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(v. 2) ARE WORKING HOURS COMPLEMENTS $^{\text {IN PRODUCTION? }}$| Lin Shao, Faisal Sohail and Emircan Yurdagul |
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# ARE WORKING HOURS COMPLEMENTS IN PRODUCTION? 

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# ARE WORKING HOURS COMPLEMENTS IN PRODUCTION? 


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This paper studies the degree of complementarity in working hours among coworkers in production. Using matched employer-employee data, we first present facts on the withinestablishment relationship between wages and hours worked that are consistent with the presence of complementarities in working hours. Next, we estimate the elasticity of substitution in working hours and find it to be 0.69 in the aggregate and between 0.52 and 1.04 across industries. We validate our elasticity estimates by showing that industries with higher elasticities exhibit greater flexibility in hours. Our results suggest that working hours are gross complements in production rather than perfect substitutes, as is typically assumed. An accounting exercise using our model estimations suggests that hours-wage penalties, due to complementarities in hours, can explain 5 to 30 percent of the gender wage gap over the life-cycle.


JEL Classification: E23, J22, J23, J31

Keywords: Labor supply, Complementarities, Wages, Production, Flexibility, Coordination
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# Are Working Hours Complements in Production?* 

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October 2022


#### Abstract

This paper studies the degree of complementarity in working hours among coworkers in production. Using matched employer-employee data, we first present facts on the within-establishment relationship between wages and hours worked that are consistent with the presence of complementarities in working hours. Next, we estimate the elasticity of substitution in working hours and find it to be 0.69 in the aggregate and between 0.52 and 1.04 across industries. We validate our elasticity estimates by showing that industries with higher elasticities exhibit greater flexibility in hours. Our results suggest that working hours are gross complements in production rather than perfect substitutes, as is typically assumed. An accounting exercise using our model estimations suggests that hours-wage penalties, due to complementarities in hours, can explain 5 to 30 percent of the gender wage gap over the life-cycle.


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## 1 Introduction

An implicit assumption in most macroeconomic models of production is that the number of working hours of different workers are perfect substitutes. However, the production process requires coordination between workers who work on different tasks. With this in mind, the assumption of perfect substitutability between working hours becomes less natural. Indeed, a consequence of the need to coordinate tasks is the need to coordinate hours worked. For instance, is a worker who works 40 hours a week equally productive regardless of whether her coworkers work 25,40 or 60 hours? Recent work suggests that the answer is no. Specifically, Bick et al. (2020) and Yurdagul (2017) document an aggregate, non-linear relationship between hours and wages, whereby workers earn higher wages when they work near the modal hours in the economy. Working hours being complements in production would imply precisely this relationship, as workers would be more productive and therefore earn higher wages when they work a similar number of hours to their coworkers.

Knowing whether and to what extent working hours are complements is essential for understanding the labor supply decisions of individuals and their response to policies. Indeed, the degree of substitutability between working hours will constrain the ability of an individual to respond to changes in income taxes, child-care provision or to idiosyncratic shocks. Yet, despite its relevance, there has been little work exploring whether the working hours of workers are complements in production. In this paper, we use matched employer-employee data from the Canadian Workplace and Employee Survey (WES) to document such evidence.

The WES, unlike many other employer-employee linked data, is ideally suited to study complementarity in working hours within the same workplace as it includes information on both working hours - measured by usual weekly hours rather than contracted hours - and average hourly wage. Moreover, the unit of a workplace in the data is an establishment which allows us to focus on the hours worked that are most relevant to coworkers while also providing a well-defined proxy of a production unit compared to firm-level data. Our analysis of the WES is divided into three parts. First, we document a series of new facts that support the
presence of complementarity in working hours. We begin by showing that individual working hours are significantly positively correlated with their average coworkers' hours. Next, we document that wages within establishments exhibit the same "hump-shaped" pattern across hours worked as observed in the aggregate data. We also find that the hours at which wages are maximized - that is, the peak of the hump shape - are strongly positively correlated with the median (or average) hours worked in the establishment. Finally, by tracking employees in the same establishment over time, we show that movements away from either the median or the wage-maximizing hours are associated with significant wage reductions. These results are robust to controlling for observable characteristics and consistent with a production process where the working hours of workers are complements.

In the second part of our analysis, which is our main contribution, we estimate the degree of working hours complementarities in production. To do this, we propose a simple CES production function that features imperfect substitutability between working hours and use the WES to estimate the elasticity of substitution between the working hours of different workers. The CES production function delivers an endogenous non-linear wage schedule. In particular, as observed in the data, wages are highest at an intermediate level of hours and decline if workers either increase or decrease their hours worked. The endogenous relationship between hours and wages underpins our estimation strategy, which follows an extensive literature by applying the optimal conditions of firms to the data in order to estimate the parameters of production functions (e.g., Katz and Murphy, 1992 and Acemoglu, 2002). ${ }^{1}$

In the aggregate, we estimate the elasticity of substitution between working hours to be 0.69 , implying that workers' hours are gross complements in production. We also document significant heterogeneity in this elasticity across industries. Our estimates range from 0.52 to 1.04 , with working hours being gross complements in all but one industry. We exploit the heterogeneity across industries to validate our estimation results. Specifically, we document an intuitive positive relationship between proxy measures of hours flexibility and our

[^1]estimated elasticities.
Our model has implications for how hours heterogeneity across workers can contribute to observed wage inequality. A potential dimension through which this channel might be at play is gender, given the significant differences in the hours distribution between men and women. By comparing the observed gender wage gap to a counterfactual gap which removes wage penalties that arise due to complementarities, we find that around $14 \%$ of the gender wage gap can be explained by variation in hours under complementarities.

Taken together, our findings challenge a canonical assumption about the nature of production, namely, that the working hours of workers are perfect substitutes. We provide strong evidence supporting coworker hours being gross complements and provide estimates for the degree of complementarities across industries and in the aggregate. Such complementarity has important implications not only for wage inequality, as we show, but also for research on labor supply and the efficacy of policies that aim to influence it.

Related Literature This paper connects to several strands of literature. First, working hours being gross complements in production may act as effective constraints on labor supply. This relates to a growing literature that has studied the implications of constraints on working hours while remaining largely indifferent as to the source of these constraints.Altonji and Paxson (1988) and Chetty et al. (2011) show that the (in)ability to choose working hours has important implications for job mobility and estimates of labor supply elasticities, respectively. More recently, Labanca and Pozzoli (2021) use Danish administrative data to study the impact of hours constraints on labor supply decisions following a tax reform. In line with our analysis, they argue that hours constraints within firms - as measured by variation in working hours - result from firms' technology rather than workers' preferences. Our analysis which uses information on the relationship between wages and hours, suggests that complementarities in working hours are a feature of the production process and may generate the apparent constraints on hours studied in the literature.

Naturally, this paper relates and contributes to the literature studying the degree of complementarity and coordination between workers. The production function underpinning our estimation is a general version of a Leontief production function studied in Becker and Murphy (1992) and is most closely related to Yurdagul (2017), which studies the flexibility motive behind entrepreneurship. Rogerson (2011) introduces the notion of coordinated working hours by imposing (exogenous) constraints on labor supply on the worker side rather than on the firm side as in this paper. Cubas et al. (2019) use worker-level data to estimate an occupation-specific measure of work schedule coordination. Their measure focuses on a strict type of hours complementarity. Namely, the extent to which occupations feature workers working at the same time rather than the same number of hours. In contrast, we focus on workers working the same number of hours and our matched employer-employee data allows us to estimate this more general form of complementarity in the same workplace, the estimation of which is not feasible using worker-level data.

In a related paper, Battisti et al. (2021) use matched employer-employee data from Italy to estimate the elasticity of substitution between the working time of workers - measured as the number of days. Consistent with our findings, their estimated elasticity indicates that the working days of coworkers are gross complements, and they argue that this complementarity has important implications for estimating the Frisch elasticity. In contrast, we focus on complementarity in working hours and utilize information on hourly wages. Further, Battisti et al. (2021) employ a structural model with a frictional labor market to estimate the degree of complementarities in working days while we employ a relatively parsimonious estimation strategy that only specifyies a production function and labor market structure. ${ }^{2}$ Despite these differences, their empirical analysis, like ours, uses within-firm/establishment variation in working time and its relationship with earnings to argue that working times for coworkers are complements in production.

[^2]Our findings are also relevant to the literature on labor market sorting. As noted in Bonhomme (2021), complementarities are crucial in driving sorting patterns. For example, Calvo et al. (2021) document complementarity between partners' hours in home production and shows that this shapes sorting in marriage and labor markets. Shao et al. (2022) show that complementarity in workers' hours impacts matching between heterogeneous firms and workers. They find that complementarity in hours and the implied sorting amplify existing heterogeneity across firms, impacting income inequality and welfare.

