, this implies that there exists a unique $\tilde{x} \in (\underline{x}_a, \overline{x}_a)$ such that h_1 decreases for $x \in [\underline{x}_a, \tilde{x}]$ and increases for $x \in [\tilde{x}, \overline{x}_a]$. This finally implies that there exists a unique $x_0^B(\sigma) \in (\underline{x}_a, \overline{x}_a)$ such that $h_1(x_0^B(\sigma)) = 0$.

Let $\hat{u}_S^{\Delta V_B(a_I^{**})=0}(\sigma) = \sqrt{x_0^B(\sigma)}$ and $\hat{u}_B^{\Delta V_B(a_I^{**})=0} = (\gamma-\sigma)/(2(1+\gamma)\hat{u}_S^{\Delta V_B(a_I^{**})=0}(\sigma))$. By construction, the parametric curve $\tilde{\mathcal{C}} = (\hat{u}_B^{\Delta V_B(a_I^{**})=0}, \hat{u}_S^{\Delta V_B(a_I^{**})=0})$, $\sigma \in [0,\gamma]$, is the frontier of the set of (u_B, u_S) such that $\Delta V_B(a_I^{**}) \leq 0$ when $a_I^{**} \geq 0$ and $(\sigma+2)u_B - u_S \geq 0$. Since $x_0^B(\sigma)$ is continuous and differentiable with respect to σ and the function $\sigma \mapsto (\hat{u}_B^{\Delta V_B(a_I^{**})=0}, \hat{u}_S^{\Delta V_B(a_I^{**})=0})$ is injective, the curve $\tilde{\mathcal{C}}$ divides the (u_B, u_S)-space into two connected subsets, one in which the buyer surplus increases, the other in which it decreases.

Impact on Developers. Let $V_S^I(a_I)$ be the developer surplus under vertical integration when $\beta_I = 1$ and the developer fee is set at some value a_I. Let V_S^S be the developer surplus under separation. Let $\Delta V_S(a_I) = V_S^I(a_I) - V_S^S$. Computations show that

$$\Delta V_S(a_I) = \frac{(\sigma - 2u_S(2u_B+u_S)+4)^2}{2(\sigma+4)^2(1-2u_Bu_S)^2} a_I \left(a_I - \frac{4(\sigma+2)u_S(v+r)}{\sigma - 2u_S(2u_B+u_S)+4}\right).$$

Using $\sigma = \gamma - 2(1+\gamma)u_Bu_S$, we rewrite $\sigma - 2u_S(2u_B+u_S)+4 = 0$ as $4+\gamma-2u_S^2-2u_Su_B(\gamma+3) = 0$ and denote the unique positive solution of this second degree polynomial equation in u_S by

$$\overline{h}_S(u_B) = \frac{1}{2}\left(\sqrt{2(\gamma+4)+(\gamma+3)^2u_B^2} - (\gamma+3)u_B\right).$$

We thus have $\sigma - 2u_S(2u_B+u_S)+4 > 0 \Leftrightarrow u_S < \overline{h}_S(u_B)$. The following facts are easily established: (i) $\overline{h}_S$ is strictly decreasing and strictly convex; (ii) $\overline{h}_S(0) = \sqrt{2+\gamma/2} > 0$; (iii) $\overline{h}_S(u_B) = u_B \Leftrightarrow u_B = u_S = 1/\sqrt{2} \Leftrightarrow \sigma = -1$. This implies that, in the (u_B, u_S)-space, and for $\sigma \geq 0$, $\overline{h}_S$ is strictly decreasing and above the 45°-degree line.

First Sufficient Condition. Therefore, if $a_I^{**} < 0$ (which amounts to $u_B > \underline{h}(u_S)$, with $\underline{h}$ below the 45°-degree line), then we also have $\sigma - 2u_S(2u_B+u_S)+4 > 0$ (which amounts to $u_S < \overline{h}_S(u_B)$, with $\overline{h}_S$ above the 45°-degree line). As a consequence, we obtain a first sufficient condition

$$a_I^{**} < 0 \Rightarrow \Delta V_S(a_I^{**}) > 0.$$

Second Sufficient Condition. Next, we want to show the existence of another sufficient condition

$$\sigma - 2u_S(2u_B+u_S)+4 < 0 \Rightarrow \Delta V_S(a_I^{**}) > 0.$$

Notice that $\sigma - 2u_S(2u_B+u_S)+4 < 0$ (which amounts to $u_S > \overline{h}_S(u_B)$ and thus implies $u_S > u_B$) implies $a_I^{**} > 0$ (because $a_I^{**} < 0$ amounts to $\underline{h}(u_S) < u_B$ and thus implies $u_S < u_B$). Therefore, $\sigma - 2u_S(2u_B+u_S)+4 < 0$ implies $\Delta V_S(a_I^{**}) > 0$.

It remains to show that the condition $\sigma - 2u_S(2u_B + u_S) + 4 < 0$ is compatible with the conditions for concavity. The condition $u_S > \overline{h}_S(u_B)$ can be equivalently expressed in terms of (u_S, σ) as $u_S > \tilde{u}_S(\sigma) \equiv \sqrt{(\gamma(\sigma+2) + 3\sigma + 4)/(2(\gamma+1))}$ with $\sigma \in [0, \gamma]$. Simple computations show then that $\tilde{u}_S^2(\sigma) \geq \underline{x}(\sigma)$ and $\tilde{u}_S^2(\sigma) \leq \overline{x}(\sigma)$ for all σ.

INTERMEDIATE REGION. Last, we establish that when $a_I^{**} > 0$ and $\sigma - 2u_S(2u_B + u_S) + 4 > 0$, $\Delta V_S(a_I^{**})$ is strictly negative. From the computations made to analyze the buyer surplus, $a_I^{**} > 0$ is equivalent to $u_S^2 > \underline{x}_a(\sigma)$. Simple computations show that $\sigma - 2u_S(2u_B + u_S) + 4 > 0$ amounts to $u_S^2 < \tilde{x}_a(\sigma) \equiv (\gamma(\sigma+2) + 3\sigma + 4)/(2(\gamma+1))$. Computations show that $\Delta V_S(a_I^{**}) = 0$ amounts to $h_2(x) \equiv (\sigma+2)(3\sigma+4)^2(\gamma-\sigma)(\gamma(\sigma+2) - \sigma(2\sigma+7) - 4) + 2(\gamma+1)^2(\sigma(\sigma+4)(\sigma(19\sigma+65) + 88) + 128)x^2 - (\gamma+1)(3\sigma+4)x(\gamma(\sigma+2)(3\sigma(3\sigma(\sigma+7)+46)+88) + \sigma(\sigma(\sigma(31\sigma+251)+720) + 880) + 384) = 0$, with $x = u_S^2$ and $x \in [\underline{x}_a, \tilde{x}_a]$. The previous expression is a strictly convex second degree polynomial in x with a strictly positive discriminant. Hence, it admits two reals roots. Computations show that $h_2(\underline{x}_a) < 0$ and $h_2(\tilde{x}_a) < 0$, which proves that $h_2(x) < 0$ for all $x \in [\underline{x}_a, \tilde{x}_a]$. Hence, if $a_I^{**} > 0$ and $\sigma - 2u_S(2u_B + u_S) + 4 > 0$, then $\Delta V_S(a_I^{**}) < 0$.

A.8. Pricing Policy under Vertical Integration with Efficiency Gains

We consider here the constrained outcome with efficiency gains in the running example. Let $\Delta = r_0 - r > 0$ denote the efficiency gain. The integrated platform's problem writes as follows

$$\begin{aligned} \max_{(\beta_I, a_I)} \quad & \pi_1^I(r_0, \beta_I r_0, a_I) \\ \text{s.t.} \quad & \pi_2^I(\beta_I r_0, r_0, a_I) \geq \pi_2^E(r, r_0, a_I) \end{aligned}$$

In the running example, the constraint $\pi_2^I(\beta_I r_0, r_0, a_I) \geq \pi_2^E(r, r_0, a_I)$ can be rewritten as $\beta_I \geq \overline{\beta}_I$ with

$$\overline{\beta}_I = 1 - \left(1 - \frac{r}{r_0}\right)\frac{8 + \sigma(8+\sigma)}{8(1+\sigma)}.$$

Let us then define: $\overline{a}_I = \arg\max_{a_I} \pi_1^I(r_0, \overline{\beta}_I r_0, a_I)$; the unconstrained outcome $(\beta_I^*, a_I^*) = \arg\max_{(\beta_I, a_I)} \pi_1^I(r_0, \beta_I r_0, a_I)$; $\overline{a}_I(r_0) = \arg\max_{a_I} \pi_1^I(r_0, r_0, a_I)$.

CONDITION FOR CONCAVITY. Up to the fact that the integrated firm's sharing parameter is now r_0, the maximization problem $\max_{(\beta_I, a_I)} \pi_1^I(r_0, \beta_I r_0, a_I)$ is the same as the one described in Section 4. Hence, the Hessian is negative definite under the same conditions as those stated in Appendix A.5.

CURVE $\beta_I^* = 1$. Up to the fact that the integrated firm's sharing parameter is now r_0, the solution of the relaxed problem is the same as in Section 4.1. The curve describing the set of parameters (u_B, u_S) such that $\beta_I^* = 1$ is therefore described by the parametric curve $(\hat{u}_B^{\beta_I^*=1}(\sigma), \hat{u}_S^{\beta_I^*=1}(\sigma))$ for $\sigma \in [0, \gamma]$ (see Appendix A.5).

CURVE $\beta_I^* = \overline{\beta}_I$. Computations show that $\beta_I^* - \overline{\beta}_I$ has the sign of $r(16(-8 + (3u_B + u_S)(u_B + 3u_S)) + 16(-12 + 5u_B^2 + 14u_B u_S + 5u_S^2)\sigma + 5(-20 + 9u_B^2 + 22u_B u_S + 9u_S^2)\sigma^2 + 9(-2 + (u_B + u_S)^2)\sigma^3)(8+\sigma(8+\sigma)) - 8v(1+\sigma)(4+3\sigma)(-16 - 3\sigma(4+\sigma) + u_S^2(12+\sigma(11+3\sigma)) + u_B u_S(20+\sigma(13+3\sigma))) + r_0(-512 + u_B^2(4+3\sigma)(8+\sigma(8+\sigma))(12+\sigma(11+3\sigma)) + \sigma(-1280 + u_S^2\sigma(4+3\sigma)(12+\sigma(11+3\sigma)) - 2\sigma(656+\sigma(340+\sigma(86+9\sigma)))) + 2u_B u_S(320+\sigma(768+\sigma(764+\sigma(384+\sigma(91+9\sigma))))))$. Expressing u_B as a function of σ, computations show that the previous expression has the sign of $-16vx(1+\gamma)(1+\sigma)(4+3\sigma)(-((4+3\sigma)(8+\sigma(5+\sigma))) - \gamma(12+\sigma(11+3\sigma)) + 2x(1+\gamma)(12+\sigma(11+3\sigma))) + r(8+\sigma(8+\sigma))(4x^2(1+\gamma)^2(4+3\sigma)(12+\sigma(11+3\sigma)) + (\gamma-\sigma)^2(4+3\sigma)(12+\sigma(11+3\sigma)) - 4x(1+\gamma)(128+\gamma(4+3\sigma)(12+\sigma(11+3\sigma)) + \sigma(272+\sigma(212+\sigma(73+9\sigma))))) + r_0(4x^2(1+\gamma)^2\sigma^2(4+3\sigma)(12+\sigma(11+3\sigma)) + (\gamma-\sigma)^2(4+3\sigma)(8+\sigma(8+\sigma))(12+\sigma(11+3\sigma)) - 4x(1+\gamma)(512+\gamma(2+\sigma)^2(4+3\sigma)(12+\sigma(11+3\sigma)) + \sigma(1600+\sigma(2080+\sigma(1444+\sigma(556+\sigma(109+9\sigma)))))))$, where $x = u_S^2$. The previous expression is a polynomial of degree 2 in x. Computations show that, if

$r_0 - r$ is not too large, this polynomial has two distinct real roots, one of which is positive and the other negative. To find out which one is positive, we compute their values when $r_0 = r$. Indeed, in this case, $\overline{\beta}_I = 1$ and thus the curves $\beta_I^* = \overline{\beta}_I$ and $\beta_I^* = 1$ are the same. The values of the two roots when $r_0 = r$ must thus be those found when studying the $\beta_I^* = 1$ curve. The computations are not reported here for the sake of brevity. Denote by $\hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma)$ the square root of the positive root of the polynomial and let $\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma) = (\gamma - \sigma)/(2(\gamma+1)\hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$. Then, the set of (u_B, u_S) such that $\beta_I^* = \overline{\beta}_I$ is described by the parametric curve associated to the parametric equations $(u_B = \hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma), u_S = \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$ for all $\sigma \in [0, \gamma]$.

