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DO NONSTRATEGIC CONSIDERATIONS MATTER FOR BEHAVIOR IN GAMES? AN EXPERIMENTAL STUDY INFORMED BY DIRECT-SUM DECOMPOSITIONS OF GAME

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Do Nonstrategic Considerations Matter for Behavior in Games? An Experimental Study Informed by Direct-sum Decompositions of Games*

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February 1, 2023

Abstract

Experimental studies have shown that Nash equilibrium has clear limitations in regard to its ability to describe how people behave in games. In this paper, we use direct-sum decomposition proposed by Candogan et al. (2011) to decompose any normal-form finite game into the *strategic* and the *nonstrategic* components. How does individual behavior react to changes in the nonstrategic component? Nash equilibrium, as any other strategic solution, is invariant to changes in the nonstrategic component. *Mutual-Max-Sum*, a new solution concept defined in this paper, depends *only* on the nonstrategic component, identifies the most relevant strategy profile in this component and it is invariant to changes in the strategic component. We design 3x3 games, informed by the direct-sum decomposition of games, to empirically test, whether and when, manipulations in the nonstrategic component affect individual behavior and whether *Mutual-Max-Sum* is behaviorally relevant. We find that changes in the nonstrategic component affect individual behavior but that *Mutual-Max-Sum* is mostly behaviorally irrelevant except when it coincides with the Pareto outcome of the game. We conclude that Candogan et al. (2011)'s decomposition is informative about individual behavior in games.

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1 Introduction

Since economics in general, and game theory in particular, adopted the use of laboratory experiments, hundreds of experimental studies have shown that Nash equilibrium theory has clear limitations in regard to its ability to describe how people behave in strategic environments, see for example Thaler (1988), Nagel (1995), McKelvey and Palfrey (1992), Goeree and Holt (2001), Arad and Rubinstein (2012) among many others. If it is not only equilibrium thinking, then what determines individual behavior in games? Extensions of individual preferences to the so called social or interdependent preferences, e.g. Sobel (2005), and models of bounded rationality, e.g. Crawford et al. (2013), have been put forward to explain individual behavior in games. Yet, the determinants of individual behavior in games are not fully understood.

In this paper, we take a novel approach. We analyze the different pieces of information or considerations contained within a game and study their impact in individual behavior. In particular, we use the direct-sum decomposition of games, proposed by Candogan et al. (2011), to connect individual behavior and different behavioral rules to the different components of a game. Candogan et al. (2011) defined a specific direct-sum decomposition for any finite games in strategic form: games are decomposed into the strategic and nonstrategic components. The appealing attribute of this particular decomposition is that the strategic component, also referred to as the normalized game, captures all strategic considerations, while the nonstrategic component, what is left, captures all nonstrategic considerations. In other words, this decomposition is the only one that separates and filters out the strategic and nonstrategic information in two different components (see footnote 5 to understand the connection of this particular decomposition with other existing decompositions). Non-cooperative games are solved using mainly strategic solution concepts. Among those, the canonical solution concept is the Nash equilibrium, which only takes the strategic information of the game into account, fully contained in the strategic component. Therefore, from a game theory point of view, only the strategic component will be key in terms of predicting individual behavior and therefore individual behavior should remain constant in strategically equivalent games, i.e. games with the same strategic component, as defined in Candogan et al. (2011). What about considerations included in the nonstrategic component? They may indeed play a role in players' decision-making. In this paper we address whether the nonstrategic component of a game is relevant to behavior and if so, when.

To illustrate all these ideas, take the Prisoner's Dilemma (PD) game, and three additional modifications of this game, all shown in Figure 1. The four games have the same unique

Prisoner's Dilemma I

	C	NC
C	6 6	7 3
NC	3 7	4 4

=

Strategic Component

	C	NC
C	-0.5 -0.5	0.5 -0.5
NC	-0.5 0.5	0.5 0.5

+

Nonstrategic Component

	C	NC
C	6.5 6.5	6.5 3.5
NC	3.5 6.5	3.5 3.5

Prisoner's Dilemma II

	C	NC
C	7.5 7.5	8.5 1.5
NC	1.5 8.5	2.5 2.5

=

Strategic Component

	C	NC
C	-0.5 -0.5	0.5 -0.5
NC	-0.5 0.5	0.5 0.5

+

Nonstrategic Component

	C	NC
C	8 8	8 2
NC	2 8	2 2

Prisoner's (non) Dilemma III

	C	NC
C	1.5 1.5	2.5 7.5
NC	7.5 2.5	8.5 8.5

=

Strategic Component

	C	NC
C	-0.5 -0.5	0.5 -0.5
NC	-0.5 0.5	0.5 0.5

+

Nonstrategic Component

	C	NC
C	2 2	2 8
NC	8 2	8 8

Prisoner's (non) Dilemma IV

	C	NC
C	4.75 4.25	5.75 4.75
NC	4.25 5.25	5.25 5.75

=

Strategic Component

	C	NC
C	-0.5 -0.5	0.5 -0.5
NC	-0.5 0.5	0.5 0.5

+

Nonstrategic Component

	C	NC
C	5.25 4.75	5.25 5.25
NC	4.75 4.75	4.75 5.25

Figure 1: Four Examples of the Prisoner's Dilemma Game

Nash equilibrium prediction, hereinafter referred to as *NE*, given by (NC, NC), which can be achieved by eliminating the strictly dominated strategy of C. Moreover, following Candogan et al. (2011), each of the four games can be decomposed into their strategic and nonstrategic components, as shown in Figure 1. For an easy illustration of how to decompose a game, we will start with the calculation of the values of the nonstrategic component and we will then get the strategic component values taking the difference between the original game and the nonstrategic component values, as it is a direct-sum decomposition. In particular, in game I and for the row player, fix column player's strategy C or NC and summing own payoffs (6+7) or (3+4) and dividing by 2, we obtain row player's values of 6.5 or 3.5 for the nonstrategic component when the column player plays C or NC, respectively. Similarly, we can perform the same calculations to obtain the nonstrategic component's values for the column player fixing strategies for the row player. The strategic component is then obtained by subtracting to each of the payoffs of the original game the value in the nonstrategic component.

On the one hand, looking at the strategic component, note that all four games have exactly the same strategic component, which can be interpreted as a game on itself, and therefore, the four original PD games, as well as the four games represented by their respective strategic components will have the same *NE* prediction. The same equivalence is true for other strategic solution concepts, such as *Quantal Response Equilibrium* (McKelvey and Palfrey, 1995) and level-*k* thinking rules (Stahl and Wilson, 1994, 1995; Nagel, 1995; Costa-Gomes et al., 2001; Camerer et al., 2004). For any games that have the same strategic component, given that this component captures all strategic considerations, Candogan et al. (2011) define them as strategically equivalent.

On the other, the nonstrategic component can be also interpreted as a game, although it is clear that there is no meaningful strategic consideration in this component because both strategies yield the same payoff for any player. Therefore, in the game represented by the nonstrategic component, all strategy profiles are Nash equilibria.

Does the addition or manipulation of a nonstrategic component affect behavior in the original game? In other words, is individual behavior constant in strategically equivalent games? This is the initial question we address in this paper. Although we have not taken these particular four games into the laboratory, we expect, as many readers will, that the answer will be positive. Jessie and Kendall (2022) and Kendall (2022) showed that individual behavior is significantly affected when manipulating the nonstrategic component of a game using different 2×2 games and stag-hunt games, respectively. Our empirical findings are consistent with their results. However, we have added to their work by delving deeper into the analysis of the nonstrategic component, by defining a new solution concept, and by using carefully designed 3×3 games in two important ways. First, we show *when* the manipulation of the nonstrategic component will affect individual behavior *most* and second, we also show *how* individual behavior will be affected, i.e., which behavioral rule individuals will follow. The answer to these two questions in short is: individual behavior will be affected most when manipulations of the nonstrategic component change the Pareto optimality of different outcomes in the original game, and individuals will mostly follow a behavioral rule that has efficiency concerns.

We start analyzing the nonstrategic component. Going back to the example: What are the relevant considerations in the nonstrategic component? It is obvious that the four outcomes given by the four strategy combinations in the nonstrategic components in Figure 1 can be partially ordered by Pareto optimality, see for example Mock (2011).[1] Most importantly, there is a unique strong Pareto optimal (*PO*) outcome, which coincides with the prediction by the social-welfare maximization or altruistic rule, hereafter *A* rule, which maximizes the sum of players' payoffs, as described by Charness and Rabin (2002).[2] Needless to say, and scanning all the four matrices shown by the nonstrategic components, the unique *A* is the sensible strategy profile to play. In particular, in the games represented by the nonstrategic components in I and II, (C,C) is the prediction by the *A* rule. In game III, the *A* rule selects (NC, NC), and finally, in game IV, the *A* prediction is now given by (C, NC). What are the three different modifications of the nonstrategic components doing to the original PDs in II, III and IV in Figure 1? In game II, it is exacerbating the social dilemma that exists in the original PD, described in game I, making the Pareto dominance between (C,C) over (NC,NC) more extreme. By contrast, the nonstrategic component in games III and IV *destroys* the social dilemma that existed in the original PD, such that we cannot even label these last two games PD games, as the unique *NE* is not Pareto dominated by the (C,C). These four games clearly illustrate that predictions by the *A* rule in the nonstrategic component will not

[1] Payoff dominance, defined by Harsanyi et al. (1988), is a related concept, when an outcome is Pareto dominating. According to these authors the payoff dominance principle relies on the idea that "rational individuals will cooperate in pursuing their common interests if the conditions permit them to do so". There are multiple experimental investigations of when payoff dominance is important for individual behavior, in games where Nash equilibrium and payoff dominance are in conflict, as in the classical Prisoner's Dilemma, among the oldest experiments on games going back to Deutsch (1958), but also in games with multiple Nash equilibria, where payoff dominance is one selection criteria (Cooper et al. (1990), Cooper et al. (1992), Van Huyck et al. (1990), Van Huyck et al. (1991), Straub (1995), Haruvy and Stahl (2007) or Crawford et al. (2008)).

[2] In the nonstrategic component, a strong *PO* and *A* or the social-welfare maximization rule will select the same strategy profile(s), which we will refer to as the *A* rule. Jessie and Kendall (2022) refer to this as the outcome with positive values. However, in the original game, strong or weak *PO* and *A* rules will not necessarily fully coincide. In particular, any *A* profile will always be strong *PO* but there can be strong *PO* profiles that are not *A*. Of all the strong *PO* profiles, we will focus on *A* profile.

necessarily coincide with the predictions by the *A* in the original game. In particular, in game IV, the prediction by *A* in the game represented by the nonstrategic component selects (C,NC) but in the original game IV, the *A* rule selects (NC,NC), so the same behavioral rule can select different strategy profiles in the nonstrategic component and the original game. Consequently, to identify the importance of the nonstrategic component, separating the *A* rule predictions in the original and in the nonstrategic component game is crucial and an important contribution of this paper. We will now go on to explain this contribution using the examples in Figure 1.

We define a new solution concept for two-player games, which we shall call the *Mutual-Max Sum*, *MMS* for short. The *MMS* identifies, in the original game, the predicted strategy profile(s) identified by the *A* rule in the nonstrategic component. In particular, the *MMS* selects strategy profile(s) where players choose their strategies by maximizing the sum of the *other* player's payoffs. The *MMS* solution concept may be understood as an empathetic player who chooses her strategy maximizing the sum of the *opponent's* payoffs, as if the other player would not be able to do so by herself. Going back to the four games in Figure 1, the *MMS* for players 1 and 2 in PD I would choose C, because this strategy would yield a payoff of 6+7=13 for the other player (if she chose NC, then this strategy would yield a payoff of 3+4=7 for the other player). Similarly, in game II, the *MMS* profile would select (C, C), as this maximizes the sum of payoffs for the other player. However, in game III, the *MMS* prediction is given by (NC, NC) and by (C, NC) in game IV.

What are the appealing features of the *MMS* solution? We show that, in the original game, the *MMS* will always identify the *A* profile(s) in the nonstrategic component (Proposition 1). An important advantage of the *MMS* solution concept is that no decomposition is required to identify the relevant profile(s) of the nonstrategic component. Interestingly, as the *NE* is indifferent between any of the strategy profiles in the nonstrategic component, the *MMS* is also indifferent between any of the strategy profiles in the strategic component. Consequently, the *NE* captures the essence of the strategic component while being indifferent between any of the strategy profiles in the nonstrategic component, and the *MMS* captures the essence of the nonstrategic component while being indifferent between any of the strategy profiles in the strategic component.

In addition to the *NE* and *MMS*, how do other solution concepts or behavioral rules depend on strategic and nonstrategic components? Let us focus on *A* and/or *PO* selection rules. First, note that a *PO* criterion can select multiple strategy profiles, so we will focus our attention on the *A* rule, which will be unique to most of our games of interest and will be strong *PO* by definition (see footnote 2). Second, we show that, in the original game, *A* profiles depend on both components and that, in principle, in the original game we can separate the predictions of these main three solution concepts: *NE*, *MMS* and *A*. This is a very important result of our study, showing that *MMS* predictions do not necessarily coincide with the predictions by *A*. For example, in games I and II, *NE* is separated from *A* and *MMS* but the last two coincide. In game III, *NE*, *MMS* and *A* are all confounded. Finally, in game IV, *MMS* predictions are different from predictions by *NE* and *A* but the last two coincide. To perfectly separate the three different rules, we then proceed to design 3×3 games to test whether *MMS* is behaviorally relevant. The question of interest in this regard is: when manipulating or adding a nonstrategic component, is *MMS* a good indicator of how manipulations in the nonstrategic component affect individual behavior? This is a relevant question because the *MMS* identifies the altruistic profile, and the most sensible strategy profile, in the nonstrategic component. An important limitation of Jessie and Kendall (2022) and Kendall (2022) is that *MMS* predictions are always confounded with the *A* behavioral rule's predictions. We show that this confound is crucial when assessing how behaviorally relevant the *MMS* predictions are.

