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JEL Classification: D71, D72, D74, F51

Keywords: Mechanism design without transfers, Border dispute settlement, Voting, Approval voting

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Locally optimal transfer free mechanisms for border dispute settlement

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18th March 2023

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1 Introduction

Decisions about how to draw the border between two neighboring countries and, correspondingly, which citizenship the residents of the contested region should have, are often at the origin of international conflict and war.[1] While the conflicting parties often claim the entire contested region for themselves as an integral part of their national territory, splitting up a region peacefully is a theoretical option[2].

From a welfare perspective, a method for the settlement of a border dispute should take into account the preferences of the local residents concerned. This is not a trivial task because individual preferences about citizenship are not directly observable to others. Similarly, the intensity of citizenship preferences is only known privately. No national or international institution can claim to know for sure how important it is for a specific person to be citizen of one country rather than another.[3]

What additionally complicates the process of determining a border is that not all borders make equal sense from an economic or purely logistic perspective. A country should ideally be geographically connected to facilitate production, the provision of public goods, travel and transportation. Other geographical factors such as the location of rivers or mountains may add further restrictions. Actually these constraints are a main reason why it makes little sense to let everybody simply be a citizen of his preferred country at the place where he lives.

This paper asks whether there exists a way to efficiently settle border disputes under asymmetric information when there are obvious geographical constraints regarding the way in which borders can be drawn. The paper uses mechanism design theory to address the problem of selecting one border from a given feasible set. Since I rule out mobility, the location of the border

[1] Wikipedia (2023) lists an impressive number of territorial disputes some of which lasted for over a century. One such case is the conflict between Italy and the Vatican over the street Passetto di Borgo in the vicinity of the Vatican City which began in 1870 and ended in 1991.

[2] A historic example is the definition of the Danish-German border in 1920 which followed a referendum-based procedure that was laid out in the treaty of Versailles (for details see Schlürmann, 2019). The new border split up the region of Schleswig which previously was a part of Germany but inhabited by many citizens who preferred Danish citizenship.

[3] The volatile survey results regarding Catalonian independence make clear how difficult it can be to predict individual or aggregate preferences for citizenship (Centre d'Estudis d'Opinio, 2020, p. 10).

determines the allocation of citizenship.[4] Thus, the problem is one of assigning each citizen in the contested region either to one country A or another country B. For the sake of realism, I only consider mechanisms which do not make use of contingent financial transfers to elicit private information. Transfers that are conditioned on individual messages are a powerful tool in theory, but so far, they play no practical role in real world border dispute settlements.[5] The framework permits to also address the problem of secessions. In that case, country B would be the new country seceding from country A.

The first main finding of this paper is that a simple decision mechanism implements a constrained optimal social choice in dominant strategies. This mechanism asks all individuals in the contested region to report their preferred citizenship and then selects one border from the feasible set that maximizes the number of individuals who live in their preferred country. When type distributions differ across individuals, the mechanism needs to be adjusted accordingly, giving more weight to citizens with stronger conditional expected valuations.

While this class of mechanisms requires detailed knowledge about the locations of voters in combination with their individual reports, a simple approval voting mechanism implements the same social choice function as the unweighted mechanism without requiring this detailed information. That mechanism selects a frontier that is approved by a maximum number of local citizens. In an extension of the basic model, I show that a strategically simple approval voting mechanism can also optimally deal with some specific cases in which the designer has no knowledge about the set of feasible frontiers. This is the second main result.

The third main result is that not all indirect simple majority voting mechanisms are suited to replace the optimal direct mechanism. Although the welfare maximizing solution is a Condorcet winner, a sequence of votes can lead to suboptimal outcomes when voters act strategically. Accordingly, the practice of voting on frontiers or secessions must be put into question and alternative mechanisms should be considered.

[4] Actually, moving people is considered illegal under international law (United Nations, 2017).

[5] When financial transfers are possible, a Vickrey Clarke Groves (VCG) mechanism can implement an ex post efficient social choice function in dominant strategies (provided that the revenue is allocated to an unconcerned third party). Similarly, with transfers, an ex post efficient social choice function can be implemented in a Bayesian Nash equilibrium with a d'Aspremont Gerard-Varet mechanism.

The paper also addresses the case where the local residents do not only care about where they live themselves, but also have preferences about the location of the border as such. In this case, voting outcomes under political competition turn out to be potentially inefficient while the simple direct mechanism put forward in this paper still performs optimally.

I would like to clearly spell out some limits of the present analysis. First, it is important to note that this paper is an exercise in mathematical allocation theory that does not shed any light on the legality of rivalling claims to any specific territory. The paper studies solutions to a given allocation problem but it cannot help to decide whether the underlying claims are legally right or wrong. Second, several aspects that may play an important role when countries determine their borders are not considered here. The list includes in particular (i) externalities that the choice of a border may have on those who live in- and outside the disputed territory, e.g. because of tax base effects or security concerns, (ii) costs that may be associated with uncertainty generated by some collective choice mechanisms including in particular potential adverse effects on private and public investment and (iii) severe adverse incentive effects that may arise when an international order puts existing frontiers into question. Related to the last point, important normative aspects of existing international law are not addressed here.[6] The present paper is an exercise in mechanism design addressing only one problem, asymmetric information about preferences and preference intensity of local residents, that is part of a larger one. Thus, it would be too early to directly draw practical normative conclusions from this analysis.

