DISCUSSION PAPER SERIES

DP17614

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POLITICAL ECONOMY AND PUBLIC ECONOMICS

CEPR

ISSN 0265-8003

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Discussion Paper DP17614
Published 27 October 2022
Submitted 04 October 2022

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Abstract

We analyze a new constitutional decision-making rule—called ”Co-Voting”—which can be described as a combination of representative democracy (or republic, where citizens delegate their decision power to a parliament) and direct democracy (or just democracy, where citizens decide through referenda). We consider a simple model in which the electorate is partially uninformed about the consequences of policies and parliament members have biased preferences regarding policy. Taking a constitutional perspective, we characterize the model primitives for which Co-Voting yields higher welfare than both direct democracy and representative democracy, which are natural benchmarks. The relative merits of Co-Voting continue to hold if proposal-making by parliament is strategic.

JEL Classification: D02, D70, D72, D82

Keywords: N/A

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Acknowledgements
We are grateful to Eric Maskin, Hans-Peter Grüner, and the participants of the 2nd ETH Democracy Workshop for valuable comments.

Republic or Democracy? Co-Voting![*,†]

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This version: September 2022

Abstract

We analyze a new constitutional decision-making rule—called "Co-Voting"—which can be described as a combination of representative democracy (or *republic*, where citizens delegate their decision power to a parliament) and direct democracy (or just *democracy*, where citizens decide through referenda). We consider a simple model in which the electorate is partially uninformed about the consequences of policies and parliament members have biased preferences regarding policy. Taking a constitutional perspective, we characterize the model primitives for which Co-Voting yields higher welfare than both direct democracy and representative democracy, which are natural benchmarks. The relative merits of Co-Voting continue to hold if proposal-making by parliament is strategic.

Keywords: direct democracy; representative democracy; constitution; voting; bias; information asymmetry

JEL Classification: D02, D70, D72, D82

[*]Today, the terms "republic" and "democracy" overlap. We borrow from Alexander Hamilton, James Madison, and John Jay and use "democracy" when we mean direct democracy and "republic" when we mean representative democracy (see e.g. Holt, 2016).

[†]We are grateful to Eric Maskin, Hans-Peter Grüner, and the participants of the 2^{nd} ETH Democracy Workshop for valuable comments. All errors are our own.

The [...] difference between a democracy and a republic [...] [is] the delegation of the government, in the latter, to a small number of citizens elected by the rest; [...] The effect [...] is, on the one hand, to refine and enlarge the public views, by passing them through the medium of a chosen body of citizens, whose wisdom may best discern the true interest of their country [...] Under such a regulation, it may well happen that the public voice, pronounced by the representatives of the people, will be more consonant to the public good than if pronounced by the people themselves, convened for the purpose. On the other hand, the effect may be inverted. Men of factious tempers, of local prejudices, or of sinister designs, may, by intrigue, by corruption, or by other means, first obtain the suffrages, and then betray the interests, of the people.

The Federalist Papers, No. 10.

1 Introduction

Democracies have two main goals when deciding on legislation or on policy. First, they should pass a law or choose a policy that expresses the citizens' preferences, as the principle of *representativeness* requires. Second, democracies should aggregate the available information relevant for the decision at hand to avoid making suboptimal choices; this is required by *efficiency*. However, the constitutional design of many countries makes it difficult to attain both goals at the same time. Direct democracies—where citizens vote in referenda for important decisions—are, in principle, better able to achieve representativeness. Representative democracies—where citizens delegate their decision power to a parliament for a given period—are, in principle, better able to achieve efficiency.

The trade-offs between direct democracy and representative democracy appear to be obvious, as spelled out in the above quote from the Federalist Papers. Asking the opinion of citizens on complicated issues may reflect the will of the people, given their beliefs, but may also lead to undesirable outcomes since most of the voters do not have the time and resources to fully understand the issues at hand (Downs, 1957). This is why, in a representative democracy, citizens delegate their decision power by electing the members of the legislature. Ideally, these members of parliament are expected to study the issues at hand thoroughly and thus to take better-informed decisions. However, a second problem arises, as the median member of parliament may have other preferences than the median citizen. This can occur for a variety of reasons, e.g. the lack of proportionality of the electoral system or the influence of lobbying groups on legislators. In such cases, representative democracy will lead to decisions that do not reflect the will of the people.

The gap between the will of the people and the will of parliament members can be illustrated by Brexit. When the referendum took place on 23 June 2016, a majority of members of the House of Commons opposed Brexit, in contrast to a majority of the electorate, as was reflected in the voting outcome. This means that parliament members' positions were biased relative to those of the citizens, which is our first main assumption. On the other hand, it is also fair to assume that citizens were less informed than members of parliament about what Brexit really meant (see e.g. Renwick et al., 2018). Our second main assumption is that voters may be less informed than parliament members about the consequences of choosing particular policies.

A new decision-making rule

The tension between representativeness and efficiency has been discussed for decades. In particular, it has triggered a great amount of research in economics and political science—see Section 2. At a practical level, the constitutional design of many democracies offers ways to ease this tension. In countries like France, Denmark, and Spain, a ratification procedure is in place for certain agreements made by the parliament or by the government: The citizens must ratify them before the agreed policy can be implemented. In other countries like the UK, the parliament must ratify some decisions directly taken by the people. In either case, however, gridlock may arise (see e.g. Jones, 2001; Binder, 1999, 2004).[1]

Consider, for example, the case in which a parliament votes first, followed by a vote by the citizenry. Usually, both the parliament and the citizens retain some form of veto power: an alternative suggested to replace the status quo can only be implemented if *(i)* parliament chooses this alternative in a parliamentary vote and accepts to put it to a vote by the whole citizenry, and *(ii)* citizens approve the alternative when they vote. Hence, the parliament's veto power originates from its proposal-making power. Citizens, on the other hand, have power insofar as they can reject—but not amend—the parliament's proposal. By design, this double-step decision-making rule does not balance the parliament's preference bias against the citizens' lack of information, as no compromise between the two is possible. This makes gridlock a likely outcome. While beyond this example there is a variety of decision-making procedures where both the parliament and the citizenry vote, none of them is designed to strike such a balance.

[1] The inability to simultaneously attain representativeness and efficiency has called in the past few years for reforms of democratic decision-making procedures—and, if possible, for innovations in collective decision processes. At the same time, since many representative democracies already allow direct consultation of the citizenry, the number of referenda has risen in representative democracies, indicating the wish to restore citizen control over decisions that many felt were in the hands of the elite (see e.g. Demange, 2019; Correa-Lopera, 2018). But this does not seem to have solved the problem. For the drawbacks of referenda, see Buisseret and Van Weelden (2021) and the references in the literature section.

In this paper, we propose and examine a decision-making rule—called *Co-Voting*—that does strike the above balance between bias and information.[2] It is developed for representative democracies and works as follows. Suppose that a collective decision has to be taken between two alternatives. Then, the parliament and the entire citizenry (or a randomly-selected subset of citizens) vote simultaneously and the two decisions are weighted according to a pre-defined key. The resulting decision is implemented.

To be specific, consider two given alternatives, say the status quo (p_0) and some amendment (p). In representative democracy, the (voting) weight of parliament is one, while the weight of citizens is zero. In direct democracy, the weight of parliament is zero, while the weight of citizens is one. By contrast, Co-Voting prescribes the outcome to be determined as follows: p will be implemented if and only if the share of citizens who voted for p *plus* the share of members of parliament who voted for p is at least one; otherwise p_0 will be maintained. This means that Co-Voting can be seen as a (convex) combination of representative democracy and direct democracy in which each of the two bodies (parliament and citizenry) has some positive—and in our case, equal—weight.

To the best of our knowledge, Co-Voting has never been implemented in practice for democratic decisions, so its performance at a practical level is untested. Yet, it is worth mentioning that a voting procedure that resembles Co-Voting is used for the presidential election in Germany. To elect the president, a federal convention must be constituted, which consists of all members of parliament plus an equal number of state electors. These electors can be members of regional parliaments who are not members of the Bundestag, and also comprise "regular" citizens—such as artists or well-known athletes—appointed by the regional parliaments. To be elected, a candidate needs a majority of votes of the entire federal convention.[3] Our goal is to put forward theoretical reasons why Co-Voting may be a useful democratic decision-making procedure from a constitutional perspective.

Model and results

For our analysis, we consider a basic model of a large citizenry and a (large) parliament in which *(i)* a fraction of citizens is not informed about the consequences of new proposals on a given policy dimension, and *(ii)* parliament members have biased preferences regarding this policy dimension compared to citizens. These two features suffice to generate a trade-off between representativeness and efficiency. Given this setup, consider now *one* given arbitrary proposal and assume that the decision-making scheme can be made contingent on the decision at hand. Then, either focusing on citizens and ignoring parliament (i.e., direct

[2]See Gersbach (2017) for a first verbal description.

[3]See `https://de.wikipedia.org/wiki/Wahl_des_deutschen_Bundespr\%C3\%A4sidenten_2017#Wahlergebnis` (in German, retrieved 13 January 2020).

democracy) or focusing on parliament and ignoring citizens (i.e., representative democracy) will always yield as good an outcome as Co-Voting from the perspective of citizens. Direct democracy performs at least as well as Co-Voting if the parliament's bias is too great relative to the citizens' lack of information. Otherwise, representative democracy performs at least as well as Co-Voting.

The picture changes dramatically if we take a constitutional perspective and the same decision-making rule has to be used for decisions whose details are unknown yet and which cannot be used to determine which decision-making rule should be employed. This is the usual approach for the constitutional design of democracies. We then show that for reasonable sets of potential proposals, Co-Voting will yield decisions that are better from an (ex-ante) welfare perspective than those implemented either by direct democracy or representative democracy. It thus follows that under such a veil of ignorance, Co-Voting may have the potential to improve democratic decision-making.

Finally, we also demonstrate that Co-Voting has important strategic consequences for proposal-making. Specifically, we show that if the median parliament member has the right to choose the proposal that will be pitted against the status quo, parliament members will only propose policy alternatives that are not bound to be rejected. Then, from a welfare perspective, Co-Voting will yield better policies in expectation than representative democracy, and it will never yield worse policies than direct democracy.[4]

Organization

The remainder of the paper is organized as follows. In Section 2 we discuss our contribution to existing strands of the literature. In Section 3 we illustrate our main insights by an example. In Section 4 we outline the model and set up notation. In Section 5 we analyze the voting outcome under different decision-making systems for a fixed policy proposal or amendment. Section 6 extends the comparative analysis to the case of multiple or unknown policy proposals. In Section 7 we endogenize a policy proposal by the parliament. Section 8 concludes. The proofs are in the appendix.

[4] Of course, democratic societies may have different preferences regarding the type of collective decision rules they want to be governed by. Also, our suggestion is not tailored to a citizenry like the Swiss—for whom direct democracy rules are an entrenched part of culture. Our suggestion might be more useful for representative democracies that want to introduce more direct-democracy elements in the collective decision-making process.