To this end, we design a laboratory experiment to address the two questions mentioned above. First, is individual behavior constant in strategically equivalent games, when the only difference resides in the nonstrategic component? Second, is *MMS* behaviorally relevant, particularly when separated from *A* rule predictions?

For the design of the experiment, we start with the direct-sum decomposition of games of normal-form by Candogan et al. (2011), which decomposes the game into the strategic and nonstrategic, and at the same time the strategic into the potential and harmonic components. We add to this decomposition the one proposed by Jessie and Saari (2015), which decomposes the nonstrategic into the behavioral and kernel components. This combination yields a four-component direct-sum decomposition of games: potential, harmonic, behavioral and kernel components. Following (Candogan et al., 2011), we use three different classes of games: harmonic games (those without a potential component), potential games (those without a harmonic component) and constant-sum games (games that have both potential and harmonic components). These further decompositions are useful to see when different behavioral rules' predictions will be differentiated. Thus, they will be important to understand the experimental design of the games. Harmonic games are useful for separating predictions by *MMS* from predictions by *NE*, although they are limited by the fact that predictions by *MMS* and *A* are fully confounded. Constant-sum games are the most useful for separating *NE* and *MMS* predictions, as their predictions will always be separated (Proposition 2). Finally, potential games are the most useful for separating *MMS*, *A* and *NE* predictions. With regard to the decomposition by Jessie and Saari (2015), we use it in order to keep constant the kernel component in all variations, in contrast to Jessie and Kendall (2022), and change only the behavioral component. This is important because there is work showing that underlying stakes can also impact individual behavior, see for example Esteban-Casanelles and Gonçalves (2020).

In the empirical test, we find that individual behavior may indeed show very important differences in strategically equivalent games, when changes occur only in the nonstrategic component. Although Nash equilibrium is a strong predictor of individual behavior in our games, changes in the nonstrategic component can clearly change individual behavior, which is consistent with Jessie and Kendall (2022). More specifically, when comparing individual behavior in games that are strategically equivalent (with the same strategic component) but that differ in their nonstrategic component (more particularly, in their behavioral component keeping the kernel component constant), individual behavior is statistically different for at least one of the player roles, and many times it is statistically different for both player roles. Moreover and most importantly, how does individual behavior change? Which rule do individuals follow? We find that *MMS* predictions gain most relevance over the Nash equilibrium for individual behavior only when they fully coincided with the *A* rule predictions. As in the designs by Jessie and Kendall (2022) and Kendall (2022), *MMS* predictions *always* coincided with the *A* rule predictions, led them to conclude that the strategy profile that is strong Pareto in the behavioral component, in our definition, the *MMS*, is very important for individual behavior. By contrast, in our design, where *MMS* can be separated from *A* and Pareto concerns, comparing observed individual behavior directly but also through the use of a mixture-of-types econometric model estimation, we confirmed that *NE* and Pareto efficient outcomes are important attractors for individual behavior, while we found little evidence to support the idea that *MMS* is generally a behaviorally relevant rule.

We conclude that Candogan et al. (2011) is useful to inform about individual behavior in games. As carefully noted in footnote 7 in by Candogan et al. (2011), the nonstrategic component can affect efficiency in games. They further mention that the nonstrategic component is of interest mainly through its effect on the efficiency or Pareto optimality properties of games. We have elaborated on this idea by empirically showing that changes in the nonstrategic component will affect the efficiency in the original game depending on the class of games. Furthermore, we empirically show that when these changes affect the Pareto optimality (*A* rules' prediction) in the original game, that is when individual behavior will be most affected. Nevertheless, *MMS* is an inherently different behavioral rule to the *PO* and *A* rules. Overall, when *MMS* does not coincide with the *PO* or *A* behavioral rules in the original game, we found little evidence for its relevance in individual behavior. To summarize, going back to the four different versions of PD in Figure 1, our results would imply that, while individual behavior would follow the *MMS* prediction in games I, II and III, in game IV the *MMS* prediction would not explain much of the individual behavior.

The paper is organized as follows. Section 2 shows the four direct-sum decomposition of games, adding the decomposition by Jessie and Saari (2015) to the one by Candogan et al. (2011). This section also relates the decomposition to different behavioral rules, and to different classes of games. Section 3 describes the experimental design and procedures to empirically test whether and when the manipulations of the nonstrategic component will affect individual behavior and whether *MMS* is a behaviorally relevant rule. Section 4 shows the results and finally, Section 5 concludes.

2 Theoretical Framework

2.1 Preliminaries

We first introduce the general framework for two-person normal form games and their corresponding bimatrix representation.

Let $\mathscr{G} = \langle I, S, T, (u_i)_{\{i=1,2\}} \rangle$ be a two-person finite normal form game, where $I = \{1,2\}$ is the set of players, $S = \{s_1, ..., s_h\}$ and $T = \{t_1, ..., t_h\}$ are the sets of strategies for players 1 and 2, respectively, and $u_i : S \times T \to \mathbb{R}$ is player i $(i = 1,2)$ payoff function. A pair (s_i, t_j) $(i, j = 1, ..., h)$ denotes a strategy profile. A mixed strategy for player i $(i = 1,2)$ is a probability measure over her possible pure strategies, $\sigma \in \Delta(S)$ and $\tau \in \Delta(T)$. We will focus on games where players have the same number of strategies, although all results are easily generalizable to games in which players have a different number of strategies.

Game $\mathscr{G}$ can be written as a bimatrix square game (A, B). Matrix A corresponds to player 1's payoffs with elements a_{ij} $(i, j = 1, ..., h)$ where $a_{ij} = u_1(s_i, t_j)$. Matrix B corresponds to player 2's payoffs with elements b_{ij} $(i, j = 1, ..., h)$, where $b_{ij} = u_2(s_i, t_j)$. Since our study focuses on two-person games, we will use matrix notation where appropriate.

2.2 Direct-Sum Decomposition of Games

We start showing the direct-sum decomposition of games, proposed by Candogan et al. (2011) and then we add the decomposition of the nonstrategic component, proposed by Jessie and Saari (2015).[3] This combination leads to a four-component direct-sum decomposition, which is important to understand the underlying reasoning behind the experimental design, in particular, the games. Although in this section we will differentiate between the game and its corresponding components, note that each component can be understood as a payoff matrix of an independent game.[4]

Candogan et al. (2011) started normalizing the game by eliminating the nonstrategic information. In particular, the nonstrategic component is computed by taking the average of each player's own payoffs for each of their opponents' strategies. Then, in order to get the strategic component, this average is subtracted from the payoffs in the game, such that in the strategic component the sum of one player's payoffs, given the other players' strategies, is always zero. They further proposed a canonical direct-sum decomposition of the strategic component into two components: *potential* and *harmonic*.

Jessie and Saari (2015) build on Candogan et al. (2011) focusing on the nonstrategic component, which in turn was decomposed into what they called *behavioral* and *kernel* components. Although they defined the decomposition for 2×2 games, it is easily generalizable to $h \times h$ games.

The combination of the two proposed decomposition of bimatrix games yields the four-component decomposition represented in Figure 2.

We will now describe the four-component decomposition for a bimatrix square game.

First, we consider the **nonstrategic component**. Denote the column vector of ones by $\mathbf{1}$ and its transpose by $\mathbf{1}^{\mathbf{T}}$. The nonstrategic component is then computed as follows:

$$(A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}}) = ((\frac{1}{h})\mathbf{1}\mathbf{1}^{\mathbf{T}}A, (\frac{1}{h})B\mathbf{1}\mathbf{1}^{\mathbf{T}})$$

Then, we can further decompose the nonstrategic component into the kernel component, and the behavioral component, denoted by $(A^{\mathscr{K}}, B^{\mathscr{K}})$ and $(A^{\mathscr{B}}, B^{\mathscr{B}})$, respectively.

The *kernel component* is a matrix of payoffs computed by taking the average of all payoffs for each player of game (A, B). Formally, the kernel component can be computed as follows:

[3] Candogan et al. (2011)'s decomposition was based on the *Helmholtz decomposition theorem*, which enables the decomposition of a flow on a graph into three components: globally consistent, locally consistent (but globally inconsistent), and locally inconsistent components, which are the potential, harmonic and nonstrategic components, respectively. For a more detailed theoretical description see Section 3 and for its application see Section 4 in Candogan et al. (2011). Jessie and Saari (2015)'s decomposition was based on the mathematics of symmetry groups and representation theory.

[4] The strategic and nonstrategic components correspond to the classes of nonstrategic and μ-normalized games introduced by Abdou et al. (2022) see Definition 2.2 and Section 2.3 for details.

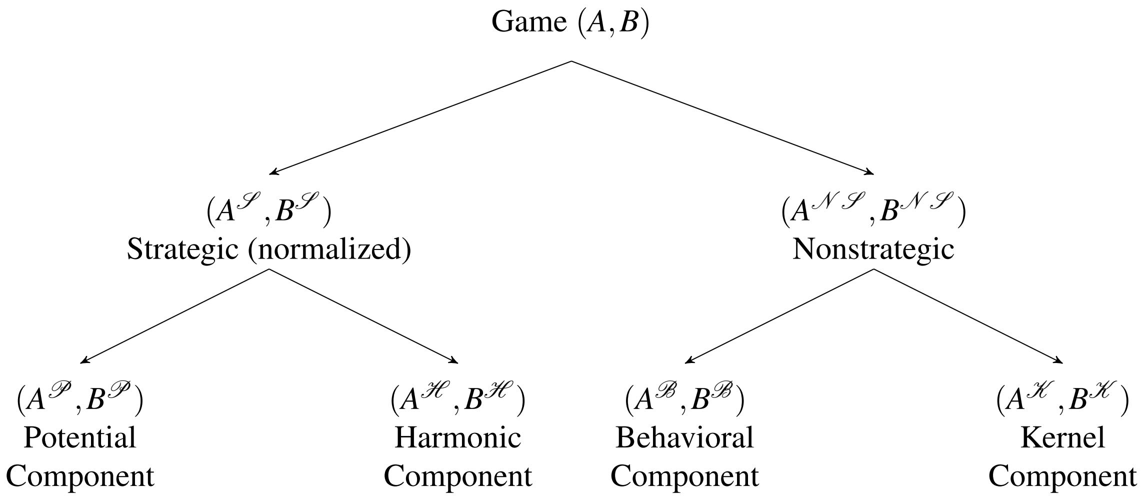

Figure 2: Diagram of Four-Component Direct-Sum Decomposition of Games

$$(A^{\mathscr{K}},B^{\mathscr{K}}) = ((\frac{1}{h^2})\mathbf{1}\mathbf{1}^{\mathbf{T}}A\mathbf{1}\mathbf{1}^{\mathbf{T}},(\frac{1}{h^2})\mathbf{1}\mathbf{1}^{\mathbf{T}}B\mathbf{1}\mathbf{1}^{\mathbf{T}})$$

This component can be interpreted as an "inflationary term" or underlying stakes that can vary by player.

The *behavioral component* is obtained as the difference between the nonstrategic and the kernel components.

$$(A^{\mathscr{B}},B^{\mathscr{B}}) = (A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}}) - (A^{\mathscr{K}},B^{\mathscr{K}})$$

All rows of matrix $A^{\mathscr{B}}$ and all columns of matrix $B^{\mathscr{B}}$ have equal payoffs, meaning that both players are strategically indifferent between their strategies in the behavioral component. Also, since this component is normalized, there must always be at least one positive payoff in each row (and column). Therefore, strategy profiles in the behavioral component can be ordered according to Pareto optimality, Mock (2011). In particular, we are interested in the strategy profile selected by the strong *PO* or *A* rule on this component. This is also the case overall for the nonstrategic component.

There are two explanatory comments that we would like to make. First, throughout the paper we will use nonstrategic and behavioral components interchangeably, as the kernel component is just a constant term for each player (and we make sure we keep this term constant in all our manipulations in the experimental design). Second, given what our paper reveals, our preference would be to change the term "behavioral component" to "efficiency component", as it can affect the efficiency or Pareto optimality of the outcomes in the original game, which is when it becomes most relevant for individual behavior. However, given this name was originally proposed by Jessie and Saari (2015), we decided to follow their labeling of these components.

Second, we consider the decomposition of the **strategic component**. We start by identifying the strategic component, which can be obtained as the difference between (A,B) and its nonstrategic component $(A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}})$.

$$(A^{\mathscr{S}}, B^{\mathscr{S}}) = (A - (\frac{1}{h})\mathbf{1}\mathbf{1}^{\mathbf{T}}A, B - (\frac{1}{h})B\mathbf{1}\mathbf{1}^{\mathbf{T}}).$$

Then, the potential and the harmonic components are obtained by first calculating the following matrices: $M = \frac{1}{2}(A^{\mathscr{S}} + B^{\mathscr{S}})$, $D = \frac{1}{2}(A^{\mathscr{S}} - B^{\mathscr{S}})$ and $\Gamma = \frac{1}{2h}(A\mathbf{1}\mathbf{1}^{\mathbf{T}} - \mathbf{1}\mathbf{1}^{\mathbf{T}}B)$.
The *potential component* is, then:

$$(A^{\mathscr{P}}, B^{\mathscr{P}}) = (M + \Gamma, M - \Gamma)$$

while the *harmonic component* is:

$$(A^{\mathscr{H}}, B^{\mathscr{H}}) = (D - \Gamma, -D + \Gamma)$$

These two components are also normalized, hence $\mathbf{1}^T A^{\mathscr{P}} = 0, B^{\mathscr{P}}\mathbf{1}$=0 and $\mathbf{1}^T A^{\mathscr{H}} = 0, B^{\mathscr{H}}\mathbf{1}$=0.

