DISCUSSION PAPER SERIES

DP17368

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MACROECONOMICS AND GROWTH

CEPR

ISSN 0265-8003

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Discussion Paper DP17368
Published 07 June 2022
Submitted 06 June 2022

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www.cepr.org

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JEL Classification: O44, Q01, Q54

Keywords: Solow-Swan growth model, Baumol model, anthropogenic climate change, mitigation, price-driven economic growth

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June 6, 2022

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The existence of an environmental limit changes fundamentally the nature of economic growth. When atmospheric greenhouse gases reach a predetermined absolute threshold, further growth requires a permanently expanding, resource-intensive mitigation effort. We incorporate anthropogenic climate change and its mitigation into the Solow-Swan growth model and show that if the rate of technical progress in mitigation fails to exceed a critical value, the economy behaves as described by Baumol (1967). Economic growth in this regime is then driven by technological progress in mitigation and the dynamics of its relative price. Even in the extreme case that long-run growth of produced output converges to zero, the growth rate of GDP measured in terms of produced goods does not.

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1 Introduction

The Solow-Swan growth model (Solow, 1956; Swan, 1956) continues to serve as the workhorse of long-run macroeconomic analysis for students and researchers alike. It provides immediate intuition for the role of simple neoclassical principles in long-run economic growth and highlights important limitations of the framework. Yet recent concerns about climate change and an upper bound on the level of atmospheric greenhouse gases consistent with planetary survival raise fundamental doubts about the usefulness of the neoclassical growth model (Dasgupta, 2021). Such concerns were first raised by the "limits to growth" literature a half-century ago (Forrester, 1971; Meadows et al., 1972) but were widely dismissed as alarmist (Solow, 1973) and are still ignored in the benchmark Solow-Swan model used to organize our thinking and teach our students. If the absolute capacity of the planet to accommodate human economic activity is limited, the Solow-Swan framework is seriously compromised because it focuses on steady states that cannot exist.

Like Mankiw et al. (1992), this paper takes Solow (1956) and Swan (1956) seriously. It adapts the canonical Solow-Swan growth model in a straightforward way to account for anthropogenic climate change and shows that the canonical growth model can accommodate hard environmental constraints. Greenhouse gases (primarily carbon dioxide, but also methane, nitrous oxide, ozone, and flourinated gases) accumulate in the atmosphere and lead to environmental collapse if they exceed a critical threshold. Based on this assumption, the economy must either scale back production or mitigate its impact on atmospheric greenhouse gas concentration at an exponentially growing rate.

Relative growth rates of technology in goods production and mitigation are key determinants of the long-run behavior of this economy. If the rate of technological progress in mitigation exceeds that of goods production, the economy can grow without limits related to the environmental constraint. Output, consumption, and capital all expand in the long run at a rate converging to that of technological progress in the production of goods. Limits to growth do emerge, however, if productivity in mitigation grows more slowly than in goods. In this case, growth in output converges to the rate of technical progress in mitigation. In fact, long-run quantitative growth is impossible in the absence of technological progress in mitigation. In all cases, the economy's steady state is characterized by long-run economic growth when measured in terms of the produced good.

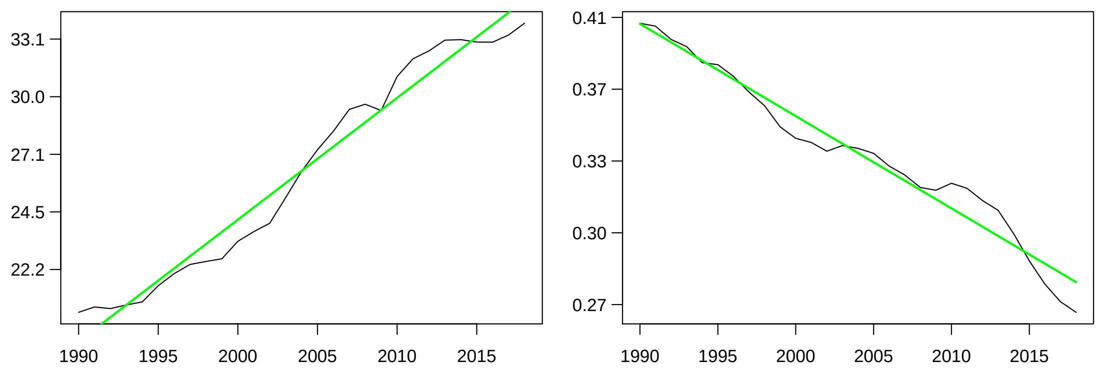
The limited growth scenario contains new and yet unexplored implications for the Solow-Swan model. First, if technical progress in mitigation is the limiting factor, mitigation must approach a share of one hundred percent of GDP, while a vanishing share of inputs are used in material production. For similar reasons, this is a familiar and central feature of Baumol's (1967) "service disease" and its effects on consumption and production patterns. Second, economic growth in value terms is a necessary outcome if the government produces abatement and purchases production factors in competitive markets. As in Baumol (1967), intersectoral factor mobility induces secular relative price changes favoring the good with lower productivity growth over time. Even in the extreme case of zero long-run quantitative growth, efforts to respect environmental constraints necessitate pure price-driven GDP growth, measured in terms of the produced good. Growing factor income delivers the resources necessary for government mitigation efforts. This is not true in a "mandate" economy that compels the private sector to produce its own mitigation or purchase it in the market. In this case, GDP growth measured in terms of the numeraire must approach zero. Third, we show that the "golden rule" savings rate is unaffected by the transition to the Baumol regime. The environmental limit does not imply scaling back capital accumulation. Finally, we confirm that population growth exacerbates the impact of the environmental limit. High population growth accelerates the transition to the Baumol economy and reduces its long-run level of per capita income. If production involves a fixed factor like land, positive population growth can drive consumption per capita to zero in the long-run limit.

In Section 2 we outline the reasoning underpinning our approach and how it builds on the existing literature on climate change and economic growth. Section 3 describes the Solow-Swan-Baumol economy and Section 4 presents our central results for long-run growth paths in the Baumol regime. In Section 5 we explore the consequences of a fixed factor of production. In the penultimate section of the paper, we present a quantitative evaluation of our model, based on Nordhaus's (2017) benchmark parametrization. Section 7 concludes.

2 Growth and Greenhouse Gases

The reduction of global greenhouse gas emissions is necessary to avoid the most disastrous consequences of climate change in this century (IPCC, 2022). If the volume of goods and services produced is to grow at all, the attendant production of greenhouse gases must be mitigated. This mitigation can either take the form of *decarbonization*, the reduction of greenhouse gas emissions occurring in the course of economic activity, or *abatement*, the use of resource-intensive carbon dioxide removal technologies such as afforestation, bioenergy with carbon capture and storage (BECCS) and direct air CO_2 capture and storage (DACCS).[1] Figure 1 shows that the emissions intensity of global GDP has fallen steadily as service sectors expand, energy efficiency improves, and the share of energy from renewable sources increases. Despite this progress, overall global emissions continue to rise and according to current projections, decarbonization on its own will not suffice for meeting global climate goals. Consequently, all pathways described in IPCC (2022) for limiting global warming to below 2°C include the large-scale use of abatement technologies. We show that decarbonization and abatement have similar but distinct implications for the economy's growth path. Decarbonization is associated with the reallocation of production factors from carbon-intensive to green technologies in a factor-neutral fashion.[2] In contrast, abatement requires the use of a resource intensive technology which diverts factors of production away from conven-

[1] We use the terms decarbonization and abatement in accordance with this definition throughout. Another form of mitigation not considered here is adaptation as examined in Fried (2021).
[2] For example, capital and labor can both be employed in a solar collector *instead* of a coal fired-power plant.



(a) **Annual CO_2 equivalent emissions (Gt).** (b) **Kg of CO_2 equivalent per USD of GDP.**

Figure 1: **Annual global greenhouse gas emissions, 1990-2020.** GDP is measured in purchasing power parity of 2017 US-Dollars. Both figures have logarithmic scales. Source: World Bank

tional economic activities.[3] We show below that this subtle difference has important consequences for GDP measurement and relative price dynamics.

