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Abstract

We study the effects of monetary and fiscal policies when both money and government bonds provide liquidity services. Because money is the unit of account, the price of money is the inverse of the price level. If prices are sticky, so is the price of money in terms of goods, and this is one important reason why money is liquid and attractive. By contrast, the price of government bonds is free to jump and often does, especially in response to news about changes in fiscal policy and the supply of bonds. Those movements in government bond prices affect available liquidity, and therefore aggregate demand, inflation and output. Under these conditions, bond-financed fiscal expansions can be contractionary, causing deflation and a temporary recession. To avoid those effects, changes in bond supply must be matched by changes in money supply and in the interest rate on money. We conclude that in a liquidity-dependent world, fiscal and monetary policies are joined at the hip.

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Joined at the hip:
monetary and fiscal policy
in a liquidity-dependent world

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March 2022

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I. Introduction

In mid-April 2021, the Federal Reserve paid 0.10% interest on reserves held by commercial banks at the Fed. At around the same time, the interest rate on short-maturity Treasury bills oscillated around 0.02%.[1] If yield is (an admittedly imperfect) proxy for liquidity, with assets that provide larger liquidity services managing to pay lower interest rates, then in mid-April Treasury bills were more liquid than bank reserves. Or, in the standard parlance of macroeconomics, government bonds were more liquid than money.

From the point of view of the holder (the private sector), in an environment of low interest rates bonds become money-like. Krishnamurthy and Vissing-Jorgensen (2012) write:

> Money is a medium of exchange for buying goods and services, has high liquidity, and has extremely high safety in the sense of offering absolute security of nominal repayment. Investors value these attributes of money and drive down the yield on money relative to other assets. We argue that a similar phenomenon affects the prices of Treasury bonds. The high liquidity and safety of Treasuries drive down the yield on Treasuries relative to assets that do not to the same extent share these attributes.

Krishnamurthy and Vissing-Jorgensen go on to provide detailed econometric evidence to justify these claims.

But that is not the end of the story. From the point of view of the issuer (the government), in an environment of low interest rates money becomes bond-like: financing government operations by issuing bonds is very similar to financing government operations by issuing bank reserves (money). Here is what a recent IMF working paper has to say on the subject:

> In modern times, money finance must be through bank reserves. The problem with relying on financing through massive excess reserves is that the financial system simply does not have any use for them… Bank reserves play an important role in the payments system but technological developments have enabled quadrillions of dollars, euros, and yen to be transferred every year without more than a small quantity of reserves being needed. Reserves in excess of this amount are simply deadweight on banks' balance sheets. Reserves suffer in comparison with treasury securities because they can only be held by banks, cannot be used as collateral efficiently like securities, and are subject to capital requirements. Treasury securities may be held by anyone residing anywhere, serve as collateral and support global capital markets, and are not subject to capital requirements when held by real money investors. Thus, if we compare the two instruments, beyond a minimum threshold required for payments, securities sell at a premium to reserves and are a more efficient and less costly way for the sovereign to finance itself. (Stella, Singh and Bhargava, 2021).

[1] Take, for instance, the 1-Month Treasury Constant Maturity Rate at https://fred.stlouisfed.org/series/DGS1MO.

So, in a low interest rate environment, there are plenty of similarities between money and government bonds. But there remains one key difference: the way their prices are set.[2] Because money is the unit of account, the price of money is the inverse of the price level. If prices are sticky, so is the price of money in terms of goods — and this is an important part of what makes money so liquid and attractive. Keynes emphasized this point in the *General Theory*, putting forth what one of us (Calvo 2012) has labelled the "price theory of money":

> …the fact that contracts are fixed, and wages are somewhat stable in terms of money, *unquestionably* plays a large part in attracting to money so high a liquidity premium (Keynes, 1936).

By contrast, the price of bonds in terms of goods is free to jump all over the place — and often does, even in the case of bonds as safe and liquid as US Treasuries. Recall the infamous 2013 "taper tantrum" and the short-lived but intense repo-market tremors of September 2019 and March 2020, all of which involved spikes in the yields of US Treasuries or, what is the same, sharp drops in the price of those bonds. Again in March-April 2021, swings in bond prices have occurred as markets digest the implications of the most recent US stimulus package.[3]

This difference matters a great deal for a number of monetary and fiscal policy issues. Imagine a policy in which the government writes checks and mails them to households, as the US and many other governments have been doing during the pandemic. Whether families will actually spend that money or save it for a rainy day is of course at the center of current debates over the effects of fiscal policy. What is not always taken into account is that the answer to this question depends crucially on how the government finances those checks.

If it prints money to fund the resulting additional budget deficit, then households will find themselves with more money in their pockets.[4] Given that goods prices are sticky in terms of money, at least in the short run, then the real stock of money will rise. If banks and households do not wish to hold this larger quantity of real money, the only way they can all get rid of it (in the aggregate, that is) is to spend more, so that goods prices will then rise. So a policy of money-financed checks can trigger an output boom and (eventually) higher inflation.

If the authorities are not comfortable with this outcome, then they can always increase the interest rate paid on money in order to induce commercial banks to hold larger reserves at the central bank. This is what happened, in a nutshell, during the Great Financial Crisis of 2007-09 and again during the Covid-19 Crisis. Remunerated bank reserves rose massively, as central banks created reserves to purchase all kinds of assets, both private and public.

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[2] See Calvo (2021) for a detailed discussion of this point and its implications.

[3] See "US Treasury bond wobble heightens concerns over health of $21tn market". *Financial Times*, 3 March 2021. https://www.ft.com/content/1deec2b3-59d4-4f90-b752-fefd2a88b5b2.

[4] Budget deficits would actually be financed by using central bank reserves, which can only be held by commercial banks. These banks, in turn, would "create" the additional money that would end up in the accounts of the public. But this wrinkle is not important for the story here. That is why we assume away commercial banks, so that households have their own monetary accounts at the central bank, which they can use for transactions

The story is very different if the additional checks issued to households are financed via bond issuance. Now the private sector will find itself with a larger nominal (and, at least for an instant, real) stock of government bonds in its portfolio. If it is happy to hold those bonds, then nothing else need happen. But if the private sector gets a case of the shivers, in order to reduce the real stock of bonds outstanding it does not have to buy more goods so that the price level will rise. It is enough for the private sector to begin selling those bonds in exchange for other assets.

The immediate consequence is that the price of government bonds will fall or (what is the same) the yield on those bonds will rise. It could even happen that the price of bonds falls sufficiently so that the actual value of the total stock outstanding does not change. In that case, overall liquidity in the hands of the public would be the same as it was before the new policy, and the checks need not have any effects on consumption, output or inflation, so fiscal policy would be entirely neutral!

Again, it could well be that the central bank is not comfortable with that outcome. Perhaps it does not want the yield on government bonds to rise, or their price to fall, because it fears the impact that might have on the balance sheets of financial intermediaries and, indirectly, on financial stability. Or perhaps the central bank is in fact hoping that the checks will stimulate aggregate demand, because inflation has been running below target. If so, the central bank can issue central bank reserves, buy the newly issued government bonds, and cause their price to rise and their yield to drop. But that, of course, would amount to money-financing of the checks!

That is the logic behind the title of this paper. In a world in which government bonds are held for liquidity purposes, fiscal policies can have unwanted effects — or no effects at all! To make sure they have the desired effects, certain kinds of fiscal policies (for instance, the issuance of checks to households) may have to be coupled with certain kinds of monetary policies (the issuance of bank reserves). In this sense, monetary and fiscal policies are *joined at the hip*.

We explore these issues in a bare-bones, conventional model. To generate liquidity services (or, more precisely, *transactional* liquidity) we assume a liquidity-in-advance constraint, so that households must have sufficient transactional liquidity (money and bonds) in their pockets in order to consume. This approach has limitations, but one great advantage: it is extremely simple and well understood. Since we are not making any claims about the origins of liquidity demand, but only about its implications, no great harm is done by relying on a standard model.[5]

Also to keep matters simple, we do away with commercial banks and equate money with central bank reserves. Readers uncomfortable with that idea can think of our money as a central bank-issued digital currency that households and firms hold in accounts at the central bank, and which can be used for all the standard transactions.[6]

[5] The main results of this paper go through if we instead assume that money and bonds enter the utility function, as long as the elasticity of substitution between consumption and transactional liquidity is sufficiently low (of course, a transactional-liquidity-in-advance constraint arises in the limit when this elasticity goes to zero). Recently there has been renewed interest in models in which money enters the utility function. See, for instance, Piazzesi, Rogers and Schneider (2021) and Diba and Loisel (2021).
[6] As in Piazzesi, Rogers and Schneider (2021).

