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Responsible Investment and Responsible Consumption

Abstract

To reduce a negative externality, socially responsible households can invest responsibly (SRI), consume responsibly (SRC), or do both. Which is better? In a closed microeconomic model with intertwined product and capital markets, we analyze how responsible households should use SRI and SRC to maximize their impact. Both strategies reduce the externality as long as investors are risk-averse and the products have no perfect substitutes. Responsible households gain the highest impact when using SRC in equal proportion to SRI. A mere focus on SRC is never efficient. SRI plays a role in any green strategy. The financial performance of green investments is determined by the responsible households' mix between SRI and SRC.

JEL Classification: G00

Keywords: Socially responsible investment (SRI), ethical investment, socially responsible consumption (SRC), sustainable investment, Sustainable consumption, green investment, Divestment, ESG, SPI

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Responsible Investment and Responsible Consumption*

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July 6, 2022

Abstract

To reduce a negative externality, socially responsible households can *invest* responsibly (SRI), *consume* responsibly (SRC), or do both. Which is better? In a closed microeconomic model with intertwined product and capital markets, we show that the greatest responsible impact is achieved by a proportional reduction in both investment (exit) and consumption (boycott). The proportional reduction is optimal because it prevents price changes and corresponding market responses that would partly offset the responsible choice. Therefore, a mere focus on SRC is never efficient. SRI should always play a role in the responsible choice. The relative financial performance of green investments is determined by the ability of responsible households to commit to the efficient exit and boycott strategy.

Keywords: Socially responsible investment (SRI), socially responsible consumption (SRC), sustainable investment, green finance, ESG, SPI.

JEL-Classification: G30, G23, D62, D64, D16, M14.

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1 Introduction

Today, the need for a more sustainable economy is on the top of the agenda of policy makers, investors, and consumers. However, responsible households may wonder if their individual investment and consumption choices have an actual impact on reducing the negative externality. Reallocation of investments from dirty to clean companies is supposed to affect the funding costs, and thus, the supply from firms.[1] However, forgoing high-yielding stocks and diversification opportunities could lead to a loss of household income. Simply discarding dirty stocks from investment portfolios might not even be effective, as other investors may compensate for the reduction. Similar arguments apply to responsible consumption. Giving up dirty goods narrows down consumption choices, and scorned goods may be consumed by less scrupulous households who substitute for the responsible choice. As households are usually both investors and consumers, the question arises of how responsible households should act to achieve the optimal reduction of a negative externality?

Our model provides the first theoretical analysis of the fundamental question of whether households should better exit dirty investments (SRI) or if they should focus on boycotting dirty consumption (SRC). Responsible households can choose to contribute to a negative externality abatement by affecting the supply side via their investment in the primary capital market and the demand side via their consumption choices. The responsible choice has indirect effects through market prices and the corresponding impact on the other households' consumption and investment decisions. Moreover, we explicitly acknowledge that both markets are intertwined: the cost of capital influences product prices as much as product market profits affect investment yields. When choosing how to exert their impact, responsible households anticipate the responses of other market participants.

Our results show that both SRI and SRC are effective. Both have a direct, symmetric and proportional impact in reducing a negative externality. However, to avoid the partial offset of their choices, households should not try to shift asset returns and product prices.

[1] In this paper we will use the terms "clean" ("dirty") to refer to a production that causes low (high) negative externalities. The wording suggests an environmental externality, but the model also applies to social or other externalities.

Instead, their responsible choice has the greatest impact if they reduce investment in proportion to their consumption reduction. Consequently, a reduction in consumption alone, without a reduction of dirty industry investment, is not efficient. The optimal impact *level* chosen by households depends on economic uncertainty, risk aversion, relative product preferences, and the commitment mechanism available to responsible households.

Responsible households face a series of commitment problems. First of all, it is individually rational to choose the same investment and consumption level as standard households. To be able to have an impact, we therefore assume that responsible households can coordinate their choices.[2] Second, at the time of consumption, the goods are already produced. It is too late to influence the level of production. Therefore, responsible households need to commit ex ante to some consumption level that is anticipated at the production stage. Third, although there are no secondary markets in our model, in reality, shares are typically bought on stock exchanges. This creates a similar time consistency problem for investments as for consumption. At the time of the share purchase, it is too late to influence the firms' investment and output levels. Therefore, households can only influence firm behavior if they can commit to an investment level ex-ante. If such a credible commitment is feasible, the anticipation of prices on the secondary market influences the prices and volumes at the IPO. In our basic model setup, we assume that responsible households can credibly commit to a certain investment and consumption level ex-ante. Later on, in Section 3, we discuss different commitment devices such as taxes, social norms and peer pressure and their impact on the optimal responsible behavior.

In our model, firms produce a good that entails a negative externality.[3] Households first invest into firms in the primary capital market, then spend the investment return on consumption. A fraction of households is "responsible" and takes the externality of their decision into account, at least partially.[4] Firms use the invested money to produce goods,

[2]This is a common assumption in the literature on responsible investors (see e. g. Oehmke and Opp, 2020; Broccardo et al., 2020). It is necessary for our Proposition 3. In contrast, Propositions 1 and 2 are more general. They also apply to altruistic households.

[3]For a positive externality, the problem is similar to public good provision by private households, as in Bergstrom et al. (1986).

[4]If responsible households had political clout, they could convince the policy maker to introduce a Pigovian tax on the good with the negative externality. In this paper, we assume that this political path

then sell the goods on a competitive market, earn the revenue, and suffer from a profit shock. Households collect profits from firms.

Our study makes three contributions to the literature on green finance. *First*, our paper addresses the substantial skepticism toward the effectiveness of green household investments in perfect capital markets (Brest et al., 2018; Krahnen et al., 2021).[5] Although arbitrageurs can partially offset green household decisions, inefficient diversification prevents full replacement of their choices. Therefore, both – SRI and SRC – are effective in reducing the externality. Their relative effectiveness depends on the risk aversion and the consumption preferences of standard households, which alter the market response to price changes. This result implies that each household has the same impact, regardless of whether the fraction of responsible households is large or small.

Second, and most importantly, we show that responsible households can fully eliminate the partial offset of their sacrifice by complementing responsible investment with proportional boycott of consumption. Intuitively, withholding the amount of capital from a dirty firm that becomes redundant due to a commitment to reduce dirty consumption forces the firm to scale back production while leaving the equilibrium prices and returns unchanged. In contrast, any disproportionate effort in one market would affect the equilibrium returns and prices, thereby changing the investment and consumption decisions of standard households whose adjustments would substitute, at least partially, for the responsible household's choices rendering them less efficient. Interestingly, this proportionality result does not depend on the level of risk aversion or the substitutability of the product.[6] If risk aversion is low, it is cheap to reduce investment in the dirty industry, but other investors also have a low risk aversion, and thus readjust their investment accordingly. Therefore, the cost is low, but the effect is also low. The same argument holds for

is not an option. One reason might be that responsible households do not have a majority or that the externality occurs globally, whereas the political system is implemented at regional levels.

[5]An example of the public discourse in this vain is the interview with Bill Gates published in the Financial Times, in which he states that "Divestment, to date, probably has reduced about zero tonnes of emissions." https://www.ft.com/content/21009e1c-d8c9-11e9-8f9b-77216ebe1f17.

[6]SRI would not have an impact on corporate behavior in the absence of risk or risk aversion because even though responsible investors eschew dirty stocks, their investments would be perfectly substituted by less ethical investors. Similarly, responsible consumption choices would be entirely offset if goods were perfect substitutes. We discuss the importance of households' need for diversification in Section 4.1.

the other exogenous parameters. In a robustness check, we show that the proportionality result even remains to hold if standard and responsible households have heterogeneous preferences on risk and product substitutability.

Third, we show that the relative financial performance of responsible investments depends on how well households are able to coordinate responsible behavior in both markets. First, we describe the optimal coordinated choice with perfect commitment. If commitment is not feasible, we show that a sacrifice can be implemented with the help of peer pressure ("shame & blame"). If households can only commit to SRC but not SRI, green investments financially underperform dirty shares, and vice versa.

Literature. Socially responsible households in our model want to behave pro-socially as proposed in Bénabou and Tirole (2010): They are willing to sacrifice utility to reduce a negative externality. Although the assumption of altruistic households would yield the same results for our first two propositions, we focus on coordinated choices of rational agents in the discussion of social welfare. The willingness of responsible investors to forgo financial performance to invest according to their social preferences is well documented (Renneboog et al., 2011; Riedl and Smeets, 2017; Barber et al., 2021). The research on responsible consumption, however, is mainly focused on consumer behavior (Nguyen et al., 2019; White et al., 2019) and how more sustainable production can increase the demand for goods. On the contrary, our article contributes to the literature by analyzing the interplay between responsible investment, production, and consumption.

The choices of responsible households in our model reflect the moral concept of direct consequentialism, as discussed in Moisson (2021) because households are concerned with the direct impact of their investment and consumption decisions. Chowdhry et al. (2018) analyze conditions under which the joint financing of purely profit-oriented and socially responsible investors improve social outcomes. Similarly, Oehmke and Opp (2020) analyze the impact of socially responsible investment on the production choices of financially constrained firms. We complement this literature on SRI by showing that if households can also commit to reduce the consumption of goods produced, SRI is also effective in reducing production quantities through increased funding costs.

Similar to Heinkel et al. (2001) the risk-sharing friction alters the equilibrium funding costs of firms in our setup such that a decreased investment from responsible households results in a lower production. As a result, market prices increase, which also increases the marginal yield on investment, so that more standard investors are willing to invest, partially compensating for the responsible investment reduction. This effect has been documented by Zerbib (2022) who show that the focus of investors on SRI stocks improves the relative financial performance of sin stocks. The key contribution of our model is to show that responsible households can fully prevent this attenuation of their responsible investment reduction by also committing to consume less from dirty firms.

Broccardo et al. (2020) analyze the relative effectiveness of investment exit or consumer boycott and compare it with the efficiency of strategies that directly influence management decisions (voice), which they find to be the most effective strategy if responsible investors own the majority of shares. On the contrary, we argue that investors are also typically consumers, and thus investment exit and boycott are not substitutes, but complement each other in a responsible strategy. Because households can do both simultaneously: consume less *and* invest less, exit and boycott become efficient. We show that responsible households can directly reduce the externality without allowing other market participants to offset their choices by not providing the capital needed to produce the amount of product, which they also boycott. This implies that retail investors can have a responsible impact even if they do not have a voice or are in the minority.

These synergies between SRI and SRC have been largely neglected in the literature. A remarkable exception is Albuquerque et al. (2019), who analyze how SRI can increase firm profitability due to an increase in the loyalty of their customer base. In our model, responsible households are also more sensitive to price changes in dirty goods, but we focus on the question of how responsible households can maximize their responsible impact on abatement with their consumption and investment choices.

The remainder of the paper is organized as follows. Section 2 introduces the model, including socially responsible households, and presents our main results. Section 3 investigates how responsible households can coordinate and commit to a sacrifice. Section 4 analyzes the robustness of our results towards alternative assumptions (heterogeneity in preferences, alternative production functions). Section 5 concludes. All proofs are in the Appendix A. Appendix B discusses alternative sources and structures of risk.

2 The Model

Setting. Consider an economy with a continuum of households of mass 1, each endowed with an initial wealth of w_0. Households first take an investment decision, earn a stochastic return, and then decide how much to consume of the produced good. Goods are produced by a continuum of firms. With an aggregate investment of I, a firm can produce a quantity of $Q = I/c$, where c is the production cost parameter.[7] After production, the firm liquidates its assets and recovers $(\lambda + \varepsilon)\, I$. Here, λ gives the average liquidation value per unit of investment, and $\varepsilon \sim N(0, \sigma)$ is a normally distributed exogenous shock with zero mean and standard deviation σ. This shock introduces risk into the profits of firms. It is meant as a short cut for asset price risk.[8] The firms' profit is

$$\Pi = P\, Q + (l + \varepsilon)\, I = P\, I/c + (\lambda + \varepsilon)\, I, \tag{1}$$

where P is the price that clears the product market. The firms' profits are shared among investors, i. e., households that have invested in the firm. Given an aggregate investment I, a household that has invested i owns a fraction i/I of the shares. The household has a claim on the fraction $i\, \Pi/I$ of firm profits. Households decide individually. Because household types have homogeneous preferences, only symmetric equilibria exist. We can, thus, omit an index for each single household. The *gross* risk-free rate is r_f. In addition to consuming the good, households derive utility from money that they can use to consume another good (the numéraire) outside the focus of this model.

