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A growing empirical literature documents a non-monotonic relationship between finance and growth. We investigate this finding in a Schumpeterian endogenous-growth model with search frictions and congestion effects in credit and innovation markets. Financial development eases the financing of innovation but exacerbates congestion effects in R&D. Conversely, policies that promote R&D aggravate financial bottlenecks. Once general equilibrium feedback effects are taken into account, the interplay between the two congestion frictions generates a non-linear relationship between finance and productivity growth. We show that, for a calibration chosen to mimic the actual US economy, the interplay between credit and innovation frictions results in a negative impact of finance on growth. This impact is however quantitatively small – consistent with the observation that, in the last century, most developed economies have experienced a widespread expansion of the financial sector yet almost constant, or slowly declining, growth rates of GDP (save for financial crises, pandemics or wars).

JEL Classification: E40, E50

Keywords: growth, Finance, search frictions

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Abstract

A growing empirical literature documents a non-monotonic relationship between finance and growth. We investigate this finding in a Schumpeterian endogenous-growth model with search frictions and congestion effects in credit and innovation markets. Financial development eases the financing of innovation but exacerbates congestion effects in R&D. Conversely, policies that promote R&D aggravate financial bottlenecks. Once general equilibrium feedback effects are taken into account, the interplay between the two congestion frictions generates a non-linear relationship between finance and productivity growth. We show that, for a calibration chosen to mimic the actual US economy, the interplay between credit and innovation frictions results in a negative impact of finance on growth. This impact is however quantitatively small – consistent with the observation that, in the last century, most developed economies have experienced a widespread expansion of the financial sector yet almost constant, or slowly declining, growth rates of GDP (save for financial crises, pandemics or wars).

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> Some academic researchers, pharma executives, and other experts have decried this explosion of [clinical] trials as a counterproductive glut motivated more by the race for money than good science and warned *that many of these efforts may not finish because of a lack of participants.*[1] (Kaiser, 2018)

Introduction

A long tradition in economics, exemplified by Solow (1956) and anchored in the Age of Enlightenment, views science and innovation as the main driving forces of long-run growth. The role played by financial development in this process is not trivial. To be true, research is costly so that, if inventions require investment in R&D beyond the means of innovators, financial development affects growth positively. However, the link between finance and grow is not one-way as financial institutions are costly to develop and maintain and their profitability is, by essence, affected by growth prospects. Furthermore, the finance-growth nexus cannot be linear as, over the last century, per-capita GDP in the United States has been growing, except for historical accidents, at an average 2% annual rate while the development of the financial sector has accelerated. Something is thus hindering, and maybe even reversing, the contribution of finance to growth: our paper explores the possibility that the interplay of congestion externalities in finance and in R&D might be the culprit.

Our formalization thus relies on the interaction between two frictional markets: the markets finance and ideas. In a world where innovation itself entails no friction or efficiency loss and finance constitutes the sole friction firms encounter, removing the sole hurdle standing in the way of innovators necessarily enhances growth. This is the traditional mechanism underlying many policy recommendations for financial liberalization.[2] By contrast with the utopian Solowian world in which innovations are instantly and freely provided to researchers by a *deus ex machina*, R&D takes time and effort and its success rate is unpredictable. Plus there is no presumption, unlike what is commonly assumed, that more science and more research are always better for growth: there might be other bottlenecks and congestion effects, interacting with those stemming from search-and-matching innovation frictions, that might be exacerbated by financial liberalization. For instance, proliferation

[1] Emphasis added.

[2] They are akin to labor-market reform policies advocated on the basis of the Diamond-Mortensen-Pissarides model: if search and matching frictions on the labor market as the sole hindrance to unemployment, a "better" functioning labor market is all it takes to reduce unemployment.

of clinical trials for checkpoint inhibitors in cancer immunotherapy might be "too much of a good thing" (Kaiser, 2018). Similarly, seemingly further removed from growth theory but in fact anchored in Jevons' (1866) paradox, Braess (1968) warns that traffic could be impeded by the addition of a new road — a remark at the core of the "Lewis-Mogride position" that postulates that punctual improvements in a road network often shift congestion to another traffic node, thereby negating the original effort and possibly exacerbating overall road congestion. Our starting point is thus that models that ignore research and innovation bottlenecks adopt, without nuance, a Renaissance-inspired belief in the unlimited scope for progress. Is it reasonable to assume that the more resources we pour into innovation the faster we will grow on average? Shall we not eventually lack individuals with the ability to do research?; or run out of the less skilled workers who provide infrastructure complementary to research (e.g., building or maintenance)? Or patients to enroll into clinical trials? Our answer is: possibly.

We investigate this conundrum in an endogenous growth model with search frictions in both credit and innovation markets. In our world, all growth is innovation-led and, partly in line with the Solowian view of the world, there is a fixed number of ideas ready to be "fetched" by scientists. Entrepreneurs do not have the wealth (or ability) to self-finance innovation and need to look for financiers. We show that all else equal, there is a negative relationship between growth and tightness in both innovation and credit markets. But once all feedback effects are taken into account, financial deepening has a non-monotonic effect on long-run growth: after a certain threshold more finance, i.e. less tightness in credit markets, increases congestion in the ideas market and growth might fall.

We then turn to a normative analysis and compare the competitive equilibrium of our model with the constrained efficient allocation. Following the search tradition, we derive a set of "modified" Hosios conditions. In our model, firms are created at average economy-wide productivity. Individual firms' investment in R&D and more liquidity can boost aggregate productivity. This externality, together with the congestion externalities standard in a search and matching framework, is a source of inefficiency. We show that entry in financial and innovation markets is efficient once innovators and financiers are compensated for their contribution to growth. Moreover, the social planner internalizes the interactions between the two congestion externalities, i.e. that the marginal contribution of finance to growth is lower when innovation markets are tighter.

The remainder of the paper is organized as follows. After a literature review in the next subsection, Section 1 illustrates the links between growth and finance in a simple partial equilibrium model. Section 2 deals with this issue in a model with double search frictions in the innovation and financial markets. Section 3, derives the constrained efficient allocation in our model. Section 4 assess the robustness of our main findings. Finally, Section 5 concludes.

Literature review

The finance-growth nexus has been the subject of an extensive empirical literature. If substantial historical evidence finds a positive effects of finance on growth (see e.g., King and Levine, 1993, Levine, 1997, Rajan and Zingales, 1998 and Beck et al., 2000), more recent studies (Popov, 2018, Arcand et al., 2015, Aghion et al., 2019) suggest the existence of non-linearities. Specifically, it has been shown that, beyond a certain threshold, financial development has no effect or could even be detrimental for growth. In theory (see Levine, 2005 and Aghion et al., 2018 and references therein), well functioning financial systems can promote growth by improving resource allocation, fostering innovation or by facilitating monitoring and pooling of risky projects. At the same time, "too much finance"could lead to a misallocation of talents to less productive sectors of the economy (Tobin, 1984) or an increase in financial fragility (Minsky, 1974 and Rajan, 2005). Aghion et al. (2019) document an inverted-U relationship between credit constraints and productivity growth at a sectoral level in France. To account for this empirical observation, they propose a theory according to which better access to credit facilitates innovation but at the same time allows less efficient incumbent firms to remain longer on the market. In the same spirit, Malamud and Zucchi (2019) propose a theoretical model where financing frictions affects differently entrant and incumbent firms and hence and the composition of growth. Our paper proposes a different, and not necessarily exclusive, explanation for the non-linear relationship between finance and productivity growth: the interplay between congestion frictions in the financial sector and in the market of ideas.

In our model, financiers provide funds to entrepreneurs to invest in R&D. All else equal, through this channel finance has a positive effect on growth. In this respect, our work contributes to the literature on innovation-led growth (see e.g. Aghion et al. 2005, Laeven et al., 2015, Chiu et al., 2017, Aghion et al., 2018). However, we depart from that literature in two important respects: i) we model bottlenecks in R&D by introducing search frictions in innovation markets, in the spirit as Silveira and Wright (2010) ii) we incorporate search frictions in financial markets. Our modeling of finance, borrowed from Wasmer and Weil (2004), embraces the view of Jaffee and Stiglitz (1990) according to which credit markets are better described as customer markets where borrowers have a single relationship with lenders. Cipollone and Giordani (2019a) and Cipollone and Giordani (2019b) provide some recent empirical support to a search and matching view of financial markets.

The advantages of our approach is twofold. First, in our model, the degree of financial development, or credit market tightness, is endogenous. Better growth prospects induce entry by financiers, thereby capturing a two-way feedback effect from growth to finance (see Robinson, 1952). Second, the interaction between congestions in two markets, i.e. financial and innovation, can generate a non-linear relation between finance and growth as observed in the data.

Our work is related to work by Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2015) and Chiu et al. (2017) who study, among others, the interactions between multiple trading frictions. In particular, the latter paper also deals with innovation-led growth in the presence of financial frictions though it restricts liquidity through a collateral constraint. In addition, Berentsen et al. (2012) integrate a search-theoretic model of money into an endogenous growth framework. They show that the welfare costs of inflation are larger taking into consideration search frictions in the innovation process, while more efficiency in financial markets has a positive but limited effect on growth.

Empirically, Bloom et al. (2020) document a sharp decline in research productivity and a substantial rise in research effort in many sectors. Gordon (2016) suggests that inequality, education, demographic and fiscal factors are fours possible forces holding back productivity growth. In our model, search frictions in the innovation market aim at broadly capturing hurdles in the production function of ideas, i.e. success in R&D requires both resources and time. Our paper provides a theoretical explanation behind the empirical observation that "ideas are getting harder to find".

1 Growth: a partial-equilibrium accounting framework

We start from a simple partial-equilibrium accounting framework of growth in which firms need an exogenous amount of time and effort to finance innovation. We will endogenize the required time and effort in the full model.

Assume that the life cycle of firms can be decomposed into four stages, illustrated by the timeline in Figure 1:

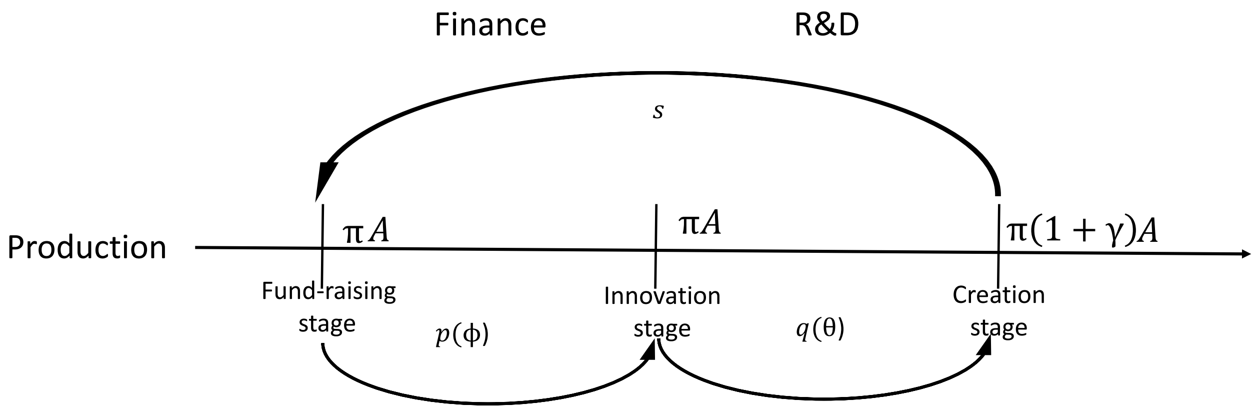

Figure 1: Markets and transitions through different stages.

- First, a newly-created firm immediately produces flow output π (without the need for workers), and suffers a concomitant flow production cost we set to π for simplicity.[3] It might be convenient to think of a firm as a robot or an automated production line.

- Second, a newly-created firm needs to find an intermediary (e.g., a bank or a venture capitalist) before it can look for an upgraded blueprint for their production line . We call p the instantaneous probability a firm meets a banker.

[3] All costs and benefits below are to be understood as deflated by average aggregate productivity, whose endogenous growth rate will be determined below.

- Third, after the firm has met its banker, it looks for an innovator who knows how to upgrade its robot, but not its production cost, by a factor $1+\gamma$, so that output net of production cost is $\pi\gamma$ after the upgrade. We call q the probability that the firms finds an innovator.[4]

- Lastly, the upgraded firm is destroyed with an exogenous instantaneous separation probability s.

The firm thus spends, in expectation, $1/p$ units of time looking for a bank, then $1/q$ units of time looking for an innovator, and finally $1/s$ units of time producing at the upgraded profit level until it is destroyed. The fraction of the firm's expected lifetime spent at high productivity is thus

$$\frac{1/s}{1/p+1/q+1/s}=\frac{1}{1+s/p+s/q}. \tag{1}$$

This fraction equals 1 if there is no destruction ($s=0$), or if meeting a bank *and* innovator is instantaneous ($p=q=\infty$). As shown in the appendix, this fraction also measures the steady-state proportion of firms who have met an innovator.

As a result, the growth rate of average productivity is simply the fraction of upgraded firms times the magnitude of the productivity jump γ stemming from each innovation, namely:

$$g=\frac{1}{1+s/p+s/q}\gamma\leq\gamma. \tag{2}$$

If credit and innovation are found instantly (i.e., if $p=q=\infty$), the growth rate reaches the growth rate of innovation γ, as in the Solow (1956) model. We will refer to γ as the *potential growth rate*. If either credit or innovation is found with delay (p or q below infinity), the growth rate falls short of its potential γ. Obviously, the growth rate is zero and the economy stagnates if it is impossible to meet the bank required to find innovators ($p=0$) or the innovators themselves ($q=0$). The positive relationship between g and p, given q, implied by equation (2) is depicted in Figure 2 as an upward-sloping GG curve.

Figure 2 illustrates that in partial equilibrium, with p and q are taken as exogenous, the positive effect of "financial liberalization" (higher p) on growth is strongest when p and g are small because the GG curve flattens and tends asymptotically to $\gamma/(1+s/q)$ as p becomes large. Put differently, more finance enables the economy to close the gap between actual (g) and potential (γ) growth which, by equation (2), is largest at low growth rates. These

[4] Thus all meeting probabilities are computed from the perspective of the firm.

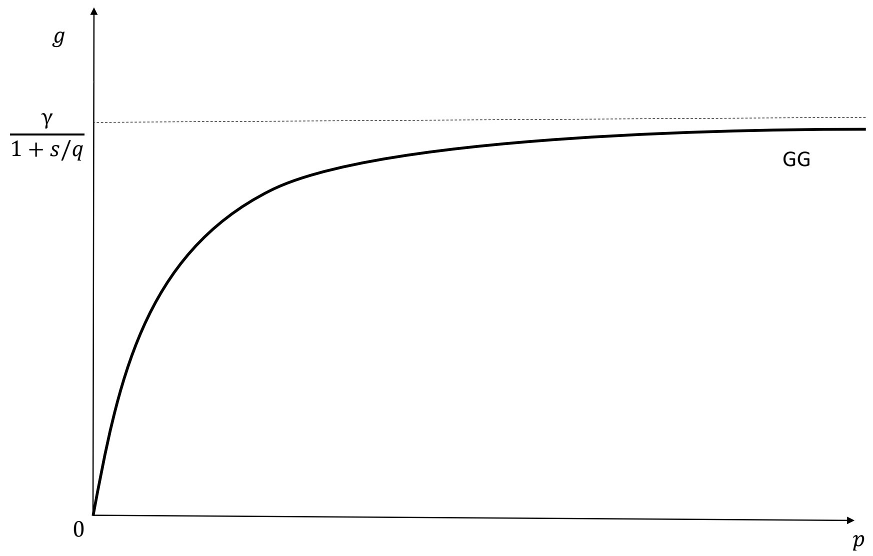

Figure 2: Growth and finance (p and q exogenous)

accounting results apply of course as well to changes in q given p: mitigate one of the two, or both, factors that delay innovation, and growth rises in equilibrium.

Moreover, as illustrated in Figure 3, a rise in the magnitude of the productivity jump γ stemming from innovation always raises, given p and q, the growth rate — although the growth gap g/γ entailed by finite p and q remains constant.

These partial equilibrium results are in line with those of a large literature but, as we show in the next section, they are fragile. They ignore that credit and innovation bottlenecks interact in equilibrium because both the ease of finding credit and the speed of innovation depend on the relative numbers of banks, firms and innovators which have found it worth their while to enter their respective markets. Keeping the probabilities p and q exogenous when they are in fact the equilibrium reflection of endogenous market tensions is as misleading as, say, evaluating the impact of a new bridge while ignoring that the removal of one bottleneck will attract extra traffic that will in the end exacerbate road congestion down the road. Building a bridge across the straight of Messina is of little use if Sicilian roads are not upgraded.

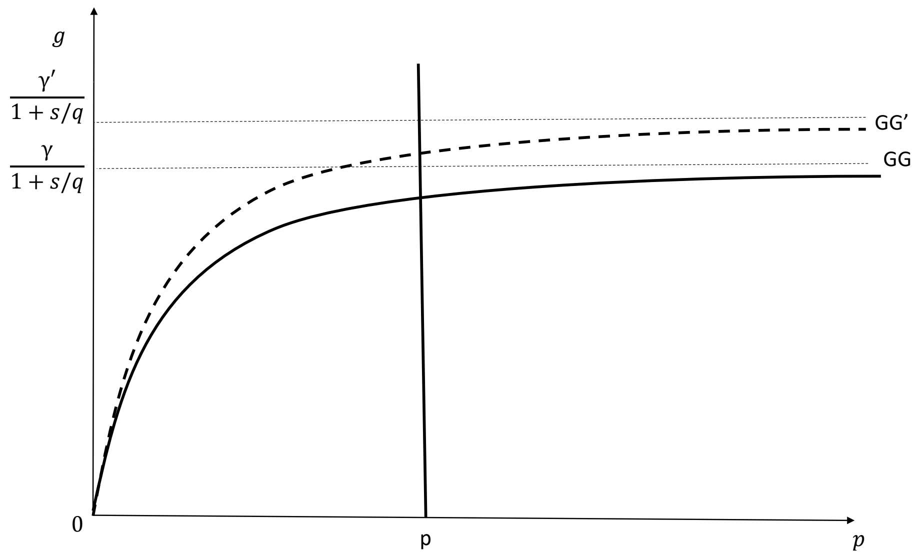

Figure 3: Larger innovation (p and q exogenous)

2 Equilibrium growth with credit and innovation frictions

To formalize these ideas, we turn the foregoing accounting framework into a *bona-fide* growth model by introducing search-and-matching frictions in credit and innovation markets. The matching probabilities p and q thus become endogenous and reflect market tensions, and are determined in equilibrium together with the growth rate.

