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Mario Forni, Luca Gambetti, Marco Lippi and Luca Sala

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
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Abstract

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JEL Classification: N/A

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Mario Forni - mario.forni@unimore.it
Dipartimento di Economia "Marco Biagi" and CEPR

Luca Gambetti - luca.gambetti@uab.cat
Universitat Autònoma de Barcelona, BSE, Università di Torino abd CCA

Marco Lippi - marco.lippi@eief.it
Einaudi Institute for Economics and Finance, Rome

Luca Sala - luca.sala@unibocconi.it
Università Bocconi, IGIER and Baffi Carefin

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Validating DSGE Models through Dynamic Factor Models

Mario Forni*

Università di Modena e Reggio Emilia, CEPR and RECent

Luca Gambetti†

Universitat Autònoma de Barcelona, BSE, Università di Torino abd CCA

Marco Lippi

Einaudi Institute for Economics and Finance

Luca Sala

Università Bocconi, IGIER and Baffi Carefin

May 31, 2022

Abstract

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JEL classification: C32, E32.

Keywords: DSGE models, validation, structural VAR, structural factor model, news shocks.

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) and Structural Vector Autoregressions (SVAR) models play a complementary role in modern macroeconomic analysis. In particular, DSGE model analysis relies on SVAR evidence for several purposes. First, SVARs are employed to guide the construction of DSGE models. Indeed, the empirical analysis of the transmission mechanism of a particular shock can provide useful insights about what type of frictions should be included in the theoretical setting. For instance, a hump-shaped response of consumption or investment to technology shock estimated in a SVAR may be suggestive of the presence of habit-formation in consumption or investment adjustment costs.

Second, SVARs are used as an empirical tool to validate economic theories (see Canova (2007)). Validation essentially entails the comparison of the theoretical responses with those estimated in SVARs where the shock of interest is identified using a limited set of restrictions implied by the DSGE model. If the empirical and theoretical impulse response functions are similar, under some reasonable criterion, then the transmission mechanism of the shock implied by the DSGE is successfully validated. Otherwise, some features of the model must be modified. For example, Galì (1999) investigates the response of hours to technology shocks estimated in a SVAR to assess the empirical support of RBC and New Keynesian models (see also Christiano et al. (2007)). Canova and Paustian (2011) assesses the validity of models with rule-of-thumb consumers and show that, to match the SVAR response of consumption to government spending shocks, an unrealistically high share of rule-of-thumb consumers is required.

Third, SVAR models are used in impulse response functions matching estimation, see Guerron-Quintana et al. (2017) and the vast literature cited therein. Within this approach, the DSGE model parameters are estimated by minimizing the distance between the implied theoretical responses and the SVAR responses of a given shock of interest.

In this paper, we urge the use of Structural Dynamic Factor Models (SDFM) as empirical tool to complement DSGE models analysis. The most compelling reason is that the log-linear solution of a DSGE model has a factor model structure. This ensures consistency between the theoretical and the empirical model. Furthermore, we also claim that the SDFM are better suited for this analysis than SVAR models since the latter may be subject to two major issues which can seriously undermine the comparison between the data and the theory.

First, there is no guarantee that the variables used in the SVAR contain the information needed to estimate the shock and its impulse response functions. This problem is often referred to as nonfundamentalness or informational deficiency (Forni et al. (2019)). When the variables are not sufficiently informative, the theoretical responses may differ substantially from those of the SVAR, not because the theory is incorrect, but because the SVAR is incorrect. This problem has received much attention in the literature and at least two solutions have been put forward. Fernández-Villaverde et al. (2007) provides a verifiable condition to understand whether a set of variables included in the DSGE admits a SVAR representation. Forni et al. (2019) provides a DSGE model-based measure to assess whether a given VAR specification can be used to estimate a single shock of interest and its impulse response functions. A third solution is suggested here: using a factor model in place of a VAR solves the problem, since factor models are not affected by informational deficiency ((Forni et al., 2009),Forni et al. (2020)).

But there is a second problem which the literature has largely ignored: the presence of measurement errors. Many macroeconomic variables, like GDP or prices, are unquestionably measured with error. If this is the case, the impulse response functions obtained from a SVAR are biased, see Lippi (2020). Not only, in presence of measurement error, the methods discussed above to establish the consistency between the SVAR and the DSGE model are no longer valid. The presence of measurement error, apart from distorting the IRF itself, brings the problem of information back.[1]

To understand why the SDFM model performs well, notice that the log-linear representation of the DSGE model postulates a VAR for the state variables in terms of the structural shocks. So if the state variables were observed, the empirical strategy closest to the DSGE model would be simply to identify structural shocks with a VAR for the state variables. This is obviously unfeasible because the state variables are generally not observed. The SDFM provides however a consistent estimator, the principal components, of the factors. The factors are statistical objects with the property of spanning the same space spanned by the state variables of the model. This implies that using a VAR for the factors, as it is done in the factor model, is equivalent to the best strategy, i.e. using a VAR for the states. This is the ultimate reason why the factor model is successful in estimating the structural shocks and the impulse response functions. With the factor model approach, on the one hand, the measurement error is no longer an issue because the factors are free of measurement error; on the other hand, the lack of information is not an issue, since the shocks are estimated using all

[1]A measurement error is often added to DSGE variables to be consistent with the data when estimating the model using likelihood function techniques. This is perfectly fine. The problem with the measurement error arises when using SVARs to compare impulse response functions.

the relevant information, i.e. the information contained in the state variables of the model.

We start off our analysis by illustrating theoretically the link between DSGE models and factor models. Using artificial data generated from a modified version of the DSGE model in Blanchard et al. (2013) we evaluate the validity of the factor model as a tool for assessing DSGE models. The responses obtained using the SDFM are very accurate, almost identical to the theoretical ones. On the contrary, when using SVAR models accurate estimates are obtained only when there is no measurement error and the variables are informationally sufficient for the shock.

As an application, we validate a New Keynesian theory of the transmission of technology shocks. We focus on both news and surprise technology shocks. The theoretical model is the same model used in the Monte Carlo simulations. The surprise and news shocks are identified assuming that they are the only two shock driving TFP in the long run. Furthermore, while the surprise shock affects TFP on impact, the news shock does not. The identification is taken from Beaudry and Portier (2006) and it is in line with the model's restrictions.

As far as the news shock is concerned, the theoretical responses of real economic activity variables are quite in line with the empirical ones. The model slightly over-predicts the effects on consumption, but for GDP investment and hours the responses are similar both in terms of shape and magnitude. However, the model does a poor job in terms of inflation and interest rate. Indeed while in the data the shock generates a large drop in both variables, in the model the two variables barely react. The finding confirms those in in Kurmann and Otrok (2017) about the difficulty of the New Keynesian model in generating the observed dynamics of inflation following a news shock.

The theoretical dynamics of the surprise shock are similar to those estimated in the factor model. The main inconsistency is again the larger response of consumption in the model. Particularly striking is the response of hours. The model captures well the change in the sign of the response, negative for the first quarters after the shock and then turning positive. Unlike for the news shock, here the response of inflation in the model is very similar to the empirical one.

We also compare our baseline validation with that performed using a SVAR. The SVAR would lead to very different conclusions. As far as news shocks are concerned, the estimated responses of real variables are much larger than those of the theoretical model. So one would conclude, unlike using the factor model, that the DSGE model actually underestimates the true responses. For the surprise shock, the SVAR evidence suggests specifying the surprise shock as a temporary shock to TFP, opposite to the factor model evidence. The result is particularly interesting as it illustrates how conclusions can be radically different using factor models or SVARs.

The remainder of the paper is organized as follows. Section 2 presents a theoretical discussion of the consequence of the presence of measurement error and nonfundamentalness in SVAR. Section 3 presents the results of the simulations. Section 4 discusses the results. Section 5 concludes.

2 DSGE and dynamic factor models

In this Section we discuss the relationship between DSGE models and Structural Dynamic Factor Models (DFM), and present the results of some Monte Carlo simulations.

2.1 The dynamic factor model

Let x_t be a n-dimensional vector of economic variables. A rigorous definition of a High-Dimensional Dynamic Factor Model requires that the vector x_t is part of an infinite-dimensional vector, so that we can make assumptions by letting n tend to infinity, see Forni et al. (2000), Stock and Watson (2002a,b), Bai and Ng (2002). Here, making reference to the version in Forni et al. (2009), we limit ourselves by recalling the main features of the model.

We assume that the variables x_{it} are co-stationary, possibly after detrending, and can be represented as

$$x_{it} = \chi_{it} + \xi_{it}, \quad i = 1, \dots, \infty, \tag{1}$$

where the following assumption hold.

(DFM1) The variables ξ_{it}, called *idiosyncratic components*, are *weakly correlated* across different i's. The formal condition is an asymptotic one: the eigenvalues of the variance covariance matrix of the ξ's are bounded as n goes to infinity. The ξ's are interpreted as containing local-sectoral variables plus measurement errors, thus mainly measurement errors for aggregates like consumption, GDP, the general industrial production index. With this interpretation some cross covariance may occur among the ξ's.

(DFM2) The variables χ_{it} are called the *common components*. Given t, the χ's, for $i \in \mathbb{N}$, span a finite-dimensional space, whose dimension is r. This implies that there exists an r-dimensional vector F_t, weakly stationary, such that

$$\chi_{it} = \lambda_{i1} F_{1t} + \cdots + \lambda_{ir} F_{rt} = \Lambda_i F_t \quad \text{or} \quad \chi_t = \Lambda F_t, \tag{2}$$

where χ_t is the n-dimensional vector of the χ's and Λ, the *factor loading* matrix, is $n \times r$.

The coordinates of F_t are called the *static factors* and (2) the *static representation* of the common components. Moreover, the factors F_t are *pervasive*, in that all of them affect, with a few possible exceptions, all the variables x_{it} and have a non-singular covariance matrix.

(DFM3) The idiosyncratic components are orthogonal to the factors at all leads and lags. Thus ξ_{it} is orthogonal to χ_{js} for all i, j, t and s.

We postulate that the common components χ_{it} and the factors F_t , are driven by a q-dimensional vector of *structural macroeconomic, or common, shocks* u_t, with $q \leq r$. Precisely:

(DFM4) The r-dimensional vector F_t has the VAR representation[2]

$$Q(L)F_t = \varepsilon_t = Su_t, \tag{3}$$

where $Q(L)$ is a stable polynomial matrix of order p with $Q(0) = I$ and S is a $r \times q$ matrix of constants.

Equation (3) implies that the structural shocks belong to the information space spanned by the VAR residuals, so that the factors are *informationally sufficient* for u_t (i.e. the structural shocks are fundamental for F_t).

By inverting (3) and using (2), we get the impulse response function representation

$$x_t = \Lambda Q(L)^{-1} S u_t + \xi_t = \Phi(L) u_t + \xi_t. \tag{4}$$

where $\Phi(L)$ is the matrix of structural impulse response functions.

2.2 Factor model representation of a DSGE model

Suppose that the data generating process is a DSGE model which admits the following the state-space representation, known as the ABCD representation (see Fernández-Villaverde et al. (2007)),

$$s_t = As_{t-1} + Bu_t \tag{5}$$

$$\chi_t = Cs_{t-1} + Du_t \tag{6}$$

where u_t is a q-dimensional vector of structural shocks, χ_t is a n-dimensional vector of economic variables, s_t is an m-dimensional vector of stationary state variables ($q \leq m$), A, B, C and D are conformable matrices of parameters, and B has a left inverse B^{-1} such that $B^{-1}B = I_q$.

[2]The existence of a VAR representation for the factors is perfectly compatible with cointegration among the variables in the panel, see Forni et al. (2020).

