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ENTRY MISTAKES

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ABSTRACT

Entry Mistakes*

Frequently, aspiring entrants have only limited information about their potential rivals' entry decisions. As a result, the outcome of the entry game may be that more firms enter than the market can sustain; or, at least, that unnecessary entry investments are made. We refer to these outcomes as 'entry mistakes'. We consider two models of non-coordinated entry. In these models, entry mistakes occur because of lags in observing rivals' entry decisions (grab-the-dollar entry) or because entry investments take time (war-of-attrition entry). The wide-body aircraft industry in the late 1960s is presented as supporting evidence for the models' assumptions. We also discuss the welfare implications of non-coordinated free entry. Both models predict that entry incentives are excessive (resp. insufficient) when duopoly profits are high (resp. low). If entry costs are high, however, entry incentives are excessive under war-of-attrition entry but insufficient under grab-the-dollar entry.

JEL Classification: L1, L6, L9
Keywords: entry, entry regulation, welfare, aircraft manufacturing

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NON-TECHNICAL SUMMARY

Frequently, aspiring entrants have only limited information about their potential rivals' entry decisions. As a result, the outcome of the entry game may be that more firms enter than the market can sustain; or, at least, that unnecessary entry investments are made. We refer to these outcomes as 'entry mistakes'.

An episode of an entry mistake is provided by the market for wide-body aircraft in the late 1960s. In April 1966, F Kolk from American Airlines sent the three major US aircraft manufacturers a proposal for a new jet plane of larger size than existing aircraft, but smaller than the recently announced Boeing 747. Although the initiative came from a particular airline, it was clear that AA's preferences were shared by several other major airlines. With Boeing busy with the development of the 747, this left McDonnell Douglas and Lockheed as the potential contenders to enter the market 'created' by AA's proposal.

Lockheed decided to enter the race in January 1967. Soon after, McDonnell Douglas also decided to enter. It is unclear how much McDonnell Douglas knew of Lockheed's decision to enter, or of how credible that decision was. What seems clear is that it was common knowledge that the market was not big enough for two different designs to survive.

Within a few weeks' difference, in September 1997, the two manufacturers presented a first proposal ('paper plane'). The airlines studied the proposals for a few months and concluded that there was not a clear winner.

The first bids were submitted in February 1968. McDonnell Douglas won the first order (from American Airlines). Lockheed then received a series of orders, thus getting way ahead of McDonnell Douglas. The last of the first set of large orders was from United Airlines, announced for 25 April 1968.

Many expected United to choose Lockheed and McDonnell to drop out of the market. As it happened, United chose McDonnell Douglas, thus cementing the market into a duopoly that would eventually lead both Lockheed and McDonnell Douglas to huge losses.

While the Lockheed-McDonnell Douglas duopoly was certainly the result of an entry mistake, one may wonder whether its source was lack of entry co-ordination or simply an incorrect forecast of market profitability (demand growth, intensity of competition). This paper argues that there was indeed a problem of entry coordination.

It begins by highlighting some of the crucial characteristics of the market which led to such an outcome, namely: i) observation lags are significant; ii) it takes time for firms to enter; iii) entry decisions are taken 'simultaneously'; and iv) entry costs are significant in size and mostly sunk in nature.

The paper then proposes two models that are consistent with these assumptions and with the possibility that entry mistakes (two firms entering in a market with room for only one) may result from equilibrium play by rational agents. In these models, entry mistakes occur because of lags in observing rivals' entry decisions (grab-the-dollar entry) or because entry investments take time (war-of-attrition entry).

Finally, we discuss the welfare implications of non-coordinated free entry. Both of the models considered predict that entry incentives are excessive (resp. insufficient) when duopoly profits are high (resp. low). If entry costs are high, however, then entry incentives are excessive under war-of-attrition entry, but insufficient under grab-the-dollar entry.

The first set of results (entry incentives as a function of the intensity of duopoly competition) seem to parallel the main results obtained for the case of sequential entry. In fact, the 'business stealing effect', first pointed out by Mankiw and Whinston (1986), seems to be a very robust effect. The second set of results (high entry costs) shows that simultaneous entry implies different results from sequential entry.

1 Introduction

Much of the Industrial Organization literature on entry assumes that firms' decisions are sequential, i.e., firm i decides whether to enter after observing firm $i-1$'s decision. Examples include models of firm location (Prescott and Visscher (1977)), capacity choice (Eaton and Ware (1987)), and technology choice (McLean and Riordan (1989)). But arbitrating an order of entry, as sequential-entry models implicitly do, amounts to assuming a "considerable feat of coordination" between potential entrants (Farrell (1987)). In most real-world examples, firms have, at most, limited information about their potential rivals' entry decisions. As a result, the outcome of the entry game may be that more firms enter than the market can sustain, an outcome that we refer to as an "entry mistake."[1]

An alternative framework that accounts for the possibility of entry mistakes is the assumption that firms make their entry decisions simultaneously and that entry is an instantaneous decision. It can be shown that the symmetric equilibrium of this alternative game involves firms playing mixed-strategies, entering with probability p, $0 < p < 1$. Since firms randomize independently, entry mistakes (excessive or insufficient entry) occur with positive probability.

Still another alternative is to assume that entry investments are made gradually: entry only occurs when all of the entry cost is paid; and firms can stop investing at any time during the process. It can be shown that the symmetric equilibrium of this alternative game implies that with positive probability two firms will enter; and that, with probability one, total entry investment is greater than what is necessary for one firm to enter. Again, entry mistakes occur (now with probability one).

If entry costs are low or if they are not sunk, then entry mistakes are not of great importance. In particular, Dixit and Shapiro (1986) show that there exists a dynamic equilibrium of the simultaneous, instantaneous entry game in which, by means of successive entry and exit, the number of active firms converges to n, the

[1]The type of entry mistake we describe here results from *lack of coordination* between potential entrants. There are obviously many other sources of entry mistakes, including incorrect forecast of the market demand or of production costs (cf Section 2). Moreover, our notion of entry mistake refers to the equilibrium level of entrants, not the social optimum number of entrants (cf Section 5).

equilibrium number of active firms in the sequential-entry game. If entry costs are significant and sunk, however, then entry mistakes are equally significant, both from the perspective of firm profitability and from the perspective of social welfare.[2]

This paper addresses several of these issues. In Section 2, we discuss an example which suggests that the paradigm of simultaneous entry is more reasonable than that of sequential entry.[3] Moreover, we argue that the example fits the assumptions of the theoretical models to be presented later, namely the assumptions that: (i) observation lags are significant, (ii) entry investments take time, (iii) entry decisions are simultaneous, and (iv) entry costs are large and sunk.

Section 3 introduces the first paradigm of simultaneous (or non-coordinated) entry, based on the "grab-the-dollar" game. We argue that discrete time is a good reduced form of continuous time with observation lags. Moreover, we show that the probability of an entry mistake is positive and increasing with the length of the observation lag.

Section 4 presents an alternative model of simultaneous entry, based on the war-of-attrition game. This game assumes that entry investments take time to complete. As under grab-the-dollar entry, there is a positive probability of entry by two firms. Moreover, even if only one firm invests all the way, the other will have incurred in an entry mistake insofar as it sunk a fraction of the entry cost without having entered.

Section 5 discusses the welfare implications of free entry when firms cannot coordinate their decisions. It shows that, depending on various parameters, free entry may induce either excessive or insufficient entry. Specifically, both models of simultaneous entry predict that entry incentives are excessive (resp. insufficient) when duopoly profits are high (resp. low). However, if entry costs are high, then entry incentives are excessive under war-of-attrition entry but insufficient under grab-the-

[2]Cabral (1993) generalizes Dixit and Shapiro's (1986) analysis, showing that, when sunk costs are significant, the number of active firms may converge to different values of n. More generally, if payoff functions have persistent effects ("experience advantages," of which sunk costs are a particular case) then entry mistakes have a permanent effect on market structure.

[3]In the paper, we use the term sequential entry to designate coordinated entry. Likewise, we use the term simultaneous entry to designate non-coordinated entry. With respect to the latter, note that, strictly speaking, it is entry decisions that are simultaneous, not entry itself.

dollar entry.