2.1 Market tensions and the growth rate

Assume, in the spirit of in Wasmer and Weil (2004), that both finance and innovation are subject to search-and-matching frictions.

Suppose the probability p that a firm meets a banker depends negatively on the credit market tension[5] ϕ defined, *from the firm's standpoint*, as the ratio of the number of firms searching for banks to the number of banks searching for firms:

$$p = p(\phi), \qquad p'(\cdot) < 0. \tag{3}$$

with $p(0) = \infty$ and $p(\infty) = 0$. The reciprocal probability of a bank finding a firm, $\phi p(\phi)$, is

[5]Throughout the paper we will use the terms "tension" and "tightness" interchangeably.

increasing in credit market tension ϕ.[6]

Assume, furthermore, that the probability a firm meets an innovator depends negatively on the innovation market tension θ defined, *again from the firm's standpoint*, as the ratio of the number of firms searching for innovators to the number of innovators looking for firms:

$$q = q(\theta), \qquad q'(\cdot) < 0, \tag{4}$$

with $q(0) = \infty$ and $q(\infty) = 0$.

Using equation 2, the growth rate of productivity is therefore negatively related to both credit and innovation market tensions:

$$g = \frac{\gamma}{1 + s/p(\phi) + s/q(\theta)} \leq \gamma. \tag{5}$$

All else equal, the tighter the credit or innovation market, the fewer the firms with an upgraded productivity, and the smaller the aggregate average rate of growth of productivity. These are obviously partial equilibrium description as ϕ and θ are endogenous variables.

2.2 Equilibrium credit market tension under free entry

Equilibrium credit tension depends on the attractiveness of entry into the market and thus, for bank and firm, on the balance of costs and benefits of operation. The costs flow from expensive and time-consuming searches incurred while seeking a match, while the benefits stem from the output upgrade afforded by the eventual match between firm and innovator, namely:

$$\frac{q(\theta)}{r - g + q(\theta)} \left(\frac{\pi\gamma}{r - g + s} - \frac{n}{q(\theta)} \right) := S[q(\theta), g; \gamma], \tag{6}$$

where r is the (subjective) interest rate of risk neutral agents, and n denotes the flow cost of searching for an innovator.[7] The term in parenthesis on the left hand side is the expected present discount value of the output upgrade enjoyed until the destruction of the firm, net of search of the cost of searching for an innovator, and measured at the time bank and firm

[6]Those properties follow from assuming a constant-returns-to-scale matching function. See the appendix for all mathematical details.

[7]We omit for simplicity from the arguments of the $S(\cdot)$ function variables for which we will not perform comparative statics experiments.

meet. It is discounted by a factor $q/(r-g+q)$, which measures the expected value, at the time of the meeting with the banker, of one unit of good at the random time of the meeting with an innovator.[8] The higher $q(\theta)$, the shorter and thus cheaper the search for an innovator, and hence the higher the the expected discounted profits ($S_q > 0$). Similarly, the faster the economy grows, or the larger the innovation, the larger the profits ($S_g > 0$ and $S_\gamma > 0$). Note that $S[q(\theta), g; \gamma]$ depends on *two* endogenous variables: the innovation market friction *and* the growth rate, while the maximal potential growth rate, γ, is exogenous

Under Nash-bargaining between firm and bank, parties split the surplus of their match according to their exogenous bargaining weights $(\omega, 1-\omega)$. If entry in the credit market is unfettered for both banks and firms,[9] profits are driven to zero in equilibrium so that the costs each party incurs to find a match must equal its share of the surplus of the the match:[10]

$$\frac{c}{p(\phi)} = \omega S[q(\theta), g; \gamma], \tag{7}$$

$$\frac{k}{\phi p(\phi)} = (1-\omega) S[q(\theta), g; \gamma], \tag{8}$$

where c denotes the flow search cost incurred by the firm to search for a bank, and k the flow search cost of the bank.

These two free-entry condition immediately imply that, under Nash-bargaining, equilibrium credit market tension equals

$$\phi^* = \frac{\omega}{1-\omega}\frac{k}{c}, \tag{9}$$

so that the equilibrium probability a firm finds a bank is

$$p^* = p\left[\frac{\omega}{1-\omega}\frac{k}{c}\right], \tag{10}$$

which defines a vertical line PP at $p = p^*$ in (p, g) space. The credit market is tight and the matching probability correspondingly low when firm drive a hard bargain with banks (ω

[8] We assume throughout the paper that $r > \gamma$. This guarantees by equation (5) that $r > g$ in equilibrium, so that all discounted sums in the paper are properly defined.

[9] In order to fix the maximum scale of output without frictions, we keep the number of innovators constant, in the same way as the number of workers is kept fixed, for instance, in Mortensen and Pissarides (1994) or in Wasmer and Weil (2004).

[10] If bargaining fails, banks and firms remain in their unmatched status with zero value (their outside option) because of free entry. As a result, the surplus is simply by the expected present discounted value of the output upgrade stemming from a match, given in expression (6).

high), if their flow search cost c is low, or the banks' search cost k is high — since all three factors attract more firms and/or fewer banks to the credit market.

Credit market tightness, ϕ^*, and thus p^*, is independent of θ and g, as in Wasmer and Weil (2004). Note that we will introduce below fixed search costs for banks which break this simplicity and make equilibrium credit market tension dependent on the endogenous growth rate.

2.3 Equilibrium growth rate

Now that we have computed the equilibrium credit-matching probability, we need to characterized the equilibrium innovation-matching probability - since the growth rate depends on both p and q.

To that effect, notice that the free-entry condition (7) can be inverted to yield a relationship between the innovation-matching probability q, and the credit-matching probability p and the growth rate g:[11]

$$q = \frac{(r-g)\frac{c}{\omega p} + n}{\frac{\pi\gamma}{r-g+s} - \frac{c}{\omega p}} := Q(p,g). \tag{11}$$

The *spillover function* $Q(\cdot,\cdot)$ summarizes the crucial interaction between credit and innovation markets that was missing from the accounting, partial-equilibrium framework of section 1.[12] For each credit matching probability and growth rate, it provides the innovation matching probability that is consistent with zero firm profits under free-entry. Unsurprisingly, $Q_p < 0$ and $Q_g < 0$: if the firm finds a bank faster or the growth rate rises, the expected profitability of the firms rises so that the probability of finding an innovator must fall concomitantly — else equilibrium profits would not be zero as free entry requires. For the same reason, lower credit flow search costs c or an increase in the magnitude γ of the innovation (which both raise the profitability of the firm) shift the spillover function $Q(p,g)$ down, and thus lower q, for given p and g.

Using the information provided by the spillover function (11) into the definition of the growth rate (5), we obtain a relationship between the growth rate and the credit matching probability under free entry:

$$g = \frac{\gamma}{1 + s/p + s/Q(p,g)}. \tag{12}$$

[11]We could as well use the other free-entry condition (8) as it is implied by the combination of equations (7) and (9).

[12]For simplicity, the dependence of Q on exogenous variables not shifted in the paper not spelled out.

This equation defines in (p, g) space a GG curve whose shape we will characterize shortly. Its intersection with the PP curve, which is vertical at $p = p^*$ as derived in equation (10), provides the equilibrium value g^* of the growth rate and thus also, using equation (11), the equilibrium innovation matching probability.

Before we proceed, it is useful to introduce two restrictions on parameters that guarantee the existence of the equilibrium and will be imposed throughout the paper:

Condition 1 $r > \gamma$,

Condition 2 $\Psi \leq \omega \frac{\pi\gamma}{r-g^0+s}$,

where $g^0 \equiv \frac{\gamma}{\frac{s}{p}+1}$ is the growth rate it would prevail in economy with frictionless innovation markets and $\Psi \equiv \frac{c}{p}$.

Proposition 1 *Under the two parameter assumptions above the equilibrium exists.*

Proof: *The first assumption guarantees by equation* (2) *that r>g in equilibrium, so that all discounted sums in the paper are finite. The second assumption ensures, by equation* (11)*, that Q is positive since it implies that* $\omega((\pi\gamma)/(r-g^0+s)) < \omega((\pi\gamma)/(r-g+s))$*. The condition states that at the lowest possible value of tightness in the innovation market (* $\theta = 0$*) a firm can enter and make more profits than the expected vacancy costs of entering in the financial market* (Ψ). •

Implicitly, the second condition defines a minimum p for which the equilibrium exists, we denote this as p_{min}. To characterize the shape of the GG curve, it is useful to establish what happens for extreme values of p:

Lemma 1 *The GG curve goes through the origin. It has an horizontal asymptote at* $g_\infty > 0$*, with* $0 < g_\infty < \gamma$ *when* $p \to \infty$.

When $p = 0$, it is impossible to meet a bank, and there is no growth since a firm cannot by assumption finance the search for innovators on its own. When the match with a bank occurs instantly ($p = \infty$), the difficulty of finding an innovator is the only brake to growth so that, from the free entry condition (11), $q = n(r-g+s)/(\pi\gamma)$ while, from (12), $g = \gamma/(1+s/q) < \gamma$. The asymptotical growth rate g_∞ is the positive root of the quadratic equation obtained by combining these two conditions, and the (positive) limit value of q

follows immediately. The following proposition enables us to gauge when the maximum growth rate is achieved:

Proposition 2 (Maximum growth rate) *Let $\mu = -Q_p p/p > 0$ denote the elasticity of q with respect to p along the spillover function. Then g is maximum when the ratio of the expected time spent looking for a bank over the the expected time spent looking for a firm equals to µ , i.e.,$1/p/1/q = \mu$.*

Proof: *From equation (14), and the fact that $Q_g < 0$, g is maximal when $1/p + 1/Q(p,..)$ is minimal. Since there is, because of free-entry in the credit market, a negative spillover between congestion in credit and innovation markets ($Q_p < 0$), a one-percent increase in $1/p$ is associated with a one-percent fall in $1/q$. These two conflicting effects balance out in levels, and the growth rate reaches a maximum, when the condition of the proposition is satisfied. It can be verified that this condition characterizes a global maximum.*•

Proposition 2 is consistent with two possibilities. Either the GG curve is rising monotonically from 0 to g^{∞} — in which case the maximum growth rate described by the proposition is reached at the asymptote g^{∞} when $p = \infty$, i.e., when credit matching is instantaneous. Or the GG curve is hump-shaped, first rising with p, then reaching above g^{∞} the maximum described by 2, and finally declining towards the horizontal asymptote at g^{∞} .

In the benchmark symmetrical case when the flow cost of searching for a bank, c, equals the flow cost ωn of searching for an innovator borne by the firm it is straightforward to establish that the GG curve is hump-shaped, i.e. the relation between finance and growth is indeed non-monotonic as shown in the following proposition:[13]

Proposition 3 (hump-shaped GG curve) *Suppose $c = \omega n$. Then the GG curve is hump-shaped. The growth rate is maximal and the total expected search time is minimal when expected credit and innovation search times are equal $1/p = 1/q$.*

Proof: *See the Appendix.*•

[13] The remainder $(1-\omega)n$ of the cost of finding an innovator is borne, after Nash-bargaining, by the bank the firm has met.

The proof, provided in the Appendix, exploits the fact that, if the GG curve has a hump, the growth rate, which depends negatively on the *total* expected search time $1/p+1/q$, must be insensitive to a first order to a change in p. For that to be the case, an infinitesimal increase (decrease) in the expected credit search time $1/q$ must be met by an exactly offsetting decrease (increase) in the expected innovation search time that leaves, therefore, $1/p+1/q$ constant. In the symmetric case $c = \omega n$, this occurs when $1/p = 1/q$, i.e., when credit and innovation expected search times are equal. This equality determines, through the spillover function (11), the point on the GG curve at which the growth rate is maximal and total expected search time is minimal. The role played by relative search costs in this result will be explained in more details in the the next subsection.

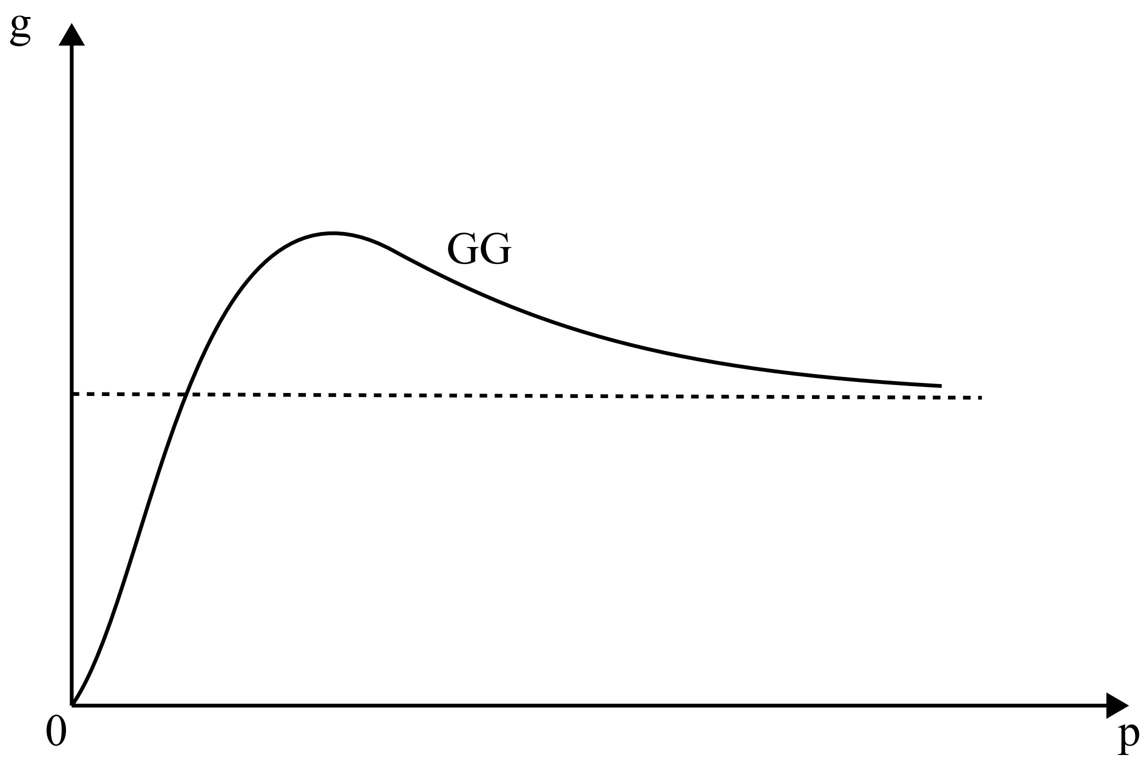

Figure 4: Hump-shaped GG curve

To understand the intuition behind this result, illustrated in Figure 4, think of the problem of driving from the Italian mainland to Catania in Sicily. This involves confronting congestion twice: first to cross the straight of Messina (currently by ferry) and second on Sicilian roads to Catania. Suppose the cost of time waiting for a ferry is the same as the cost of time driving on congested Sicilian roads (this hypothesis is analogous to our symmetrical cost assumption). Will an increase in the number of ferries or the construction of bridge across the straight reduce total travel time to Catania? It all depends on relative congestion. If the main bottleneck is on the mainland it will. If it is on Sicilian roads, it won't. Total expected travel time will be minimized when expected time spent on the continent and on the island are equalized.

The GG curve is hump-shaped because a rise in p has two conflicting effects on growth: on the one hand, by making financing easier, it increases the proportion of firms that can search for innovators — which contributes to raising the growth rate. On the other hand, as evidenced by the negative derivative Q_p of the spillover function, a rise in p is associated with a fall in q that contributes to lowering the growth rate — because easier credit attracts more firms relative to banks, thereby raising the innovation market tension and lowering p up to the point where zero profit are reestablished. Which of these two effect dominates depend on the magnitude of p. When it is close to zero, the first effect dominates. When p is large, the second effect dominates. This simple result can be generalized to the case when $c \neq \omega n$, as established by the next proposition:

Proposition 4 (Hump-shaped GG curve general case) *The GG curve is hump-shaped if*

$$n^2 < \frac{c}{\omega}\left[(r - g_\infty)\Pi_\infty + n\right], \tag{13}$$

where $\Pi_\infty \equiv \frac{\pi\gamma}{r - g^\infty + s}$.

Proof: *See the Appendix* •

This condition is always satisfied when $n \approx 0$, with $g_\infty \approx 0$ and $\Pi_\infty \approx \pi\gamma/(r+s)$. It is is always violated if $n \to \infty$, since $g_\infty \to \gamma$ and $\Pi_\infty \to \pi\gamma/(r-\gamma+s)$ — as the left-hand side goes to infinity faster than the right-hand side. The condition above is trivially satisfied if the flow costs of searching for the bank are greater than the flow costs of searching for innovators, i.e. $c > \omega n$.[14]

A simple approximation. To better grasp the gist of these results and why relative search costs in the two markets play an important role it is useful to revert to some back of the

[14] This follows from $c > \omega n > \omega n \left[\frac{n}{[(r-g_\infty)\Pi_\infty + n]}\right]$, since the term in square brackets is a number smaller than one.

envelope calculations. For values of interest rates close to the growth rate of the economy, $r \approx g$, the free entry condition simplifies as

$$1/q + 1/p = \frac{\pi\gamma}{sn} + (1 - \frac{c}{\omega n})(1/p).$$

The left-hand side of the expression captures total waiting time in the two markets, a quantity that has a direct impact on growth. The equilibrium growth rate is then approximately

$$g = \frac{\gamma}{1 + s(1/q + 1/p)} = \frac{\gamma}{1 + s(\frac{\pi\gamma}{sn} + (1 - \frac{c}{\omega n})\frac{1}{p})} := g(p).$$

The two equations above make clear that the link between finance, i.e. $\frac{1}{p}$, and growth is directly related to the relative magnitude of search costs in the two markets since via its impact on total waiting time. Specifically, $g_p > 0 \; iff \; c/\omega < n$. Before we proceed, it is useful to note that the non-negativity requirement on q imposes that

$$p \geqq \frac{c/\omega}{\pi\gamma/s} := p_{min}.$$

so that the equilibrium growth rate evaluated at the minimum p can be expressed as

$$g(p_{min}) = \gamma/(1 + \pi\gamma\omega/c).$$

while for $p \to \infty$,

$$g(\infty) = \gamma/(1 + \pi\gamma/n).$$

We can then distinguish between three cases:

1. If $c/\omega < n$, the growth rate is increasing in p. Its minimum value is $g(p_{min})$, and its maximum value is $g(\infty)$.

