No. 1735

STAGE FINANCING AND THE ROLE OF CONVERTIBLE DEBT

Francesca Cornelli and Oved Yosha

FINANCIAL ECONOMICS

Centre for Economic Policy Research

ISSN 0265-8003

STAGE FINANCING AND THE ROLE OF CONVERTIBLE DEBT

Francesca Cornelli and Oved Yosha

Discussion Paper No. 1735
November 1997

Centre for Economic Policy Research
90–98 Goswell Rd
London EC1V 7DB
Tel: (44 171) 878 2900
Fax: (44 171) 878 2999
Email: cepr@cepr.org

CEPR Discussion Paper No. 1735

November 1997

ABSTRACT

Stage Financing and the Role of Convertible Debt*

Venture capital financing is characterized by extensive use of convertible debt and stage financing. The paper shows why convertible debt is better than a simple mixture of debt and equity in stage financing situations. When the venture capitalist retains the option to abandon the project, the entrepreneur has an incentive to engage in 'window dressing' or short-termism, i.e. to bias positively the short-term performance of the project, in order to reduce the probability that the project will be liquidated. With a convertible debt contract, such behaviour reduces the likelihood of liquidation, but increases the probability that the venture capitalist will convert debt into equity, reducing the entrepreneur's profits. With convertible debt, therefore, the entrepreneur will not engage in as much short-term behaviour in terms of signal manipulation in comparison to a situation where only straight debt-equity financing is used.

JEL Classification: G24, G32
Keywords: venture capital, convertible debt, short-termism, window dressing

Francesca Cornelli
Finance Department
2304 SH–DH
The Wharton School
Philadelphia
PA 19104–6367
USA
Tel: (1 215) 898 8049
Fax: (1 215) 898 6200
Email: cornelli@finance.wharton.
upenn.edu

Oved Yosha
Berglas School of Economics
Tel Aviv University
Tel Aviv 69978
ISRAEL
Tel: (972 3) 640 7242
Fax: (972 3) 640 9908
Email: yosha@econ.tau.ac.il

*This paper is produced as part of a CEPR research programme on *Finance in Europe: Markets, Instruments and Institutions*, supported by a grant from the Commission of the European Communities under its Human Capital and Mobility Programme (no. ERBCHRXCT940653). The authors thank Leonardo

Felli, Martin Hellwig, Ronen Israel, Matthew Jackson, Rob Johnson, Dan Maldoon, Kjell Nyborg, Manju Puri, Artur Raviv, Yossi Spiegel, Josef Zechner, and participants at various seminars for helpful comments and suggestions. They thank the Institut d'Anàlisi Econòmica (F Cornelli), the London Business School (O Yosha) and IGIER (both authors) for their hospitality. The project was supported by an LBS Research Fellowship Grant.

Submitted 22 August 1997

NON-TECHNICAL SUMMARY

Because of the high levels of uncertainty and failure risk of new ventures, a widely used financing technique is the infusion of capital over time. The venture capitalist who provides the funds retains the option to abandon the venture at any stage whenever the forward-looking net present value of the project is negative. Each financing round is usually related to a significant stage in the development process, such as completion of design, pilot production, first profitability results, or the introduction of a second product. At every stage new information about the venture is released.

Stage financing is appealing to the venture capitalist who preserves a valuable option. First, the option to abandon is essential because an entrepreneur will almost never stop investing in a failing project as long as others are providing capital. Second, since abandonment of the project by the venture capitalist typically forces the entrepreneur to liquidate the project, the option to abandon creates incentives for the entrepreneur to maximize value and meet goals. There is empirical evidence that when the fraction of intangible assets in a project is high, agency costs are likely to be high and monitoring is more valuable. The number of stage financing rounds then increases with the fraction of intangible assets.

Stage financing may not always work as a value enhancing motivating factor. It might induce the entrepreneur to aim for short-term success rather than long-term value creation. In other words, the entrepreneur may try to improve short-term performance to make sure that the project will be refinanced. The paper argues that convertible debt, which is widely used in venture capital financing, can attenuate the incentives to engage in 'window dressing' on the part of entrepreneurs. By shifting resources to improving short-term signals the entrepreneur reduces the probability of liquidation but, at the same time, increases the probability that in the event of refinancing the venture capitalist will decide to exercise the debt conversion option, becoming the owner of a substantial fraction of the venture. If the terms of conversion are set in advance to be sufficiently favourable to the debt holder (the venture capitalist), the entrepreneur faces a situation where good news about the project entails a reduction in their profit because of debt conversion by the venture capitalist. As a consequence the entrepreneur will not engage in as much short-term behaviour in terms of signal manipulation in comparison to a situation where only straight debt-equity financing is used.

The debt conversion option, at a prespecified conversion ratio that is sufficiently favourable to the entrepreneur, can be interpreted in terms of 'renegotiation design' (a concept suggested by Aghion, Dewatripont and Rey in a recent paper). They show that if the bargaining power in various renegotiation contingencies can be allocated in advance in a credible manner, the first-best allocation can be implemented despite non-verifiability of a commonly observed signal. In the model in this paper, agreeing on a debt conversion option at a predetermined price is roughly equivalent to agreeing on the degree of bargaining power that the venture capitalist will have after observing the non-verifiable signal.

An analogous effect occurs if the entrepreneur issues warrants to the venture capitalist. If the terms of conversion of warrants to regular shares are set in advance and are sufficiently favourable to the venture capitalist, the entrepreneur will not engage in as much short-term behaviour in terms of signal manipulation in comparison to a situation where only straight debt-equity financing is used. The paper explains how the model admits warrants in lieu of, or in addition to, convertible debt. Convertible preferred equity is also commonly used in venture capital financing. Until conversion, a convertible preferred stock promises a fixed dividend (and hence is similar to convertible debt). Unlike debt, failure to pay the dividend does not trigger liquidation; rather, the unpaid dividends accrue and must be paid before any dividends are paid out to common stock holders. The model applies, with minor changes, to convertible preferred equity as well. Other major advantages of preferred shares are favourable tax treatment and the fact that preferred shares carry votes with them. Since the model abstracts from taxes and control rights, it would make little difference if convertible preferred equity rather than convertible debt were used.

In the model here, with a straight debt-equity contract, the entrepreneur will always engage in signal manipulation. To underline the damaging effect of such behaviour the paper constructs the model so that if the entrepreneur is unable to commit to not manipulating the signal, the project will not be financed, even if total expected profits without signal manipulation are positive. The paper then shows how to construct a contract with convertible debt such that the entrepreneur will not engage in signal manipulation and, consequently, the venture capitalist will finance the project.

Venture capital convertible debt contracts do not always rely on voluntary debt conversion, but rather on automatic conversion agreements, where debt is converted if certain profit, sales, or performance milestones are attained. Such features fit the model of this paper perfectly, where the central property of the convertible debt contract is the threat that, if the project performs too well in the short term, debt will be converted. Another characteristic of the model is that the optimal capital structure for the project consists of a mixture of debt, equity, and convertible securities, which is very common in real life situations. The paper establishes that such a mixture of instruments is superior to straight debt-equity financing, but does not actually determine the optimal mix of financing instruments.

Most existing explanations for the use of convertible debt are centered on asymmetries of information between managers and investors regarding the firm's value and riskiness. These explanations are mainly suitable for publicly traded convertible securities, but do not fit very well with venture capital financing, which is centred on a close relationship between an entrepreneur and a venture capitalist, and where information regarding the project's quality is gradually revealed to both parties.

1 Introduction

Because of the great uncertainty and high failure risk of new ventures, a widely used financing technique is the infusion of capital over time. The venture capitalist who provides the funds retains the option to abandon the venture at any stage whenever the forward looking net present value of the project is negative. Each financing round is usually related to a significant stage in the development process, such as completion of design, pilot production, first profitability results, or the introduction of a second product. At every stage new information about the venture is released (Sahlman 1990).

Stage financing is appealing to the venture capitalist who preserves a valuable option. First, the option to abandon is essential because an entrepreneur will almost never stop investing in a failing project as long as others are providing capital (Admati and Pfleiderer 1994). Second, since abandonment of the project by the venture capitalist typically forces the entrepreneur to liquidate the project, the option to abandon creates incentives for the entrepreneur to maximize value and meet goals. Gompers (1995) points out that when the fraction of intangible assets in a project is high, agency costs are likely to be high and monitoring is more valuable. He then shows that the number of stage financing rounds increases with the fraction of intangible assets.

Stage financing may not always work as a value enhancing motivating factor. It might induce the entrepreneur to aim for short-term success rather than long-term value creation. In other words, the entrepreneur may try to improve short-term performance in order to make sure that the project will be refinanced (Sahlman 1988, Hellmann 1993, von Thadden 1995).[1] We argue that convertible debt, which is widely used in venture capital financing, can attenuate the incentives to engage in "window dressing" on the part of entrepreneurs. By shifting resources to improving short-term signals the entrepreneur reduces the probability of liquidation but, at the same time, increases the probability that

[1] Narayanan (1985) and Stein (1989) focus on short-termistic behavior of managers whose salaries depend on short-term profitability or stock price performance.

in the event of refinancing the venture capitalist will decide to exercise the debt conversion option, becoming the owner of a substantial fraction of the venture. If the terms of conversion are set in advance to be sufficiently favorable to the debt holder (the venture capitalist), the entrepreneur faces a situation in which good news about the project entail a reduction in his profit because of debt conversion by the venture capitalist. As a consequence he will not engage in as much short-termistic signal manipulation in comparison to a situation where only straight debt-equity financing is used.