An important result of our analysis is that the degree of complementarity in the production process varies across industries. Specifically, we find that the manufacturing and construction sectors feature a higher degree of hours complementarity than service sectors, especially low-skill service sectors such as retail. While this finding may seem intuitive, this is the first paper providing empirical evidence to support it. This cross-industry difference in complementarity is also related to the literature on structural change and the evolution of gender inequality, which argues that women have a comparative advantage in the service sector. Hence, a structural shift from manufacturing to services benefits women more than men (Ngai and Petrongolo, 2017). Our results suggest that, as women value flexibility more than men (Goldin, 2014), this comparative advantage could stem from the relative flexibility in hours afforded in service sectors. This result complements existing literature which emphasizes the comparative advantage based on sector-specific skill requirements (Rendall, 2018).

Our analysis of the role of hours heterogeneity on the gender wage gap relates to the reduced-form analysis in Goldin (2014) and Goldin (2015) which argue that the penalty for working shorter hours contributes to the gender wage gap. ${ }^{3}$ In contrast to these studies, we do not rely on aggregated measures of penalties but can identify both short- and long-hour wage penalties experienced by individuals due to hours heterogeneity within an establishment

[^3]and the presence of complementarities in hours. Our analysis suggests that part of the gender wage gap is technological and influenced by complementarities in hours worked. This analysis complements Cubas et al. (2019) which finds that the requirement to work at the same time contributes significantly to the gender wage gap.

Finally, the findings in this paper provide insights that may be leveraged by policymakers. First, heterogeneity in the degree of complementarity in working hours implies that changes in income tax or other labor supply policies will have differential impacts depending on the industry. Second, taking into account the inherent complementarities in working hours may be particularly important when considering policies related to labor supply over the lifecycle or by gender. For instance, Ameriks et al. (2020) find that older workers would work longer if they could choose their hours. Mas and Pallais (2017, 2020) argue that alternative work arrangements, particularly those that provide more flexibility, play an important role in workers' labor supply decisions. Complementarities in working hours introduce a trade-off for employers if they consider providing more flexible contracts. Indeed, if coworker hours are complements, employees may be less productive, on average, when they work different hours. Policies or alternative work arrangements that intend to address gender inequity or labor force participation through increased flexibility need to internalize this trade-off.

The rest of this paper is organized as follows. In Section 2, we describe our data and present evidence for the presence of complementarities between the hours of coworkers. In Section 3 we propose a production function and use it to estimate the degree of complementarity between hours within firms, and validate our estimation by relating proxy measures of work schedule coordination to our estimates across industries. In Section 4 we explore implications of complementarities in working hours for inequality, focusing on the gender wage gap. We conclude in Section 5 .

## 2 Evidence of Complementarities in Working Hours

Our empirical analysis uses matched employer-employee data from the Canadian Workplace and Employee Survey (WES). In this section, we use the WES to present novel evidence supporting the presence of complementarities in working hours.

The WES is an annual survey of Canadian establishments and their workers with a longitudinal design. The survey tracks surviving employers for six years from 1999 to 2006. In even-numbered years, a sample of employees from each employer is interviewed and followed for two years. A maximum of twenty-four employees are sampled, and in workplaces with fewer than four employees, all employees are interviewed. We restrict attention to individuals aged between 25 and 64 and, we exclude those who usually work less than 10 hours a week or earn less than half the federal minimum wage. ${ }^{4}$ Our final sample includes just over 120,000 employer-employee-year observations.

Importantly, and in contrast to many other matched employer-employee datasets, we observe both the usual weekly hours worked and average hourly wage of employees, which makes the WES particularly well-suited to addressing our primary research question. Our measure of hourly wage includes overtime pay, commissions, and tips. However, our results are robust to an alternative measure of wages that excludes such extra pay. We also observe employee occupation, education and a number of other demographic characteristics such as age, marital status, immigration status and parenthood. ${ }^{5}$

### 2.1 Correlations Between Own and Coworker Hours

Complementarities between working hours would imply a positive correlation between the hours of individual workers and those of their coworkers. This is due to the sorting of workers with similar desires into the same establishments and the wage schedules of individual workers

[^4]pushing workers towards their peers' hours. Indeed, the crude correlation between a worker's hours and the average hours of her coworkers in our data is 0.33 .

While suggestive, the unconditional positive correlation need not be due to complementarities but instead may be driven by other factors such as sorting of similar workers into production units or may be explained by characteristics of establishments. To control for such (observable) factors, we conduct a more formal evaluation of the correlation between the hours of a worker and those of her coworkers by estimating the following regression,

$$
\begin{equation*}
\log \left(h_{i s t}\right)=\alpha+\gamma \log \left(\bar{h}_{s_{-i}}\right)+\delta X_{i}+\eta Y_{s}+\mathbf{B}_{t}+\epsilon_{i s t}, \tag{1}
\end{equation*}
$$

where $h_{i s t}$ are hours worked by worker $i$ employed by establishment $s$ in year $t$ and $\bar{h}_{s_{-i}}$ is the average hours worked among $i$ 's coworkers in establishment $s . X_{i}$ is a vector of individuallevel control variables which include a quadratic in age, dummy variables for educational status (college degree or not) as well as indicators for marital and immigration status. $Y_{s}$ is a vector of establishment-level controls that includes establishment age, size and industry and the average establishment wage. $\mathbf{B}_{t}$ captures year fixed effects.

Table 1: Correlation between Own and Coworker Hours

|  | $(1)$ | $(2)$ | $(3)$ |
| :--- | :---: | :---: | :---: |
| Average Coworker Hours | 0.345 | 0.285 | 0.277 |
|  | $(0.014)$ | $(0.014)$ | $(0.014)$ |
| Individual Controls |  |  |  |
| Establishment Controls | N | Y | Y |
| Average Wage | N | N | Y |
| $N$ | 120420 | 118336 | Y |
| $R^{2}$ | 0.118336 |  |  |
|  |  | 0.190 | 0.192 |

Notes: The table reports the coefficient $\gamma$ from estimating equation (1). The regressions include a set of controls for worker and establishment characteristics, as indicated in the table. Robust standard errors are reported in the parentheses.

Table 1 reports the coefficient $\gamma$ - the elasticity of worker $i$ 's hours with respect to those of her coworkers. Under all specifications, we document a significant and positive correlation
between own and coworker hours with an estimated elasticity between 0.28 and 0.35 . In other words, having coworkers that work longer (shorter) hours is associated with one's own hours being longer (shorter) - a key implication of a production process where working hours are complementary. Having said this, we do not interpret this finding to be conclusive in establishing the presence of working hours complementarities. For example, we cannot rule out workers sorting on unobserved individual or establishment characteristics. Instead, we take this evidence to be suggestive, which, combined with the evidence on earnings that we present below, strengthens the case for the hours of workers being complements in production.

### 2.2 Wage-Hour Profiles within Establishments

If working hours are complements in production, we expect workers to be less productive if they work longer or shorter hours than their coworkers. This lower productivity should be reflected in workers' earnings with relatively lower wages for workers working shorter or longer hours. Such a relationship between wages and hours has been previously documented in the aggregate by Yurdagul (2017) and Bick et al. (2020) using US labor force surveys. ${ }^{6}$ These papers' findings point to wage penalties for working either short or long hours among all workers in the economy. To test whether such penalties are present within establishments, we use the WES to estimate the following specification,

$$
\begin{equation*}
\log \left(w_{i s t}\right)=\alpha+\left(\sum_{h \in H} \gamma_{h} \mathbb{I}_{i, h}\right)+\delta X_{i}+\mathbf{A}_{s}+\mathbf{B}_{t}+\epsilon_{i s t}, \tag{2}
\end{equation*}
$$

where $w_{i s t}$ is the hourly wage of worker $i$ in establishment $s$ at time $t$. The indicator variable $\mathbb{I}_{i, h}$ is equal to one if an individual works $h$ hours. We partition weekly hours into a set $H$ by grouping hours in 5 -hour bins. We choose the category $40-44$ as the reference category of hours worked as most workers work these hours. Then, the coefficients $\gamma_{h}$ capture the relative wage penalty/premium from working either more or less than the reference hours

[^5]bin. $X_{i}$ is a vector of individual-level control variables which include a quadratic function of age, dummy variables for educational status (college degree or not) as well as indicators for marital and immigration status. $\mathbf{A}_{s}$ and $\mathbf{B}_{t}$ are establishment and year fixed effects, respectively.

