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JEL Classification: D72

Keywords: Elections, Politicians, Polarization

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Polarization and Electoral Balance[☆]

David K. Levine[1], Andrea Mattozzi[2]

Abstract

We study a model of electoral competition in which two politicians with different office motivations set party platforms and both politicians and grassroots can provide electoral effort. While the underlying structure of the model is asymmetric, we show that both parties have an equal chance of winning the election. In equilibrium, however, only the most office motivated politician matters for policy polarization and welfare: a kind of Gresham's law for politicians. The greater this office motivation, the greater is polarization and the lower is welfare. Less interest in politics means also greater polarization and lower welfare.

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1. Introduction

The outcomes of U.S. national elections are evenly balanced between parties. From the beginning of the roughly 200 years since it began until the current time, the Democratic party has won presidential elections roughly half the time.[3] Such a strong empirical regularity calls out for a strong explanation and the obvious explanation is that party platforms adjust so as to give both parties an equal chance.[4] One such explanation is the standard Downsian model - the Hotelling model applied to politics. In that model office oriented politicians locate themselves at the median voter position to maximize their chance of winning. Unfortunately this model makes the strong counterfactual prediction that there is no policy polarization - that is both parties choose the same centrist platform, which is problematic given the empirical evidence showing increasing polarization in U.S. national elections.[5]

Reflecting on polarization we think a suitable model is one where there are two dimensions of platforms: the range of policies on which parties are willing to take a stand and the vigor with which they will be pursued. Competition over the first dimension leads in a natural way not to equal size of parties (the Democratic party is larger than the Republican party) but to equal cost of turning out voters (the Republican party makes up for its smaller size because it can turn out its voters more easily). Subsequently politicians compete over the vigor with which they pursue issues creating polarization in order to shift the cost of turning out voters from themselves to party activists: the grassroots.

In our model, two political parties compete in a single election. Each party has a politician, grassroots and voters. Politicians are office motivated, grassroots care about the platform and voters care about both the platform and

[3]For smaller jurisdictions in the US, party affiliations seem to be determined by preferences over national issues, so we do not expect parity there. Furthermore, our main focus is on elections dominated by two parties, as it is typically the case with first past the post electoral systems.

[4]Note that the issue here is the frequency with which the two parties win elections and not, as in Levine and Martinelli (2022), why elections may be close.

[5]See for example Hall (2019).

polarization.[6] The model is fundamentally asymmetric because the politicians differ in how strongly office motivated they are. They act as Stackelberg leaders moving first, choosing a platform, then choosing a level of electoral effort. Grassroots move second and also choose a level of electoral effort. Voters move last turning out in response to the chosen platform and the effort provided by the politician and grassroots.

As indicated, our key assumption is that platforms have two dimensions: policy limits and the strength of policy. A policy limit measures stands on the issues: for or against abortion, gun control, taxes, government spending, immigration and so forth. For a right party, for example, broader policy limit corresponds to Ronald Reagan's "larger tent:" a wider range of policies acceptable to middle-of-the-road voter are emphasized. By contrast the strength of policy measures the extent to which the policies will be implemented: it may be that few if any laws or other changes are proposed, or it may be that extensive effort are proposed to implement policies. In our narrative we will show that there is convergence, as in the Hotelling model, on policy limits. Note that this does not mean that implemented policies will converge. In fact, parties will implement different policies and this coupled with divergence in policy strengths will lead to polarization. To take an example: the Republican party has historically been anti-abortion and the Democratic party pro-choice: we take these as the policy limits for the two parties. A debate over particular medical procedures in the third trimester of pregnancy we would regard involving low strength and relatively little polarization, while a debate over whether abortion should be unrestricted or banned under all circumstances we regard as involving high strengths and a high degree of polarization.

In this model, depending on platforms, voters will choose to support one of the two parties and, if mobilized by electoral effort, will vote for that party. While the division of support in the population does not need to be symmetric, in the resulting equilibrium both parties are equally likely to win, platforms

[6] Our model captures in the simplest way the fact that political parties are typically made up of several layers with leaders at the top, grassroots organizers and turnout brokers in the middle, and voters at the bottom.

are polarized, grassroots dissipate rent by exerting electoral effort and suffer the consequences, and politicians make no effort. In fact, politicians have an incentive to choose polarizing platforms precisely to induce effort from grassroots that substitutes for their own. Furthermore, the degree of polarization is determined by the most office motivated politician - a kind of Gresham's law for politicians. The greater this office motivation, the greater is polarization and the lower is welfare. Furthermore, less interest in politics, due, perhaps, to relatively good economic times, also leads to greater polarization and lower welfare.