2 Relation to the Literature

The elements of the fundamental trade-off we examine—preference bias versus asymmetric information—have been addressed and justified extensively in the literature. On the one hand, let us focus on the preference bias. The assumption that the citizens' and the elected officials' preferences differ in representative democracies is the focus of the public choice literature following Buchanan and Tullock (1962). Various strands of the political economic literature have since also addressed this principal-agent relationship (see e.g. the textbook by Drazen (2000)) and discussed this assumption. There are many reasons why representatives' preferences, and hence policy, can differ from those of the citizens.

First, the perspective of candidates might change once they have experienced the reality of office-holding so that their preferences shift away from the preferences of the voters—and away from their own (past) preferences as candidates. Second, interest groups can try and influence members of the legislature by offering campaign contributions or career opportunities. A large literature has examined these channels of influence and how they generate diverging interests between voters and the government—we refer to Gersbach (2014) for a short survey. Third, some groups of the citizenry may exhibit lower turnout rates for parliamentary elections (see e.g. Palfrey and Rosenthal, 1983; Ledyard, 1984; Palfrey and Rosenthal, 1985, for models of costly voting). Young people, for instance, tend to participate less. This might be due to lack of interest, of information, or of foresight, for example.[5] Fourth, the interests of elected officials and voters do not coincide when poorly informed citizens do not correctly identify their best representative.[6]

The divergence between citizens' preferences and implemented policies can be lessened if citizens are often directly empowered to take decisions. Studies have shown that in systems with direct democratic institutions public policy is more likely to reflect the preferences of the median voter (see e.g. Gerber, 1996b), since citizens can affect policymaking and policy outcomes both directly (initiatives and referenda) and indirectly (policy proposals in parliament to preempt referenda, see Bowler and Donovan (2004)).[7] We add to the existing literature by showing how preference bias can be balanced by including the views of the citizenry through Co-Voting.[8]

[5] See `https://www.swissinfo.ch/eng/democratic-duty_should-we-worry-about-low-voter-turnouts-in-switzerland-/44248880`, retrieved 17 January 2020.

[6] For the statistical methods to measure the representativeness of a parliament or government, see Achen (1978).

[7] An extended discussion on this topic is given in Wüest and Lloren (2016).

[8] We do not consider elections, so we do not take into account the strategic incentives generated by the information asymmetry between citizens and politicians nor how such asymmetry arises in the first place (see e.g. Bernhardt et al., 2007; Heidhues and Lagerlöf, 2003; Kartik et al., 2015, among others).

On the other hand, let us focus on the second element defining the main trade-off of our model, viz. asymmetry of information. An extensive strand of literature has examined the voters' difficulty to make informed decisions in general. Madison (1787) and Sieyès (1789) were among the first to develop a rationale for representative democracy based on informational advantages of delegated office-holders. The more recent literature has addressed the sources of this informational advantage. Roemer (1994) and Cukierman and Tommasi (1998) emphasize how particular policies affect voting outcomes. Schultz (1996, 2002), Martinelli (2001), and Jensen (2009) focus on knowledge about the conditions for particular policies to yield good outcomes or bad ones. In political science, research has also indicated that most voters have very limited knowledge of politics and, therefore, rely on information cues or shortcuts to make voting decisions (see e.g. Sniderman et al., 1991; Zaller, 1992; Lupia, 1994).[9] Our analysis entails that some degree of delegation of decisions to parliaments is useful from a constitutional perspective.

For our analysis of Co-Voting, we compare this new decision-making procedure with both representative democracy and direct democracy. The theoretical literature comparing directly different forms of democracy is comparatively sparse. In their classical work, Maskin and Tirole (2004) study three different decision-making rules: direct democracy, representative democracy, and judiciary, and when it is optimal to apply which governance mode. Schultz (2003), Kessler (2005), and Correa-Lopera (2018) analyzed the relative merits of representative democracy and direct democracy under uncertainty about the socially optimal policy. Bihan (2018) suggests different model environments and highlights the circumstances when direct democracy or representative democracy performs better. We add to this literature by exploring the performance of a new decision-making rule that aims at combining the strengths of representative and direct democracy in a *flexible* way.[10]

[9]However, if voters take their cues primarily from parties and legislators they vote for in elections, see e.g. Hobolt (2007), Kriesi (2005), then the outcomes of direct democratic votes may simply mirror those that would have been produced by representative decision making.

[10]Many aspects have to be considered when comparing different forms of democracy, and within the broad classes of representative and direct democracy, there are many different sub-forms and subconceptions. First, different forms of democracy may produce differences in growth-promoting policies (Persson, 2005) and direct democracy may favorably affect fiscal and economic performance (Feld et al., 1999; Matsusaka, 1995; Feld and Matsusaka, 2003; Feld and Savioz, 1997). In direct democracies, citizens may obtain direct benefits from their possibility to participate in collective decision-making—called *procedural utility*. Frey et al. (2004, 2011); Besley and Coate (2008) show that policy outcomes on specific issues may substantially differ from what the majority prefers when citizens have only one vote to decide on a bundle of issues. Spiegel and Cukierman (2000) investigate what happens when one can choose simple direct democracy to approximate the outcomes from representative democracy and how the divergence can be characterized in terms of the political environment. Coffman (2016) provides conditions under which representative democracy implements the choices made by citizens in direct democracy. Prato and Strulovici (2016) develop a model to study the effects of direct democracy institutions on the incentives and selection of elected officials and find a negative impact on politicians' vote in such institutions, which may dominate any direct benefit.

Our paper also contributes to the literature that shows how proposal-making in legislatures is affected by referenda. Leemann and Wasserfallen (2016) argue that referenda partially correct the misalignment of preferences between representatives and the public. Gerber (1996a) study policy proposals when there is a possibility of a referendum and when there is not. Matsusaka (2005) survey theoretical aspects of direct democracy and describe when it is optimal to use referenda. Our analysis identifies circumstances under which Co-Voting can outperform referenda in direct democracies.

Finally, from a broader perspective our setup can be reinterpreted as a parliamentary system with two chambers—or decision bodies in general—who differ in both preferences and information. There is a large literature on bicameralism (see e.g Diermeier and Myerson, 1999; Cutrone and McCarty, 2006; Coakley, 2014). From this perspective, we contribute new insights to the optimal design of a constitutional system that comprises two chambers whose members differ along more than one dimension.

3 A Motivating Example

Our insights can be illustrated by an example. A society of unit mass has to decide between the status quo, p_0, and a policy proposal, $p \in \{p_1, p_2, p_3\}$. From an ex-ante perspective, any of the three potential policies (p_1, p_2, and p_3) is equally likely to be proposed. Individuals—no matter whether they are regular citizens or members of parliament—can be of four types, say t_1, t_2, t_3, and t_4, depending on their preferences regarding the proposals in relation to the status quo. Table 1 depicts these preference types. We use 0 to denote that the individual type prefers the status quo over the new policy, while 1 denotes the opposite.

	t_1	t_2	t_3	t_4
p_1	0	0	1	0
p_2	0	1	1	0
p_3	1	1	1	0

Table 1: Preferences for different types.

The society consists of the electorate and a parliament. For simplicity we assume that both decision-making bodies have unit mass. As is standard, we proceed on the assumption that the law of large numbers can be applied to a continuum of random variables. The electorate and the parliament differ in two respects, which are justified in Section 4. First, within the electorate, the first three types (t_1, t_2, t_3) are equally likely, with the probability for a citizen to belong to type t_4 being equal to zero. This means that the utilitarian optimal decision from the *electorate's perspective* is not to implement policy p_1 and to implement policies p_2 and p_3, if given a binary choice between each of these policies and the status quo. In the parliament, the first three types (t_1, t_2, and t_3) are also equally likely, but the probability of a parliament member being of type t_4 is assumed to be slightly higher than one half to generate a conflict between parliament and citizens. Citizens of type t_4 prefer the status quo to any other policy choice. This means that, compared to the citizens, the parliament's preferences are biased and in this example in favor of the status quo.

The second dimension on which parliament differs from the electorate is the accuracy of beliefs about which alternative is the best one. All parliament members know precisely which alternative will have which consequences and they vote according to their preferences. In contrast, we assume that for each given proposal p, all citizens of types t_1, t_2, and t_3 have wrong beliefs about the consequences of alternatives and thus about their preferences when they had correct information. This is depicted in Table 2, in which the circled numbers denote the type who has the wrong beliefs, e.g., 1 means that individuals of the corresponding type believe that policy proposal p should be implemented, but if they had the correct information, they would prefer the status quo to remain in place.

	t_1	t_2	t_3	t_4
p_1	0	1	1	0
p_2	0	1	0	0
p_3	0	1	1	0

Table 2: Believed preferences for different citizen types.

Then we consider three different decision-making rules to be applied for deciding between a given p and p_0. In direct democracy (DD), all citizens vote for one alternative (according to their believed preference) and p is implemented if and only if it collects at least half of the votes. This condition can be formulated as

$$\sigma_{DD} \geq \frac{1}{2},$$

where σ_{DD} denotes the share of votes in favor of p within the electorate. In representative democracy (RD), all members of parliament vote for one alternative (according to their preference) and p is implemented if and only if it collects at least half of the votes from the parliament members. This condition can be formulated as

$$\sigma_{RD} \geq \frac{1}{2},$$

where σ_{RD} denotes the share of votes in favor of p in the parliament. In Co-Voting (CV), all citizens vote for one alternative (according to their believed preference) and all members of parliament vote for one alternative (according to their preference), and p is implemented if and only if

$$\frac{1}{2} \cdot \sigma_{DD} + \frac{1}{2} \cdot \sigma_{RD} \geq \frac{1}{2}.$$

That is, Co-Voting is a convex combination of direct democracy and representative democracy.

Table 3 presents the outcomes under each of the above decision-making rules, as well as the utilitarian socially optimal decision (SO) for the electorate. A checkmark means that p will be implemented. Otherwise, the status quo will prevail.

	DD	RD	CV	SO
p_1	✓			
p_2				✓
p_3	✓		✓	✓

Table 3: Outcomes by decision-making rule.

Some comments are in order. First, while direct democracy and representative democracy only implement the socially optimal solution in one scenario (p_3 for DD and p_1 for RD), Co-Voting achieves this goal in two scenarios (for p_1 and p_3). Second, none of the three voting procedures implements the utilitarian optimal solution in all scenarios. Third, if the socially optimal solution for a *given* policy proposal is implemented by Co-Voting, it is always implemented by either DD or RD: p_3 is implemented by DD and Co-Voting, while p_1 is implemented by RD and Co-Voting. Fourth and last, Co-Voting can be preferable to RD and DD from a welfare perspective if either there is uncertainty about the policy proposal or if the chosen procedure will be used for a (large) number of policy decisions. This can be the case if we take a constitutional perspective and have to choose among RD, DD, and Co-Voting without knowing the degree of conflict and the difference of information levels between citizens and parliament and the set of alternatives.

