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A Theory of Social Impact Bonds

Abstract

Social impact bonds (SIBs) are an innovative financing mechanism for public goods. In a SIB, an investor provides capital to a service provider for a social intervention. The investor receives a return from the government based on the outcome of the intervention relative to a predetermined benchmark. We describe the basic structure of a SIB and provide some descriptive statistics for these financial instruments. We then consider a formal model of SIBs and examine their ability to finance positive net present value projects that traditional debt finance cannot. We find that SIBs expand the set of implementable projects if governments are pessimistic (relative to the private sector) about the probability an intervention would succeed or if the government is particularly averse to paying costs associated with a project that does not generate offsetting benefits. As various public programs include both these features, we conclude that SIBs are a real innovation in public finance and should be considered for projects when traditional debt finance has been rejected.

JEL Classification: H41, P16, G12

Keywords: Public Goods, Fixed Income Securities, impact Investing, Social Impact Bonds

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1 Introduction

Impact investing, financial investing that values positive social or environmental impact and traditional financial returns, is a growing field in asset management.[1] In 2020, professionals who considered environmental and social factors in their investment choices managed (USD) 16.6 trillion of assets, representing a 38% increase over the previous two years (Aitken et al., 2020). One type of impact investing is the social impact bond (SIB), an arrangement where investors in a project receive financial returns based on project outcomes, specifically the accomplishment of prespecified social objectives.

In a traditional social impact bond, a government attracts financing from an investor. The investor passes the financing on to a service provider who implements a project designed to solve a specific social problem. If the project meets specified social goals, e.g., test scores for an education intervention, the investor receives, from the government, the initial investment back plus interest payments. In many of these arrangements, investors receive no money back if the goals are not met. For payments to be made, a third-party assessor must certify that the intervention achieved the prespecified goals. Given the complexity of these arrangements, an additional entity, known as the intermediary, often coordinates the various parties to the SIB arrangement.[2]

Social impact bonds differ from outcome- or performance-based contracts (James, 2012), where service providers receive additional payments for achieving better outcomes. These arrangements have no outside investor. Consequently, some upfront government payments are necessary given the limited budgets of non-profit service providers. Moreover, non-profit service providers likely have less capacity to bear risk than private investors, limiting the extent to which the risk of project failure can be transfered to the service provider.

Social impact bonds have been growing since their initial launch in September of 2010. For example, 2017 saw the launch of 49 unique SIBs. As of 2020 over USD 570 million has

[1] For academic work on impact investing see Bialkowski and Starks (2016) or the survey of Renneboog et al. (2008).

[2] Many reports describe the structure of SIBs. For one example, see Gustafsson-Wright et al. (2015).

been invested in social impact bonds.[3]

Despite the growth of social impact bonds as a vehicle for financing public programs, with the exception of Pauly and Swanson (2017), we have been unable to find a theoretical analysis of the benefits of SIBs. Our paper addresses this gap. Within the context of an economic model, we compare SIBs with debt finance and characterize the conditions under which SIBs can expand the set of projects that a government can finance.

We find that social impact bonds have advantages over traditional debt finance. Specifically, when the government is unduly pessimistic or particularly averse to states of the world when project costs cannot be offset with project benefits, then SIBs can finance a set of positive net present value projects that debt finance cannot. We conclude that SIBs are a useful form of finance and should be considered for projects when debt finance has been rejected.

The intuition for our main results are straightforward. First, when governments are pessimistic about the probability a project will be successful, a social impact bond that pays off only when a project is successful will have low ex ante expected financing costs. However, an optimistic private sector will view this bond as a good investment opportunity. Second, when the government is averse to states of the world where the project fails and it must make interest payments associated with the project, it will prefer a financing arrangement that has it make larger payments when the project is successful in exchange for lower payments if the project fails.

There are important reasons to believe that the private sector may be more optimistic than the government about project success. First, most SIBs fund projects that are novel approaches to address persistent social problems. As such, governments lack direct experience with these projects and are likely uninformed about their probability of success. However, service providers have often accumulated evidence on the benefits of their programs and

[3]Statistics are authors' calculations based on data from the Social Finance Impact Bond Global Database, https://sibdatabase.socialfinance.org.uk/, see Social Finance (2019), and the Oxford Government Outcomes Lab Impact Bonds Database, https://golab.bsg.ox.ac.uk/knowledge-bank/indigo/ .

investors who take the time to review studies on these projects may become better informed than the government about project success. Second, if we allow for an environment with many investors with a range of beliefs, we only require that at least one investor is more optimistic than the government for the SIB to expand the set of projects that can be financed.

The "It's All About Me" social impact bond is a good example of how experience led to an investor being more informed than the government about a social program (Gentleman, 2013). The goal of this SIB was to increase adoption rates among children in foster care. The SIB was spearheaded by Jim Clifford who, along with his wife, had successfully adopted nine children. Their experience convinced them that a program with careful matching of children with adoptive parents and easily accessible parenting support would lead to higher adoption rates. Local authorities agreed to pay £54,000 per successful adoption reflecting the high cost of caring for foster children.

There are also important reasons that governments might be particularly averse to paying out costs associated with a project that does not yield benefits. Politicians have career concerns and paying for a failed project can make them look less competent, reducing their re-election probabilities. Moreover, they may appear to be incompetent even if, ex ante, the project was a good investment choice but bad luck led to an adverse outcome. Consequently, politicians will try to distance themselves from being associated with failed interventions.

For example, as noted in Meyer and Kirby (2011), SIBs may have had a historical antecedent in the U.S.S. Monitor, the first U.S. "ironclad" ship launched during the Civil War. The design for the ship was so novel that the U.S. Navy would pay for the cost of the ship only on the condition that the Monitor triumph over Confederate ships in an actual battle (Tierney, 2011). Perhaps unnerved by the ship's revolutionary design, and concerned about funding a fiasco, the Navy's Ironclad Board moved to insure itself.

Our paper relates to a variety of literatures. Pauly and Swanson (2017) conduct a theoretical analysis of social impact bonds. Like that paper, our paper contains an equivalency result, i.e., debt finance and SIBs are equivalent in a world with perfect information and no

risk aversion. However, we differ from Pauly and Swanson in how we break the equivalency. Specifically, they consider models where effort is unobservable and SIBs can incentivize effort. We consider a model where there is asymmetric information about the probability of project success and where governments, because of electoral concerns, are particularly averse to states of the world where they must pay costs associated with a project in excess of benefits.

This equivalency approach links to the theoretical literature on corporate social responsibility (CSR) and impact investing. There, Small and Zivin (2005) argue firms that are engaged in CSR and those that are not will be priced the same when firm-provided public goods are substitutes for non-profit-provided services. In contrast, Baron (2008) allows for unobservable effort and consumer and manager affinity for providing public goods and demonstrates that some level of CSR is optimal for the firm. However, SIBs differ from CSR. With CSR the firm is ostensibly providing a public good. With a SIB, a for-profit firm provides financing, but a not-for-profit entity carries out the service.

SIBs also differ from traditional impact investing where investors invest in for-profit firms taking into account the social impact of these firms' activities. In this growing literature, Barber et al. (2021) show that impact investors are willing to accept lower financial returns, about 4-5 percentage points lower, and are more likely to be foundations and public pensions. Oehmke and Opp (2020) explain impact investing via financing constraints. Chowdhry et al. (2019) consider conditions under which it is optimal for a profit-motivated investor to give an equity claim to an impact investor and to, at times, condition these claims on social outcomes. These papers substantially increase understanding of impact investing. We expand on this literature by explicitly including a government sector and addressing the question as to why the government would prefer to finance some social projects with a SIB when traditional debt is a viable option.