In our study of the macroeconomic consequences of climate change and its mitigation, we relate to a literature on integrated assessment models (IAMs), that explicitly address bidirectional feedback between economic activity and the climate system. Material goods production causes greenhouse gas emissions which accumulate in the atmosphere. By decreasing the earth's reflectivity, accumulated greenhouse gases cause climate change which feeds back into economic activity through a damage function. The optimal response to the problem of climate change is characterized as balancing the costs of avoiding present emissions against future expected discounted damages caused by climate change.[4] Optimal policy responses are sensitive to two modeling choices: the social discount rate and the damage function.[5] The social discount rate involves trading off the well-being of current against future generations, a philosophical question without a clear answer (Ramsey, 1928; Stern, 2007). Parameterization of a damage function requires a clear stance on the path of climate change and its effects on economic activity over many decades. In the presence of exponential growth, damages based on historical data often appear trivial compared to future incomes. At the same time, the associated uncertainty is significant, and many growth scenarios include catastrophic and irreversible changes in the climate system.[6]

[3] Not all technologies can be cleanly separated into one of these categories. Nevertheless, we consider the simple distinction useful for our analysis. Van der Ploeg and Rezai (2019) make a similar distinction between sequestration and the substitution of renewable for fossil energy sources.

[4] The literature on IAMs includes parsimonious models that capture the relevant interactions with a small number of parameters and allow (partial) analytical characterizations of the model dynamics (among others, see DICE (Nordhaus, 2017), Golosov et al. (2014) and van der Ploeg and Rezai (2019)). A second strand of the literature focuses on larger quantitative models with more complex climate systems, multiple sectors, different types of emissions, or spatial differences. See for example FUND (Waldhoff et al., 2014), PAGE (Hope, 2011), GCAM (Calvin et al., 2019) and REMIND (Luderer et al., 2015).

[5] See Stern (2007), Pindyck (2013) and Heal (2017).

[6] For example, non-linear effects (Burke et al., 2015), growth effects of damages (Piontek et al., 2019) or different impacts on investment versus consumption (Casey et al., 2021). On the role of uncertainty itself, see Aengenheyster et al. (2018).

To sidestep these questions and focus on the consequences of mitigation policy, we propose a modification of the Solow-Swan model with an exogenous upper bound on the concentration of greenhouse gases in the earth's atmosphere.[7] The upper bound leads to a feedback between economic activity and cumulative greenhouse gas emissions without having to specify a damage function. It can be understood as an *environmental limit* in the sense of Dasgupta (2021), the outcome of a political agreement such as the Paris accord, or an exogenously mandated national carbon budget. Moreover, since we study a Solow-type economy, the savings rate is constant and the social discount rate does not enter the model solution. This simplicity creates space for strong analytic results regarding positive outcomes for the economy resulting from a minimal set of assumptions. Rather than ruling out material growth, we show how the resource costs of mitigation in the face of an absolute upper bound on cumulative greenhouse gas emissions changes, but does not eliminate, the potential for economic growth. Solving for an optimal policy is a relatively straightforward exercise that we leave to future research. In the quantitative application in Section 6, we draw a link to IAMs by introducing an explicit damage function.

Our modeling strategy incorporates both decarbonization as well as abatement as components of a CO_2 mitigation strategy. Consistent with Nordhaus (2017), we introduce decarbonization as an exogenous decrease of emissions intensity of output over time. This admits the possibility of long-run economic growth without unbounded growth of emissions.[8] In contrast, abatement is a technology that requires the diversion of production factors from use in conventional economic activity, and is not explicitly modeled in Nordhaus (2017), but has been incorporated into a number of IAMs (Kalkuhl et al., 2015; van der Ploeg and Rezai, 2019).

From a modeling perspective, the two approaches to mitigation, decarbonization and abatement, are closely related to an earlier literature on economic growth and pollution. Brock and Taylor (2010) show that their growth model with polluting and non-polluting technologies is isomorphic to a model in which all production causes pollution but there is a cleanup technology. The potential for long-run growth depends on the relative productivity growth of non-polluting technology in the first case and of the cleanup technology in the second.[9] We establish a similar isomorphism in our model. In a Cobb-Douglas setup, only a composite of technical progress in decarbonization and abatement is relevant for long-run consumption possibilities. Importantly, the isomorphism applies only to the quantity of goods consumed, not to the dynamics of relative prices, deployment of factors across sectors, and measured GDP. The Baumol effect and price-driven economic growth arise when a resource intensive cleanup technology is available as an alternative to exogenous

[7] The assumption of an upper bound on cumulative emissions follows Kalkuhl and Brecha (2013) and Llavador et al. (2011).

[8] Directed technical change studied in Acemoglu et al. (2012) can replicate this transition as an endogenous phenomenon. Allocating research effort to the production of clean inputs reduces the emissions intensity of output over time. Sufficient substitutability of clean and dirty inputs is necessary for the possibility of long-run growth without environmental collapse. See also Acemoglu et al. (2016) and Aghion et al. (2019).

[9] Stokey (1998) finds similar results in an optimal growth model. Aghion et al. (1998) extend the analysis to a Schumpterian growth model and provide an detailed discussion of the literature investigating pollution and sustainable growth.

technical progress in decarbonization.

Our approach is related to an extensive literature on resource economics. The distance to the environmental limit can be understood as a natural resource that is continuously depleted. Building on Hotelling's (1934) contribution, the seminal research of Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974) establish that one necessary condition for positive long-run growth is an elasticity of substitution between the scarce resource and other factors of production greater than unity. The emissions intensity of output is exogenous in our framework, which implies that no substitution away from the natural resource can occur. Yet, positive production and indeed sustained growth is possible even if the environmental limit binds. The reason is that the natural resource can be replenished through abatement, which qualitatively changes economic dynamics. This option does not exist in the standard setting of resource economics.

More recently, Hassler et al. (2021) study directed technical change in input efficiency in a model with fossil fuels as a depletable resource. In the absence of directed technical change, the factor share of fossil fuels approaches 100 percent in the limit and their result resembles our finding for resource-intensive abatement. Directing technical change towards improvements in fuel efficiency, however, returns the economy to a balanced growth path in their framework. A similar finding could be expected in our framework under conditions of directed technical change.

Finally, our work is related to a literature on unbalanced economic growth. The seminal contributions of Baumol and his coauthors emphasize the effects of unequal rates of technological progress in different sectors on relative prices and long-run growth.[10] If sectoral output shares are stable over time, Baumol's cost disease arises; the share of total inputs used by technologically stagnant sectors approaches unity and the long-run growth rate declines. Nordhaus (2008) empirically assesses and finds strong support for Baumol's cost disease. Subsequent research has established that this effect has accompanied the structural transformation of advanced economies from manufacturing to services during the second half of the last century.[11] Central to this result is an elasticity of substitution between manufactured goods and services less than one. In our framework the elasticity of substitution between abatement and physical production is effectively zero, giving rise to a strong form of Baumol's cost disease. Acemoglu and Guerrieri (2008) study how unequal output elasticities with respect to the factors of production across sectors cause unbalanced growth with secular trends in factor shares and reallocation of production factors. In the analysis that follows, we emphasize the outsized role of technological progress by restricting our attention to a model with identical output elasticities in the material production and abatement sectors.

[10] See Baumol and Bowen (1965), Baumol (1967) and Baumol et al. (1985).

[11] A large literature studies the relative importance of non-homothetic preferences and unequal technological growth rates. See Kongsamut et al. (2001), Herrendorf et al. (2013), Herrendorf et al. (2015) and Boppart (2014). López et al. (2003) point out that the costs of pollution can be another reason for structural change towards a service economy. Ngai and Pissarides (2007) study approximate balanced growth in economies with unequal rates of technological progress across sectors.