This decomposition separates the cyclical and the acyclical parts of the strategic component giving rise to the harmonic and the potential components, respectively. Therefore, by construction, the harmonic part is a zero-sum payoff matrix. Consequently, starting from any of its payoff profiles, there exists a deviation for a single player that strictly increases her payoff until the same payoff profile is reached again. By contrast this iteration always ends in the potential component.[5]

This completes the introduction of the four-component direct-sum decomposition of games. From now on, when a particular component is a matrix of zeros, we say that it lacks this particular component. To illustrate the calculation of the four-component direct-sum decomposition, please find a detailed step by step calculation for a particular game, as well as the 11 experimental games we use later in the empirical test decomposed into the four direct-sum components in the Online Appendix A.

[5] Kalai and Kalai (2013) proposed a decomposition of a two-person normal-form game into an identical common-interest component, which is potential, and a zero-sum component which is not necessarily a harmonic component. Clearly, this decomposition is in line with the decomposition of the strategic component proposed by Candogan et al. (2011) whenever each cell of payoffs of the auxiliary matrix Γ is zero. Hwang and Rey-Bellet (2020) show that any two-person normal form game can be uniquely decomposed into a zero-sum normalized game, a zero-sum equivalent potential game, and an identical interest normalized game.

2.3 Decomposition and Solution Concepts: the Usual Suspects and *Mutual-Max-Sum*

A solution concept or behavioral rule can be understood as a prediction of how agents will play a game. We start by listing the most common solution concepts used to explain individual behavior in games and then introduce a new solution concept.

The canonical solution concept is the *NE*. A strategy profile is said to be a *NE* if no player can gain by altering its strategy, given the existing strategies of other players. Thus, a *NE* represents a best response by any player to the given strategies of other players. Among the non-equilibrium solution concepts, the level-k thinking model excels. In the so-called level-k model, each player $k = 0, 1, \ldots$ corresponding to the number of steps of reasoning the player is able to perform. Thus, a level-*0* agent chooses her strategies randomly while a level-*1* agent assumes her opponent will act as a level-*0* agent and best responds. Alternatively, level-*1* players 1 and 2 sum their own payoffs across columns and rows, respectively, and take the strategy that yields the maximum sum of payoffs.[6]

Other solution concepts can be better understood as if they were selected by an external observer whose aim is to identify the best outcomes for the two players. Pareto optimality or efficiency (*PO*) stands out as the most popular criterion. With a weak *PO*, any change will make at least one player no better off, but may not make any party worse off. With a strong *PO*, any change will make at least one player worse off. Often there will be multiple strategy combinations that lead to *PO* outcomes. The most salient *PO* outcome is the *altruistic*, or social welfare maximizing behavioral rule (Charness and Rabin, 2002), *A*, one which can be viewed as an implicit agreement between players who select the strategy profile that maximizes the sum of their payoffs. So, when choosing her strategy, the *A* behavioral rule simply sums her own and opponent's payoffs in each cell of the payoff matrix, and applies the maxmax operator. In such a solution, rather than trying to predict her decision, both players implicitly assume that the other player is also altruistic (Costa-Gomes et al., 2001). Finally, also following Costa-Gomes et al. (2001), we consider both the *Pessimistic* and the *Optimistic* behavioral rules. The *Pessimistic* (*P*) can be understood as a conservative player who, when choosing her strategy, maximizes her minimum payoff. The *Optimistic* behavioral rule (*O*) on the other hand, when choosing her strategy, maximizes her maximum payoff.

We now introduce a novel solution concept that we call, *Mutual-Max-Sum* (*MMS*). This solution can be understood as the reciprocal behavior that may take place in bilateral encounters between empathetic players. Thus, each player when choosing her strategy considers,

[6] $k = 2$ or higher are similarly defined such that level-k best response to level-k-*1* behavior.

not her own payoffs, but instead the payoffs of her opponent.[7]

DEFINITION 1. *Let $\mathscr{G}$ be a two-person normal-form game. A strategy profile $(\widetilde{s},\widetilde{t}) \in S \times T$ is Mutual-Max-Sum if:*

$$\widetilde{s} \in \arg\max_{s_i \in S} \sum_{t_j \in T} u_2(s_i,t_j) \text{ and } \widetilde{t} \in \arg\max_{t_j \in T} \sum_{s_i \in S} u_1(s_i,t_j).$$

Note that this is not an equilibrium concept, as players are not mutually best responding to each other. Indeed, individuals choose their strategies independently of the behavior of their opponent but we assume both players are doing this in order to define the *MMS* profile.

In an interesting and useful result, demonstrated in Appendix B, we relate *MMS* in the original game and the A payoff profile(s) in the behavioral component. In particular, the *MMS* in the (A,B) always identifies exactly the payoff profile(s) that is A in the behavioral component.

PROPOSITION 1. *Let (A,B) be a bimatrix game and let $(A^{\mathscr{B}},B^{\mathscr{B}})$ be its behavioral component. Then the MMS solution(s) of (A,B) will coincide with the A payoff profile(s) of $(A^{\mathscr{B}},B^{\mathscr{B}})$.*

It is worth noting two points. First, this result allows us to apply the *MMS* solution to game (A,B) and identify the most relevant payoff profile(s) in the behavioral component without having to decompose the game. Second, although the *MMS* solution may not be unique, in our experimental exercise we will only consider the case where this is unique or the trivial case, where the behavioral component is zero, such that any strategy combination is trivially *MMS*.

All these behavioral rules that we have presented are defined in the original game (A,B). Now, if we were to make changes to any of the two main strategic or nonstrategic components, would their predictions change? We can identify strategic, nonstrategic, and mixed behavioral rules based on their dependence on the strategic and nonstrategic components. On the one hand, *NE* and any level-k behavioral rules are strategic rules, such that any change in the nonstrategic component will never affect their predicted behavior. On the other hand, *MMS* is a nonstrategic rule, such that it is invariant to any changes in the strategic component.

[7]This definition shares similarities with the *mutual-max* solution defined by Rabin (1993). The difference relies on the fact that for the selection of a strategy in the mutual-max solution each player considers the maximum payoff of her opponent while in the *MMS* solution each player selects the maximum *sum* of the payoffs of her opponent. As it is the case for the *mutual-max* solution, the *MMS* does not satisfy invariance to the deletion of dominated strategies. However, in contrast to the *mutual-max* solution, the *MMS* satisfies the invariance to affine transformations.

Finally, predictions by *PO*, *A*, *P* and *O* rules can be affected by any changes in any of the two main components, meaning that we will refer them as mixed behavioral rules.

Looking at this in more detail, we can see that the *NE* prediction in the original game (A,B) will always coincide with its prediction in the strategic component. This is also the case for any behavioral rule that is strategic. In other words, as the strategic component contains all the strategic information of the original game, the strategic solutions remain invariant between the original game and the strategic component. Furthermore, the predictions by every strategic behavioral rule would be trivially indifferent for any of the strategy profiles in the nonstrategic components. Interestingly, the *MMS* solution is the mirror image such that its prediction in the original game (A,B) will always coincide with its prediction in the nonstrategic component and, further, its prediction in the strategic component will be trivially indifferent for any of the strategy profiles. In short, the strategic component isolates all the strategic considerations, while the nonstrategic component isolates all the nonstrategic considerations. These results are summarized in the following remark.

REMARK 1. *(i) The Nash equilibria of* $(A^{\mathscr{S}},B^{\mathscr{S}})$ *coincides with the Nash equilibria of* (A,B)*, while every strategy profile of* $(A^{\mathscr{S}},B^{\mathscr{S}})$ *is a Mutual-Max-Sum. (ii) The Mutual-Max-Sum solution* $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$ *coincides with the Mutual-Max-Sum of* (A,B)*, while every strategy profile in the* $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$ *is a Nash equilibrium.*

Finally, note that we cannot make any similar statements for mixed rules, such as the *PO*, *A*, *P* and *O* rules. They can select different strategy profiles for different components and, therefore, their predictions in the original game are not invariant to changes in any of the components.

2.4 Decomposition and Classes of Games: Harmonic, Potential and Constant-Sum Games

Candogan et al. (2011), see their Theorem 5.1, allows to reformulate the classes of potential and harmonic games in terms of their components as follows: the absence of a harmonic component defines a potential game, while the absence of a potential component defines a harmonic game. A *potential game* admits at least one Nash equilibrium in pure strategies, while a square *harmonic game* admits only a uniformly mixed Nash equilibrium.[8] Any other

[8]As pointed out by Candogan et al. (2011), harmonic games have appeared in earlier publications but have not been defined as a class.

class of games always has both components. Looking at these other games with both components, we will focus on *constant-sum* games. These are games of conflict where the sum of all players' payoffs remains constant for each strategy profile, meaning that the gain for one player is always at the expense of her opponent. All constant-sum games have both potential and harmonic components, except the class of matching pennies (rock-paper-scissors for three strategies).[9]

Our main objective is to understand whether and when the manipulations in the nonstrategic component will affect individual behavior and also whether the *MMS* solution is relevant to behavior. Therefore, we use this classification of games to assess when the *MMS* solution will make a different prediction from other relevant behavioral rules. To proceed, we start by saying that any two solution concepts or behavioral rules are separa*ble* if they can provide a different predicted probability of choosing each of the strategies, while they are separat*ed* if they never coincide. Below is a table that summarizes the separability between *MMS*, *NE*, and *A* by the classes of games.

Table 1: Separability between *MMS*, *NE* and *A*, by Class of Game

	MMS vs NE	MMS vs A
Harmonic Games	Separated	Not separable
Constant-sum Games	Separated	Not separable*
Potential Games	Separable	Separable

* In CSG all strategy profiles are *A*.

Harmonic games. The unique Nash equilibrium prediction in these games is the uniformly mixed strategy profile. Therefore, the *MMS* prediction is always separated in these games. However, such a prediction cannot be separated by the *A* behavioral rule. Consequently, harmonic games will not be useful when it comes to separating the *MMS* solution from the *A* behavioral rule.

Constant-sum games. These games are particularly interesting because *NE* and *MMS* are always perfectly separated as shown by the proposition below. However, they will not to be useful when it comes to separating *MMS* predictions from *A* rule predictions.

[9]Furthermore, a constant-sum game can be transformed into a *zero-sum* game by subtracting half of the value of the constant from each payoff in the initial game so that in the former the kernel is positive instead of 0. Zero-sum games generalize the *generalized rock-paper-scissors games* whose decomposition was analyzed by Candogan et al. (2011).

PROPOSITION 2. *Let (A, B) be a constant-sum game with a unique Nash equilibrium in pure strategies and a unique MMS solution. Then, the NE and the MMS solution will never coincide.*

Potential games. These games offer the highest degree of separability. Interestingly, manipulating the behavioral component we can lead to three situations:

(1) The *MMS* coincides with a *NE* of the game, in which case we would say that it *reinforces* the strategic behavior.

(2) The *MMS* points out to a different strategy profile such that it does not coincide with the *NE*.

(3) The *MMS* can be separated from the *A*.

As summarized in Table 1, although it is relatively easy to separate *MMS* predictions from *NE* predictions, it is not trivial to separate *MMS* predictions from *A* rule's predictions, as illustrated by the work by Jessie and Kendall (2022). In our design, we do separate them, and we show that this separation leads to very different interpretation of the results on the importance of the nonstrategic component.

3 Experimental Study

Do nonstrategic considerations affect individual behavior or is individual behavior constant in strategically equivalent games? Do individuals follow *MMS* predictions? As these are empirical questions, we carried out a laboratory experiment. Potentially, we could use existing empirical studies and games to answer this question. However, the games in existing studies were not designed with our research questions in mind, and, as such, they would not provide the most informative answer. Therefore, we designed our own games guided by the four direct-sum decompositions of games.

3.1 Procedures

Using the ORSEE system (Greiner, 2015), we recruited 200 subjects for the experiment. The laboratory sessions, which lasted around 1 hour and a half, were conducted using the computer software z-tree (Fischbacher, 2007). The 5 sessions, with around 40 participants each, took place in April 2022 in the Laboratory of Experimental Analysis (Bilbao Labean) at the University of the Basque Country UPV/EHU.[10]

We started with general instructions that informed subjects that payments would depend on their own and other participants' decisions in the same session, as well as on luck. After that, the participants were given detailed instructions explaining the task in hand, including examples of games, how their own and the other players' decisions could affect the payments and how they were going to be matched. Before subjects started the task, we posed a set of three questions to ensure the correct understanding of the payoff-matrix representation of games and payments. Online Appendix C includes a translated version of the instructions.

All of the subjects played the same eleven 3×3 normal-form two-player games in the same order, twice, once as a row player and once as a column player, leading to a total of 22 decisions per subject.

When the subjects had finished the 22 decisions, the computer randomly matched subjects in pairs and selected one game per pair, in each of the two parts (the first 11 decisions and the second 11 decisions). This ensured that each subject was paid for one game played in each of the two player roles. After we informed subjects about their payments, the subjects completed a non-incentivized questionnaire regarding demographic data, risk preferences following Eckel and Grossman (2002), and a cognitive reflection test. Table 2 shows the descriptive statistics for all these variables. The majority of the subjects were aged between 18 and 22, with a higher presence of women (64%). This is consistent with there being a higher proportion of women studying social sciences, particularly Business Administration and Management, Law and Economics, which represents more than 60%. We also requested free-format responses regarding their explanations of how they made their choices and their expectations of how other subjects made their choices. To finish the session, each subject was paid privately according to the two games selected plus a 3 euros attendance fee. The average payment was 17.06 euros, with a standard deviation of 3.71.