Section 6 concludes the paper.

2 Wide-body aircraft: The Lockheed-McDonnell Douglas story

One of the most fascinating events in the business history of aircraft manufacturing occurred during the second half of the sixties, the time when wide-body jet-liners first appeared.[4]

In 1965–66, Boeing and PanAm set up an agreement whereby PanAm would secure the first order of a new large aircraft to be developed by Boeing, the B-747. A few days after the companies signed a formal contract, F. Kolk from American Airlines sent Boeing and the other manufacturers a proposal for a new aircraft of larger size than the existing ones but smaller than the proposed 747. With Boeing busy with the development of the 747, this left McDonnell Douglas and Lockheed as the potential contenders to enter the market "created" by AAs proposal.[5]

Not long after Kolk's proposal, the U.S. government announced it was dropping its plans to support the development of the supersonic transport (SST) project.[6] Lockheed, which had invested heavily into the SST project, decided to transfer that effort, and the staff involved in it, to working on Kolk's proposal. The credibility of Lockheed's decision was strengthened by hiring one of Boeing's senior engineers (Lockheed experience in civil aeronautics was limited). Despite Lockheed's move and the perception that the market would likely not hold two firms, McDonnell Douglas decided, three months later, to enter the race with its own design.

The first seriously credible signal that the companies were committed to the market was received in September 1967, when specific design proposals were sent to the main airlines. The proposals appeared within a few weeks difference: McDonnell Douglas had closed the initial gap by developing the first draft of blueprints in six

[4]Table 1 presents a summary of the main chronological events surrounding this process.

[5]Although the initiative came from American Airlines, several other airlines agreed that the 747 was too big for their needs.

[6]In fact, funding was only discontinued in May 1971; cf Bluestone et al. (1981), p. 65.

Table 1: Wide-body aircraft. Chronology of main events surrounding the McDonnell Douglas and Lockheed entry process in the late sixties.

Year	Month	Event
1965	December	Boeing and PanAm agree on development of 747.
1966	April	Pan Am orders 25 B-747's (Pratt & Whitney engine).
		American Airlines proposes alternative wide-body specs.
	December	SST project canceled.
1967	January	Lockheed decides to enter:
		Allocates SST engineering staff to WB project;
		Hires Boeing's senior engineer R. Hopps.
	April (?)	McDonnell Douglas decides to enter.
	September	Lockheed ready with proposal.
		McDonnell Douglas follows a few weeks later.
	October - - January	Airlines study proposals (airframes and engines).
1968	February	15th: First bids submitted.
		19th: American Airlines orders 25 DC-10's.
	March	29th: TWA and Eastern order 94 L-1011's;
		Air Holdings, Ltd. orders 50 L-1011's.
	April	2nd: Delta orders 24 L-1011's.
		United proposes Lockheed to supply L-1011
		with GE engine; negative answer.
		25th: United orders 30 DC-10's (GE engines).
1975		L-1011 receives no orders.
1980		Orders for DC-10 drop dramatically.

months (as opposed to Lockheed's nine months). From then on, the game was a war of attrition as much as it had been a preemption game up until then.

Between October 1967 and January 1968, the main airlines studied the proposals. The aircraft designs were (unsurprisingly) similar in their main characteristics (length, width, wing span, capacity, range, speed, engine thrust, seat-mile costs). From an engineering point of view, Lockheed was generally preferred; but most airline executives had greater trust in McDonnell Douglas as a company: there was no clear winner. The process was further complicated by the general disagreement regarding the choice of engine manufacturers: some airlines preferred a GE engine, whereas others favored Rolls-Royce.

The first bids were submitted in mid-February 1968. The bids were reportedly within two hundred thousand dollars of each other. On February 19, American Airlines announced the first order—to McDonnell Douglas. Despite this first setback, Lockheed did dot give up; and, by means of some aggressive marketing, it managed to secure the next three large orders. The pressure was now on McDonnell Douglas to leave the market while it was time to do it. In fact, their contract with American Airlines gave McDonnell Douglas the option of canceling the order if within ninety days two other airlines hadn't ordered the DC-10.

Attention was now centered around United Airlines, the next airline expected to place an order. Although all previous orders had specified Rolls-Royce engines, United preferred the GE engine. A proposal was made to Lockheed for supplying the combination L-1011/GE. Lockheed's response was negative, perhaps because it sensed it had won the race and gained sufficient bargaining power, as no airline would want to place an order on McDonnell Douglas' nearly orphaned design. As it turned out, on 25 April 1968 United placed an order with McDonnell Douglas, thus cementing the market into a duopoly that would eventually lead both Lockheed and McDonnell Douglas to huge losses.[7]

■ **Discussion.** While the Lockheed-McDonnell Douglas duopoly was certainly the result of an entry mistake, one may wonder whether its source was lack of entry coordination or simply an incorrect forecast of market profitability (demand growth, intensity of competition). In fact, Newhouse (1988) claims that by the mid-sixties "nearly everyone was unreservedly optimistic, or professed to be" (p. 11). However, he later adds that "none of the suppliers felt that the market was large enough

[7] After a good start in terms of orders, the L-1011 suffered from production delays caused by the bankruptcy of Rolls Royce, its engine supplier. As a result, many potential buyers switched to the B-747 and DC-10. Lockheed received no orders for the L-1011 in 1975 and very few in the ensuing years. It finally discontinued production in 1981.

McDonnell Douglas eventually received almost twice as many orders for the DC-10 as Lockheed for the L-1011. However, adverse publicity surrounding crashes of DC-10 aircraft in Chicago, Mexico City and Antarctica led to a sharp decline in orders (6 in the first half of 1980 from 24 in the first half of 1979). Production was discontinued soon after. Cf Bluestone et al. (1981).

to sustain both airplanes" (p. 149).[8] An alternative explanation to a coordination mistake or a calculation mistake is that the firms held opposite priors with respect to their relative probability of success in the market. However, by January 1968, it was common knowledge to both airlines and manufacturers that the former's' preferences were split between the two designs, i.e., that there was no clear winner.

In the next two sections, we present theoretical models that are consistent with the possibility of entry mistakes resulting from equilibrium play by rational agents. In order to justify some of the assumptions on which those models are based, we now summarize some of the relevant characteristics of the aircraft example.

First, observation lags seem to be important: by the time Lockheed presented the result of its initial efforts (the first credible signal of its commitment to entry), McDonnell Douglas had already been working on the project for several months.

Second, the example shows that it takes time for firms to enter: Lockheed spent nine months, and McDonnell Douglas six months, before it produced a first proposal to show the airlines; and this was just the first step: developing the first set of plans, building a prototype, and setting up production plants may take as long as four years.

Third, entry decisions seem to be taken "simultaneous". In fact, McDonnell Douglas decided to enter about three months after Lockheed. However, on a scale of years (the total length of the entry process), three months do not amount to much.

Finally, entry costs are significant in size and mostly sunk in nature. It is difficult to find good estimates for the cost of developing a new aircraft, but the order of magnitude is billions of dollars (compare with the price of an individual aircraft, something in the order of 15 million dollars). Sunkness of the development costs results from the specificity of most of the design and prototype work.[9]

[8]Bluestone et al. (1981) claim that "by 1979 there were only 160 L-1011s in service worldwide compared to 276 DC-10s. Given the costs of development, the break-even point for these aircraft is said to be in the vicinity of 300 planes" (p. 9).

[9]As it happened, Lockheed applied some of the technology developed in the SST project to the design of the L-1011: not all design costs are completely sunk.

3 Entry with an observation lag

Consider a new market for a durable good. From time T on, a measure one of consumers are each willing to pay μ for one unit of the good. There are two potential entrants. Entry is instantaneous and implies a sunk cost σ. Fixed costs are zero and marginal cost constant (zero, with no further loss of generality).

A distinctive feature of the model is that entry decisions are not immediately observed. Specifically, a firm that enters at time t is only observed by its rival and by consumers with a lag $\lambda > 0$. In other words, at time t firm i (and consumers) can only observe firm j's actions up to time $t - \lambda$.