The debt conversion option, at a pre-specified conversion ratio that is sufficiently favorable to the entrepreneur, can be interpreted in terms of "renegotiation design," a concept suggested by Aghion, Dewatripont, and Rey (1994). They show that if the bargaining power in various renegotiation contingencies can be allocated in advance in a credible manner, the first best allocation can be implemented despite non-verifiability of a commonly observed signal. In our model, agreeing on a debt conversion option at a predetermined price is roughly equivalent to agreeing on the degree of bargaining power that the venture capitalist will have after observing the non-verifiable signal.

An analogous effect occurs if the entrepreneur issues warrants to the venture capitalist. If the terms of conversion of warrants to regular shares are set in advance and are sufficiently favorable to the venture capitalist, the entrepreneur will not engage in as much short-termistic signal manipulation in comparison to a situation where only straight debt-equity financing is used. We explain how our model admits warrants in lieu of or in addition to convertible debt. Convertible preferred equity is also commonly used in venture capital financing. Until conversion, a convertible preferred stock promises a fixed dividend (and hence is similar to convertible debt). Unlike debt, failure to pay the dividend does not trigger liquidation; rather, the unpaid dividends accrue and must be paid before any dividends are paid out to common stock holders. Our model applies, with minor changes, to convertible preferred equity as well. Other major advantages of preferred shares are the favorable tax treatment and the fact that preferred shares carry votes with them. Since our model abstracts from taxes and control rights, it would make little difference if we used convertible preferred equity rather than convertible debt.[2]

In our model, with a straight debt-equity contract the entrepreneur will always engage in signal manipulation. To underline the damaging effect of such behavior we construct the model so that if the entrepreneur cannot commit not to manipulate the signal, the project will not be financed, even if the total expected profits without signal manipulation are positive. We then show how to construct a contract with convertible debt such that the entrepreneur will not engage in signal manipulation and, consequently, the venture capitalist will finance the project.

Venture capital convertible debt contracts sometimes do not rely on voluntary debt conversion, but rather on automatic conversion agreements, where debt is converted if certain profit, sales, or performance milestones are attained.[3] Such features fit exactly our model, where the central property of the convertible debt contract is the threat that, if the project performs too well in the short term, debt will be converted. Another characteristic of our model is that the optimal capital structure for the project consists of a mixture of debt, equity, and convertible securities which is very common in real life situations. We establish that such a mixture of instruments is superior to straight debt-equity financing, but we do not actually determine the optimal mix of financing instruments.[4]

Existing explanations for the use of convertible debt are centered on asymmetries of information between managers and investors regarding the firm's value and riskiness (Green 1984, Brennan and Schwartz 1988, Harris and Raviv 1985, Stein 1992, and Nyborg 1995). These explanations are mainly suitable for publicly traded convertible secu-

[2]For an analysis of transfers of control in venture capital financing situations see, for example, Chan, Siegel, and Thakor (1990) and Trester (1996).

[3]Gompers (1996, Table 4) reports, for a sample of 50 convertible preferred equity financings by venture capital firms, that 38 percent of the contracts included automatic conversion covenants prior to the initial public offering, where conversion would take place if annual income were to exceed some pre-specified amount.

[4]The optimal mix of debt, equity, and convertible debt depends, among other things, on the bargaining power of the parties, which determines how the surplus (created by the presence of convertible debt) is split among them.

rities, but do not fit very well with venture capital financing which is centered on a close relationship between an entrepreneur and a venture capitalist, and where information regarding the project's quality is gradually revealed to both parties.[5] The exceptions are Green (1984), where convertible debt affects the inclination of the entrepreneur to engage in risky projects, and is therefore particularly useful in start-up financing situations, and Gompers (1996) who also focuses on the use of convertible debt in venture capital financing.

Of related interest is the literature analyzing long term financing contracts with information arising in the short term (a typical characteristic of venture capital financing). Both Hellmann (1993) and von Thadden (1995) study situations where the entrepreneur might lose sight of the big picture, focusing too narrowly on meeting short-term goals. von Thadden (1995) shows that a contract that resembles a long-term credit line can reduce such short-termistic behavior in stage financing situations, while Hellmann (1993) concentrates on an optimal debt-equity mix. Bagwell and Zechner (1993) analyze a situation where managers of a division of a firm can exert effort to influence a divestiture decision by the top management, which is a different context in which an agent attempts to influence the decision of a principal to discontinue a project. They study the role of the capital structure in reducing the cost of such behavior. Most of this literature focuses on the choice of the debt-equity mix, while our focus is on how to improve on any given debt-equity contract by using convertible debt. The exceptions are Marx (1993) and Berglöf (1994). Marx (1993) argues that when the venture capitalist is risk averse, convertible preferred equity dominates both only equity and only debt financing by generating the right incentives for the venture capitalist to intervene in the project as a response to poor performance. Berglöf (1994) focuses on the role of convertible securities in mitigating the distributional conflicts associated with a future sale of the firm.

In the next section we present the model under the assumption that debt-equity con-

[5]See Sahlman (1988) and Gorman and Sahlman (1989). Gompers (1996) argues that venture capital convertible debt contracts are quite different from convertible debt contracts in large public corporations.

tracts cannot be renegotiated, and show how the introduction of convertible debt generates an improvement for both parties. In Section 3 we relax this restriction, showing that when renegotiation of debt-equity contracts is allowed, convertible debt contracts are strictly better than straight debt-equity in most cases, and equivalent to debt-equity contracts in some cases. In Section 4 we further extend the model by introducing an additional decision for the entrepreneur—whether or not to exert effort which affects long term profits. The basic results are unaffected by this generalization of the model. Section 5 concludes.

2 The Model

At time 0 an entrepreneur considers undertaking a project with uncertain returns. There are two possible states of nature: with probability 1/2 the state is "good", which we index $y = y_h$, and with probability 1/2 the state is "bad," $y = y_l$. If $y = y_i$, the project will generate a (stochastic) output, π_i, distributed exponentially with density $\lambda_i e^{-\lambda_i \pi_i}$ on the interval $[0, \infty)$. Let $\lambda_h < \lambda_l$, i.e. $1/\lambda_h$, the mean of π_h, is larger than $1/\lambda_l$, the mean of π_l.

The entrepreneur has no capital and, therefore, will ask a venture capitalist to lend him the required funds. The total amount of money necessary for the project is $I_1 + I_2$, of which I_1 must be invested at time 0, and I_2 at time 1 provided that the project is not liquidated.

Information and signal manipulation

At time 1, a signal, x, about the distribution of y is realized and is observed by the entrepreneur and the venture capitalist. The signal can be, for example, the short-term performance of the project, observed by both parties who update their expectations about the quality of the project. The signal can take three values: x_b (bad), x_m (medium), and x_g (good). A bad signal is a sure indication that the project is bad (i.e. that the state of nature is y_l), while a good signal indicates with certainty that $y_i = y_h$. Most of the time, however, a medium signal is observed, leaving room for uncertainty regarding the state of nature. In other words, short-term performance can be extremely good or bad so there is no doubt regarding the long-run prospects of the project, or it may be satisfactory with some uncertainty remaining. The signal is observed by both parties but is not verifiable. Therefore, it is not possible at time 0 to write a contract contingent on the signal x.

Between time 0 and time 1, the entrepreneur can engage in "window-dressing," artificially improving short-term performance. By artificially we mean that such activity does not affect the probability of a good outcome ($y_i = y_h$) in the long-run, which remains 1/2, but simply reduces the probability that the project is revealed to be bad already at time 1. Let us call such activity signal manipulation and denote it $a = 1$, while $a = 0$ denotes no signal manipulation. The venture capitalist cannot observe whether the entrepreneur engages in signal manipulation.[6]

Table 1 displays the joint distribution of signal and output when $a = 0$ (no signal manipulation) and when $a = 1$ (signal manipulation). The parameter $\epsilon \in (0, 1/3)$ is exogenous and captures the extent to which the entrepreneur can manipulate the signal. By manipulating the signal the entrepreneur reduces the probability of x_b from 1/3 to ϵ and increases the probability of x_g from ϵ to 1/3. Furthermore, signal manipulation reduces the probability of y_h conditional on observing x_m (in comparison to the probability of y_h conditional on x_m when there is no signal manipulation). This is perfectly intuitive: when the venture capitalist knows that the entrepreneur has manipulated the signal, he relies less on short-term performance. Finally, note that signal manipulation does not affect the ex-ante probability of y_h which remains 1/2.

To simplify the exposition, the joint distribution in Table 1 is such that x_b and x_g are precise signals, i.e. after observing them the venture capitalist and the entrepreneur are certain that the state of nature is, respectively, bad or good. However, this feature is not

[6] We assume that the entrepreneur can manipulate the signal costlessly. In Section 4 we introduce both a direct cost of signal manipulation (born by the entrepreneur) and an indirect cost—signal manipulation comes at the expense of effort devoted to the improvement of long-run performance. The results do not change in a meaningful way.

Table 1: Joint distribution of output (y) and signal (x) with no signal manipulation ($a = 0$) and with signal manipulation ($a = 1$). The state can be bad, with low output (y_l), or good, with high output (y_h), and the signal can be bad (x_b), medium (x_m), or good (x_g). The parameter ϵ is exogenous.

	No signal manipulation ($a = 0$)			With signal manipulation ($a = 1$)		
	x_b	x_m	x_g	x_b	x_m	x_g
Low output: y_l	$\frac{1}{3}$	$\frac{1}{6}$	0	ϵ	$\frac{1}{2} - \epsilon$	0
High output: y_h	0	$\frac{1}{2} - \epsilon$	ϵ	0	$\frac{1}{6}$	$\frac{1}{3}$

necessary for our result. For example, if we modified Table 1 by letting the probability of the events "x_b and y_h" and "x_g and y_l" be $\alpha > 0$ (rather than zero), the same results would go through provided that α is not too high.[7]

For our main result, we need to assume that the ratio λ_l/λ_h is "large enough," i.e. that the profitability of the project in the good state, relative to the bad state, is sufficiently big. More precisely, we assume that

$$\frac{\lambda_h}{\lambda_l} e^{\frac{\lambda_l}{\lambda_h} - 1} \geq 4 - 6\epsilon \tag{1}$$

which holds for λ_l/λ_h large enough since $\lim_{y \to \infty} \frac{1}{y} e^{y-1} = \infty$. To give a sense of the required order of magnitude for this ratio, $\lambda_l/\lambda_h \geq 4$ is a sufficient (but not necessary) condition for Proposition 2 to hold. After presenting the proposition, we provide the intuition for requirement (1).