3.2 Experimental Design: Player Roles, Games, Behavioral Rule Predictions and Separability

The specific structure of the experiment was as follows. The computer randomly divided the participants into two types, *Type* 1 and *Type* 2. *Type* 1 subjects started the first eleven decisions playing as row players and, then, in the second part of the task, they played as

[10] The CEISH-UPV/EHU Ethics Committee issued a favorable report for carrying out the experiment. Ref.: M10_2022_102

Table 2: Descriptive Statistics

Variables	Mean Values	Stand. Dev.
Women	0.635	
Age	20.75	2.817
Spanish	0.96	
University Entry Grade (out of 10)	7.831	2.269
Business and Economics Degree	0.625	
Distribution over risk choices:		
1.5€ with 0.50 or 1.5€ with 0.50	0.350	
1.3€ with 0.50 or 1.8€ with 0.50	0.170	
1.1€ with 0.50 or 2.1€ with 0.50	0.195	
0.9€ with 0.50 or 2.4€ with 0.50	0.080	
0.7€ with 0.50 or 2.7€ with 0.50	0.070	
0.6€ with 0.50 or 2.8€ with 0.50	0.020	
0.4€ with 0.50 or 2.9€ with 0.50	0.020	
0€ with 0.50 or 3€ with 0.50	0.095	
Cognitive reflection test:		
Q1. Percent correct answer	0.295	
Q1. Percent intuitive answer	0.210	
Q2. Percent correct answer	0.375	
Q2. Percent intuitive answer	0.370	
Q3. Percent correct answer	0.600	
Q3. Percent intuitive answer	0.280	

Notes: Women is a dummy variable which takes a value of 1 if the subject is female. Age is referred to in years. Spanish is a dummy variable which takes a value of 1 if the subject is Spanish. University Entry Grade is normalized to a grade out of 10. Risk Choices are ordered from safest to riskiest and was elicited via Eckel and Grossman (2002). Finally, the cognitive reflection test includes questions from Toplak et al. (2014). The questions are as follows: 1. If John can drink one barrel of water in 6 days, and Mary can drink one barrel of water in 12 days, how long would it take them to drink one barrel of water together? (correct answer 4 days; intuitive answer 9); 2. Jerry received both the 15th highest and the 15th lowest mark in the class. How many students are in the class? (correct answer 29 students; intuitive answer 30); 3. A man buys a pig for 60, sells it for 70, buys it back for 80, and sells it finally for 90. How much has he made? (correct answer 20; intuitive answer 10).

column players. *Type* 2 subjects played the opposite way round, first as column players and then as row players. The subjects were never informed about their types or even about the existence of types, but at the beginning of the experimental task they were told they would be presented with 11 payoff-matrices, one at a time. Only when these 11 decisions had been taken were they told that they would be presented with an additional set of 11 payoff matrices. The subjects did know there would be participants playing as row and column players, but they were not explicitly told that the total of 22 matrix payoffs came from the same 11 games. In order to facilitate the reading of the games, we showed all the games to all subjects from the perspective of row players, transposing the games when the subject was a column player. There were no time restrictions for making decisions.

When designing the games, the main goal was to separate *MMS* predictions from the predictions of other behavioral rules, particularly the predictions by the *NE* and *A* behavioral rules. Therefore, we chose 3×3 normal-form games instead of 2×2 normal-form games, as 2×2 games make it impossible to perfectly separate out the predictions of *three* different behavioral rules.

Figure 3 displays the eleven 3×3 normal-form two-player games designed for the experiment. We presented the games to the subjects in a randomized order, but in the same order to all subjects.[11]

By design no game has dominated strategies in pure strategies. The eleven games can be separated into 3 different sets of games. G1 to G3 are strategically equivalent harmonic games, where G2 and G3 have a behavioral component, and the *MMS* points towards a different strategy profile each, while G1 has no behavioral component. G4 and G5 are the two experimental constant-sum games we designed. These are interesting because by definition the predictions of *NE* and *MMS* are always fully separated. Finally, G6 to G8 and G9 to G11 are the two sets of strategically equivalent potential games. Both sets have the same structure. The first game has no behavioral component, meaning that the behavioral component is composed of all 0s, and the *NE* and *A* behavioral predictions coincide in the same strategy profile. Then, in the second game of each set, we added a behavioral component where the *MMS*, *A* and *NE* predictions are all separated. Finally, in the last game of each potential set, we increased the magnitude of the behavioral component to obtain a game where the *MMS* prediction will also coincide with the *A* rule's prediction. However, these two are separated from the *NE* predictions, which is Pareto dominated.

[11]The actual order of the games was G5, G9, G7, G11, G2, G6, G8, G4, G10, G3, G1. The goal of randomizing was to prevent the subjects from observing the similarity in some particular games, such as the harmonic game with no behavioral component and the harmonic game with the addition of a behavioral component.

Harmonic Games

G1

6.62* 6.56*	7.62* 5.56*	5.62* 7.56*
5.62* 7.56*	6.62* 6.56*	7.62* 5.56*
7.62* 5.56*	5.62* 7.56*	6.62* 6.56*

G2

6.62* 11.56*	7.62* 0.56*	5.62* 7.56*
0.62* 12.56*	1.62* 1.56*	2.62* 5.56*
12.62* **10.56***	10.62* 2.56*	11.62* 6.56*

G3

6.62* 1.56*	7.62* 5.56*	5.62* 12.56*
0.62* 2.56*	1.62* 6.56*	2.62* 10.56*
12.62* 0.56*	10.62* 7.56*	**11.62*** **11.56***

Constant-sum Games

G4

4.55* 5.45*	3.77 6.23	1.67 8.33
5.82 4.18	3.15 6.85	**7.76** **2.24**
4.89 5.11	9.78 0.22	0.98 9.02

G5

3.11 6.89	4.80 5.20	**6.67** **3.33**
3.95 6.05	3.97* 6.03*	1.90 8.10
7.72 2.28	5.74 4.26	0.15 9.85

Potential Games. First set

G6

8.53 7.18	7.63 4.05	8.59 6.66
8.29 6.82	8.43 4.75	8.03 5.97
5.11 5.49	12.55* 10.71*	7.08 6.87

G7

9.68 8.44	8.78 1.45	9.74 8.00
9.73 8.08	9.87 2.15	**9.47** **7.31**
2.52 6.75	9.96* 8.11*	4.49 8.21

G8

8.33 6.98	7.43 0.05	8.39 10.86
12.49 6.62	12.63 0.75	**12.23** **10.17**
1.11 5.29	8.55* 6.71*	3.08 11.07

Potential Games. Second Set

G9

5.56 7.53	5.22 6.88	5.72 7.41
6.32* 9.15*	5.32 7.84	4.85 7.40
4.03 5.82	6.30 7.78	6.18 7.69

G10

5.86 6.63	5.52 7.28	**6.02** **7.91**
6.12* 8.25*	5.12 8.24	4.65 7.90
3.93 4.92	6.20 8.18	6.08 8.19

G11

9.86 3.53	9.52 6.58	**10.02** **11.71**
2.32* 5.15*	1.32 7.54	0.85 11.70
3.73 1.82	6.00 7.48	5.88 11.99

Notes: For each game, outcomes compatible with the *NE* play are denoted by *, those compatible with the *MMS* are in **bold**, and those compatible with the *A* play are underlined. For simplicity purposes, *MMS* is only shown when the behavioral component of the game is non-zero, so is not shown in G1, G6, and G9. In the experiment, we show only two decimals as in the figure. The actual values of the games are displayed in Figure 4 in Appendix A.

Figure 3: Experimental Games

With regard to the actual chosen payoff numbers, we opted for having three digit numbers in order to increase separability between different behavioral rules and also avoid round numbers. Then, the choice was to either opt for points that we then translated into euros or present the payoffs in euros directly (and use decimals). We opted for the latter for simplicity.

Table 3: Predicted Strategies by Different Behavioral Rules for the 11 Games

Rules	Roles	*G1*	*G2*	*G3*	*G4*	*G5*	*G6*	*G7*	*G8*	*G9*	*G10*	*G11*
NE: Nash Equilibrium	R	1,2,3	1,2,3	1,2,3	1	2	3	3	3	2	2	2
	C	1,2,3	1,2,3	1,2,3	1	2	2	2	2	1	1	1
MMS: Mutual-Max Sum	R	1,2,3	3	3	2	1	1,2,3	2	2	1,2,3	1	1
	C	1,2,3	1	3	3	3	1,2,3	3	3	1,2,3	3	3
A: Altruistic	R	1,2,3	3	3	1,2,3	1,2,3	3	1	2	2	3	1
	C	1,2,3	1	3	1,2,3	1,2,3	2	1	3	1	2	3
PO: Pareto Optimality	R	1,2,3	1,2,3	1,3	1,2,3	1,2,3	3	1,3	1,2,3	2	2,3	1,3
	C	1,2,3	1	1,3	1,2,3	1,2,3	2	1,2	1,2,3	1	1,2	3
L1: Level-1	R	1,2,3	1,2,3	1,2,3	1	2	3	3	3	2	2	2
	C	1,2,3	1,2,3	1,2,3	2	1	2	2	2	2	2	2
P: Pessimistic	R	1,2,3	3	2	1	2	3	3	3	2	2	2
	C	1,2,3	3	3	1	2	2	2	2	2	2	1
O: Optimistic	R	1,2,3	2	1	3	3	3	1	3	2	2	3
	C	1,2,3	1	1	2	1	2	2	2	1	2	3

Notes: The table reports the strategies predicted by all the behavioral rules we consider; 1, 2 and 3 refer to the first, second and third strategies, respectively. In a few instances, a behavioral rule is indifferent between multiple strategies, so we assume the behavioral rule will predict any of those strategies with equal probability.

Table 3 shows the predicted strategies by different behavioral rules. We can comment on the predicted choices by the *MMS*. In the games with no behavioral component, such that these games only have the strategic component and the kernel component, we can observe that *MMS* is indifferent between any of the strategies (see games G1, G6 and G9, both in Figure 3 and in Figure 4). In any other case, i.e., when the behavioral component is positive, then the games are designed such that the *MMS* will have a unique prediction.

Table 4: Separation between Different Behavioral Rules

	MMS	*NE*	*A*	*PO*	*L1*	*P*	*O*
MMS	0	**0.6061**	**0.3636**	**0.5606**	**0.6061**	**0.5455**	**0.5455**
NE	0.6061	0	0.4848	0.3788	0.1515	0.1818	0.3636
A	0.3636	0.4848	0	0.1970	0.4848	0.4242	0.3636
PE	0.5606	0.3788	0.1970	0	0.4091	0.4697	0.3182
L1	0.6061	0.1515	0.4848	0.4091	0	0.2121	0.3030
P	0.5455	0.1818	0.4242	0.4697	0.2121	0	0.3636
O	0.5455	0.3636	0.3636	0.3182	0.3030	0.3636	0

Notes: The table reports the proportions of decisions across all 22 decisions in which the different behavioral rules predict different strategies. The minimum possible separation value is 0, which occurs when two behavioral rules prescribe the same strategy in all 22 decisions, and the maximum possible separation value is 1, which occurs when the two models predict a different strategy in each of the 22 decisions. When one behavioral rule's prediction is 1,2,3, meaning playing each of the strategies with equal probability, and another behavioral rule's prediction is 1,2, meaning playing the first two strategies with equal probability, the separation value is equal to 1/3, as these two behavioral rules can be separated only 1/3 of the times, particularly, when a subject plays the third strategy.

Finally, we are able to measure how successful we were in separating *MMS* predictions from the predictions of any other behavioral rules. Table 4 shows the separation between different behavioral rules. The values in the table represent the proportion of games $\times$ player roles, i.e. decisions in which the predictions of two behavioral rules are separated. The numbers can take any value between 0 (no separation at all, such that two behavioral rules predict exactly the same strategy in each of the 22 decisions) and 1 (full separation, such that two behavioral rules predict a different strategy in each of the 22 decisions). The most interesting row in the table is the one referring to the *MMS*, shown in bold, as the main goal when designing the games was to have the highest separation between *MMS* and the rest of the behavioral rules. All separation values for *MMS* are above 50%, as desired, with the exception of the separation between *MMS* and *A* behavioral rules, which is the hardest to separate. This is closely linked to the results shown in Table 1, as harmonic games and constant-sum games are not qualified to separate predictions by *A* and *MMS*.

As far as the separability between other behavioral rules is concerned, we can conclude that these games are far from ideal in terms of separating predictions by *NE* and $L1$, predictions by *NE* and *P*, and predictions by *PO* and *A*, with perfect confounds between some of these behavioral rules and particular classes of games. However, the goal was to separate *MMS* from *NE* and from *A* and *PO* rules. We will come back to this when interpreting the empirical results.

4 Results

We will start off by showing some preliminary analysis testing for whether different sessions can be pooled and for the effects of player role order, as half of the subjects played first as row players and then as column players, while the other half played in the reverse order. We will then analyze how the subjects played game by game to understand whether the manipulation and addition of a behavioral component affect individual behavior. Finally, we will carry out mixture-of-types model estimations, across all games and by sets of games, to get conclusions about the empirical relevance of *MMS*.

4.1 Preliminaries: Testing for the Effects of Player Role Order

We held 5 different sessions and in each of them we had subjects playing the games in each of the two roles.