Once in the market (i.e., once entry is observed), a firm sets its price and consumers decide whether or not to accept it. If the firm enters before $T - \lambda$, then the price is set at time T.

In what follows, we will concentrate our analysis on the firms' entry decisions before time $T - \lambda$. During this phase, the payoff structure of the entry game can be summarized as follows. If the two firms enter, then they receive a payoff of $-\sigma$ (at time of entry); in fact, they will be competing a la Bertrand at time T and receive zero revenues. Otherwise, a firm that enters at time t receives a payoff (discounted to the time of entry) of $\pi(t) \equiv e^{-(T-t)}\mu - \sigma$ (monopoly revenue μ, starting at time T, minus cost σ, paid at time t). We normalize time so that $e^{-T}\mu - \sigma = 0$; that is, at time zero a firm is indifferent between entering as a monopolist or not entering.[10]

[10]There is an alternative set of assumptions which is payoff equivalent to the one considered in the text: Suppose that consumers arrive in the market gradually: a fraction $\Phi(t)$ by time t. As before, each consumer is willing to buy one unit for μ. Once a firm's entry has been observed, the firm sets a price and consumers decide whether or not to accept it.

The crucial assumption under this alternative is that consumers do not discount the future. As a result, consumers do not accept any price above marginal cost before a period λ elapses after observing entry by the first firm; i.e., before time $t + 2\lambda$, assuming the first firm enters at time t. This is so because, in the likelihood that a second firm enters in the period $[t, t + \lambda]$, price will equal marginal cost from $t + \lambda$ (or before) onwards. If, by time $t + 2\lambda$, no second entry is observed, then the initial entrant sets $p = \mu$ and its offer is accepted by all consumers, present and future. In this alternative setting, payoff in case of single entry, discounted to the time of entry, is given by

$$\pi(t) = \mu \left(e^{-2\lambda} \int_{-\infty}^{t+2\lambda} \phi(\tau) d\tau + \int_{t+2\lambda}^{\infty} e^{t-\tau} \phi(\tau) d\tau \right) - \sigma(t),$$

where $\phi(t) = \frac{d\Phi(t)}{dt}$.

Clearly, in a subgame perfect equilibrium, a firm which observes at time t that its rival has entered before $t - \lambda$ will not enter. Therefore, entry by two firms only occurs if entry times differ by less than λ. Moreover, the entry strategy can be summarized by a cdf $F(t)$, the probability of entering by time t given that it has not been observed by then that the rival has entered. Finally, notice that in a symmetric equilibrium a firm must be indifferent between entering and not entering.[11]

Intuitively, the existence of observation lags implies the possibility that two firms enter the market despite the fact there is only room for one. Although firms want to avoid a duopoly, they cannot avoid the possibility that one enter at time t and the other at time $t' < t + \lambda$, in ignorance of the entry decision by the first firm, thus leading to an "entry mistake." In fact, the main point of this section is that, in the limit where the observation lag is very small, the probability of an entry mistake of this kind is also very small; and that, if the observation lag is positive, then the probability of an entry mistake is also positive. Moreover, this probability is increasing in the length of the observation lag.

The way we present this point is the following: First, we argue that a discrete-time model is a good approximation to the continuous-time model presented above, where the length of the observation lag is proportional to the width of the discrete-time grid. Second, we show that the probability of an entry mistake in the discrete-time model is increasing in the width of the grid.

3.1 Discrete time as an approximation.

The model of entry with an observation lag is quite difficult to analyze. In this section, we show that a similar model in discrete time provides a good approximation to the continuous-time model. The advantage of the discrete-time model is that it is much easier to solve and analyze. In particular, it is possible to compute the probability that an entry mistake occurs.[12]

[11]If not, then both firms would want to enter at some time t; but if the two firms enter, then they receive negative profits.

[12]The results of this subsection may be of more general interest in that they confirm the intuition that simultaneous moves in discrete time are a good reduced form of a continuous-time model when there are observation lags. See, for example, Fudenberg et al. (1983, Section 4).

The discrete-time model is the following. Players must decide whether or not to enter at each moment $k\varepsilon$, where $k \in \mathbb{Z}$ and $\varepsilon > 0$. If a firm enters alone, then it receives $\pi(t) - \sigma$. If two firms enter together, then both lose σ.[13]

Let $F^i(t)$, $i = C, D$, be the cumulative probability of entry up to time t in the equilibrium of the continuous- and discrete-time games, respectively.[14] The following result suggests that the discrete-time model is indeed a good reduced form of the continuous-time model.

Proposition 1 *If* $\varepsilon = 2\lambda$, *then, for* $t < T - \lambda$, $F^D(k\varepsilon + \lambda) = F^C(2k\lambda)$, *for all integer* k.

Figure 1 depicts the cumulative distributions in the continuous-time and discrete-time games for the particular case when $\pi(t) = 2 - \exp(-t^2/8)$, $\sigma = 1$, $\varepsilon = 2$ and $\lambda = 1$. The figure illustrates Proposition 1: beginning at $t = \lambda$, the equilibrium cumulative distributions of the continuous- and discrete-time models coincide every ε.

The intuition for the result is the following. Assume that $F^D(0) = F^C(\lambda) = F^C(1)$, as is the case in the figure. Consider firm i's decision of entering at time $t = \varepsilon = 2\lambda = 2$. First, notice that the payoff from entering the market alone is the same in both models (because the profit function is the same). Second, the probability that firm j will enter simultaneously, conditional on no entry having occurred before,

[13] This game generalizes the game of grab-the-dollar, first proposed by by Gilbert and Stiglitz (1979) as a stylized model of non-coordinated entry into a new market. Suppose there are two potential entrants into a natural monopoly (i.e., a market where discounted duopoly profits, net of entry costs, are negative). The value of the new market, in case of monopoly, is 2. If two firms enter, then profits are zero. Assuming a sunk entry cost of 1, then an entrant gets a net payoff of 1, if it is the only one to "grab the dollar," or -1, if two firms enter simultaneously. Entry can occur in any period $t = 1, 2, \ldots$.

It can be shown that the unique symmetric equilibrium of this game consists of firms entering with probability 1/2 in each period if no firm has entered in the past (and entering with probability zero if the rival firm entered in the past). In this equilibrium, the probability that each firm becomes a monopolist is 1/3. The probability that the a duopoly emerges (an entry mistake) is also 1/3.

In the remainder of the paper, we refer to the model of this section as grab-the-dollar entry, as opposed to the model in the next section, which we refer to as war-of-attrition entry.

[14] As shown in the proof of Proposition 1, there exists a continuum of equilibria in the continuous-time game. The one we select is the unique equilibrium such that $F(t)$ is continuous and firms are indifferent between entering and not entering for all positive t. This equilibrium is the only robust equilibrium in the sense described in the proof.

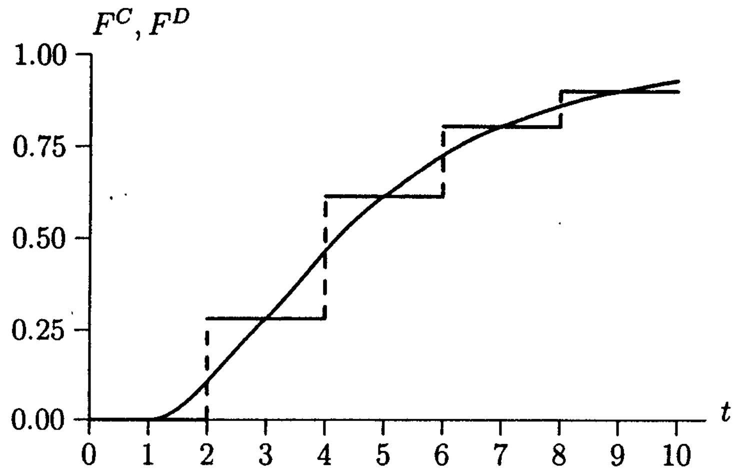

Figure 1: Cumulative probability of entry in discrete- and continuous-time games. $\pi(t) = 2 - \exp(-t^2/8)$, $\sigma = 1$, $\varepsilon = 2$, $\lambda = 1$.

is given by $\left(F^D(t) - F^D(t-\epsilon)\right) / \left(1 - F^D(t-\epsilon)\right) = \left(F^D(2) - F^D(0)\right) / \left(1 - F^D(0)\right)$ in the discrete-time model; in the continuous-time model, this probability is given by the probability that firm j will have entered between $t - \lambda$ and t (an event that firm i cannot observe at time t) plus the probability that firm j will enter between t and $t + \lambda$ (not having observed firm i's decision). In total, this probability is given by $\left(F^C(t+\lambda) - F^C(t-\lambda)\right) / \left(1 - F^C(t-\lambda)\right) = \left(F^C(3) - F^C(1)\right) / \left(1 - F^C(1)\right)$. But then $F^D(0) = F^D(t - \varepsilon) = F^C(t - \lambda) = F^C(1)$ implies that $F^D(2) = F^D(t) = F^C(t + \lambda) = F^C(3)$, and the complete result follows by induction. In other words, if firm j enters within λ time of firm i it is *as if* it had entered simultaneously. In equilibrium, this probability must be the same as the probability of firm j's entry at time t in the discrete-time game.