Production and consumption of the good generate an externality. Let x_{C} denote the externality per unit of consumption. Examples are the hazards of passive smoking, the carbon emissions from driving a car, etc. Let x_{P} denote the per-unit externality from production. Examples are the ecological impact of intensive agriculture, the pollution and emissions of production plants, and so forth. Due to the linear production technology,

[7] The simplification of a linear production function keeps the model simple and allows us to get tractable results. We discuss in Section 4.2 that any production function $Q(I)$ can be linearly approximated locally. Based on this approximation we add a qualification to our results regarding the specific characteristics of a general production function.

[8] The liquidation value can also be interpreted as the firm's stock price in a multi-period model, so that the noise simply represents the price risk. Alternatively, one could consider a shock that stems from uncertainty in the production process. We discuss this in Section B in the Appendix.

input and output are proportional, hence it does not matter whether the externality occurs in the production process or with consumption. We can simply denote $x = x_{\mathrm{C}}/c + x_{\mathrm{P}}$ as the aggregate externality, with

$$x\, I = x_{\mathrm{C}}\, Q + x_{\mathrm{P}}\, I = x_{\mathrm{C}}\, I/c + x_{\mathrm{P}}\, I. \tag{2}$$

The aggregate x can be positive or negative, in general, but we will concentrate on the negative externality case. This makes the good a "dirty" good. Households are endowed with w_0 and have a nested utility function. We assume CARA utility to keep our model simple and focus on the interaction between the two markets in altering the impact on the externality. This implies that our investment and consumption decisions are separable and the level of consumer wealth does not affect their optimal consumption nor investment.[9]

$$U = u\big(m + a\, q - b\, q^2/2 - x\, I\big) \quad \text{with} \quad u(c) = -e^{-\rho\, c} \tag{3}$$

where the quantity of the dirty good consumed is q, the preference for the product is a, and b measures its substitutability. Furthermore, m is the amount of money left to consume another good (the numéraire) that does not cause externalities. Note that $m = w_1 - q\, P = (w_0 - i)\, r_f + i\, r - q\, P$ because households optimally choose how much of their investment returns $w_1 = (w_0 - i)\, r_f + i\, r$ to consume in the form of the dirty good and the numéraire. The coefficient ρ gives the absolute risk aversion. As in (2), I is the aggregate investment volume of the firm and x is the externality.

There are two types of households. A fraction $1 - \gamma$ are *standard* households that do not care about the externality in their individual decisions. They invest i_S and consume q_S to maximize their expected utility. A fraction γ is called *responsible*. We denote the optimal investment and consumption levels in an equilibrium without any responsible households ($\gamma = 0$) as i_0 and q_0, which we will use as a reference point for responsible choices. Responsible households want to reduce the negative externality. In exchange for such an impact, responsible households are willing potentially to sacrifice some of their utility. The investment and consumption reduction may be partially undone by the other market participants.

[9] The assumption of an exponential utility function is common (Broccardo et al., 2020). If risk aversion decreased with the level of the original endowment, as well as the realized profits from investment, the optimal decision of households to invest would be altered, which would add further interesting effects to our results that we leave aside in this setup.

First, we answer the question, if responsible households want to have an impact on the externality, how should they reduce their consumption and investment in an efficient way in order to yield the highest impact at the lowest cost of utility? Given the optimal behavior and responses of standard households, we therefore analyze the *effectiveness* of each responsible choice in reducing the externality. Second, we derive the *efficient* combination of responsible investment for any target impact level. Finally, we analyze the optimal target impact level, to which responsible households commit if they can coordinate their choices in both markets. We continue with a discussion of different mechanisms that would either ensure coordination on these optimal responsible choices or that would result in deviating outcomes.

To analyze the efficient responsible choice for any target impact level we assume for now that responsible households can ex ante coordinate on and commit to some investment level and some consumption level. At the time of the respective markets, the optimal choice is different, but households can no longer reconsider. In fact, because responsible households have the same preferences as standard households and suffer from the externality in the same way, it can be seen that access to commitment technology is the only conceptual difference between standard and responsible households.[10]

To reduce the externality, responsible households can affect the supply side through their investment decision $i_R = I_0 - \Delta i$, where Δi denotes the *exit* of the dirty investment, which is the difference between responsible investment and the optimal investment absent any responsible households I_0. Responsible households can also affect the demand side through their choice of consumption $q_R = q_0 - \Delta q$, where Δq denotes responsible *boycott*, which is the difference between responsible consumption and optimal consumption absent any responsible households q_0.

The timeline of the model is given in Figure 1.

Product Market Equilibrium. To solve the model by backward induction, we start with the product market. Resulting from the earlier investment decision, a standard

[10]The ability to coordinate is the necessary condition to ensure that responsible investors want to have an impact in equilibrium (Oehmke and Opp, 2020).

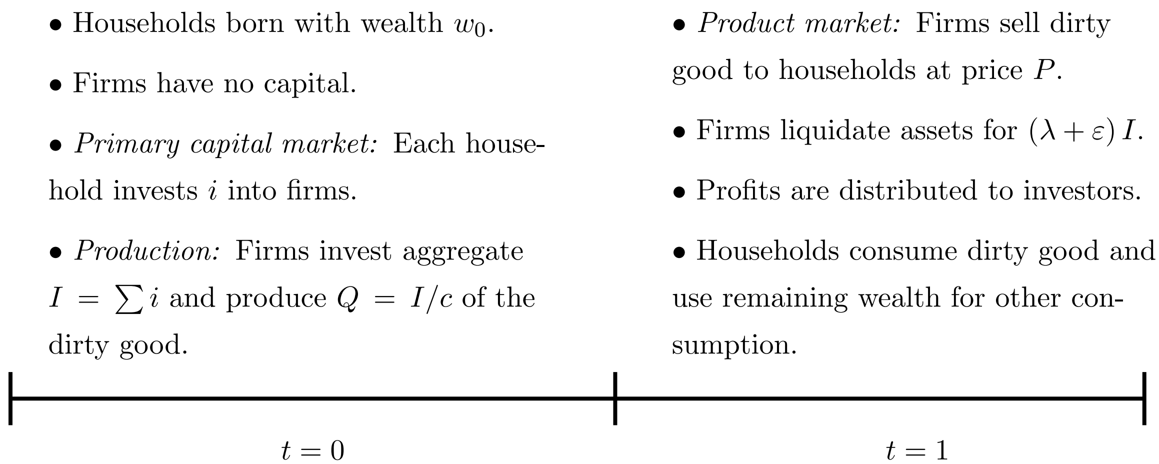


Figure 1: Sequence of Events

household has wealth w_1 at date $t = 1$, potentially different from that of responsible households. Given w_1, each standard household decides how much of the good to buy in $t = 1$. The budget constraint is $w_1 = m_S + P\, q_S$. Its objective function is therefore

$$U_S = -e^{-\rho\,(w_1 - P\, q_S + a\, q_S - b\, q_S^2/2)}\, e^{\rho\, x\, I} \tag{4}$$

The first-order condition is

$$\frac{\partial U_S}{\partial q_S} = \rho\,(a - b\, q_S - P)\, e^{-\rho\,(w_1 - P\, q_S + a\, q_S - b\, q_S^2/2)}\, e^{\rho\, x\, I} = 0, \tag{5}$$

and we obtain

$$q_S^* = \frac{a - P}{b}. \tag{6}$$

The optimal consumption depends on the price of the good and the preferences for it, but is independent of the initial wealth w_1. If responsible households behaved like standard households, aggregate consumption would be $Q = (a - P)/b$. The aggregate demand is therefore

$$\begin{aligned} Q &= (1-\gamma)\,\frac{a - P}{b} + \gamma\, q_R, \quad \text{or equivalently} \\ P &= a - b\,\frac{Q - \gamma\, q_R}{1 - \gamma}. \end{aligned} \tag{7}$$

Due to the nested structure of the utility function, neither the households' risk aversion nor return expectations play a role for their consumption choices. The inner utility function yields a linear demand function for the good. Responsible consumption (Δq) reduces the market price of the dirty good, which makes it more attractive for standard households.

Capital Market Equilibrium. Now we proceed to date $t = 0$ in the backward induction. Anticipating consumption choices at $t = 1$, standard households decide how much to invest in the firm and in other assets. Each household is marginal; therefore, they take the aggregate investment I as given. With $Q = I/c$, the return r from an individual investment is

$$\begin{aligned} r &= \frac{\Pi}{I} = \frac{P\,Q + (\lambda + \varepsilon)\,I}{I} = \frac{P}{c} + (\lambda + \varepsilon) \\ &= \frac{1}{c}\left(a - b\,\frac{Q - \gamma\,q_R}{1 - \gamma}\right) + (\lambda + \varepsilon). \end{aligned} \tag{8}$$

The return r decreases in the aggregate investment I. A higher investment leads to a higher output, which depresses the prices and thus the firms' profits. Due to the shock ε, the return r is normally distributed with mean $P/c + \lambda$ and standard deviation σ/c. Substituting (6) into (4), we obtain the *standard* household's expected utility

$$\mathrm{E}U_S = -e^{-\rho\,\frac{(a-P)^2}{2\,b}}\,e^{\rho\,x\,I}\int -e^{-\rho\,w_1}\,f(w_1)\,dw_1. \tag{9}$$

If *standard* households invest i_S of the initial wealth, they receive a random $r\,i_S$ from that investment, in addition to the safe return $(w_0 - i_S)\,r_f$. The wealth of the household at $t = 1$ is $w_1 = (w_0 - i_S)\,r_f + r\,i_S$. The household chooses i_S to maximize expected utility

$$\begin{aligned} \mathrm{E}U_S &= -e^{-\rho\,\frac{(a-P)^2}{2\,b}}\,e^{\rho\,x\,I}\int -e^{-\rho\,((w_0-i_S)\,r_f + r\,i_S)}\,f(r)\,dr \\ &= -e^{-\rho\,\frac{(a-P)^2}{2\,b}}\,e^{\rho\,x\,I}\,e^{-\rho\,\left((w_0-i_S)\,r_f + i_S\,(P/c+\lambda - i_S\,\rho\,\sigma^2/2)\right)} \end{aligned} \tag{10}$$

and accordingly chooses

$$\begin{aligned} i_S^* &= \frac{c}{b + c^2\,\rho\,\sigma^2}\left(a - c\,(r_f - \lambda) + \gamma\,b\,\frac{q_R - {}^{i_R}\!/_{c}}{1-\gamma}\right) \quad \text{and thus} \\ q_S^* &= \frac{1}{b + c^2\,\rho\,\sigma^2}\left(a - c\,(r_f - \lambda) - \gamma\,c^2\,\rho\,\sigma^2\,\frac{q_R - {}^{i_R}\!/_{c}}{1-\gamma}\right). \end{aligned} \tag{11}$$

We see that, if responsible households consume (invest) less, standard households react by consuming more (less) and investing less (more).

Defining Q_0 and I_0 as the equilibrium quantities produced and invested in the absence of responsible households, we can rewrite the responsible choices in terms of reductions in consumption Δq and investment Δi as

$$Q^* = Q_0 - \Delta Q = Q_0 - \gamma \left(\varphi\, \Delta q + (1-\varphi)\, {}^{\Delta i}/_{c}\right) \quad \text{with} \tag{12}$$

$$\varphi = \frac{b}{b + c^2\, \rho\, \sigma^2} \quad \text{and}$$

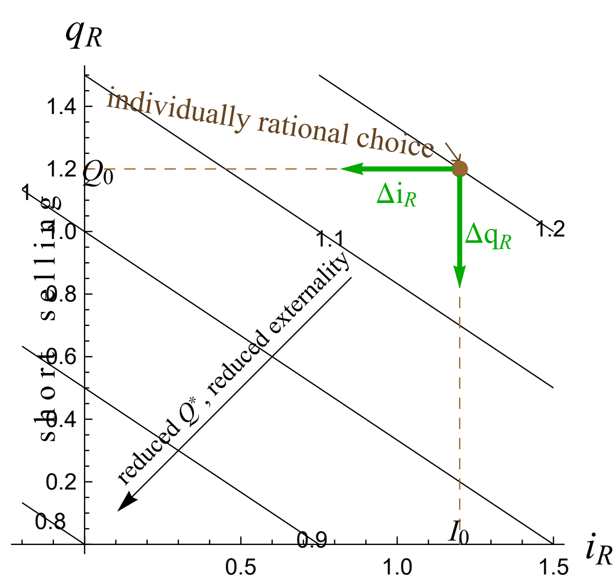
$$I^* = I_0 - \Delta I = I_0 - \gamma\, c \left(\varphi\, \Delta q + (1-\varphi)\, {}^{\Delta i}/_{c}\right) \tag{13}$$

(shown in Appendix A). The aggregate negative externality $x\, I^*(\Delta i, \Delta q)$ that arises in equilibrium is a function of the responsible choices Δi and Δq.