3 A Solow-Swan-Baumol Growth model with Greenhouse Gases

3.1 Technology of goods and mitigation production

The Solow-Swan model (Solow 1956, Swan 1956) serves as the analytic framework. In the spirit of Baumol (1967), we assume a two-good economy: produced goods Q serve as either private consumption or investment, while *abatement* B is a pure public good that reverses the negative environmental consequences of that production - i.e. the emission of greenhouse gases. Goods and abatement compete for scarce capital K and labor L, using production technologies that are identical up to a multiplicative factor:

$$Q_t = K_{Qt}^{\alpha} \left(A_{Qt} L_{Qt}\right)^{1-\alpha} \quad \text{and} \quad B_t = K_{Bt}^{\alpha} \left(A_{Bt} L_{Bt}\right)^{1-\alpha}. \tag{1}$$

Put differently, both goods are produced using a standard Cobb-Douglas production function with labor-augmenting technological progress, while A_Q and A_B grow at exogenous and nonnegative rates a_Q and a_B , respectively.[12] We follow Baumol (1967) and assume that factors of production can be deployed costlessly in either sector and are fully employed:[13]

$$K_t = K_{Qt} + K_{Bt} \quad \text{and} \quad L_t = L_{Qt} + L_{Bt}. \tag{2}$$

3.2 Households

Worker/households supply labor inelastically at any point in time in amount L_t , which grows at exogenous rate $n \geq 0$. They own the capital stock, that evolves as the difference between gross investment and depreciation:

$$\dot{K}_t = I_t - \delta K_t, \tag{3}$$

where the rate of depreciation of capital δ lies in the unit interval. Investment in this closed economy equals private savings, a fixed fraction of after-tax income:[14]

$$I_t = s(Y_t - T_t). \tag{4}$$

Y_t is GDP measured at factor costs and T_t represents lump-sum taxes to be defined below. The remainder is consumed:

$$C_t = (1 - s)(Y_t - T_t). \tag{5}$$

[12] Identical output elasticities in the two sectors highlight the importance of technological change and are not essential for the results we present. Allowing for different Cobb-Douglas technologies implies an additional factor bias in the sense of the Rybczynski Theorem (see Jones, 1965) for which we have no a priori intuition. See Acemoglu and Guerrieri (2008) for an analysis of a two-sector growth model with different output elasticities. We have also solved a version of the model with Leontief technology in abatement. The main results are unaffected in this case.
[13] In his original analysis of the service disease, Baumol modeled a single factor of production, labor.
[14] We show below that after-tax income equals Q_t.

3.3 Factor markets and factor price determination

Factor prices are determined in competitive spot markets for output, labor, and capital rental services. With the produced output good as a numeraire, competitive remuneration of labor and capital implies that

$$w_t = (1-\alpha)K_{Qt}^{\alpha}A_{Qt}^{1-\alpha}L_{Qt}^{-\alpha} \quad \text{and} \quad r_t = \alpha K_{Qt}^{\alpha-1}\left(A_{Qt}L_{Qt}\right)^{1-\alpha} \tag{6}$$

where w_t and r_t are the wage and the rental rate of capital in period t, respectively.

Because abatement is a pure public good, it will not be produced voluntarily by the private sector. In the benchmark version of this model, the government supplies abatement itself, hiring workers and renting capital at market prices in order to produce it. In practice, not all abatement will be supplied by the government and enter GDP as public consumption, however; in regulatory regimes private firms may be compelled to offset their emissions, and demand for private production of abatement arises that enters as an intermediate input to production. We consider this alternative case in Section 4.4. Under the assumption that the government chooses factor inputs to minimize cost of producing any level of abatement B, capital intensity is equal in both sectors:

$$\frac{K_{Bt}}{L_{Bt}} = \frac{K_{Qt}}{L_{Qt}} \tag{7}$$

and the economy will scale without any structural implications across sectors. Let γ_t be the fraction of labor and capital dedicated to abatement at time t and $1-\gamma_t$ the fraction dedicated to the production of material output; it follows that

$$Q_t = (1-\gamma_t)K_t^{\alpha}\left(A_{Qt}L_t\right)^{1-\alpha} \quad \text{and} \quad B_t = \gamma_t K_t^{\alpha}\left(A_{Bt}L_t\right)^{1-\alpha}. \tag{8}$$

In this closed economy, GDP is also equal to the sum of total expenditures of private agents and the government, measured in units of the produced good:[15]

$$Y_t = Q_t + p_t B_t \tag{9}$$

where p_t is the price of abatement in terms of material goods. Abatement is produced by the government and enters GDP at factor cost:

$$p_t = \frac{w_t\gamma_t L_t + r_t\gamma_t K_t}{B_t} = \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha} \tag{10}$$

where the last equality follows from the fact that the marginal rate of transformation under the

[15] Baumol (1967) employed an alternative, nonstandard definition: $\bar{Y}_t = W_1 Q_t + W_2 B_t$ where W_1 and W_2 are arbitrary and constant weights. Our definition highlights the very effect that he stressed: the value of the good produced by the low-productivity sector increases secularly relative to the other. We discuss issues related to the measurement of GDP and its growth rate in Section 4.4.

assumed conditions is $\frac{dQ_t}{dB_t} = \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha}$.[16] At any time t the government sets a lump-sum tax T_t to finance its expenditure

$$T_t = p_t B_t, \tag{11}$$

establishing that after-tax income indeed equals Q_t.

3.4 Environmental limit and mitigation

On the environmental side of the model, greenhouse gases accumulate in the atmosphere according to[17]

$$\dot{G}_t = \sigma_t^{-1} Q_t - B_t \tag{12}$$

where σ_t is the marginal output associated with an additional unit of greenhouse gas emissions. Conversely, σ_t^{-1} measures the marginal CO_2 emissions intensity of produced output.

In addition to abatement, the economy can mitigate its impact on atmospheric CO_2 through *decarbonization*. By decarbonization we mean the reduction of emissions intensity of output over time. This can be due to improvements in energy efficiency or substitution to less CO_2 intensive production processes. This measure of "CO_2 efficiency" of economic output, σ_t grows at exogenous rate a_σ, which we call the rate of decarbonization. In the following, we define *mitigation* as the combination of abatement and decarbonization. The cost of abating emissions caused by one unit of output is $\frac{p}{\sigma}$ and grows at rate $(1-\alpha)(a_Q - a_B) - a_\sigma$. That is, the cost of abatement per unit of output might be declining even while the cost of abatement per kilogram of CO_2 is growing over time.

As in the standard Solow-Swan model, preferences are not modeled explicitly. We simply assume a strict upper bound $\bar{G}$ that we call the *environmental limit*:

$$G_t \leq \bar{G} \quad \forall t. \tag{13}$$

In this simple form, the environmental limit represents the maximum sustainable level of greenhouse gases consistent with avoiding environmental collapse (Dasgupta, 2021). Alternatively, it might stand for some exogenous and possibly socially optimal level of greenhouse gases, or the outcome of an arbitrary political agreement. The government is assumed to impose an abatement policy that ensures that the environmental limit is not violated:

$$B_t = \begin{cases} 0 & if \quad G_t < \bar{G} \\ \sigma_t^{-1} Q_t & if \quad G_t = \bar{G}. \end{cases} \tag{14}$$

[16] This follows from combining Cobb-Douglas technology and factor price equalization.
[17] Ignoring natural sinks for greenhouse gases simplifies our analysis, but has no qualitative consequences for the model's implications.

4 Equilibrium growth paths

We now exploit the simplicity of the Solow-Swan-Baumol setup and characterize two regimes which govern the economy's law of motion. The *Solow-Swan regime* obtains as long as the environmental limit is not binding. In order to highlight the stark contrast between the two regimes, the government does not engage in any abatement at all in this regime.[18] Once this limit is reached, the government implements the minimal amount of abatement necessary to ensure $G = \bar{G}$. We call this regime the *Baumol regime*, in which no further production of goods is possible without abatement: $Q_t = \sigma_t B_t$.