Panels (a) and (b) of Figure 1 report the results from estimating equation (2). Panel (a) plots the wage-hours profile - that is, the coefficients $\gamma_{h}$ - when we exclude establishment fixed-effects but include controls for establishment size, age, and industry. Consistent with existing evidence from worker-level data, the estimated wage-hours profile features a kinked or hump-shaped pattern, with relatively long and short hours exhibiting lower wages than those obtained in intermediate hours.

Note that while the median (and modal) weekly hours worked in Canada is 40 hours, the hours at which wages are maximized within establishments is around 50 hours. As will be clear in the next section, a production function in which coworkers' hours are complements does not necessitate that wages are maximized at the median, modal, or even average hours worked in the establishment. Indeed, through the lens of our theoretical framework, wages are maximized when hours are exactly equal to a particular moment of the hours distribution in a firm. This moment will be positively correlated with the mean and the median of the hours distribution but will ultimately depend on the elasticity of substitution between hours. With this in mind, we will consider multiple measures of usual hours worked within an establishment in the empirical analysis that follows.

Panel (b) plots the coefficients $\gamma_{h}$ when establishment fixed effects are included. This captures the wage-hours profile within establishments - a measure that, to our knowledge, is novel. Importantly, we find that short and long hours penalties exist within establishments. Indeed, workers earn around $5 \%$ lower wages when working either 25 or 60 hours per week relative to those working around 40 hours. Compared to Panel (a), which excludes establishment fixed effects, the penalties from working short or long hours are relatively smaller but still statistically significant.

While a hump-shaped wage-hours profile is consistent with complementarities in working hours, there may be other, unrelated drivers of the observed short and long hours penalties within establishments. For instance, short hours penalties may reflect a (time) startup cost of working, while long hours penalties may be due to diminishing returns. A more specific litmus test for the presence of complementarities is to ask whether the hours at which wages are maximized relate to the usual hours worked within an establishment. Complementarities in working hours would imply a positive relationship between the wage-maximizing hours and usual hours worked. We test this prediction and find a significant positive correlation between the wage-maximizing hours and two measures of usual establishment hours: the median and average hours worked. Indeed, the correlation between median (average) hours and the hours at which wages are highest is 0.84 (0.79) in the WES. ${ }^{7}$

Another important and related implication of complementarities in working hours is that deviating from the hours that most workers work impacts one's productivity. In particular, the more a worker deviates from her coworkers, the less productive she will be per hours worked. To study whether this is observed in the WES, we test whether deviations from the median or wage-maximizing hours result in wage penalties and whether these penalties increase as the magnitude of deviation increases. We do this by estimating the following,

$$
\begin{equation*}
\log \left(w_{i s t}\right)=\alpha+\left(\sum_{h \in H} \gamma_{\Delta h} \mathbb{I}_{i, \Delta h}\right)+\delta X_{i}+\mathbf{A}_{s t}+\epsilon_{i s t}, \tag{3}
\end{equation*}
$$

where $\mathbb{I}_{\Delta h}$ indicates deviations of a worker's own hours from a reference level of hours which is either the median or wage-maximizing hours in an establishment. $\mathbf{A}_{s t}$ is an interaction of establishment and year fixed effects and all other regressors are as in equation (2).

Panels (c) and (d) of Figure 1 report the results from estimating this alternative regression. Panel (c) shows the relationship between relative wages and deviations from the median hours worked in an establishment. We find that wages are the highest when workers

[^6]

Figure 1: Relationship between wages and hours using within-establishment variation Notes: Panels (a) and (b) the coefficient $\gamma_{h}$ from estimating variants of equation (2). In Panel (a), we do not include establishment fixed effects $\mathbf{A}_{s}$, but instead include a set of establishment characteristics including establishment size and age dummies as well as industry fixed effects. Panel (b) reports the coefficient $\gamma_{h}$ when establishment fixed effects are included. Panels (c) and (d) report the coefficient $\gamma_{\Delta h}$ from estimating equation (3) when the reference hours are the establishment median and establishment wage-maximizing hours, respectively. The shaded area represents the $95 \%$ confidence interval of the coefficient using robust standard errors.
work around 10 hours longer than the establishment median hours, with larger deviations from this level resulting in larger wage reductions.

Panel (d) shows the relationship between relative wages and deviations from the wagemaximizing hours in an establishment. By construction, wages are the highest when workers remain at those hours that deliver the maximal hourly wage. Consistent with complementarities in hours, wage penalties monotonically increase as workers' hours deviate from the wage-maximizing hours. Furthermore, the wage decrease is roughly symmetric regardless of
whether a worker works longer or shorter hours relative to the wage-maximizing hours.

### 2.3 Changes in Wages when Hours Change

A concern with the above analysis of hours and wages is that the relationships we document may be driven by unobserved individual characteristics. While we cannot fully address this concern, we can control for the fixed unobservable traits of workers by exploiting the short panel nature of the WES. Related to the intuition above, if coworkers' hours are complements in production, the same worker should earn lower wages if they deviate further from the hours of their coworkers between two periods. By tracking workers over time, we control for the time-invariant unobserved characteristics of workers.

We test the relationship between changes in wages and hours worked at the individual level in the WES. More formally, for each worker $i$ working in establishment $s$ in year $t$, we first compute the absolute $\log$ difference between their own hours, $h_{i, t}$, and the median or wage-maximizing hours, $\bar{h}_{s, t}:\left|\log h_{i, t}-\log \bar{h}_{s, t}\right|$. We then compute the changes in this measure between period $t$ and $t+1: \Delta h_{i s t}=\left|\log h_{i, t+1}-\log \bar{h}_{s, t+1}\right|-\left|\log h_{i, t}-\log \bar{h}_{s, t}\right|$. A positive (negative) value for the difference in differences, $\Delta h_{i s t}$, indicates that worker $i$ moved further (closer) from the reference hours between period $t$ and $t+1$. Similarly, we can compute the corresponding changes in wages between $t$ and $t+1, \Delta w_{i s t}=\left(\log w_{i, t+1}-\log w_{i, t}\right)$.

The presence of complementarities would imply a negative correlation between $\Delta h_{i s t}$ and changes in wages $\Delta w_{i s t}$. That is, workers who move further away from their establishments' median or wage-maximizing hours suffer wage losses while those who move closer experience wage gains.

We examine this relationship estimating the following,

$$
\begin{equation*}
\Delta w_{i s t}=\alpha+\left(\sum_{\Delta h_{i s t} \in \mathcal{H}} \gamma_{\Delta h_{i s t}} \mathbb{I}_{\Delta h_{i s t}}\right)+\delta X_{i}+\mathbf{A}_{s t}+\epsilon_{i s t} \tag{4}
\end{equation*}
$$

where the indicator variable $\mathbb{I}_{\Delta h_{i s t}}$ is equal to one if the relative change in difference hours
over time is $\Delta h_{\text {ist }}$. We partition this "difference in difference" measure into a set $\mathcal{H}$ by grouping hours changes into $10 \%$ bins. We choose the 0 to $+10 \%$ bin as the reference group since most workers fall into this category of hours changes. All other regressors are as in equation (3).


Figure 2: Dynamic Changes in Hours and Wages
Notes: The figure reports the coefficient $\gamma_{\Delta h_{i s t}}$ from estimating equation (4). Panels (a) and (b) report this coefficient when the reference hours are the establishment median and establishment wage-maximizing hours, respectively. The shaded area represents the $95 \%$ confidence interval of the coefficient using robust standard errors.

Figure 2 reports the coefficient $\gamma_{\Delta h_{i s t}}$ when using the median (Panel (a)) or wagemaximizing hours (Panel (b)) as the reference hours $\bar{h}_{s, t}$. For both reference hours, we find a clear negative correlation between changes in wages and changes in hours worked over time - in line with the presence of complementarities in hours worked. Indeed, moving $10 \%$ further from the median and wage-maximizing hours results in a $2 \%$ and $5 \%$ penalty in wages, respectively.

### 2.4 Discussion

The evidence presented in this section is consistent with the presence of complementarities in working hours in production. However, it is not necessarily the only mechanism that can explain the patterns we document. Here, we briefly discuss alternative mechanisms while reiterating the evidence supporting complementarities.

First, the positive correlation between one's own and their coworkers' hours could also be due to individual preferences. Rogerson (2011) proposes that a motive for working hours coordination may arise from individuals' preferences for coordinating leisure time - in other words, there may exist complementarities in leisure. Although we control for observable characteristics of individuals in our reduced-form analysis, these characteristics do not entirely capture individuals' underlying tastes over market (and non-market) hours.