The slope of the parametric curve $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma), \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$ when $r_0 = r$ is given by $-A/B$ where $A = (((4+3\sigma)(8+\sigma(5+\sigma)) + \gamma(12+\sigma(11+3\sigma)))(176+\sigma(264+\sigma(185+66\sigma+9\sigma^2)))$ and $B = (12+\sigma(11+3\sigma))(768+\gamma(464+\sigma(792+\sigma(571+9\sigma(22+3\sigma)))) + \sigma(1584+\sigma(1352+\sigma(569+3\sigma(38+3\sigma))))))$. Since both A and B are positive, the slope of the parametric curve is negative when $r_0 = r$. By continuity, this shows that, if $r_0 - r$ is not too large, the parametric curve $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma), \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$ is downward sloping in the (u_B, u_S)-space. Simple computations also show that: $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(0))^2 = (3(r+v)\gamma^2)/((1+\gamma)(2r(8+3\gamma) - (r_0 - v)(8+3\gamma) + (64(2r - r_0 + v)^2 + 48(2r - r_0 + v)^2\gamma + 9(r_0+v)^2\gamma^2)^{1/2})$; $(\hat{u}_S^{\beta_I^*=\overline{\beta}_I}(0))^2 = (2r(8+3\gamma) - (r_0 - v)(8+3\gamma) + (64(2r - r_0 + v)^2 + 48(2r - r_0 + v)^2\gamma + 9(r_0+v)^2\gamma^2)^{1/2})/(12(r+v)(1+\gamma)))$; $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\gamma))^2 = 0$; and $(\hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\gamma))^2 = ((8(r+v)(1+\gamma)(4+3\gamma)(16+3\gamma(4+\gamma)) - 2(256+\gamma(640+\gamma(656+\gamma(340+\gamma(86+9\gamma)))))\Delta))/((4+3\gamma)(12+\gamma(11+3\gamma))(8(r+v)(1+\gamma) - \gamma^2\Delta)))$, where $\Delta = r_0 - r$.

BINDING CONSTRAINTS. Let us first notice that the two parametric curves $(\hat{u}_B^{\beta_I^*=1}(\sigma), \hat{u}_S^{\beta_I^*=1}(\sigma))$ and $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma), \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$, for $\sigma \in [0, \gamma]$, cannot cross each other when $r_0 > r$, for otherwise we would have $\overline{\beta}_I = 1$, which is impossible. Computations then show that $\hat{u}_S^{\beta_I^*=1}(\gamma) > \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\gamma)$. Since $\hat{u}_B^{\beta_I^*=1}(0) = \hat{u}_B^{\beta_I^*=\overline{\beta}_I}(0) = 0$, this implies that the parametric curve $(\hat{u}_B^{\beta_I^*=1}(\sigma), \hat{u}_S^{\beta_I^*=1}(\sigma))$ is above the parametric curve $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma), \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$ in the (u_B, u_S)-space.

To conclude, we have the following in the (u_B, u_S)-space: the constraint $\beta_I \leq 1$ is binding at optimum above the parametric curve $(\hat{u}_B^{\beta_I^*=1}(\sigma), \hat{u}_S^{\beta_I^*=1}(\sigma))$ for $\sigma \in [0, \gamma]$; none of the constraints $\beta_I \leq 1$ and $\beta_I \geq \overline{\beta}_I$ are binding at the optimum between the two parametric curves; the constraint $\beta_I \geq \overline{\beta}_I$ is binding at the optimum below the parametric curve $(\hat{u}_B^{\beta_I^*=\overline{\beta}_I}(\sigma), \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\sigma))$ for $\sigma \in [0, \gamma]$.

CURVE $\overline{a}_I = 0$. Straightforward but tedious computations show that $\overline{a}_I \geq 0$ amounts to $8r(4+3\sigma)(-2u_B(2+\sigma)(4+3\sigma) + u_S(32+3\sigma(16+\sigma(7+\sigma)))) + u_S(-8v(4+3\sigma)(32+3\sigma(16+\sigma(7+\sigma))) + \Delta\,(-512+\sigma(4+\sigma)(-256+\sigma(4+\sigma)(-20\ +9\sigma)))) + u_B(4+3\sigma)(16v(2+\sigma)(4+3\sigma) + \Delta(256+\sigma(384+\sigma\ (184+3\sigma(12+\sigma))))) \geq 0$. Let us now work with (σ, u_S) by using $u_B = (\gamma - \sigma)/(2u_S(1+\gamma))$. Introducing also $x = u_S^2$, $\overline{a}_I \geq 0$ amounts to $16r(4+3\sigma)(-((\gamma - \sigma)(2+\sigma)(4+3\sigma)) + x(1+\gamma)(32+3\sigma(16+\sigma(7+\sigma)))) - 16v(4+3\sigma)(-((\gamma-\sigma)(2+\sigma)(4+3\sigma)) + x(1+\gamma)(32+3\sigma(16+\sigma(7+\sigma)))) + \Delta\ (2x(1+\gamma)(-512+\sigma(4+\sigma)(-256+\sigma\ (4+\sigma)(-20+9\sigma))) + (\gamma - \sigma)(4+3\sigma)(256+\sigma(384+\sigma(184+3\sigma(12+\sigma))))) \geq 0$. The left-hand side is linear and decreasing in x. It is therefore positive when $x < \tilde{x}(\sigma) = (((\gamma-\sigma)(4+3\sigma)(-16r(2+\sigma)(4+3\sigma) + 16v(2+\sigma)(4+3\sigma) + \Delta(256+\sigma(384+\ \sigma(184+3\sigma(12+\sigma))))))/(-2(1+\gamma)(8\ r(4+3\sigma)(32+3\sigma(16+\sigma(7+\sigma))) + 8v(4+3\sigma)(32+3\sigma(16+\sigma(7+\sigma))) -\ \Delta(-512+\sigma(4+\sigma)(-256+\sigma(4+\ \sigma)(-20+9\sigma))))))$.

Let $\hat{u}_S^{\overline{a}_I=0}(\sigma) = \sqrt{\tilde{x}(\sigma)}$ and $\hat{u}_B^{\overline{a}_I=0}(\sigma) = (\gamma - \sigma)/(2(1+\gamma)\sqrt{\tilde{x}(\sigma)})$. By construction, $\overline{a}_I > 0$ above the parametric curve $(\hat{u}_B^{\overline{a}_I=0}(\sigma), \hat{u}_S^{\overline{a}_I=0}(\sigma))$, $\sigma \in [0, \gamma]$, in the (u_B, u_S)-space. Computations then show that the slope of the parametric curve is positive when Δ goes to 0 and that the parametric curve is below the 45°-degree line. By continuity, the slope will remain strictly positive and the parametric curve below the 45°-degree line for all σ when Δ is sufficiently close to 0.

CURVE $\beta_I^* r_0 = r$. Following the same reasoning, one can show that there exists a downward sloping parametric curve $(u_B^{\beta_I^* r_0=r}(\sigma), u_S^{\beta_I^* r_0=r}(\sigma))$, $\sigma \in [0,\gamma]$, such that, in the (u_B, u_S)-space, $\beta_I^* r_0 > r$ above this curve and $\beta_I^* r_0 < r$ below this curve.

A.9. *Pricing Policy under Vertical Integration with Coordination Motives*

We consider the situation with coordination motives studied in Section 6. First, it is straightforward to show that the Hessian matrix associated to the unconstrained problem does not depend on α_B. Hence, the Hessian is negative definite under the same conditions as those stated in Appendix A.5.

The optimal pricing policy associated to the unconstrained problem is as follows: β_I^* and a_I^* are given by (A.22) and (A.23) respectively in which $v+r$ is replaced by $v+r+\alpha_B$. Therefore, the curves $a_I^* = 0$ and $\beta_I^* = 1$ are the same as in Appendix A.5.

The constraint $\tilde{\pi}_2^I(\beta_I, 1, a_I) \geq \pi_2^E(1, 1, a_I)$ writes as follows

$$\beta_I \geq \tilde{\beta}_I \equiv 1 - \alpha_B \frac{(3\sigma+4)}{4r(\sigma+1)}. \tag{A.25}$$

Let us then define $\tilde{a}_I = \arg\max_{a_I} \tilde{\pi}_1^I(1, \tilde{\beta}_I, a_I)$ and $\tilde{a}_I(1) = \arg\max_{a_I} \tilde{\pi}_1^I(1, 1, a_I)$.

Straightforward but tedious computations show that $\beta_I^* \geq \tilde{\beta}_I$ amounts to $3\sigma^3(4(r+v)(u_S(u_B+u_S)-1) + \alpha_B(-3u_B^2 - 2u_Bu_S + u_S^2 + 2)) + \sigma^2(4(r+v)(2u_S(8u_B + 7u_S) - 15) + \alpha_B(-45u_B^2 - 46u_Bu_S + 11u_S^2 + 40)) + 4\sigma((r+v)(33u_Bu_S + 23u_S^2 - 28) + \alpha_B(-20u_B^2 - 23u_Bu_S + 3u_S^2 + 20)) + 16(r+v)(5u_Bu_S + 3u_S^2 - 4) - 16\alpha_B(3u_B^2 + 5u_Bu_S - 4) \geq 0$.

Let us now work with (σ, u_S) by using $u_B = (\gamma-\sigma)/(2u_S(1+\gamma))$. Introducing also $x = u_S^2$, $\beta_I^* \geq \tilde{\beta}_I$ amounts to $8(\gamma+1)r(\sigma+1)x(-\gamma(\sigma(3\sigma+11)+12) - ((3\sigma+4)(\sigma(\sigma+5)+8)) + 2(\gamma+1)(\sigma(3\sigma+11)+12)x) + 8(\gamma+1)(\sigma+1)vx(-\gamma(\sigma(3\sigma+11)+12) - ((3\sigma+4)(\sigma(\sigma+5)+8)) + 2(\gamma+1)(\sigma(3\sigma+11)+12)x) + \alpha_B(-(3\sigma+4)(\sigma(3\sigma+11)+12)(\gamma-\sigma)^2 + 4(\gamma+1)^2\sigma(\sigma(3\sigma+11)+12)x^2 + 4(\gamma+1)x(\gamma(\sigma+2)(\sigma(3\sigma+11)+12) + \sigma(\sigma+6)(\sigma(3\sigma+11)+20)+64)) \geq 0$. The left-hand side is a strictly convex second order polynomial in x, which takes a strictly negative value at $x=0$. That polynomial admits two roots $\overline{x}(\sigma)$ and $\underline{x}(\sigma)$ with $\underline{x}(\sigma) < 0 < \overline{x}(\sigma)$. Therefore, $\beta_I^* \geq \tilde{\beta}_I$ if and only if $x \geq \overline{x}(\sigma)$. Using $\alpha_B = \alpha(v+r)$ to simplify further, we have $\overline{x}(\sigma) = A/B$ with $A = \sigma^3(-\alpha(3\gamma+29)+6\gamma+44) + \sigma^2(-17\alpha\gamma - 86\alpha + 28\gamma + 126) + 2\sigma(-\alpha(17\gamma+60)+23\gamma+76) + 8(1-\alpha)(3\gamma+8) + 3(2-\alpha)\sigma^4 + [\alpha(3\sigma+4)(\sigma(3\sigma+11)+12)^2((\alpha+4)\sigma+4)(\gamma-\sigma)^2 + (\alpha(\gamma(\sigma+2)(\sigma(3\sigma+11)+12) + \sigma(\sigma+6)(\sigma(3\sigma+11)+20)+64) - 2(\sigma+1)(\gamma(\sigma(3\sigma+11)+12) + (3\sigma+4)(\sigma(\sigma+5)+8)))^2]^{1/2}$ and $B = 2(\gamma+1)(\sigma(3\sigma+11)+12)((\alpha+4)\sigma+4)$.