Proposition 1 (Effectiveness) *Both, the reduction in investment Δi and in consumption Δq are effective measures to reduce the aggregate externality in equilibrium. The marginal rate of transformation between a reduction in investment (SRI) and a reduction in consumption (SRC) is*

$$MRT_{\Delta i, \Delta q} = -\frac{(1-\varphi)/c}{\varphi} = -c\, \frac{\rho\, \sigma^2}{b}.$$

Figure 2: Impact of Reducing q_R or i_R



Parameters are $c = 1, \rho = 1/3, a = 2, b = 1/2, x = 1, \gamma = 1/3,\ \lambda = 0,\ \sigma = 1$ and $r_f = 1$.

Figure 2 illustrates the proposition, showing that i_R and q_R have a nearly symmetric impact. In the numerical example, $Q_0 = 1.2$, and because $c = 1$, also $I_0 = 1.2$. If there were only standard households ($\gamma = 0$), the externality would be $1.2\,x$. The parallel lines are iso-impact lines; their slope equals the MRT. If responsible households reduce investment or consumption, also the aggregate externality $x\,I$ goes down.

The marginal rate of transformation reflects the amount of dirty consumption that responsible households can additionally consume if they give up a unit of dirty investment and hold their impact level constant. It measures the relative effectiveness between SRI and SRC. Both a reduction in investment i_R and in consumption q_R have a comparable impact. The effectiveness $(1 - \varphi)/c$ of a reduction in investment i_R increases in asset risk σ, and risk aversion ρ. The effectiveness φ of a reduction in consumption q_R increases in the product specificity b.

Only at the extreme, in the absence of any risk or risk aversion, investment reduction Δi has no effect. Standard households compensate completely for the reduced investment. However, a reduction in consumption Δq still results in a reduction in aggregate consumption. Δq decreases the expected profits of the firm, so the equilibrium investment is reduced. Standard households do not compensate for this by consuming more. An analogous effect holds if $b = 0$. In this case, a reduction in consumption Δq has no direct effect on the externality but a reduction in investment in the primary capital market reduces the overall production of the good, increasing prices, and therefore indirectly reducing consumption.

Efficient Combination of Responsible Choices. We now analyze which combination of consumption reduction Δq and investment reduction Δi optimizes the trade-off of responsible households between expected utility and impact. Proposition 1 says that if a group of households wants to reduce the externality to some target, it can do so by reducing investment, consumption, or both. Formally, the marginal rate of substitution (MRS), defined by the household's utility function, must equal the marginal rate of transformation (MRT), defined by the standard households' reaction. The next proposition proves that, in order to keep utility as high as possible, it is optimal to reduce investment and consumption in proportion, such that expected yields and prices remain unchanged.

Proposition 2 (Efficiency) *The efficient responsible behavior satisfies*

$$\frac{q_R^*}{i_R^*} = \frac{\Delta q^*}{\Delta i^*} = \frac{1}{c}. \tag{14}$$

It is optimal to reduce investment and consumption by the same factor, such that the equilibrium price and asset return remain unchanged.

The proposition is relatively robust to modifications in the model, see the discussion in Section 4. The rationale for Proposition 2 is that any disproportionate change is offset at least partially by the market response of standard households and is therefore less effective. To see this, we first consider the expected equilibrium return for given Δi and Δq

$$E[r^*] = E[r]_0 - \gamma \frac{b\, c\, \rho\, \sigma^2}{b + c^2\, \rho\, \sigma^2} \left(\Delta q - {}^{\Delta i}\!/_{c}\right). \tag{15}$$

The expected return increases if responsible households reduce their investment and decreases if responsible households consume less of the dirty good. Similarly, the equilibrium price for the good is

$$P^* = P_0 - \gamma \frac{b\, c^2\, \rho\, \sigma^2}{(b + c^2\, \rho\, \sigma^2)} \left(\Delta q - {}^{\Delta i}\!/_{c}\right). \tag{16}$$

The equilibrium price of the dirty good decreases (increases) if responsible households reduce their consumption (investment). Therefore, if responsible households efficiently reduce their dirty investment and consumption such that $\Delta q^* = \Delta i^*/c$, both the market price $P^* = P_0$ and the expected return $E[r^*] = E[r_0]$ are the same as in the absence of responsible households. Consequently, the standard households' consumption level q_S^* and investment level i_S^* remain unchanged. Because this is an efficiency result, it describes the combination of investment and consumption, not the levels. Also the motivation of responsible households is irrelevant. Up to here, responsible households could be altruists, or coordinate on some action as a group.

In principle, responsible households could reduce consumption relatively more than they reduce investment. This would imply that the price for the good would decrease, and standard households would buy more of it. However, the financing cost for the firm would increase and standard households would invest less. Responsible households could also do the opposite, reduce investment relatively more than consumption. Then the standard households' reaction would go in the opposite direction. Proposition 2 states that neither of this disproportionate reductions are efficient. The same impact can be achieved with a smaller utility reduction.

The rationale also applies to any level of risk and risk aversion or marginal utility of consumption. If the utility loss of a reduction in investment Δi is low, for example, because risk aversion or risk itself is low, it is also very easy for standard households to substitute for the reduced investment. The same is true for a reduction in q_R. Therefore, the greatest impact results from a proportional reduction in both investment and consumption.

Because responsible households optimally withdraw the investment proportionally to their consumption reduction so that prices and yields do not change, efficient behavior is also independent of γ. If responsible households moved prices, then optimal behavior might change if more households act responsibly. However, because the efficient responsible choice does not influence prices, proportional reduction is optimal regardless of how many households act responsibly (γ) and also of the total size of their responsible choice. We discuss the robustness of Proposition 2, and of the according intuitions, in Section 4.

Figure 3: Expected Utility of Responsible Households

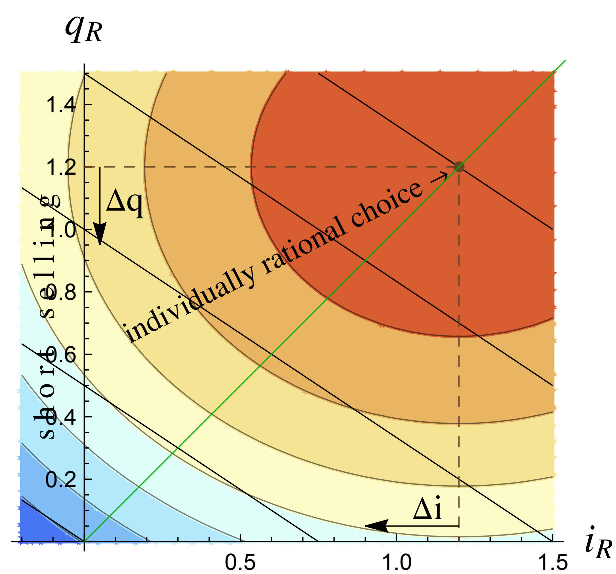


Figure 3 illustrates Proposition 2. It shows how the expected utility of individual responsible households depends on q_R and i_R (warm colors stand for high utility). The individual utility is maximized at $\Delta i = \Delta q = 0$. If responsible households want to have an impact, they must deviate from this point towards the origin. For each impact level, the utility is maximized on the green line, given by $q_R^* = i_R^*/c$, and thus also by $\Delta q^* = \Delta i^*/c$.

Optimal Level of Responsible Choices. We now analyze how much responsible households must sacrifice and what impact level they want to achieve optimally. Responsible households differ from standard households because they coordinate to implement the collectively optimal choice for themselves. The individual impact of each individual responsible household is infinitesimally small. By reducing consumption and investment, each household can only achieve a marginal impact on the externality, but suffers a discrete utility loss. Thus, it is rational from the individual perspective to neglect the negative externality. However, from a collective perspective, households can benefit from responsible behavior. They profit not only from their own infinitesimal impact, but also from that of other responsible households. The responsible household's willingness to sacrifice marginal utility therefore increases in the aggregate impact that responsible households are able to achieve.

Corollary 1 *To achieve an exogenous impact goal G, responsible households optimally sacrifice (in the absence of short-selling)*

$$\Delta q = \min\left(\frac{G}{c\,\gamma}, q_0\right) \text{ and } \Delta i = \min\left(\frac{G}{\gamma}, I_0\right) \tag{17}$$

where $q_0 = \frac{a-c(r_f-\lambda)}{b+c^2\,\rho\,\sigma^2}$ and $I_0 = c\frac{a-c(r_f-\lambda)}{b+c^2\,\rho\,\sigma^2}$ define the optimal individual consumption and investment choices in the absence of responsible households, which constrain the feasible responsible choice.

The required sacrifice to reach a fixed goal decreases in the proportion of responsible households.[11] Fewer responsible households require higher individual responsible choices. If there are too few responsible households ($\gamma < \frac{G}{c\,(\varphi\,q_0+(1-\varphi)I_0)}$), the goal cannot be achieved even if all responsible households abstain completely from investment and consumption.

[11] An example of such a goal is the $1.5°C$ goal of the Paris Agreements.

But what if responsible households can set their own goal? We assume that responsible households differ from standard households only because they are able to coordinate choices and thereby, at least partially, internalize the negative externality. Therefore, responsible households maximize their expected utility by taking into account the market responses of standard households, as well as the impact of their own choices on the externality. Instead of maximizing (5), the responsible objective can be summarized as

$$\mathrm{E}U_R = -e^{-\rho\,(w_R - P^*\,q_R + a\,q_R - b\,q_R^2/2)}\,e^{\rho\,x\,I^*}\,e^{-\rho\,\left((w_0 - i_R)\,r_f + i_R\,(P^*/c + \lambda - i_R\,\rho\,\sigma^2/2)\right)}. \tag{18}$$

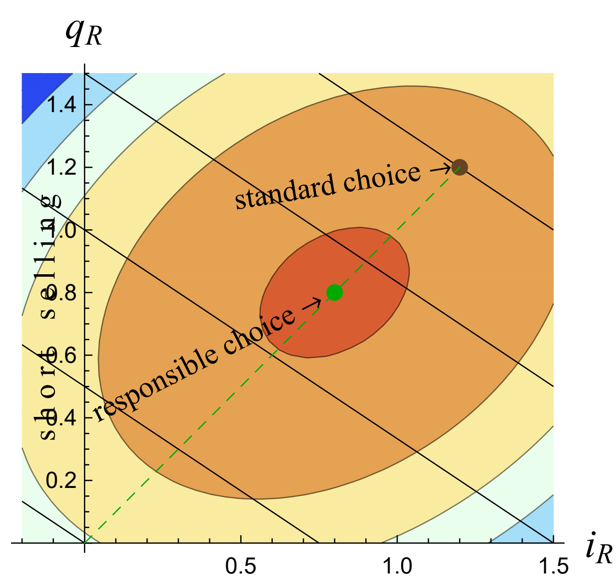
where P^* and I^* are given in (16) and (13) respectively. This yields our third result.

Proposition 3 (Coordinated Choices) *If $\gamma\,x \leq a/c - r_f$, responsible households optimally coordinate on*

$$\Delta q^* = c\,\frac{\gamma\,x}{b + \rho\,\sigma^2} \quad \textit{and} \quad \Delta i^* = c^2\,\frac{\gamma\,x}{b + \rho\,\sigma^2}. \tag{19}$$

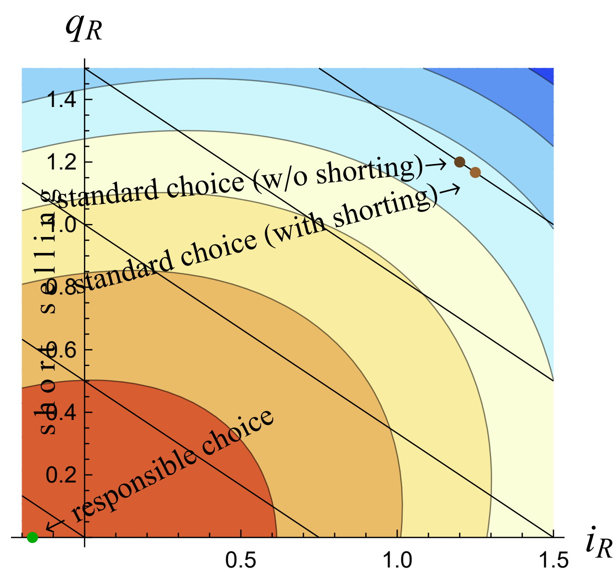
In the absence of short-selling, if $\gamma\,x > a/c - r_f$, households consume nothing and invest nothing.