We start off by testing whether we can pool all 5 sessions, both overall, and by player role order. Table 5 shows the p-values for the two-sided t-test performed for the overall participants in each session, and for the subsets of participants corresponding to each player type in the experiment. We cannot reject the null hypothesis of no significant differences between each of the sessions and the rest. Therefore, we are able to pool all 5 sessions.

Table 5: Poolability of Sessions: p-value of two-sided T-test

H_0	Overall	Type 1	Type 2
S1 = Rest	0.4685	0.5135	0.7130
S2 = Rest	0.6178	0.4663	0.1477
S3 = Rest	0.8487	0.5470	0.7383
S4 = Rest	0.3022	0.4600	0.4718
S5 = Rest	0.9742	0.1450	0.1276

Notes: The null hypotheses are $H_0 : \mu_1 = \mu_2$ where μ_1 and μ_2 correspond to the means of the distributions of the strategy choices for participants for a given session and for the remaining sessions jointly, respectively. For p-values lower than the significance level, the null hypothesis is rejected in favor of the alternative, $H_1 : \mu_1 \neq \mu_2$. S1, S2,..., S5 refer to different sessions, while Rest refers to the remaining sessions pooled together. Type 1 and 2 refer to those subjects who played first as row players and then as column players and then the other way round, respectively. Results are robust to using non-parametric tests (Mann-Whitney U test).

Due to the two-part design of the experiment, and two types of subjects (Type 1 and Type 2, as described in Section 3.2), we next check whether there was any kind of effect from player role order when participants chose their strategies, i.e., whether subjects behaved differently when they started playing as row players instead of as column players. Table 6 displays p-value for the two-sided t-test. We cannot reject the null hypothesis of equal behavior across different player role orders for either of the sessions individually, nor for all of them as a whole. This allows us to use the data for each subject in both roles, and not only in the first role they performed the task. Consequently, we are able to use 200 observations per game.

Table 6: Player Role Order Effects: p-value of two-sided T-test

H_0	Overall	Session 1	Session 2	Session 3	Session 4	Session 5
Type 1 = Type 2	0.8246	0.4704	0.8805	0.7800	0.5416	0.3020

Notes: The null hypotheses are $H_0 : \mu_1 = \mu_2$ where μ_1 and μ_2 correspond to the means of the distributions of the strategy choices for participants labeled as type 1 (started as a row player and then as a column players) and type 2 (started as a column player and then as a row player), respectively, for a given session. For p-values lower than the significance level, the null hypothesis is rejected in favor of the alternative, $H_1 : \mu_1 \neq \mu_2$. Results are robust to using non-parametric tests (Mann-Whitney U test).

4.2 Individual Behavior Game by Game: Is Individual Behavior Constant in Strategically Equivalent Games?

We start by analyzing individual behavior game by game. Table 7 shows the frequencies of play, by all 200 participants, of each of the three strategies in each of the player roles game by game. The strategies that are in the *NE* profile, *MMS* and *A* are denoted by *, in bold and underlined, respectively, for each game. For simplicity, we have only marked the *MMS* prediction when the behavioral component is non-zero. For example, in G7 the *NE* profile is $(3,2)$, the *MMS* solution is the strategy profile $(2,3)$, and the *A* strategy profile is $(1,1)$.

Table 7: Frequencies of Strategy Choices, by Player Role and by Game

	Row Players			Column Players		
	1	2	3	1	2	3
G1	<u>0.225</u>*	<u>0.545</u>*	<u>0.230</u>*	<u>0.205</u>*	<u>0.570</u>*	<u>0.225</u>*
G2	0.105*	0.200*	<u>**0.695**</u>*	<u>**0.310**</u>*	0.165*	0.525*
G3	0.300*	0.420*	<u>**0.280**</u>*	0.105*	0.255*	<u>**0.640**</u>*
G4	<u>0.805</u>*	<u>**0.160**</u>	<u>0.035</u>	<u>0.775</u>*	<u>0.205</u>	<u>**0.020**</u>
G5	<u>**0.265**</u>	<u>0.700</u>*	<u>0.035</u>	<u>0.360</u>	<u>0.605</u>*	<u>**0.035**</u>
G6	0.030	0.015	<u>0.955</u>*	0.040	<u>0.920</u>*	0.040
G7	<u>0.100</u>	**0.060**	0.840*	<u>0.105</u>	0.770*	**0.125**
G8	0.055	<u>**0.135**</u>	0.810*	0.045	0.735*	<u>**0.220**</u>
G9	0.045	<u>0.900</u>*	0.055	<u>0.565</u>*	0.345	0.090
G10	**0.075**	0.795*	<u>0.130</u>	0.455*	<u>0.365</u>	**0.180**
G11	<u>**0.380**</u>	0.555*	0.065	0.380*	0.235	<u>**0.385**</u>

Notes: 1, 2, 3 denote the first, second, and third strategies of the game respectively for each role. For each game, strategies in the *NE* strategy profile are denoted by *, those in the *MMS* strategy profile are in **bold**, and those in the *A* strategy profile are <u>underlined</u>. For simplicity purposes, *MMS* is only shown when the behavioral component of the game is non-zero, so in G1, G6, and G9 is not shown.

A straightforward way to analyze whether manipulation of the behavioral component affects individual behavior is to compare individual behavior across subsets of strategically equivalent games (G1-G2-G3, G6-G7-G8 and G9-G10-G11). We check whether the observed differences are significant or not by performing paired two-sided t-tests between strategically equivalent games. Table 8 shows the corresponding p-value for each of those tests.

Table 8: Behavioral Effects, p-value of (paired) two-sided t-test

	Row Players	Column Players
Games	Harmonic	
G1-G2	$4.48e-15^{***}$	0.0110^{**}
G1-G3	0.7441	$1.03e-12^{***}$
	Potential 1	
G6-G7	$6.73e-05^{***}$	0.6068
G6-G8	$5.87e-05^{***}$	$9.32e-06^{***}$
	Potential 2	
G9-G10	0.2258	0.0020^{***}
G9-G11	$2.43e-11^{***}$	$3.06e-09^{***}$

Notes: The null hypotheses are $H_0 : \mu_1 = \mu_2$ where μ_1 and μ_2 correspond to the means of the distributions of the strategy choices for each of the two games considered (first column), respectively. For p-values lower than the significance level, the null hypothesis is rejected in favor of the alternative, $H_1 : \mu_1 \neq \mu_2$. $^{*}p \leq 10\%$, $^{**}p \leq 5\%$, $^{***}p \leq 1\%$. Results are robust to using non-parametric tests (Mann-Whitney U test).

We start with the three strategically equivalent harmonic games, G1 to G3, where each of them has a unique uniformly mixed NE. First, for neither of the two roles the observed frequencies are equal to the theoretical predictions (of $1/3$ for each strategy), as the subjects playing in both player roles show a bias towards the central strategy. This bias is consistent with experimental work on related zero-sum games, see Rubinstein et al. (1997), Rosenthal et al. (2003) and Crawford and Iriberri (2007). Second, in G2 and G3, once a positive behavioral component is added such that there is a unique MMS prediction, the strategy choice frequencies change either both or for at least one of the player roles. In more detail, for the row player, the frequency of playing the third strategy increases from 0.230 to 0.695 (increment of 200%) and to 0.280 (increment of 22%) in G2 and G3, respectively. For the column player, the significance of the effect is similar. The observed frequency of playing the MMS strategy increases from 0.205 to 0.310 in G2 (increment of 55%) and from 0.225 to 0.640 in G3 (increment of 184%). As shown by the p-values in Table 8, the change in the strategy choices from G1 to G2 is significant for both player roles, while the change in strategy choices from G1 to G3 is only significant for the column player. Therefore, the addition of a behavioral component with a unique MMS (which is at the same time an A strategy profile) strategy profile does indeed modify individual behavior in harmonic games. However, it is worth remembering that in harmonic games, the *MMS* will always coincide with *A* rule predictions, so we cannot conclude that *MMS* is relevant for behavior.

We observe a similar pattern for the two strategically equivalent sets of potential games. For both sets, we start with a game with no behavioral component, G6 and G9, where the *NE* and the *A* solutions coincide, such that the observed frequencies are clearly the highest: 0.955 and 0.920 in G6, and 0.900 and 0.565 in G9, for row and column roles, respectively. In the first modification, G7 and G10, where all three behavioral rules are perfectly separated, we are able to observe that the frequencies of the *NE* strategies decrease, while the strategy choices by *MMS* and *A* increase. In G7 and G10, when *MMS* and *A* are directly competing with each other, *A* seems to get slightly higher adherence. In more detail, *NE* predicted strategy changes from 0.955 to 0.84 and from 0.92 to 0.77 for G7 and from 0.90 to 0.795 and from 0.565 to 0.455 for G10, for each of the player roles, respectively. Finally, in the second modification, G8 and G11, when the behavioral components are added, *MMS* and *A* fully coincide and compete with the *NE* prediction, and the frequency of play for the strategies prescribed by the *NE* decrease even further, bringing the frequency of play by *MMS* and *A* rules' predictions close to the frequency of *NE*. In more detail, we observe that the play for the *NE* strategies decreases down to 0.810 and 0.735 in G8, and down to 0.555 and 0.380 in G11, for row and column players, respectively. Table 8 shows that these behavioral changes are significant across strategically equivalent games for both player roles in the second modification, while for the first modification the behavioral changes are only significant for one of the player roles. Thus, for potential games, we conclude that *MMS* is most relevant for behavior when it coincides with the predictions of the *A* type.

Finally, it is worth remembering that the constant-sum games we considered for the experiment were independent games of each other. By contrast with the harmonic and potential games, we did not modify and add any behavioral component.[12] Despite this, we can remark an important aspect of the observed behavior. For both games, G4 and G5, the strategy in the *NE* strategy profile was by far the highest observed choice with frequencies between 0.605 to 0.805, which is in line with the results in Rey-Biel (2009) (please see the next section to note the lack of separability between *P* rule and *NE* rules in constant-sum games). Interestingly, for row players the strategy predicted by the *MMS* profile is the second highest observed frequency, while it is the lowest for the column role.

To sum up, adding a behavioral component where we have a unique *MMS* seems to affect individual behavior because the observed behavior between strategically equivalent games

[12] Note that if we start with a constant-sum game and modify the behavioral component, the resulting game will no longer be a constant-sum game.

changes significantly. However, and more importantly, these changes are most relevant when the *MMS* and *A* behavioral rules predictions coincide, leading us to conclude that it is not the *MMS* itself which has the most impact on behavior but the *A* behavioral rule. This is an important contribution to the findings of Jessie and Kendall (2022) and Kendall (2022). This result will be more clearly confirmed in the following section.

4.3 Mixture-of-types Model Estimation: Do Individuals Follow the *MMS* Behavioral Rule?

Mixture-of-types models, which are probabilistic models for representing the presence of sub-populations within an overall population, are useful to understand the prevalence of each behavioral rule on the subject sample. In this section we carry out two types of mixture-of-types models estimation, First using all 11 games and then using each of the sets of classes of games. It should be noted that among the different classes, potential games offer the greatest separation between the three main behavioral rules we considered in this study: *MMS*, *NE* and *A* models.

We assume that a subject i employing rule k follows type-k's predicted decision with probability $(1-\varepsilon_k)$ but with a probability of making a mistake of $\varepsilon_k \in [0,1]$. In such a case, the individual would play each of the three available strategies uniformly at random. As in most mixture-of-types model applications, we assume that the errors are identically and independently distributed across games and that they are type-specific. The first assumption facilitates the statistical treatment of the data, while the second considers that some behavioral rules may be more difficult to follow and thus make more errors than others.

The likelihood of a particular individual of a particular type can be constructed as follows. First, let $P_k^{g,j}$ be type-k's predicted choice probability for strategy j in game g. Some rules may predict more than one strategy in a particular game. This characteristic is reflected in the vector $P_k^g = (P_k^{g,1}, P_k^{g,2}, P_k^{g,3})$ with $\sum_j P_k^{g,j} = 1$. When multiple strategies belong to the predicted set, the predicted choice probabilities are defined as choosing uniformly randomly over the predicted set. For each individual in each game, we observe the chosen strategy and whether it is consistent with k. Let $x_i^{g,j} = 1$ if strategy j is chosen by subject i in game g in the experiment and $x_i^{g,j} = 0$ otherwise. The likelihood of observing a sample $x_i = (x_i^{g,j})_{g,j}$ given type k and subject i is then:

$$L_i^k(\varepsilon_k | x_i) = \prod_g \prod_j \left[(1-\varepsilon_k) P_k^{g,j} + \frac{\varepsilon_k}{3} \right]^{x_i^{g,j}} \tag{1}$$

Second, the likelihood function is given by the sum of all the behavioral types that are considered. We include $K = 7$ behavioral models: *MMS*, *NE*, *A*, *PO*, *L1*, *P* and *O*; where p_k assigns probabilities $p = (p_1, p_2, ..., p_K)$ to each behavioral rule. Finally, and as we are interested in the behavioral rule's frequency at the sample of subjects in the experiment, we sum the log likelihood over all 200 subjects.

$$lnL(p,\varepsilon|x_i) = \sum_i ln \sum_k p_k L_i^k(\varepsilon_k|x_i) \tag{2}$$

The output from these models are the estimated frequencies for each of the behavioral models we consider, $p = (p_1, p_2, ..., p_K)$, as well as their noise levels, $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_K)$.

