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Abstract

Blanket exclusion of "brown" stocks is seen as the best way to reduce their negative externalities, by starving them of capital and hindering their expansion. We show that a more effective strategy may be tilting -- holding a brown stock if it is best-in-class, i.e. has taken a corrective action. While such holdings allow the firm to expand, they also encourage the corrective action. We derive conditions under which tilting dominates exclusion for externality reduction. If the corrective action is unobservable to the market, the investor is unable to tilt even if she has perfect information -- doing so would lead her to hold a company that has taken the action but the market thinks it has not, leading to accusations of greenwashing. Even if managers can costlessly disclose a signal of their actions, they will only do so under certain circumstances, and even a manager intending to take the action will only disclose a noisy signal.

JEL Classification: D62, G11, G34

Keywords: Socially responsible investing, sustainable investing, Externalities, Exclusion, Divestment, tilting, exit, governance

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Socially Responsible Divestment*

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Abstract

Blanket exclusion of "brown" stocks is seen as the best way to reduce their negative externalities, by starving them of capital and hindering their expansion. We show that a more effective strategy may be tilting – holding a brown stock if it is best-in-class, i.e. has taken a corrective action. While such holdings allow the firm to expand, they also encourage the corrective action. We derive conditions under which tilting dominates exclusion for externality reduction. If the corrective action is unobservable to the market, the investor is unable to tilt even if she has perfect information – doing so would lead her to hold a company that has taken the action but the market thinks it has not, leading to accusations of greenwashing. Even if managers can costlessly disclose a signal of their actions, they will only do so under certain circumstances, and even a manager intending to take the action will only disclose a noisy signal.

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Responsible investing – the practice of incorporating environmental, social, and governance ("ESG") factors into investment decisions – is becoming increasingly mainstream. In 2006, the United Nations established the Principles for Responsible Investment ("UN PRI"), a commitment to invest responsibly, which was signed by 63 investors managing a total of $6.5 trillion. By the end of 2021, this had grown to 4,375 investors, representing $121 trillion.

One goal of responsible investing is to improve risk-adjusted returns, by incorporating ESG factors that are not fully priced by the market. However, critics argue that enhancing returns is simply investing, not responsible investing (e.g. Mackintosh, 2022). The more distinctive goal is to improve companies' ESG performance through two channels. The first is by engaging with a company, through voting or private discussions with management. The second is through divesting from "brown" companies that exert negative externalities, and/or simultaneously investing in "green" companies – indeed, the first of the UN PRI's principles is "We will incorporate ESG issues into investment analysis and decision-making processes". Doing so increases the cost of capital of brown companies, hindering their expansion, while helping green firms to grow.

Under this channel, the most powerful investment strategy is blanket exclusion of industries an investor deems as irresponsible. Nobel Peace Prize winner Desmond Tutu called for outright divestment from the fossil fuel industry, similar to the anti-apartheid divestment campaign from South Africa in the 1980s. 1,500 institutions, collectively managing $40 trillion (including Harvard University, the State of Maine, and the Norwegian Sovereign Wealth Fund) have publicly committed to divest from fossil fuels.[1] Practitioners and the general public hold investors accountable for their holdings of brown firms. In 2020, Extinction Rebellion protesters dug up a lawn outside Trinity College, Cambridge in protest of its endowment's investment in fossil fuel companies, and many asset owners evaluate asset managers according to whether they manage a "net zero" portfolio. Beyond climate, Morningstar's "globe" ratings of funds are based on the Sustainalytics ESG ratings of the stocks they hold and are thus boosted by divesting from brown stocks; Hartzmark and Sussman (2019) find that fund flows are significantly influenced by these ratings.[2] Academic studies of greenwashing by asset managers similarly analyze their portfolio holdings (e.g. Gibson et al. (2022), Kim and Yoon (2021), and

[1] Source: Global Fossil Fuel Divestment Commitments Database. https://divestmentdatabase.org/.

[2] While some rating providers industry-adjust their ratings, others do not, so the average rating of a "brown" industry is lower than of a "green" industry.

Liang, Sun, and Teo (2021)). Gibson et al. (2022) define the goal of responsible investing as "to direct capital towards companies that make the world more sustainable"; under this goal, the average ESG rating of portfolio companies is indeed the relevant measure.

However, this argument considers only one channel through which divestment can affect a company's real actions – the primary markets channel, whereby divestment affects the terms at which a company raises new capital. As the survey of Bond, Edmans, and Goldstein (2012) points out, stock market trading – and thus divestment strategies – can also have real effects through a secondary markets channel. Specifically, trading leads to the stock price reflecting a manager's real actions, thus rewarding or punishing him for taking them. Even if a firm is in an irremediably brown sector, which unavoidably produces negative externalities, the manager may be able to take corrective actions to mitigate these externalities. Blanket exclusion fails to reward such actions because the firm is divested no matter what. Thus, it may be optimal for a responsible investor to pursue a "tilting" strategy, where she tilts away from a brown industry but is willing to hold firms that are best-in-class, i.e. take corrective actions.

We build a model in which responsible investment affects firm behavior through both above channels. There is a single brown firm that emits negative externalities. The firm's manager can take a non-contractible corrective action, such as investing in clean energy, that reduces both externalities and also firm value. The firm also raises capital which it uses to fund an expansion, increasing both firm value and externalities. The firm's manager is concerned with both fundamental value and the stock price; the latter may arise through takeover threat, termination threat, or reputational concerns.

The firm is owned by a continuum of risk-averse, profit-motivated, atomistic investors ("households") and a risk-neutral responsible investor. The responsible investor is able to take large positions and have price impact, and so we refer to her as a blockholder. Her objective is to minimize the externalities produced by the brown firm. To do so, she announces an investment strategy that depends on whether the brown firm takes a corrective action. Under exclusion, the blockholder never holds the firm; under tilting, she invests if and only if it takes the action. In the core model, we assume that the blockholder can commit to her investment strategy. For example, some funds advertise themselves as boycotting certain industries; deviation will lead to investor withdrawals and potentially regulatory action. Other funds state that they have a best-in-class strategy which involves investing in leaders in controversial industries. Deviating from this and excluding entire industries may lead to investors withdrawing to cheaper passive funds that pursue exclusion, increase tracking error or reduce risk-adjusted returns.

We show that the optimal divestment strategy balances two forces. On the one hand, since the brown firm continues to produce negative externalities even under the corrective action, the blockholder wishes to minimize its size. She does so through blanket exclusion – by holding none of the brown firm's shares, they have to be held entirely by risk-averse households, who require a risk premium for doing so. This minimizes the stock price, as in Heinkel, Kraus, and Zechner (2001), and thus the new funds the firm can raise. On the other hand, the investor wishes to incentivize the manager to take the action. Exclusion provides no such incentives, since the firm is always divested. Tilting rewards the manager for taking the action – by buying shares, the blockholder reduces the number that must be held by households, thus increasing the stock price.

Intuitively, the blockholder's strategy is analogous to an incentive contract. Exclusion corresponds to paying the manager a flat salary, which minimizes the cost to the firm but provides no incentives. Tilting incentivizes the action, but is costly – in a contracting setting, the cost is the monetary value of the incentive; in a responsible investment setting, the cost is financing the expansion of a brown firm. This analogy highlights how exclusionary strategies may be suboptimal, despite being widely advocated – they are tantamount to giving the manager zero incentives.

We show that the optimal investment strategy involves tilting if the corrective action is effective at reducing the externality, because incentivizing the action is particularly important compared to stifling capital raising. Tilting is also optimal if the action is less costly and if the manager's stock price concerns are high, as then the blockholder does not need to offer large rewards (in the form of share purchases) to incentivize the action; thus, the additional expansion and externalities created are low. These results suggest that exclusion may be optimal for industries such as controversial weapons[3], where it is relatively difficult to reduce the harm produced. In contrast, tilting may be preferred for fossil fuels, where managers can take corrective actions such as investing in clean energy, and the net cost of these actions may not be high – while developing clean energy requires substantial investment, it also generates significant future cash flows.

One might also think that exclusion is optimal if the firm is raising a significant amount

[3] "Controversial" weapons are are typically defined as those covered by international conventions and treaties, such as the Anti-Personnel Mines Treaty 1997 and the Convention on Cluster Munitions 2008.

of new capital, because it is particularly important to stifle capital raising. However, this turns out to not always be the case; it depends on how the effect of the action scales with firm size. One assumption is for the action to be multiplicative in firm size, and thus have a greater mitigating impact in a large firm – for example, reducing the per-unit amount of pollution has a greater impact in a firm that produces more. If so, then there is a force in the opposite direction – if the firm is raising a significant amount of new capital, it becomes even more important to induce the corrective action. Overall, the amount of capital raised has an ambiguous effect on the optimal investment strategy; for similar reasons, the profitability of the investment opportunity has an ambiguous effect. However, if the action is additive in firm size, then exclusion is indeed optimal when the brown firm is raising more capital and has superior investment opportunities.

We show that firm value is always lower under tilting than exclusion. Tilting implements the corrective action, which is costly to firm value, and this always outweighs the fact that tilting boosts the stock price and allows the firm to invest more. Thus, if the parameters are such that the investor optimally chooses divestment, her social objective (the desire to minimize externalities) does not come at the expense of firm value. However, it would be incorrect to conclude that there is no trade-off between financial and social value. While there is indeed no conflict from the investor's perspective, there remains a trade-off from the firm's perspective. The only way that the firm can reduce externalities is to take the corrective action, which always erodes firm value. In contrast, the investor can reduce externalities by an exclusion strategy, which does not involve the costly action.

We extend the model to the case in which the corrective action is unobservable, so the investor is unable to condition her investment on it; instead, she can make it contingent on a noisy signal of the action. The noisier the signal, the greater the reward the investor needs to offer to induce the action, and the more likely she is to choose exclusion. This result highlights a new benefit of ESG disclosure – it allows investors to induce corrective actions without having to promise large amounts of capital, thus financing the expansion of brown firms.

Importantly, even if the blockholder can gather perfect information about the manager's action at an arbitrarily small cost, she may not do so. It may seem that such information will allow her to induce the action at lower cost, i.e. promising a lower investment – since the blockholder will always make the investment if the manager has taken the action, he will do so even if the investment is small. However, the blockholder may end up buying a company that has taken the action even though the noisy public signal suggests that it has not. Doing so may lead to the blockholder being accused of greenwashing – buying a brown company even though, in the eyes of the market, it has not taken any corrective action. If the blockholder suffers a sufficiently large reputational cost from buying such a company, she will not base her purchases on her private information. This reduces her incentives to gather it in the first place, and may deter her from inducing the corrective action.

If the manager is able to increase the precision of the public signal through disclosure, he will do so if his stock price concerns are sufficiently high, as then he benefits from the blockholder's purchases if he has taken the action. It might seem that he will disclose a perfect signal, so that he will be given full credit for his action. However, this turns out not to be the case – he discloses a noisy signal, so that the blockholder has to promise a large investment to induce the action.

A common criticism of divestment is that arbitrageurs can buy brown stocks at depressed prices, reversing the impact of divestment. In our final extension, an arbitrageur appears with positive probability, and has a single objective of maximizing trading profits. The arbitrageur buys half the shares that are not purchased by the blockholder, thus reducing the impact of her trading decisions. On the one hand, this makes tilting less effective – since arbitrageur partially offsets the blockholder's trades, he needs to promise an even larger purchase to induce the corrective action, making tilting more expensive to implement. On the other hand, the arbitrageur makes exclusion less effective, since she buys up underpriced stock and reduces the impact of exclusion on the cost of capital. Since the arbitrageur buys half of the free float, his impact is greater on exclusion (where the blockholder's trade is zero and the free float is the total shares outstanding) than on tilting. Thus, the greater the probability of the arbitrageur, the more likely the blockholder is to tilt.