Intuitively, by choosing costly effort politicians provide an "head-start" to their grassroots in the election. The magnitude of this head-start is inversely related to polarization, which is costless for office motivated politicians. Hence, enough platform polarization effectively redirect the burden of effort away from candidates and onto their grassroots. The required level of polarization is pinned down by the valuation of the politician who has the most to gain from winning and may otherwise be most at-risk of exerting effort in the election.[7]

We emphasize that a unique element of our model is the idea that a subset of voters are not simply driven like sheep by politicians but engage in self-organization: the grassroots matter. Perhaps nowhere was this more apparent than in the 2016 US Presidential election where the Democratic party was blindsided by grass-roots organizations that got out the vote for Donald Trump in the absence of much Republican party effort. In the subsequent Congressional election this effort was matched, despite all gerrymandering, by an even more striking Democratic grass-roots movement. The history of politics is full of grassroots movements ranging from labor unions to social clubs and just as party leaders put effort into turning out the vote so do the grassroots. There are many existing models of platform competition and while all have a role for both politicians and voters, few have a role for grassroots.[8] In addition, in

[7] Alternatively, politicians' office motivation could be interpreted as policy motivation stemming from polarization on other non-modeled issues where the politicians' positions cannot change. A more office motivated politician is therefore a more ideological one and our results would suggest that polarization may propagate from non-pliable to pliable issues.

[8] Notable exceptions are Miller and Schofield (2007) and Venkatesh (2020). None of these

our model voters' turnout is a function of both policy platforms and electoral effort. In light of the increasing importance of GOTV campaigns in political elections,[9] our grassroots model provides an empirically grounded alternative to the valence competition models that follow Stokes (1963) critique to the Downsian model.[10]

In models such as Herrera, Levine and Martinelli (2008), Callander (2008), and Hirsch (2022) politicians have both office motivation and ideological motivation. In particular Callander (2008) uses a signaling model very unlike the model here but does have a result similar to ours: office motivated candidates drive out policy oriented candidates. To explain balanced elections, however, such models must assume that political parties are symmetric - a fact that in a sense is the one that needs to be explained. We assume instead that politicians are purely office motivated and place the ideological motivation with the grassroots. There is a great deal of evidence for this. The triangulation of Clinton and Blair was widely criticized as opportunistic. George H. W. Bush conveniently switched positions on abortion when it advanced his political career, Boris Johnson thought that Brexit was a terrible idea before he was for it, and of course Trump was a liberal New York Democrat before becoming a conservative Republican. A related model assuming purely office motivated politicians in the valence competition literature is Ashworth and Bueno De Mesquita (2009). Their model, however, is a symmetric one in which politicians have an incentive for more polarized platforms because it causes a volatile electorate to focus on issues rather than valence competition, and so increases rents to politicians. We focus instead on the substitutability between the campaign effort of candidates and grassroots to turnout voters in the context of a simple four-stage asymmet-

works, however, considers substitutability between candidates and grassroots effort and its effect on polarization.

[9]See Green and Gerber (2019) for voting mobilization and grassroots movements in US. Enos, Fowler and Vavreck (2013) provide evidence that GOTV, by increasing the differences between voters and nonvoters, may lead to an increase in political inequality.

[10]See Carillo and Castanheira (2008), Eyster and Kittsteiner (2007) and Meirowitz (2007) for models with "endogenous" valence and Groseclose (2001), Aragones and Palfrey (2002), and Kartik and McAfee (2007) for the case of exogenous valence advantage. A review of this literature can be found in Iaryczower and Mattozzi (2013).

ric contest that leads to simple and sharp results concerning equilibrium.

What narrative can we tell? Hall (2019) and particularly McCarthy, Poole and Rosenthal (2006) cite evidence that polarization fell during the twentieth century up until about 1980 then started rising.[11] In our model the more voters are interested in politics the less polarization there will be. In the US for example the earlier part of the century was fraught with depression, war, and then cold war. After 1980 the economy boomed, war and cold war were on the way out, and indeed the entire world started experiencing a wave of prosperity unprecedented in history. It makes sense then that in the face of reduced risk and increased prosperity, concern about politics declined and consequently polarization rose. There is some reason to believe that in the late 19th Century the UK faced few serious challenges: yet there was great polarization over the issue of self-rule for Ireland. Through the lenses of our simple model, these increases in polarization could be related to political leaders striving to transfer the cost of electoral effort to the voters. The less concerned about political issues voters are, the greater polarization is needed to goad them into action.

2. The Model

We study a multi-stage political contest with two parties $k = \{L, R\}$. There are two politicians, one in each party, a mass of potential voters, and each party has a representative grassroots member.[12] In the first two stages of the game politicians choose two dimensions defining the campaign platform, in the second two stages politicians and grassroots choose electoral effort, which transforms campaign platforms into electoral support, and finally the outcome of the election is determined by voters.