2. If $c/\omega > n$, g is decreasing in p. Its minimum value is $g(\infty)$, and its maximum value is $g(p_{min})$.

3. If $c/\omega = n$, g is independent of p, and it is equal to the following expression

$$g(p_{min}) = \gamma/(1 + \frac{\pi\gamma}{c/\omega}) = \gamma/(1 + \frac{\pi\gamma}{n}) = g(\infty).$$

The intuition behind these results is the following. Increasing the probability of meeting a financier always leads to a fall in q as per the free-entry condition. So the firm spends on average less time looking for credit, more looking for an innovation. When the cost of looking for credit, corrected by the firm's bargaining power, is lower than the cost of looking for an innovation (case 1), the profit of the firms would fall if the total search time were to stay the same or *a fortiori* increase. But that's inconsistent with zero profits and some firms will exit the market. As a result, the total search time falls and growth must rise. The same explanation holds, *mutatis mutandis*, in case 2 and explains why, in case 3, the total search time remains constant when p varies. In terms of our road traffic analogy, suppose that bridge tolls are less expensive than road tolls on Sicily (case 1). Reducing traffic jams on the first (cheaper) bridge intensifies traffic jams on the second (expensive) road, thereby increasing the total cost of traveling. In the long-run, higher expenses will push some trucks off the road and eventually ease traffic congestion on the whole route.

2.3.1 Comparative statics

Let us look at some qualitative comparative statics. In the following we will evaluate the equilibrium effects of lower search costs for financiers, higher search costs for firms and an increase in the productivity jump. We show that once general equilibrium effects are taken into account the results deviate from what shown in Section 1, i.e. the relationship between finance, innovation and growth is non-monotonic.

More finance A lower search cost k for banks reduces equilibrium credit market tension ϕ by encouraging bank entry. This raises the equilibrium credit matching probability p^*, thereby shifting the PP curve to the right with an unchanged GG curve. The effect on the equilibrium growth rate is positive, left of the hump of the GG curve, if p is initially small and the increase in k is small enough. Right of the hump, however, if p is initially small, a rise in p *lowers* the growth rate.

Higher search costs for firms, c, have too a benign effect on equilibrium credit market tension. However in this case, the probability of a match in the innovation market increases. In equilibrium, higher costs discourage firms entry in both markets and innovation tension ϑ decreases .[15] Graphically, the GG curve shifts upward and the PP curve to the right. The final effect on growth is positive.

[15] Here we assumed that $p = p_0\phi^{-\eta}$, so that $\frac{c}{p} = \frac{c^{1-\eta}}{p}\left(\frac{\omega k}{1-\omega}\right)^{\eta}$, and hence q and p are increasing in c.

Table 1: Parametrization

Parameter	Value	Parameter	Value
r	3.5%	$1/\phi$	0.06
s	4%	γ	0.023
ω	0.5	c	0.166
π	2.75	n	0.331
		k	2.758

Larger innovation What happens if the productivity jump γ due to innovation increases? Such an increase has two counteracting effects: a direct positive effect on g and an indirect negative effect through a tightening in the innovation market. Higher benefits from the output upgrade incentivize entry in the financial markets from both firms and banks thereby leaving the equilibrium financial market tightness unchanged. At the same time, more firms will find profitable to search for innovators thereby creating congestion in that market and impinging on growth. Graphically, the GG curve could shift either upward or downward, depending on parameters values.[16] Thus, differently from our partial equilibrium example, the final effect on the equilibrium growth rate is non monotonic.

2.3.2 A simple numerical exercise

The purpose of this subsection is to evaluate the equilibrium properties of our model with a simple calibration exercise. Table 1 summarizes our parametrization.

The basic unit of time is one year. The risk-free rate, r, is 3.5% and the separation rate, s, is set to an annualized 4%, as in Petrosky-Nadeau and Wasmer (2015). We assume a symmetric bargaining power for banks and firms, ω=.5. The target durations in credit markets (for creditors) and innovation markets (for firms) are, respectively, slightly below 1 months and 2 years, respectively. The first number together with our target for credit market tightness, ϕ, implies a duration in credit markets for firms slightly above 1 year, as in Wasmer and Weil. While the second number is in line with the average time for patent approvals for 2020 according to data published by the USPTO. The productivity jump, γ is set to target an annual growth rate, g, of 2%. For simplicity, we assume that the flow costs of searching for banks, c equals the flow cost of searching for innovators borne by the firms, ωn, while k is set

[16]For the calibration outlined in the subsection below, an increase in the productivity jump has positive effect on equilibrium growth.

Table 2: Lower frictions in credit and innovation markets

	Benchmark	Low credit frictions	Low innovation frictions	Low frictions in both markets
g	2.00%	2.00%	2.07%	2.12%
$\frac{1}{q}$	2 yr	3.4 yr	1.03 yr	1.75 yr

to target a fraction of employed in the financial sector over total employment of 6% according to the Bureau of Labor Statistics data for 2020. The value for π is chosen to normalize total discounted output net of production costs to 1. Finally, tightness in innovation market, ϑ, is set to match the fraction of employed in scientific research and development services over total employed of 0.5% in 2020 (BLS)

Table 2 reports the equilibrium growth rate and duration in the innovation market predicted by our model in 4 different cases: our current calibration, "low" credit market frictions, "low" innovation market frictions and low frictions in both markets. To decrease the level of credit market frictions we let $p_0 \to \infty$ while to reduce search time in the innovation markets, we double the search costs for innovators, n.[17]

Before commenting on the results of Table 2, it is good to recall that given our chosen calibration (c=ωn), as shown in section 2.3, the maximum growth rate would be achieved for p=q. In our case, with p>q, the economy is on the right of the hump in a rather flat region. Moreover, as exemplified by our back of the envelope calculations at the end of the same section, when the real rate is close to the growth rate of the economy, the latter is relatively insensitive to access to credit. The main take away from this simple exercise is that for a calibration close to US data, changes in credit markets have only a marginal negative effect on growth as they mainly exacerbate bottlenecks in the innovation market. Similarly, the second column of the table shows also that this holds trues also for lower frictions in the innovation market. However, the last column in the table illustrates that a combined reduction of frictions in both markets would result in a higher growth rate. This can be construed as substantiating the OECD view that innovation is the outcome of a "system" favorable to growth rather than the result of isolated pro-growth measures. [18] All in all, the results above show that for a calibration chosen to mimic the current US economy, financial

[17]An increase in the search costs for innovators reduce a firm profitability. To keep profit at zero, the probability of meeting innovators must increase (equation 11).

[18]For example, the OECD's 2015 innovation strategy argues that policy makers can promote innovations by focusing on five areas of action among which education and training systems as well as a business environment supporting investment in knowledge based capital.

factors play only a moderate role for growth. In light of what shown in the previous section, this should perhaps not come as a surprise when the real and growth rate are close to each other and search costs in both markets are symmetric. More generally, potential growth, as captured by the productivity jump γ plays an important role. Specifically, is it possible to show[19] that the elasticity of the growth rate with respect to finance is a multiple of the factor $\frac{\gamma-g}{\gamma}$. That is, as long as potential growth is close to the actual growth rate, growth is relatively insensitive to changes in financial factors.

Innovation and growth contribute to societal well-being and the non-monotonical relation between finance and growth translates on the interlink between welfare and financial development. Figure 5 illustrates this point plotting the long-run discounted value of output net of search cost, as a function of credit matching probability, p for our benchmark calibration.[20]. Two observations are in order. First, the welfare's shape mimics the GG curve and it's hump-shaped in p. Second, for the chosen calibration, credit market tightness is too low, i.e. less matching speed in credit market would ease congestion in the innovation market thereby resulting in a welfare improvement. Quantitatively, an instantaneous matching in credit markets ($p_0 \to \infty$) would lead to 3.4% deterioration in long-run output compared to its maximum value.

[19]These results are available upon request

[20]The dotted vertical line in the figure indicates p_{min}

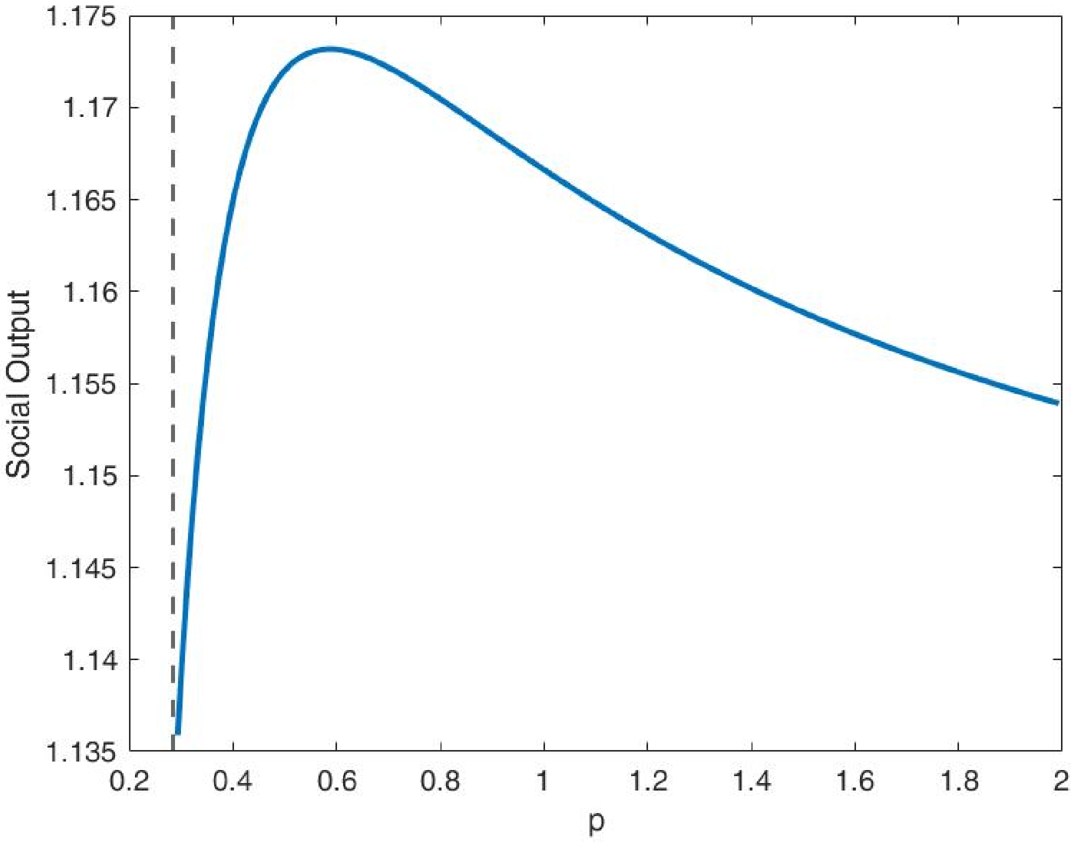

Figure 5: Welfare and finance

3 Efficient finance and R&D

As shown in the previous section, there may be *too much* finance in equilibrium: easier access to credit may dampen growth when the relaxation of credit fictions exacerbates innovation frictions. In this section, we formalize what is *too much* by taking into account the effects of different distortions on social output. Specifically, we now turn to describing the efficiency properties of the competitive equilibrium of our model by characterizing the constrained efficient allocation. In what follows, we assume the following functional forms for the two matching probabilities in credit and innovation market, respectively:

$$q(\theta) \equiv \frac{\mu(\mathscr{F}_1, \mathscr{I}_1)}{\mathscr{F}_1} = \mu\left(1, \theta^{-1}\right) = q_0 \theta^{-\varepsilon},$$
$$p(\phi) \equiv \frac{m(\mathscr{F}_0, \mathscr{B}_0)}{\mathscr{F}_0} = m\left(1, \theta^{-1}\right) = p_0 \phi^{-\eta}.$$

As a preliminary it is useful to express the competitive equilibrium in a more compact way explicitly taking into account a compensation, w, for innovators.

3.1 Block bargaining and competitive equilibrium

So far we have assumed, for ease of exposition, that innovators are not compensated for their work.[21] Suppose now that, after a successful match, innovators receive a wage w negotiated through Nash bargaining. Bargaining in the financial and innovation markets occurs independently. In the former, firms and banks do not take into account the effect of credit repayment ρ on wages.[22] In the innovation market, the newly formed bank-firm pair bargains with a innovator. If we define the joint bank-firm value $\hat{J}_i$ as:

$$\hat{J}_i = \hat{B}_i + \hat{F}_i \;\; i = 0, ..3,$$

where $\hat{B}_i$ and $\hat{F}_i$ represent, respectively the value of a bank and a firm at each stage, the negotiated wage for the innovator solves the following maximization problem:

$$w = \arg\max \left(\hat{J}_2 - \hat{J}_1\right)^{1-\alpha} \left(\hat{I}_2 - \hat{I}_1\right)^{\alpha},$$

[21] Alternatively, one could assume that innovators receive an exogenous wage.

[22] This assumption keeps the solution for credit repayment unchanged from what we found in the previous sections.

in which $\alpha \in (0,1)$ denotes the bargaining weight of innovators. As shown in the appendix, the solution of this problem is a surplus sharing rule:

$$w = \alpha\left(\pi\gamma + \theta n_k - \left(r - g + s\right) K\left(\phi\right)\right),$$

where $K\left(\phi\right) \equiv \frac{c}{p} + \frac{k}{p\phi}$ measures total discounted search costs in the financial market and $n_k \equiv \left(r-g\right) K\left(\phi\right) + n$ encompasses not only recruiting costs n in the innovation market but also the annuitized value of search costs in the financial market. The wage is increasing in innovators productivity ($\pi\gamma$) and the innovation market tightness (θn_k). This general equation encompasses the case $w = 0$, considered so far, if innovators had no bargaining power $\alpha = 0$. For α sufficiently small, all the properties of the equilibrium we described in the previous section stand still.

By summing-up the two free-entry conditions in the financial market, taking account the wage rule above, we can express the equilibrium of our model in a compact way by two equations:

$$\frac{n_k}{q} = \frac{(1-\alpha)\left(\pi\gamma - \left(r-g+s\right) K\left(\phi\right)\right) - \alpha\theta n_k}{\left(r-g+s\right)},$$

and g expressed by equation(12).

3.2 Constrained efficient allocation

Consider now the problem of a social planner maximizing the presented discounted value of output net of search cost

$$\max_{\mathscr{I}_1,\mathscr{B}_0,\mathscr{F}_0,\mathscr{F}_1,A} \Omega^{SP} = \int e^{-rt}\Pi A dt$$

$$st: \dot{A} = g\left(\mathscr{I}_1,\mathscr{B}_0,\mathscr{F}_0,\mathscr{F}_1\right) A$$

$$\dot{\mathscr{F}_1} = -q\left(\theta\right)\mathscr{F}_1 + p\left(\phi\right)\mathscr{F}_0$$

$$\dot{\mathscr{I}_1} = s\left(1-\mathscr{I}_1\right) - \theta q\left(\theta\right)\mathscr{I}_1,$$

where

$$\Pi = \left[\pi\gamma\left(1-\mathscr{I}_1\right) - c\mathscr{F}_0 - k\mathscr{B}_0 - n\mathscr{F}_1\right],$$

$$g\left(\mathscr{I}_1,\mathscr{B}_0,\mathscr{F}_0,\mathscr{F}_1\right) = \frac{\gamma}{\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)}.$$

$\mathscr{I}_i$, $\mathscr{B}_i$ and $\mathscr{F}_i$ denote the number of innovators, banks and firms, respectively, in different stages, A is the aggregate average productivity and the matching frictions impose a technological constraint on the social planner. Congestion externalities are a standard market failure in matching models. Here, the endogeneity of the growth rate entails a second source of externalities since individual investment in R&D and more liquidity can boost aggregate productivity. An inspection of the following two first-order conditions will make this point more clear:

$$e^{-rt} kA = \Phi p_{\mathscr{B}_0} \mathscr{F}_0 + \Xi g_{\mathscr{B}_0} A, \tag{14}$$

$$e^{-rt} cA = \Phi \left(\mathscr{F}_0 p_{\mathscr{F}_0} + p \right) + \Xi g_{\mathscr{F}_0} A, \tag{15}$$

where Ξ and Φ are the co-states associated, respectively, to the law of motion of technology and the flow of matched firms (the notation $\frac{\partial f(x)}{dx} = f_x$ applies). At an optimum, the social planner equalizes the discounted flow costs of searching in the financial market to the expected social benefits from an additional bank-firm match where the latter takes into account the effects of successful matches on growth ($g_{\mathscr{B}_0}$ and $g_{\mathscr{F}_0}$). The following proposition characterized a constrained efficiency condition in the financial and innovation market.