Table 9: Estimation Results

	All 11 Games			Harmonic			CSG			Potential	
	p_k	ε_k		p_k	ε_k		p_k	ε_k		p_k	ε_k
Rules	(1)	(2)	Rules	(3)	(4)	Rules	(5)	(6)	Rules	(7)	(8)
NE	0.42	0.29	$NE = L1$	0.00	–	$NE = P$	0.82	0.33	NE	0.45	0.26
MMS	0.02	0.54	$MMS = A$	0.18	0.23	MMS	0.00	–	MMS	0.05	0.56
A	0.07	0.34				$A = PO$	0.00	–	A	0.00	–
PO	0.05	0.12	PO	0.02	0.23				PO	0.17	0.16
$L1$	0.12	0.35				$L1$	0.17	0.40	$L1$	0.17	0.26
P	0.28	0.23	P	0.58	0.46				P	0.13	0.12
O	0.04	1.00	O	0.22	0.96	O	0.00	–	O	0.03	0.88
LL	3491.04			1191.40			603.45			1647.28	

Notes: The table reports the estimation results for the uniform error specification for all 11 games, in columns 1 and 2, for the three harmonic games, in columns 3 and 4, for the two constant-sum games in columns 5 and 6 and for the 6 potential games, in columns 7 and 8. Columns 1, 3, 5 and 7 present the estimated frequencies of each behavioral model, while columns 2, 4, 6 and 8 show the estimated error for each of the behavioral models. All models are identifiable in all 11 games and in the 6 potential games. In the harmonic games, *NE* and *L*1 are confounded, as well as *MMS* and *A*. In the constant-sum games, *NE* and *P* are confounded, as well as *A* and *PO*.

The main takeaway from Table 9 is that the *MMS* behavioral rule has negligible relevance when it comes to explaining individual behavior. This is confirmed in columns 1, 5 and 7. Of course, if we focus on the harmonic class of games, we can misleadingly interpret that the *MMS* appears to be an important rule in explaining behavioral data. However, it is important to note that in the harmonic class of games, *MMS* predictions are totally confounded with the *A* rule predictions. By contrast, in constant-sum games, where *NE* and *MMS* are fully separated, and more importantly, in potential games, where *MMS* is separable from *NE*, *A* and *PO* rules' predictions, we can see that the relevant behavioral rules are *NE* and those with efficiency concerns. We therefore conclude that the *MMS* behavioral rule is *most* relevant for explaining individual behavior when it coincides with either *NE* and with behavioral rules with efficiency concerns (*A* and *PO*).

Two additional comments. First, with regard to other behavioral rules on top of the three main we have focused on, and consistent with existing work, other two behavioral rules are important: $L1$ and P rules. $L1$ rules are important for constant-sum and potential games, while P is behaviorally very relevant for harmonic and constant-sum games. Second, although we have mentioned that A is a refinement of *PO* and is our main focus, we have kept both *PO* and A in the econometric specification as there is some separability. If we include only A, then, as expected, most of the behavior explained before by *PO* is now explained by A. These results are available upon request.

5 Conclusions

In this paper, we empirically test two main questions. First, whether and when changes in the nonstrategic component of games of normal form are relevant for individual behavior. Second, after defining the *MMS*, whether *MMS* predictions are behaviorally relevant when they are clearly separated from *NE* and A behavioral rules. As they are empirical questions, we carry out a laboratory experiment.

Regarding the first question, and consistent with the work by Jessie and Kendall (2022) and Kendall (2022), we find that additions and manipulations of nonstrategic component indeed can change individual behavior, particularly when the Pareto optimality of different outcomes is changed in the original game. In other words, individual behavior can vary in strategically equivalent games. Regarding the second question, in relation to *MMS*, which captures the most important considerations of the nonstrategic component of the game, we do not find that it is in general a behaviorally relevant rule. How useful is then the *MMS*? From a theoretical point of view, it is the *only* behavioral rule that depends only on the nonstrategic component. Moreover, on many occasions *MMS* predictions will be coinciding with the A behavioral rule, which shows Pareto efficiency concerns and indeed could change the efficiency of different outcomes, as mentioned by Candogan et al. (2011). In those cases, *MMS* would be most relevant but this is the case only when this confound occurs. *MMS* can be separated from the A rule predictions and when it is the case, we find little empirical evidence of the *MMS* relevance to explain individual behavior. We conclude that the decomposition proposed by Candogan et al. (2011) is a useful tool to predict which behavioral rule will be important for explaining individual behavior in games.

We see two avenues for further research. First, the empirical analysis could also be applied to games with more than 3 strategies, as this would expand the possibility of separating out more than the three behavioral rules we have focused on: NE, MMS and A. Second, and more challenging, the analysis could be extended to games with more than two players, where a potential re-definition of MMS is needed.

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A Decomposition of Games: An Example and the Case for the 11 Experimental Games

We start by showing an example of how to compute the four-components of the decomposition of a game. We specifically decompose G4, and Figure 4 displays the decomposition of the 11 experimental games.[13]

G4 is represented in matrix notation as follows:

$$A = \begin{pmatrix} 5.45 & 6.23 & 8.33 \\ 4.18 & 6.85 & 2.24 \\ 5.11 & 0.22 & 9.02 \end{pmatrix}; \quad B = \begin{pmatrix} 4.55 & 3.77 & 1.67 \\ 5.82 & 3.15 & 7.76 \\ 4.89 & 9.78 & 0.98 \end{pmatrix}$$

Recall that we can decompose a game starting either from the *strategic* component or from the *nonstrategic* components. We will start by obtaining the nonstrategic components. The kernel component is obtained as $A^{\mathscr{K}} = (\frac{1}{h^2})\mathbf{11^T}A\mathbf{11^T}$, and $B^{\mathscr{K}} = (\frac{1}{h^2})\mathbf{11^T}B\mathbf{11^T}$, for row and column player, respectively, which can be computed, alternatively, as follows:

$$k_1 = \frac{5.45+6.23+8.33+4.18+6.85+2.24+5.11+0.22+9.02}{9} = 5.29$$
$$k_2 = \frac{4.55+3.77+1.67+5.82+3.15+7.76+4.89+9.78+0.98}{9} = 4.71$$

With the kernel values, which are the average of each player's payoffs, the resulting matrix of the kernel component, for each player, is:

$$A^{\mathscr{K}} = \begin{pmatrix} k_1 & k_1 & k_1 \\ k_1 & k_1 & k_1 \\ k_1 & k_1 & k_1 \end{pmatrix} = \begin{pmatrix} 5.29 & 5.29 & 5.29 \\ 5.29 & 5.29 & 5.29 \\ 5.29 & 5.29 & 5.29 \end{pmatrix}; \quad B^{\mathscr{K}} = \begin{pmatrix} k_2 & k_2 & k_2 \\ k_2 & k_2 & k_2 \\ k_2 & k_2 & k_2 \end{pmatrix} = \begin{pmatrix} 4.71 & 4.71 & 4.71 \\ 4.71 & 4.71 & 4.71 \\ 4.71 & 4.71 & 4.71 \end{pmatrix}$$

As the game is a 3×3 game, we have 3 behavioral values for each player. To obtain each value, for each of the opponent's strategies, we just compute the average payoff, keeping constant the opponent's strategy, and subtract the own kernel value. That is,

$$b_1^1 = \frac{5.45+4.18+5.11}{3} - 5.29 = -0.38$$
$$b_1^2 = \frac{6.23+6.85+0.22}{3} - 5.29 = -0.86$$

[13] In both cases, for simplicity, we rounded up all values to two decimals, as in the experiment.

$$b_1^3 = \frac{8.33 + 2.24 + 9.02}{3} - 5.29 = 1.24$$

Analogously we obtain the behavioral values for column player: $b_2^1 = -1.38$, $b_2^2 = 0.87$, and $b_2^3 = 0.51$.

The matrices of the behavioral component, given the behavioral values obtained above, are:

$$A^{\mathscr{B}} = \begin{pmatrix} b_1^1 & b_1^2 & b_1^3 \\ b_1^1 & b_1^2 & b_1^3 \\ b_1^1 & b_1^2 & b_1^3 \end{pmatrix} = \begin{pmatrix} -0.38 & -0.86 & 1.24 \\ -0.38 & -0.86 & 1.24 \\ -0.38 & -0.86 & 1.24 \end{pmatrix}; \ B^{\mathscr{B}} = \begin{pmatrix} b_2^1 & b_2^1 & b_2^1 \\ b_2^2 & b_2^2 & b_2^2 \\ b_2^3 & b_2^3 & b_2^3 \end{pmatrix} = \begin{pmatrix} -1.38 & -1.38 & -1.38 \\ 0.87 & 0.87 & 0.87 \\ 0.51 & 0.51 & 0.51 \end{pmatrix}$$

To obtain the strategic component, we can either normalize the original game or take the differences between the original game and the sum of the nonstrategic component. The strategic component of the game is:

$$A^{\mathscr{S}} = \begin{pmatrix} 0.54 & 1.80 & 1.80 \\ -0.73 & 2.42 & -4.29 \\ 0.20 & -4.21 & 2.49 \end{pmatrix}; \ B^{\mathscr{S}} = \begin{pmatrix} 1.22 & 0.44 & -1.66 \\ 0.24 & -2.43 & 2.18 \\ -0.33 & 4.56 & -4.24 \end{pmatrix}$$

Once we obtain the strategic component, denoted by $A^{\mathscr{S}}$ and $B^{\mathscr{S}}$ for row and column player, respectively, we can compute the potential and harmonic components. To do so, we need to calculate first three auxiliary matrices: $M = \frac{1}{2}(A^{\mathscr{S}} + B^{\mathscr{S}})$, $D = \frac{1}{2}(A^{\mathscr{S}} - B^{\mathscr{S}})$, and $\Gamma = \frac{1}{2h}(A\mathbf{1}\mathbf{1}^{\mathbf{T}} - \mathbf{1}\mathbf{1}^{\mathbf{T}}B)$. In our case,

$$M = \frac{1}{2}\begin{pmatrix} 0.54 + 1.22 & 1.80 + 0.44 & 1.80 - 1.66 \\ -0.73 + 0.24 & 2.42 - 2.43 & -4.29 + 2.18 \\ 0.20 - 0.33 & -4.21 + 4.56 & 2.49 - 4.24 \end{pmatrix} = \begin{pmatrix} 0.88 & 1.12 & 0.07 \\ -0.25 & -0.01 & -1.05 \\ -0.06 & 0.17 & -0.87 \end{pmatrix}$$

$$D = \frac{1}{2}\begin{pmatrix} 0.54 - 1.22 & 1.80 - 0.44 & 1.80 + 1.66 \\ -0.73 - 0.24 & 2.42 + 2.43 & -4.29 - 2.18 \\ 0.20 + 0.33 & -4.21 - 4.56 & 2.49 + 4.24 \end{pmatrix} = \begin{pmatrix} -0.34 & 0.68 & 1.73 \\ -0.49 & 2.42 & -3.24 \\ 0.26 & -4.39 & 3.36 \end{pmatrix}$$

To obtain the matrix Γ we need first two more auxiliaries matrices, denoted by Γ^A and Γ^B, when $\Gamma^A = A\mathbf{1}\mathbf{1}^{\mathbf{T}}$, and $\Gamma^B = \mathbf{1}\mathbf{1}^{\mathbf{T}}B$

$$\Gamma^A = \begin{pmatrix} 0.54+1.80+1.80 & 0.54+1.80+1.80 & 0.54+1.80+1.80 \\ -0.73+2.42-4.29 & -0.73+2.42-4.29 & -0.73+2.42-4.29 \\ 0.20-4.21+2.49 & 0.20-4.21+2.49 & 0.20-4.21+2.49 \end{pmatrix} = \begin{pmatrix} 4.13 & 4.13 & 4.13 \\ -2.61 & -2.61 & -2.61 \\ -1.53 & -1.53 & -1.53 \end{pmatrix}$$

$$\Gamma^B = \begin{pmatrix} 1.22+0.24-0.33 & 0.44-2.43+4.56 & -1.66+2.18-4.24 \\ 1.22+0.24-0.33 & 0.44-2.43+4.56 & -1.66+2.18-4.24 \\ 1.22+0.24-0.33 & 0.44-2.43+4.56 & -1.66+2.18-4.24 \end{pmatrix} = \begin{pmatrix} 1.14 & 2.58 & -3.71 \\ 1.14 & 2.58 & -3.71 \\ 1.14 & 2.58 & -3.71 \end{pmatrix}$$

Then,

$$\Gamma = \frac{1}{2h}(\Gamma^A - \Gamma^B) = \frac{1}{6} \begin{pmatrix} 4.13-1.14 & 4.13-2.58 & 4.13+3.71 \\ -2.61-1.14 & -2.61-2.58 & -2.61+3.71 \\ -1.53-1.14 & -1.53-2.58 & -1.53+3.71 \end{pmatrix} = \begin{pmatrix} 0.50 & 0.26 & 1.31 \\ -0.62 & -0.86 & 0.18 \\ -0.44 & -0.68 & 0.36 \end{pmatrix}$$

Finally, the potential component is obtained as $(M+\Gamma, M-\Gamma)$ and the harmonic component as $(D-\Gamma, -D+\Gamma)$.