This paper is related to the theoretical literature on responsible investing. Heinkel, Kraus, and Zechner (2001) show that divestment reduces the stock price by increasing the shares that must be held by risk-averse investors. Davies and Van Wesep (2018) demonstrate that the resulting lower price raises the number of shares granted to the manager if his equity-based pay is fixed in dollar terms, paradoxically rewarding him. Oehmke and Opp (2020) show that responsible investing is only effective if responsible investors are affected by externalities regardless of whether they own the emitting companies, and if they can co-ordinate. Pedersen, Fitzgibbons, and Pomorski (2021) focus on the asset pricing implications of responsible investing and solve for the ESG-efficient frontier. The above papers do not involve new financing and investment, so the lower stock price from divestment has no real effects. Pastor, Stambaugh, and Taylor (2021) model how greater taste for green companies increases their valuation and reduces equilibrium expected returns. While firms make investment decisions, they are financed by internal cash flow and so there is no primary markets channel through which the stock price affects investment.[4] The above papers do not model externalities or study different strategies pursued by responsible investors; instead, investors' demands are automatic given their tastes. Landier and Lovo (2020) find that the more money investors put into ESG funds, the more important it is for an industry to reduce its externalities to obtain financing. The only existing argument against divestment of which we are aware is that it hinders an investor's ability to engage, as modelled by Broccado, Hart, and Zingales (2021). However, many investors rarely engage – their expertise may be stock selection rather than engagement, or they lack the substantial financial resources needed. For example, Engine No. 1 spent $30 million electing three climate-friendly directors onto Exxon's board.

Relative to the above literature, a unique feature of our model is the incorporation of a secondary market channel through which responsible investing affects externalities. This is related to models on "governance through exit", such as Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011), where investor trading causes a manager's actions to be reflected in the stock price. Those papers do not feature primary markets channels through which trading may have real effects; in addition, they are not models of responsible investing as investors' objective is to maximize trading profits and firms produce no externalities.

Some empirical studies question either the effectiveness or justification of exclusion as a responsible investing strategy. Teoh, Welch, and Wazzan (1999) show that the anti-apartheid campaign to divest from South Africa had a negligible effect on company valuations. Berk and van Binsbergen (2021) calculate that ESG-motivated exclusion has little effect on the long-term cost of capital; in our model, tilting can work by boosting the short-term stock price. Gantchev, Giannetti, and Li (2022) show that divestment following negative environmental and social ("E&S") incidents, and the threat of exit, disciplines managers to improve E&S performance. Such actions are only possible if the investor is willing to hold brown firms in

[4]Instead, the stock price affects investment because investors dislike holding brown stocks, and so demand a higher return for holding them, which reduces the short-term stock price. However, the stock price does not affect the fundamental value created by the investment, unlike in our model where it affects the amout of new capital raised.

the first place. Turning to the justification of exclusion, Cohen, Gurun, and Nguyen (2021) show that the fossil fuel industry produces more green patents than nearly any other sector, suggesting that companies within this industry can take corrective actions.

1 The Model

1.1 Players and Timing

We consider a single firm with a risk-neutral manager. The firm is in a "brown" industry and thus emits externalities, which will be specified later. The initial number of shares is normalized to one. The financial market consists of a continuum of risk-averse, profit-motivated, atomistic investors, indexed by $i \in [0, 1]$, and a risk-neutral responsible investor that aims to minimize the externalities produced by the firm. The responsible investor has the ability to take large positions and thus have price impact, and so we refer to it as a blockholder ("B").

There are four dates $t \in \{0, 1, 2, 3\}$. At $t = 0$, B announces an investment strategy $x(a)$ that depends on a publicly-observable action $a \in \{0, 1\}$ taken by the firm. We will sometimes refer to action $a = 1$ as the "corrective action" or "becoming greener", such as a fossil fuel company investing in clean energy. The strategy $x(0) = x(1) = 0$ represents "exclusion", where B never holds the firm regardless of its action; the strategy $\{x(0) = 0,\ x(1) > 0\}$ represents "tilting", where B tilts away from the stock – she does not hold it if $a = 0$, but is willing to own a strictly positive amount if $a = 1$. As we show below, in equilibrium, B's strategy will be either exclusion or tilting.

Initially, we assume that B can commit to the investment strategy. For example, an asset manager can launch a fund with a stated investment strategy to exclude brown firms, such as the Vanguard ESG Developed World All Cap Equity Index Fund. Subsequently deviating from such a strategy will lead to investor withdrawals, and may prompt regulatory action – for example, the UK's Financial Conduct Authority has forced funds to remove "sustainability" labels from their name due to not investing in accordance with their stated strategy. Alternatively, an asset manager can launch a fund with a tilting strategy, which generally avoids brown firms but is willing to hold them if they are "best-in-class" in their industry, such as Royal London Asset Management's range of sustainable funds.[5] Such funds claim to add value

[5] There are some passive funds that engage in tilting, such as the Legal & General Future World Climate

through undertaking active management and analyzing individual companies within a sector. Failing to hold any firms in a controversial industry may also lead to investor withdrawals as it would be cheaper to hold a passive fund that pursues an exclusionary strategy. In addition, avoiding entire industries will increase tracking error and may reduce risk-adjusted returns; Hong and Kacperczyk (2009) find that alcohol, tobacco, and gaming (often viewed as brown industries) significantly outperform their peers, and Bolton and Kacperzcyk (2021) document higher returns for stocks that emit more carbon dioxide. In ongoing work, we are analyzing the case in which B is unable to commit to her investment strategy. Atomistic investors such as households cannot commit to a strategy as they are not accountable to either a regulator or clients; in any case commitment is irrelevant because they are atomistic.

At $t = 1$, the manager takes action $a \in \{0, 1\}$. Choosing the corrective action ($a = 1$) reduces the firm's externality and decreases firm value by c, net of any benefit. We have $c > 0$: the corrective action is detrimental to firm value, otherwise it would automatically be taken without the need for responsible investment. For example, while investing in clean energy will generate future cash flows, the present value of these cash flows may not exceed the cost of the investment. After taking the action, the firm issues $q \in (0, 1)$ additional shares to finance an investment project. At $t = 2$, investors trade claims to the firm's terminal value. At $t = 3$, the firm generates both a terminal cash flow and negative externalities.

1.2 Firm Value and Externalities

The firm's terminal value is specified as:

$$V = \widetilde{A} + rI - ca \tag{1}$$

where $\widetilde{A} \sim N(\mu, \sigma)$ represents the random return generated by the firm's assets in place. The cost associated with the manager's corrective action is captured by ca and the gross return from the new investment is given by rI with $r > 1$. The firm finances the investment solely by issuing new shares so that $I = pq$. To focus on the main economic mechanism – the blockholder's trade-off between providing incentives for the action and less capital for brown investments – we take the firm's issuance decision q as given. For example, q may be limited by

Change Equity Factors Index Fund, which tracks the FTSE Russell All-World ex CW Climate Balanced Factor Index.

the amount of equity that can be raised without the agency costs of outside equity becoming too severe. With fixed q, lower demand for the firm's stock by the blockholder reduces the stock price p and thus the level of investment $I = pq$.

In our setting, investment has constant returns to scale – the firm can invest any amount I, with a constant gross return of r. Thus, a divestment-induced decrease in the stock price p has a linear effect on investment $I = pq$. The opposite assumption would be that the firm has a single lumpy investment project which requires $\overline{I}$ dollars of investment. In this case, p (and thus divestment) cannot change the level of investment along the intensive margin. It could only have an effect on the extensive margin, which in turn would require q to be endogenous so that the firm might choose not to raise capital and invest at all. This setting would be less appealing for two reasons. First, it would lead to "bang-bang" solutions as the firm invests either 0 or $\overline{I}$. If the firm is already choosing not to raise capital and invest, a further decrease in p would have no effect on investment; conversely, if the firm is already choosing to invest, a further increase in p is ineffective. Second, and more importantly, there would be a substantial loss of tractability. The per-share value of the firm is given by

$$v \equiv \frac{V}{1+q}, \tag{2}$$

and so q affects both the numerator (through affecting investment and thus aggregate firm value) and denominator. Thus, if q were endogenous, it would not be solvable in closed form. An intermediate assumption would be diminishing returns to scale, in which case a lower p will always reduce the level of investment but the effect is less than one-for-one. We choose the constant returns to scale assumption because it is more tractable, and also because it is the setting in which divestment is most likely to be effective; despite this, we will show that divestment is not always optimal.

The firm's operations generate a negative externality $f(\widetilde{A}, rI, a)$ to society. The externality depends on the firm's assets in place $\tilde{A}$, the payoff from investment rI, and the corrective action a. We assume that the action is expected to reduce the negative externality, $\mathbb{E}[f(\widetilde{A}, rI, 1)] < \mathbb{E}[f(\widetilde{A}, rI, 0)]$, and that greater investment increases the externality by increasing the size of the firm, that is, $\frac{\partial f(\widetilde{A}, I, a)}{\partial I} > 0$. As a result, there are two ways in which the blockholder's investment strategy can reduce externalities. The first is by increasing the cost of capital and thus constraining the amount of externality-producing investment that the manager undertakes. This is typically the stated rationale for divestment strategies. The second is by directly rewarding the manager for taking the corrective action.

1.3 Manager's Problem

The manager's utility function depends on the equilibrium stock price p and the per-share firm value v:

$$U_m = \omega p + (1-\omega)v, \tag{3}$$

with $\omega \in [0,1]$. The concern for the short-term stock price ω is standard in the literature and can arise from a number of sources introduced by prior research, such as takeover threat (Stein, 1988), termination threat (Edmans, 2011), or concern for managerial reputation (Narayanan, 1985; Scharfstein and Stein, 1990). Another common justification is that the firm intends to raise equity at $t = 2$ (Stein, 1996).[6] We do not include this as a source of $\omega > 0$ because we explicitly model equity issuance. Indeed, we show that even if the manager is fully aligned with the firm's long-term value ($\omega = 0$), he will still care about the stock price p as it will affect the terms at which he will raise equity. While prior papers typically group equity issuance together with other justifications for $\omega > 0$, separating out equity issuance is important in our model as it has a different implication for the channels through which the blockholder can reduce externalities. As we will show, exclusion is more effective if the firm is raising more equity; tilting is more effective if ω is higher.

At $t = 1$, the manager solves:

$$\max_{a\in\{0,1\}} \mathbb{E}[U_m], \tag{4}$$

where the expectation is taken over $\widetilde{A}$. Importantly, the manager takes the blockholder's investment policy $x(a)$ as given when choosing a.

1.4 Financial Market

The blockholder commits to a demand schedule $x(a)$. Households maximize a standard mean-variance objective with constant absolute risk aversion parameter $\gamma > 0$. When submitting

[6] An additional source of stock price concerns is if the manager plans to sell ω shares at $t = 2$, as in Stein (1989).

their demands, households condition on the action a and the stock price p:

$$\max_{x_i} \mathbb{E}[x_i(v-p)|a,p] - \frac{\gamma}{2} Var(x_i(v-p)|a,p). \tag{5}$$

Their demand function is thus given by:

$$x_i = \frac{\mathbb{E}[v|a,p] - p}{\gamma Var(v|a,p)}. \tag{6}$$

Market clearing requires that total demand equals supply:

$$x(a) + \int_0^1 x_i di = 1 + q. \tag{7}$$

Solving this equation for p yields:

$$p = \mathbb{E}[v|a,p] - \gamma Var(v|a,p)\,(1 + q - x(a)) \tag{8}$$

with $\mathbb{E}[v|a,p] = \frac{\mu + rI - ca}{1+q}$, $Var(v|a,p) = \frac{\sigma^2}{(1+q)^2}$, and $I = qp$.