In what follows, we generalize the insights provided by the above example to the case of arbitrary distributions of preferences of both citizens and parliament members.

4 Model

4.1 Setup

We consider a society endowed with a constitutional rule according to which it will take one or more decisions about whether or not to implement changes in the status quo. Any such change must take place along a one-dimensional compact policy space $[-A, A] \subseteq \mathbb{R}$, with $A > 0$. A policy proposal is an element of $[-A, A]$ and is denoted by p. For simplicity, we assume that the status-quo policy is 0. The society is composed of citizens—the *electorate*. There is also a *parliament*. Both the electorate and the parliament are of unit mass. All individuals, no matter whether they are parliament members or regular citizens, have standard quadratic preferences over elements of $[-A, A]$ which are characterized by their peak. That is, the preferences of an individual with peak i regarding policy p are described by his/her utility function

$$u_i(p) = -(p - i)^2. \tag{1}$$

The citizens' cumulative distribution function of peaks is denoted by F, with f denoting the corresponding probability density function. The probability distribution function of peaks for parliament members is denoted by G, with g denoting the corresponding probability density function. At this point, we do not need to specify any relation between F and G. We will do this in Section 6, where we will assume that that parliament members are biased towards positive policies compared to citizens.[11] The model works in the same way if parliament is biased in the opposite direction.

The electorate and parliament also differ insofar that all parliament members are better informed about the consequences of policies than citizens. Specifically, we assume that every parliament member knows—or can learn at negligible cost—the consequences of a policy for their utility. That is, for them a policy labeled as p enters as policy point p in their utility function. In contrast, a citizen only learns this fact with probability q, with $q \in [0, 1]$, in which case s/he is informed. With the remaining probability $1 - q$, a citizen will instead believe that a policy labeled as p may be associated with different consequences than with policy point p. In this second case, the citizen is uninformed. For simplicity, we assume that being informed or uninformed is independent of preferences. There are many reasons why some some citizens are uninformed. The most obvious is rational ignorance,

[11] One standard way of assuming this would be to impose first-order stochastic dominance, i.e., to assume that $F(i) \geq G(i)$ for all $i \in [-A, A]$. In this case, the parliament's peaks are more to the right than the citizens' peaks.

as an individual voter has a negligible influence on voting outcomes. Other possibilities arise when citizens receive information exclusively from biased media, participate mainly in echo chambers of like-minded individuals, or when individuals interpret information according to personal cues. Parameter q measures the degree to which the whole electorate is uninformed, ranging from fully informed ($q = 1$) to fully uninformed ($q = 0$). Formally, we assume that an uninformed voter associates some random variable $X := X(p)$ with each policy p, and as is standard, we build on expected utility theory. We then assume that

$$E[X] = \mu(p) < \infty$$

and

$$E[X^2] = \sigma^2(p) < \infty$$

for some functions $\mu(p)$ and $\sigma^2(p)$ satisfying $\mu(0) = \sigma^2(0) = 0$ and

$$\mu(p) \cdot p > 0 \text{ for } p \neq 0, \tag{2}$$

which in particular implies that $\mu(p) \neq 0$ when $p \neq 0$.

The above means that the first two moments of the random variable X depend on the announced policy p. The function $\mu(p) - p$ describes the informational bias, while $\sigma^2(p)$ is related to the information noise level. Note that for the status quo, there is neither bias nor noise and that Condition (2) implies that in expected terms, uninformed voters are able to tell whether p is to the right of to the left of the status quo. As examples for the latter assumption, it is often the case that voters can recognize whether a policy proposal tends to be tougher or weaker on immigration, whether it entails more or less redistribution, or whether it implies more free trade or more protectionism. Yet, they are unable to tell the *extent* to which these policies will have such consequences, and can only recognize the *direction* of the expected impact. The events according to which different voters are informed or not are stochastically independent.

4.2 Socially optimal solution

Our goal is to examine different decision-making rules as to how well they implement the electorate's preferences, so we disregard parliament members' preferences for the societal calculus.[12] To define optimality for a *given* policy proposal, we then take a utilitarian perspective. This leads to the following notion of societal welfare when policy $p \in [-A, A]$

[12] The assumption that the parliament has unit mass is made for simplicity, but it is reasonable to assume that the weight of parliament members in total welfare is negligible.

is adopted:

$$W(p) = -\int_{-\infty}^{\infty} (p-i)^2 f(i) di.$$

Accordingly, to maximize social welfare in utilitarian terms, policy p should be implemented if and only if[13]

$$W(p) > W(0).$$

Let now

$$E_F := \int_{-\infty}^{\infty} i f(i) di \tag{3}$$

denote the average peak within the electorate. If $E_F \leq 0$ ($E_F \geq 0$), the average peak is to the left (right) of the status-quo policy, which is 0. Note that since F may be skewed, the median voter's peak may be to the left of E_F or to its right, as well as to the left of 0 or to its right. Then we can easily observe that

$$W(p) > W(0) \Leftrightarrow \begin{cases} E_F > \frac{p}{2} & \text{if } p \geq 0, \\ E_F < \frac{p}{2} & \text{if } p \leq 0. \end{cases} \tag{4}$$

The above expression is very intuitive. Consider the case $p > 0$ (the case $p < 0$ can be explained similarly). Clearly, any individual with peak to the left of 0 prefers the status quo to remain in place over implementing policy p. It is also clear that any individual with peak to the right of p prefers policy p over the status quo. What about those individuals whose peak is between 0 and p? Since the preferences represented by (1) are single-peaked, there is a number, $p/2$, such that all those with a peak to the left of $p/2$ prefer the status quo over policy p and all those with a peak to the right of $p/2$ prefer policy p over the status quo (see also Propositions 1 and 2 below). From a utilitarian perspective, it is then easy to see that the relative position of E_F with respect to $p/2$ determines whether policy p is socially preferred to the status quo—if $E_F > p/2$, in which case the citizen preferences are sufficiently biased towards large peak values—or not—if $E_F < p/2$, in which case the citizen preferences are sufficiently biased towards low peak values.

For a given E_F, Expression (4) enables us to partition the interval of all policies $[-A, A]$ into three smaller intervals, with the middle interval containing the policies that are socially preferred to the status quo and the other two intervals containing the remaining policies. The latter are *not* preferred to the status quo. This implies, in turn, that given F and hence E_F, we can characterize the decisions that are optimal from a social perspective among all policies $p \in [-A, A]$ by the middle interval, which we denote as (L^o, R^o) (o stands

[13] As a non-essential tie-breaking rule, we therefore assume that the status quo should prevail if it yields the same aggregate utility as policy p.

for "optimal"). In Figure 1 we show this partition graphically for the case where $E_F \leq 0$.[14] Note that $0 \notin (L^o, R^o)$, i.e., the status quo never belongs to the middle interval.

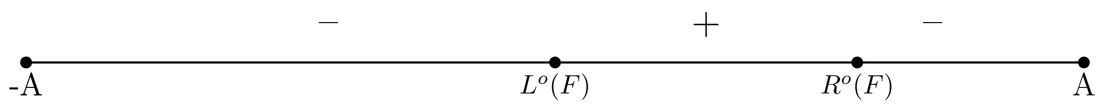

Figure 1: The case $E_F \leq 0$. Sign + means that the policies in this interval should be implemented according to our welfare measure, while sign − means that the policies in this interval should not be implemented.

To compute the above-mentioned middle interval, we can use Expression (4). First, assume that $E_F \leq 0$. Then $L^o(F) = 2E_F \leq 0$ and $R^o(F) = 0$. This means that policy proposals in the interval $[-A, 2E_F]$ should not be implemented, policy proposals in the interval $(2E_F, 0)$ should be implemented, and policy proposals in the interval $[0, A]$ should not be implemented. Second, assume that $E_F \geq 0$. Then $L^o(F) = 0$ and $R^o(F) = 2E_F \geq 0$. This means that policy proposals in the interval $[-A, 0]$ should not be implemented, policy proposals in the interval $(0, 2E_F)$ should be implemented, and policy proposals in the interval $[2E_F, A]$ proposals should not be implemented.

4.3 The constitutional rule

Next we introduce different decision-making rules at the constitutional level. These rules (or constitutions) are characterized by parameter s, with $s \in [0, 1]$. For each s, we let the constitution specify the decision-making rule that specifies the following course of events for *all* policy proposals $p \in [-A, A]$:

(a) All citizens vote in favor of either p or p_0, and simultaneously

(b) all members of parliament either vote in favor of p or p_0.

(c) Then p is implemented if and only if

$$(1 - s) \cdot \sigma_{DD} + s \cdot \sigma_{RD} \geq \frac{1}{2},$$

where we use σ_{DD} to denote the share of votes cast in favor of p within the electorate and σ_{RD} to denote the share of votes cast in favor of p within the parliament.[15]

[14] The case $E_F \geq 0$ is similar.

[15] In general, one can define a decision-making rule for each $s \in [0, 1]$ but the main insights can be conveyed with $s \in \{0, 1/2, 1\}$. Similarly, using qualified majority rules instead of simple majority rules does not affect the main logic of our results.

Clearly, $s = 1$ corresponds to representative democracy and $s = 0$ corresponds to direct democracy, which serve as benchmarks for our analysis of Co-Voting. Co-Voting considers $s = 1/2$, so it can be seen as a convex combination of direct democracy and representative democracy.

5 Analysis

In this section we characterize the voting decisions of parliament members (henceforth MP) and citizens. We assume that all individuals cast a vote and hence do not abstain (or that the decision to turn out is exogenously given and independent of preferences), and that they do not use weakly dominated strategies. In our setup with two alternatives, viz. p and p_0, this means that our equilibrium notion is Nash equilibrium of the underlying one-shot game, given the value of s, in which all individuals vote (sincerely) for the alternative from which they expect higher utility. As a further, non-essential tie-breaking rule, we assume that an individual votes for the status quo if s/he is indifferent between policy p and p_0. We also proceed on the assumption that policy proposal p is exogenously given. In Section 7 we will assume this away and consider endogenously chosen policies p.

We start with the analysis of MPs' decision. We do not consider the case $p = 0$, since it is trivial.

Proposition 1. *Consider a parliament member with peak j. Then,*

(i) if $p > 0$, s/he votes for p if and only if $j > \frac{p}{2}$, and

(ii) if $p < 0$, s/he votes for p if and only if $j < \frac{p}{2}$.

Proof. See Appendix. □

Accordingly, we obtain

$$\sigma_{RD} = \begin{cases} 1 - G\left(\frac{p}{2}\right) & \text{if } p \geq 0, \\ G\left(\frac{p}{2}\right) & \text{if } p < 0. \end{cases} \tag{5}$$

Note that the share of votes is *not* continuous at the status-quo policy 0, unless $G(0) = 1/2$.