Consider the parametric curve defined as follows: $(\hat{u}_B^{\beta_I^*=\tilde{\beta}_I} = (\gamma-\sigma)/(2(1+\gamma)\sqrt{\overline{x}(\sigma)}), \hat{u}_S^{\beta_I^*=\tilde{\beta}_I} = \sqrt{\overline{x}(\sigma)})$ for $\sigma \in [0,\gamma]$. Computations show that $\hat{u}_B^{\beta_I^*=\tilde{\beta}_I}(\gamma) = 0 < \hat{u}_S^{\beta_I^*=\tilde{\beta}_I}(\gamma)$ and that $0 < \hat{u}_B^{\beta_I^*=\tilde{\beta}_I}(0) < \hat{u}_S^{\beta_I^*=\tilde{\beta}_I}(0) \Leftrightarrow \alpha \leq 1$. The slope of the parametric curve for $\alpha = 0$ is given by (A.24). By continuity, it remains strictly negative for all σ when α is sufficiently close to 0.

Last, straightforward but tedious computations show that $\tilde{a}_I \geq 0$ amounts to $(8ru_B(2+\sigma)(4+3\sigma) + u_B(4+3\sigma)(8v(2+\sigma) + \alpha_B(4+\sigma)(8+3\sigma)) - 4ru_S(32+3\sigma(16+\sigma(7+\sigma))) + u_S(-\alpha_B(4+\sigma)(16+\sigma(20+3\sigma)) - 4v(32+3\sigma(16+\sigma(7+\sigma))))) \leq 0$. Let us now work with (σ, u_S) by using $u_B = (\gamma-\sigma)/(2u_S(1+\gamma))$. Introducing also $x = u_S^2$, $\tilde{a}_I \geq 0$ amounts to $\alpha_B(4+\sigma)(-((\gamma-\sigma)(4+3\sigma)(8+3\sigma)) + 2x(1+\gamma)(16+\sigma(20+3\sigma))) + 8r(-((\gamma-\sigma)(2+\sigma)(4+3\sigma)) + x(1+\gamma)(32+3\sigma(16+\sigma(7+\sigma)))) + 8v(-((\gamma-\sigma)(2+\sigma)(4+3\sigma)) + x(1+\gamma)(32+3\sigma(16+\sigma(7+\sigma)))) \leq 0$. The left-hand side is linear and increasing in x. It is therefore positive when $x > \tilde{x}(\sigma) = ((\gamma-\sigma)(4+3\sigma)(8r(2+\sigma) + 8v(2+\sigma) + \alpha_B(4+\sigma)(8+3\sigma)))/(2(1+\gamma)(64(2(r+v)+\alpha_B) + 96(2(r+v)+\alpha_B)\sigma + 4(21(r+v)+8\alpha_B)\sigma^2 + 3(4(r+v)+\alpha_B)\sigma^3))$.

Let $\hat{u}_S^{\tilde{a}_I=0}(\sigma) = \sqrt{\tilde{x}(\sigma)}$ and $\hat{u}_B^{\tilde{a}_I=0}(\sigma) = (\gamma-\sigma)/(2(1+\gamma)\sqrt{\tilde{x}(\sigma)})$. By construction, $\tilde{a}_I > 0$ above the parametric curve $(\hat{u}_B^{\tilde{a}_I=0}(\sigma), \hat{u}_S^{\tilde{a}_I=0}(\sigma))$, $\sigma \in [0,\gamma]$, in the (u_B, u_S)-space. Computations then show that the slope of the parametric curve when α_B goes to 0 is $(2(2+\sigma)(4+3\sigma)(256+\sigma(640+3\sigma(200+\sigma(80+11\sigma)))+\gamma(64+3\sigma(48+\sigma(46+\sigma(20+3\sigma))))))/((32+3\sigma(16+\sigma(7+\sigma)))((8+3\sigma(4+\sigma))(32+3\sigma(16+\sigma(9+2\sigma)))-\gamma(64+3\sigma(48+\sigma(46+\sigma(20+3\sigma))))))$, which is positive. Moreover, when α_B goes to 0, we have $\hat{u}_B^{\tilde{a}_I=0}(\sigma)^2 - \hat{u}_S^{\tilde{a}_I=0}(\sigma)^2 = ((\gamma-\sigma)(1+\sigma)(16+3\sigma(4+\sigma))(48+\sigma(68+3\sigma(9+\sigma))))/(4(1+\gamma)(2+\sigma)(4+3\sigma)(32+3\sigma(16+\sigma(7+\sigma))))$, which is nonnegative since $\gamma \geq \sigma$. This thus shows that the parametric curve is below the 45°-degree line when $\alpha_B = 0$. By continuity, the slope will remain strictly positive and the parametric curve below the 45°-degree line for all σ when α_B is sufficiently close to 0.

Figure 16 summarizes the main features of the optimal pricing policy with coordination motives.

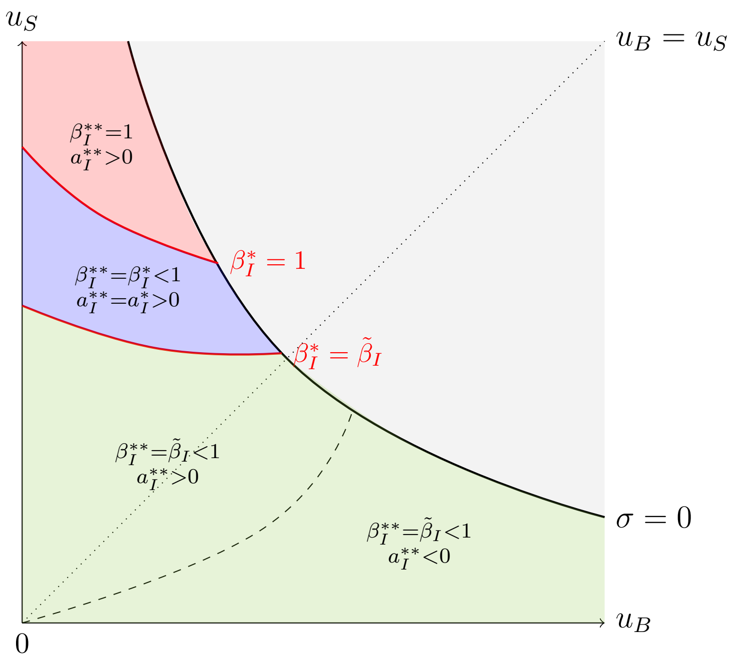


Figure 16: The integrated platform's optimal pricing policy (β_I^{**}, a_I^{**}) in the running example with coordination motives.

B. Online Appendix (Not For Publication)

B.1. Impact of Vertical Integration with Efficiency Gains: Polar Cases

Let $\Delta = r_0 - r > 0$ denote the efficiency gain. Remind that the integrated platform's problem writes as follows

$$\begin{aligned} \max_{(\beta_I, a_I)} \quad & \pi_1^I(r_0, \beta_I r_0, a_I) \\ \text{s.t.} \quad & \pi_2^I(\beta_I r_0, r_0, a_I) \geq \pi_2^E(r, r_0, a_I) \\ & 0 \leq \beta_I \leq 1. \end{aligned}$$

In the running example, the constraint $\pi_2^I(\beta_I r_0, r_0, a_I) \geq \pi_2^E(r, r_0, a_I)$ can be rewritten as $\beta_I \geq \overline{\beta}_I$ with

$$\overline{\beta}_I = 1 - \left(1 - \frac{r}{r_0}\right)\frac{8 + \sigma(8+\sigma)}{8(1+\sigma)}.$$

Let us then define: $\overline{a}_I = \arg\max_{a_I} \pi_1^I(r_0, \overline{\beta}_I r_0, a_I)$; $(\beta_I^*, a_I^*) = \arg\max_{(\beta_I, a_I)} \pi_1^I(r_0, \beta_I r_0, a_I)$; $\overline{a}_I(r_0) = \arg\max_{a_I} \pi_1^I(r_0, r_0, a_I)$.

Proof of Proposition 4 (Buyer-skewed Network Effects, $u_S = 0$). For later reference, note that when $u_S = 0$ the integrated platform sets $\beta_I = \overline{\beta}_I$ and $a_I = \overline{a}_I$. Simple computations show that

$$\overline{a}_I = -\frac{u_B\,(16(v+r)(2+\gamma)(4+3\gamma) + (16+3\gamma(4+\gamma))(8+\gamma(8+\gamma))\Delta)}{8(4+3\gamma)((4+\gamma)^2 - 2u_B^2(2+\gamma))}.$$

Moreover, when $u_S = 0$, a necessary condition to ensure that the maximization problem $\max_{(\beta_I, a_I)} \pi_1^I(r_0, \beta_I r_0, a_I)$ is concave is $u_B \leq \bar{u}_B = \sqrt{\frac{2(2+\gamma)(32+\gamma(32+9\gamma))}{(4+3\gamma)(12+\gamma(11+3\gamma))}}$.

Impact on Manufacturer M_2. Let $\pi_2^I(\beta_I r_0, r_0, a_I)$ be M_2's profit under vertical integration when the sharing parameter is equal to β_I and the developer fee is set at some value a_I. Let π_2^S be M_2's profit under separation. Let $\Delta\pi_2(\beta_I, a_I) = \pi_2^I(\beta_I r_0, r_0, a_I) - \pi_2^S$. Computations show that

$$\text{(B.1)} \quad \Delta\pi_2(\beta_I, a_I) = \frac{2(2+\gamma)[v(4+3\gamma) - r_0(\gamma - 4\beta_I(1+\gamma)) - a_I u_B(4+3\gamma)]^2}{(4+\gamma)^2(4+3\gamma)^2} - \frac{2(2+\gamma)(v+r)^2}{(4+\gamma)^2}.$$

One can show that the term in brackets in the right-hand side of (B.1) is nonnegative, for otherwise M_2's markup is negative under integration. Therefore, $\Delta\pi_2(\beta_I, a_I)$ has the same sign as

$$f(\beta_I, a_I) = \frac{(v(4+3\gamma) - r_0(\gamma - 4\beta_I(1+\gamma)) - a_I u_B(4+3\gamma))}{(4+3\gamma)} - (v+r).$$

Then computations show that

$$\text{(B.2)} \quad f(\overline{\beta}_I, \overline{a}_I) = \frac{16(v+r)u_B^2(2+\gamma)(4+3\gamma) - (4+\gamma)\left[4\gamma(2+\gamma)(4+\gamma) - u_B^2(8+3\gamma)(8+\gamma(8+\gamma))\right]\Delta}{8(4+3\gamma)((4+\gamma)^2 - 2(2+\gamma)u_B^2)}.$$

For all $u_B \leq \bar{u}_B$, computations show that the denominator in (B.2) is positive. Therefore $f(\beta_I, \overline{a}_I)$ has the same sign as its numerator. Then, simple computations show that the numerator of $f(\beta_I, \overline{a}_I)$ is increasing in u_B, is negative when $u_B = 0$ and positive when $u_B = \bar{u}_B$. It follows that there exists a unique $\hat{u}_B \in (0, \overline{u}_B)$ such that $f(\overline{\beta}_I, \overline{a}_I)$, and thus $\Delta\pi_2(\overline{\beta}_I, \overline{a}_I)$ as well, is negative if and only if $u_B < \hat{u}_B$.

Impact on Buyers. Let $V_B^I(\beta_I r_0, a_I)$ be the buyer surplus under vertical integration when the sharing parameter is equal to β_I and the developer fee is set at some value a_I. Let V_B^S be the buyer surplus under separation. Let $\Delta V_B(\beta_I r_0, a_I) = V_B^I(\beta_I r_0, a_I) - V_B^S$.