The stock price p is the certainty equivalent per-share value of the firm. The second term represents the risk discount, which is increasing in risk $Var(\cdot)$, the risk aversion of households γ, and the number of shares held by households $1 + q - x(a)$. An increase in the blockholder's demand raises the stock price by reducing the number of shares that risk-averse investors need to hold.

1.5 Blockholder's Problem

The blockholder chooses the investment strategy $x(a)$ to minimize the expected externality:

$$\min_{x(a)} \mathbb{E}[f(\widetilde{A}, rI, a)]. \tag{9}$$

We assume that $0 \leq x(a) \leq 1 + q$. The assumption $x(a) \geq 0$ results from short-sale constraints, which are standard in the blockholder exit literature (e.g. Admati and Pfleiderer, 2009; Edmans, 2009); without short-sales constraints, blockholders have no special role as any investor can exit, regardless of her initial stake. However, this assumption is not necessary for our results. If short-sales are possible, all the results continue to apply except that B need not be a blockholder – she can be any large investor that can commit to an investment strategy.[7] Similarly, $x(a) \leq 1+q$ means that the blockholder cannot buy more than the entire firm, i.e. households cannot short sell. If this assumption is relaxed, our results become stronger as the blockholder has a greater ability to reward the corrective action.

2 Optimal Investment Strategies

We solve the model by backwards induction. We first re-write the equilibrium stock price as a function of the blockholder's strategy and the corrective action. We take the expression for the equilibrium stock price in equation (8), plug in $I = pq$ and solve for p:

$$p(a) = \frac{\mu - ca - \left(1 - \frac{x(a)}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{10}$$

The intuition is as follows. In the absence of an investment decision, the stock price is the certainty equivalent firm value divided by the number of shares $(1+q)$. One may think that investment should add an additional term to firm value in the numerator. However, since the value of the investment is $rqp(a)$, it effectively reduces the number of shares by rq in the denominator.[8]

To ensure that $p(a)$ is positive, we assume that $\mu > \gamma\sigma^2 + c$ so that expected firm value is not outweighed by the risk premium and the cost of the corrective action, and that $1+q-rq > 0$ so the effective number of shares does not turn negative. The second condition can be rewritten $r < \frac{1+q}{q}$. Intuitively, if r is sufficiently large, then households demand more shares when the price is higher, since their funds will be invested in a very profitable investment opportunity, leading to an upward-sloping demand curve.

[7] We would only need a limit on the maximum possible short-sales to prevent the stock price in equation (8) from turning negative.

[8] To see this, we have:

$$\begin{aligned} \text{Market value of firm} &= \text{Certainty equivalent fundamental value of firm} \\ p(1+q) &= \text{Certainty equivalent assets in place} + prq \\ p(1+q-rq) &= \text{Certainty equivalent of assets in place} \end{aligned}$$

We next solve for the manager's optimal choice of a. He takes the corrective action if $\mathbb{E}[U_m|a=1] \geq \mathbb{E}[U_m|a=0]$. Plugging in the expressions for $p(a)$ and $\mathbb{E}[v]$ derived above shows that this inequality is satisfied if and only if:

$$x(1) - x(0) \geq \frac{c(1+q)}{\gamma\sigma^2\left[\omega + (1-\omega)z\right]} \equiv \overline{\Delta}_x \tag{11}$$

where

$$z \equiv \frac{rq}{1+q} \in (0,1). \tag{12}$$

The action $a = 1$ has two effects on the manager's objective function. First, it incurs a cost c which reduces fundamental value and thus the stock price; the latter in turn lowers investment and further reduces fundamental value. Second, it increases the blockholder's demand from $x(0)$ to $x(1)$, raising the stock price $p(a)$ and thus investment and firm value. Thus, the manager takes the corrective action if the second force is sufficiently strong, i.e. she pursues a tilting strategy where $x(1) - x(0)$ is sufficiently high.

The last step is to solve for the blockholder's optimal policy $x(a)$. The previous assumptions $\mathbb{E}[f(\widetilde{A}, rI, 1)] > \mathbb{E}[f(\widetilde{A}, rI, 0)]$ and $\frac{\partial f(\widetilde{A}, rI, a)}{\partial I} > 0$, and the fact that $p(0)$ increases with $x(0)$, imply that the blockholder optimally sets $x(0) = 0$. It immediately follows from the inequality above that the blockholder can implement the corrective action by setting $x(1) \geq \overline{\Delta}_x$ and that she can implement the brown action by setting $x(1) \in [0, \overline{\Delta}_x)$. We assume that

$$c \leq \gamma\sigma^2\left[\omega + (1-\omega)z\right] \tag{13}$$

so that the constraint $x(1) \leq 1 + q$ does not bind in the main model. (In the extensions, this condition will differ, and we will state the new condition required). Proposition 1 states the results; all omitted proofs are given in the Appendix.

Proposition 1 *(Blockholder's strategy): The blockholder's optimal strategy is given as follows:*

1. *If* $\mathbb{E}[f(\widetilde{A}, rI(a=0; x=0), 0)] < \mathbb{E}[f(\widetilde{A}, rI(a=1; x=\overline{\Delta}_x), 1)]$*, the optimal strategy is exclusion, i.e.* $x(1) = x(0) = 0$*, and the manager chooses* $a = 0$*;*

2. *If* $\mathbb{E}[f(\widetilde{A}, rI(a=0; x=0), 0)] \geq \mathbb{E}[f(\widetilde{A}, rI(a=1; x=\overline{\Delta}_x), 1)]$*, the optimal strategy is tilting, i.e.* $x(1) = \overline{\Delta}_x$ *and* $x(0) = 0$*, and the manager chooses* $a = 1$*.*

The threshold $\overline{\Delta}_x$ is defined in equation (11).

The intuition is as follows. The blockholder's investment strategy $x(a)$ is analogous to an incentive contract provided to a manager, except that incentives are not provided by cash, but through purchasing shares which raises the stock price.[9] A higher stock price in turn increases the manager's objective function in two ways – first, directly as the manager places weight ω on the stock price and second, indirectly by increasing the amount of investment and thus fundamental value (on which the manager places weight $1-\omega$). As in a compensation model, it is optimal to give the lowest possible reward upon $a = 0$. In a contracting setting with limited liability, this involves zero pay; in an investment setting with short-sales constraints, this involves zero demand. Whether to reward $a = 1$ depends on whether the benefits of the action exceed the costs. In a contracting setting, the cost is the financial cost of pay. In our investment setting, the cost is that positive demand increases the stock price, raising investment and thus the externality. This analogy highlights the drawback of exclusion strategies, despite them being practiced by many investors – they are tantamount to giving the manager zero reward for desirable actions.

We now consider two functional forms for the externality. The first is where the externality is multiplicative in firm size. For example, if the corrective action involves developing a less polluting technology, this is implemented firm-wide and thus has a larger effect on larger firms. The second is where the action is additive in firm size. For example, it may be to increase board diversity, which helps contribute to social inequality in boardrooms in aggregate regardless of the size of the firm.

2.1 Multiplicative Externality

In this subsection, we use a multiplicative functional form for the externality:

$$f(\widetilde{A}, rI, a) = \lambda(\widetilde{A} + rI)(1 - \xi a) \tag{14}$$

[9] A second difference is that an incentive contract is contingent upon output, whereas the blockholder's strategy is contingent upon the action a. We only require the action a to be publicly observable, but not contractible. Section 3 studies the case in which a is not publicly observable.

so that the blockholder chooses $x(a)$ to minimize $\mathbb{E}[f] = \lambda(\mu + rI)(1 - \xi a)$.[10] The parameter $\lambda > 0$ scales the externality and $0 < \xi < 1$ determines the efficacy of the corrective action.

The blockholder chooses tilting, and sets $x(1) = \overline{\Delta}_x$, if the expected externality is lower with the corrective action. Evaluating $\mathbb{E}[f]$ at $a = 1$ and $a = 0$ leads to the following condition for tilting:

$$x(1) = \overline{\Delta}_x \Leftrightarrow \xi \geq \frac{rq\left(p\left(1\right) - p\left(0\right)\right)}{\mu + rqp\left(1\right)}. \tag{15}$$

Plugging in the expression for $p(a)$ in equation (10) leads to Proposition 2.

Proposition 2 *(Blockholder's strategy, multiplicative externality): The blockholder's optimal strategy is tilting and the manager chooses $a = 1$ if and only if*

$$\xi \geq \overline{\xi}_{mult} \equiv \frac{(1-\omega)c}{(1-\omega)c + (\frac{\mu}{z} - \gamma\sigma^2)(\omega + \frac{z}{1-z})}. \tag{16}$$

Otherwise, it is exclusion and the manager chooses $a = 0$. The tilting strategy involves $x(1) = \overline{\Delta}_x$. $\overline{\xi}_{mult}$ is increasing in (c, γ, σ) and decreasing in (ω, μ). If $\omega = 0$, then $\overline{\xi}_{mult}$ is decreasing in (r, q). If $\omega \in (0, 1)$, then $\overline{\xi}_{mult}$ is hump-shaped in (r, q).

The intuition is as follows. The blockholder chooses tilting if the corrective action is sufficiently effective at reducing externalities (ξ is sufficiently high). Then, the most effective way to reduce externalities is to incentivize the manager to take the action through purchasing shares if he does so, rather than to starve the firm of funds through blanket exclusion. The threshold $\overline{\xi}_{\text{mult}}$ is lower (i.e. tilting is more likely to be optimal) if the manager has greater stock price concerns (ω is high[11]) and the corrective action is less costly (c is low). This reduces the number of shares $\overline{\Delta}_x$ that B needs to purchase to induce the corrective action, meaning that doing so is possible without raising investment by much.

Tilting is also preferred if the firm is large (high μ) and risk σ and risk aversion γ are small, because this increases the stock price and thus the amount of investment. In addition, high μ increases assets in place. Both factors lead to greater firm value and thus higher externalities.

[10] We assume that the externality does not depend on the ca term. We implicitly assume that the firm has cash on hand to pay this cost and does not need to disinvest to do so.

[11] One might think that there is a force in the opposite direction – higher ω means a lower weight $(1 - \omega)$ on fundamental value, and so the manager has less incentive to increase the stock price to increase the amount raised by the new investment. However, the cost of the investment c also affects fundamental value, so a lower weight on fundamental value makes the manager more willing to pay the cost.

Since the action a has a multiplicative effect on the externality, the greater the firm value, the greater the benefit from the action. As we will shortly show, $\overline{\xi}$ is independent of these parameters under an additive functional form for the externality.

Finally, the capital raised by the firm (q) and the profitability of the investment (r) have a non-monotonic effect on $\overline{\xi}_{\text{mult}}$. One might think that these parameters should have an unambiguous effect – the greater the capital raised, the more important the cost of capital channel, and thus the more valuable exclusion is to increase the cost of capital. However, there is a force in the opposite direction – the greater the capital raised, the more important it is to induce the corrective action to reduce the externalities from the new investment. If q and r are sufficiently high, such a large amount of capital is raised that this second force dominates and further increases in q and r make tilting more effective.

If $\xi > \overline{\xi}_{\text{mult}}$, i.e. the blockholder chooses tilting, then her holdings of the brown firm upon the corrective action are higher if γ, σ, and ω are low. If risk and risk aversion are small, then changes in demand have small effects on the stock price; thus, a large change is needed to incentivize $a = 1$. Similarly, if the manager has little concern ω for the stock price, a large increase is necessary to induce the action.

2.2 Additive Externality

In this subsection, we set $f(\widetilde{A}, rI, a) = \lambda \left(\widetilde{A} + rI - \xi a\right)$. Proceeding as above leads to Proposition 3.