Figure 4: Expected Utility of Responsible Households, $\gamma = 1/3$, $x = 1$



We illustrate the optimal responsible choice in Figure 4. In the figure, $\gamma = 1/3$ of households are responsible, and the externality is $x = 1$. If responsible households collectively reduce q_R by Δq and i_R by Δi, they profit from the reduced externality. As shown in Proposition 2, it is efficient for responsible households to reduce investment and consumption proportionally.

Figure 5: Expected Utility of Responsible Households, $\gamma = 1/3$, $x = 11/3$



An increase in the externality x or the fraction of responsible households moves the collectively optimal point towards the origin. For $\gamma\, x = a/c - r_f$, the origin is reached. For $\gamma\, x > a/c - r_f$, it is collectively optimal to consume nothing, $q_R^* = 0$, and (if possible) to short-sell shares,

$$i_S^* = -\frac{c\,(1-\gamma)\,(c\,(r_f + x\,\gamma) - a)}{b\,(c^2\,(1-\gamma) + 2\,\gamma) + c^2\,(1-\gamma)\,\rho\,\sigma^2} < 0.$$

With short selling, investment and consumption are obviously no longer reduced in proportion. Therefore, the responsible behavior now also affects the equilibrium price P. Standard households react by investing more and consuming less. Because standard households have reduced their consumption, the externality is further reduced. Figure 5 illustrates this: if consumption and investment are not reduced proportionally, the impact is relatively small. In the figure, the two brown points (with and without short-selling) are seemingly on the same iso-impact line.

Social Welfare. If households coordinate on the optimal choice $\Delta i^*, \Delta q^*$ the obtained aggregate impact is

$$x\,\Delta I^*(\gamma) = \frac{\gamma^2\,c^2\,x^2}{b + c^2\,\rho\,\sigma^2} \tag{20}$$

which is increasing and convex in the proportion of responsible households γ. The impact per household increases in the fraction of responsible households so that each individual responsible household is also willing to forego more investment and consumption. If all households fully internalize the externality, $\gamma = 1$, they act like a social planer. The externality problem vanishes and aggregate social welfare becomes

$$W(1) = W_{\text{FB}} = u\Big(\frac{(a - c\,(r_f - \lambda + x))^2}{2(b + \rho\,c^2\,\sigma^2)} + r_f\,w_0\Big). \tag{21}$$

On the contrary, without any responsible households $\gamma = 0$, the welfare obtained is

$$W(0) = u\Big(\frac{(a - c\,(r_f - \lambda + x))^2}{2(b + \rho\,c^2\,\sigma^2)} + r_f\,w_0 - \frac{1}{2}\frac{c^2\,x^2}{b + \rho\,c^2\,\sigma^2}\Big) \tag{22}$$

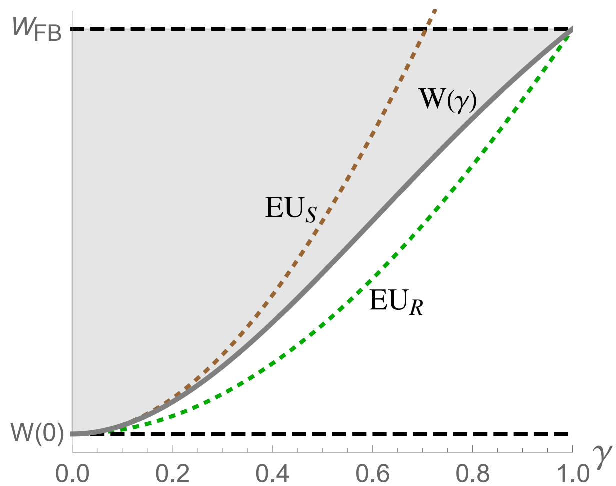
Households suffer in the aggregate because they do not price in the externality. Social welfare with a given proportion of responsible households equals the corresponding equilibrium level of aggregated utilities

$$\begin{aligned} W(\gamma) &= \gamma \mathrm{E}U_R^*(\gamma) + (1-\gamma)\mathrm{E}U_S^*(\gamma) \quad \text{with} \\ \mathrm{E}U_R^*(\gamma) &= u\Big(\frac{(a - c\,(r_f - \lambda + x))^2}{2(b + \rho\,c^2\,\sigma^2)} + r_f\,w_0 - \frac{1}{2}\frac{c^2\,x^2}{b + \rho\,c^2\,\sigma^2} + \frac{1}{2}x\,\Delta I^*(\gamma)\Big) \quad \text{and} \\ \mathrm{E}U_S^*(\gamma) &= u\Big(\frac{(a - c\,(r_f - \lambda + x))^2}{2(b + \rho\,c^2\,\sigma^2)} + r_f\,w_0 - \frac{1}{2}\frac{c^2\,x^2}{b + \rho\,c^2\,\sigma^2} + x\,\Delta I^*(\gamma)\Big). \end{aligned} \tag{23}$$

Coordination of responsible choices allows for an increase in aggregate social welfare, but this welfare gain is not equally distributed. Standard households gain more from the sacrifice $x\,\Delta I^*$ because they free-ride on the externality reduction without changing their optimal investment and consumption. Responsible households also gain from the coordinated choice but they have to sacrifice consumption and investment to reach this impact.

We illustrate the social welfare gain of higher proportions of responsible households in Figure 6. The dashed lines represent the minimum $W(0)$ and maximum $W(1) = W_{FB}$ achievable social welfare. The gray area summarizes the social welfare losses due to

Figure 6: Social Welfare as a Function of Responsible Households γ, $x = 1$



the externality induced overproduction. The bold gray line depicts the social welfare $W(\gamma)$. Social welfare increases in the proportion γ of households acting responsibly. The responsible choice raises the individual utility of both responsible (dotted green line) and standard (dotted brown line) households. However, the gain in expected utility, however, is higher for standard households, who free-ride on the sacrifice of responsible households. If all households act responsibly, they implement the first-best social welfare.

3 Commitment Devices

Until now, we have simply assumed that responsible households can commit ex-ante to a desired consumption and investment level, but we did not discuss how. We now discuss possible commitment mechanisms and their impact on our results. In Section 3.1, we assume that responsible households are part of the same political entity and can implement a tax on investment, and a tax on consumption. Tax revenue can be redistributed between responsible households in a lump-sum way. In Section 3.2, responsible households only have access to a dissipative tax. They can shame & blame one another for dirty investment and consumption. In Section 3.3, we discuss the outcome if households can only credibly commit to a reduction in one market but not in the other. We show that such a one-sided commitment affects the equilibrium market prices and, therefore, yields empirical predictions.

3.1 Taxation

The γ responsible households can be interpreted as members of a country or some other political entity. They could then vote to implement Pigovian taxes τ_I on investment and τ_Q on consumption. If there are no administrative costs, investment and consumption are perfectly observable and if taxes are completely redistributed to responsible households, the collectively optimal decision of Proposition 3 can be implemented.

For concreteness, assume that within the political entity, households pay $P + \tau_Q$ instead of P for the good. In addition, they only get a return of $P/c + \lambda - \tau_I$ instead of $P/c + \lambda$ outside the entity. The aggregate tax revenue is then $T = \tau_Q\, q_R + \tau_I\, i_R$, it is redistributed within the entity. Then the individually rational choice within the entity is

$$\begin{aligned} i_S^* &= \frac{c}{b + c^2\,\rho\,\sigma^2}\left(a - c\,(r_f - \lambda) + \gamma\,\frac{b\,\tau_I - c\,\rho\,\sigma^2\,\tau_Q}{c\,\rho\,\sigma^2}\right), \\ q_S^* &= \frac{1}{b + c^2\,\rho\,\sigma^2}\left(a - c\,(r_f - \lambda) - \gamma\,c\,\frac{b\,\tau_I - c\,\rho\,\sigma^2\,\tau_Q}{b}\right). \end{aligned} \tag{24}$$

Comparing these values with those of (11), we see that the collectively optimal allocation is implemented by individually rational choices if tax levels are

$$\tau_I = \frac{c^2\,\rho\,\sigma^2}{b + c^2\,\rho\,\sigma^2}\,\gamma\,x \quad \text{and} \quad \tau_Q = c\,\frac{b}{b + c^2\,\rho\,\sigma^2}\,\gamma\,x. \tag{25}$$

There are several notable properties. Taxes are optimally proportional to the size of the externality x, but also to the size of the group. This is because of the externality: households outside the entity also profit from the reduction in the externality. If γ is small, the benefits accrue mostly outside of the entity. Second, if risk aversion or risk is small, only consumption should be taxed. A tax on investment would be futile, as standard investors would compensate for the reduction in investment. Symmetrically, if b is small, only investment should be taxed. Bringing the consumption tax and the investment tax on the same scale by dividing through c, the sum is an invariant, $\tau_Q/c + \tau_I = \gamma\,x$.

3.2 Shame & Blame

In the absence of political institutions, responsible households can also be interpreted as members of a social group of size γ. Due to their potential connection, members can observe the choices of others (e. g., through social networks). To reduce the externality, the members of the group can shame & blame each other for investing in the dirty industry or for the consumption of dirty goods. Assume that exerting shame & blame is free, but leads to a reduction in the utility of the blamed household by $\gamma\, g_I$ for each dollar invested and $\gamma\, g_C$ for each unit consumed.[12] It is equivalent to a tax that is not restituted to the group. Shame & blame, thus, directly affects the expected utility of each household, similar to a warm glow (see Andreoni, 1990) with a negative sign.

This constitutes a fundamental deviation from our benchmark setting. Until now, responsible households were assumed to have the same utility functions as standard households and only differ in their ability to coordinate their choices. Thus, free coordination and taxation allow responsible households to reach the first-best welfare if all households acted responsibly. In this section we assume responsible households cannot coordinate on the responsible choice itself. However, they can credibly commit to shaming & blaming each other to alter their utility functions. The rationale is to create additional costs of dirty investment and consumption. That way, responsible households are induced to consume and invest less. Instead of choosing responsible quantities, the social group can coordinate on penalties in the form of shame & blame. Because of the additional costs necessary to create commitment, the first-best social welfare (the investment and consumption amount chosen by a social planner) cannot be reached anymore even if all households act responsible.[13]

Without loss of generality, assume that the reduction in utility is proportional to γ. The rationale is that with an increasing social group size, the probability of being shamed

[12]The factor γ expresses the fact that a larger group can exert more shame. The disutility can also be interpreted as a feeling of guilt. However, shame & blame can be chosen endogenously, while guilt, as part of the utility function, can be seen as exogenous.