Our first step is to show that $V_B^I(\overline{\beta}_I r_0, \overline{a}_I)$ is increasing in u_B. Omitting notations, we have

$$\text{(B.3)} \qquad \frac{dV_B^I}{du_B}(\overline{\beta}_I r_0, \overline{a}_I) = \frac{\partial V_B}{\partial u_B} + \frac{\partial V_B}{\partial \beta_I}\frac{d\overline{\beta}_I}{du_B} + \frac{\partial V_B}{\partial a_I}\frac{d\overline{a}_I}{du_B}.$$

Computations show that the first term in Equation (B.3) is positive, the second term is nil since $d\overline{\beta}_I/du_B = 0$ and the third term is positive. This shows that $V_B^I(\overline{\beta}_I r_0, \overline{a}_I)$ is increasing in u_B, and so is $\Delta V_B(\overline{\beta}_I r_0, \overline{a}_I)$ consequently.

Then, we notice that $\Delta V_B(\overline{\beta}_I r_0, \overline{a}_I)$ is positive when $u_B = \overline{u}_B$. Therefore, there exists $\hat{u}_B \leq 0$ such that $\Delta V_B(\overline{\beta}_I r_0, \overline{a}_I)$ is nonnegative if and only if $u_B \geq \hat{u}_B$.[40]

Impact on Developers. When $u_S = 0$, the developers surplus is simply given by $V_S^I(\beta_I r_0, a_I) = a_I^2/2$ and therefore $\Delta V_S(\overline{\beta}_I r_0, \overline{a}_I) \geq 0$.

PROOF OF PROPOSITION 5 (DEVELOPER-SKEWED NETWORK EFFECTS, $u_B = 0$). When $u_B = 0$, a necessary condition to ensure that the maximization problem $\max_{(\beta_I, a_I)} \pi_1^I(r_0, \beta_I r_0, a_I)$ is concave is $u_S \leq \bar{u}_S = \sqrt{\frac{2(2+\gamma)(32+\gamma(32+9\gamma))}{(4+3\gamma)(12+\gamma(11+3\gamma))}}$.

We know from Section 5.2 that, first, M_2's participation constraint is binding when $u_S \leq \hat{u}_S^{\beta_I^* = \overline{\beta}_I}(\gamma)$ and, second, that the constraint $\beta_I \leq 1$ is binding when $u_S \geq \hat{u}_S^{\beta_I^*=1}(\gamma)$. We also know that $\hat{u}_S^{\beta_I^* = \overline{\beta}_I}(\gamma) < \hat{u}_S^{\beta_I^*=1}(\gamma)$. When $u_B = 0$, the integrated platform's optimal pricing policy is therefore given by

$$(\beta_I^I, a_I^I) = \begin{cases} (\overline{\beta}_I, \overline{a}_I) & \text{if } u_S \leq \hat{u}_S^{\beta_I^* = \overline{\beta}_I}(\gamma), \\ (\beta_I^*, a_I^*) & \text{if } \hat{u}_S^{\beta_I^* = \overline{\beta}_I}(\gamma) < u_S \leq \hat{u}_S^{\beta_I^*=1}(\gamma), \\ (1, \overline{a}_I(r_0)) & \text{if } u_S > \hat{u}_S^{\beta_I^*=1}(\gamma), \end{cases}$$

where

$$\overline{\beta}_I = \frac{8r(1+\gamma) - \gamma^2\Delta}{(8(1+\gamma)\ (r+\Delta)},$$

$$\overline{a}_I = \frac{8u_S(r+v)(4+3\gamma)(32+3\gamma(16+\gamma(7+\gamma))) - u_S(-512+\gamma(4+\gamma)(-256+\gamma(4+\gamma)\ (-20+9\gamma)))\Delta}{8(4+\gamma)^2(4+3\gamma)^2 - 16u_S^2(32+\gamma(64+\gamma(40+7\gamma)))},$$

$$\beta_I^* = \frac{v(4+3\gamma)(-16-3\gamma(4+\gamma)+u_S^2(12+\gamma(11+3\gamma))) + r(64+\gamma(96+\gamma(52+9\gamma))) + (64+\gamma(96+\gamma(52+9\gamma)))\Delta}{2(2+\gamma)(32+\ \gamma(32+9\gamma)) - u_S^2(4+3\gamma)(12+\gamma(11+3\gamma)))(r+\Delta)}$$

$$a_I^* = \frac{u_S(4+3\gamma)(12+\gamma(11+3\gamma))(r+v+\Delta)}{2(2+\gamma)(32+\gamma(32+9\gamma)) - u_S^2(4+3\gamma)(12+\gamma(11+3\gamma))},$$

$$\overline{a}_I(r_0) = \frac{u_S(4+3\gamma)(32+3\gamma(16+\gamma(7+\gamma)))(r+v+\Delta)}{(4+\gamma)^2(4+3\gamma)^2 - 2u_S^2(32+\gamma(64+\gamma(40+7\gamma)))}.$$

Impact on Manufacturer M_2. Let $\pi_2^I(\beta_I r_0, r_0, a_I)$ be M_2's profit under vertical integration when the sharing parameter is equal to β_I and the developer fee is set at some value a_I. Let π_2^S be M_2's profit under separation. Let $\Delta\pi_2(\beta_I, a_I) = \pi_2^I(\beta_I r_0, r_0, a_I) - \pi_2^S$. We have

$$\pi_2^I(\beta_I r_0, r_0, a_I) = \frac{2(2+\gamma)(v(4+3\gamma) - a_I u_S \gamma + r_0(4(1+\gamma)\beta_I - \gamma))^2}{(4+\gamma)^2(4+3\gamma)^2}$$

[40] One can additionally find conditions under which $\hat{u}_B$ is positive. If this is the case, $\Delta V_B(\overline{\beta}_I r_0, \overline{a}_I)$ is negative in the neighborhood of $u_B = 0$.

and

$$\pi_2^S = \frac{2(r+v)^2(2+\gamma)}{(4+\gamma)^2}.$$

Tedious calculations then show that: (i) $\pi_2^I(\beta_I^I r_0, r_0, a_I^I)$ is decreasing in u_S when $u_S \le \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\gamma)$; (ii) $\pi_2^I(\beta_I^I r_0, r_0, a_I^I)$ is increasing in u_S when $\hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\gamma) < u_S \le \hat{u}_S^{\beta_I^*=1}(\gamma)$; (iii) $\pi_2^I(\beta_I^I r_0, r_0, a_I^I)$ is decreasing in u_S when $u_S > \hat{u}_S^{\beta_I^*=1}(\gamma)$; (iv) $\Delta\pi_2(\beta_I^I, a_I^I) < 0$ when $u_S = 0$. Together, since $\Delta\pi_2(\beta_I^I, a_I^I)$ is continuous in u_S, these observations show that $\Delta\pi_2(\beta_I^I, a_I^I)$ is positive for some parameters values only if $\Delta\pi_2(\beta_I^I, a_I^I) > 0$ when $u_S = \hat{u}_S^{\beta_I^*=1}(\gamma)$. Put differently, there exists an interval $[\underline{u}_S^{M_2}, \bar{u}_S^{M_2}]$ such that: (i) if $\Delta\pi_2(\beta_I^I, a_I^I) > 0$ when $u_S = \hat{u}_S^{\beta_I^*=1}(\gamma)$, then, $\Delta\pi_2(\beta_I^I, a_I^I) > 0$ for all $u_S \in (\underline{u}_S^{M2}, \bar{u}_S^{M2})$, (ii) $\underline{u}_S^{M_2} \le \hat{u}_S^{\beta_I^*=1}(\gamma) \le \bar{u}_S^{M_2}$, (iii) if $\Delta\pi_2(\beta_I^I, a_I^I) \le 0$ when $u_S = \hat{u}_S^{\beta_I^*=1}(\gamma)$, then, $\underline{u}_S^{M_2} = \hat{u}_S^{\beta_I^*=1}(\gamma) = \bar{u}_S^{M_2}$.

Impact on Buyers. When $u_B = 0$, calculations show that the buyer surplus under integration is given by

$$\text{(B.4)}\quad V_B^I(\beta_I r_0, a_I) = \frac{1}{(4+\gamma)^2(4+3\gamma)^2}(2+\gamma)(2a_I u_S v(4+3\gamma)^2 + v^2(2+\gamma)(4+3\gamma)^2 + a_I^2 u_S^2(16+\gamma(16+\gamma)) + r_0^2(16+16\beta_I\gamma(1+\gamma)+\gamma(16+\gamma)+\beta_I^2(1+\gamma)(16+\gamma(16+9\gamma))) + 2r_0(v(4+3\gamma)^2(1+\beta_I+\beta_I\gamma) + a_I u_S(16+\gamma(16+\gamma+8\beta_I(1+\gamma))))).$$

The buyer surplus under separation is given by

$$V_B^S = \frac{(v+r)^2(2+\gamma)^2}{(4+\gamma)^2}.$$

Tedious calculations then show that: (i) $\Delta V_B^I(\overline{\beta}_I r_0, \overline{a}_I)$ is increasing in u_S, (ii) $\Delta V_B^I(\overline{\beta}_I r_0, \overline{a}_I)$ is positive when $u_S = \bar{u}_S$, and (iii) that the sign of $\Delta V_B^I(\overline{\beta}_I r_0, \overline{a}_I)$ depends on the values of γ and Δ when $u_S = 0$. Indeed, when $u_S = 0$, we have

$$\Delta V_B^I(\overline{\beta}_I r_0, \overline{a}_I) = \frac{2+\gamma}{(64(1+\gamma)(4+\gamma)^2(4+3\gamma)^2)}(64(r+v)^2(1+\gamma)(2+\gamma)(4+3\gamma)^2 - 16(r+v)(1+\gamma)(4+3\gamma)^2(-8+\gamma^2)\Delta + (1024+\gamma(2048+\gamma(1088+\gamma(-64+\gamma(-112+\gamma(16+9\gamma))))))\Delta^2)).$$

The previous equation shows that, for instance, when Δ is small $\Delta V_B^I(\overline{\beta}_I r_0, \overline{a}_I)$ is positive.

From the three observations above, we conclude that there exists a cutoff $\tilde{u}_S^B$ such that $\Delta V_B(\beta_I^I r_0, a_I^I)$ is nonnegative if and only if $u_S \ge \tilde{u}_S^B$.

Impact on Developers. When $u_B = 0$, calculations show that the developer surplus under integration is given by

$$V_S^I(\beta_I r_0, a_I) = \frac{(2u_S(v(2+\gamma)+r_0(1+\beta_I(1+\gamma))) - a_I(4+\gamma-2u_S^2))^2}{2(4+\gamma)^2}$$

and the developer surplus under separation is given by

$$V_S^S = \frac{2u_S^2(2+\gamma)^2(v+r)^2}{(4+\gamma)^2}.$$

As is standard in models with linear demands, $V_S^I(\beta_I r_0, a_I)$ is proportional to the square of the developers demand. It follows that $\Delta V_S(\beta_I r_0, a_I) = V_S^I(\beta_I r_0, a_I) - V_S^S$ has the sign of $(2u_S(v(2+\gamma)+r_0(1+\beta_I(1+\gamma))) - a_I(4+\gamma-2u_S^2)) - 2u_S(2+\gamma)(v+r)$.

Tedious calculations then show that: (i) $\Delta V_S(\beta_I^I r_0, a_I^I)$ is negative when $u_S \leq \hat{u}_S^{\beta_I^*=\overline{\beta}_I}(\gamma)$, (ii) $\Delta V_S(\beta_I^I r_0, a_I^I)$ is increasing in u_S when $u_S \geq \hat{u}_S^{\beta_I^*=1}(\gamma)$, and (iii) that the sign of $\Delta V_S^I(r_0, \overline{a}_I(r_0))$ depends on the values of γ and Δ when $u_S = \overline{u}_S$. Indeed, when $u_S = \overline{u}_S$, simple calculations show that $\Delta V_S^I(r_0, \overline{a}_I(r_0))$ has the sign of

$$\text{(B.5)} \quad -(r+v)(4+3\gamma)(32+3\gamma(16+\gamma(7+\gamma)))(64-\gamma(16-3\gamma(20+3\gamma(5+\gamma)))) \\ +(4+\gamma)(6144+\gamma(22016+\gamma(33728+\gamma(29008+\gamma(15316+\gamma(5021+9\gamma(106+9\gamma)))))))\Delta,$$

which can be positive or negative depending on the values of γ and the efficiency gain.