In summary, the discrete-time game is a good approximation of the continuous time game when the width of the discrete grid equals twice the observation lag. This corresponds to the possibility of firm j entering without firm i observing it, or, alternatively, of firm i entering without firm j observing it.

3.2 Observation lags and entry mistakes.

The question we are now interested in addressing is, how do entry mistakes depend on the extent of observation lags. Denote by P the probability of an entry mistake

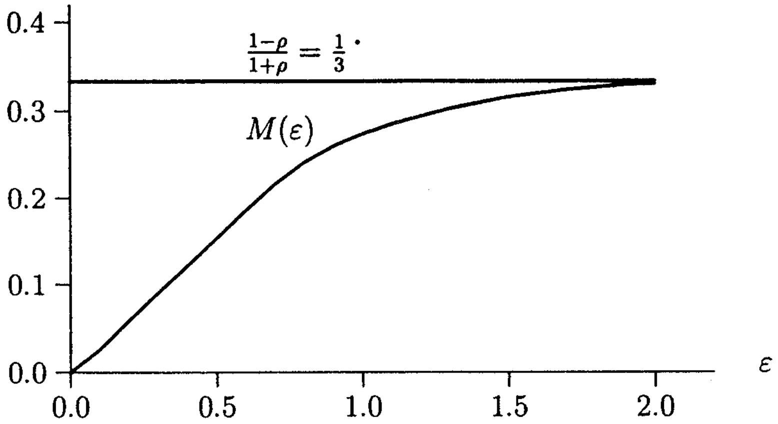

Figure 2: Probability of an entry mistake as a function of ε ("observation lag"). $\pi(t) = 2 - \exp(-t^2)$; $\sigma = 1$.

in the play of the symmetric mixed-strategy equilibrium of the game presented in the previous section (the generalized grab-the-dollar game). The following result shows that entry mistakes are more likely when observation lags are longer.[15]

Proposition 2 *(a)* $P(\varepsilon) < P(2\varepsilon)$

(b) $\lim_{\varepsilon \to 0} P(\varepsilon) = 0$

(c) $\lim_{\varepsilon \to \infty} P(\varepsilon) = \dfrac{\bar{\pi} - \sigma}{\bar{\pi} + \sigma}$.

Figure 2 illustrates the main ideas of Proposition 2 for the specific case $\pi(t) = 2 - \exp(-t^2)$, $\sigma = 1$. The probability of an entry mistake is increasing in the value of ε, ranging from zero ($\varepsilon = 0$) to $\frac{\bar{\pi}-\sigma}{\bar{\pi}+\sigma}$ ($\varepsilon = \infty$). For the particular values considered in this example, the maximum value of $P(\varepsilon)$ is $1/3$. However, other functions might generate different limit values. In fact, if $\pi(t) = \alpha - \exp(-t^2)$, then the maximum value of $P(\varepsilon)$ ranges from zero ($\alpha = 1$) to one ($\alpha \to \infty$). In words, the probability of an entry mistake can, in principle, take any value between zero and one.[16]

The same is not true, however, of the expected *value* of entry mistakes. Denote by V the expected value of entry mistakes as a fraction of maximum monopoly profits, i.e., V is equal to P, the probability that a second firm will enter, times

[15] Sutton (forthcoming, Proposition 11.3) presents a result similar to part (b) of Proposition 2.

[16] Holden and Riis (1994) show that the probability of an entry mistake is bounded away from zero even if ε is made arbitrarily small. However, the payoff structure they consider is different from the one in this paper.

σ, the cost of that second entry, divided by $\bar{\pi}$, $V = \frac{\sigma}{\bar{\pi}} P$. From Proposition 2, we conclude that V is, at most, $\frac{\bar{\pi}-\sigma}{\bar{\pi}+\sigma} \cdot \frac{\sigma}{\bar{\pi}}$. This is maximum when $\frac{\sigma}{\bar{\pi}} = \sqrt{2} - 1$, yielding expected costs that are approximately 17% of gross monopoly profits ($\bar{\pi}$), or 29% of net monopoly profits ($\bar{\pi} - \sigma$).

■ **Summary.** The main point of this section is that, when entry is instantaneous but there are observation lags, then there is a positive probability of an entry mistake. The example of wide-body aircraft suggests that observation lags may indeed be important. For example, from the time Lockheed first decided to enter the market until the moment when the L1011's first blueprints were disclosed, more than nine months elapsed.

If entry is instantaneous and observation lags are very small, then the probability of an entry mistake is also very small. In the next section we show that, when observation lags are negligible but entry is not instantaneous (it takes time to enter), then the probability of an entry mistake is, again, positive.

4 Entry as a war of attrition

One of the characteristics suggested by the aircraft example is that entry takes time: developing the first set of plans, building a prototype, and setting up production plants may take as long as four years. During this period, firms will continuously invest at a positive, though possibly variable, rate. The model presented in the previous section assumes that entry is an instantaneous process. We now consider an alternative model where entry takes time.

Assume, as in the previous section, that the payoff from being a monopolist is given by $\pi(t)$, t being the time of entry, whereas the entry cost is σ. We now assume, for simplicity, that there are no observation lags. The main difference between grab-the-dollar entry and war-of-attrition entry is that, in the former, entry investments are instantaneous, whereas in the latter it takes time for the entry investment σ to take place. Specifically, assume that entry investments are made at a constant rate of 1 per unit of time, so that it takes σ time to complete the necessary initial investment. Assume that firms start investing at time t_0, and, with no further loss of generality, make $t_0 = 0$. Denote by $\pi = \pi(\sigma)$ monopoly profits at the time of completion of the entry investment. We assume that $\sigma < \pi$.[17]

There are two important assumptions in this model. First, the investment rate is assumed constant. A more reasonable alternative is to make total cost a convex function of the investment rate. However, considering industries such as aircraft manufacturing, it seems reasonable to assume a fixed maximum rate of investment. For example, it is likely that, in the short run, the number of engineers working on the development of a new aircraft cannot be increased.

A potentially more problematic assumption is that both firms start investment at a given date t_0. One possible defense of this assumption is that the payoff from entering increases discountinuously at time t_0 from a prohibitively low value to a reasonable one.[18] If we make the equilibrium assumption that a firm does not start investing at all if it is starting late with respect to its rival, then the unique symmetric equilibrium calls for firms to start investing at date t_0.

If the payoff functions are continuous in t, then the above argument would imply that firms start investing at time time t_0 such that $\pi(t_0 + \sigma) = \sigma$. However, this is by no means the only possible equilibrium. An alternative subgame equilibrium hypothesis is that, if firm i starts investment before time t_0, then firm j follows immediately after, whereas, if firm i starts investment after time t_0, then the previous equilibrium assumption holds.[19] Such strategies would imply that it is a symmetric

[17] A similar application of the war of attrition to Economics is Fudenberg and Tirole's (1986) theory of exit. In fact, our model can be interpreted as one in which firms exit from an entry race. Our assumption that $\sigma < \pi$ is similar to Fudenberg-Tirole's assumption that eventually the market can sustain two firms. Note however that, differently from us, Fudenberg and Tirole assume incomplete information. Other related papers on the war of attrition are Ghemawat and Nalebuff (1985), Krishna and Morgan (1997), Bulow and Klemperer (1987).