In the first stage politicians choose the first campaign platform dimension: a *policy limit* $q_k \in [-1, 1]$ indicating the *range* of policies that are acceptable for a voter to support that politicians' party in case the voter turns out. Define

[11] Hall (2019) provides by way of explanation the idea that it is far more costly to run for office, so only extremists win. Without rejecting this, we observe that this does not explain why parties alternate in power and why voters should be more polarized.

[12] These might also be interest groups.

the *left policy limit* $q_\ell = \min\{q_L, q_R\}$ with corresponding left policy party and the *right policy limit* $q_r = \max\{q_L, q_R\}$ with corresponding right policy party. In case $q_L = q_R$ the left policy party is L and the right policy party is R. The actual policy or policies of the left policy party implemented after the election will be chosen in the range $[-1, q_\ell]$ and those chosen by the right policy party will be in the range $[q_r, 1]$.[13] We assume that there is a unit mass of voters and that they have types in $[-1, 1]$ distributed according to a continuous distribution and each voter supports the party whose policy range is closest to its type.

In the second stage politicians choose the second campaign platform dimension: the *strength* of their policy $x_k \in \Re^+$. This measure the extent to which the campaign platform will be implemented for given policy limit. We measure polarization according to total strength of the two parties $V = x_k + x_{-k}$.[14]

In the third stage politicians choose electoral effort e_k. Finally, in the fourth stage the grassroots, a subset of voters whose interests are the same as the voters, choose their own electoral effort E_k. Efforts mobilize voters depending on their party affiliation and we assume that effort of politicians and grassroots are perfect substitutes.[15] The party that mobilizes the most voters $\lambda_k(e_k + E_k)$ wins the election, where $\lambda_k > 0$ captures the *efficacy* of these efforts for mobilization and it depends on policy limits as we will specify below.[16] The politician of party k receives a reward $B_k > 0$ for winning and nothing for losing. This office motivation represents how much rent the politician expects to get from the

[13]The reversal of party roles when $q_L > q_R$ reflects the idea that if a left party (say) faces an opponent with a limit that is further left it becomes discredited and the politician suffers a large loss. In this case each party wishes to reverse position, as happened in the realignment between the Democratic and Republic Party after the Democrats attempted to poach northern voters from the Republic party by shifting its position on civil rights.

[14]We could also include differences in policy limits, but as we will show the two parties choose $q_L = q_R$, that is, the ranges of acceptable policy are symmetric and hence including policy limits will not matter.

[15]This greatly simplifies computations because the marginal cost of effort by the grassroots does not depend upon the effort of the politician. We indicate below that the basic ideas carry over with imperfect substitutes.

[16]Our modeling of voters' mobilization follows standard assumptions adopted by the existing group-turnout models. See Levine and Mattozzi (2020) for a recent formal model of voter mobilization and a review of the literature. In addition, and differently from existing models, here we assume that policy affects mobilization too.

office, either from power or from money. We presume that the cost to society of this type of self-interested behavior by politicians exceeds the benefit to the politician: certainly this is the usual moral and economic view of rent-seeking by politicians.[17] Hence we view B_k as a measure of how office motivated the candidate is. Letting p_k be the probability that party k wins, the expected utility of a politician is $u_k = p_k B_k - e_k$.

The voters and grassroots of party k, by contrast, care only about platforms. Voters supporting party k will vote for k if they are mobilized. We denote by $A > 0$ how strongly voters feel about politics. If a voter is a party k supporter, the utility of the k platform winning is Ax_k and of the $-k$ platform winning is $-Ax_{-k}$. That is, voters prefer greater implementation of the policies they like and less implementation of the policies they dislike. The grassroots as indicated are a subset of voters so receive expected utility $v_k = A\left(p_k x_k - (1-p_k)x_{-k}\right) - E_k$.

Finally, we assume that it is easier to mobilize many voters in parties with a larger support.[18] Let z_j denote the mass of voters supporting the party with platform $j \in \{\ell, r\}$. Then the efficacy of effort $\lambda_j = H_j(z_j) > 0$ is a strictly increasing continuous function of the mass of supporters and we assume that for some intermediate population the efficacies are the same, that is, for some $0 < z^* < 1$ we have $H_\ell(z^*) = H_r(1 - z^*)$, and for convenience we normalize the efficacy of effort at this point to one.

With respect to welfare, we regard the politician as having measure zero. By the welfare of the grassroots we mean the average expected utility between the two parties, and similarly for the welfare of the voters. Since the gains to one party are cancelled by the loss to the other the grassroots welfare is the negative of the average effort while the welfare of the voters is 0.