In the spirit of the Solow-Swan model, we are silent on the optimality of the transition between the two regimes and focus on long-run outcomes. Characterization of this transition would require an explicit Ramsey analysis with respect to issues raised in Section 2. Furthermore, the transition is likely to involve large and convex adjustment costs. We are also silent on the paths of technical progress of the two technologies, except to say that they grow at respective exogenous rates a_Q and a_B and assume levels at t given by $A_{Qt} = A_{Q0}e^{a_Q t}$ and $A_{Bt} = A_{B0}e^{a_B t}$. At the point of transition, the levels of these indicators of technical progress assume utmost importance, because they determine the level of sustainable consumption and investment (residual output) going forward. By treating A_{Qt} and A_{Bt} as exogenous, we highlight the importance of follow-up analysis that explicitly incorporates directed technical change.

4.1 Growth path for $G_t < \bar{G}$: Balanced growth à la Solow-Swan

In the Solow-Swan regime, abatement is zero, $B_t = 0$ and the model above is identical to the standard Solow-Swan setup. Greenhouse gases accumulate according to (12) but they do not affect dynamics if $G_t < \bar{G}$. The economy remains on its balanced growth path, as long as it has not reached the environmental limit. As in the standard model, the steady state of capital in efficiency units $k_t = \frac{K_t}{A_{Qt}L_t}$ is

$$k^* = \left(\frac{s}{a_Q + \delta + n}\right)^{\frac{1}{1-\alpha}}. \tag{15}$$

If the economy is on the balanced growth path, physical output, the capital stock, and GDP all grow at rate $a_Q + n$:[19]

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{Y}_t}{Y_t} = a_Q + n, \tag{16}$$

with all factors employed in the material production sector.

$$K_{Qt} = K_t; \quad L_{Qt} = L_t; \quad \gamma_t = L_{Bt} = K_{Bt} = B_t = 0. \tag{17}$$

The rate of greenhouse gas accumulation is proportional to the CO_2 emissions intensity of material

[18] Judging from the current state of policy inertia, this might not actually be such a bad approximation to reality.
[19] We use the standard "dot notation" for the first derivative with respect to time.

production:

$$\frac{\dot{G}_t}{G_t} = \sigma_t^{-1}\frac{Q_t}{G_t} \tag{18}$$

and the regime is in place as long as $G_t < \bar{G}$.

In the following, we assume $a_Q + n \geq a_\sigma$.[20] In this case, exponential growth in Q_t in the Solow-Swan regime implies that G_t grows without bound, but this is ruled out by the environmental limit. At some time, say $\bar{t}$, the environmental constraint $G_t = \bar{G}$ becomes binding, unbounded growth is no longer possible, and the Solow-Swan regime ends.

4.2 Growth path when $G_t = \bar{G}$: Unbalanced growth à la Baumol

At time $\bar{t}$, the government abatement effort kicks in and $\sigma_t B_t = Q_t$ must hold for all $t \geq \bar{t}$. Produced output falls discretely by γ_t% at $t = \bar{t}$, as γ_t% of the capital and labor endowments are diverted to abatement. Thereafter the economy assumes characteristics familiar from Baumol's (1967) economy under conditions of differential sectoral productivity growth. A sufficient condition for Baumol's cost disease to arise is that the output share of the sector with slower technical progress does not converge to zero.[21] Our environmental interpretation of the model provides economic foundations for government intervention which ensures that this condition is met.[22]

To see this, first note that the condition $\sigma_t B_t = Q_t$ pins down γ_t, the share of resources devoted to abatement:

$$\gamma_t \sigma_t K_t^\alpha \left(A_{Bt} L_t\right)^{1-\alpha} = (1-\gamma_t) K_t^\alpha \left(A_{Qt} L_t\right)^{1-\alpha} \tag{19}$$

or

$$\gamma_t = \frac{1}{1 + \sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}} \tag{20}$$

Three distinct cases emerge in the Baumol regime:

1. $a_B + \frac{1}{1-\alpha} a_\sigma = a_Q$: γ_t, the share of factors used for abatement, is constant and determined by the initial relative productivities.
2. $a_B + \frac{1}{1-\alpha} a_\sigma > a_Q$: the share of factors used for abatement converges to zero, $\gamma_t \to 0$.
3. $a_B + \frac{1}{1-\alpha} a_\sigma < a_Q$: the share of factors used for abatement converges to one, $\gamma_t \to 1$.

Because it has the most salient implications for the economy's growth path, we focus on the last case in the next section. In the first and second cases, the environmental limit imposes no long-run constraints on growth and its effects on economic dynamics are unremarkable. For details see Appendix A. For now, notice that the second and third case reflect Baumol's (1967) conclusion

[20] This condition is sufficient but not necessary for the Solow-Swan regime to end in finite time.

[21] See Baumol et al. (1985) who find that sectoral output shares were approximately constant and unrelated to the rate of productivity growth. The literature on structural change finds that an elasticity of substitution between goods and services below unity is sufficient to generate Baumol's cost disease (Boppart, 2014).

[22] Baumol (1967) points out that a government intervention could impose constant output shares, but he provides no reason why this might be desirable.

that in the long run, all factors of production ultimately are diverted to the low-growth sector if its output share does not vanish.

At the point of the transition, the price of abatement is given by equation (10). Changing relative productivity levels imply that the price of abatement grows at a constant rate:

$$\frac{\dot{p}_t}{p_t} = (1-\alpha)(a_Q - a_B). \tag{21}$$

Finally, it is useful to decompose GDP growth at time t, g_{Yt}, into its components deriving from growth in material production and growth in abatement valued at market prices, which is the sum of growth rates of the valuation and the quantity of that abatement:

$$g_{Yt} \equiv \frac{\dot{Y}_t}{Y_t} = \frac{Q_t}{Y_t}\left[\frac{\dot{Q}_t}{Q_t}\right] + \frac{p_t B_t}{Y_t}\left[\frac{\dot{p}_t}{p_t} + \frac{\dot{B}_t}{B_t}\right]. \tag{22}$$

4.3 When mitigation is a limit on growth: $a_B + \frac{1}{1-\alpha}a_\sigma < a_Q$

In the most interesting case, productivity growth in the mitigation technologies is slower than productivity growth in the produced goods sector. One motivation for this assumption is that many negative emissions technologies, like afforestation or BECCS, cannot be scaled up indefinitely because they are limited by the efficiency of natural processes involved and the amount of available land. In the extreme case of $a_B = a_\sigma = 0$, as argued by Dasgupta (2021) and the degrowth literature more generally (e.g. Hickel and Kallis, 2020), there is a strict upper bound on the possibility of mitigating environmental damages caused by economic activity.[23] As we have seen, the case $a_B + \frac{1}{1-\alpha}a_\sigma < a_Q$ implies that abatement absorbs all factors of production in the long run. This powerful result is an inevitable consequence of the hard environmental limit combined with slow technological progress in mitigation.

Under these conditions, the aggregate dynamics of capital, consumption, and output are determined by the following system:

$$Q_t = (1-\gamma_t)K_t^\alpha (A_{Qt}L_t)^{1-\alpha}, \tag{23}$$

$$B_t = \gamma_t K_t^\alpha (A_{Bt}L_t)^{1-\alpha}, \tag{24}$$

$$\gamma_t = \frac{1}{1+\sigma_t\left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}}, \tag{25}$$

$$\dot{K}_t = sQ_t - \delta K_t. \tag{26}$$

For purposes of visualization, we redefine the model's variables in terms of efficiency units of labor

[23] Georgescu-Roegen (1971) argues for an even tighter constraint on total cumulative output that can be produced on the planet.

in abatement, denoted using a tilde for any variable X_t, as $\tilde{x}_t = \frac{X_t}{\sigma_t^{\frac{1}{1-\alpha}} A_t^B L_t}$.[24] We denote the long-run limit of a variable $\tilde{x}$ with a star, analogous to the steady state in the Solow-Swan regime, i.e. $\tilde{x}^* = \lim_{t\to\infty} \tilde{x}_t$. We emphasize that $\tilde{x}^*$ is a limiting point and *not* a steady state. Unlike a steady state, the behavior of the economy at this limiting point is not defined, and the notion of convergence under unbalanced growth must be taken more literally than that associated with a steady state.[25]

Net material output per effective capita $\tilde{q}_t$ available for consumption or capital formation is given by

$$\tilde{q}_t = \frac{Q_t}{A_{Bt} L_t \sigma_t^{\frac{1}{1-\alpha}}} = (1-\gamma_t)\tilde{k}_t^\alpha \sigma_t^{-1} \left(\frac{A_{Qt}}{A_{Bt}}\right)^{1-\alpha} = \gamma_t \tilde{k}_t^\alpha. \tag{27}$$

Capital in terms of efficiency units of labor in abatement accumulates according to:

$$\dot{\tilde{k}}_t = s\gamma_t \tilde{k}^\alpha - \left(\delta + n + a_B + \frac{1}{1-\alpha} a_\sigma\right) \tilde{k}_t. \tag{28}$$

This is the familiar accumulation equation of the standard Solow model, with $a_B + \frac{1}{1-\alpha} a_\sigma$ representing the overall technological growth rate and augmented by the term $\gamma_t = \frac{1}{1+\sigma_t \left(A_{Bt}/A_{Qt}\right)^{1-\alpha}}$, which shifts productivity over time.