[13]The outcome is the same as with responsible investors that have altruistic motives hard-wired into their utility functions. However, we focus on endogenous motives because we want to discuss optimal penalties. Shame & blame is a conceptual way out.

or the intensity of the shame increases. The utility function of a responsible household becomes

$$U_R = u\big(m + a\,q_R - b\,q_R^2/2 - x\,I - \gamma\,g_I\,i_R - c\,\gamma\,g_C\,q_R\big). \tag{26}$$

where, again, $m = (w_0 - i_R)\,r_f + i_R\,r - q_R\,P$. The direct disutility from shame & blame is zero in two limiting cases. If $g_I = 0$ (or $g_C = 0$), households do not shame & blame each other for investing (consuming), and if $i_R = 0$ (or $q_R = 0$), responsible households abstain completely from dirty investment (consumption). Only in these extremes, shame & blame induces no direct costs. As responsible households gradually reduce their investment and consumption as in Proposition 3, shame & blame reduces the responsible households' utility. This additional reduction in utility creates a centrifugal force. These forces make any mix of SRI and SRC inefficient if households can only commit to responsible behavior by shaming and blaming each other. To see this, consider first the demand for the product of responsible households

$$q_R^* = \frac{a - P - c\,\gamma\,g_C}{b}, \tag{27}$$

toned down due to disutility g_C. Similarly, the investment for a given share price is

$$i_R^* = \frac{P/c - r_f - \gamma\,g_I}{\rho\,\sigma^2}. \tag{28}$$

In combination with the market clearing conditions on the product market and the stock market, and the demand of standard households, we obtain an aggregate quantity

$$Q^* = \frac{a - c\,(r_f + \gamma^2\,(g_I + g_C))}{b + c^2\,\rho\,\sigma^2}. \tag{29}$$

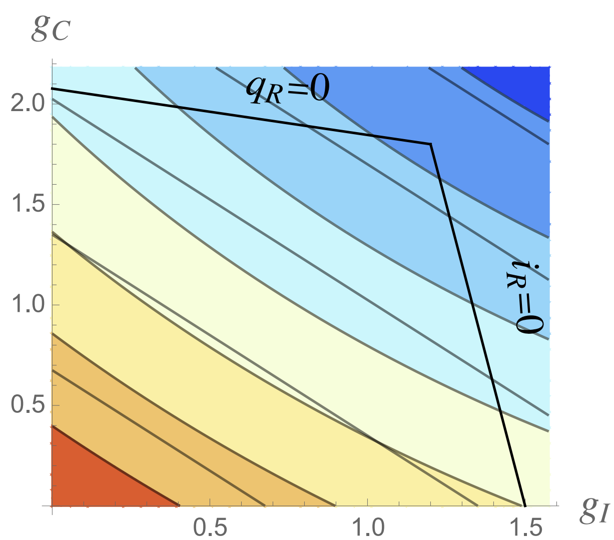
The disutility of investment g_I and consumption g_C has an identical effect on aggregate production Q^*. If ρ is low, an increase in g_I has a strong effect on investment by responsible households i_R^*, but the standard households are also more willing to substitute by investing more. The role of risk aversion is canceled out. The same holds for larger b.

However, the disutility created from shame & blame has an important implication for the optimal individual choice. As households can no longer choose the actual investment and consumption quantities, but only the additional costs g_I and g_C, the nature of the optimization problem changes. The expected utility of responsible households, which is concave in investment and consumption amounts, is convex in the shaming cost factors as we prove in Appendix A. For a given target impact on the externality, the utility-maximizing choice of shame therefore becomes a corner solution.

In contrast to Proposition 2 it is no longer optimal to commit to a proportional reduction in investment and consumption. Instead, responsible households focus on *either* investment or consumption reduction. For small impact levels, the optimal shame & blame strategy focuses either on reducing investment (by an increase in g_I), or on reducing consumption (by an increase in g_C), but not both. For large impact levels, it consists of either $q_R = 0$ or $i_R = 0$, and a reduction of the other amount. The rationale is the mentioned centrifugal force due to the direct disutility induced by guilt.

Figure 7 shows the iso-impact lines as black diagonals. Each line has a slope of -1. The impact is lowest at the origin and increases towards the north-east. The figure

Figure 7: Expected Utility of Responsible Households with Shame & Blame



also shows the expected utility of responsible households, depending on the disutility of investment g_I and of consumption g_C. In the numerical example, for $g_I = 1.2$, investment i_R by responsible households becomes zero. For $g_C = 1.8$, consumption q_R by responsible households becomes zero. Increasing g_I or g_C above these levels does not have additional impact.

Also the insights from Proposition 3 change. Collectively, the optimal choice does not depend continuously on the fraction γ and the externality x. The optimal shame & blame policy is binary. For low levels of γ and x, it is optimal to do nothing and set $g_I = g_C = 0$. At the point $2\,\gamma\,x = a/c - r_f$, it becomes optimal to set g_I and g_C at a level that *completely* prevents responsible households from dirty investment and dirty consumption. In a sense, the choice of responsible households is always extreme. Either they behave like standard households (for $2\,\gamma\,x < a/c - r_f$), or they neither invest nor consume (for $2\,\gamma\,x > a/c - r_f$). In the second case, they shame & blame one another, but because they neither invest nor consume, this does not create any direct disutility.

The size of the group becomes crucial for the socially optimal choice. If the group size is too small, given the externality, responsible households do not want to coordinate on reducing the externality. If the group size is large enough, responsible households will credibly commit to reduce the externality without any direct disutility from shaming and blaming. Coordination implies a commitment on a shame & blame threat that deters any dirty investment or consumption of the group members.

3.3 One-sided Commitment

Commitment to the coordinated choice can be achieved in several ways. Coordination could be more feasible in one market than in the other. On the one hand, investing in a green fund might offer a natural way to coordinate SRI. However, the individual investment decision is difficult to observe. Consumption behavior, on the other hand, could be easily observed, so peer pressure-based coordination might be more feasible for SRC. Our model implies testable empirical implications for the case where households can better coordinate in one market than in the other.

If responsible households can only commit to $\Delta q > 0$ but not to a reduction in investment, the optimal responsible choice becomes

$$\Delta q = x\,\gamma\,\frac{c}{b + (1-\gamma^2)\,c^2\,\rho\,\sigma^2} \tag{30}$$

We use (15) and (16) to derive the implications of household behavior and obtain

$$P = P_0 - \frac{\gamma}{1-\gamma}\,\frac{b\,c^2\,\rho\,\sigma^2}{b + c^2\,\rho\,\sigma^2}\,\Delta q \quad \text{and}$$

$$E[r] = E[r_0] - \frac{\gamma}{1-\gamma}\,\frac{b\,c\,\rho\,\sigma^2}{b + c^2\,\rho\,\sigma^2}\,\Delta q.$$

Hypothesis 1 *If responsible households can coordinate on SRC, but fail to coordinate on SRI, green investments yield a higher financial performance than dirty shares.*

One-sided consumption boycott decreases investment yields and market prices. In response, standard households consume more, but invest less than in the absence of responsible households.

The opposite occurs if households can only commit to green investments. If responsible households can only commit to Δi but not to Δq the optimal responsible choice becomes

$$\Delta i = x\,\gamma\,\frac{c^2}{b\,(1-\gamma^2)+c^2\,\rho\,\sigma^2}. \tag{31}$$

We then obtain

$$P = P_0 + \gamma\,\frac{b\,c^2\,\rho\,\sigma^2}{b+c^2\,\rho\,\sigma^2}\,\frac{\Delta i}{c} \quad \text{and}$$
$$E[r] = E[r_0] + \gamma\,\frac{b\,c\,\rho\,\sigma^2}{b+c^2\,\rho\,\sigma^2}\,\frac{\Delta i}{c}.$$

Hypothesis 2 *If responsible households focus on SRI, neglecting SRC, dirty investments outperform green shares.*

The expected return of the dirty investment increases in equilibrium, as does the price of the dirty good. Standard households therefore invest more but consume less than in the absence of responsible households. This result may explain why empirical papers obtain contradictory results on the relative performance of green investments over time and in different countries.

4 Robustness: Heterogeneity and Fixed Costs

We have made several modeling choices. In reality, some of our assumptions may not hold. In this section, we formally discuss the effect of household preference heterogeneity and the robustness of our results toward changes in the production function. We show that propositions 1 and 2 hold in essence. If responsible households individually optimize their investment and consumption, they are at a stationary point. Hence, if they reduce investment and consumption, their loss in utility is of the second order, whereas their effect on the externality is of the first order. Therefore, if they want to reduce the externality, they should always adapt both investment and consumption. By reducing both, responsible households can avoid the equilibrium price and yield changes that would result in attenuating market responses of standard households, which partially offset the responsible choices. Proposition 3 is a benchmark proposition for specific modeling assumptions and will have to be modified.

4.1 Heterogeneity

A major simplification in our model is the assumption that households have homogeneous preferences. More realistically, we could interpret standard investors as large institutional investors. These might have better diversification opportunities, and therefore, in effect a smaller degree of risk aversion. In addition, the dirty good might be a regional product. Standard households could then be interpreted as international consumers on the product market, caring less about consuming the good. Such a heterogeneity of the preferences of the households may affect our results. For example, if responsible households are more risk averse, they choose a lower level of investment. On the other hand, once responsible households reduce their investment, the less risk-averse standard households will react stronger and increase their investment more because they care less about the increased risk exposure. In this section, we analyze the aggregate effects of possible heterogeneity in preferences. We use indices for preference parameters: ρ_S is the risk aversion of standard households, ρ_R that of responsible households, etc. For a given consumption q_R and investment i_R of responsible households, prices and yields adjust so that standard households choose

$$
\begin{aligned}
q_S^* &= \frac{1}{b_S + c^2\,\rho_S\,\sigma^2}\left(a_S - c\,(r_f - \lambda) + c\,\gamma\,\rho_S\,\sigma^2\,\frac{i_R - c\,q_R}{1-\gamma}\right) \quad \text{and} \\
i_S^* &= \frac{c}{b_S + c^2\,\rho_S\,\sigma^2}\left(a_S - c\,(r_f - \lambda) - b_S\,\frac{\gamma}{c}\,\frac{i_R - c\,q_R}{1-\gamma}\right), \quad \text{so} \\
Q^* &= \frac{(1-\gamma)\,(a_S - c\,(r_f - \lambda)) + \gamma\,(b_S\,q_R + c\,\rho_S\,\sigma^2\,i_R)}{b_S + c^2\,\rho_S\,\sigma^2}
\end{aligned}
$$

and $I^* = c\,Q^*$, the same as (43) and (11) with the only difference that variables a, b and ρ now carry that subscript S. Plugging this into the utility function of the responsible households, the first-order condition yields

$$q_R^* = \frac{1}{(1-\gamma)\,\frac{b_R}{b_S} + 2\,\gamma} \left(\left((1-\gamma)\,\frac{\rho_R}{\rho_S} + 2\,\gamma \right) \frac{i_R^*}{c} + (1-\gamma)\,\frac{a_R - a_S}{b_S} \right). \tag{32}$$

This implies that for $a_R = a_S$, q_R^* and i_R^* are proportional. In addition, the reduction in consumption Δq and investment Δ_i by responsible households is therefore proportional, as in Proposition 2. If $a_R > a_S$ ($a_R < a_S$), then a constant is added to (subtracted from) consumption. This implies that responsible households optimally lower their investment more (less) relative to the reduction in consumption to reach their target impact. There are a few more special cases. If $b_R = b_S$, then the first fraction becomes $1/(1+\gamma)$. If $\rho_R = \rho_S$, then the inner bracket becomes $1+\gamma$. Now we discuss the three sources of heterogeneity in sequence, first $\rho_R \neq \rho_S$, then $a_R \neq a_S$, and finally $a_R \neq a_S$.

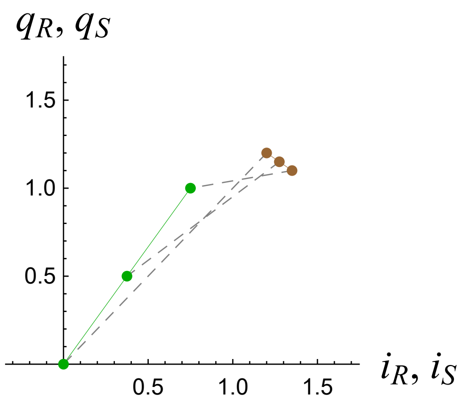
Heterogeneous Risk Aversion. We first analyze the effect of different degrees of risk aversion. Responsible households may have fewer opportunities to diversify and, therefore, may be more risk averse than standard households such that it is reasonable to assume $\rho_R \neq \rho_S$. We focus on the case $\rho_R > \rho_S$. Equation (32) turns into

$$q_R^* = \frac{1}{c}\,\frac{(1-\gamma)\,\frac{\rho_R}{\rho_S} + 2\,\gamma}{1+\gamma}\,i_R^*. \tag{33}$$

The intuition is as follows. If the risk aversion of responsible households is higher than that of standard households, reducing investment is relatively costly for responsible households and will be nearly completely offset by standard households. However, in equilibrium, responsible households will not invest much anyway due to their high-risk aversion. Hence, it is efficient to reduce investment only a little, but also from a low starting level. In the end, this leads to the proportionality result. Both investment and consumption are reduced in proportion.