From the observations above, we conclude that there exists a cutoff $\tilde{u}_S^S$ such that $\Delta V_S(\beta_I^I r_0, a_I^I)$ is nonnegative if and only if $u_S \geq \tilde{u}_S^S$.

B.2. Impact of Vertical Integration with Coordination Motives: Polar Cases

PROOF OF PROPOSITION 6. As shown in Appendix A.9, when $u_S = 0$, $\beta_I^* = \tilde{\beta}_I$ and $a_I^* = \tilde{a}_I$. Moreover, when $u_S = 0$, a necessary condition to ensure that the maximization problem of platform I is concave is $u_B^2 \leq \bar{u}_B^2 = 2/3(3+4/(4+3\gamma)+\gamma/(12+\gamma(11+3\gamma)))$.

Impact on Manufacturer M_2. Let $\Delta\pi_2(\beta_I, a_I) = \tilde{\pi}_2^I(\beta_I, 1, a_I) - \pi_2^S$. Simple computations show that $\Delta\pi_2(\beta_I, a_I)$ has the sign of $3(v+\alpha_B)(1+\gamma)+r(-1+4\beta_I)(1+\gamma)-a_I u_B(4+3\gamma)$. Evaluating this expression at $a_I = \tilde{a}_I$ and $\beta_I = \tilde{\beta}_I$ and differentiating with respect to u_B, we find that $d\Delta\pi_2(\tilde{\beta}_I, \tilde{a}_I)/du_B$ has the sign of $(u_B(4+\gamma)^2(4+3\gamma)(8r(2+\gamma)+8v(2+\gamma)+\alpha_B(4+\gamma)(8+3\gamma)))/(2((4+\gamma)^2)^2 - 2u_B^2(2+\gamma))$, which is positive. Put differently, $\Delta\pi_2(\tilde{\beta}_I, \tilde{a}_I)$ is increasing in u_B. Then, computations show that, when $\alpha_B > 0$, $\Delta\pi_2(\tilde{\beta}_I, \tilde{a}_I)$ evaluated at $u_B = 0$ and at $u_B = \bar{u}_B$ is negative and positive respectively. Therefore, by continuity, there exists $\hat{u}_B^{\Delta\pi_2=0} > 0$ such that $\Delta\pi_2(\tilde{\beta}_I, \tilde{a}_I)$ is negative if and only $u_B < \hat{u}_B^{\Delta\pi_2=0}$.

Impact on Buyers. Let V_B^S and $V_B^I(\tilde{\beta}_I, \tilde{a}_I)$ denote respectively the buyers surplus under separation and integration. Let $\Delta V_B^I(\tilde{\beta}_I, \tilde{a}_I) = V_B^I(\tilde{\beta}_I, \tilde{a}_I) - V_B^S$. Computations show that $d\Delta V_B^I(\tilde{\beta}_I, \tilde{a}_I)/du_B$ is equal to $(u_B(2+\gamma)(4+\gamma)(8r(2+\gamma)+8v(2+\gamma)+\alpha_B(4+\gamma)(8+3\gamma))(4r(2+\gamma)(4+\gamma)+4v(2+\gamma)(4+\gamma)+\alpha_B(3u_B^2(2+\gamma)^2+(4+\gamma)^2)))/(4(-2u_B^2(2+\gamma)+(4+\gamma)^2)^3)$. Computations then show that the denominator in the previous expression is positive when $u_B \leq \bar{u}_B$. Therefore, $d\Delta V_B^I(\tilde{\beta}_I, \tilde{a}_I)/du_B \geq 0$ and $\Delta V_B^I(\tilde{\beta}_I, \tilde{a}_I)$ is increasing in u_B. Then, computations show that, when $\alpha_B > 0$, $\Delta V_B^I(\tilde{\beta}_I, \tilde{a}_I)$ evaluated at $u_B = 0$ and $u_B = \bar{u}_B$ is negative and positive respectively. Therefore, by continuity, there exists $\hat{u}_B^{\Delta V_B^I=0} > 0$ such that $\Delta V_B^I(\tilde{\beta}_I, \tilde{a}_I)$ is negative if and only $u_B < \hat{u}_B^{\Delta V_B^I=0}$.

Impact on Developers. When $u_S = 0$, the developers surplus is simply given by $V_S^I(\beta_I, a_I) = a_I^2/2$ and therefore $\Delta V_S(\tilde{\beta}_I, \tilde{a}_I) \geq 0$.

PROOF OF PROPOSITION 7. As shown in Appendix A.9, when $u_B = 0$, there exists thresholds $\hat{u}_S^{\beta_I^*=\tilde{\beta}_I}$ and $\hat{u}_S^{\beta_I^*=1}$ such that the solution of platform I's maximization problem under integration is: $\beta_I^{**} = \tilde{\beta}_I$ and $a_I^{**} = \tilde{a}_I$ when $u_S \leq \hat{u}_S^{\beta_I^*=\tilde{\beta}_I}$; $\beta_I^{**} = \beta_I^*$ and $a_I^{**} = a_I^*$ when $\hat{u}_S^{\beta_I^*=\tilde{\beta}_I} < u_S \leq \hat{u}_S^{\beta_I^*=1}$; $\beta_I^{**} = 1$ and $a_I^{**} = \tilde{a}_I(1)$ when $u_S \geq \hat{u}_S^{\beta_I^*=1}$. Moreover, when $u_B = 0$, a necessary condition to ensure that the maximization problem of platform I is concave is $u_S^2 \leq \bar{u}_S^2 = 2(2+\gamma)(32+\gamma(32+9\gamma))/(4+3\gamma)(12+\gamma(11+3\gamma))$.

Impact on Manufacturer M_2. Computations show that $\Delta\pi_2(\beta_I, a_I)$ has the sign of $3(v+\alpha_B) - a_I u_S \gamma + 3(v+\alpha_B)\gamma + r(-1+4\beta_I)(1+\gamma)$, which is increasing in β_I and decreasing in a_I. Since, following integration and when $u_B = 0$, β_I (weakly) decreases and a_I increases, we have that $\Delta\pi_2(\beta_I^{**}, a_I^{**}) \leq 0$.

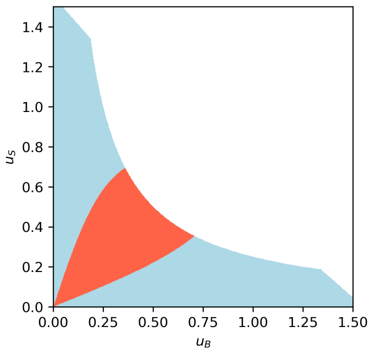
Impact on Buyers. In the following, we are going to show that $\Delta V_B^I(\beta_I^{**}, a_I^{**})$ is increasing in u_S. Differentiating with respect to u_S, we have $d\Delta V_B^I(\beta_I^{**}, a_I^{**})/du_S = \partial V_B^I/\partial u_S +$

$(\partial V_B^I/\partial \beta_I)(d\beta_I^{**}/du_S) + (\partial V_B^I/\partial a_I)(da_I^{**}/du_S)$, omitting arguments for the sake of conciseness. Then, computations show: (i) $\partial V_B^I/\partial u_S = (1/((4+\gamma)^2(4+3\gamma)^2))2a_I(2+\gamma)((v+\alpha_B)(4+3\gamma)^2 + a_I u_S(16+\gamma(16+\gamma)) + r(16+\gamma(16+\gamma+8\beta_I(1+\gamma))))$, which is positive since $a_I = a_I^{**} \geq 0$ when $u_B = 0$; (ii) $\partial V_B^I/\partial \beta_I = (1/((4+\gamma)^2(4+3\gamma)^2))2r(1+\gamma)(2+\gamma)(16(v+\alpha_B+r\beta_I) + 8(r+a_I u_S + 3(v+\alpha_B) + 2r\beta_I)\gamma + 9(v+\alpha_B+r\beta_I)\gamma^2)$, which is positive; (iii) $\partial V_B^I/\partial a_I = (1/((4+\gamma)^2(4+3\gamma)^2))2u_S(2+\gamma)((v+\alpha_B)(4+3\gamma)^2 + a_I u_S(16+\gamma(16+\gamma)) + r(16+\gamma(16+\gamma+8\beta_I(1+\gamma))))$, which is positive. Then, computations show that $d\beta_I^*/du_S = (2u_S(r+v+\alpha_B)(4+3\gamma)(12+\gamma(11+3\gamma))(64+\gamma(96+\gamma(52+9\gamma))))/(r(u_S^2(4+3\gamma)(12+\gamma(11+3\gamma)) - 2(2+\gamma)(32+\gamma(32+9\gamma)))^2) > 0$ and, since $d\beta_I^{**}/du_S = 0$ when $\beta_I^{**} \neq \beta_I^*$, we thus have $d\beta_I^{**}/du_S \geq 0$ for all u_S. Similarly, computations show that $da_I^{**}/du_S \geq 0$ for all u_S. Together, this shows that $\Delta V_B^I(\beta_I^{**}, a_I^{**})$ is increasing in u_S. Then, computations show that, when $\alpha_B > 0$, $\Delta V_B^I(\beta_I^{**}, a_I^{**})$ is negative when $u_S = 0$ and $\Delta V_B^I(\beta_I^{**}, a_I^{**})$ is positive when $u_S = \bar{u}_S$. Therefore, by continuity, there exists $\hat{u}_S^{\Delta V_B^I=0} > 0$ such that $\Delta V_B^I(\beta_I^{**}, a_I^{**}) < 0$ if and only if $u_S < \hat{u}_S^{\Delta V_B^I=0}$.

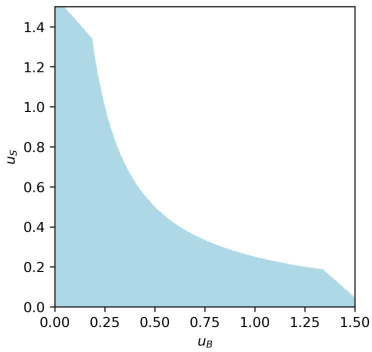
Impact on Developers. Computations show that $\Delta V_S^I(\beta_I, a_I)$ has the sign of $a_I(-4 + 2u_S^2 - \gamma) + 2ru_S(-1+\beta_I)(1+\gamma)$ which is increasing in β_I and decreasing in a_I if $2u_S^2 < 4+\gamma$. Computations show that the last inequality holds when $u_S < \hat{u}_S^{\beta_I^*=1}$. Since β_I decreases and a_I increases following integration when $u_B = 0$, we therefore have that $\Delta V_S^I(\beta_I^{**}, a_I^{**}) \leq 0$ when $u_S < \hat{u}_S^{\beta_I^*=1}$. When $u_S \geq \hat{u}_S^{\beta_I^*=1}$, $\Delta V_S^I(\beta_I^{**}, a_I^{**})$ has the sign of $-((u_S(r+v+\alpha_B)(-4+2u_S^2 - \gamma)(4+3\gamma)(32+3\gamma(16+\gamma(7+\gamma))))/(-(4+\gamma)^2(4+3\gamma)^2 + 2u_S^2(32+\gamma(64+\gamma(40+7\gamma)))))$. Simple calculations then show that this last expression is increasing in u_S and negative in $u_S = \bar{u}_S$. Therefore, $\Delta V_S^I(\beta_I^{**}, a_I^{**}) < 0$ when $u_S \geq \hat{u}_S^{\beta_I^*=1}$.