Having said this, coordination of hours due to individuals' preferences cannot account for the (within establishment) non-linear relationship between hours and wages that we document. Mechanisms behind the non-linear wage-hours profile could include the presence of fixed costs to production (leading to a short hours penalty) and diminishing returns to working hours (leading to a long hours penalty). Eden (2021) emphasizes the link between productivity and working time, highlighting that productivity increases both with working time (as skills are accumulated) and restfulness (through leisure). Eden argues that this trade-off is important for understanding the optimal number of days in a workweek. Such a trade-off could generate a non-linear relationship between absolute levels of working hours and wages. However, it would not necessarily generate the static and dynamic, non-linear patterns we document between relative hours (that is, hours relative to the median or wagemaximizing hours) and wages.

Overall, complementarities in working hours in production and alternative mechanisms such as those discussed here are not mutually exclusive. However, in the novel evidence presented here using the WES and related literature, there is significant support for the presence of working hours complementarities. For instance, Bick et al. (2020) suggest that the need to coordinate working hours may generate complementarities in working hours and could explain the (economy-wide) non-linear relationship between working hours and wages. To support this idea, they show that the modal (and mean) hours worked in Denmark are lower than in the US and, consistent with complementarities in working hours, the hours at
which wages are maximized in Denmark are also lower than in the US. ${ }^{8}$
Labanca and Pozzoli (2021) explore the determinants of constraints on working hours and test whether firms' technology or individual preferences drive such constraints. They argue that constraints on hours are driven by the technologies utilized by firms rather than the collective choice of individuals coordinating on leisure. Their findings indicate that firms' technologies, rather than individuals' tastes, are more important for understanding constraints on hours - constraints that would arise naturally under a production process featuring complementarities in worker hours.

Motivated by these findings in the literature and those presented here, in the next section, we move beyond the reduced-form analysis of variation in hours and wages and use this variation to estimate the degree of complementarities in working hours in a more structural framework.

## 3 Estimating Complementarities in Working Hours

In this section, we first propose a model, which we then use to estimate the parameter that governs complementarities in working hours.

### 3.1 Production Function

The model we use to estimate the degree of complementarity in working hours is intentionally simple and limited to the description of the production function and a labor market structure. Firms' production is represented by: $Y=z L^{\eta}$, where $L$ denotes the effective labor input and $z$ is broadly defined as productivity, which can capture non-labor variables at the firm level such as technology, capital, or intermediate inputs.

[^7]In contrast to standard neoclassical models of production, where the effective labor input of a firm is the sum of total hours worked, we follow Yurdagul (2017) and allow for complementarities between the hours of workers. In particular, we assume that the labor input of workers is aggregated in a non-linear manner so that the aggregate labor input depends on the distribution of hours worked in a firm. The labor input $L$ is given by,

$$
L=\left(\int_{i \in N} x_{i} l_{i}^{\rho} d i\right)^{\frac{1}{\rho}}\left(\int_{i \in N} x_{i} d i\right)^{1-\frac{1}{\rho}}
$$

where $N$ is the set of workers, and $\left\{l_{i}\right\}_{i \in N}$ is their hours worked. Without loss of generality, we assume the working hours range from 0 and 1 . The contribution of each worker to the labor input is scaled up by their efficiency units $x_{i}>0$. To abstract from indices of workers, one can rewrite the aggregation in terms of the measure of workers employed at each level of hours worked,

$$
\begin{equation*}
L=\left(\int_{x \in B_{x}} \int_{0}^{1} x \mu(l, x) l^{\rho} d l d x\right)^{\frac{1}{\rho}}\left(\int_{x \in B_{x}} \int_{0}^{1} x \mu(l, x) d l d x\right)^{1-\frac{1}{\rho}} \tag{5}
\end{equation*}
$$

where $\mu(l, x)$ is the measure of workers with efficiency $x$ working $l$ hours. The parameter $\rho$ determines the elasticity of substitution $\left(\frac{1}{1-\rho}\right)$ between hours of different workers and is our key parameter of interest to be estimated.

In order to estimate $\rho$, we must first provide additional structure on the economic environment. To this end, we assume that labor markets are segmented by firm type, $z$, and all firms within a sub-market compete for workers in perfectly competitive labor markets. ${ }^{9}$ Under this market structure, firms take the wage schedule $w_{z}(l, x)$ in their market as given

[^8]and choose the measure, $\mu(l, x)$, of workers with a given hour-efficiency combination to hire in order to maximize their static profits:
$$
\pi=\max _{\mu(l, x)} z L^{\eta}-\int_{x \in B_{x}} \int_{0}^{1} w_{z}(l, x) \mu(l, x) l d l d x
$$
where $L$ is given by (5).
The first order condition of the firm's maximization problem returns an expression for the equilibrium wage schedule,
\[

$$
\begin{equation*}
w_{z}(l, x)=\eta z x L^{\eta-1} E\left(l^{\rho}\right)^{\frac{1}{\rho}}\left[\frac{1}{\rho} \frac{l^{\rho-1}}{E\left(l^{\rho}\right)}+\left(1-\frac{1}{\rho}\right) l^{-1}\right], \tag{6}
\end{equation*}
$$

\]

where the right-hand side is the marginal productivity of a worker with efficiency units of $x$ working $l$ hours. $E\left(l^{\rho}\right) \equiv\left(\int_{x \in B_{x}} \int_{0}^{1} x \mu(l, x) l^{\rho} d l d x\right) \div\left(\int_{x \in B_{x}} \int_{0}^{1} x \mu(l, x) d l d x\right)$ is a weighted average of $l^{\rho}$, and $L$ is the aggregate labor input. ${ }^{10}$

Equation (6) shows that a worker's wage depends not only on her own hours but those of her coworkers. Indeed, for $\rho \in(-\infty, 1)$ the maximum hourly wage, for each efficiency group, is achieved at the same level of hours $l^{*}=E\left(l^{\rho}\right)^{\frac{1}{\rho}}$ and wages decrease as working hours move away from $l^{*} .{ }^{11}$ This wage schedule would also generate a channel through which coworkers' hours are positively correlated - as observed in the data. If $\rho=1$, so that workers' hours are perfect substitutes in production, the hourly wage for the same $x$-type workers will be the same regardless of their hours. On the other hand, as $\rho \rightarrow-\infty$, so that the production function approaches Leontief, the marginal product for all workers will be zero if they do not work $l^{*}$ hours. As illustrated by these two extremes, different values of $\rho$ represent different relationships between wages and hours worked, which we use to estimate this parameter.

[^9]
### 3.2 Estimation Strategy

Estimating $\rho$ directly from equation (6) requires information on firm level productivity $z$, the returns to scale parameter $\eta$ as well as a measure of worker efficiency $x$. This poses a challenge since there are no natural counterparts to these measures in the WES. However, we can eliminate $z$ and $\eta$ by normalizing worker earnings $\left(w_{z}(l, x) l\right)$ relative to average earnings of her establishment $\bar{W} \equiv \eta z L^{\eta-1} E\left(l^{\rho}\right)^{\frac{1}{\rho}} \mathbb{E}(x)$ where $\mathbb{E}(x)$ is the mean worker efficiency across all workers in the firm. Then, relative worker earnings for a worker $i$ is independent of measures that are fixed within an establishment and given by,

$$
\begin{equation*}
\tilde{W}_{i} \equiv \frac{w_{z}\left(l_{i}, x_{i}\right) l_{i}}{\bar{W}}=\frac{x_{i}}{\mathbb{E}(x)}\left[\frac{1}{\rho} \frac{l_{i}^{\rho}}{E\left(l^{\rho}\right)}+1-\frac{1}{\rho}\right] . \tag{7}
\end{equation*}
$$

While simpler, estimating $\rho$ from (7) requires information on worker type $x$ which is unobserved. Instead, we construct a proxy for worker efficiency, $X$, using a linear function of education, gender and age:

$$
X_{i}=\theta_{0}+\theta_{1} \text { Education }_{i}+\theta_{2} \text { Gender }_{i}+\theta_{3} \text { Age }_{i},
$$

where Education ${ }_{i}$, Gender $_{i}$, and $\mathrm{Age}_{i}$ are dummy variables which indicate whether worker $i$ has a college degree, is male, or over 45 years of age, respectively. In turn, we compute the efficiency of worker $i$ relative to the firm average using $\tilde{X}_{i} \equiv \frac{X_{i}}{\mathbb{E}(X)}$ so that the expression for relative earnings becomes,

$$
\tilde{W}_{i}=\tilde{X}_{i} \times\left[\frac{1}{\rho} \frac{l_{i}^{\rho}}{E\left(l^{\rho}\right)}+1-\frac{1}{\rho}\right] .
$$