Our solution concept is subgame perfect equilibrium.

[17] We do not explicitly model these costs viewing them as some orders of magnitude less than the costs and benefits of economic policies.

[18] For lobbying effort it may be the other way around, but Levine and Mattozzi (2020) show that in important elections larger parties have lower costs of turning out voters.

3. The Equilibrium

Define the most office motivate politician w to be such that $B_w \geq B_{-w}$, that is the one who gets the greatest rent from office-holding. Note that if there is a tie both politicians are by definition most office motivated. By a *least polarized equilibrium* we mean any equilibrium with the least polarization V in the set of equilibria.

Theorem 1. *In any equilibrium $q_w = q_{-w} = q^*$ and both parties have an equal chance of winning. Furthermore, neither politician k provides any effort, each gets expected utility $B_k/2$, with probability one polarization is at least $V \geq B_w/(2A)$ and grassroots welfare is the expected value of $-AV/2$. There is an equilibrium in which $x_w = 0$ and $x_{-w} = B_w/(2A)$, so in a least polarized equilibrium, polarization is exactly $V = B_w/(2A)$. Least polarized equilibria are in second stage pure strategies and $x_w \leq B_{-w}/A$, $x_{-w} \leq B_w/(2A)$.*

The theorem contains a number of insights.

First, it is an Hotelling type of result: policy limits are uniquely chosen to equalize the efficacy of effort. While the endogenous division of support in the population does not need to be symmetric in equilibrium, still each party has an equal chance of winning the election. Note that this does not mean that implemented policies will converge. In fact, the left policy party will implement policies in the range $[-1, q^*]$ and the right policy party in the range $[q^*, 1]$.[19]

Second, politicians choose not to provide effort. While effort by grassroots and politicians are substitutes, polarization and grassroots effort are strategic complements: politicians create polarization to motivate voters and avoid providing their own effort.

Third, the total welfare of grassroots and voters is proportional to the negative of polarization. Hence we may unambiguously identify greater polarization with lower welfare. This gives particular meaning to least polarized equilibria as

[19] We can think of the policy range as being the views of different politicians in the party. In the current US Senate the most right wing Democrats, Sinema and Manchin, represent the left policy limit, while the most left wing Republicans, Collins and Murkowski, represent the right policy limit - and in fact the positions of those four are not terribly different, so policy limits do equalize, although nobody would claim that the US Senate is not polarized.

these are exactly the welfare maximizing equilibria, and indeed, the politicians are themselves indifferent as to the level of polarization.[20]

Fourth, turning to least polarized equilibria, only the most office motivated politician matters for polarization and the greater his office motivation, the higher polarization. Intuitively, the politician who has the most to gain from winning will be the one with stronger incentives to exploit polarization in order to better motivate the grassroots in mobilizing voters. This mechanism will be the stronger, the less the grassroots care about issues.

Finally, turning from welfare to distribution, we observe $x_w \leq B_{-w}/A$, that is lower values of B_{-w} restrict the ability of the most office motivated politician w to increase the strength of his policy platform. This is bad for his party's grassroots and voters and good for the grassroots and voters of the other party. That is to say, if the most office motivated politician chooses to maximize the strength of his policy platform (that is he takes an extreme policy position), grassroots and voters in the party led by the other politician are better off the less office motivated their own leader is: the race to dissipate rents will be less fierce. Similarly lower values of B_w restrict the possibility of taking extreme positions of the less office motivated politician .

3.1. Fourth Stage Game

We start with the game between the grassroots. We define the disadvantaged group d by $\lambda_{-d}(AV + e_{-d}) \geq \lambda_d(AV + e_d)$.

Proposition 1. *In equilibrium if $AV < (\lambda_{-k}/\lambda_{-k})e_{-k} - e_k$ then $p_k = 0$. Otherwise let $L = \lambda_d(AV + e_d) - \max_k\{\lambda_{-k}e_{-k}, \lambda_k e_k\}$. Then*

$$p_d = (1/2)\left(2L/(AV\lambda_{-d}) - (L/(AV))^2/(\lambda_{-d}\lambda_d)\right).$$

[20]In fact politicians may also be mildly averse to polarization. First, they are concerned with their legacy - highly polarizing politicians are less likely to be remembered fondly. In a similar vein less polarization means that the opposition is less likely to engage in character assassination. Real assassination could be also an issue: both the Lincoln and McKinley assassinations appear to have been motivated in part by the highly polarized political atmosphere. The recent attack on the US Congress is another example of violence against politicians that arose from political polarization.

If $\lambda_k = \lambda_{-k}$ *and* $e_k = e_{-k}$ *then* $v_k = -Ax_{-k}$.