A key mechanism in our model is a desire for governments to avoid states of the world where they must pay costs associated with a project (e.g., financing costs), but receive no

benefits from the project (e.g., a failed educational initiative). This mechanism relates to the literature on optimal debt management (e.g. IMF and the World Bank Staff 2001; Missale 1997; Filardo et al. 2012; Vajs 2014), which stresses the need to incorporate long-term debt in country portfolios to avoid the risk of funding crises. While we do not consider projects large enough that failure could lead to a public debt crisis, we show how careful structuring of a SIB can reduce the government's (or politician's) risk exposure.

Interestingly, this approach parallels the static theory of optimal corporate capital structure, e.g., Scott Jr. (1976). In this theory, firms issue equity despite its tax disadvantages (Miller, 1977) to lower the probability of financial distress.[4] Similarly, to avoid high debt payments in bad states of the world a government would wish to issue equity. However, the government cannot issue ownership shares. SIBs can provide an equity-like instrument that allows interest payments to depend on the state of the world. The benefit of conditioning interest payments on the state of the world is also stressed in the literature on state-contingent sovereign debt, which argues that governments should issue bonds where interest payments depend on the state of the macroeconomy (IMF and the World Bank Staff, 2017).

Finally, our work relates to the theoretical literature on public-private partnerships (PPSs): Maskin and Tirole (2008); Iossa and Martimort (2015); Hart (2003); Reich (2000). However, there is a key difference. With a public-private partnership, the government contracts with a for-profit firm to provide a service. With a SIB, typically a non-profit firm provides the service and the for-profit firm provides the contingent financing. Therefore, the typical rationale for PPPs, to incentize the service provider, is not directly applicable to a SIB as the service provider's payment does not typically depend on project outcomes.[5]

The next section provides background on social impact bonds and section 3 provides descriptive statistics. Section 4 describes the main model of the paper. Section 5 derives and discusses our key insights about SIBs. Section 6 extends the model for the cases where the SIB has higher interest rates and higher administrative costs. Section 7 concludes.

[4]See also Fischer et al. (1989); Hennessy and Whited (2005); Marsh (1982); Stein (1992).
[5]See Warner (2013) for aditionally discussion of the differences between PPSs and SIBs.

2 Background on Social Impact Bonds

A social impact bond is "a multi-stakeholder partnership in which private investors, philanthropic funders and impact investors—not governments—take on the financial risk of expanding preventive programs that help poor and vulnerable people" (Callanan et al., 2012). All SIBs require four core agents to function: the issuer, investors, service providers, and a third-party assessor. The issuer is the entity that issues the SIB. The issuer is most often a public entity, i.e., the government, so much so that hereafter we will use "issuer" and "government" interchangeably. In traditional public financing of social programs, the government provides up-front funding to service providers and shoulders the financial risk associated with the funded program. Under a SIB arrangement, private investors provide the up-front capital and the government agrees to repay them for the program's cost (plus an agreed-upon rate of return) if the program achieves predefined outcomes. This structure allows the government to (1) pay only for successful programs, (2) pay with a portion of the realized benefits from the program (e.g., cost savings), and (3) align payment with the realization of benefits. Private investors provide up-front funding for the administration, execution, and evaluation of the program. Investors accept the risks associated with program failure in exchange for a commensurately higher rate of return than that of traditional public debt. Private investors could include impact investors (both small- and large-scale) and philanthropic individuals or organizations.[6]

2.1 Examples

2.1.1 Peterborough Impact Bond

The world's first social impact bond, the Peterborough Impact Bond, was pioneered in the United Kingdom in 2010 as part of an effort by the Ministry of Justice to reduce reconviction rates among nonviolent offenders (Disley et al., 2015). The bond funded work by

[6]See, among others, Gustafsson-Wright et al. (2015) and Lower-Basch (2014) for descriptions of the functioning of SIBs.

the One Service to operate at Peterborough Prison and provide three cohorts of 1,000 male prisoners with pre- and post-release support in housing, training and employment, parenting, substance abuse, and mental health. Social Finance, a non-profit organization, acted as the intermediary to coordinate the project and raised capital from a total of 17 investors (principally charitable trusts and foundations) who would receive outcome payments if re-conviction rates fell by an average of 7.5% or more across the targeted cohorts.

The Peterborough Impact bond was highly successful on several fronts. First, re-conviction rates fell by an average of 9% across the cohorts, triggering repayment in full for all investors plus a return of 3% interest per annum.[7] Second, the program led to sustained, ongoing services that continued to operate at Peterborough after the end of the pilot. Third, assessments of the program reported that the service provider found that the SIB funding, with its focus on outcomes-based payments, enabled it to alter services to achieve the stated objectives and respond to feedback from program monitors. We use the term successful, here, to describe a SIB where the underlying project achieves outcomes sufficient enough to trigger payments to the investors. We note that, depending on the return paid to investors, the SIB may be more costly than direct financing. However, a project that does not trigger payments may be successful from the government's point of view, as it has avoided paying for a failed project.

2.1.2 The ABLE Project for Incarcerated Youth

The ABLE Project for Incarcerated Youth was established in 2013 with the goal of reducing recidivism among incarcerated youth at Riker's Island jail and has been described by Olson et al. (2013) and MDRC (2015). At the program's inception, almost 50% of 18-year-old men released from Riker's returned to jail within two years. The intervention provided young men aged 16 through 21 job training, transitional employment, and job placement services. A

[7] Originally intended to operate until 2017, the Peterborough Impact Bond was ended early in 2015 to avoid any duplication in services after roll-out of the nationwide Transforming Rehabilitation program that mandated statutory supervision for short-sentenced offenders. Outcome payments were ultimately based on average recidivism rate reduction across the first two cohorts (9%).

successful outcome was defined as a 10% reduction in the number of days participants were jailed during the subsequent year. Goldman Sachs and Bloomberg Philanthropies invested (USD) 9.6 million to fund the program and could have earned up to (USD) 11.7 million in repayment (22% rate of return) from the City of New York's Department of Corrections. Preliminary results found no statistically significant reduction in recidivism between those who received the intervention and a matched control group. Therefore, while the program was intended to last four years, it shut down early and investors were not repaid.

2.1.3 Massachusetts Juvenile Justice Pay for Success Initiative

As described in Rangan and Chase (2015), the Commonwealth of Massachusetts launched in 2014 the Massachusetts Juvenile Justice Pay for Success Initiative to address recidivism among young adults. An estimated 55% of young adults who are on probation or age out of the juvenile justice system will be imprisoned at least once within three years. The program targets roughly one thousand individuals and aims to reduce re-incarceration rates by 40% and increase job readiness and employment. Massachusetts partnered with Roca Inc., a non-profit focused on addressing recidivism, to provide outreach and skill-building interventions. Third Sector Capital Partners acted as the intermediary and raised (USD) 28 million from numerous private investors to cover 85% of Roca's costs upfront. The project was expected to run for six years and the investors, Goldman Sachs and the Kresge Foundation for Living Cities, will only be repaid if the aforementioned incarceration reduction outcomes are achieved, at an interest rate of 3-5% depending on the investor. Incarcerating one inmate costs approximately (USD) 55,000 per annum, so reducing prison populations through social programs could lead to significant savings for Massachusetts taxpayers. Reducing incarceration by 40% is estimated to generate (USD) 22 million in budgetary savings, and a 55% reduction would yield (USD) 33 million in savings.[8]

[8]For more information on this SIB see https://www.thirdsectorcap.org/portfolio/massachusetts-juvenile-justice-pfs-initiative/ .