This menu of assets gives rise to a menu of possible monetary policy frameworks. We focus on a policy regime that targets the quantity of money (central bank reserves) and the interest rate on those reserves, as the world´s main central banks have been doing since the Great Financial Crisis. Given this, the interest rate on the "pure" bond adjusts to clear markets.

Our first set of results has to do with the effects of fiscal policies, defined as increases in government transfers to households financed by issuing long-maturity bonds. The conventional wisdom in policy circles is that such a policy should be expansionary, raising aggregate demand and output.[7] We show that there are some situations in which the conventional wisdom is borne out. But there are others in which these helicopter drops of bonds are neutral, or even recessionary! It all depends on expectations and timing details.

In our model an unanticipated and permanent bond issue is neutral. The price of bonds drops so that total transactional liquidity, consumption and output are unchanged. The analogy is with an unanticipated and permanent step increase in the money supply in the context of perfectly flexible prices. No monetary economist is surprised to hear such a policy has zero real effects.

An unanticipated and transitory bond supply increase, on the other hand, has the expected expansionary effects — at least for a while. The intuition is that bond prices fall, but by less than the increase in nominal (and real, given predetermined bond prices) bond supply. So liquidity goes up, and so do aggregate demand and output.

This logic is reversed in the case of other fiscal policies. An anticipated and permanent future step increase in bond supply causes a drop in bond prices in the short run (when bond supply is still unchanged), and therefore a liquidity squeeze and a recession. The very realistic policy of having the fiscal authority issue bonds gradually for a finite period of time, returning at the end of that period to a policy of fixed nominal bond supply, also has a recessionary impact. Bond prices fall more quickly than bond supply increases, causing the value of government bonds outstanding to decline. The resulting drop in liquidity is, once again, recessionary.

So the moral of the story turns out to be simple: when bond prices are free to react to expected changes in fiscal policy, they can move in ways that curtail available liquidity, and therefore push down aggregate demand and output. That cannot happen in the case of conventional monetary policy, because the price of money in terms of goods is fixed whenever prices are sticky.

An important caveat is in order. We are not claiming that all fiscal policies have non-standard effects. It is bond-financed increases in government transfers to the private sector that can have non-standard effects or no effects at all.[8] But of course, much of the recent fiscal expansion in response to the Covid-19 crisis consisted of sharp increases in government transfers, so our exercise is not without policy relevance.

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[7] Although in our model, which features lump sum taxes, in the absence of liquidity effects, temporary changes in transfers financed by issuing bonds should have no real effects, because Ricardian equivalence obtains.

[8] And money-financed fiscal expansion also has standard effects in our model.

What can monetary policy do in response? Casual observation of recent events suggests that liquidity provision is often targeted at stabilizing bond prices. That is what the Fed seems to have done, for instance, during recent episodes of volatility in the Treasury and Repo markets.

An instance of this liquidity provision occurred at the outset of Covid-19 crisis, when yields rose sharply and bid-offer spreads widened dramatically. Market stress was intensified by several technical features of the market, such as the limited collateral base of dealers and the weakness of centralized trading arrangements (Duffie, 2020). But it is unlikely that the episode of intense market stress would have occurred in the absence of huge expected fiscal deficits and the associated anticipated massive issue of Treasury bonds.

In response to market stress, the Federal Reserve purchased $1 trillion of Treasuries in the three-week period from March 16, and then continued to buy at a high rate. Vissing-Jorgensen (2021) argues that "Fed purchases were causal for driving down yields" and that these "purchase effects" were due to acute liquidity needs on the part of sellers.

Given the apparent importance of monetary policy responses to bond market turmoil, our second set of results describes the monetary policies that must accompany expected bond helicopter drops in order to ensure those bonds issues will not have contractionary effects on liquidity, consumption and output. The model confirms the conjecture that to avoid that unwanted outcome, the central bank must cut the interest rate on money and also expand the money supply in specific ways that depend on the timing details of the bond helicopter drop.

For instance, following the announcement of a future and permanent increase in the supply of government bonds, the required monetary policy involves gradually increasing the nominal money supply —followed by a step decrease in nominal (and real) money just as bond supply takes a step increase.

So our story features a kind of fiscal dominance, but not the traditional kind. Monetary policy is not compelled to finance the budget deficit, as it often happens in emerging markets. Monetary policy is compelled to stabilize government bond prices, to prevent fluctuations in those prices from having unwanted effects on output and inflation. The motivation is different, but an outside and uninformed observer could easily conclude this is case of fiscal dominance.

The next section lays out the basic model, while section III explores the effects of fiscal shocks. Section IV considers complementary monetary policies and Section V concludes.

II. The basic model

Consider the simplest possible model, with an infinitely lived individual, time-separable utility index and a constant rate of time discount ρ. Time is continuous and the economy is closed. There is one good and two assets: money and a long-term government bond.

We can describe the economy through a few equations (derivations are in the appendix).[9] All variables are in log deviations from steady state levels (at which, for simplicity, all variables will be constant over time). First, ℓ_t is liquidity, defined as

$$\ell_t - \bar{\ell} \equiv \alpha(m_t - \bar{m}) + (1-\alpha)(q_t - \bar{q}) + (1-\alpha)(z_t - \bar{z}) \tag{1}$$

where m_t is real money holdings, z_t real holdings of a long-term bond, q_t is the price of that bond in terms of money and $\alpha \in (0,1)$ is a parameter. Steady-state levels are denoted by overbars. We assume bonds take the form of perpetuities that yield liquidity services but pay no coupon.[10]

Next is a simile of a liquidity-in-advance constraint, which we assume always binds:

$$y_t - \bar{y} = c_t - \bar{c} \leq \ell_t - \bar{\ell} \tag{2}$$

where c_t is household consumption and y_t is output, equal to household consumption in a closed economy with no investment and no government consumption. Again, overbars denote steady-state levels. With the liquidity-in-advance constraint pinning down consumption and output, the standard Euler equation (derived in the appendix) determines the "pure" interest rate i_t.

For a given level of liquidity, agents must decide how much money and how many bonds to hold. That decision is summarized by the portfolio balance equation

$$\rho(m_t - \bar{m}) - \rho(q_t - \bar{q}) - \rho(z_t - \bar{z}) = i_t^m - \dot{q}_t, \tag{3}$$

where i_t^m is the interest rate on money. The ratio of money to bonds (or the difference, in logs) depends on relative returns.[11] Two identities complete the demand side of the model:

$$\dot{m}_t = \mu_t - \pi_t \tag{4}$$

$$\dot{z}_t = \zeta_t - \pi_t \tag{5}$$

where μ_t is nominal money growth, ζ_t is nominal bond growth, and π_t the rate of price inflation.

The supply side is simply the Calvo-Phillips equation, as in Calvo (1983) or Galí (2015).

$$\dot{\pi}_t = \rho\pi_t - \kappa(y_t - \bar{y}) \tag{6}$$

[9] All variables except for interest and growth rates are in logs and overbars denote steady state values.
[10] The assumption that the government bonds are not instant-maturity bonds is crucial, because it allows the price of the bonds to deviate from unity, and that price plays a key role in the results below. The assumption that long-maturity bonds yield liquidity services is realistic. In the United States, for instance, 10-year Treasuries are often used as collateral in repo operations, in what looks a lot like a liquidity-in-advance constraint.
[11] See proof in Appendix before equation (A15).

Finally, we specify the policy regime: i_t^m, μ_t and ζ_t are policy instruments. The "pure" interest rate i_t is endogenous, as is q_t, the price of zero-coupon bonds. The policy of fixing the interest on money and making asset purchase decisions (QE) that set the nominal stocks of bonds and money, is not unlike what the Fed and other central banks have been doing since the Global Financial Crisis.