B.3. *Additional Simulations - Impact of Vertical Integration with Efficiency Gains*

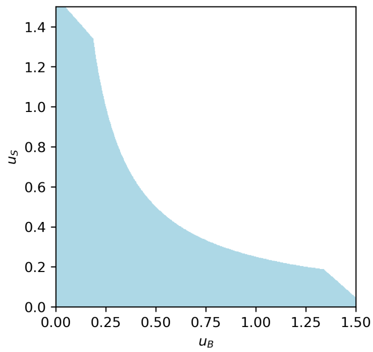
Last, we provide several additional simulations for the various settings discussed in the main text.[41]



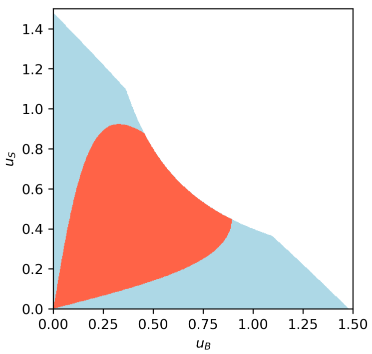
(a) $\gamma = 1$, $r_0 - r = 0$



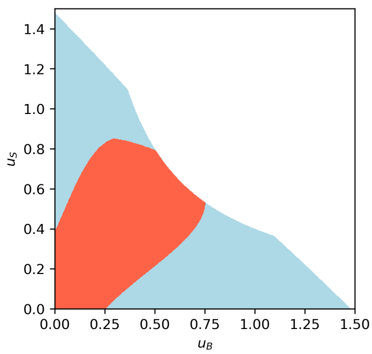
(b) $\gamma = 1$, $r_0 - r = 0.3$



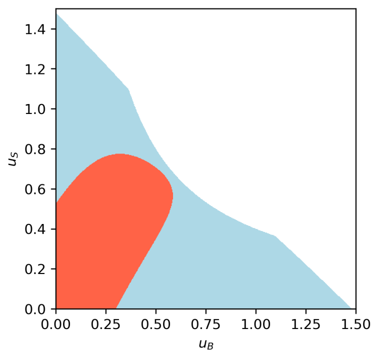
(c) $\gamma = 1$, $r_0 - r = 0.6$



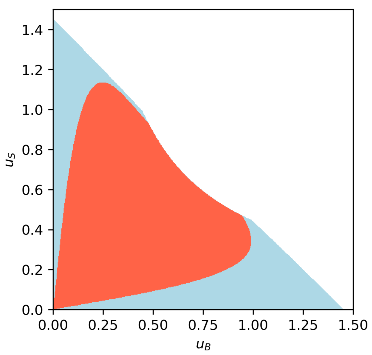
(d) $\gamma = 4$, $r_0 - r = 0$



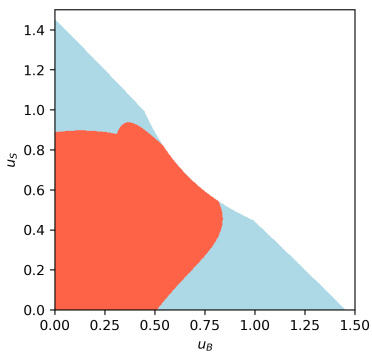
(e) $\gamma = 4$, $r_0 - r = 0.3$



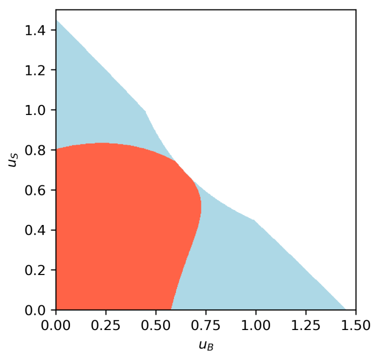
(f) $\gamma = 4$, $r_0 - r = 0.6$



(g) $\gamma = 8$, $r_0 - r = 0$



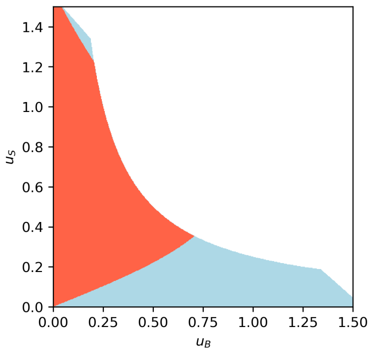
(h) $\gamma = 8$, $r_0 - r = 0.1$



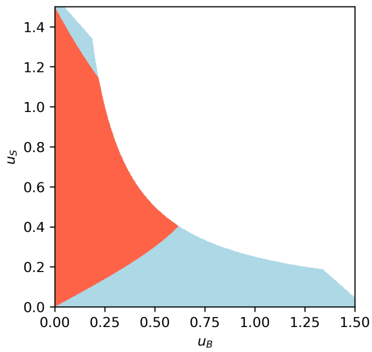
(i) $\gamma = 8$, $r_0 - r = 0.2$

Figure 17: Impact of vertical integration on buyer surplus (V_B) for different degrees of substitutability (γ) and levels of efficiency gains ($r_0 - r$): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).

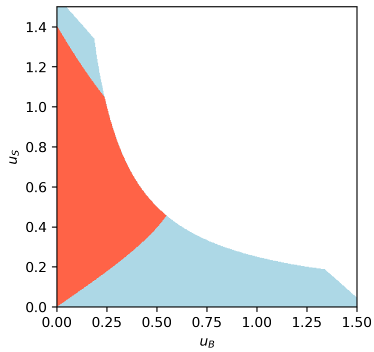
[41] The Python code of the simulations is available on the authors' webpages. We use the following value for parameter v: $v = 2$.



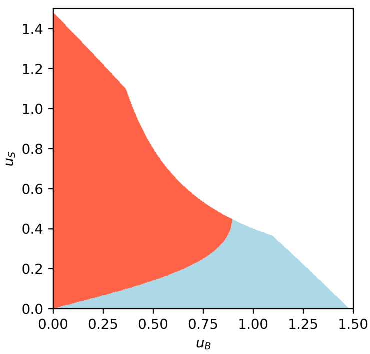
(a) $\gamma = 1$, $r_0 - r = 0$



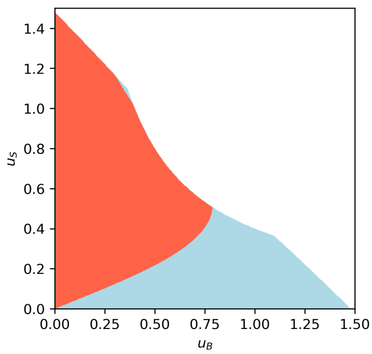
(b) $\gamma = 1$, $r_0 - r = 0.3$



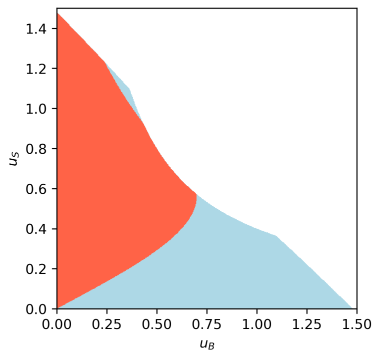
(c) $\gamma = 1$, $r_0 - r = 0.6$



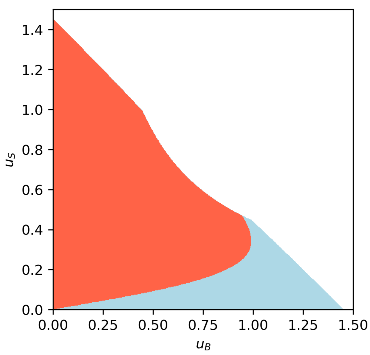
(d) $\gamma = 4$, $r_0 - r = 0$



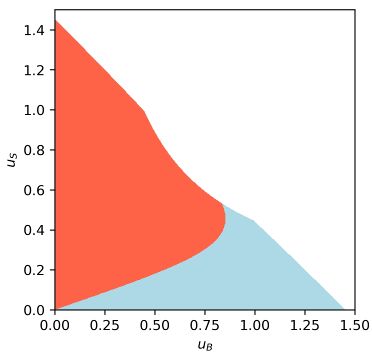
(e) $\gamma = 4$, $r_0 - r = 0.3$



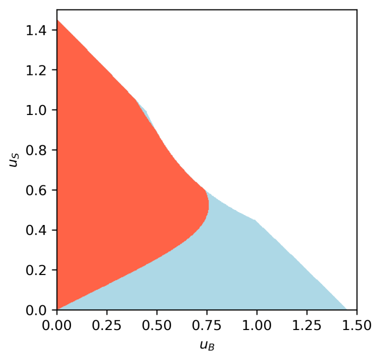
(f) $\gamma = 4$, $r_0 - r = 0.6$



(g) $\gamma = 8$, $r_0 - r = 0$

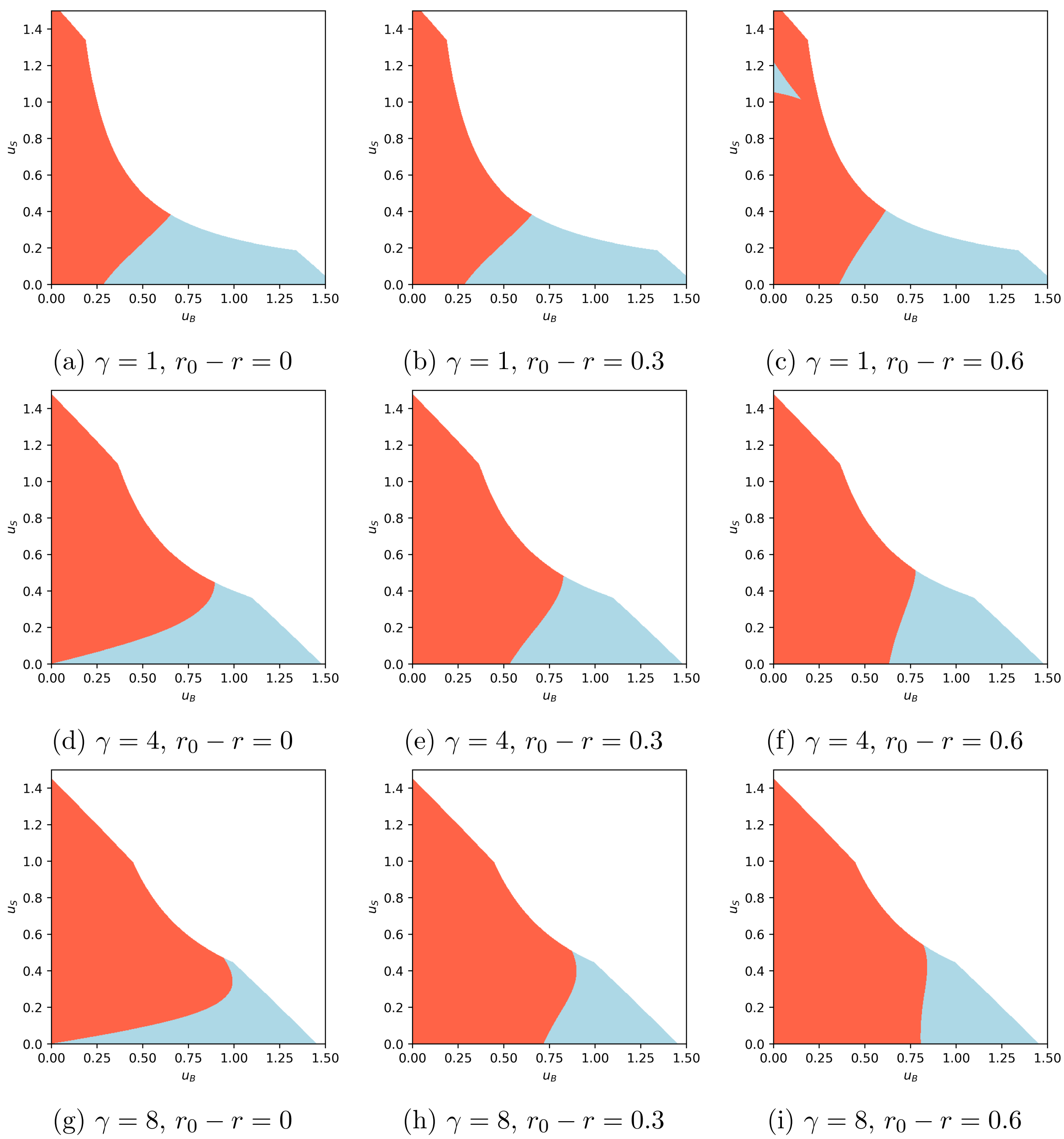


(h) $\gamma = 8$, $r_0 - r = 0.3$



(i) $\gamma = 8$, $r_0 - r = 0.6$

Figure 18: Impact of vertical integration on developer surplus (V_S) for different degrees of substitutability (γ) and levels of efficiency gains ($r_0 - r$): V_S decreases (resp. increases) following integration in the red area (resp. in the blue area).

