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JEL Classification: N/A

Keywords: Brand Dilution, Brand Licensing, Double-sided Moral Hazard, Reciprocal Effect

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An ugly face, really?
A theoretical investigation into the causes of brand dilution*

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1 Introduction

Leveraging brand equity is a major strategy for companies striving for growth. By extending their brands beyond the original product category, companies can enter new markets and create new businesses. Recent decades have seen the launch of myriad brand extensions: approximately 90% of new products are extensions of existing brand names (Keller et al. 2008, Hariharan et al. 2014).

Sometimes the extension products are developed in house. However, entering into a new business may require resources and competencies that the brand owner does not have and that may be too expensive to acquire or develop internally. In these cases, companies may seek the benefits of specialization by choosing venture partners and entering into licensing agreements with them. This strategy is used widely, to the point that revenues from brand licensing are counted in the hundreds of billions.[1]

The costs and benefits of brand extensions are clear. On the one hand, companies can transfer the reputation and image of the parent brand onto the new products,[2] creating positive spillover effects on demand. Besides, a successful extension may improve the parent brand's name. On the other hand, an unsuccessful extension may damage the original brand and reduce the demand for the company's core products. The literature refers to the former case as positive reciprocal effect, or brand enhancement, and to the latter as negative reciprocal effect, or brand dilution (Keller and Aaker 1992, Gürhan-Canli and Maheswaran 1998).

Understanding when brand extension is more or less likely to dilute a brand name is an important matter in management and marketing, and the literature has devoted considerable attention to it (e.g. Loken and John 1993, John et al. 1998, Swaminathan et al. 2001, Keller and Sood 2003). Specifically regarding the mode of extension, the empirical literature has found that brand dilution tends to be associated with licensing more than with internal development (Choi 2001, Colucci et al. 2008, Jayachandran et al. 2013, Bărbulescu Robinson et al. 2015). Though much of the existing evidence is anecdotal,[3] the correlation seems robust and is often given a causal interpretation: a

[1] See e.g. LIMA (2020) and Bărbulescu Robinson et al. (2015). Top global licensors include the Walt Disney Company, Hasbro and Warner Bros (Global, 2021). Historically, the use of brand licensing dating back to the fashion industry in the 1950s, when Christian Dior, Chanel and Pierre Cardin started licensing their names. In that industry, brand licensing is still widespread today (Saviolo and Giannelli, 2001).

[2] The parent brand is an existing brand used to create a new product called the brand extension (Aaker and Keller 1990, John et al. 1998).

[3] An infamous example is the brand name of Pierre Cardin – once the epitome of brand extension *via* licensing, with more than 500 licensing agreements including cigarettes, baseball caps and toilet-seat covers. Another example

switch from internal development to licensing, that is to say, is taken to cause a higher risk of brand dilution (see e.g. Quelch 1985, Raugust 2012). The argument is that licensing may create scope for opportunistic behaviors on the part of the licensee that may damage the parent brand, whereas a brand owner that develops the new product in house never loses control of the extension and thus cannot cause any harm to the parent brand.

However, drawing causal inferences from empirical correlations is problematic. In the absence of experimental or quasi-experimental evidence, theory is needed to support causal claims. To the best of our knowledge, though, no formal theory that substantiates or denies such claims has been articulated so far. In fact, the choice of the mode of extension is an issue nearly untouched in the theoretical management literature.[4]

This paper fills that gap. It develops a theoretical model where the choice of whether to engage in brand extension, the choice of the organizational mode (i.e., internal development or licensing), and the investments in the quality of the core and extension products, which ultimately determine the likelihood of brand enhancement or brand dilution, are all endogenously determined. Using this model, we show that when the brand owner is nearly indifferent between internal development and licensing, *a switch to licensing always increases the quality of the extension product and thus reduces the likelihood of brand dilution.*

Evidently, this finding does not support the conventional wisdom mentioned above. It implies that the correlation observed in the data does not arise because licensing causes brand dilution; rather, it must arise because companies find it profitable to resort to brand licensing when the risk of brand dilution is higher. This finding has the potential to turn the managerial implications of the empirical evidence upside down.

The mechanism behind our result is based on two simple and general properties of the model's equilibrium. First, firms always under-invest in the quality of both the core and the extension products compared to the first-best solution (which is obtained under vertical integration, or with complete contracts). The reason is that with incomplete contracts the benefits of specialization may be obtained only at the cost of contractual inefficiencies that reduce the incentives to invest

is Yves Saint Laurent, which managed 60 licensing contracts in 2001 and then cut them back to 15 the next year (Corbellini and Saviolo, 2014).

[4]In their empirical research on the determinants of the mode of extension, Colucci et al. (2008) propose and test hypotheses based on transaction costs economics but do not put forward any formal model.

in quality. The under-investment result implies that increasing the quality of either product leads to higher profits. Second, the investment in the quality of the core product is always higher under in-house development than under licensing. The intuition here is that under licensing the brand owner internalizes only partially the positive externality that the reputation of the brand exerts on the demand for the licensee's product.

These properties imply that if the quality of the extension product were the same under both organizational modes, internal development would be more profitable. For the two modes of extension to be equally profitable, the quality of the extension product must be higher under licensing.

This result has various counterintuitive implications. Consider, for instance, the impact of an increase in the strength of the demand spillovers mentioned above on the choice of the organizational mode. Since the weakness of licensing is that those spillovers are internalized only partially, it might seem that stronger spillovers should make in-house development relatively more profitable. In fact, however, we find that stronger spillovers may cause a move from in-house development to licensing rather than the other way around.

After developing the model, we present a detailed analysis of the effects of changes in the exogenous variables – including the magnitude of the reciprocal effect, the size of the extension market, the perceived distance between the core and extension products, and their technological distance – and we then discuss the managerial and empirical implications of the results. Here, a final remark is in order. Our model regards brand dilution as a deliberate strategic choice. This marks a notable difference with Aaker (1990)'s view of brand dilution as an unintended consequence of a poorly executed extension strategy – "the ugly" face of brand extension. In fact, brand extension can be profitable even if it results in brand dilution. Thus, companies may willingly pursue extensions that are likely to damage their brand names in order to "monetize" the brand.

The remainder of the paper is organized as follows: Section 2 presents the main ingredients of the model and reviews the literature that supports our modeling choices. Section 3 presents the assumptions of the baseline model in a detailed way. Section 4 solves the model for the case of a single extension product, contrasting the case of in-house development and licensing. Section 5 analyzes the causes of brand dilution and the empirical correlations generated by the model. Section 6 analyzes several extensions of the baseline model. Section 7 discusses the theoretical and empirical implications of the results and concludes the paper.

2 Elements of the model

In this section, we discuss the main elements of the model and review the empirical literature that underpins our modeling choices.

It is important to stress from the outset that we do not aim here at providing micro-foundations for the demand externalities mentioned in the introduction and discussed in greater detail below. Starting from the seminal work of Wernerfelt (1988), the industrial organization literature has devoted considerable effort to understanding how brand names can convey information to rational consumers and hence affect their behavior.[5] Our contribution addresses a different issue; it focuses on the consequences of the demand externalities rather than on their causes. We build on the evidence provided by the empirical marketing literature, which has abundantly documented the direct and reciprocal spillovers and their main properties, and simply posit that these effects exist. For our purposes, consumers may be thought of as "black boxes," which may be driven by purely psychological or behavioral mechanisms. Integrating rigorous micro-foundations of consumer behavior into the analysis is a task left for future research.

Given this modeling strategy, our model has three main ingredients: the demand externalities associated with brand extension, the benefits of specialization that create the incentive to license, and the double-sided moral hazard problem that emerges under licensing. We now present each ingredient in turn, reviewing the empirical evidence that supports our modeling assumptions.

2.1 Demand externalities

As noted, there are two types of demand spillovers: direct (i.e., from the brand owner's core product to the extension product) and reciprocal (i.e., the other way around).[6] We assume that the direct spillover is always positive: creating and maintaining brand value increases the demand for the extension product. As the marketing literature has abundantly shown, the extent to which consumers transfer their brand perceptions from the original incarnation of the parent brand (the core product) to its new application (the extension product) depends on the perceived similarity between the two (see e.g., Aaker and Keller (1990), Park et al. (1991), Broniarczyk and Alba

[5] See e.g., Tadelis (1999), Cabral (2000), Choi (2001), Hakenes and Peitz (2008), Cabral (2009), Miklós-Thal (2012), Rasmusen (2016), and the references therein. The industrial organization literature often refers to brand extension as of "umbrella branding."

[6] In our model the core product is the category representative of the parent brand.

(1994), Völckner and Sattler (2006), Miniard et al. (2018)). Accordingly, we shall posit that the direct effect decreases with the *perceived distance* between the products.

What distinguishes brand extension from other frameworks where direct demand externalities also arise is the reciprocal effect that the extension has on the demand for the core product. In this respect, the empirical literature has shown that the reciprocal effect may have either sign, depending on the quality of the products as perceived by consumers. A high-quality extension strengthens the parent brand and increases the sales of established products, leading to an enhancement in brand equity (Czellar 2003, Völckner and Sattler 2006, Salinas and Pérez 2009, Pina et al. 2013, Michel and Donthu 2014). Conversely, a low-quality extension not only entails a low demand for the extension product but may also damage the sales of the parent brand's existing products, thereby leading to a dilution of brand equity (Aaker and Keller 1990, Loken and John 1993, Keller and Sood 2003).

Furthermore, the literature has shown that the magnitude of the reciprocal effect, like that of the direct effect, depends on the perceived distance between the core and the extension products. In other words, higher similarity leads to a fuller transfer of consumer perceptions in both directions. For example, consumer acceptance of Gucci sneakers may have a noticeable impact on the willingness to buy Gucci handbags, but the quality of Ferrari clothing is unlikely to appreciably affect the demand for Ferrari cars. This is true both when the reciprocal effect is positive (Keller and Aaker 1992, Gürhan-Canli and Maheswaran 1998, Swaminathan et al. 2001, Pina et al. 2013) and when it is negative (Gürhan-Canli and Maheswaran 1998, Keller and Sood 2003, Sood and Keller 2012).

Accordingly, we shall allow for the possibility that the reciprocal effect may be either positive or negative. Whatever its sign, its magnitude will depend on the perceived distance between the products.

2.2 Benefits of specialization

The second ingredient of the model are the benefits of specialization. We assume that the brand owner is less efficient than the specialized licensee in supplying the extension product. The notion that delegating certain tasks to external, specialized agents may have efficiency advantages over in-house production is so common in the management literature that it hardly needs elaboration.

The benefits of specialization have been abundantly documented in many areas of economics and management and lie at the heart of the transaction-cost theory of the firm (Coase 1937, Willamson 1991).

The literature on brand extension, in particular, has shown that the efficiency advantages are larger, the higher the dissimilarity in the manufacturing and marketing technologies of the core and extension products (Colucci et al., 2008). In keeping with these findings, we assume that the licensee's greater efficiency depends on the *technological distance* between the products. The technological distance may or may not be related to the perceived distance.