From the ABCD representation we derive another representation to make explicit and clear the link between the DSGE model and the factor model discussed in the previous sub-section. From equations (5) and (6) we get

$$\chi_t = Gf_t \tag{7}$$

where $G = \begin{pmatrix} DB^{-1} & C - DB^{-1}A \end{pmatrix}$ and $f_t = \begin{pmatrix} s_t' & s_{t-1}' \end{pmatrix}'$. The vector f_t has the VAR representation

$$f_t = \tilde{A}f_{t-1} + \tilde{B}u_t \tag{8}$$

with $\tilde{A} = \begin{pmatrix} A & 0_m \\ I_m & 0_m \end{pmatrix}$, $\tilde{B} = \begin{pmatrix} B \\ 0_m \end{pmatrix}$, I_m is the m-dimensional identity matrix and 0_m a $m \times m$ matrix of zeros. Unlike representation (2)-(3), representation (7) is not necessarily minimal (i.e., the representation of the model with the smallest number of factors), since the covariance matrix of f_t, Σ_f, may have reduced rank $r < 2m$. We can easily derive the minimal representation. If (7) is not minimal, then there exist a $2m \times r$ matrix P such that $f_t = PF_t$, where F_t has a nonsingular covariance matrix. Equations (7)-(8) reduce to the minimal representation

$$\chi_t = \Lambda F_t \tag{9}$$

$$F_t = QF_{t-1} + Su_t \tag{10}$$

where $\Lambda = GP$, $Q = P^{-1}\tilde{A}P$, $S = P^{-1}\tilde{B}$, P^{-1} being a left inverse of P. There are two important remarks to notice about representation (9)-(10). First, $F_t = P^{-1}f_t$ spans the same information space as $f_t = (s_t' \ s_{t-1}')'$, so that F_t contains all the relevant information about the DSGE dynamics. Second, F_t follows the VAR representation (10), where, in general, the residuals have reduced rank, i.e. $q < r$.

By inverting the VAR for F_t, we get the MA representation

$$\chi_t = \Phi(L)u_t = \Lambda(I - QL)^{-1}Su_t, \tag{11}$$

where $\Phi(L)$ is the matrix of impulse-response functions. Assuming that the economic variables are observed with error, we obtain

$$x_t = \Lambda F_t + \xi_t = \Phi(L)u_t + \xi_t, \tag{12}$$

where ξ_t is a vector of measurement errors. By comparing (4) and (12) it is seen that, when the variables are observed with error, the linearized DSGE model has a factor model representation with $Q(L) = I - Q$.

Notice that the existence of a factor model representation is always ensured independently of the values of the parameters of the matrices A, B, C and D. This is not the case for the existence of a VAR representation for χ, see Section 3. This points to the factor model as the natural empirical model to validate and complement DSGE analysis.

In practice, one can compare the empirical impulse response functions estimated with the factor model for a given shock to those of the DSGE. Under the null that the DSGE is the DGP, the two should be similar. If not, this suggests some sort of misspecification of the DSGE model.

2.3 Identification and estimation of the factor model

As already observed, Λ and F_t are not unique; however, under the above assumptions $\chi_t = \Lambda F_t = \Lambda Q(L)^{-1}\varepsilon_t$ is unique. To get an MA representation with q orthonormal shocks we observe that the covariance matrix of the VAR residuals can be represented as $\Sigma_\varepsilon = VMV'$, where M is the $q \times q$ diagonal matrix having on the diagonal the q non-zero eigenvalues of Σ_ε and V is the $r \times q$ matrix having on the columns the corresponding eigenvectors. Then, defining $W = VM^{-1/2}$, $v_t = W'\varepsilon_t$ and $R = VM^{1/2}$ we get the representation

$$\chi_t = \Lambda Q(L)^{-1} R v_t. \tag{13}$$

The above representation is a fundamental MA representation with orthonormal shocks. Starting from the above representation we can get the structural shocks as $u_t = H'v_t$, where H is a $q \times q$ orthogonal matrix (see Rozanov (1967), pp. 56-7; see also Section 3.2 in Forni et al. (2009)). The corresponding matrix of impact effects S is obtained as $S = RH$. Lastly, the determination of the matrix H can be obtained by imposing restrictions on the impulse response functions of the χ's, i.e.

$$\Phi(L) = \Lambda Q(L)^{-1} RH,$$

such as zero impact or long-run effects, just in the same way as in standard VAR analysis. Of course, we can identify a single shock along with its impulse response function by limiting ourselves to determining just a single column of the matrix H.

Coming to estimation, we first transform the variables to get a stationary vector x_t and estimate the number of static factors to get $\hat{r}$. Then we estimate the static factors themselves by means of the first $\hat{r}$ ordinary principal components of the x's, and the factor loadings by means of the associated eigenvectors. Precisely, let $\hat{\Sigma}_x$ be the sample variance-covariance matrix of x_t: our estimated loading matrix $\hat{\Lambda}$ is the $n \times r$ matrix having on the columns the normalized eigenvectors corresponding to the first largest $\hat{r}$ eigenvalues of $\hat{\Sigma}_x$, and our estimated factors are $\hat{F}_t = \hat{\Lambda}' x_t$.[3]

[3]Notice that the factors F_t and the loadings Λ are not identified, since given any non-singular $r \times r$

Second, we set a number of lags $\hat{p}$ and run a VAR($\hat{p}$) with $\hat{F}_t$ to get estimates of $Q(L)$ and the residuals ε_t, say $\hat{Q}(L)$ and $\hat{\varepsilon}_t$.

As a third step, having an estimate $\hat{q}$ of the number of dynamic factors, we obtain an estimate of the non-structural representation (13). Let $\hat{\Sigma}_\varepsilon$ be the sample covariance matrix of $\hat{\varepsilon}_t$. We first estimate the matrices V and M, by computing the largest $\hat{q}$ eigenvalues and the corresponding eigenvectors of $\hat{\Sigma}_\varepsilon$; then we compute $\hat{R} = \hat{V}\hat{M}^{1/2}$ and $\hat{v}_t = \hat{M}^{-1/2}\hat{V}'\hat{\varepsilon}$.

Finally, we obtain an estimate of H by imposing suitable identification restrictions on the estimated impulse response functions. The estimates of the structural impulse response functions is given by $\hat{\Phi}(L) = \hat{\lambda}\hat{Q}(L)^{-1}\hat{S}$, where $\hat{S} = \hat{R}\hat{H}$. The structural shocks are estimated as $\hat{u}_t = \hat{H}'\hat{v}_t$.

For the consistency of this estimation procedure see Forni et al. (2009), Proposition 3.

To get the confidence bands, we bootstrap the estimated VAR residuals $\hat{\varepsilon}_t$ and use $\hat{\Lambda}$ and $\hat{A}(L)$, along with the initial conditions $\hat{F}_1, \ldots, \hat{F}_p$, to construct the artificial series χ_t^1. Then we add the estimated idiosyncratic components $\hat{\xi}_t = x_t - \hat{\chi}_t$ to get the artificial series $x_t^1 = \chi_t^1 + \hat{\xi}_t$ and estimate the model to get the IRFs $\hat{\Phi}^1(L)$. We repeat the procedure m times to get $\hat{\Phi}^j(L)$, $j = 1, \ldots, m$. Finally we take suitable percentiles of the IRFs distribution for each horizon.[4]

2.4 Simulations

We assess the validity of the factor model as an empirical tool to validate DSGE models using two simulations. More specifically, we investigate whether the factor model is able to correctly capture the effects of the news shock using a version of the DSGE model studied in Blanchard et al. (2013). To be consistent with the vast majority of DSGE models we assume, unlike the original version, that information is perfect.

The economic model is a medium scale DSGE equipped with all the frictions that are considered necessary to capture the persistence of macro data: habit persistence,

matrix M, we have $\chi_t = \Lambda^* F_t^*$, where $\Lambda^* = \Lambda M^{-1}$ and $F_t^* = MF_t$. Hence, strictly speaking, we do not estimate F_t and Λ but a basis of the space spanned by F_t and the corresponding factor loading matrix. This however is not a problem in the present context since we are only interested in the product $\chi_t = \Lambda F_t$, which is identified.

[4]Estimation of the DFM entails the estimation of a VAR for the factors, which are stationary. One might wonder if this does not lead to cointegration problems. The answer is no. The reason is that in the DFM the spectral density matrix of the factors is singular. For singular vector variables, unlike the standard non-singular case, (a) I(1) variables (in our case the cumulated sum of the factors) are always cointegrated; despite this, (b) a (finite order) VAR for I(0) variables (in our case the factors) does exist, except very special cases. For a broader discussion see Forni et al. (2020). The first simulation in Section 2.4.1 can be regarded as an illustration of this point.

adjustment costs to investment, sticky prices, sticky wages, etc. While we defer all the details of the model to Appendix, the process for total factor productivity (TFP) deserves a brief discussion here to understand how the news shock is defined and modeled. We assume that TFP, denoted a_t, follows the process:

$$a_t = a_{t-1} + P_t + T_t \tag{14}$$

$$P_t = \phi P_{t-1} + \epsilon_{t-4} \tag{15}$$

$$T_t = \rho T_{t-1} + \eta_t \tag{16}$$

where P_t is the news component of TFP and ϵ_t is the news shock with four periods of anticipation. T_t is the surprise component of technology, driven by the surprise shock η_t. A number of parameters is calibrated following Blanchard et al. (2013) and Kurmann and Otrok (2017), see Table 2 in the Appendix. The parameters of the shock processes are estimated, see Appendix and Table 3.

As mentioned above, the news shock and its IRFs are taken as our target for the validation exercise.

2.4.1 Large samples

We first assess the performance of the DFM in large samples. First, from the DSGE model we generate all states and stationary endogenous variables. TFP is taken in first differences. The shocks are Gaussian i.i.d. with variance equal to the variance resulting from the estimates of the DSGE model (see Appendix). To estimate the factor model, we need a large number of time series. For this purpose, we generate additional series. Each new series is constructed as a linear combination of the stationary model variables (states and endogenous variables) with coefficients randomly drawn from a uniform distribution with support between -1 and 1. We then add an i.i.d. Gaussian measurement error ξ_{it} to each variable χ_{it}. The measurement errors are independent of each other and independent of the variables. We scale the size of measurement errors in such a way that the ratio $k_i = \mathrm{Var}(\chi_{it})/\mathrm{Var}(x_{it})$, the fraction of the variance of the observed stationary series accounted for by the true series, is $k_i = 0.9$. We apply this procedure to get 100 artificial datasets of length $T = 5000$ and cross-sectional dimension $N = 2000$.

To identify the news shock we impose the following restrictions: (a) the surprise shock is the only shock having a non-zero impact effect on TFP, implying that the news shock has a zero impact effect on TFP; (b) the news and the surprise shocks are the only shocks affecting TFP 40 periods after the shock.[5] These restrictions are

[5] Using longer horizons for identification does not change the results.

sufficient to identify both technology shocks and are fully consistent with the model. We normalize the size of the shock by imposing that the estimated and the theoretical responses of TFP at horizon 40 are equal.

The test by Alessi et al. (2010), henceforth ABC, indicates 15 static factors in all datasets. So, we set $r = 15$. We also set $q = 7$, the true number of common shocks. We include 4 lags in the VAR for the factors, $p_F = 4$. With this parameter specification, we estimate the factors using the first 15 principal components, identify the shock as described above and estimate the IRFs for all artificial data sets.

Figure 1 plots the results of the simulation. Between the blue dashed lines lies the 68% of the empirical distribution of the IRFs estimated with the DFM, the blue solid line being the theoretical IRF. The DFM performs extremely well, with the theoretical IRFs always laying within the confidence bands.[6]

2.4.2 Small samples

Turning to small samples, we set $T = 235$ and $N = 228$, which are the time-series and cross-sectional dimensions of the dataset we will use in our empirical application. The data are generated as above, but now for each dataset, we use the ABC test for the number of static factors and the Akaike Information Criterion (AIC) to select the number of lags in the VAR for the factors. As for the number of dynamic factors, again we stick to the true number of factors $q = 7$.

Figure 2 reports the results. Again, the blue dashed lines include the 68% of the empirical distribution of the IRFs estimated with the DFM, the blue solid line is the theoretical IRF. The DFM performs well as before, with the theoretical IRFs always laying within the confidence bands. Of course, the bands are much wider than the large sample case.