Proposition 5 (Modified Hosios) *The decentralized solutions for credit and innovation tightness maximize net social welfare if:*

$$1 - \omega = \eta \frac{k\left(r - g^*\right)}{k\left(r - g^*\right) - \eta \Pi^* g_{\mathscr{B}_0}} > \eta, \tag{16}$$

$$\alpha = \varepsilon + \frac{\tau}{\hat{w}} > \varepsilon,$$

where $g^* = g\left(\theta^*, \phi^*\right)$ *denotes the efficient level of growth and*

$$\hat{w} \equiv \frac{\pi\gamma + \theta^* n_k - \left(r - g - s\right) K\left(\phi^*\right)}{\left(r + s - g^*\right)},$$

$$\tau \equiv \frac{g_{\mathscr{I}_1}}{\theta^* q} \left[\Pi^* + \frac{\left(k - \eta p \phi K\left(\phi^*\right)\right)}{g_{\mathscr{B}_0}} \left(s + \theta^* q\right) \right],$$

$$\Pi^* \equiv \frac{\theta^* s q}{\left(s + \theta^* q\right)} \left[\frac{\pi\gamma}{s} - \left(1 + \frac{r - g}{q}\right) K\left(\phi^*\right) - \frac{n_k}{q} \right],$$

$$g_{\mathscr{B}_0} \equiv \frac{g^2 s \eta}{\gamma} \frac{\phi^*}{q} \left(\frac{s + \theta^* q}{\theta^* s} \right),$$

Proof: *See the appendix.* •

In the absence of growth externalities, the first efficiency condition above would collapse into the usual Hosios condition, $(1-\omega) = \eta$ which states that at an optimum, the share of surplus accruing to banks $(1-\omega)$ in a match should equal to the elasticity of the matching function with respect to the corresponding search input (η). Disregarding the growth effects, when the elasticity η is high, searching firms in the financial market creates more congestion than unmatched banks and the social planner would like to give a higher share of surplus to the banks to reduce that cost. On the other hand, when η is too low, there are too many banks searching for firms. With endogenous growth, more banks searching have also a positive effect on growth, thereby giving a further incentive to the social planner to stimulate entry by new banks by setting $(1-\omega) > \eta$. Another way of looking at this result, comes by noting that total search costs , $i.e.$ $K\left(\phi\right) \equiv \frac{k}{\phi p(\phi)} + \frac{c}{p(\phi)}$ are minimized at ϕ^K :

$$\phi^K = \frac{1-\eta}{\eta}\frac{k}{c} > \frac{\omega}{(1-\omega)}\frac{k}{c} = \phi^*.$$

Taking into account growth, the optimal level of credit market tightness, ϕ^* is lower than the value that would minimize search costs, ϕ^K, since the social planner internalizes the negative effects of the trading friction on growth. Note that ϕ^K also corresponds to the optimal credit market tightness in absence of endogenous growth. The social planner internalizes banks' contribution to growth and the interactions between the two searching frictions. This becomes more clear noting that we can rewrite the term in the denominator of equation (16) as :

$$\Pi^* g_{B_0} = \frac{g^2 s\eta\phi^*}{\gamma}[\frac{\pi\gamma}{s} - \left(1 + \frac{r-g}{q}\right)K\left(\phi^*\right) - \frac{n_k}{q})].$$

That is, when there is more congestion in innovation markets (a lower q) banks marginal contribution to growth is lower. Therefore the share of surplus accruing to bank should be lower to disincentivize banks' entry, in other words, the tighter are innovation markets the lower the optimal size of financial sector.

Turning to the innovation market, the second efficiency condition conveys a similar message. The optimal share of surplus going to innovators (α) should be greater than their contribution in terms of matching (ε) since their entry has also a positive effect on the average growth rate. This implies that the optimal level of innovation market tightness $\left(\theta^*\right)$ is lower compared to a case without endogenous growth.

4 Robustness

In this section, we evaluate the robustness of our findings to different model assumptions. First, we modify the original model to allow for a direct feedback, absent heretofore, from growth onto credit market tension. To that effect, we introduce the assumption that entering in financial markets entails a fix licensing cost for banks. Second, we show that relationship between finance and growth is monotonic if markets of ideas are frictionless.

4.1 Fixed entry cost for banks

Assume that a bank suffers a fixed entry cost K, say a licensing cost, at the time it enters the credit market to offer its services to firms. The annuity value of that cost in a growing economy is $(r-g)K$, and it adds up to the flow search cost k paid by the bank. Since the growth rate is endogenous in our model, the effect of the fixed entry cost K depends on its impact on the equilibrium growth rate—an effect absent when growth is exogenous. This introduces into our model a direct feedback from growth into finance that we now investigate.

We assume that the fixed market-entry cost K is paid by the bank each and every time it starts searching for a firm—i.e. that upon exogenous separation from the firm or in case of negotiation failure it reverts to its zero pre-entry value. As a result, the fixed cost K does not affect the surplus of the bank-firm match, and the free entry conditions (7) and (8) become

$$\frac{c}{p(\phi)} = \omega S[q(\theta), g; \gamma], \tag{17}$$

$$\frac{k+(r-g)K}{\phi p(\phi)} = (1-\omega) S[q(\theta), g; \gamma]. \tag{18}$$

Note the way we introduce fixed costs of entry differs from the approach taken, for instance, in Wasmer and Weil (2004). There, the fixed cost K is paid once and for all when entering the market, and endogenous separation or failure of Nash-bargaining does not entail exit from the market and the ensuing need to repay the entry cost to resume search. As a result, the value of a bank that has just separated equals the value of a firm which has just entered — i.e., K as required under perfect competition. The fixed entry cost K therefore affects the surplus of the match between bank and firm — both because it changes the exit value of the bank to a positive number (K), and because it improves the firm's outside option during Nash-bargaining. To avoid such unnecessary modeling complications, we assume instead that a separated bank, or one that has failed in negotiations with the bank, exits the market altogether and loses its costly operating license. To reenter the market and resume search, the bank must therefore repay the fixed cost K . As are result, the exit value of a bank is zero — which simplifies calculations without affecting results qualitatively.[23]

Retracing the steps taken in section 2, the conditions (17) and (18) can be rewritten as

$$q = Q(p, g; c, \gamma), \tag{19}$$

$$\phi = \frac{\omega}{1-\omega}\frac{k + (r-g)K}{c}. \tag{20}$$

The novel term $(r-g)K$ on the right hand-side of equation (20) captures the annuity value, in an economy growing at the endogenous rate g, of the fixed entry cost K. The faster the economy grows, the smaller the fixed cost looms in the bank's cost computations. The equilibrium impact of the fixed cost thus depends on whether it slows down or raises growth. In the first case, lower growth amplifies the impact of barriers to credit entry. In the latter, it mitigates them. Interestingly, equation (20) shows a direct positive effect of growth on financial deepening. Higher growth reduces the annuity value of the licensing cost, thereby inducing entry of new banks and reducing the tightness of credit markets.

4.1.1 Equilibrium

Equation (20) implies that the equilibrium credit matching probability obeys

$$p = p\left[\frac{\omega}{1-\omega}\frac{k + (r-g)K}{c}\right]. \tag{21}$$

Since $p'(\cdot) < 0$, and provided $r > g$ (a condition always satisfied in equilibrium), this equation defines, given K, an upward sloping PP curve in (p, g) space—as shown in Figure 6.[24] When $K = 0$, the PP curve is vertical. Raising K shifts the PP curve to the left and flattens it. Furthermore, since the surplus S does not depend on K, neither does the spillover function

[23] This approach is similar to that of Pissarides (2009) in a labor setting where fixed training costs are incurred each time a match occurs.

[24] When $p \to \infty$, the PP curve has a horizontal asymptote at $(k + rK)/K := g_p^{\infty}$. The minimal value of p occurs when $g = 0$.

(19) — so that the equation and position of the GG curve defined in (12) are unaffected by the introduction of the fixed entry cost K.

We can therefore represent the equilibrium with fixed entry costs K in Figure 6.

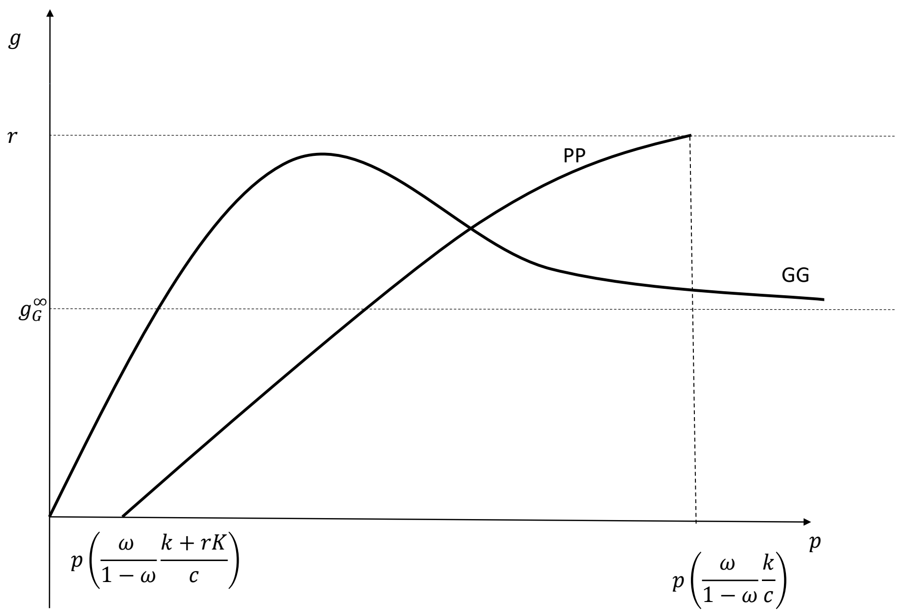

Figure 6: Equilibrium growth and credit matching probability ($K > 0$)

The feedback from growth to credit tightness, captured by the new PP curve could potentially result in multiplicity of equilibria, as depicted in figure 7. The following proposition rules out this possibilities and establishes the uniqueness of the equilibrium under standard assumptions on the matching probabilities.[25]

Proposition 6 *In the presence of fixed-entry costs in financial market, if an equilibrium exists it is unique.*

Proof: *See the Appendix.* •

As in our benchmark model, a policy intervention which lowers costs for financial intermediaries stimulates entry and decreases credit market tightness. On the margin, lowering licensing costs has a stronger impact on market tightness the lower the initial growth rate

[25] Here, we assume the same functional forms for the two matching probabilities defined in the previous section.

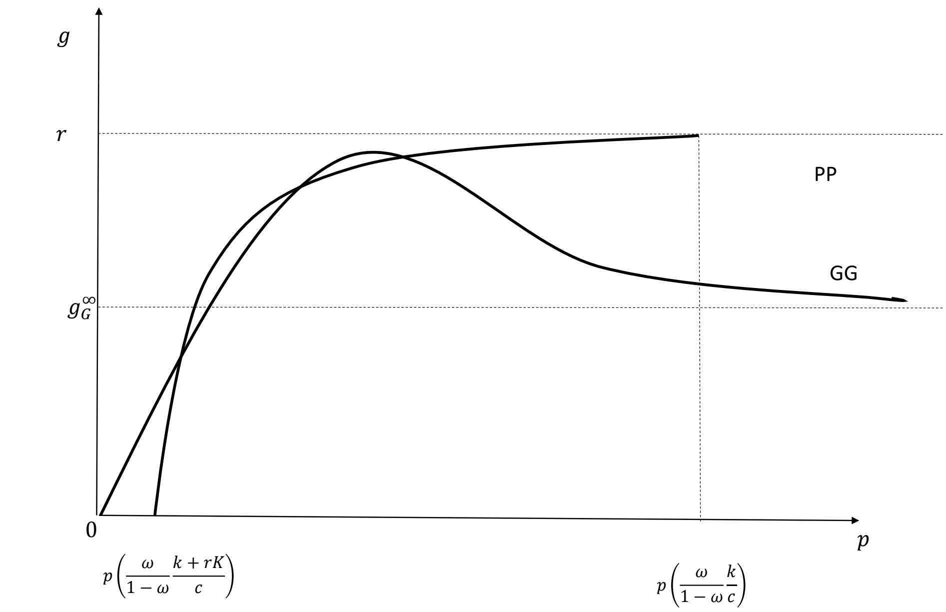

Figure 7: Multiple Equilibria

and for higher level of interest rates. As before, the interaction between credit and innovation frictions makes the policy have a negative effect on congestion in the innovation market, and the equilibrium effect on growth is ambiguous. Graphically, a reduction in search costs shifts the PP curve to the right while lowering licensing costs steepens the curve and moves it to the right (see Figures 8 and 9).

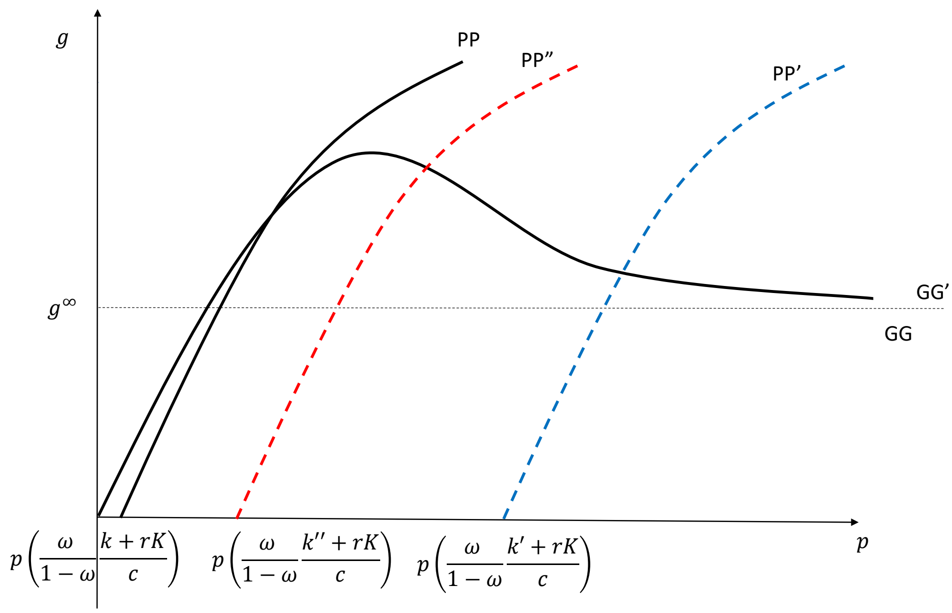

Figure 8: More finance with fixed entry cost

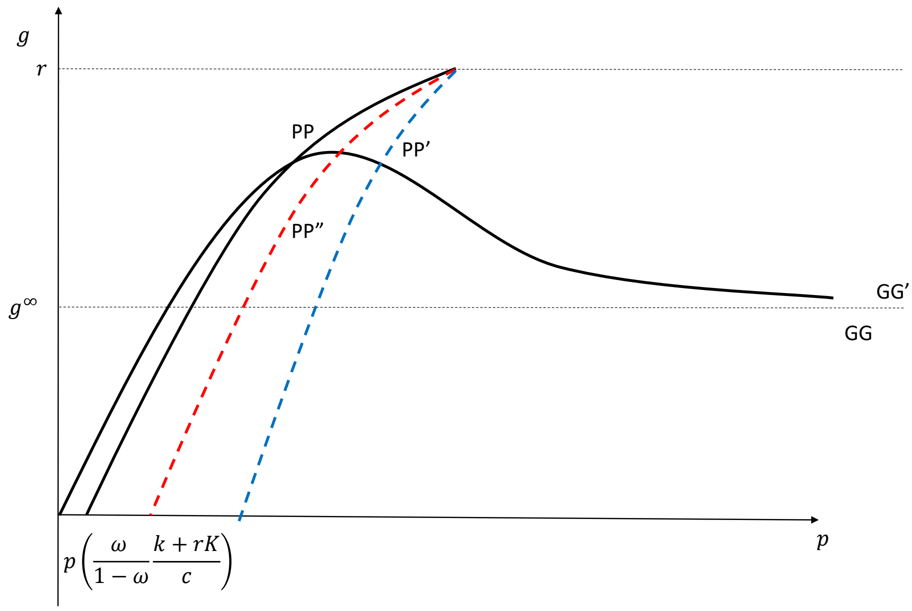

Figure 9: Smaller bank entry cost

Search frictions only in one market

In the foregoing analysis, the interaction between search frictions in financial and innovation markets generates a non-monotonic relationship between growth and finance. To shed more light on this channel, this section presents a simplified version of our growth model where R&D is frictionless.

Specifically, assume that a firm needs a financial intermediary to find an upgraded blueprint to boost its productivity. Finding a financiers requires effort and time but, once the firm has met a bank, an innovator is found costlessly and instantly. As shown in the appendix, the equilibrium of this simple model can be summarized by two equations

$$\begin{aligned} BB &: \quad \frac{k}{\phi p(\phi)} = \frac{(1-\omega)}{(r-g+s)}\left[\pi\gamma + c - \frac{\omega}{1-\omega}\frac{k}{\phi}\right], \\ GG &: \quad g = \frac{\gamma}{\frac{s}{p(\phi)}+1}. \end{aligned}$$

The first equation represents a free-entry condition in the financial sector, the second captures the average growth rate of the economy when firms meet innovators instantaneously after having secured financing. Both curves are downward sloping in the (ϕ, g) plane, as shown in figure 10 [26].

In this simplified set-up, a more efficient matching between firms and financiers modeled as an increase in p_0 makes the first curve shifts to the left, i.e. for given g the discounted search cost for banks decreases thereby inducing more banks to enter and thus reducing credit tightness. The GG^{ϕ} curve moves upward, since finance has a positive direct effect on the share of innovating firms. An improvement in credit markets therefore results unequivocally into higher growth and less tightness in financial markets. Finance is always good for growth because it is the only hindrance to innovation. This is the traditional mechanism underlying simplistic policy recommendations for financial liberalization. Similarly, if only innovation markets were frictional while access to credit were free, more developed R&D markets would always be beneficial for growth.

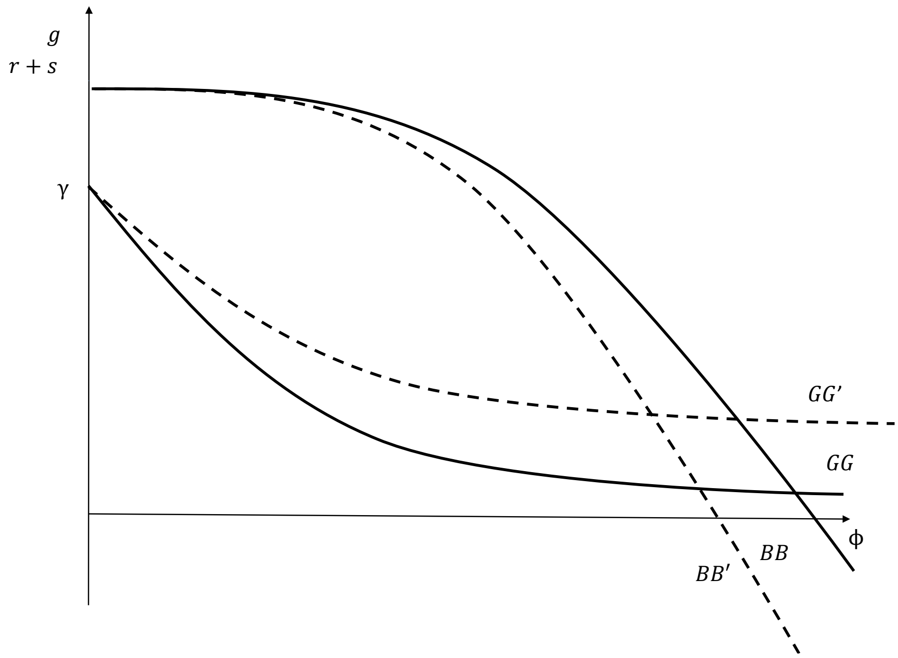

Figure 10: More efficient credit markets when innovation markets are frictionless

[26]The existence of an equilibrium is ensured by the condition $r > \gamma$.