Figure 8 illustrates the effects. The individually rational choice of responsible households would have $q_R = 1$ and $i_R = 0.75$. Standard households have a lower risk aversion. Therefore, they invest considerably more but consume about the same. The aggregate

Figure 8: Expected Utility of Responsible Households, $\rho_R > \rho_S$



Parameters are $\rho_S = 1/3$, $\rho_R = 2/3$, and $\gamma = 50\%$, everything else as in Figure 2.

output is

$$I = c\,\frac{a - c\,(r_f - \lambda)}{b + c^2\,\rho_S\,\sigma^2}\,\frac{b\left(1 - \gamma + 2\,\frac{\rho_S}{\rho_R}\,\gamma\right) + (1+\gamma)\left(1 - \gamma + \frac{\rho_S}{\rho_R}\,\gamma\right)c^2\,\rho_S\,\sigma^2}{b\left(1 - \gamma + 2\,\frac{\rho_S}{\rho_R}\,\gamma\right) + (1+\gamma)\,c^2\,\rho_S\,\sigma^2} = 1.05. \tag{34}$$

The product price is $P = 1.4$, and the expected return $r = P/c + \lambda$ is the same because $c = 1$ and $\lambda = 0$ in the example. Now, if responsible households reduce the externality efficiently, they follow (33) and reduce investment and consumption in proportion. Because they initially had a lower share of investment, their reduction in investment is less palpable. Consequently, the price P increases and also the expected investment return r increases. Standard households react by investing more and consuming less, the brown point moves.

Hypothesis 3 *If responsible households are more (less) risk averse than standard households, an increased (decreased) responsible choice increases (decreases) the return of dirty investments and decreases (increases) the return of clean investments.*

At the extreme point in the figure, $i_R = q_R = 0$. Aggregate output has dropped to

$$I = c\,\frac{a - c\,(r_f - \lambda)}{b + c^2\,\rho_S\,\sigma^2}\,(1 - \gamma) = 0.6. \tag{35}$$

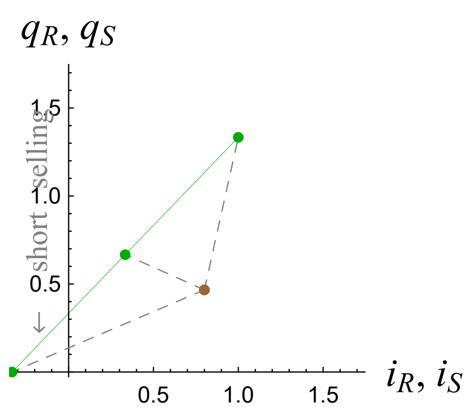
The product price is at $= 1.45$. The externality $x\,I$ is reduced by less than half because of the reaction of standard households. When comparing (34) with (35), we see that the impact of responsible households on the aggregate externality is smaller (larger) than their share γ if $\rho_R > \rho_S$ ($\rho_R < \rho_S$). In summary, compared to Figure 3, the green line is steeper, but it still goes through the origin. Responsible households should still reduce investment and consumption proportionally.

Heterogeneous Product Preferences. Similar to the better diversified international investors, international standard households may have a lower preference for the local product, $a_S < a_R$. Simplifying (32) The optimal combination of i_R and q_R now satisfies

$$q_R^* = \frac{1}{c}\, i_R^* + \frac{1-\gamma}{1+\gamma}\,\frac{a_R - a_S}{b}. \tag{36}$$

Due to preference heterogeneity, it is no longer optimal for responsible households to reduce i_R and q_R in the same proportion. If $a_R > a_S$, the investment should be reduced more than proportionally.

Figure 9: Expected Utility of Responsible Households, $a_R > a_S$



Parameters are $a_S = 1.5$, $a_R = 2$, and $\gamma = 50\%$, everything else as in Figure 2.

We illustrate this point in Figure 9, which shows the expected utility of responsible households with a stronger preference for the good $a_R = 2 > a_S = 1.5$. The individually utility-maximizing investment is $i_R^* = 1$, and consumption is at $q_R^* = 1.33$. Standard households invest about as much but consume considerably less. The aggregate investment is at $I^* = 0.9$, with an externality corresponding to it.

If responsible households efficiently reduce the externality, they follow (36). $\Delta i_R = c\,\Delta q_R$, but because investment was initially at a relatively low level, it is reduced more than proportionally. The product price is constant at

$$P = \frac{c}{b + c^2\,\rho\,\sigma^2}\left(b\,(r_f - \lambda) + c\,\rho\,\sigma^2\,\frac{a_S + \gamma\,a_R}{1+\gamma}\right),$$

also the expected return $r = P/c + \lambda$ does not change. Consequently, standard households do not change their choice of i_S^* and q_S^*.

At some point, responsible investment i_R^* has fallen to zero, but the consumption of responsible households q_R^* is still positive. If responsible households now want to reduce the externality even more, they need to short-sell the dirty shares. This is not surprising: a large a_R means that responsible households like consuming the dirty product a lot. The maximum impact is reached for $q_R^* = 0$ and $i_R^* = -0.33$, which leads to an aggregate output of $I^* = 0.233$.

Heterogeneous Product Substitutability. The substitutability parameter b may also differ between the two groups of investors. For example, responsible investors may find it easier to switch between producers such that $b_R > b_S$, or vice versa. Proposition 1 refers exclusively to the substitutability of standard households, so that the marginal rate of technical substitution becomes

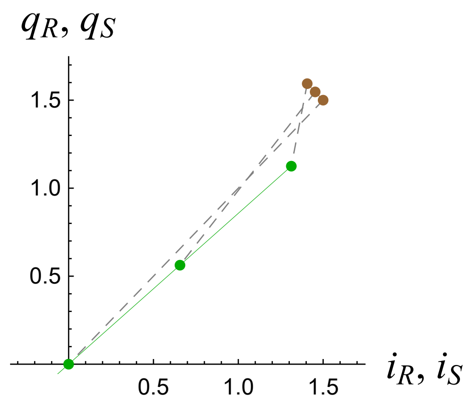
$$\frac{\rho\,\sigma^2}{b_S}\frac{1}{c}.$$

The impact of consumption reduction Δq disappears if *standard* households find it very easy to switch to another product. Proposition 2 also changes. Simplifying (32), it is still optimal for responsible households to reduce investment and consumption in proportion. However, the relation is now as follows

$$q_R^* = \frac{1}{c}\,\frac{1+\gamma}{(1-\gamma)\,\frac{b_R}{b_S} + 2\,\gamma}\,i_R^*. \tag{37}$$

In Figure 10, we have $b_S = 1/3$, and $b_R = 0.5$. The rational choice for each household is $q_R^* = 1.125$ and $i_R^* = 1.3125$. Standard households invest about the same and consume considerably more. The aggregate investment is at $I^* = 1.36$. Now as responsible households want to lower aggregate investment, they reduce consumption and investment in proportion, which leads to an increase in the price P and the expected return $r = P/c + \lambda$. Hence, standard households react by consuming less and investing more.

Figure 10: Expected Utility of Responsible Households, $b_R > b_S$



Parameters are $b_S = 1/3$, $b_R = 1/2 > b_S$, and $\gamma = 50\%$, everything else as in Figure 2.

Hypothesis 4 *If responsible households find it easier than standard households to substitute the dirty product, an increased (decreased) responsible choice increases (decreases) the return of dirty investments.*

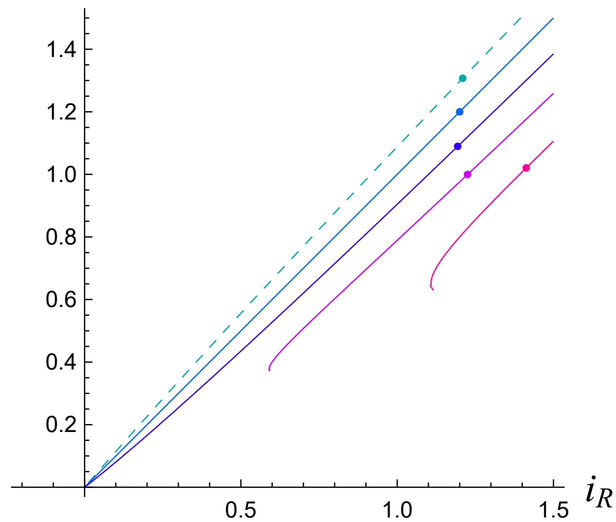
At the origin, $q_R^* = 0$ and $i_R^* = 0$, and the aggregate investment is $I^* = 0.75$. Although 50% of households are responsible, and they reduce consumption and investment to zero, the aggregate externality decreases by less than 50% because of the reaction of standard households.

4.2 Fixed Costs

Assume that, to produce a quantity Q, an investment of $I_F + c\,Q$ is needed. Pollution may stem from consumption or production, and the initial investment I_F may also cause emissions. The aggregate emissions will still be linear in Q, $X = X_0 + x\,Q$. The solution for the responsible households' optimization is still in closed form, but equations become rather messy, so let us turn to a simulation. Figure 11 shows the curves of the optimal combinations of q_R and i_R for different levels of fixed costs I_F. Points give the individually optimal levels.

We see that for $I_F = 0$, the efficient combination of q_R and i_R is a (blue) line through the origin, as stated in Proposition 2. It has a slope of $1/c$. For a fixed investment of

Figure 11: Impact of Fixed Costs



Parameters are $c = 1, \rho = 1/3, a = 2, b = 1/2, x = 1, \gamma = 1/2$, $\lambda = 0$, $\sigma = 1$ and $r_f = 1$. Furthermore, $I_F = 0.0$ (blue), $I_F = 0.1$ (deep purple), $I_F = 0.2$ (purple), and $I_F = 0.3$ (red). The dashed curve has $I_F = -0.1$.

$I_F = 0.1$ (deep purple), the efficient combination is a slightly convex curve. Starting from the individually optimal point, it is initially optimal to reduce consumption more than proportionally. The curve goes through the origin also for general parameter constellations. Therefore, on average, responsible households should reduce investment and consumption proportionally.

For higher levels of fixed costs ($I_F = 0.2$, purple), there is an additional effect. If the responsible households do not invest ($i_R = 0$), the investment of the standard households is insufficient to cover the fixed costs. Consequently, the industry collapses, there is no investment, and there is no consumption. For $I_F = 0.3$ (red), the effect is even more pronounced.

In the numerical example, we have not taken the liquidation values into account. Depending on the nature of the fixed costs, the resale value V may be relatively high. In that case, only the difference (net fixed costs) $I_F - V/r_f$ matters. It depends on the risk-free rate, and on the depreciation of the fixed asset. The effects of Figure 11 are mitigated accordingly.

Proposition 4 (Generalization of Proposition 2) *If the depreciation of fixed assets is negligible, it is efficient for responsible households to reduce investment and consumption proportionally, as in Proposition 2. If depreciation is noticeable, responsible households should reduce investment and consumption in proportion* on average*. For minor impact levels, they should reduce consumption more.*

We provide the formal proof in Appendix A. If depreciation is noticeable, responsible consumption is, then, relatively more effective for small levels of impact than responsible investment. For large levels of depreciation, if standard households cannot stem the net fixed costs on their own, responsible households can halt production by not investing.

5 Conclusion

Ethical concerns are increasingly important to retail investors. However, investment, production, and consumption decisions are intertwined and should not be studied separately. We raise the question of whether household investors should worry about ethics when investing money. Alternatively, households could focus on more sustainable consumption. We develop a tractable closed microeconomic model with interlinked capital and goods markets that allows us to analyze the optimal choice of responsible households. We show that responsible concerns matter, regardless of whether they occur when investing (socially responsible investment, SRI, exit) or when consuming (socially responsible consumption, SRC, boycott) as long as standard households ("the others") are risk averse and find it difficult to replace the dirty good. However, to achieve the greatest impact at the lowest possible utility loss, responsible households must reduce their dirty consumption proportionally to their divestment from dirty firms (Proposition 2). A disproportional reduction would be substituted for by other market participants, at least partially, and is therefore suboptimal. If responsible households can coordinate, their commitment to proportional consumption and investment reduction increases with the size of the externality and with their group size. The proportionality implies that responsible households do not influence product prices and capital returns.

We have assumed that production and consumption come with a negative externality. However, the formalism also holds for the opposite sign. Hence, if the product bears a positive externality, responsible households should increase their consumption and investment by the same factor. A green financial strategy should then also be part of responsible households' behavior. They should divest from dirty industries and overinvest in clean industries.