4.3.1 Long-run limit of the limited growth regime

We now study the behavior of our economy in the long run as γ_t approaches unity. From equation (27), we can see that the "modified Baumol-Solow-Swan model" converges to the following intensive-form production function in terms of labor efficiency units in mitigation:

$$\tilde{q}^* = \left(\tilde{k}^*\right)^\alpha \tag{29}$$

with the associated accumulation equation:

$$\dot{\tilde{k}}^* = s\left(\tilde{k}^*\right)^\alpha - \left(\delta + n + a_B + \frac{1}{1-\alpha} a_\sigma\right) \tilde{k}^*. \tag{30}$$

Since we consider a long-run limit, $\dot{\tilde{k}}^* = 0$ and the growth rate of capital is $g_K = n + a_B + \frac{1}{1-\alpha} a_\sigma$. Figure 2 depicts this modified Baumol-Solow-Swan model using the familiar textbook diagram. The long run limit of this economy is given by the intersection of the concave intensive-form savings function with the capital widening line with slope $a_B + a_\sigma(1-\alpha) + \delta + n$. Output in intensive form (in terms of efficiency units of labor in mitigation) can be read off the intensive-form production function. Comparative statics are identical to the Solow-Swan model. Material output per capita

[24] If mitigation technology grows faster than technology of material production, efficiency units have to be defined in terms of the productivity of the material sector to obtain finite limits.

[25] At the limit point, $\gamma = 1$ implies zero material output. As the economy approaches this limit, however, output does not converge to zero, but continues to grow at a positive rate.

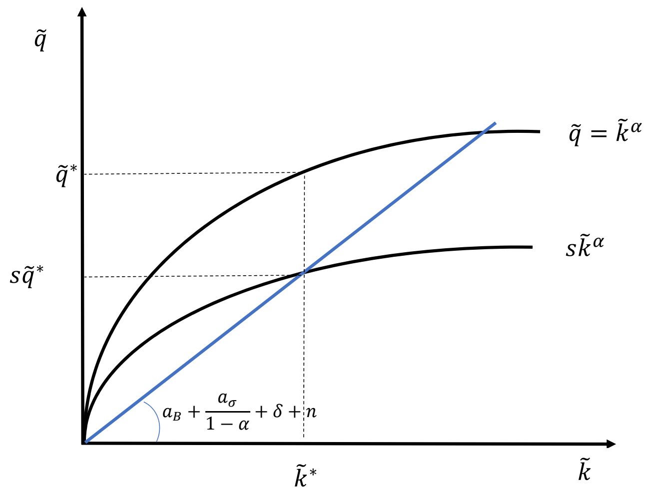


Figure 2: The long-run limit of the Baumol regime in the Solow-Swan-Baumol model

is $\sigma_t^{\frac{1}{1-\alpha}} A_{Bt} \left(\tilde{k}^*\right)^\alpha$ which grows at rate $a_B + \frac{1}{1-\alpha}a_\sigma$, as does consumption per capita. The effort to respect the environmental limit implies a permanent decline in sustainable levels of consumption and material production.

Figure 3 displays an illustrative growth path for material output. Upon reaching the environmental limit, the economy experiences a drop in the production of material output, as resources are shifted towards the abatement sector. The extent of this decline depends on the relative levels of productivity at the point of regime change. In the aftermath, output growth is low and declines even more over time, as efficiency gains in the material sector have less and less impact. In the long run, the growth rate of material production output per capita reaches $a_B + \frac{1}{1-\alpha}a_\sigma$, reflecting technical progress in both abatement and decarbonization.

The growth rates of key exogenous and endogenous variables in the model are summarized in Table 1 and constitute a central contribution of this paper: Growth in quantities is augmented by price-driven GDP growth, that is, growth in the value of GDP in terms of the produced good. As a general result, the long-run growth rate of GDP at market prices, g_Y^*, is given by:

$$\lim_{t\to\infty} g_{Yt} = \lim_{t\to\infty} \left\{ \frac{Q_t}{Y_t}(a_B + \frac{1}{1-\alpha}a_\sigma + n) + \frac{p_t B_t}{Y_t}\left[(1-\alpha)(a_Q - a_B) + a_B + \frac{\alpha}{1-\alpha}a_\sigma + n\right]\right\}$$
$$= (1-\alpha)(a_Q + n) + \alpha\left(a_B + \frac{1}{1-\alpha}a_\sigma + n\right) = g_P + g_B. \tag{31}$$

In contrast, the growth rate of GDP with constant weights, $\hat{g}_Y^*$, converges to

$$\lim_{t\to\infty} g_{\bar{Y}t} = \lim_{t\to\infty} \left\{ \frac{W_1 Q_t}{\bar{Y}_t}(a_B + \frac{1}{1-\alpha}a_\sigma + n) + \frac{W_2 B_t}{\bar{Y}_t}\left(a_B + \frac{\alpha}{1-\alpha}a_\sigma + n\right)\right\}$$
$$= 1 * \left(a_B + \frac{1}{1-\alpha}a_\sigma + n\right) + 0 * \left(a_B + \frac{\alpha}{1-\alpha}a_\sigma + n\right) = g_Q, \tag{32}$$

Growth rate	Value
g_Q (production technology)	a_Q
g_B (abatement technology)	a_B
g_σ (decarbonization)	a_σ
g_B^* (abatement)	$a_B + \frac{\alpha}{1-\alpha} a_\sigma + n$
g_Q^* (produced goods)	$a_B + \frac{1}{1-\alpha} a_\sigma + n$
g_Y^* (GDP)	$g_P + g_B$
$g_{\bar{Y}}^*$ (GDP, constant weights)	g_Q
g_p (price of abatement)	$(1-\alpha)(a_Q - a_B)$

Table 1: **Growth rates in the Baumol regime** ($\frac{a_\sigma}{1-\alpha} + a_B < a_Q$).
Starred values indicate long-run limits.

a result that is identical to Baumol's (1967) finding. When $a_Q > a_B + \frac{\alpha}{1-\alpha} a_\sigma$, the growth rate of GDP at market prices exceeds the growth rate of material production in the limit. To understand this result, note that technological progress in goods production has two effects. First, it increases productivity. Second, it leads to more greenhouse gas emissions. To respect the environmental limit, the government must divert more and more resources to the abatement sector. However, these resources are becoming increasingly more productive in the production of goods. This means the government has to pay ever-higher factor prices; because public consumption enters GDP at cost, rising factor prices increase the value of government-supplied abatement, which in turn leads to long run GDP growth. Abatement approaches a share of one hundred percent of GDP.

4.3.2 Savings, mitigation and the golden rule

If $a_Q > a_B + \frac{\alpha}{1-\alpha} a_\sigma$, our model predicts that virtually all resources are diverted to the abatement effort in the long run, while a vanishing share of resources is employed in the production technology causing emissions. Since it is the production of goods that makes the abatement effort necessary, this growth path might appear inefficient; the "degrowth" position maintains that it is better to scale back production instead. This could be achieved by reducing the savings rate, capital accumulation and productive capacity. It is thus crucial to understand the effects of savings and abatement on long-run consumption possibilities in our model.