3 Validation through SVAR models

As mentioned in the introduction, common practice in existing studies is to use SVAR to complement DSGE analysis. In this section we spend some words to warn about the potential danger of this practice arising from lack of information of the SVAR specification and the presence of measurement error in the observable variables.

[6]Notice that the cumulated factors are cointegrated, since we have 15 variables driven by 7 shocks. Estimating a VAR for the factors produces excellent results, as discussed in footnote 4.

3.1 Potential pitfalls with SVARs

Assume that the true model is (11) and the data $x_t = \chi_t$ are free of measurement error. Let $\tilde{x}_t$ be a subset of x_t used in a SVAR to estimate the shock of interest u_{it} and its impulse response functions. In order for the comparison between the theoretical IRFs of u_{it} with those obtained from a SVAR to be meaningful, it must be the case that the structural shock and its impulse response functions can be obtained using the appropriate combination of the Wold residuals and the Wold impulse response functions of the VAR. In other words, $\tilde{x}_t$ has to be *informationally sufficient* (see Forni et al. (2019)) for u_{it}, or equivalently, u_{it} must be *fundamental* for $\tilde{x}_t$. When this is not the case, obviously the comparison between the SVAR and the theoretical IRF is meaningless.

Forni et al. (2019) show that $\tilde{x}_t$ is informationally sufficient for u_{it} when the unexplained-variance ratio in the regression of u_{it} on the model-implied Wold innovations, δ_i, is zero (exact sufficiency) or close to zero (approximate sufficiency). So, when δ_i is close to zero, then one can use the SVAR estimated responses to validate the DSGE. Furthermore, when δ_i is close to zero for all of the shocks in u_t, then the SVAR is able to recover all the elements in u_t. This case is equivalent to the poor man's condition of Fernández-Villaverde et al. (2007) being satisfied.

Unfortunately, the absence of measurement error is an unrealistic assumption for many macroeconomic aggregates, like prices or GDP. When some of the variables are affected by measurement error, the results in Forni et al. (2019) and Fernández-Villaverde et al. (2007) no longer hold. This means that there is no useful guide to understand which variables are informationally sufficient for the shock of interest.

A second consequence is that, when variables are measured with error, the impulse response functions obtained from the SVAR are biased, the error contaminating dynamically the effects of macroeconomic shocks. So, in presence of measurement error, even the responses obtained from a SVAR which would be informationally sufficient absent the measurement error, would deliver wrong IRFs.

The elementary example below is sufficient to give an idea of the consequences of measurement errors for SVAR analysis. Let the economic model consist of only one variable χ_t, one unit-variance structural shock u_t and the structural equation

$$\chi_t = (2.5 + 1.2\,L)u_t \tag{17}$$

Suppose that χ_t is measured with an error η_t, which is a white noise process with $\sigma^2_\eta = 2.31$, orthogonal to the white noise u_t at all leads and lags, so that we observe

$$x_t = (2.5 + 1.2\,L)u_t + \xi_t. \tag{18}$$

It is easily seen that

$$x_t = (3 + L)v_t, \tag{19}$$

where v_t is a unit-variance white noise. The question is: what is v_t? For example, if χ_t is the rate of change of productivity and u_t the technology shock, can we say that v_t is just $u_t + e\omega_t$ for some e, so that we can claim that, after all, v_t is the technology shock plus a measurement error? The answer is an emphatic no. From (18) and (19) we obtain

$$v_t = \frac{2.5 + 1.2\,L}{3 + L} u_t + \frac{1}{3 + L} \xi_t. \tag{20}$$

Thus v_t is a moving average including all past values of u_t and ξ_t, not a combination of their current values only. The situation is much worse in multivariate models. Lippi (2020) shows that, in general, the structural shocks estimated with the SVAR are not only contaminated dynamically by the measurement error, they are also contaminated by other structural shocks. For instance, what is supposedly estimated to be a technology shock, turns actually out to be a dynamic combination of the technology shock, the measurement error and other non-technology shocks.

3.2 Simulations

We use few simulations to illustrate the problems discussed above. We use the same DSGE model as in the previous section and the same identificantion conditions.

3.2.1 Large Samples

Simulation 1 - The first simulation exercise assesses the ability of SVARs to recover the true impulse response functions in the most favorable case: informational sufficiency and no measurement errors. Using the theoretical model, we generate 100 different datasets with $T = 5000$ observations. Shocks are drawn from a Normal distribution, with zero mean and variance equal to the estimated one. For each dataset we estimate a VAR with 12 lags including the following seven variables: TFP, GDP, consumption, investment, hours worked, inflation, interest rate. We follow the prevailing practice in empirical VAR analysis and estimate the VAR in levels. Using these variables in the DSGE model, the measure of informational sufficiency for the news shock is $\delta = 0.0096$, so that the information in the 7-dimensional vector used in the VAR is sufficient to recover both the news shock and the corresponding IRFs. In Figure 3 we report the 68% bands of the empirical distribution of the estimated impulse responses (dashed lines), together with the true ones (solid lines). As expected, the VAR does extremely well. The theoretical responses always lie within the bands, which are extremely tight, owing to the large sample size.

Simulation 2 - In the second simulation, we use the same VAR specification but we add measurement errors. Let $\tilde{\chi}_{it}$, $i = 1, \ldots, 7$, be the 7 variables used in the previous simulation, and let $\chi_{it} = \tilde{\chi}_{it}$ if $\tilde{\chi}_{it}$ is I(0), $\chi_{it} = \tilde{\chi}_{it} - \tilde{\chi}_{i,t-1}$ if $\tilde{\chi}_{it}$ is I(1). We add a stationary measurement error to each variable χ_{it}: $x_{it} = \chi_{it} + \xi_{it}$ and cumulate those series entering the VAR in levels. Thus, the measurement error is I(0) for variables I(0) and I(1) for variables I(1). We scale the size of the measurement error by the ratio $k_i = \frac{Var(\chi_{it})}{Var(x_{it})} = 0.9$. As in the previous section, we generate 100 dataset of length $T = 5000$. A VAR with 12 lags is estimated and the news shock is identified as before. Figure 4 reports the 68% bands of the empirical distribution of the estimated IRFs (dashed lines), together with the true ones (solid lines). The figure clearly shows that there are important biases in the estimated IRFs, the true responses laying outside the bands in many cases. The bias is particularly severe for hours worked at horizons 0 to 5 periods, and for the interest rate on impact.

Simulation 3 - We now assess the implications of information insufficiency for the estimated SVARs. To do so, we consider a specification including TFP and investment without measurement error. The specification is chosen only for illustrative purposes. With this 2-dimensional vector, the informational sufficiency measure for the news shock is $\delta = 0.96$, meaning that a VAR with TFP and investment provides virtually no information to recover the news shock. The simulations are identical to the previous ones: 100 datasets of length $T = 5000$ and the same scheme to identify the news shock. Figure 5 reports the 68% bands of the empirical distribution of the estimated IRFs (dashed lines), together with the true ones (solid lines). The bias is huge and the VAR responses are totally misleading. In particular, the VAR largely overestimates the response of investment to the news shock. As for TFP, we observe the reverse: the SVAR underestimate the true response although the bias is less severe than for investment. When both problems are present (results not shown here), the results are even worse.

3.3 Small Samples

We repeat simulations 2 and 3 using small samples, i.e. by setting $T = 235$ and $N = 228$ and report the results in Figures 6 and 7. Again we show the 68% bands of the empirical distribution of the estimated impulse responses (dashed red lines), together with the true ones (solid lines). For the sake of comparison we also add the 68% bands of the empirical distribution obtained from the DFM (dashed blue lines).

The distortions observed in large samples for the SVAR are present also in small samples. In particular, in the 7-variable VAR all real economic activity variables are overestimated. Similarly, In the 2-variable model, the responses of TFP and investment are largely overestimated.

4 Application: Technology shocks in the New Keynesian model

In this Section we validate the transmission mechanism of TFP news and surprise shocks of the New Keynesian model discussed before. Kurmann and Otrok (2017) show that the the New Keynesian model has an hard time in replicating the impulse response functions of inflation and interest rate obtained in a SVAR model. Here we revisit the application through the lenses of a factor model and we extend the analysis to consider also the surprise technology shock.

The TFP process is the one presented in equations (14)-(16) and it is set in order to match the empirical impulse response functions of the factor model, giving the model the best chance to fit the data.

4.1 Data and number of factors

We consider the US quarterly dataset of Ng and McCracken (2016) over the sample 1960:Q1-2019:Q2 and add the TFP variables from Fernald (2012), that are needed to identify the shocks. In total we have 228 series. All series have been transformed to reach stationarity.

We start by testing for the number of static and dynamic factors. The ABC test for the number of static factors delivers $r = 5$ or $r = 10$. The log version of the test by Hallin and Liška (2007) for the number of dynamic shocks q, delivers $q = 6$. In the baseline specification therefore we set $r = 10$ and $q = 6$. We use four lags in the VAR for the factors, $p_F = 4$.

4.2 Results

The factor model is estimated using the standard FGLR procedure described above. We identify the news and surprise shocks using the same restrictions used above which for clarity we report here. (a) The surprise shock is the only shock having a non-zero impact effect on TFP, therefore the news shock has a zero impact effect on TFP. (b) The news and the surprise shocks are the only shocks affecting TFP 40 periods after the shock. These restrictions are implied by the model.

Figures 8 and 9 report the results for the news shock. Black solid lines represent the point estimates of the factor model, gray areas are the 68% confidence bands, purple circled lines are the theoretical responses implied by the DSGE. For the news shock, we normalize the size of the shock by imposing that the response of TFP at horizon 40 is the same in the two models, and equal to the empirical one. For the surprise shock, we normalize the responses by imposing that the impact of TFP is the same in the two models and equal to the empirical one.

As far as the news shocks is concerned, the theoretical responses of real economic activity variables are quite in line with the empirical ones. The model slightly over-predicts the effects on consumption, but for GDP, investment and hours the responses are similar both in terms of shape and magnitude.

The model, consistently with the data, is able to generate the so-called news driven business cycle. TFP is unchanged for the first four periods, slowly increasing afterwards, but investment, consumption, GDP and hours immediately increase. Agents anticipate future increase in TFP and this, due to the presence of habit persistence, investment adjustment costs and variable capacity utilization, move the four variables in the same direction, a necessary requirement for a shock generating cyclical fluctuations.

However the model does a poor job in terms of inflation and interest rate. Indeed, while in the data the shock generates a large drop in both variables, in the model the two variables barely react. The result confirms the finding in Kurmann and Otrok (2017), obtained using a SVAR, about the difficulty of the New Keynesian model in generating the observed dynamics of inflation following a news shock.

The theoretical dynamics of the surprise shock, see Figure 9, match pretty well those estimated in the factor model. Again, an inconsistency is represented by the larger response of consumption in the model. Particularly striking is the response of hours. The model captures well the change in the sign of the response, negative for the first quarters after the shock and then turning positive. Unlike for the news shock, here the response of inflation in the model is very similar to the empirical one.

We turn now the attention to the variance decomposition reported in Table 1. According to the DSGE model, news shocks are dominant for fluctuations in real variables while surprise shocks play a more modest role. The news shock explains 69% of GDP variance after eight quarters while the surprise shock only 15%. As in Blanchard et al. (2013) the shock explains the bulk of consumption fluctuations while it is less important for investment.

The picture is very different by inspecting the numbers obtained with the factor model. Here the role of the two shocks is much more balanced, each of the shock explaining around 30% of GDP fluctuations. Still, the news shock is more important than the surprise shock for fluctuations in consumption and investment, but quantitatively its role is more limited relative to the theoretical model. The news shock explains around half of the fluctuations in consumption and about 40% of the fluctuations in investment.

Summing up, while the theory does a quite good job in terms of surprise shock dynamics, it misses important features of the transmission mechanisms of news shock. First, the response of nominal variables are at odds with those observed in the data. Second, the model places too much importance on news shock.

4.3 SVAR results

For the sake of comparison, and to illustrate potential differences in the conclusions about model validity, we compare the model responses to those estimated in a SVAR.