5 Conclusion

In the last century, most developed economies have experienced a widespread expansion of the financial sector yet almost constant growth rates of GDP (save for wars and financial or health crises). In this paper, we build a parsimonious endogenous growth model with search frictions in credit and innovation markets to shed more light into this empirical observation. In our model, higher growth induces firms and banks to entering in the market and can increase the size of the financial sector. All else equal, tight credit and innovation markets have a negative effect on growth. However, we show that once all general equilibrium effects are taken into account, financial deepening beyond a certain threshold is harmful for growth. Finance has a non-monotonic effect on long-run growth since there are other bottlenecks than money hindering innovation, i.e. a too big financial sector can create congestion in the innovation markets. For a calibration chosen to mimic the actual US economy, with relatively good functioning credit and innovation markets, the effects of finance on growth are negative but only marginal.

We look at this issue also from a normative perspective and show that entry in both markets is efficient once innovators and financiers are compensated for their contribution to growth and that the social planner internalizes the interactions between the two congestion frictions.

Financial development could also have an impact on the speed of convergence to the world technological frontier. We explore this possibility in a companion paper where less developed countries strive to reach the technological frontier by imitation in a multi-country version of our model.

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Appendix A The model

This section describes in more detail our model.

A.1 Matching functions and probabilities

We assume the following functional forms:

$$q(\theta) \equiv \frac{\mu(\mathscr{F}_1, \mathscr{I}_1)}{\mathscr{F}_1} = \mu\left(1, \theta^{-1}\right) = q_0 \theta^{-\varepsilon}$$
$$p(\phi) \equiv \frac{m(\mathscr{F}_0, \mathscr{B}_0)}{\mathscr{F}_0} = m\left(1, \theta^{-1}\right) = p_0 \phi^{-\eta}$$

where $\mu(\cdot)$ and $m(\cdot)$ are constant return to scale technologies producing matches in the innovation and the credit markets, respectively.

A.2 The value of an innovator

The Bellman equations describing the steady-state values of the innovator over the four stages are

$$\left(r-g\right)\hat{I}_1 = -\ell + \theta q(\theta)\left[\hat{I}_2 - \hat{I}_1\right]$$
$$\left(r-g\right)\hat{I}_2 = s\left[\hat{I}_3 - \hat{I}_2\right]$$
$$\hat{I}_3 = \hat{I}_1$$

where ℓ is the search cost for innovators. There is no free-entry in this market and the total number of inventors is normalized to 1

$$\mathscr{I} = \mathscr{I}_1 + \mathscr{I}_2 = 1.$$

Then, the dynamics of the flow of inventors searching for entrepreneurs can be expressed as

$$\dot{\mathscr{I}}_1 = s(1-\mathscr{I}_1) - \theta q(\theta)\mathscr{I}_1.$$

Abstracting from the very short-run $\dot{\mathscr{I}}_1 = 0$:

$$\mathscr{I}_1 = \frac{s}{s + \theta q(\theta)}.$$

A.3 Value of a firm

The values of a firm along the balanced path in the four stages are described by

$$\begin{aligned}
(r-g)\hat{F}_0 &= -c + p(\phi)\left[\hat{F}_1 - \hat{F}_0\right] \\
(r-g)\hat{F}_1 &= q(\theta)\left[\hat{F}_2 - \hat{F}_1\right] \\
(r-g)\hat{F}_2 &= \pi\left(1+\gamma\right) - \pi - \rho + s\left[\hat{F}_3 - \hat{F}_2\right] \\
\hat{F}_3 &= \hat{F}_0.
\end{aligned}$$

In the fund-raising stage, firms produce πA, pay a flow cost cA to search for banks and production costs πA. Once the match is created, in stage 1, they will still produce πA (and sustain production costs πA) and with probability $q(\theta)$, they will meet an innovator and have access to a better technology, where $\theta = \frac{\mathscr{F}_1}{\mathscr{I}_1}$, i.e. the ratio between entrepreneurs and available innovators, represents tightness in the market of idea and $q'(\theta) < 0$. In the next stage, 2, they will produce $\pi\left(1+\gamma\right)A$ and repay the contracted amount ρA to the bank, and sustain costs πA. Matches are exogenously destroyed with probability s at the end of that stage. With free entry, $\hat{F}_0 = 0$ and

$$\begin{aligned}
\hat{F}_1 &= \frac{c}{p(\phi)} \\
\hat{F}_1 &= \frac{q(\theta)}{r-g+q(\theta)}\frac{\pi\gamma-\rho}{r-g+s} \\
\frac{c}{p(\phi)} &= \frac{q(\theta)}{r-g+q(\theta)}\frac{\pi\gamma-\rho}{r-g+s}.
\end{aligned}$$

A.4 Value of a bank

Similarly, we can express the Bellman equations describing the evolution of the bank values along the balanced growth path as:

$$
\begin{aligned}
(r-g)\hat{B}_0 &= -k + \phi p(\phi)\left[\hat{B}_1 - \hat{B}_0\right] \\
(r-g)\hat{B}_1 &= -n + q(\theta)\left[\hat{B}_2 - \hat{B}_1\right] \\
(r-g)\hat{B}_2 &= \rho + s\left[\hat{B}_3 - \hat{B}_2\right]
\end{aligned}
$$

where k and n are search costs in the financial and innovation markets, respectively.

With free-entry:

$$
\begin{aligned}
\hat{B}_1 &= \frac{k}{\phi p(\phi)} \\
\hat{B}_1 &= -\frac{n}{r-g+q(\theta)} + \frac{q(\theta)\rho}{(r-g+q(\theta))(r-g+s)} \\
\frac{k}{\phi p(\phi)} &= -\frac{n}{r-g+q(\theta)} + \frac{q(\theta)\rho}{(r-g+q(\theta))(r-g+s)}.
\end{aligned}
$$

If we instead assume that banks need to pay a fix cost K upon entry, $\hat{B}_0 = \hat{B}_3 = K$,

$$
\begin{aligned}
\hat{B}_1 &= \frac{(r-g)K + k}{\phi p(\phi)} \\
\hat{B}_2 &= \frac{\rho + sK}{r-g+s} \\
\hat{B}_1 &= \frac{-n + q(\theta)\hat{B}_2}{r-g+q(\theta)} \\
&= \frac{-n + q(\theta)\hat{B}_2}{r-g+q(\theta)} + \frac{q(\theta)}{r-g+q(\theta)}\frac{\rho+sK}{r-g+s},
\end{aligned}
$$

by equating forward and backward Bellman equations, we get:

$$
\frac{(r-g)K + k}{\phi p(\phi)} = \frac{-n}{r-g+q(\theta)} + \frac{q(\theta)}{r-g+q(\theta)}\frac{\rho}{r-g+s}.
$$

A.5 Bargaining

No entry-costs

Banks and firms share the surplus

$$S=\frac{q(\theta)}{r-g+q(\theta)}\frac{\pi\gamma}{r-g+s}-\frac{n}{r-g+q(\theta)}$$

according to the rule:

$$\rho=\arg\max\left(\hat{B}_1-\hat{B}_0\right)^{(1-\omega)}\left(\hat{F}_1-\hat{F}_0\right)^{\omega}.$$

Then we have

$$(1-\omega)\hat{F}_1=\omega\hat{B}_1$$
$$(1-\omega)\left[\frac{q(\theta)}{r-g+q(\theta)}\frac{\pi\gamma-\rho}{r-g+s}\right]=\omega\left(\frac{q(\theta)\rho}{\left(r-g+q(\theta)\right)\left(r-g+s\right)}-\frac{n}{r-g+q(\theta)}\right)$$
$$\rho=(1-\omega)\pi\gamma+\omega\frac{n\left(r-g+s\right)}{q(\theta)}.$$

Furthermore, using the backward definition of $\hat{F}_1$ and $\hat{B}_1$, it follows that

$$(1-\omega)\frac{c}{p(\phi)}=\omega\frac{k}{\phi p(\phi)}$$
$$\phi=\frac{\omega}{1-\omega}\frac{k}{c}.$$

Entry costs

When there are fixed entry costs in the banking sector, we assume that they do not affect the outside option of the bank

$$\rho=\arg\max\left(\hat{B}_1\right)^{(1-\omega)}\left(\hat{F}_1-\hat{F}_0\right)^{\omega}.$$

As a result, the surplus is unaffected:

$$S=\frac{q(\theta)}{r-g+q(\theta)}\frac{\pi\gamma}{r-g+s}-\frac{n}{r-g+q(\theta)}.$$

The equilibrium tightness is determined using the backward definition of $\hat{F}_1$ and $\hat{B}_1$:

$$\phi=\frac{\omega}{1-\omega}\frac{\left(r-g\right)K+k}{c}.$$

A.6 Flow equations and technology evolution

The number of entrepreneurs in the four stages evolves according to the following equations:

$$
\begin{aligned}
\dot{\mathscr{F}}_2 &= -s\mathscr{F}_2 + q(\theta)\mathscr{F}_1 \\
\dot{\mathscr{F}}_1 &= -q(\theta)\mathscr{F}_1 + p(\phi)\mathscr{F}_0 \\
\dot{\mathscr{F}}_0 &= s\mathscr{F}_2 - p(\phi)\mathscr{F}_0.
\end{aligned}
$$

Thus, abstracting from the very short run $\dot{\mathscr{F}}_i = 0$, we have

$$
\begin{aligned}
\mathscr{F}_1 &= \frac{s}{q(\theta)}\mathscr{F}_2 \\
\mathscr{F}_0 &= \frac{s}{p(\phi)}\mathscr{F}_2.
\end{aligned}
$$

Since

$$
(\mathscr{F}_1 + \mathscr{F}_0 + \mathscr{F}_2) = \mathscr{F},
$$

it follows that

$$
\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)\mathscr{F}_2 = \mathscr{F}
$$

and thus that

$$
\frac{\mathscr{F}_2}{\mathscr{F}} = \frac{1}{\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)}.
$$

Note that the law of motion of technology in discrete time would be

$$
\begin{aligned}
A_{t+1} &= \frac{(\mathscr{F}_1 + \mathscr{F}_0)}{\mathscr{F}}A_t + \frac{\mathscr{F}_2}{\mathscr{F}}A_t(1+\gamma) \\
\frac{A_{t+\Delta} - A_t}{\Delta} &= \frac{\left(\mathscr{F}_1 + \mathscr{F}_0 + \mathscr{F}_2(1+\gamma) - \mathscr{F}\right)}{\mathscr{F}}A_t \\
\frac{A_{t+\Delta} - A_t}{\Delta} &= \frac{\mathscr{F}_2}{\mathscr{F}}\gamma A_t,
\end{aligned}
$$

so that, in continuous time, letting $\Delta \to 0$, we have:

$$g \equiv \frac{\dot{A}}{A} = \frac{\gamma}{\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)}.$$

A.7 Equilibrium conditions

A.7.1 *No fixed-costs*

To sum up, the equilibrium conditions of this economy read:

$$\begin{aligned}
\frac{k}{\phi p\left(\phi\right)} &= \frac{(1-\omega)\, q\left(\theta\right)}{\left(r-g+q\left(\theta\right)\right)} \left\{\frac{\pi\gamma}{\left(r-g+s\right)} - \frac{n}{q\left(\theta\right)}\right\} \\
\frac{c}{p\left(\phi\right)} &= \frac{\omega q\left(\theta\right)}{r-g+q\left(\theta\right)} \left\{\frac{\pi\gamma}{\left(r-g+s\right)} - \frac{n}{q\left(\theta\right)}\right\} \\
\phi &= \frac{\omega}{1-\omega}\frac{k}{c} \\
\rho &= (1-\omega)\,\pi\gamma + \omega\frac{n\left(r-g+s\right)}{q\left(\theta\right)} \\
\frac{\dot{A}}{A} \equiv g &= \frac{\gamma}{\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)}.
\end{aligned}$$

The equilibrium can be represented in a more compact way using the following GG and PP curves:

$$\begin{aligned}
GG &: \quad g = \frac{\gamma}{\left(\frac{s}{Q(p,g)} + \frac{s}{p} + 1\right)} \\
PP &: \quad p = P\left(\frac{\omega}{1-\omega}\frac{k}{c}\right),
\end{aligned}$$

where $q = \frac{(r-g)\frac{c}{\omega p}+n}{\frac{\pi\gamma}{r-g+s}-\frac{c}{\omega p}} := Q(p,g)$.

A.7.2 Fixed entry costs

With fixed entry costs, the equilibrium conditions become:

$$\frac{k+\left(r-g\right)K}{\phi p\left(\phi\right)} = \frac{(1-\omega)\,q\left(\theta\right)}{\left(r-g+q\left(\theta\right)\right)}\left\{\frac{\pi\gamma}{\left(r-g+s\right)}-\frac{n}{q\left(\theta\right)}\right\}$$

$$\frac{c}{p\left(\phi\right)} = \frac{\omega q\left(\theta\right)}{r-g+q\left(\theta\right)}\left\{\frac{\pi\gamma}{\left(r-g+s\right)}-\frac{n}{q\left(\theta\right)}\right\}$$

$$\rho = (1-\omega)\,\pi\gamma+\omega\frac{\left(r-g+s\right)}{q\left(\theta\right)}n$$

$$\phi = \omega\frac{k+\left(r-g\right)K}{c\left(1-\omega\right)}$$

$$\frac{\dot{A}}{A} \equiv g = \frac{\gamma}{\left(\frac{s}{q(\theta)}+\frac{s}{p(\phi)}+1\right)}.$$

The equations defining the GG and PP curves become, accordingly:

$$GG \quad : \quad g = \frac{\gamma}{\left(\frac{s}{Q(p,g)}+\frac{s}{p}+1\right)}$$

$$PP \quad : \quad PP: p = P\left(\frac{\omega}{1-\omega}\frac{k+(r-g)K}{c}\right),$$

where $q = \frac{(r-g)\frac{c}{\omega p}+n}{\frac{\pi\gamma}{r-g+s}-\frac{c}{\omega p}} := Q(p,g)$.

Appendix B Growth and finance

As a preliminary to the proof of the next two propositions, it is useful to define $P = 1/p$ and $Q = 1/q$. From the definition of the growth rate, we have

$$P+Q = (\gamma/g-1)/s := M(g). \tag{B.1}$$

If GG has a hump, it must be that (locally) $dP = -dQ$ so that $dg = 0$.

From the free-entry condition:

$$cP = \frac{\omega}{1+Q(r-g)}[\frac{\pi\gamma}{r-g+s}-nQ] \tag{B.2}$$

or, in logs,

$$\log c + \log P = \log \omega - \log[1 + Q(r-g)] + \log[\frac{\pi\gamma}{r-g+s} - nQ] \tag{B.3}$$

We can now derive the condition under which $dP = -dQ$ and $dg = 0$ according to the (log) free entry condition. To do so, let's keep g constant and totally differentiate so that:

$$\frac{dP}{P} = -\frac{(r-g)dQ}{1+Q(r-g)} - \frac{ndQ}{\frac{\pi\gamma}{r-g+s} - nQ}. \tag{B.4}$$

Hence $dP = -dQ$ and $dg = 0$ if and only if

$$\frac{1}{P} = \frac{(r-g)}{1+Q(r-g)} + \frac{n}{\frac{\pi\gamma}{r-g+s} - nQ} \tag{B.5}$$

$$= \frac{(r-g)}{1+Q(r-g)} + \frac{n\omega/cP}{1+Q(r-g)} \tag{B.6}$$

or

$$1 = \frac{P(r-g) + n\omega/c}{Q(r-g)+1} \tag{B.7}$$

or

$$P - Q = \frac{1 - n\omega/c}{r-g} := N(g). \tag{B.8}$$

The hump thus occurs at (P, Q, g) that are defined by the three equations (1), (2) and (8). Now (1) and (8) can be solved, given g, to provide:

$$P = \frac{M(g)+N(g)}{2} \tag{B.9}$$

$$Q = \frac{M(g)-N(g)}{2}. \tag{B.10}$$

Substituting these values in B.2 provides a single equation in g, and thus the value of the g at hump. Existence condition for the hump boils down to checking under which condition this equation has a solution below r.