2.3 Double-sided moral hazard

The final key ingredient of the analysis is the double-sided moral hazard problem that emerges under brand licensing. Generally speaking, a double-sided moral hazard problem arises when (*i*) both parties of a relationship make investments that can increase the value from the relationship; (*ii*) these investments are not perfectly verifiable and hence are non-contractible; and (*iii*) the interests of the two parties are not coincident. Under these conditions, incomplete contracts cannot achieve the first best and must trade off the parties' incentives to invest as best as they can. Since the parties' interests cannot be aligned perfectly due to the incompleteness of contracts, opportunistic behaviors inevitably emerge. The resulting inefficiencies represent the comparative dis-advantage of licensing.

Clearly, the conditions that create a double-sided moral hazard problem do occur in brand extension cases.[7] The literature has shown that the licensor makes important investments in reputation and brand image, and that the licensee in turn invests considerable resources in the design, manufacturing and distribution of the extension product (Raugust, 2012). It has also shown that neither agent can perfectly monitor the activity of the other, which makes those investments largely non-contractible, and that this double-sided moral hazard influences the licensing contract structure (Jayachandran et al., 2013). Finally, it is evident that the licensor and the licensee have different

[7] The litigation between Calvin Klein and its licensee Warnaco Group neatly illustrates the kind of opportunistic behaviors that may arise in these circumstances. In 2000, Calvin Klein charged Warnaco Group with brand equity dilution for breaching the jeanswear licensing and distribution contract by distributing products through warehouse clubs that the brand owner considered unacceptable channels. The same year, Warnaco filed countersuit, accusing Calvin Klein of ineffective brand advertising and thus of damaging its business (Fournier and Boer, 2002). In this example, moral hazard is evidently double sided.

goals.[8]

Similar double-sided moral hazard problems have been analyzed in the industrial organization literature on franchising and technological licensing (e.g., Bhattacharyya and Lafontaine (1995), Choi (2001), Hernández-Murillo and Llobet (2006), Arora et al. (2013), Tauman and Zhao (2018) and the references therein). However, the case of brand licensing is different because the brand owner earns profits from both the core and the extension product, but only the profits from the extension markets can be shared with the licensee. Furthermore, the presence of the reciprocal effect has been ignored in the literatures on franchising and patent licensing. This effect implies that the brand owner must incentivize the licensee's effort not only to increase revenues from the extension but also to preserve its revenues from the core business. These differences change the nature of the double-sided moral-hazard problem and call for a specific analysis of the contractual arrangements that may arise in brand licensing.

3 Model assumptions

In this section, we detail the assumptions of the baseline model.

3.1 Supply

There are two kinds of products: the core product, which is manufactured and marketed by the brand owner, and a non-core, extension product. (The analysis readily extends to multiple extensions, as discussed below.)

The non-core product can only be brought to the market under the parent brand name, either directly by the brand owner, or else indirectly, *via* licensing agreements. A number of specialized licensees can supply the product; the brand owner plays one potential licensee against the other and thus has all the bargaining power. (We shall relax this assumption in Section 6.)

The perceived quality of the core and the extension products is determined by the efforts exerted by their respective suppliers at various stages of the production process, such as design,

[8] The final goal of both is to maximize their respective profits. As for the intermediate goals, the licensor aims to nurture and strategically orient the brand, searching for an exclusive image and product positioning that increases brand awareness. The licensee, on the other hand, aims to exploit consumers' brand awareness in order to push commercial diffusion of the extension.

manufacturing, or marketing. We denote these efforts by e_C and e_E, respectively.[9]

The cost of exerting effort on the core product is $\frac{1}{2}\beta e_C^2$. As for the extension product, the cost is $\frac{1}{2}\beta e_E^2$ under licensing,[10] when the product is supplied by a specialized licensee, and $\frac{1}{2}\theta\beta e_E^2$ under internal development. These costs are fixed; variable production costs are normalized to zero. The parameter $\theta > 1$ captures the brand owner's reduced efficiency outside of its core business. It may be interpreted as the technological distance between the core and non-core products.

3.2 Demand

To eschew confounding strategic effects, we assume that the markets for the core and the extension products are both monopolistic. We also assume that the products are neither substitutes nor complements, so the demand for each does not depend on the other's price. (This distinguishes brand extension from e.g. the choice of the product line, where the different products are substitutes.) However, demands are related because of the direct and reciprocal demand spillovers discussed above. The magnitude of these effects depends on the distance between the products, in the space of consumer perceptions. We capture such perceived distance by a dissimilarity parameter, α, that is taken to range from 0 to 1 (a normalization).[11]

To otain closed-form solutions, we posit specific functional forms for demand. In particular, the demand for the extension product q is taken to be linear:

$$q = \rho\left[e_E + (1-\alpha)e_C - p\right], \tag{1}$$

where p is the price and ρ is a scale parameter that measures the size of the non-core market. Equation (1) says that the demand for the extension is affected both by its own quality e_E and by the quality of the core product e_C. The dependence on e_C reflects the direct effect that the brand name has on the demand for the extension product. This effect, which is always positive, is stronger, the closer the two products in the space of consumer perceptions. Therefore, all else

[9]As a rule, we use latin letters to denote endogenous variables and greek letters for exogenous parameters. The one exception is profits, which are denoted by π.

[10]The possibility that β may be different for the core and the extension products is analytically equivalent to changes in the parameters μ and ρ introduced below.

[11]The perceived distance α may be correlated with the technological distance θ. In our analysis, however, we treat the two parameters as independent. When they change in a correlated way, one can just combine our results on the effects of separate changes in α and θ.

equal, the demand for the extension product decreases with its perceived distance from the core product. The linear specification of demand (1) yields a quadratic profit function, which implies that the efforts e_C and e_E are complements. This complementarity is an important feature of the model.

As for the core product, we assume that it is demanded by $\mu > 0$ symmetric consumers, each of whom purchases either 0 or 1 units and has a willingness to pay of $e_C + \lambda(1 - \alpha)e_R$, where

$$e_R = e_E - \varepsilon \tag{2}$$

is the gap between the quality of the extension and a benchmark quality level ε. (The benchmark is exogenous in baseline model, but in an extension we shall allow it to depend on the quality of the core product.) This specification yields a rectangular demand curve, so the profit from the core product is a linear function of the efforts e_C and e_E.[12] The parameter μ represents the size of the market for the core product. The parameter λ instead measures the relevance of the reciprocal spillover effect.[13] We assume that it ranges between 0 and 1, so that the brand owner's effort on the core product always remains the main determinant of its demand. A summary of notation used is presented in Table 1.

Three further remarks are in order. First, the sign of the reciprocal effect is endogenous. It may be positive (brand enhancement) or negative (brand dilution), depending on whether e_E is high or low. Second, like the direct effect, the reciprocal effect depends on the perceived distance between the products. Thus, the relevance of the reciprocal effect (whatever its sign) is lower for more distant products (higher α). Third, in the limiting case $\lambda = 0$ the reciprocal effect vanishes, and our model becomes similar to existing models of franchising or patent licensing. What distinguishes brand licensing is the possibility, which arises when $\lambda > 0$, that the extension may enhance or dilute the parent brand.

[12]This simplifies the analysis without destroying the strategic effects of interest, as the complementarity of the effort levels e_C and e_E is already guaranteed by our specification of the demand for the extension product.

[13]Among other things, this may depend on the size of the extension market ρ. If this is so, then the parameters λ and ρ will co-vary.

Table 1: Notation

Parameters

Notation	Domain	Meaning
α	[0,1]	Perceived distance
β	normalized to 1	Cost of effort
θ	[1,+∞)	Technological distance
μ	(0,+∞)	Size of the core market
ρ	[0,1]	Size of the non-core market
λ	[0,1]	Magnitude of reciprocal effect
ε	(0,+∞)	Benchmark level of quality

Variables

Notation	Meaning
e_C	Quality of the core product
e_E	Quality of the extension product
$e_R = e_H - \varepsilon$	Reciprocal effect
q	Demand of the extension product
p	Price of the extension product
π_B	Brand owner's profit
π_L	Licensee's profit
s	Royalty rate
F	Fixed fee

3.3 Timing

The timing of choices is as follows. First, the brand owner chooses whether to engage in brand extension and in which way. If it opts for in-house development, the brand owner then simply makes all remaining choices (effort levels and prices). If instead the brand owner engages in licensing, it sets the terms of the licensing contract offered to the licensee. We assume that effort levels are not contractible,[14] so payments may depend only on output q. Initially, we restrict attention to pure royalty contracts that specify a non-negative royalty rate $s \geq 0$ per unit of output (among the extensions, we shall allow for two-part tariffs $\{s, F\}$ where F is a fixed fee, so that the licensing revenue is $sq + F$). Given the brand owner's offer, the licensee then chooses whether to sign the contract or not. If the contract is signed, the brand owner and the licensee simultaneously and independently choose e_C and e_E. Finally, the brand owner and the licensee set the prices of the products they supply. This sequence of moves is depicted in Figure 1.

[14] If efforts levels were verifiable and contractible, firms could achieve the vertically integrated, efficient solution.

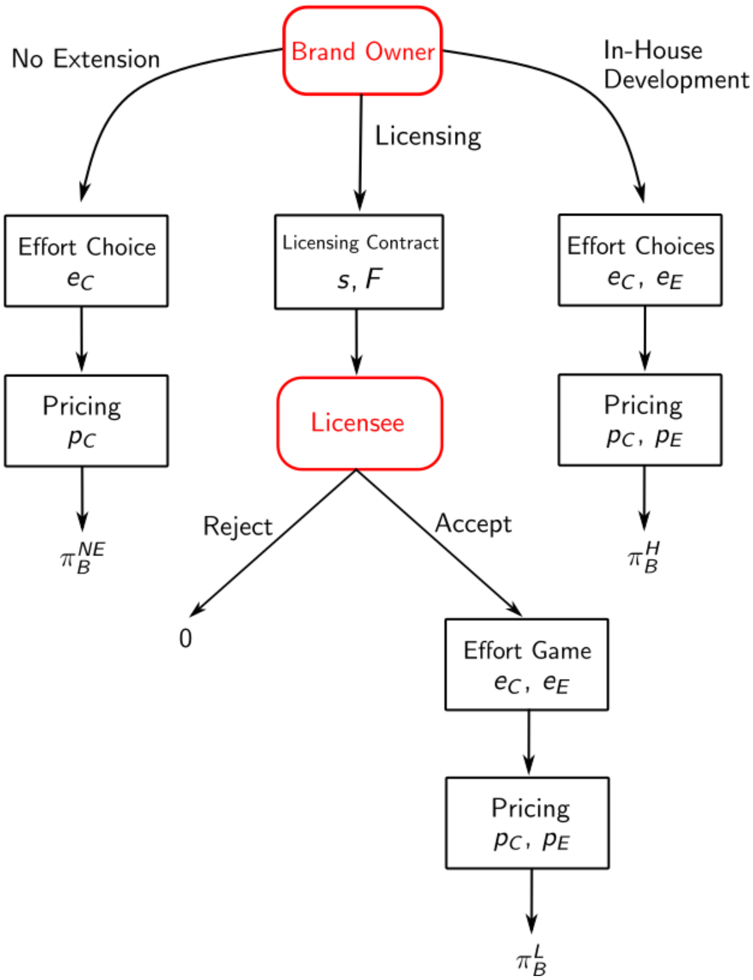


Figure 1: Timing of the game

4 Equilibrium

In this section, we derive the model's equilibrium for each organizational mode and present some preliminary comparative statics results. After deriving the equilibrium for each case, in the next section we shall turn to the choice of the optimal mode of extension.