2.1.4 Additional Examples

While prison recidivism is a popular issue area for social impact bonds, over the past decade these mechanisms have been used in many other contexts that highlight their versatility and customizability.[9] For example, the Educate Girls Development Impact Bond, operating in Rajasthan, India, aimed to address issues of educational access and attendance among schoolgirls. Payments for the Educate Girls bond were structured around two performance measures: learning levels and enrollment rates. The program achieved 160% of its target learning outcomes and threshold investors were repaid their principal investments in full and received an impressive 15% return, which is set to be reinvested in future development programs.[10]

Another social impact bond, issued in Cameroon in 2017, focuses on ophthalmic health.[11] The Cameroon Cataract Development Bond is a financing vehicle to fund as many as 18,000 cataract surgeries over five years. This bond structures its payment thresholds around three outcome categories: volume of completed surgeries, quality of completed surgeries, and financial sustainability of the surgical hospital. Scheduled to conclude in 2023, it is currently on target to meet all three outcomes goals, and the highest potential return to investors is 8%.

[9] Responding to the COVID-19 threat, Palandjian and Brest (2020) argue SIBs are a useful mechanism to finance pandemic response.

[10] Results for the Educate Girl's SIB are obtained from UBS Optimus Foundation (2018) available at: https://instiglio.org/educategirlsdib/about-the-dib/ .

[11] Oroxom et al. (2018) provide a detailed description of the Cameroon bond. Their paper is available at: https://golab.bsg.ox.ac.uk/knowledge-bank/case-studies/cameroon-cataract-bond/ .

3 Data and Descriptive Statistics

3.1 Data Description

We obtain data from the Social Impact Bond Global Database as compiled and maintained by Social Finance and the Oxford Government Outcomes Lab Impact Bonds Database.[12] The Social Finance database contains data through March 2019. The Oxford database is regularly updated and intended to include all issued SIBs. We have data for 221 projects on the following variables for each SIB project: target population, location, launch date, duration of bond, capital raised, maximum outcome payments, investors, outcomes funder, service provider, and intermediary (if applicable). In constructing the dataset for this paper, we convert all local currencies to 2020 USD and include a variable categorizing each SIB location country by its World Bank income classification.

While the Social Finance database includes information on metrics and outcomes when this information is publicly available, key data are missing for many SIB projects in our database. For example, 51 projects have undisclosed values for capital raised and 68 projects have unspecified investor counts. Of all the projects, only eight completed projects have a published return on investment.

3.2 Descriptive Statistics

Table 1 provides descriptive statistics for completed and ongoing social impact bond projects. The vast majority (91%) of SIB projects are located in high-income countries, with the highest concentration in Europe and Central Asia (63%) followed by North America (16%) and East Asia and the Pacific (9%) (Table 1). Only seven SIB projects have been located in Latin America (Argentina, Colombia and Peru), eight in Sub-Saharan African (Cameroon, Kenya, Mali, Nigeria, South Africa, the Congo, and Uganda), eight in the Middle East, and

[12]These databases are available at: https://sibdatabase.socialfinance.org.uk/ and https://golab.bsg.ox.ac.uk/knowledge-bank/indigo/ respectively.

Table 1: Social Impact Bond Project Distribution by Issue Area, Country Income Classification, and Region

Project Characteristic	**Count**	**Percent**
Issue Area		
Workforce Development	61	28%
Child and Family Welfare	40	18%
Health	36	16%
Housing/Homelessness	32	14.5%
Education and Early Years	32	14.5%
Criminal Justice	17	8%
Poverty and Environment	3	1%
Country Income Classification		
High income	202	91%
Upper-middle income	8	4%
Lower-middle income	8	4%
Low income	3	1%
Region		
Europe and Central Asia	140	63%
North America	36	16%
East Asia and Pacific	19	9%
Sub-Saharan Africa	8	4%
Middle East and North Africa	8	4%
Latin America and the Caribbean	7	3%
South Asia	3	1%

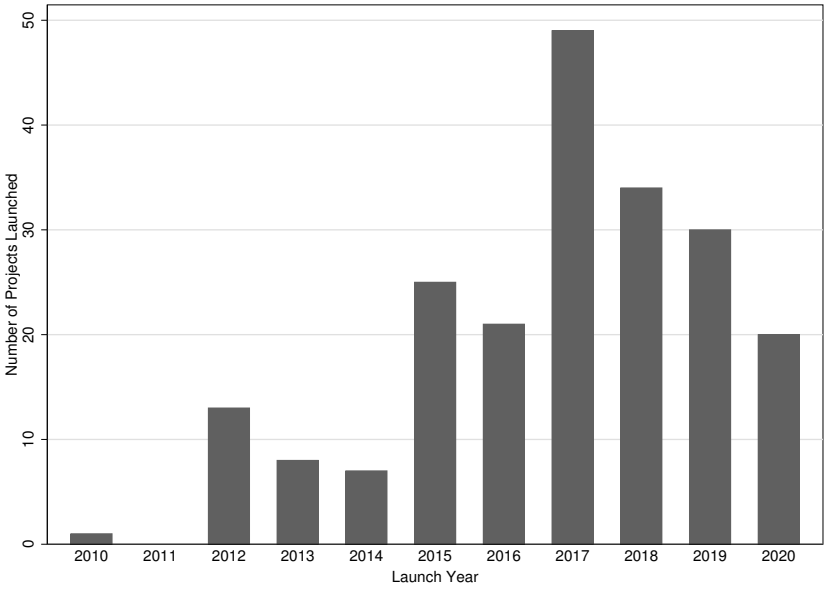
Source: Authors' calculations based on the Social Finance Impact Bond Global Database and the Oxford Government Outcomes Lab Impact Bonds Database.

three in South Asia (all in India). These projects cover broad issue areas, from affordable housing programs to medical procedures to sanitation infrastructure to wildlife conservation programs. Most projects, however, fall under the umbrellas of workforce development, child and family welfare, health, housing and homelessness, or education. One possible explanation for such concentration in these issue areas is that they often involve shorter-term intervention programs. Programs with shorter durations may lend themselves to earlier evaluation and faster repayment for investors.

As demonstrated in figure 1, following their initial launch with the Peterborough Impact Bond in 2010, new social impact bond projects began gaining traction. The number of new

bonds peaked in 2017, with 49 launching in that year alone. In 2018 the number of new bonds dropped to 34 new bonds – but was still the second highest number on record. Additionally, 2019 saw 30 launches and 2020 saw 20 launches consistent with a decline in SIB issuance. Whether the drop in recent years will be indicative of future trends in the creation of new SIBs is unclear.

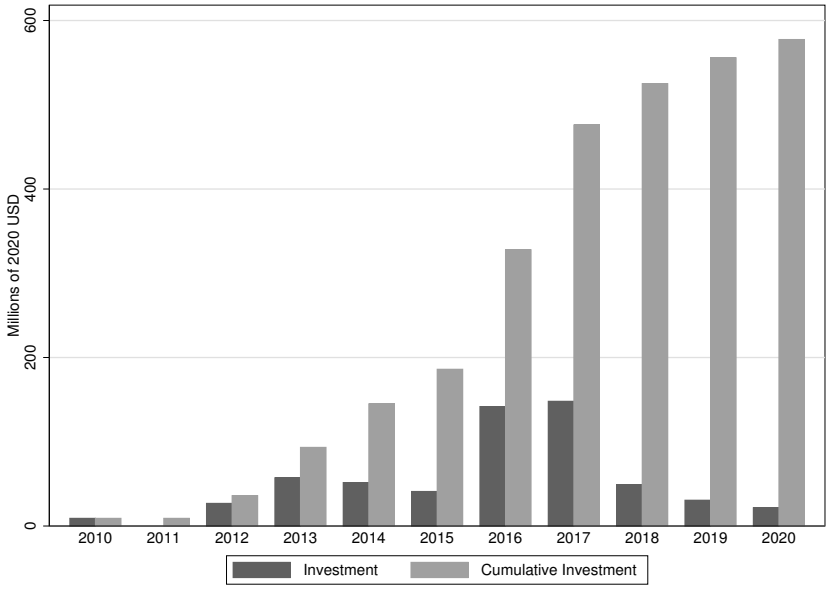
Figure 1: SIB Projects by Launch Year (2010 - 2020)



Source: Authors' calculations based on the Social Finance Impact Bond Global Database and the Oxford Government Outcomes Lab Impact Bonds Database.