As for the citizens' voting decision, it is useful to define

$$t(p) := \begin{cases} \frac{\sigma^2(p)}{2\mu(p)} & \text{if } p \neq 0, \\ 0 & \text{if } p = 0. \end{cases} \tag{6}$$

Proposition 2. *Consider a citizen with peak i. If s/he is informed, then*

(i) if $p > 0$, s/he votes for p if and only if $i > \frac{p}{2}$, and

(ii) if $p < 0$, s/he votes for p if and only if $i < \frac{p}{2}$.

If s/he is uninformed, then

(iii) if $p > 0$, s/he votes for p if and only if $i > t(p)$, and

(iv) if $p < 0$, s/he votes for p if and only if $i < t(p)$.

Proof. See Appendix. □

It then follows that

$$\sigma_{DD} = \begin{cases} q \cdot \left[1 - F\left(\frac{p}{2}\right)\right] + (1-q) \cdot \left[1 - F\left(t(p)\right)\right] & \text{if } p > 0, \\ q \cdot F\left(\frac{p}{2}\right) + (1-q) \cdot F\left(t(p)\right) & \text{if } p < 0. \end{cases} \tag{7}$$

As in the case of parliament members, the share of citizens who vote in favor of the proposal is generically not continuous at $p = 0$. The voters' aggregate decision then depends on two elements. First, which is the share of uninformed citizens and what are their beliefs? Second, what is the distribution of peaks for all citizens no matter the information they possess?

6 Comparing Decision-making Rules

In this section, we first compare the conditions for which a given policy proposal p will be chosen over the status quo in the three different decision-making rules under scrutiny. Namely, these rules are representative democracy ($s = 1$), direct democracy ($s = 0$), and Co-Voting ($s = 1/2$). Subsequently we assume that all the policies are equally likely to be proposed and investigate the conditions under which Co-Voting is preferable for society to the other two decision-making rules. This is done both for general preference distribution functions F and G and general random variable $X(p)$, but also for a particular, insightful case. Throughout our analysis, A is assumed to be large enough, to have all left and right endpoints of the intervals L^z and R^z are internal. We also proceed on the following assumption on $X(p)$:

Assumption 1. *The function $t(p)$ is increasing in p.*

Assumption 1 is mostly technical in nature but facilitates our analysis substantially. It is also sensible, as it requires that uncertainty about the consequences of policies increases more than $\mu(p)$ when p shifts further away from the status quo. This appears to be the case often, as large policy changes tend to be associated with more uncertainty (see e.g. Callander, 2011).

6.1 A given policy proposal

Due to the similar way in which they are defined, we can analyze the outcome under each of the three decision-making rules for a given policy proposal p at once. We only need to distinguish two cases, depending on whether $p > 0$ or $p < 0$. The case $p = 0$ is trivial and is thus not considered.

6.1.1 Policies $p > 0$

For $s \in [0, 1]$, Conditions (5) and (7) imply that policy p will be implemented under the decision-making rule defined by parameter s if and only if

$$\Phi(p) := s \cdot G\left(\frac{p}{2}\right) + (1 - s) \cdot \left[q \cdot F\left(\frac{p}{2}\right) + (1 - q) \cdot F(t(p))\right] < \frac{1}{2}. \tag{8}$$

On the one hand, due to Assumption 1, function $\Phi(p)$ is increasing in p. We will use this property later. On the other hand, for a given p, $\Phi(p)$ and thus the left-hand side of Equation (8) are linear in s. This implies that, for a given p, either $s = 0$ or $s = 1$ must yield the maximum social welfare among all decision-making rules defined by parameter $s \in [0, 1]$.

Corollary 1. *For a given $p > 0$, either representative or direct democracy yields the best possible outcome from a social welfare perspective among all decision rules characterized by $s \in [0, 1]$.*

Proof. See Appendix. □

Two comments are in order. First, beyond direct democracy and representative democracy, other decision-making rules, and Co-Voting, in particular, can also yield the best possible outcome. Second, there are constellations of the primitives of the model for which no decision-making rule will choose the policy that is preferable from a utilitarian welfare perspective.

6.1.2 Policies $p < 0$

For policies $p < 0$, policy p will be implemented by the decision-making rule defined by parameter s if and only if

$$\frac{1}{2} < \Phi(p), \tag{9}$$

where $\Phi(p)$ has been defined in Condition (8).

The following corollary can then be proved following the lines of the proof of Corollary 1.

Corollary 2. *For a given $p < 0$, either representative democracy or direct democracy yields the best possible outcome in terms of social welfare among all decision rules characterized by $s \in [0, 1]$.*

6.2 Arbitrary policy proposals

We now compare the performance of the three decision-making rules regarding the ability to implement the utilitarian optimal decision under the assumption that the same decision-making rule will be applied for all $p \in [-A, A]$. Alternatively, one can assume that at the moment in which a decision-making procedure is put into place, it is uncertain which policy will be proposed. Formally, given a decision-making rule defined by parameter $s \in [0, 1]$, which we denote by ρ_s, define

$$\pi^s := \mu\left(\{p \in [-A, A] : \rho_s \text{ implements } p \Longleftrightarrow W(p) > W(0)\}\right), \tag{10}$$

where μ denotes the Lebesgue measure. For convenience, we denote $\pi^r = \pi^1$, $\pi^d = \pi^0$, and $\pi^c = \pi^{1/2}$.

We are now in a position to introduce an order to compare decision-making rules.[16]

Definition 1. *Let $s, s' \in [0, 1]$. Then we say that the decision-making rule ρ_s dominates the decision-making rule $\rho_{s'}$ if and only if $\pi^s \geq \pi^{s'}$.*

If the inequality is strict, we say that domination between rules is strict. Because either $\Phi(0) \geq 1/2$ or $\Phi(0) \leq 1/2$, it is easy to see that continuity of $\Phi(p)$ implies that one of the following two cases must hold.

First, assume that there is some policy $p > 0$ such that Condition (8) holds for this policy. Then, this same condition must also hold for any other p', such that $0 < p' < p$. This

[16] Other orders yield the same insights regarding the relative merits of Co-Voting compared to direct democracy and representative democracy.

is due to the monotonicity of the probability distribution functions and that of function $t(p)$, which ensure that $\Phi(p)$ is an increasing function of p, as we have already discussed. Moreover, this latter property implies that Condition (9) cannot hold for any $p' < 0$.

Second, assume that there is some policy $p < 0$ such that Condition (9) holds for this policy. Then, as in the previous case, this same condition must also hold for any p', such that $p < p' < 0$. Moreover, Condition (8) cannot hold for any $p' > 0$.

From the above it therefore follows that for any $s \in [0, 1]$, the set of policies that are approved under decision-making rule ρ_s is an open interval of $[-A, A]$ with either its supremum or its infimum being zero. To illustrate this fact, Figure 2 shows the partition for each decision-making rule.

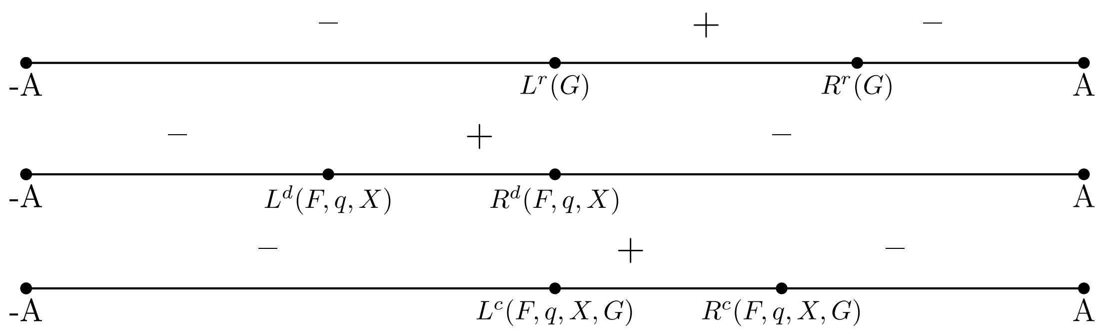

Figure 2: In the top line, the subdivision indicates the outcome under representative democracy. In the middle line, the subdivision indicates the outcome under direct democracy. In the bottom line, the subdivision indicates the outcome under Co-Voting. A "+" sign indicates that p will be implemented by the corresponding decision-making rule, while a "−"sign indicates that the status-quo policy 0 will remain in place.

Any of the partitions in Figure 2 can be characterized by the boundaries of the middle interval. For representative democracy, we have denoted the middle interval by (L^r, R^r), for direct democracy, by (L^d, R^d), and for Co-Voting, by (L^c, R^c). These six parameters can be computed as described next. For simplicity, we proceed for the remainder of the paper on the assumptions that

$$F(0), G(0) \neq \frac{1}{2}$$

and

$$F(0) + G(0) \neq 1.$$

These conditions ensure that the middle intervals are non-empty for each of the three decision-making rules.

6.2.1 Co-Voting

First we consider Co-Voting. On the one hand, if the boundary condition

$$\frac{1}{2} \cdot G(0) + \frac{1}{2} \cdot F(0) < \frac{1}{2}$$

holds, then $L^c = 0$ and $R^c = R^c(F, q, X, G)$ solves the following equation:

$$\frac{1}{2} \cdot G\left(\frac{R^c}{2}\right) + \frac{1}{2} \cdot \left(q \cdot F\left(\frac{R^c}{2}\right) + (1-q) \cdot F(t(R^c))\right) = \frac{1}{2}.$$

On the other hand, if the boundary condition

$$\frac{1}{2} \cdot G(0) + \frac{1}{2} \cdot F(0) > \frac{1}{2}$$

holds, then $R^c = 0$ and $L^c = L^c(F, q, X, G)$ solves the following equation:

$$\frac{1}{2} \cdot G\left(\frac{L^c}{2}\right) + \frac{1}{2} \cdot \left(q \cdot F\left(\frac{L^c}{2}\right) + (1-q) \cdot F(t(L^c))\right) = \frac{1}{2}.$$