The paper is related to the seminal work of Alesina and Spolaore (1997) who analyze the optimal partitioning of a set of locations in a local public good setup and show that politically stable borders can be inefficient (see Spolaore, 2022, for a recent survey of the ensuing literature). This research focuses on an elementary trade off: having more countries increases the fixed cost of government, but it also better tailors public goods to the preferences of local citizens. What distinguishes the present work from this line of research is (i) that it focuses on an information aggregation problem and (ii) that it

[6] While international law protects the integrity of existing states against attempts of a secession of part of the country, secessions have sometimes been recognized by other states after they occurred. This looks somewhat inconsistent. With a view to the mentioned incentive effects, it may make sense to make the hurdle for such an ex-post recognition particularly or prohibitively high if the seceded part is integrated into another country. These aspects are not addressed in the present analysis.

considers all possible one-stage decision mechanisms.

The paper also contributes to the literature on optimal mechanisms without transfers (examples include Börgers 2004, Schmitz and Tröger, 2012, Gershkov, Moldovanu, and Shi 2017, Grüner and Tröger, 2019). From a theoretical perspective, it fills a gap in the mechanism design literature, addressing a general class of allocation problems: collective decisions regarding a vector of individual specific binary outcomes with a given set of feasible outcome vectors. Binary voting is one special case of this setup which obtains when the feasibility restriction is that the individual outcome has to be the same for everyone. The mechanism put forward here then turns into the simple majority rule. Thus, the well-known simple and qualified majority rules are special cases of the mechanisms that are put forward here.

2 The model

Consider two countries A and B and a contested region R that is populated by I citizens. A collective choice has to be made about how to allocate the citizens of R to the countries A and B. Call an individual outcome x_i, where the outcome $x_i = 0$ means that i becomes a citizen of country A, and $x_i = 1$ means that i becomes a citizen of country B. An overall outcome is a vector $x \in \{0,1\}^I =: \mathbf{X}$. There are feasibility constraints, captured by the feasible set of allocations $\mathbf{F} \subseteq \mathbf{X}$. An example for a simple meaningful feasibility set is $\mathbf{F} = \{x \in \mathbf{X} | \, x_1 \leq x_2 \leq ... \leq x_n\}$. This restriction makes sense when the n citizens are located on the unit interval $[0,1]$ (representing e.g. a valley) with individual locations $l_1 < l_2 \leq ... \leq l_n$. The restriction obtains when two countries must be connected. More complex meaningful feasibility sets arise in a two- or three-dimensional landscape.

Individuals' preferences for citizenship are fully represented by a privately known type $\theta_i \in \Theta_i$ that represents the value of living in country B instead of living in country A. Thus, a negative valuation indicates a preference for country A, and a positive valuation a preference for country B. Denote the vector of types by $\theta = (\theta_1, ..., \theta_n) \in \Theta := \Theta_1 \times ... \times \Theta_I$. Valuations are drawn independently from a joint distribution $\phi(\theta) = \phi_1(\theta_1) \times ... \times \phi_n(\theta_n)$.

A social choice function maps all relevant information into an outcome: $x = f(\theta)$. I restrict the analysis of direct mechanisms to deterministic, transfer-free mechanisms. I define *realized social welfare* as

$$W(x,\theta)=\sum_{i=1}^{I}\theta_i x_i, \tag{1}$$

and, for a given social choice function $f(\theta)$, the *expected social welfare* as $E_\theta W(f(\theta),\theta)$.

I require that a social choice function must be implemented in dominant strategies. Thus, an ex-ante welfare maximizing planner solves:

$$\begin{aligned}
&\max_{f(\theta)} E_\theta W(f(\theta),\theta) \qquad (2)\\
&\quad s.t.\\
&f(\Theta_1\times ...\times\Theta_I)\subseteq \mathbf{F},\\
&f_i(\theta_i,\theta_{-i})\geq f_i(\theta_i',\theta_{-i}) \text{ if } \theta_i>0 \text{ for all } i,\theta_i,\theta_i',\theta_{-i},\\
&f_i(\theta_i,\theta_{-i})\leq f_i(\theta_i',\theta_{-i}) \text{ if } \theta_i<0 \text{ for all } i,\theta_i,\theta_i',\theta_{-i}.
\end{aligned}$$

3 The generalized majority mechanism

Consider first the case where individual types are drawn independently from identical symmetric distributions so that $\phi_i(\theta_i)=\phi_i(-\theta_i)=\phi_j(\theta_j)$ for all i, $j=1,...,I$. A straightforward way to address the design problem in this case is to maximize the number of individuals who live in what they claim to be their strictly preferred country, hereafter called the *number of fits.* For a given vector of announced types $\hat{\theta}$ and a given allocation x, the *number of fits* is

$$S\left(x,\hat{\theta}\right):=\sum_{i=1}^{n}\frac{sgn\left(\hat{\theta}_i\left(x_i-\frac{1}{2}\right)\right)+1}{2}. \tag{3}$$

There may be more than one outcome that maximizes the number of fits. To select one of them, I define the *lexical outcome* in some (finite) set $\tilde{\mathbf{F}}\subseteq\mathbf{F}$ as the (unique) first ranked outcome in $\tilde{\mathbf{F}}$ according to the lexicographic order based on its components. According to this criterion, one discards outcomes that put the first individual country in country B if there are other outcomes in $\tilde{\mathbf{F}}$ that put that individual in country A, and so on.

Definition 1 *The S-mechanism asks all individuals to report their types. It selects the lexical outcome in the set* $\tilde{\mathbf{F}} \subseteq \mathbf{F}$ *of alloations that maximize the number of fits* $S\left(x, \hat{\theta}\right)$.