[18] American Airlines announcement of the desired specifications for a new aircraft may be an instance of this. No manufacturer would be willing start developing a new aircraft before having an idea of what the market demanded.

[19] In order to model that "firm j follows immediately after," we need to introduce observation lags again, although now the length of such lag (λ), may be arbitrarily small. The subgame we have in mind is one in which, if firm i starts investing at time $t < t_0$, then firm j starts investing at time $t + \lambda$, the moment it observes firm i started investing. Firm j invests all the way to $t + \lambda + \sigma$, whereas firm i drops out before completion of the entry investment. Notice that this is indeed a subgame equilibrium. If firm i decides to carry on the entry investment all the way, firm j will

equilibrium for both firms to enter at time t_0, where t_0 can be arbitrarily chosen. Likewise, we concentrate our attention on the symmetric equilibrium where firms start investing at time t_0, taking the starting time as given.

We begin by deriving the symmetric equilibrium strategies. Let $F(t)$ be the probability that a given firm will have dropped out of the entry race by time t given that the rival has not dropped out. Make the equilibrium hypothesis that firms are indifferent between any exit time.Since dropping out at time zero yields a payoff of zero, the indifference condition becomes:

$$F(t)(\pi - \sigma) + \big(1 - F(t)\big)(-t) = 0.$$

The left-hand side is the expected value for a firm that decides to invest until time t even if its rival does not drop out of the market. With probability $F(t)$, the rival will indeed drop out before time t, in which case the firm receives the net payoff $\pi - \sigma$. With probability $\big(1 - F(t)\big)$, the rival will not drop out before time t, in which case the firm does drop out, receiving a payoff of $-t$, the entry cost already sunk by then.

Solving for $F(t)$, we get

$$F(t) = \frac{t}{\pi - \sigma + t}. \tag{1}$$

Notice that $F(\sigma) = \frac{\sigma}{\pi} < 1$, that is, in equilibrium each firm will invest all the way up to σ even if its rival does not drop out of the entry race. Just like in grab-the-dollar entry, entry mistakes (too many entrants) occur with positive probability.

Moreover, even if only one firm eventually survives the entry race, the "losing" firm will have incurred, with probability one, strictly positive entry costs. In other words, the expected value of entry mistakes includes the probability of two firms entering (as in the grab-the-dollar entry) plus the expected value of entry investments by the firm that drops out.

Specifically, define the relative expected value of entry mistakes, V as the expected value of entry costs in excess of σ (the minimum entry cost for the entry

only be able to observe its completion at time $t + \sigma + \lambda$, which is the time at which firm j will have completed its own entry investment.

of one firm), divided by π. In the case of grab-the-dollar entry, as we saw in the previous section, this is simply σ times the probability that a duopoly emerges divided by π. In the case of war-of-attrition entry, as we have just argued, one must also include the costs incurred by firms that drop out of the entry race before they complete the necessary investment for entry.

Proposition 3 *Under war-of-attrition entry,*

(a) The expected value of entry mistakes as a percentage of monopoly profits is given by

$$V = \frac{\sigma}{\pi}\left(1 - \frac{\sigma}{\pi}\right).$$

(b) The probability of a duopoly is given by

$$P = \left(1 - \frac{\sigma}{\pi}\right)^2.$$

In words, the probability of a duopoly is decreasing in the ratio between entry costs and monopoly profits. The expected value of entry mistakes, in turn, is increasing in that ratio for low values and decreasing for high values. The maximum value of entry mistakes (25% of monopoly profits, or 50% of net monopoly profits) is obtained for $\frac{\sigma}{\pi} = \frac{1}{2}$.[20]

5 Free entry and welfare

What are the social welfare implications of free entry when there can be entry mistakes? Previous papers (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987) compared the equilibrium outcome with the second-best optimum. The second-best optimum assumes the central planner can choose the number of entrants but takes as given the competition between active firms (or lack of thereof). Mankiw and

[20]This result has an alternative interpretation in terms of entry times. We assumed entry starting at time t_0. How would the expected value of entry mistakes change if entry were to start at a slightly later date, say, $t_0' > t_0$. Since $\pi(t)$ is increasing, the value of π (monopoly profits at the time firms finish entry investments) is now greater, and $\frac{\sigma}{\pi}$ lower. Therefore, delaying the start of the entry race increases the expected value of entry mistakes if and only if $\frac{\sigma}{\pi} > 1/2$; and increases the probability of a duopoly for any value of $\frac{\sigma}{\pi}$.

Whinston (1986) and Suzumura and Kiyono (1987) considered the case when there is an infinite number of potential entrants, oligopoly profits are given by $\pi(n)$ (where n is the number of active firms), and entry is *sequential*.

For the purpose of comparing the cases of sequential and simultaneous entry, we consider, as it were, the "common denominator" between the cases considered in the sequential- and simultaneous-entry models. We suppose there are two potential entrants, monopoly profits are π, duopoly profits are γ ($0 \leq \gamma \leq \pi/2$), and the cost of entry is σ ($0 \leq \sigma \leq \pi$). Moreover, let gross social surplus (gross of entry costs) be $\mu\pi$ under monopoly and $\nu\pi$ under duopoly. We require $\mu = \nu$ when $\gamma = \pi/2$ (perfect collusion with homogeneous product) and $\nu > \mu > 1$ when $\gamma < \pi/2$.

■ **Sequential entry.** For comparison, we begin by presenting a result for the sequential-entry, or coordinated-entry, model. In this case, the equilibrium number of firms is one if $\gamma < \sigma < \pi$, and two if $\gamma > \sigma$. The social optimum number of firms is two if $\nu\pi - 2\sigma > \mu\pi - \sigma$, one otherwise. Comparing the equilibrium and the social optimum number of firms, we get:

Proposition 4 *Under sequential entry,*

(a) For $\sigma < \pi/2$, there is excess entry if γ is large enough.

(b) For a given small σ, there is insufficient entry if γ is small enough.

This result parallels some of the results in Mankiw and Whinston (1986) (see also Suzumura and Kiyono, 1987). They consider the more general case of N potential entrants and show that, if the firms' margins are positive and if entry does not make the market much more competitive, then incentives to enter are excessive. The intuition for this result is that much of the benefits a firm receives from entering an industry correspond to a transfer of profit from the incumbent firms. This private benefit does not correspond to a social benefit, thus leading to an externality in the direction of excessive entry incentives. Mankiw and Whinston (1986) refer to this as the *business stealing* effect. Their result corresponds to part (a) of Proposition 4.

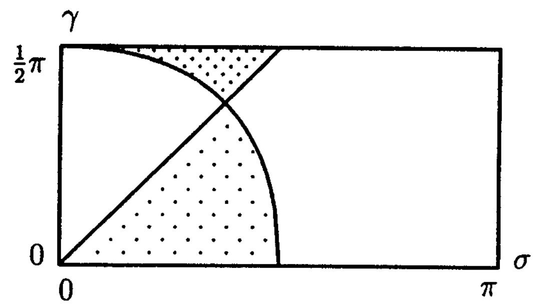

Figure 3: Coordinated entry with linear demand. Excessive entry in dark-shaded area. Insufficient entry in light-shaded area. Second best in non-shaded area.

Part (b) states that the business stealing effect is not the end of the story. If the entry of a second firm makes competition much more intense, then an opposite externality occurs: even though a second firm would increase social welfare, competition is so intense that the private benefit from entry does not cover the entry cost.

Figure 3 depicts the regions of excessive and insufficient entry in the (σ, γ) space for the case of a linear oligopoly (linear demand and linear cost functions). Parts (a) and (b) of Proposition 4 can be readily checked. It is also interesting to see that, for extreme values of the entry cost, the equilibrium solution yields the social welfare second best.

■ **Grab-the-dollar entry.** Let us now consider the case of simultaneous entry, beginning with grab-the-dollar entry. The model we consider here is slightly different from the model presented in Section 3. It is less general in that we assume π to be constant. On the other hand, it allows for the possibility of duopoly profits being positive.