Proof. Notice that this is an all pay auction with complete information, linear costs, and (possibly) head starts. If party k wins the election for certain, a grassroots k gets $Ax_k - E_k$ and if party k loses for certain, grassroots k gets $-Ax_{-k} - E_k$. Hence the benefit of winning over losing is AV, and k will provide this much effort to get a certain win over a certain loss. As usual in the all pay auction we think of the grassroots as "bidding" of a total amount of effort. It follows that the amount that k is willing to bid is $\lambda_k(AV + e_k)$ since e_k units of effort are provided by the politician for free and the efficacy of effort is λ_k. Hence for $\lambda_{-k}e_{-k} \geq \lambda_k e_k$ in this linear all-pay auction $-k$ has a head-start advantage of $\lambda_{-k}e_{-k} - \lambda_k e_k$.

Now assume without loss of generality that $\lambda_{-d}(AV + e_{-d}) \geq \lambda_d(AV + e_d)$ and refer to d as the disadvantaged party. From standard results on complete information all-pay auctions (see for example Hillman and Riley (2006) and Baye, Kovenock and De Vries (1998)) there is a unique equilibrium, both grassroots adopt mixed strategies with a known structure.[21] The lowest bid of $-k$ is $\lambda_{-k}e_{-k}$ and the most k is willing to bid is $\lambda_k(AV + e_k)$. Hence if $\lambda_{-k}e_{-k} > \lambda_k(AV + e_k)$ it follows that k loses for certain. This gives the first result concerning p_k. Otherwise define $\underline{b} = \max_k\{\lambda_{-k}e_{-k}, \lambda_k e_k\}$. On $(\underline{b}, \lambda_d(AV + e_d)]$ grassroots k plays a uniform mixture with density height f_k, and has an atom at $\lambda_k e_k$ of F_k. This atom is calibrated so as to make the opponent indifferent between low and high bids.

The values of f_k, F_k are easily computed. A bid of $b_k > \underline{b}$ wins AV with probability $f_{-k}(b_k - \underline{b})$ and costs b_k/λ_k with probability 1. Indifference of k then implies that $f_{-k} = 1/(AV\lambda_k)$. It follows that if we define $L = \lambda_d(AV + e_d) - \underline{b}$ then $F_k = 1 - f_k L = 1 - L/(AV\lambda_{-k})$.

Note that if d has the head-start advantage so $\lambda_d e_d \geq \lambda_{-d}e_{-d}$ then $L = \lambda_d AV$ so $F_{-d} = 1 - L/(AV\lambda_d) = 0$ and $-d$ has no atom in this case. From this we may compute the probability of d winning. With probability F_d the disadvantaged grassroots opt out and loses for sure. Otherwise with probability $1 - F_d$

[21] See Levine and Mattozzi (2020) for details.

with probability F_{-d} the advantaged grassroots opts out and the disadvantaged grassroots wins for sure, or with probability $1 - F_{-d}$ both bid uniformly on $(\underline{b}, \lambda_d(AV + e_d)]$, implying that each has a 50% chance of winning. Hence

$$p_d = (1 - F_d)\left(F_{-d} + (1/2)(1 - F_{-d})\right) =$$

$$(1/2)(1 - F_d)\left(1 + F_{-d}\right) =$$

$$(1/2)(L/(AV\lambda_{-d}))\left(2 - L/(AV\lambda_d)\right) =$$

$$(1/2)\left(2L/(AV\lambda_{-d}) - (L/(AV))^2/(\lambda_{-d}\lambda_d)\right).$$

Finally, if $\lambda_k = \lambda_{-k}$ and $e_{-k} = e_k$ the contest is symmetric so there is complete rent dissipation so that $v_k = -Ax_{-k}$. □

3.2. Third Stage Game

For the third stage game we define an equilibrium as *peaceful* if $e_k = e_{-k} = 0$ with probability one. Otherwise we call the equilibrium *contested.* Let G_k, G_{-k} denote the equilibrium strategies for the politicians choice of effort in the third stage of the game contingent on the earlier platform choices, $u_k(G_k, G_{-k})$ their expected utility and $p(G_k, G_{-k})$ the probability of winning.

Proposition 2. *For any λ_k, λ_{-k} if $AV > (\max_k \lambda_k / \min_k \lambda_k) \max B_k$ in any Nash equilibrium of the subgame starting in the third stage there is a unique equilibrium and it is peaceful in the third stage.*

The computations in the proof use the fact that effort by grassroots and politicians are perfect substitutes. However, the key idea is that greater polarization causes the grassroots to make greater effort, and so it reduces the marginal benefit of politician effort: this idea remains valid even without perfect substitutes. Hence, when there is "enough" polarization as exactly computed here, politicians choose not to provide effort.