In the general case where individual valuations are drawn from different distributions, the S-mechanism can be weighted. I define individual probabilities of positive and negative valuations as follows:

$$\beta_i^+ := \int_{\theta_i > 0} \phi_i(\theta_i)\, d\theta_i,$$

$$\beta_i^- := \int_{\theta_i < 0} \phi_i(\theta_i)\, d\theta_i,$$

Whenever β_i^+ or respectively β_i^- are both nonzero for all individuals, I speak of a *nontrivial environment.* In a nontrivial environment, the conditional valuations are

$$G_i^+ := \frac{\int_{\theta_i > 0} \phi_i(\theta_i)\, \theta_i d\theta_i}{\beta_i^+},$$

$$G_i^- := \frac{\int_{\theta_i < 0} \phi_i(\theta_i)\, \theta_i d\theta_i}{\beta_i^-},$$

and the absolute conditional valuation is

$$G_i(\theta_i) := \begin{cases} G_i^+ & \theta_i > 0 \\ 0 & \theta_i = 0 \\ -G_i^- & \theta_i < 0 \end{cases}.$$

An environment is called *symmetric* if for all i, j

$$G_i^+(\theta_i) = -G_i^-(\theta_i) = G_j^+(\theta_i).$$

The weights $G_i(\theta_i)$ permit to define the *weighted number of fits* as:

$$\tilde{S}\left(x, \hat{\theta}\right) := \sum_{i=1}^{n} \frac{sgn\left(\hat{\theta}_i\left(x_i - \frac{1}{2}\right)\right) + 1}{2} G_i(\theta_i). \tag{4}$$

Definition 2 *The weighted S-mechanism asks all individuals to report their types. It selects the lexical outcome in the set* $\tilde{\mathbf{F}} \subseteq \mathbf{F}$ *of allocations that maximize the weighted number of fits.*

To study optimality, it is important to note that incentive compatibility of a social choice function $f(\theta)$ requires that for all players the interim probability to live in country B is a step function of θ_i. This is why this mechanism can be replaced by a mechanism that just asks for the sign of the valuation.

Lemma 1 *Consider a revelation mechanism Γ implementing the social choice function $f(\theta)$. Let $\pi_i(\theta_i, f(\cdot)) = E_{\theta_{-i}}(f(\theta_i, \theta_{-i}))$ be the interim probability to live in country B. The social choice function $f(\theta)$ is Bayesian incentive compatible if and only if there are values $\pi_i^- \leq \pi_i^+$ so that $\pi_i(\theta_i, f(\cdot))$ satisfies*

$$\pi_i(\theta_i, f(\cdot)) = \begin{cases} \pi_i^- & \text{if } \theta_i < 0 \\ \pi_i^+ & \text{if } \theta_i > 0 \end{cases}$$

and

$$\pi_i(0, f(\cdot)) \in [\pi_i^-, \pi_i^+] .$$

The social choice function is dominant strategy implementable only if this condition holds.

Proof: The "if" part is obvious. "Only if" part: Whenever the first condition does not hold for some i, then, for at least one type with nonzero valuation, i can increase his expected payoff by choosing a different message with the same sign. The condition $\pi_i^- \leq \pi_i^+$ needs to hold because otherwise i can increase his expected payoff by misreporting all nonzero types. The second condition must hold because otherwise i can increase his expected payoff for negative or for positive types. $\square$

Note that the interim probabilities π_i^-, $\pi_i(0, f(\cdot))$ and π_i^+ need not be the same for different individuals.

I introduce some more notation:

- Call $\sigma(\theta) := (sgn(\theta_1), ..., sgn(\theta_I))$ the *realized sign profile*. Let $\sigma_i(\theta_i) := sgn(\theta_i)$. The *set of possible sign profiles* is $\Sigma := \{1, 0, -1\}^I$ with elements s.

- Call $\Delta(\mathbf{F})$ the *set of probability distributions over* $\mathbf{F}$.

- Call $\rho : \Sigma \to \Delta(\mathbf{F})$ a *profile-assignment rule.* Note that a profile assignment rule can be interpreted as an indirect mechanism that only permits signals in favor of citizenship in country A or country B and abstentions.

- Call $\sigma^{-1}(s) \subset \Theta$ the *set of type profiles with sign profile* s. Define $\sigma_i^{-1}(s_i) \subset \Theta_i$ as the set of individual i´s types with sign s_i.

- For any given sign profile s and any $\theta \in \sigma^{-1}(s)$ I define the following distribution function

$$\varsigma(\theta, s) = \frac{\phi(\theta)}{\int_{\sigma^{-1}(s)} \phi\left(\tilde{\theta}\right) d\tilde{\theta}}.$$

For any given social choice function $f(\theta)$, one can construct an associated profile assignment rule in the following way.

Definition 3 *Call $\rho_{f(\cdot),s}(x) \in \Delta(\mathbf{F})$ the density function that assigns the density $\varsigma\left(\tilde{\theta}, s\right)$ to the outcome $f\left(\tilde{\theta}\right)$. The profile assignment rule that maps profile s into $\rho_{f(\cdot),s}(x)$ corresponds to $f(\theta)$.*

Now the following holds:

Lemma 2 *If some direct revelation mechanism implements the social choice function $f(\theta)$ in Bayesian Nash equilibrium (dominant strategy equilibrium), then the corresponding profile assignment rule $\rho_{f(\cdot)}(s)$ implements a social choice function $g(\theta)$ in Bayesian Nash equilibrium (dominant strategy equilibrium) that yields identical interim payoff for all types.*

Proof: Similar to Grüner and Tröger (2019), Lemma 1: Consider some direct mechanism $\Gamma = (\Theta_1, ..., \Theta_I, f(\theta))$ with a Bayesian truthtelling equilibrium. When individual i chooses to report a positive (negative) valuation under the profile assignment rule $\rho_{f(\cdot),s}(x)$, and expects the other citizens to report the sign of their valuations truthfully, it realizes the same interim probability of living in country B as any other citizen i type with a positive (negative) valuation does in the original equilibrium of the mechanism Γ. Formally, for a given sign s_i and for all $\tilde{\theta}_i \in \sigma_i^{-1}(s_i)$

$$\begin{aligned} & E_{\theta_{-i}} \rho_{f(\cdot),(s_i,\sigma_{-i}(\theta_{-i}))}(x_i) \\ = \ & E_{\theta_i,\theta_{-i}|\theta_i \in \sigma_i^{-1}(s_i)} f_i(\theta_i,\theta_{-i}) \\ = \ & E_{\theta_{-i}} f_i\left(\tilde{\theta}_i,\theta_{-i}\right). \end{aligned}$$

The first equality follows from the definition of $\rho_{f(\cdot),(s)}$, and the second equality follows from Lemma 1.