As shown in figure 2, total investment in social impact bonds has increased almost every year since their inception in 2010 with the Peterborough Impact Bond (in 2011 there were no new bonds). From 2012 to 2017 we see steady increases in total investment, with new investment peaking in 2017. Although the growth rate of investment levels has slowed in recent years, cumulative global investment in SIBs currently stands at (2020 USD) 577 million. Cumulative global investment is 21% higher than it was in 2017 when new investment peaked.

Figure 2: Global SIB Investment Level (2010-2020)



Source: Authors' calculations based on data from the Social Finance Impact Bond Global Database and the Oxford Government Outcomes Lab Impact Bonds Database.

Table 2 provides summary statistics for some of the key variables in the data set. The average investment in a social impact bond is (USD) 3.46 million, although the largest project involved a (USD) 32.4 million dollar investment and the smallest a (USD) 60,000 investment. Projects tend to have larger investment values in the U.S. than they have in the United Kingdom or globally. Outcome payments average about (USD) 6.3 million but clearly vary by project. A typical project has multiple investors, with the average number of investors being about three. Finally, average maturity of these bonds is 4.57 years, although the maturity is longer in the U.S. at roughly 5.67 years.

Table 2: SIB Project Investment Data by Country (2010 – 2020)

Country	Capital Raised (2020 USD Millions)	Max Outcomes Payments (2020 USD Millions)	Number of Investors	Time to Maturity (Years)
Global	3.46	6.3	3.37	4.57
	[0.06 - 32.4]	[0.11 - 37.7]	[1 - 59]	[1-30]
UK	1.68	5.19	2.43	4.44
	[0.13 - 9.2]	[0.11 - 28.8]	[1 - 17]	[2 - 10]
US	9.80	10.75	5.17	5.67
	[0.15 - 32.4]	[0.15 - 37.7]	[1 - 35]	[2 - 30]
Other	2.78	5.49	3.57	4.24
	[0.06 - 28.5]	[0.24 - 20.9]	[1 - 59]	[1 - 10]

Source: Authors' calculations based on data from the Social Finance Impact Bond Global Database and the Oxford Government Outcomes Lab Impact Bonds Database. Average values listed, min and max in brackets.

4 Model

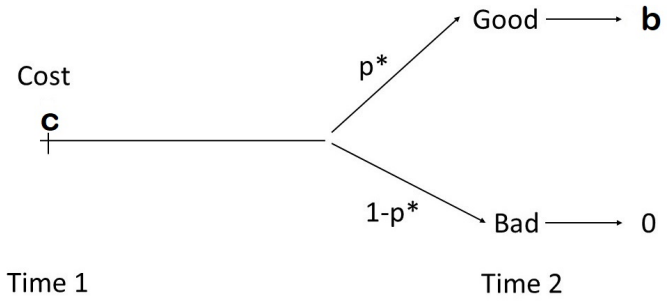
This section sets forth our model of social impact bonds. It first describes a project the government could potentially finance. Next, it describes the objectives of the government and the investor who may finance the project. Then we define the financing instruments: a debt contract and a SIB. Finally, we describe the incentive compatibility conditions that are required for the government to provide payment and the investor to provide financing sufficient to implement the project.

4.1 Project

Consider a situation where the government is deciding to finance a public project. The project costs c, this period, to implement and has a potential benefit of b next period. Next period there are two possible states of the world: the good state and the bad state. In the good state the government receives the benefit b; in the bad state the government receives zero. The good state of the world occurs with unknown probability denoted by p^*. We assume that $\frac{p^*b}{c} > 1 + r$ so that, ex ante, implementing the project has a positive net

expected present value. Figure 3 summarizes the project.

Figure 3: Potential Project



In the model, the probability of project success is exogenous. This assumption has two important implications. The first implication is that we abstract away from a potential justification for SIBs, namely that they increase provider effort and consequently increase the probability a project succeeds. We make this assumption because, as noted in the introduction, previous work has considered the relationship between SIBs and effort and because we wish to distinguish between SIBs and performance-based contracts, which are clearly intended to incentivize the service provider. The second implication is that we do not consider the possibility that a SIB can create perverse incentives for the government, i.e. the government might partially sabotage a project to avoid paying the success payment. We find this latter possibility unlikely. In most of the SIBs we have reviewed the government is invested in solving the social problem the SIB addresses and governments are likely to suffer political consequences if they actively try to prevent the project from helping those the SIB is designed to help.

4.2 Financing

There are two possible mechanisms to finance the project. The first, debt finance, constrains the government's payments to be constant across the two states. The second, a social impact bond, allows the payments to be conditional on the benefits the government receives and therefore on the state of the world.

4.3 Agents

The model contains two types of agents: a risk-neutral investor and the government.[13] The risk-neutral investor will only invest in a project if she expects to get a return greater than or equal to her required return. Her required return is given by

$$r - \omega. \tag{1}$$

Here r is the expected return on comparable government debt and ω represents the discount an investor is willing to take to invest in a socially responsible project.[14] In section 6.1 we consider the possibility that investors require a higher return on SIBs than comparable government debt. The investor also believes that the project will succeed with probability p, which may or may not equal p^* (the true probability).

The second type of agent is the government. The government maximizes the expected present value of benefits minus costs of implementing the project. However, the government has an added preference for avoiding what we call unfunded costs.[15] It is specifically averse to experiencing states of the world where it must pay costs associated with a project (e.g., debt payments) when no corresponding benefits materialize. While we do not currently model why this is the case, one potential explanation is that failed projects can be portrayed as wasteful spending by political opponents, reducing a politician's re-election probability. The magnitude of this aversion is captured with the parameter ϕ.[16]

[13] We model the investor as risk neutral because returns are intended to depend on the causally estimated benefit of a novel social program. It is reasonable that these outcomes are uncorrelated with the stochastic discount factor of the investor. Moreover in section 6.1, we relax this assumption by allowing the SIB to require a higher return than traditional government debt.

[14] We include ω here to clarify that the investor's required return may be less than on typical government debt because the project has some social benefit. However in the model ω plays no independent role and the model could equivalently be solved by letting r be the required return and setting ω equal to zero.

[15] We use the term unfunded costs because when the project succeeds its benefits are high enough so that the project costs, the debt payments, can be funded from or paid out of the realized benefits.

[16] For example, Rogoff and Sibert (1988) consider a model where politicians have unobserved competency. Spending money with little benefit reveals the politician's incompetency and results in her being voted out of office.