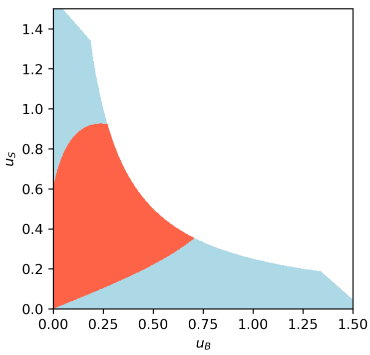


(a) $\gamma = 1$, $r_0 - r = 0$ (b) $\gamma = 1$, $r_0 - r = 0.3$ (c) $\gamma = 1$, $r_0 - r = 0.6$

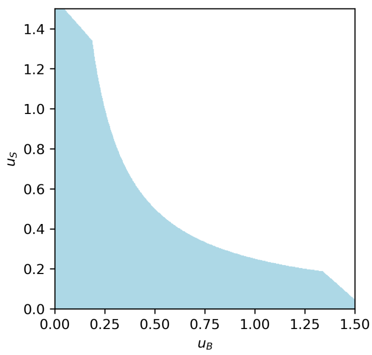
(d) $\gamma = 4$, $r_0 - r = 0$ (e) $\gamma = 4$, $r_0 - r = 0.3$ (f) $\gamma = 4$, $r_0 - r = 0.6$

(g) $\gamma = 8$, $r_0 - r = 0$ (h) $\gamma = 8$, $r_0 - r = 0.3$ (i) $\gamma = 8$, $r_0 - r = 0.6$

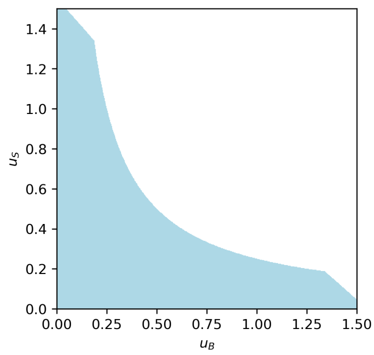
Figure 19: Impact of vertical integration on the non-integrated manufacturer's profit (π_2) for different degrees of substitutability (γ) and levels of efficiency gains ($r_0 - r$): π_2 decreases (resp. increases) following integration in the red area (resp. in the blue area).



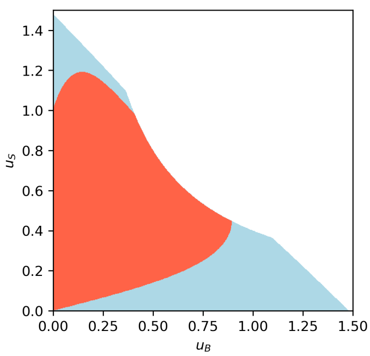
(a) $\gamma = 1$, $r_0 - r = 0$



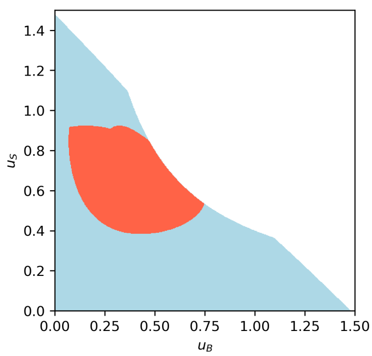
(b) $\gamma = 1$, $r_0 - r = 0.3$



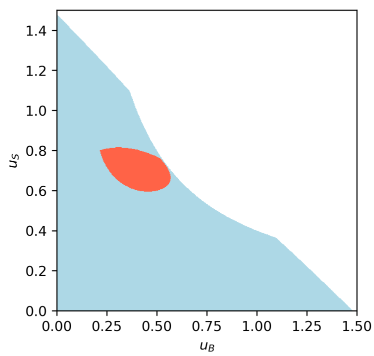
(c) $\gamma = 1$, $r_0 - r = 0.6$



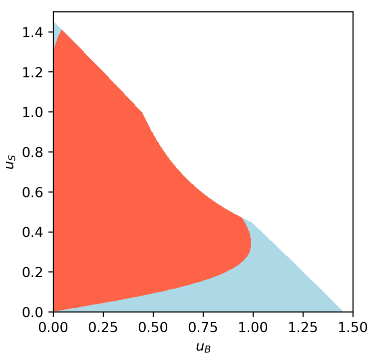
(d) $\gamma = 4$, $r_0 - r = 0$



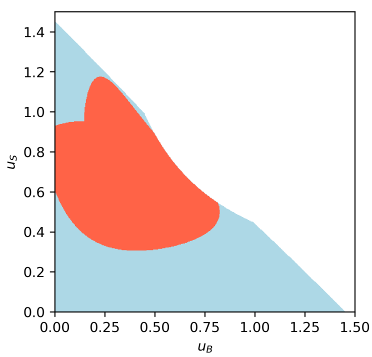
(e) $\gamma = 4$, $r_0 - r = 0.3$



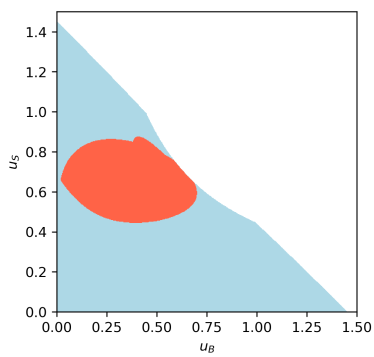
(f) $\gamma = 4$, $r_0 - r = 0.6$



(g) $\gamma = 8$, $r_0 - r = 0$



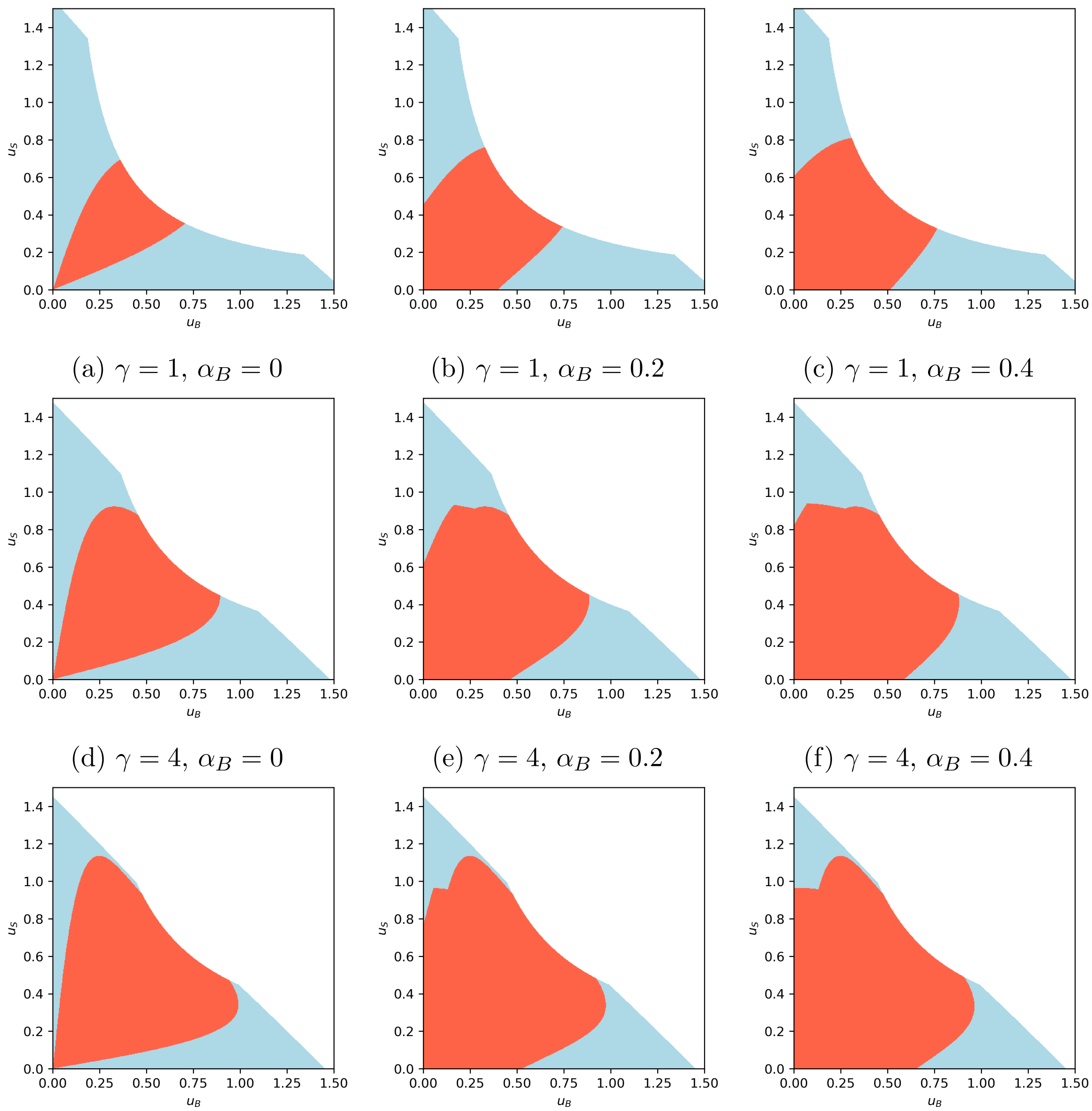
(h) $\gamma = 8$, $r_0 - r = 0.3$



(i) $\gamma = 8$, $r_0 - r = 0.6$

Figure 20: Impact of vertical integration on total welfare (W) for different degrees of substitutability (γ) and levels of efficiency gains ($r_0 - r$): W decreases (resp. increases) following integration in the red area (resp. in the blue area).

B.4. Additional Simulations - Impact of Vertical Integration with Coordination Motives

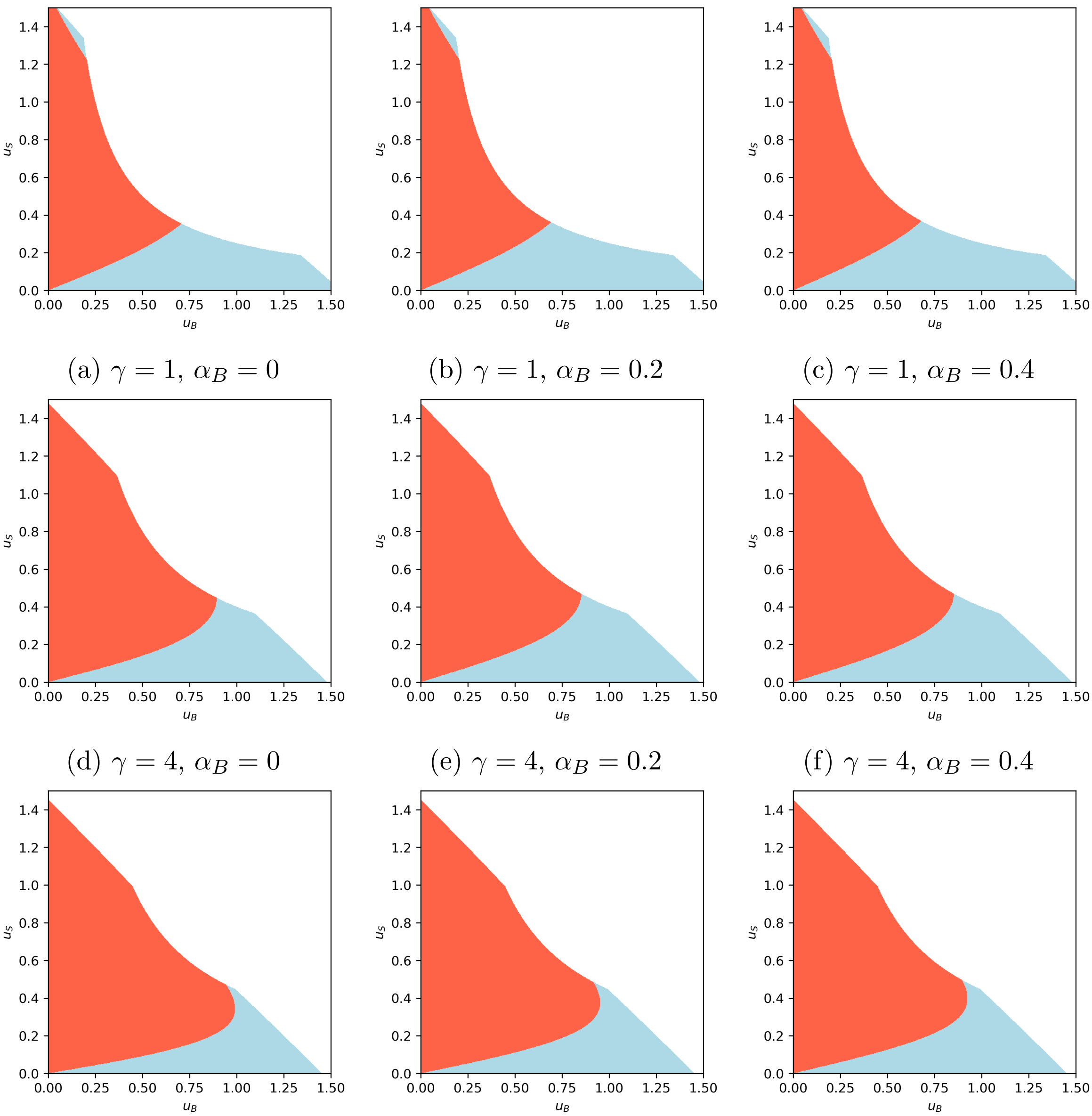


(a) $\gamma = 1$, $\alpha_B = 0$ (b) $\gamma = 1$, $\alpha_B = 0.2$ (c) $\gamma = 1$, $\alpha_B = 0.4$