Figures 10 and 11 plot the SVAR responses (point estimates dashed yellow lines and 68% confidence bands) and the model responses (lines with purple circles) for the news and surprise shock, respectively. Using the SVAR, one would conclude that the model substantially under-predicts the responses of real economic activity variables, in particular GDP consumption and investment, to a news shock.

But even more strikingly, the SVAR estimates the surprise shock to be temporary, instead of permanent. So, the SVAR evidence would suggest to specify a stationary process for the surprise shock. In addition, the responses of real activity variables to the surprise shock are much smaller than in the model, suggesting that the model does a poor job in matching the dynamics of the surprise shock. The result is particularly interesting in that it illustrates how conclusions can be radically different using factor models or SVARs.

As for the variance decomposition (Table 1, bottom panel), the SVAR attributes a disproportionally large role to to news shocks for business cycle fluctuations and a very small role to surprise technology shocks. At an horizon of 8 quarters, the news shock explains around 60% and 74% of the variance of GDP and consumption respectively, while the surprise shock explains only around 8% of the variance of the two variables. The former result is in line with DSGE predictions (but in contrast to what found with the DFM).

5 Conclusions

There is, by now, a widespread agreement about the importance of using time series models to validate and to guide the construction of DSGE models. The reason is that time series models are much less restricted than DSGE models and the comparison of the empirical and theoretical impulse response functions might provide useful information about the empirical support of the transmission mechanisms embedded in the DSGE. To do so, the literature has largely relied on SVARs models.

In this paper, we argue that Dynamic Factor Models are better suited to be the empirical counterpart of the DSGE model. The most compelling reason is that DSGE models have a factor model structure. This ensures consistency between the theoretical model and the empirical one. In addition, factor models are not subject to two major problems: the information set misspecification, i.e. what variables to put in the VAR, and measurement errors. The two can create important distortions in the estimation of impulse response functions. This means that there is the risk of rejecting the model using SVARs even when the model is true. We show this using a set of simulations.

As an application, we test a theory of news shocks in a medium-scale DSGE model. We show that the model qualitatively matches the responses quite well, although quantitatively tend to overestimate the empirical responses of GDP, consumption and hours worked to the news shock.

References

Alessi, L., M. Barigozzi, and M. Capasso (2010). Improved penalization for determining the number of factors in approximate static factor models. *Statistics and Probability Letters 80*, 1806–1813.

Bai, J. and S. Ng (2002). Determining the number of factors in approximate factor models. *Econometrica 70*, 191–221.

Beaudry, P. and F. Portier (2006). Stock prices, news, and economic fluctuations. *American Economic Review 96*, 1293–1307.

Blanchard, O. J., G. Lorenzoni, and J. L'Huillier (2013). News, noise, and fluctuations: An empirical exploration. *American Economic Review 103*, 3045–70.

Canova, F. (2007). *Methods for applied Macroeconomics.* Princeton, New Jersey: Princeton University Press.

Canova, F. and M. Paustian (2011). Business cycle measurement with some theory. *Journal of Monetary Economics 58*(4), 345–361.

Christiano, L. J., M. Eichenbaum, and R. Vigfusson (2007). Assessing Structural VARs. In *NBER Macroeconomics Annual 2006, Volume 21*, pp. 1–106.

Fernald, J. (2012). A quarterly, utilization-adjusted series on total factor productivity. *Federal Reserve Bank of San Francisco WP Series 2012-19*.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, T. J. Sargent, and M. W. Watson (2007). ABCs (and Ds) of understanding VARs. *American Economic Review 97*, 1021–1026.

Forni, M., L. Gambetti, M. Lippi, and L. Sala (2020). Common components structural VARs. *CEPR Working Paper No. 15529*.

Forni, M., L. Gambetti, and L. Sala (2019). Structural VARs and noninvertible macroeconomic models. *Journal of Applied Econometrics 34*(2), 221–246.

Forni, M., D. Giannone, M. Lippi, and L. Reichlin (2009). Opening the black box: Structural factor models versus structural VARs. *Econometric Theory 25*, 1319–1347.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2000). The Generalized Dynamic Factor Model: Identification and estimation. *The Review of Economics and Statistics 82*, 540–554.

Galì, J. (1999). Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *The American Economic Review 199*, 249–271.

Guerron-Quintana, P., A. Inoue, and L. Kilian (2017). Impulse response matching estimators for DSGE models. *Journal of Econometrics 196*(1), 144–155.

Hallin, M. and R. Liška (2007). Determining the number of factors in the general dynamic factor model. *Journal of the American Statistical Association 102*, 603–617.

Kurmann, A. and C. Otrok (2017). News shocks and inflation: Lessons for new keynesians. Working paper.

Lippi, M. (2020). Validating DSGE models with SVARs and High-Dimensional Dynamic Factor Models. Working paper. Einaudi Institute for Economics and Finance, Rome.

Ng, S. and M. McCracken (2016). FRED-MD: a monthly database for macroeconomic research. *Journal of Business and Economic Statistics*, 574–589.

Rozanov, Y. A. (1967). *Stationary Random Processes.* San Francisco: Holden Day.

Stock, J. H. and M. W. Watson (2002a). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association 97*, 1167–1179.

Stock, J. H. and M. W. Watson (2002b). Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics 20*, 147–162.

Tables

DSGE

Variables	News				Surprise			
DSGE								
	h=0	h=4	h=8	h=40	h=0	h=4	h=8	h=40
TFP	0.00	0.32	7.90	71.88	100.00	99.68	92.1	28.12
GDP	30.94	57.99	67.58	82.98	14.37	17.53	18.05	12.96
Consumption	77.12	80.24	82.22	87.25	13.6	14.55	14.46	11.36
Investment	9.63	23.02	38.73	71.68	2.76	7.12	11.55	12.07
Hours	30.56	69.44	78.60	83.14	15.45	1.95	2.37	2.33
Inflation	1.14	2.42	2.23	6.26	8.31	16.37	18.01	11.01
Interest rate	0.12	0.72	1.09	2.68	0.76	3.49	5.72	5.69
Factor model								
Variables	News				Surprise			
	h=0	h=4	h=8	h=40	h=0	h=4	h=8	h=40
TFP	0.00	4.19	3.61	36.59	77.77	67.08	65.95	38.98
GDP	4.80	10.79	29.83	33.24	43.90	35.73	37.82	32.58
Consumption	11.39	42.87	51.49	47.92	10.81	13.56	12.57	8.67
Investment	0.39	23.14	43.00	51.72	6.07	11.99	15.36	19.89
Hours	3.05	17.60	24.51	30.03	4.47	8.42	9.37	8.82
Inflation	55.44	61.71	59.92	46.90	15.01	14.89	13.20	10.18
Interest rate	44.66	34.65	28.49	31.46	0.40	5.97	4.88	6.10
VAR								
Variables	News				Surprise			
	h=0	h=4	h=8	h=40	h=0	h=4	h=8	h=40
TFP	0.00	0.39	0.40	37.18	100.00	90.55	86.42	43.38
GDP	7.18	39.03	59.54	80.54	6.57	7.51	8.74	4.16
Consumption	54.51	66.51	74.08	82.92	2.32	6.71	7.15	2.10
Investment	6.65	30.48	43.73	63.69	0.23	4.32	5.73	4.12
Hours	0.04	13.72	29.72	33.90	17.47	4.10	2.62	5.25
Inflation	26.16	38.62	39.16	30.55	2.70	2.78	6.61	18.05
Interest rate	10.63	8.01	5.96	10.35	3.27	3.10	3.18	10.40

Table 1: Variance decomposition at horizon h.

Figures

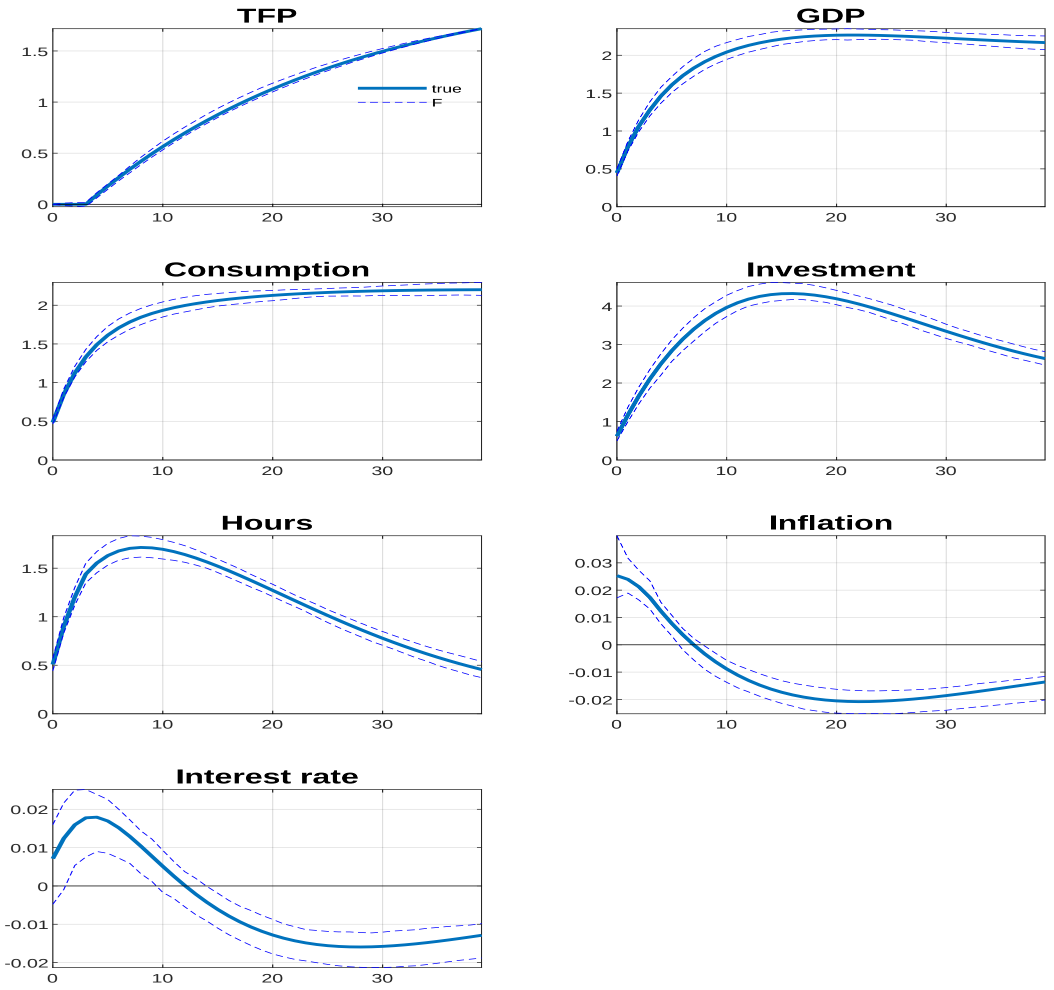

Figure 1: Identifying a news shock - Factor model with $r = 15$, $q = 7$, $p_F = 4$. Bold line: true response. Dashed lines: 68% percentiles of the distribution of 100 simulations. $N = 2000$, $T = 5000$.