A Proofs

Proof of Proposition 1: The equilibrium for a given responsible investment i_R and consumption level q_R is defined by the following system of equations

$$\frac{\partial \mathrm{E}U_S}{\partial q_S} = \big(P - (a - b\, q_S)\big)\rho = 0 \tag{38}$$

$$\frac{\partial \mathrm{E}U_S}{\partial i_S} = \big(r_f - \lambda + i_S\, \rho\, \sigma^2 - \frac{P\, Q}{I}\big) = 0$$

$$I = \gamma\, i_R + (1-\gamma)\, i_S$$

$$Q = \gamma\, q_R + (1-\gamma)\, q_S \tag{39}$$

$$Q = {}^{I}\!/_{c} \tag{40}$$

Using $Q = {}^{I}\!/_{c}$, the first order condition with respect to i_S yields

$$i_S^* = \frac{P/c + \lambda - r_f}{\rho\, \sigma^2}. \tag{41}$$

In the aggregate, the capital market must clear:

$$I^* = \gamma\, i_R + (1-\gamma)\, \frac{P/c + \lambda - r_f}{\rho\, \sigma^2}. \tag{42}$$

Substituting the aggregate demand (7) and $I^* = c\, Q^*$ we obtain the equilibrium quantity

$$Q^* = \frac{(1-\gamma)\,(a - c\,(r_f - \lambda)) + \gamma\,(b\, q_R + c^2\, \rho\, \sigma^2\, {}^{i_R}\!/_{c})}{b + c^2\, \rho\, \sigma^2}. \tag{43}$$

the equilibrium investment is $I^* = c\, Q^*$ the equilibrium prices are given by

$$P^* = \frac{c^2\, \rho\, \sigma^2\, a\, + b\, c\,(r_f - \lambda)}{b + \rho\, c^2\, \sigma^2} - \frac{\gamma}{1-\gamma}\, \frac{b\, c^2\, \rho\, \sigma^2}{(b + c^2\, \rho\, \sigma^2)}\, \big(q_R - {}^{i_R}\!/_{c}\big) \tag{44}$$

and

$$E[r]^* = \frac{c\,\rho\,\sigma^2\,(a + c\,\lambda) + b\,r_f}{b + c^2\,\rho\,\sigma^2} - \frac{\gamma}{1-\gamma}\,\frac{b\,c\,\rho\,\sigma^2}{b + c^2\,\rho\,\sigma^2}\,\big(q_R - {}^{i_R}/_{c}\big). \tag{45}$$

The individual investment and consumption choices of standard households as provided in (11). Alternatively, using the definition $q_R = q_0 - \Delta q$ and $i_R = i_0 - \Delta i$ we can express the equilibrium investment as

$$I = i_S - \gamma \Delta i. \tag{46}$$

Therefore, we can write the equilibrium quantities in terms of boycott Δq and exit Δi as

$$\begin{aligned}
q_S &= \frac{1}{b + c^2\,\rho\,\sigma^2}\,\big(a - \ c(r_f - \lambda) - \gamma\,c^2 \rho\,\sigma^2\,(\Delta q - {}^{\Delta i}/_{c}\big)\\
i_S &= \frac{c}{b + c^2\,\rho\,\sigma^2}\,\big(a - c\,(r_f - \lambda) - \gamma\,b\,(\Delta q - {}^{\Delta i}/_{c}\big)\\
Q &= \frac{1}{b + c^2\,\rho\,\sigma^2}\,\big(a - c\,(r_f - \lambda) - \gamma\,(b\,\Delta q + c^2 \rho\,\sigma^2\,{}^{\Delta i}/_{c}\big)\\
I &= \frac{c}{b + c^2\,\rho\,\sigma^2}\,\big(a - c\,(r_f - \lambda) - \gamma\,(b\,\Delta q + c^2 \rho\,\sigma^2\,{}^{\Delta i}/_{c}\big)\\
P &= \frac{1}{b + c^2\,\rho\,\sigma^2}\,\big(c^2\,\rho\,\sigma^2\,a + b\,c\,(r_f - \lambda) - \gamma\,b\,c^2\,\rho\,\sigma^2\,(\Delta q - {}^{\Delta i}/_{c})\big).
\end{aligned}$$

We now describe the equilibrium without any responsible households and elaborate how responsible households can have an impact that shifts this equilibrium toward pollution abatement. In an economy without responsible households ($\gamma = 0$), the equilibrium quantity would be

$$Q_0 = \frac{a - c\,(r_f - \lambda)}{b + c^2\,\rho\,\sigma^2}$$

and $I_0 = c\,Q_0$ accordingly. The expected equilibrium return is then

$$E[r_0] = \frac{c\,\rho\,\sigma^2\,(a + c\,\lambda) + b\,r_f}{b + c^2\,\rho\,\sigma^2}. \tag{47}$$

The equilibrium price for the good is

$$P_0 = \frac{c}{b + \rho\,c^2\,\sigma^2}\,\big(c\,\rho\,\sigma^2\,a\ + b\,(r_f - \lambda)\big). \tag{48}$$

Using equation (13) we can write the aggregate impact of responsible choices as

$$\begin{aligned}
x\Delta I(\Delta i, \Delta q) &= x(\gamma\,c\,(\varphi\,\Delta q + (1-\varphi)\,{}^{\Delta i}/_{c}), \quad \text{and}\\
\mathrm{MRT}_{\Delta i, \Delta q} &= -\frac{\partial x \Delta I}{\partial \Delta i}\Big/\frac{\partial x \Delta I}{\partial \Delta q} = -\frac{{}^{(1-\varphi)}/_{c}}{\varphi} = -\frac{c\,\rho\,\sigma^2}{b}.
\end{aligned}$$

is the marginal rate of transformation for impact creation. ■

Proof of Proposition 2: The idea of the proof is in Figure 3. The slope of the diagonal iso-impact lines (Proposition 1) is $\frac{di_R}{dq_R} = -c\,\rho\,\sigma^2/b$. The implicit function theorem gives the slope of the iso-utility curves as

$$\frac{\frac{\partial \mathrm{E}\,U_R}{\partial i_R}}{\frac{\partial \mathrm{E}\,U_R}{\partial q_R}} = -\frac{\rho\,\sigma^2}{b}\,\frac{b\,((1+\gamma)\,i_R - 2\,c\,q_R\,\gamma) - c\,(1-\gamma)\,(a - c\,(r_f - \lambda - x\,\gamma + \rho\,\sigma^2\,i_R))}{\rho\,\sigma^2\,c\,(c\,q_R\,(1+\gamma) - 2\,i_R\,\gamma) - (1-\gamma)\,(a - b\,q_R - c\,(r_f - \lambda + x\,\gamma))}. \quad (49)$$

The utility function is a positive monotonic transformation of a quadratic function with the Hessian matrix

$$H = \begin{pmatrix} -\rho^2\,\sigma^2\,\frac{b\,(1+\gamma)+c^2\,(1-\gamma)\,\rho\,\sigma^2}{(1-\gamma)(b+c^2\,\rho\,\sigma^2)} & \frac{2\,b\,c\,\gamma\,\rho^2\,\sigma^2}{(1-\gamma)(b+c^2\,\rho\,\sigma^2)} \\ \frac{2\,b\,c\,\gamma\,\rho^2\,\sigma^2}{(1-\gamma)(b+c^2\,\rho\,\sigma^2)} & -b\,\rho\,\frac{b\,(1-\gamma)+c^2\,(1+\gamma)\,\rho\,\sigma^2}{(1-\gamma)(b+c^2\,\rho\,\sigma^2)} \end{pmatrix}.$$

The determinant is

$$\frac{b\,(1+\gamma)\,\rho^3\,\sigma^2}{1-\gamma}.$$

Hence the matrix is negative definite, hence each iso-impact line touches an iso-utility curve at one unique point, and this point is the optimum. Entering $i_R = c\,q_R$ into (49) gives a slope of $-c\,\rho\,\sigma^2/b$, which is identical to the slope of the iso-impact lines. This proves that the optimum satisfies $i_R = c\,q_R$. Because also $I_0 = c\,Q_0$, consequently $\Delta i = c\,\Delta q$. Note that one could have used Lagrange multipliers. This would have lead to algebraically intractable equations, with the same outcome. ■

Proof of Proposition 3: The derivative of (18) with respect to i_R and q_R yields

$$\frac{\partial \mathrm{E}U_R}{\partial q_R} = \rho\,b\,\frac{(1-\gamma)(b\,q_R - a + c(r_f - \lambda + x\,\gamma)) + c\,\rho\,\sigma^2\,(c\,q_R\,(1+\gamma) - 2\,i_R\gamma)}{(1-\gamma)(b + c^2\,\rho\,\sigma^2)} = 0 \quad (50)$$

$$\frac{\partial \mathrm{E}U_R}{\partial i_R} = \rho\,\sigma\,\frac{(1-\gamma)(\rho\,c^2\,\sigma^2\,i_R - a\,c + c^2\,(r_f - \lambda + x\,\gamma)) + b\,(i_R(1+\gamma) - 2\,c\,q_R\,\gamma))}{(1-\gamma)(b + c^2\,\rho\,\sigma^2)} = 0 \quad (51)$$

Solving (50) for q_R we obtain

$$q_R(i_R) = \frac{(1-\gamma)(a - c(r_f - \lambda + x\,\gamma)) + 2\,i_R\gamma\,c\,\rho\,\sigma^2}{(1-\gamma)b + (1+\gamma)\rho\,c^2\,\sigma^2} \quad (52)$$

Inserting (52) into (51) we obtain

$$\frac{(1+\gamma)\,\rho\,\sigma(b\,i_R - \,a\,c + c^2\,(r_f - \lambda + x\,\gamma) + r_R\,\rho\,c^2\sigma^2)}{b(1-\gamma) + (1+\gamma)\rho\,c^2\,\sigma^2)} = 0. \quad (53)$$

Solving (53) for i_R^* and $q_R^* = q_R(i_R^*)$ we obtain the optimal investment and consumption quantities of responsible households,

$$q_R^* = \frac{a - c\,(r_f - \lambda + \gamma\, x)}{b + \rho\,\sigma^2} \quad \text{and} \quad i_R^* = c\,\frac{a - c\,(r_f - \lambda + \gamma\, x)}{b + \rho\,\sigma^2}. \tag{54}$$

Subtracting the responsible choice from the standard choice $\Delta i = i_0^* - i_R^*$ and $\Delta q = q_0^* - q_R^*$, we obtain the optimal responsible choice as displayed in Proposition 3. ■

Proofs for Section 3.2 (Shame & Blame): In the absence of free coordination, responsible households cannot credibly commit to a certain investment or consumption amount. Instead, the equilibrium outcomes become a function of the shame costs g_C and g_I. Given these costs, the equilibrium is defined as

$$\begin{aligned}
i_S &= c\frac{a - c\left(r_f + \gamma^2\,(g_C - g_I\,\frac{b}{\rho\,\sigma^2})\right)}{b + \rho\sigma^2} \\
q_S &= \frac{a - c\left(r_f + \gamma^2\,(g_I - \;g_C\frac{\rho\,\sigma^2}{b})\right)}{b + \rho\sigma^2} \\
i_R &= i_S - c\,\gamma\frac{c}{\rho\,\sigma^2}\,g_I \\
q_R &= q_S - \gamma\,\frac{c}{b}\,g_C, \quad \text{resulting in} \\
Q &= \frac{a - c\,(r_f + (\gamma^2\,(g_I + g_C)))}{b + \rho\,\sigma^2}.
\end{aligned}$$

The slope of the diagonal iso-impact lines is $\frac{dg_I}{dg_C} = -1$. Increasing the shame has an identical impact on the externality for each choice. The implicit function theorem gives the slope of the iso-utility curves as

$$-\frac{\frac{\partial \mathrm{E}\,U_R}{\partial g_I}}{\frac{\partial \mathrm{E}\,U_R}{\partial g_C}} = -\frac{b\,(\rho\,\sigma^2(c(\gamma((2-\gamma)\gamma g_C + g_I + x) + r_f) - a) + bc\gamma(1-\gamma)^2 g_I)}{\rho\,\sigma^2\,(-ab + bc(\gamma(g_C + (2-\gamma)\gamma g_I + x) + r_f) + c(1-\gamma)^2\gamma g_C\,\rho\,\sigma^2)}. \tag{55}$$

The utility function is positive monotonic transformation of a quadratic function with the Hessian

$$H = \begin{pmatrix} \frac{c^2\gamma^2\left(b(1-\gamma)^2 + \rho\,\sigma^2\right)}{\sigma^2(b+\rho\,\sigma^2)} & -\frac{c^2(2-\gamma)\gamma^3\rho}{b+\rho\sigma^2} \\ -\frac{c^2(2-\gamma)\gamma^3\rho}{b+\rho\,\sigma^2} & \frac{c^2\gamma^2\rho\left(b+(1-\gamma)^2\rho\,\sigma^2\right)}{b(b+\rho\,\sigma^2)} \end{pmatrix}.$$

The determinant has

$$\frac{c^4(1-\gamma)^2\gamma^4\rho}{b\sigma^2}$$

as the main factor. Hence, the matrix is positive definite such that the iso-impact line touches an iso-utility curve at the two corners, and one of the corner solutions is the optimum.