Finally, we can substitute $E\left(l^{\rho}\right)$ with an observable measure that does not depend on $\rho$ by recognizing that wages for each efficiency group in a firm are maximized when hours worked are $l^{*}=E\left(l^{\rho}\right)^{\frac{1}{\rho}}$. This allows us to replace $E\left(l^{\rho}\right)^{\frac{1}{\rho}}$ with the observed hours $\tilde{l}^{*}$ that return the highest hourly wage in each establishment. Then, for each worker, we compute
the hours relative to this establishment-specific reference hour as $\tilde{h}_{i}=\frac{l_{i}}{\widetilde{\imath}_{\imath}^{*}}$, which delivers our estimating equation:

$$
\begin{equation*}
\tilde{W}_{i}=\tilde{X}_{i} \times\left[\frac{1}{\rho} \tilde{h}_{i}^{\rho}+1-\frac{1}{\rho}\right] \tag{8}
\end{equation*}
$$

where $\tilde{W}_{i}$ are the earnings of worker $i$ relative to the average firm earnings, $\tilde{X}_{i}$ is a proxy for relative worker efficiency and $\tilde{h}_{i}$ are the hours worked of worker $i$ relative to the wage maximizing hours. We use this equation to estimate the substitution parameter $\rho$.

### 3.3 Results

We estimate equation (8) using a non-linear least squares regression and report our estimation results in Table 2. ${ }^{12}$ Panel A reports the estimates of $\rho$ for the aggregate sample for three alternative proxy measures of worker skill $x_{i}$. When considering only education as a proxy for skill, we estimate $\rho$ to be around -0.44, implying an elasticity of substitution of around 0.69. Including gender and age leads to a very similar elasticity. Importantly, regardless of the proxy for $x_{i}$, we find that the elasticity of substitution between working hours is below 1 - that is, working hours are gross complements in production.

We also separately estimate the working hours elasticity of substitution by industry by restricting our sample to a particular industry and then estimating the substitution parameter, $\rho$, using education, age, and gender to proxy for worker skill, $x_{i}$. Panel B of Table 2 shows the results for each of the 14 industry groups in the WES. Except for one industry, our estimates imply working hours to be gross complements in production. Indeed, even in "communications and other utilities", where the elasticity of substitution is around 1.04, the $95 \%$ confidence interval does not preclude working hours being gross complements. ${ }^{13}$

Our results show significant heterogeneity in the extent of complementarity in working

[^10]Table 2: Estimation Results

Panel A: Aggregate

Proxy for $x_{i}$ using only Education
Proxy for $x_{i}$ using only Education and Gender
Proxy for $x_{i}$ using only Education, Gender and Age

| Substitution Parameter, $\rho$ |  | Elasticity of Substitution $\frac{1}{1-\rho}$ |  |
| :---: | :---: | :---: | :---: |
| Estimate | 95\% CI | Estimate | 95\% CI |
| -0.443 | [-0.476,-0.410] | 0.693 | [0.677,0.709] |
| -0.459 | [-0.491,-0.426] | 0.686 | [0.671,0.701] |
| -0.459 | [-0.491,-0.426] | 0.686 | [0.671,0.701] |

Panel B: Industry

|  | Substitution Parameter, $\rho$ |  | Elasticity of Substitution $\frac{1}{1-\rho}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Estimate | 95\% CI | Estimate | 95\% CI |
| Secondary product manufacturing | -0.943 | [-1.269,-0.616] | 0.515 | [0.441,0.619] |
| Primary product manufacturing | -0.707 | [-0.932,-0.482] | 0.586 | [0.518,0.675] |
| Construction | -0.683 | [-0.799,-0.566] | 0.594 | [0.556,0.638] |
| Transportation, warehousing, wholesale | -0.646 | [-0.749,-0.543] | 0.608 | [0.572,0.648] |
| Labor intensive tertiary manufacturing | -0.621 | [-0.823,-0.418] | 0.617 | [0.548,0.705] |
| Education and health services | -0.604 | [-0.675,-0.533] | 0.624 | [0.597,0.653] |
| Real estate, rental and leasing operations | -0.483 | [-0.660,-0.307] | 0.674 | [0.603,0.765] |
| Forestry, mining, oil, and gas extraction | -0.478 | [-0.658,-0.297] | 0.677 | [0.603,0.771] |
| Capital intensive tertiary manufacturing | -0.403 | [-0.683,-0.123] | 0.713 | [0.594,0.890] |
| Business services | -0.388 | [-0.546,-0.229] | 0.721 | [0.647,0.813] |
| Finance and insurance | -0.248 | [-0.379,-0.118] | 0.801 | [0.725,0.895] |
| Retail trade and consumer services | -0.182 | [-0.290,-0.074] | 0.846 | [0.775,0.931] |
| Information and cultural industries | -0.178 | [-0.345,-0.010] | 0.849 | [0.743,0.990] |
| Communication and other utilities | +0.040 | [-0.138, +0.219] | 1.042 | [0.878,1.281] |

Panel C: Establishment Size

| Substitution Parameter, $\rho$ <br> Estimate |  |  | Elasticity of Substitution $\frac{1}{1-\rho}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| -0.403 | $[-0.477,-0.330]$ |  | 0.713 | $[0.677,0.752]$ |
| -0.448 | $[-0.497,-0.399]$ |  | 0.691 | $[0.668,0.715]$ |
| -0.495 | $[-0.548,-0.442]$ |  | 0.669 | $[0.646,0.693]$ |

Notes: The table reports estimates of the substitution parameter, $\rho$ and the corresponding elasticity of substitution $\frac{1}{1-\rho}$ along with $95 \%$ confidence intervals, as estimated from equation (8) using non-linear least squares. Panel A reports these estimates for the aggregate sample. Panel B and C report the same measures by establishment industry and establishment size, respectively. To limit the influence of outliers, we trim the top and bottom 1 percent of the ratios $\tilde{W}_{i}$ and $\tilde{h}_{i}$ from equation (8).
hours. Secondary and primary product manufacturing features the highest degree of complementarities with an estimated elasticity of substitution between working hours of 0.52 and 0.59 , respectively.

Low-skill services sectors such as retail trade and consumer services, and information and cultural industries exhibit the lowest (statistically significant) degree of complementarities with an elasticity of substitution of around 0.85 . In contrast, education and health services, which require higher skills, feature higher complementarities with an elasticity of 0.62.

The manufacturing sector generally features stronger complementarities in working hours than service sectors. ${ }^{14}$ This suggests that the manufacturing sector allows less flexibility in terms of allowing workers to choose a "non-standard" number of working hours than the service sector. Such flexibility may give women, who tend to value flexibility more than men (Goldin, 2014), a comparative advantage in service industries. Indeed, our findings complement existing works such as Ngai and Petrongolo (2017) and Rendall (2018) that highlight women's comparative advantage in the service sector. These estimates suggest that this comparative advantage could stem from a relative flexibility in service sector production. We further study how our estimates of complementarities vary with flexibility and female employment in the next section, which aims to validate our estimates.

Finally, Panel C of Table 2 reports the estimated substitution parameter $\rho$ and corresponding elasticity of substitution by establishment size. We consider three size categories; small establishments with under 10 employees, medium sized establishments with between 10 and 99 employees, and large establishments with over 100 employees. Our estimated elasticities indicate that complementarities are increasing with establishment size with an estimated elasticity of 0.71 in small establishments compared to 0.69 and 0.67 for larger establishments.

Although these point estimates are not statistically different across size groups, we consider higher levels of substitutability of hours in smaller establishments to be intuitive and

[^11]consistent with existing evidence. For example, Elfenbein et al. (2010) show that employees in smaller workplaces undertake a larger number of activities related to the business which may decrease the need to coordinate with coworkers. Related to this, Molina-Domene (2018) shows that workers in larger firms undertake fewer tasks and are more likely to be specialists raising the need to coordinate with coworkers resulting in stronger complementarities in working time between different workers.

### 3.4 Validation

Here, we explore whether our estimates of complementarities are systematically related to measures of hours coordination across industries. In particular, we use three measures to proxy for worker coordination at the industry level and test whether industries with a lower degree of complementarities in working hours also feature lower levels of coordination in work schedules. The three measures are i) the share of workers that work flexible hours, ii) the standard deviation of hours worked, and iii) the share of female workers in an industry. Figure 3 plots the industry average of these measures and the industry elasticity of substitution.