Assume that in steady state $i = \rho$, so that the steady state inflation rate is zero, as is i^m. In addition, in steady state the authorities are neither creating nor destroying nominal money or bonds, so $\mu = \zeta = 0$. Finally and intuitively, output is at its steady state level, so that

$$c = \bar{c} = y = \bar{y} \tag{7}$$

and

$$\bar{q} + \bar{z} - \bar{m} = \log(1-\alpha) - \log\alpha \tag{8}$$

With some algebra (see the Appendix) the economy can be reduced to:

$$\dot{s}_t = \rho(s_t - \bar{s}) + i_t^m + \zeta_t - \mu_t \tag{9}$$

$$\dot{\pi}_t = \rho\pi_t - \kappa(m_t - \bar{m}) - \kappa(1-\alpha)(s_t - \bar{s}) \tag{10}$$

$$\dot{m}_t = \mu_t - \pi_t \tag{11}$$

where $(s_t - \bar{s})$ is the difference (in logs) between bonds and money:

$$(s_t - \bar{s}) \equiv (q_t - \bar{q}) + (z_t - \bar{z}) - (m_t - \bar{m}) \tag{12}$$

Differential equations (10) and (11) are a 2x2 system in m_t and π_t, with s_t as an exogenous variable. The appendix shows that, given that π_t is a "jumpy" variable and m_t is a state variable, the system is saddle-path stable. The associated phase diagram, with inflation on the vertical axis and real money balances on the horizontal axis, and assuming for $\mu = 0$, is shown below.

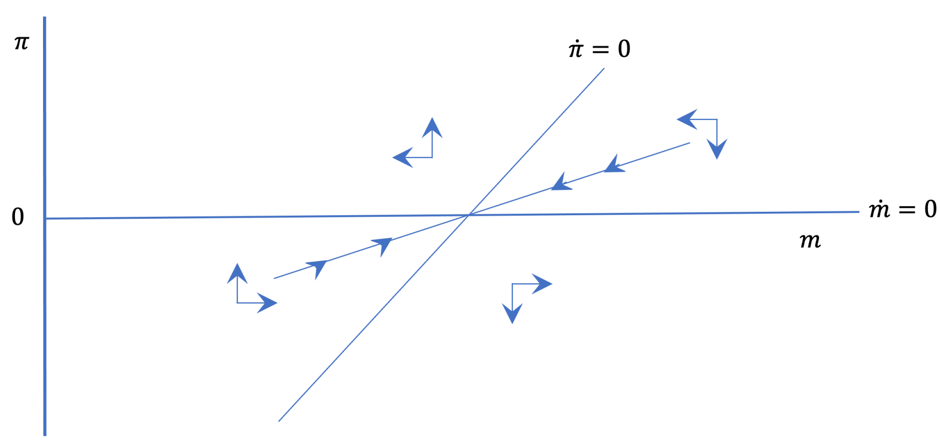

Figure 1

III. The effects of shocks to the supply of government bonds

How does the economy react when the fiscal authority carries out transfers to citizens financed by issuing zero coupon bonds? Contrary to conventional wisdom, such transfers can be neutral or contractionary. But the devil is in the details. Let us see where and how.

Unanticipated and permanent increase in the stock of bonds

First, consider the effects of an unexpected and permanent increase in the supply of government bonds. Start from steady state and the expectation of $i_t^m = \mu_t = 0$ for all $t \geq 0$ and $\zeta_t = 0$ for all $t > 0$, and assume that z rises from $\bar{z}$ to $\bar{z}' > \bar{z}$. Note that what increases is the nominal stock of bonds, but since the price level is predetermined, real bonds rise by the same amount. Now, steady state demand for bonds has not changed, so the higher supply prompts a lower price: $\bar{q}$ falls to $\bar{q}'$, and $\bar{q} + \bar{z} = \bar{q}' + \bar{z}'$ remains the same. In short: an unexpected and permanent helicopter drop of zero-coupon bonds causes a drop in their price, with no other effects.

Note, however, that if bond holdings were not uniformly distributed across consumers (a case that in this paper we will not explore) neutrality may not hold. For example, in a realistic scenario in which bonds are mostly held by financial institutions as backstop for repos, an increase in the supply of government bonds to finance income transfers to consumers (as it occurred during the pandemic) could lower the market value of the backstop and possibly make those financial institutions subject to runs, which is what we saw in the Great Financial Crisis. If the effect were large enough, income transfers of that sort could trigger central bank bond purchases to stabilize their price: a simple example of *de facto* lack of central bank independence from fiscal policy.

Anticipated and permanent increase in the stock of bonds

Fiscal policies are typically announced before they are implemented. So in the interest of realism, consider next an anticipated and permanent helicopter drop of bonds. Suppose at time 0 agents expect that nominal money balances will remain constant throughout, and nominal bonds outstanding will too, except for a step increase at time $T > 0$. Because the price level is predetermined, at time T the real stock of bonds outstanding will jump up as well. Assume finally that $i_t^m = \mu_t = 0$ always and that $\zeta_t = 0$ as well except for the discontinuity at T.

To sort out what happens, go back to the asset market equilibrium condition (3):

$$\dot{q}_t = \rho(q_t - \bar{q}) + \rho(z_t - \bar{z}) - \rho(m_t - \bar{m}) \tag{13}$$

where we have set $i_t^m = 0$. In the time interval [0, T) the difference $(z_t - \bar{z}) - (m_t - \bar{m}) = 0$ is unchanged, since that difference depends only on the nominal stocks of money and bonds, and those nominal stocks are fixed in that interval. We therefore have

$$\dot{q}_t = \rho(q_t - \bar{q}) \tag{14}$$

Because this differential equation is unstable, at time T the price of bonds must be at its steady state level —otherwise it would not converge. Moreover, that price cannot jump down at time T, because that would inflict a capital loss on bondholders, and arbitrage should prevent that. So it must be the case that $q_T + \bar{z}' = \bar{q}' + \bar{z}'$, where $\bar{z}'$ is the post-shock stock of bonds outstanding and $\bar{q}'$ is the new steady state price of bonds. It follows that q_t must jump down at time 0 and then fall gradually between 0 and T so that equation (14) will hold exactly at time T. Hence, it is a terminal condition — the requirement that q_t be at a lower level at T— that drives dynamics.

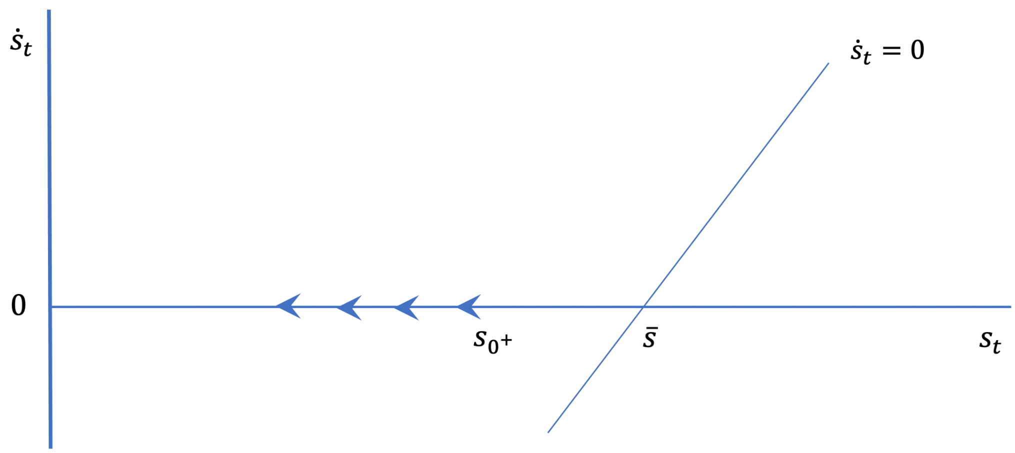

Figure 2

Recalling the definition of s_t in equation (12), and given the trajectory of q_t, it follows that s_t must also drop on impact at time 0, and then continue to decline until it jumps back to its initial steady state at time T. Of course, the jump at T occurs not because of a movement in q_t, but because the stock of bonds, z_t, jumps at that time.

To see what happens to inflation and real balances, turn to the phase diagram in Figure 3. If $s_t < \bar{s}$, the $\dot{\pi}_t = 0$ schedule shifts to the right. On impact, it goes from $\dot{\pi}_t = 0$ to $\dot{\pi}_t{}' = 0$. Thereafter it keeps shifting farther to the right, until it jumps back to its original position (i.e., $\dot{\pi}_t = 0$) at time T. But that does not change the qualitative conclusion that between times 0 and the instant before T the system is guided by the dynamics emanating from a steady state such as point *B*.[12]

So at time 0 the system drops from *A* to *D*. Thereafter it follows the arrows corresponding to the steady state at *B*, until at time T it reaches the saddle path that goes to the steady state at *A*. Between the time of the shock and the moment when the economy crosses the horizontal axis, inflation is negative but rising. The Calvo equation in its original form can be written as

$$\kappa(y_t - \bar{y}) = \rho\pi_t - \dot{\pi}_t \tag{15}$$

So if $\pi_t < 0$ and $\dot{\pi}_t > 0$, it follows that $y_t < \bar{y}$. Unambiguously, the economy is in a recession.