To ensure subgame perfection we solve the model backwardly, starting from the pricing stage.

4.1 Prices

Equilibrium prices are easy to determine. In the market for the core product, the brand owner extracts all consumer surplus by setting a price equal to the consumers' willingness to pay, $e_C + \lambda(1-\alpha)e_R$. Thus, the brand owner nets a profit of $\mu\,[e_C + \lambda(1-\alpha)e_R]$. In the market for the extension product, the seller charges the monopoly price

$$p^M = \frac{e_E + (1-\alpha)e_C + s}{2}. \tag{3}$$

The corresponding output and profits are, respectively:

$$q^M = \rho\frac{e_E + (1-\alpha)e_C - s}{2}, \tag{4}$$

$$\pi^M = \rho\frac{[e_E + (1-\alpha)e_C - s]^2}{4}. \tag{5}$$

(Under internal development, price, output and profits from the extension are given by the above formulas with $s = 0$.) Note that $\frac{\partial^2 \pi^M}{\partial e_C \partial e_E} = \frac{\rho}{2}(1-\alpha) > 0$, confirming that efforts e_C and e_E are complements.

4.2 Effort levels

Moving backwards, we next analyze quality investments. Here we must distinguish between the case of internal development and licensing.

4.2.1 In-house development

When the extension is developed internally, the brand owner's overall profit is

$$\pi_B = \mu\,[e_C + \lambda(1-\alpha)e_R] + \rho\frac{[e_E + (1-\alpha)e_C]^2}{4} - \frac{1}{2}\theta\beta e_E^2 - \frac{1}{2}\beta e_C^2. \tag{6}$$

The first term on the right-hand side is the profit from the core business, the second term is the profit from the extension product, and the last terms are the costs of the investments in the quality of the products.

Plainly, the profit function is homogeneous of degree one in μ, ρ and β, so we can normalize setting $\beta = 1$. We also assume that $\rho < 1$ to ensure concavity of the profit function.[15]

The optimal effort levels e_C and e_E are then determined by the following first-order conditions:

$$e_C = \mu + (1-\alpha)\frac{\rho}{2}\left[e_E + (1-\alpha)e_C\right], \tag{7}$$

$$\theta e_E = \frac{\rho}{2}\left[e_E + (1-\alpha)e_C\right] + (1-\alpha)\lambda\mu. \tag{8}$$

These conditions say that for both products, the marginal costs of quality provision (the left-hand sides) must be equal to the marginal benefits (the right-hand sides). The marginal benefits comprise two terms. The first is the impact of a product's quality on the profit obtained from that product, the second is the impact on the profit obtained from the other product. These latter terms reflect the demand spillovers, and as such are proportional to the similarity between the core and non-core product, $(1-\alpha)$. In addition, the second term in (8) is proportional to λ, the intensity of the reciprocal effect.

Solving the system of first-order conditions gives the optimal effort levels e_C^H and e_E^H. The explicit formulas are reported in Appendix A, where we prove the following:

Proposition 1. *Under in-house development, both effort levels e_C^H and e_E^H are decreasing in α and θ and increasing in λ, μ and ρ.*

The intuition is as follows. First, an increase in θ makes it more costly for the brand owner to increase the quality of the extension product and hence reduces the corresponding investment. Since efforts are complements, this reduces also the investment in the core product. Next, an increases in μ or ρ increases the size of the markets, and hence the benefits from quality investments. The complementarity between the efforts implies that an increase in the size of either market increases the investment in the quality of both products. By the same logic, an increase in the perceived distance α, which reduces the effective size of the market for the extension, decreases both investment levels. Finally, an increase in λ raises the brand owner's incentive to invest in the extension, and hence also in the core product (once again, by the complementarity of the efforts).

Note that by setting $\theta = 1$ in the above formulas, one obtains the first-best solution from the viewpoint of the firms, where industry profits are maximized. This is the solution that would

[15] Besides guaranteeing concavity, the assumption that ρ is not too large captures the notion that the brand owner obtains most of its revenue from, and devotes most of its efforts to, its core business.

arise under vertical integration, or with complete contracts. Thus, it follows immediately from Proposition 1 that under internal development both efforts are below their first-best levels.

Corollary 1. *Under in-house development, the equilibrium exhibits under-investment in the quality of both products.*

Substituting the equilibrium efforts e_C^H and e_E^H into (6) one finally obtains the brand owner's profit under in-house development, π_B^H. The explicit formula is reported in Appendix B.

4.2.2 Licensing

Next consider the case where the brand owner engages in licensing. Given the timing of moves, we first calculate the equilibrium efforts for any possible royalty rate s, and we then determine the optimal rate s^*.

The effort game. Under brand licensing, the brand owner and the licensee simultaneously choose their effort levels to maximize their respective profits

$$\pi_B = \mu \left[e_C + \lambda(1-\alpha)e_R \right] + s\rho \frac{(1-\alpha)e_C + e_E - s}{2} - \frac{1}{2}e_C^2, \tag{9}$$

where the middle term in (9) is the licensing revenue sq^M, and

$$\pi_L = \rho \frac{\left[(1-\alpha)e_C + e_E - s\right]^2}{4} - \frac{1}{2}e_E^2. \tag{10}$$

We look for the Nash equilibrium of this stage of the game. The best-response functions are given by the following conditions:

$$e_C = \mu + (1-\alpha)\frac{\rho}{2}s, \tag{11}$$

$$e_E = \frac{\rho}{2}\left[(1-\alpha)e_C + e_E - s\right]. \tag{12}$$

The difference between (7) and (11) is due to the fact that the brand owner now internalizes the direct spillover effect only partially, i.e., only to the extent that it increases the revenue from licensing.[16] On the other hand, there are three differences between (12) and (8). First, the marginal cost of effort is lower (i.e., e_E rather than θe_E), reflecting the superior efficiency of the specialized licensee. Second, the licensee does not internalize the reciprocal effect, and thus the perceived marginal benefit of its effort comprises only the direct effect. Third, the presence of the royalty rate reduces the output of the non-core product, and hence the marginal benefit of increasing its quality.

Given the best-response functions, it is easy to find the Nash equilibrium efforts for any level of the royalty rate s. Condition (11) immediately gives $e_C(s)$. Substituting it into (12), one gets:

$$e_E(s) = \rho \frac{(1-\alpha)\mu - \left[1-(1-\alpha)^2\frac{\rho}{2}\right] s}{2-\rho}, \tag{13}$$

if the expression on the right-hand side is positive, i.e., if s is not too high; otherwise, $e_E(s)$ vanishes. The comparative statics of $e_C(s)$ and $e_E(s)$ can be obtained directly from these formulas:

Lemma 1. *Under licensing, for each level of the royalty rate s both effort levels $e_C(s)$ and $e_E(s)$ are independent of λ, decreasing in α, and increasing in μ and ρ. Furthermore, $e_C(s)$ is increasing and $e_E(s)$ is decreasing in s.*

The effects of changes in the perceived distance α and the size parameters μ and ρ are the same as under internal development, and for the same intuitive reasons. Under licensing, however, efforts are independent of θ. They are also independent of λ, as the reciprocal effect is not internalized by the brand owner at this stage. (It will be internalized in the choice of the royalty rate, which is analyzed below.)

The comparative statics with respect to s is intuitive. An increase in the royalty rate reduces the output of the extension product and hence the licensee's incentive to exert effort. The incentive may even vanish if the royalty rate is too high. On the other hand, an increase in the royalty rate raises the brand owner's effort, as the brand owner obtains a higher return from expanding the demand for the extension product.

Comparing the incentives to exert efforts under licensing with the first-best, which as noted coincides with the internal-development equilibrium with $\theta = 1$, one immediately obtains the

[16] Even though s will be determined later, one can already see that the internalization is partial as $s < e_E + (1-\alpha)e_C$, an inequality that must hold as otherwise the output level would vanish.

following:

Corollary 2. *Under licensing, the equilibrium exhibits under-investment in the quality of both products.*

Thus, both organizational modes lead to under-investment in quality. This implies that higher effort levels are good for industry profits.

The royalty rate. Moving one step backwards, let us now consider the choice of the royalty rate s. This involves a complex trade-off. First, there is the standard trade-off between profit margins and volumes, which arises because the licensing revenue is s times q^M, and the volume q^M decreases with s. In addition, when setting the royalty rate the brand owner anticipates the impact of s on the effort levels $e_C(s)$ and $e_E(s)$. We know from Lemma 1 that a higher royalty rate increases the brand owner's effort e_C but reduces the licensee's effort e_E. This entails another non trivial trade-off, as both efforts are below their respective first-best levels (Corollary 2).

In our fully specified model, the optimal royalty rate can be calculated explicitly:[17]

$$s^* = 2\mu(1-\alpha)\frac{2(1-\lambda)+(1-\alpha)^2\lambda\rho}{8-(1-\alpha)^2\rho\,(2+\rho)}. \tag{14}$$

In view of condition $\rho < 1$, it is easy to verify that $s^* > 0$ and that the associated monopoly output q^M is also strictly positive. From (14), the following comparative statics results immediately follow:

Lemma 2. *The optimal royalty rate s^* is decreasing in the size of the reciprocal effect, λ, and in the perceived distance between the core and non-core product, α. It is increasing in the size of the core and non-core markets, μ and ρ.*

The reason why the optimal royalty rate depends negatively on the reciprocal effect parameter λ is simple. As λ increases, the reciprocal demand spillover gets larger. As a consequence, the brand owner wants to incentivize the licensee's effort more strongly. This is achieved by reducing s, at the cost of discouraging its own investment.

As for the size parameters μ and ρ, we know that an increase in either one has a positive effect on both effort levels. As a consequence, the willingness to pay for the extension product increases, allowing the brand owner to increase the royalty rate.

[17] This is obtained by plugging $e_C(s)$ and $e_E(s)$ into (9) and maximizing with respect to s.

The effect of the dissimilarity parameter α is more complex. An increase is α reduces both the spillover effects on demand and the size of the non-core market. As we have just seen, the former effect is positive, the latter negative. As it turns out, the stronger effect is the negative one, so the optimal royalty rate is a decreasing function of α.