$$A^{\mathscr{P}} = \begin{pmatrix} 0.88+0.50 & 1.12+0.26 & 0.07+1.31 \\ -0.25-0.62 & -0.01-0.86 & -1.05+0.18 \\ -0.06-0.44 & 0.17-0.68 & -0.87+0.36 \end{pmatrix} = \begin{pmatrix} 1.38 & 1.38 & 1.38 \\ -0.87 & -0.87 & -0.87 \\ -0.51 & -0.51 & -0.51 \end{pmatrix}$$

$$B^{\mathscr{P}} = \begin{pmatrix} 0.88-0.50 & 1.12-0.26 & 0.07-1.31 \\ -0.25+0.62 & -0.01+0.86 & -1.05-0.18 \\ -0.06+0.44 & 0.17+0.68 & -0.87-0.36 \end{pmatrix} = \begin{pmatrix} 0.38 & 0.86 & -1,24 \\ 0.38 & 0.86 & -1.24 \\ 0.38 & 0.86 & -1.24 \end{pmatrix}$$

$$A^{\mathscr{H}} = \begin{pmatrix} -0.34-0.50 & 0.68-0.26 & 1.73-1.31 \\ -0.49+0.62 & -0.49+0.86 & -3.24-0.18 \\ 0.26+0.44 & -4.39+0.68 & 3.36-0.36 \end{pmatrix} = \begin{pmatrix} -0.84 & 0.42 & 0.42 \\ 0.14 & 3.29 & -3.42 \\ 0.71 & -3.70 & 3.00 \end{pmatrix}$$

$$B^{\mathscr{H}} = \begin{pmatrix} 0.34+0.50 & -0.68+0.26 & -1.73+1.31 \\ 0.49-0.62 & 0.49-0.86 & 3.24+0.18 \\ -0.26-0.44 & 4.39-0.68 & -3.36+0.36 \end{pmatrix} = \begin{pmatrix} 0.84 & -0.42 & -0.42 \\ -0.14 & -3.29 & 3.42 \\ -0.71 & 3.70 & -3.00 \end{pmatrix}$$

The final decomposition for G4 is:

$$\begin{aligned}
\begin{pmatrix} 5.45,4.55 & 6.23,3.77 & 8.33,1.67 \\ 4.18,5.82 & 6.85,3.15 & 2.24,7.76 \\ 5.11,4.89 & 0.22,9.78 & 9.02,0.98 \end{pmatrix} &= \begin{pmatrix} 1.38,0.38 & 1.38,0.86 & 1.38,-1.24 \\ -0.87,0.38 & -0.87,0.86 & -0.87,-1.24 \\ -0.51,0.38 & -0.51,0.86 & -0.51,-1.24 \end{pmatrix} \\
&+ \begin{pmatrix} 0.84,-0.84 & 0.42,-0.42 & 0.42,-0.42 \\ 0.14,-0.14 & 3.29,-3.29 & -3.42,3.42 \\ 0.71,-0.71 & -3.70,3.70 & 3.00,-3.00 \end{pmatrix} \\
&+ \begin{pmatrix} -0.38,-1.38 & -0.86,-1.38 & 1.24,-1.38 \\ -0.38,0.87 & -0.86,0.87 & 1.24,0.87 \\ -0.38,0.51 & -0.86,0.51 & 1.24,0.51 \end{pmatrix} \\
&+ \begin{pmatrix} 5.29,4.71 & 5.29,4.71 & 5.29,4.71 \\ 5.29,4.71 & 5.29,4.71 & 5.29,4.71 \\ 5.29,4.71 & 5.29,4.71 & 5.29,4.71 \end{pmatrix}
\end{aligned}$$

Harmonic Games

G1 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G1

6.62 6.56	7.62 5.56	5.62 7.56
5.62 7.56	6.62 6.56	7.62 5.56
7.62 5.56	5.62 7.56	6.62 6.56

Potential

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Harmonic

0 0	1 -1	-1 1
-1 1	0 0	1 -1
1 -1	-1 1	0 0

Behavioral

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Kernel

6.62 6.56	6.62 6.56	6.62 6.56
6.62 6.56	6.62 6.56	6.62 6.56
6.62 6.56	6.62 6.56	6.62 6.56

G2 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G2

6.62 11.56	7.62 0.56	5.62 7.56
0.62 12.56	1.62 1.56	2.62 5.56
12.62 10.56	10.62 2.56	11.62 6.56

Potential

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Harmonic

0 0	-1 1	1 -1
1 -1	0 0	-1 1
-1 1	1 -1	0 0

Behavioral

5 0	-5 0	0 0
5 -5	-5 -5	0 -5
5 5	-5 5	0 5

Kernel

6.56 6.62	6.56 6.62	6.56 6.62
6.56 6.62	6.56 6.62	6.56 6.62
6.56 6.62	6.56 6.62	6.56 6.62

G3 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G3

1.56 6.62	5.56 7.62	12.56 5.62
2.56 0.62	6.56 1.62	10.56 2.62
0.56 12.62	7.56 10.62	11.56 11.62

Potential

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Harmonic

0 0	1 -1	-1 1
-1 1	0 0	1 -1
1 -1	-1 1	0 0

Behavioral

0 -5	0 0	0 5
-5 -5	-5 0	-5 5
5 -5	5 0	5 5

Kernel

6.62 6.56	6.62 6.56	6.62 6.56
6.62 6.56	6.62 6.56	6.62 6.56
6.62 6.56	6.62 6.56	6.62 6.56

Constant-Sum Games

G4 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G4

4.55 5.45	3.77 6.23	1.67 8.33
5.82 4.18	3.15 6.85	7.76 2.24
4.89 5.11	9.78 0.22	0.98 9.02

Potential

0.38 1.38	0.86 1.38	-1.24 1.38
0.38 -0.87	0.86 -0.87	-1.24 -0.87
0.38 -0.51	0.86 -0.51	-1.24 -0.51

Harmonic

0.84 -0.84	-0.42 0.42	-0.42 0.42
-0.14 0.14	-3.29 3.29	3.42 -3.42
-0.71 0.71	3.70 -3.70	-3.00 3.00

Behavioral

-1.38 -0.38	-1.38 -0.86	-1.38 1.24
0.87 -0.38	0.87 -0.86	0.87 1.24
0.51 -0.38	0.51 -0.86	0.51 1.24

Kernel

4.71 5.29	4.71 5.29	4.71 5.29
4.71 5.29	4.71 5.29	4.71 5.29
4.71 5.29	4.71 5.29	4.71 5.29

G5 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G5

3.11 6.89	4.80 5.20	6.67 3.33
3.95 6.05	3.97 6.03	1.90 8.10
7.72 2.28	5.74 4.26	0.15 9.85

Potential

0.70 -0.64	0.61 -0.64	-1.32 -0.64
0.70 0.95	0.61 0.95	-1.32 0.95
0.70 -0.31	0.61 -0.31	-1.32 -0.31

Harmonic

-2.45 2.45	-0.67 0.67	3.13 -3.13
-0.03 0.03	0.08 -0.08	-0.06 0.06
2.48 -2.48	0.59 -0.59	-3.07 3.07

Behavioral

0.64 -0.70	0.64 -0.61	0.64 1.32
-0.95 -0.70	-0.95 -0.61	-0.95 1.32
0.31 -0.70	0.31 -0.61	0.31 1.32

Kernel

4.22 5.78	4.22 5.78	4.22 5.78
4.22 5.78	4.22 5.78	4.22 5.78
4.22 5.78	4.22 5.78	4.22 5.78

Potential Games: First Set

G6 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G6

$8.529\overline{4}$ $7.182\overline{7}$	$7.62\overline{4}$ $4.05\overline{1}$	$8.592\overline{7}$ $6.656\overline{1}$
$8.28\overline{7}$ $6.82\overline{4}$	$8.432\overline{7}$ $4.742\overline{7}$	$8.026\overline{1}$ $5.982\overline{7}$
$5.112\overline{7}$ $5.492\overline{7}$	$12.552\overline{7}$ $10.706\overline{1}$	$7.08\overline{3}$ $6.87\overline{1}$

Potential

$0.280\overline{5}$ $0.682\overline{7}$	$-0.62\overline{4}$ $-2.44\overline{8}$	$0.343\overline{8}$ $0.156\overline{1}$
$0.03\overline{8}$ $0.32\overline{4}$	$0.183\overline{8}$ $-1.757\overline{2}$	$-0.222\overline{7}$ $-0.527\overline{2}$
$-3.136\overline{1}$ $-1.007\overline{2}$	$4.303\overline{8}$ $4.206\overline{1}$	$-1.1\overline{6}$ $0.37\overline{1}$

Harmonic

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Behavioral

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Kernel

8.25 6.50	8.25 6.50	8.25 6.50
8.25 6.50	8.25 6.50	8.25 6.50
8.25 6.50	8.25 6.50	8.25 6.50

G7 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G7

$9.680\overline{5}$ $8.442\overline{7}$	$8.77\overline{5}$ $1.45\overline{1}$	$9.743\overline{8}$ $7.996\overline{1}$
$9.72\overline{8}$ $8.08\overline{4}$	$9.873\overline{8}$ $2.142\overline{7}$	$9.467\overline{2}$ $7.312\overline{7}$
$2.523\overline{8}$ $6.752\overline{7}$	$9.963\overline{8}$ $8.106\overline{1}$	$4.49\overline{2}$ $8.2\overline{1}$

Potential

$0.280\overline{5}$ $0.682\overline{7}$	$-0.62\overline{4}$ $-2.44\overline{8}$	$0.343\overline{8}$ $0.156\overline{1}$
$0.03\overline{8}$ $0.32\overline{4}$	$0.183\overline{8}$ $-1.757\overline{2}$	$-0.222\overline{7}$ $-0.527\overline{2}$
$-3.136\overline{1}$ $-1.007\overline{2}$	$4.303\overline{8}$ $4.206\overline{1}$	$-1.1\overline{6}$ $0.37\overline{1}$

Harmonic

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Behavioral

1.15 1.26	1.15 -2.60	1.15 1.34
1.44 1.26	1.44 -2.60	1.44 1.34
-2.59 1.26	-2.59 -2.60	-2.59 1.34

Kernel

8.25 6.50	8.25 6.50	8.25 6.50
8.25 6.50	8.25 6.50	8.25 6.50
8.25 6.50	8.25 6.50	8.25 6.50

G8 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G8

$8.330\overline{5}$ $6.982\overline{7}$	$7.42\overline{5}$ $0.05\overline{1}$	$8.393\overline{8}$ $10.856\overline{1}$
$12.4\overline{8}$ $6.62\overline{4}$	$12.633\overline{8}$ $0.742\overline{7}$	$12.22\overline{7}$ $10.172\overline{7}$
$1.113\overline{8}$ $5.292\overline{7}$	$8.553\overline{8}$ $6.706\overline{1}$	$3.08\overline{2}$ $11.07\overline{1}$

Potential

$0.280\overline{5}$ $0.682\overline{7}$	$-0.62\overline{4}$ $-2.44\overline{8}$	$0.343\overline{8}$ $0.156\overline{1}$
$0.03\overline{8}$ $0.32\overline{4}$	$0.183\overline{8}$ $-1.757\overline{2}$	$-0.222\overline{7}$ $-0.527\overline{2}$
$-3.136\overline{1}$ $-1.007\overline{2}$	$4.303\overline{8}$ $4.206\overline{1}$	$-1.1\overline{6}$ $0.37\overline{1}$

Harmonic

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Behavioral

-0.20 -0.20	-0.20 -4.00	-0.20 4.20
4.20 -0.20	4.20 -4.00	4.20 4.20
-4.00 -0.20	-4.00 -4.00	-4.00 4.20

Kernel

8.25 6.50	8.25 6.50	8.25 6.50
8.25 6.50	8.25 6.50	8.25 6.50
8.25 6.50	8.25 6.50	8.25 6.50

Potential Games: Second Set

G9 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G9

5.56 7.53	5.22 6.88	5.72 7.41
$6.32\overline{3}$ 9.15	$5.32\overline{3}$ 7.84	$4.85\overline{3}$ 7.40
$4.02\overline{6}$ 5.82	$6.29\overline{6}$ 7.78	$6.17\overline{6}$ 7.69

Potential

0.06 0.03	-0.28 -0.62	0.22 -0.09
$0.82\overline{3}$ 1.65	$-0.17\overline{6}$ 0.34	$-0.64\overline{6}$ -0.10
$-1.47\overline{3}$ -1.68	$0.79\overline{6}$ 0.28	$0.67\overline{6}$ 0.19

Harmonic

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Behavioral

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Kernel

5.50 7.50	5.50 7.50	5.50 7.50
5.50 7.50	5.50 7.50	5.50 7.50
5.50 7.50	5.50 7.50	5.50 7.50

G10 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G10

5.86 6.63	5.52 7.28	6.02 7.91
$6.12\overline{3}$ 8.25	$5.12\overline{3}$ 8.24	$4.65\overline{3}$ 7.90
$3.92\overline{6}$ 4.92	$6.19\overline{6}$ 8.18	$6.07\overline{6}$ 8.19

Potential

0.06 0.03	-0.28 -0.62	0.22 -0.09
$0.82\overline{3}$ 1.65	$-0.17\overline{6}$ 0.34	$-0.64\overline{6}$ -0.10
$-1.47\overline{3}$ -1.68	$0.79\overline{6}$ 0.28	$0.67\overline{6}$ 0.19

Harmonic

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Behavioral

0.30 -0.90	0.30 0.40	0.30 0.50
-0.20 -0.90	-0.20 0.40	-0.20 0.50
-0.10 -0.90	-0.10 0.40	-0.10 0.50

Kernel

5.50 7.50	5.50 7.50	5.50 7.50
5.50 7.50	5.50 7.50	5.50 7.50
5.50 7.50	5.50 7.50	5.50 7.50

G11 = **Potential** + **Harmonic** + **Behavioral** + **Kernel**

G11

9.86 3.53	9.52 6.58	10.02 11.71
$2.32\overline{3}$ 5.15	$1.32\overline{3}$ 7.54	$0.85\overline{3}$ 11.70
$3.72\overline{6}$ 1.82	$5.99\overline{6}$ 7.48	$5.87\overline{6}$ 11.99

Potential

0.06 0.03	-0.28 -0.62	0.22 -0.09
$0.82\overline{3}$ 1.65	$-0.17\overline{6}$ 0.34	$-0.64\overline{6}$ -0.10
$-1.47\overline{3}$ -1.68	$0.79\overline{6}$ 0.28	$0.67\overline{6}$ 0.19

Harmonic

0 0	0 0	0 0
0 0	0 0	0 0
0 0	0 0	0 0

Behavioral

4.30 -4.00	4.30 -0.30	4.30 4.30
-4.00 -4.00	-4.00 -0.30	-4.00 4.30
-0.30 -4.00	-0.30 -0.30	-0.30 4.30

Kernel

5.50 7.50	5.50 7.50	5.50 7.50
5.50 7.50	5.50 7.50	5.50 7.50
5.50 7.50	5.50 7.50	5.50 7.50

Notes: The figure displays the 11 experimental games used in the experiments and the corresponding four-components decomposition. In some games and components the actual value is periodic, $1.23\overline{4}$ denotes the periodicity of the third decimal.