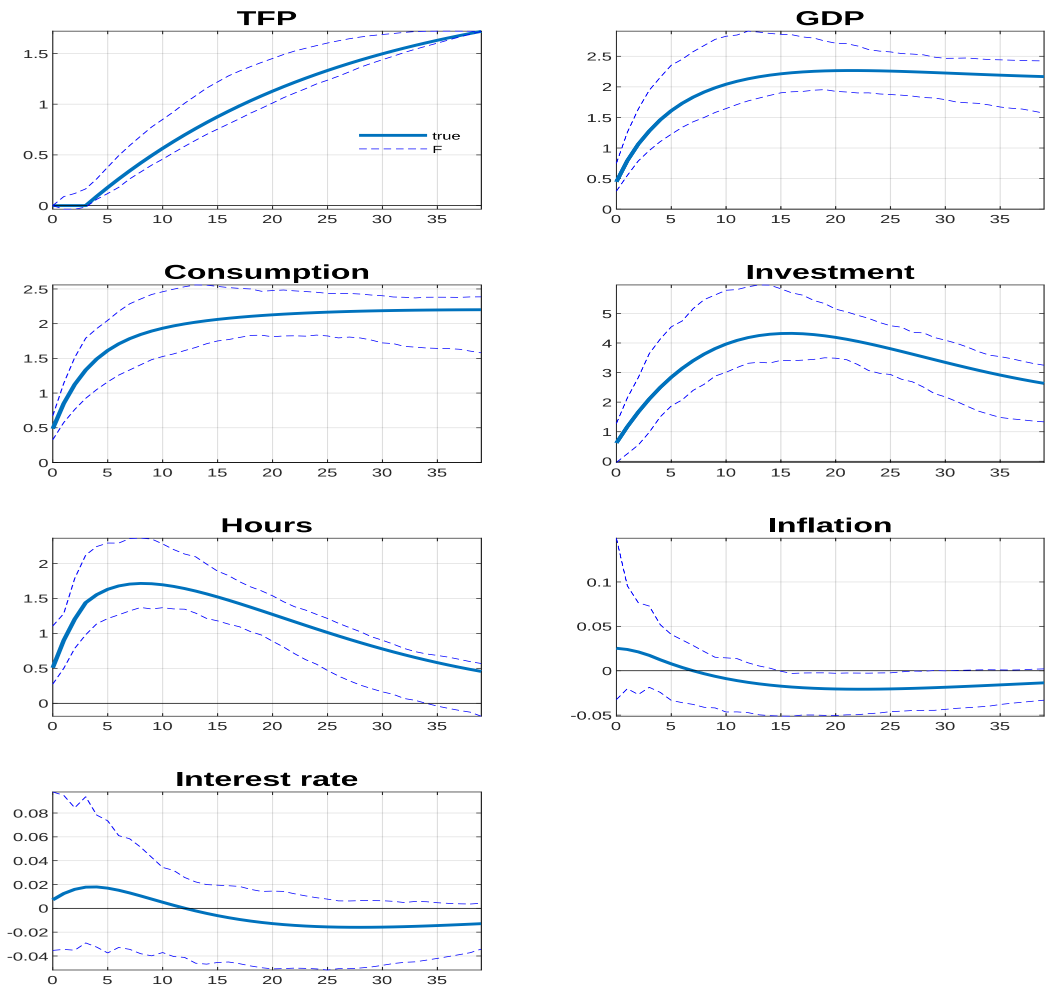

Figure 2: Identifying a news shock - Factor model. Bold line: true response. Dashed blue lines: 68% percentiles of the distribution of 100 simulations, DFM. The DFM is estimated with $q = 7$, r chosen with ABC test, p_F chosen with Akaike. Sample: $T = 235$. $k = 0.9$.

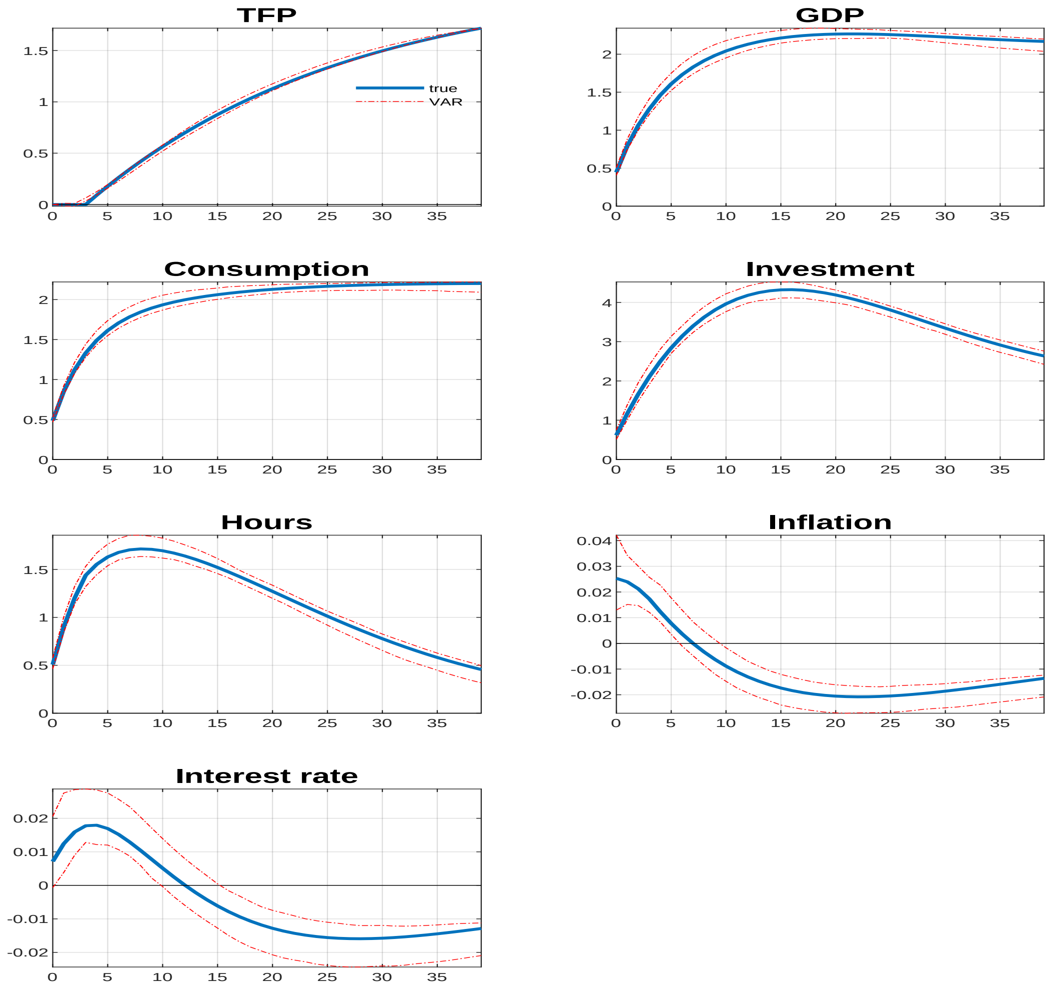

Figure 3: Identifying a news shock. 7-dimensional VAR ($\delta = 0.0086$) with 12 lags, no measurement error. Bold line: true response. Dashed lines: 68% percentiles of the distribution of 100 simulations. $T = 5000$.

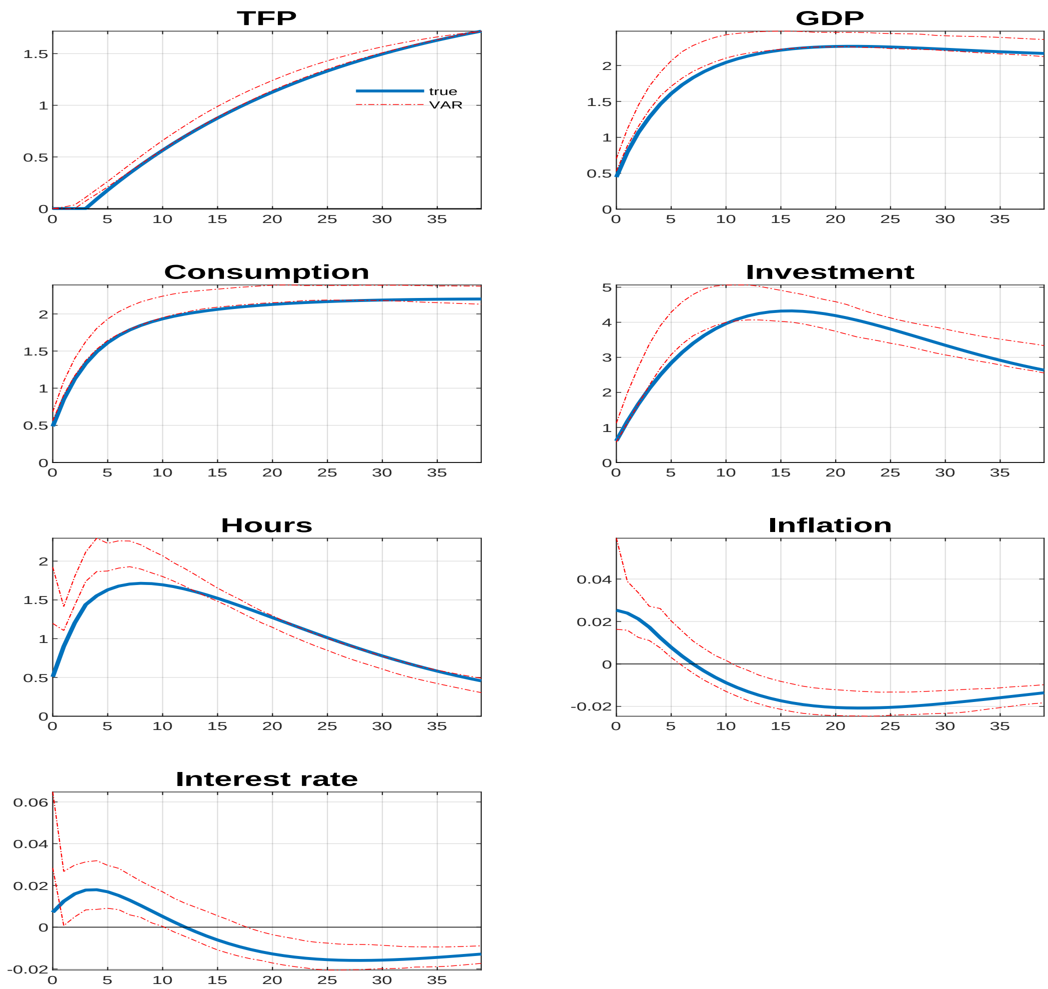

Figure 4: Identifying a news shock. 7-dimensional VAR ($\delta = 0.0086$) with 12 lags, measurement error: $k = 0.9$. Bold line: true response. Dashed lines: 68% percentiles of the distribution of 100 simulations. $T = 5000$.

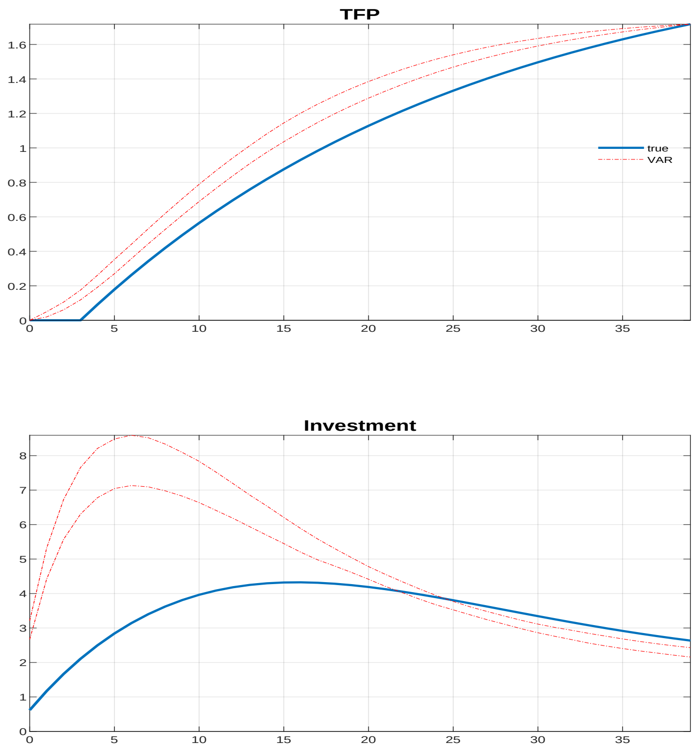

Figure 5: Identifying a news shock. 2-dimensional VAR ($\delta = 0.96$) with 12 lags, no measurement error. Bold line: true response. Dashed lines: 68% percentiles of the distribution of 100 simulations. $T = 5000$.