The equilibrium probability of entry is given by the indifference condition $p\gamma + (1-p)\pi = \sigma$, whence $p = (\pi - \sigma)/(\pi - \gamma)$. We assume the central planner can control the value of p (by means of entry taxes and subsidies) but not the outcome of monopoly of duopoly competition. We thus say there is excessive entry if and only if the derivative of social welfare with respect to p is negative. Computing this derivative yields the following result:

Proposition 5 *Under grab-the-dollar entry,* *(a) For a given small σ, there is excess entry if γ is large enough.*

(b) For a given small σ, there is insufficient entry if γ is small enough.

(c) There is insufficient entry if σ is large enough.

(d) For a given large σ, there is excess entry if δ is large enough.

Proposition 5 parallels some of the results in Proposition 4. As before, part (a) (resp. part (b)) states that, if duopoly profits are very high (resp. very low), then there are excessive (resp. insufficient) incentives to enter.

However, there are also important differences. Parts (c) and (d) introduce a set of considerations that are absent from the sequential-entry model. Part (c) states that if entry costs are large, then entry incentives are too small. The intuition for this result is that, from society's viewpoint, the probability of an entry mistake, p^2, is very small: entry mistakes are a second-order effect. In fact, since σ is close to π, p is close to zero. However, for a given firm, *conditional on entering*, the probability of an entry mistake is p, a first-order effect. For this reason, the risk of an entry mistake is disproportionally large from a private perspective, implying a *delay effect*, in the terminology of Bolton and Farrell (1986): firms are too cautious (too slow) in their entry decision, from the perspective of social welfare.

However, if the discount factor is very close to one (part (d)), then time is not a very important factor from a social welfare point of view. As a result, the delay effect is of little importance. The main relevant effect, from a social welfare perspective, is the possibility of an entry mistake — the *duplication effect*, in the terminology of Bolton and Farrell (1986). Firms are too eager to enter, from a social welfare perspective.[21]

[21] Proposition 5 is related to some of the results in Bolton and Farrell (1986). The latter compare the centralized and the decentralized solutions to an entry problem similar to the one we consider. (The entry model in Bolton and Farrell (1986) explicitly considers the possibility of payoff differences between the two potential entrants.) In addition to characterizing the delay and the duplication effects, Bolton and Farrell (1986) present conditions under which one of the solutions is superior to the other one. Our analysis is complementary to theirs. Implicitly, we assume that the government does not have the option of centralizing the entry decision, but can influence the entry rate (by means of entry subsidies or taxes). In this case, the relevant question is whether entry incentives under free entry are excessive or insufficient.

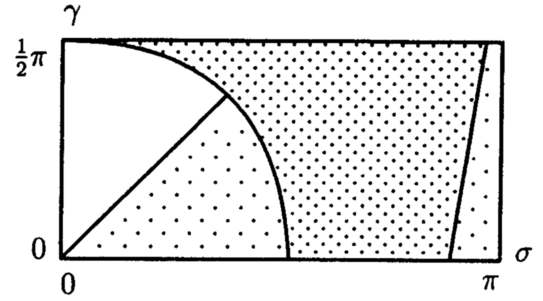

Figure 4: Grab-the-dollar entry with linear demand (and $\delta = .99$). Excessive entry in dark-shaded area. Insufficient entry in light-shaded area. Second best in non-shaded area.

Figure 4 depicts the regions of excessive and insufficient entry in the (σ, γ) space in the case of a linear oligopoly (linear demand and linear cost functions). Parts (a) to (c) of Proposition 5 can be readily checked.

Figure 4 also shows that the comparative statics with respect to the value of σ are "non-monotonic." Specifically, there exist values of γ such that, by increasing σ, we move from a region of insufficient entry to one of excessive entry and then back to one of insufficient entry. In fact, for high values of δ, this is true for any low value of γ. The idea is that, as the entry cost increases, both the private and the social incentives for entry decrease. However, these marginal benefits do not vary in the same proportion. In fact, they do not vary linearly: as the value of σ reaches very high values and the probability of entry becomes very small, the social marginal benefit of additional entry becomes very large, thus reversing the effect of excessive entry for intermediate values of σ.

■ **War-of-attrition entry.** Consider now the case of war-of-attrition entry. The model we consider here is slightly more general than the one presented in Section 4 in that we allow for positive duopoly profits. For $t < \sigma - \gamma$, the equilibrium cdf $F(t)$ is the same as when duopoly profits are zero (Equation 1). However, if a firm invests up to $\sigma - \gamma$, that firm will invest to the end will probability one. In fact, at $t = \sigma - \gamma$, the remaining entry cost is equal to duopoly profits.

The question of whether entry incentives are excessive or insufficient is addressed

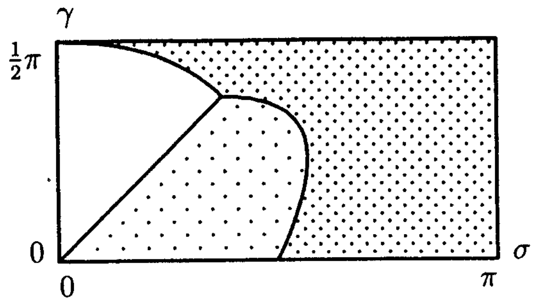

Figure 5: War-of-attrition entry with linear demand. Excessive entry in dark-shaded area. Insufficient entry in light-shaded area. Second best in non-shaded area.

in the following way. We assume that the cost of entry is given by $\tau\sigma$, where τ is a policy variable controlled by the government: $\tau > 1$ corresponds to an entry tax, whereas $\tau < 1$ represents an entry subsidy. We then say that free entry is excessive if and only if the derivative of social welfare with respect to τ, at $\tau = 1$, is positive. Computing such derivative we get the following result:

Proposition 6 *Under war-of-attrition entry,*

(a) There is excess entry if γ is large enough.

(b) For a given small σ, there is insufficient entry if γ is small enough.

(c) There is excess entry if σ is large enough.

(d) If γ is small and σ is such that equilibrium entry is second best, then a small increase in γ leads to insufficient entry.

Parts (a) and (b) are similar to those in Propositions 4 and 5. In fact, the intuition implicit in Mankiw and Whinston's (1986) "business stealing" effect seems to be quite robust. Parts (c) and (d) highlight the main differences between war-of-attrition entry and grab-the-dollar entry. First, if entry costs are very high, then entry incentives are excessive. In fact, since one firm is certain to enter, there is no great social benefit in encouraging entry when the potential cost of an entry mistake is so high. This contrasts with the case of grab-the-dollar entry, where there is a positive probability that no firm will enter.

Second, the relation between private and social incentives may be non-monotonic with respect to γ (whereas, under grab-the-dollar entry, non-monotonicity occurred with respect to σ). Specifically, there exist values of σ such that entry incentives are excessive for γ low, insufficient for intermediate γ, and again excessive for high γ. The intuition for this result is the following. At $\gamma = 0$, the derivative of social surplus with respective to γ is zero: a little profits does not reduce social surplus very much. However, even though an increase in γ has a second-order effect on social surplus, it has a first order effect on the firms' incentives to enter. In particular, many instances of firms dropping out of the race just before completing the investment σ are now changed to completion. This in turn has a first-order effect on social welfare, for the difference in terms of social surplus between monopoly and duopoly is strictly positive.

6 Concluding remarks

The main point of this paper is that coordination entry mistakes may happen. In many industries, the model of sequential entry is not an appropriate description of reality, as it implicitly assumes too much coordination between potential entrants. Having said that, one must also admit that the models of simultaneous entry we have considered assume too little coordination, in fact, no coordination at all. Most likely, reality is somewhere between the extremes of sequential entry and the models of simultaneous entry considered in this paper.

Specifically, the models of sequential entry and grab-the-dollar entry can be thought of as extremes of a more general model in which there is some, but not total, coordination between potential entrants. Consider the following three-stage incomplete information model. In the first stage, Nature generates each firm's payoff from being a monopolist, $\pi_i = 2 + \alpha\eta_i$, $i = 1, 2$. In the second stage, Nature decides whether η_i is firm i's private information (probability $1 - \theta$) or whether it is common knowledge to the two players (probability θ). Finally, in the third stage players simultaneous choose whether or not to enter. Assume that the two η_i are independently drawn from the smooth cdf $F(\cdot)$, symmetric about $F(0) = \frac{1}{2}$.