Equilibrium effort levels Substituting s^* into $e_C(s)$ and $e_E(s)$ one finally obtains the equilibrium efforts under licensing, e_C^L and e_E^L. The explicit expressions are reported in Appendix C, which also shows the following:

Proposition 2. *Under licensing, both effort levels e_C^L and e_E^L are decreasing in α and increasing in μ and ρ. Moreover, e_C^L is decreasing and e_E^L is increasing in λ.*

The comparative statics now accounts for both the direct effects considered in Lemma 1 and the indirect effects that operate *via* the royalty rate s^*. For parameters α, μ and ρ, the indirect effect either reinforces the direct effect or is dominated by it, so the overall impact of these parameters is the same as in Lemma 1. As for the intensity of the reciprocal effect λ, however, the direct effect is nil, so only the indirect effect matters. Combining Lemmas 1 and 2, it follows immediately that the effect is positive for e_H and negative for e_C.

By plugging the equilibrium efforts e_C^L and e_E^L into the profit function (9) one obtains the brand owner's equilibrium profit under licensing, π_B^L. The explicit expression is reported in Appendix D.

4.2.3 No extension

Finally, we consider the case in which the extension is not carried out. In this case, the willingness to pay for the core product reduces to e_C, and the brand owner's profit to $\mu e_C - \frac{1}{2}e_C^2$. The optimal choice of effort then is $e_C = \mu$, yielding an equilibrium profit of $\pi_B^{NE} = \frac{1}{2}\mu^2$.

5 The causes of brand dilution

To understand the causes of brand dilution, we must analyze the determinants of the quality of the extension e_E, as brand dilution is when e_E is lower than the benchmark ε. Propositions 1 and 2 tell us how the exogenous parameters of the model affect e_E as long as the brand owner sticks to a given organizational mode. To complete the picture, we must determine (i) how the choice of the organizational mode is affected by changes in the model's parameters, and (*ii*) what happens to effort levels when the brand owner switches from one organizational mode to the other.

We now consider each of these issues, starting from the latter.

5.1 Organizational modes and quality

Proposition 3. *The brand owner's effort on the core product* e_C *is always higher under internal development than under licensing:* $e_C^H > e_C^L$*.*

Analytically, the result follows immediately from the fact that $e_C^H - e_C^L = (1-\alpha)\, q^M > 0$. The intuition is that under in-house development the brand owner fully internalizes the externality that the core product exerts on the extension product, whereas under licensing the internalization is only partial.

As for the quality of the extension, the impact of the organizational mode is generally ambiguous. To make some progress, it may be useful to consider certain relevant "margins," where different options are equally profitable to the brand owner. For ease of reference, we shall call *extensive margin* the locus of parameter values where the brand owner is just indifferent between engaging in brand extension or not:

$$\pi_B^{NE} = \max\left[\pi_B^H, \pi_B^L\right], \tag{15}$$

and *intensive margin* the locus where the brand owner is just indifferent between internal development and licensing:

$$\pi_B^H = \pi_B^L \; (>\pi_B^{NE}). \tag{16}$$

5.1.1 The extensive margin

Proposition 4. *If licensing is the best organizational mode at the extensive margin (i.e., if* $\pi_B^L = \pi_B^{NE} \geq \pi_B^H$*), then brand extension entails brand dilution:* $e_E < \varepsilon$*.*

The logic behind this result is very simple.[18] The contribution of the extension to the profit of the brand owner is twofold. On the one hand, the extension product generates a profit on its own, which under licensing is always positive as it coincides with the licensing revenue sq^M. On the other hand, the extension changes the profit from the core business. To a first-order approximation, the

[18] A proof is provided in Appendix E.

change is $\mu\lambda(1-\alpha)e_R$,[19] and therefore its sign coincides with that of the reciprocal effect $e_R = e_E - \varepsilon$. At the extensive margin, the total effect of the extension on the brand owner's profit is nil. Since the profit from the extension market is always positive, the effect on the profit from the core business must be negative. But this implies that we must have $e_R < 0$, i.e., brand dilution.

Intuitively, any brand licensing that in equilibrium results in brand enhancement is definitely profitable – a win-win solution. For licensing to be profit neutral, it must entail brand dilution. By continuity, Proposition 4 implies that some profitable extensions must necessarily result in brand dilution.

The result does not necessarily hold when instead the best option at the extensive margin is internal development, i.e., if $\pi_B^H = \pi_B^{NE} > \pi_B^L$. In this case, the direct profit from the extension, $\pi^M - \frac{1}{2}e_E^2$, can be negative. But even though there might be brand enhancement at the extensive margin, this possibility arises only for certain parameter values.[20] This means that even if the extension product is developed internally, the extension may well entail brand dilution and still be profitable.

These results are noteworthy. They imply that rather than being an unforeseen effect of a poorly executed extension strategy, brand dilution can be viewed as a deliberate strategic choice.[21] It may be tempting to interpret this conclusion in a dynamic way. In other words, one may think of a two-stage strategy, where companies build brand reputation first and then monetize the brand. Since our model is timeless, however, we shall not further pursue this dynamic interpretation here.

5.1.2 The intensive margin

Proposition 5. *At the intensive margin (i.e., when $\pi_B^H = \pi_B^L$), a switch from internal development to licensing always increases the quality of the extension e_E and reduces the likelihood of brand*

[19] This is an approximation because the extension affects also e_C. The approximation is exact for $s \approx 0$, when the effect of the extension on e_C is negligible.

[20] Intuitively, the possibility arises when the extension is carried out with the exclusive purpose of enhancing the value of the brand through a positive reciprocal effect. For this strategy to be profitable, however, the reciprocal effect must be sufficiently strong (high λ) and the market for the extension product must be sufficiently small (low ρ). Appendix F expands on this point.

[21] Examples of licensed extension whose contribution to the parent brand image is doubtful, if not definitely harmful, include Lacoste casualwear and accessories, Black&Decker small domestic appliances or Samsonite outerwear collection, e.g. Aaker (1990). A more recent example is Ferrari's first ready-to-wear collection, in June 2021 (Limei Hoang (2021) *When Luxury Brands Delve Into Other Categories, Does It Fail Or Thrive?*. Luxury Society: https://www.luxurysociety.com/en/articles/2021/08/when-luxury-brands-delve-other-categories-does-it-fail-or-thrive,retrieved on February 28, 2022.).

dilution.

This results follows from two properties of our model: the under-investment result (Corollaries 1 and 2), and the fact that the investment in the quality of the core product is always higher under in-house development than under licensing (Proposition 3).[22] In the light of these properties, the relative profitability of the two organizational modes, internal development and licensing, depends on the following factors. First of all, the under-investment result implies that all else equal, total industry profits are higher when the quality of the extension product is higher. Suppose, however, that the quality of the extension product is the same under both modes of extension, so that this effect is neutralized. Still, total industry profits might differ for several other reasons. On the one hand, the cost of delivering said given quality is lower for the specialized licensee than for the brand owner. On the other hand, under internal development the quality of the core product is higher, and a higher quality is associated with higher profits (because of the under-investment result). In our model, these two effects offset each other. However, two additional forces reduce the brand owner's profit under licensing. First, when the royalty rate is positive, industry profits are eroded by the pricing distortions that arise in a vertical relationship. Second, the brand owner must share these profits with the licensee.

This implies that if the quality of the extension product were the same under both modes of extension, the brand owner's profit would be higher under internal development. It follows that for the two modes of extension to be equally profitable, the quality of the extension product must be higher under licensing.

Proposition 5 runs counter to the conventional wisdom that brand dilution is caused by brand licensing. The fallacy of the conventional wisdom is to give a causal interpretation to the association between brand licensing and brand dilution that has been observed in the empirical literature. In fact, such a correlation may arise for other reasons, as we shall show below.

5.2 When and how to extend the brand

We now move back to the first stage of the game, where the brand owner chooses whether or not to engage in brand extension, and, if yes, in which way. Plainly, this choice rests on the comparison of

[22] The proof is relegated in G

the profit levels π_B^{NE}, π_B^H, and π_B^L. In this subsection, we analyze the way in which the comparison depends on the parameters of the model.

For some parameters, the analysis is obvious. It may be easily verified that the size of the core market μ has a multiplicative effect on all equilibrium profits and thus does not affect the relative profitability of the different options. Also, it is evident that an increase in ε generally reduces the profitability of brand extension, without affecting the relative profitability of internal development and licensing. Finally, an increase in the technological distance θ reduces the profitability of in-house development, without affecting the relative profitability of the other two options.

The impact of the other parameters is less obvious and is analyzed below.

5.2.1 The magnitude of the reciprocal effect

Holding the other parameters of the model constant, the effect of changes in the intensity of the reciprocal effect, λ, is as follows:[23]

Proposition 6. *Suppose that the technological distance is sufficiently high. Then, the brand owner develops the extension internally if λ is small, licenses the brand to a specialized licensee for intermediate values of λ, and does not engage in brand extension if λ is large.*

On the face of it, this result (which is illustrated in Figure 2 below) may sound surprising. The comparative advantage of licensing is the greater technological efficiency of the licensee compared to the brand owner; its comparative dis-advantage, on the other hand, lies in the more limited internalization of the demand externalities between the products. It is therefore tempting to conjecture that an increase in λ, which amplifies the indirect demand externality, should make licensing relatively less profitable than internal development.

In fact, the opposite holds. To understand why this is so, it may be useful to paraphrase Proposition 6 as follows: "An increase in λ makes licensing relatively more profitable than internal development at the intensive margin, and relatively less profitable than no extension at the extensive margin." From this viewpoint, it appears that the first part of the result follows immediately from Proposition 5. This proposition says that at the intensive margin, a switch to licensing causes an

[23]Some of the intervals mentioned in Proposition 6 may be empty. For example, in-house development may never be optimal if θ is very large, and brand extension may never be optimal if ε is very large. Similar remarks apply to Propositions 7 and 8 below.

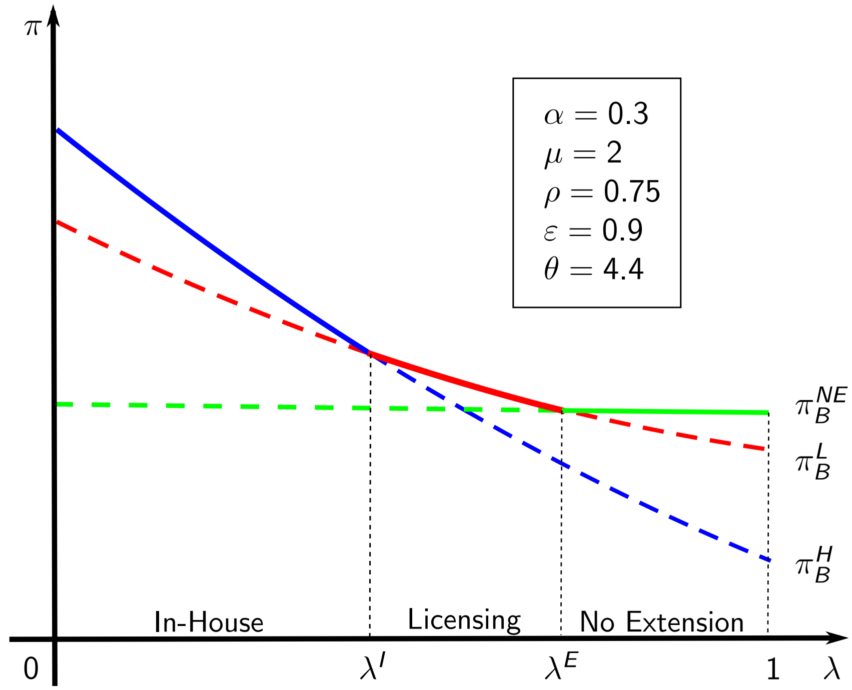


Figure 2: Profits under internal development, licensing and no extension, as a function of λ.

upward jump in e_E. The result then follows from the fact that the upward jump is more valuable, the higher is λ. The second part of the result is instead a consequence of Proposition 4. This proposition says that at the extensive margin licensing entails brand dilution. The result then follows from the fact that such brand dilution is more damaging, the higher is λ.