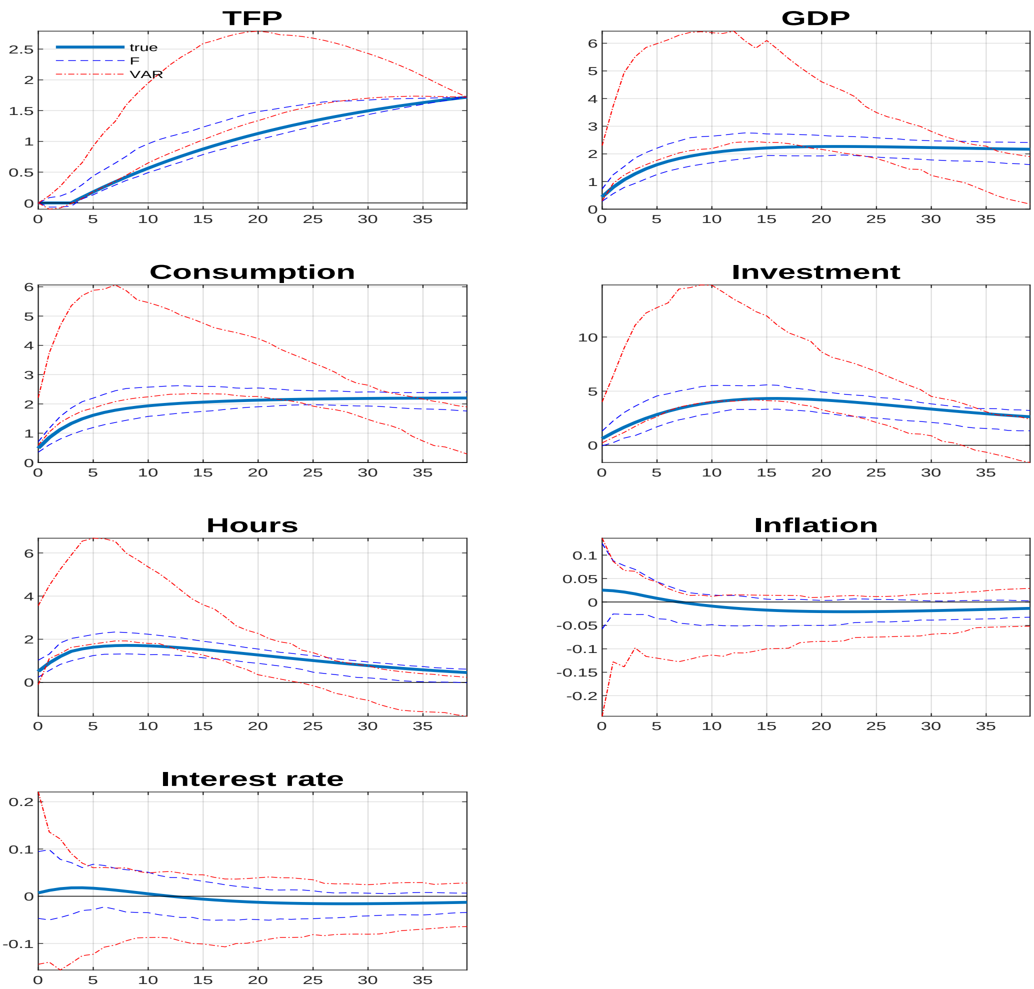

Figure 6: Identifying a news shock - Factor model vs 7-dimensional VAR ($\delta = 0.0086$). Bold line: true response. Dashed blue lines: 68% percentiles of the distribution of 100 simulations, DFM. The DFM is estimated with $q = 7$, r chosen with ABC test, p_F chosen with Akaike. Dotted red lines: 68% percentiles of the distribution of 100 simulations, VAR. The VAR is estimated with p_{VAR} chosen with Akaike. Sample: $T = 235$, $N = 228$. $k = 0.9$.

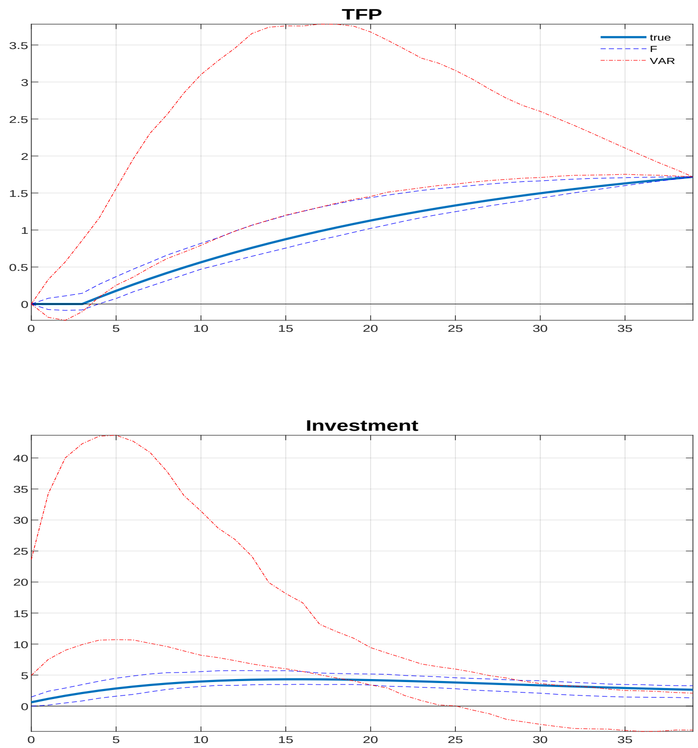

Figure 7: Identifying a news shock - Factor model vs bivariate VAR ($\delta = 0.96$). Bold line: true response. Dashed blue lines: 68% percentiles of the distribution of 100 simulations, DFM. The DFM is estimated with $q = 7$, r chosen with ABC test, p_F chosen with Akaike. Dotted red lines: 68% percentiles of the distribution of 100 simulations, VAR. The VAR is estimated with p_{VAR} chosen with Akaike. Sample: $T = 235$, $N = 228$. $k = 0.9$.

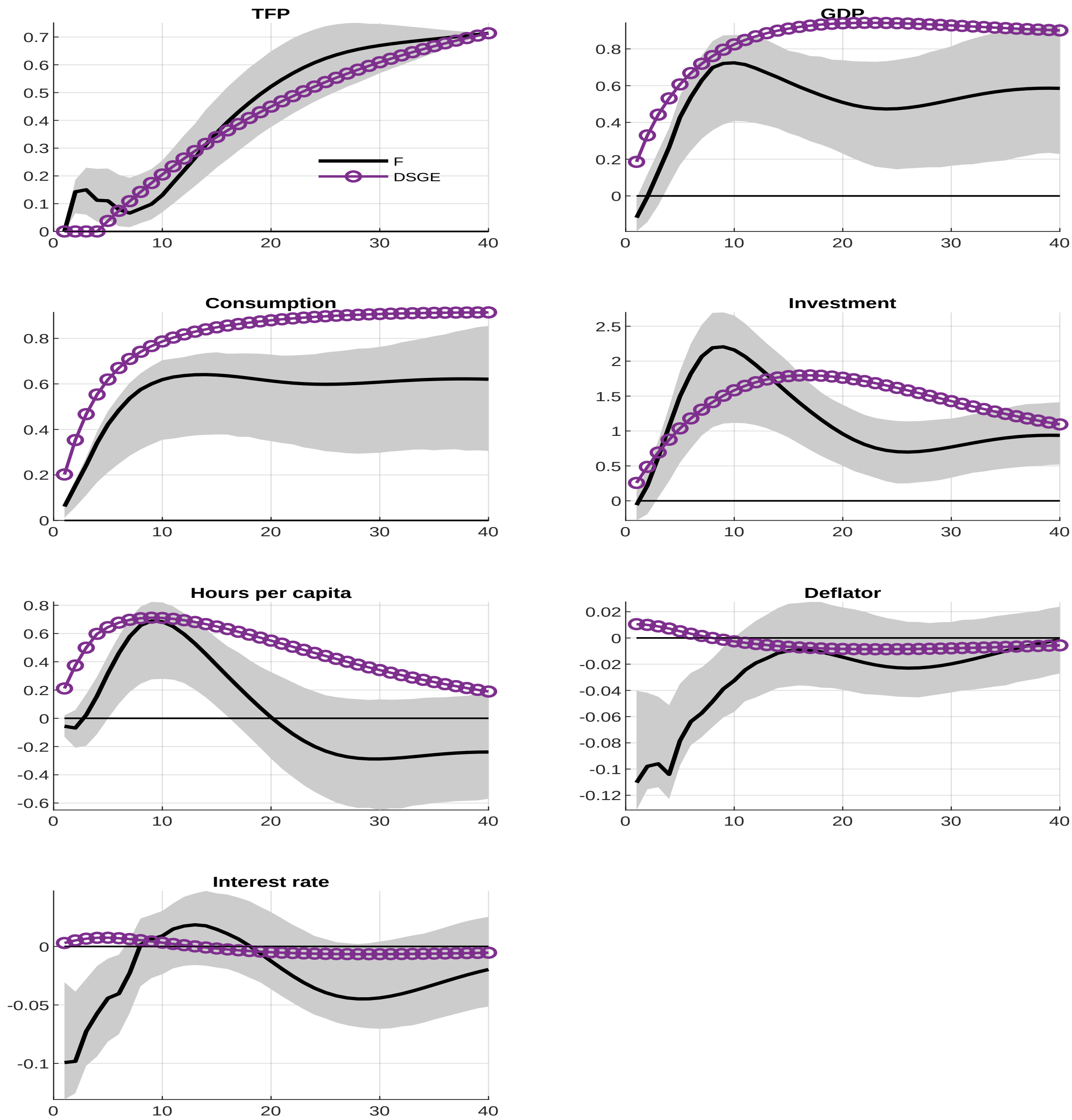

Figure 8: Identifying a news shock: DFM vs. DSGE. The DFM is estimated with $r = 10$, $q = 6$, $p_F = 4$. Bold line: response from the DFM. Gray area: 68% bootstrapped confidence intervals from the DFM.