Deviations to another announcement about the sign of the valuation yield the same interim probabilities of being granted citizenship of country B as a deviation to any valuation with another sign under the direct mechanism Γ. Formally, for a given alternative sign $\hat{s}_i \neq s_i$ and for all $\hat{\theta}_i \in \sigma_i^{-1}(\hat{s}_i)$

$$\begin{aligned} & E_{\theta_{-i}} \rho_{f(\cdot),(\hat{s}_i,\sigma_{-i}(\theta_{-i}))}(x_i) \\ = \ & E_{\theta_i,\theta_{-i}|\theta_i \in \sigma_i^{-1}(\hat{s}_i)} f_i(\theta_i,\theta_{-i}) \\ = \ & E_{\theta_{-i}} f_i\left(\hat{\theta}_i,\theta_{-i}\right). \end{aligned}$$

Thus, for a given sign $s_i \neq \hat{s}_i$ and for all $\tilde{\theta}_i \in \sigma_i^{-1}(s_i)$ and for all $\hat{\theta}_i \in \sigma_i^{-1}(\hat{s}_i)$

$$\begin{aligned} E_{\theta_{-i}} \rho_{f(\cdot),(s_i,\sigma_{-i}(\theta_{-i}))}(x_i) \ & \geq \ E_{\theta_{-i}} \rho_{f(\cdot),(\hat{s}_i,\sigma_{-i}(\theta_{-i}))}(x_i) \\ & \Leftrightarrow \ E_{\theta_{-i}} f_i\left(\tilde{\theta}_i,\theta_{-i}\right) \geq E_{\theta_{-i}} f_i\left(\hat{\theta}_i,\theta_{-i}\right). \end{aligned}$$

Thus, there is a truthtelling Bayesian equilibrium with identical interim payoffs and welfare.

The same type of argument applies to dominant strategy equilibria. $\square$

Based on this Lemma, one can restrict the further welfare analysis to the comparison of profile assignment rules.

Proposition 3 *(i) All weighted S-mechanisms have an ex-post equilibrium in which agents report their types truthfully.*

(ii) All weighted S-mechanisms implement an ex-post efficient social choice.

(iii) The weighted S-mechanism is a solution to the planner's problem (2).

Note that (iii) implies that the unweighted S-mechanism is a solution to the planers problem (2) in all symmetric environments.

Proof: (i) Equilibrium: Consider w.l.o.g. individual $i = 1$ with $\theta_1 > 0$, i.e. an individual preferring to live in country B. Consider some vector of reports $\hat{\theta} = \left(\theta_i, \hat{\theta}_{-i}\right)$ and a result that maximizes $S\left(x, \left(\theta_i, \hat{\theta}_{-i}\right)\right)$. An alternative individual report $\hat{\theta}_i \neq \theta_i$ with $\hat{\theta}_i > 0$ does not change $S\left(x, \left(\hat{\theta}_i, \hat{\theta}_{-i}\right)\right)$ for any $\hat{\theta}_{-i}$. Thus, the individual does not gain from a different message with the same sign.

Consider a false report $\hat{\theta}_i \leq 0$. By reporting $\hat{\theta}_i < 0$ instead of the true $\theta_i > 0$, the following two conditions hold for all $\hat{\theta}_{-i}$ and for all x_{-i} satisfying $(0, x_{-i}), (1, x_{-i}) \in \mathbf{F}$.

$$S\left((1, x_{-i}), \left(\theta_i, \hat{\theta}_{-i}\right)\right) = S\left((1, x_{-i}), \left(\hat{\theta}_i, \hat{\theta}_{-i}\right)\right) + 1 = -1,$$

$$S\left((0, x_{-i}), \left(\theta_i, \hat{\theta}_{-i}\right)\right) = S\left((0, x_{-i}), \left(\hat{\theta}_i, \hat{\theta}_{-i}\right)\right) - 1.$$

Thus, the probability that individual i is allocated to country A weakly increases and reporting $\hat{\theta}_i < 0$ is suboptimal.

(ii) Ex-post efficiency: Consider any realization of θ and the corresponding outcome of the mechanism $f(\theta)$. A Pareto-improvement implies that all individuals who live in their preferred country continue to live there and that others who did not live in their preferred country now do. This is not possible because the social choice already maximizes $S(x,\theta)$ on $\mathbf{F}$. Thus, there is no Pareto superior outcome relative to the outcome of the S-mechanism.

(iii) Welfare: According to Lemma 1, it suffices to consider profile assignment rules with truthful reports of signs that differ from the one which is associated with the S-mechanism. No such rule can yield higher expected welfare than the S-mechanism because (i) in equilibrium, the profile assignment rule that is associated with S mechanism transmits the entire sign profile to the planner and (ii) not knowing the size of valuations but only their sign, the planner cannot do better than by maximizing $\tilde{S}(x, \theta)$ for all θ. □

The welfare results are intuitive. Conditional on the true sign profile $\sigma(\theta)$ the optimal choice in $\mathbf{F}$ is the one that maximizes the weighted number of individuals who live in their preferred country. In a symmetric environment, the best use, the planner can make of the realized sign profile $\sigma(\theta)$ is to maximize the number of individuals who live in their preferred country. Any other incentive compatible profile assignment rule does not transmit more information to the planner than the sign profile. This is why no other profile assignment rule can deal in a better way with that profile.