Subsequently, the government's expected utility is given by

$$E(U) = \\ -(1+\phi)c_1 + \frac{1}{1+r}\left[q\left(b - c_2^g\right) - (1-q)c_2^b\right] - \frac{\phi}{1+r}\left[q * max(0, c_2^g - b) + (1-q) * max(0, c_2^b)\right] \tag{2}$$

In this function c_1 are any costs the government pays in the first period, c_2^g are costs the government pays in the second period if the good state of the world occurs, c_2^b are costs the government pays in the second period if the bad state of the world occurs, $\frac{1}{1+r}$ is the government's discount factor, and q is the government's belief regarding the probability that the good state of the world will occur. As with the investor, this probability q may not necessarily equal the true probability p^*. Finally, note that when the government chooses not to finance the project, its utility equals zero.

To grasp this utility function, note that the first term captures government costs in the first period. The cost, c_1, is multiplied by $1 + \phi$ because there are no benefits in the first period to offset any costs and therefore, given the government's aversion to unfunded costs, government utility falls by an additional factor ϕ. The second term represents the expected present discounted value of net benefits from the project. The third term represents the expected present discounted value of unfunded costs. The use of the max function ensures that when benefits b exceed costs in the good state of the world, c_2^g, the unfunded cost is exactly zero.

In this model we assume that the subjective probabilities regarding project success can differ between investors and the government. In our SIB descriptions, in section 2, we stressed that SIBs tend to finance projects with novel approaches for addressing social problems. Consequently, it is not surprising that individuals will differ on their expectations for project success. Moreover, service providers often pilot their programs to obtain evidence on their impact. Investors who review this evidence may then become more optimistic than the

government regarding project success.

4.4 Financial Instruments

Two instruments are available to finance the project: traditional debt finance and a social impact bond.

Definition 1. *Debt finance* is a contract that requires the government to pay the investor $c(1+i)$ in each state of the world. i is the interest rate on the debt contract.

Definition 2. A *social impact bond* (SIB) is a contract that requires the government to pay the investor prespecified values $c_2^g \geq 0$ in the good state of the world and $c_2^b \geq 0$ in the bad state of the world. Additionally, consistent with the SIBs we observe in the data, we require that $c_2^g \geq c_2^b$.

4.5 Incentive Compatibility

A project is *government incentive compatible* if the government would choose to implement the project with an available financing instrument. The project is *investor incentive compatible* if the investor is willing to provide financing for the project. For the investor, incentive compatibility requires that her expected return be greater than the required return given by equation (1). For the government, incentive compatibility requires that the expected utility of implementing the project is greater than the utility of not implementing the project, 0.

Definition 3. A project is *implementable* if it is incentive compatible for both the investor and the government.

5 Results

This section examines the conditions under which the government can implement potential projects with debt finance and the conditions under which a social impact bond can expand

the set of projects that can be financed.

5.1 Debt Finance

Theorem 1. *A debt contract with interest rate i can implement the project if and only if* $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq r - \omega$.

Proof. See appendix.

To understand this result, note that the condition requires that the government's expected return on the project be greater than the required return of the investor. However, note that the expected cost of the project to the government is not just c but $c[1 + \phi(1 - q)]$. This value is the government's real cost because, based on its beliefs, with probability $1 - q$ the government will have to make debt payments in excess of received benefits, which has an additional cost of ϕ per dollar of unfunded costs. However, when the expected benefit of the project is high enough to cover the government's interest costs while accounting for its aversion to unfunded costs, the government is willing to finance the project with debt.

Remark 1. When the government has no preference for avoiding states of the world with unfunded costs, i.e., $\phi = 0$, then any project where $\frac{qb}{c} - 1 \geq r - \omega$ can be implemented. This result is the conventional result that the government should finance any project for which the expected return is greater than the rate at which it can borrow. Note also in this case ($\phi = 0$), if the project cannot be funded with debt finance then $\frac{qb}{c} < 1 + r - \omega$. Because we assume that the project is exante beneficial, i.e., $\frac{p^*b}{c} > 1 + r$, then when $\phi = 0$, the projects that cannot be implemented by debt finance are ones where $\frac{qb}{c} < \frac{p^*b}{c}$ or $q < p^*$. Therefore we can conclude, in this case, that, if the government will not fund the project with debt finance, then the government is unduly pessimistic in that its subjective expectation of the probability of success is less than the true probability.

Remark 2. When $\frac{qb}{c[1+\phi(1-q)]} - 1 > r - \omega$, the project is implementable with any interest rate i where $\frac{qb}{c[1+\phi(1-q)]} - 1 > i > r - \omega$. The higher the interest rate, the larger is the share of

the expected benefit from the project that is paid out to the investor.

5.2 Social Impact Bond

Now we consider the projects that can be implemented under a social impact bond, and specifically the capacity of a SIB to expand the set of implementable projects.

Theorem 2. *A social impact bond is investor incentive compatible if and only if* $\frac{pc_2^g+(1-p)c_2^b}{c} - 1 \geq r - \omega$ *and is government incentive compatible if and only if* $\frac{qb}{c} \geq \frac{qc_2^g+(1-q)(1+\phi)c_2^b}{c}$.

Proof. See appendix.

Theorem 3. *A project is implementable using a social impact bond whenever* $p\frac{b}{c} \geq (r-\omega+1)$. *Moreover if* $p \geq q$, *this condition will hold for any project that is implementable with a social impact bond.*

Proof. See appendix

To develop some intuition for this last result, note that if the investor believes the good state of the world will occur with probability p, the left-hand side of this condition is the expected return on the contract that pays all the project benefits to the investor in the good state and pays zero in the bad state. The government would accept that contract because it would set government utility to exactly zero. Therefore, whenever this condition holds we can implement the project.

To understand why, when $p \geq q$, this condition must hold for any implementable project, consider the contract $c_2^g = \frac{c(r-\omega+1)}{p}$ and $c_2^b = 0$. This contract has the benefit of completely eliminating unfunded costs and exactly compensating the investor by giving her exactly the required return. Moreover, when $p \geq q$ the investor values payments in the good state relatively more than the government because the investor views these states as more likely. As such it is best for the government to transfer all its payments to the good state. Therefore, from the government's point of view this is the best contract, and if it cannot implement the

project no other contract will. Now note that the government would accept this contract if it gives a non-negative expected return to the government. The government will get a positive expected return if $q(b - \frac{c(r-\omega+1)}{p}) \geq 0$, which leads to the condition of theorem 3.

An important implication of theorem 3 is that when at least one investor is as optimistic as the government, then any project with positive expected value from the point of view of the government can be implemented.

With only debt finance, theorem 1 demonstrates that only projects where the expected return $\frac{qb}{c} \geq (1 + r - \omega) * [1 + \phi(1 - q)]$ will be implementable. We can think about the government's aversion to unfunded costs as creating a wedge between the expected return on a project and the cost of debt finance. The size of the wedge is proportional to ϕ, the magnitude of the government's aversion to unfunded costs, and to $1-q$, the probability that the government experiences unfunded costs.

Because a social impact bond can eliminate these unfunded costs, if the government can find an investor that is at least as optimistic as it is, it can fund any project where its expected return is greater than the interest rate on debt, i.e., $\frac{qb}{c} \geq (1 + r - \omega)$. Because if $\frac{qb}{c} \geq (1 + r - \omega)$ and the investor has a success belief $p \geq q$, then $\frac{pb}{c} \geq (1 + r - \omega)$. As such, the SIB solves the inefficiency caused by the government's aversion to unfunded costs. In fact, if the inefficiency is large enough (for example if ϕ is large) a SIB could implement a project that debt finance cannot, even if the private sector is more pessimistic than the government.

Remark 3. When $\phi = 0$ and $p \leq q$, any project that is implementable with a SIB is also implementable with debt finance.