Panel (a) shows that industries with a larger share of workers with flexible working hours also feature higher elasticities of substitution and, therefore a lower degree of complementarity in working hours. ${ }^{15}$ Indeed, the correlation between workers' shares and elasticity is 0.59. Industries with lower shares of flexible hour workers, such as labor-intensive or primary product manufacturing, feature a higher degree of complementarity (a lower elasticity of substitution) than industries with higher shares of flexible hour workers, such as information and cultural industries. This is intuitive since greater flexibility in choosing one's working hours may indicate less need for workers to coordinate their work schedule and hence a lower degree of complementarity in working hours.

Next, we examine how the industry's standard deviation of working hours relates to our

[^12]

Figure 3: Elasticity of Substitution and Coordination Measures Across Industries
Notes: Each panel of the figure plots the industry-specific estimate of the elasticity of substitution on the horizontal axis and a different measure of coordination on the vertical axis. Panel (a) plots the share of workers with flexible hours, Panel (b) plots the within-industry standard deviation of hours worked, and Panel (c) plots the share of female workers. The correlation between elasticity of substitution and coordination measure is reported in the panel. The sample excludes "Communication and other utilities" since our estimate does not establish at the $5 \%$ significance level whether working hours are either gross complements or substitutes. The correlation between elasticity of substitution and share of flexible hours, the share of female workers, and the standard deviation of working is $0.32,0.26$, and 0.19 , respectively, when all 14 sectors are included. The descriptions of the WES industry codes can be found in Table A.3.
estimates of working hour complementarities. Consistent with Labanca and Pozzoli (2021), we view a more significant variation in hours as indicating a production process that requires lower coordination of hours. We expect to see a positive relationship between the degree of substitution in working hours and dispersion across industries. Panel (b) plots these two measures and shows that, as expected, there is a positive correlation (0.40) between our estimated elasticity of substitution and the standard deviation of working hours.

Finally, we consider the share of female workers as a proxy for the degree of flexibility in hours in production. Since female workers tend to work fewer hours and choose more flexible occupations, a higher share of female workers in an industry may proxy for the degree of flexibility in working hours (Goldin, 2014). ${ }^{16}$ Panel (c) shows that there exists a strong positive correlation (0.51) between the degree of working hours substitutability and the share of female workers across industries.

Importantly, the measures of hours coordination considered here are not directly related to variation in earnings across workers, which underpinned our estimation strategy. Taken together, the findings in Figure 3 support our estimated measures of complementarities in

[^13]hours.

## 4 Implications for the Gender Wage Gap

The presence of complementarities in working hours has wide-ranging implications on, among other aspects, wage inequality, the response of labor supply to policies and shocks, and the design of alternative work arrangements. In this section, we explore its implications, particularly for the gender wage gap.

Although the majority of the gender wage gap remains unexplained by observable characteristics, heterogeneity in hours worked is often cited as an important contributor (see, for example, Blau and Kahn, 2017). ${ }^{17}$ For instance, Goldin (2014) discusses how the wage penalty for part-time work lowers women's average wages as they tend to work shorter hours. We reassess the role of hours on the gender wage gap through the lens of our theoretical model. When combined with estimates of $\rho$, our production function allows us to directly compute both short- and long-hour penalties at the individual level in an establishment. This is in contrast to existing studies, such as Goldin (2015), which use aggregated measures of wage-hour penalties. Instead, our parsimonious structural framework allows us to construct counterfactual wages for each individual that remove wage penalties due to working hours. Comparing the observed gender wage gap to one constructed using these counterfactual wages allows us to quantify the role of hours in driving the gender wage gap.

Notice wage penalties arise under the presence of complementarities in working hours if workers deviate from their coworkers' hours. However, if hours are perfect substitutes (i.e., $\rho=1$ ), differences in hours will not generate wage penalties (among identical workers). Accordingly, the degree of complementarities in working hours and the distribution of hours will be crucial in shaping the role of hours on the gender wage gap.

Revisiting equation (6) allows us to construct a measure of wages that removes wage

[^14]penalties while leaving the other components of workers' wages unchanged. ${ }^{18}$ Recall, when $\rho<1$, wages (for each worker type in an establishment) are maximized at $l^{*}=E\left(l^{\rho}\right)^{\frac{1}{\rho}}$ and decrease as worker hours deviate from $l^{*}$. As such, our model predicts that the ratio of the actual wage of a worker $i$ in establishment $j, w_{j}\left(l_{i}, x_{i}\right)$, and the maximum attainable wage in that establishment, $w_{j}^{*}\left(l_{i}, x_{i}\right)$, is
$$
\frac{w_{j}\left(l_{i}, x_{i}\right)}{w_{j}\left(l_{j}^{*}, x_{i}\right)}=\frac{l_{j}^{*}}{l_{i}}\left[\frac{1}{\rho} \frac{l_{i}^{\rho}}{l_{j}^{* \rho}}+\left(1-\frac{1}{\rho}\right)\right] .
$$

Using this ratio, we can compute an alternative wage which removes the effects of the short- and long-hour penalties for each individual $i$ in an establishment. This counterfactual wage is,

$$
\begin{equation*}
w_{i}^{*}=w_{i} \frac{l_{i}}{l_{j}^{*}}\left[\frac{1}{\rho} \frac{l_{i}^{\rho}}{l_{j}^{* \rho}}+\left(1-\frac{1}{\rho}\right)\right]^{-1} \tag{9}
\end{equation*}
$$

This counterfactual wage adjusts each worker's wages by removing the penalties arising from hours deviations while still taking into account any wage components not attributable to the individual's hours (due to unobserved ability, for example). In other words, it eliminates the component of wages that correspond to penalties resulting from working either relatively short or long within a given establishment.

We use equation (9) to study the contribution of hours (and the corresponding wage penalties) on the gender wage gap. The extent to which counterfactual wages, $w_{i, j}^{*}$ differ from $w_{i, j}$ depend on i) the degree to which hours deviate, $\frac{l_{i, j}}{l_{j}^{*}}$ and ii) the elasticity parameter $\rho .{ }^{19}$

Panel (a) of Figure 4 reports the distribution of $\frac{l_{i, j}}{l_{j}^{*}}$ across individuals and establishments.

[^15]As with the absolute level of hours worked, the relative working hours of men and women feature significant differences, with male hours being longer and more concentrated around the wage-maximizing hours than women's. Further, for both genders, a relatively small share of workers work longer than the wage-maximizing hours. As such, we expect the long-hours penalty to play a relatively small role in generating overall wage penalties and the gender wage gap.


Figure 4: Distribution of relative hours and wage penalties, by gender
Notes: Panel (a) plots the distribution of worker hours relative to the establishment's wage-maximizing hours $\left(\frac{l_{i, j}}{l_{j}^{t}}\right)$ by gender. Panel (b) plots the average wage penalty by gender, where wage penalty for each individual $i$ in establishment $j$ is calculated as $\left(1-\frac{w_{i, j}}{w_{i, j}^{*}}\right)$.

Given the observed variation in relative hours, the presence of complementarities in hours will introduce positive wage penalties for all workers. Panel (b) plots the average wage penalties over the life-cycle, separately by gender. Consistent with the relative hours distribution in Panel (a), the imputed penalties for women are higher than those for men throughout the life-cycle. Women's wage penalties are roughly twice those of men, with the largest difference occurring during the late 30 's and early 40 's - a point in the life-cycle during which the gender wage gap also peaks as documented by Goldin (2014) in the US. Wage penalties for women rise again in the late 50 's and beyond, but so do penalties for men with relatively little changes in differences in penalties.

Next, we compare the observed gender wage gap to the gender wage gap derived from


Figure 5: The gender wage gap and the role of hours penalties
Notes: Panel (a) plots the observed (solid lines) and counterfactual (dashed lines) gender gap wage gap over the life-cycle by cohort in the WES. The gender wage gap is computed as one minus the ratio of average female to average male earnings. Panel (b) plots the difference between the observed and counterfactual gender wage gaps as a share of the observed gender wage gap.
counterfactual wages. Panel (a) of Figure 5 reports the level of the observed and counterfactual wage gaps by age and cohort. Consistent with existing literature, the gender wage gap appears to be inverse- U shaped over the life-cycle, peaking in the late 30 's. However, given the WES is available for a relatively small number of years, it is difficult to establish a clear distinction between life-cycle and cohort effects. The counterfactual wage gap, which results from removing wage penalties, is lower at each point in the life-cycle, suggesting that the penalties resulting from hours heterogeneity widen the gender wage gap.