[12] We are taking the liberty here and in the next temporary-policy exercises of calling *B* steady state even though it shifts over time before reaching *T*.

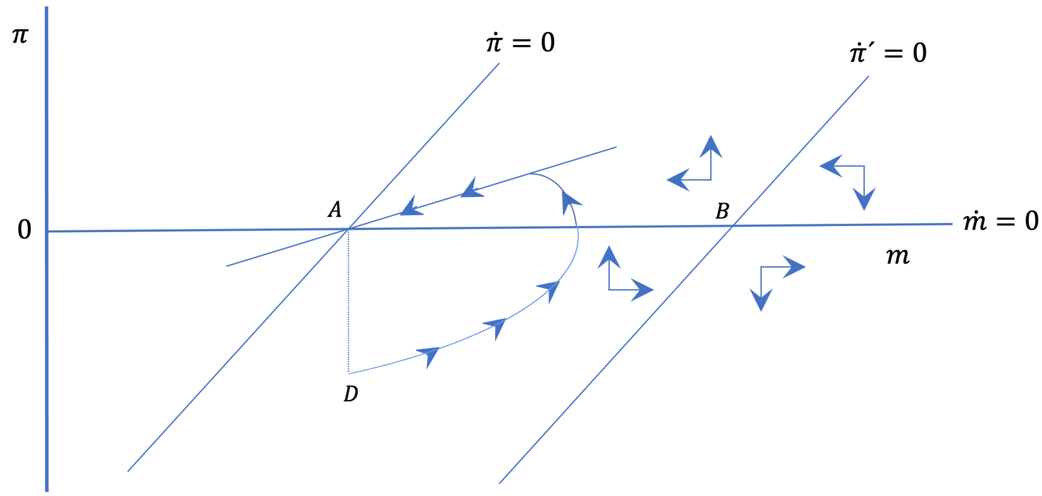

Figure 3

Once the system crosses the horizontal axis, there follows a period in which $\pi_t > 0$ and $\dot{\pi}_t > 0$, so the output gap can be positive or negative. But after time T, as the economy travels along the saddle path, $\pi_t > 0$ and $\dot{\pi}_t < 0$, so that $y_t > \bar{y}$. A naïve observer, noting that the economy enters a boom just as government begins to drop bonds, could conclude that the policy is expansionary. But that would neglect the fact that the expectation of the drop previously caused a recession!

Unanticipated and transitory increase in the flow of bonds

Next consider the effects of an unanticipated and transitory increase in ζ_t. Starting at 0 the fiscal authority issues bonds at a constant speed for an interval of time of length T , and then stops:

$$\zeta_t = \begin{cases} \zeta > 0 & \text{for} \quad 0 \leq t < T \\ 0 & \text{for} \quad t \geq T \end{cases} \tag{16}$$

To sort out what happens, it helps to begin by asking what is the trajectory of s_t. If $i_t^m = \mu_t = 0$ always, which we assume, equation (9), which guides the evolution of s_t, can be written as

$$\dot{s}_t = \rho[s_t - (\bar{s} - \zeta\rho^{-1})] \tag{17}$$

Between 0 and T the $\dot{s}_t = 0$ schedule shifts to the left, as depicted above. It follows that s_t drops on impact and then gradually rises, so that it can be back at its initial steady state at time T.

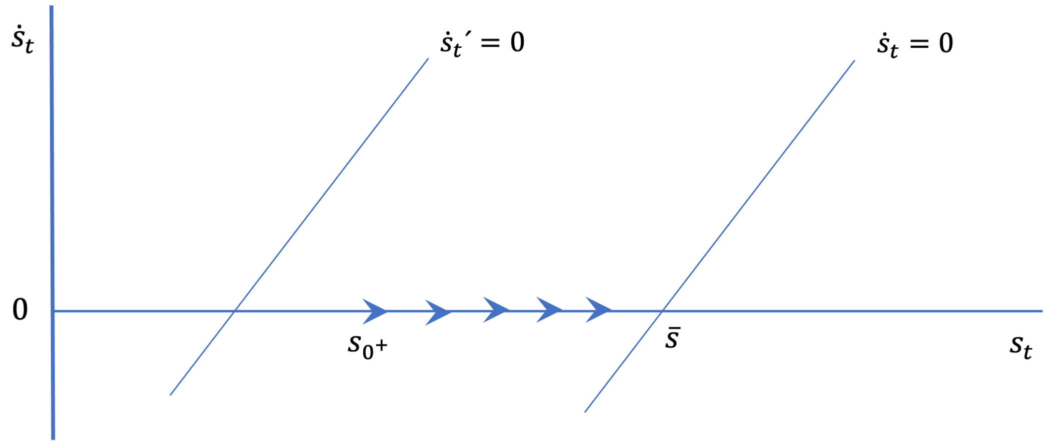

Figure 4

What happens to inflation, real money balances and output? We can infer this from equations (10), (11), Figure 4, and from the phase diagram with inflation and money balances (Figure 5). Between times 0 and T, $s_t < \bar{s}$. Recalling (10), we can see the shock causes the $\dot{\pi}_t = 0$ schedule to shift to the right. On impact, it goes from $\dot{\pi}_t = 0$ to $\dot{\pi}_t{}' = 0$. The difference with the previous exercise is that now between times 0 and T, $\dot{\pi}_t{}' = 0$ gradually shifts back to the left, until it is in its original position at time T. Still, between times 0 and the instant before T the system is guided by the dynamics emanating from a steady state such as that at *B*.

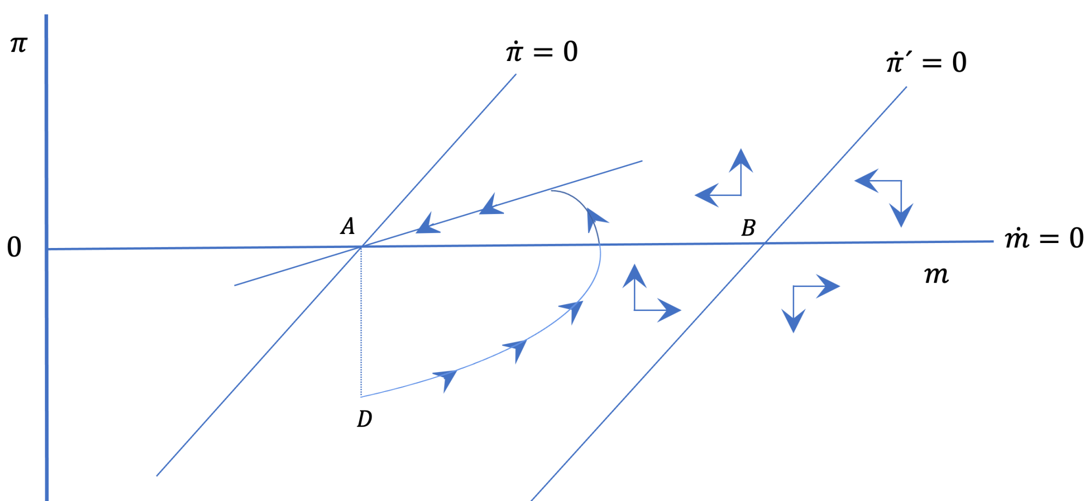

Figure 5

So at time 0 the system drops from *A* to point *D*. Thereafter it follows the arrows corresponding to the steady state at *B*, until at time T it reaches the saddle path that takes it back to point *A*. Again the economy goes through an initial recession. After the fiscal expansion has ended, there is a boom, with inflation positive (but falling) and output above its natural rate.

A naïve observer could once again be misled by this sequence of events. For instance, an opponent of bond-financed fiscal expansion could argue that now that the helicopter drops have ended, "confidence is back" and "growth can resume". That of course is the wrong conclusion. The boom in the final part of the cycle is part-and-parcel of the earlier recession. There is no confidence regained, since fiscal sustainability is not the issue here.

The intuition for these contractionary results follows from the behavior of q_t, the price of the zero-coupon bonds. Both the once-and-for-all anticipated helicopter drop and the temporary-but-sustained helicopter drop of bonds cause a fall in the money price of those bonds. As a result, the ratio of bond-liquidity to money-liquidity also falls (s_t goes down temporarily).