More specifically, the free-entry condition is:

$$cP[1+Q(r-g)] = \omega\Pi - n\omega Q \tag{B.11}$$

or

$$[cP + n\omega Q] + cPQ(r-g) - \omega\Pi = 0. \tag{B.12}$$

Now, at the hump,

$$Q = P - N(g) = P + \frac{n\omega/c - 1}{r-g}, \tag{B.13}$$

so that

$$PQ = P^2 + \frac{n\omega/c - 1}{r-g} P. \tag{B.14}$$

Using the last two equations to eliminate Q and PQ from the free-entry condition yields:

$$\left[cP + n\omega[P + \frac{n\omega c - 1}{r-g}] \right] + c \left[P^2 + \frac{n\omega/c - 1}{r-g} P \right] (r-g) - \omega\Pi = 0. \tag{B.15}$$

Collecting terms, we get another quadratic equation in P, with coefficients depending on $r-g$:

$$c(r-g)P^2 + 2n\omega P + n\omega \frac{n\omega/c - 1}{r-g} - \omega \frac{\pi\gamma}{r-g+s} = 0. \tag{B.16}$$

Its discriminant is

$$\begin{aligned}
\Delta &= 4n^2\omega^2 - 4c(r-g)\left(n\omega \frac{n\omega/c - 1}{r-g} - \omega \frac{\pi\gamma}{r-g+s} \right) \\
&= 4n^2\omega^2 - 4cn\omega\,(n\omega/c - 1) + 4c(r-g)\omega \frac{\pi\gamma}{r-g+s} \\
&= 4cn\omega + 4c(r-g)\omega \frac{\pi\gamma}{r-g+s} > 0
\end{aligned}$$

Proposition 7 (hump-shaped GG curve) *Suppose $c = \omega n$. Then the GG curve is hump-shaped. The growth rate is maximal and the total expected search time is minimal when expected credit and innovation search times are equal $1/p = 1/q$.*

Proof: *In the special case $n\omega/c = 1$, $N(g) = 0$ and $P = Q = M(g)/2$ which can be inserted into the free-entry condition B.2 to provide the value of g at the hump.*

$$\begin{aligned}
cP\left(1 + P(r-g)\right) &= \omega \left(\frac{\pi\gamma}{r-g+s} - nP \right) \\
cP\left(1 + P(r-g)\right) &= \omega\Pi - cP
\end{aligned}$$

For a given g, the equation above provides a quadratic expression in P with the following *two roots*

$$P_1 = \frac{-\sqrt{c^2 + c\Pi\omega\left(r-g\right)} - c}{c\left(r-g\right)}$$
$$P_2 = \frac{\sqrt{c^2 + c\Pi\omega\left(r-g\right)} - c}{c\left(r-g\right)}$$

It's clear that if $\left(r-g\right) > 0$, $P_1 < 0$ *and* $P_2 > 0$. *Using the fact that* $P = \frac{M(g)}{2}$, *we have*

$$P_2 = \frac{-c + \sqrt{c^2 + c\frac{\pi\gamma\omega(r-g)}{r-g+s}}}{c\left(r-g\right)} = \frac{M\left(g\right)}{2}.$$

The left-hand side of this equation evaluated at 0 *and* r *is*

$$\frac{M\left(g\right)}{2} = \frac{\frac{\gamma}{g} - 1}{s}$$
$$M\left(r\right) = \frac{\frac{\gamma}{r} - 1}{2s} < 0$$
$$M\left(0\right) = \infty.$$

Similarly, the right-hand side is

$$P_1\left(0\right) = \frac{-c + \sqrt{c^2 + cr\omega\frac{\pi\gamma}{r+s}}}{cr} > 0$$
$$P_1\left(r\right) = \lim_{g\to r} \frac{\sqrt{c^2 + c\frac{\pi\gamma}{r-g+s}\omega\left(r-g\right)} - c}{c\left(r-g\right)}$$
$$= \lim_{g\to r} \frac{\left(-\frac{1}{2}\right)\left(c^2 + c\frac{\pi\gamma}{r-g+s}\omega\left(r-g\right)\right)^{-\frac{1}{2}}\left(c\frac{\pi\gamma}{r-g+s}\omega\right)}{-cg} = \frac{\left(-\frac{1}{2}\right)c\frac{\pi\gamma}{s}\omega}{-c^2 r} = \frac{\pi\gamma\omega}{2scr}.$$

So there is always a solution between 0 and r, $i.e.$ *the left-hand and right-hand sides cross at least once in that interval since:*

$$\begin{aligned} P_1\left(0\right) &< M\left(0\right) \\ P_1\left(r\right) &= \frac{\pi\gamma\omega}{2scr} > 0 > \frac{\gamma - r}{rs} = M\left(r\right). \end{aligned}$$

•

Proposition 8 (hump-shaped GG curve general case) *The GG curve is hump-shaped if*

$$n^2 < \frac{c}{\omega}\left[(r-g_\infty)\Pi_\infty + n\right]. \tag{B.17}$$

Proof: *For ease of exposition, it is useful to divide the proof into few steps.*

Step 1: Positive P *The quadratic equation in* P *(derived above)*

$$c(r-g)P^2 + 2n\omega P + n\omega\frac{n\omega/c-1}{r-g} - \omega\frac{\pi\gamma}{r-g+s} = 0$$

has two roots P_1 *and* P_2 *whose sum and product are*

$$\begin{aligned}
P_1 + P_2 &= -\frac{2n\omega}{c(r-g)} < 0 \\
P_1P_2 &= \frac{\omega\left(n\frac{n\omega/c-1}{r-g} - \frac{\pi\gamma}{r-g+s}\right)}{c(r-g)}.
\end{aligned}$$

If

$$\begin{aligned}
n\frac{n\omega/c-1}{r-g} - \frac{\pi\gamma}{r-g+s} &< 0 \Rightarrow \\
n\frac{n\omega/c-1}{r-g} - \Pi &< 0,
\end{aligned}$$

then one of the two roots is positive.

Step 2: Sufficient condition *The condition above can be simplified as*

$$\begin{aligned}
n\omega - c &< \frac{c}{n}\left(r-g\right)\Pi \\
n^2 &< \frac{c}{\omega}\left[(r-g)\Pi + n\right].
\end{aligned}$$

This condition involves an endogenous variable (g)*, but a sufficient condition for it to hold is*

$$n^2 < \frac{c}{\omega}\left[(r-g_\infty)\Pi_\infty + n\right]$$

where g_∞ *is the horizontal asymptote of the GG curve when* $p \to \infty$*.*

Step 3: $r-g>0$ *Finally, we need to check that* $r-g>0$*. Let's denote with* P_1 *the positive root, recall that*

$$P_1(g)=\frac{M(g)+N(g)}{2}.$$

The righthand side of this equation is

$$\begin{aligned}\frac{M(g)+N(g)}{2}&=\frac{1-n\omega/c}{r-g}+\frac{\frac{\gamma}{g}-1}{s}\\ M(r)+N(r)&=\infty\\ M(0)+N(0)&=\infty.\end{aligned}$$

The lefthand side is:

$$\begin{aligned}P_1(0)&=\frac{-n\omega+\sqrt{cn\omega+cr\omega\frac{\pi\gamma}{r+s}}}{cr}>0\\ P_1(r)&=\lim_{g\to r}\frac{-n\omega+\sqrt{cn\omega}}{c(r-g)}=\infty.\end{aligned}$$

Hence there must be at least a solution in between 0 and r*.*

•

Proposition 9 (Uniqueness of the hump) *If the GG curve has an hump, it's unique.*

Proof: *Recall the equation describing the GG curve:*

$$g=\frac{\gamma}{1+s/p+s/Q(p,g)},\tag{B.18}$$

where the function $Q(p,g)$ *is derived implicitly from the free-entry condition:*

$$q=\frac{(r-g)\frac{c}{wp}+n}{\frac{\pi\gamma}{r-g+s}-\frac{c}{wp}}:=Q(p,g)$$

with $Q_p<0$ *and* $Q_g<0$*. This implies after straightforward differentiation that the sign of the slope of the GG curve* (dg/dp) *is the same as the sign of*

$$A:=1/p^2+Q_p/q^2\tag{B.19}$$

since

$$
\begin{aligned}
G_p &= \frac{dg}{dp} = \frac{\gamma s}{\left(\frac{s}{q}+\frac{s}{p}+1\right)^2}\left(\frac{q_p}{q^2}+\frac{1}{p^2}\right) = A(p)\frac{\gamma s}{\left(\frac{s}{q}+\frac{s}{p}+1\right)^2} \\
G_{pp} &= \frac{dg}{dpdp} = 2\frac{\gamma s^2}{\left(\frac{s}{q}+\frac{s}{p}+1\right)^3}A(p)^2 \\
&\quad + \frac{\gamma s}{\left(\frac{s}{q}+\frac{s}{p}+1\right)^2}A_p
\end{aligned}
$$

where $A_p = \frac{q_{pp}}{q^2} - \frac{2q_p^2}{q^3} - \frac{2}{p^3}$. *At the hump,* p^*, $A(p^*) = 0$ *so that*

$$
G_{pp} = \frac{\gamma s}{\left(\frac{s}{q}+\frac{s}{p^*}+1\right)^2}A_p(p^*).
$$

With some algebra, it is possible to show that at the hump the GG function is concave, i.e.

$A_p < 0$

$$
\begin{aligned}
A_p &= \frac{q_{pp}}{q^2} - \frac{2q_p^2}{q^3} - \frac{2}{p^3} \\
\text{/ Recall } q_{pp} &= -2q_p\left(\frac{1}{p} - \frac{\Psi_p}{(\omega\Pi - \Psi)}\right) \text{ /} \\
&= -\frac{2q_p}{q^2}\left(\frac{1}{p} - \frac{\Psi_p}{(\omega\Pi - \Psi)}\right) - \frac{2q_p^2}{q^3} - \frac{2}{p^3} \\
\text{/ Recall } \frac{q_p}{q^2} &= -\frac{1}{p^2} \text{/} \\
&= -\frac{2q_p}{q^2}\left(\frac{1}{p} - \frac{\Psi_p}{(\omega\Pi - \Psi)} + \frac{q_p}{q}\right) - \frac{2}{p^3} \\
&= \frac{2}{p^2}\left(\frac{1}{p} - \frac{\Psi_p}{(\omega\Pi - \Psi)} + \frac{q_p}{q} - \frac{1}{p}\right) \\
&= \frac{2}{p^2}\left(\frac{q_p}{q} - \frac{\Psi_p}{(\omega\Pi - \Psi)}\right) \\
\text{/ Recall } \frac{q_p}{q} &= \omega\Psi_p \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)\left(\omega n + \left(r-g\right)\Psi\right)} \text{/} \\
&= \frac{2}{p^2}\left(\omega\Psi_p \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)\left(\omega n + \left(r-g\right)\Psi\right)} - \frac{\Psi_p}{(\omega\Pi - \Psi)}\right) \\
&= \underbrace{\left(\frac{2}{p^2}\right)}_{+}\underbrace{\frac{\Psi_p}{(\omega\Pi - \Psi)}}_{-}\underbrace{\left(\frac{\omega n + \omega\left(r-g\right)\Pi}{\omega n + \left(r-g\right)\Psi} - 1\right)}_{+} < 0
\end{aligned}
$$

We can show that the last term is positive using the no free-entry condition:

$$
\Psi = \omega\beta\left(\Pi - \frac{n}{q}\right) < \omega\beta\Pi < \omega\Pi
$$

where

$$
\begin{aligned}
\beta &= \frac{q}{r-g+q} < 1 \\
\Pi &= \frac{\pi\gamma}{r-g+s} \\
\Psi &= \frac{c}{p}.
\end{aligned}
$$

We used the following expressions:

$$
\begin{aligned}
q &= \frac{\omega n + \left(r-g\right)\Psi}{(\omega\Pi - \Psi)} \\
\Psi_p &= -\frac{c}{p^2} < 0,\ \Psi_{pp} = 2\frac{c}{p^3} > 0, \frac{\Psi_{pp}}{\Psi_p} = -\frac{2}{p} \\
q_p &= \Psi_p \frac{\left(r-g\right)(\omega\Pi - \Psi) + \omega n + \left(r-g\right)\Psi}{(\omega\Pi - \Psi)^2} = \omega\Psi_p \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)^2} < 0 \\
\frac{q_p}{q} &= \omega\Psi_p \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)^2} \frac{(\omega\Pi - \Psi)}{\omega n + \left(r-g\right)\Psi} = \omega\Psi_p \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)\left(\omega n + \left(r-g\right)\Psi\right)} \\
q_{pp} &= \omega\Psi_{pp} \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)^2} + 2\omega\Psi_p^2 \frac{\left(r-g\right)\Pi + n}{(\omega\Pi - \Psi)^3} = \\
&= q_p \frac{\Psi_{pp}}{\Psi_p} + \frac{2q_p\Psi_p}{(\omega\Pi - \Psi)} = \frac{-2q_p}{p} + \frac{2q_p\Psi_p}{(\omega\Pi - \Psi)} = -2q_p\left(\frac{1}{p} - \frac{\Psi_p}{(\omega\Pi - \Psi)}\right) > 0.
\end{aligned}
$$

•

Proposition 10 *In the presence of fixed-entry costs in financial market, If the equilibrium exists is unique.*

Proof: *As a preliminary, it is useful to rewrite the PP curve by explicitly expressing g as a function of p:*

$$
\begin{aligned}
PP \quad &: \quad p = p_0 \left[\frac{\omega}{1-\omega} \frac{k + (r-g)K}{c}\right]^{-\eta} \Rightarrow \\
g &= \frac{1}{K}\frac{c}{\omega}(1-\omega)\left(\frac{1}{c}\frac{\omega}{1-\omega}(k+Kr) - \left(\frac{p}{p_0}\right)^{-\frac{1}{\eta}}\right) \Rightarrow \\
g &= \frac{(k+Kr)}{K} - \frac{1}{K}\frac{c}{\omega}(1-\omega)\left(\frac{p}{p_0}\right)^{-\frac{1}{\eta}}.
\end{aligned}
$$

It is straightforward to show that PP is an increasing and concave curve in the (p, g) plane:

$$
\begin{aligned}
PP_p &= \left(\frac{1}{\eta}\right)\frac{1}{K}\frac{c}{\omega p_0}(1-\omega)\left(\frac{p}{p_0}\right)^{-\frac{1}{\eta}-1} > 0 \\
PP_{pp} &= -\left(\frac{1}{\eta}\right)\left(\frac{1}{\eta}+1\right)\frac{1}{K}\frac{c}{\omega p_0^2}(1-\omega)\left(\frac{p}{p_0}\right)^{-\left(\frac{1}{\eta}+2\right)} < 0.
\end{aligned}
$$

Let's now define the $RR(p)$ curve as the vertical difference between PP and GG:

$$RR(p) \equiv PP - GG = \frac{(k+Kr)}{K} - \frac{1}{K}\frac{c}{\omega}(1-\omega)\left(\frac{p}{p_0}\right)^{-\frac{1}{\eta}} - \frac{\gamma}{1+s/p+s/\frac{(r-g)\frac{c}{\omega p}+n}{\frac{\pi\gamma}{r-g+s}-\frac{c}{\omega p}}}.$$

It follows that

$$RR_p = PP_p - GG_p.$$

First, notice that if PP and GG cross after the hump, i.e. $RR(p) = 0$ for $p > p^*$, then $RR_p > 0$ since GG is downward sloping. The GG is downward sloping and we cannot have multiple crossing on that side. Things are different if the two curves cross for $p < p^*$. At the hump

$$RR_p(p^*) = \left(\frac{1}{\eta}\right)\frac{1}{K}\frac{c}{\omega p_0}(1-\omega)\left(\frac{p^*}{p_0}\right)^{-\frac{1}{\eta}-1} > 0.$$

At $p = 0$,

$$\lim_{p\to 0} RR_p(0) = \left(\frac{1}{\eta}\right)\frac{1}{K}\frac{c}{\omega p_0}(1-\omega)\left(\frac{p}{p_0}\right)^{-\frac{1}{\eta}-1} + \frac{\gamma s}{\left(\frac{s}{q}+\frac{s}{p}+1\right)^2}\left(\frac{q_p}{q^2}+\frac{1}{p^2}\right) = \infty.$$

Furthermore RR_p is monotonic and decreasing for $p < p^*$ so that:

$$RR_{pp} = GG_{pp} - \left(\frac{1}{\eta}\right)\left(\frac{1}{\eta}+1\right)\frac{1}{K}\frac{c}{\omega p_0^2}(1-\omega)\left(\frac{p}{p_0}\right)^{-\left(\frac{1}{\eta}+2\right)} < 0.$$

Since as we have already shown that GG is concave at the hump. This means that for points at the left of the hump, when both PP and GG are increasing, the PP curve is always steeper than GG, i.e. there cannot be more than one crossing point. •

Appendix C Block bargaining

Let's assume that the joint value bank-firm pair bargain with a innovator. If we define:

$$J_i = B_i + F_i.$$

Then, in different stages:

$$
\begin{aligned}
\left(r-g\right)\hat{J}_2 &= \pi\gamma - w - s\hat{J}_2 \\
\left(r-g\right)\hat{J}_1 &= -n + q\left(\theta\right)\left[\hat{J}_2 - \hat{J}_1\right] \\
K\left(\phi\right) \equiv \frac{k}{\phi p\left(\phi\right)} &+ \frac{c}{p\left(\phi\right)} = \hat{J}_1.
\end{aligned}
$$

In the innovation market, the negotiated wage for the innovator is the solution to

$$
\begin{gathered}
w = \arg\max\left(\hat{J}_2 - \hat{J}_1\right)^{1-\alpha}\left(\hat{I}_2 - \hat{I}_1\right)^{\alpha} \\
\Rightarrow \alpha\left(\hat{J}_2 - \hat{J}_1\right) = (1-\alpha)\left(\hat{I}_2 - \hat{I}_1\right) \\
\alpha\left(\hat{J}_2 - \hat{J}_1\right) = (1-\alpha)\left(\hat{I}_2 - \hat{I}_1\right) \\
(1-\alpha)\left(\hat{I}_2 - \hat{I}_1\right) = \alpha\left(\hat{J}_2 - \hat{J}_1\right).
\end{gathered}
$$