6.2.2 Direct democracy

Second, consider direct democracy. On the one hand, if the boundary condition

$$F(0) < \frac{1}{2}$$

holds, then $L^d = 0$ and $R^d = R^d(F, q, X)$ solves the following equation:

$$q \cdot F\left(\frac{R^d}{2}\right) + (1-q) \cdot F(t(R^d)) = \frac{1}{2}.$$

On the other hand, if the boundary condition

$$F(0) > \frac{1}{2}$$

holds, then $R^d = 0$ and $L^d = L^d(F, q, X)$ solves the following equation:

$$q \cdot F\left(\frac{L^d}{2}\right) + (1-q) \cdot F(t(L^d)) = \frac{1}{2}.$$

6.2.3 Representative democracy

Third and last, consider representative democracy. On the one hand, if the boundary condition

$$G(0) < \frac{1}{2}$$

holds, then $L^r = 0$ and $R^r = R^r(G)$ solves the following equation:

$$G\left(\frac{R^r}{2}\right) = \frac{1}{2}.$$

On the other hand, if the boundary condition

$$G(0) > \frac{1}{2}$$

holds, then $R^r = 0$ and $L^r = L^r(G)$ solves the following equation:

$$G\left(\frac{L^r}{2}\right) = \frac{1}{2}.$$

6.2.4 Comparing the three rules

According to the above, the three middle intervals are defined in terms of distributions F and G, function $t(p)$, and parameter q. Finding which of the three decision-making rules yields higher welfare is then equivalent to determining which of these rules yields a partition whose three intervals $(-,+,-)$ match the sign of the intervals shown in Figure 1 for a subset of $[-A, A]$ of a larger Lebesgue measure. Formally, in the case of Co-Voting, we obtain that

$$\pi^c = |\min\{L^o, L^c\} + A| + |\min\{R^o, R^c\} - \max\{L^o, L^c\}| + |A - \max\{R^o, R^c\}|$$

is the Lebesgue measure of the set of policy proposals where Co-Voting yields the socially optimal decision. One can define π^r and π^d analogously for representative democracy and direct democracy, respectively. Because each of the three middle intervals always has an endpoint that is zero—and hence coincides with the status quo—, finding necessary and sufficient conditions in terms of the primitives of the model that guarantee

$$\pi^c \geq \max\{\pi^r, \pi^d\} \tag{11}$$

is the following combinatorial problem: First, we need to fix one endpoint of each middle interval to zero. Second, we solve for the other endpoint to obtain closed expressions for π^c, π^r, π^d. Third and last, we impose Condition (11) to obtain the desired condition.[17] Hence, although the number of configurations of the middle intervals that we have to consider is finite, it can be very large for arbitrary distributions F and G and arbitrary functions $t(p)$. For an illustration, consider the configuration in Figure 3, and assume further that $R^r(G) < R^o(F)$ and $R^c(F, q, X, G) < R^o(F)$.

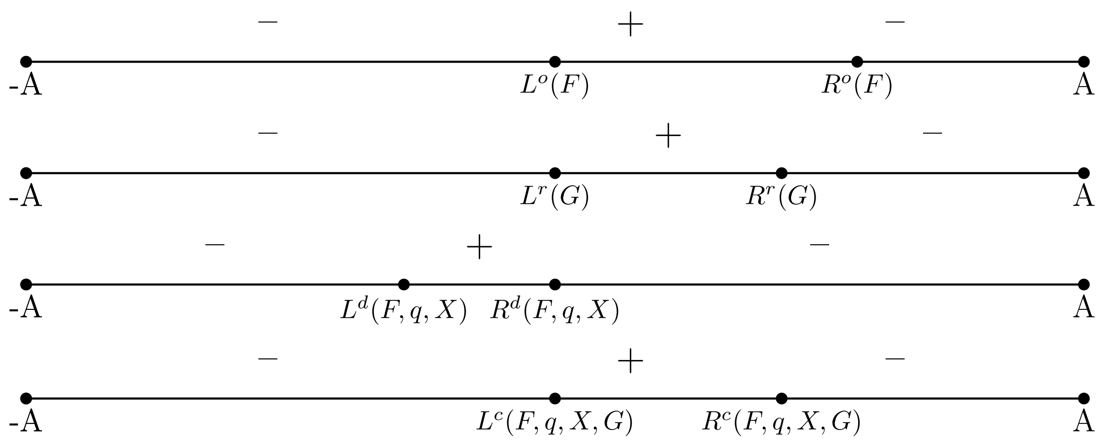

Figure 3: On the top line, it is assumed that $L^o(F) = 0$ and $R^o(F) > 0$, on the second line from the top, it is assumed that $L^r(G) = 0$ and $R^r(G) > 0$. On the third line, $R^d(F, q, X) = 0$ and $L^d(F, q, X) < 0$. On the last line, $L^c(F, q, X, G) = 0$ and $R^c(F, q, X, G) > 0$.

In this configuration, we have

$$
\begin{aligned}
\pi^r &= 2A + R^r(G) - R^o(F), \\
\pi^d &= 2A + L^d(F, q, X) - R^o(F), \\
\pi^c &= 2A + R^c(F, q, X, G) - R^o(F, q, X, G).
\end{aligned}
$$

Then, Condition (11) can be rewritten simply by plugging the values of the endpoints of the middle intervals set out in the above three equations.[18]

In general, we have the following result:[19]

Theorem 1. *There is a uniquely determined set of necessary and sufficient conditions in terms of F, G, q, and X for which Condition* (11) *holds.*

[17]The same applies if we want impose that Condition (11) must hold strictly, in which case Co-Voting will be strictly preferred to the other two decision-making rules.

[18]Note that it is possible for R^x to be either larger than A or smaller than $-A$. In the former case, we take $R^x = A$ and in the latter we take $R^x = -A$.

[19]In the proof of Theorem 1 the maximum total number of configurations is shown to be 96.

Proof. See Appendix. □

6.2.5 Uniform preference distributions and informational bias

Theorem 1 is very general as it imposes a very mild structure on the problem at hand. In this section, we add further structure by making a number of assumptions on F, G, and X. These assumptions have a nice interpretation and allow an application of the main insights behind Theorem 1. For our application we let $U([k_1, k_2])$ denote a uniform random variable on $[k_1, k_2]$. We also let $\delta(p)$ denote a (delta) random variable on $[-A, A]$ whose outcome is p with probability one.

Assumption 2. *Given $e, b \geq 0$ and $b \leq \frac{e}{3}$,*

$$X(p) \sim \begin{cases} U([p-b-e, p-b+e]) & \text{if } p > e+b, \\ \delta(p) & \text{otherwise.} \end{cases}$$

Assumption 2 deals with the electorate's informational deficit. It means that when gauging a new policy proposal that is to the right of the status quo, uninformed citizens have a bias towards believing that such a policy is closer to the status quo than it really is. The bias is defined by parameter b and can exist for a variety of reasons as discussed above. For its part, parameter e captures the noise level regarding the information about p. If $e = 0$, there is no noise and uninformed citizens believe with probability one that the policy labelled p corresponds to the actual policy $p - b$. Policies that are either very close to the status quo 0 or to its left are always gauged perfectly by all citizens. This is the case when policies left to the status quo have been in place in the past, while policies to the right of the status quo have never been implemented. One example is a policy space $[-A, A]$ that captures integration into the multinational political and/or economic world—the more the policy is to the right, the higher the level of integration level. The condition $b \leq \frac{e}{3}$ makes sure that $t(p)$ function is increasing in p.

Assumption 3. *Let $B, w > 0$ be such that*

$$(2 + w)B < A. \tag{12}$$

Then,

$$F \sim U([B, 2B])$$

and

$$G \sim U([(1 + w)B, (2 + w)B]).$$

Assumption 3 addresses the preference bias of parliament members relative to citizens. Assuming that preferences are uniformly distributed, it implies that parliament members' preferences are biased to the right, with Bw being the size of such bias. It also implies that all citizens and parliament members prefer policies to the right of the status quo. Hence, all decision-making rules will prescribe that policies to the left of the status quo will *not* be implemented. This means that the relative merits of each decision-making rule in terms of welfare will depend on whether or not this rule prescribes the socially optimal decision for choices between the status quo and policies that are to the right of the status quo. Finally, Condition (12) ensures that no individual has a preferred policy that is beyond the rightmost policy A.

The above means that the primitives of this model specification are B, q, b, e, w, although henceforth we assume $B = 1$. We can do this normalization without loss of generality. We also proceed on the assumption that

$$1 > e + b, \tag{13}$$

which guarantees that Condition (2) holds, i.e., that citizens can tell whether a policy is to the right or to the left of the status quo.

We then obtain the following result:

Theorem 2. *Under Assumptions 2 and 3, if Condition* (13) *holds, Co-Voting yields higher welfare than representative democracy and direct democracy if and only if*

$$|3 - R^c| \leq |3 - R^d| \quad \text{and} \quad |3 - R^c| \leq |3 - R^r|,$$

where

$$R^d = R^d(w, q, e, b) = \frac{6b + 9 - 3qb + \sqrt{(6b + 9 - 3qb)^2 - 12(3(1-q)b^2 + (1-q)e^2 + 9b)}}{6}$$

and

$$R^c = R^c(w, q, e, b) =$$
$$\frac{9b - 3bq + 6(3 + w) + \sqrt{(9b - 3bq + 6(3 + w))^2 - 24((1-q)e^2 + 3(1-q)b^2 + 6b(3 + w))}}{12}.$$

Moreover, the set of 4-tuples $(w, q, e, b) \in [0, 1]^4$ satisfying both inequalities strictly has positive Lebesgue measure.

Proof. See Appendix. □

Theorem 2 characterizes the situations in which Co-Voting dominates both representative democracy and representative democracy by means of two inequalities involving parameters w, q, e, b. It shows that despite the simplicity of Assumptions 2 and 3, such inequalities involve complicated expressions that can be difficult to interpret. Hence, to make more sense of Theorem 2 we derive in what follows a series of comparative static results on the set of parameters w, q, e, b that yield more clear insights about the merits of Co-Voting.

Our first result shows that if the information noise level e is high enough and the informational bias level b is low enough—and zero, in particular—, Co-Voting dominates both benchmark voting rules if and only if the preference bias w is not very large.

Proposition 3. *For any fixed $q \in (0, 1/2)$, there exist thresholds $e(q) := 1/\sqrt{2(1-q)} < 1$ and $b(q) > 0$ so that for any $e \geq e(q)$ and $b \leq b(q)$, Co-Voting dominates both direct democracy and representative democracy if and only if*

$$0 \leq w \leq w(q),$$

where $w(q) > 0$.

Proof. See Appendix. □

Proposition 3 addresses low, or even zero, values of b, with values of e that are large enough. The latter parameter captures the level of the noise (or variance) that uninformed citizens must deal with when trying to gauge the extent of policies that are sufficiently to the right of the status quo. Since $q < 1/2$, and therefore more than half of the citizens are uninformed, assuming a large value of e guarantees that citizens alone are unable to reach socially optimal decisions. For such a setup, if the preference bias w is not very large, Co-Voting can effectively balance lack of information on the part of the citizens with the more informed parliament members. If w is too large, however, Co-Voting outcomes are too biased themselves and citizens are better off with direct democracy.

Our second result analyzes the performance of Co-Voting compared to direct democracy and representative democracy depending on varying levels of information noise level e.

Proposition 4. *For any fixed $q \in (0, 1)$, there are thresholds $w_1(q) > 0, w_2(q) < 0$ and $b(q) > 0$ so that for any $w_1(q) < w \leq w_2(q)$ and $b \leq b(q)$, Co-Voting dominates both direct democracy and representative democracy if and only if*

$$e_l(q) \leq e \leq e_h(q),$$

with $0 < e_l(q) < e_h(q)$.