Proof of Proposition 4: The proof is algebraically messy but straightforward. First, we solve the standard households' first-order conditions, market clearing conditions, and the production equation $c\,Q = I - I_F$ for q_S, i_S, Q, I and P. The equilibrium for a given responsible investment i_R and consumption level q_R is defined by equations (38) to (39) and $Q = (I - I_F)c$. Using this modified production function we can rewrite the first order condition with respect to i_S as

$$i_S^* = \frac{P/c + \lambda - r_f}{\rho\,\sigma^2} - \frac{I_F}{I^*}\frac{P/c}{\rho\,\sigma^2}. \tag{56}$$

In the aggregate, the capital market must clear,

$$I^* = \gamma\, i_R + (1-\gamma)\,\frac{\frac{P}{c}\,\frac{I^*-I_F}{I^*} + \lambda - r_f}{\rho\,\sigma^2}. \tag{57}$$

This resembles our benchmark condition except for the factor $(I^* - I_F)/I^*$. We can set $Q^* = \frac{I^*-I_F}{c}$ and insert the aggregate demand (7). The equilibrium quantity Q^* is now implicitly defined by the quadratic function

$$\begin{aligned} b\,(Q^*)^2 + (c\,Q^* + I_F)^2\rho\,\sigma^2 = {} & Q^*((1-\gamma)(a - c(r_f-\lambda)) + \gamma(b\,q_R + c\,\rho\,\sigma^2\,i_R)) \\ & + I_F(i_R\,\gamma\,\rho\,\sigma^2 - (1-\gamma)(r_f - \lambda)). \end{aligned} \tag{58}$$

Note that in the limit $I_F \to 0$ the equilibrium quantity approaches our benchmark solution (43). We solve the equilibrium for given q_R and i_R to obtain the general equilibrium market responses for any responsible choice,

$$q_S^* = \frac{1}{2\,(1-\gamma)} \left(\frac{(1-\gamma)(a - c\,(r_f - \lambda)) - \gamma\, c^2 \rho\, \sigma^2\, (q_R - {}^{i_R}\!/_c)}{b + c^2 \rho\, \sigma^2} - \gamma\, q_R + \frac{\sqrt{\Omega}}{b + c^2 \rho\, \sigma^2} - 2(1-\varphi)^{I_F}\!/_c \right) \tag{59}$$

$$i_S^* = \frac{c}{2\,(1-\gamma)} \left(\frac{(1-\gamma)(a - c\,(r_f - \lambda)) + \gamma\, b\, (q_R - {}^{i_R}\!/_c)}{b + c^2 \rho\, \sigma^2} - \gamma\, i_R + \frac{\sqrt{\Omega}}{b + c^2 \rho\, \sigma^2} + 2\,\varphi\, {}^{I_F}\!/_c \right)$$

$$Q^* = \frac{1}{2} \left(\frac{(1-\gamma)(a - c\,(r_f - \lambda)) + \gamma\, (b\, q_R + c^2 \rho\, \sigma^2\, {}^{i_R}\!/_c)}{b + c^2 \rho\, \sigma^2} + \frac{\sqrt{\Omega}}{b + c^2 \rho\, \sigma^2} - 2(1-\varphi)^{I_F}\!/_c \right) \tag{60}$$

$$I^* = \frac{c}{2} \left(\frac{(1-\gamma)\big(a - c\,(r_f - \lambda)\big) - \gamma\, (b\, q_R + c^2 \rho\, \sigma^2\, {}^{i_R}\!/_c)}{b + c^2 \rho\, \sigma^2} + \frac{\sqrt{\Omega}}{b + c^2 \rho\, \sigma^2} + 2\,\varphi\, I_F \right) \tag{61}$$

$$P^* = \frac{1}{2} \left(\frac{\big(c^2 \rho\, \sigma^2\, a + b\, c\, (r_f - \lambda)\big) - \frac{\gamma}{1-\gamma}\, b\, c^2\, \rho\, \sigma^2\, (q_R - {}^{i_R}\!/_c)}{b + c^2 \rho\, \sigma^2} \right) + \frac{b}{2\,(1-\gamma)} \left(((1-\gamma)^a\!/_b - \gamma\, b\, q_R) - \frac{\sqrt{\Omega}}{b + c^2 \rho\, \sigma^2} + 2\,(1-\varphi)^{I_F}\!/_c \right) \tag{62}$$

where

$$\Omega = \frac{4 I_F}{c} \left((1-\gamma) a + \gamma b q_R + \frac{b I_F}{c} \right) (b + c^2 \rho\, \sigma^2) + \left((1-\gamma)(a - (r_f - \lambda)) + \gamma\, b q_R + 2\, b\, \frac{I_F}{c} + c^2 \rho\, \sigma^2 \frac{i_R}{c} \right)^2$$

is an auxiliary variable. Note that the first term in each bracket resembles the benchmark equilibrium values without fixed costs.

We first calculate the marginal rate of transformation for exiting one unit of investment in units of boycott as (here we make use of the fact, that the externality is a linear function of investment I^*, where the fixed part I_0 does not affect the marginal effect of the responsible choice).

$$\text{MRT}_{i_R, q_R} = -\frac{\frac{\partial I^*}{\partial i_R}}{\frac{\partial I^*}{\partial q_R}} = -\frac{c\, \rho\, \sigma^2}{b} \frac{(1-\gamma)(a + c(r_f - \lambda) + \gamma\, (b\, q_R - c^2\, \rho\, \sigma^2 {}^{i_R}\!/_c) - \sqrt{\Omega}}{2\, c((I_F - \gamma i_R) \rho\, \sigma^2 + (1-\gamma)(r_f - \lambda))} \tag{63}$$

At the extreme, if responsible households fully exit and boycott the MRT becomes

$$\text{MRT}_{i_R \to 0, q_R \to 0} = -\frac{c\, \rho\, \sigma^2}{b} \frac{(1-\gamma)(a + c(r_f - \lambda)) - \sqrt{\Omega_0}}{2\, c(I_F \rho\, \sigma^2 + (1-\gamma)(r_f - \lambda))} \tag{64}$$

with

$$\Omega_0 = \frac{4 I_F}{c} \left((1-\gamma) a + \frac{b I_F}{c} \right) (b + c^2\, \rho\, \sigma^2) + \left((1-\gamma)(a - (r_f - \lambda)) + 2\, b\, \frac{I_F}{c} \right)^2 .$$

To prove that complete boycott $q_R = 0$ and exit $i_R = 0$ are always a part of the optimal strategy, we will now prove that $\mathrm{MRT}_{i_R\to 0, q_R\to 0} = \mathrm{MRS}_{i_R\to 0, q_R\to 0}$. In other words, we prove that also with fixed costs, the iso-impact lines touch the iso-utility curve at the unique point $i_R \to 0, q_R \to 0$ such that proportional reduction is optimal at this point.

To calculate the marginal rate of substitution consider the expected utility of responsible households given the general equilibrium responses as described above

$$\mathrm{E}U_R = u\Big(aq_R - bq_R^2/2 - P^*(q_R, i_R)q_R - bq_R^2 - i_R(r_f - \lambda) + r_f\, w_0 + i_R P^*(q_R, i_R)\frac{Q^*(q_R, i_R)}{I^*(q_R, i_R)} - i_R^2\,\rho\,\sigma^2\Big)$$

For brevity we leave aside the asterisk for the equilibrium values for market responses in the following notion. Using the implicit function theorem we solve for the marginal rate of substitution,

$$\mathrm{MRS}_{i_R, q_R} = -\frac{\frac{\partial \mathrm{E}U_R}{\partial i_R}}{\frac{\partial \mathrm{E}U_R}{\partial q_R}} = -\frac{2a - 2bq_R + \frac{2i_R Q P'(q_R)}{I} + \frac{2i_R P Q'(q_R)}{I} - \frac{2i_R P Q I'(q_R)}{I^2} - 2q_R P'(q_R) - 2P}{\frac{2i_R Q P'(i_R)}{I} - 2q_R P'(i_R) + \frac{2i_R P Q'(i_R)}{I} - \frac{2i_R P Q I'(i_R)}{I^2} - 2i_R \rho\,\sigma^2 + \frac{2PQ}{I} - 2r_f} \tag{65}$$

Setting $i_R = 0$ and $q_R = 0$ this simplifies to

$$\mathrm{MRS}_{i_R\to 0, q_R\to 0} = -\frac{(a - P^*(0,0))}{\frac{P(0,0)^* \cdot Q^*(0,0)}{I^*(0,0)} - r_f}. \tag{66}$$

Inserting (60), (61) and (62) at the point $i_R = 0$ and $q_R = 0$ we obtain

$$\mathrm{MRS}_{i_R\to 0, q_R\to 0} = -\frac{c\,\rho\,\sigma^2}{b}\frac{(1-\gamma)(a + c(r_f - \lambda)) - \sqrt{\Omega_0}}{2\,c(I_F\rho\,\sigma^2 + (1-\gamma)(r_f - \lambda))} \tag{67}$$

which is identical to 64. Hence, the efficient combination of q_R and i_R is always a curve through the origin (if it is defined there). ■

B Productivity Shocks

Let us consider shocks in production volumes. To fix ideas, assume that with an investment of I, the output is $Q = I/c + \varepsilon$, where ε is a normally distributed random variable with mean 0 and standard deviation σ. Assume that there are only standard households with a utility function as before, see (3) on page 8. Therefore, they still demand

$q = (a - P)/b$ for a given price P. Hence for an aggregate quantity of Q, the product market clears at $P = a - b\,Q$. The firm's profits are

$$\Pi = P\,Q = (a - b\,Q)\,Q.$$

Households maximize their expected utility from investment,

$$-\exp(-\rho\,(i\,r + (w_0 - i)\,r_f)) \times \text{other terms}$$

where $r = \Pi/I$ is the return from their investment. Hence they maximize

$$\int -\exp(-\rho\,(i\,\frac{(q - b\,(I/c + \varepsilon))\,(I/c + \varepsilon)}{I} + (w_0 - i)\,r_f))\,f(\varepsilon)\,d\varepsilon.$$

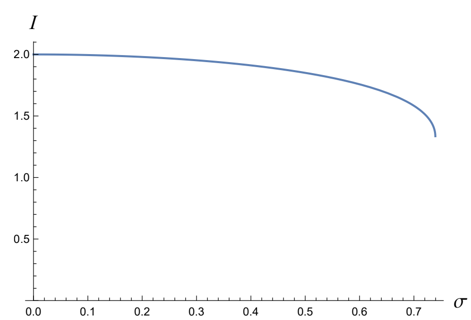
The first order condition yields

$$i^* = I\,\frac{a^2c - 2bc(2r_fI + b\sigma^2) - \sqrt{(ac - 2bI)^2(a^2 - 4br_fI) + 4b^4c^2\sigma^4}}{2bc(a^2 - 4br_fI)\rho\,\sigma^2}.$$

The capital market clears when $i^* = I$. Solving for I yields

$$I = \frac{c}{2b}\Big(a - cr_f(1 - 2b\rho\,\sigma^2)^2 + \sqrt{4b^2\rho(2b + a^2\rho)\sigma^4 - 4b(b + a^2\rho)\sigma^2 + (a - cr_f(1 - 2b\rho\,\sigma^2)^2)^2}\Big).$$

Just to see that this result makes sense, the following figure shows the equilibrium investment volume I as a function of risk σ.



For zero risk, investment starts at the level $I = c\,(a - c\,r_f)/b$ and decreases for higher levels of risk. At some level, the industry collapses.

For our purpose, we would now need to introduce two classes of households, and solve for the equilibrium quantity depending on γ, i_R and q_R. This is possible, but terms become more and more intractable. In addition, there is a conceptual problem. In our original model, responsible households can anticipate the market price, and thus anticipate their consumption level. Now if quantities and prices fluctuate, a pre-commited quantity may not be desirable any longer, or in fact, it may not even be feasible. This conceptual problem does not appear in our original modeling choice.

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