Proof. Setting $\phi = 0$ in theorem 2 we obtain that a social impact bond is government incentive compatible only if $q\frac{b}{c} > \frac{qc_2^g + (1-q)c_2^b}{c}$. Recall, also from theorem 2, that the SIB is only investor incentive compatible if $\frac{pc_2^g + (1-p)c_2^b}{c} - 1 > r - \omega$. When $q \geq p$ these two conditions imply $\frac{qb}{c} - 1 \geq r - \omega$, which is the implementability condition from theorem 1 for debt finance. □

Remark 3 implies that for social impact bonds to add value by increasing the set of implementable projects, we must either have investors who are more optimistic than the government or have a government that is averse to unfunded costs.

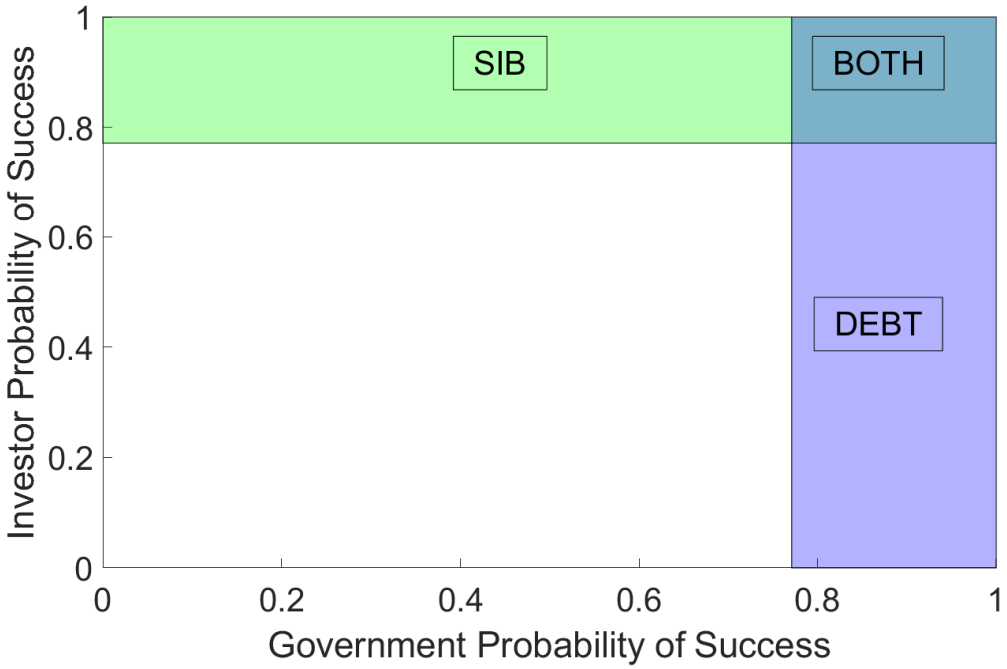
5.3 Examples

Figure 4 presents a specific example to illustrate the role that government and investor beliefs play in the implementability of the project. We parameterize this example using an aversion to unfunded costs $\phi = 0$, a benefit-cost ratio $\frac{b}{c} = 1.4$, and the investor's required return $r - \omega = 0.08$. The y-axis gives the investor's belief for the probability that the project will succeed. The x-axis gives the government's probability belief that the project will succeed. The upper-right square, labeled BOTH, indicates that for projects where the government and investors are very optimistic about project success both debt and a SIB can implement the project. Now moving to the rectangle in the upper left of the diagram, we have the region where only the SIB can implement the project. In this region the government is unwilling to finance the project with debt because the government believes that the project is likely enough to fail that it is not worth financing at the 8% interest rate. However, as long as the investor is sufficiently optimistic, a SIB can implement the project. Because the investor thinks the project is likely to succeed, the SIB meets her required return. Finally, note that debt implementability is insensitive to the investor's belief about project success. This results occurs because debt guarantees the investor the same payment no matter the state of the world.[17,18]

[17]Note from theorem 3, when $p < q$, there are some projects implementable with a SIB that do not satisfy the condition $p \geq c(r + \omega - 1)/b$. Therefore DEBT should be interpreted as DEBT definitely while SIB can be interpreted as SIB only.

[18]Because we assume that the project has ex ante benefits in excess of costs there is no risk that the SIB will implement a project that is not worth implementing. However, an interesting question that we do not consider here regards the possibility that SIBs will implement bad projects, with ex ante costs in excess of expected benefits, when investors are overly optimistic.

Figure 4: Disagreement about the Probability of Success



Note: This figure indicates the range of investor and government subjective probabilities of project success where projects can be implemented with debt (DEBT), can be implemented with both debt and a social impact bond (BOTH), and can be implemented with a social impact bond only (SIB). The parameters used are $\phi = 0$, $\frac{b}{c} = 1.4$, and $r - \omega = 0.08$.

Next, Figure 5 uses a similar example to illustrate the role that aversion to unfunded costs has on the implementability of the project. Again, the benefit-cost ratio $\frac{b}{c} = 1.4$ and an investor's required return $r - \omega = 0.08$. In this figure, we set the investor's probability of success (p) equal to the government's probability of success (q) and consider a range of values for the aversion to unfunded costs (ϕ) from zero to three. Any project for which the investor and the government are sufficiently optimistic can be implemented by a social impact bond. For SIBs, the aversion to unfunded costs is irrelevant because the SIB does not require the government to make payments in the bad state (when unfunded costs could occur). However, as the aversion to unfunded costs rises, fewer and fewer projects can be implemented with debt finance. Because the government risks unfunded costs, it will only be willing to use debt to finance projects for which it is highly optimistic.

Figure 5: The Effect of Unfunded Costs

Note: This figure indicates the range of ϕ (aversion to unfunded costs) where the project can be implemented with debt, a social impact bond (SIB), or both, given the government and the investor's belief that the project succeeds. Here we assume that the government and investor share the same belief. The parameters used are $\frac{b}{c} = 1.4$ and $r - \omega = 0.08$.

6 Model Extensions

In this section we consider two extensions to the model. First, we consider the case where the social impact bond must pay a higher interest rate than traditional debt. Second, we consider the case where the social impact bond, due to administrative costs, leads to higher project costs than implementing the project through traditional debt. In both cases we find that the benefits of the SIB are mitigated but not eliminated.

6.1 Higher Relative Interest Rates on the Social Impact Bond

As social impact bonds are relatively new, they may be viewed as riskier than traditional debt. Moreover, they are riskier than traditional debt insofar as the payments to investors depend on an uncertain outcome (whether or not the project produces a positive benefit).

Consequently, one might expect that investors in SIBs would require a higher interest rate than investors in traditional debt. In this section we consider the impact of higher interest rates on the appeal of SIBs. We find that while higher interest rates mitigate the attractiveness of SIBs they do not eliminate their usefulness. In fact, as long as the expected return on the project, using the investor's success probability, is positive, there is a range of aversion to unfunded costs (ϕ) for which the social impact bond can implement the project and traditional debt cannot. This result holds even in the case where the government is as optimistic as the investor about the likelihood of project success. The following theorem provides the basis for this conclusion.

Theorem 4. *Let r' be the investor's required return on the SIB. If $q < 1$ and $p\frac{b}{c} \geq (1 + r')$, so that the project is implementable with a SIB, then a range of values of ϕ, $\phi > \frac{q\frac{b}{c} - (1+r)}{(1+r)(1-q)}$ exist where the SIB can finance the project and traditional debt cannot. Moreover for $\phi > \frac{r'-r}{(1+r)}\left(\frac{b/c}{\frac{b}{c}-(1+r')}\right)$ the government success probability q required to implement the debt contract will be higher than the investor success probability $p = \frac{(1+r')}{b/c}$ that is required to ensure the project can be implemented with the SIB.*

Proof. See appendix.