Figure 4: Decomposition of the 11 Experimental Games

B Proofs

A two-person game $\mathscr{G}$ can be written as a $h \times h$ bimatrix game (A,B) with $a_{ij} = u_1(s_i,t_j)$ $(i,j = 1,...,h)$ and $b_{ij} = u_2(s_i,t_j)$ $(i,j = 1,...,h)$. To be specific,

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1h} \\ \vdots & \ddots & \vdots \\ a_{h1} & \cdots & a_{hh} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & \cdots & b_{1h} \\ \vdots & \ddots & \vdots \\ b_{h1} & \cdots & b_{hh} \end{pmatrix}$$

Proof of Proposition 1. A strategy profile $(\widetilde{s_i},\widetilde{t_j}) \in S \times T$ in game $\mathscr{G}$ is *mutual-max-sum MMS* if:

$$\widetilde{s_i} \in \arg\max_{s_i \in S} \sum_{t_j \in T} u_2(s_i,t_j) \text{ and } \widetilde{t_j} \in \arg\max_{t_j \in T} \sum_{s_i \in S} u_1(s_i,t_j).$$

We now define the following matrices $\widetilde{A}$, $\widetilde{B}$ as follows: the players add their opponent's payoffs for each of their own strategies.

$$\widetilde{A} = \begin{pmatrix} \sum_j b_{1j} & \cdots & \sum_j b_{1j} \\ \vdots & \ddots & \vdots \\ \sum_j b_{hj} & \cdots & \sum_j b_{hj} \end{pmatrix} \text{ and } \widetilde{B} = \begin{pmatrix} \sum_i a_{i1} & \cdots & \sum_i a_{ih} \\ \vdots & \ddots & \vdots \\ \sum_i a_{i1} & \cdots & \sum_i a_{ih} \end{pmatrix}$$

The *MMS* can be alternatively defined by requiring from each player to choose the strategies with the highest payoff in matrices $\widetilde{A}$ and $\widetilde{B}$.

Next, the nonstrategic component $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$ can be displayed as follows:

$$A^{\mathscr{N}\mathscr{S}} = \frac{1}{h}\begin{pmatrix} \sum_i a_{i1} & \cdots & \sum_i a_{ih} \\ \vdots & \ddots & \vdots \\ \sum_i a_{i1} & \cdots & \sum_i a_{ih} \end{pmatrix} \text{ and } B^{\mathscr{N}\mathscr{S}} = \frac{1}{h}\begin{pmatrix} \sum_j b_{1j} & \cdots & \sum_j b_{1j} \\ \vdots & \ddots & \vdots \\ \sum_j b_{hj} & \cdots & \sum_j b_{hj} \end{pmatrix}$$

Observe that $\widetilde{B} \propto A^{\mathscr{N}\mathscr{S}}$ and $\widetilde{A} \propto B^{\mathscr{N}\mathscr{S}}$. Since the altruistic solution in $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$ is computed by selecting the strategy profile that maximizes the sum of both players payoffs, then we can state that the *MMS* solution in (A,B), computing by selecting the strategies with the highest payoffs in $\widetilde{A}$ and $\widetilde{B}$, coincides with the altruistic solution in $(A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}})$. Finally, the behavioral component is defined by,

$$(A^{\mathscr{B}},B^{\mathscr{B}}) = (A^{\mathscr{N}\mathscr{S}},B^{\mathscr{N}\mathscr{S}}) - (A^{\mathscr{K}},B^{\mathscr{K}}).$$

Given that subtracting the kernel component $(A^{\mathscr{K}},B^{\mathscr{K}})$ means subtracting a fixed amount for each player then the *altruistic* solution in $(A^{\mathscr{N}\mathscr{S}}, B^{\mathscr{N}\mathscr{S}})$ coincides with the *altruistic* solution in $(A^{\mathscr{B}}, B^{\mathscr{B}})$ and consequently with the *MMS* solution in (A,B). $\square$

Proof of Proposition 2. Let (A,B) be a *constant-sum game* in which an amount $C > 0$ is to be divided between players 1 and 2. W.l.o.g. let (a_{11}, b_{11}) be the Nash equilibrium and the MMS outcome of the game.
First, we show that $a_{1j} > a_{i1}$, $i = j \neq 1$.

Since (a_{11}, b_{11}) is the NE outcome, necessarily $b_{1j} \leq b_{11}$ and $a_{i1} \leq a_{11}$ for $i,\ j = 2,...,h$. By definition of the constant-sum-game, $a_{1j} + b_{1j} = a_{11} + b_{11} = C$. As $b_{1j} \leq b_{11}$ we have $a_{11} \leq a_{1j}$ and by NE outcome we have that $a_{i1} \leq a_{11} \leq a_{1j}$. Therefore, by transitivity we have:

$$a_{1j} \geq a_{i1},\ i = j \neq 1 \tag{3}$$

Second, given (a_{11}, b_{11}) is the *MMS* outcome, by definition we have:

$$\begin{aligned}
\text{for each } i &= 2,...,h,\ \sum_{j=1}^{h} b_{1j} > \sum_{j=1}^{h} b_{ij} \text{ , and} \\
\text{for each } j &= 2,...,h,\ \sum_{i=1}^{h} a_{i1} > \sum_{i=1}^{h} a_{ij}
\end{aligned}$$

Summing over these two expressions we obtain:

$$(h-1)\sum_{j=1}^{h} b_{1j} + (h-1)\sum_{i=1}^{h} a_{i1} > \sum_{j=1}^{h} b_{2j} + ... + \sum_{j=1}^{h} b_{hj} + \sum_{i=1}^{h} a_{i2} + ... + \sum_{i=1}^{h} a_{ih}$$

Considering that for each i, j, $b_{ij} = C - a_{ij}$ and substituting it in the previous expression, by simple algebraic manipulations, we obtain:

$$\sum_{i=2}^{h} a_{i1} > \sum_{j=2}^{h} a_{1j}$$

which contradicts condition (3). $\square$

C English Translation of Experimental Instructions

[The original experimental instructions were in Spanish.]
[These general instructions were read aloud and provided in paper only.]

THANK YOU FOR PARTICIPATING IN OUR EXPERIMENT!

Let's start the experiment. From now on, you are not allowed to talk, watch what other participants are doing or walk around the classroom. Please turn off and put away your mobile phone. If you have any questions or need help, raise your hand and one of the researchers will come and talk to you. Please do not write over these instructions. If you do not comply with these rules, YOU WILL BE ASKED TO LEAVE THE EXPERIMENT WITH NO PAYMENT. Thank you.

The University of the Basque Country UPV/EHU and the research projects have provided the funds for this experiment. **You will receive 3 Euros for coming on time.** Additionally, if you follow the instructions correctly you have the chance to win more money. This is a group experiment. The amount you can earn depends on your decisions, the decisions of other participants, as well as on chance. Different participants can earn different amounts.

No participant will be able to identify any other participant by his or her decisions or by his or her earnings in the experiment. We, the researchers, will be able to observe at the end of the experiment the earnings of each participant, but we will not associate the decisions you have made with the names of any participant.

EARNINGS:

During the experiment you will be able to earn experimental points. At the end, each experimental point will be exchanged for Euros, exactly 1 experimental point is worth 1 Euro. In addition, we will round up decimals to the nearest tenth.

Everything you earn will be paid to you in cash in a strictly private manner at the end of the experimental session. **Your final earnings will be the sum of the 3 Euros you receive for participating plus whatever you earn during the experiment.**

If, for example, you get a total of 25.19 experimental points you will get a total of 28.20 Euros (3 Euros as payment for participating and 25.20 Euros from converting the 25.19 experimental points to 25.20 Euros).

If, for example, you get 0.20 experimental points you will get 3.20 Euros (3+0.20 = 3.20).

If, for example, you get 12.83 experimental points you will get 15.90 Euros (3+12.90 = 15.90).

Before starting the experiment, we will explain in detail what kind of decisions you can make and how you can get experimental points.

[From now on, the instructions were read aloud and they were only provided on the computer screen.]

DETAILED INSTRUCTIONS OF THE EXPERIMENT:

This experiment consists of several rounds of decisions. In each of the rounds, you will be paired with a randomly chosen participant from this session. From now on, we will refer to you as "You" (in red) and the other participant as "Other Participant" (in blue) in these instructions.

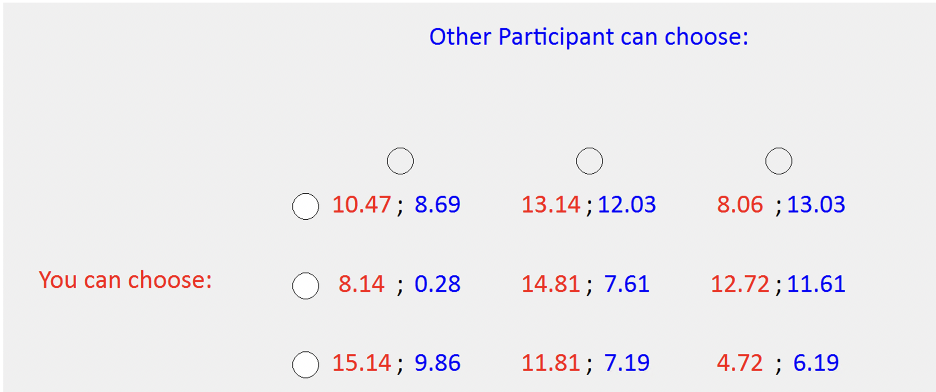
In each round you will see a table and you will have to make a decision, choosing from three possible options. Each decision will be presented in the form of a table similar to the one below (but each time with different values). You will see the corresponding table each time you have to choose an option. Each row of the table corresponds to an option you can choose and the red numbers are the possible experimental points you can earn.

The other participant will also have to choose, independently from you, between her options, which correspond to the columns of the table and the blue numbers are the possible experimental points that the other participant can earn. That is, you choose rows, while the other participant chooses columns. However, to simplify things, the experiment is programmed in such a way that all participants - including the person you are paired with - see their decisions just as in our example. That is, each of you will be presented with your possible actions in the rows of the table.

When choosing, you will not know the option chosen by the other participant, and when the other participant is choosing among her options she will not know the option you have chosen either.

The amount of experimental points you can get in each of the rounds depends on the option you have chosen and the option the other participant has chosen.

The experimental points table you see is an example of what you will see in each of the rounds.



Example 1: if this round is chosen at random and you take the first choice (row) and the other participant takes the second choice (column), you will get 13.14 experimental points and the other participant 12.03 experimental points.

Example 2: if this round is chosen at random and you take the third option (row) and the other participant takes the first option (column), you will get 15.14 experimental points and the other participant 9.86 experimental points.

These are just two examples to better understand how to read the table, as well as to better understand how decisions affect the experimental points you can earn, but are not intended to suggest which decisions you should make.

To make your decision, click on the white button next to the option you want to make. The button will then turn red to indicate which option you have selected. Once you have chosen an option, the choice is not final and you can change it as many times as you like by clicking on another button, until you click on the "OK" button that will appear in the lower right corner of each screen. Once you have clicked "OK" your choice will be final and you will move on to the next round. You will not be able to move on to the next round until you have chosen an option and clicked "OK". You will not have any time restrictions. Take as much time as you need in each round.

Summary:

- Your experimental points will be in red and the other participant's experimental points will be in blue.

- You will participate in several different rounds. In each round you will be paired with a random participant and the experimental points table will be different.

- In each round, you can choose between three different options (rows) and the experimental points you earn depend on which option you have chosen, which option the other participant has chosen, as well as whether that round is randomly chosen at the end of the experiment.

We will start the experiment in a few moments. Before we begin, you will see an example again and you will have to answer several questions. If you have any questions or need help at any point during the experiment, please raise your hand and one of the researchers will come and talk to you.

[From now on, the instructions were not read aloud and they were provided on the computer screen.]

UNDERSTANDING TEST:

To make sure you understand the game, on the next screen we will ask you to answer some questions about the game.

[The table displayed on the screen was the same as the one shown above.]

- Write here <u>your points earned</u> in this round if you choose your second choice and the other participant chooses her third choice, if this round is randomly selected for your payment. **[Correct answer: 12.72]**
- Write here <u>the points earned by the other participant</u> if you choose your third choice and the other participant chooses her second choice, if this round is chosen for payment. **[Correct answer: 7.19]**

DECISION SCREEN:

We will now show you 11 tables, one at a time, and will ask you to make a choice from each table.

At the end of the experiment, we will choose one of the 11 tables at random and pay you for that table.

Click OK to start viewing the tables.

[Once each participant made choices for the first 11 tables they were shown the following instructions on the screen.]

DECISION SCREEN:

We will now show you other 11 tables, one at a time, and will ask you to make a choice on each table. After these 11 tables the experiment ends.

At the end of the experiment, we will choose one of these 11 tables at random and pay you for that table.

Click OK to start viewing the tables.