Proposition 3 *(Blockholder's strategy, additive externality): The blockholder's optimal strategy is tilting if*

$$\xi \geq \overline{\xi}_{add} \equiv \frac{c}{\frac{1}{z}\frac{\omega}{1-\omega} + 1}. \tag{17}$$

Otherwise, it is exclusion. The tilting strategy involves $x(1) = \overline{\Delta}_x$. $\overline{\xi}_{add}$ *is increasing in* (c, r, q), *decreasing in* ω, *and independent of* (μ, γ, σ).

We discuss only the comparative statics that differ from the multiplicative case. Now, exclusion is unambiguously more preferred if the capital raised by the firm (q) and the profitability of the investment (r) are sufficiently high. When the firm is raising more capital, it is particularly important to stifle capital raising. Since the effect of the additive action is independent of the amount of capital raised, there is no opposing force. In contrast, μ, γ, and σ no longer have an effect on the optimal strategy, since the effect of the additive action is independent of the amount of new investment and thus the stock price. For the remainder of the paper, we focus on the case of multiplicative externalities, although highlight the results that hold for both cases.

2.3 Firm Value

Proposition 4 compares expected firm value under tilting or exclusion, to study whether the blockholder's desire to minimize externalities comes at the expense of firm value. The analysis holds irrespective of the functional form for the externalities.

Proposition 4 *(Firm value comparison): The expected value of the firm under tilting is always lower than under exclusion:*

$$E\left[V|Tilting\right] = E\left[V|Exclusion\right] - \frac{\omega c}{\omega + (1-\omega) z}. \tag{18}$$

On the one hand, tilting induces the corrective action which reduces firm value by c; on the other hand, tilting leads to a higher stock price which allows the firm to invest more in the positive-NPV project. Proposition 4 shows that first force is always greater than the second – firm value is always lower under tilting than under exclusion – for any strictly positive ω. The intuition is as follows. If $\omega > 0$, the manager is concerned about the stock price. Thus, he will take the corrective action partly due to his stock price concerns, rather than because the action increases firm value by allowing the firm to invest more, and so he will take the action even if it reduces firm value. The action increases firm value if and only if $rq\left[p(1) - p(0)\right] > c$, but it increases the manager's payoff if and only if $rq\left[p(1) - p(0)\right] > \frac{c}{\frac{1+q}{rq}\frac{\omega}{1-\omega}+1}$. Since the manager only places weight $(1-\omega)$ on fundamental value, he does not fully internalize the cost of the action. Since the blockholder chooses $x(1)$ so that the manager is exactly indifferent between $a = 1$ and $a = 0$, in equilibrium we have $rq\left[p(1) - p(0)\right] = \frac{c}{\frac{1+q}{rq}\frac{\omega}{1-\omega}+1} < c$, and so the action always reduces firm value.

[AE: I've rewritten this. This isn't so much the trade-off from the investor's perspective, as she doesn't care about firm value (only trading profits); instead I talk about the trade-off from the social planner / society's perspective] Practitioners debate whether there is a trade-off between

financial and social value. In our setting, financial value corresponds to firm value V, and social value corresponds to the negative of externalities $-f$. From the firm's perspective, there is always a trade-off between financial and social value – actions taken to increase social value are costly to financial value. However, Proposition 4 shows that, from society's perspective, there need not be a trade-off. If exclusion is the optimal strategy, then the presence of a blockholder with purely social objectives is not at the expense of financial value – indeed, the blockholder's investment strategy leads to greater financial value (relative to tilting) even though the blockholder is unconcerned with financial value. However, it would be incorrect to conclude that there is no trade-off from the firm's perspective – forcing the firm to take the corrective action would automatically reduce financial value. Instead, the absence of the trade-off from society's perspective arises because the blockholder can reduce externalities more by starving the firm of capital rather than encouraging it to take the costly action. Overall, whether *investors'* pursuit of social objectives reduces firm value has no bearing on whether *firms* face this trade-off.

Finally, since the blockholder chooses endogenously whether to tilt or exclude a firm, there are conditions under which tilted firms have both higher firm value and lower externalities than excluded firms. This positive correlation between financial and social value may lead to conclusions that there is no trade-off between these objectives, when the positive correlation is driven by selection – the investor is choosing to tilt in companies that are more valuable to begin with. Corollary 1 demonstrates this result.

Corollary 1 *(No trade-off):*

(i) Let i denote a firm in which the blockholder optimally tilts, and j denote a firm that she optimally excludes. Let i and j differ only in (μ, ξ). There exists $\xi^* < 1$ such that if $\xi_i > \xi^*$ and $\mu_i - \mu_j > c\frac{\omega(1-z)}{\omega(1-z)+z}$, then firm i has a higher expected value and lower expected externalities than firm j.

(ii) The blockholder's trading profits under tilting are greater than under exclusion.

We start with part (i). If μ_i is sufficiently higher than μ_j, then the value of firm i is higher than firm j; if the corrective action is sufficiently powerful, then externalities are also lower. However, this correlation is driven by selection – the blockholder endogenously chooses to tilt in firms in which the corrective action is powerful, and if such firms are also more valuable firms, then it will seem that financial value and and social value coincide. However, there remains a trade-off between both objectives, since if the blockholder chose to exclude firm i, its value would be higher.

We now move to part (ii). In the model, the blockholder's objective function is to minimize externalities. A more general objective function would involve a weighted sum of the blockholder's trading profits and (the negative of) externalities – the blockholder does not receive per-share firm value, but firm value minus the price paid. Under exclusion, the blockholder's profit is zero as she owns no shares. Under tilting, the blockholder's profits are positive. Since households are risk-averse, they are only willing to hold a strictly positive amount if the stock price is less than the fundamental value of the firm, and so the blockholder earns trading profits. Intuitively, buying shares upon the corrective action not only rewards the action, but also gives the blockholder a return for bearing risk. Thus, there is no trade-off between social value and the blockholder's trading profits.

3 Unobservable Corrective Action

In this section, we consider the case in which the corrective action is not publicly observable. As a result, B cannot condition her holdings on a. Instead, there is a public signal $s \in \{0, 1\}$, which is correlated with the action (such as an ESG rating). The precision of this signal is given by

$$\tau \equiv \Pr[s = a|a] \in [0.5, 1). \tag{19}$$

This signal is publicly observed at $t = 2$, before trade in the secondary market takes place. The blockholder is able to condition her holdings on this signal, $x(s) \geq 0$. Households have rational expectations and correctly conjecture the manager's equilibrium corrective action.

We proceed in two steps. First, we take the signal precision as given and analyze how τ affects the optimal investment strategy. Second, we endogenize the signal precision and allow the manager to choose τ ex ante.

3.1 Optimal Investment Strategies

Following the same steps as in the baseline model, the manager takes the corrective action if and only if:

$$x(1) - x(0) \geq \frac{1}{2\tau - 1} \frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2[\omega + (1-\omega)z]} \equiv \widehat{\Delta}_x(\tau). \tag{20}$$

As in the baseline model, the manager chooses the corrective action if it leads to the blockholder buying a sufficiently large amount. Importantly, the threshold $\widehat{\Delta}_x$ is decreasing in signal precision τ: $\frac{\partial \widehat{\Delta}_x}{\partial \tau} < 0$. Intuitively, B has to provide the manager stronger incentives to take the corrective action when the public signal is less precise. If $\tau = 1/2$, the signal is uninformative about the corrective action. Since the blockholder is unable to reward the action, it follows that $\mathbb{E}[U_m|a = 1] < \mathbb{E}[U_m|a = 0]$ and so the manager always chooses $a = 0$.

The blockholder optimally chooses tilting if (i) the expected externality with $a = 1$ and $x(1) = \widehat{\Delta}_x$ is lower than that under $a = 0$ and $x(1) = 0$, and (ii) $\widehat{\Delta}_x(\tau) \leq 1+q$. The equivalent of condition (13), to ensure $x(1) \leq 1 + q$, is $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$. Under this assumption, the blockholder's optimal strategy is given by Proposition 5:

Proposition 5 *(Unobservable corrective action): The blockholder's optimal strategy is tilting and the manager chooses $a = 1$ if and only if*

$$\xi \geq \overline{\xi}(\tau) \equiv 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\left(\frac{\tau}{2\tau-1} \frac{1-\omega}{\omega + \frac{z}{1-z}} - 1\right)}. \tag{21}$$

Otherwise, it is exclusion and the manager chooses $a = 0$. The tilting strategy involves $x(1) = \widehat{\Delta}_x(\tau)$. $\overline{\xi}(\tau)$ is increasing in (c, γ, σ) and decreasing in (ω, μ, τ).

As in the baseline model, B chooses tilting if the effectiveness of the corrective action exceeds a threshold. This threshold is decreasing in signal precision τ. A higher τ means that it is less costly for the blockholder to implement the corrective action, and so it makes tilting preferable to exclusion. Note that $\overline{\xi}(1/2) = 1$: if the signal is pure noise, then the blockholder always chooses exclusion.

Proposition 5 thus highlights a new benefit of superior ESG disclosure. Common arguments are that ESG disclosure is valuable to allow investors to allocate capital according to ESG performance, and to hold managers to account. Both of these channels operate here, but there is an additional force – by allowing investors to allocate capital according to ESG performance, they can induce corrective actions without having to commit to a significant investment in a brown firm, thus making them more likely to implement the corrective action in the first place.

3.2 Blockholder Private Information

We now allow the blockholder to gather private information on the manager's corrective action a, after the action has been taken but before she makes her trading decision. We assume the cost of information acquisition is arbitrarily small, and if the blockholder is indifferent between acquiring information and remaining uninformed, she prefers the latter. Other market participants remain uninformed about a, although they continue to observe the public signal s.

If the blockholder acquires private information on a, we assume that she can commit to an investment that conditions on a. Since both the manager and blockholder know when $a = 1$, the blockholder can commit to rewarding the manager by purchasing stock if $a = 1$. If the blockholder reneges among this commitment, then she will be unable to induce the action in any of the firms that she holds stakes in going forwards.[12] If the blockholder does not acquire information, then he can only commit to an investment strategy that is based on the realization of the public signal s. If the blockholder acquires a stake in the firm (i.e., $x > 0$), but the public signal indicates that the corrective action has not been taken by the firm, that is, $s = 0$, then the blockholder incurs a reputational cost of $g > 0$. This cost results from accusations of greenwashing – the blockholder has invested in a brown firm despite there being no public evidence that it has taken a corrective action.[13] The blockholder objective function is to minimize the sum of the firm's externalities, her cost of information acquisition, and her reputational costs.

Proposition 6 *(Blockholder private information):*

(i) *If* $\tau \geq \frac{1}{2} + \frac{1}{2}\frac{1-\omega}{\frac{1+z}{1-z}+\omega}$*, the blockholder remains uninformed and chooses tilting if and only if* $\xi > \overline{\xi}(\tau)$*, as in Proposition 5.*

[12] The same logic means that B is able to commit to acquiring information. If she reneges on this commitment, she will be uninformed about a and thus will not be able to reward the manager by purchasing stock if $a = 1$. Allowing for a divestment strategy that conditions both on a and s would not change the result since the action a is perfectly predictable in equilibrium.

[13] Note that the "public" is different from the atomistic investors who have rational expectations about the actual actions taken by the firm's manager. For example, this may include the blockholder's current or future clients.

(ii) Suppose $\frac{1}{2} < \tau < \frac{1}{2} + \frac{1}{2}\frac{1-\omega}{\frac{1+z}{1-z}+\omega}$ *and let*

$$\overline{\xi}_{in}(g) \equiv \frac{(1-\omega)c + \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{(1-\omega)c + \left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)}. \tag{22}$$

$$\overline{\xi}_{un}(g) \equiv 1 - \frac{\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{c\left[\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right]}. \tag{23}$$

Then, there exists $g^* > 0$ *that satisfies* $\overline{\xi}(\tau) = \overline{\xi}_{in}(g^*)$ *such that:*

(a) *If* $g \geq g^*$*, the blockholder remains uninformed and chooses tilting if and only if* $\xi > \overline{\xi}(\tau)$.