[7] A high value of α renders the good signal not so good after all.

Stage financing

After observing the signal, the venture capitalist decides whether to refinance the project and supply I_2 to the entrepreneur, or liquidate. For simplicity, we assume that the payoff to both parties following liquidation is zero. The venture capitalist will refinance if his interim payoff is positive. If the project is refinanced, the output is realized at time 2, and is distributed to the entrepreneur and the venture capitalist according to the contract that has been agreed upon. In principle, the venture capitalist could commit at time 0 to supply both amounts I_1 and I_2, irrespectively of the signal x. Committing the entire amount necessary for a project is not common in venture capital financing situations, where the ex-ante risk of failure is often too high compared to the required investment. We, therefore, assume that

$$\frac{1}{2}\frac{1}{\lambda_l} + \frac{1}{2}\frac{1}{\lambda_h} - (I_1 + I_2) < 0, \tag{2}$$

namely, committing the entire amount $I_1 + I_2$ up front is not profitable. Notice that the (ex-ante) payoff in (2) is the same for $a = 0$ and $a = 1$. This is because signal manipulation affects only the (interim) probabilities of the states of nature conditional on the observed signal, but not the true (ex-ante) probabilities. Requirement (2) implies that stage financing is the only viable method for undertaking the venture.

Debt-equity financing

The venture capitalist can provide funding to the entrepreneur in the form of debt, equity, or a combination of the two. We denote a debt-equity contract by (d, s), where d is the amount of debt that the entrepreneur owes the venture capitalist, and s is the fraction of the enterprise's equity owned by the venture capitalist. For the sake of clarity, in this section we restrict attention to debt-equity contracts that cannot be renegotiated at time 1. In Section 3 we relax this restriction, explaining when a debt-equity contract will be renegotiated. The essence of the analysis goes through also when debt-equity contracts can be renegotiated at time 1. Studying the model with the possibility of renegotiation develops our intuition for why convertible debt solves the window-dressing problem.

Consider the debt-equity contract by (d, s). The magnitudes d and s are determined at time 0 (prior to the choice of a by the entrepreneur), and do not change throughout the life-time of the venture. At time 0, I_1 is supplied. At time 1, if the project is not liquidated after observing the signal, I_2 is supplied. At time 2, the output is divided according to d and s. The promised amount d may be larger or smaller than $I_1 + I_2$, the total amount of financing.[8] We assume $s \in [0, 1)$, i.e. the entrepreneur must own some shares. This is easily justified on the grounds that otherwise, the entrepreneur will not make any effort for the project to succeed in the long-run.[9] Of course, the debt-equity contract must be acceptable to both parties, namely, d and s must be such that, given the level of a optimally chosen by the entrepreneur, the ex-ante payoff to both parties is non-negative. We provide a detailed analysis later.[10]

Convertible debt

Alternatively, the venture capitalist and the entrepreneur can sign a convertible debt contract which we characterize in the following way. At time 0, the initial debt and equity

[8] For example, $d > I_1 + I_2$ reflects a positive interest rate.

[9] In Section 4 we introduce an additional choice of effort by the entrepreneur, that affects long-run (time 2) profits. We will show that, indeed, $s < 1$.

[10] Since x is not verifiable, it is not possible to write in the contract at time 0 that I_2 will depend on x. Further, also d and s must be specified in the contract at time 0. It can be shown (but we omit here the detailed explanation for simplicity) that both parties will prefer to specify d and s at time 0, taking into account the entire loan $I_1 + I_2$. The intuition relies on the fact that one of the two parties may end up being held up at time 1. If the entrepreneur has enough bargaining power, then at date 1 the venture capitalist may be "locked in." For example, if the entrepreneur has all the bargaining power, then at time 1 he will offer a new debt-equity contract which guarantees the venture capitalist just enough (expected) revenue to cover I_2. The venture capitalist will accept the offer since I_1 is sunk. Foreseeing this, the venture capitalist will prefer to have d and s specified at time 0. A similar logic can be applied to the entrepreneur if his participation in the project requires an arbitrarily small amount of effort at time 0 (as we will assume in Section 4). Then, at time 1 the entrepreneur is "locked in"—since his investment in effort is sunk—and if the venture capitalist has enough bargaining power he can extract part of the entrepreneur's surplus. As a result, the entrepreneur will prefer to specify d and s at time 0. Therefore, defining the cum interest debt, d, relative to the entire amount of financing, $I_1 + I_2$, and s as a share of the ex-post profits of the project, is without loss of generality.

holdings by the venture capitalist are $d_0 > 0$ and $s_0 \geq 0$. Let $d_0 - d > 0$ of the debt be convertible into equity at the conversion ratio γ satisfying $s - s_0 = \frac{1}{\gamma}(d_0 - d)$, where s and d are the post-conversion debt and equity positions after *all* the convertible debt is converted, and $s - s_0$ is the fraction of the equity purchased. A special case is 100 percent initial debt financing ($s_0 = 0$). Another special case is that all the debt is convertible ($d = 0$). A contract with convertible debt is, therefore, characterized by $(d_0, s_0, \gamma, d_0 - d)$.

At time 0, I_1 is supplied. At time 1, after observing the signal, the venture capitalist decides whether to liquidate or refinance. He can refinance without converting any debt, in which case the output is divided at time 2 according to d_0 and s_0, as in the case of a straight debt-equity contract. Alternatively, he can refinance and convert any amount smaller or equal to $d_0 - d$ at the pre-determined conversion ratio γ.[11] Denote by $\hat{d} \geq d$ and $\hat{s} \in [s_0, s]$ the post-conversion debt and equity positions after converting some amount $d_0 - \hat{d} \leq d_0 - d$ at the conversion rate γ, namely, $\hat{s} - s_0 = \frac{1}{\gamma}(d_0 - \hat{d})$. At time 2, the output is divided according to the new debt and equity positions, $\hat{d}$ and $\hat{s}$. We provide a detailed analysis later, where we also show that renegotiating γ is not possible since any change in γ, after observing the signal x, entails a loss to one of the parties.

The analysis proceeds in the following way. Given the contract that is in effect, the entrepreneur decides whether to manipulate the signal so as to maximize his ex-ante payoff. We will show that for any straight debt-equity contract, the entrepreneur will choose to manipulate the signal. To underline the loss of welfare due to the manipulation of the signal, we will assume that the parameters are such that if there is signal manipulation, debt-equity financing will not take place in the first place. Moreover, we will show that

[11]The deadline for conversion is irrelevant as long as it is prior to the total resolution of uncertainty, at time 2. Since no income accrues to the shareholders before time 2, the venture capitalist has no interest in converting debt before observing the signal x. The time of debt conversion in the model seems, therefore, to capture a realistic feature of venture capital financing, namely, that the decision whether to convert debt, and how much to convert, is taken at a time when there is still much uncertainty remaining regarding the success of the venture. At the end of this section we will show that it is never optimal to set the deadline at time 2, i.e. after the resolution of uncertainty.

for the same parameter values there is a convertible debt contract, $(d_0, s_0, \gamma, d_0 - d)$, such that the entrepreneur will choose not to engage in signal manipulation and, therefore, the project will be financed.

The liquidation decision

The entrepreneur never wants to liquidate the project once it has started since he provides no financing and, because of limited liability, he always obtains a positive payoff as long as $s < 1$.[12] The driving force behind our result is the conflict of interest between the entrepreneur, who always wants to proceed with the project, and the venture capitalist who wants to refinance the project only if there are favorable interim news regarding the probability of success. One could imagine that the venture capitalist will choose to continue the project if the interim performance is above a minimum threshold. For simplicity, we have modeled the interim performance with the discrete signal, x. We, therefore, assume that the parameters are such that if the signal is bad the venture capitalist will liquidate the project, while if it is medium or good he will provide additional financing. Also, the decision whether or not to convert debt will depend on the interim performance. To ensure that the venture capitalist will liquidate following x_b, we assume that the (interim) expected profit of the project, conditional on a bad signal, is negative,

$$\frac{1}{\lambda_l} - I_2 < 0, \tag{3}$$

[12] Moreover, we can imagine that any investment by the entrepreneur in human capital is firm-specific and, hence, sunk.

which is true with or without window dressing.[13] To ensure that the (interim) expected profit of the project, conditional on a medium signal, is positive we assume that

$$\left(\frac{1/2-\epsilon}{2/3-\epsilon}\right)\frac{1}{\lambda_l}+\left(\frac{1/6}{2/3-\epsilon}\right)\frac{1}{\lambda_h}-I_2>0. \tag{4}$$

Condition (4) implies that total (interim) expected profits are positive when $a = 1$, and therefore also when $a = 0$. Condition (4) thus ensures that there exists a debt-equity contract such that, following the signal x_m, the venture capitalist's (interim) expected share of the output, if the project is continued, is positive (we spell out the details later). In sum, following x_b the venture capitalist liquidates the project, while following x_m, and hence also following x_g, he may refinance it.