4 Approval voting

The mechanism put forward so far requires that the designer knows the identity and location of all individuals. Otherwise, he cannot evaluate whether or not a frontier maximizes the number of fits. In this section, I show that an alternative indirect mechanism requires less detailed information on the side of the designer.

Consider the following mechanism. All voters can approve some subset $F_i \subseteq \mathbf{F}$. The mechanism implements the lexical outcome in the set of allocations that are approved by a maximum number of voters. As an example, consider the case where no border is approved by any voter. In this case all citizens are allocated to country A if $x = (0, ..., 0) \in \mathbf{F}$. The same holds if all borders in $\mathbf{F}$ are approved by all voters.

The approval voting mechanism has a dominant strategy equilibrium in which all players approve an allocation if and only if it puts them in their preferred country. Any strategy that involves not approving one of the individually optimal borders x is dominated by the one that replicates the strategy but approves border x as well. This increases the payoff in potential situations where the own statement about this border is pivotal and it leaves the payoff unaffected otherwise. For the same reason, approving an individually suboptimal border would be suboptimal.

The indirect approval voting mechanisms replicates the social choice function of the direct S-mechanism. In contrast to the S-mechanism, approval voting does not require that the designer possesses any knowledge about citizens' location. All that is necessary is to collect the sets of approved allocations from all citizens living in R. This may be easier to implement in cases where the set of feasible allocations has not too many elements.

A disadvantage of the approval voting mechanism is that it permits the use of signals F_i that are inconsistent. These are the ones that involve a contradiction regarding the preferences of individual i. A contradiction arises in the cases where $\theta_i \neq 0$ when an individual approves one frontier that puts it in country A and another frontier that puts it in country B. In the present setup, these signals are superfluous since they are not used in equilibrium.[7]

An important advantage of the approval voting mechanism is that it can deal elegantly with some cases in which the set $\mathbf{F}$ is not known to the designer. To see why, consider the following modification of the baseline model. For some given set $\check{\mathbf{F}} \subset \mathbf{X}$, the payoffs are

$$u_i\left(x, \theta_i\right) = \begin{cases} \sum_{i=1}^{I} \theta_i x_i, & \text{if } x \in \mathbf{X} \setminus \check{\mathbf{F}} \\ \sum_{i=1}^{I} \theta_i x_i - z, & \text{if } x \in \check{\mathbf{F}} \end{cases}$$

with

$$-z < \min_{\theta_i}\left(\Theta_i\right) =: \underline{\mathrm{u}} < 0.$$

Consequently, for all $x \in \check{\mathbf{F}}$, all individuals in region R receive a payoff that lies below the lowest possible payoff of individuals when $x \in \mathbf{X} \setminus \check{\mathbf{F}}$. Thus, one can interpret the set $\mathbf{X} \setminus \check{\mathbf{F}}$ as the set of feasible allocations in the sense that only these allocations do not yield payoffs that are Pareto-dominated by another outcome independently of the realization of the vector of types.

In what follows, I show that a Bayesian Nash equilibrium exists, in which voters approve exactly those elements in $\mathbf{X} \setminus \check{\mathbf{F}}$ which put them in their preferred country. In other words, they disapprove those allocations that are not feasible and also those that are feasible but do not put them in their preferred country. To see why this constitutes an equilibrium, consider that an individual faces three possible payoffs: a payoff below $\underline{\mathrm{u}}$ that obtains if $x \in \check{\mathbf{F}}$, the payoff zero for being put into country A if $x \in \mathbf{X} \setminus \check{\mathbf{F}}$, and the payoff $\theta_i \geq \underline{\mathrm{u}}$ for being put into country B if $x \in \mathbf{X} \setminus \check{\mathbf{F}}$. Approving all allocations that yield the maximum payoff in $\{0, \theta_i\}$ and not approving any element in $\check{\mathbf{F}}$ must be part of any undominated strategy. The reason is a pivotality argument similar to the one that has already been made above. The decision to approve an individually optimal outcome is either irrelevant or, in case of pivotality, increasing the payoff. The same holds for the decision to disapprove an outcome in $\check{\mathbf{F}}$. Approving an outcome in in $\mathbf{X} \setminus \check{\mathbf{F}}$ that does not put an individual in the prefered country only makes sense if there is a risk that the equilibrium outcome may otherwise lie in in $\check{\mathbf{F}}$. This risk does not exist if every individual adheres to the above strategy. This is why a

[7] Still, they add complexity to the mechanism. This problem can in principle be fixed with a direct mechanism. However, replacing the indirect mechanism by a direct one again requires that the designer knows where all individuals are located.

strategy profile that is based on this strategy constitutes a Bayesian Nash equilibrium. In this equilibrium in undominated strategies any individual can be sure that outcomes in $\check{\mathbf{F}}$ do not realize. The result is a constrained welfare maximum.[8]

The following proposition describes the equilibrium. It additionally states that the S-mechanism may instead lead to other, suboptimal equilibrium allocations.