By (1) and (12), total liquidity can be written as

$$\ell_t - \bar{\ell} = (m_t - \bar{m}) + (1-\alpha)(s_t - \bar{s}) \tag{18}$$

We know $m_t - \bar{m}$ is fixed on impact, and $s_t - \bar{s}$ drops in both cases. It follows that total liquidity drops, and because of the liquidity-in-advance constraint, so must aggregate demand and output. The resulting deflation is the only way the real value of liquidity can eventually be rebuilt. But it is a gradual and painful process, in that it involves a protracted recession.

Unanticipated and temporary increase in the stock of bonds

Lest readers conclude that helicopter drops of bonds are always contractionary, finally consider an unanticipated but temporary step increase in the stock of zero coupon bonds. At time 0 agents learn that nominal bond supply will rise immediately and remain at the new higher level until $T > 0$. At that time, nominal bond supply will return to its initial level.

Recalling (12), the definition of s_t, it is easy to see that whatever happens to that variable on impact and in the short run, it must be back to its initial steady state level at T, since relative demands for money and bonds will not have changed in equilibrium. On impact m_t is unchanged and z_t jumps up (the nominal stock of bonds rises while the price level is predetermined).

What happens to q_t? Intuition suggests that q_t should fall: the supply of bonds is going up, so their price should go down. The key observation is that the initial (impact) drop in q_t must be less than the initial upward jump up in z_t, for only in that way will s_t continue to rise after time 0 and find itself at a level such that it can jump back down to its initial steady state level at T, the time when the step reduction in nominal (and real) bond supply will take place.[13]

[13] Note also that the price of bonds cannot jump at T, because that would amount to a huge capital gain on bonds that will naturally be arbitraged away.

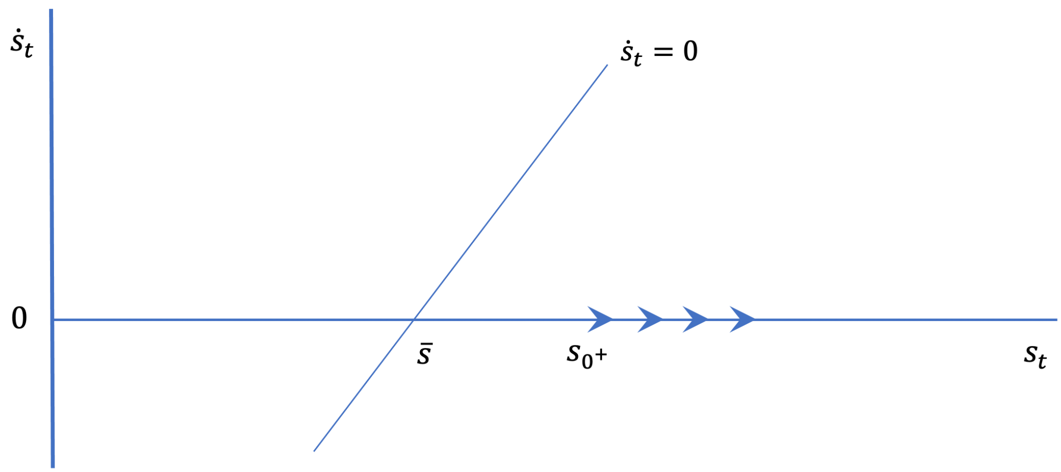

Figure 6

What happens to inflation, real money balances and output? We can describe the dynamics of the 2x2 system using the phase diagram below. If $s_t > \bar{s}$, the $\dot{\pi}_t = 0$ schedule shifts to the left. On impact, it goes from $\dot{\pi}_t = 0$ to $\dot{\pi}_t{}' = 0$. Then it keeps shifting farther to the left, until it jumps back to its original position at time T. Between times 0 and the instant before T, the system is guided by the dynamics emanating from a steady state such as point *B*.

So at time 0 the system jumps from *A* to *D*. Thereafter it follows the arrows corresponding to the steady state at *B*, until at T it reaches the saddle path that takes it back to the steady state at *A*.

Between the time of the shock and the moment when the system crosses the horizontal axis, inflation is positive but falling. By arguments analogous to those used before, if $\pi_t > 0$ and $\dot{\pi}_t < 0$, it follows that $y_t > \bar{y}$. The economy is in a boom. Once the system crosses the horizontal axis, there follows a period in which $\pi_t < 0$ and $\dot{\pi}_t < 0$, so the output gap can have either sign. But after time T, as the economy travels along the saddle path, $\pi_t < 0$ and $\dot{\pi}_t > 0$, so that $y_t < \bar{y}$.

The temporary helicopter drop of government bonds does produce an output boom — but at the cost of an eventual recession. The recession arrives around the time the bond supply expansion is to be undone, which could trigger calls for the continuation of the "expansionary" fiscal policy. Of course, this policy exercise is not very realistic, because governments that issue a large stock of bonds one year do not typically retire every last penny worth of those bonds T periods later.

The intuition as to why this kind of helicopter drop induces an initial boom, while the others caused initial recessions, has to do with the behavior of liquidity. In the first two cases the increase in bond supply is either eventual (at time T) or gradual, but since bond pricing is forward-looking, the price jumps down right away. Total available bond liquidity (quantity times price) drops —and so do aggregate demand and output. In this case that logic is reversed. The supply of bonds rises today, but because the increase is to be undone at time T, the drop in bond prices is less than proportional. Total bond liquidity goes up, and so do aggregate demand and output.

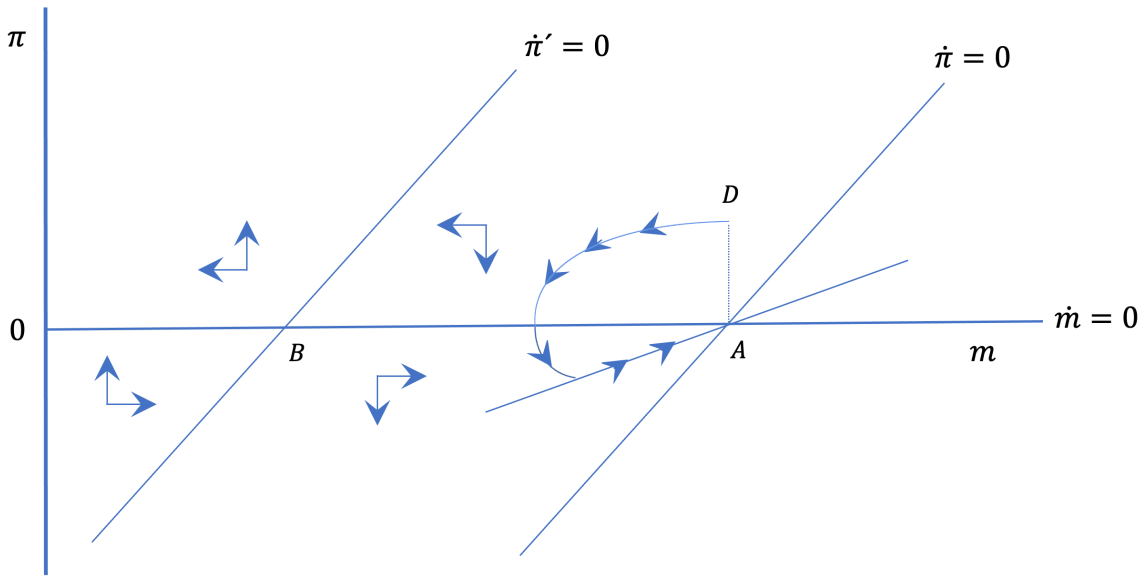

Figure 7

IV. **What is to be done?**

Monetary and fiscal authorities do have the tools at their disposal to avoid unwanted effects on inflation and output from the different kinds of helicopter drops of bonds. But what needs to be done depends on the details of the specific policy shock. For simplicity and brevity, we focus only on those fiscal policies that cause an initial recession.

Anticipated and permanent increase in the stock of bonds

Start from the anticipated future step (helicopter) increase in bond-holdings. Can the recessionary effects of that fiscal expansion be avoided? A monetary policy that achieves such an aim must satisfy three requirements.

First, given the liquidity-in-advance constraint, for output and inflation to be at their steady state levels between times 0 and T, liquidity must be constant throughout.

Second, during that time interval the price q_t has to decline, so that it will be at its new steady state level at time T.

Third, since q_t will fall, something else must rise to ensure the constancy of liquidity. By the very nature of this exercise, bond-holdings z cannot move. That leaves money balances as the only candidate. And since inflation needs to be zero, real balances can change only if nominal balances are changing.