How much of the gender wage gap is due to heterogeneity in hours and the resulting wage penalties due to complementarities? Panel (b) answers this question by reporting the difference between observed and counterfactual gender wage gaps. It shows that hours heterogeneity accounts for between 5 to 30 percent of the gender wage gap over the lifecycle. The largest contribution to the gender wage gap heterogeneity in hours, and the accompanying wage penalties, occurs in the early 30 's, accounting for $30 \%$ of the observed gender wage gap. This contribution tends to decline with age until reaching a minimum of around during the early 60 's. However, as with the gender wage gap itself, the effects of time, age, and cohort are difficult to disentangle without a longer sample period. On average, we
find the hours contribute to around $14 \%$ of the gender wage gap; put differently, if we remove the wage penalties that arise due to complementarities in working hours, the gender wage gap will shrink by $14 \%$ or around 2 percentage points (relative to the $13.3 \%$ gender wage gap documented in Pelletier et al. (2019) in Canada in 2018.).

Equation 9 makes clear that heterogeneity in the degree of complementarities in working hours, as governed by $\rho$, also influences the magnitude of wage penalties resulting from hours heterogeneity and hence the contribution of hours for the gender wage gap. To illustrate this, we compute the share of the gender wage gap explained by hours separately for each industry in the WES and plot this share in Figure 6. Industries that feature stronger substitutability in working hours across worker hours also feature a smaller role for hours in the gender wage gap. This is intuitive since strong substitution between working hours implies that deviations in working hours will not lead to large wage penalties. Indeed, given a preference for shorter hours, women may internalize this and sort into industries with highly substitutable hours in order to avoid large penalties from deviations (Panel (c) of Figure 3).


Figure 6: Gender-wage gap and hours penalties, across sectors
Notes: The figure plots industry-specific estimates of the elasticity of substitution. $\frac{1}{1-\rho}$, and the share of the gender wage gap explained by hours penalty in each industry.

The idea that working hours contribute to the gender wage gap is consistent with the analysis in Goldin (2014) and Goldin (2015). We explicitly quantify the short- and long-hour wage penalties experienced by individuals for deviating from coworkers' hours in the presence
of complementarity. In contrast, existing works such as Goldin (2015) have used reduced form analysis based on aggregated (average) penalties. Importantly, our analysis suggests that part of the gender wage gap can be attributed to production technology, namely the presence of complementarities in hours worked. In this regard, our results complement Cubas et al. (2019), which show that the gender wage gap can be explained by, in part, the need to work at the same time as one's coworkers.

In contrast, existing works have used reduced form analysis based on aggregated (average) penalties as in Goldin (2015).

Taken together, our analysis suggests that hours heterogeneity under complementarities in hours contributes significantly to the gender wage gap and wage inequality more generally.

## 5 Conclusion

This paper uses matched employer-employee data from Canada to study complementarities between coworkers' hours in production. We provide novel reduced-form evidence that is consistent with the presence of such complementarities. Specifically, we find (i) coworker hours are strongly positively correlated with individual hours, (ii) wages within establishments peak at intermediate hours and feature penalties at extreme levels of hours, (iii) workers that move further away from (closer to) the median or the wage-maximizing hours in an establishment face wage reductions (increases).

We then use a simple model to estimate the degree of complementarities in working hours. The economy-wide elasticity of substitution is estimated to be 0.69 , implying that working hours are gross complements in production. Although our industry-specific estimates exhibit significant variation, we find that hours are gross complements for almost all sectors in our data. To validate our estimates, we show that the estimated elasticities of substitution are positively correlated to proxy measures of the degree of coordination in working hours across industries.

Finally, we explore the implications of complementarities in working hours for the gender wage gap. In particular, we investigate the role of heterogeneity in hours in accounting for the gender wage gap. By removing individual-level wage penalties that arise under complementarities in hours, we show that the hours heterogeneity accounts for around $14 \%$ of the overall gender wage gap.

Overall, our results challenge a canonical assumption about the nature of production, namely, that the working hours of workers are perfect substitutes. We provide strong evidence supporting coworker hours being gross complements. Such complementarity has important implications for research on labor supply and the efficacy of policies that aim to influence it.

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## Appendix for:

# Are Working Hours Complements in Production? 

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## A Additional Figures and Tables

Table A.1: Summary Statistics, WES

|  | Obs |  | Mean |
| :--- | :---: | :---: | :---: |
| Age | Std. Dev. |  |  |
| Usual Weekly Hours Worked | 129,037 | 42.78 | 9.89 |
| Hourly Wage (CAD) | 126,613 | 37.57 | 8.17 |
| Hourly Wage (no extra earnings, CAD) | 105,005 | 20.96 | 12.94 |
| High School Graduate | 129,037 | 0.85 | 11.44 |
| College Graduate | 129,037 | 0.36 | 0.50 |
| Establishment Age | 126,911 | 21.77 | 22.13 |
| Total \# Employees in Firm | 129,037 | 402.17 | 1088.71 |
| Gross operating revenue (mil. CAD) | 129,037 | 37.08 | 139.05 |
| Average Usual Weekly Hours in Establishment | 128,828 | 37.11 | 6.50 |
| Average Hourly Wage in Establishment | 128,828 | 20.71 | 9.75 |

Notes: The table reports a number of summary statistics from the WES sample.


Figure A.1: Aggregate Relationship between Hours and Wages, Data from US and Canada Worker Surveys
Notes: The figure plots the aggregate relationship between weekly hours worked and wages. In particular, it plots the coefficient $\gamma_{h}$ as estimated from the following regression,

$$
\log \left(w_{i}\right)=\alpha+\left(\sum_{h \in H} \gamma_{h} \mathbb{I}_{i, h}\right)+\delta X_{i}+\epsilon_{i}
$$

where $\log \left(w_{i}\right)$ is the log hourly wages of individual $i . X_{i}$ is a vector of individual-level controls which includes demographic controls including gender, race and education dummies, a quadratic in years of experience as well as state, year, and industry fixed effects. The indicator variable $\mathbb{I}_{i, h}$ is equal to one if an individual works $h$ hours. Weekly hours $h$ are partitioned into a set $H=\{10-14,15-19, \ldots, 65-69,70-99\}$. As most workers work 40 hours, the category $40-44$ hours is the omitted (reference) category. Data from the US is from 1991 to 2018 Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). Data from Canada is from the 1997 to 2018 Canadian Labor Force Surveys (LFS). Additional details on the sample construction can be found in Shao et al. (2022). The bottom horizontal axis reports results from the CPS. The top horizontal axis is shifted by 5 hours and reports results from the LFS. The shaded regions indicate the $95 \%$ confidence interval.


Figure A.2: Wage-Maximizing Hours and Average/Median coworker Hours
Notes: The figure illustrates the relationship between establishment-level wage-maximizing hours and average/median hours. Each dot represents a group of establishments with the same wage-maximizing hours. Due to data confidentiality restrictions, the panels only display binned groups with more at least ten observations. The horizontal axis reports the wage-maximizing hours while the vertical axis reports the mean value of establishment-level average hours (Panel (a)) and median hours (Panel (b)) in each group.

Table A.2: Estimated Coefficient of Proxy Measure of Worker Efficiency, $\widetilde{X}_{i}$

|  | $(1)$ | $(2)$ | $(3)$ |
| :--- | :---: | :---: | :---: |
| Coefficient on Education, $\theta_{1}$ | 0.142 | 0.155 | 0.173 |
|  | $(0.003)$ | $(0.003)$ | $(0.003)$ |
| Coefficient on Gender, $\theta_{2}$ | - | 0.203 | 0.211 |
|  | - | $(0.003)$ | $(0.003)$ |
| Coefficient on Age, $\theta_{3}$ | - | - | 0.097 |
|  | - | - | $(0.003)$ |

Notes: The table reports the coefficient $\theta_{i}$ from estimating equation (8) using the aggregate WES sample. Robust standard errors are reported in the parentheses.