In the long-run limit, consumption *per capita* is given by:

$$\sigma_t^{\frac{1}{1-\alpha}} A_{Bt}(1-s)\tilde{q}^* = \sigma_t^{\frac{1}{1-\alpha}} A_{Bt}(1-s) \left(\frac{s}{a_B + \frac{1}{1-\alpha} a_\sigma + n + \delta} \right)^{\frac{\alpha}{1-\alpha}}. \quad (33)$$

Material output and consumption per capita are positively related to productivity in the abatement sector (A_{Bt}) and the "CO_2 efficiency" of production (σ_t). The levels of A_{Bt} and σ_t jointly determine effective total factor productivity.[26] In a growing economy with a binding environmental constraint,

[26] This isomorphism between ex-post abatement and decarbonization of production processes can also be found in

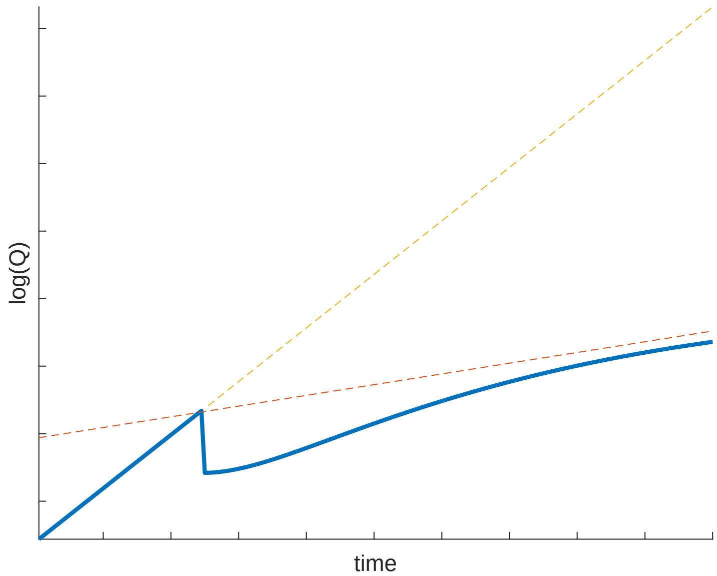


Figure 3: Growth path for Q_t. The yellow dashed line is the balanced growth path in the absence of the environmental limit. The red dashed line is the long-run growth path in the Baumol regime.

the wealth of nations is determined by productivity in mitigation activities. This can be seen by defining the variable $A_t \equiv A_{Bt}\sigma_t^{\frac{1}{1-\alpha}}$ which grows at rate $a \equiv a_B + \frac{1}{1-\alpha}a_\sigma$.

As in the Solow model, the savings rate plays a dual role for per-capita consumption. Lower savings directly increase current consumption, but reduce capital accumulation and future consumption. As in Phelps (1961), this gives rise to a golden rule savings rate which maximizes consumption in effective per capita units. The abrupt need for abatement leads to a decline in effective total factor productivity. Does this reduction in effective productivity imply that the economy should reduce the savings rate? The answer is no. Given that the government supplies all abatement, the golden rule savings rate is given by

$$s_{Gold} = \alpha, \tag{34}$$

the same consumption-maximizing savings rate in the standard Solow-Swan model. In order to reach the golden rule consumption level, the economy must follow the growth path derived in Section 4.3. That is, price-driven growth is a necessary outcome if the government pursues the objective of maximizing consumption per capita while purchasing the necessary production factors for abatement on competitive markets.[27]

Brock and Taylor (2010). Abatement is driven by Harrod-neutral (labor-augmenting) technical progress, while decarbonization is equivalent to Hicks-neutral technical progress. Under Cobb-Douglas technologies, they are equivalent up to a scaling factor.

[27] In this setup, government spending on abatement would be recorded as consumption in the income and product accounts, but our result would also be consistent with a broader interpretation of investment to include abatement, in which the savings rate would be expanded to include this activity as well.

4.4 Alternative implementation: Outsourced Abatement or Emission Mandates

In this section, we discuss briefly an alternative policy arrangement that implements the sustainable path in the Baumol regime. Instead of producing abatement itself, the government simply compels private firms to offset emissions by purchasing abatement services from other firms or producing them in-house.[28] The production side of the economy remains identical to the economy described above. This "mandate economy" is capable of achieving the same path for material production and abatement as described above. The difference in measured GDP at market prices, however, could not be more striking.

Consider the static optimization problem of a firm operating under a government mandate to offset its emissions. Its profits are given by

$$\begin{aligned} \Pi_t^Q &= \max K_t^\alpha (A_{Qt} L_t)^{1-\alpha} - w_t L_t - r_t K_t - p_t B_t \\ &\text{s.t.} \\ &K^\alpha (A_{Qt} L_t)^{1-\alpha} \leq \sigma_t B_t. \end{aligned} \tag{35}$$

This yields factor demands characterized by:

$$w_t = (1-\alpha)(1-\sigma_t^{-1} p_t) K_t^\alpha A_{Qt}^{1-\alpha} L_t^{-\alpha} \quad \text{and} \quad r_t = \alpha(1-\sigma_t^{-1} p_t) K^{\alpha-1} (A_{Qt} L_t)^{1-\alpha}. \tag{36}$$

The firm purchases B_t units of abatement from a distinct economic sector producing under competitive conditions. The objective function of an abatement-producing firm is:

$$\Pi_t^B = p_t K_t^\alpha (A_{Bt} L_t)^{1-\alpha} - w_t L_t - r_t K_t. \tag{37}$$

Optimality implies:

$$w_t = (1-\alpha) p_t K_t^\alpha (A_{Bt})^{1-\alpha} L_t^{-\alpha} \quad \text{and} \quad r_t = \alpha p_t K_t^{\alpha-1} (A_{Bt} L_t)^{1-\alpha}. \tag{38}$$

Using the fact that factor shares are equal in both sectors and combining the equations in (36) and (38), cost of abatement per unit of output is:

$$\sigma_t^{-1} p_t = \frac{1}{1 + \sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}}. \tag{39}$$

In the case that $\sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha} \to 0$, $\sigma_t^{-1} p_t \to 1$. Conversely, if $\sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha} \to \infty$, then $\sigma_t^{-1} p_t \to 0$.

[28] In the following, we treat these two alternatives as equivalent.

Consider the definition of GDP as the sum of value added in each sector:

$$Y_t = (Q_t - p_t B_t) + p_t B_t. \tag{40}$$

While its price rises secularly when $a_Q > a_B$, abatement no longer enters GDP directly, but rather only as an intermediate input. In fact, as $\sigma_t^{-1} p_t \to 1$, $Y_t = Q_t = \sigma_t^{-1} p_t Q_t = p_t B_t$. In this case, all value added originates in the abatement sector in the limit. While measured GDP growth remains positive, it is entirely driven by technical progress in mitigation, $a = a_B + \frac{a_\sigma}{1-\alpha}$. This result contrasts starkly with price-driven GDP growth that emerges when the government purchases abatement in the market.[29]

5 Population growth and land in production

Central to the analysis thus far is the assumption that abatement is produced at constant returns to scale in capital and labor. In reality, the feasibility of large-scale deployment of negative emissions technologies has been called into question. In particular, the scalability of technologies related to biomass production, traditional afforestation, or BECCS, require land (or ocean) surface, which is in fixed supply. Furthermore, a limit to the absolute quantity of abatement has dire consequences in the presence of population growth: If abatement cannot be scaled up with population, the amount of emissions per head must fall. If decoupling of production and emissions proceeds too slowly, output and consumption must decline to zero in the long-run.