We assume that, conditional on $a = 0$, the ex-ante expected profits of the project are positive. Since by (3), the venture capitalist always liquidates the project after observing x_b, ex-ante expected profits (conditional on $a = 0$) are

$$\frac{1}{6}\frac{1}{\lambda_l}+\frac{1}{2}\frac{1}{\lambda_h}-I_1-\frac{2}{3}I_2>0. \tag{5}$$

In other words, with the introduction of stage financing, if there is no signal manipulation the project is worth financing so, in particular, there is a debt-equity contract such that the venture capitalist would agree to lend I_1 at time 0. We further assume that if the signal is manipulated ($a = 1$), the ex-ante expected profits of the project are negative,

$$(\frac{1}{2}-\epsilon)\frac{1}{\lambda_l}+\frac{1}{2}\frac{1}{\lambda_h}-I_1-(1-\epsilon)I_2<0, \tag{6}$$

and the venture capitalist will not agree to provide I_1 at time 0. By manipulating the

[13] We do not actually need such a strong assumption. The venture capitalist will choose not to refinance if *his* payoff, not the entire expected output, is lower than I_2. For simplicity, we make the stronger assumption (3) which allows us to claim that for any debt-equity contract, the venture capitalist liquidates the project after observing a bad signal.

signal, the entrepreneur reduces to ϵ the probability that the project will be liquidated, but in so doing he also reduces the benefit of introducing stage financing (i.e. he reduces the value of the option to abandon the project) rendering the project nonviable. However, we are now going to show that the entrepreneur, once debt-equity financing has been obtained, will always want to manipulate the signal.

Signal manipulation with debt-equity financing

Consider the debt-equity contract (d, s). Assuming limited liability and zero bankruptcy costs, the ex-post payoffs (after the state of nature is revealed but before profits are realized) to the venture capitalist and the entrepreneur when $y = y_i$, $i = l, h$, are

$$\begin{aligned}
\mathcal{VC}_i &= \int_0^d \pi_i \lambda_i e^{-\lambda_i \pi_i} d\pi_i + \int_d^\infty [d + s(\pi_i - d)]\lambda_i e^{-\lambda_i \pi_i} d\pi_i \\
&= \tfrac{1}{\lambda_i}[1 - e^{-\lambda_i d}(1-s)], \\
\mathcal{E}_i &= \int_d^\infty (1-s)(\pi_i - d)\lambda_i e^{-\lambda_i \pi_i} d\pi_i \\
&= \tfrac{1}{\lambda_i} e^{-\lambda_i d}(1-s).
\end{aligned} \tag{7}$$

Interim payoffs following the signal x_j are weighted averages of ex-post payoffs, minus I_2 for the venture capitalist (at time 1, I_1 is already sunk):

$$\begin{aligned}
E[\mathcal{VC}]_j &= q_{lj}(a)\tfrac{1}{\lambda_l}\left[1 - e^{-\lambda_l d}(1-s)\right] + q_{hj}(a)\tfrac{1}{\lambda_h}\left[1 - e^{-\lambda_h d}(1-s)\right] - I_2, \\
E[\mathcal{E}]_j &= q_{lj}(a)\tfrac{1}{\lambda_l} e^{-\lambda_l d}(1-s) q_{hj}(a)\tfrac{1}{\lambda_h} e^{-\lambda_h d}(1-s),
\end{aligned} \tag{8}$$

where $q_{ij}(a)$ is the probability of y_i conditional on the signal x_j, given a. For instance, if $a = 0$, $q_{hm}(0) = \frac{1/2-\epsilon}{2/3-\epsilon}$ and $q_{lm}(0) = \frac{1/6}{2/3-\epsilon}$. The ex-ante payoff of the entrepreneur is the weighted average of the interim payoffs. The ex-ante payoff of the venture capitalist is the weighted average of the interim payoffs minus the investment I_1. Of course, the ex-ante payoffs take into account the liquidation decision of the venture capitalist, namely the fact that if the signal is x_b the project is terminated.

From (5) and (6) it follows that the venture capitalist may finance the project (i.e., provide seed financing I_1) only if he anticipates that $a = 0$. However, the entrepreneur cannot commit not to manipulate the signal (since a is not observable).

Proposition 1 *For any debt-equity contract, the entrepreneur will manipulate the signal, i.e., in equilibrium* $a = 1$.

Proof. We begin by showing that $a = 0$ cannot occur in equilibrium. Consider (d, s) such that if $a = 0$ the venture capitalist refinances the project after observing x_m, namely, his payoff in (8) is positive for $a = 0$. Suppose that (by way of contradiction) in equilibrium $a = 0$. If the entrepreneur indeed chooses $a = 0$ his ex-ante payoff is

$$E[\mathcal{E}]_{a=0} = \frac{1}{6}\frac{1}{\lambda_l}e^{-\lambda_l d}(1-s) + \frac{1}{2}\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s). \tag{9}$$

This payoff is constructed as follows. With probability 1/3 the signal is x_b and the payoff to the entrepreneur is zero. With the remaining probability the signal is x_m or x_g and the venture capitalist will refinance the project. Then, $y_i = y_l$ occurs with probability 1/6, generating the payoff $\frac{1}{\lambda_l}e^{-\lambda_l d}(1-s)$, and $y_i = y_h$ occurs with probability 1/2, generating the payoff $\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s)$. If the entrepreneur deviates choosing $a = 1$ (while the venture capitalist believes that $a = 0$), he gets

$$E[\mathcal{E}]_{a=1} = (\frac{1}{2} - \epsilon)\frac{1}{\lambda_l}e^{-\lambda_l d}(1-s) + \frac{1}{2}\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s), \tag{10}$$

which is surely higher. Therefore $a = 0$ is not optimal for the entrepreneur, and cannot occur in equilibrium.

We have shown that if the venture capitalist refinances the project after observing x_m, $a = 0$ cannot occur in equilibrium. Let us consider the other possibility, namely that (d, s) is such that the venture capitalist liquidates the project after observing x_m, and suppose (again, by way of contradiction) that the entrepreneur chooses $a = 0$. The payoff to the entrepreneur is

$$E[\mathcal{E}]_{a=0} = \epsilon \frac{1}{\lambda_h} e^{-\lambda_h d}(1-s). \quad (11)$$

By deviating to $a = 1$ (while the venture capitalist believes that $a = 0$) the entrepreneur increases his payoff to

$$E[\mathcal{E}]_{a=1} = \frac{1}{3}\frac{1}{\lambda_h} e^{-\lambda_h d}(1-s). \quad (12)$$

Therefore $a = 0$ can never occur in equilibrium.

We now show, using the same logic, that if (for some reason) the project is financed at time 0 with a straight debt-equity contract, in equilibrium the entrepreneur manipulates the signal ($a = 1$). Consider (d, s) such that if $a = 1$ the venture capitalist refinances the project after observing x_m, namely, his payoff in (8) is positive for $a = 1$. If the entrepreneur chooses $a = 1$ he obtains the payoff in (10) while if he chooses $a = 0$ he obtains the payoff in (9) which is smaller. Similarly, for (d, s) such that if $a = 1$ the venture capitalist liquidates the project after observing x_m, if the entrepreneur chooses $a = 1$ he obtains the payoff in (12) while if he chooses $a = 0$ he obtains the smaller payoff in (11).[14]
qed

We have demonstrated that if the venture capitalist agrees to finance the project, $a = 1$ is optimal for the entrepreneur, whereas $a = 0$ is not. However, we assumed that if $a = 1$ the project is not worth financing in the first place (see (6)). Therefore, the project, which is profitable provided there is no signal manipulation, will not be financed with a straight debt-equity contract because straight debt-equity fails to provide the incentives for no signal manipulation, rendering the project nonviable.

[14]This feature of the model resembles Stein (1989) where managers try to inflate the price of the company shares. Although the market is perfectly able to anticipate this behavior, the managers will nevertheless engage in signal manipulation since they take market expectations as given.

The role of convertible debt in preventing signal manipulation

We will show that with an appropriately designed financing scheme that combines equity, debt, and convertible debt, the project will be financed, and the entrepreneur will not engage in signal manipulation. Consider a convertible debt contract, $(d_0, s_0, \gamma, d_0 - d)$, where d_0 and s_0 are the initial debt and equity positions, and γ is the conversion ratio. The debt and equity positions after conversion of an amount of convertible debt $d_0 - \hat{d} \leq d_0 - d$, are $\hat{d}$ and $\hat{s}$,where $\hat{s} - s_0 = \frac{1}{\gamma}(d_0 - \hat{d})$. Therefore, the ex-post payoffs to the venture capitalist and the entrepreneur when $y = y_i$, $i = l, h$, are

$$\begin{aligned} \mathcal{VC}_i &= \tfrac{1}{\lambda_i}[1 - e^{-\lambda_i[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})], \\ \mathcal{E}_i &= \tfrac{1}{\lambda_i}e^{-\lambda_i[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s}). \end{aligned} \tag{13}$$

If the venture capitalist converts the entire amount of convertible debt, then $\hat{s} = s$ and $\hat{d} = d$. The interim payoffs are weighted averages of such ex-post payoffs (minus I_2 for the venture capitalist).

The conversion ratio, γ, is not renegotiable after the signal x is observed. At time 1, since the entrepreneur and the venture capitalist are risk neutral, there can be no mutual gain from changing the terms of the contract—any such change will benefit one party at the expense of the other. For example, following x_g, a small increase in γ renders conversion more expensive, benefiting the entrepreneur, while a decrease in γ benefits the venture capitalist. A large change in γ may alter the venture capitalist's decision whether to convert debt (following x_m or x_g). If it does, then the venture capitalist's expected share of the output must increase. This follows from the fact that the venture capitalist is free to choose whether to convert debt or not, so he will always choose the action that maximizes his payoff. By risk neutrality, an increase in the venture capitalist's expected share of the output implies that the entrepreneur's expected share must decrease. Thus, there can be no renegotiation of γ.