(b) *If* $g < g^*$*, the blockholder chooses exclusion if* $\xi < \overline{\xi}_{in}(g)$*, informed tilting if* $\overline{\xi}_{in}(g) < \xi < \overline{\xi}_{un}(g)$*, and uninformed tilting if* $\xi > \overline{\xi}_{un}(g)$*. In this case* $\overline{\xi}_{in}(g) < \overline{\xi}(\tau) < \overline{\xi}_{un}(g)$*, with* $\lim_{g\nearrow g^*}\overline{\xi}_{in}(g) = \overline{\xi}(\tau) = \lim_{g\nearrow g^*}\overline{\xi}_{un}(g)$.

(c) *The equilibrium expected externality is increasing in* g.

Overall, Proposition 6 shows that the blockholder is less likely to acquire information when the cost from greenwashing accusations g is large. Absent reputational concerns, it is efficient for the blockholder to acquire private information as she does not need to promise as large a purchase to induce the corrective action – since the manager knows that the blockholder will have observed that he has taken the action, he will be willing to do so even if the promised purchases are low. However, by committing to condition her strategy on a, the blockholder exposes herself to the risk that she will end up purchasing shares if $a = 1$ even if $s = 0$ – investing in a company that the public thinks has taken no corrective action. If the cost of greenwashing actions is sufficiently large, the blockholder is less likely to acquire private information which, in turn, increases the cost of inducing the corrective action and deters her from doing so in the first place.

In reality, many responsible investors claim to undertake detailed analysis to gather private information on companies' social performance. Indeed, one might think that doing so makes them more effective, since they can hold firms more accountable for their social performance. However, contrary to their claims, they have no incentive to gather private information, as they are unable to trade on such information if they are evaluated on how their investments vary with publicly observable signals.

3.3 Optimal Disclosure

In this section, we return to the case in which a is unobservable to all investors, and allow the manager to choose τ ex ante to maximize his expected payoff. We assume that if the manager is indifferent between different values of τ, he chooses the lowest possible τ of $\frac{1}{2}$. Loosely speaking, if the manager is different between disclosure and non-disclosure, he chooses non-disclosure as this would be strictly optimal if disclosure were costly. We also assume the choice of τ is made public, so that the blockholder can condition her divestment strategy on τ.

The equivalent of condition (13), to ensure $x(1) \leq 1 + q$, is $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$. Under this assumption, Proposition 7 gives the manager's optimal disclosure policy and shows how it affects the blockholder's optimal investment strategy.

Proposition 7 *(Optimal disclosure policy). If and only if*

$$\xi \geq 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\frac{1-\omega}{\omega+\frac{z}{1-z}}} \equiv \overline{\xi}_{disc}, \tag{24}$$

then the manager chooses $\tau^* = \max\{\widehat{\tau}(\xi), \tau^{min}\} \in \left(\frac{1}{2}, 1\right)$ *where* $\widehat{\tau}(\xi)$ *satisfies* $\xi = \overline{\xi}(\tau)$ *and* τ^{min} *satisfies* $\widehat{\Delta}_x(\tau^{min}) = 1 + q$*, the blockholder chooses tilting, and the manager chooses* $a = 1$*. Otherwise, the manager chooses* $\tau = \frac{1}{2}$*, the blockholder chooses exclusion, and the manager chooses* $a = 0$*. The threshold* $\overline{\xi}_{disc}$ *decreases in* (μ, ω)*, it increases in* (c, γ, σ)*, and it is hump-shaped in* (r, q)*.*

The manager discloses information (i.e. chooses $\tau > \frac{1}{2}$), and the blockholder chooses tilting, if and only if the corrective action is sufficiently effective. The threshold for ξ decreases in the manager's stock price concerns. This is because disclosure increases the stock price, as long as the manager has also taken the action.[14] Since managers only disclose if they take the action, one might think that they should choose full disclosure ($\tau = 1$) so that their action is always reflected in the public signal ($s = 1$). In contrast, the manager deliberately discloses noisy signals, so that the blockholder has to promise a high investment $x(1)$ upon the action in

[14] One might think that there is a force in the opposite direction – higher ω means a lower weight $(1 - \omega)$ on fundamental value, and so the manager has less incentive to increase the stock price to increase the amount raised by the new investment. However, the cost of the investment c also affects fundamental value, so a lower weight on fundamental value makes the manager more willing to pay the cost.

order to induce it. Indeed, $\widehat{\tau}(\xi)$ is the minimum disclosure that persuades the blockholder to implement the action.

The model considers a blockholder who chooses optimally between tilting and exclusion strategies. Stepping outside the model, if there was a probability that the blockholder only implements exclusion strategies (e.g. due to lack of sophistication, or its clients believing that exclusion is the best way to invest responsibly), then the greater this probability, the more likely it is for the blockholder to choose minimal disclosure ($\tau = \frac{1}{2}$). Thus, if the economy contains more responsible investors that are open to adopting a tilting strategy, this would encourage firms to disclose more information about their ESG activities, in turn reinforcing investors' incentives to adopt the tilting strategy.

4 Presence of Arbitrageur

A common criticism of divestment strategies is they allow arbitrageurs to buy brown firms at depressed prices, both generating profit for themselves and also attenuating the impact of divestment on prices. This section extends the model to incorporating an arbitrageur, A, who is purely profit-motivated like households, and is risk-neutral and can take large stakes and have price impact like the blockholder. We return to the case in which the action a is publicly observable; this simplifies the analysis as it means that firm value (which is net of c, if $a = 1$) is publicly observable.

With probability $\eta > 0$, A arrives after B has announced her investment strategy and the manager has taken action a. The presence of the arbitrageur is public information. He trades an amount y at $t = 2$ to maximize $\Pi_A(y) = y(v - p)$. The equivalent of condition (13), to ensure $x(1) \leq 1 + q$, is $c \leq \gamma\sigma^2[\omega + (1-\omega)z]\left(1 - \frac{\eta}{2}\right)$. Under this assumption, the solution is given in Proposition 8:

Proposition 8 *(Arbitrageur). If the arbitrageur is present, his trading volume and profit are given by*

$$y^*(x) = \arg\max_y \Pi_A(y) = \frac{1+q-x}{2} \tag{25}$$

$$\Pi_A(y^*(x)) = \left(\frac{1}{2}\frac{1+q-x}{1+q}\right)^2 \gamma\sigma^2 \tag{26}$$

and, conditional on x, the stock price is given by

$$p\left(x,a,y^*\left(x\right)\right)=\frac{\mu-ca-\left(1-\frac{\eta}{2}\right)\left(1-\frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{27}$$

The blockholder's optimal strategy is tilting and the manager chooses $a=1$ if and only if

$$\xi\geq\overline{\xi}_{arb}\equiv\frac{\left(1-\omega\right)c}{\left(1-\omega\right)c+\left(\frac{\mu}{z}-\left(1-\frac{\eta}{2}\right)\gamma\sigma^2\right)\left(\omega+\frac{z}{1-z}\right)}. \tag{28}$$

Otherwise, it is exclusion and the manager chooses $a=0$. The tilting strategy involves

$$x\left(1\right)=\frac{\left(1+q\right)c}{\left(1-\frac{\eta}{2}\right)\gamma\sigma^2\left[\omega+(1-\omega)z\right]}. \tag{29}$$

As is standard, the arbitrageur buys half of the freely-floating shares not acquired by the blockholder, as shown in equation (25). Comparing (27) with (10), there is an additional $\left(1-\frac{\eta}{2}\right)$ term in the numerator, which multiplies the term containing x and means that the blockholder's trade has a lower effect on the stock price. Intuitively, if the arbitrageur is present, she buys half of the free float, so any impact of the blockholder investing (or choosing not to invest) is halved. As a consequence, equation (29) contains an additional $\left(1-\frac{\eta}{2}\right)$ term in the denominator – since the blockholder has smaller price impact, she must promise a higher purchase to induce the manager to take the corrective action, which makes tilting less effective and more expensive to implement. On the other hand, exclusion becomes even less effective because the arbitrageur partially reverses the impact of exclusion on the stock price and the cost of capital. Since the arbitrageur buys half of the free float, his impact is decreasing in the blockholder's trade. Thus, while the arbitrageur makes both exclusion and tilting less effective, the impact is greater on exclusion as the blockholder's trade is zero. Thus, the threshold in (28) is decreasing in η – the greater the probability of the arbitrageur appearing, the more likely the blockholder is to tilt.

5 Conclusion

This paper has analyzed the optimal investment strategy of a responsible investor whose goal is to minimize the externalities emitted by a brown firm. While exclusion – never investing in the brown firm – minimizes the stock price and thus the amount of externality-enhancing investment that the firm can undertake, it provides no incentives for a brown firm to undertake a corrective action as it will be excluded regardless. Tilting provides incentives to take the action, at the cost of providing capital to a brown firm and allowing it to expand. The optimal strategy is for the investor to choose tilting if the action is effective at reducing externalities and comes at little cost to firm value, and also if the manager's stock price concerns are high, as then the blockholder does not need to promise a high investment to persuade the manager to take the action.

We extend the model to the case in which the corrective action is not observable, but a noisy signal is, and the investor can condition her holdings only on the signal. The noisier the signal, the greater the reward the investor needs to offer to induce the action, and the more likely she is to choose tilting. Indeed, even if the blockholder herself can perfectly observe the manager's action, she will still choose to ignore it and follow an exclusion strategy if the public signal is sufficiently noisy. If the manager has discretion on the signal, he will choose to disclose some information if his stock price concerns are sufficiently high, as the blockholder will buy if he has taken the corrective action, increasing the stock price. However, he will only disclose a noisy signal, so that the investor has to promise high investment upon the corrective action in order to induce it. Even if the blockholder has the option to acquire private information about the manager's action at an arbitrarily small cost, she may refrain from doing so if she suffers a sufficiently large reputational cost from investing in a company that has taken a corrective action but the public is unaware of this fact. Finally, if there is an arbitrageur who buys underpriced stock, exclusion becomes relatively less effective compared to tilting as the arbitrageur offsets the negative effect of exclusion on the stock price. Thus, the greater the probability of the arbitrageur being present, the more likely the blockholder is to tilt.

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A Proofs

Proof of Proposition 1. The blockholder's objective function is given by $\mathbb{E}[f(\widetilde{A}, rI, a)]$ with $I = qp(a)$. The equilibrium stock price, as a function of a, is given by:

$$p(a) = \begin{cases} \frac{\mu - \gamma\sigma^2 + \frac{x(1)}{1+q}\gamma\sigma^2 - c}{1+q-rq} & \text{if } a = 1 \\ \frac{\mu - \gamma\sigma^2}{1+q-rq} & \text{if } a = 0. \end{cases} \tag{30}$$

If $a = 1$, then the realized, and thus the expected, externality increases in $x(1)$ through its impact on $p(1)$. As a result, the blockholder's objective given $a = 1$ is minimized at the smallest possible value that implements $a = 1$, $x(1) = \overline{\Delta}_x$. It follows that the blockholder implements $a = 1$ by choosing $x(1) = \overline{\Delta}_x$ if and only if:

$$x(1) = \overline{\Delta}_x \Leftrightarrow \mathbb{E}[f(\widetilde{A}, rqp(0), 0)] \geq \mathbb{E}[f(\widetilde{A}, +rqp(1; x(1) = \overline{\Delta}_x), 1)]. \tag{31}$$

Otherwise, the blockholder is better off implementing $a = 0$ and sets $x(1) = x(0) = 0$. ■

Proof of Proposition 2. We start with the condition under which the blockholder prefers $a = 1$ provided in the main text:

$$x(1) = \overline{\Delta}_x \Leftrightarrow \xi \geq \frac{rq\,(p\,(1) - p\,(0))}{\mu + rqp\,(1)}. \tag{32}$$

Evaluating $p(a)$ at $a \in \{0, 1\}$ and using $x(1) = \overline{\Delta}_x$ leads to:

$$\overline{\xi}_{\text{mult}} = \frac{\frac{\overline{\Delta}_x}{1+q}\gamma\sigma^2 - c}{\frac{\mu}{z} - \gamma\sigma^2 + \frac{\overline{\Delta}_x}{1+q}\gamma\sigma^2 - c} = \frac{c\,(1-\omega)}{\left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right) + c\,(1-\omega)} \tag{33}$$

where we have used $\overline{\Delta}_x = \frac{c(1+q)}{\gamma\sigma^2[\omega + (1-\omega)z]}$.