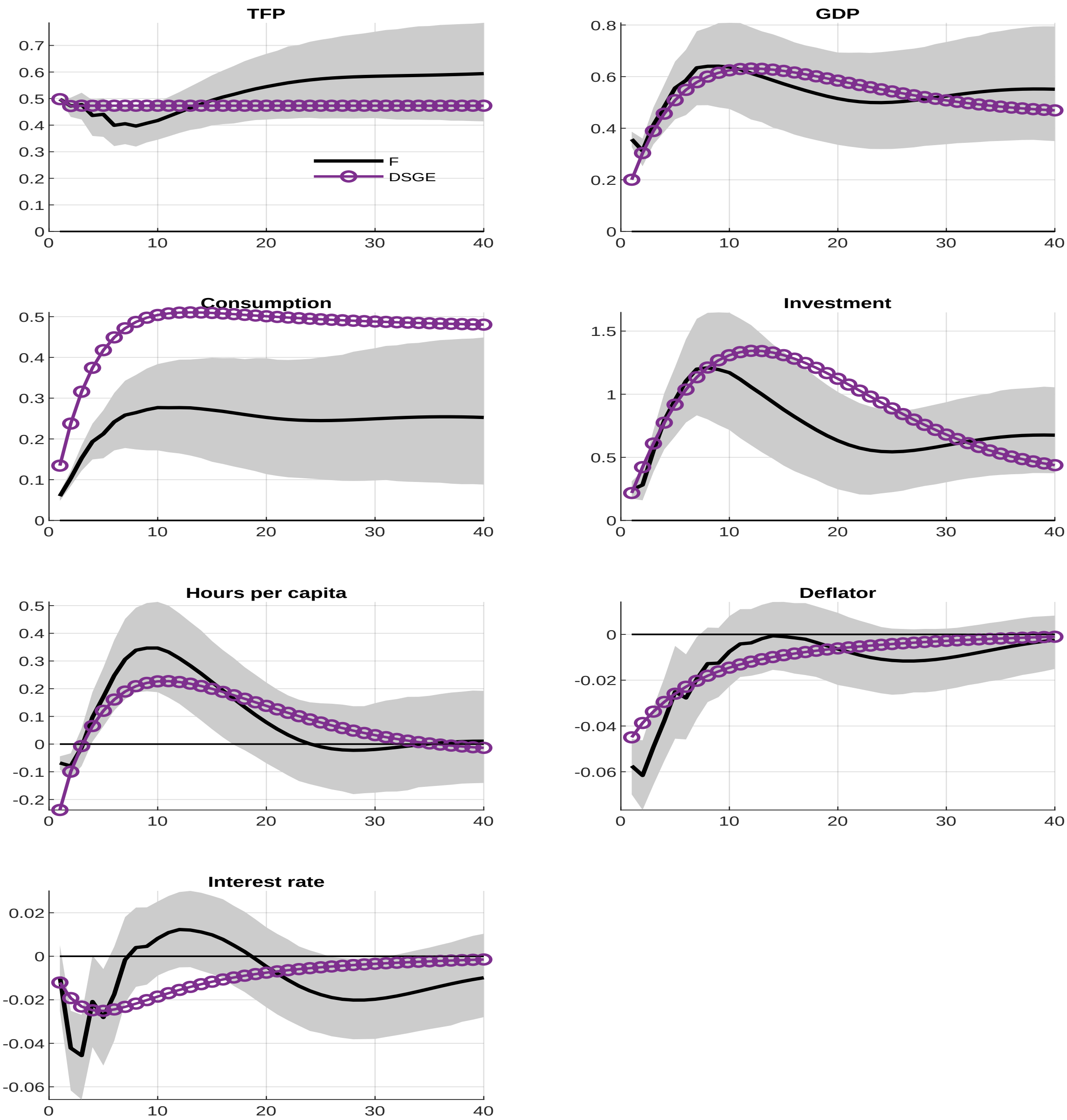

Figure 9: Identifying a surprise shock: DFM vs. DSGE. The DFM is estimated with $r = 10$, $q = 6$, $p_F = 4$. Bold line: response from the DFM. Gray area: 68% bootstrapped confidence intervals from the DFM.

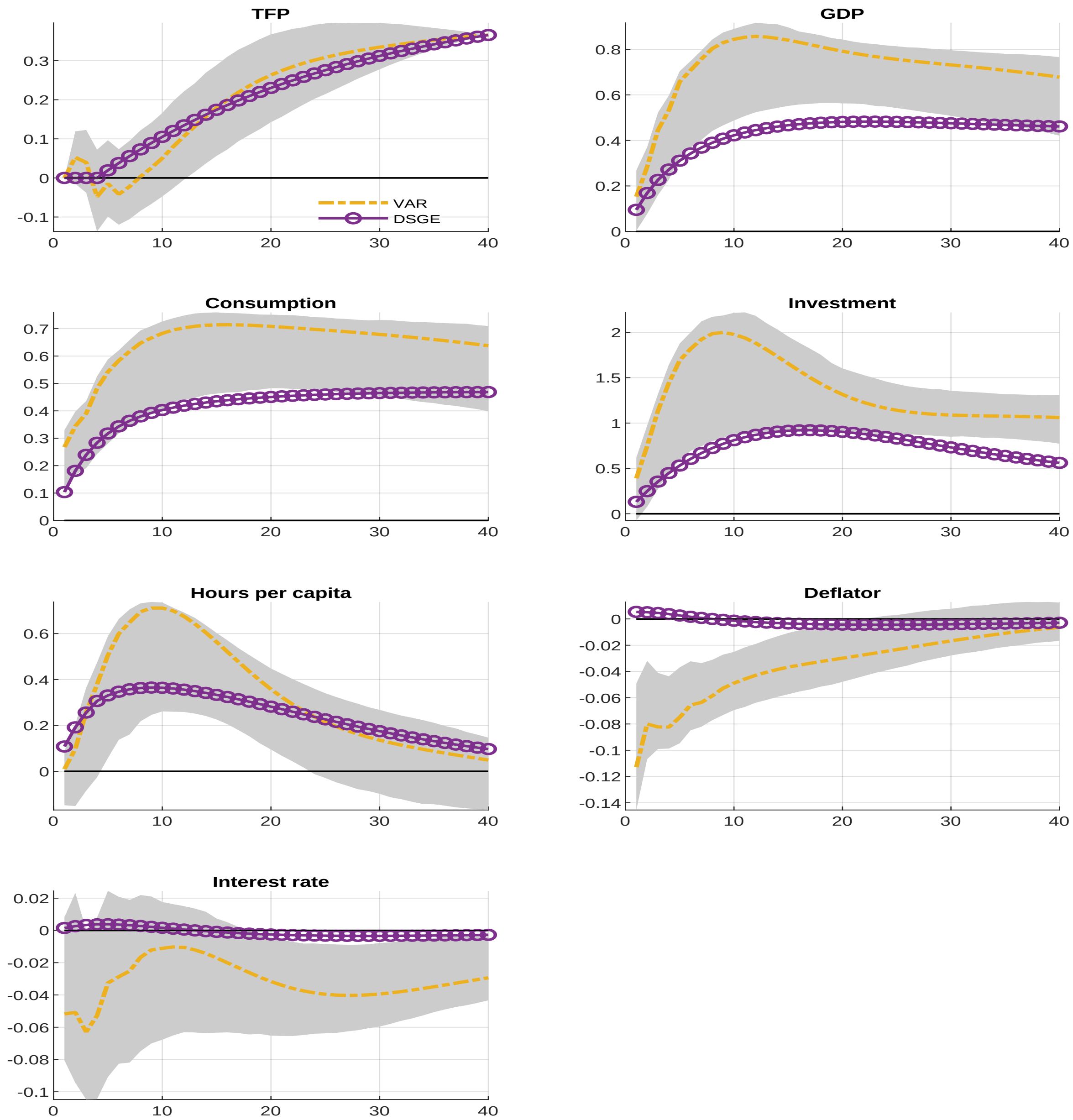

Figure 10: Identifying a news shock: VAR vs. DSGE. The VAR is estimated with $p_{VAR} = 4$. Bold line: response from the VAR. Gray area: 68% bootstrapped confidence intervals from the VAR.

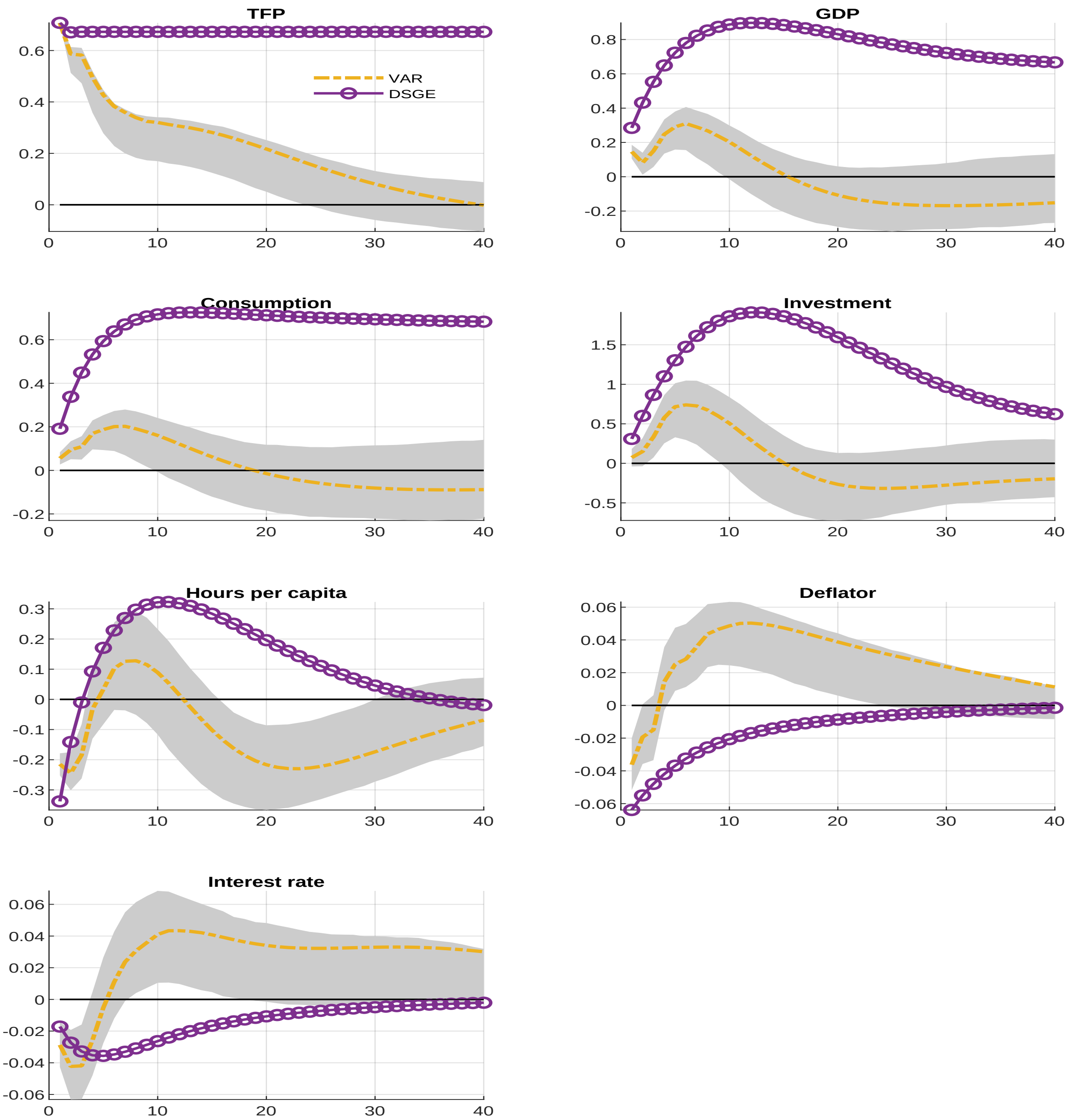

Figure 11: Identifying a surprise shock: VAR vs. DSGE. The VAR is estimated with $p_{VAR} = 4$. Bold line: response from the VAR. Gray area: 68% bootstrapped confidence intervals from the VAR.

Appendix

The model follows closely Blanchard et al. (2013). The representative household has the utility function:

$$E_t\left[\sum_{t=0}^{\infty}\beta^t\left(\log\left(C_t-hC_{t-1}\right)-\frac{1}{1+\varsigma}\int_0^1 N_{jt}^{1+\varsigma}dj\right)\right],$$

where C_t is consumption, the term hC_{t-1} captures internal habit formation, and N_{jt} is the supply of specialized labor of type j. The household budget constraint is

$$P_tC_t+P_tI_t+T_t+B_t+P_tC(U_t)\bar{K}_{t-1}=R_{t-1}B_{t-1}+Y_t+\int_0^1 W_{jt}N_{jt}dj+R_t^kK_t,$$

where P_t is the price level, T_t is a lump sum tax, B_t are holdings of one period bonds, R_t is the one period nominal interest rate, Y_t are aggregate profits, W_{jt} is the wage of specialized labor of type j, N_{jt}. R_t^k is the capital rental rate.

Households choose consumption, bond holdings, capital utilization, and investment each period so as to maximize their expected utility subject to the budget constraint and a standard no-Ponzi condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires $B_t=0$.

The capital stock $\bar{K}_t$ is owned and rented by the representative household and the capital accumulation equation is

$$\bar{K}_t=(1-\delta)\bar{K}_{t-1}+D_t\left[1-G(I_t/I_{t-1})\right]I_t,$$

where δ is the depreciation rate, D_t is a stochastic investment-specific technology parameter, and G is a quadratic adjustment cost in investment

$$G(I_t/I_{t-1})=\chi(I_t/I_{t-1}-\Gamma)^2/2,$$

where Γ is the long-run gross growth rate of TFP. The model features variable capacity utilization: the capital services supplied by the capital stock $\bar{K}_{t-1}$ are $K_t=U_t\bar{K}_{t-1}$, where U_t is the degree of capital utilization and the cost of capacity utilization, in terms of current production, is $C(U_t)\bar{K}_{t-1}$, where $C(U_t)=U_t^{1+\zeta}/(1+\zeta)$.