We will now construct a convertible debt contract, $(d_0, s_0, \gamma, d_0 - d)$, such that in equilibrium the entrepreneur will not manipulate the signal ($a = 0$) and, therefore, the project will be financed. The contract has the feature that the venture capitalist converts the entire amount $d_0 - d$ if he observes x_g, and converts nothing if he observes x_m. Given this behavior of the venture capitalist, the entrepreneur finds it optimal to choose $a = 0$, and thus, the venture capitalist agrees to provide seed financing at time 0. This result is summarized in the following proposition:

Proposition 2 *There is a convertible debt contract, $(d_0, s_0, \gamma, d_0 - d)$, such that in equilibrium there is no signal manipulation and the project is financed.*

A detailed proof is provided in the Appendix. We provide here the main steps of the construction, and their economic significance.[15]

The proof consists of constructing a convertible debt contract such that in equilibrium: (a) The venture capitalist agrees to provide I_1 at time 0; (b) the venture capitalist liquidates the project after observing x_b but not otherwise; (c) the venture capitalist does not convert any debt after observing x_m; (d) the venture capitalist converts the entire amount $d_0 - d$ after observing x_g; (e) the entrepreneur chooses $a = 0$. The choice of γ, the conversion ratio, plays a key role in the construction. If γ is low, converting debt into equity is cheap and the venture capitalist will always convert debt if he decides to refinance the project. If γ is high, the venture capitalist will never convert debt. The fact that γ is not too high guarantees that when the venture capitalist chooses to convert debt into equity after observing a good signal, conversion takes place at a very convenient rate which hurts the entrepreneur (in other words, the venture capitalist is buying underpriced equity). This threat of debt conversion at a low price induces the entrepreneur to refrain from engaging in signal manipulation.

[15]Typically, there will be many such contracts. The choice among them will depend on the relative bargaining power of entrepreneur and venture capitalist which will determine the division of surplus between them.

Recall that (3) implies that after observing a bad signal, x_b, the venture capitalist liquidates the project (requirement (b)). This constitutes the basic motivation for the entrepreneur to manipulate the signal, rendering straight debt-equity financing nonviable.

The decision of the venture capitalist how much debt to convert is a solution to the maximization of his interim payoff with respect to $\hat{s}$. After observing x_g the venture capitalist maximizes

$$E[\mathcal{VC}]_g = \frac{1}{\lambda_h}\left[1 - e^{-\lambda_h[d_0-\gamma(\hat{s}-s_0)]}(1-\hat{s})\right] \tag{14}$$

with respect to $\hat{s}$, and after observing x_m he maximizes

$$\begin{aligned} E[\mathcal{VC}]_m \;=\; & \tfrac{1/6}{2/3-\epsilon}\,\tfrac{1}{\lambda_l}\left[1 - e^{-\lambda_l[d_0-\gamma(\hat{s}-s_0)]}(1-\hat{s})\right] \\ & +\tfrac{1/2-\epsilon}{2/3-\epsilon}\,\tfrac{1}{\lambda_h}\left[1 - e^{-\lambda_h[d_0-\gamma(\hat{s}-s_0)]}(1-\hat{s})\right] \end{aligned} \tag{15}$$

with respect to $\hat{s}$. Notice that in these expressions we have substituted for $\hat{d}$ using $\hat{s} - s_0 = \frac{1}{\gamma}(d_0 - \hat{d})$.

The convertible debt contract that we construct in the Appendix has the property that for any given s_0, γ is chosen so that[16]

$$\frac{1}{\lambda_h} - \gamma(1 - s_0) = 0. \tag{16}$$

Condition (16) ensures that for any initial equity position, s_0, the price of conversion, γ, is sufficiently low for the venture capitalist to convert the entire amount $d_0 - d$ following a good signal, x_g, i.e., (16) is maximized at $\hat{s} = s$ (requirement (d)). This constitutes the "threat" that prevents the entrepreneur from manipulating the signal.

We also want to ensure that when $a = 0$ (no signal manipulation), the venture capitalist does not convert any debt after observing a medium signal, x_m, i.e. that (15) is maximized at $\hat{s} = s_0$ (requirement (c)). This creates the motivation for the entrepreneur not to

[16]The construction does not rely on "razor's edge" arguments, and goes through if this equality is not satisfied exactly.

manipulate the signal. In the Appendix we show that, given (16), the expression in (15) is strictly convex with respect to $\hat{s}$, and is maximized either when $\hat{s} = s$ (the maximal possible amount is converted) or when $\hat{s} = s_0$ (no debt is converted). The latter is optimal for the venture capitalist if

$$\begin{aligned} \frac{1}{6}\frac{1}{\lambda_l}\left[1 - e^{-\lambda_l[d_0-\gamma(s-s_0)]}(1-s)\right] \quad &+ \quad (\tfrac{1}{2}-\epsilon)\frac{1}{\lambda_h}\left[1 - e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s)\right] - I_2 \leq \\ \frac{1}{6}\frac{1}{\lambda_l}\left[1 - e^{-\lambda_l d_0}(1-s_0)\right] \quad &+ \quad (\tfrac{1}{2}-\epsilon)\frac{1}{\lambda_h}\left[1 - e^{-\lambda_h d_0}(1-s_0)\right] - I_2. \end{aligned} \tag{17}$$

The initial equity position of the venture capitalist, s_0, and the conversion ratio, γ, implied by condition (16) are such that converting a small amount has no effect on his payoff in the bad state and a negative effect on his payoff in the good state. That is, given the conversion ratio, he would have actually liked to increase his debt position at the margin (which, of course, he cannot do). If the amount of convertible debt is sufficiently large, further debt conversion starts becoming profitable for the venture capitalist at the margin. Condition (17) ensures that the latter effect dominates. An important implication of this step in the proof is that for the contract to be effective, the amount of convertible debt must be sufficiently big.

To ensure that, given the behavior of the venture capitalist, the entrepreneur prefers not to manipulate the signal (requirement (e)), we need the condition

$$\begin{aligned} &\frac{1}{6}\frac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + (\tfrac{1}{2}-\epsilon)\frac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0) + \epsilon\frac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s) \geq \\ &(\tfrac{1}{2}-\epsilon)\frac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + \frac{1}{6}\frac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0) + \frac{1}{3}\frac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s). \end{aligned} \tag{18}$$

This condition compares the entrepreneur's ex-ante payoff without signal manipulation (the left hand side) to his ex-ante payoff with signal manipulation, given that the venture capitalist converts debt when he observes x_g but not when he observes x_m. The intuition for (18) is quite simple. By manipulating the signal the entrepreneur reduces the probability of liquidation (which is to his advantage), at a cost: signal manipulation increases the probability of a good signal inducing the venture capitalist to convert debt. If it is advantageous for the venture capitalist to convert debt, the entrepreneur is worse off. When this effect is sufficiently strong the entrepreneur will not manipulate the signal.

The last condition that has to be verified is the ex-ante participation condition for the venture capitalist (requirement (a)),[17]

$$\begin{aligned} &\tfrac{1}{6}\tfrac{1}{\lambda_l}[1 - e^{-\lambda_l d_0}(1 - s_0)] \ + \ \left(\tfrac{1}{2} - \epsilon\right)\tfrac{1}{\lambda_h}[1 - e^{-\lambda_h d_0}(1 - s_0)] \\ &\quad + \epsilon\tfrac{1}{\lambda_h}[1 - e^{-\lambda_h[d_0 - \gamma(s - s_0)]}(1 - s)] - I_1 - \tfrac{2}{3}I_2 \ \geq \ 0, \end{aligned} \tag{19}$$

where the first three terms correspond to the three outcomes that occur with positive probability following x_m or x_g when $a = 0$ (see Table 1); I_2 is multiplied by the probability that refinancing will occur when $a = 0$.

Conditions (17), (18), and (19) depend on the parameters λ_l and λ_h. Assumption (1), that the difference between $1/\lambda_h$ and $1/\lambda_l$ is "large enough," ensures that these conditions hold. The intuition is quite simple. The contract must be such that the venture capitalist chooses not to convert debt when he observes x_m (i.e. when the state of nature y_l is more likely) and to convert debt when he observes x_g (i.e. when the state of nature y_h is more likely) which is more easily achieved when profits are very different across states of nature. Furthermore, debt conversion must constitute a sufficiently large threat for the entrepreneur, which is more likely to happen if expected profits in the good state—when debt conversion occurs—are very high, namely, when the venture capitalist buys underpriced equity the loss to the entrepreneur is very big.

In conditions (17), (18), and (19) the post-conversion debt level, d, is eliminated using $s - s_0 = \frac{1}{\gamma}(d_0 - d)$. The three inequalities depend on the parameters d_0, s_0, s, and γ. Condition (16) pins down γ for any value of s_0. In the Appendix it is shown that, after substituting for γ using (16), values of $d_0 > 0$, $s_0 \in [0, 1]$, and $s \in (s_0, 1]$ can be found so that

[17] The entrepreneur will always agree to participate since he is not committing any resources of his own.

conditions (17), (18), and (19) hold strictly. This is the sense in which the construction does not rely on "razor's edge" arguments. In particular, condition (16) may hold only approximately without affecting the validity of the proof.

The role played by the nonverifiability of x should now be apparent. If x were verifiable then the parties could agree at time 0 on a contingent debt-equity capital structure, as a function of the realization of x. The above convertible debt contract could then be mimicked by a contingent debt-equity contract. Nonverifiability of x rules out such a contract, highlighting the fundamental role of the debt conversion option which is to adjust the debt-equity structure to new information that is revealed during the life-time of the project. This feature bears resemblance to provisions allowing the provider of financing to take control of the project at some interim stage in response to nonverifiable information, a feature that is very common in incomplete contracts environments. Here, as well as in von Thadden (1995), the contract allows the provider of financing to alter the financial structure of the contract (to change the output allocation rule) at some interim stage in response to nonverifiable information. Furthermore, securities where debt conversion occurs automatically in response to a verifiable performance variable such as profits or sales (Gompers 1996), are none other than contingent debt-equity contracts that resemble exactly the contracts we would obtain in our model when x is verifiable.