(d) $\gamma = 4$, $\alpha_B = 0$ (e) $\gamma = 4$, $\alpha_B = 0.2$ (f) $\gamma = 4$, $\alpha_B = 0.4$

(g) $\gamma = 8$, $\alpha_B = 0$ (h) $\gamma = 8$, $\alpha_B = 0.2$ (i) $\gamma = 8$, $\alpha_B = 0.4$

Figure 21: Impact of vertical integration on buyer surplus (V_B) for different degrees of substitutability (γ) and levels of the gain for buyers when manufacturers adopts the same operating system (α_B): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).

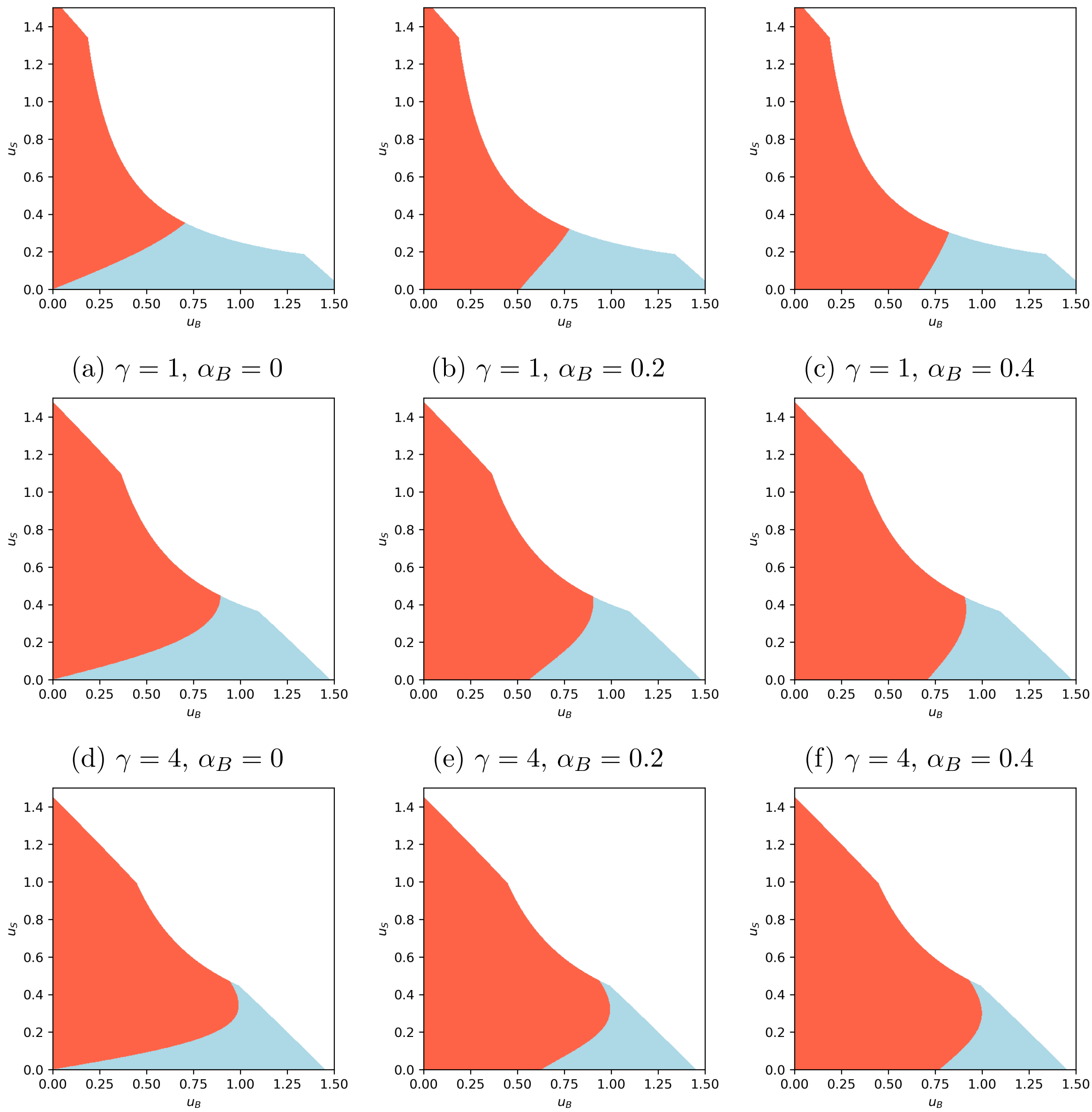


(a) $\gamma = 1$, $\alpha_B = 0$ (b) $\gamma = 1$, $\alpha_B = 0.2$ (c) $\gamma = 1$, $\alpha_B = 0.4$

(d) $\gamma = 4$, $\alpha_B = 0$ (e) $\gamma = 4$, $\alpha_B = 0.2$ (f) $\gamma = 4$, $\alpha_B = 0.4$

(g) $\gamma = 8$, $\alpha_B = 0$ (h) $\gamma = 8$, $\alpha_B = 0.2$ (i) $\gamma = 8$, $\alpha_B = 0.4$

Figure 22: Impact of vertical integration on developers surplus (V_S) for different degrees of substitutability (γ) and levels of the gain for buyers when manufacturers adopts the same operating system (α_B): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).

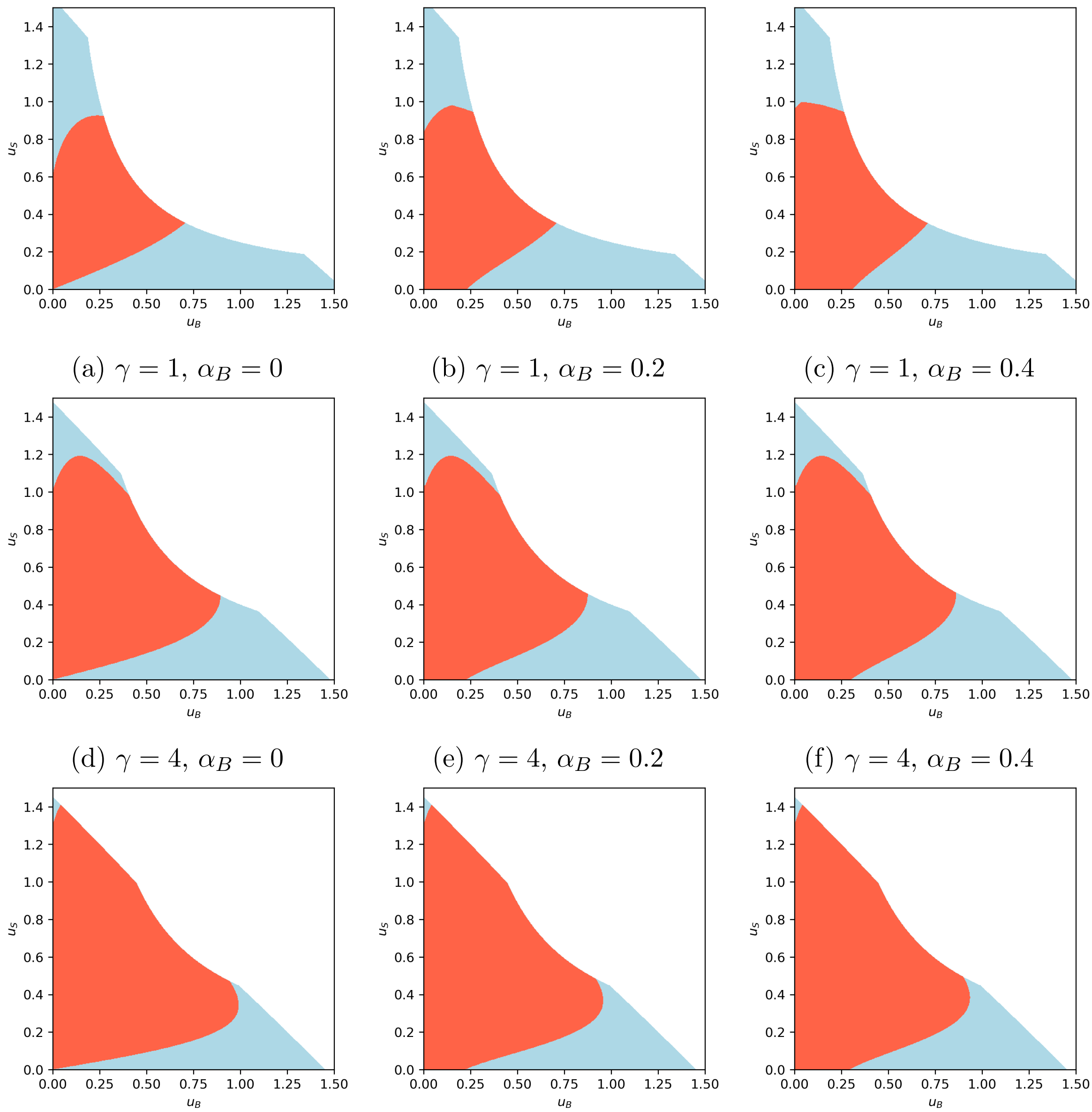


(a) $\gamma = 1$, $\alpha_B = 0$ (b) $\gamma = 1$, $\alpha_B = 0.2$ (c) $\gamma = 1$, $\alpha_B = 0.4$

(d) $\gamma = 4$, $\alpha_B = 0$ (e) $\gamma = 4$, $\alpha_B = 0.2$ (f) $\gamma = 4$, $\alpha_B = 0.4$

(g) $\gamma = 8$, $\alpha_B = 0$ (h) $\gamma = 8$, $\alpha_B = 0.2$ (i) $\gamma = 8$, $\alpha_B = 0.4$

Figure 23: Impact of vertical integration on the non-integrated manufacturer's profit (π_2) for different degrees of substitutability (γ) and levels of the gain for buyers when manufacturers adopts the same operating system (α_B): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).



(a) $\gamma = 1$, $\alpha_B = 0$ (b) $\gamma = 1$, $\alpha_B = 0.2$ (c) $\gamma = 1$, $\alpha_B = 0.4$

(d) $\gamma = 4$, $\alpha_B = 0$ (e) $\gamma = 4$, $\alpha_B = 0.2$ (f) $\gamma = 4$, $\alpha_B = 0.4$

(g) $\gamma = 8$, $\alpha_B = 0$ (h) $\gamma = 8$, $\alpha_B = 0.2$ (i) $\gamma = 8$, $\alpha_B = 0.4$

Figure 24: Impact of vertical integration on total welfare (W) for different degrees of substitutability (γ) and levels of the gain for buyers when manufacturers adopts the same operating system (α_B): V_B decreases (resp. increases) following integration in the red area (resp. in the blue area).