The condition that the technological distance be sufficiently high (the precise lower bound may be found in Appendix F) ensures that licensing can be an optimal strategy. If the condition fails, different patterns may emerge. For example, the brand owner may find it optimal to develop the extension internally both when λ is small and when it is large, and to not engage in brand extension for intermediate values of λ. In this case, there would be two extensive margins: high and low. Brand extension entails brand dilution at the low margin, brand enhancement at the high one. For high values of λ the brand owner suffers a loss in the extension market,[24] but brand extension is still profitable because it increases the demand for the core product. Extending the brand then

[24] This follows by the same logic that underlies Proposition 4. If the extension entails brand enhancement at the extensive margin, then the direct contribution of the extension to the brand owner's profit, $\pi^M - \frac{1}{2}e_E^2$, must be negative.

becomes as a sort of investment in advertising.[25]

5.2.2 The size of the extension market

Next, consider the effect of changes in ρ, the size of the market for the extension product. Holding the other parameters constant, when ρ varies we have:

Proposition 7. *The brand owner does not engage in brand extension if ρ is small, develops the extension internally for intermediate values of ρ, and licenses the brand to a specialized licensee if ρ is large.*

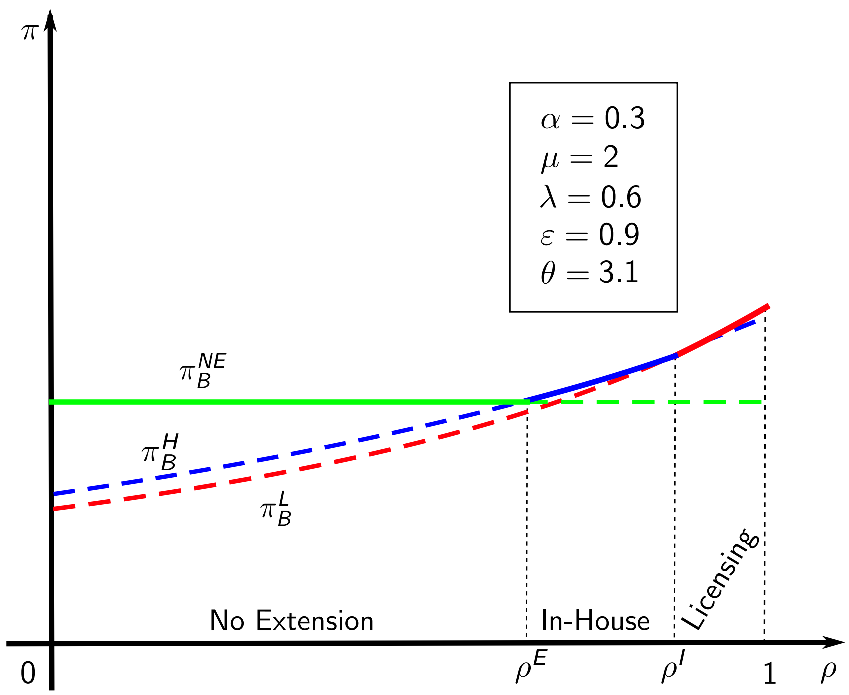


Figure 3: Profits under internal development, licensing and no extension, as a function of ρ.

The first part of the result (which is illustrated in Figure 3) is obvious. When the non-core market is small, the incentive to invest in the quality of the extension is small as well. As a consequence, it is more likely that the extension entails brand dilution, and hence that it is not

[25] Pirelli's tennis balls is a case in point. In the '70s and '80s, Pirelli's tennis balls division kept generating losses. Yet, production was never discontinued because the management believed that Pirelli tennis balls would contribute to maintain and enhance the brand name. More recently, Giorgio Armani has introduced, amongst its extensions, flowers and chocolates to strengthen even further the Armani brand image (Editorial staff (2003) *Giorgio Armani ha aperto uno store a Monaco*. Pambianco News: https://www.pambianconews.com/2003/12/18/giorgio-armani-ha-aperto-uno-store-a-monaco-8134/, retrieved on February 28, 2022.)

profitable. The second part of the result may instead sound surprising if one reasons on the basis of the internalization-of-demand-externalities logic. But in fact, the correct intuition is again provided by Proposition 5: a higher ρ makes the quality of the extension more important and thus raises the relative profitability of licensing, which generates the higher quality at the relevant margin.

5.2.3 The perceived distance

Finally, consider the effect of the perceived distance between the core and non-core products. Holding the other parameters constant, when α varies we have:

Proposition 8. *The brand owner develops the extension internally if α is small, licenses the brand to a specialized licensee for intermediate values of α, and does not engage in brand extension if α is large.*

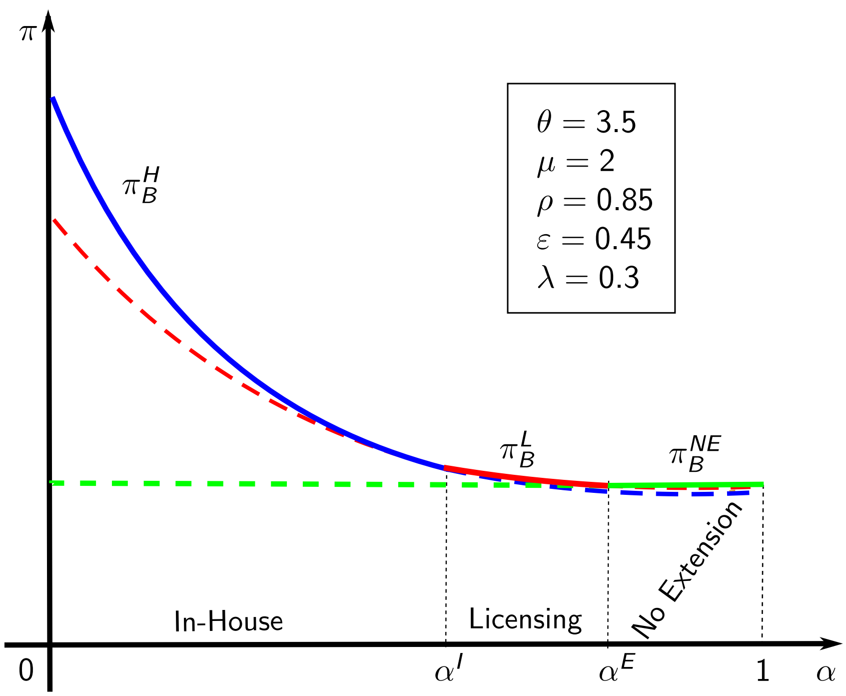


Figure 4: Profits under internal development, licensing and no extension, as a function of α.

This result (which is illustrated in Figure 4) combines elements from Propositions 6 and 7. On the one hand, an increase in α is similar to a decrease in ρ in that it reduces the size of the extension market. On the other hand, an increase in α is similar to a decrease in λ in that it reduces the intensity of the reciprocal effect. Proposition 8 shows how these effects interact to determine the overall impact of the perceived distance.

5.3 What causes brand dilution?

We are now ready to answer the main question of this paper: What causes brand dilution? And, in particular, does licensing cause brand dilution?

To address these issues, we analyze how the exogenous parameters affect the quality of the extension product e_E and hence the likelihood of brand dilution. Since licensing is not an exogenous variable, however, it cannot exert a causal effect on another endogenous variables, i.e., brand dilution. In this respect, what the model determines is whether brand licensing and brand dilution co-vary or counter-vary with the exogenous parameters, and hence whether they tend to be associated positively or negatively.

5.3.1 The technological distance

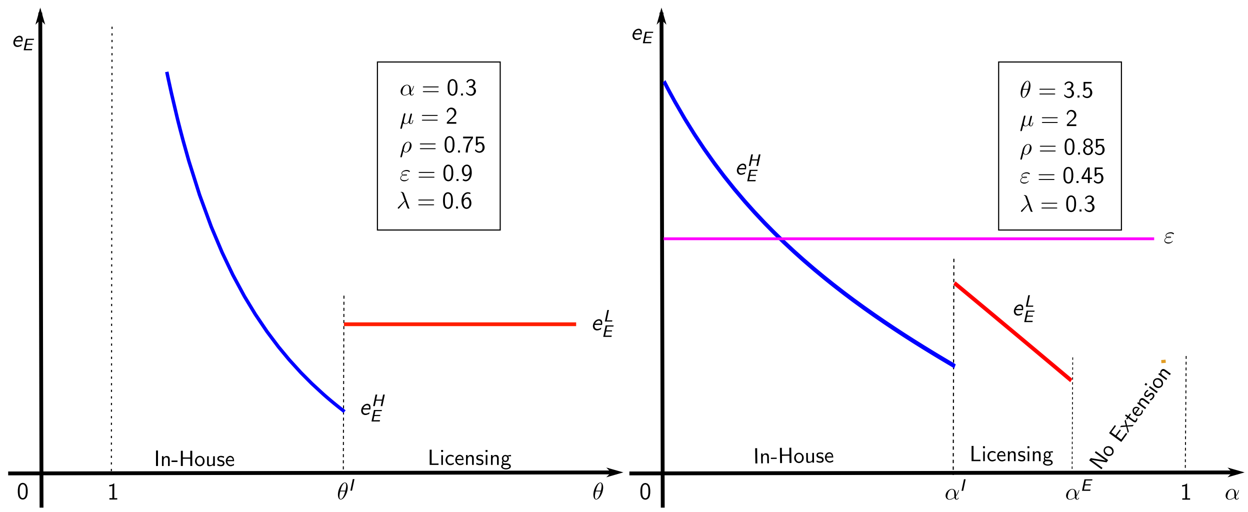
To begin with, let us consider the impact of the technological distance θ. Figure 5a shows that when θ is small, the best option is to develop the extension internally;[26] in this case, e_E decreases with θ. As θ gets larger, however, it becomes preferable to resort to licensing. At the switching point, e_E jumps up, in accordance with Proposition 5. As θ increases further, e_E then stays constant, as the brand owner engages in brand licensing, and e_E^L is independent of θ.