These three requirements immediately rule out a standard Taylor rule, in which the interest responds to deviations of inflation and output from their steady state levels, as an effective tool in achieving the desired goal. We will see below that manipulating the interest rate on money changes the speed with which the price q_t falls. But fall it must, and that means ceteris paribus liquidity will be contracting. Unless nominal and real balances are increased, of course, but that increase cannot be achieved via a Taylor rule alone. Additional QE-like policies, which target the quantity of money (or central bank reserves, if you prefer) are needed.

Recall the asset market equilibrium condition that governs the evolution of q_t (equation (3)):

$$\dot{q}_t = i_t^m + \rho(q_t - \bar{q}) + \rho(z_t - \bar{z}) - \rho(m_t - \bar{m}) \tag{19}$$

If in addition we require that liquidity be constant at its steady state level, and given that $(z_t - \bar{z}) = 0$ throughout, this becomes, recalling equation (1),

$$\dot{q}_t = i_t^m + \frac{\rho}{\alpha}(q_t - \bar{q}) \tag{20}$$

So the evolution of bond prices depends on the interest rate on money and on bond prices themselves. It is easy to check that the higher the interest rate on money, the larger the initial drop in q_t.[14] If the central bank wanted to prevent a crash in bond prices at time 0, it could move the interest rate to prevent that. But notice that interest rate would have to be negative! Otherwise q_t would not begin falling at time 0, when $q_0 = \bar{q}$.

Countries like Sweden, Switzerland and Denmark have paid negative rates on excess bank reserves, so this is not out of the question. But there are limits on how negative that rate can be. To avoid wading into that controversial terrain, here we simply adopt the criterion of minimizing the initial drop subject to the zero lower bound on the nominal interest rate. So, for simplicity, we assume $i_t^m = 0$ throughout. Given this assumption and the terminal condition $q_T = \bar{q}'$, there is only one initial q_t —call it q_{0^+}— that satisfies the solution to differential equation (20).

This implies that the price of bonds will fall on impact. So to keep total liquidity constant, the central bank must engineer a step increase in nominal and real balances, equal to

$$m_{0^+} - \bar{m} = -\left(\frac{1-\alpha}{\alpha}\right)(q_{0^+} - \bar{q}) \tag{21}$$

---

[14] Supposing that i^m is constant, (20) can be solved forward to yield

$$q_T = e^{\frac{\rho}{\alpha}T} q_{0^+} + \left(\frac{\alpha}{\rho} i^m - \bar{q}\right)\left(e^{\frac{\rho}{\alpha}T} - 1\right)$$

where q_T is the level the price of bonds must reach at time T, and q_{0^+} the level the price of bonds jumps to immediately after the shock becomes known at time 0. For a given q_T, it is clear that the larger i^m is, the lower must q_{0^+} be to ensure the above equation holds. If $i^m = 0$, then

$$q_{0^+} - \bar{q} = (q_T - \bar{q})\, e^{-\frac{\rho}{\alpha}T} < 0$$

Recalling equation (1) and also recalling that q_t will fall gradually after time 0, constancy of liquidity is ensured if

$$\dot{m}_t = \mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\dot{q}_t \tag{22}$$

Hence, nominal balances have to keep rising to keep liquidity constant. Combining equation (20) (with $i_t^m = 0$) and (22) to eliminate $\dot{q}_t$ yields

$$\mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\frac{\rho}{\alpha}(q_t - \bar{q}) > 0 \tag{23}$$

which is the policy rule. Given that $q_t < \bar{q}$ in $t \in (0, T)$, $\mu_t > 0$ during that time period.

Finally, note from (20) (with $i_t^m = 0$) and (23) that

$$\ddot{m}_t = -\left(\frac{1-\alpha}{\alpha}\right)\frac{\rho}{\alpha}\dot{q}_t > 0 \tag{24}$$

That is, real balances increase at an increasing rate between time 0 and T.

Now, the instant before time T real money balances will be above their steady state level. Since the price level is predetermined and cannot jump, the only way to mop up this excess money is for the central bank to buy it through an open market operation involving the "pure" bond (recall the path of zero-coupon bonds is pinned down by the initial policy announcement). At time T the step increase in bond supply will also take place, so the ratio of bonds to money will rise.

In short: there is a monetary policy that can avoid unwanted fluctuations in aggregate demand, output and inflation following the announcement of a future helicopter drop in government bonds. That policy involves keeping the interest rate on money at zero during the transition and gradually increasing the nominal money supply during that time interval— followed by a step decrease in nominal money just as zero-coupon bond supply takes a step increase at T.

This monetary policy bears more than a passing resemblance to what we have seen in practice, and across many central banks, during the Great Financial Crisis and more recently during the Covid-19 crisis: ultra-low interest rates accompanied by QE operations that cause the money supply to expand, followed by an eventual "normalization" that undoes the previous monetary expansion. In spite of what looks like a very aggressive monetary policy, inflation is zero!

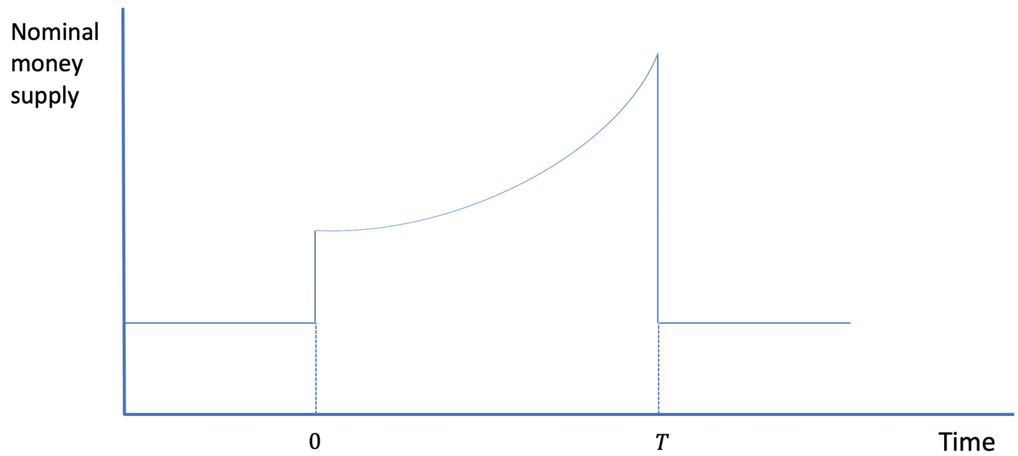

Figure 8

Unanticipated and transitory increase in the flow of bonds

Consider, finally, the temporary increase in ζ_t, the rate of nominal bond issuance. Recall that this policy also induces an initial recession, followed by a boom. Can monetary policy prevent these fluctuations in output (and inflation)?

Between times 0 and T, the nominal stock of zero-coupon bonds is growing (by assumption). If inflation is zero, as required to keep output constant, the real stock of bonds z_t must be going up as well. But the value of total bonds demanded in the new steady state after time T will the same as in the original steady state. Since the price of bonds cannot jump after time 0, it follows that at T the price of bonds must be such that $q_T z_T = \bar{q}\bar{z}$. Since $z_T > \bar{z}$, it follows that $q_T < \bar{q}$. So the bond price jumps down at time 0, and then continues to fall until time T.

What must monetary policy do in response? As before, keeping output constant requires keeping liquidity constant. That in turn requires a discrete jump up in money supply at time 0, and then a gradual mopping up of that money between 0 and T. The difference between this case and the previous case is that now there is no step change in the stock of bonds at time T, so there cannot be a step change in money holdings at that time either. If nominal and real balances go up at time 0 to compensate for the drop in the price of bonds, then the stock of real money must decline in the transition, so that it will be back at its steady state level at time T.

The required jump up in money supply at time 0 is given by:

$$m_{0^+} - \bar{m} = -\left(\frac{1-\alpha}{\alpha}\right)(q_{0^+} - \bar{q}) \tag{25}$$

Taking time derivatives of (1) we see that keeping liquidity constant between 0 and T implies

$$\dot{m}_t = \mu_t = -\left(\frac{1-\alpha}{\alpha}\right)(\zeta + \dot{q}_t) \tag{26}$$

Because we require $\dot{m}_t = \mu_t < 0$, it must be the case that $\zeta + \dot{q}_t > 0$. That is, the total value of bonds outstanding must be rising between 0 and T. That is intuitive, since the total value of bonds falls discretely at time 0, but is unchanged across steady states.