Warrants

In the above analysis we restricted attention to $d_0 > 0$ (strictly positive initial debt) and $d \geq 0$ (non-negative post-conversion debt). Suppose, for example, that $d_0 = 0$ (no initial debt). Then $s - s_0 = \frac{1}{\gamma}(d_0 - d)$ is interpreted as follows: The venture capitalist acquires $s - s_0$ shares at the exercise price γ dollars per share, paying a total of $|d|$ dollars.[18] We can think of the entrepreneur issuing to the venture capitalist a certain number of warrants at

[18]Since $s - s_0 > 0$ and $d_0 = 0$, d must be a negative number. The convention $d < 0$ is not convenient to work with; we use it here only to highlight that the analysis for warrants is identical to that of convertible debt.

time 0, with the initial contract specifying that the venture capitalist, after observing the signal x, can convert warrants into shares at the predetermined exercise price γ.

The entrepreneur may issue convertible debt as well as warrants. Then, the venture capitalist may, for example, convert all the initial debt and exercise some of the warrants (i.e. the initial debt is $d_0 > 0$, and $d < 0$). To exactly apply our model we would need to assume that the debt conversion ratio and the exercise price of warrants are both γ. Allowing for different conversion ratios would not, however, substantially alter the analysis. There are interesting issues that arise, such as when it would make sense to use a contract that combines convertible debt and warrants, when the venture capitalist would prefer to convert debt rather than exercise the warrant conversion option, and whether we should expect the conversion ratios to differ. These questions are left for future work.

The deadline for conversion

Until now we have been assuming that the deadline for the conversion was strictly before time 2, and we argued that this was a realistic feature. Here we want to show that, indeed, it is not optimal for the parties to set the deadline for conversion at time 2, i.e. after full realization of uncertainty. The argument is simple: if the deadline for conversion were at time 2, then the venture capitalist would wait until time 2, when he will have more information, to decide whether to convert debt and how much to convert. This decision would, therefore, be independent of the signal $\tilde{x}$ (that is observed at time 1). But then the logic behind Proposition 1 applies: Since the threat of debt conversion at a favorable conversion ratio in response to a good signal is no longer present, the entrepreneur has an incentive to manipulate the signal in order to reduce the probability of liquidation. As a consequence, the project will not be financed. The parties have a common interest to set the deadline for debt conversion strictly before time 2. Then, as we have seen, they can write a convertible debt contract such that the entrepreneur will not manipulate the signal and, hence, the project will be undertaken.

3 Allowing for Renegotiation of Debt-Equity Contracts

In Section 2 we showed that a convertible debt contract is better than a debt-equity contract since it reduces the incentives of the entrepreneur to manipulate the signal. We assumed that the debt-equity contract could not be renegotiated at the interim stage after the signal is observed by the parties. We will show now that most of the time there is, indeed, no scope for such renegotiation. Interestingly, the cases where renegotiation may happen in equilibrium reinforce the intuition, presented briefly at the end of the previous section, that convertible debt contracts improve upon debt-equity contracts because they impose renegotiation in a specific way.

Consider a debt-equity contract, (d, s), and suppose renegotiation of the contract is allowed at time 1, after observing the signal x. If the venture capitalist observes a medium or high signal his interim payoff is positive, and he will refinance the project with no renegotiation. This follows from the linearity of the payoffs (risk neutrality). Renegotiation will happen if after observing a medium or high signal the interim payoff of the venture capitalist is negative. Since, by (4), the total surplus from continuation is positive, renegotiation that induces the venture capitalist not to liquidate creates surplus that can be split between the parties. In other words, there is a Pareto improving allocation that can be implemented via a new debt-equity contract $(\tilde{d}, \tilde{s})$.

There are two cases to consider: (a) The venture capitalist wants to liquidate after observing x_m but not after observing x_g (the other way around is not possible), and (b) the venture capitalist wants to liquidate after observing both x_m and x_g. Consider case (a). We want to show that, as in Proposition 1, in equilibrium $a = 1$, implying that a convertible debt contract is strictly better than the debt-equity contract with renegotiation.

We turn to the proof of this result. Let us first check that $a = 1$ is an equilibrium. Since we are assuming that there is renegotiation following x_m, the interim payoff of the venture capitalist is negative:

$$E[\mathcal{VC}]^m_{a=1} = \frac{1/6}{2/3-\epsilon}\frac{1}{\lambda_h}\left[1-e^{-\lambda_h d}(1-s)\right]+\frac{1/2-\epsilon}{2/3-\epsilon}\frac{1}{\lambda_l}\left[1-e^{-\lambda_l d}(1-s)\right]-I_2 \ < \ 0. \quad (20)$$

Renegotiation will entail a new debt-equity contract, $(\tilde{d},\tilde{s})$, such that

$$E[\mathcal{VC}]^m_{a=1} = \frac{1/6}{2/3-\epsilon}\frac{1}{\lambda_h}\left[1-e^{-\lambda_h \tilde{d}}(1-\tilde{s})\right]+\frac{1/2-\epsilon}{2/3-\epsilon}\frac{1}{}\lambda_l\left[1-e^{-\lambda_l \tilde{d}}(1-\tilde{s})\right]-I_2 \ \geq \ 0. \quad (21)$$

Since the expression in (21) is larger than that in (20), we must have

$$\frac{1}{\lambda_i}e^{-\lambda_i d}(1-s) > \frac{1}{\lambda_i}e^{-\lambda_i \tilde{d}}(1-\tilde{s}), \quad (22)$$

for $i = l, h$. In equilibrium, the ex-ante payoff to the entrepreneur is

$$E[\mathcal{E}]_{a=1} = (\frac{1}{2}-\epsilon)\frac{1}{\lambda_l}e^{-\lambda_l \tilde{d}}(1-\tilde{s}) + \frac{1}{6}\frac{1}{\lambda_h}e^{-\lambda_h \tilde{d}}(1-\tilde{s}) + \frac{1}{3}\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s). \quad (23)$$

By deviating to $a = 0$ he obtains

$$E[\mathcal{E}]_{a=0} = \frac{1}{6}\frac{1}{\lambda_l}e^{-\lambda_l \tilde{d}}(1-\tilde{s}) + (\frac{1}{2}-\epsilon)\frac{1}{\lambda_h}e^{-\lambda_h \tilde{d}}(1-\tilde{s}) + \epsilon\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s) \quad (24)$$

which, using (22), is strictly smaller. To ensure that $a = 0$ is not an equilibrium, we follow an analogous procedure: using (22), we get that the interim payoff of the entrepreneur, for any renegotiated debt-equity contract, $(\tilde{d},\tilde{s})$, is larger when $a = 1$.

We turn to case (b), namely when there is renegotiation after observing both x_m and x_g. If the renegotiated debt-equity contract, $(\tilde{d},\tilde{s})$, is the same after x_m and x_g then, by analogous arguments to those made in the previous paragraph, only $a = 1$ is an equilibrium. The renegotiated contract need not, however, be the same after observing x_m and after observing x_g. For example, due to exogenous circumstances the bargaining power of the parties may differ according to the realization of the signal so the renegotiated contract would be different in each case. In that event $a = 0$ may be an equilibrium. To show how this can be achieved, consider a convertible debt contract (d_0, s_0, γ, d) that satisfies Proposition 2 (i.e., such that $a = 0$ is an equilibrium), and assume that the renegotiation conditions are such that after x_m the renegotiated contract is (d_0, s_0) and after x_g it is (d, s) where $s = s_0 + \frac{1}{\gamma}(d_0 - d)$. These renegotiated debt-equity contracts yield identical payoffs to those obtained with the convertible debt contract. Therefore, the equilibrium behavior of the entrepreneur must be the same, i.e. $a = 0$. In this particular case, allowing for renegotiation has the effect that certain debt-equity contracts are equivalent in terms of signal manipulation and payoffs to a convertible debt contract. Notice, however, that for renegotiation to improve upon a debt-equity contract we are relying on exogenous circumstances, i.e. the terms of renegotiation (such as outside options, bargaining power, etc.) must be such that the final allocations happen to be exactly those achieved under the convertible contract debt contract constructed in the previous section, whereas with convertible debt we can improve upon any debt-equity contract.

The relation to Aghion, Dewatripont, and Rey (1994) should now be very apparent. They show that if the parties can write in the contract how to conduct the renegotiation in future contingencies, the inefficiency arising from the non-verifiability of interim signals can be eliminated.[19] The intuition here is analogous: There is a particular type of renegotiation that would eliminate the inefficiency (i.e. entail $a = 0$). We may be so lucky that such renegotiation would take place spontaneously. But in most cases this would not happen, so we must write in the contract how this renegotiation will take place. The debt conversion clause can be interpreted as a particular manner of implementing contractually the optimal type of renegotiation. In fact, the convertible debt contract gives a great deal of bargaining power to the venture capitalist (he decides whether to convert debt), but it also sets limits to his power since the terms of conversion, i.e. γ, are set in advance so as not to be too

[19] Nöldeke and Schmidt (1997) have applied this idea in the framework of a model with relationship specific investment.

unfavorable to the entrepreneur.

4 Long-term effort

Until now we have focused on the entrepreneur's choice of whether to manipulate the signal, identifying a financing structure that prevents the entrepreneur from doing so. The literature has often focused on the "long-term effort" of entrepreneurs (and managers), namely, on their contribution to profits in the long-run. For expositional reasons, our analysis has abstracted from this feature, but the basic short-run versus long-run trade-off is implicit in our model, as we will now demonstrate. One may easily imagine that the entrepreneur, in order to undertake short-term effort (signal manipulation) would reduce long-term effort. This shift in resources away from long-term effort constitutes an additional cost of short-termism.[20] We now incorporate in the model such a choice for the entrepreneur, in the simplest possible way, to emphasize that it does not affect our result.