We formalize these results by introducing land D_{it} as a fixed factor of production:

$$Q_t = K_{Qt}^{\alpha} \left(A_{Qt} L_{Qt}\right)^{1-\alpha-\xi} D_{Qt}^{\xi} \quad \text{and} \quad B_t = K_{Bt}^{\alpha} \left(A_{Bt} L_{Bt}\right)^{1-\alpha-\xi} D_{Bt}^{\xi}. \tag{41}$$

The total supply of land is fixed at unity: $D_{Qt} + D_{Bt} = 1$. Apart from this extension, the model is unchanged. The assumption of equal factor shares guarantees that both technologies are scaled without factor bias on the equilibrium growth path.

We continue to focus on the case in which conventional technological progress in material goods production exceeds that of mitigation ($a_Q > a_B + \frac{a_\sigma}{1-\alpha}$). To highlight the main result of this section, we limit attention to the long run as $\gamma_t \to 1$.[30] The limiting dynamics of the capital stock are determined by

$$\dot{K}_t = s\sigma_t K_t^{\alpha} (A_{Bt} L_t)^{1-\alpha-\xi} - \delta K_t. \tag{42}$$

[29] This result holds irrespective of the numeraire chosen and is thus immune from the "Gerschenkron effect" (Gerschenkron, 1947). It is easy to show that despite the secular rise of the price of mitigation, the growth of the Fischer exact index of economic activity in either economy in time t is given by $g_{Yt} = (1-\gamma_t) g_{Qt} + \gamma_t g_{Bt}$ and $\lim_{t\to\infty} g_{Yt} = a_B + \frac{a_\sigma}{1-\alpha}$. See also Duernecker et al. (2021).

[30] The standard Solow-Swan model with land as a production factor is well-understood; see Romer (2012) or Weil (2012).

This is the same accumulation equation as in Romer's (2012) version of the Solow-Swan model with land in production and productivity in mitigation replacing productivity in conventional production. It follows that the growth rate of capital, output and consumption on the balanced growth path is given by[31]

$$g_K = g_Q = g_C = \frac{(1-\alpha-\xi)(n + a_B + \frac{1}{1-\alpha-\xi}a_\sigma)}{1-\alpha}. \tag{43}$$

Notice that these are growth rates for capital and output in levels. Equation (43) implies that consumption per capita grows at the rate

$$\frac{(1-\alpha-\xi)(a_B + \frac{1}{1-\alpha-\xi}a_\sigma) - \xi n}{1-\alpha}. \tag{44}$$

If $n > \frac{1-\alpha-\xi}{\xi}(a_B + \frac{1}{1-\alpha-\xi}a_\sigma)$, consumption per capita must fall at a constant rate along the long-run growth path. That is, productivity growth in mitigation places an upper bound on sustainable population growth in the presence of a fixed factor.

6 Solow-Swan-Baumol meets Nordhaus: A quantitative exercise

In this section, we subject our model to the "smell test" by incorporating features commonly found in more complex, commonly used IAMs. The goal of this section is to show that the economic intuition developed analytically is robust and applies in more general frameworks encountered in the climate change literature. We add the following features to the model: i) an emission limit that declines over time, mimicking recent political agreements and ii) economic damages resulting from climate change.

Since we do not model optimizing agents, the reduction of CO_2 emissions is not the outcome of a welfare maximization problem, but is rather exogenously specified. It takes the form $\bar{G}_t$ for $t > 0$ and is twice differentiable with respect to time. To capture the recent political agreements on greenhouse gas reduction in a parsimonious way, we assume that an agreement is implemented at time $\bar{t}$ and the emissions limit declines at a constant rate after that time:

$$\ddot{\bar{G}}_t = -\delta_G \dot{\bar{G}}_t. \tag{45}$$

This functional form implies that cumulative CO_2 emissions approach a constant value in the long run. The limit is set such that emissions are continuous at time $\bar{t}$ and do not decline discretely.[32] We assume that the agreement is binding at all times after $\bar{t}$ which implies

$$\dot{G}_t = \dot{\bar{G}}_t \quad \forall t > \bar{t}. \tag{46}$$

[31] See Romer (2012) for a proof.

[32] This implies $\bar{G}_{\bar{t}} = G_{\bar{t}}$ and $\dot{\bar{G}}_{\bar{t}} = \dot{G}_{\bar{t}}$.

Our amendments of the environmental side of the model largely follow Nordhaus' (2017) version of the DICE model, which is a widely used benchmark for IAMs. To capture damages caused by rising temperatures, we define net output as

$$\tilde{Q}_t = \Omega_t Q_t. \tag{47}$$

That is, a share $(1 - \Omega_t)$ of output is used to undo damage caused by climate change such as the destruction caused by more frequent extreme weather events. The damage function $(1 - \Omega_t)$ is conceptually different from resources used for abatement as they only offset the current degradation but have no effect on atmospheric CO_2 concentration. Only net output is available for consumption and investment, which are given by $C_t = (1 - s)\tilde{Q}_t$ and $I_t = s\tilde{Q}_t$ respectively. Economic damages or losses $(1 - \Omega_t)$ are increasing and convex in the global temperature anomaly $\mathcal{T}_t$, defined as the increase above average pre-industrial levels:

$$\Omega_t = 1 - \psi \mathcal{T}_t^2. \tag{48}$$

To keep the model parsimonious, we choose a simpler climate module than that used by Nordhaus (2017). The global temperature anomaly $\mathcal{T}_t$ is a linear function of cumulative greenhouse gas emissions:

$$\mathcal{T}_t = \nu G_t. \tag{49}$$

We parameterize the model using standard values from the growth and climate literature. We provide more details and a table of parameter values in Appendix B. We consider three scenarios and display their transition paths of the economy in Figure 4.

In the first scenario, emissions continue to rise over time as no abatement is undertaken ($\gamma = 0$). Even in the absence of abatement, however, the production technology becomes less emissions-intensive. Until the end of the century, cumulative emissions roughly triple and the temperature rises to 3.0°C above its pre-industrial value. Damages $1 - \Omega$ reach 3% of GDP. This reflects the relatively low costs associated with climate change in Nordhaus' (2017) standard parameterization. As a result, technological progress easily outweighs these costs. Output and consumption grow exponentially and almost unaffected by the climate damages.

In the second scenario, there is a binding agreement as described in the calibration above and the economy follows an emissions path compatible with 1.5°C of warming until the end of the century. Emissions decline rapidly and are close to zero by 2100. The reduction in emissions is achieved through a gradual increase in abatement which reaches 15% of GDP. In line with the analysis above, the relative price of abatement grows exponentially: as production becomes more and more efficient but emissions targets tighten, the abatement technology becomes increasingly valuable. In this scenario, damages remain small at less than 1% of output. As more resources are diverted to abatement, consumption and goods production decrease further and further below the path without abatement. By 2100, consumption per capita is approximately 10% lower.