It follows that

$$
\begin{gathered}
(1-\alpha)\left(\hat{I}_2 - \hat{I}_1\right) = \alpha\left(\hat{J}_2 - \hat{J}_1\right) \\
(1-\alpha)\left(r-g\right)\left(\hat{I}_2 - \hat{I}_1\right) = \alpha\left(r-g\right)\left(\hat{J}_2 - \hat{J}_1\right) \\
(1-\alpha)\left[w - s\left(\hat{I}_2 - \hat{I}_1\right) - \left(r-g\right)\hat{I}_1\right] = \alpha\left[\pi\gamma - w - s\left(\hat{J}_2 - \hat{J}_1\right) - \left(r-g+s\right)\hat{J}_1\right] \\
(1-\alpha)\left[w - \left(r-g\right)\hat{I}_1\right] = \alpha\left[\pi\gamma - w - \left(r-g\right)\hat{J}_1\right] \\
w = \alpha\left(\pi\gamma - \left(r-g+s\right)\hat{J}_1\right) + (1-\alpha)\hat{I}_1
\end{gathered}
$$

$$
\begin{aligned}
\left(r-g\right)\hat{I}_1 &= \theta q(\theta)\left[\hat{I}_2 - \hat{I}_1\right] = \\
&= \theta q(\theta)\frac{\alpha}{(1-\alpha)}\left[\hat{J}_2 - \hat{J}_1\right] \\
&= \theta q(\theta)\frac{\alpha}{(1-\alpha)}\left[\frac{\left(r-g+q\right)K\left(\phi\right)+n}{q} - \hat{J}_1\right] \\
&= \theta q(\theta)\frac{\alpha}{(1-\alpha)}\left[\frac{\left(r-g+q\right)K\left(\phi\right)+n}{q} - K\left(\phi\right)\right] \\
&= \theta\frac{\alpha}{(1-\alpha)}\left[\left(r-g\right)K\left(\phi\right)+n\right]
\end{aligned}
$$

$$
\begin{aligned}
\left(r-g\right)\hat{I}_1 &= \theta q(\theta)\left[\hat{I}_2-\hat{I}_1\right] = \\
&= \theta q(\theta)\frac{\alpha}{(1-\alpha)}\left[\hat{J}_2-\hat{J}_1\right].
\end{aligned}
$$

Therefore

$$
\begin{aligned}
w &= \alpha\left(\pi\gamma-\left(r-g+s\right)\hat{J}_1\right)+(1-\alpha)\hat{I}_1 \\
&= \alpha\pi\gamma+(1-\alpha)\hat{I}_1-\alpha\left(r-g+s\right)K\left(\phi\right) \\
&= \alpha\pi\gamma+(1-\alpha)\theta\frac{\alpha}{(1-\alpha)}\left(\left(r-g\right)K\left(\phi\right)+n\right)-\alpha\left(r-g+s\right)K\left(\phi\right) \\
&= \alpha\left(\pi\gamma+\theta\left(r-g\right)K\left(\phi\right)+\theta n-\left(r-g+s\right)K\left(\phi\right)\right) \\
&= \alpha\left(\pi\gamma+\theta n_k-\left(r-g+s\right)K\left(\phi\right)\right)
\end{aligned}
$$

where $n_k \equiv \left(r-g\right)K\left(\phi\right)+n$. Plugging w in the value functions:

$$
\begin{aligned}
\hat{J}_2 &= \frac{\pi\gamma-\alpha\left(\pi\gamma+\theta n_k-\left(r-g+s\right)K\left(\phi\right)\right)}{\left(r-g+s\right)} \\
\hat{J}_2 &= \frac{n_k}{q}+K\left(\phi\right) \\
\frac{n_k}{q}+K\left(\phi\right) &= \frac{(1-\alpha)\pi\gamma-\alpha\theta n_k+\alpha\left(r-g+s\right)K\left(\phi\right)}{\left(r-g+s\right)} \\
\frac{n_k}{q} &= \frac{(1-\alpha)\left(\pi\gamma-\left(r-g+s\right)K\left(\phi\right)\right)-\alpha\theta n_k}{\left(r-g+s\right)}.
\end{aligned}
$$

Appendix D Efficiency in financial and innovation markets

The social planner solves the following problem

$$
\begin{aligned}
\max_{\mathscr{I}_1,\mathscr{B}_0,\mathscr{F}_0,\mathscr{F}_1,A} \Omega^{SP} &= \int e^{-rt}\Pi A dt \\
st:\ \dot{A} &= g\left(\mathscr{I}_1,\mathscr{B}_0,\mathscr{F}_0,\mathscr{F}_1\right)A \\
\dot{\mathscr{F}}_1 &= -\mu\left(\mathscr{F}_1,\mathscr{I}_1\right)+m\left(\mathscr{F}_0,\mathscr{B}_0\right) \\
\dot{\mathscr{I}}_1 &= s\left(1-\mathscr{I}_1\right)-\mu\left(\mathscr{F}_1,\mathscr{I}_1\right)
\end{aligned}
$$

where

$$\mu(\mathscr{F}_1, \mathscr{I}_1) = q_0 \mathscr{F}_1^{1-\varepsilon} \mathscr{I}_1^{\varepsilon}$$
$$m(\mathscr{F}_0, \mathscr{B}_0) = p_0 \mathscr{F}_0^{1-\eta} \mathscr{B}_0^{\eta}$$
$$q(\theta) = \frac{\mu(\mathscr{F}_1, \mathscr{I}_1)}{\mathscr{F}_1}$$
$$p(\phi) = \frac{m(\mathscr{F}_0, \mathscr{B}_0)}{\mathscr{F}_0}$$

$$\Pi = \left[\pi\gamma(1-\mathscr{I}_1) - c\mathscr{F}_0 - k\mathscr{B}_0 - n\mathscr{F}_1\right]$$
$$g(\mathscr{I}_1, \mathscr{B}_0, \mathscr{F}_0, \mathscr{F}_1) = \frac{\gamma}{\left(\frac{s}{\frac{\mu(\mathscr{F}_1,\mathscr{I}_1)}{\mathscr{F}_1}} + \frac{s}{\frac{m(\mathscr{F}_0,\mathscr{B}_0)}{\mathscr{F}_0}} + 1\right)}$$

This is the Hamiltonian

$$\begin{aligned}
H &= e^{-rt}\left[\pi\gamma(1-\mathscr{I}_1) - c\mathscr{F}_0 - k\mathscr{B}_0 - n\mathscr{F}_1\right] A \\
&+ \Phi\left[-\mu(\mathscr{F}_1, \mathscr{I}_1) + m(\mathscr{F}_0, \mathscr{B}_0)\right] + \Psi\left[s(1-\mathscr{I}_1) - \mu(\mathscr{F}_1, \mathscr{I}_1)\right] \\
&+ \Xi\left[gA\right]
\end{aligned}$$

with following first-order conditions:

$$\frac{\partial H}{\partial \mathscr{B}_0} = 0 : -e^{-rt} kA + \Phi m_{\mathscr{B}_0} + \Xi g_{\mathscr{B}_0} A = 0 \tag{D.20}$$
$$\frac{\partial H}{\partial \mathscr{F}_0} = 0 : -e^{-rt} cA + \Phi m_{\mathscr{F}_0} + \Xi g_{\mathscr{F}_0} A = 0 \tag{D.21}$$
$$\dot{\Phi} = -\frac{\partial H}{\partial \mathscr{F}_1} : \dot{\Phi} = e^{-rt} nA + (\Phi + \Psi)\mu_{\mathscr{F}_1} - \Xi g_{\mathscr{F}_1} A \tag{D.22}$$
$$\dot{\Psi} = -\frac{\partial H}{\partial \mathscr{I}_1} : \dot{\Psi} = e^{-rt} \pi\gamma A + (\Phi + \Psi)\mu_{\mathscr{I}_1} + s\Psi - \Xi g_{\mathscr{I}_1} A \tag{D.23}$$
$$\dot{\Xi} = -\frac{\partial H}{\partial \mathscr{A}} : \dot{\Xi} = -e^{-rt}\Pi - \Xi g \tag{D.24}$$

where:

$$
\begin{aligned}
m_{\mathscr{B}_0} &= \eta \frac{m}{\mathscr{B}_0} = \eta p \phi \\
m_{\mathscr{F}_0} &= \left(1-\eta\right) \frac{m}{\mathscr{F}_0} = \left(1-\eta\right) p \\
\mu_{\mathscr{F}_1} &= \left(1-\varepsilon\right) \frac{\mu}{\mathscr{F}_1} = \left(1-\varepsilon\right) q \\
\mu_{\mathscr{I}_1} &= \frac{\varepsilon \mu}{\mathscr{I}_1} = \varepsilon q \theta \\
p_{\mathscr{B}_0} &= \eta \frac{p}{\mathscr{B}_0} \\
p_{\mathscr{F}_0} &= -\eta \frac{p}{\mathscr{F}_0} \\
q_{\mathscr{I}_1} &= \varepsilon \frac{q}{\mathscr{I}_1} \\
q_{\mathscr{F}_1} &= -\varepsilon \frac{q}{\mathscr{F}_1}
\end{aligned}
$$

$$
\begin{aligned}
g_{\mathscr{B}_0} &= \frac{g s p_{\mathscr{B}_0}}{p^2 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = \frac{g s \eta}{p \mathscr{B}_0 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = \frac{g^2 s \eta}{p \mathscr{B}_0 \gamma} \\
g_{\mathscr{F}_0} &= \frac{g s p_{\mathscr{F}_0}}{p^2 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = \frac{-g s \eta}{p \mathscr{F}_0 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = \frac{-g_{\mathscr{B}_0}}{\phi} \\
g_{\mathscr{I}_1} &= \frac{g s q_{\mathscr{I}_1}}{q^2 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = \frac{g s \varepsilon}{q \mathscr{I}_1 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = g_{\mathscr{B}_0} \frac{\varepsilon}{\eta} \frac{p}{q} \frac{\mathscr{B}_0}{\mathscr{I}_1} \\
g_{\mathscr{F}_1} &= \frac{g s q_{\mathscr{F}_1}}{q^2 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = -\frac{g s \varepsilon}{q \mathscr{F}_1 \left(\frac{s}{q} + \frac{s}{p} + 1\right)} = \frac{-g_{\mathscr{I}_1}}{\theta}.
\end{aligned}
$$

From the first equation:

$$-e^{-rt}kA + \left(\Phi m_{\mathscr{B}_0} + A\Xi g_{\mathscr{B}_0}\right) = 0 \Rightarrow$$

$$-e^{-rt}kA + \left(A\Xi g_{\mathscr{B}_0} + \Phi\eta p\phi\right) = 0$$

$$\Phi = \frac{e^{-rt}k - \Xi g_{\mathscr{B}_0}}{\eta p\phi} A \Rightarrow$$

$$\dot{\Phi} = \frac{-\left(e^{-rt}rk + \dot{\Xi} g_{\mathscr{B}_0}\right)A + \dot{A}\left(e^{-\hat{r}t}k - \Xi g_{\mathscr{B}_0}\right)}{\eta p\phi}$$

$$\dot{\Phi} = \frac{-\left(e^{-rt}rk + \dot{\Xi} g_{\mathscr{B}_0}\right)A + gA\left(e^{-\hat{r}t}k - \Xi g_{\mathscr{B}_0}\right)}{\eta p\phi}$$

$$\dot{\Phi} = \frac{-e^{-rt}rkA + gAe^{-\hat{r}t}k - g_{\mathscr{B}_0}A\left(g\Xi + \dot{\Xi}\right)}{\eta p\phi}$$

$$\dot{\Phi} = -e^{-rt}\frac{\left(r-g\right)k - \Pi g_{\mathscr{B}_0}}{\eta p\phi}A.$$

From the second first-order condition:

$$-e^{-rt}cA + \Phi m_{\mathscr{F}_0} + A\Xi g_{\mathscr{F}_0} = 0 \Rightarrow$$

$$-e^{-rt}cA + A\Xi g_{\mathscr{F}_0} + \Phi\left(1-\eta\right)p = 0$$

$$\Phi = \frac{e^{-rt}cA - A\Xi g_{\mathscr{F}_0}}{\left(1-\eta\right)p} \Rightarrow$$

$$\dot{\Phi} = -e^{-rt}\frac{\left(r-g\right)c - \Pi g_{\mathscr{F}_0}}{\left(1-\eta\right)p}A$$

And the last two:

$$\dot{\Phi} = e^{-rt}nA + (\Phi+\Psi)\mu_{\mathscr{F}_1} - \Xi Ag_{\mathscr{F}_1} \Rightarrow$$

$$\dot{\Phi} = e^{-rt}nA + \Psi(1-\varepsilon)q + \Phi(1-\varepsilon)q - A\Xi g_{\mathscr{F}_1}$$

$$\dot{\Psi} = e^{-rt}\pi\gamma A + (\Phi+\Psi)\mu_{\mathscr{I}_1} + s\Psi - A\Xi g_{\mathscr{I}_1} \Rightarrow$$

$$\dot{\Psi} = e^{-rt}\pi\gamma A + \Phi\varepsilon q\theta + \left(s+\varepsilon q\theta\right)\Psi - A\Xi g_{\mathscr{I}_1}.$$

We can then derive the optimal credit market tightness from:

$$\dot{\Phi} = -e^{-rt}\frac{(r-g)\,k - \Pi g_{\mathscr{B}_0}}{\eta p\phi}A$$
$$\dot{\Phi} = -e^{-rt}\frac{(r-g)\,c - \Pi g_{\mathscr{F}_0}}{(1-\eta)\,p}A$$

$$\frac{(r-g)\,k}{\eta p\phi} - \frac{\Pi g_{\mathscr{B}_0}}{\eta p\phi} = \frac{(r-g)\,c}{(1-\eta)\,p} + \frac{\Pi g_{\mathscr{B}_0}}{(1-\eta)\,p\phi}.$$

Therefore

$$\frac{(r-g)\,k}{\eta p\phi} - \frac{\Pi g_{\mathscr{B}_0}}{\eta p\phi} = \frac{(r-g)\,c}{(1-\eta)\,p} + \frac{\Pi g_{\mathscr{B}_0}}{(1-\eta)\,p\phi}$$
$$(1-\eta)(r-g)\,k - (1-\eta)\,\Pi g_{\mathscr{B}_0} = \eta\left((r-g)\,c\phi + \Pi g_{\mathscr{B}_0}\right)$$
$$\phi^{SP} = \frac{(1-\eta)(r-g)\,k - \Pi g_{\mathscr{B}_0}}{\eta (r-g)\,c}.$$

Note that the usual Hosios condition, $(1-\eta) = \omega$ does not imply $\phi^{SP} = \phi^{CE}$. We can derive a modified Hosios condition

$$\phi^{SP} = \phi^{CE} = \frac{\omega}{1-\omega}\frac{k}{c}$$

$$\frac{(1-\eta)(r-g)\,k - \Pi g_{\mathscr{B}_0}}{\eta (r-g)\,c} = \frac{\omega}{1-\omega}\frac{k}{c}$$
$$(1-\eta)(1-\omega)(r-g)\,k - (1-\omega)\,\Pi g_{\mathscr{B}_0} = \omega k\eta (r-g)$$
$$\left((1-\eta)(1-\omega) - \omega\eta\right)(r-g)\,k = (1-\omega)\,\Pi g_{\mathscr{B}_0}$$
$$k(r-g)(1-\omega-\eta) = (1-\omega)\,\Pi g_{\mathscr{B}_0}$$
$$(1-\omega)\left(\Pi g_{\mathscr{B}_0} - k(r-g)\right) = -\eta k (r-g)$$
$$(1-\omega) = \eta\frac{k(r-g)}{k(r-g) - \Pi g_{\mathscr{B}_0}} > \eta$$

From

$$\frac{\eta\Phi}{A} = \frac{e^{-rt}k - \Xi g_{\mathscr{B}_0}}{p\phi}$$
$$(1-\eta)\frac{\Phi}{A} = \frac{e^{-rt}c - \Xi g_{\mathscr{F}_0}}{p}$$

given that $g_{\mathscr{F}_0} = \frac{-g_{\mathscr{B}_0}}{\phi}$. It follows that

$$\begin{aligned}
\Phi &= \frac{A}{p} e^{-rt} \left[\frac{k}{\phi} + c - \frac{\Xi g_{\mathscr{B}_0}}{\phi} + \frac{\Xi g_{\mathscr{B}_0}}{\phi} \right] \\
\Phi &= A e^{-rt} K\left(\phi\right) \\
\dot{\Phi} &= -r e^{-rt} K\left(\phi\right) A + \dot{A} e^{-rt} K\left(\phi\right) \\
&= -e^{-rt} K\left(\phi\right) A \left(r - g\right)
\end{aligned}$$

where total search costs are measured by

$$K\left(\phi\right) = \frac{k}{p\phi} + \frac{c}{p}.$$

We can then use this expression for Φ to derive Ξ :

$$\begin{aligned}
\frac{\eta\Phi}{A} &= \frac{e^{-rt} k - \Xi g_{\mathscr{B}_0}}{p\phi} \\
e^{-rt} k - \frac{\Phi\eta p\phi}{A} &= \Xi g_{\mathscr{B}_0} \\
e^{-rt} k - \eta p\phi e^{-rt} K\left(\phi\right) &= \Xi g_{\mathscr{B}_0} \\
e^{rt}\Xi &= \frac{\left(k - \eta p\phi K\left(\phi\right)\right)}{g_{\mathscr{B}_0}} = \frac{\left(1-\eta\right) k - \eta c\phi}{g_{\mathscr{B}_0}}.
\end{aligned}$$

Finally, we can rearrange the last two first-order conditions:

$$\begin{aligned}
\dot{\Phi} &= e^{-rt} n A + \Psi\left(1-\varepsilon\right) q + \Phi\left(1-\varepsilon\right) q - A\Xi g_{\mathscr{F}_1} \\
\dot{\Psi} &= e^{-rt} \pi\gamma A + \Phi\varepsilon q\theta + \left(s + \varepsilon q\theta\right)\Psi - A\Xi g_{\mathscr{I}_1}.
\end{aligned}$$

From the first equation:

$$
\begin{aligned}
\dot{\Phi} &= e^{-rt}nA + (\Phi+\Psi)\,\mu_{\mathscr{F}_1} - A\Xi g_{\mathscr{F}_1}\\
-(r-g)\,e^{-rt}K(\phi)\,A &= e^{-rt}nA + (\Phi+\Psi)\,(1-\varepsilon)\,q - A\Xi g_{\mathscr{F}_1}\\
-(r-g)\,e^{-rt}K(\phi)\,A &= e^{-rt}nA + \left(Ae^{-rt}K(\phi)+\Psi\right)(1-\varepsilon)\,q - A\Xi g_{\mathscr{F}_1}\\
\Psi &= -e^{-rt}\left(\frac{nA+(r-g)\,K(\phi)\,A}{(1-\varepsilon)\,q} + AK(\phi)\right) + \frac{A\Xi g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}\\
\Psi &= -e^{-rt}\left(\frac{n+(r-g)\,K(\phi)}{(1-\varepsilon)\,q} + K(\phi)\right)A + \frac{A\Xi g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}\\
\dot{\Psi} &= \left(\frac{n+(r-g)\,K(\phi)}{(1-\varepsilon)\,q} + K(\phi)\right)e^{-rt}\left(Ar-\dot{A}\right) + \frac{g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}\left(A\dot{\Xi}+\dot{A}\Xi\right)\\
\dot{\Psi} &= \left(\frac{n+(r-g)\,K(\phi)}{(1-\varepsilon)\,q} + K(\phi)\right)e^{-rt}\,(r-g)\,A + \frac{g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}A\left(-e^{-rt}\Pi - g\Xi + g\Xi\right)\\
\dot{\Psi} &= \left(\frac{n+(r-g)\,K(\phi)}{(1-\varepsilon)\,q} + K(\phi)\right)e^{-rt}\,(r-g)\,A - \frac{g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}e^{-rt}\Pi A.
\end{aligned}
$$