Proof. See Appendix. □

Proposition 4 addresses low—or even zero—values of b and low values of w. The latter parameter captures the extent to which parliament members' preferences are biased, compared to those of the citizens. For such a setup, if information noise level e is neither very large nor very small, Co-Voting can effectively balance the citizens' lack of information with the more informed parliament members. Yet, these parliament members have biased preferences. If e is too large, Co-Voting outcomes are too noisy, and citizens are better off with representative democracy. If e is too small, Co-Voting outcomes are too biased, as they rely on parliament members. Then, citizens are better off with direct democracy.

For the last result of this section, we assume that $e \approx 0$, i.e., there is no noise (or variance) in what uninformed citizens believe. There is only a perception bias. Such a bias is captured by parameter b. For simplicity, we also assume that $q = 1/2$, i.e., 50% of the citizens are uninformed. We then obtain the following result:

Proposition 5. *For given $b > 0$, Co-Voting dominates both direct democracy and representative democracy if*

$$w_l(b) \leq w \leq w_h(b),$$

where $0 < w_l(b) < w_h(b)$. Moreover, for $w \in [w_l(b), w_h(b)]$,

(i) the margin in social welfare by which Co-Voting dominates direct democracy is first increasing in w and then decreasing, and

(ii) the margin in social welfare by which Co-Voting dominates representative democracy in w is first increasing in w and then decreasing.

The above result shows that even if there is no information noise, i.e. $e = 0$, Co-Voting can be desirable from a social welfare perspective for *any* value of the information bias b. It suffices that the extent to which the preferences of parliament members are biased, i.e. w, is neither very large nor very small. If w is too large, citizens are better off under direct democracy, as both Co-Voting and representative democracy yield outcomes that are too biased. If w is too small, citizens are better off under representative democracy, as both Co-Voting and direct democracy yield outcomes that are often wrong, being based on wrong beliefs. Recall that we have assumed that half of the citizenry is uninformed, i.e., $q = 1/2$. Finally, items *(i)* and *(ii)* from Proposition 5 indicate that increasing w from low values increases the relative merits of Co-Voting, but that increasing such parameter further decreases such merits. It thus implies that there is some w that maximizes the difference in social welfare between Co-Voting and each of the two benchmark decision-making procedures separately.

7 Endogenous Policy Proposals

In the previous section, we assumed for simplicity that all policies are equally likely, but the results carry over qualitatively to distributions that are less uniform but have full support on $[-A, A]$. Such preference distributions can emerge if, e.g., preferences themselves are subject to future shocks. Yet, for given aggregate preferences, the institutional design of proposal-making can lead to very different distributions of policies that can be pitted against the status quo. In this section we consider the standard case where parliament chooses which policy alternative p will be put to vote against the status quo. For the voting decision between the (endogenously chosen) policy proposal and the status quo, one can then either use Co-Voting ($s = 1/2$), direct democracy ($s = 0$), or representative democracy ($s = 1$). For each $s \in \{0, 1/2, 1\}$, we consider the following dynamic game:

1. Stage 1: The parliament makes policy proposal p, with $p \in [-A, A]$.
2. Stage 2: A decision between p and the status quo is taken according to the decision-making rule defined by parameter s.

Since all members of parliament have single-peaked preferences, we obtain a simple characterization of the proposals that will be made by parliament depending on the decision-making rule that is in place. From our analysis of Section 5, we know that parliament only wants to implement policies from the interval (L^r, R^r). This interval depends on G, which is the distribution of peaks among parliament members. Hence, a necessary condition for a policy to be proposed by parliament is that it belongs to (L^r, R^r). Under the assumption that arbitrary distributions G must be considered, this condition is also sufficient if the decision-making procedure is representative democracy. But it is neither sufficient under direct democracy nor under Co-Voting. In either case, parliament members will only propose policy alternatives that are not bound to be rejected. It suffices to assume that proposal-making decisions followed by votes are sequentially rational and that all else being equal, proposal-makers prefer to propose a winning proposal to a losing proposal.

That is, we have shown the following result:

Proposition 6. *The parliament will propose a policy proposal p with the following properties:*

(i) $p \in [L^r, R^r]$ *in the case of representative democracy,*

(ii) $p \in [L^r, R^r] \cap [L^d, R^d]$ *in the case of direct democracy,*

(iii) $p \in [L^r, R^r] \cap [L^c, R^c]$ *in the case of Co-Voting.*

Some comments are in order. First, we note that $0 \in [L^r, R^r] \cap [L^c, R^c] \cap [L^d, R^d]$. This means that although parliament will always make a proposal, it will propose the status quo in some cases. This is equivalent to no proposal being made. In general, what proposal is made will then depend on who the median legislator is—when a non-qualified majority of votes is required—and, hence, on the specifics of distribution G. For this reason we assume from now on that any policy proposal as determined in Proposition 6 is equally likely to be made by parliament. This is in line with our analysis from the previous section and corresponds to the ideas that *(i)* institutions are in place for a long period of time over which many policies will have to be proposed, and *(ii)* from a constitutional perspective, institutions must be chosen under the veil of ignorance. We divide our analysis into two polar situations and we show that Co-Voting performs well in both situations. This adds to the potential of Co-Voting as a democratic decision-making rule.

7.1 Almost informed voters

First, we assume that voters are relatively well informed. Specifically, we consider:

Assumption 4. *Given* $e \geq 0$*,*

$$X(p) \sim \begin{cases} U([p-e, p+e]) & \text{if } |p| > e, \\ \delta(p) & \text{otherwise.} \end{cases}$$

Assumption 4 implies that when gauging a new policy proposal which is not close to the status quo, uninformed citizens cannot tell well what the consequences of p will be. Parameter e simply defines the noise level of information about p. Policies that are close to the status quo are, however, assessed perfectly by all citizens.

As in Section 6, we assume that parliament members' and citizens' peaks (or preferences) are distributed uniformly, and that the distribution for parliament members is biased to the right by some amount B.[20]

[20]Other parametric distributions yield similar insights.

Assumption 5. *Let $B > 0$ such that $3B \leq A$. Then,*

$$F \sim U([-3B, 2B])$$

and

$$G \sim U([-2B, 3B]).$$

We obtain the following result:

Proposition 7. *Let $B > 0$ be given and assume that $q = 1/2$. Under Assumptions 4 and 5, if the parliament has the right to make proposals, there is $e(B) > 0$ such that if $e \leq e(B)$,*

(i) Co-Voting strictly dominates representative democracy, and

(ii) Co-Voting dominates direct democracy.

Proof. See Appendix. □

That is, if all citizens are either informed or almost informed, Co-Voting is preferable to representative democracy from a social welfare perspective and cannot do worse—in expected welfare terms—than direct democracy. We stress that as e goes to zero, direct democracy approaches the socially optimal decision, since no mistakes will occur on the part of the voters. In such cases, Co-Voting will *merely* replicate direct democracy.

7.2 Almost uninformed voters

For the second polar case, we maintain Assumption 5 on the citizens' and parliament members' preference distributions, and we consider the following assumption on information:

Assumption 6.

$$X(p) \sim \begin{cases} U([0, A]) & \text{if } p > 0, \\ U([-A, 0]) & \text{if } p < 0, \\ \delta(p) & \text{if } p = 0. \end{cases}$$

Assumption 6 means that uninformed voters cannot tell the extent of policies, and can only tell their sign.

We obtain the following result:

Proposition 8. *Under Assumptions 5 and 6, if the parliament has the right to make proposals, there is $B(A)$ such that if $B \leq B(A)$, then* *(i) Co-Voting strictly dominates representative democracy, and*

(ii) Co-Voting dominates direct democracy.

Proof. See Appendix. □

According to the above result, when uninformed citizens are clueless about the extent of policies and parliament members have biased preferences, Co-Voting is preferable to representative democracy from a social welfare perspective and cannot do worse—in expected welfare terms—than direct democracy. Yet, the reason is very different from the above case of almost informed voters. Uninformed voters being clueless entails that no other policy than the status quo is implemented in direct democracy. When Co-Voting is in place, some policies might be approved, which could result in a welfare increase compared to the status quo. These policies are never going to be proposed by parliament. Instead, parliament will only propose socially undesirable policies.

7.3 Other potential benefits of Co-Voting

In this section we have so far analyzed the performance of Co-Voting relative to direct democracy and representative democracy when policy-proposal is in the hands of parliament. We have shown that Co-Voting is robust against its strategic considerations. In real-world political environments, other strategic actors beyond parliament members might affect the fundamentals/primitives of our model. The media could strategically determine the values of b (information bias) and e (information noise), or even of q (the share of uninformed citizens). In turn, lobbying groups could strategically determine the value of w (preference bias). It turns out that Co-Voting can also be appealing as a (constitutional) decision-making rule in such cases, as it can be robust against both types of strategic considerations simultaneously. By definition, direct democracy is robust against lobbying groups, but it can be manipulated by media. As to representative democracy, it is robust against the media, but it can be manipulated by lobbying groups. The robustness of Co-Voting hinges on the fact that it is a convex combination of both representative democracy and direct democracy.[21]

[21] Khoshnama (2021) shows that Co-Voting remains appealing even if citizens differentially abstain.

8 Conclusion

We have provided the first formal analysis of a Co-Voting, a new decision-making rule, by analyzing a simple model that generates a fundamental trade-off between representative democracy and direct democracy. Our results have put forward some theoretical reasons why Co-Voting could be used in real-world environments to mitigate the deficiencies of standard democratic decision-making rules from a constitutional perspective when the parliament's and the electorate's preferences are misaligned, on the one hand, and when a significant share of the population lacks information about the consequences of a given policy, on the other hand.

For our analysis, we have imposed by design equal weight between parliament and the electorate. While this is a sensible assumption, other weights could also be considered, and some of them might be preferable from a social perspective. Our analysis also side-passed the individual incentives to become informed. These aspects might be very relevant for the assessment of Co-Voting as a decision-making rule, and must therefore be considered in future research.

Finally, several other aspects of Co-Voting are promising for future research. For instance, one could decide to endogenize the weights of parliament and the electorate in Co-Voting. One possibility would be to give all citizens the choice to either vote as a regular citizen or to delegate his/her voting power to the parliament as a whole, say, by increasing the weight parliament is given in Co-Voting.

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Appendix

Proof of Proposition 1. An MP with peak j prefers p over $p_0 = 0$ if and only if

$$u_j(p) = -(p-j)^2 > u_j(p_0) = -j^2. \tag{14}$$

The above inequality is equivalent to $p(p-2j) < 0$. Hence, if $p > 0$, it must be the case that $j > \frac{p}{2}$ for the MP to vote in favor of alternative p. If $p < 0$, it must be the case that $j < \frac{p}{2}$ for the MP to vote in favor of alternative p.