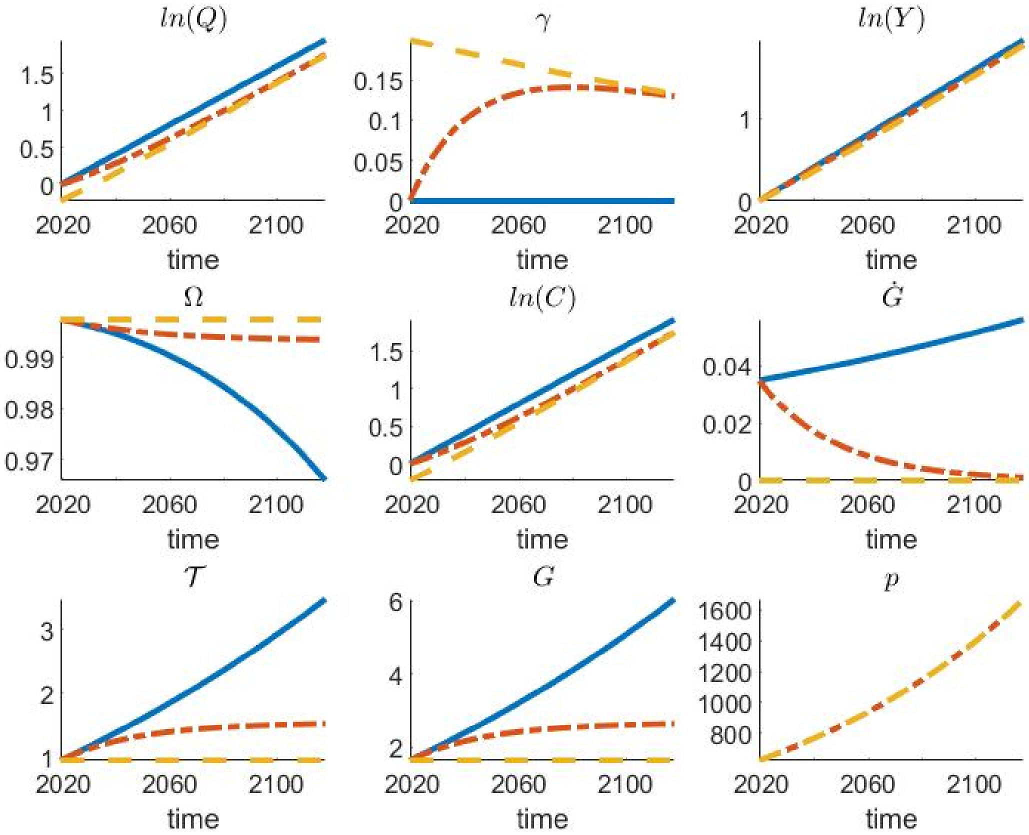


Figure 4: Growth paths in the quantitative model. Blue line: no emissions reduction. Red dash-dotted line: reduction compatible with 1.5°C warming. Yellow dashed line: zero net emissions.

In the third scenario, emissions decline to zero immediately; there is no further global warming and no further increase in damages. This policy is associated with an initial drop of 20% in produced goods and consumption. Over time, as goods production becomes less carbon-intensive and abatement technology improves, the gap between the zero emissions path and the zero abatement path shrinks. However, it remains sizable in the time span considered. Importantly, GDP at market prices is almost unaffected. As resources are used for abatement, they continue to contribute to a growing GDP.

As in Nordhaus (2017), climate damages are small relative to the exponentially growing output, even with warming in excess of 3.0°C. Engaging in costly abatement appears wasteful in terms of consumption possibilities. A large literature has recently emerged, however, which takes issue with the benign assessment emerging from the baseline DICE model.[33] Recent political agreements aim for temperature increases in the range of 1.5°C to 2.0°C until the end of the century. This would imply a growth path similar to the red-dashed line, implying a much larger abatement effort relative to "business as usual" scenario.

7 Conclusion

The strength and ultimate utility of the Solow-Swan model is its clarity: it lays out clearly the implications of minimalist assumptions on technology and behavior for long-run economic growth. Our simple adaptation of the Solow-Swan model highlights resource-intensive mitigation as a central feature of economic growth under an environmental limit or constraint, and provides a benchmark for teaching and understanding economic growth under these conditions.

Price-driven growth is essential for enabling government supplied mitigation, as it directly reflects the market consequence of government acquisition of production factors for public good provision. A typical form of Baumol's cost disease emerges as the slowly growing abatement sector employs a continuously increasing share of production factors. This result points to the importance of directed technical change in altering the long-run growth path of these economies. The power of Baumol's message extends to the choice of mandates versus markets. Paradoxically, the "market solution" requires active engagement of government - as opposed to a "mandate economy" that simply forces firms to adopt the desired level of mitigation and depresses measured economic growth in the long run.

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Appendix

A Alternative cases in the Baumol regime

In this section we characterize the dynamics in the Baumol regime in two cases not discussed in the main text: i) equal growth rates in both sectors and ii) faster productivity growth in the mitigation

A.1 Costly green growth: $a_Q = a_B + \frac{a_\sigma}{1-\alpha} = a$

Consider now the case in which the two technologies grow at the same rate. In this case the ratio of the two productivities and the share of resources devoted to each sector are constant as soon as the economy reaches the environmental constraint:

$$\gamma = \frac{1}{1 + \sigma_t \left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}}. \tag{50}$$

The economy exhibits balanced growth in this case, with steady state level of capital per efficiency unit of labor

$$k^* = \left(\frac{\gamma s}{a + \delta + n}\right)^{\frac{1}{1-\alpha}}. \tag{51}$$

Furthermore, all quantities grow asymptotically at rate $a + n$. That is, the economy behaves like a standard Solow economy, except for the presence of the parameter γ in steady state value of capital. This parameter captures the fact that in the steady state, a positive fraction of resources must be devoted to the abatement sector. The relative price of abatement is constant.

Reaching the environmental constraint corresponds to a one-time downward shift in the production function, as resources are redeployed immediately for abatement. Asymptotically the economy reaches its previous growth rate. The new growth path permanently lies below the growth path before the environmental constraint was reached.

A.2 Outgrowing environmental constraints: $a_Q < a_B + \frac{a_\sigma}{1-\alpha}$

Now consider the case in which the mitigation technology grows at a faster rate than the production technology. The share of resources used in the abatement sector approaches zero in the long run.

$$\lim_{t\to\infty} \gamma_t = \frac{1}{1+\sigma_t\left(\frac{A_{Bt}}{A_{Qt}}\right)^{1-\alpha}} = 0. \tag{52}$$

Capital per efficiency unit of labor returns to the same limit as a standard Solow-Swan economy

$$k^* = \left(\frac{s}{a_Q+\delta+n}\right)^{\frac{1}{1-\alpha}} \tag{53}$$

As above, the environmental constraint shifts back the production function, as resources are now required for abatement. However, as technological progress reduces the resources necessary to mitigate the environmental damage and the production function gradually shifts back. As a result, output falls temporarily as γ_t% of resources are used for abatement. As the mitigation technology becomes more productive, however, the growth rate of produced output recovers and output attains once again its initial growth path.

B Calibration details

Baseline parameter values along with the calibration targets are given in Table 2. We choose standard values for all parameters present in the Solow model. A second set of parameters can be mapped directly to Nordhaus (2017). In particular, we choose the same rate of decarbonization a_σ and the quadratic term of the damage function ψ. For the lack of a better source, we set the rate of technological progress in abatement a_B equal to the rate at which resource costs of abatement decline in Nordhaus (2017), even though the definitions of abatement do not match exactly across models. We calibrate the remaining parameters using 2019 data on output, emissions and temperature changes. Initial productivity of conventional technology A_{Q2019} is set to match global GDP of \$87tr.[34] We choose the sensitivity of temperature to cumulative CO_2 emissions ν to generate the observed 0.95°C increase in global mean temperatures above pre-industrial times

[34] We assume that the initial capital stock K_{2019} equals the steady state value of the Solow model without climate change.

caused by cumulative emissions of $G_{2019} = 1.65$ GtC. The initial value of σ_{2019} generates an emissions intensity of 0.4kg/$GDP. The relative productivity of mitigation is set to generate initial output costs of full sequestration of 20% following van der Ploeg and Rezai (2019). Finally, we choose δ_G to be consistent with a 1.5°C warming until the end of the century.

Parameter	Symbol	Value	Target
Capital share	α	0.33	standard value
Savings rate	s	30%	standard value
Capital depreciation rate	δ	3%	standard value
Technological progress in production	a_Q	2%	standard value
Technological progress in abatement	a_B	0.5%	Nordhaus (2017)
Rate of decarbonization in production	a_σ	1.5%	Nordhaus (2017)
Rate of emissions reduction	δ_G	5%	1.5°C warming in 2100
Quadratic damage term	ψ	0.227%	Nordhaus (2017)
Slope of temperature wrt CO_2	ν	$0.58 * 10^{-12}$	Warming in 2019: 0.95°C
Efficiency of production	A^Q_{2019}	$36 * 10^{12}$	World GDP: $87tr
Efficiency of abatement	A^M_{2019}	$2.4 * 10^9$	van der Ploeg and Rezai (2019)
Emissions intensity of output	σ^M_{2019}	$4 * 10^{-4}$	Emissions per GDP 0.4kg/$

Table 2: Baseline parameterization