If the upward jump is relatively small (as is the case in Figure 5a), then in a dataset where the variation is driven by heterogeneity in the technological distance θ, one should observe a positive correlation between brand licensing and brand dilution. But of course, licensing cannot be regarded as the cause of brand dilution. On the contrary, the positive association arises because licensing becomes the better organizational mode precisely when the concern for brand dilution is higher.

5.3.2 The perceived distance

A similar picture is obtained when the parameter that varies is the perceived distance α. Starting from internal development when α is small, as α increases the intensive margin is passed first and the extensive margin next. Figure 5b shows that e_E decreases with α both before and after the

[26] The figure considers the case where $\pi_B^L > \pi_B^{NE}$. Remember that both π_B^L and π_B^{NE} are independent of θ.



(a) Technological distance.

(b) Perceived distance.

Figure 5: Effort levels on the extension product as a function of the technological (θ) and perceived (α) distance.

upward jump at the intensive margin. When the extensive margin is reached, e_E ends up below the benchmark ε, in accordance with Proposition 4.

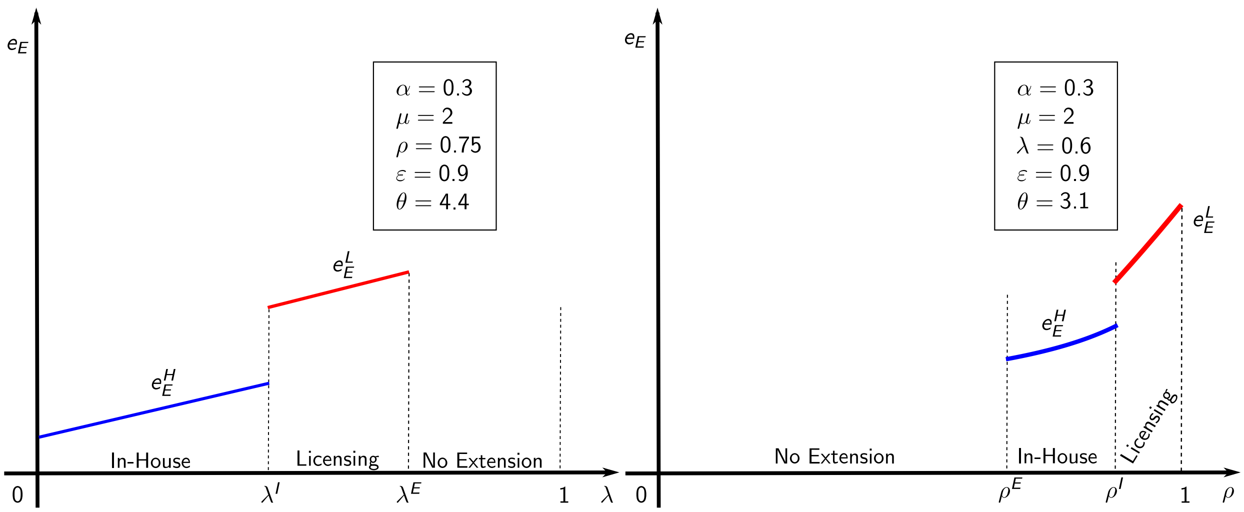
If the upward jump is small, one should observe a positive correlation between brand licensing and brand dilution in a dataset where the variation is driven by heterogeneity in the perceived distance α. But again, no causal inference may be drawn.

5.3.3 The intensity of the reciprocal effect

Changes in the intensity of the reciprocal effect λ paint a different picture. Figure 6a represents the case where the technological distance is sufficiently large that Proposition 6 applies. The figure shows that for each organizational mode, the quality of the extension e_E improves as λ increases. And since licensing prevails for high values of λ, it is now associated with a lower risk of brand dilution. Therefore, if the main source of heterogeneity in the data were the size of the reciprocal effect, the model would generate a negative correlation between brand licensing and brand dilution.

5.3.4 The size of the extension market

The same conclusion holds true for changes in the size of the extension market, ρ. The logic is exactly the same as for changes in λ. Figure 6b illustrates.



(a) Intensity of reciprocal effect.

(b) Size of extension market.

Figure 6: Effort levels on the extension product as a function of the intensity of the reciprocal effect (λ) and size of the extension market (ρ).

5.3.5 Summing up

So, what causes brand dilution? In our model, brand dilution is more likely to occur when the technological distance is large, when the intensity of the reciprocal effect is weak, when the size of the extension market is small, and when the perceived distance is large. These results have interesting managerial implications and provide testable hypotheses for the empirical analysis. Both will be further discussed in the concluding section.

In our model, however, licensing does not cause brand dilution. Any correlation between licensing and brand dilution that may be observed in the data is generated by the co-movements of the organizational choice and the investment in the quality of the extension caused by changes in the exogenous parameters. The correlation may have either sign. If the main source of the variability in the data is heterogeneity in λ or ρ, then brand licensing will be associated with less brand dilution. If instead the main source of the variability is heterogeneity in α or θ, then brand licensing will be associated with more brand dilution. The empirical evidence suggests that the latter case is probably the more relevant one. But in any case, neither variable has a causal effect on the other.

6 Robustness

In this section, we briefly report on several extensions of the baseline model. A more detailed analysis of these extensions may be found in the on-line appendix available at: xxx.

6.1 Endogenous quality benchmark

In the baseline model, the benchmark level of quality is an exogenous parameter, ε. We have analyzed a variant of the model where the benchmark is an increasing function of the quality of the core product:

$$e_R = e_E - (\varepsilon + \gamma e_C) \tag{17}$$

where $\gamma > 0$ is a parameter. The algebra is more cumbersome, but the qualitative results are almost identical to the baseline model. Under both modes of extension, an increase in γ reduces the brand owner's incentive to exert effort and thus makes brand extension less profitable. However, it does not affect the relative profitability of licensing and internal development.

The main change with respect to the baseline model is an additional effect that now arises at the intensive margin. Since a switch from internal development to licensing reduces e_C (Proposition 3), this now tends to increase e_R. This additional effect runs counter to the conclusion of Proposition 5 but is always dominated by the other effects at work in our model. Thus, Proposition 5 continues to hold. The comparative statics analysis of Section 5 does not change either.[27]

6.2 Multiple extensions

In another robustness check, we have considered the case of a number of potential extension products $i \in N = \{1, 2, ..., n\}$. In this case, the reciprocal effect is the sum of the effects generated by all extensions:

$$e_R = \sum_{i \in \mathcal{E}} \lambda_i (1 - \alpha_i) (e_{E,i} - \varepsilon_i) \tag{18}$$

where $\mathcal{E} \subseteq N$ is the set of the products for which the extension is realized.

Naturally, the brand owner may choose to develop some extensions internally and resort to licensing for others. The determinants of this choice are the same as in the case of a single extension.

[27] The calculations are available here.

The more products are developed in house, the better the internalization of the demand externality, and hence the higher the effort on the core product, e_C.

Propositions 4 and 5 now hold product by product. When the brand owner is just indifferent between entering into a licensing agreement for product i or not, the contribution of product i to the reciprocal effect is negative: $e_{E,i} < \varepsilon_i$. When instead the brand owner is just indifferent between extending product i internally or *via* licensing, a switch to licensing increases the quality of product i and hence reduces the likelihood of brand dilution.

6.3 Two-part tariffs

In the baseline model, the brand owner is restricted to licensing contracts that involve the payment of royalties proportional to the output of the extension product. In another robustness check, we have allowed the brand owner to use two-part tariffs $\{s, F\}$, where F is a fixed fee.[28] In this case, the licensing revenue is $sq + F$. We restrict s to be non-negative, as negative royalty rates are almost never observed in the real world (perhaps because they might open the door to various kinds of opportunistic behavior).[29]

If the brand owner has all the bargaining power, it makes a take-it-or-leave-it offer and thus sets the fixed fee so that the licensee just breaks even, $\pi_L = 0$. This condition pins down the fixed fee F for any value of the royalty rate. The royalty rate is then chosen so as to maximize the brand owner's profit.

As long as the optimal royalty rate is positive, all our results continue to hold. For some parameter values, however, the optimal contract involves $s^* = 0$, i.e., fixed fee licensing. In this case, there is no distortion in the monopoly price. While Proposition 4 continues to hold, Proposition 5 may now fail. That is, if $s^* = 0$ then for some parameter values e_E may jump down rather than up at the intensive margin.[30]

This is an important caveat to keep in mind. Note, however, that pure fixed-fee contracts are

[28] In the absence of uncertainty, more complex schedules would be redundant: see, for instance, Bhattacharyya and Lafontaine (1995); Bousquet et al. (1998).

[29] For example, the licensee might inflate volumes by secretly re-purchasing the product from its buyers.

[30] To be precise, if the equilibrium royalty rate were $s^* = 0$ even in the absence of a non-negativity constraint, the result that at the intensive margin one must have $e_E^L \geq e_E^H$ would continue to hold (with a strict inequality if the brand owner does not have all the bargaining power). It is when $s^* = 0$ because of the non-negativity constraint on s that Proposition 5 may fail.

rarely observed in brand licensing: see e.g. Raugust (2012).[31]

6.4 Bargaining

Finally, we have relaxed the assumption that the brand owner makes take-it-or-leave-it offers by applying instead the Nash bargaining solution, which allows for any distribution of the bargaining power among the parties. With two-part tariffs, changes in the bargaining power do not affect the equilibrium royalty rate. They affect only the fixed fee: instead of transferring all rents from the extension to the brand owner, the fixed fee now splits these rents between the two parties according to their respective bargaining power. The effect on the relative profitability of the various options is obvious: as the licensee's bargaining power increases, licensing becomes less profitable .

This conclusion applies also to the case of pure royalty licensing. In this case, the equilibrium royalty rate may depend on the parties' bargaining power, but the qualitative conclusions of the analysis remain the same as in the baseline model.[32]

7 Conclusions

This paper has proposed a comprehensive theoretical framework that allows one to delve into various economic and managerial issues related to brand extension and brand licensing. The analysis has led us to question the opinion, commonly held by scholars and practitioners alike, that brand dilution is an unintended consequence of a poorly executed extension project, and that licensing always increases the risk of brand dilution. The analysis has also cast doubts on another common opinion, namely, that internal development is to be preferred when "stakes are high."

On the first point, the analysis has identified cases where the optimal strategy necessarily entails brand dilution. This is so, in particular, when a licensed extension is just profitable. However, even internally developed extensions that are much more than barely profitable may knowingly result

[31] The calculations are available here.