Proposition 4 *Consider a symmetric environment in which all allocations in some set $\check{\mathbf{F}} \subset \mathbf{X}$ are strictly Pareto dominated by all other allocations. A strategically simple approval voting mechanism has a welfare maximizing Bayesian Nash equilibrium in undominated strategies. The S-mechnism may instead have suboptimal Bayesian Nash equilibria in undominated strategies. When only undominated allocations are put up for an approval vote, the voting game has a unique equilibrium in dominant strategies that maximizes social welfare.*

Proof It remains to be shown that the S-mechanism may instead lead to other, suboptimal equilibrium allocations. The proof is by example. To see why, consider the case with $n = 2$ and the full set of feasible allocations $X = \{0,1\}^2$. Let $\Theta_i = \{-1, 1\}$. Assume that the allocation $(1, 0)$ yields utility $-2 < min_{\theta_i}(\Theta_i)$ for both players. All other allocations yield utility $\theta_i x_i$. Consider the case where β_1^+ is close to one and assume that player 1 announces his type sincerely. Player 2 with type $\theta_2 = -1$ announcing his type sincerely ends up with the allocation $(1, 0)$ with probability β_1^+, yielding the payoff -2. Announcing type $\theta_2 = 1$, he instead realizes a payoff of -1 with the same probability. Thus, his best reply is to always signal a preference for country B. A best reply of player 1 to this strategy is to always announce the true type. $\square$

[8] Note that the approval voting mechanism is strategically simple in the sense of Börgers and Li (2019). All that a player needs to know to fix his strategy is that other players do not play weakly dominated strategies. Based on this consideration, the equilibrium strategy is optimal independently of a player's strategy beliefs as long as they are compatible with some utility belief.

5 Voting mechanisms

Those who favor restructuring existing frontiers often argue in favor of some sort of plebiscite in order to settle the issue. This raises the question whether the optimal S-mechanism can be replaced by some more familiar form of collective decision making. Specifically, I will focus on mechanisms that involve some form of binary majority voting.

There are many ways in which one can organize a voting procedure for the choice from the set **F**, in particular when it has more than two elements. I distinguish three prominent setups:

(i) a direct democracy with sequential binary votes on the entire set **F**,
(ii) political competition with perfectly informed candidates, and
(iii) political competition with imperfectly informed candidates.

The two competitive setups involve the selection of policy proposals by the candidates. An advantage compared to the approval voting mechanism is that voters only face two alternatives which reduces complexity. The key question that needs to be addressed is whether candidates have incentives to propose policies that are in line with social welfare maximization.

In all three setups, I stick to simple majority rule as the rule for each vote that takes place. Since this rule cannot account for preference intensities and in order to give voting a fair chance, I assume that the environment is symmetric in what follows. Still, the setup permits that $\beta_i^+(\theta_i)$ and $\beta_i^-(\theta_i)$ are not the same, i.e. it permits asymmetric probability distributions.

It is useful to briefly discuss the benchmark case where voters vote naively in the sense that they act as if all their votes were decisive. Consider the case of a binary vote in a direct democracy in which voters may abstain. It is easy to see that any unique welfare maximizing solution wins against all other alternatives if all voters vote for their preferred alternative if they have one and abstain otherwise. Under the same assumption on voting behavior, any sequence of binary votes which covers the entire set of feasible allocations **F** leads to unique welfare maximizing solution. Obviously, it is necessary to vote or on the entire set of options in **F**.[9]

[9] Considering only a subclass of feasible partitions can exclude the optimum. Moreover, there are examples where one single vote leads "away" from the optimal frontier location. To see why, consider a linear world with one border, seven citizens and valuations

6 Strategic sequential voting

This section deals with the more interesting case where voters act strategically when they vote sequentially. Consider a sequential voting game with a known finite sequence of binary votes. The winning alternative in each voting round enters the next round against one of the alternatives which have not yet been put up for a vote if there is at least one left. The alternative selected in the last round is implemented and counts for payoffs. The sequence of votes is deterministic and known ex ante.

With more than two voters, any full-information or Bayesian voting game under simple majority rule has trivial equilibria in which all voters vote for the same alternative. The same holds for any sequential voting game where unanimous voting on all stages constitutes a Nash equilibrium. In order to rule out these implausible equilibria, I restrict the following analysis to trembling hand perfect equilibria. Importantly, a sequential voting game may have trembling hand perfect equilibria in which the implemented social choice function does not yield a constrained welfare maximum. The S-mechanism has the advantage that its unique trembling hand perfect equilibrium always selects a welfare maximum.

Proposition 5 *Consider a sequence of binary votes that includes all feasible alternatives. There exists a nontrivial environment in which the voting game has a trembling hand perfect equilbrium that does not maximize social welfare.*

Proof A sequential Bayesian voting game is a signaling game with potentially many equilibria. In a first step, I simplify the analysis and consider the limit case in which voters' valuations $\theta_i \in \{-1, 1\}$ are common knowledge, i.e. where β_i^+ or β_i^- are zero. In the limit case, an example of a suboptimal trembling hand perfect equilibrium can be constructed. In a second step, I will show that the analysis extends to a nontrivial setup with $\beta_i^+, \beta_i^- > 0$ for all i.

The proof of the first claim (existence of an environment with an inefficient trembling hand perfect equilibrium in a full information case) is by

$(1, 1, 1, -1, -1, -1, -1)$. Consider a local vote amongst the first five individuals about moving the frontier form the right of position 5 (status quo) the to left of position 1. This referendum moves the first five individuals to country B although all seven individuals should be in country A. While the move improves welfare, moving the frontier in the opposite direction would increase welfare even further.

example. I consider a full information benchmark case with four homogenous groups of citizens $g = 1, ..., 4$. Group 1 has $2\hat{n}$ members, where $\hat{n} \geq 1$, group 2 has $2\hat{n} + z$ members, where $z \in \{1, ..., \hat{n}\}$, Group 3 has $\hat{n}$ members, group 4 has z members.[10] Members of groups 1 and 4 prefer living in country A, i.e. for them $\beta_i^+ (\theta_i) = 0$. All others prefer living in country B, i.e. for them $\beta_i^- = 0$. Expected conditional valuations are $G_i^+ = -G_i^- = 1$ where they exist. The set of feasible allocations is $\{x_1, x_2, x_3\}$:

1. x_1: Everybody lives in country B.
2. x_2: Only members of groups 1 and 3 live in country A.
3. x_3: Everybody lives in country A.