It immediately follows from the expression for $\overline{\xi}_{\text{mult}}$ that $\frac{\partial\overline{\xi}_{\text{mult}}}{\partial\mu} < 0$, $\frac{\partial\overline{\xi}_{\text{mult}}}{\partial\gamma} > 0$, and $\frac{\partial\overline{\xi}_{\text{mult}}}{\partial\sigma} > 0$. For the effect of c, we can divide the expression above by c to see that $\frac{\partial\overline{\xi}_{\text{mult}}}{\partial c} > 0$. For the comparative statics with respect to ω, we re-write the expression as:

$$\overline{\xi}_{\text{mult}} = \frac{c}{\left(\frac{\mu}{z} - \gamma\sigma^2\right) g_1(\omega) + c} \tag{34}$$

with $g_1(\omega) \equiv \left(\omega + \frac{z}{1-z}\right)\frac{1}{1-\omega}$ and $\frac{\partial g_1'}{\partial \omega} = \frac{1}{(1-\omega)^2(1-z)} > 0$ because $z \in (0,1)$. It follows that $\frac{\partial \overline{\xi}_{\text{mult}}}{\partial \omega} < 0$ if $\omega \in [0,1)$. If $\omega = 1$, then $\overline{\xi}_{\text{mult}} = 0$.

For the comparative statics with respect to z, and thus (r, q), we re-write the expression above as:

$$\overline{\xi}_{\text{mult}} = \frac{(1-\omega)\,c}{g_2(z) + (1-\omega)\,c} \tag{35}$$

with $g_2(z) = \left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)$. If $\omega = 1$, then $\overline{\xi}_{\text{mult}}$ does not depend on z. If $\omega < 1$, then the sign of $\frac{\partial \overline{\xi}_{\text{mult}}}{\partial z}$ is the opposite of $g_2'(z)$, which is equal to:

$$g_2'(z) = \frac{\mu - \gamma\sigma^2}{(1-z)^2} - \frac{\omega\mu}{z^2}. \tag{36}$$

Also note that $g_2''(z) > 0$, $\lim_{z\to 0} g'(z) = -\infty$ if $\omega > 0$ and $\lim_{z\to 0} g'(z) > 0$ if $\omega = 0$, and that $\lim_{z\to 1} g'(z) = \infty$. It follows that $g_2(z)$ is U-shaped in z if $\omega > 0$ and that it is increasing in z if $\omega = 0$. As a result, $\overline{\xi}_{\text{mult}}$ is hump-shaped in (r, q) if $\omega > 0$ and decreasing in (r, q) if $\omega = 0$. ■

Proof of Proposition 3. The expected externality given $a \in \{0, 1\}$ is given by:

$$\mathbb{E}[f|a = 1] = \lambda\left(\mu + \frac{z}{1-z}\left(\mu - \gamma\sigma^2 - c + \frac{\gamma\sigma^2}{1+q}\overline{\Delta}_x\right) - \xi\right) \tag{37}$$

and

$$\mathbb{E}[f|a = 0] = \lambda\left(\mu + \frac{z}{1-z}\left(\mu - \gamma\sigma^2\right)\right). \tag{38}$$

It follows that $\mathbb{E}[f|a = 1] \leq \mathbb{E}[f|a = 0]$ is equivalent to

$$\xi \geq \frac{z}{1-z}\left(\frac{\gamma\sigma^2}{1+q}\overline{\Delta}_x - c\right) \equiv \overline{\xi}_{\text{add}} \tag{39}$$

where $\overline{\Delta}_x = \frac{c(1+q)}{\gamma\sigma^2[\omega+(1-\omega)z]}$. Plugging in this expression leads to

$$\overline{\xi}_{\text{add}} = \frac{cz}{(1-z)}\left(\frac{1}{\omega + (1-\omega)z} - 1\right) = \frac{c}{\frac{1}{z}\frac{\omega}{(1-\omega)} + 1}. \tag{40}$$

It immediately follows that $\overline{\xi}_{\text{add}}$ increases in c, r, and q and that it decreases in ω. ■

Proof of Proposition 4. Expected firm value under exclusion is given by:

$$\begin{aligned} E\left[V|Exclusion\right] &= \mu + rqp\left(0\right) \\ &= \mu + rq\left[\frac{\mu - \gamma\sigma^2}{1+q-rq}\right] \\ &= \mu\frac{1}{1-z} - \gamma\sigma^2\frac{z}{1-z} \end{aligned}$$

Expected firm value under tilting is given by:

$$\begin{aligned} E\left[V|Tilting\right] &= \mu + rqp\left(1\right) - c \\ &= \mu + rq\left[\frac{\mu - \gamma\sigma^2 + \frac{c}{\omega+(1-\omega)\frac{rq}{1+q}} - c}{1+q-rq}\right] - c \\ &= \mu\frac{1}{1-z} - \gamma\sigma^2\frac{z}{1-z} - c\frac{\omega}{\omega+(1-\omega)z} \end{aligned}$$

We thus have

$$E\left[V|Tilting\right] = E\left[V|Exclusion\right] - c\frac{\omega}{\omega+(1-\omega)z}.$$

■

Proof of Corollary 1. We start with part (i). Under exclusion, expected firm value and externalities are given as follows:

$$\begin{aligned} E\left[V_i|Exclusion\right] &= \mu_i + r_iq_ip_i\left(0\right) = \frac{\mu_i - \gamma\sigma_i^2 z_i}{1-z_i} \\ E\left[f_i|Exclusion\right] &= \lambda_i E\left[V_i|Exclusion\right]. \end{aligned}$$

Under tilting, expected firm value and externalities are given by:

$$\begin{aligned} E\left[V_i|Tilting\right] &= \mu_i + r_iq_ip_i\left(1\right) - c_i = \frac{\mu_i - \gamma\sigma_i^2 z_i}{1-z_i} - c_i\frac{\omega_i}{\omega_i+(1-\omega_i)z_i} \\ E\left[f_i|Tilting\right] &= \lambda_i(E\left[V_i|Tilting\right] + c_i)(1-\xi_i). \end{aligned}$$

We thus have $E\left[V_i|Tilting\right] > E\left[V_j|Exclusion\right]$ and $E\left[f_i|Tilting\right] < E\left[f_j|Exclusion\right]$ if and only if

$$\frac{\lambda_i}{\lambda_j}(E\left[V_i|Tilting\right]+c_i)(1-\xi_i) < E\left[V_j|Exclusion\right] < E\left[V_i|Tilting\right]$$

Suppose that firm i and j differ only in (μ, ξ), with all other parameters constant. We have $\lambda_i(E\left[V_i|Tilting\right]+c_i)(1-\xi_i) < \lambda_j E\left[V_j|Exclusion\right]$ if and only if:

$$\frac{\lambda_i}{\lambda_j}(E\left[V_i|Tilting\right]+c_i)(1-\xi_i) < E\left[V_j|Exclusion\right] \Leftrightarrow$$
$$\frac{\frac{(\mu_i-\mu_j)}{1-z}+c\frac{(1-\omega)}{\omega z^{-1}+(1-\omega)}}{\frac{\mu_i-\gamma\sigma^2 z}{1-z}+c\frac{(1-\omega)}{\omega z^{-1}+(1-\omega)}} < \xi_i.$$

Note that the left-hand side is strictly smaller than 1 since $\mu_j > \gamma\sigma^2 + c > z\gamma\sigma^2$.

The condition $E\left[V_j|Exclusion\right] < E\left[V_i|Tilting\right]$ is equivalent to:

$$\frac{\mu_i-\gamma\sigma^2 z}{1-z} - c\frac{\omega}{\omega+(1-\omega)\,z} > \frac{\mu_j-\gamma\sigma^2 z}{1-z} \Leftrightarrow \mu_i-\mu_j > c\frac{\omega(1-z)}{\omega+(1-\omega)z}.$$

We now move to part (ii). Under exclusion, the blockholder's trading profits are zero as she owns no shares. Under tilting, her profits are:

$$E\left[\Pi|Tilting\right] = x\left(1\right)\left(v\left(1\right)-p\left(1\right)\right) = x\left(1\right)\left(\frac{\mu+rqp\left(1\right)-c}{1+q}-p\left(1\right)\right).$$

This is positive if:

$$x\left(1\right)\left(\frac{\mu+rqp\left(1\right)-c}{1+q}-p\left(1\right)\right) > 0 \Leftrightarrow$$
$$p\left(1\right) < \frac{\mu-c}{1+q-rq} \Leftrightarrow$$
$$\frac{\mu-c-\left(1-\frac{x(1)}{1+q}\right)\gamma\sigma^2}{1+q-rq} < \frac{\mu-c}{1+q-rq} \Leftrightarrow$$
$$x(1) < 1+q \Leftrightarrow$$
$$c < \gamma\sigma^2\left[\omega+(1-\omega)z\right]$$

which holds due to inequality (13). ■

Proof of Equation (20). The equilibrium stock price given public signal s is given by:

$$p(\widehat{a}, s) = \frac{\mu - c\widehat{a} - \left(1 - \frac{x(s)}{1+q}\right)\gamma\sigma^2}{1 + q - rq}, \tag{41}$$

where $\widehat{a}$ denotes the action conjectured by households.

If the manager chooses $a = 1$, his expected utility is given by:

$$\mathbb{E}[U_m|a = 1] = \omega\left[\tau p(\widehat{a}, 1) + (1-\tau)p(\widehat{a}, 0)\right] + (1-\omega)\frac{\mu + rq\left[\tau p(\widehat{a}, 1) + (1-\tau)p(\widehat{a}, 0)\right] - c}{1+q}.$$

If he chooses $a = 0$, his expected utility is given by:

$$\mathbb{E}[U_m|a = 0] = \omega\left[\tau p(\widehat{a}, 0) + (1-\tau)p(\widehat{a}, 1)\right] + (1-\omega)\frac{\mu + rq\left[\tau p(\widehat{a}, 0) + (1-\tau)p(\widehat{a}, 1)\right]}{1+q}.$$

Conditional on tilting, the manager chooses $a = 1$ if and only if $\mathbb{E}[U_m|a = 1] \geq \mathbb{E}[U_m|a = 0]$, which is equivalent to the condition in equation (20). ■

Proof of Proposition 5. For $\tau \in (1/2, 1)$, the blockholder chooses tilting, $(x(1) = \widehat{\Delta}_x, x(0) = 0)$ if (i) the expected externality with $a = 1$ and $x(1) = \widehat{\Delta}_x$ is lower than that under $a = 0$ and $x(1) = 0$, and (ii) $x(1) \leq 1 + q$. It follows from the expression for $\widehat{\Delta}_x(\tau)$ that condition (ii) is equivalent to $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$. Otherwise, she chooses exclusion and sets $x(1) = x(0) = 0$. Suppose $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$, then the blockholder chooses tilting if:

$$\begin{aligned}
&\left[\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)\right](1-\xi) \leq \mu + rqp(0,0) \Leftrightarrow \\
&\qquad 1 - \xi \leq \frac{\mu + rqp(0,0)}{\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)} \Leftrightarrow \\
&\qquad 1 - \xi \leq \frac{\mu + \frac{z}{1-z}\left(\mu - \gamma\sigma^2\right)}{\mu + \frac{z}{1-z}\left(\mu - c - \left(1 - \frac{\tau\hat{\Delta}_x}{1+q}\right)\gamma\sigma^2\right)} \Leftrightarrow \\
&\qquad \xi \geq 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 - c + \gamma\sigma^2\frac{\tau\hat{\Delta}_x}{1+q}} \Leftrightarrow \\
&\qquad \xi \geq 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\left(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega+\frac{z}{1-z}} - 1\right)} \equiv \overline{\xi}(\tau).
\end{aligned}$$

It immediately follows that $\overline{\xi}(\tau)$ increases in (c, γ, σ) and decreases in (μ, τ). Moreover, it decreases in ω because $\frac{1-\omega}{\omega+\frac{z}{1-z}}$ decreases in ω. For $\tau = 1/2$, the blockholder always chooses exclusion. ■

Proof of Proposition 6. We start by calculating the blockholder's payoff in different scenarios, assuming that $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$ so that she can implement tilting, which is shown in Proposition 5. If $c > \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$, then the blockholder always chooses exclusion. First, if the blockholder chooses exclusion, then the manager chooses $a = 0$, and the blockholder's payoff is independent of his private information and given by

$$\Pi_{exclusion} = -\lambda \left[\mu + rqp(0)\right].$$

In particular, the blockholder never acquires information if he intends to use exclusion.