Suppose the entrepreneur can make two types of effort: short-term effort, a_1, which has exactly the same effect as a in Table 1, and long term effort, a_2. If $a_2 = 1$ the probabilities are exactly those in Table 1, while if $a_2 = 0$ the ex-ante probability of y_h is reduced by ϕ and the ex-ante probability of y_l increases by ϕ. A simple way of introducing this change is to assume that ϕ is subtracted from the probability of the event "x_m and y_h" and is added to the probability of the event "x_m and y_l", whether a_1 is equal to 0 or 1. Naturally, this implies that $\phi < 1/6$. Moreover, we assume that both a_1 and a_2 involve a cost c that is born by the entrepreneur, and that the entrepreneur can only exert one of the two types of effort (or none).[21]

Assumption (2) is preserved, namely, stage financing is necessary. Let us for a moment ignore signal manipulation (i.e. let us assume that $a_1 = 0$) and focus on the choice of a_2.

[20] Hellwig (1994) analyzes the choice of the optimal debt-equity mix when effort has two dimensions.

[21] Alternatively, one could assume that c is high enough so that the entrepreneur will never choose to exert both types of effort.

From the perspective of joint profit maximization, it is optimal that $a_2 = 1$ if the expected benefits exceed the cost:

$$\phi(\frac{1}{\lambda_h} - \frac{1}{\lambda_l}) > c. \tag{25}$$

The entrepreneur will exert long time effort if *his* payoff increases by more than the cost of exerting long-term effort (recall that we are assuming that $a_1 = 0$):

$$\phi(1-s)(\frac{1}{\lambda_h}e^{-\lambda_h d} - \frac{1}{\lambda_l}e^{-\lambda_l d}) > c. \tag{26}$$

A necessary condition for (26) is $s < 1$. If (25) is satisfied then, the venture capitalist will want to induce the entrepreneur to exert long-term effort, and will insist on a debt-equity contract with $s < 1$ that satisfies (26). The need to induce the entrepreneur to exert long-term effort thus provides the justification for our assumption in previous sections that in any debt-equity contract, the venture capitalist does not own 100 percent of the equity of the project.[22]

Let us now introduce the possibility of signal manipulation. We preserve condition (3)—the venture capitalist liquidates the project after observing x_b regardless of a_1 and a_2. Condition (4) is, instead, modified so that the venture capitalist does not necessarily liquidate after observing x_m, even if $a_1 = 1$ and $a_2 = 0$ which is the combination of effort choices for which interim expected profits following x_m are lowest:

$$\frac{1/2+\phi-\epsilon}{2/3-\epsilon}\frac{1}{\lambda_l} + \frac{1/6-\phi}{2/3-\epsilon}\frac{1}{\lambda_h} - I_2 > 0. \tag{27}$$

Condition (5) is preserved: without signal manipulation ($a_1 = 0$) but with long-term effort ($a_2 = 1$) the project will be financed. However, we assume that with signal manipulation

[22] An alternative to equity participation could be a wage contingent on long-term performance. The same problems of "window-dressing" would then arise.

($a_1 = 1$) but without long-term effort ($a_2 = 0$) the project will not be financed:

$$(\frac{1}{2} - \epsilon + \phi)\frac{1}{\lambda_l} + (\frac{1}{2} - \phi)\frac{1}{\lambda_h} - I_1 - (1-\epsilon)I_2 \ < \ 0, \tag{28}$$

Conditions (5) and (28) together imply that the absence of signal manipulation is necessary for the project to be financed.

In this version of the model we obtain a result that is analogous to that in Proposition 1: With debt-equity financing only $a_1 = 1$ and $a_2 = 0$ is an equilibrium, provided ϕ is low enough. We now sketch the proof. Suppose that (d, s) are such that the venture capitalist refinances the project after observing x_m. Then, if in equilibrium $a_1 = 0$ and $a_2 = 1$, the ex-ante payoff of the entrepreneur would be

$$E[\mathcal{E}]_{a_1=0,a_2=1} = \frac{1}{6}\frac{1}{\lambda_l}e^{-\lambda_l d}(1-s) + \frac{1}{2}\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s) - c. \tag{29}$$

By (26), deviating to $a_1 = a_2 = 0$ is not worthwhile, but by deviating to $a_1 = 1$ and $a_2 = 0$ the entrepreneur obtains the payoff

$$E[\mathcal{E}]_{a_1=1,a_2=0} = (\frac{1}{2} - \epsilon + \phi)\frac{1}{\lambda_l}e^{-\lambda_l d}(1-s) + (\frac{1}{2} - \phi)\frac{1}{\lambda_h}e^{-\lambda_h d}(1-s) - c, \tag{30}$$

which is higher if ϕ is small enough. This is quite intuitive: if the long term effort is very important then manipulating the signal may not be worthwhile, but if it is not, then the entrepreneur will choose to manipulate the signal reducing long-term profits.

By similar arguments, analogous to those made in previous sections, we can show that in equilibrium $a_1 = 1$ and $a_2 = 0$ and, hence, the project will not be financed at time 1. Convertible debt solves this problem in exactly the same manner. The conditions ensuring that the venture capitalist converts debt following x_g and does not convert debt following

x_m remain the same. The condition ensuring that $a_1 = 1$ and $a_2 = 0$ becomes

$$\begin{gathered}\tfrac{1}{6}\tfrac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + (\tfrac{1}{2}-\epsilon)\tfrac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0) + \epsilon\tfrac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s) \geq \\ (\tfrac{1}{2}-\epsilon+\phi)\tfrac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + (\tfrac{1}{6}-\phi)\tfrac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0) + \tfrac{1}{3}\tfrac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s),\end{gathered} \tag{31}$$

and is more easily satisfied than (18). The intuition is that we have added a cost to manipulating a signal (the reduction in long-term profits) which makes it easier to induce the entrepreneur not to deviate from the desired behavior.

5 Concluding Remarks

We have argued that convertible debt can be better than a simple mixture of debt and equity in stage financing situations. When the venture capitalist retains the option to abandon the project if in the medium term he receives a negative signal, the entrepreneur has an incentive to engage in "window dressing" or short-termism, i.e. to bias the signal towards positive values, in order to reduce the probability that the project will be liquidated. Convertible debt reduces the incentives to engage in short-termism through the threat of debt conversion. In practice, debt may or may not be converted into shares depending on the information received at the interim stage of the venture.

Existing explanations for the use of convertible debt are centered on asymmetries of information between managers and investors regarding the firm's value and riskiness. These explanations are more suitable for publicly traded convertible securities, but do not fit very well with venture capital financing which involve a close relationship between an entrepreneur and the venture capitalist, and where information regarding the project's quality is gradually revealed to both parties.

The fundamental role of the debt conversion option is to adjust the ownership (debt-equity) structure to this new information. This feature resembles provisions allowing the provider of financing to take control of the project at some interim stage in response to nonverifiable information, a feature that is very common in incomplete contracts environments. In our model, in Nöldeke and Schmidt (1997), as well as in von Thadden (1995), the contract allows the provider of financing to alter the financial structure of the contract (to change the output allocation rule) at some interim stage in response to nonverifiable information. The need to commit in advance to the terms of debt conversion is in line with Aghion, Dewatripont, and Rey (1994) who show that by specifying in the contract the bargaining procedure to be used in future renegotiations, the efficient outcome can be achieved.

Appendix: Proof of Proposition 2

Let

$$\frac{1}{\lambda_h} - \gamma(1 - s_0) = 0. \tag{32}$$

The construction does not rely on "razor's edge" arguments, and goes through if this equality is not satisfied exactly. The intuition is the one given above: we want to choose a conversion ratio such that the venture capitalist will convert all debt after observing x_g and no debt at all after observing x_m. We will show that for any s_0, the value of γ defined in (16) satisfies such conditions (but it is not the only value of γ that does).

From (3) it follows that after observing a bad signal, x_b, the venture capitalist liquidates the project.

An implication of (32) is that after observing a good signal, x_g, the venture capitalist converts the entire amount of convertible debt. To see this, recall that the total amount of convertible debt prescribed by the contract is $d_0 - d$, and consider the interim payoff of the venture capitalist, after observing x_g and converting the amount $d_0 - \hat{d} \leq d_0 - d$,

$$E[\mathcal{VC}]_g = \frac{1}{\lambda_h}\left[1 - e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})\right], \tag{33}$$

where $\hat{s} = s_0 + \frac{1}{\gamma}(d_0 - \hat{d})$. The derivative of the above payoff with respect to $\hat{s}$ is $e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]}\left[\frac{1}{\lambda_h} - \gamma(1 - \hat{s})\right]$ which, by (32), is strictly positive for any $\hat{s} > s_0$, namely, the venture capitalist converts the maximal permissible amount of debt.