If we again keep $i^m = 0$ between 0 and T, recalling (19) the evolution of q_t is given by

$$\dot{q}_t = \frac{\rho}{\alpha}(q_t - \bar{q}) + \frac{\rho}{\alpha}(z_t - \bar{z}) \tag{27}$$

Combining (26) and (27) to eliminate $\dot{q}_t$ yields the policy rule for the central bank in $t \in (0, T)$:

$$\mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\left[\zeta + \frac{\rho}{\alpha}(q_t - \bar{q}) + \frac{\rho}{\alpha}(z_t - \bar{z})\right] \tag{28}$$

Finally, taking time derivatives of (27) and (28) we obtain

$$\ddot{m}_t = -\left(\frac{1-\alpha}{\alpha}\right)\frac{\rho}{\alpha}(\zeta + \dot{q}_t) < 0 \tag{29}$$

That is, real balances decrease at a decreasing rate between times 0 and T.

In summary, the monetary policy that prevents the fluctuations in inflation and output associated with sustained (and temporary) helicopter drops of government bonds involves keeping interest rates on money at zero, engineering an initial step increase in money, and then gradually mopping up the newly created real balances — all of it to keep liquidity constant throughout.

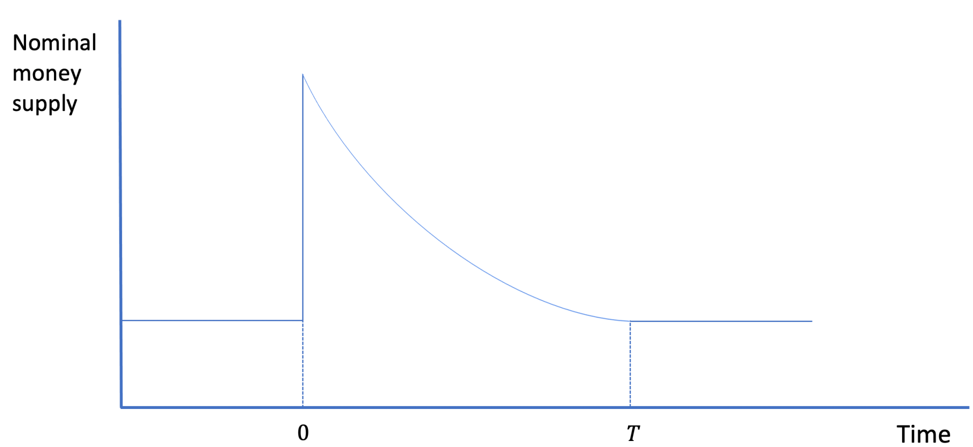

Figure 9

V. Concluding remarks

Because money is the unit of account, the price of money is the inverse of the price level. If prices are sticky, so is the price of money in terms of goods. And, as Keynes emphasized in what Calvo (2012) has labelled the "price theory of money", this is an important reason why money is attractive. By contrast, the price of government bonds is free to jump and often does, especially in response to news about current and future changes in fiscal policy and the supply of bonds.

Those movements in bond prices affect total liquidity available to agents, and hence can affect aggregate demand, inflation an output. To avoid such effects, expected changes in bond supply must be matched by expected changes in money supply and in the interest rate on money. We conclude that in a liquidity-dependent world, fiscal and monetary policies are *joined at the hip*.

One special feature of our analysis is that the only fiscal policies we have considered consist of transfers to households, whether bond- or money-financed. Of course, not all fiscal policies consist just of transfers. Government consumption and investment directly increase aggregate demand. If those expenditures are financed via bond issuance, the larger bond supply would still affect bond prices, and potentially affect overall liquidity and consumption as well.

So, the effects we have emphasized in this paper would still be present, but they would push in the opposite direction than would direct government expenditure and its conventional effects. Which would dominate and what would happen to aggregate demand would then be an empirical question, dependent on parameter values and the composition of public expenditure.

The analysis presented in this paper has relevance for current debates about the effectiveness of fiscal stimulus and, whether the Biden administration´s $1.9 trillion fiscal stimulus, coming on the heels of other large stimulus packages carried out by the Trump White House, is too large and will cause overheating and the return of inflation.

Of course, the model we have developed here is far too stylized for detailed policy analysis, but a theme that emerges is indeed policy-relevant: what matters for the control of aggregate demand is the joint stance of fiscal and monetary policy —in particular, the perceived commitment (whether illusory or real) on the part of the Federal Reserve to stabilize asset prices in general, and the price of Treasuries in particular. Our model suggests that without that perceived commitment, a policy of fiscal transfers to households need not be expansionary, and in some circumstances could even be contractionary.

We have illustrated the implications of "jumpy" bond prices (in contrast to "sticky" money prices) by focusing on current fiscal and monetary policy debates. But the point is more general. Keynesian macroeconomics is built upon the very realistic premise of nominal wage and price stickiness, which in turn is what endows monetary policies with real effects. In a simple world in which money and liquidity are synonymous (so that there is only one source of transactional liquidity), and in which goods prices are sticky in terms of money, by controlling the money supply (or the cost of holding money), the authorities enjoy a great deal of control over the economy.

But in a more complex and realistic world in which other assets like government bonds also provide transactional liquidity services and, crucially, in which goods prices are not sticky in terms of the prices of those other assets, financial and macro dynamics can be quite different to what conventional, off-the-shelf models suggest.

In this paper we have ignored the possibility of using government bonds to bypass price stickiness, even though these bonds enter the liquidity-in-advance constraint and are, therefore, transactional liquid assets that could sustain the liquidity necessary for full employment if their prices (in terms of the unit of account) were high enough. Calvo (2022) shows that if those bonds are employed for price haggling under low-cost frictions, full employment would be sustainable and dominate Keynesian unemployment equilibrium discussed in this paper. Resolving which setup dominates in practice is an eminently empirical issue. However, given that the realm of price stickiness is mostly outside the financial sector, our conjecture is that the assumptions of the present paper constitute a good first approximation.[15]

[15] However, Calvo (2022) argues that price haggling through alternative transactional assets may be a relevant assumption in currency substitution models where the two transactional liquid assets are an emerging market domestic currency and a reserve currency like the US dollar.

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Appendix

The representative individual maximizes the objective function

$$U = \int_0^\infty \log C_t e^{-\rho t}\, dt \tag{A1}$$

where C_t is consumption and ρ the subjective rate of discount, subject to the budget constraint

$$\dot{B}_t + Q_t \dot{Z}_t + \dot{M}_t = Y_t + \Psi_t + \ (i_t - \pi_t)B_t - \pi_t Q_t Z_t + (i_t^m - \pi_t)M_t - C_t, \tag{A2}$$

where Z_t is the real stock of zero-coupon bonds, Q_t is the price of those bonds, B_t is the real stock of a "pure" government bond, i_t is the nominal rate of interest on that "pure" government bond, M_t is the real stock of money, i_t^m is the interest rate paid on money, π_t is the rate of inflation, Y_t is output (all of which accrues to the household) and Ψ_t is real lump-sum government transfers that ensure the government fiscal balance equals zero.[16]

If we define total real asset-holdings as $A_t = B_t + Q_t Z_t + M_t$, (A2) can be written as

$$\dot{A}_t = Y_t + \Psi_t + \ (i_t - \pi_t)A_t - \left(i_t - \frac{\dot{Q}_t}{Q_t}\right) Q_t Z_t - (i_t - i_t^m)M_t - C_t, \tag{A3}$$

The government budget constraint is

$$\dot{B}_t + Q_t \dot{Z}_t + \dot{M}_t = \Psi_t + (i_t - \pi_t)B_t - \pi_t Q_t Z_t + (i_t^m - \pi_t)M_t \tag{A4}$$

The presence of lump-sum government transfers (or taxes), coupled with the fact that government can borrow as much as it wishes as long as it satisfies (A4), plus the corresponding no-Ponzi game condition, means that in the absence of liquidity effects this model would feature Ricardian equivalence.

Consolidating the representative household´s budget constraint (A2) and the government´s budget constraint (A4) we obtain $C_t = Y_t$, as should be in a closed economy with no investment.