[32] In the bargaining process, the disagreement payoff is nil for the licensee and is $\max\left[\pi_B^H, \pi_B^{NE}\right]$ for the brand owner. The royalty rate must then maximize the Nash product

$$\left(\pi_B^L - \max\left[\pi_B^H, \pi_B^{NE}\right]\right)^{\eta} (\pi_L)^{1-\eta}$$

where $\eta \in (0, 1)$ is the brand owner's relative degree of bargaining power. The most notable change with respect to the baseline case (which is re-obtained for $\eta = 0$) is that when $\max\left[\pi_B^H, \pi_B^{NE}\right] = \pi_B^H$ the royalty rate depends on the technological distance θ. In particular, since in this case the brand owner's disagreement payoff worsens as the technological distance θ increases, the equilibrium royalty rate decreases with θ.

in brand dilution. This indicates that brand dilution is not to be eschewed at all costs, and that, when it occurs, it need not be the outcome of a backfired strategy. Rather, brand dilution may be a deliberate choice – a monetization of the brand.

In this perspective, managers should consider the brand as an asset on which not only to invest but also, possibly, divest. Brand dilution represents a viable opportunity to divest, cashing-in the brand value. On the other hand, brand extension can also represent a form of pure investment. Our analysis has indeed identified cases where the brand owner suffers profit losses on the extension product and yet carries out the extension in order to benefit from brand enhancement. In these cases, the extension is used as a costly device to promote the value of the brand.

Lacking the time dimension, in our model the investment and divestment phases are not distinct in time but in the space of products. That is, managers invest in brand value by exerting effort on the core product and on those extensions that entail brand enhancement; they divest by pursuing brand extension beyond the point where it starts entailing brand dilution. In practice, however, the invest/divest dichotomy often involves time in a crucial way. A dynamic extension of the model is needed to account for this. It would be useful, for instance, to assess at what stage of a brand life-cycle managers should opt for brand dilution as a profitable strategy.

On the second point, our results refute the idea of a causal relationship from brand licensing to brand dilution. We have shown that, in fact, a switch to licensing always increases the quality of the extension for those products for which internal and licensed development are almost equally profitable options. For these products, licensing systematically results either in less brand dilution, or in more brand enhancement than internal development. Therefore, rather than regarding licensing as a possible cause of brand dilution, managers should consider it as a countermeasure to defuse the reduction in brand value that certain profitable extensions might entail.

While these findings challenge the conventional wisdom that licensing causes brand dilution, they also clarify why it could emerge. Our model can indeed produce a positive correlation between brand licensing and brand dilution even in the absence of any causal link. This is so, in particular, if the main source of variation in the observable data is changes in the perceived and technological distance between the core and the extension products. In these cases, the positive correlation arises because brand licensing becomes the more profitable strategy precisely when the risk of brand dilution is higher.

Another contribution of the paper is to conceptually distinguish different facets of brand extension that are often conflated together both in the scholarly debate and in managerial practice. The analysis has shown that such different factors may instead have different, possibly even opposite, implications.

Consider, for instance, the commonly held view that internal development is to be preferred when "stakes are high." A natural interpretation of "high stakes" is that the potential market for the extension product is big. However, our analysis has provided only partial support to the common view. The reason for this is that in our model there are two parameters that correlate with the size of the extension market: the scale parameter ρ, and the perceived distance α (less distant products being in higher demand than more distant ones, because they benefit more strongly from the reputation and image of the parent brand). The impact of these parameters on the optimal mode of extension is quite different, though. The analysis has shown that the brand owner should engage in licensing when the extension market is big because of a high ρ but should resort to internal development when the extension market is big because of a small α. Thus, looking at size only may be misleading. Managers should try to identify the reason why the extension market is big or small and react differently to different factors of market size. Likewise, scholars who use the size of the extension market as an explanatory variable in empirical research should expect mixed results. More definite results can be obtained by disentangling the different effects.

As another example, consider the demand spillovers, which play such a crucial role in brand extension. Since these externalities are better internalized under in-house development, it is tempting to think that stronger spillovers should make in-house development relatively more profitable than licensing. But again, our analysis has uncovered a more complex picture.

Specifically, we have distinguished between the demand externalities running from the core to the extension product and those running in the opposite direction. The magnitude of the former is captured by the perceived distance α; that of the latter by the reciprocal effect parameter λ.[33] As it turns out, these parameters have opposite effects. Stronger spillovers from the core product to the extension product (lower α) make in-house development relatively more profitable, in accordance with the internalization-of-demand-externalities intuition. However, stronger spillovers from the extension product to the core product (higher λ) make in-house development relatively

[33] To be precise, the magnitude of the reciprocal effect depends on both α and λ.

more profitable. This runs counter to the above intuition, highlighting once again the importance of disentangling different effects.

In sum, this paper has shown that the best way to perform brand extension, and its impact of the likelihood of brand dilution, depend on more factors, and in a subtler way, than is often thought. This may provide food for thought for both managers and empirical researchers.

There is clearly much scope for further research in this area, and we hope that this paper may stimulate it.

Appendices

A Proof of Proposition 1

The solution to the system of first-order conditions (7)-(8) is

$$
\begin{aligned}
e_C^H &= \frac{\mu\left\{2\theta-\rho\left[1-(1-\alpha)^2\lambda\right]\right\}}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}, && (19)\\
e_E^H &= \frac{(1-\alpha)\mu\left\{\lambda\left[2-(1-\alpha)^2\rho\right]+\rho\right\}}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}. && (20)
\end{aligned}
$$

The concavity of the profit function of the brand owner under in-house development is guaranteed by the sufficient condition $\rho < 1$, which we assume to hold throughout. By taking the first-order partial derivatives w.r.t α we obtain

$$
\begin{aligned}
\frac{\partial e_C^H}{\partial\alpha} &= -\frac{2(1-\alpha)\mu\rho(\theta+\lambda)(2\theta-\rho)}{\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}^2}, && (21)\\
\frac{\partial e_E^H}{\partial\alpha} &= -\frac{\mu\left\{\theta\lambda\left[(1-\alpha)^2\rho-2\right]^2+\theta\rho\left[(1-\alpha)^2\rho+2\right]+\lambda\rho\left[3(1-\alpha)^2\rho-2\right]-\rho^2\right\}}{\left\{\theta\left[(1-\alpha)^2\rho-2\right]+\rho\right\}^2}. && (22)
\end{aligned}
$$

Simple inspection reveals that (21) is always negative in the admissible parameter constellation. As for (22), by performing the substitution $(1-\alpha)^2 = X \in [0.1]$, it is easy to observe that the numerator has two roots, namely

$$
X_1 = \frac{4\theta\lambda-\rho(\theta+3\lambda)+\sqrt{\rho(\theta+\lambda)(-16\theta\lambda+\theta\rho+9\lambda\rho)}}{2\theta\lambda\rho} \qquad (23)
$$

$$
X_2 = \frac{4\theta\lambda-\rho(\theta+3\lambda)-\sqrt{\rho(\theta+\lambda)(-16\theta\lambda+\theta\rho+9\lambda\rho)}}{2\theta\lambda\rho}, \qquad (24)
$$

We prove that none of these roots lies in $[0,1]$, which implies that the original derivative does not change sign in that interval. As a preliminary observation, a necessary condition for the roots to be real is that $\theta \leq \frac{9\lambda\rho}{16\lambda-\rho}$. If they are real, furthermore, $X_1 > X_2$. Let us start by X_1 and prove that, if it is real, it must be negative. Assume that $4\theta\lambda-\rho(\theta+3\lambda) < 0$, then by using some algebra one obtains that $X_1 > 0 \Leftrightarrow -4\theta\lambda(2\theta-\rho)(2\lambda+\rho) > 0$, which is a contradiction. Assuming that $4\theta\lambda-\rho(\theta+3\lambda) > 0$ implies $\theta > \frac{3\lambda\rho}{4\lambda-\rho}$, with $4\lambda-\rho > 0$, which is not compatible the roots to be real.

So we conclude that if X_1 is real, it must be negative. As a consequence $X_2 (< X_1)$ is negative too. A simple numerical substitution then confirms that (22) is negative in $[0, 1]$

Similarly, the derivatives

$$\frac{\partial e_C^H}{\partial \theta} = -\frac{(1-\alpha)^2 \mu\rho \left\{\lambda \left[2-(1-\alpha)^2\rho\right]+\rho\right\}}{\left\{\theta\left[(1-\alpha)^2\rho-2\right]+\rho\right\}^2}, \tag{25}$$

$$\frac{\partial e_E^H}{\partial \theta} = -\frac{(1-\alpha)\mu \left[2-(1-\alpha)^2\rho\right]\left\{\lambda\rho+\left[2-(1-\alpha)^2\rho\right]\right\}}{\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}^2}, \tag{26}$$

are also negative in the admissible parameter space. Next, consider the derivatives with respect to λ

$$\frac{\partial e_C^H}{\partial \lambda} = \frac{(1-\alpha)^2\mu\rho}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}, \tag{27}$$

$$\frac{e_E^H}{\partial \lambda} = \frac{(1-\alpha)\mu\left[2-(1-\alpha)^2\rho\right]}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}. \tag{28}$$

It is immediate to observe that are always positive. Likewise, the derivatives w.r.t. μ

$$\frac{\partial e_C^H}{\partial \mu} = \frac{2\theta-\rho\left[1-(1-\alpha)^2\lambda\right]}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}, \tag{29}$$

$$\frac{e_E^H}{\partial \mu} = \frac{(1-\alpha)\left\{\lambda\left[2-(1-\alpha)^2\rho\right]+\rho\right\}}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}, \tag{30}$$

are positive as well, as are the ensuing derivatives w.r.t. ρ

$$\frac{\partial e_C^H}{\partial \rho} = \frac{2(1-\alpha)^2\theta\mu(\theta+\lambda)}{\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}^2}, \tag{31}$$

$$\frac{e_E^H}{\partial \rho} = \frac{2(1-\alpha)\mu(\theta+\lambda)}{\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}^2}. \tag{32}$$

■

B Profits under in-house development

The value of the brand owner's profit under in-house development, when effort levels are set optimally, is

$$\pi_B^H = \frac{\mu^2 \left\{2\theta + 2(1-\alpha)^2\lambda^2 - \rho\left[1-(1-\alpha)^2\lambda\right]^2\right\}}{2\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}} - (1-\alpha)\varepsilon\lambda\mu. \tag{33}$$

C Proof of Proposition 2

The equilibrium values of the efforts under licensing are the following, respectively for the brand owner and for the licensee.

$$e_C^L = \mu + \frac{(1-\alpha)^2\mu\rho\left\{2-\lambda\left[2-(1-\alpha)^2\rho\right]\right\}}{8-(1-\alpha)^2\rho(\rho+2)}, \tag{34}$$

$$e_E^L = \frac{(1-\alpha)\mu\rho\left\{1+\frac{\left[2-(1-\alpha)^2\rho\right]\left\{2-\lambda\left[2-(1-\alpha)^2\rho\right]\right\}}{8-(1-\alpha)^2\rho(\rho+2)}\right\}}{2-\rho}. \tag{35}$$

The comparative statics are drawn from the investigation of the following first-order partial derivatives.