Proof. We may assume $e_k \leq B_k$ since it could not be optimal to bid more than the value of the prize. We may assume without loss of generality that

$AV > (\lambda_{-k}/\lambda_{-k})B_w$. By Proposition 1 this implies that for $\lambda_{-d}(AV + e_{-d}) \geq \lambda_d(AV + e_d)$, $L = \lambda_d(AV + e_d) - \max_k\{\lambda_{-k}e_{-k}, \lambda_k e_k\}$ and

$$p_d = (1/2)\left(2L/(AV\lambda_{-d}) - (L/(AV))^2/(\lambda_{-d}\lambda_d)\right).$$

From this we may compute the derivative

$$\frac{\partial p_d}{\partial L} = (1/V)\left(1/(A\lambda_{-d}) - \left(L/(A^2V)\right)/(\lambda_{-d}\lambda_d)\right).$$

Since $L \leq \max_k \lambda_k AV$ we have

$$-(1/V)\left(\max_k \lambda_k/A\right)/(\lambda_{-d}\lambda_d) \leq \frac{\partial p_d}{\partial L} \leq (1/V)\left(1/(A\lambda_{-d})\right).$$

Moreover $-\max_k \lambda_k \leq \partial L/\partial e_k \leq \lambda_d$, so

$$-(1/(AV))\max_k \lambda_k\left(\max_k \lambda_k\right)/(\lambda_{-d}\lambda_d) \leq \frac{\partial p_d}{\partial e_k} \leq (1/(AV))\max_k \lambda_k/\lambda_{-d}.$$

From this it follows that

$$(1/(AV))\max_k \lambda_k\left(\max_k \lambda_k\right)/(\lambda_{-d}\lambda_d) \geq \frac{\partial p_{-d}}{\partial e_k} \geq -(1/(AV))\max_k \lambda_k/\lambda_{-d}.$$

Summarizing, for any λ_d, λ_{-d} we have $\partial p_k/\partial e_k \leq (\max_k \lambda_k/\min_k \lambda_k)/(AV)$.

The expected utility for politician k for e_k is $u_k(e_k, G_{-k}) = B_k \int p(e_k, G_{-k})dG_{-k} - e_k$. Differentiating under the integral sign we get

$$\frac{\partial u_k}{\partial e_k} = B_k \int \frac{\partial p(e_k, G_{-k})}{\partial e_k} dG_{-k} - 1 \leq B_w\left(\max_k \lambda_k/\min_k \lambda_k\right)/(AV) - 1$$

so that for $AV > (\max_k \lambda_k/\min_k \lambda_k)\, B_w$ the only equilibrium is for both politicians to choose $e_k = 0$. □

3.3. Second Stage Game

Proposition 3. *For any λ_k, λ_{-k} in any Nash equilibrium of the subgame starting in the second stage the third stage is peaceful.*

Proof. Let $\overline{e}_k$ be the expected effort in some Nash equilibrium of the subgame

starting in the third stage, and let q_k, q_{-k} be the winning probabilities. From Proposition 2 k can guarantee a peaceful equilibrium by choosing $x_k \geq \overline{V}$. By Proposition 1 this results in a unique fourth stage equilibrium with winning probabilities p_k, p_{-k}. Hence $q_k B_k - \overline{e}_k \geq p_k B_k$ and $(1 - q_k)B_{-k} - \overline{e}_{-k} \geq (1 - p_k)B_{-k}$. Dividing the first inequality by B_k and the second by B_{-k} and adding we see that

$$1 - \overline{e}_k / B_k - \overline{e}_{-k} / B_{-k} \geq 1.$$

This is possible if and only if $\overline{e}_k = \overline{e}_{-k} = 0$, which is to say the third stage is peaceful. □

3.4. First Stage Game

Proposition 4. *In any Nash equilibrium of the game in the first stage* $q_k = q_{-k} = q^*$.

Proof. As the third stage equilibrium will be peaceful politician expected utility is $p_k B_k$ so this is in fact (normalizing payoffs dividing by B_k) a zero-sum game in the first period. Moreover, from Proposition 3 the third stage equilibrium will be peaceful for any realization of λ_k, λ_{-k} . From Proposition 1 the advantaged party has $\lambda_{-d} \geq \lambda_d$, $L = \lambda_d AV$, and

$$p_d = \lambda_d / \lambda_{-d} - (1/2) \left(\lambda_d / \lambda_{-d} \right)^2 .$$