To understand the logic of this result return to Figure 5. The line at the bottom of the box that defines the region of beliefs that ensures the project is implementable with a SIB is given by the equation $p = \frac{(1+r')c}{b}$. The increase in the required interest rate from r to r' raises this line. By assumption $p > \frac{(1+r')c}{b}$ so the SIB can implement the project. The line at the bottom of the region of beliefs where traditional debt can implement the project is given by $q = \frac{(1+\phi)(1+r)}{\frac{b}{c}+\phi(1+r)}$. As ϕ gets large this value asymptotes to 1. So there must be a level of ϕ where q, the government's success probability, is not sufficient to implement the project via debt. Moreover, because we assume that $p > \frac{(1+r')c}{b}$ and p is at most equal to 1, we have that $\frac{(1+r')c}{b} < 1$. Therefore for some ϕ the line at the bottom of the region of beliefs where traditional debt can implement the project must lay above the line that determines

the bottom of the region where beliefs are sufficient to implement the project with a SIB.

6.2 Higher Relative Project Costs for the Social Impact Bond

A noted limitation of social impact bonds is that the administrative costs are high relative to traditional debt finance (see, for example, Gustafsson-Wright et al., 2015). These costs include the bespoke nature of the contracts that require more labor to write and review and the need to decide and agree upon the outcome metrics. Additionally, the project must fund an assessor to verify if the project achieved the positive outcomes that trigger payments to investors.

To understand how these higher costs affect the usefulness of the SIB we consider the situation where the project costs more to implement using a SIB than using traditional debt finance. Again we find that these higher costs mitigate but do not eliminate the usefulness of the SIB. If the government's aversion to unfunded costs is large enough projects will exist where it is worthwhile to pay the extra costs of implementing the project via a SIB. We make this conclusion via the following theorem.

Theorem 5. *Let c' be the cost required to fund a project via the SIB. If $q < 1$ and $p\frac{b}{c'} \geq (1+r)$, so that the project is implementable with a SIB, then a range of values $\phi > \frac{q\frac{b}{c}-(1+r)}{(1+r)(1-q)}$ exist where the SIB can finance the project and traditional debt cannot. Moreover, for $\phi > \frac{\left(\frac{c'}{c}-1\right)b}{b-c'(1+r)}$, the government success probability q required to implement the debt contract will be higher than the investor success probability $p = \frac{(1+r)}{b/c'}$ that is required to ensure the project can be implemented with the SIB.*

Proof. See appendix.

The logic behind this theorem is the same as the previous theorem. The increase in costs shifts up the line that determines the smallest investor success probability that will allow the project to be implemented using a SIB. As long as this belief is less than 1, there will be an aversion to unfunded costs large enough to eliminate the ability of the project to

be funded with traditional debt while it can still be funded with a SIB. Moreover, for any government success probability there is always a friction large enough to render the project unimplementable.[19]

7 Conclusion

In this paper we formulate a novel theory of social impact bonds. Social impact bonds allow the government to finance some positive net present value projects that cannot be financed with traditional debt finance. SIBs solve two inefficiencies. The first inefficiency arises when the government is unduly pessimistic about the possible success of the project. The second inefficiency arises when political realities lead politicians to be overly concerned about paying for underperforming projects. By allowing the payouts to depend on the project's success and the beliefs of the investor, the SIB can enable the financing of positive net present value projects when one or both of these inefficiencies prevent the project from being financed through traditional debt finance. As both these features are common in public investment, SIBs represent a useful innovation in public finance and should be considered when traditional debt financing has been rejected.

Looking forward, additional features of SIBs are worthy of exploration. One such feature is the role an investor can play in affecting the probability a project succeeds. Another important issue to explore is the SIB transaction cost relative to traditional debt finance and the resulting appropriate scale of social impact bonds. Finally, the implementation of a SIB-financed project may alter beliefs about the probable success of the project, enabling the project to be financed by debt finance in the future. A dynamic model would be needed to explore this avenue.

[19]Note that for simplicity we assume that administrative costs can be financed with the debt instrument and paid back when the state of the world is revealed.

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Appendix: Proofs of Theorems

Theorem 1. *A debt contract with interest rate i can implement the project if and only if* $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq r - \omega$.

Proof. First, if the debt contract can implement the project, then the contract is incentive compatible for the government. So it must be true that $b \geq c_2^g$, otherwise at least two terms in the government's utility would be negative and the rest are at most zero and therefore the government's utility would be less than zero. Given that $b \geq c_2^g$, the government's expected utility is given by $\frac{1}{1+r}\left[q\left(b - c_2^g\right) - (1-q)c_2^b\right] - \frac{\phi}{1+r}\left[(1-q)c_2^b\right]$. Substituting in the debt contract $c(1+i) = c_2^g = c_2^b$, we have that the government's expected utility equals $\frac{1}{1+r}\left[qb - c(1+i)\right] - \frac{\phi}{1+r}\left[(1-q)c(1+i)\right]$. Implementability implies that this utility is greater than or equal to zero, which implies that $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq i$. Moreover, implementability implies that the project is incentive compatible for the investor, which requires $i \geq r - \omega$. Combining this condition with the previous condition yields $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq r - \omega$.

To show that if the stated condition holds we can implement the project, consider the debt contract with $i = r - \omega$. Because the implementability constraint for the investor is that the expected return be greater than or equal to $r - w$, the contract is incentive compatible for the investor. Moreover if $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq r - \omega$ then $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq i$, which implies that $qb > c[1+\phi(1-q)](1+i)$. Consequently, $qb - c(1+i) - \phi(1-q)c(1+i) > 0$ and therefore the government's utility is greater than zero. Note that again we know that $b \geq c_2^g = c * (1 + r - \omega)$ because the project is assumed to have a positive net expected present value so $b > c(1+r)/p^*$. □

Theorem 2. *A social impact bond is investor incentive compatible if and only if* $\frac{pc_2^g + (1-p)c_2^b}{c} - 1 \geq r - \omega$ *and is government incentive compatible if and only if* $\frac{qb}{c} \geq \frac{qc_2^g + (1-q)(1+\phi)c_2^b}{c}$.

Proof. Because the social impact bond pays the investor c_2^g in the good state and c_2^b in the bad state of the world, and the investor believes the good state will occur with probability

p, then her expected return equals $\frac{pc_2^g+(1-p)c_2^b}{c}$ for the initial investment of c. This is greater than the required return if and only if $\frac{pc_2^g+(1-p)c_2^b}{c} > 1+r-\omega$.

As in the proof of the previous theorem, if a SIB is incentive compatible for the government it must be true that $b \geq c_2^g$. Given that $b \geq c_2^g$, the government's expected utility is given by $\frac{1}{1+r}\left[q\left(b-c_2^g\right)-(1-q)c_2^b\right] - \frac{\phi}{1+r}\left[(1-q)c_2^b\right]$. Utility is non-negative when $qb - qc_2^g - (1-q)c_2^b - \phi(1-q)c_2^b \geq 0$. Rearranging we get $q\frac{b}{c} \geq \frac{qc_2^g+(1-q)(1+\phi)c_2^b}{c}$.