The investment-specific shock $d_t=\log D_t$ follows the stochastic process:

$$d_t=\rho_d d_{t-1}+\varepsilon_{dt}.$$

ε_{dt} and all the variables denoted with ε from now on are i.i.d. shocks.

Consumption and investment are in terms of a final good which is produced by competitive final good producers using the CES production function

$$Y_t=\left(\int_0^1 Y_{jt}^{\frac{1}{1+\mu_{pt}}}dj\right)^{1+\mu_{pt}}$$

which employs a continuum of intermediate inputs. Y_{jt} is the quantity of input j employed and μ_{pt} captures a time-varying elasticity of substitution across goods, where $\log(1+\mu_{pt}) = \log(1+\mu_p) + m_{pt}$ and m_{pt} follows the process $m_{pt} = \rho_p m_{pt-1} + \varepsilon_{pt} - \psi_p \varepsilon_{pt-1}$.

The production function for intermediate good j is

$$Y_{jt} = (K_{jt})^{\alpha} (A_t L_{jt})^{1-\alpha},$$

where K_{jt} and L_{jt} are, respectively, capital and labor services employed. The technology parameter $a_t = \log(A_t)$ follows the process

$$a_t = a_{t-1} + P_t + T_t \tag{21}$$

$$P_t = \phi P_{t-1} + \epsilon_{t-4} \tag{22}$$

$$T_t = \rho T_{t-1} + \eta_t \tag{23}$$

where P_t is the part of TFP driven by the news shock ϵ_{t-4} and T_t is the part of TFP driven by the surprise shock η_t.

Blanchard et al. (2013) treat explicitly the constant term in TFP growth by letting $A_t = \Gamma^t e^{a_t}$, but calibrate $\Gamma = 1$.

Intermediate good prices are sticky with price adjustment as in Calvo, 1983. Each period intermediate good firm j can freely set the nominal price P_{jt} with probability $1-\theta_p$ and with probability θ_p is forced to keep it equal to P_{jt-1}. These events are purely idiosyncratic, so θ_p is also the fraction of firms adjusting prices each period.

Labor services are supplied to intermediate good producers by competitive labor agencies that combine specialized labor of types in $[0,1]$ using the technology

$$N_t = \left[\int_0^1 N_{jt}^{\frac{1}{1+\mu_{wt}}} dj\right]^{1+\mu_{wt}},$$

where $\log(1+\mu_{wt}) = \log(1+\mu_w) + m_{wt}$ and m_{wt} follows the process $m_{wt} = \rho_w m_{wt-1} + \varepsilon_{wt} - \psi_w \varepsilon_{wt-1}$.

The presence of differentiated labor introduces monopolistic competition in wage setting as in Erceg, Henderson and Levin, 2000. Specialized labor wages are also sticky and set by the household. For each type of labor j, the household can freely set the price W_{jt} with probability $1-\theta_w$ and has to keep it equal to W_{jt-1} with probability θ_w.

Market clearing in the final good market requires

$$C_t + I_t + C(U_t)\bar{K}_{t-1} + G_t = Y_t.$$

Market clearing in the market for labor services requires $\int L_{jt} dj = N_t$.

Government spending is set as a fraction of output and the ratio of government spending to output is $G_t/Y_t = \psi + g_t$, where g_t follows the stochastic process

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt}.$$

Monetary policy follows the interest rate rule

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\left(\gamma_\pi \pi_t + \gamma_y \hat{y}_t\right) + q_t,$$

where $r_t = \log R_t - \log R$ and $\pi_t = \log P_t - \log P_{t-1} - \pi$, π is the inflation target, $\hat{y}_t$ is defined below and q_t follows the process

$$q_t = \rho_q q_{t-1} + \varepsilon_{qt}.$$

The model is solved and a log-linear approximation around a deterministic steady-state is computed.

Given that TFP is non-stationary, some variables need to be normalized to ensure stationarity. We define $\hat{c}_t$ as

$$\hat{c}_t = \log(C_t/A_t) - \log(C/A),$$

where C/A denotes the value of C_t/A_t in the deterministic version of the model in which A_t grows at the constant growth rate Γ. Analogous definitions apply to the quantities $\hat{y}_t, \hat{k}_t, \hat{k}_t, \hat{\imath}_t$. The quantities N_t and U_t are already stationary, so $n_t = \log N_t - \log N$, and similarly for u_t. For nominal variables, it is necessary to take care of non-stationarity in the price level, so: $\hat{w}_t = \log(W_t/(A_t P_t)) - \log(W/(AP)), r_t^k = \log(R_t^k/P_t) - \log(R^k/P), m_t = \log(M_t/P_t) - \log(M/P), r_t = \log R_t - \log R, \pi_t = log(P_t/P_{t-1}) - \pi$.

Finally, for the Lagrange multipliers: $\hat{\lambda}_t = \log(\Lambda_t A_t) - \log(\Lambda A), \hat{\phi}_t = \log(\Phi_t A_t/P_t) - \log(\Phi A/P)$. Φ_t is the Lagrange multiplier on the capital accumulation constraint. The hat is only used for variables normalized by A_t.

The first order conditions can be log-linearized to yield

$$\begin{aligned}\hat{\lambda}_t \quad = \quad & \frac{h\beta\Gamma}{(\Gamma - h\beta)(\Gamma - h)} E_t \hat{c}_{t+1} - \frac{\Gamma^2 + h^2\beta}{(\Gamma - h\beta)(\Gamma - h)} \hat{c}_t + \frac{h\Gamma}{(\Gamma - h\beta)(\Gamma - h)} \hat{c}_{t-1} + \\ & + \frac{h\beta\Gamma}{(\Gamma - h\beta)(\Gamma - h)} E_t[\Delta a_{t+1}] - \frac{h\Gamma}{(\Gamma - h\beta)(\Gamma - h)} \Delta a_t\end{aligned}$$

$$\hat{\lambda}_t = r_t + E_t[\hat{\lambda}_{t+1} - \Delta a_{t+1} - \pi_{t+1}]$$
$$\hat{\phi}_t = (1-\delta)\beta\Gamma^{-1}E_t[\hat{\phi}_{t+1} - \Delta a_{t+1}] + (1-(1-\delta)\beta\Gamma^{-1})E_t[\hat{\lambda}_{t+1} - \Delta a_{t+1} + r^k_{t+1}]$$
$$\hat{\lambda}_t = \hat{\phi}_t + d_t - \chi\Gamma^2\left(\hat{\imath}_t - \hat{\imath}_{t-1} + \Delta a_t\right) + \beta\chi\Gamma^2 E_t\left(\hat{\imath}_{t+1} - \hat{\imath}_t + \Delta a_{t+1}\right)$$
$$r^k_t = \zeta u_t$$
$$m_t = \alpha r^k_t + (1-\alpha)\hat{w}_t$$
$$r^k_t = \hat{w}_t - \hat{k}_t + n_t$$

Log-linearizing the accumulation equation for capital and the equation for capacity utilization, yields

$$\begin{aligned}
\hat{k}_t &= u_t + \hat{\bar{k}}_{t-1} - \Delta a_t \\
\hat{\bar{k}}_t &= (1-\delta)\Gamma^{-1}\left(\hat{\bar{k}}_t - \Delta a_t\right) + \left(1-(1-\delta)\Gamma^{-1}\right)d_t + \hat{\imath}_t.
\end{aligned}$$

Approximating and aggregating the intermediate goods production function over producers and using the final good production function yields

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)n_t$$

Market clearing in the final good market yields

$$(1-\psi)\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{\imath}_t + \frac{R^kK}{PY}u_t + g_t$$

C/Y, I/Y and $R^kK/(PY)$ are all equilibrium ratios in the deterministic version of the model in which A_t grows at the constant rate Γ.

Aggregating individual optimality conditions for price setters yields the Phillips curve

$$\pi_t = \beta E_t\pi_{t+1} + \kappa m_t + \kappa m_{pt}$$

where $\kappa = (1-\theta_p\beta)(1-\theta_p)/\theta_p$.

Finally, aggregating individual optimality conditions for wage setters yields

$$\begin{aligned}
\hat{w}_t = &\frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \frac{1}{1+\beta}(\pi_t + \Delta a_t) + \frac{\beta}{1+\beta}E_t(\pi_{t+1} + \Delta a_{t+1}) - \\
&-\kappa_w\left(\hat{w}_t - \zeta n_t + \hat{\lambda}_t + \kappa_w m_{wt}\right)
\end{aligned}$$

where $\kappa_w = \frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w(1+\beta)\left(1+\zeta\left(1+\frac{1}{\mu_w}\right)\right)}$.

The log-linear model is estimated using Bayesian methods. Some parameters were calibrated using the mean values estimated in Blanchard et al. (2013) and some using the values used in Kurmann and Otrok (2017). Table 2 reports the calibrated parameters.

Variables used in the estimation are the growth rates of output, consumption, investment and real wages, hours, the inflation rate and the federal funds rate. The choice of priors is very close to the one used in Blanchard et al. (2013). Exception is made for the AR coefficients of the shocks, assumed here to be Normal with mean equal to 0 and standard deviation equal to 0.5 (0.4 for the coefficient ρ related to the transitory technology component) and for σ_d assumed here to be distributed as an Inverse Gamma with mean equal to 5 and standard deviation equal to 1.5.

We use a simulated annealing procedure to obtain the mode of the posterior distribution. Table 3 summarizes the priors and the mode estimates of the parameters.[7]

[7]Results using the mean of the posterior distribution are equivalent to those using the mode.

Calibrated parameters		
ζ (elasticity of k utilization)	2.07	BLL
χ (I adj. cost)	5.5	BLL
h (habit persistence)	0.75	KO
ς (inverse Frish elast.)	3.98	BLL
θ_w (W stickiness)	0.87	BLL
θ_p (P stickiness)	0.88	BLL
γ_π (π in Taylor rule)	1.003	BLL
γ_y (Y gap in Taylor rule)	0.0044	BLL
μ_p (SS P markup)	0.3	BLL
μ_w (SS W markup)	0.05	BLL
α (coeff. in prod. function)	0.19	BLL
Γ (TFP growth)	1	BLL
ψ (G/Y)	0.22	BLL
δ (K depreciation)	0.025	BLL
β (discount factor)	0.99	BLL

Table 2: Calibrated parameters following results in Blanchard et al. (2013) (last column: BLL) or Kurmann and Otrok (2017) (last column: KO).

Estimated parameters		
Parameter	Prior	Mode
ρ_r (i smoothing)	$Beta(0.5, 0.2)$	0.74
ρ (technology surprise)	$\mathcal{N}(0.0\ , 0.4)$	-0.05
ϕ (technology news)	$Beta(0.5, 0.2)$	0.96
ρ_q (monetary)	$\mathcal{N}(0.0\ , 0.5)$	0.24
ρ_d (I specific)	$\mathcal{N}(0.0, 0.5)$	0.66
ρ_p (P markup)	$\mathcal{N}(0.0, 0.5)$	0.87
ρ_w (W markup)	$\mathcal{N}(0.0, 0.5)$	0.97
ρ_g (G)	$\mathcal{N}(0.0, 0.5)$	0.99
ψ_p (MA in P mkup)	$Beta(0.5, 0.2)$	0.63
ψ_w (MA in W mkup)	$Beta(0.5, 0.2)$	0.98
σ_ε (permanent tech.)	$I\Gamma(0.5, 1.0)$	0.09
σ_η (temporary tech.)	$I\Gamma(1.0, 1.0)$	0.76
σ_q (monetary)	$I\Gamma(0.15, 1.0)$	0.20
σ_d (I specific)	$I\Gamma(5.0, 1.5)$	4.91
σ_p (p markup)	$I\Gamma(0.15, 1.0)$	0.13
σ_w (w markup)	$I\Gamma(0.15, 1.0)$	0.37
σ_g (gov exp.)	$I\Gamma(0.5, 1.0)$	0.48
Posterior value at mode		-1324.64

Table 3: Parameter estimates - mode.