The unique expected welfare maximizing alternative is x_2. It allocates $4\hat{n} + z$ citizens in line with their preferences, while alternative x_1 allocates $3\hat{n} + z$ citizens in line with what they prefer, and x_3 only $2\hat{n} + z$ citizens. Figures 1 and 2 illustrate the example in a two-dimensional plane for $\hat{n} = 2$ and $z = 1$.

Consider a sequential voting game in which first there is a vote on the two alternatives x_1 and x_2, and second, the winning alternative is entering a vote against x_3.

The following is an equilibrium in undominated strategies: In the second round, everybody votes for his preferred alternative if there is one. In the second round vote amongst the alternatives $\{x_2, x_3\}$ indifferent voters (the ones in groups 1 and 3) vote for x_3. Note that nobody is indifferent in the second round vote amongst the alternatives $\{x_1, x_3\}$. In the first round, all members of groups 1 and 4 vote for alternative x_2. All others vote for alternative x_1.

Consider the votes that can possibly take place in the second round. In the second round vote on $\{x_1, x_3\}$, $3\hat{n}+z$ voters vote in favor of x_1 and $2\hat{n}+z$ vote in favor of x_3. Alternative x_1 wins that vote. In the second round vote on $\{x_2, x_3\}$, $2\hat{n} + z$ voters vote in favor of x_2 and $3\hat{n} + z$ vote in favor of x_3. Note that no one is ever pivotal along the equilibrium path.

[10] This is a simple special case of what will be needed below. Let group i have n_i members. The equilibrium below requires $n_2 + n_3 > n_1 + n_4$ (first and second stage requirement) and $n_1 + n_3 + n_4 > n_2$ (the other second stage vote) and $n_1 + n_2 > n_2 + n_3 \Leftrightarrow n_1 > n_3$ and $n_1 + n_2 > n_1 + n_4 \Leftrightarrow n_2 > n_4$ (optimality of allocation x_2).

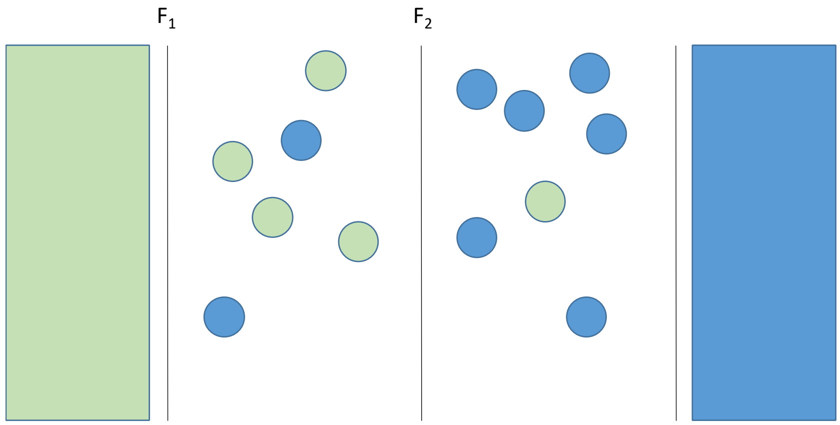

Figure 1: Green dots indicate citizens who prefer to be citizens of country A, blue dots those who prefer to be citizens of country B. F1, F2, and F3 are the three feasible borders.

I now show that this strategy profile is also a trembling hand perfect equilibrium. To see why, consider a sequence of totally mixed strategies where each player must play his seven non-equilibrium strategies with probability $\varepsilon > 0$ and his equilibrium strategy with probability $1 - 7\varepsilon$. In the two second round votes this yields a small probability for pivotality for all players, thus requiring that the second-round vote is in line with voter preferences. This is the case.

The first period vote may also be pivotal. Not that, for any given probability $\varepsilon > 0$, and from the perspective of any player, there is a positive but small probability that the outcome of the second round vote on $\{x_1, x_3\}$ is x_3 and that the outcome of the second round vote on $\{x_2, x_3\}$ is x_2. Taking the almost certain second round outcomes x_1 and x_3 into account, all voters who prefer x_1 to x_3 must vote for x_1 in the first voting round and all voters who prefer x_3 to x_1 must vote for x_2. Again, this is the case.

This completes the analysis of the full information case. It remains to consider "close" asymmetric information environments. Consider some specific nontrivial environment with the following properties: (i) For voters in groups 1 and 4, the type is $\theta_i = 1$ with probability $1 - \delta$ (hereafter called their likely type) and $\theta_i = -1$ with probability δ. (ii) For members of groups 2 and 3 the type is $\theta_i = -1$ with probability $1 - \delta$ (again called their likely

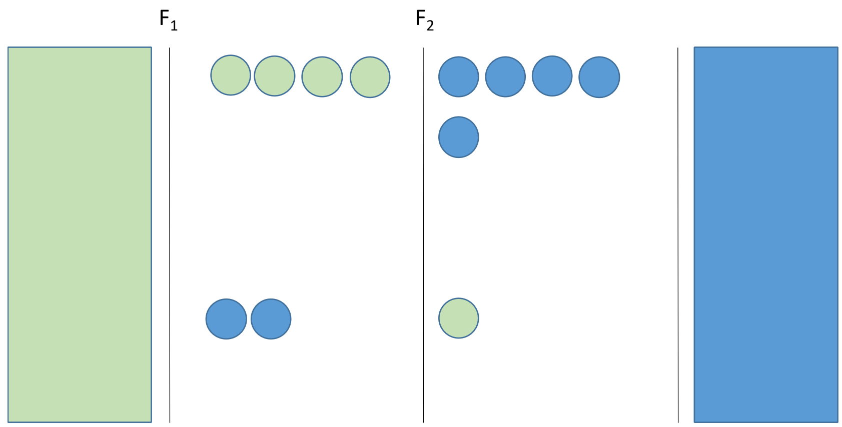

Figure 2: The four groups of citizens.