Second, if the blockholder is uninformed about a and chooses tilting then he must be conditioning the divestment strategy on the public signal s. Therefore, he never suffers reputation costs and the payoff from tilting is

$$\Pi_{tilting}^{un} = -\lambda \left[\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)\right](1-\xi).$$

Third, if the blockholder is informed about a and chooses tilting he has two options. First, if he chooses to condition the divestment strategy on a then his expected payoff is

$$\Pi_{tilting}^{in} = -\lambda\left(\mu + rqp(1)\right)(1-\xi) - (1-\tau)\, g.$$

Second, if in spite of being informed he conditions his strategy on signal s then his expected payoff is $\Pi_{tilting}^{un}$. Therefore, the blockholder has no incentives to acquire information if it is not being used.

Overall, if the blockholder prefers uninformed tilting over exclusion if and only if $\Pi_{tilting}^{un} > \Pi_{exclusion} \Leftrightarrow \xi > \overline{\xi}(\tau)$. He prefers informed tilting over exclusion if and only if $\Pi_{tilting}^{in} >$

$\Pi_{exclusion} \Leftrightarrow$

$$
\begin{aligned}
-\lambda\left(\mu+rqp(1)\right)(1-\xi)-(1-\tau)\,g \geq -\lambda\left[\mu+rqp(0)\right] &\Leftrightarrow \\
\xi \geq \frac{rq\left[p(1)-p(0)\right]+\frac{(1-\tau)g}{\lambda}}{\mu+rqp(1)} &\Leftrightarrow \\
\xi \geq \frac{\frac{z}{1-z}\left[-c+\frac{x(1)}{1+q}\gamma\sigma^2\right]+\frac{(1-\tau)g}{\lambda}}{\mu+\frac{z}{1-z}\left[\mu-c-\left(1-\frac{x(1)}{1+q}\right)\gamma\sigma^2\right]} &\Leftrightarrow \\
\xi \geq \frac{-c+\frac{x(1)}{1+q}\gamma\sigma^2+\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}}{\frac{\mu}{z}-\gamma\sigma^2-c+\frac{x(1)}{1+q}\gamma\sigma^2} &\Leftrightarrow \\
\xi \geq \overline{\xi}_{\text{in}}\left(g\right) \equiv \frac{(1-\omega)\,c+\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega+\frac{z}{1-z}\right)}{(1-\omega)\,c+\left(\frac{\mu}{z}-\gamma\sigma^2\right)\left(\omega+\frac{z}{1-z}\right)}&.
\end{aligned}
$$

Notice that $\overline{\xi}\left(\tau\right) > \overline{\xi}_{\text{in}}\left(g\right) \Leftrightarrow$

$$
\begin{aligned}
\overline{\xi}\left(\tau\right) > \overline{\xi}_{\text{in}}\left(g\right) &\Leftrightarrow \\
1-\frac{\frac{\mu}{z}-\gamma\sigma^2}{\frac{\mu}{z}-\gamma\sigma^2+c(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega+\frac{z}{1-z}}-1)} > \frac{(1-\omega)\,c+\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega+\frac{z}{1-z}\right)}{(1-\omega)\,c+\left(\frac{\mu}{z}-\gamma\sigma^2\right)\left(\omega+\frac{z}{1-z}\right)} &\Leftrightarrow \\
\frac{\frac{\mu}{z}-\gamma\sigma^2}{\frac{\mu}{z}-\gamma\sigma^2+c(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega+\frac{z}{1-z}}-1)}c\left[\frac{\tau}{2\tau-1}\left(1-\omega\right)-\frac{1}{1-z}\right] > \frac{(1-\tau)\,g}{\lambda}\frac{1-z}{z}\left(\omega+\frac{z}{1-z}\right) &\Leftrightarrow \\
\left(1-\overline{\xi}\left(\tau\right)\right)c\left[\frac{\tau}{2\tau-1}\left(1-\omega\right)-\frac{1}{1-z}\right] > \frac{(1-\tau)\,g}{\lambda}\frac{1-z}{z}\left(\omega+\frac{z}{1-z}\right)&
\end{aligned}
$$

Notice $\frac{\tau}{2\tau-1}\left(1-\omega\right)-\frac{1}{1-z} > 0 \Leftrightarrow \tau < \frac{1}{2-(1-z)(1-\omega)}$. Thus,

$$
\begin{aligned}
&\overline{\xi}\left(\tau\right) > \overline{\xi}_{\text{in}}\left(g\right) \Leftrightarrow \\
&\quad \tau < \frac{1}{2-\left(1-z\right)\left(1-\omega\right)} \text{ and } \overline{\xi}\left(\tau\right) < \overline{\xi}_{\text{un}}\left(g\right)
\end{aligned}
$$

where

$$
\overline{\xi}_{\text{un}}\left(g\right) \equiv 1-\frac{\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega+\frac{z}{1-z}\right)}{c\left[\frac{\tau}{2\tau-1}\left(1-\omega\right)-\frac{1}{1-z}\right]}
$$

The blockholder prefers informed tilting over uninformed tilting if and only if $\Pi_{tilting}^{in} >$

$\Pi^{un}_{tilting} \Leftrightarrow$

$$
\begin{aligned}
-\lambda\left(\mu + rqp(1)\right)(1-\xi) - (1-\tau)\,g &> -\lambda\left[\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)\right](1-\xi) \Leftrightarrow \\
p(1) + \frac{1}{rq}\frac{(1-\tau)\,g}{\lambda}\frac{1}{1-\xi} &< \tau p(1,1) + (1-\tau)p(1,0) \Leftrightarrow \\
\frac{\mu - c - \left(1 - \frac{x(a=1)}{1+q}\right)\gamma\sigma^2}{1+q-rq} + \frac{1}{rq}\frac{(1-\tau)\,g}{\lambda(1-\xi)} &< \tau\frac{\mu - c - \left(1 - \frac{x(s=1)}{1+q}\right)\gamma\sigma^2}{1+q-rq} + (1-\tau)\frac{\mu - c - \gamma\sigma^2}{1+q-rq} \Leftrightarrow \\
\frac{x(a=1)}{1+q}\gamma\sigma^2 + \frac{1+q-rq}{rq}\frac{(1-\tau)\,g}{\lambda(1-\xi)} &< \tau\frac{x(s=1)}{1+q}\gamma\sigma^2 \Leftrightarrow \\
\frac{\frac{c(1+q)}{\gamma\sigma^2[\omega+(1-\omega)z]}}{1+q}\gamma\sigma^2 + \frac{1-z}{z}\frac{(1-\tau)\,g}{\lambda(1-\xi)} &< \tau\frac{\frac{1}{2\tau-1}\frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2[\omega+(1-\omega)z]}}{1+q}\gamma\sigma^2 \Leftrightarrow \\
\frac{c}{\omega+(1-\omega)z} + \frac{1-z}{z}\frac{(1-\tau)\,g}{\lambda(1-\xi)} &< \frac{\tau}{2\tau-1}(1-\omega)(1-z)\frac{c}{\omega+(1-\omega)z}
\end{aligned}
$$

That is, $\Pi^{in}_{tilting} > \Pi^{un}_{tilting} \Leftrightarrow$

$$
\tau < \frac{1}{2-(1-z)(1-\omega)} \text{ and } \xi < \overline{\xi}_{\text{un}}(g).
$$

We consider two cases:

1. Suppose $\overline{\xi}(\tau) < \overline{\xi}_{\text{in}}(g)$. There are three sub cases:

   (a) If $\xi < \overline{\xi}(\tau)$ then the blockholder prefers exclusion over both informed and uninformed tilting and hence he never becomes informed and always chooses exclusion.

   (b) If $\overline{\xi}(\tau) < \xi < \overline{\xi}_{\text{in}}(g)$ then the blockholder prefers uninformed tilting over exclusion, and exclusion over informed tilting. Therefore, the blockholder never becomes informed and he choose tilting.

   (c) If $\overline{\xi}_{\text{in}}(g) < \xi$, then exclusion is an inferior strategy. Recall $\overline{\xi}(\tau) < \overline{\xi}_{\text{in}}(g)$ implies either $\tau \geq \frac{1}{2-(1-z)(1-\omega)}$, in which case we have $\Pi^{in}_{tilting} < \Pi^{un}_{tilting}$, or $\overline{\xi}_{\text{un}}(g) < \overline{\xi}(\tau)$, which given $\overline{\xi}(\tau) < \overline{\xi}_{\text{in}}(g) < \xi$, implies $\overline{\xi}_{\text{un}}(g) < \xi$, i.e., $\Pi^{in}_{tilting} < \Pi^{un}_{tilting}$. Either way, the blockholder remains uninformed.

   We conclude, if $\overline{\xi}(\tau) < \overline{\xi}_{\text{in}}(g)$ then the blockholder remains uninformed. He chooses exclusion if and only if $\xi < \overline{\xi}(\tau)$. Notice that if $\tau < \frac{1}{2-(1-z)(1-\omega)}$ then $\overline{\xi}(\tau) > \overline{\xi}_{\text{in}}(0)$, and hence, there is $g^* > 0$ that satisfies $\overline{\xi}(\tau) = \overline{\xi}_{\text{in}}(g^*)$, such that $\overline{\xi}(\tau) < \overline{\xi}_{\text{in}}(g) \Leftrightarrow g > g^*$. Notice that if $\overline{\xi}(\tau) = \overline{\xi}_{\text{in}}(g^*)$ and $\tau < \frac{1}{2-(1-z)(1-\omega)}$, then it must be $\overline{\xi}(\tau) = \overline{\xi}_{\text{un}}(g^*)$

2. Suppose $\overline{\xi}(\tau) > \overline{\xi}_{\text{in}}(g)$. There are three sub cases:

   (a) If $\xi < \overline{\xi}_{\text{in}}(g)$ then the blockholder prefers exclusion over both informed and uninformed tilting and hence he never becomes informed and always chooses exclusion.

   (b) If $\overline{\xi}_{\text{in}}(g) < \xi < \overline{\xi}(\tau)$ then the blockholder prefers informed tilting over exclusion, and exclusion over uninformed tilting. Therefore, the blockholder becomes informed and chooses tilting.