Each politician by choosing $q_k = q^*$ can guarantee $\lambda_k / \lambda_{-k} \geq 1$ implying that $p_k \geq 1/2$. This proves, despite the lack of continuity, that the minimax Theorem holds. Moreover, if the opponent chooses any other strategy than $q_{-k} = q^*$ then $p_k > 1/2$. Hence the zero sum game has a unique equilibrium in which each politician chooses $q_k = q^*$. □

Note that the result of Proposition 3 does not depend on politicians moving simultaneously in the first period.[22]

[22] We can consider a sequential version of the first stage in which one politician is chosen at random to go first. The first politician is L or R according to their type, but the second politician may undercut the first by switching sides. if the second mover preempts the first by

The result is driven by the reversal of positions when one party attempts to build a supermajority. This is a real phenomenon. In the 1960s the Democrats, the party of the right, temporarily built a supermajority by moving to the left of the Republicans on civil rights. The Republicans responded with Nixon's "Southern strategy" becoming the party of the right and forcing the Democrats to be the party of the left by stealing all of their right-wing (Southern) voters. The changing position of Democratic politicians such as Senator Robert Byrd on race indicate the malleability of political beliefs. The shifting of roles like this is a complicated dynamical phenomenon which we try to capture in this simple static model of real-time realignment.

3.5. The Third Stage Revisited

We now know from Proposition 4 that $q_k = q_{-k} = q^*$ and so $\lambda_k = \lambda_{-k} = 1$. We now reexamine what happens in the third stage.

Proposition 5. *With $\lambda_k = \lambda_{-k} = 1$ in the third stage game a peaceful equilibrium exists if and only if $AV \geq \max B_k/2$. Furthermore if $AV > \max B_k$ it is the only equilibrium. If $AV < \min\{\min B_k, \max B_k/2\}$ then there is no pure strategy equilibrium. In any equilibrium $u_k(G_k, G_{-k}) \geq B_k - B_{-k} - AV$. If $B_{-k} \leq B_k$ then in any contested equilibrium $u_{-k}(G_{-k}, G_k) < B_{-k}/2$.*

Notice that if polarization is low the equilibrium must be in mixed strategies: the existence of such an equilibrium is not in question as it follows from the Glicksberg fixed point theorem. While little is known about the structure of such equilibria, what matters is that Proposition 5 establishes a key fact: any contested equilibrium must be less advantageous to the less office motivated politician than a peaceful equilibrium. This is key since, as the first part of Proposition 5 establishes, by creating enough polarization in the first stage a peaceful equilibrium in the second stage can be guaranteed.

switching sides, then the first mover has the option to respond by also switching sides: they can choose any q (if the second mover is R who chose to go ℓ) to the right of what R chose. If we have two L (or two R) politicians the voters who are served by both affiliate with the closest one.

Proof. By Proposition 1 the objective function is given by

$$u_k(e_k, e_{-k}) = B_k p(e_k, e_{-k}) - e_k = (1/2)B_k \left(1 + \text{sign}(e_k - e_{-k}) \left(\frac{e_{-k} - e_k}{AV}\right)^2\right) - e_k.$$

A peaceful equilibrium exists if and only if when $-k$ provides no effort $e_{-k} = 0$ it is optimal for k also to provide no effort. In this case the objective function is convex in e_k so the optimum is either to provide no effort and get $B_k/2$ or provide AV units of effort and get $B_k - AV$. It follows that there is a peaceful equilibrium $e_k = e_{-k} = 0$ if and only if $AV \geq \max B_j/2$, $j \in \{k, -k\}$.

The uniqueness of peaceful equilibrium is given in Proposition 2.

Turning to the existence of contested pure strategy equilibria, we observe that if both politicians provide the same level of effort $\partial u_k/\partial e_k = -1$ so this is an equilibrium only if it is peaceful. If both provide different levels of effort then the one k providing higher effort is on the convex part of the utility function so must provide effort $e_{-k} + AV$ and win for sure. This implies that $-k$ loses for sure so it must be that $e_{-k} = 0$. If this is to be optimal for $-k$ it must be that

$$\frac{\partial u_{-k}(0, AV)}{\partial e_{-k}} = \frac{B_{-k}}{AV} - 1 \leq 0,$$

that is $AV \geq B_j$ for one of the $j \in \{k, -k\}$. We turn to utility. Since $-k$ will never provide more than B_{-k} units of effort k can win for certain by providing an effort of $B_{-k} + AV$ yielding an expected utility of $B_k - B_{-k} - AV$.Finally, suppose that $B_k > B_{-k}$ and let $c_k(e_k) = e_k/B_k$ denote the linear cost of exerting effort relative to the value of the prize B_k and $c_k(G_k)$ the expected cost where the expectation is taken with respect to the equilibrium distribution of effort choice G_k. From optimality of G_k and symmetry we have

$$p(G_k, G_{-k}) - c_k(G_k) \geq p(G_{-k}, G_{-k}) - c_k(G_{-k}) = 1/2 - c_k(G_{-k}).$$