□

Proof of Proposition 2. If citizen with peak i is informed, which happens with probability q, then his/her voting behavior is described by Proposition 1. If s/he is uninformed, which then happens with probability $1-q$, the voting decision is more involved. The utility from the status-quo is

$$E[u_i(0)] = -i^2$$

and the expected utility from the proposal p is

$$E[u_i(X(p))] = -\int_{-\infty}^{\infty}(l-i)^2 h(l)dl = -i^2 - \sigma^2(p) + 2i\mu(p),$$

where $h(l)$, with $l \in \mathbb{R}$, is used to denote the density function of random variable $X(p)$. Accordingly,

$$E[u_i(X(p))] > E[u_i(0)] \Longleftrightarrow 2i\mu(p) > \sigma^2(p). \tag{15}$$

Hence, if $p > 0$, it must be the case that $i > \frac{\sigma^2(p)}{2\mu(p)} = t(p)$ for the citizen to vote in favor of alternative p. If $p < 0$, it must be the case that $i < \frac{\sigma^2(p)}{2\mu(p)} = t(p)$ for the citizen to vote in favor of alternative p. Note that we have used Condition (2).

□

Proof of Corollary 1. The result of the corollary follows almost directly once we note that the left-hand side of Inequality (8) is linear in s. We repeat it for completeness,

$$s \cdot G\left(\frac{p}{2}\right) + (1-s)\cdot\left[q \cdot F\left(\frac{p}{2}\right) + (1-q)\cdot F(t(p))\right] < \frac{1}{2}. \tag{16}$$

To see this, we distinguish two main cases.

First, assume that policy p is preferred to the status quo from a welfare perspective. On the one hand, if

$$G\left(\frac{p}{2}\right) \leq q \cdot F\left(\frac{p}{2}\right) + (1-q) \cdot F(t(p)),$$

taking $s = 1$ ensures that Inequality (16) holds if such an inequality also holds for some $s \in [0, 1)$. If the latter condition is not satisfied, none of the voting procedures considered implements the utilitarian optimal solution and they all yield the same outcome. On the other hand, if

$$G\left(\frac{p}{2}\right) \geq q \cdot F(\frac{p}{2}) + (1-q) \cdot F(t(p)),$$

then taking $s = 0$ ensures that Inequality (16) holds if such an inequality also holds for some $s \in (0, 1]$. As before, if the latter condition is not satisfied, then none of the voting procedures considered implement the utilitarian optimal solution (and they all yield the same outcome).

Second, assume that the status quo is preferred to policy p from a social perspective. Then, one can use the above logic to see that if Inequality (16) does not hold for some $s \in (0, 1)$, it will not hold for $s = 0$ or $s = 1$ either.

□

Proof of Theorem 1. We want to find the necessary and sufficient conditions, such that $\pi^c \geq \pi^r$ and $\pi^c \geq \pi^d$. As mentioned in the main text, we have to consider eight different cases for which one of the endpoints of the three middle intervals must be zero. It turns out that two of these cases are impossible. In particular, because Co-Voting is a convex combination of $s = 0$ and $s = 1$ it is impossible that

- $L^d(F, q, X) = 0$, $L^r(G) = 0$ and $R^c(s, f, q, X, G) = 0$, and
- $R^d(F, q, X) = 0$, $R^r(G) = 0$ and $L^c(s, F, q, X, G) = 0$.

This leaves us with six possible configurations:

- $L^d(F, q, X) = 0$, $L^r(G) = 0$ and $L^c(s, f, q, X, G) = 0$.
- $L^d(F, q, X) = 0$, $R^r(G) = 0$ and $L^c(s, f, q, X, G) = 0$.
- $L^d(F, q, X) = 0$, $R^r(G) = 0$ and $R^c(s, f, q, X, G) = 0$.
- $R^d(F, q, X) = 0$, $L^r(G) = 0$ and $L^c(s, f, q, X, G) = 0$.
- $R^d(F, q, X) = 0$, $L^r(G) = 0$ and $R^c(s, f, q, X, G) = 0$.
- $R^d(F, q, X) = 0$, $R^r(G) = 0$ and $R^c(s, f, q, X, G) = 0$.

For each configuration, we have to consider two cases, depending on whether E_F is positive or negative. This means that we have to distinguish twelve cases. For each case, we further have to consider a number of subcases, because the relative positions of the endpoints of each of the three middle intervals defines a further subdivision of the policy space, indicating where each rule is performing optimally and where it fails. There are at most two subcases for each comparison, totalling at most $12 \cdot 2^3 = 96$ configurations. Summing up the lengths of the intervals where the rule performs optimally yields the efficiency—in terms of Lebesgue measure—of that voting rule, i.e., it yields π^c, π^r, π^d. By comparing these numbers, we can derive the necessary and sufficient conditions under which Co-Voting performs better than direct democracy and representative democracy, i.e., under which $\pi^c \geq \pi^r$ and $\pi^c \geq \pi^d$.

$\square$

Proof of Theorem 2. We proceed in three steps. First, we derive an expression for $t(p)$ and show that it is increasing in p. Second, we use this expression to find the endpoints of the middle intervals. Third, using the expressions for the endpoints, we investigate when Co-Voting performs (strictly) better than representative democracy and direct democracy. Although in the main text, we assume $B = 1$ for simplicity (and without loss of generality), we now proceed with an arbitrary value of $B \geq \frac{e+b}{2} > 0$, where $F \sim U([B, 2B])$. The same comment applies throughout the appendix.

Step 1: Technical derivations

Under the assumptions of the theorem, we obtain

$$F(x) = \begin{cases} 0 & \text{if } x \in [-A, B], \\ \frac{x-B}{B} & \text{if } x \in [B, 2B], \\ 1 & \text{if } x \in [2B, A], \end{cases}$$

and

$$G(x) = \begin{cases} 0 & \text{if } x \in [-A, (1+w)B], \\ \frac{x-(1+w)B}{B} & \text{if } x \in [(1+w)B, (2+w)B], \\ 1 & \text{if } x \in [(2+w)B, A]. \end{cases}$$

Now recall that

$$t(p) = \frac{\sigma^2(p)}{2\mu(p)}.$$

Since $p > e + b$,

$$\mu(X(p)) = p - b,$$

so that

$$\sigma^2(X(p)) = \int_{p-b-e}^{p-b+e} x^2 \frac{1}{2e} dx = \frac{1}{6e} x^3|_{p-b-e}^{p-b+e} = \frac{6(p-b)^2 e + 2e^3}{6e} = \frac{3(p-b)^2 + e^2}{3}.$$

This implies that

$$t(p) = \frac{3(p-b)^2 + e^2}{6(p-b)}.$$

We observe that $\frac{\partial t(p)}{\partial p} > 0$ for $p \geq e + b$. If $p < e + b$, it follows trivially that $\sigma^2(X(p)) = p^2$ and $\mu(X(p)) = p$, in which case

$$t(p) = \frac{p}{2}.$$

We have thus proved that $t(p)$ is an increasing function in p over the whole interval $[-A, A]$, and thus it satisfies Assumption 1.

Step 2: Finding the middle intervals' endpoints

We are now in a position to investigate the middle intervals, which will allow us to determine whether or not Condition (11) holds in terms of the primitives of this more structured model. We stress that these primitives are B, w, q, e, b.

We start by noting that

$$E_F = \int_{-\infty}^{\infty} i f(i) di = \frac{3}{2} B > 0.$$

This implies that for the socially optimal solution, the endpoints of the middle interval are

$$L^o = 0$$

and

$$R^o = 3B. \tag{17}$$

Second, in the case of representative democracy, we have

$$L^r(G) = 0,$$

while $R^r(G)$ must solve the equation $G(\frac{x}{2}) = \frac{1}{2}$. This means that

$$R^r(G) = (3 + 2w)B. \tag{18}$$

Note that here we assume A is large enough, namely, $A > (3 + 2w)B$.

Third, in the case of direct democracy, we have

$$L^d(F, q, X) = 0,$$

while $R^d(F, q, X)$ is the policy proposal p that solves the equation

$$qF(\frac{x}{2}) + (1-q)F(t(x)) = \frac{1}{2}.$$

Due to (13), it must be the case that p is a solution x of

$$q\left(\frac{\frac{x}{2} - B}{B}\right) + (1-q)\left(\frac{\frac{3(x-b)^2+e^2}{6(x-b)} - B}{B}\right) = \frac{1}{2},$$

assuming the following two conditions $B \leq x \leq 2B$ and $B \leq t(x) \leq 2B$.

The latter equation is equivalent to

$$3x^2 - (6b + 9B - 3qb)x + 3(1-q)b^2 + (1-q)e^2 + 9bB = 0.$$

If we let

$$u := 6b + 9B - 3qb$$

and

$$v := 3(1-q)b^2 + (1-q)e^2 + 9bB,$$

there are two solutions $x = \frac{u \pm \sqrt{u^2 - 12v}}{6}$. However, only the largest solution is valid, because the function $qF(x/2) + (1-q)F(t(x))$ is increasing in x. To sum up,

$$R^d(F, q, X) = \frac{6b + 9B - 3qb + \sqrt{(6b + 9B - 3qb)^2 - 12(3(1-q)b^2 + (1-q)e^2 + 9bB)}}{6}. \tag{19}$$

Finally, we calculate the endpoints of the middle interval for the case of Co-Voting. It follows that

$$L^c(F, q, X, G) = 0$$

and $R^c(F, q, X, G)$ is the solution x to

$$\frac{1}{2} \cdot G\left(\frac{x}{2}\right) + \frac{1}{2} \cdot \left(qF\left(\frac{x}{2}\right) + (1-q)F(t(x))\right) = \frac{1}{2}.$$

The latter equation is equivalent to

$$\left(\frac{\frac{x}{2}-(1+w)B}{B}\right)+q\left(\frac{\frac{x}{2}-B}{B}\right)+(1-q)\left(\frac{\frac{3(x-b)^2+e^2}{6(x-b)}-B}{B}\right)=1.$$

By simplifying the latter, we obtain

$$6x^2-(9b-3bq+6(3+w)B)x+(1-q)e^2+3(1-q)b^2+6bB(3+w)=0.$$

We let

$$m:=9b-3bq+6(3+w)B$$

and

$$n:=(1-q)e^2+3(1-q)b^2+6bB(3+w).$$

One can verify that the solution to this equation is equal to $\frac{m+\sqrt{m^2-24n}}{12}$. That is,

$$R^c(F,q,X,G)=\frac{9b-3bq+6(3+w)B+\sqrt{(9b-3bq+6(3+w)B)^2-24((1-q)e^2+3(1-q)b^2+6bB(3+w))}}{12}. \tag{20}$$

Step 3: Comparing decision-making rules

As mentioned in the main text, Condition (11) holds if and only if

$$|R^o-R^c|\leq|R^o-R^d| \tag{21}$$

and

$$|R^o-R^c|\leq|R^o-R^r|, \tag{22}$$

where R^o, R^c, R^d, R^r are given by Equations (17)–(20). Finally, take $B=1$, $q=0.1$, $w=0.04$, $e=0.79$ and $b=0.03$. Both Inequalities (21) and (22) hold strictly, so by continuity, the set of points satisfying both inequalities has non-zero measure. □

Before proceeding to the proofs of Propositions 3, 4, 5, we will compute derivatives of the right endpoints with respect to various parameters. These derivations will be useful in the proofs.