If we define $n_k \equiv n+(r-g)\,K(\phi)$ and use the expressions above in $\dot{\Psi} = -\frac{\partial H}{\partial \mathscr{I}_1}$:

$$
\begin{gathered}
\dot{\Psi} = e^{-rt}\pi\gamma A + \Phi\varepsilon q\theta + (s+\varepsilon q\theta)\,\Psi - A\Xi g_{\mathscr{I}_1}\\
\left(\frac{n_k}{(1-\varepsilon)\,q} + K(\phi)\right)e^{-rt}\,(r-g)\,A - \frac{g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}e^{-rt}\Pi A =\\
e^{-rt}\pi\gamma A + \left(Ae^{-rt}K(\phi)\right)\varepsilon q\theta\\
+(s+\varepsilon q\theta)\left(-e^{-rt}\left(\frac{n_k}{(1-\varepsilon)\,q} + K(\phi)\right)A + \frac{A\Xi g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}\right)\\
-A\Xi g_{\mathscr{I}_1}
\end{gathered}
$$

$$
\begin{gathered}
\left(\frac{n_k}{(1-\varepsilon)\,q} + K(\phi)\right)e^{-rt}\,(r-g)\,A - \frac{g_{\mathscr{F}_1}}{(1-\varepsilon)\,q}e^{-rt}\Pi A =\\
e^{-rt}\pi\gamma A + \left(Ae^{-rt}K(\phi)\right)\varepsilon q\theta\\
+(s+\varepsilon q\theta)\left(-e^{-rt}\left(\frac{n+(r-g)\,K(\phi)}{(1-\varepsilon)\,q} + K(\phi)\right)A - \frac{A\Xi g_{\mathscr{I}_1}}{\theta\,(1-\varepsilon)\,q}\right)\\
-A\Xi g_{\mathscr{I}_1}
\end{gathered}
$$

$$
\begin{gathered}
\left(\frac{n_k}{(1-\varepsilon)\,q}+K\left(\phi\right)\right)\left(r-g\right)+\frac{g_{\mathscr{I}_1}}{\theta\,(1-\varepsilon)\,q}\Pi= \\
\pi\gamma+K\left(\phi\right)\varepsilon q\theta \\
-\left(s+\varepsilon q\theta\right)\left(\frac{n_k}{(1-\varepsilon)\,q}+K\left(\phi\right)+\frac{e^{rt}\Xi g_{\mathscr{I}_1}}{\theta\,(1-\varepsilon)\,q}\right) \\
-g_{\mathscr{I}_1}e^{rt}\Xi
\end{gathered}
$$

$$
\begin{aligned}
\frac{n_k}{(1-\varepsilon)\,q}\left(r-g+s\right)=\pi\gamma-\left(r-g-s\right)K\left(\phi\right)-\varepsilon q\theta\frac{n_k}{(1-\varepsilon)\,q} \\
-\frac{g_{\mathscr{I}_1}}{\theta\,(1-\varepsilon)\,q}\Pi-e^{rt}\Xi g_{\mathscr{I}_1}\left[\frac{s+\varepsilon q\theta+\theta\,(1-\varepsilon)\,q}{\theta\,(1-\varepsilon)\,q}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{n_k}{(1-\varepsilon)\,q}\left(r-g+s\right)=\pi\gamma-\left(r-g-s\right)K\left(\phi\right)-\varepsilon q\theta\left[\frac{n_k}{(1-\varepsilon)\,q}\right] \\
-\frac{g_{\mathscr{I}_1}}{\theta\,(1-\varepsilon)\,q}\Pi-e^{rt}\Xi g_{\mathscr{I}_1}\left[\frac{s+\theta q}{\theta\,(1-\varepsilon)\,q}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{n_k}{(1-\varepsilon)\,q}\left(r-g+s\right)=\pi\gamma-\left(r-g-s\right)K\left(\phi\right)-\varepsilon\theta\left[\frac{n_k}{(1-\varepsilon)}\right] \\
-\frac{g_{\mathscr{I}_1}}{\theta\,(1-\varepsilon)\,q}\left[\Pi+e^{rt}\Xi\left(s+\theta q\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{n_k^{SP}}{q}&=\frac{\pi\gamma-\left(r-g-s\right)K\left(\phi\right)}{\left(r-g+s\right)} \\
&-\varepsilon\frac{\pi\gamma+\theta n_k-\left(r-g-s\right)K\left(\phi\right)}{\left(r-g+s\right)} \\
&-\frac{g_{\mathscr{I}_1}}{\theta q}\left[\Pi+\frac{\left(k-\eta p\phi K\left(\phi\right)\right)}{g_{\mathscr{B}_0}}\left(s+\theta q\right)\right]
\end{aligned}
$$

or, in a more compact form,

$$
\frac{n_k^{SP}}{q}=\frac{\pi\gamma-K\left(\phi\right)\left(r+s-g\right)}{\left(r+s-g\right)}-\left(\varepsilon\hat{w}+\tau\right)
$$

where

$$\hat{w} \equiv \frac{\pi\gamma + \theta n_k - (r-g-s) K(\phi)}{(r+s-g)}$$
$$\tau \equiv \frac{g_{\mathscr{I}_1}}{\theta q}\left[\Pi + \frac{(k-\eta p\phi K(\phi))}{g_{\mathscr{B}_0}}(s+\theta q)\right].$$

Recall the competitive equilibrium:

$$\frac{n_k^{CE}}{q} = \frac{\pi\gamma - K(\phi)(r+s-g)}{(r+s-g)} - \alpha\frac{\pi\gamma + \theta n_k - (r-g-s)K(\phi)}{(r+s-g)}.$$

SP and CE coincide if:

$$\alpha\hat{w} = \varepsilon\hat{w} + \tau$$
$$\alpha = \varepsilon + \frac{\tau}{\hat{w}} > \varepsilon.$$

This is the second modified Hosios condition.

Note: Some useful algebra

$$\frac{g_{\mathscr{I}_1}}{g_{\mathscr{B}_0}} = \frac{\varepsilon}{\eta}\frac{p}{q}\frac{\mathscr{B}_0}{\mathscr{I}_1}$$
$$= \frac{\varepsilon}{\eta}\frac{p}{q}\frac{\mathscr{F}_0}{\mathscr{F}_1}\frac{\theta}{\phi} = \frac{\varepsilon}{\eta}\frac{p}{q}\frac{q}{p}\frac{\theta}{\phi} = \frac{\varepsilon}{\eta}\frac{\theta}{\phi}$$
$$\frac{g_{\mathscr{I}_1}}{\theta q} = g_{\mathscr{B}_0}\frac{\varepsilon}{\eta}\frac{p}{q}\frac{\mathscr{B}_0}{\mathscr{I}_1} = \frac{q\theta}{p\phi}g_{\mathscr{B}_0}\frac{\varepsilon}{\eta}\frac{p}{q} = g_{\mathscr{B}_0}\frac{\theta}{\phi}\frac{\varepsilon}{\eta}$$

$$g_{\mathscr{B}_0} = \frac{g^2 s\eta}{p\mathscr{B}_0}$$
$$g_{\mathscr{F}_0} = \frac{-g_{\mathscr{B}_0}}{\phi}$$
$$g_{\mathscr{I}_1} = g_{\mathscr{B}_0}\frac{\varepsilon}{\eta}\frac{p}{q}\frac{\mathscr{B}_0}{\mathscr{I}_1}.$$

Recall that:

$$\begin{aligned}
(1-\mathscr{I}_1) &= \frac{\theta q}{s+\theta q}\\
\mathscr{I}_1 &= \frac{s}{s+\theta q}\\
\mathscr{F}_1 = \theta\mathscr{I}_1 &= \frac{\theta s}{s+\theta q}\\
\mathscr{F}_0 = \frac{q}{p}\mathscr{F}_1 &= \frac{q}{p}\left(\frac{\theta s}{s+\theta q}\right)\\
\mathscr{B}_0 &= \frac{q}{p\phi}\left(\frac{\theta s}{s+\theta q}\right)\\
\frac{\mathscr{B}_0}{\mathscr{I}_1} &= \frac{q\theta}{p\phi},
\end{aligned}$$

so that

$$\begin{aligned}
\Pi &= \left[\pi\gamma\,(1-\mathscr{I}_1) - c\mathscr{F}_0 - k\mathscr{B}_0 - n\mathscr{F}_1\right]\\
&= \frac{1}{s+\theta q}\left[\pi\gamma\theta q - c\frac{q\theta s}{p} - k\frac{q\theta s}{p\phi} - ns\theta\right]\\
&\frac{\theta sq}{(s+\theta q)}\left[\frac{\pi\gamma}{s} - \frac{c}{p} - \frac{k}{p\phi} - \frac{n}{q}\right]\\
&= \frac{\theta sq}{(s+\theta q)}\left[\frac{\pi\gamma}{s} - K - \frac{n}{q}\right]\\
&= \frac{\theta sq}{(s+\theta q)}\left[\frac{\pi\gamma}{s} - \left(1+\frac{r-g}{q}\right)K - \frac{n_k}{q}\right].
\end{aligned}$$

Appendix E Only one friction

E.1 Only financial frictions

The value of a bank

The Bellman equations describing the steady-state values of the bank over the two stages are:

$$\begin{aligned}
(r-g)\,\hat{B}_0 &= -k + \phi p\,(\phi)\,[\hat{B}_1 - \hat{B}_0]\\
(r-g)\,\hat{B}_1 &= \rho + s\,[\hat{B}_0 - \hat{B}_1]
\end{aligned}$$

where k is a search cost in the financial market. With free-entry:

$$\begin{aligned}
\hat{B}_1 &= \frac{k}{\phi p(\phi)} \\
\hat{B}_1 &= \frac{\rho}{(r-g+s)} \\
\frac{k}{\phi p(\phi)} &= \frac{\rho}{(r-g+s)}.
\end{aligned}$$

The value of an Innovator/Firm

Similarly, the value of a firm on the balanced growth path is:

$$\begin{aligned}
(r-g)\hat{F}_0 &= -c + p(\phi)\left[\hat{F}_1 - \hat{F}_0\right] \\
(r-g)\hat{F}_1 &= \pi\gamma - \rho + s\left[\hat{F}_0 - \hat{F}_1\right]
\end{aligned}$$

or

$$(r-g)\left[\hat{F}_1 - \hat{F}_0\right] = \pi\gamma - \rho + c - (s+p)\left[\hat{F}_1 - \hat{F}_0\right].$$

There is no free-entry in this market and the total number of Firms is normalized to 1:

$$\mathscr{F} = \mathscr{F}_1 + \mathscr{F}_0 = 1.$$

Then, the dynamics of the flow of firms searching for banks can be expressed as

$$\dot{\mathscr{F}}_0 = s(1-\mathscr{F}_0) - p\mathscr{F}_0.$$

Abstracting from the very short-run so that $\dot{\mathscr{F}} = 0$:

$$\begin{aligned}
\mathscr{F}_0 &= \frac{s}{s+p} \\
\mathscr{F}_1 &= \frac{p}{s+p}.
\end{aligned}$$

Nash Bargaining

Under Nash bargaining, the surplus is shared according to the bargaining weights

$$(1-\omega)\left[\hat{F}_1 - \hat{F}_0\right] = \omega\hat{B}_1.$$

After some algebra, we have:

$$
\begin{gathered}
(1-\omega)\left(r-g\right)\left[\hat{F}_1-\hat{F}_0\right]=\omega\left(r-g\right)\hat{B}_1 \\
(1-\omega)\left[\pi\gamma-\rho+c-\left(s+p\right)\left(\hat{F}_1-\hat{F}_0\right)\right]=\omega\left[\rho-s\hat{B}_1\right] \\
(1-\omega)\left[\pi\gamma-\rho+c-p\left(\hat{F}_1-\hat{F}_0\right)\right]-s\left(1-\omega\right)\left(\hat{F}_1-\hat{F}_0\right)=\omega\rho-s\omega\hat{B}_1 \\
(1-\omega)\left[\pi\gamma-\rho+c-p\left(\hat{F}_1-\hat{F}_0\right)\right]=\omega\rho \\
(1-\omega)\left[\pi\gamma-\rho-\left(r-g\right)\hat{F}_0\right]=\omega\rho \\
\rho=(1-\omega)\left[\pi\gamma-\left(r-g\right)\hat{F}_0\right].
\end{gathered}
$$

Moreover

$$
\begin{aligned}
\left(r-g\right)\hat{F}_0 &= -c+p\left(\phi\right)\left[\hat{F}_1-\hat{F}_0\right]=-c+p\left(\phi\right)\frac{\omega\hat{B}_1}{1-\omega} \\
\left(r-g\right)\hat{F}_0 &= -c+p\frac{\omega}{1-\omega}\frac{k}{\phi p} \\
\left(r-g\right)\hat{F}_0 &= -c+\frac{\omega}{1-\omega}\frac{k}{\phi}.
\end{aligned}
$$

Plugging the expression above in the equilibrium ρ :

$$
\rho=(1-\omega)\left[\pi\gamma+c-\frac{\omega}{1-\omega}\frac{k}{\phi}\right].
$$

Therefore, the free entry condition in the banking sector can be rewritten as

$$
\begin{aligned}
\frac{k}{\phi p\left(\phi\right)} &= \frac{\rho}{\left(r-g+s\right)} \\
\frac{k}{\phi p\left(\phi\right)} &= \frac{(1-\omega)}{\left(r-g+s\right)}\left[\pi\gamma+c-\frac{\omega}{1-\omega}\frac{k}{\phi}\right].
\end{aligned}
$$

Equilibrium

The equilibrium is summarized by the two expressions below:

$$
\begin{aligned}
BB &: \frac{k}{\phi p\left(\phi\right)}=\frac{(1-\omega)}{\left(r-g+s\right)}\left[\pi\gamma+c-\frac{\omega}{1-\omega}\frac{k}{\phi}\right] \\
GG &: g=\frac{\gamma}{\frac{s}{p}+1}.
\end{aligned}
$$

Both curves are downward sloping in the ϕ, g plane. Taking limits of the GG curve, we find:

$$\begin{aligned} g_{GG}^{0} &= \gamma \\ g_{GG}^{\infty} &= 0, \end{aligned}$$

while, for the BB curve, we have:

$$\begin{aligned} g_{BB}^{\infty} &= -\infty \\ g_{BB}^{0} &= r+s. \end{aligned}$$

E.2 Perfect credit, frictions in R&D

Value of a firm

The Bellman equations describing the values of the firm along the balanced growth path in the two stages are:

$$\begin{aligned} (r-g)\,\hat{F}_0 &= -n + q(\theta)\left[\hat{F}_1 - \hat{F}_0\right] \\ (r-g)\,\hat{F}_1 &= \pi\gamma - w + s\left[\hat{F}_0 - \hat{F}_1\right]. \end{aligned}$$

The free-entry condition is

$$\frac{n}{q(\theta)} = \frac{\pi\gamma - w}{r-g+s}.$$

Value of an innovator

Similarly, for innovator:

$$\begin{aligned} (r-g)\,\hat{I}_0 &= -m + \theta q(\theta)\left[\hat{I}_1 - \hat{I}_0\right] \\ (r-g)\,\hat{I}_1 &= w + s\left[\hat{I}_0 - \hat{I}_1\right] \\ (r-g)\left[\hat{I}_1 - \hat{I}_0\right] &= w + m - \left(s + \theta q(\theta)\right)\left[\hat{I}_1 - \hat{I}_0\right]. \end{aligned}$$

Nash-Bargaining

$$(1-\alpha)\left[\hat{I}_1-\hat{I}_0\right]=\alpha\hat{F}_1$$
$$(1-\alpha)\left(r-g\right)\left[\hat{I}_1-\hat{I}_0\right]=\alpha\left(r-g\right)\hat{F}_1$$
$$(1-\alpha)\left(r-g\right)\left[w+s\left[\hat{I}_0-\hat{I}_1\right]-\left(r-g\right)\hat{I}_0\right]=\alpha\left(r-g\right)\left[\pi\gamma-w-s\hat{F}_1\right]$$
$$(1-\alpha)\left[w+s\left[\hat{I}_0-\hat{I}_1\right]-\left(r-g\right)\hat{I}_0\right]=\alpha\left[\pi\gamma-w-s\hat{F}_1\right]$$
$$w=\alpha\pi\gamma+(1-\alpha)\left(r-g\right)\hat{I}_0.$$

Furthermore

$$\begin{aligned}\left(r-g\right)\hat{I}_0 &= -m+\theta q(\theta)\left[\hat{I}_1-\hat{I}_0\right]=-m+\theta q(\theta)\frac{\alpha\hat{F}_1}{1-\alpha}\\ \left(r-g\right)\hat{I}_0 &= -m+\theta q(\theta)\left[\hat{I}_1-\hat{I}_0\right]=-m+\theta q(\theta)\frac{n}{q(\theta)}\frac{\alpha}{1-\alpha}\\ &= -m+n\theta\frac{\alpha}{1-\alpha}.\end{aligned}$$

Then

$$w=\alpha\pi\gamma+(1-\alpha)\left[-m+n\theta\frac{\alpha}{1-\alpha}\right].$$

Equilibrium

$$\begin{aligned}FF^{\theta} &: \frac{n}{q(\theta)}=\frac{(1-\alpha)\left(\pi\gamma+m\right)-\alpha n\theta}{r-g+s}\\ GG^{\theta} &: g=\frac{\gamma\pi}{\frac{s}{q(\theta)}+1}.\end{aligned}$$

In this case, too, increased slackness on innovation markets has a positive effect on growth.