By subtraction we have

$$p(G_k, G_{-k}) - 1/2 \geq c_k(G_k) - c_k(G_{-k}). \tag{3.1}$$

Reversing the role of the two politicians we also have

$$p(G_{-k}, G_k) - 1/2 \geq c_{-k}(G_{-k}) - c_{-k}(G_k)$$

or since one politician's chance of winning is the other's chance of losing

$$p(G_k, G_{-k}) - 1/2 \leq c_{-k}(G_k) - c_{-k}(G_{-k}).$$

Together with 3.1 this gives

$$c_{-k}(G_k) - c_{-k}(G_{-k}) \geq c_k(G_k) - c_k(G_{-k}) = (B_{-k}/B_k)\left(c_{-k}(G_k) - c_{-k}(G_{-k})\right).$$

Since $B_{-k}/B_k < 1$ it follows that $c_{-k}(G_k) - c_{-k}(G_{-k}) \geq 0$ hence it must also be the case that $c_k(G_k) - c_k(G_{-k}) \geq 0$. From equation 3.1 then $p(G_k, G_{-k}) \geq 1/2$. If $p(G_k, G_{-k}) > 1/2$ then certainly $u_{-k}(G_{-k}, G_k) < B_{-k}/2$. Suppose instead that $p(G_k, G_{-k}) = 1/2$. This implies that if one politician provides zero effort for certain both do so, so that the equilibrium would be peaceful. Hence both provide positive effort with positive probability so both get less than $B_j/2$, $j \in \{k, -k\}$. □

3.6. Second Stage Game Revisited

We can now prove the main theorem. Recall that $B_w \geq B_{-w}$. We know from Proposition 2 we should have a peaceful equilibrium, so there must be enough polarization for this to be true. The key new idea here is to find a peaceful equilibrium in the third stage. We do so by observing that if the politician w with greater office motivation chooses not to polarize, that is, $x_w = 0$ then the politician with less office motivation who stands to lose in a contested election can block w from a contested election by providing enough polarization, so this will be an equilibrium. Here are the details of the proof:

Proof. From Proposition 2 the equilibrium is peaceful, and from Proposition 5 both parties have an equal chance of winning giving the politicians expected utility $B_k/2$ and with probability one $V \geq B_w/(2A)$. Grassroots welfare is also given in that result. Second, we show that the lower bound on polarization is

achieved by showing that indeed $x_w = 0$ and $x_{-w} = B_w/(2A)$ can be supported as an equilibrium. To do this, we assign a peaceful equilibrium to the third stage whenever one exists. We apply Proposition 5 to see that for any level of polarization equal to or greater than the equilibrium level there is a peaceful equilibrium in the second stage. In particular on the equilibrium path politician k gets $B_k/2$. Since deviations by w only increase polarization no advantage is derived. Moreover, $-w$ gets $B_{-w}/2$ and by Proposition 5 there is no equilibrium of the third stage game which gives greater expected utility than this. Hence this constitutes a subgame perfect equilibrium. Third, polarization can be equal to $B_w/(2A)$ with probability one if and only if the politicians employ pure strategies in the second stage. Finally, w can choose a platform of $x_w = 0$ and by Proposition 5 receive at least $B_w - B_{-w} - Ax_{-w}$ in the third stage. As this must be less than or equal to the equilibrium expected utility of $B_w/2$ we see that $x_{-w} \geq B_w/(2A) - B_{-w}/A$. In a least polarized equilibrium $x_{-w} = B_w/(2A) - x_w$. Substituting for x_{-w} in the inequality we have $B_w/(2A) - x_w \geq B_w/(2A) - B_{-w}/A$ giving the stated upper bound on x_w. Finally, $x_w \geq 0$ and the equality gives $x_{-w} \leq B_w/(2A)$. □

4. Conclusion

We have examined a model in which platforms have two dimensions: a range of policies that are acceptable for supporters and the effort made to implement those policies. We showed how substitutability between grassroots and politicians efforts lead to polarization: politicians create polarization to motivate voters and avoid providing their own effort. This polarization coexists with equal probability that the two parties win, despite underlying asymmetries, due to a Hotelling like effect over the range of acceptable policies. Moreover, the more office motivated politicians are, the more they will exploit polarization. Since the consequences on voters depend on the most office motivated politician, our model suggests that high variance in potential candidates can be pernicious as much as one bad apple can spoil the entire barrel. This is especially true in good times when a great deal of polarization is needed to motivate voters.

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