   (c) If $\overline{\xi}(\tau) < \xi$, then exclusion is an inferior strategy. Recall $\overline{\xi}(\tau) > \overline{\xi}_{\text{in}}(g)$ implies $\tau < \frac{1}{2-(1-z)(1-\omega)}$ and $\overline{\xi}_{\text{un}}(g) > \overline{\xi}(\tau)$. Therefore, in this case, $\overline{\xi}_{\text{in}}(g) < \overline{\xi}(\tau) < \overline{\xi}_{\text{un}}(g)$. The blockholder chooses informed tilting if $\xi < \overline{\xi}_{\text{un}}(g)$, and uninformed tilting if $\xi > \overline{\xi}_{\text{un}}(g)$.

   We conclude that if $\overline{\xi}(\tau) > \overline{\xi}_{\text{in}}(g)$ then the blockholder chooses exclusion if $\xi < \overline{\xi}_{\text{in}}(g)$, informed tilting if $\overline{\xi}_{\text{in}}(g) < \xi < \overline{\xi}_{\text{un}}(g)$, and uninformed tilting if $\xi > \overline{\xi}_{\text{un}}(g)$.

Finally, suppose $\xi < \overline{\xi}(\tau)$. Notice that the amount of externalities under exclusion is lower than under informed tilting if and only if $\xi < \overline{\xi}_{\text{in}}(0)$. Therefore, if $\xi < \overline{\xi}_{\text{in}}(0)$ then g has no impact on the externalities in equilibrium. If $\overline{\xi}_{\text{in}}(0) < \xi < \overline{\xi}(\tau)$ then larger g increases the externalities in equilibrium by increasing the likelihood of exclusion in a region where informed tilting generates lower externalities.

Second, suppose $\xi > \overline{\xi}(\tau)$. Notice that the amount of externalities under informed tilting is lower than under uninformed tilting if and only if $\xi < \overline{\xi}_{\text{un}}(0)$. Therefore, if $\xi > \overline{\xi}_{\text{un}}(0)$ then g has no impact on the externalities in equilibrium. If $\overline{\xi}(\tau) < \xi < \overline{\xi}_{\text{un}}(0)$ then larger g increases the externalities in equilibrium by increasing the likelihood of uninformed tilting in a region where informed tilting generates lower externalities. ■

Proof of Proposition 7. We have shown before that $\overline{\xi}(\tau)$ is a decreasing function of τ. Moreover, $\lim_{\tau\to1}\overline{\xi}(\tau) < 1$. If $\lim_{\tau\to1}\overline{\xi}(\tau) > \xi$ then the blockholder chooses exclusion regardless of τ. In this case, the manager chooses $\tau = \frac{1}{2}$. Suppose $\lim_{\tau\to1}\overline{\xi}(\tau) \leq \xi$, there exists

$\widehat{\tau}(\xi) \in (\frac{1}{2}, 1)$ such that, $\xi \geq \overline{\xi}(\tau) \Leftrightarrow \tau \geq \widehat{\tau}(\xi)$. Moreover, suppose that $\widehat{\Delta}_x(\tau) \leq 1+q$. We can write the expected payoff and stock price as functions of τ as follows

$$\mathbb{E}[p(\tau)] = \frac{\mu - \gamma\sigma^2 + \left(\frac{\tau\widehat{\Delta}_x(\tau)}{1+q}\gamma\sigma^2 - c\right)\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)}}{1+q-rq} \tag{42}$$

and

$$\mathbb{E}[v(\tau)] = \frac{\mu + rq\mathbb{E}[p(\tau)] - c\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)}}{1+q}. \tag{43}$$

The manager's expected utility can be rewritten as:

$$\begin{aligned}
\mathbb{E}[U_m(\tau)] &= \omega\mathbb{E}[p(\tau)] + (1-\omega)\mathbb{E}[v(\tau)] \\
&= [\omega + (1-\omega)z]\,\mathbb{E}[p(\tau)] + (1-\omega)\frac{\mu - c\cdot\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)}}{1+q} \\
&= [\omega + (1-\omega)z]\left(\frac{\mu-\gamma\sigma^2}{1+q-rq} + \frac{\frac{\tau\widehat{\Delta}_x(\tau)}{1+q}\gamma\sigma^2 - c}{1+q-rq}\cdot\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)}\right) + (1-\omega)\frac{\mu - c\cdot\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)}}{1+q} \\
&= [\omega + (1-\omega)z]\,\frac{\mu-\gamma\sigma^2}{1+q-rq} + (1-\omega)\frac{\mu}{1+q} \\
&+ \left([\omega + (1-\omega)z]\,\frac{\frac{\tau\widehat{\Delta}_x(\tau)}{1+q}\gamma\sigma^2 - c}{1+q-rq} - (1-\omega)\frac{c}{1+q}\right)\cdot\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)} \\
&= [\omega + (1-\omega)z]\,\frac{\mu-\gamma\sigma^2}{1+q-rq} + (1-\omega)\frac{\mu}{1+q} \\
&+ \frac{c}{1+q}\left(\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right)\cdot\mathbf{1}_{\tau\geq\widehat{\tau}(\xi)}
\end{aligned}$$

Notice that $\frac{\tau}{2\tau-1}$ decreases in τ. Thus, the manager chooses $\tau = \widehat{\tau}(\xi)$ if $\frac{\widehat{\tau}(\xi)}{2\widehat{\tau}(\xi)-1}(1-\omega) - \frac{1}{1-z} > 0$, and $\tau = \frac{1}{2}$ otherwise. Notice that

$$\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z} > 0 \Leftrightarrow \tau < \frac{1}{1+\omega+(1-\omega)z}$$

Thus, the manager chooses $\tau = \widehat{\tau}(\xi)$ if $\widehat{\tau}(\xi) < \frac{1}{1+\omega+(1-\omega)z}$, and $\tau = \frac{1}{2}$ otherwise.

Next, we plug in $\tau = \frac{1}{1+\omega+(1-\omega)z}$ into $\widehat{\Delta}_x(\tau)$ to check whether the blockholder's position is less than $1+q$. It follows that $\widehat{\Delta}_x\left(\frac{1}{1+\omega+(1-\omega)z}\right) \leq 1+q$ is equivalent to $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$. In this case, the blockholder can afford to implement tilting at $\tau = \frac{1}{1+\omega+(1-\omega)z}$. If instead

$c > \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$, then the blockholder cannot implement tilting for any $\tau < \frac{1}{1+\omega+(1-\omega)z}$ because $\widehat{\Delta}_x(\tau)$ is decreasing in τ. Hence, the manager chooses $\tau = \frac{1}{2}$.

Suppose $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$ and recall that $\widehat{\tau}(\xi)$ satisfies $\xi = \overline{\xi}(\tau)$, and since $\overline{\xi}(\tau)$ is a decreasing function,

$$\widehat{\tau}(\xi) < \frac{1}{1+\omega+(1-\omega)z} \Leftrightarrow$$
$$\xi > \overline{\xi}\left(\frac{1}{1+\omega+(1-\omega)z}\right)$$

Next, we use the expression for $\overline{\xi}$ to re-write the condition above as:

$$\xi > 1 - \frac{z^{-1}\mu - \gamma\sigma^2}{z^{-1}\mu - \gamma\sigma^2 + c\frac{(1-z)(1-\omega)}{z+\omega-z\omega}} \equiv \overline{\xi}_{\text{disc}}$$

The right-hand side of this condition increases in c, γ, σ and it decreases in μ, ω. It is hump-shaped in z, and thus in r, q.

Finally, we solve for the lowest value of $\tau \in \left(\frac{1}{2}, \frac{1}{1+\omega+(1-\omega)z}\right)$ that satisfies $\widehat{\Delta}_x(\tau^{\min}) = 1 + q$. This leads to $\tau^{\min} = \frac{1}{2}\left(1 + \frac{c(1-\omega-(1-\omega)z)}{\gamma\sigma^2(\omega+(1-\omega)z)}\right)$. For any $\xi \geq \overline{\xi}(\tau^{\min})$, the manager sets $\tau^* = \tau^{\min}$ because any $\tau < \tau^{\min}$ would lead to exclusion. ■

Proof of Proposition 8. Given x, a, and y, the stock price is given by:

$$p(x,a,y) = \frac{\mu - ca - \left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{44}$$

Thus, A's profit is given by:

$$\begin{aligned}
\Pi_A(y) &= y\left(v(x,a,y) - p(x,a,y)\right) \\
&= y\left(\frac{\mu + rqp(x,a,y) - ac}{1+q} - p(x,a,y)\right) \\
&= y\left(\frac{\mu - (1-q-rq)\,p(x,a,y) - ac}{1+q}\right) \\
&= y\left(\frac{\mu - \left[\mu - ca - \left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2\right] - ac}{1+q}\right) \\
&= y\frac{\left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2}{1+q}
\end{aligned}$$

and so his trade is given by:

$$y^*(x) = \arg\max_y \Pi_A(y) = \frac{1+q-x}{2}$$

which yields a profit of

$$\Pi_A(y^*(x)) = \left(\frac{1}{2}\frac{1+q-x}{1+q}\right)^2\gamma\sigma^2.$$

Thus, B expects the stock price to be

$$\begin{aligned}
p(x,a,y^*(x)) &= (1-\eta)\frac{\mu - ca - \left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq} + \eta\frac{\mu - ca - \left(1 - \frac{x+y^*(x)}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{x}{1+q} - \eta\frac{y^*(x)}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{x}{1+q} - \eta\frac{\frac{1+q-x}{2}}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{\eta}{2}\right)\left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq}
\end{aligned}$$

The manager chooses $a = 1$ if and only if

$$\begin{aligned}
\omega p\left(x\left(1\right),1\right)+\left(1-\omega\right)\frac{\mu+rqp\left(x\left(1\right),1\right)-c}{1+q} &> \omega p\left(x\left(0\right),0\right)+\left(1-\omega\right)\frac{\mu+rqp\left(x\left(0\right),0\right)}{1+q} \\
\left[\omega+(1-\omega)z\right]\left[p\left(x\left(1\right),1\right)-p\left(x\left(0\right),0\right)\right] &> (1-\omega)\frac{c}{1+q} \\
x\left(1\right)-x\left(0\right) &> \frac{\left(1+q\right)c}{\left(1-\frac{\eta}{2}\right)\gamma\sigma^2\left[\omega+(1-\omega)z\right]}
\end{aligned}$$

B chooses tilting if and only if

$$\begin{aligned}
\xi &\geq \frac{rq\left(p\left(1\right)-p\left(0\right)\right)}{\mu+rqp\left(1\right)} \\
&= rq\frac{\frac{-c+\left(1-\frac{\eta}{2}\right)\left(\frac{\Delta_x}{1+q}\right)\gamma\sigma^2}{1+q-rq}}{\mu+rq\frac{\mu-c-\left(1-\frac{\eta}{2}\right)\left(1-\frac{\Delta_x}{1+q}\right)\gamma\sigma^2}{1+q-rq}} \\
&= \frac{-c+\left(1-\frac{\eta}{2}\right)\left(\frac{\Delta_x}{1+q}\right)\gamma\sigma^2}{\frac{\mu}{z}-c-\left(1-\frac{\eta}{2}\right)\left(1-\frac{\Delta_x}{1+q}\right)\gamma\sigma^2} \\
&= \frac{\left(1-\omega\right)c}{\left(1-\omega\right)c+\left(\frac{\mu}{z}-\left(1-\frac{\eta}{2}\right)\gamma\sigma^2\right)\left(\omega+\frac{z}{1-z}\right)}.
\end{aligned}$$

The condition $x(1) \leq (1+q)$ is equivalent to $c \leq \gamma\sigma^2[\omega+(1-\omega)z]\left(1-\frac{\eta}{2}\right)$. If $c > \gamma\sigma^2[\omega+(1-\omega)z]\left(1-\frac{\eta}{2}\right)$, then the blockholder cannot implement tilting and chooses $x(1)=x(0)=0$.

■