To ensure that when $a = 0$ (no signal manipulation) the venture capitalist does not convert any debt after observing x_m, consider the interim payoff of the venture capitalist when $a = 0$, after observing x_m and converting the amount $d_0 - \hat{d} \leq d_0 - d$,

$$\begin{aligned} E[\mathcal{VC}]_m \;=\; & \tfrac{1/6}{2/3-\epsilon}\,\tfrac{1}{\lambda_l}\left[1 - e^{-\lambda_l[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})\right] \\ & + \tfrac{1/2-\epsilon}{2/3-\epsilon}\,\tfrac{1}{\lambda_h}\left[1 - e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})\right]. \end{aligned} \tag{34}$$

The derivative of this payoff with respect to $\hat{s}$ is

$$\begin{aligned} \frac{dE[\mathcal{VC}]_m}{d\hat{s}} &= \frac{1/6}{2/3-\epsilon}\, e^{-\lambda_l[d_0-\gamma(\hat{s}-s_0)]}\left[\frac{1}{\lambda_l}-\gamma(1-\hat{s})\right] \\ &\quad +\frac{1/2-\epsilon}{2/3-\epsilon}\, e^{-\lambda_h[d_0-\gamma(\hat{s}-s_0)]}\left[\frac{1}{\lambda_h}-\gamma(1-\hat{s})\right]. \end{aligned} \tag{35}$$

When $\hat{s} = s_0$, using (32), the derivative in (35) is strictly negative. As $\hat{s}$ increases the derivative increases and becomes positive. Once it has become positive the derivative remains positive as $\hat{s}$ increases further. Therefore, the venture capitalist's payoff in (15) is maximal either when $\hat{s} = s$ (the maximal possible amount is converted) or when $\hat{s} = s_0$ (no debt is converted). The latter is optimal for the venture capitalist if (17) is satisfied, which can be rewritten as

$$\frac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0) \geq \frac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + \frac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s) \tag{36}$$

To ensure that, given the behavior of the venture capitalist, the entrepreneur prefers not to manipulate the signal, we need the condition

$$\begin{aligned} &\left[\frac{1}{6}\frac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + \left(\frac{1}{2}-\epsilon\right)\frac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0)\right] + \epsilon\frac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s) \geq \\ &\left[\left(\frac{1}{2}-\epsilon\right)\frac{1}{\lambda_l}e^{-\lambda_l d_0}(1-s_0) + \frac{1}{6}\frac{1}{\lambda_h}e^{-\lambda_h d_0}(1-s_0)\right] + \frac{1}{3}\frac{1}{\lambda_h}e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s). \end{aligned} \tag{37}$$

This condition compares the entrepreneur's ex-ante payoff without signal manipulation (the left hand side) to his ex-ante payoff with signal manipulation, given that the venture capitalist converts debt when he observes x_g but not when he observes x_m.

The ex-ante participation condition for the venture capitalist is[23]

$$\begin{aligned} &\frac{1}{6}\frac{1}{\lambda_l}[1-e^{-\lambda_l d_0}(1-s_0)] + \left(\frac{1}{2}-\epsilon\right)\frac{1}{\lambda_h}[1-e^{-\lambda_h d_0}(1-s_0)] \\ &\quad + \epsilon\frac{1}{\lambda_h}[1-e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s)] - I_1 - \frac{2}{3}I_2 \geq 0, \end{aligned} \tag{38}$$

[23]The entrepreneur will always agree to participate since he is not committing any resources of his own.

where the first three terms correspond to the three outcomes that occur with positive probability following x_m or x_g when $a = 0$ (see Table 1); I_2 is multiplied by the probability that refinancing will occur when $a = 0$.

Fix $s_0 < 1$. This value of s_0 will not change through the remainder of the construction. As d_0 increases, the left hand side of (38) approaches $\frac{1}{6}\frac{1}{\lambda_l} + \left(\frac{1}{2} - \epsilon\right)\frac{1}{\lambda_h} + \epsilon\frac{1}{\lambda_h} - I_1 - \frac{2}{3}I_2$ which, by (5), is strictly positive. Therefore, for any $s \in [s_0, 1]$ there is d_0^s such that if $d_0 \geq d_0^s$, (38) holds. Let $d_0^\circ = \max_{s\in[s_0,1]} d_0^s$. From now on, we restrict attention to $d_0 \geq d_0^\circ$. We are, therefore, free to choose $s \in [s_0, 1]$ without violating (38).

We turn to (36). Substitute for γ, using (32), divide through by $\frac{1}{\lambda_h}e^{-\lambda_h d_0}(1 - s_0)$, and rearrange to get

$$(1-s)e^{\frac{s-s_0}{1-s_0}} \leq [1 - \frac{\lambda_h}{\lambda_l}e^{-d_0(\lambda_l-\lambda_h)}](1-s_0). \tag{39}$$

Let (36), and therefore also (39), hold with equality (this requirement will be relaxed later). Now consider (37). Substitute for γ, using (32), substitute for $(1-s)e^{\frac{s-s_0}{1-s_0}}$, using (39), rearrange dividing through by $\frac{1}{6}\frac{1}{\lambda_l}e^{-\lambda_l d_0}$, to obtain

$$(4-6\epsilon)(1-s_0) - e^{\frac{\lambda_l}{\lambda_h}\frac{s-s_0}{1-s_0}}(1-s) \leq 0. \tag{40}$$

It is easily verified that the left hand side is a strictly convex function of s, reaching its minimum at $\bar{s}$ satisfying $1 - \bar{s} = \frac{\lambda_h}{\lambda_l}(1 - s_0)$; hence $\bar{s} \in (s_0, 1)$.

Consider the inequality (39) when $s = \bar{s}$. Since the right hand side increases with d_0, approaching unity, while the left hand side is strictly smaller than unity, as $\bar{s} \in (s_0, 1)$, it follows that there is $\bar{d}_0$ such that if $d_0 \geq \bar{d}_0$, (39), and therefore also (36), hold. Let $d_0^* = \max(d_0^\circ, \bar{d}_0)$.

Summarizing, if $s = \bar{s}$ and $d_0 \geq d_0^*$, we know that (38), (39), and therefore also (36), hold. The "degrees of freedom" exploited so far are as follows: $s_0 \in [0, 1)$ was chosen arbitrarily, pinning down γ by (32), s was set equal to $\bar{s}$, d_0 has to be larger than some minimal value, d_0^*, that can be calculated. Given d_0, the post-conversion debt level, d, is determined by $\bar{s} - s_0 = \frac{1}{\gamma}(d_0 - d)$.

It remains to be verified that (40) holds. Substitute $\bar{s}$ for s, using $1 - \bar{s} = \frac{\lambda_h}{\lambda_l}(1 - s_0)$ (see above), and rearrange to obtain $[(4-6\epsilon) - \frac{\lambda_h}{\lambda_l} e^{\frac{\lambda_l}{\lambda_h}-1}] \leq 0$ which, noting that $\lim_{y\to\infty} \frac{1}{y}e^{y-1} = \infty$, holds for $\frac{\lambda_l}{\lambda_h}$ large enough (greater than 4 is a sufficient condition, given the particular joint probability distributions in Table 1), which completes the proof of Proposition 2.

References

AGHION, P., DEWATRIPONT, M. AND P. REY 1994, "Renegotiation Design with Unverifiable Information," *Econometrica,* **62**, pp.257–282.

ADMATI, A. AND P. PFLEIDERER 1994, "Robust Financial Contracting and the Role of Venture Capitalists," *Journal of Finance*, **49**, pp.371–402.

BAGWELL, L. AND J. ZECHNER 1993, "Influence Costs and Capital Structure," *Journal of Finance*, **48**, pp.975–1008.

BERGLÖF, E. 1994, "A Control Theory of Venture Capital Finance," *Journal of Law, Economics, and Organization*, **10**, pp.247–267.

BRENNAN, M. AND E. SCHWARTZ 1988, "The Case for Converibles," *Journal of Applied Corporate Finance*, **1**, pp.55–64.

CHAN, Y.S., SIEGEL, D., AND A. THAKOR 1990, "Learning, Corporate Control, and Performance in Venture Capital Contracts," *International Economic Review,* **31**, 365–381.

GOMPERS, P. 1995, "Optimal Investment, Monitoring, and the Staging of Venture Capital," *Journal of Finance*, **50**, pp.1461–1490.

GOMPERS, P. 1996, "A Clinical Examination of Convertible Securities in Venture Capital Investments," mimeo, Harvard University.

GORMAN, M. AND A. SAHLMAN 1989, "What do Venture Capitalists Do?" *Journal of Business Venturing*, **4**, pp.231–248.

GREEN, R. 1984, "Investment Incentives, Debt and Warrants," *Journal of Financial Economics*, **13**, pp.115–136.

HARRIS, M. AND A. RAVIV 1985, "A Sequential Signaling Model of Convertible Debt Call Policy," *Journal of Finance*, **41**, pp.815–830.

HELLMANN, T. 1993, "Financial Structure and Control in Venture Capital," mimeo., Stanford University.

HELLWIG, M. 1994, "A Reconsideration of the Jensen-Meckling Model of Outside Finance," WWZ-Discussion Papers 9422.

MARX, L. 1993, "Negotiation and Renegotiation of Venture Capital Contracts," mimeo., Northwestern University.

NARAYANAN, M.P. 1985, "Managerial Incentives for Short-Term Results," *Journal of Finance*, **40**, pp.1469–1484.

NÖLDEKE, G. AND K. SCHMIDT 1997, "Sequential Investments and Options to Own," mimeo, University of Munich.

NYBORG, K. 1995, "Convertible Debt as Delayed Equity: Forced versus Voluntary Conversion and the Information Role of Call Policy," *Journal of Financial Intermediation*, **4**, pp.358–395.

SAHLMAN, W. 1988, "Aspects of Financial Contracting in Venture Capital," *Journal of Applied Corporate Finance*, **1**, pp.23-36.

SAHLMAN, W. 1990, "The Structure and Governance of Venture Capital Organization," *Journal of Financial Economics*, **27**, pp.473-521.

STEIN, J. 1989, "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *Quarterly Journal of Economics*, **104**, pp.655-669.

STEIN, J. 1992, "Convertible Bonds as Backdoor Equity Financing," *Journal of Financial Economics*, **32**, pp.3-21.

VON THADDEN, E. 1995, "Long-Term Contracts, Short-Term Investment and Monitoring," *Review of Economic Studies*, **62**, pp. 557-575.

TRESTER, J. 1994, "Venture Capital Contracting under Asymmetric Information," Working Paper 94-06, The Wharton School.