Information leaks between the two potential entrants may serve as a coordination device. In fact, a natural equilibrium of this game is the following: If both η_i are common knowledge, then the player with a higher η_i enters.[22] If η_i only is common knowledge, then player i enters if and only if $\eta_i \geq 0$, and player j enters if and only if $\eta_i < 0$. Finally, if both η_i are private information, then let each player i enter if and only if $\eta_i \geq 0$. One can easily show that these strategies constitute a Bayesian equilibrium of the incomplete information game. Moreover, the limit of this equilibrium as $\alpha \to 0$, that is, as the game approaches one of complete information, corresponds to a *correlated* equilibrium of the complete information model (cf Milgrom and Weber (1985)).

This is a very simplistic and in some ways unrealistic model. For example, the probability of an information leak is exogenously given, whereas we would expect firms to invest more heavily in information gathering when the value of the market and/or the cost of entry are greater. However, it serves to show that the models of sequential and simultaneous entry can be thought of as two extreme cases: sequential entry corresponds to the case when $\theta = 1$, whereas simultaneous entry corresponds to $\theta = 0$ (in which case the incomplete information game corresponds to Harsanyi's (1973) purification argument). Which model is more realistic is a matter for empirical investigation. Most likely, reality is somewhere in between the extremes of the two abovementioned models, i.e., $0 < \theta < 1$.

Likewise, the models of grab-the-dollar entry and war-of-attrition entry are somewhat extreme in the way they treat entry investments. Examples like that of wide-body aircraft suggest that reality is a combination of both instantaneous decisions and time-lasting ones.

[22]Ties can be resolved in any way as the probability of their occurrence is nil.

Appendix

Proof of Proposition 1: Consider first the continuous-time model. Make the equilibrium hypothesis that the equilibrium is symmetric and that the cumulative distribution characterizing each firm's strategy, $F^C(t)$, is continuous. Make the additional equilibrium hypothesis that each firm is indifferent between entering and not entering, at all times $0 \leq t < T - \lambda$. [23]

We then have

$$\sigma = \pi(t)\frac{1 - F^C(t+\lambda)}{1 - F^C(t-\lambda)} \tag{2}$$

or

$$F^C(t+\lambda) = \frac{\pi(t) - \sigma + F^C(t-\lambda)}{\pi(t)}, \tag{3}$$

for $0 \leq t < T - \lambda$. On the right-hand-side of (2), the second term represents the probability that the rival firm will enter during the period $[t-\lambda, t+\lambda]$ conditional on not having entered by $t - \lambda$. Equation (3) yields a unique solution for $F^C(t)$. In fact, $F^C(t) = 0$ for $t < 0$ generates the values of $F^C(t)$ for $t \in [\lambda, 2\lambda]$. In particular, $F^C(\lambda) = 1 - \sigma/\pi(0) = 0$. Hence, $F^C(t) = 0$ for $t \in [0, \lambda]$. Given this, all values of $F^C(t)$ can be obtained.

An alternative equilibrium hypothesis would be that each firm is indifferent between entering and not entering only after t_0, $0 < t_0 \leq \lambda$, and strictly prefers not to enter before t_0. ($t_0 > \lambda$ cannot be an equilibrium, since expected payoff from entering after $t = 0$ and before $t_0 - \lambda$ would be positive.) By the same reasoning as above, the indifference condition for $t_0 \leq t \leq \lambda + t_0$ implies equilibrium values of $F^C(t)$ for $\lambda + t_0 < t < 2\lambda + t_0$, and recursively, the values of $F^C(t)$ for $(2k+1)\lambda + t_0 < t < 2(k+1)\lambda + t_0$ can be derived. However, the equilibrium conditions have no implications regarding the value of $F^C(t)$ for $t_0 < t < t_0 + \lambda$ other than $F^C(t) \leq F^C(t_0 + \lambda)$. In particular, the solution derived for $t_0 = 0$ is consistent with the equilibrium hypothesis. Other solutions would also be consistent with each particular values t_0, but these would not be consistent with the equilibrium hypothesis $t_0 = 0$. More specifically, the solution for $t_0 = 0$ is the only that is consistent with all equilibrium hypotheses $t_0 \in [0, 1]$. The same reasoning would apply for discontinuous functions $F^C(t)$.

Consider now the discrete-time model: players can only move (simultaneously) at times $\{k\varepsilon\}_{k \in \mathbb{Z}}$. At each point, conditional on entry not having occurred before, the probability of entry in a symmetric equilibrium, $p(t)$, solves the indifference condition

$$\sigma = \Big(1 - p(t)\Big)\,\pi(t)$$

or

$$p(t) = 1 - \frac{\sigma}{\pi(t)}$$

The cumulative probability of entry is recursively given by

$$F^D\left(k\varepsilon\right) = F\Big((k-1)\varepsilon\Big) + \Big(1 - F\Big((k-1)\varepsilon\Big)\Big)\,p(k\varepsilon)$$

[23] Clearly, for $t < 0$, it is a dominant strategy not to enter with probability one, so that $F(0) = 0$.

$$F^D(k\varepsilon) = F\left((k-1)\varepsilon\right) + \left(1 - F\left((k-1)\varepsilon\right)\right)\left(1 - \frac{\sigma}{\pi(k\varepsilon)}\right),$$

or

$$F^D(k\varepsilon) = \frac{\pi(k\varepsilon) - \sigma + F^D\left((k-1)\varepsilon\right)}{\pi(k\varepsilon)}.$$

Comparing to (3), the result follows. ■

Proof of Proposition 2: (a) Consider two games with the same $\pi(\cdot)$, but different ε: $\varepsilon_2 = 2\varepsilon_1$. The equilibrium of the first game is characterized by a series of entry probabilities $p_1, p_2, p_3, \ldots$. The equilibrium of the second game, in turn, is characterized by the series of entry probabilities $p_2, p_4, p_6, \ldots$.

Let us compare the probability that each game ends in an entry mistake by time ε_2 conditional on the game ending by that time. In the case of the second game, this is given by

$$m_2 = \frac{p_2^2}{1-(1-p_2)^2}.$$

(The denominator equals the probability that the game ends during the first entry round.)

Likewise, for the first game we have

$$m_1 = \frac{p_1^2 + (1-p_1)^2 p_2^2}{1-(1-p_1)^2(1-p_2)^2}.$$

Straightforward calculation indicates that $m_2 > m_1$ if and only if $p_2 > p_1$, which in turn follows from the assumption that $\pi(\cdot)$ is strictly increasing.

Finally, the the proof of part (a) follows from an induction argument: Conditional on the game not having ended by time $k\varepsilon_2$, the probability of it ending in an entry mistake by time $(k+1)\varepsilon_2$ is greater for the second game than it is for the first one.

(b) For a small μ_1, define t' by $\pi(t') = \mu_1$. The game ends by t' with probability greater than $1 - \mu_2$, where μ_2 can be made arbitrarily small by an appropriate choice of a small ε. (Notice that there exists a $t'' < t'$ such that $\pi(t'')$ is strictly positive and so is the probability of entry in all entry moments between t'' and t'.) Since the payoff from being a monopolist is very small and the expected value of the game is zero, it must be the case that the probability of an entry mistake is also very small, specifically, a value μ_3 that tends to zero when μ_1 and μ_2 tend to zero. Since μ_1 and μ_2 can be made arbitrarily small by choosing ε arbitrarily small, the result follows.[24]

(c) As the value of ε becomes arbitrarily large, the relevant values of $\pi(t)$ for the purpose of computing $p_1, \ldots$ all converge to $\bar{\pi} \equiv \lim_{t\to\infty} \pi(t)$. Therefore, the probability of entry at each $k\varepsilon$ converges to $\bar{p}$, which solves

$$(1-\bar{p})\bar{\pi} = \sigma,$$

[24] I am grateful to Paul Klemperer for a suggestion that simplified this part of the proof significantly. A result similar to this appears as Proposition 11.3 in Sutton (forthcoming).

or

$$\bar{p} = 1 - \frac{\sigma}{\pi}.$$

In the limit, the probability of an entry mistake is thus given by

$$\bar{M} = \bar{p}^2 + (1-\bar{p})^2\bar{p}^2 + (1-\bar{p})^2(1-\bar{p})^2\bar{p}^2 + \ldots$$

Substituting, expression (c) results. ■

Proof of Proposition 3: The probability of duopoly is given by

$$\left(1 - F(\sigma)\right)^2 = \left(1 - \frac{\sigma}{\pi}\right)^2,$$

which is equal to the expression in the text.