The derivatives w.r.t. α are

$$\frac{\partial e_C^L}{\partial\alpha} = -\frac{2(1-\alpha)\mu\rho\left\{16-\lambda\left\{16-(1-\alpha)^2\rho\left[16-(1-\alpha)^2\rho(\rho+2)\right]\right\}\right\}}{\left[8-(1-\alpha)^2\rho(\rho+2)\right]^2}, \tag{36}$$

$$\frac{\partial e_E^L}{\partial\alpha} = -\frac{\mu\rho\left\{\lambda\left\{(1-\alpha)^2\rho\left\{\rho\left\{3(1-\alpha)^4\rho^2+2[3(\alpha-2)\alpha+1](1-\alpha)^2\rho-48(\alpha-2)\alpha-52\right\}+88\right\}-32\right\}+(1-\alpha)^2\rho\left\{\rho\left[20-(1-\alpha)^2\rho(\rho+2)\right]-8\right\}-32\right\}}{(\rho-2)[8-(1-\alpha)^2\rho(\rho+2])^2} \tag{37}$$

It is easy to ascertain that (36) is always negative in the admissible parameter constellation. By contrast, (37) is cumbersome, yet it can be ascertained that all of its roots lie outside the $[0,1]$ interval and that, therein the derivative is negative.[34]

The derivatives w.r.t. μ are

[34] See the online Appendix.

$$\frac{\partial e_C^L}{\partial \mu} = 1 + \frac{(1-\alpha)^2 \rho \left\{2 - \lambda \left[2 - (1-\alpha)^2 \rho\right]\right\}}{8 - (1-\alpha)^2 \rho (2+\rho)}, \tag{38}$$

$$\frac{\partial e_E^L}{\partial \mu} = \frac{(1-\alpha)\rho \left\{1 - \frac{\left[2-(1-\alpha)^2\rho\right]\left\{2-\lambda\left[2-(1-\alpha)^2\rho\right]\right\}}{8-(1-\alpha)^2\rho(\rho+2)}\right\}}{2-\rho}. \tag{39}$$

As in the preceding case, it is immediate that $\frac{\partial e_C^L}{\partial \mu}$ is positive. The derivative $\frac{\partial e_E^L}{\partial \mu}$ has five roots, instead. Besides $\alpha = 1$, the others are

$$\alpha = 1 + \frac{\sqrt{2}}{2}\sqrt{\frac{1}{\lambda} + \frac{4}{\rho} - \frac{\sqrt{8\lambda\rho - 16\lambda + \rho^2}}{\lambda\rho}}, \quad 1 - \frac{\sqrt{2}}{2}\sqrt{\frac{1}{\lambda} + \frac{4}{\rho} - \frac{\sqrt{8\lambda\rho - 16\lambda + \rho^2}}{\lambda\rho}}, \tag{40}$$

$$\alpha = 1 + \frac{\sqrt{2}}{2}\sqrt{\frac{1}{\lambda} + \frac{4}{\rho} + \frac{\sqrt{8\lambda\rho - 16\lambda + \rho^2}}{\lambda\rho}}, \quad 1 - \frac{\sqrt{2}}{2}\sqrt{\frac{1}{\lambda} + \frac{4}{\rho} + \frac{\sqrt{8\lambda\rho - 16\lambda + \rho^2}}{\lambda\rho}}. \tag{41}$$

All the roots are real as long as $0 < \lambda \leq \frac{\rho^2}{16-8\rho}$ and, if they are so, the first root in (40) is larger than 1, while the second root is negative. Furthermore, the absolute value of the first root in (40) is smaller than that of the first one in (41), and likewise for the second root. This confirms that the derivative does not change sign in $[0, 1]$, and a numerical evaluation reveals that it is positive in that interval.

Let us now move to the derivatives w.r.t. ρ.

$$\frac{\partial e_C^L}{\partial \rho} = \frac{2(1-\alpha)^2 \mu \left\{(1-\alpha)^2 \rho^2 + 8 - \lambda \left(8 - (1-\alpha)^2 \rho (8 - (2 - (2-\alpha)\alpha)\rho)\right)\right\}}{\left(8 - (1-\alpha)^2 \rho (\rho+2)\right)^2} \tag{42}$$

$$\frac{\partial e_E^L}{\partial \rho} = \frac{-4(1-\alpha)\mu\left\{\lambda\left((1-\alpha)^2\rho\left\{-(1-\alpha)^2\rho^3+2[2-(2-\alpha)]^2\rho^2-8[3-2(2-\alpha)\alpha]\rho+32\right\}-16\right)+2(1-\alpha)^2\rho^2\{6-[2-(2-\alpha)\alpha]\rho\}-16\right\}}{\{16+(1-\alpha)^2\rho^3-4[3-(2-\alpha)\alpha]\rho\}^2} \tag{43}$$

The first expression is clearly positive in the admissible parameter space. The second is more involved, however it can be shown that it is always positive as well.[35]

[35] See the online Appendix.

Last, we consider the derivatives of the effort levels w.r.t. λ.

$$\frac{\partial e_C^L}{\partial \lambda} = -\frac{(1-\alpha)^2\mu\rho\left[2-(1-\alpha)^2\rho\right]}{8-(1-\alpha)^2\rho(\rho+2)}, \tag{44}$$

$$\frac{\partial e_E^L}{\partial \lambda} = \frac{(1-\alpha)\mu\rho\left[2-(1-\alpha)^2\rho\right]^2}{(2-\rho)\left[8-(1-\alpha)^2\rho(2+\rho)\right]}. \tag{45}$$

It is straightforward to observe that the first is always negative and the second always positive in the parameter space. ■

D Equilibrium licensing profits

The profits under licensing area

$$\pi_B^L = \frac{\mu\left\{\mu\left\{2+\rho\left[2(1-\alpha)^2\lambda-1\right]\right\} - \frac{\mu\rho\left[2(1-\alpha)(1-\lambda)-(1-\alpha)^3\lambda\rho\right]^2}{8-(1-\alpha)^2\rho(2+\rho)}\right\}}{2(2-\rho)} - (1-\alpha)\varepsilon\lambda \tag{46}$$

E Proof of Proposition 4

The parametric configuration describing the extensive margin under licensing is implicitly defined by the equation

$$\pi_B^L = \pi_B^{NE}. \tag{47}$$

By solving this equation for μ we obtain

$$\mu^E = \frac{2\varepsilon\lambda(2-\rho)\left[8-(1-\alpha)^2\rho(\rho+2)\right]}{(1-\alpha)\rho\left\{\lambda\left\{\lambda\left[2-(1-\alpha)^2\rho\right]^2-2(1-\alpha)^2\rho^2+8\right\}+4\right\}}. \tag{48}$$

It is easy to ascertain that the difference between the effort of the licensee and ε, evaluated at (48), namely

$$e_E^L - \varepsilon\Big|_{\mu=\mu^E} = -\frac{\varepsilon\left\{4-\lambda^2\left[2-(1-\alpha)^2\rho\right]^2\right\}}{4+\lambda\left\{8+\lambda\left[2-(1-\alpha)^2\rho\right]^2-2(1-\alpha)^2\rho^2\right\}}, \tag{49}$$

is negative in all the admissible parameter space.

F Internal development: Extensive margin(s)

The extensive margin, if the extension is developed in-house, is defined by

$$\pi_B^H = \pi_B^{NE}. \tag{50}$$

This equation has two roots in λ, namely

$$\lambda^E = \frac{(1-\alpha)\varepsilon\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}-(1-\alpha)^2\mu\rho-\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}\sqrt{\frac{(1-\alpha)^2\left\{(1-\alpha)^2\mu^2\rho+\varepsilon^2\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}-2(1-\alpha)\varepsilon\mu\rho\right\}}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}}}{(1-\alpha)^2\mu[2-(1-\alpha)^2\rho]}, \tag{51}$$

and

$$\hat{\lambda}^E = \frac{(1-\alpha)\varepsilon\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}-(1-\alpha)^2\mu\rho+\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}\sqrt{\frac{(1-\alpha)^2\left\{(1-\alpha)^2\mu^2\rho+\varepsilon^2\left\{\theta\left[2-(1-\alpha)^2\rho\right]-\rho\right\}-2(1-\alpha)\varepsilon\mu\rho\right\}}{\theta\left[2-(1-\alpha)^2\rho\right]-\rho}}}{(1-\alpha)^2\mu[2-(1-\alpha)^2\rho]}. \tag{52}$$

The roots are real so long as $\theta \geq \frac{\rho(\alpha\mu-\varepsilon-\mu)^2}{\varepsilon^2[2-(1-\alpha)^2\rho]}$. If real, the first root is lesser than the second. It is a matter of algebra to obtain that

$$e_E^H - \varepsilon\Big|_{\lambda=\lambda^E} = -\frac{\sqrt{\frac{(1-\alpha)^2\{(1-\alpha)^2\mu^2\rho+\varepsilon^2\{\theta[2-(1-\alpha)^2\rho]-\rho\}-2(1-\alpha)\varepsilon\mu\rho\}}{\theta[2-(1-\alpha)^2\rho]-\rho}}}{(1-\alpha)}, \tag{53}$$

which is negative.

The second root may be lesser than 1,[36] in which case the difference in efforts, there evaluated, is

$$e_E^H - \varepsilon\Big|_{\lambda=\hat{\lambda}^E} = \frac{\sqrt{\frac{(1-\alpha)^2\{(1-\alpha)^2\mu^2\rho+\varepsilon^2\{\theta[2-(1-\alpha)^2\rho]-\rho\}-2(1-\alpha)\varepsilon\mu\rho\}}{\theta[2-(1-\alpha)^2\rho]-\rho}}}{(1-\alpha)}, \tag{54}$$

clearly positive.

[36] This requires that ρ takes relatively low, see the online Appendix for the details.

G Proof of Proposition 5

The locus of the intensive margin is implicitly defined by the equation

$$\pi_B^H = \pi_B^L. \tag{55}$$

The solution to this equation w.r.t. θ is

$$\theta^I = \frac{2\lambda^2\left[2-(1-\alpha)^2\rho\right]\left\{\rho\left[6-2(2-\alpha)\alpha-\rho\right]+8\right\}-8\lambda\rho\left\{4-\left[2-(2-\alpha)\alpha\right]\rho-4\right\}\rho^2}{\rho\left\{8+2\lambda\left\{(1-\alpha)^2\rho\left\{\rho\left[2-(1-\alpha)^2\rho\right\}+4\right\}-8-\lambda^2\left[2-(1-\alpha)^2\rho\right]^3\right\}+(1-\alpha)^2\rho^3-8\rho\right\}}. \tag{56}$$

The difference of the in-house and licensing efforts, evaluated at θ^I is

$$e_E^H - e_E^L\Big|_{\theta=\theta^I} = -\frac{2(1-\alpha)\mu\rho\left\{2-\lambda\left[2-(1-\alpha)^2\rho\right]\right\}}{\left[8-(1-\alpha)^2\rho(2+\rho)\right]\left\{2-\lambda\left[(1-\alpha)^2\rho\right]+\rho\right\}}, \tag{57}$$

which is clearly negative in the admissible parameter space.

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