First, the derivatives of the socially optimal right endpoint are:

$$\frac{\partial R^o}{\partial e}=\frac{\partial R^o}{\partial b}=\frac{\partial R^o}{\partial w}=0. \tag{23}$$

Second, the derivatives of the representative democracy right endpoint are:

$$\frac{\partial R^r}{\partial e} = \frac{\partial R^r}{\partial b} = 0, \frac{\partial R^r}{\partial w} = 2B. \tag{24}$$

Third, the derivatives of the direct democracy right endpoint are:

$$\frac{\partial R^d}{\partial w} = 0, \frac{\partial R^d}{\partial e} = \frac{2e(1-q)}{12\sqrt{u^2-12v}}, \frac{\partial R^d}{\partial b} = \frac{6-3q}{12} + \frac{2u(6-3q)-12(6(1-q)b+9B)}{12\sqrt{u^2-12v}}, \tag{25}$$

where u and v are defined in the proof of the theorem 2.

Fourth and last, the derivatives of the Co-Voting right endpoint are:

$$\begin{aligned}
\frac{\partial R^c}{\partial w} &= 6B/12 + \frac{2m \cdot 6B + 6bB}{2 \cdot 12\sqrt{m^2-24n}}, \\
\frac{\partial R^c}{\partial b} &= \frac{9-3q}{12} + \frac{2m \cdot (9-3q) + 6(1-q)b + 6B(3+w)}{2 \cdot 12\sqrt{m^2-24n}}, \\
\frac{\partial R^c}{\partial e} &= \frac{-2 \cdot 24(1-q)e}{2 \cdot 12\sqrt{m^2-24n}},.
\end{aligned} \tag{26}$$

where m and n are defined in the proof of the theorem 2.

Proof of Proposition 3. We provide a proof for any $B > 0$. Take $e = \frac{B}{\alpha}$ for some α that is defined later, $w = 0.01$ and $b = 0$. Then

$$R^c = B\frac{18.06 + \sqrt{18.06^2 - 24(1-q)/\alpha^2}}{12}, R^d = B\frac{9 + \sqrt{9^2 - 12(1-q)/\alpha^2}}{6},$$

and $R^r = 3.02B$. Using these identities, $|R^c - R^o| < |R^d - R^o|$ and $|R^c - R^o| < |R^r - R^o|$ become equivalent to

$$\left|\frac{18.06 + \sqrt{18.06^2 - 24t}}{12} - 3\right| < \left|\frac{9 + \sqrt{9^2 - 12t}}{6} - 3\right|$$

and

$$\left|\frac{18.06 + \sqrt{18.06^2 - 24t}}{12} - 3\right| < 0.02.$$

To see this, it suffices to cancel B and use t to denote $\frac{1-q}{\alpha^2}$. One can then show that if $t \in [0.2, 0.5]$, both inequalities are satisfied. That is, by taking

$$\alpha = \sqrt{\frac{1-q}{0.5}}$$

and applying properties of derivatives of the right endpoints with respect to information noise level e, (25), (26), namely that $\frac{\partial R^c}{de} < 0$ and $\frac{\partial R^d}{de} > 0$, we obtain that the statement holds for any

$$e > e(B, q) = \frac{B}{\sqrt{2(1-q)}}.$$

By the continuity of functions R^c and R^d in b, we obtain that there is an upper bound on b, denoted by $b(B, q) > 0$, below which Co-Voting dominates both representative democracy and direct democracy. By continuity of R^c and R^d in w, we obtain that there exists $w(B, q) > 0.01$ such that for any $w < w(B, q)$, Co-Voting dominates both vrepresentative democracy and direct democracy. This completes the proof of the proposition.

□

Proof of Proposition 4. We prove the proposition for any $B > 0$. Take $e = B$ and $b = 0$. Then the right endpoints are

$$R^c = B\frac{6(3+w) + \sqrt{(6(3+w))^2 - 24(1-q)}}{12}$$

and

$$R^d = B\frac{9 + \sqrt{9^2 - 12(1-q)}}{6}.$$

First, we note that $|R^c - R^o| < |R^r - R^o| = 2wB$ holds for any $q > 0$. This holds because for $q = 0$, equality holds, while R^r is decreasing function in q and R^c is increasing in q. $|R^c - R^o| < |R^d - R^o|$ is equivalent to

$$\frac{6(3+w) + \sqrt{(6(3+w))^2 - 24(1-q)}}{12} - 3 < 3 - \frac{9 + \sqrt{9^2 - 12(1-q)}}{6},$$

after cancelling B. Solving w then yields $g(q) \leq w \leq f(q)$ for some positive function f. For example, for $q = 0.5$, $f(0.5) \approx 0.08$. We take $w_1(q) = g(q)$ and $w_2(q) = f(q)$.

From the continuity of R^c and R^d as functions of b, we obtain that there is an upper bound on b, $b(q) > 0$, so that for any $b \leq b(q)$, Co-Voting dominates both representative democracy and direct democracy. We can also easily verify that for high enough values of e, representative democracy dominates Co-Voting. From the continuity of R^c and R^d in e and the properties of derivatives, (26), we conclude that there exist values $e_l(q)$ and $e_h(q)$, so that $0 \leq e_l(q) \leq e_h(q)$ and that for each $e \in [e_l(q), e_h(q)]$, Co-Voting dominates both representative democracy and direct democracy. This completes the proof of the proposition. □

Proof of Proposition 5. The proof follows from the proof of Proposition 4, by plugging in $e = 0$ and $B = 1$, and from the properties of the derivatives of the endpoint functions in a variable w, (25), (24), (26). $\square$

Proof of Proposition 7. We proceed similarly to the proof of Theorem 2.

Step 1: Technical derivations

Under the assumptions of the proposition, we obtain

$$F(x) = \begin{cases} 0 & \text{if } x \in [-A, -3B], \\ \frac{x-2B}{5B} + 1 & \text{if } x \in [-3B, 2B], \\ 1 & \text{if } x \in [B, A], \end{cases}$$

and

$$G(x) = \begin{cases} 0 & \text{if } x \in [-A, -2B], \\ \frac{x-3B}{5B} + 1 & \text{if } x \in [-2B, 3B], \\ 1 & \text{if } x \in [3B, A]. \end{cases}$$

Now recall that

$$t(p) = \frac{\sigma^2(X(p))}{2\mu(p)}.$$

When $|p| > e$,

$$\mu(X(p)) = p,$$

and hence

$$\sigma^2(X(p)) = \frac{3p^2 + e^2}{3}.$$

That is,

$$t(p) = \frac{3p^2 + e^2}{6p}.$$

We observe that $\frac{\partial t(p)}{\partial p} > 0$. If $|p| < e$, it follows trivially that $\sigma^2(X(p)) = p^2$ and $\mu(X(p)) = p$, in which case

$$t(p) = \frac{p}{2}.$$

We have thus proved that $t(p)$ is an increasing function in p over the whole interval $[-A, A]$. Thus, it satisfies Assumption 1.

Step 2: Finding the middle intervals' endpoints

We are now in a position to investigate the middle intervals that will allow us to determine whether or not Condition (11) holds in terms of the primitives of this more structured model. We stress that these are B, w, q, e, b.

We start by noting that

$$E_F = \int_{-\infty}^{\infty} i f(i) di = \frac{1}{5B}\frac{1}{2}i^2\Big|_{-3B}^{2B} = -\frac{B}{2} < 0.$$

This means that the endpoints of the middle interval are computed as $R^o = 0$ for the socially optimal decisions and

$$L^o = -B. \tag{27}$$

In determining the endpoints of the middle interval in the case of representative democracy, we obtain that

$$L^r(G) = 0$$

and $R^r(G)$ solves the equation in the unknown x,

$$G\left(\frac{x}{2}\right) = \frac{1}{2}.$$

Accordingly,

$$R^r(G) = B. \tag{28}$$

In determining the endpoints of the middle interval in the case of direct democracy, we obtain that

$$R^d(F, q, X) = 0,$$

while $L^d(F, q, X)$ solves the following equation in the unknown p:

$$qF\left(\frac{p}{2}\right) + (1-q)F(t(p)) = \frac{1}{2}.$$

Suppose now that e is sufficiently smaller than B. Then, the above solution p must also solve the following equation in the variable x:

$$q\left(\frac{\frac{x}{2} - 2B}{5B} + 1\right) + (1-q)\left(\frac{\frac{3x^2+e^2}{6x} - 2B}{5B} + 1\right) = \frac{1}{2}.$$

This is equivalent to

$$3x^2 + 3Bx + (1-q)e^2 = 0.$$

There are two solutions $x = \frac{-3B \pm \sqrt{9B^2 - 12(1-q)e^2}}{6}$, but only the smaller is valid. Hence,

$$L^d(F, q, X) = \frac{-3B - \sqrt{9B^2 - 12(1-q)e^2}}{6}. \tag{29}$$

Finally, we calculate the endpoints that correspond to Co-Voting. We have

$$L^c(F, q, X, G) = 0$$

and $R^c(F, q, X, G)$ solves the following equation in the unknown x:

$$\frac{1}{2}G\left(\frac{x}{2}\right) + \frac{1}{2}\left(qF\left(\frac{x}{2}\right) + (1-q)F(t(x))\right) = \frac{1}{2},$$

which is equivalent to

$$\left(\frac{\frac{x}{2} - 3B}{5B}\right) + 1 + q\left(\frac{\frac{x}{2} - 2B}{5B} + 1\right) + (1-q)\left(\frac{\frac{3x^2+e^2}{6x} - 2B}{5B} + 1\right) = 1,$$

by plugging in the functional form of G, F and t. Solving this equation with $q = \frac{1}{2}$ and arbitrarily low e, we obtain that

$$L^c(F, q, X, G) < 0. \tag{30}$$

Step 3: Welfare assessment

By plugging $e = 0$ in the formulas of endpoints (see (27)–(30)), we see that parliament will propose the status quo under Co-Voting. This yields higher welfare for citizens, compared to a proposal $p = B$ that will be implemented under representative democracy. Finally, direct democracy yields the same outcome as Co-Voting.

$\square$

Proof of Proposition 8. In this case, we obtain

$$t(p) = \frac{\int_0^A x^2 dx/A}{A} = \frac{A}{3} \text{ for } p > 0,$$

and symmetrically

$$t(p) = -\frac{A}{3} \text{ for } p < 0.$$

If A is sufficiently large compared to B, then, under direct democracy, no new proposal will be implemented if q is large enough, since both $L^d(F, q, X)$ and $R^d(F, q, X)$ are zero. Under Co-Voting, by contrast, some new proposals improving welfare could be implemented if they were proposed. However, parliament will never propose such policies, since $[L^r, R^r] \cap [L^c, R^c] = \{0\}$. Finally, all the policies that will be proposed (and implemented) under representative democracy are less preferable than the status quo from a welfare perspective. This completes the proof of the proposition.

$\square$