Maximization by the representative individual is also subject to the constraint

$$C_t \leq L_t \tag{A5}$$

where liquidity is defined as

[16] Without this lump-sum rebate assumption, even models with only one currency would exhibit a direct link between fiscal and monetary policy. Thus, deactivating such a link helps to identify other channels through which monetary and fiscal policies interact, due to the existence of a second transactional asset.

$$L_t = \frac{M_t^{\alpha}(Q_t Z_t)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \tag{A6}$$

The Hamiltonian for the household´s problem is

$$\mathcal{H} = \log C_t + \lambda_t \left[Y_t + \Psi_t + (i_t - \pi_t) A_t - \left(i_t - \frac{\dot{Q}_t}{Q_t} \right) Q_t Z_t - (i_t - i_t^m) M_t - C_t \right] + \lambda_t \gamma_t (L_t - C_t) \tag{A7}$$

where λ_t is the co-state associated with state variable A_t and γ_t is the multiplier associated with the liquidity-in-advance constraint.

First-order conditions are

$$\frac{1}{C_t} = \lambda_t (1 + \gamma_t) \tag{A8}$$

$$\left(i_t - \frac{\dot{Q}_t}{Q_t} \right) Q_t Z_t = \gamma_t (1-\alpha) L_t \tag{A9}$$

$$(i_t - i_t^m) M_t = \gamma_t \alpha L_t \tag{A10}$$

$$\dot{\lambda}_t = -\lambda_t (i_t - \pi_t - \rho) \tag{A11}$$

Taking logs of (A8) yields

$$-\log C_t = \log \lambda_t + \log(1 + \gamma_t) \tag{A12}$$

Next, taking time derivatives (small case variables denote logs) and combining with (A11) we have the Euler condition for this problem:

$$\dot{c}_t = (i_t - \pi_t - \rho) - (1 + \gamma_t)^{-1} \dot{\gamma}_t \tag{A13}$$

The last term on the RHS is non-standard, and it comes from the liquidity-in-advance constraint (see Reis, 2017). Given the existence of a (binding) liquidity constraint and a policy regime that targets the interest rate paid on money and the quantity of money (or of central bank reserves, if you prefer), the Euler equation does not play any role in the determination of output, inflation or asset prices. Once these variables have been determined, the Euler equation pins down the path of the interest rate on the "pure" bond, which does not yield liquidity services.

Next, combine (A9) and (A10) to obtain

$$\alpha \left(i_t - \frac{\dot{Q}_t}{Q_t} \right) Q_t Z_t = (1-\alpha)(i_t - i_t^m) M_t \tag{A14}$$

Now take logs of (A14) and do a first-order approximation around the steady state, noting that in steady state $i_t - \dot{q}_t = i_t - i_t^m = \rho$.[17] Then multiply through by ρ, rearrange and you obtain

$$\rho(m_t - \overline{m}) - \rho(q_t - \overline{q}) - \rho(z_t - \overline{z}) = i_t^m - \dot{q}_{t,} \tag{A15}$$

where overbars denote steady state levels. This is expression that appears in the text as (3). Next, take (A9) and (A10) and use them in (A6), the definition of liquidity:

$$L_t = \frac{1}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}} \frac{\gamma_t^\alpha \alpha^\alpha L_t^\alpha}{(i_t - i_t^m)^\alpha} \; \frac{\gamma_t^{1-\alpha}(1-\alpha)^{1-\alpha} L_t^{1-\alpha}}{\left(i_t - \frac{\dot{Q}_t}{Q_t}\right)^{1-\alpha}} \tag{A16}$$

Some manipulation and simplification yields

$$\gamma_t = (i_t - i_t^m)^\alpha \left(i_t - \frac{\dot{Q}_t}{Q_t}\right)^{1-\alpha} \tag{A17}$$

so that

$$\bar{\gamma} = \rho^\alpha \rho^{1-\alpha} = \rho \tag{A18}$$

To log linearize, first take logs of (A17) and then carry out a first-order linear approximation, again recalling that in steady state γ_t, $(i_t - i_t^m)$ and $(i_t - \dot{q}_t)$ all equal ρ.[18] Then, multiply through by ρ to obtain

$$(\gamma_t - \bar{\gamma}) = \alpha(i_t - i_t^m - \rho) + (1-\alpha)(i_t - \dot{q}_t - \rho) \tag{A19}$$

which also appears in the text, this time as (4).

In logs, the equations of motion for money and bonds are:

$$\dot{m}_t = \mu_t - \pi_t \tag{A20}$$

$$\dot{z}_t = \zeta_t - \pi_t \tag{A21}$$

which appear in the text as (4) and (5), and where μ_t is nominal money growth and ζ_t is nominal bond growth. Adding these two to asset market equilibrium condition (A15) yields

$$\dot{q}_t + \dot{z}_t - \dot{m}_t = i_t^m + \zeta_t - \mu_t + \rho(q_t - \overline{q}) + \rho(z_t - \overline{z}) - \rho(m_t - \overline{m}) \tag{A22}$$

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[17] Such an approximation uses the fact that for a differentiable function $f(x)$, $f(\bar{x} + \Delta x) - f(\bar{x}) \cong f'(\bar{x})\Delta x$, where again overbars denote steady state levels.

[18] This approximation also uses the fact that for a differentiable function $f(x)$, $f(\bar{x} + \Delta x) - f(\bar{x}) \cong f'(\bar{x})\Delta x$, where overbars denote steady state levels.

Define

$$(s_t - \bar{s}) \equiv (q_t - \bar{q}) + (z_t - \bar{z}) - (m_t - \bar{m}) \tag{A23}$$

Given this definition, differential equation (A22) becomes (to obtain (A24), just replace (A23) in (A22) and rearrange):

$$\dot{s}_t = \rho(s_t - \bar{s}) + i_t^m + \zeta_t - \mu_t \tag{A3}$$

which appears as (9) in the text.

In logarithms, liquidity ℓ_t as defined in (A6) is

$$\ell_t \equiv \alpha m_t + (1-\alpha) q_t + (1-\alpha) z_t \tag{A25}$$

Evaluate this equation at the steady state:

$$\bar{\ell} = \alpha \bar{m} + (1-\alpha)\bar{q} + (1-\alpha)\bar{z} \tag{A26}$$

Subtract one equation from the other and obtain

$$\ell_t - \bar{\ell} \equiv \alpha(m_t - \bar{m}) + (1-\alpha)(q_t - \bar{q}) + (1-\alpha)(z_t - \bar{z}) \tag{A27}$$

which corresponds to equation (1) in the text.

In a closed economy with no investment and no government consumption, plus and lump-sum subsidies equal to seigniorage from money and zero-coupon bond creation, market clearing requires $y_t - \bar{y} = c_t - \bar{c}$ always. Therefore, the liquidity-in-advance constraint is

$$y_t - \bar{y} \leq \ell_t - \bar{\ell} \tag{A28}$$

which always binds. Bringing these two together, assuming the constraint binds and using the definition of $(s_t - \bar{s})$ in (A23) we have:

$$y_t - \bar{y} = (m_t - \bar{m}) + (1-\alpha)(s_t - \bar{s}) \tag{A29}$$

Next recall the Calvo-Phillips equation (in the text it is equation (6)):

$$\dot{\pi}_t = \rho \pi_t - \kappa(y_t - \bar{y}) \tag{A30}$$

Combining (A27) and (A28) yields:

$$\dot{\pi}_t = \rho\pi_t - \kappa(m_t - \overline{m}) - \kappa(1-\alpha)(s_t - \bar{s}) \tag{A31}$$

which appears as (10) in the text.

Equations (A20) and (A29) are a 2x2 system of differential equations which in matrix form is

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{m}_t \end{bmatrix} = \Omega \begin{bmatrix} \pi_t \\ m_t \end{bmatrix} + \begin{bmatrix} \kappa\overline{m} - \kappa(1-\alpha)(s_t - \bar{s}) \\ \mu_t \end{bmatrix} \tag{A32}$$

where

$$\Omega = \begin{bmatrix} \rho & -\kappa \\ -1 & 0 \end{bmatrix} \tag{A33}$$

Therefore $\mathrm{Det}\ (\Omega) = -\kappa < 0$, and $\mathrm{Tr}\ (\Omega) = \rho > 0$. It follows that one of the eigenvalues of Ω is positive and the other is negative. Since π_t is a "jumpy" variable and m_t is a state variable, we conclude that the 2x2 system is saddle-path stable, as asserted in the text.

The discussion in the text focuses on paths that satisfy *necessary conditions* for rational expectations equilibria, which are locally unique. This includes ruling out discontinuous jumps of endogenous variables that would give rise to capital gains. Moreover, the analysis is *local* in the sense that results are claimed to hold on sets that are sufficiently close to the steady state. There are no rigorous proofs of existence except when differential equations give rise to a saddle path configuration. For instance, in equation (A32) if the second term on the RHS is nil.