# Table A.3: WES and NAICS Industry Codes 

| WES Industry Code | WES Industry Description | NAICS Industry Description and Code |
| :---: | :---: | :---: |
| 1 | Forestry, mining, oil, and gas extraction | Forestry and Logging (113), Support Activities for Forestry (1153), Oil and Gas Extraction (211), Mining (except Oil and Gas) (212), Support Activities for Mining and Oil and Gas Extraction (213) |
| 2 | Labor intensive tertiary manufacturing | Food Manufacturing (311), Beverage and Tobacco Product Manufacturing (312), Textile Mills (313), Textile Product Mills (314), Clothing Manufacturing (315), Leather and Allied Product Manufacturing (316), Furniture and Related Product Manufacturing (337), Miscellaneous Manufacturing (339) |
| 3 | Primary product manufacturing | Wood Product Manufacturing (321), Paper Manufacturing (322), Petroleum and Coal Products Manufacturing (324), Non-Metallic Mineral Product Manufacturing (327), Primary Metal Manufacturing (331) |
| 4 | Secondary product manufacturing | Chemical Manufacturing (325), Plastics and Rubber Products Manufacturing (326), Fabricated Metal Product Manufacturing (332) |
| 5 | Capital intensive tertiary manufacturing | Printing and Related Support Activities (323), Machinery Manufacturing (333), Computer and Electronic Product Manufacturing (334), Electrical Equipment, Appliance and Component Manufacturing (335), Transportation Equipment Manufacturing (336) |
| 6 | Construction | Prime Contracting (231), Trade Contracting (232), Construction of Buildings (236), Heavy and Civil Engineering Construction (237), Specialty Trade Contractors (238) |
| 7 | Transportation, warehousing, wholesale | Farm Product Wholesaler-Distributors (411), Petroleum Product WholesalerDistributors (412), Food, Beverage and Tobacco Wholesaler-Distributors (413), Personal and Household Goods Wholesaler-Distributors (414), Motor Vehicle and Parts Wholesaler-Distributors (415), Building Material and Supplies Wholesaler-Distributors (416), Machinery, Equipment and Supplies Wholesaler-Distributors (417), Miscellaneous Wholesaler-Distributors (418), Wholesale Agents and Brokers (419), Air Transportation (481), Rail Transportation (482), Water Transportation (483), Truck Transportation (484), Transit and Ground Passenger Transportation (485), Pipeline Transportation (486), Scenic and Sightseeing Transportation (487), Support Activities for Transportation (488), Warehousing and Storage (493) |
| 8 | Communication and other utilities | Utilities (221), Postal Service (491), Couriers and Messengers (492), Waste Management and Remediation Services (562) |
| 9 | Retail trade and consumer services | Motor Vehicle and Parts Dealers (441), Furniture and Home Furnishings Stores (442), Electronics and Appliance Stores (443), Building Material and Garden Equipment and Supplies Dealers (444), Food and Beverage Stores (445), Health and Personal Care Stores (446), Gasoline Stations (447), Clothing and Clothing Accessories Stores (448), Sporting Goods, Hobby, Book, and Music Stores (451), General Merchandise Stores (452), Miscellaneous Store Retailers (453), Nonstore Retailers (454), Amusement, Gambling, and Recreation Industries (713), Accommodation (721), Food Services and Drinking Places (722), Repair and Maintenance (811), Personal and Laundry Services (812) |
| 10 | Finance and insurance | Monetary Authorities - Central Bank (521), Credit Intermediation and Related Activities (522), Securities, Commodity Contracts, and Other Financial Investments and Related Activities (523), Insurance Carriers and Related Activities (524), Funds and Other Financial Vehicles (526) |
| 11 | Real estate, rental and leasing operations | Real Estate (531), Rental and Leasing Services (532), Lessors of Nonfinancial Intangible Assets (except Copyrighted Works) (533) |
| 12 | Business services | Professional, Scientific, and Technical Services (541), Management of Companies and Enterprises (551), Administrative and Support Services (561) |
| 13 | Education and health services | Educational Services (611), Ambulatory Health Care Services (621), Hospitals (622), Nursing and Residential Care Facilities (623), Social Assistance (624), Grantmaking and Giving Services (8132), Social Advocacy Organizations (8133), Civic and Social Organizations (8134), Business, Professional, Labor, Political, and Similar Organizations (8139) |
| 14 | Information and cultural industries | Publishing Industries (except Internet) (511), Motion Picture and Sound Recording Industries (512), Broadcasting and Telecommunications (513), Information Services and Data Processing Services (514), Performing Arts, Spectator Sports, and Related Industries (711), Museums, Historical Sites, and Similar Institutions (712) |


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[^1]:    ${ }^{1}$ Although our data is at the establishment level, we assume that all establishments in a firm share a production function. As such, we use the terms firm and establishment interchangeably.

[^2]:    ${ }^{2}$ As will be clear below, our estimation only requires workers' wages to be proportional to their marginal product (as in Katz and Murphy (1992), for example) and thus can accommodate alternative market structures, such as those with constant wage markdowns, which might result from firms having labor market power.

[^3]:    ${ }^{3}$ Also related is Erosa et al. (2016) also explore the role of labor supply for the gender wage gap through the lens of a quantitative life-cycle framework which emphasizes the role of fertility and human capital accumulation.

[^4]:    ${ }^{4}$ Our sample restrictions follow Bick et al. (2020). Further, the prevalence of low wage and/or low hours workers is relatively low. For example, only $1.2 \%$ of private employees work either below 10 hours or earn below half the minimum wage in the 2015 Canadian Labor Force Survey (LFS) - a nationally representative survey of the Canadian labor force.
    ${ }^{5}$ Summary statistics are reported in Table A. 1 in the Appendix.

[^5]:    ${ }^{6}$ Bick et al. (2020) and Yurdagul (2017) use data from the Current Population Survey (CPS) and Survey of Income and Program Participation, respectively.

[^6]:    ${ }^{7}$ Figure A. 2 in the Appendix shows a strong positive relationship between establishment-level wagemaximizing hours and average or median hours worked. A simple linear regression between average (median) hours and wage-maximizing hours gives a statically significant slope coefficient of 0.88 (0.94).

[^7]:    ${ }^{8}$ Figure A. 1 in the Appendix compares the aggregate wage-hours profile estimated from the US CPS and the Canadian Labor Force Survey (LFS). The figure shows that (economy-wide) wages peak at higher hours in the US than in Canada. Consistent with complementarities in working hours, the average hours worked in the US are also higher than the average hours worked in Canada. For example, in 2015, US workers worked around 3 hours longer ( 41.5 vs. 38.2 ).

[^8]:    ${ }^{9}$ The purpose of the partitioning is simply to accommodate heterogeneous equilibrium wage schedules between firms since perfect competition implies uniform wages within each sub-market. We could also allow for a more general segmentation whereby firms and workers are randomly allocated into an arbitrary number of sub-markets, with perfect competition within each sub-market. Even though we abstract from the possibility of imperfect competition in labor markets (e.g. Berger et al., 2022), our estimation only requires wages to be proportional to workers' marginal product and can accommodate constant wage markdowns which might result from firms having labor market power.

[^9]:    ${ }^{10}$ Our estimation relies on variation within firms rather than between firms. Hence, for notational convenience, we omit indexing the variables $L, E\left(l^{\rho}\right), \mu(l, x), Y, \pi$ by firms' type although they are all specific to a firm.
    ${ }^{11}$ Notice, $l^{*}$ depends on $\rho$ and would in general be different from the median or average hours worked in a firm - as in the data.

[^10]:    ${ }^{12}$ We jointly estimate the coefficients $\theta_{i}$ for the proxy measure of worker skills along with the substitution parameter $\rho$. Estimates of $\theta_{i}$ are reported in Table A. 2 in the Appendix.
    ${ }^{13}$ Table A. 3 in the Appendix lists the NAICS industry codes that comprise each WES industry category.

[^11]:    ${ }^{14}$ The estimated elasticities for all manufacturing and service industries are 0.61 and 0.69 , respectively.

[^12]:    ${ }^{15}$ The share of flexible workers in an industry is the share of workers giving an affirmative response to the question Do you work flexible hours? An affirmative answer implies that workers can vary their daily start and stop times as long as they work a full workweek. That is, on a daily basis, a "flexible" worker may not work the same number of hours as their coworkers nor work at the same time as them.

[^13]:    ${ }^{16}$ The correlation between the share of flexible hour workers and female workers is 0.35 .

[^14]:    ${ }^{17}$ Pelletier et al. (2019) documents recent trends in the gender wage gap in Canada and finds that, in 2018, around $63 \%$ of the wage gap between men and women was not accounted for by observable characteristics.

[^15]:    ${ }^{18}$ In particular, when we adjust a worker's hours, we do not modify the effective labor aggregate $L_{j}$, or the expectation term $E_{j}\left(l^{\rho}\right)$ of the firm in order to directly identify the hours penalty term in the baseline wage equation.
    ${ }^{19}$ In this analysis, setting the firm-specific reference hours worked to be $l_{j}^{*}$ allows us to compute a uniform measure of hours-wage penalties across all workers. Adjusting the wages of a worker by setting her hours equal to an alternative reference point would generally result in wage penalties for each individual that are not clearly interpreted. For instance, if we set hours equal to an alternative $\tilde{l}$ that is constant across workers, then the wage penalties might increase or decrease for different workers relative to the baseline, depending on their original hours, and the actual wage maximizing hour $l_{j}^{*}$ in their firm.