To show the converse, note that $q\frac{b}{c} \geq \frac{qc_2^g+(1-q)(1+\phi)c_2^b}{c}$ implies that $b \geq c_2^g + \frac{(1-q)(1+\phi)c_2^b}{q}$. Now, because $c_2^b \geq 0$, then $b \geq c_2^g$. Therefore the government's expected utility is given by $\frac{1}{1+r}\left[q\left(b-c_2^g\right)-(1-q)c_2^b\right] - \frac{\phi}{1+r}\left[(1-q)c_2^b\right]$. Because $q\frac{b}{c} \geq \frac{qc_2^g+(1-q)(1+\phi)c_2^b}{c}$ implies $qb - qc_2^g - (1-q)c_2^b - \phi(1-q)c_2^b \geq 0$, the government's expected utility is non-negative. □

Theorem 3. *A project is implementable using a social impact bond whenever $p\frac{b}{c} \geq (r-\omega+1)$. Moreover if $p \geq q$, this condition will hold for any project that is implementable with a social impact bond.*

Proof. Based on the previous theorem, a project is government incentive compatible if and only if $q\frac{b}{c} \geq \frac{qc_2^g+(1-q)(1+\phi)c_2^b}{c}$. Consider the contract $c_2^{g^*} = \frac{c(r-\omega+1)}{p}$ and $c_2^{b^*} = 0$. For this contract the government's implementability constraint simplifies to $q\frac{b}{c} > q\frac{(r-\omega+1)}{p}$ or $p\frac{b}{c} > (r-\omega+1)$. Therefore, when this condition holds, a social impact bond exists that can implement the project.

To show that when $p \geq q$ the stated condition will hold for any project that is implementable with a SIB, we seek to find the contract that maximizes the set of projects that will satisfy the government's implementability constraint. To do this, we consider a social impact bond contract that makes the investor indifferent between investing and not investing in the SIB, i.e., her incentive compatibility constraint holds as an equality. Therefore, $pc_2^{g^*} + (1-p)c_2^{b^*} = c(r-\omega+1)$, which implies $c_2^{b^*} = \frac{c(r-\omega+1)-pc_2^{g^*}}{1-p}$. From the previous theorem, this contract is incentive compatible for the government if and only if $q\frac{b}{c} > q\frac{c_2^{g^*}}{c} + \frac{(1-q)(1+\phi)}{c}\left[\frac{c(r-\omega+1)-pc_2^{g^*}}{1-p}\right]$ or $q\frac{b}{c} > \left[q - \frac{p(1-q)(1+\phi)}{1-p}\right]\frac{c_2^{g^*}}{c} + \frac{(1-q)(1+\phi)}{1-p}(r-\omega+1)$. When

$p \geq q$, the first term on the right-hand side of the government's incentive compatibility condition $\left[q - \frac{p(1-q)(1+\phi)}{1-p}\right] \frac{c_2^{g^*}}{c}$ is negative. Because we will maximize the set of implementable projects by making the right-hand side of the government's compatibility constraint as small as possible, we can maximize the set of implementable projects by making $c_2^{g^*}$ as large as possible, which gives $c_2^{g^*} = \frac{c(r-\omega+1)}{p}$ and $c_2^{b^*} = 0$. In this case the implementability constraint simplifies to $q\frac{b}{c} > q\frac{(r-\omega+1)}{p}$ or $p\frac{b}{c} > (r - \omega + 1)$. □

Theorem 4. *Let r' be the investor's required return on the SIB. If $q < 1$ and $p\frac{b}{c} \geq (1 + r')$, so that the project is implementable with a SIB, then a range of values of ϕ, $\phi > \frac{q\frac{b}{c}-(1+r)}{(1+r)(1-q)}$ exist where the SIB can finance the project and traditional debt cannot. Moreover for $\phi > \frac{r'-r}{(1+r)}\left(\frac{b/c}{\frac{b}{c}-(1+r')}\right)$ the government success probability q required to implement the debt contract will be higher than the investor success probability $p = \frac{(1+r')}{b/c}$ that is required to ensure the project can be implemented with the SIB.*

Proof. From theorem 1 the debt contract can implement the project if and only if $\frac{qb}{c[1+\phi(1-q)]} \geq (1 + r)$. This condition holds if and only if $q \geq \frac{(1+r)[1+\phi(1-q)]}{b/c}$, and therefore traditional debt cannot implement the project when $q < \frac{(1+r)[1+\phi(1-q)]}{b/c}$ which holds when $\phi > \frac{q\frac{b}{c}-(1+r)}{(1+r)(1-q)}$. Therefore when this condition holds the project cannot be implemented with a debt contract but by assumption, $p\frac{b}{c} \geq (1 + r')$, and so the project can be implemented with a SIB. Additionally traditional debt can finance the project if and only if $q \geq \frac{(1+r)[1+\phi(1-q)]}{b/c}$ which implies $q \geq \frac{1+r}{b/c}\frac{(1+\phi)}{1+\phi\frac{(1+r)}{b/c}}$. The corresponding lower bound on the required investor's success probability given by theorem 3 is $p \geq \frac{1+r'}{b/c}$. We have that $\frac{1+r}{b/c}\frac{(1+\phi)}{1+\phi\frac{(1+r)}{b/c}} > \frac{1+r'}{b/c}$ when $(1+r)(1+\phi) > (1 + r')\left[1 + \phi\frac{(1+r)}{b/c}\right]$ or $r + (1 + r)\phi > r' + \phi\frac{(1+r')(1+r)}{b/c}$ or $\phi(1 + r)\left[\frac{\frac{b}{c}-(1+r')}{\frac{b}{c}}\right] > r' - r$ which gives $\phi > \frac{r'-r}{(1+r)}\left[\frac{\frac{b}{c}}{\frac{b}{c}-(1+r')}\right]$. □

Theorem 5. *Let c' be the cost required to fund a project via the SIB. If $q < 1$ and $p\frac{b}{c'} \geq (1+r)$, so that the project is implementable with a SIB, then a range of values $\phi > \frac{q\frac{b}{c}-(1+r)}{(1+r)(1-q)}$ exist where the SIB can finance the project and traditional debt cannot. Moreover, for $\phi > \frac{\left(\frac{c'}{c}-1\right)b}{b-c'(1+r)}$, the government success probability q required to implement the debt contract will be*

higher than the investor success probability $p = \frac{(1+r)}{b/c'}$ *that is required to ensure the project can be implemented with the SIB.*

Proof. Following exactly the proof of theorem 4, traditional debt cannot implement the project when $q < \frac{(1+r)[1+\phi(1-q)]}{b/c}$, which holds when $\phi > \frac{q\frac{b}{c}-(1+r)}{(1+r)(1-q)}$. Therefore when this condition holds the project cannot be implemented with a debt contract but by assumption, $p\frac{b}{c'} > (1+r)$, and so the project can be implemented with a SIB. Additionally, traditional debt can finance the project if and only if $q \geq \frac{(1+r)[1+\phi(1-q)]}{b/c}$, which implies $q \geq \frac{(1+r)(1+\phi)}{\frac{b}{c}+\phi(1+r)}$. The corresponding lower bound on the required investor's success probability given by theorem 3 is $p > \frac{1+r}{b/c'}$. We have that $\frac{(1+r)(1+\phi)}{\frac{b}{c}+\phi(1+r)} > \frac{1+r}{b/c'}$ when $\frac{(1+\phi)}{\frac{b}{c}+\phi(1+r)} > \frac{1}{b/c'}$ or $\frac{b}{c'}(1+\phi) > \frac{b}{c} + \phi(1+r)$ or $(1+\phi) > \frac{c'}{c} + \phi(1+r)\frac{c'}{b}$ or $\phi(1-(1+r)\frac{c'}{b}) > \frac{c'}{c} - 1$ or $\phi > \frac{\left(\frac{c'}{c}-1\right)b}{b-c'(1+r)}$. □