The expected value of entry mistakes consists of two components. Entry mistakes occur either because there is a duopoly or because one of the firms incurred entry costs before dropping out of the entry race. The first term is given by $\sigma \cdot (1 - \frac{\sigma}{\pi})^2 \cdot \frac{1}{\pi}$. The second one, in turn, is

$$\frac{1}{\pi} \cdot \int_0^\sigma 2f(s)\left(1 - F(s)\right) s\, ds.$$

Substituting $F(s) = s/(\pi - \sigma + s)$, $f(s) = (\pi - \sigma)/(\pi - \sigma + s)^2$, we get

$$\begin{aligned} 2\frac{(\pi-\sigma)^2}{\pi}\int_0^\sigma \frac{s}{\pi-\sigma+s}\,ds &= 2\frac{(\pi-\sigma)^2}{\pi}\left[-\frac{s}{2}(\pi-\sigma+s)^{-2} - \frac{1}{2}(\pi-\sigma+s)^{-1}\right]_0^\sigma \\ &= \frac{\pi-\sigma}{\pi}\frac{\sigma^2}{\pi^2} \\ &= (1-\frac{\sigma}{\pi})\frac{\sigma}{\pi}^2. \end{aligned}$$

Finally, the expected value of entry mistakes under war-of-attrition entry is given by

$$V = \frac{\sigma}{\pi}\left(1 - \frac{\sigma}{\pi}\right)^2 + \left(1 - \frac{\sigma}{\pi}\right)\left(\frac{\sigma}{\pi}\right)^2,$$

which is equal to the expression in the text. ■

Proof of Proposition 4: In equilibrium, two firms will enter if and only if $\gamma > \sigma$. From a social welfare perspective, a duopoly is better than a monopoly if and only if $\nu\pi - 2\sigma > \mu\pi - \sigma$. Suppose that $\gamma = \pi/2$. Then, if $\sigma < \pi/2$, two firms will enter. However, $\gamma = \pi/2$ implies $\mu = \nu$. Therefore, a monopoly is better than a duopoly for any $\sigma > 0$. This proves part (a) by continuity. Suppose now that $\gamma = 0$ and σ is small but positive. In equilibrium, only one firm enters the market. However, since σ is small and ν is strictly greater than μ, a duopoly is better than a monopoly. This proves part (b) by continuity. ■

Proof of Proposition 5: Social welfare is given by the equation

$$W = p^2(\nu\pi - 2\sigma) + 2p(1-p)(\mu\pi - \sigma) + (1-p)^2\delta W,$$

or

$$W = \frac{p^2(\nu\pi - 2\sigma) + 2p(1-p)(\mu\pi - \sigma)}{1-(1-p)^2\delta}$$

The derivative with respect to p is given by

$$\begin{aligned}\frac{\partial W}{\partial p} &= \frac{\left(2p(\nu\pi - 2\sigma) + 2p(1-2p)(\mu\pi - \sigma)\right)\left(1-(1-p)^2\delta\right)}{\left(1-(1-p)^2\delta\right)^2} - \\ &\quad - \frac{\left(2(1-p)\delta\right)\left(p^2(\nu\pi - 2\sigma) + 2p(1-p)(\mu\pi - \sigma)\right)}{\left(1-(1-p)^2\delta\right)^2}\end{aligned}$$

A necessary and sufficient condition for $\frac{\partial W}{\partial p} > 0$ is therefore

$$\begin{aligned}\xi &\equiv \left(2p(\nu\pi - 2\sigma) + 2p(1-2p)(\mu\pi - \sigma)\right)\left(1-(1-p)^2\delta\right) - \\ &\quad - \left(2(1-p)\delta\right)\left(p^2(\nu\pi - 2\sigma) + 2p(1-p)(\mu\pi - \sigma)\right) > 0.\end{aligned}$$

The equilibrium probability of entry is given by the minimum between 1 and the value from the indifference condition

$$p\gamma + (1-p)\pi = \sigma,$$

or

$$p = \min\left(1, \frac{\pi - \sigma}{\pi - \gamma}\right). \tag{4}$$

Suppose that $\gamma = \pi/2$ and $\sigma < \pi/2$. From (4), $p = 1$. It follows that

$$\begin{aligned}\xi &= 2(\nu\pi - 2\sigma) - 2(\mu\pi - \sigma) \\ &= -2\sigma < 0,\end{aligned}$$

which, by continuity, proves part (a). Suppose that $\gamma = \sigma = 0$. Again, (4) implies $p = 1$. It follows that

$$\xi = 2\nu\pi - 2\mu\pi = 2(\nu - \mu)\pi > 0,$$

which, by continuity, proves part (b).

Suppose that $\sigma = \pi$. From (4), $p = 0$, and

$$\xi = 2(\mu\pi - \sigma) = 2\pi(\mu - 1) > 0,$$

which, by continuity, proves part (c).

Finally, suppose that $\delta = 1$. Substituting in the expression for ξ and simplifying, we get

$$\xi = 2p^2\left((\nu\pi - 2\sigma) - (\mu\pi - \sigma)\right).$$

We thus get $\xi < 0$ if and only if $\sigma > \pi(\nu - \mu)$, which proves part (d). ■

Proof of Proposition 6: Social welfare is given by

$$W = \left(\frac{\pi - \tau\sigma}{\pi - \gamma}\right)^2 (\nu\pi - \sigma) + \left(1 - \left(\frac{\pi - \tau\sigma}{\pi - \gamma}\right)^2\right)\mu\pi - \sigma - \frac{(\pi - \tau\sigma)(\tau\sigma - \gamma)^2}{(\pi - \gamma)^2\tau}$$

The derivative with respect to τ, at $\tau = 1$, is

$$\left.\frac{\partial W}{\partial \tau}\right|_{\tau=1} = (\pi - \gamma)^{-2}\left(\sigma^2(\pi - \gamma) + \pi\gamma^2 - \sigma^2\gamma - 2\sigma\pi(\pi - \sigma)(\nu - \mu)\right) \tag{5}$$

Suppose that $\gamma = \pi/2$. This implies that $\nu = \mu$. If $\sigma > \gamma$, so that the solution is interior, we get

$$\left.\frac{\partial W}{\partial \tau}\right|_{\tau=1} = \pi > 0.$$

If $\sigma < \gamma$, then we have a corner solution. The minimum tax that would yield an interior solution is $\tau = \frac{\gamma}{\sigma}$. At that level, we have

$$\left.\frac{\partial W}{\partial \tau}\right|_{\tau=\frac{\gamma}{\sigma}} = 4\frac{\sigma^2}{\pi} > 0.$$

In either case, part (a) follows by continuity.

Substituting $\gamma = 0$ in (5), we get

$$\left.\frac{\partial W}{\partial \tau}\right|_{\tau=1} = \frac{1 + 2(\nu - \mu)}{\pi}\sigma^2 - 2(\nu - \mu)\sigma.$$

For small values of σ, the second term dominates. Since $\nu > \mu$ for $\gamma = 0$, part (b) follows by continuity.

Substituting $\sigma = \pi$ in (5), we get

$$\left.\frac{\partial W}{\partial \tau}\right|_{\tau=1} = \pi > 0,$$

and part (c) follows by continuity.

Finally, straightforward computation establishes that

$$\left.\frac{\partial^2 W}{\partial \tau\, \partial \gamma}\right|_{\tau=1,\gamma=0} = -4\frac{\sigma(\pi - \sigma)(\nu - \mu)}{\pi^2} < 0,$$

which proves part (d). ■

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