type) and $\theta_i = 1$ with probability δ. Consider a strategy profile in which the types $\theta_i = 1$ in groups 1 and 4 and the types $\theta_i = -1$ in groups 2 and 3 play their equilibrium strategy from the above full information example. Moreover, consider any collection of strategies for the unlikely types of all players. For small enough δ, the strategies of the more likely types from the full information limit case satisfy the best reply condition associated with a trembling hand perfect equilibrium. This is so because, in the limit case, all voters strictly prefer to vote for their preferred alternative in the first stage and they are indifferent or strictly prefer their equilibrium vote in the second stage votes. For any given strategy collection of the unlikely types, equilibrium payoffs of the likely types are continuous in δ. In the second stage, indifferent likely type voters remain indifferent for positive δ. In the first stage, no one is indifferent at $\delta = 0$ which is why the continuity argument also applies.

To extend this into a trembling hand perfect equilibrium, it remains to specify the equilibrium behavior of the unlikely types. Consider the voting behavior that is in favor of the preferred alternative in stage 2 if there is one. In case of indifference, one can pick any behavior. In stage 1, assume that those who favor country B vote for x_1. The same arguments as above make sure that this behavior is in line with trembling hand perfection if δ is small enough. The limit of the so described equilibrium for $\delta \to 0$ is the one that is described above. $\square$

All ex-post equilibria of the S-mechanism are trembling hand perfect. Thus, the proposition implies that the S-mechanism is more reliable as a tool to implement the welfare maximum than sequential voting procedures.

7 Two party competition

Proposals to hold referenda on secessions are often put forward political groups or parties that act in a deeply divided and competitive environment. Do these political actors have an incentive to propose welfare maximizing allocations? It turns out that parties' access to citizens' information about preferences is key.

In what follows, I assume that one party wants to maximize the number of people living in country A, whereas the other one wants to maximize the number of people living in country B. As a tie-breaking rule, I assume that indifferent voters vote for the party that shares their own country preference (alternatively, one can put an ε weight on the party winning in the utility function).

Proposition 6 *Let x=(0,...,0) and x=(1,...,1) both be feasible. (i) A game of two-party competition with perfectly informed parties and partisan voters has a Nash equilibrium in which both parties maximize social welfare. (ii) A game of two-party competition with uninformed parties and partisan voters has a unique Nash equilibrium in which party A proposes to put every individual in country A and party B proposes to put every individual in country B. The equilibrium does not maximize expected social welfare.*

Proof In a first step, consider the case of the competition of two parties with perfect information about voter types. As one can easily verify, both parties offering the same solution that maximizes the number of fits is a Nash equilibrium. In equilibrium both parties receive one half of the expected votes. Any alternative platform only gains more votes if it puts more voters who previously did not fit into their preferred country than it puts voters who previously were allocated to their preferred country into the other country. Therefore, the alternative platform would increase the number of fits which yields a contradiction. Moreover, there are no other symmetric equilibria and there are no asymmetric equilibria.

Next, consider the case where parties are imperfectly informed about voter preferences (just like anybody else). Again, assume that the two parties A and B want to maximize the size of countries A and B respectively. The parties simultaneously commit to their platform in $\mathbf{F}$. The party that wins the majority of votes implements its platform. Again, I postulate the same tie-breaking rule as above. Also assume that the two options (i) everybody lives in country A and (ii) everybody in country B are in the feasible set. Then it is an equilibrium that party A proposes to put everyone in country A and party B proposes to put everyone in country B.

Given party A's strategy, all A voters vote for party A no matter what party B proposes. Thus, for party B, the best chance to win is to put everybody in country B to maximize the number of votes that it receives from potential B voters. This platform both maximizes the chance of winning and the number of citizens living in country B, conditional on winning. Thus, it is a unique best reply It remains to prove uniqueness and suboptimality. Regarding a possible equilibrium where both parties offer non-extreme platforms, the same argument as above can be made. To prove that welfare is not maximized, it suffices to consider the case where the realized sign profile corresponds exactly to a feasible allocation in the sense that putting everybody in his preferred country is feasible. Instead, the majority decides where individuals have to live. $\square$

8 Conclusion

A mechanism for the resolution of territorial disputes that is based on simple majority voting generally results in (local) welfare losses relative to a system that is based on location data and individual reports or on approval voting. Similarly, a system that is based on competitive proposals can lead to considerable welfare losses when voters have partisan preferences. The simple direct mechanism put forward in this paper yields a superior result. In 1920, the Danish-German border was determined with a similar mechanism. The allocation of the border was based on the outcome of local referenda. Municipalities with a higher share of pro Danish (pro German) votes were more likely to be allocated to Denmark (Germany). Still, the decision was not to have any regional enclaves in either country. The 1920 border is still intact today, and it is not contested, indicating that the type of mechanism may be practically robust. There is an intuitive explanation for this robustness. The mechanism maximizes the number of citizens who are satisfied by the outcome, thus minimizing resistance, and granting legitimacy. Ultimately, this may reduce the probability of further (armed) conflict.

Several potentially relevant issues have not been addressed in the present paper, including (i) possible adverse incentive effects that arise when citizens or countries may expect that the use of force makes the use of an allocation mechanism such as the one put forward in this paper more likely, (ii) the possibility of locally correlated types, (iii) the role of voluntary ex-post mobility and (iv) the existence of sequential voting procedures with an optimal trembling hand perfect equilibrium. These issues deserve to be addressed in future research.

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